STUDENT'S TEXT

UNIT NO. 33

CSMP

MATHEMATICS FOR THE ELEMENTARY SCHOOL GRADE 6

PART I

SCHOOL MATHEMATICS STUDY GROUP

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School Mathematics Study Group

Mathematics for the Elementary School, Grade 6

Unit 33

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Student's Text, Part I

REVISED EDITION

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FOREWORD

The increasing contribution of mathematics to the culture of the modern world, as well as its importance as a vital part of scientific and humanistic education, has made it essential that the mathematics in our schools be both well selected and well taught.

With this in mind, the various mathematical organizations in the United States cooperated in the formation of the School Mathematics Study Group (SMSG). SMSG includes college and university mathematicians, teachers of mathematics at all levels, experts in education, and representatives of science and technology. The general objective of SMSG is the improvement of the teaching of mathematics in the schools of this country. The National Science Foundation has provided substantial funds for the support of this endeavor.

One of the prerequisites for the improvement of the teaching of mathematics in our schools is an improved curriculum--one which takes account of the increasing use of mathematics in science and technology and in other areas of knowledge and at the same time one which reflects recent advances in mathematics itself. One of the first projects undertaken by SMSG was to enlist a group of outstanding mathematicians and mathematics teachers to prepare a series of textbooks which would illustrate such an improved curriculum.

The professional mathematicians in SMSG believe that the mathematics presented in this text is valuable for all well-educated citizens in our society to know and that it is important for the precollege student to learn in preparation for advanced work in the field. At the same time, teachers in SMSG believe that it is presented in such a form that it can be readily grasped by students.

In most instances the material will have a familiar note, but the presentation and the point of view will be different. Some material will be entirely new to the traditional curriculum. This is as it should be, for mathematics is a living and an ever-growing subject, and not a dead and frozen product of antiquity. This healthy fusion of the old and the new should lead students to a better understanding of the basic concepts and structure of mathematics and provide a firmer foundation for understanding and use of mathematics in a scientific society.

It is not intended that this book be regarded as the only definitive way of presenting good mathematics to students at this level. Instead, it should be thought of as a sample of the kind of improved curriculum that we need and as a source of suggestions for the authors of commercial textbooks. It is sincerely hoped that these texts will lead the way toward inspiring a more meaningful teaching of Mathematics, the Queen and Servant of the Sciences.

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PREFACE

As one of its contributions to the improvement of mathematics in the schools of this country, the School Mathematics Study Group has prepared a series of sample text materials for grades 4 through 6. These are designed to illustrate a kind of mathematics curriculum that we believe appropriate for elementary schools.

This volume is a portion of these materials which were prepared by a group of 30 individuals, divided almost equally between distinguished college and university mathematicians and master elementary teachers and consultants. A strong effort has been made on the part of all to make the content of this text material mathematically sound, appropriate and teachable. Preliminary versions were used in numerous classrooms both to strengthen and to modify these judgments.

The content is designed to give the pupil a much broader concept, than has been traditionally given at this level, of what mathematics really is. There is less emphasis on rote learning and more emphasis on the construction of models and symbolic representation of ideas and relationships from which pupils can draw important mathematical generalizations.

The basic content is aimed at the development of some of the fundamental concepts of mathematics. These include ideas about:

number; numeration; the operations of arithmetic; and intuitive geometry. The simplest treatment of these ideas is introduced early. They are frequently re-examined at each succeeding level and opportunities are provided throughout the texts to explore them more fully and apply them effectively in solving problems. These basic mathematical understandings and skills are continually developed and extended throughout the entire mathematics curriculum, from grades K through 12 and beyond.

We firmly believe mathematics can and should be studied with success and enjoyment. It is our hope that these texts may greatly assist all pupils and teachers who use them to achieve this goal, and that they may experience something of the joy of discovery and accomplishment that can be realized through the study of mathematics.

Chapter 1

EXPONENTS

THE MEANING OF EXPONENT

Suppose you were asked to read a very large number such as one that told you the distance to a star or one that gave the weight of the earth in pounds. These numbers and many others like them are so very large that you would have difficulty reading them. For example, the earth's weight is about 13,000,000,000,000,000,000,000,000 pounds.

This is a very large number. Can you read it? Can you think of some way in which you might tell a friend what the weight of the earth is in pounds?

In this chapter you will learn new ways of reading and writing these large numbers. These new ideas will be used often in mathematics and science courses which you will study later.

Product Expressions and Repeated Factors

1. The sentence $7 \times 9 = 63$ shows that 63 is the product of 7 and 9. It also shows that 7 and 9 are ________ of 63. Because it names a number as a product, an expression like 7×9 is called a <u>product expression</u>. Give other product expressions for 63 if there are any.

2. Write the decimal numeral for each of the following product expressions.

(a) 3×15	(g) $3 \times 4 \times 4$
(b) 3×16	(h) $2 \times 3 \times 3$
(c) 4×20	(i) $3 \times 2 \times 2 \times 2 \times 2$
(d) 4×12	(j) $2 \times 2 \times 3 \times 5$
(e) 2×24	(k) $5 \times 5 \times 5$
(f) 3×18	(l) $3 \times 7 \times 7$

3. How many times is the factor 2 used in the product expression in (j) above? In (i)?

4. What factor is used more than once in example (h)?

5. What number is shown as a <u>repeated</u> factor in example (l)?

6. Write one or more product expressions for each of the following. Show at least one <u>repeated</u> <u>factor</u> in each product expression. The number of blanks will help you with some of them.

Example:	$16 = 4 \times 4$
	$16 = 2 \times 2 \times 2 \times 2$

(a) 27 = ____ × ____ × ____

(b) 25 = ____ × ____

(c) 36 = ____ × ____, or

36 = ____ × ____ × ____ × ____

(d) 32 = ____ × ____ × ____, or

32 = ____ × ____ × ____ × ____ × ____

(e) 20 = ____ × ____ × ____

(f) 50 = ____ × ____ × ____

(g) 28 = ____ × ____ × ____

(h) 90 = ____ × ____ × ____

(i) 75 = ____ × ____ × ____

(j) 100 = ____ × ____, or

100 = ____ × ____ × ____ × ____

(k) 72

(l) 144

(m) 1000

(n) 125

Using Exponents to Write Numerals

There is a short way to write product expressions which show repeated factors. This short way uses a numeral to tell the number of times a factor is repeated. Here are some examples.

(a) $5 \times 5 \times 5$ is shortened to 5^3.

(b) 6×6 is shortened to 6^2.

(c) $2 \times 2 \times 2 \times 2 \times 2$ is shortened to 2^5.

What we have is a new way to name numbers. The new symbols like 5^3, 6^2, and 2^5 are made up of two numerals. The upper numeral is called the <u>exponent</u> and the lower one is called the <u>base</u>.

(a) 5^3 is read "five to the third power".

(b) 6^2 is read "six to the second power".

(c) 2^5 is read "two to the fifth power".

The new names are called <u>exponent forms</u>.

(a) 5^3 is the exponent form of the expression $5 \times 5 \times 5$.

$$\underset{\text{(decimal)}}{125} = \underset{\text{(product expression)}}{5 \times 5 \times 5} = \underset{\text{(exponent form)}}{5^3}$$

(b) 6^2 is the exponent form of the expression 6×6.

$$\underset{\text{(decimal)}}{36} = \underset{\text{(product expression)}}{6 \times 6} = \underset{\text{(exponent form)}}{6^2}$$

(c) 2^5 is the exponent form of the expression $2 \times 2 \times 2 \times 2 \times 2$.

$$\underset{\text{(decimal)}}{32} = \underset{\text{(product expression)}}{2 \times 2 \times 2 \times 2 \times 2} = \underset{\text{(exponent form)}}{2^5}$$

A number which can be expressed in exponent form is called a <u>power</u> of the number named by the base. The number 125 is called the <u>third power of</u> 5. The number 36 is called the <u>second power of</u> 6, and, the number 64 is called the <u>sixth power</u> of 2. The third power of 4 is also 64. This is why a symbol like 5^3 is read

"five to the third power".

Notice that more than one base can be used in expressing some numbers as powers.

64 is the <u>third</u> power of <u>four</u> and also the sixth power of 2.

$$2^6 = 4^3.$$

1. Express each of the following in words. For 7^3, say "seven to the third power."

(a) 3^5

(b) 4^2

(c) 9^3

(d) 7^2

(e) 10^4

(f) 5^3

(g) 12^2

(h) 15^3

2. Sometimes numbers may be written in several exponent forms: In what different exponent forms is 16 written in Example (a) in the box below?

3. In what different exponent forms is 100 written in Example (e)?

4. Tell the exponent forms to be used in the blanks in the box.

(a) $16 = 4 \times 4 = 4^2$
$\quad = 2 \times 2 \times 2 \times 2 = 2^4$

(b) $36 = 6 \times 6 = 6^2$
$\quad = 2 \times 2 \times 3 \times 3 = 2^2 \times$ _____

(c) $28 = 7 \times 2 \times 2 = 7 \times$ _____

(d) $81 = 9 \times 9 = 9^2$
$\quad = 3 \times 3 \times 3 \times 3 =$ _____

(e) $100 = 10 \times 10 = 10^2$
$\quad = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$

(f) $144 = 12 \times 12 = 12^2$
$\quad = 2 \times 2 \times 2 \times 2 \times 3 \times 3 =$ _____ $\times$ _____

5. Complete the following sentences:

(a) The numeral 6^3 has exponent _____ and base _____.

(b) The numeral 6^3 is the exponent form of the product expression _______________.

(c) The numeral 6^3 is read ___________________________.

(d) The number 216 is the ____________ power of 6.

(e) In the expression 3×10^4 a _________ ___________ has been written in exponent form.

(f) If n is a counting number then the number 4^n has __________ as a factor.

(g) The numeral _________ has exponent 3 and base 4.

(h) The number 81 can be written in exponent form with base 3 and exponent ________.

(i) The third power of four has decimal numeral ________.

Exercise Set 1

1. Copy and write the product expression for each of the following exponent forms:

   Example: $7^3 = 7 \times 7 \times 7$.

   (a) 3^4 (c) 6^3 (e) 42^3 (g) 19^4

   (b) 5^2 (d) 2^5 (f) 25^2

2. Write the exponent form for each of the following product expressions:

   Example: $21 \times 21 \times 21 = 21^3$

   (a) $8 \times 8 \times 8 \times 8$ (e) $2 \times 2 \times 2 \times 2 \times 2 \times 2$

   (b) $11 \times 11 \times 11$ (f) $30 \times 30 \times 30$

   (c) $3 \times 3 \times 3 \times 3 \times 3 \times 3$ (g) $10 \times 10 \times 10 \times 10$

   (d) 17×17 (h) $12 \times 12 \times 12 \times 12 \times 12$

3. Express each number below as the product of a repeated factor. Then express it in exponent form. (Hint! If you have trouble finding a repeated factor, express the number as a product of primes.)

   Example: $125 = 5 \times 5 \times 5 = 5^3$

   (a) 81 (e) 144 (i) 5 to the third power

   (b) $6^2 \times 6^2$ (f) 64 (j) 8 to the second power

   (c) 32 (g) 625 (k) 10 to the fourth power

   (d) 343 (h) 216 (l) 2 to the fifth power

4. Write each of the following product expressions in exponent form as a power of four.

Example: $4 \times 4 \times 4 = 4^3$

(a) 4×4

(b) $4 \times 4 \times 4 \times 4$

(c) $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$

(d) 16×64

(e) $4^2 \times 4^3$

(f) $4^3 \times 2^3 \times 2$

5. Write the decimal numeral for each of the following:

Example: $6^3 = 6 \times 6 \times 6 = 36 \times 6 = 216$

(a) 5^4

(b) 17^2

(c) 4^3

(d) 9^2

(e) 10^4

(f) 26^2

(g) $5^2 \times 2^3$

(h) $3^2 \times 8^2$

Table I

A Number Decimal Numeral	B Product Expression with Repeated Factors	C Exponent Form	D Powers of Ten
(a) 10	None	10^1	First
(b) 100	10×10	10^2	
(c) 1,000	$10 \times 10 \times 10$		Third
(d) 10,000		10^4	
(e)	$10 \times 10 \times 10 \times 10 \times 10$		Fifth
(f) 1,000,000		10^6	
(g)	$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$		
(h) 100,000,000			Eighth

1. Read the numerals in Column A above and supply those that are missing.
2. In Column A each number is how many times as large as the one named above it?
3. Tell what is missing in Columns A, B, and D.
4. Compare the number of zeros in each numeral in Column A with the exponent of 10 in Column C in the same row. What is true in each comparison?
5. Do you see that 1 followed by six zeros can be expressed as 10 to the sixth power? It is written 10^6.
6. To write the decimal numeral for 10^7, we write 1 followed by how many zeros?
7. Express each of the following as a power of ten.

   (a) 100,000 (b) 100,000,000 (c) 1,000,000,000

Table II

A Decimal Numeral	B Product Expressions	C Exponent Form of B
400	4 × 100 = 4 × (10 × 10)	4×10^2
6,000	6 × 1,000 = 6 × (10 × 10 × 10)	6×10^3
500		5×10^2
90,000	9 × 10,000 =	
	7 × 1,000 = 7 × (10 × 10 × 10)	
300,000	3 × 100,000 =	
80	8 × 10	8×10^1
		2×10^6
		3×10^1
27,000	27 × 1,000 =	27×10^3
15,000,000	15 × 1,000,000 =	

1. In the table above, what product expressions are given for 400?
   How is 400 expressed in exponent form?

2. How is 6,000 expressed in exponent form?

3. Supply the numerals which are missing in the above table.

4. On Page 1, the weight of the earth was given as about 13,000,000,000,000,000,000,000,000 pounds. Express the weight in the form used in Column C in the table.

5. Did you ever hear the name "googol" used for a number? Googol is the name given to a number written as "1" followed by one hundred zeros. Express this number as a power of ten.

Exercise Set 2

1. Write each of the following in exponent form as a power of ten.

   Example: $1,000 = 10^3$

   (a) 10,000
   (b) 100
   (c) 100,000
   (d) 10
   (e) 10,000,000,000
   (f) 1,000,000

2. Write each of the following as a power of 10.

   Example: $10 \times 10 = 10^2$

   (a) $10 \times 10 \times 10 \times 10$
   (b) $10 \times 10 \times 10 \times 10 \times 10 \times 10$
   (c) $10 \times 10 \times 10$
   (d) $100 \times 100 \times 100$
   (e) $10 \times 1,000$
   (f) $1,000 \times 1,000 \times 1,000$

3. Find the decimal numeral for each of the following.

   Example: $6 \times 10^3 = 6,000$

   (a) 7×10^4
   (b) $10^3 \times 2$
   (c) 9×10^6
   (d) $10^5 \times 8$
   (e) $(3 \times 2) \times 10^2$
   (f) $(2 \times 5) \times 10^4$

4. Write each of the following in the kind of exponent form shown in exercise 3.

   Example: $5,000 = 5 \times 10^3$

   (a) 60,000
   (b) 200
   (c) 700,000
   (d) 8,000,000
   (e) 90
   (f) 300,000,000

Exercise Set 3

1. Which of the following is the largest number? Which is the smallest number? Explain your answer.

   (a) (3×4) (b) 4^3 (c) 3^4

   (d) 43 (e) 34

2. 2^6 is a number how much larger than 6^2?

3. The number 2^8 is how many times as large as the number 8^2?

4. Suppose you are offered a job which would take you 5 working days to complete. The employer offers you 7¢ the first day. Each day after, for four days, your daily wage will be multiplied by 7.

   (a) Make a table like the one below to show the amount you would earn each day. Show also your daily earnings written as a power of 7.

Day on job	Earnings each day	Earnings written as a power of 7
First	7¢	7^1
Second		
Third		
Fourth		
Fifth		

   (b) What will be your total earnings for the week?

5. Find the decimal numeral for each of the following:

   (a) 15^2 (c) 2×10

   (b) 3×4^1 (d) $2^3 \times 5^2$

6. Make the necessary computations. Then mark each of the following mathematical sentences true or false.

(a) $10^2 \neq 2 \times 10$

(b) $30^2 < 3 \times 10^2$

(c) $15^2 > 10 + 5^2$

(d) $3^4 - 2^4 = 50 + 15$

(e) $11^2 \neq 13^2 - 2^2$

(f) $6^2 \times 2^2 = 12^2$

(g) $10^2 - 9^2 = 100 - 90$

(h) $5^3 + 3^3 < 8^3$

(i) $150 - 12^2 < 10$

(j) $9^3 - 700 = 29$

(k) $8^3 - 80 = 3$

EXPANDED NOTATION

The system we use for naming numbers is the decimal system. In our system we group by tens. The word decimal comes from the Latin word "decem" which means "ten."

Just as our written language uses an alphabet of 26 symbols, the decimal system uses an "alphabet" of ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These are used as numerals for <u>digits</u>. Digits are whole numbers less than ten.

In our written language the alphabet symbols are used to form words which are used as names. In the decimal system the ten symbols for digits are used to form "words" which name larger whole numbers. These "words" are numerals made up of two, three, four, or more digit numerals.

To understand the decimal numeral system we learn how to find the meaning of "words" like 23, 8.6, .04. Let us review the way we think of decimals for whole numbers.

1. In the numeral 5555, each numeral 5 represents a different value. The place in which a 5 is written tells whether it represents 5 ones, 5 tens, 5 hundreds or 5 thousands. The meaning of each numeral is shown by the diagram in box A. Read the names of the places shown in box A.

A

5 5 5 5

5 ones	or (5×1)	
5 tens	or (5×10)	or (5×10^1)
5 hundreds	or $(5 \times 10 \times 10)$	or (5×10^2)
5 thousands	or $(5 \times 10 \times 10 \times 10)$	or (5×10^3)

2. As we go from right to left in the numeral 5555, the value represented by each 5 is how many times the value of the 5 before it? As we go from left to right the value represented by each 5 is one-tenth the value of the 5 before it.

3. The diagram in box A shows that place values are powers of ten. In the decimal system, we group not just by tens, but by powers of ten. What powers of ten are shown in box A?

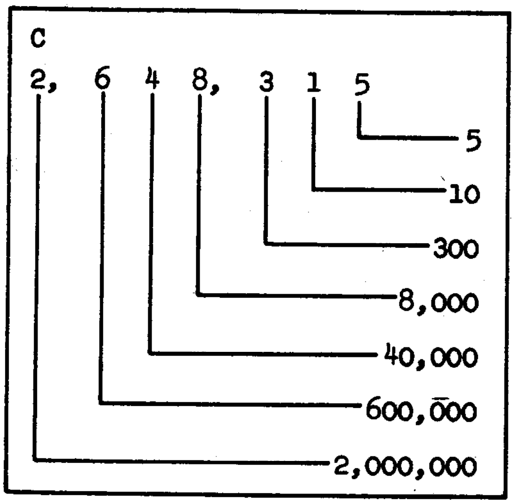
B

$$5555 = (5 \times 10 \times 10 \times 10) + (5 \times 10 \times 10) + (5 \times 10) + (5 \times 1)$$
$$= (5 \times 10^3) + (5 \times 10^2) + (5 \times 10^1) + (5 \times 1)$$

Box B shows the numeral 5555 written in expanded form or in <u>expanded notation</u>. The last line shows the <u>exponent form</u> of this expanded notation.

The numeral 2,648,315 is read "two million, six hundred forty eight thousand, three hundred fifteen."

Each group of three-place numerals is separated by a comma to make reading easier. We do not use the word "and" between each group because "and" is reserved for use in reading the decimal point in numerals such as 123.85.



4. Study the diagram in box C and tell the value represented by each digit in 2,648,315.

5. Now study the diagram in box D and tell the value represented by each digit numeral in 2,648,315. Give the value in repeated factor form and in exponent form.

D

2 6 4 8 3 1 5

5 ones		or (5×1)
1 ten	or (1×10)	or (1×10^1)
3 hundreds	or $(3\times10\times10)$	or (3×10^2)
8 thousands	or $(8\times10\times10\times10)$	or (8×10^3)
4 ten thousands	or $(4\times10\times10\times10\times10)$	or (4×10^4)
6 hundred thousands	or $(6\times10\times10\times10\times10\times10)$	or (6×10^5)
2 millions	or $(2\times10\times10\times10\times10\times10\times10)$	or (2×10^6)

$2,648,315 = (2\times10^6)+(6\times10^5)+(4\times10^4)+(8\times10^3)+(3\times10^2)+(1\times10^1)+(5\times1)$

6. Read each of the following numerals:

(a) 317 (c) 1,306 (e) 10,010 (g) 606,606

(b) 98 (d) 26,840 (f) 545,845 (h) 32,976,418

7. Write a decimal numeral for each of these:

(a) nine hundred three

(b) thirty thousand three hundred thirty

(c) eight thousand eight

(d) four hundred forty five thousand four hundred forty five

8. Express each of the following numerals in expanded notation. Give both the repeated factor form and also the exponent form.

(a) 783 (d) 200,456

(b) 3,075 (e) 73,800

(c) 81,040 (f) 5,247,600

<u>Summary</u>:

1. Grouping in the decimal system is by tens and powers of ten.
2. The decimal system has ten special symbols for the ten digits of the system.
3. In the decimal system the place values are powers of ten arranged in increasing order from right to left.
4. The place names from right to left are units (ones), tens, hundreds, thousands, ten thousands, hundred thousands, millions, and so on.

Exercise Set 4

The numeral, 234, has been written in expanded notation in three ways in the box at the right.

$$\begin{aligned} 234 &= (2\times 100) + (3\times 10) + (4\times 1) \\ &= (2\times 10\times 10) + (3\times 10) + (4\times 1) \\ &= (2\times 10^2) + (3\times 10^1) + (4\times 1) \end{aligned}$$

1. For each of the following numerals, write the expanded notation in the three ways shown in the example above.

   (a) 675 (d) 26,405

   (b) 8042 (e) 137,600

   (c) 5,168 (f) 2,987,654

2. Write the decimal numeral which is expressed in expanded notation below.

   Examples: $(6 \times 10^3) + (4 \times 10^2) + (2 \times 10^1) + (1 \times 1) = 6421$

   $(7 \times 10 \times 10) + (0 \times 10) + (8 \times 1) = 708$

   (a) $(3 \times 10^2) + (6 \times 10) + (5 \times 1)$

   (b) $(4 \times 10^2) + (7 \times 10) + (8 \times 1)$

   (c) $(3 \times 10^3) + (0 \times 10^2) + (6 \times 10) + (7 \times 1)$

   (d) $(2 \times 10^4) + (3 \times 10^3) + (5 \times 10^2) + (4 \times 10) + (0 \times 1)$

   (e) $(9 \times 10 \times 10 \times 10) + (3 \times 10 \times 10) + (5 \times 10) + (2 \times 1)$

   (f) $(6 \times 10 \times 10 \times 10 \times 10) + (0 \times 10^3) + (4 \times 10 \times 10) + (0 \times 10) + (8 \times 1)$

   (g) $(7 \times 10^4) + (3 \times 10^3) + (0 \times 10^2) + (0 \times 10^1) + (0 \times 1)$

3. Find the names for as many groups beyond the million group as you can.

Exercise Set 5

1. Name the largest and the smallest numbers which have exactly four decimal numerals, not using any zero.

2. Use the numerals 4, 5, 6, 7, and 8 to name eight different numbers with five-place numerals. Write the numerals in a column in order of the size of the numbers from smallest to largest.

3. Show that 2^5 and 5^2 do not name the same number.

4. Do 2^4 and 4^2 name the same number?

5. Make the necessary computations, then mark each of the following mathematical sentences true or false.

   (a) $4^3 = 8^2$

   (b) $4 \times 10 = 400$

   (c) $10 \times 12 < 12^2$

   (d) $10^3 > 5 \times 100$

   (e) $7 \times 2 \times 2 \times 2 > 7 \times 2^2$

   (f) $2 \times 6^2 = 3^2 \times 2^4$

   (g) $6^2 \neq 2^6$

   (h) $9^2 = 3^4$

6. Copy the following and make each one into a true mathematical sentence. Do this by writing one of the symbols in the box in each blank.

Symbols
>
<
=

   (a) 3×10^4 ____ $6 \times 5 \times 10^3$

   (b) 7×10^2 ____ $2^3 \times 100$

   (c) $2^3 \times 3^2$ ____ 4×4^2

   (d) $(6 \times 10^2) + 5^2$ ____ 5^4

   (e) $10^2 \times 7^2$ ____ 3×12^3

   (f) 3×50^2 ____ $9^2 \times 10^2$

FINDING PRODUCTS USING EXPONENT FORMS

1. Can you think of a way to find an exponent form for these product expressions?

   (a) $10^2 \times 10^2 = ?$ (c) $10^5 \times 10^3 = ?$

   (b) $2^3 \times 2^2 = ?$ (d) $7^3 \times 7^1 = ?$

   We could solve the problems above by changing the exponent forms to decimal numerals, then multiplying in the usual way and then changing back to exponent form.

$$2^3 \times 2^2 = 8 \times 4$$
$$= 32.$$

Then since

$$32 = 8 \times 4$$
$$= (2 \times 2 \times 2) \times (2 \times 2)$$
$$= 2^5$$
$$2^3 \times 2^2 = 2^5$$

It will be much quicker if we can learn to name the product in exponent form without changing to decimal numerals and back. Finding a way to do this and learning to use it is our purpose in this section.

2. In the box below are examples of the multiplication of numbers expressed in exponent forms with the same base. Study these examples. Can you discover how the exponent of the numeral for the product is obtained? The questions below the box may help.

(a)	$2^3 \times 2^4 = (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) = 2^7$
(b)	$3^2 \times 3^3 = (3 \times 3) \times (3 \times 3 \times 3) = 3^5$
(c)	$10^2 \times 10^4 = (10 \times 10) \times (10 \times 10 \times 10 \times 10) = 10^6$

3. Example (a) shows exponent forms for two numbers, 2^3 and 2^4, and for their product, 2^7. What is true about the base of all three forms? How can the exponent for the product be found from the exponents for the factors?

4. In example (b) the base in each exponent form is 3. The three exponents shown are 2, 3, and 5. What is the exponent in the product? What addition fact connects these exponents?

5. In example (c) the exponent 6 was found by counting. What operation can be used instead of counting 2 and then 4?

6. For the following examples, change each exponent form to repeated factor form. Are all the products correct?

(a) $5^3 \times 5^1 = 5^4$ (d) $10^2 \times 10^2 \times 10^2 = 10^5$

(b) $7^2 \times 7^2 = 7^4$ (e) $8^3 \times 8^2 = 8^5$

(c) $6^1 \times 6^1 \times 6^1 = 6^3$ (f) $4^2 \times 4^3 = 4^5$

7. For the following examples, write the number as a power without changing to repeated factor form.

Example: $2^4 \times 2^3 = 2^{(4+3)} = 2^7$

(a) $6^2 \times 6^1$ (d) $10^3 \times 10^2 \times 10^2$

(b) $3^3 \times 3^1 \times 3^2$ (e) $25^2 \times 25^2$

(c) $5^2 \times 5^2 \times 5^2$ (f) $100^2 \times 100^2$

8. Copy and complete each of the following. Use only exponent forms.

(a) $2^1 \times 2^3 =$ _____

(b) $18^1 \times 18^4 =$ _____

(c) _____ $\times 9^1 = 9^6$

(d) $a^1 \times a^7 =$ _____

(e) $6 \times 6^3 =$ _____

(f) $2^4 \times 2 =$ _____

(g) $3 \times$ _____ $= 3^3$

(h) $a^5 \times a =$ _____

(i) $5^4 \times$ _____ $= 5^6$

(j) $27^3 \times 27^6 =$ _____

(k) $19^5 \times$ _____ $= 19^8$

FINDING A COMMON BASE

1. Write each of the following as a power of 2.

   Example: $16 = 4 \times 4 = (2 \times 2) \times (2 \times 2) = 2^4$, or

   $16 = 2 \times 8 = 2 \times 2 \times 4 = 2 \times 2 \times 2 \times 2 = 2^4$

   (a) 64 (b) 32 (c) 128

2. Write each of the following as a power of 5.

   (a) 25 (b) 625 (c) 125

3. Write each of the following as a power of 10.

   (a) 100 (b) 10,000 (c) 1,000

One way to find an exponent form for 125×25 is first to change the decimal numerals 125 and 25 to exponent forms with the same base. Here are two examples of this method.

(a) $125 \times 25 = 5^3 \times 5^2$
$= 5^5$

(b) $49 \times 343 = 7^2 \times 7^3$
$= 7^5$

This method can be used only if the factors in the product expression are powers of the <u>same</u> number.

4. Express as powers of the same number and multiply using exponent forms.

   (a) $16 \times 4^2 =$ (d) $81 \times 3^3 =$

   (b) $7^3 \times 49 =$ (e) $64 \times 32 =$

   (c) $5^3 \times 25 =$ (f) $3 \times 9^2 =$

Exercise Set 6

1. Write each of these numbers as a power.

(a) $5^2 \times 5^8 =$

(b) $3^4 \times 3^2 =$

(c) $9 \times 9^2 =$

(d) $10^5 \times 10^5 =$

(e) $7^2 \times 7 =$

(f) $6^2 \times 6^8 =$

(g) $8^3 \times 8^4 =$

2. Change the numerals to exponent forms with the same base and multiply.

Example: $81 \times 27 = 3^4 \times 3^3 = 3^7$.

(a) $9 \times 3 =$

(b) $8 \times 2 =$

(c) $25 \times 25 =$

(d) $81 \times 9 =$

(e) $32 \times 4 =$

(f) $64 \times 16 =$

(g) $100 \times 10{,}000 =$

(h) $10{,}000 \times 10{,}000 =$

3. Think of letters of our alphabet as names of counting numbers. Express each of the following as a power.

Example: $a^2 \times a^3 = a^{(2 + 3)} = a^5$

(a) $b^2 \times b^3 =$

(b) $y^3 \times y^2 =$

(c) $4^2 \times 4^3 =$

(d) $n^3 \times n^4 =$

(e) $3^4 \times 3^7 =$

(f) $m^5 \times m^2 =$

4. Use exponent forms to shorten the multiplication process as shown in the example.

Example: $300 \times 4{,}000 = (3 \times 10^2) \times (4 \times 10^3)$

$= (3 \times 4) \times 10^2 \times 10^3 = 12 \times 10^5$

$= 1{,}200{,}000$

(a) $50 \times 700 =$

(b) $400 \times 400 =$

(c) $500 \times 60 =$

(d) $1{,}600 \times 500 =$

QUOTIENTS EXPRESSED IN EXPONENT FORM

Since $216 \div 36$ names the number 6 as a quotient we will call it a <u>quotient expression</u>.

You have learned how to find the decimal numeral for $216 \div 36$ by the division process. Suppose 216 were written in exponent form as 6^3 and 36 were written as 6^2. Is there a way to <u>divide</u> as well as multiply using exponent forms? Can we will in the blank below with an exponent form?

$$6^3 \div 6^2 = \underline{\quad ? \quad}$$

Here are examples showing two ways we might answer such a question.

<u>First Way</u>

(a) $4^5 \div 4^2 = (4 \times 4 \times 4 \times 4 \times 4) \div (4 \times 4)$

$= (64 \times 4 \times 4) \div 16 = (256 \times 4) \div 16$

$= 1024 \div 16$

$= 64$

$= 4^3$

(b) $2^7 \div 2^3 = \big((2 \times 2) \times (2 \times 2) \times (2 \times 2) \times 2\big) \div (2 \times 2 \times 2)$

$= (4 \times 4 \times 4 \times 2) \div 8$

$= (64 \times 2) \div 8$

$= 128 \div 8$

$= 16$

$= 2^4$

Second Way

(a) $4^5 \div 4^2 = (4 \times 4 \times 4 \times 4 \times 4) \div 4^2$

$= \big((4 \times 4 \times 4) \times (4 \times 4)\big) \div 4^2 = (4^3 \times 4^2) \div 4^2$

We notice that we must multiply by 4^2 and then divide the result by 4^2. But these operations undo one another, so we do not need to do either.

$(4^3 \times 4^2) \div 4^2 = 4^3$.

$4^5 \div 4^2 = 4^3$

(b) To express $2^7 \div 2^3$ as a power of 2 we use $2^n \times 2^3 = 2^7$. Since $n + 3 = 7$, $n = 4$ and $2^4 \times 2^3 = 2^7$. So $2^7 \div 2^3 = (2^4 \times 2^3) \div 2^3 = 2^4$.

Since the second way is so much shorter than the first it is the way we <u>should</u> use if we can understand it. Perhaps we can see better how it works if we first write division sentences as multiplication sentences:

(a) To find $4^5 \div 4^2$ we think $? \times 4^2 = 4^5$. Now we think the $?$ can be 4^n and write $4^n \times 4^2 = 4^5$. Since $n + 2 = 5$, $n = 3$ and $4^3 \times 4^2 = 4^5$. So

$$4^5 \div 4^2 = (4^3 \times 4^2) \div 4^2 = 4^3$$

We can now write $4^5 \div 4^2 = 4^3$

(b) If $n = 2^7 \div 2^3$ then

$$n \times 2^3 = 2^7.$$

But $2^4 \times 2^3 = 2^7$, so

$$n = 2^4 = 2^{(7 - 3)}.$$

1. Use the ideas in the above examples in explaining how to fill in each blank with an exponent form.

(a) $5^3 \times 5 =$ _____

(b) $(5^3 \times 5) \div 5 =$ _____

(c) $5^4 \div 5 =$ _____

(d) $(3^3 \times 3^2) \div 3^2 =$ _____

(e) $3^5 \div 3^2 =$ _____

(f) $10^4 \times 10^2 =$ _____

(g) $10^4 \div 10^2 =$ _____

(h) $10^4 \times$ _____ $= 10^6$

(i) $10^6 \div 10^4 =$ _____

(j) $10^{(2 + 4)} \div$ _____ $= 10^2$

Exercise Set 7

1. Write each quotient expression in exponent form.

(a) $2^5 \div 2^3 =$

(b) $4^4 \div 4^1 =$

(c) $3^6 \div 3^3 =$

(d) $10^3 \div 10^2 =$

(e) $9^2 \div 9 =$

(f) $7^3 \div 7^2 =$

(g) $10^6 \div 10^3 =$

(h) $12^5 \div 12^3 =$

2. Change the numerals to exponent forms with the same base and divide.

Example: $216 \div 36 = 6^3 \div 6^2 = 6^{(3 - 2)} = 6^1 = 6$

(a) $16 \div 4 =$

(b) $64 \div 2 =$

(c) $243 \div 9 =$

(d) $10{,}000 \div 1{,}000 =$

(e) $81 \div 27 =$

(f) $1{,}000{,}000 \div 100 =$

3. Answer the following in exponent form.

(a) $10^6 \times 10^3 =$

(b) $15^3 \div 15^2 =$

(c) $2^8 \div 2^4 =$

(d) $10 \times 10 =$

(e) $16 \div 8 =$

(f) $9^2 \times 3^3 =$

(g) $4^3 \times 64 =$

(h) $16 \div 16 =$

(i) $5^3 \div 5 =$

(j) $1^4 \div 1^2 =$

Exercise Set 8

Using Exponent Forms

1. The area of a square region is 5^6 square feet. How long is a side?

2. A rectangular region has sides which are 2^4 inches and 5^4 inches long.
   (a) Name the measure of the area of the region in any convenient way,
   (b) Write the decimal numeral for the area measure.

3. The area of the United States is about 3,600,000 square miles. If our country were a rectangular region with one side 1,000 miles long, how long would the other side be?

4. Some very small animals which can be seen only through a microscope increase in number by splitting into two of the same kind. After a certain time each of these divides into two animals and so on. Suppose one kind of such animals divides exactly every 10 minutes.
   (a) How many animals will be produced from a <u>single</u> animal in one hour?
   (b) About how long is required to produce 1,000 animals from 1 animal?

5. To go into orbit around the earth a satellite must be travelling about 18×10^3 miles an hour. In circling the earth once the satellite goes about 27×10^3 miles. How many times around the earth does the satellite go in 3 hours?

6. The nearest star is about $3,441 \times 10^{10}$ miles away. The sun is about 93×10^6 miles away.

   (a) Write the decimal numeral for 93×10^6.

   (b) About how many times as far away as the sun is the nearest star?

   (c) If the distance to the sun were used as a unit, about what would be the measure of the distance to the nearest star?

7. Light travels about 186,000 miles a second.

   (a) About how many seconds does it take light to travel from the sun to the earth? How many minutes, to the nearest minute?

   (b) Use the answers to 7 (a) and 6 (b) to find about how many minutes it takes light to travel from the nearest star to the earth.

   (c) Find out whether this is longer or shorter than one year.

Supplementary Exercise Set

1. For each whole number from 50 through 70 write the number as a product of primes, or write "prime" after the number if it is prime. If a prime factor occurs more than once rewrite the product expression using exponential forms. Here is an example: $50 = 2 \times 5 \times 5$
$= 2 \times 5^2$.

2. Express each of the following numbers as a product of powers of primes as in the example.
Example: $(2^3 \times 3^2) \times (2 \times 3^3 \times 5) = 2^4 \times 3^5 \times 5$.
(a) $2^3 \times (2 \times 3^4)$
(b) $5^2 \times (3 \times 5 \times 7)$
(c) $3^2 \times 48$
(d) $36 \times (2 \times 5^2)$
(e) 36×48
(f) $10^2 \times 6 \times 7$
(g) $10^2 \times 2^2 \times 5$
(h) 144×12
(i) The number of minutes in a day

3. Write "yes" if the second number is a factor of the first, write "no" if it is not. Do not make any long computations.
(a) $2^2 \times 3 \times 5^3$, 2^2
(b) $2^2 \times 3 \times 5^3$, 3×5^3
(c) $2^2 \times 3 \times 5^3$, 14
(d) $2^2 \times 3 \times 5^3$, 6
(e) 6×5^2, 15
(f) 60×60, 25
(g) $2 \times 3^2 \times 5 \times 7^2$, 35

4. Write T for the following sentences which are true and F for those which are false.

(a) $9 \times 10^3 < 10^4$

(b) $(3 + 4)^2 = 3^2 + 4^2$

(c) $17^5 > 17^6$

(d) $4^2 \times 10^2 > 2,500$

(e) $3^6 \div 3^2 = 3^3$

(f) $10^3 = 2^3 \times 5^3$

(g) $(m \times n)^3 = m^3 \times n^3$

(h) $(4^2)^3 = 4^2 \times 4^2 \times 4^2$

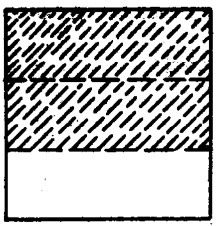
(i) $(4^2)^3 = 4^5$

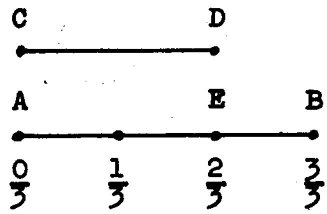
Chapter 2

RATIONAL NUMBERS

RATIONAL NUMBER

Rational numbers may be used for the measure of a region and for the measure of a line segment. Think of other uses for rational numbers. Look at the figures below.



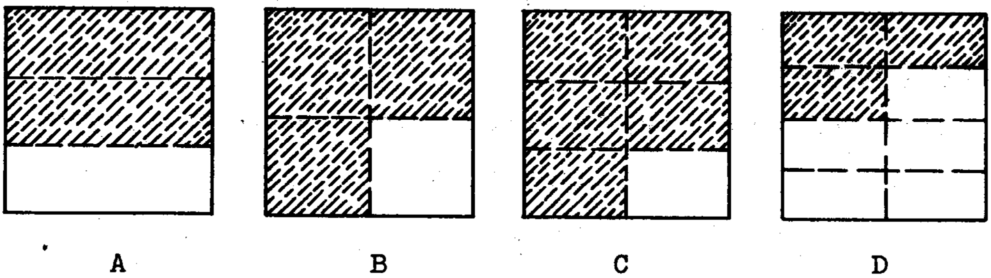


The shaded region is two-thirds of the square region. $\overline{AE}$ is two-thirds of $\overline{AB}$. Two-thirds is the measure of $\overline{CD}$.

Two-thirds may be represented by the numeral $\frac{2}{3}$, which is called a fraction. The 3 below the bar indicates that both the square region and $\overline{AB}$ have been separated into three parts of equal measure. The number 3 is called the <u>denominator</u> of the fraction.

The 2 above the bar in $\frac{2}{3}$ tells the number of thirds we are using. There are 2 thirds shaded in the square region and 2 thirds used for $\overline{AE}$. The number 2 is called the <u>numerator</u> of the fraction.

Exercise Set 1



1. Answer the following questions for each region above:

   a. Into how many parts is each separated?

   b. How many parts are shaded?

   c. If the measure of each large square region is 1, write the fraction that indicates the measure of the shaded region.

2. a. What do the denominators you wrote represent?

   b. What do the numerators represent?

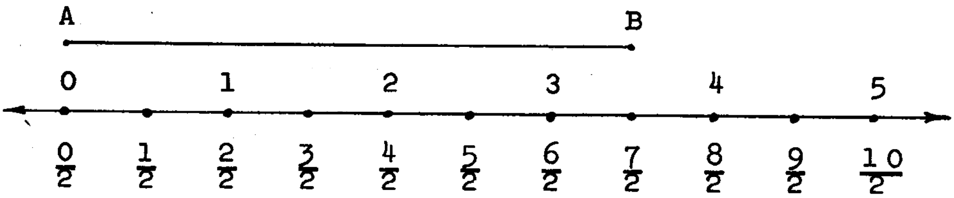
3. For each figure above, write the fraction that indicates the measure of the unshaded region.

4. a. What do the denominators you wrote for Exercise 3 represent?

   b. What do the numerators represent?

   c. $\frac{3}{4} = \frac{2}{4} + \frac{1}{4}$ is shown in the shaded region of Figure B. Write similar mathematical sentences for the shaded parts of Figures A, C, and D.

5.



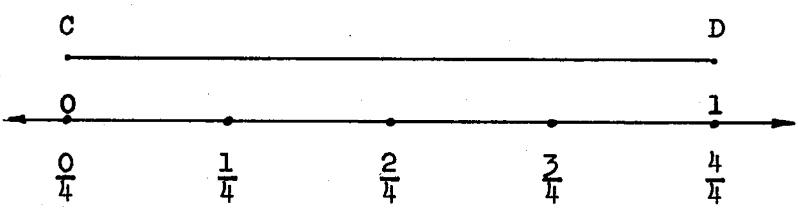
a. What fraction names the measure of $\overline{AB}$?

b. What does the denominator represent?

c. What does the numerator represent?

d. $\frac{2}{2} = \frac{1}{2} + \frac{1}{2}$. Write a mathematical sentence for your answer to Exercise 5a. Is more than one sentence possible?

6.

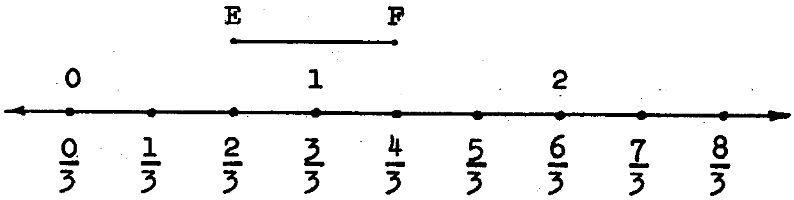


a. What fraction names the measure of line segment $\overline{CD}$?

b. What does the denominator represent?

c. $\frac{2}{4} = \frac{1}{4} + \frac{1}{4}$. Write a mathematical sentence for your answer to Exercise 6a.

7.



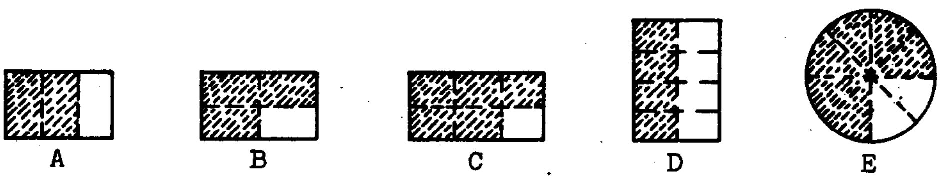
a. What is the measure of $\overline{EF}$?

b. What does the denominator represent?

c. What does the numerator represent?

d. Write a mathematical sentence about the measure of $\overline{EF}$.

8. In Figures A - E, each figure represents a region whose measure is 1. Copy and complete the chart.



	Number of congruent Parts	Number of Parts Shaded	Measure of Shaded Region
A	3	2	$\frac{2}{3}$
B			
C			
D			
E			

9. Draw simple figures to show segments or regions whose measures are the numbers: $\frac{1}{4}$, $\frac{2}{5}$, $\frac{1}{6}$, $\frac{7}{8}$.

10. a. What is the 2 called in these fractions: $\frac{2}{3}$, $\frac{2}{9}$, $\frac{2}{11}$, $\frac{2}{25}$?

    b. What are the 3, 9, 11, and 25 called in the fractions?

11. Complete the sentences below, using the Figures of Exercise 8.

    a. The denominator of the fraction $\frac{3}{4}$ shows that Figure B in Exercise 8 has been separated into ________ congruent parts.

    b. The numerator of the fraction $\frac{6}{8}$ shows that Figure E has ________ parts shaded.

    c. The denominator of the fraction $\frac{2}{3}$ shows that Figure A has been separated into ________ congruent parts.

    d. The denominator of the fraction $\frac{4}{8}$ shows that Figure D has been separated into ________ congruent parts.

Use rational numbers to answer these questions.

12. What part of a week is:

    a. 1 day

    b. 3 days

    c. 5 days

    d. 7 days

13. What part of one year is:

a. 9 months c. 6 months

b. 4 months d. 10 months

14. What part of an hour is:

a. 45 minutes c. 15 minutes

b. 30 minutes d. 10 minutes

15. What part of a pound is:

a. 4 ounces c. 12 ounces

b. 8 ounces d. 15 ounces

16. What part of a yard is:

a. 1 foot c. 3 feet

b. 2 feet d. 4 feet

17. What part of a foot is:

a. 9 inches c. 4 inches

b. 8 inches d. 12 inches

18. What part of a day is:

a. 6 hours c. 8 hours

b. 60 minutes d. 12 hours

19. What part of a mile is:

a. 2,640 feet c. 1,320 feet

b. 660 feet d. 330 feet

DIFFERENT NAMES FOR A NUMBER

Regions are measured in terms of a unit square region. We may decide what unit square region we wish to use.

Let us use the unit square region shown in Figure 1. (We may sometimes call it a unit square instead of a unit square region. You must remember, however, we mean unit square region.)

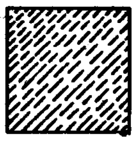


Figure 1

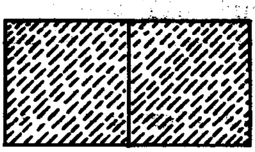


Figure 2

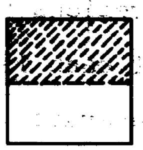


Figure 3

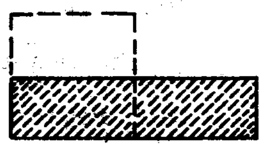
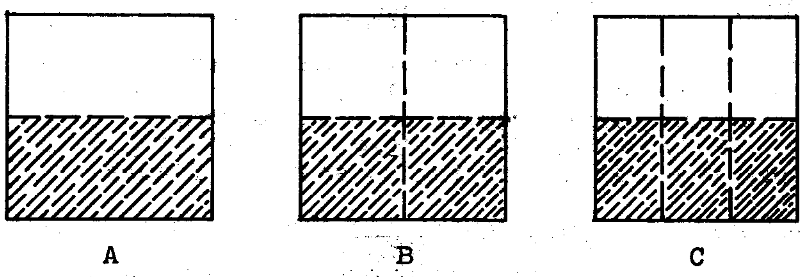


Figure 4

The shaded region of Figure 1 has the measure 1. The shaded region of Figure 2 has the measure 2 because it can be exactly covered by 2 unit squares. The shaded region of Figure 3 has the measure $\frac{1}{2}$ because it is one of the two congruent parts of a unit square.

A region may have the measure 1 and not have the same shape as the unit square of Figure 1. The shaded region of Figure 4 has measure 1. The unit square is sketched in with dotted lines to help you compare the shaded area with a unit square.



The unit square regions above are all the same size and shape. The measure of each is one.

Each region is separated into smaller congruent regions. One-half of Region A is shaded. Two-fourths of Region B is shaded. Three-sixths of Region C is shaded. The three shaded regions are the same size and shape. Their measures are equal.

The fractions $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ are all names for the same rational number.

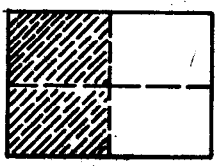
There are many other names for $\frac{1}{2}$. They may be found by drawing diagrams like A, B, and C. They may also be found by <u>multiplying the numerator and denominator of the fraction by the same number</u>.

A number has more names than you can count. Some other names for one-half are:

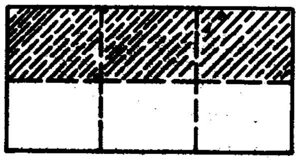
$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \ldots \frac{100}{200}, \ldots$$

Exercise Set 2

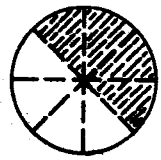
1. The shaded regions in A, B, and C suggest other names for $\frac{1}{2}$. Write these names. Write five other names for $\frac{1}{2}$.



A

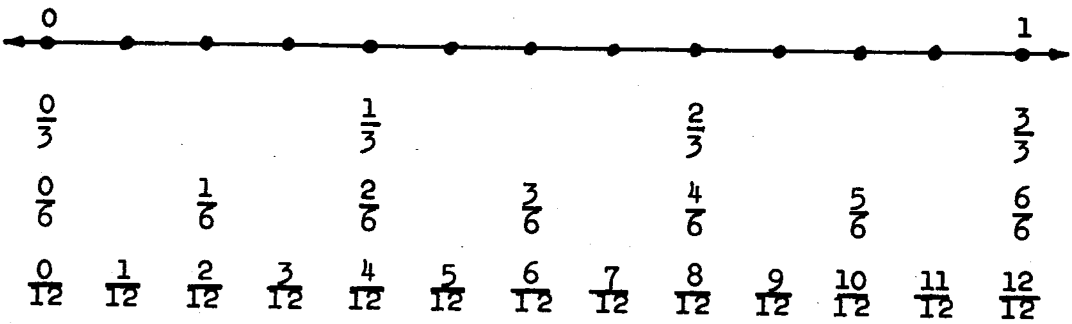


B



C

2. The number line below suggests different names for rational numbers $\frac{0}{3}$, $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$.



a. Write the names suggested on the number line for $\frac{2}{3}$.

b. Write three other names for $\frac{2}{3}$.

c. On the number line above, other names for 1 are suggested. Write these names, and three others.

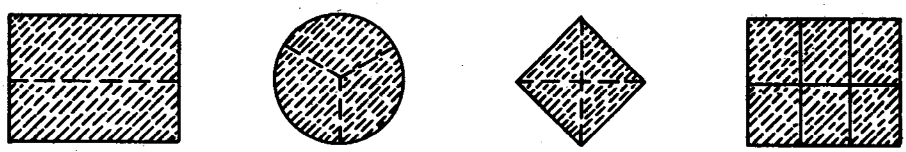
d. There is no other name for $\frac{1}{12}$ shown on the number line. Write one other name you know for $\frac{1}{12}$.

e. If the number line were extended to the point $\frac{4}{3}$, what other names would you write for $\frac{4}{3}$?

3. Draw simple figures to show

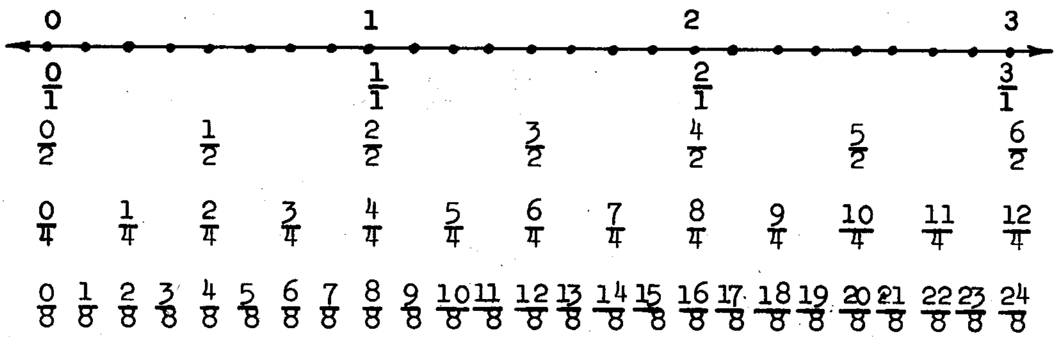
$$\frac{1}{4} = \frac{2}{8};\quad \frac{1}{3} = \frac{2}{6};\quad \frac{3}{4} = \frac{6}{8};\quad 1 = \frac{2}{2}.$$

4. Write the fractions which rename 1 as suggested by the shaded regions in the figures below.



A B C D

5.



a. Write 3 other names for $1\frac{1}{2}$.

b. Write 2 other names for $\frac{16}{8}$.

c. Write four names for the number matching the point halfway between 2 and 3.

d. If the number line were extended to the point matching 4, write the set of fractions with denominator 4 that you would write below the number line.

e. If you wrote $\frac{2}{3}$ on the number line in the row of fourths, between which two fractions would it be placed?

f. If you wrote a fraction for a number halfway between $\frac{6}{8}$ and $\frac{7}{8}$, what would it be?

6. Below are sets of names for numbers.

Set A = $\{\frac{11}{22}, \frac{2}{4}, \frac{3}{6}, \frac{3}{4}, \frac{2}{1}, \frac{20}{10}, \frac{5}{10}, \frac{11}{21}\}$

Set B = $\{\frac{13}{39}, \frac{3}{7}, \frac{2}{6}, \frac{3}{9}, \frac{4}{10}, \frac{11}{31}, \frac{4}{12}, \frac{5}{15}\}$

a. What fractions in Set A are other names for $\frac{1}{2}$?

b. What fractions in Set B are other names for $\frac{1}{3}$?

7. Which of the following rational numbers are the same as whole numbers?

$$\frac{12}{2}, \frac{17}{5}, \frac{3}{9}, \frac{71}{11}, \frac{50}{10}, \frac{9}{3}, 5\frac{1}{4}$$

8. Write the names of whole numbers between $\frac{1}{2}$ and $\frac{9}{2}$.

9. Replace n with a numeral to make the statements below true.

a. $\frac{1}{2} = \frac{2}{n}$

b. $\frac{1}{4} = \frac{n}{8}$

c. $\frac{2}{4} = \frac{16}{n}$

d. $\frac{3}{4} = \frac{n}{28}$

e. $\frac{2}{3} = \frac{8}{n}$

f. $\frac{14}{16} = \frac{n}{8}$

g. $3 = \frac{9}{n}$

h. $1 = \frac{6}{n}$

i. $\frac{7}{4} = \frac{35}{n}$

j. $\frac{24}{8} = \frac{n}{3}$

10. Which number is greater?

a. $\frac{1}{2}$ or $\frac{1}{3}$

b. $\frac{3}{4}$ or $\frac{2}{3}$

c. $\frac{4}{4}$ or $\frac{1}{2}$

d. $\frac{7}{3}$ or $\frac{2}{1}$

e. $\frac{15}{16}$ or $\frac{3}{4}$

f. $\frac{5}{8}$ or $\frac{1}{1}$

11. Mark each statement true or false.

a. $\frac{1}{2} = \frac{7}{14}$

b. $\frac{1}{3} = \frac{1}{9}$

c. $\frac{2}{3} = \frac{18}{27}$

d. $\frac{1}{4} = \frac{4}{20}$

e. $\frac{3}{4} = \frac{18}{24}$

f. $\frac{2}{6} = \frac{8}{14}$

g. $\frac{4}{6} = \frac{40}{60}$

h. $\frac{2}{8} = \frac{22}{88}$

i. $\frac{72}{12} = \frac{6}{6}$

j. $\frac{88}{22} = 4$

k. $\frac{24}{30} = \frac{4}{5}$

l. $\frac{15}{3} = 18$

12. Complete, using ">", "<", or "=" in each blank.
(Recall that ">" means "is greater than" and "<" means "is less than.")

a. $\frac{1}{2}$ _____ $\frac{1}{4}$

b. $\frac{3}{4}$ _____ $\frac{7}{8}$

c. 2 _____ $\frac{24}{16}$

d. $7\frac{1}{2}$ _____ $\frac{15}{2}$

e. $\frac{3}{4}$ _____ $\frac{8}{12}$

f. 5 _____ $\frac{60}{10}$

g. $3\frac{1}{3}$ _____ $\frac{10}{3}$

h. $\frac{2}{3}$ _____ $\frac{1}{1}$

13. Write fractions that make true statements.

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} =$ _____ = _____ = _____ = _____ = _____ = _____

$\frac{1}{1} = \frac{2}{2} = \frac{3}{3} =$ _____ = _____ = _____ = _____ = _____ = _____

$\frac{24}{12} = \frac{12}{6} =$ _____ = _____

$\frac{2}{3} = \frac{4}{6} =$ _____ = _____ = _____ = _____ = _____

14. Which of the following fractions are other names for $\frac{2}{3}$?

$$\frac{6}{8}, \frac{6}{9}, \frac{8}{12}, \frac{9}{12}, \frac{5}{6}, \frac{10}{15}, \frac{9}{15}$$

15. Which of the following fractions are <u>not</u> names for $\frac{9}{12}$?

$$\frac{2}{3}, \frac{3}{4}, \frac{24}{18}, \frac{27}{36}, \frac{6}{8}, \frac{18}{21}$$

16. Arrange in order from least to greatest.

$$\frac{1}{2}, \frac{1}{3}, \frac{7}{4}, \frac{10}{1}, \frac{5}{6}, \frac{11}{2}$$

17. Name the greater number of each pair.

a. $\frac{5}{4}$ or $\frac{6}{3}$

b. $\frac{2}{8}$ or $\frac{8}{2}$

c. $\frac{1}{1}$ or $\frac{15}{16}$

d. $\frac{17}{4}$ or $\frac{32}{5}$

e. $\frac{8}{3}$ or $\frac{11}{4}$

f. $\frac{5}{6}$ or $\frac{11}{12}$

18. $\frac{3}{5}$ may be thought of as $\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$ or as "three $\frac{1}{5}$'s" or as $\frac{2}{5} + \frac{1}{5}$ or as $\frac{6}{10}$.

$4\frac{1}{2}$ may be thought of as $4 + \frac{1}{2}$ or $9 \times \frac{1}{2}$ or as

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

3.7 may be thought of as $3 + 0.7$ or as $2.3 + 1.4$.

Write three other names for each of these numbers:

a. $\frac{1}{2}$ b. $\frac{7}{4}$ c. 2.5

FRACTIONS AND MIXED FORMS

It is easy to see that $2\frac{1}{4}$ and $\frac{9}{4}$ are names for the same number. $\frac{9}{4}$ is a fraction name and $2\frac{1}{4}$ is a mixed form for this number.

$$
\begin{aligned}
2\tfrac{1}{4} &= 2 + \tfrac{1}{4} \\
&= \tfrac{2}{1} + \tfrac{1}{4} \\
&= \tfrac{2 \times 4}{1 \times 4} + \tfrac{1}{4} \\
&= \tfrac{8}{4} + \tfrac{1}{4} \\
2\tfrac{1}{4} &= \tfrac{9}{4}
\end{aligned}
\qquad\qquad
\begin{aligned}
\tfrac{9}{4} &= \tfrac{8}{4} + \tfrac{1}{4} \\
&= 2 + \tfrac{1}{4} \\
&= 2\tfrac{1}{4}
\end{aligned}
$$

Suppose you have less familiar numbers.

$$
\begin{aligned}
18\tfrac{5}{13} &= \tfrac{18}{1} + \tfrac{5}{13} \\
&= \tfrac{18 \times 13}{1 \times 13} + \tfrac{5}{13} \\
&= \tfrac{234}{13} + \tfrac{5}{13} \\
18\tfrac{5}{13} &= \tfrac{239}{13}
\end{aligned}
$$

Consider the number $\frac{174}{8}$. How can it be written in mixed form? You know that $\frac{8}{8} = 1$.

$$
\begin{aligned}
\tfrac{174}{8} &= \tfrac{8 + 166}{8} \\
&= \tfrac{8}{8} + \tfrac{166}{8} \\
&= 1\tfrac{166}{8}
\end{aligned}
\qquad \text{or} \qquad
\begin{aligned}
\tfrac{174}{8} &= \tfrac{16 + 158}{8} \\
&= \tfrac{16}{8} + \tfrac{158}{8} \\
&= 2 + \tfrac{158}{8} \\
&= 2\tfrac{158}{8}
\end{aligned}
$$

Recall that a fraction is in <u>simplest form</u> when the numerator and denominator have no common factor, except 1. In the <u>simplest</u> mixed form for a rational number, the fraction is in simplest form and names a number less than 1. Is either mixed form above in <u>simplest</u> mixed form? To find the simplest mixed form for $\frac{174}{8}$ write the numerator, 174, in the form $(8 \times n) + r$, with r less than 8. What operation do you use to find n and r ?

$$174 = (8 \times n) + r$$

$$174 = (8 \times 21) + 6$$

$$\frac{174}{8} = \frac{(8 \times 21) + 6}{8}$$

$$= \frac{8 \times 21}{8} + \frac{6}{8}$$

$$\frac{174}{8} = 21 + \frac{6}{8} = 21\frac{6}{8} = 21\frac{3}{4}$$

<u>Exercise Set 3</u>

Write names for these numbers in the form shown. In Exercise 1, be sure $r < 6$.

1. $38 = (6 \times n) + r$
2. $55 = (3 \times n) + r$
3. $72 = (12 \times n) + r$
4. $69 = (14 \times n) + r$
5. $124 = (25 \times n) + r$
6. $347 = (18 \times n) + r$

Name these numbers in simplest mixed form. Show your work as in the examples above Exercise Set 3.

7. $\frac{50}{8}$
8. $\frac{71}{7}$
9. $\frac{41}{9}$
10. $\frac{87}{10}$
11. $\frac{145}{15}$
12. $\frac{296}{18}$

Find fraction names for these numbers.

13. $8\frac{3}{8}$
14. $9\frac{5}{8}$
15. $12\frac{3}{5}$
16. $23\frac{9}{11}$
17. $50\frac{3}{4}$
18. $29\frac{5}{7}$

ADDING AND SUBTRACTING RATIONAL NUMBERS

Recall that it is easy to add or subtract two rational numbers if they are named by fractions with the same denominator.

$$\frac{2}{5} + \frac{1}{5} = \frac{2+1}{5} = \frac{3}{5}$$

$$\frac{4}{3} - \frac{2}{3} = \frac{4-2}{3} = \frac{2}{3}$$

$$1\frac{3}{8} - \frac{5}{8} = \frac{11-5}{8} = \frac{6}{8} = \frac{3}{4}$$

To add or subtract rational numbers named by fractions with the same denominator, we add or subtract the numerators to find the numerator of the result. The denominator of the result is the same as the denominator of the two original fractions.

If the denominators of the two fraction names are not the same, one or both rational numbers are renamed so the fractions have the same denominator.

Add:

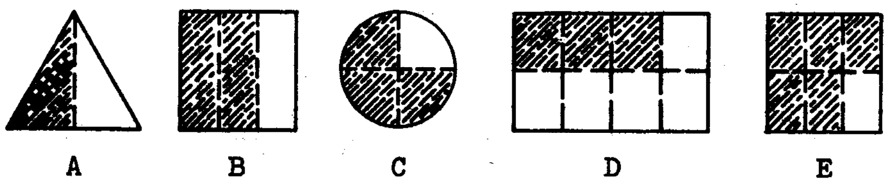
$$\begin{array}{l} \frac{2}{3} = \frac{10}{15} \\ \frac{3}{5} = \underline{\frac{9}{15}} \\ \qquad \frac{19}{15} = 1\frac{4}{15} \end{array} \qquad\qquad \begin{array}{l} 2\frac{1}{2} = 2\frac{3}{6} \\ 3\frac{1}{3} = \underline{3\frac{2}{6}} \\ \qquad 5\frac{5}{6} \end{array}$$

Subtract:

$$\begin{array}{l} \frac{5}{8} = \frac{5}{8} \\ \frac{1}{2} = \underline{\frac{4}{8}} \\ \qquad \frac{1}{8} \end{array} \qquad\qquad \begin{array}{l} 2\frac{3}{4} = 2\frac{6}{8} = 1\frac{14}{8} \\ 1\frac{7}{8} = 1\frac{7}{8} = \underline{1\frac{7}{8}} \\ \qquad\qquad \frac{7}{8} \end{array}$$

Exercise Set 4

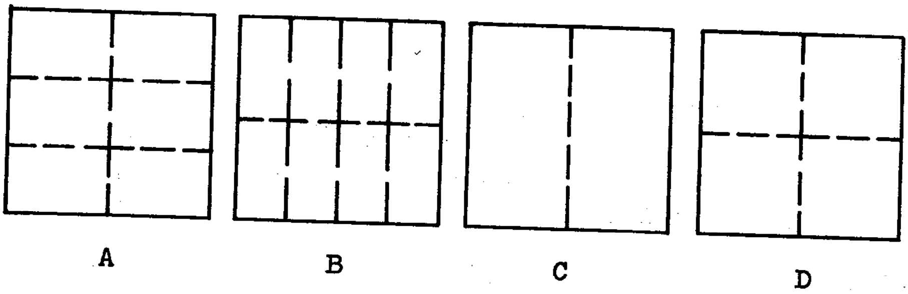
1.



a. Figure A pictures the addition fact $\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$. What addition facts are suggested by the shaded and unshaded regions of Figures B? C? D? and E?

b. Figure B also shows that $1 - \frac{2}{3} = \frac{1}{3}$ and $1 - \frac{1}{3} = \frac{2}{3}$. What subtraction facts are suggested by the shaded and unshaded regions of Figure A? Figure C? Figure D? Figure E?

2.



Choose one of the rectangular regions A, B, C, or D to find the sums of the numbers in the chart below.

	Numbers	Rectangular Region Used	Sum
a.	$\frac{1}{2}$ and $\frac{3}{8}$	B	$\frac{7}{8}$
b.	$\frac{1}{4}$ and $\frac{5}{8}$		
c.	$\frac{1}{2}$ and $\frac{1}{3}$		
d.	$\frac{1}{2}$ and $\frac{1}{4}$		

3. Recall what is meant by the "prime factorization" of a counting number. Complete:

	Fraction	Prime Factorization of Denominator	Prime Factorization of Numerator	Greatest Common Factor	Simplest Form
a.	$\frac{6}{8}$	$2 \times 2 \times 2$	2×3	2	$\frac{3}{4}$
b.	$\frac{8}{10}$				
c.	$\frac{9}{12}$				
d.	$\frac{2}{8}$				
e.	$\frac{8}{12}$				
f.	$\frac{14}{16}$				
g.	$\frac{18}{6}$				
h.	$\frac{9}{6}$				
i.	$\frac{6}{9}$				
j.	$\frac{54}{27}$				

4. Rename each pair of numbers below so the fractions have the same denominator. Use the smallest denominator possible.

a. $\frac{1}{2}$ and $\frac{2}{3}$ c. $\frac{2}{3}$ and $\frac{3}{4}$ e. $\frac{3}{4}$ and $\frac{4}{5}$

b. $\frac{5}{8}$ and $\frac{3}{4}$ d. $\frac{1}{2}$ and $\frac{7}{10}$ f. $\frac{2}{3}$ and $\frac{3}{5}$

3. Recall what is meant by the "prime factorization" of a counting number. Complete:

	Fraction	Prime Factorization of Denominator	Prime Factorization of Numerator	Greatest Common Factor	Simplest Form
a.	$\frac{6}{8}$	$2 \times 2 \times 2$	2×3	2	$\frac{3}{4}$
b.	$\frac{8}{10}$				
c.	$\frac{9}{12}$				
d.	$\frac{2}{8}$				
e.	$\frac{8}{12}$				
f.	$\frac{14}{16}$				
g.	$\frac{18}{6}$				
h.	$\frac{9}{6}$				
i.	$\frac{6}{9}$				
j.	$\frac{54}{27}$				

4. Rename each pair of numbers below so the fractions have the same denominator. Use the smallest denominator possible.

a. $\frac{1}{2}$ and $\frac{2}{3}$ c. $\frac{2}{3}$ and $\frac{3}{4}$ e. $\frac{3}{4}$ and $\frac{4}{5}$

b. $\frac{5}{8}$ and $\frac{3}{4}$ d. $\frac{1}{2}$ and $\frac{7}{10}$ f. $\frac{2}{3}$ and $\frac{3}{5}$

5. Express in simplest form:

a. $\frac{6}{8}$ d. $\frac{8}{12}$ g. $\frac{12}{8}$

b. $\frac{14}{10}$ e. $\frac{10}{15}$ h. $1\frac{4}{3}$

c. $\frac{15}{3}$ f. $\frac{7}{7}$ i. $\frac{11}{4}$

6. Find the sum of each pair of numbers and express it in its simplest form:

a. $\frac{7}{8}$ and $\frac{3}{4}$ f. $\frac{3}{4}$ and $\frac{4}{5}$

b. $\frac{5}{12}$ and $\frac{2}{3}$ g. $\frac{9}{8}$ and 6

c. $2\frac{1}{2}$ and 3 h. $7\frac{1}{2}$ and $2\frac{4}{5}$

d. $13\frac{2}{3}$ and $34\frac{3}{4}$ i. $\frac{2}{3}$ and $\frac{7}{9}$

e. $12\frac{1}{2}$ and $4\frac{2}{5}$ j. $4\frac{1}{2}$ and $5\frac{2}{3}$

7. Find n. Express n in its simplest form.

a. $n = \frac{3}{4} - \frac{1}{2}$ d. $n = 1\frac{7}{12} - \frac{3}{4}$ h. $n = 9\frac{2}{3} - 3\frac{3}{4}$

b. $n = \frac{7}{8} - \frac{3}{8}$ e. $n = 15\frac{2}{3} - 12\frac{1}{6}$ i. $n = 8\frac{1}{4} - 5\frac{2}{5}$

c. $n = \frac{7}{1} - \frac{1}{3}$ f. $n = 13\frac{5}{8} - 11$ j. $n = \frac{9}{10} - \frac{1}{2}$

g. $n = \frac{1}{3} - \frac{1}{4}$

8. Find the numbers n, p, w, and so on. Express the numbers in simplest form.

a. $\frac{3}{2} + 1 - \frac{1}{4} = n$

b. $\frac{1}{3} + \frac{4}{6} - 1 = p$

c. $\frac{1}{3} + \frac{1}{4} - \frac{1}{6} = w$

d. $2 - \frac{5}{4} + \frac{1}{2} = s$

e. $3 - 1\frac{1}{2} + \frac{3}{2} = m$

f. $9\frac{5}{8} + \frac{6}{16} - \frac{1}{2} = x$

g. $26\frac{1}{3} + 23\frac{4}{6} - 12\frac{1}{2} = y$

h. $45\frac{7}{8} - 19\frac{3}{4} - 26\frac{1}{8} = z$

USING PARENTHESES

Very early in your study of mathematics you learned that a number can have many names. $7 + 1$, 2×4, $14 - 6$, and $16 \div 2$ are all other names for 8.

Did you realize, however, that all of the different names for a number must be names for just that one number?

For what number is $6 + 3 \times 4$ another name? Is the number 36 or 18 ?

To remove any doubt about what <u>one</u> number $6 + 3 \times 4$ names, very helpful symbols called parentheses are used.

Notice that $(6 + 3) \times 4$ and $6 + (3 \times 4)$ represent two different numbers.

$$(6 + 3) \times 4 = 9 \times 4 = 36$$

$$6 + (3 \times 4) = 6 + 12 = 18$$

The use of parentheses is very helpful in writing correctly the mathematical sentences for story problems.

<u>Exercise Set 5</u>

1. Which of the following pairs of numerals name the same number?

   a. $(3 + 2) + 5$ and $3 + (2 + 5)$

   b. $(16 \div 8) \div 2$ and $16 \div (8 \div 2)$

   c. $(15 - 3) - 2$ and $15 - (3 - 2)$

   d. $2 \times (4 + 5)$ and $(2 \times 4) + 5$

   e. $\frac{3}{4} - (\frac{1}{2} + \frac{1}{4})$ and $(\frac{3}{4} - \frac{1}{2}) + \frac{1}{4}$

2. Place parentheses in the following so that

   a. $2 \times 3 + 1 = 8$

   b. $2 + 4 \times 3 = 14$

   c. $6 \times 3 - 1 = 17$

   d. $12 - 1 \times 2 = 22$

3. Write in numerals, using parentheses.

   a. Subtract the sum of $2\frac{2}{7}$, $\frac{3}{4}$, and $3\frac{1}{2}$ from 10.

   b. Divide the product of 32 and 67 by 16.

   c. Add 5×8 to the product of 4 and 7.

   d. Divide 2750 by 5 and multiply the result by 3.

Exercise Set 6

Read each problem carefully. Then write the relationships in the problem as a mathematical sentence. Solve, and write the answer in a complete sentence.

1. Sue and Tom are twins. Sue is $46\frac{3}{4}$ inches tall. Tom is $48\frac{1}{2}$ inches tall. How much taller is Tom than Sue?

2. Mary walks $\frac{7}{8}$ of a mile to school. Jane walks $\frac{3}{4}$ of a mile to school. How much farther does Mary walk than Jane?

3. In Mrs. Hardgrove's class $\frac{1}{5}$ of the class goes home for lunch and $\frac{1}{3}$ of the class eats in the cafeteria. The other boys and girls eat bag lunches in the room. What part of the class eats in the room?

4. Hale's record shop had a "$\frac{1}{4}$ off the original price" sale. What part of the original price did each record cost?

5. Peggy made a two piece playsuit for herself. The pattern required $\frac{3}{4}$ yards material for the blouse and $1\frac{1}{2}$ yards for the skirt. How much material was required for the playsuit?

6. For lunch, Ted ate $\frac{3}{4}$ of a peanut butter sandwich and $\frac{1}{2}$ of a jam sandwich. How many sandwiches did Ted eat for lunch?

7. $\frac{1}{4}$ of the student body of Oaks Junior High School attended Ward Elementary School. $\frac{5}{8}$ of the student body attended Morgan Elementary School. What part of the student body attended elementary schools other than those mentioned?

8. Mrs. Green used $\frac{1}{4}$ of a dozen eggs in a cake and $\frac{1}{6}$ of a dozen eggs in a salad dressing. What part of a dozen eggs did she have left?

9. Bob had a piece of balsa wood one foot long. He cut off two pieces $\frac{1}{2}$ and $\frac{1}{3}$ foot long for the model he was making. How many inches long was the piece he had left?

10. Janet filled $\frac{1}{4}$ of her stamp book with American stamps and $\frac{2}{3}$ of the book with stamps from other countries. What part of the book was not filled?

11. If you attend school 9 months of the year, what part of the year are you not in school?

12. Alice weighed $69\frac{1}{4}$ pounds at the end of June and $71\frac{2}{3}$ pounds at the end of July. She gained $\frac{1}{2}$ pound in August. How much did Alice gain in July and August together?

DECIMAL NAMES FOR RATIONAL NUMBERS

As you know, any rational number has many fraction names. Certain numbers can also have decimal names. This is true provided the denominator of the fraction name is 10, or 100, or 1,000.

Do you recall the meaning of numerals like 23.64 ? Think of the place value system of numeration.

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
		2	3.	6	4	

23.64 means 2 tens + 3 ones + 6 tenths + 4 hundredths or

$$23.64 = (2 \times 10) + (3 \times 1) + (\tfrac{6}{10}) + (\tfrac{4}{100})$$

Look at the last two terms.

$$\frac{6}{10} + \frac{4}{100} = \frac{60}{100} + \frac{4}{100}$$
$$= \frac{64}{100}$$

So you read 23.64 as "twenty-three and sixty-four hundredths". You can write it $23\frac{64}{100}$.

$$\text{Since } \quad 23\frac{64}{100} = 23 + \frac{64}{100}$$
$$= \frac{2300}{100} + \frac{64}{100}$$
$$= \frac{2364}{100},$$
$$23.64 = \frac{2364}{100}$$

Do you see that you can find a fraction name for 23.64 in the following way:

The digits 2364 indicate the numerator.

The place value of the last digit in the decimal 23.64 indicates the denominator.

We shall call the fraction found in this way the <u>fraction form of the decimal</u>.

Now consider the decimal 4.206. What is the numerator of its fraction form? What is the place value of the digit 6 ? What is the denominator of its fraction form? How is the fraction form for 4.206 written?

Exercise Set 7

1. Which of the following decimals have fraction forms with the same numerator?

a.	0.13	f.	0.25
b.	2.5	g.	1.3
c.	7.85	h.	0.785
d.	0.013	i.	0.025
e.	78.5	j.	13

2. Which of the decimals in Exercise 1 all have fraction forms with the same denominator?

3. Are there any two of the numerals listed in Exercise 1 that are names for the same number?

4. Write fraction names:

   a. 0.32

   b. 18.04

   c. 0.075

   d. 462.5

   e. 9.1

5. Write decimal names:

   a. $\frac{3}{10}$

   b. $\frac{48}{100}$

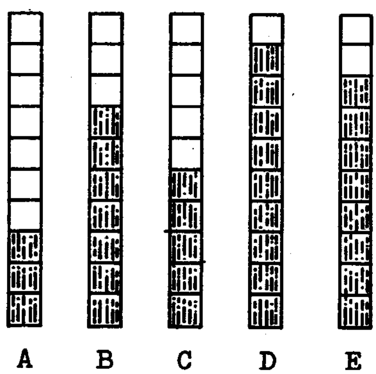
   c. $\frac{132}{100}$

   d. $\frac{38}{1000}$

   e. $\frac{78}{10}$

Exercise Set 8

Each of regions A, B, C, D, and E is a unit region.



1. a. Write the fraction that represents the measure of each shaded region.

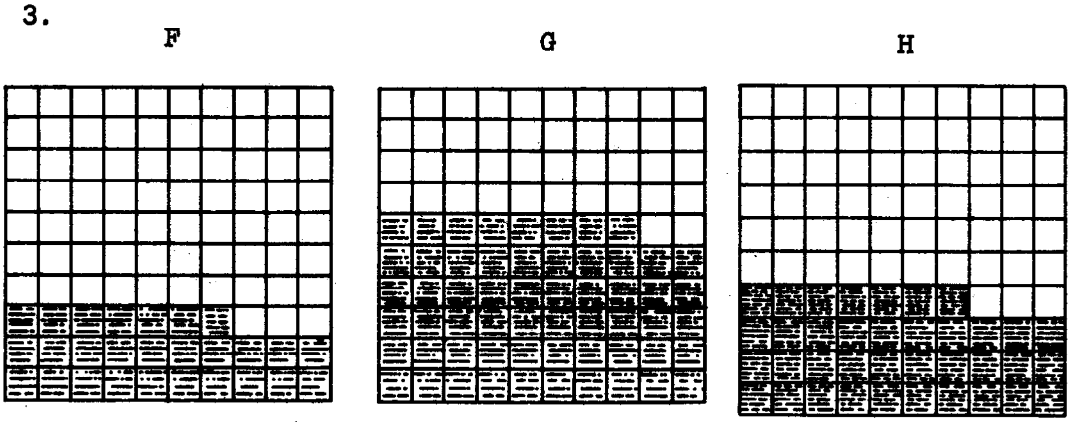
   b. Write the decimal that represents the measure of each shaded region.

2. a. E suggests the addition sentence $\frac{8}{10} + \frac{2}{10} = \frac{10}{10}$. What addition sentences are suggested by A, B, C, and D ?

b. E suggests the subtraction sentence $\frac{10}{10} - \frac{8}{10} = \frac{2}{10}$. What subtraction sentences are suggested by A, B, C, and D ?

c. Use decimals to write the subtraction sentence suggested by E. Suggested by A, B, C, and D.

3.



F, G, and H are unit regions.

a. Write the fraction that represents the measure of each shaded region.

b. Write the decimal that represents the measure of each shaded region.

4. What addition sentences are suggested by F, G, and H ? Write them a) using fractions and b) using decimals.

5. Write the subtraction sentences suggested by F, G, and H. Write them a) using fractions and b) using decimals.

6. Describe a region you would shade to show .001 of the unit square F.

7. Describe a region you would shade to show .005 of the unit square G.

8. For each decimal, write the numerator of a fraction form.

a.	0.4	d.	0.1	g.	7.25
b.	0.25	e.	0.37	h.	13.28
c.	0.01	f.	1.8	i.	4.251

9. What is the denominator of the fraction form for each decimal in Exercise 8 ?

10. Write as decimals:

a.	$\frac{45}{100}$	d.	$4\frac{38}{100}$	g.	$\frac{75}{100}$
b.	$2\frac{7}{10}$	e.	$\frac{5}{100}$	h.	$\frac{1457}{1000}$
c.	$\frac{9}{10}$	f.	$\frac{235}{100}$	i.	$1\frac{50}{1000}$

11. Write as fractions:

a.	0.65	d.	0.3	g.	0.10
b.	0.8	e.	0.07	h.	7.87
c.	1.70	f.	1.1	i.	0.123

12. Find n in each sentence:

a. $\frac{3}{4} + 2\frac{5}{6} + 3\frac{2}{3} = n$

c. $2\frac{1}{2} + \frac{4}{5} + 1 = n$

b. $3\frac{2}{3} + 2\frac{3}{4} + 1\frac{1}{2} = n$

d. $6\frac{3}{10} + 11\frac{1}{4} + \frac{4}{5} = n$

13. Arrange in order from least to greatest:

0.52, 0.056, 1.04, 0.09, 3.68, 0.1, 4.00

14. **Find n:**

a. $n = 14\frac{1}{2} - 7\frac{5}{8}$

b. $2 - 1\frac{1}{2} = n$

c. $n = 4\frac{1}{4} - 2\frac{5}{6}$

d. $n = \frac{1}{2} - \frac{1}{9}$

e. $n = 7 - \frac{4}{5}$

f. $n = 24\frac{2}{3} - 19\frac{3}{4}$

g. $79\frac{7}{8} - 25\frac{3}{8} = n$

h. $52\frac{1}{3} - 46\frac{5}{6} = n$

i. $104\frac{2}{5} - 93\frac{2}{3} = n$

j. $\frac{1}{2} - \frac{3}{10} = n$

15. **Find t:**

a. $t = 0.9 - 0.71$

b. $t = 0.72 - 0.395$

c. $t = 0.8 - 0.47$

d. $t = 0.35 - 0.2$

e. $t = 27.53 - 8.9$

f. $16.32 - 3.79 = t$

g. $1.2 - 0.09 = t$

h. $5.65 - 0.3 = t$

i. $9.7 - 3.67 = t$

j. $15 - 7.48 = t$

16. **Add:**

a. 0.5 and 0.39

b. 0.73 and 0.6

c. 14.01 and 1.9

d. 1 and 0.1

e. 2.16 and 7.8

f. 47.1 and 9.072

g. 0.07 and 4.3

h. 20.1 and 0.201

Exercise Set 9

Read the following carefully. Show the relationships in each problem using a number-line diagram. Then answer the question asked in the problem.

1. Jack ran the 50 yard dash in 8.7 seconds. Brian ran the distance in 11.0 seconds. Who won? By how many seconds?

2. During the first six months of a year, 14.8 inches of rain was recorded. During the next six months, 9.79 inches fell. How much rain fell during the year?

3. The Empire State Building in New York City is 1250 feet high. The Statue of Liberty in New York harbor is 305.5 feet high. How much higher is the Empire State Building than the Statue of Liberty?

4. The average annual rainfall of Louisiana is 57.34 inches. The average annual rainfall for Nevada is 8.6 inches. What is the difference between the annual rainfall averages of these two states?

5. The normal body temperature is 98.6°. Bill's temperature was 0.8° above normal. What was his temperature?

6. Jeff's garden is $30\frac{3}{4}$ feet long and $17\frac{1}{2}$ feet wide. How many feet of wire will it take to put a fence around it?

7. Below are the lengths of four Italian ships:

Leonardo da Vinci	761.2 ft.
Augustus	680.4 ft.
Cristoforo Columbo	700.0 ft.
Guillio Cesare	680.6 ft.

List the ships in order, from longest to shortest. Then make up three problems about the lengths of the ships.

8. The highest average annual temperature for the world was 88°F. recorded in Africa. The highest average annual temperature for the United States was 77.6°F. recorded in Florida. What is the difference between these two temperatures?

9. Pat rode his bicycle $19\frac{1}{3}$ miles one day and $15\frac{2}{5}$ miles the next day. How much farther did he ride his bicycle the first day than the second?

10. 3.2 inches of rain fell on Monday, 3.0 inches on Tuesday, and 2.4 inches on Wednesday. How many inches of rain fell on the three days?

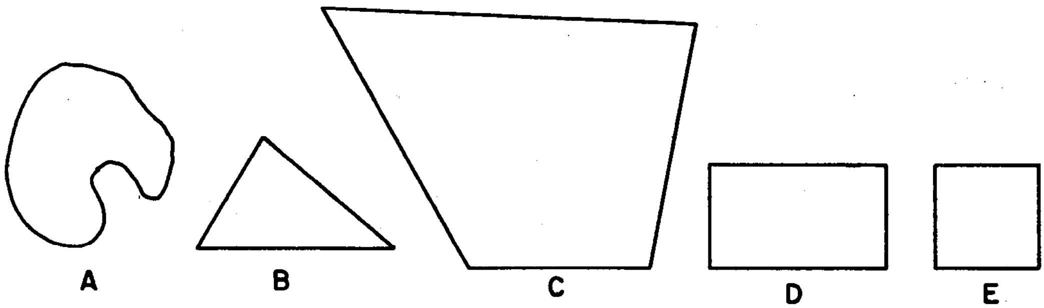
11. The British ship, Queen Elizabeth, is 1031 feet long. The Andes, another British ship, is 669.3 feet long. How much longer is the Queen Elizabeth?

12. The equatorial diameter of the world is 7,926.68 miles. The polar diameter is 7,899.99 miles. How much greater is the equatorial diameter than the polar diameter?

RECTANGULAR REGIONS: REVIEW

We are going to use rectangular regions to find a way to multiply rational numbers. Do you remember all you learned about rectangular regions?

Do you recall what a rectangle is? It is a simple closed curve which is the union of 4 segments and has 4 right angles.

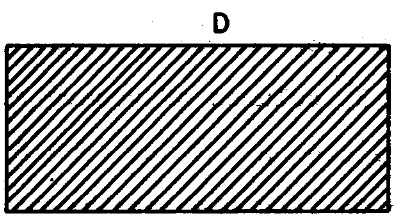


Figures A, B, C, D, and E all represent simple closed curves.

Which figures are the union of four segments?

Which figures have four right angles?

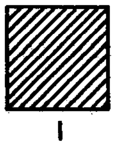
Figure D represent a rectangle. The union of the rectangle and its interior is a rectangular region.



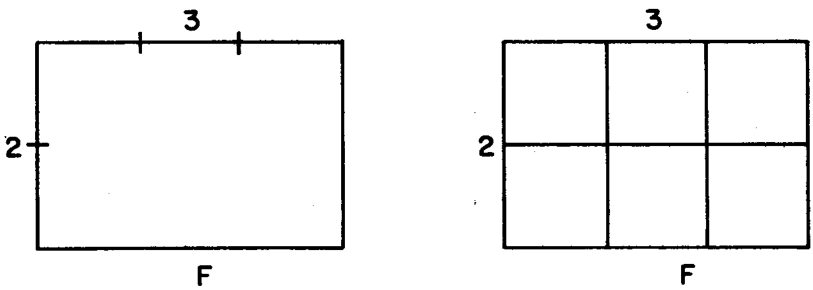
Rectangle D and the shaded part make up a rectangular region.

Figure E also represents a rectangle. It is also a square. Why?

Do you recall what kind of region is usually used to measure a rectangular region? It is customary to use a square region with sides 1 unit long.



Suppose Figure F is a rectangle, with sides 3 units and 2 units in length. How do you find the measure of the rectangular region?



By drawing lines, we can separate the rectangular region into 6 congruent square regions, each having sides 1 unit in length. These 6 square regions "cover" the rectangular region, so the measure of region F is 6.

You see there are 2 rows of square regions, with 3 in each row. Or there are 3 columns of square regions, with 2 in each column. So there are 2×3 or 3×2 square regions.

What is an easy way to find the measure of a rectangular region when the measures of its sides are whole numbers?

Exercise Set 10

1. a. Draw a rectangle 4 in. by 3 in. (This means 4 in. long and 3 in. wide.)

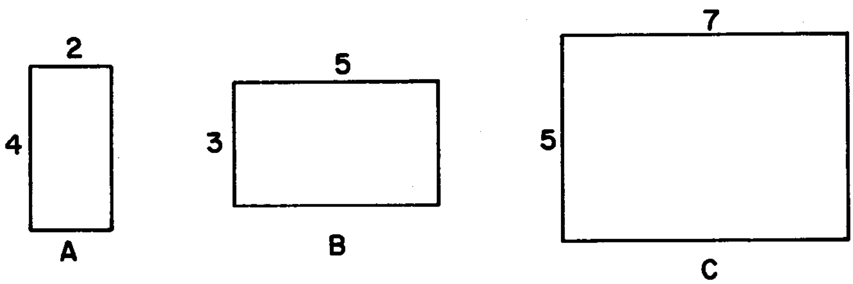
   b. Shade its interior.

   c. Draw lines to separate the rectangular region into unit square regions.

   d. Find the measure of the rectangular region.

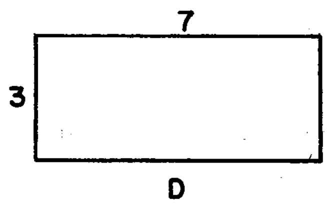
   e. What is the name of each unit square region?

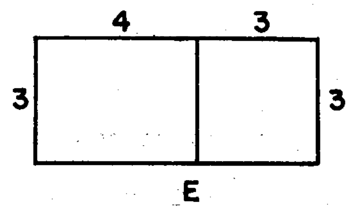
2. Suppose the rectangles A, B, and C below have sides with the measures shown. Find the measures of the rectangular regions.



3. Suppose the measure of a rectangular region is 24. What pairs of whole numbers could be the measures of its sides?

4. Rectangle D has sides whose measures are 7 and 3. What is the measure of the rectangular region?





5. Figure E is the union of two rectangular regions. Find their measures. Find the sum of their measures.

6. Do Exercises 4 and 5 show that

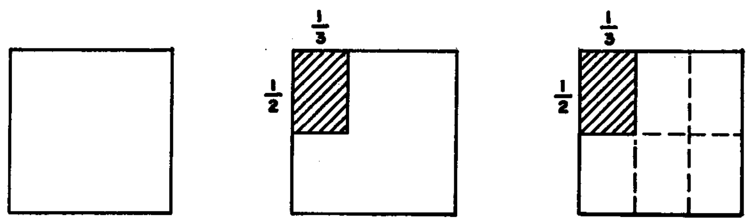
$$3 \times 7 = (3 \times 4) + (3 \times 3) \ ?$$

PRODUCTS AS MEASURES OF RECTANGULAR REGIONS

Exploration

You know how to add and subtract rational numbers. Now we shall study multiplication of rational numbers.

1. Figure A shows a unit square region. Figure B shows the same unit square region. It also shows a shaded rectangular region whose sides are $\frac{1}{2}$ unit and $\frac{1}{3}$ unit in length.



A B C

You can separate the unit square region into rectangular regions which are congruent to the $\frac{1}{2}$ by $\frac{1}{3}$ region, as shown in Figure C.

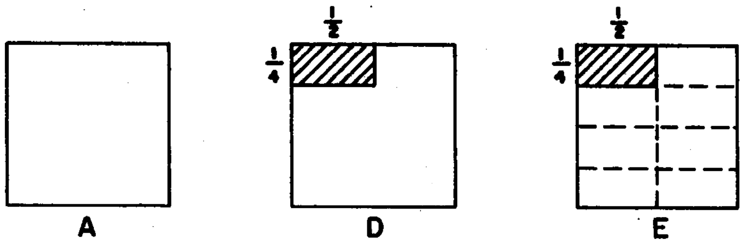
a. How many congruent regions are there?

b. What fraction names the measure of the shaded rectangular region?

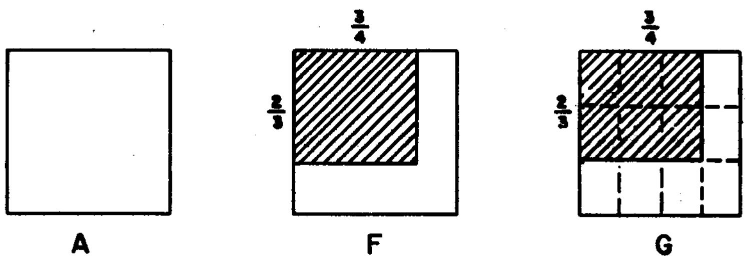
2. Figure D shows a $\frac{1}{4}$ by $\frac{1}{2}$ rectangular region shaded. Figure E shows the unit square region separated into congruent rectangular regions.

   a. How many rectangular regions congruent to the $\frac{1}{4}$ by $\frac{1}{2}$ region are there in the unit square region?

   b. What is the measure of the $\frac{1}{4}$ by $\frac{1}{2}$ shaded region?



3. Figure F shows a shaded rectangular region $\frac{2}{3}$ by $\frac{3}{4}$. Figure G shows the shaded rectangular region and also the unit square region separated into congruent rectangular regions.

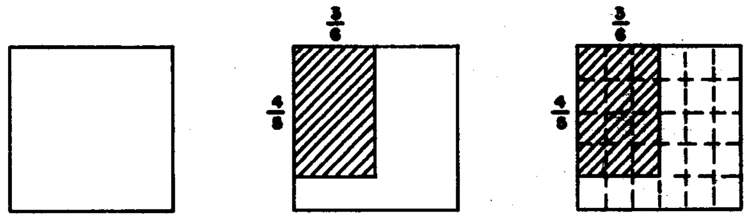


   a. How many small rectangular regions are there in the unit square region?

   b. How many are there in the shaded region?

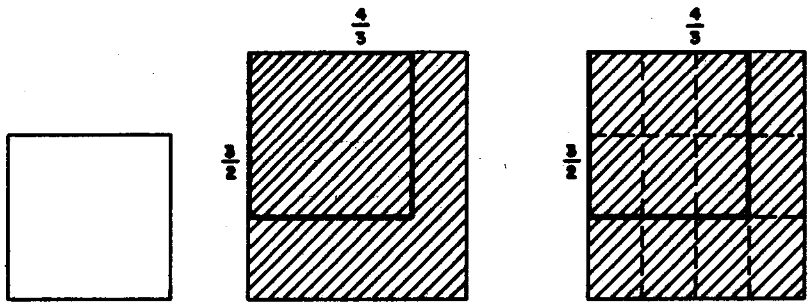
   c. What is the measure of the shaded region?

4. Figures H and I show a shaded $\frac{4}{5}$ by $\frac{3}{6}$ rectangular region. What is its measure?



A H I

5. Figure J shows a shaded rectangular region larger than the unit square region. The unit square is shown in dark lines. J is a $\frac{3}{2}$ by $\frac{4}{3}$ region. Figure K shows the shaded rectangular region and the unit square region separated into congruent rectangular regions.



A J K

a. How many small rectangular regions are there in the unit square region?

b. What is the measure of each small rectangular region?

c. How many are there in the shaded region?

d. What is the measure of the shaded region?

6. Complete this table about the shaded regions in Exercises 1 - 5.

Measures of Sides	Measure of Region
$\frac{1}{2}$ by $\frac{1}{3}$	____
$\frac{1}{4}$ by $\frac{1}{2}$	____
$\frac{2}{3}$ by $\frac{3}{4}$	____
$\frac{4}{5}$ by $\frac{3}{6}$	____
$\frac{3}{2}$ by $\frac{4}{3}$	____

7. Now consider some rectangular regions the measures of whose sides are whole numbers. Complete this table.

Measures of Sides	Measure of Region	Operation Used
2 by 3	____	______
5 by 8	____	______
7 by 6	____	______

In Exercise 7 you used multiplication to find the measures of rectangular regions, the measure of whose sides are whole numbers.

We will also call this operation multiplication when the measures of the sides are rational numbers. We will say that

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$, because a $\frac{1}{2}$ by $\frac{1}{3}$ region has measure $\frac{1}{6}$.

$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$, because a $\frac{1}{4}$ by $\frac{1}{2}$ region has measure $\frac{1}{8}$.

8. Write mathematical sentences to show the relation between the measures of the sides and the measure of the region for the other regions in Exercise 6.

The measure of a rectangular region whose sides have measures that are rational numbers is the product of those rational numbers.

Exercise Set 11

1. The regions below are unit square regions. The measure of each whole region is 1.

   For each shaded region, write

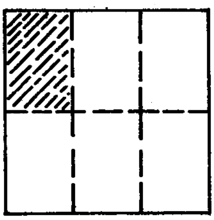
   a) the measure of each side.

   b) the measure of the region.

   c) a mathematical sentence which shows how the measures of the sides are related to the measure of the region.

   Underline the measure of each shaded region.

   The sentence for A is $\frac{1}{2} \times \frac{1}{3} = \underline{\frac{1}{6}}$.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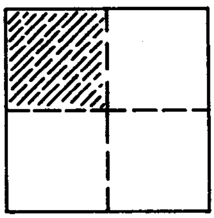
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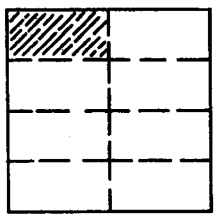
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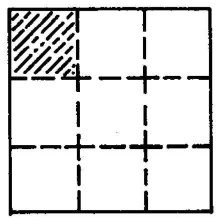
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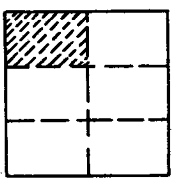
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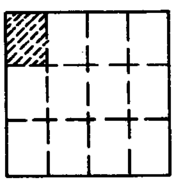
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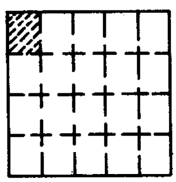
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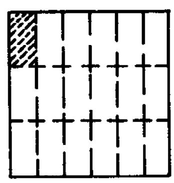
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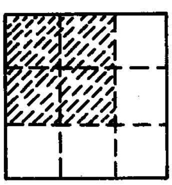
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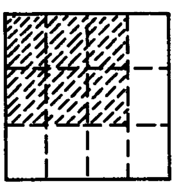
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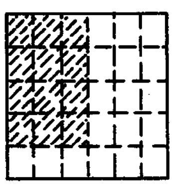
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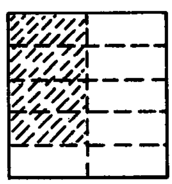

L

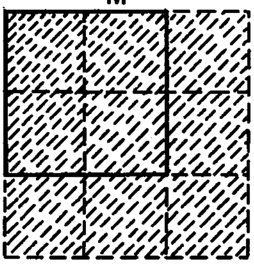
2. In each figure below, the unit square is the region bounded by solid lines. For each shaded region, write

   a) the measure of each side.

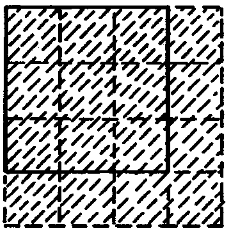
   b) the measure of the region.

   c) the mathematical sentence which shows the relation of the measures of the sides to the measure of the region. Underline the measure of each shaded region.

M



N



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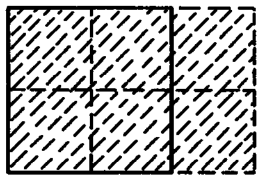


3. Draw a unit square region. Show $\frac{1}{8}$ of this region by drawing lines and shading the region.

4. What mathematical sentence describes the shaded region of Exercise 3 ?

5. Draw a unit square region. Show $\frac{4}{5}$ of this region by drawing lines and shading the region.

6. What mathematical sentence describes the shaded part of Exercise 5 ?

7. Draw a unit square region. Show $\frac{10}{6}$ of this region by drawing lines and shading the region.

8. What mathematical sentence describes the shaded part of Exercise 7 ?

RATIONAL NUMBERS AND WHOLE NUMBERS

Exploration

You can use what you know about multiplication of whole numbers to study multiplication of rational numbers. Consider the product $3 \times 2 = 6$.

1. Here are some fraction names for the numbers 2, 3, and 6:

$$2 = \frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{20}{10}$$

$$3 = \frac{3}{1} = \frac{6}{2} = \frac{12}{4} = \frac{15}{5}$$

$$6 = \frac{6}{1} = \frac{12}{2} = \frac{18}{3} = \frac{30}{5} = \frac{36}{6}$$

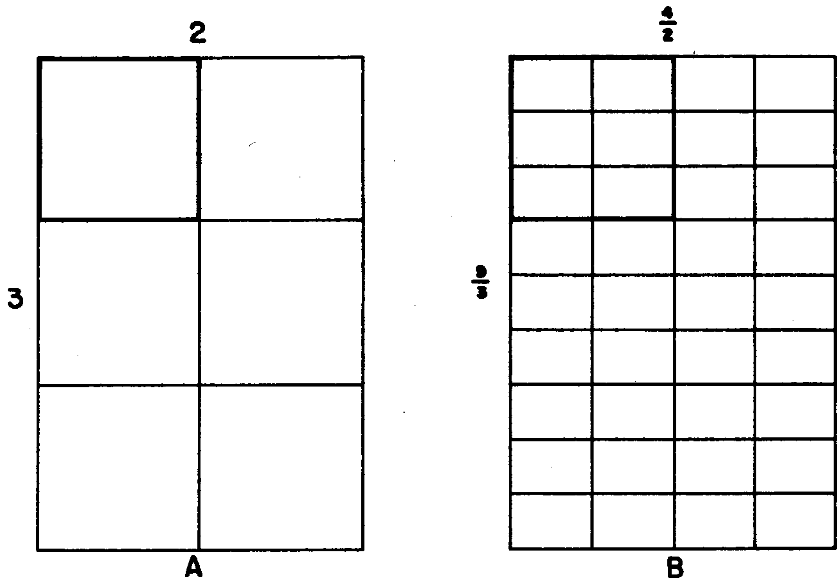
Since $3 \times 2 = 6$, should $\frac{3}{1} \times \frac{2}{1}$ be another name for 6 ?

should $\frac{6}{2} \times \frac{4}{2}$ be another name for 6 ?

should $\frac{12}{4} \times \frac{6}{3}$ be another name for 6 ?

2. a. Figure A is a 3 by 2 rectangular region.

Figure B is a $\frac{9}{3}$ by $\frac{4}{2}$ rectangular region. Should A and B have the same measure?



b. Region A is separated into unit square regions to show that its measure is _____ $\times$ _____ or _____.

c. Figure B is separated into _____ congruent rectangular regions.

d. The unit square is shown with dark lines. Each small rectangular region is _____ of the unit square region.

e. From c) and d), you know that the measure of region B is _____.

f. Does Figure B show that $\frac{9}{3} \times \frac{4}{2} = \frac{36}{6}$?

g. Compare your answers for b and f. Are the measures of regions A and B the same? Does $3 \times 2 = \frac{9}{3} \times \frac{4}{2}$?

3. Consider $2 = \frac{6}{3}$ and $3 = \frac{6}{2}$.

Since $2 \times 3 = 6$, should $\frac{6}{3} \times \frac{6}{2}$ be another name for 6 ? Try some operations with the numerators and denominators to find a fraction name for 6.

4. Is $\frac{6 + 6}{3 + 2} = 6$ a true statement?

Is $\frac{6 - 6}{3 - 2} = 6$ a true statement?

Is $\frac{6 \div 6}{3 \div 2} = 6$ a true statement?

Is $\frac{6 \times 6}{3 \times 2} = 6$ a true statement?

5. Without using a drawing, try to find the product 2×3 by using a different pair of fraction names for 2 and 3. Did you find any operation on the numerators and denominators which seemed to give a fraction name for 6 ?

COMPUTING PRODUCTS OF RATIONAL NUMBERS USING FRACTIONS

Exploration

If your work for Exercise 1 in Exercise Set 9 was correct, you wrote these mathematical sentences:

A. $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

H. $\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$

I. $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

K. $\frac{4}{5} \times \frac{3}{6} = \frac{12}{30}$

M. $\frac{4}{3} \times \frac{3}{2} = \frac{12}{6}$

O. $\frac{2}{2} \times \frac{3}{2} = \frac{6}{4}$

1. Look at sentence A again. Does $\frac{1}{6} = \frac{1 \times 1}{2 \times 3}$?

   In H: Does $\frac{1}{18} = \frac{1 \times 1}{3 \times 6}$?

   I: Does $\frac{4}{9} = \frac{2 \times 2}{3 \times 3}$?

   K: Does $\frac{12}{30} = \frac{4 \times 3}{5 \times 6}$?

   M: Does $\frac{12}{6} = \frac{4 \times 3}{3 \times 2}$?

   O: Does $\frac{6}{4} = \frac{2 \times 3}{2 \times 2}$?

2. If a and b are any counting numbers, what is the product $\frac{1}{a} \times \frac{1}{b}$?

3. If a and c are any whole numbers, and b and d are any counting numbers, what is $\frac{a}{b} \times \frac{c}{d}$?

$$\boxed{\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}}$$

Exercise Set 12

Find a single rational number for the product expressions:

1. $\frac{1}{2} \times \frac{1}{5}$
2. $\frac{1}{5} \times \frac{1}{3}$
3. $\frac{1}{2} \times \frac{1}{4}$
4. $\frac{3}{4} \times \frac{2}{5}$
5. $\frac{2}{3} \times \frac{3}{4}$
6. $\frac{4}{5} \times \frac{2}{3}$
7. $\frac{3}{5} \times \frac{4}{5}$
8. $\frac{3}{2} \times \frac{4}{5}$
9. $\frac{6}{5} \times \frac{3}{4}$
10. $\frac{1}{4} \times \frac{1}{5}$
11. $\frac{1}{5} \times \frac{1}{6}$
12. $\frac{1}{2} \times \frac{1}{6}$
13. $\frac{3}{4} \times \frac{3}{4}$
14. $\frac{2}{3} \times \frac{3}{5}$
15. $\frac{5}{6} \times \frac{2}{3}$
16. $\frac{2}{7} \times \frac{1}{3}$
17. $\frac{2}{3} \times \frac{3}{2}$
18. $\frac{5}{2} \times \frac{10}{3}$
19. $\frac{2}{5} \times \frac{2}{3}$
20. $\frac{3}{8} \times \frac{3}{4}$
21. $\frac{2}{9} \times \frac{1}{2}$
22. $\frac{7}{10} \times \frac{2}{3}$
23. $\frac{2}{5} \times \frac{3}{7}$
24. $\frac{4}{3} \times \frac{5}{3}$
25. $\frac{1}{10} \times \frac{1}{10}$
26. $\frac{1}{10} \times \frac{1}{10^2}$
27. $\frac{1}{10^2} \times \frac{1}{10^2}$

Draw rectangular regions to illustrate these products:

28. $\frac{1}{2} \times \frac{2}{3}$
29. $\frac{1}{5} \times \frac{1}{4}$
30. $\frac{4}{3} \times \frac{6}{5}$
31. $\frac{3}{4} \times \frac{3}{5}$

Exercise Set 13

1. The table shows some measures of rectangular regions. Complete the table:

	Measure of One Side	Measure of Other Side	Measure of Rectangular Region
a.	$\frac{2}{3}$	$\frac{2}{3}$	
b.	$\frac{5}{4}$	$\frac{3}{4}$	
c.	$\frac{2}{5}$	$\frac{3}{4}$	
d.	$\frac{1}{2}$	$\frac{17}{8}$	
e.	$\frac{2}{3}$	$\frac{1}{2}$	
f.	$\frac{3}{4}$	$\frac{4}{5}$	
g.	$\frac{2}{5}$	$\frac{3}{7}$	
h.	$\frac{a}{b}$	$\frac{c}{d}$	

2. Rename each of the following in mixed form or as a whole number:

a. $\frac{4}{3}$ c. $\frac{16}{9}$ e. $\frac{12}{6}$

b. $\frac{3}{2}$ d. $\frac{2}{2}$ f. $\frac{6}{4}$

3. Rename each of the following by a fraction in simplest form:

a. $3\frac{1}{2}$ c. $1\frac{1}{2}$ e. $5\frac{2}{2}$

b. $6\frac{2}{3}$ d. $3\frac{3}{4}$ f. $4\frac{3}{5}$

NAMING PRODUCTS WITH DECIMALS

Exploration

1. a. Draw a unit square.

   b. Draw dotted lines and show by shading a $\frac{1}{10}$ by $\frac{1}{10}$ region of the unit square.

   c. What is the measure of the shaded region?

   d. Write a mathematical sentence suggested by your diagram.

2. a. Draw a unit square.

   b. Draw dotted lines and show by shading a 0.1 by 0.1 region of the unit square.

   c. What is the measure of the shaded region?

   d. Write a mathematical sentence suggested by your diagram.

3. How are the mathematical sentences you wrote for Exercise 1 and 2 alike?

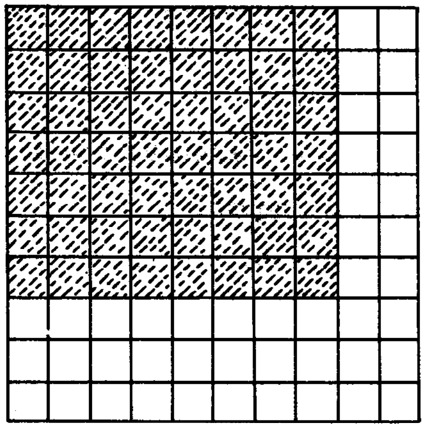
4. Finish these sentences without the use of diagrams.

   a. $\frac{1}{10} \times \frac{1}{10} =$ ____ c. $\frac{1}{10} \times \frac{1}{100} =$ ____ e. $1 \times \frac{1}{10} =$ ____

   b. $0.1 \times 0.1 =$ ____ d. $0.1 \times 0.01 =$ ____ f. $1 \times 0.1 =$ ____

   <u>The products you have just found are important for you to remember</u>. <u>You will use them often</u>.

The product 0.7×0.8 is represented in the diagram.



The shaded region is a 0.7 by 0.8 region, so its measure is 0.7×0.8.

The unit square region is separated into 10×10, or 100 congruent square regions. So the measure of each small square region is 0.01. The shaded region covers 7×8 or 56 small square regions, so its measure is 56×0.01, or 0.56.

$$0.7 \times 0.8 = 0.56$$

The product of the numerators of the fraction forms ($\frac{7}{10}$ and $\frac{8}{10}$) for 0.7 and 0.8 is 7×8 or 56. The product of the denominators is 100. A fraction with numerator 56 and denominator 100 names the same number as 0.56.

Exercise Set 14

1. Draw a unit square region. Separate it into 10×10, or 100, congruent square regions. Use this unit square region to find the products below.

   a. 0.2×0.3 c. 0.9×0.7

   b. 0.6×0.3 d. 0.5×0.4

2. The table shows measurements for rectangular regions.

Complete the table.

	Length of One Side	Length of Other Side	Perimeter	Area
a.	$1\frac{1}{2}$ in.	$\frac{3}{4}$ in.	$4\frac{1}{2}$ in.	$1\frac{1}{8}$ sq. in.
b.	1 ft.	$2\frac{1}{3}$ ft.		
c.	$\frac{2}{3}$ ft.	$2\frac{1}{4}$ ft.		
d.	7 in.	$\frac{5}{8}$ in.		
e.	0.7 mi.	0.5 mi.		
f.	$\frac{3}{4}$ mi.	6 mi.		
g.	$7\frac{1}{2}$ yd.	2 yd.		
h.	$\frac{7}{8}$ in.	$\frac{3}{4}$ in.		
i.	$\frac{1}{2}$ ft.	$4\frac{1}{3}$ ft.		
j.	0.8 in.	0.3 in.		
k.	0.12 mi.	0.5 mi.		
l.	$\frac{1}{2}$ ft.	$8\frac{1}{2}$ ft.		

PRODUCTS OF RATIONAL NUMBERS USING THE NUMBER LINE

Exploration

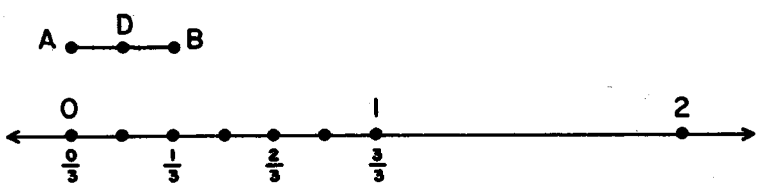
We have used the relation between the measures of the sides and the measure of a rectangular region to give a meaning to the product of any two rational numbers.

If we wish, we can always picture $\frac{1}{2} \times \frac{2}{3}$ as the measure of a $\frac{1}{2}$ by $\frac{2}{3}$ rectangular region. There are, however, other situations which lead to the same rule: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$.

We shall now study some of these other meanings for the product of rational numbers.

First let us use the number line to think about what we usually mean by "$\frac{1}{2}$ of $\frac{1}{3}$" or "$\frac{4}{5}$ of $\frac{2}{3}$".

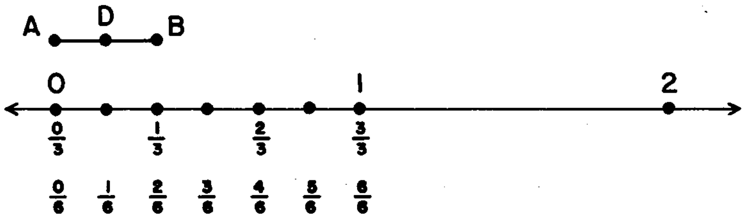
1. Begin by representing $\frac{1}{2}$ of $\frac{1}{3}$. Look at the number line and $\overline{AB}$.



$\overline{AB}$ has measure $\frac{1}{3}$.

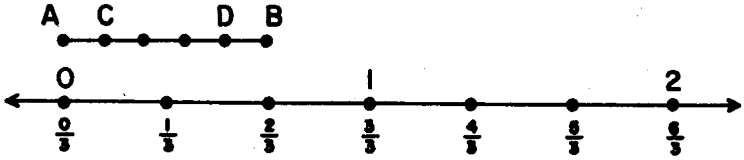
To represent $\frac{1}{2}$ of $\frac{1}{3}$, locate D to separate $\overline{AB}$ into congruent segments $\overline{AD}$ and $\overline{DB}$. $\overline{AD}$ is $\frac{1}{2}$ of $\overline{AB}$. So $m\overline{AD}$ should be $\frac{1}{2}$ of $\frac{1}{3}$.

a. $\frac{1}{2}$ of $\frac{1}{3}$ should be a number which matches D. To find this number, you need a scale with a smaller unit. What scale?



You see that D matches $\frac{1}{6}$, so the diagram shows that $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$.

2. Now consider $\frac{4}{5}$ of $\frac{2}{3}$.



a. What segment has measure $\frac{2}{3}$?

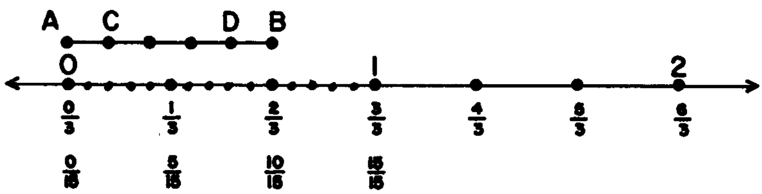
b. How do you find a segment which is $\frac{1}{5}$ of $\overline{AB}$?

c. What segment with endpoint A has measure $\frac{1}{5}$ of $\frac{2}{3}$?

d. What segment with endpoint A has measure $\frac{4}{5}$ of $\frac{2}{3}$?

e. To find $\frac{4}{5}$ of $\frac{2}{3}$, you must find a number on the number line which matches point ______.

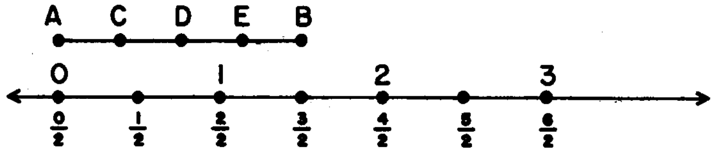
f. You need a scale marked with a smaller unit. What unit? Should separating each $\frac{1}{3}$ segment into 5 congruent segments do?



g. What number matches D?

h. If $\frac{4}{5}$ of $\frac{2}{3}$ = n, what number is n?

3. Now find $\frac{7}{4}$ of $\frac{3}{2}$ using the number line.

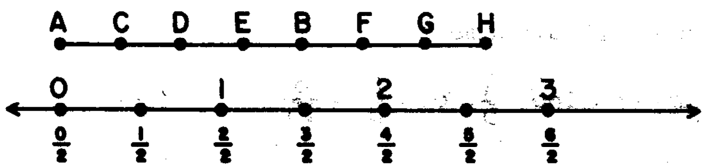


a. $\overline{AB}$ has measure ______.

b. To represent $\frac{7}{4}$ of $\frac{3}{2}$, first separate $\overline{AB}$ into _____ congruent segments. $\overline{AB}$ is separated into congruent segments by points _____, _____, and _____.

c. What segment with endpoint A has measure $\frac{1}{4}$ of $\frac{3}{2}$?

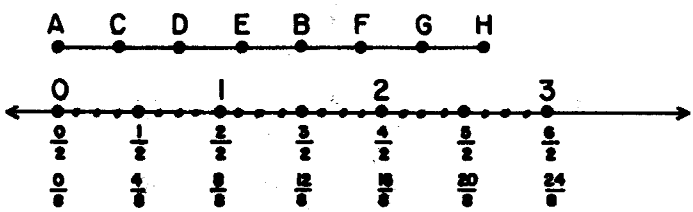
d. To represent $\frac{7}{4}$ of $\frac{3}{2}$, you need ______ of these segments. $\overline{AB}$ is not long enough to represent $\frac{7}{4}$ of $\frac{3}{2}$, so draw three more segments: $\overline{BF}$, $\overline{FG}$, and $\overline{GH}$.



e. Segment ______ has measure $\frac{7}{4}$ of $\frac{3}{2}$.

f. To find $\frac{7}{4}$ of $\frac{3}{2}$, find what number matches point _____.

g. You need a scale in smaller units. A scale of _______ will do.



h. What number matches H ?

i. $\frac{7}{4}$ of $\frac{3}{2} = n$. $n = ?$

Now look at your results for Exercises 1 - 3.

In Exercise 1 you found that $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$.

In Exercise 2 you found that $\frac{4}{5}$ of $\frac{2}{3} = \frac{8}{15}$.

In Exercise 3 you found that $\frac{7}{4}$ of $\frac{3}{2} = \frac{21}{8}$.

4. Use the rule $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$ to find

a. $\frac{1}{2} \times \frac{1}{3}$

b. $\frac{4}{5} \times \frac{2}{3}$

c. $\frac{7}{4} \times \frac{3}{2}$

How do your products compare with your results for Exercises 1 - 3 ?

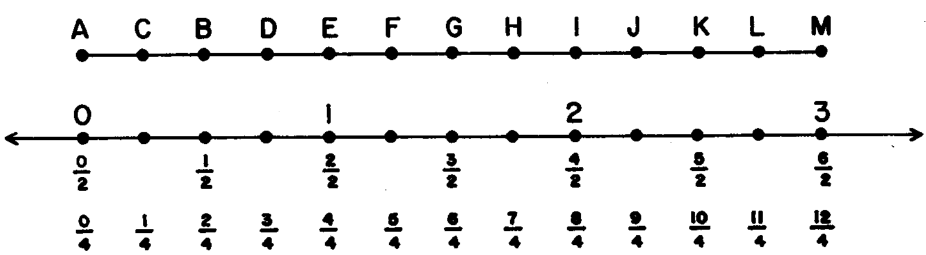
5. Are these reasonable statements?

On the number line, $\frac{4}{5}$ of $\frac{2}{3} = \frac{8}{15}$

$$\frac{8}{15} = \frac{4 \times 2}{5 \times 3} = \frac{4}{5} \times \frac{2}{3}$$

So $\frac{4}{5}$ of $\frac{2}{3} = \frac{4}{5} \times \frac{2}{3}$.

6. On the number line below, the measure of $\overline{AB}$ is $\frac{1}{2}$.



Find a segment with endpoint A whose measure is:

a. $\frac{1}{2}$ of $\frac{1}{2}$ $\qquad$ $\frac{1}{2}$ of $\frac{1}{2}$ = ______

b. $\frac{2}{2}$ of $\frac{1}{2}$ $\qquad$ $\frac{2}{2}$ of $\frac{1}{2}$ = ______

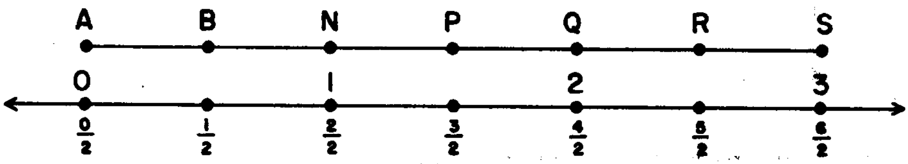
c. $\frac{3}{2}$ of $\frac{1}{2}$ $\qquad$ $\frac{3}{2}$ of $\frac{1}{2}$ = ______

d. $\frac{4}{2}$ of $\frac{1}{2}$ $\qquad$ $\frac{4}{2}$ of $\frac{1}{2}$ = ______

e. $\frac{5}{2}$ of $\frac{1}{2}$ $\qquad$ $\frac{5}{2}$ of $\frac{1}{2}$ = ______

f. $\frac{6}{2}$ of $\frac{1}{2}$ $\qquad$ $\frac{6}{2}$ of $\frac{1}{2}$ = ______

7. On the number line below, the measure of $\overline{AB}$ is $\frac{1}{2}$.



a. Find a segment with endpoint A whose measure is $\frac{1}{2} + \frac{1}{2}$. $\frac{1}{2} + \frac{1}{2}$ = ______.

b. Find a segment with endpoint A whose measure is $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$ = ______.

c. Find a segment with endpoint A whose measure is $3 \times \frac{1}{2}$. $3 \times \frac{1}{2}$ = ______.

Your answer to Exercise 6f should show that $\frac{6}{2}$ of $\frac{1}{2} = \frac{6}{4}$.

Your answer to Exercise 7c should show that $3 \times \frac{1}{2} = \frac{3}{2}$.

Are $\frac{6}{4}$ and $\frac{3}{2}$ names for the same number?

Is this true? $\frac{6}{2}$ of $\frac{1}{2} = 3 \times \frac{1}{2}$?

Exercise Set 15

1. Draw number lines and segments to show

   a. $\frac{1}{3}$ of $\frac{1}{2}$

   b. $\frac{3}{4}$ of $\frac{2}{5}$

   c. $\frac{3}{2}$ of $\frac{7}{5}$

   d. $\frac{3}{4}$ of 10

   e. $\frac{6}{2}$ of $\frac{1}{4}$

2. Find by using $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

   a. $\frac{1}{3} \times \frac{1}{2}$

   b. $\frac{3}{4} \times \frac{2}{5}$

   c. $\frac{3}{2} \times \frac{7}{5}$

   d. $\frac{3}{4} \times 10$

   e. $\frac{6}{2} \times \frac{1}{4}$

3. From your results in Exercises 1 - 2, is it true that

   $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{3} \times \frac{1}{2}$? $\frac{3}{4}$ of $\frac{2}{5} = \frac{3}{4} \times \frac{2}{5}$? $\frac{3}{2}$ of $\frac{7}{5} = \frac{3}{2} \times \frac{7}{5}$?

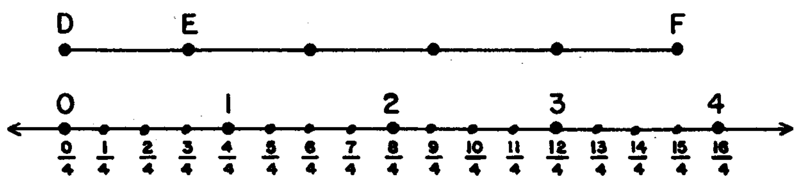
   Look at the number lines and segments in Exercises 4 - 7.

4. In the diagram below, find a segment whose measure is:

   a. $\frac{3}{4}$

   b. $5 \times \frac{3}{4}$

   c. $\frac{15}{4}$

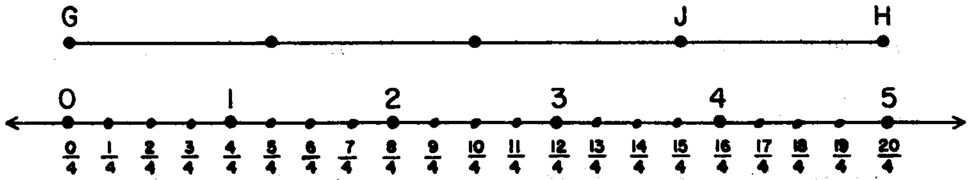


5. In the diagram below, find a segment whose measure is:

a. 5

b. $\frac{3}{4} \times 5$

c. $\frac{15}{4}$

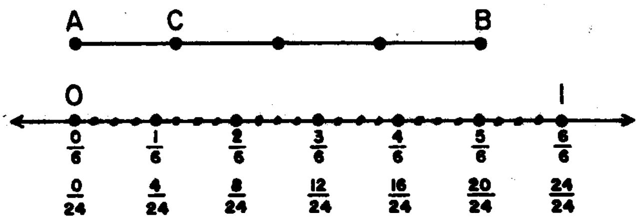


6. Find a segment whose measure is:

a. $\frac{5}{6}$

b. $\frac{1}{4} \times \frac{5}{6}$

c. $\frac{5}{24}$

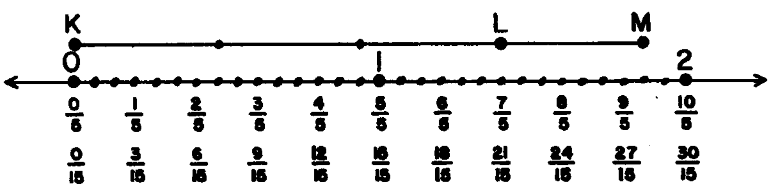


7. Find a segment whose measure is:

a. $\frac{7}{5}$

b. $\frac{4}{3} \times \frac{7}{5}$

c. $\frac{28}{15}$



Make a number line diagram to represent each problem. Then write a mathematical sentence to represent the problem and answer the question in a complete sentence.

8. The Scouts hiked from the school to a camp $3\frac{1}{2}$ miles away. They stopped to rest when they had gone $\frac{1}{3}$ of the way. How far had they walked when they stopped to rest?

9. Jane had $\frac{3}{4}$ yard of ribbon for badges. She made 4 badges of equal length. How long was each badge?

10. A 5 story building is 48 feet high. If the stories are of equal height, how far above the ground is the ceiling of the third story?

11. Bill put up shelves in his room, each $2\frac{1}{4}$ feet long. How long a board did he use for 3 shelves?

12. Sue used $\frac{3}{4}$ of a piece of toweling to make a place mat. If the piece was $\frac{2}{3}$ yard long, how long a piece did she use for the place mat?

USING MATHEMATICAL SENTENCES TO DESCRIBE PROBLEMS

Exploration

1. Richard's new foreign car travels 29 miles on one gallon of gas. How many miles will it travel on 7 gallons?

$$29 \times 7 = t$$

$$203 = t$$

Richard's car will go 203 miles on 7 gallons of gas. What relationship in the problem is expressed in the mathematical sentence?

2. Consider the problem: Mary's mother bought 7 yards of material to make two costumes. She used $3\frac{2}{3}$ yards for one costume and $2\frac{1}{6}$ yards for the other. How many yards of material did she have left?

To solve this problem, what question should you ask first? Should you ask, "How many yards did she use in all for the two costumes?"

Suppose you call this number k. Does

$$3\frac{2}{3} + 2\frac{1}{6} = k \ ?$$

What question could you answer next?

Suppose n is the number of yards left. Does $7 - k = n$? Which of these mathematical sentences is a correct representation for the problem?

$$7 - 3\frac{2}{3} + 2\frac{1}{6} = n$$

$$(7 - 3\frac{2}{3}) + 2\frac{1}{6} = n$$

$$7 - (3\frac{2}{3} + 2\frac{1}{6}) = n$$

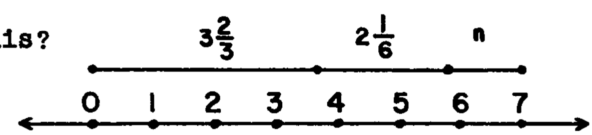
What about these sentences? Would any of them be correct also?

$$(7 - 3\frac{2}{3}) - 2\frac{1}{6} = n$$

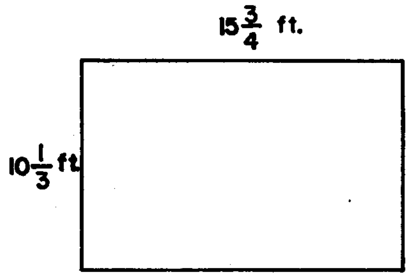
$$n + (3\frac{2}{3} + 2\frac{1}{6}) = 7$$

$$3\frac{2}{3} + 2\frac{1}{6} = 7 - n$$

Can this problem be represented by segments on the number line like this?



3. Below is a diagram of Steve's garden. He wants to put a wire fence around it. The wire costs $13\frac{1}{2}$ cents a foot. What will be the cost of the wire?



Do you have to know how much wire is needed before you can answer the question, "What will be the cost of the wire?"

Let m represent the perimeter of the garden in feet.

Does $(10\frac{1}{3} \times 2) + (15\frac{3}{4} \times 2) = m$?

What question could you answer next?

Let n = the number of cents the wire costs.

Does $13\frac{1}{2} \times m = n$?

Which of the following mathematical sentences best expresses the relationship in the problem?

$$15\frac{3}{4} \times [(2 + 10\frac{1}{3}) \times (2 \times 13\frac{1}{2})] = n$$

$$(15\frac{3}{4} \times 2) + 10\frac{1}{3} \times (2 \times 13\frac{1}{2}) = n$$

$$[(15\frac{3}{4} \times 2) + (10\frac{1}{3} \times 2)] \times 13\frac{1}{2} = n$$

Notice symbols [ ] called brackets are used to group parentheses that name only one number.

What about these sentences? Would any of them be correct also?

$$[(15\frac{3}{4} + 15\frac{3}{4}) + (10\frac{1}{3} + 10\frac{1}{3})] \times 13\frac{1}{2} = n$$

$$[(15\frac{3}{4} + 15\frac{3}{4}) \times 13\frac{1}{2}] + [(10\frac{1}{3} + 10\frac{1}{3}) \times 13\frac{1}{2}] = n$$

$$(15\frac{3}{4} + 15\frac{3}{4} + 10\frac{1}{3} + 10\frac{1}{3}) \times 13\frac{1}{2} = n$$

Exercise Set 16

Read the following carefully, write the relationship in each problem as a mathematical sentence, solve, and answer the question asked in the problem:

1. A recipe calls for $\frac{1}{4}$ cup butter. If you make only $\frac{1}{2}$ of the recipe, how much butter do you need?

2. John lives $\frac{1}{3}$ mile from school. Harry lives only half that distance from school. How far from the school does Harry live?

3. Mrs. Morgan bought half a cake for her family of four. If she made all servings the same size, what part of a cake was each person served?

Exercise Set 17

1. Peter bought $\frac{1}{2}$ pound of cheese for school lunches. The first day he used $\frac{1}{5}$ of the amount he bought. How much cheese did he use?

2. Sara is supposed to practice the piano $\frac{1}{2}$ hour each day. She practiced only $\frac{1}{3}$ of that time on Saturday. What part of an hour did she practice on Saturday?

3. Terry ate $\frac{1}{3}$ dozen cookies after lunch. He ate $\frac{1}{4}$ dozen cookies after dinner. What part of a dozen cookies did he eat?

4. The distance from the library to the city hall is $\frac{3}{4}$ of a mile. What part of a mile will you have gone if you walk $\frac{3}{4}$ of this distance?

5. Ned needs a piece of canvas $8\frac{1}{4}$ feet long. He has one piece $2\frac{3}{4}$ feet long and another piece $3\frac{1}{2}$ feet long. How much does he still need?

6. One-half the pupils of a school are going to a concert. These children will be taken in 5 buses. What part of the pupils of the school will ride in each bus?

7. A gallon of cream weighs 8.4 pounds, a gallon of milk weighs 8.6 pounds, and a gallon of water weighs 8.3 pounds. How many pounds will a gallon of cream, a gallon of milk, and a gallon of water weigh together?

8. Carol feeds her dog $\frac{3}{4}$ pounds of meat daily. She feeds him twice a day. What part of a pound of meat does the dog get at each meal.

9. The record speed for an airplane in 1960 was 1,526 miles per hour. The record speed for an automobile was 394.19 miles per hour. How much greater was the speed recorded for an airplane?

10. Mary weighs $62\frac{1}{2}$ pounds. Her brother weighs $\frac{1}{2}$ as much as she weighs. What does he weigh?

11. George lives 2.7 miles from school. He makes one round trip each day. How many miles does he walk to school each week?

12. Mrs. Marks bought a $5\frac{1}{2}$ pound roast and $\frac{3}{4}$ pound of ground meat. How much meat did she buy altogether?

13. Mr. Hayes drove 42.3 miles per hour for 3 hours. How many miles did he drive?

14. Mike's garden is $15\frac{1}{2}$ feet by $20\frac{1}{2}$ feet. What is the area of the garden?

15. Tim's dog eats $\frac{1}{4}$ pound of food in the morning and $\frac{1}{3}$ pound of food in the afternoon. How much food does Tim's dog eat during one week?

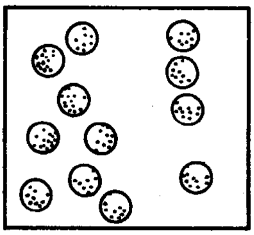
RATIONAL NUMBERS WITH SETS OF OBJECTS

You have illustrated products of rational numbers by using rectangular regions and by using segments on the number line.

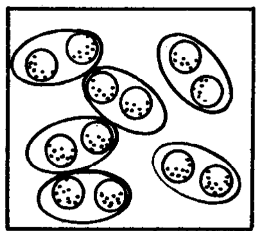
Multiplication of rational numbers can also be used to answer some kinds of questions about sets of objects.

Exploration

Picture A represents a set of golf balls. In picture B this set is separated into subsets, with the same number of balls in each subset.



A

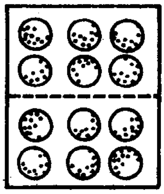


B

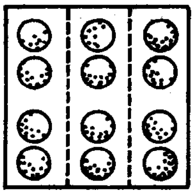
1. How many balls are in the set? What rational number represents the part of the set in each subset? How many balls are in each subset? Does the sentence $\frac{1}{6}$ of 12 balls are 2 balls describe this situation?

2. Draw pictures of the set of balls separated into fewer subsets, with the same number of balls in each subset. Can you do this in more than one way?

3. Pictures C, D, E, F, G show the balls arranged in arrays. The broken lines in each picture show the separation of the set into subsets, with the same number of balls in each subset.



C



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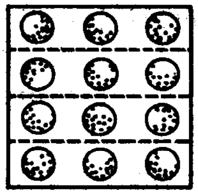

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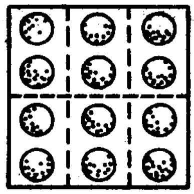

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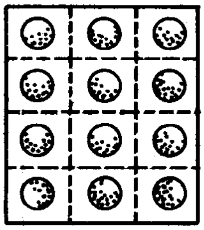


G

4. Write a sentence like the one in Exercise 1 which describes each picture.

5. Use the pictures to find

a. $\frac{2}{3}$ of 12 balls

b. $\frac{4}{6}$ of 12 balls

c. $\frac{5}{12}$ of 12 balls

d. $\frac{1}{2}$ of 12 balls

e. $\frac{2}{4}$ of 12 balls

f. $\frac{6}{12}$ of 12 balls

6. a. Is $\frac{2}{3}$ of 12 the same number of balls as $\frac{2}{3} \times \frac{12}{1}$?

   b. Is $\frac{7}{12}$ of 12 the same number as $\frac{7}{12} \times \frac{12}{1}$?

   c. Is $\frac{5}{6}$ of 12 the same number as $\frac{5}{6} \times \frac{12}{1}$?

   d. Can you use multiplication of rational numbers to find the number of objects in a part of a set?

7. Do your answers in Exercise 6 agree with what you found to be true when you used segments on the number line?

8. a. Draw a picture to represent a set of 12 licorice sticks.

   b. You wish to divide the licorice sticks among 5 boys. Draw rings around subsets to show the whole sticks each boy will get.

   c. Show on your drawing how you will divide up the remaining sticks.

   d. Write the rational number which represents the number of sticks each boy will get. Is your result equal to $\frac{1}{5} \times \frac{12}{1}$?

<u>Exercise Set 18</u>

1. Draw pictures of golf balls. Then write the mathematical sentences.

a. $\frac{4}{5}$ of 15	d. $\frac{3}{7}$ of 28
b. $\frac{7}{8}$ of 24	e. $\frac{7}{5}$ of 20
c. $\frac{9}{10}$ of 30	f. $\frac{5}{4}$ of 24

2. Find n:

a. $\frac{4}{5} \times \frac{15}{1} = n$

b. $\frac{7}{8} \times \frac{24}{1} = n$

c. $\frac{9}{10} \times \frac{30}{1} = n$

d. $\frac{3}{7} \times \frac{28}{1} = n$

e. $\frac{7}{5} \times \frac{20}{1} = n$

f. $\frac{5}{4} \times \frac{24}{1} = n$

3. Compare your results in Exercises 1 and 2.

4. Only two-thirds of the pupils in Bedford School can be seated in the auditorium, so only two-thirds of the classes may go to the assembly. There are 33 classes. How many classes may go?

5. Jim said that $\frac{9}{10}$ of his 40 tomato plants had tomatoes on them. How many plants had tomatoes? How many did not?

6. The refreshment committee estimated that $\frac{3}{4}$ of the pupils and parents would come to the class picnic. If there were 84 pupils and parents, how many people did the committee think would come?

7. A class had a supply of 1 gross of pencils (144) when school began. A month later they had 84 left. What rational number tells what part of their supply they had left?

8. Jane had 24 questions right on a test with 30 questions. What rational number tells what part of the test she answered correctly?

PROPERTIES OF RATIONAL NUMBERS

Exploration

You know several properties of the operation of multiplication with whole numbers: Commutative, Associative, Closure, Property of Zero, and Property of One. You also know the Distributive Property, which relates multiplication and addition of whole numbers.

1. Illustrate each of these properties with whole numbers. Make diagrams (using rectangular regions or segments on the number line) to illustrate the following products:

2. a. $\frac{2}{3} \times \frac{5}{4}$

   b. $\frac{5}{4} \times \frac{2}{3}$

3. a. $\frac{2}{5} \times 3$

   b. $3 \times \frac{2}{5}$

4. Find the products in Exercise 2 and Exercise 3 using $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$.

5. a. Is $\frac{2}{3} \times \frac{5}{4} = \frac{5}{4} \times \frac{2}{3}$ a true statement?

   b. Is $\frac{2}{5} \times 3 = 3 \times \frac{2}{5}$ a true statement?

6. What property for multiplication of rational numbers is suggested by Exercises 2 - 5 ?

Find s and t:

7. a. $s = (\frac{1}{4} \times \frac{2}{3}) \times \frac{2}{5}$

   b. $t = \frac{1}{4} \times (\frac{2}{3} \times \frac{2}{5})$

8. a. $s = (7 \times \frac{3}{4}) \times \frac{5}{6}$

   b. $t = 7 \times (\frac{3}{4} \times \frac{5}{6})$

9. a. What is true of the products s and t in Exercise 7?

   b. What is true of the products s and t in Exercise 8?

10. What property of multiplication of rational numbers is suggested by Exercises 7 - 8?

11. Find the numbers n and t by using drawings like the ones on Page 70 for Exercise 4.

    a. $n = \frac{1}{2} \times (\frac{2}{3} + \frac{3}{4})$

    b. $t = (\frac{1}{2} \times \frac{2}{3}) + (\frac{1}{2} \times \frac{3}{4})$

12. Which of these sentences is true in Exercise 11?

    $n > t$ $\qquad$ $n = t$ $\qquad$ $n < t$

13. State a property of rational numbers suggested by Exercises 11 - 12.

Find r and s:

14. a. $\frac{0}{8} \times \frac{3}{4} = r$

    b. $\frac{5}{6} \times \frac{0}{3} = s$

15. What property of rational numbers does Exercise 14 suggest?

16. Find r and s. Express your answer in simplest form.

a. $r = \frac{3}{3} \times \frac{3}{5}$ b. $s = \frac{7}{3} \times \frac{10}{10}$

17. Compare each product in Exercise 16 with the factors. What do you observe?

18. State a property of rational numbers suggested by Exercise 16.

19. You are used to working with rational numbers that are easy to picture. Here are some names for less commonly used rational numbers: $\frac{178}{249}$ and $\frac{31}{155}$.

a. Can you imagine a rectangular region whose sides have these measures?

b. How would you find the measure of the region?

c. What kind of number would your result be?

d. Make up two other strange rational numbers. If they were measures of the sides of a rectangular region, what kind of number would the measure of the region be?

e. What property of multiplication of rational numbers does this suggest?

Exercise Set 19

Copy and complete these multiplication charts. Express products in simplest form.

1.

x	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$
$\frac{1}{2}$	$\frac{1}{4}$			
$\frac{2}{3}$				
$\frac{3}{4}$				
$\frac{4}{5}$				

2.

x	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	1
0	0				
$\frac{1}{2}$					
$\frac{2}{3}$					
$\frac{3}{4}$					
1					

3.

x	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	
2	$\frac{2}{2}$ or 1				
3					
4					
5					

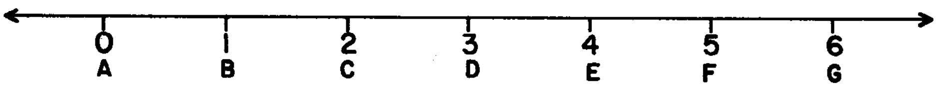
4. What property (or properties) do you find illustrated in each chart?

SIZE OF PRODUCTS

Exploration

A bug named Willie crawls along a crack in the floor. He travels 2 feet per minute.

Suppose the crack represents a number line.



1. How far does Willie crawl in 3 minutes? If he starts at A, what point does he reach in 1 minute? in 2 minutes? in $2\frac{1}{2}$ minutes? in 3 minutes?

2. For each part of Exercise 1, write a mathematical sentence which shows the relation between the time Willie crawls, his speed, and the distance he goes.

3. Between which labeled points will Willie be when he has crawled $2\frac{3}{4}$ minutes? $1\frac{1}{4}$ minutes? $\frac{2}{3}$ minute? $\frac{2}{5}$ minute?

4. Write a mathematical sentence for each part of Exercise 3.

5. What operation is indicated in each mathematical sentence in Exercises 2 and 4?

6. a. If Willie starts at A and crawls less than one minute, on which segment must he be?

   b. If Willie starts at A and crawls more than one minute, where will he be?

Exercise Set 20

1. A jet plane travels 600 miles an hour. At that rate, how far does it travel in 4 hours? in 3 hours? in $2\frac{1}{2}$ hours? in $1\frac{1}{4}$ hours? 1 hour? $\frac{3}{4}$ hour? $\frac{1}{2}$ hour? $\frac{1}{3}$ hour? $\frac{1}{10}$ hour?

   Record these facts in the table below.

JET PLANE

Number of Hours	Number of Miles per Hour	Total Number of Miles Traveled
4	600	$4 \times 600 = 2400$
3	600	
$2\frac{1}{2}$	600	
$1\frac{1}{4}$	600	
1	600	
$\frac{3}{4}$	600	
$\frac{1}{2}$	600	
$\frac{1}{3}$	600	
$\frac{1}{10}$	600	

2. What operation did you use to answer each part of Exercise 1?

3. Suppose the Scouts hike 3 miles an hour. At that rate, how far do they walk in 4 hours? In 3 hours? Make a table like the one above. Use the same numbers of hours.

4. A fast, lively turtle walks $\frac{1}{8}$ mile an hour. Make a table like the one above for the turtle. What operation do you use to find his distances?

Use your tables for Exercises 1, 3, and 4 to answer these questions.

5. Make these sentences true by putting >, <, or = in the blank.

	Plane	Scouts	Turtle
a.	4×600 ____ 600	4×3 ____ 3	$4 \times \frac{1}{8}$ ____ $\frac{1}{8}$
b.	3×600 ____ 600	3×3 ____ 3	$3 \times \frac{1}{8}$ ____ $\frac{1}{8}$
c.	$2\frac{1}{2} \times 600$ ____ 600	$2\frac{1}{2} \times 3$ ____ 3	$2\frac{1}{2} \times \frac{1}{8}$ ____ $\frac{1}{8}$
d.	$1\frac{1}{4} \times 600$ ____ 600	$1\frac{1}{4} \times 3$ ____ 3	$1\frac{1}{4} \times \frac{1}{8}$ ____ $\frac{1}{8}$
e.	1×600 ____ 600	1×3 ____ 3	$1 \times \frac{1}{8}$ ____ $\frac{1}{8}$
f.	$\frac{3}{4} \times 600$ ____ 600	$\frac{3}{4} \times 3$ ____ 3	$\frac{3}{4} \times \frac{1}{8}$ ____ $\frac{1}{8}$
g.	$\frac{1}{2} \times 600$ ____ 600	$\frac{1}{2} \times 3$ ____ 3	$\frac{1}{2} \times \frac{1}{8}$ ____ $\frac{1}{8}$
h.	$\frac{1}{3} \times 600$ ____ 600	$\frac{1}{3} \times 3$ ____ 3	$\frac{1}{3} \times \frac{1}{8}$ ____ $\frac{1}{8}$
i.	$\frac{1}{10} \times 600$ ____ 600	$\frac{1}{10} \times 3$ ____ 3	$\frac{1}{10} \times \frac{1}{8}$ ____ $\frac{1}{8}$

6. Look at the column in Exercise 5 about the jet plane.
   a. What factor is shown in each product expression?
   b. In which lines of the table is the product greater than this factor? equal to this factor? less than this factor?
   c. How do you explain your observation in b ?

7. Look at the column in Exercise 5 about the Scouts.

   a. What factor is shown in each product expression?

   b. In which lines of the table is the product greater than this factor? equal to this factor? less than this factor?

   c. How do you explain your observation in b ?

8. Answer the questions in Exercise 6, using the turtle column.

9. Examine your answers for Exercises 6, 7, and 8. Fill in >, =, < to make these sentences true. Put the same symbol in both blanks of each sentence.

   a. When one factor in a product expression _____ 1, the product _____ the other factor.

   b. When one factor in a product expression _____ 1, the product _____ the other factor.

   c. When one factor in a product expression _____ 1, the product _____ the other factor.

A NEW PROPERTY OF RATIONAL NUMBERS: RECIPROCAL PROPERTY

You have seen that some properties of whole numbers are also properties of rational numbers. Do you think that rational numbers may have some properties which whole numbers do not have?

Exploration

1. Find r in each sentence.

   a. $r = \frac{3}{4} \times \frac{4}{3}$ c. $r = \frac{5}{2} \times \frac{2}{5}$ e. $r = \frac{25}{3} \times \frac{3}{25}$

   b. $r = \frac{7}{8} \times \frac{8}{7}$ d. $r = \frac{9}{10} \times \frac{10}{9}$ f. $r = \frac{100}{7} \times \frac{7}{100}$

2. What do you notice about the product in each part of Exercise 1?

3. Write the two factors in Exercise 1-a. What do you notice about them?

4. Do you notice the same thing about the factors in Exercise 1-b through 1-f?

Find the rational number n which makes each sentence true.

5. $\frac{2}{3} \times n = 1$

6. $n \times \frac{8}{10} = 1$

7. $1 = \frac{7}{5} \times n$

8. $n \times \frac{12}{5} = 1$

$\frac{4}{3}$ is called the reciprocal of $\frac{3}{4}$

$\frac{7}{8}$ is the reciprocal of $\frac{8}{7}$.

9. What is the product when a number is multiplied by its reciprocal?

10. Find n in each sentence:

a. $n = 0 \times \frac{5}{8}$ c. $n = \frac{7}{9} \times 0$ e. $n = \frac{13}{18} \times \frac{0}{23}$

b. $n = \frac{3}{4} \times \frac{0}{2}$ d. $n = \frac{0}{3} \times \frac{8}{5}$ f. $n = 0 \times \frac{17}{10}$

11. What property of multiplication of rational numbers does Exercise 10 illustrate?

12. If possible, find a rational number n which makes each sentence true:

a. $\frac{0}{3} \times n = 0$ c. $n \times 0 = 1$

b. $n \times \frac{0}{4} = 1$ d. $0 \times n = 0$

13. Is there a rational number which does not have a reciprocal? Is there more than one such rational number?

14. Can we state this property for rational numbers? For every rational number $\frac{a}{b}$, if a is not 0, and b is not 0, $\frac{a}{b} \times \frac{b}{a} = 1$.

15. Could the property in Exercise 14 be stated this way? Every rational number except 0 has a reciprocal.

16. Think of the set of whole numbers. Can you find a whole number n such that

a. $5 \times n = 1$? b. $n \times 8 = 1$?

17. Does the set of whole numbers have the reciprocal property stated in Exercise 15?

18. Find a rational number n such that the sentence in Exercise 16a is true. Do the same for the sentence in Exercise 16b.

19. Do you see an easy way to find the reciprocal of a rational number?

20. The measure of a rectangular region is 1. Find the measure of a side, if the measure of the other side is:

a. $\frac{3}{4}$ c. $\frac{9}{4}$ e. $\frac{11}{13}$

b. $\frac{7}{10}$ d. $1\frac{1}{2}$ f. 0.25

<u>If the product of two rational numbers is 1, each number is the reciprocal of the other.</u>

<u>Exercise Set 21</u>

1. Mr. Brown bought 6.1 gallons of gas on Saturday and 7.9 gallons on Sunday. How many gallons of gas did Mr. Brown buy on the two days together?

2. One jet averaged 659.49 miles per hour on its test flight. Another jet averaged 701.1 miles per hour. How much greater was the average speed of the second plane?

3. If the average rainfall in a state is 2.7 inches per month, what will be the total rainfall for the year?

4. What is the area of a rectangular room whose sides are $12\frac{3}{4}$ feet and 15 feet long?

5. Jim is $50\frac{1}{2}$ inches tall. Sally is $48\frac{3}{4}$ inches tall. How much taller is Jim than Sally?

6. A recipe called for $1\frac{3}{4}$ cups oatmeal, and $1\frac{1}{2}$ cups flour, and $1\frac{1}{3}$ cups raisins. How many cups of dry ingredients were called for in the recipe?

7. Joel has planted $\frac{3}{4}$ of his garden in vegetables. $\frac{1}{2}$ of this section is planted in tomatoes. What part of the whole garden is planted in tomatoes?

8. If 1 day is $\frac{1}{7}$ of a week, what part of the week is 12 hours?

9. Allen drinks $1\frac{1}{2}$ cups of milk three times a day. How many cups of milk does he drink in one week?

10. Eddie's house is $\frac{3}{8}$ mile from school. How far does he walk each day if he makes two round trips? How far does he walk each week?

11. For the summer, Rick and Sam cut lawns for the neighbors. Together they charged 3 dollars an hour. They worked $7\frac{3}{4}$ hours each day. How much did they earn in one day? in one week (5 days)?

12. Bill is $2\frac{1}{3}$ years older than Bob. Bob is $3\frac{1}{2}$ years older than Jack. How much older is Bill than Jack?

13. The measurements of the sides of a rectangular sheet of metal are 17.2 inches and 9.8 inches. What is the area of the sheet in square inches?

14. California had 3.6 inches of rain in January, 5.1 inches in February, 5.8 inches in March, and 4.4 inches in April. The total amount for the year was 23.0 inches. How much rain fell during the other eight months?

15. Mrs. Jackson baked $2\frac{1}{2}$ dozen cookies. For lunch Helen ate $\frac{1}{4}$ dozen, Janet ate $\frac{1}{3}$ dozen, Dotty ate $\frac{1}{6}$ dozen, and Ellen ate $\frac{1}{12}$ dozen. How many dozen cookies were left.

COMPUTING PRODUCTS OF RATIONAL NUMBERS USING DECIMALS

Exploration
(Extending Decimal Notation)

1. You know that 45.687 means

$(4 \times 10) + (5 \times 1) + (6 \times \frac{1}{10}) + (8 \times \frac{1}{100}) + (7 \times \frac{1}{1000})$

Suppose you see the numeral 45.6872. What number does the digit 2 represent? What should be the place-value name? In the decimal system of numeration, the value of each place is $\frac{1}{10}$ the value of the place on its left. For example,

$$0.1 = \frac{1}{10}$$
$$0.01 = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$$
$$0.001 = \frac{1}{10} \times \frac{1}{100} = \frac{1}{1,000}.$$

So the value of the next place is $\frac{1}{10} \times \frac{1}{1,000}$, or $\frac{1}{10,000}$. The digit 2 in the numeral above represents $2 \times \frac{1}{10,000}$, or 2 ten-thousandths.

2. Suppose the numeral is 45.68723. What number does the 3 represent? What should be the place-value name?

3. Does the chart below agree with your answer?

Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths
					4	5.	6	8	7	2	3

Exercise Set 22

1. Express the meaning of the numeral 97.04682 in expanded notation.

2. Jack wrote his answer this way:

$$(9 \times 10) + (7 \times 1) + (0 \times \frac{1}{10}) + (4 \times \frac{1}{100}) + (6 \times \frac{1}{1000}) +$$
$$(8 \times \frac{1}{10,000}) + (2 \times \frac{1}{100,000})$$

Bill wrote this:

$$(9 \times 10) + (7 \times 1) + (0 \times \frac{1}{10}) + (4 \times \frac{1}{10^2}) + (6 \times \frac{1}{10^3}) +$$
$$(8 \times \frac{1}{10^4}) + (2 \times \frac{1}{10^5})$$

Whose answer was correct?

3. Show that $0.2761 = \frac{2761}{10,000}$ by first writing 0.2761 in expanded notation.

4. Write the decimal name for

 a. 234 thousandths

 b. 17 ten-thousandths

 c. 346 hundred-thousandths

5. Write the fraction form for

 a. 0.2567

 b. 0.01682

 c. 32.5678

6. Suppose you wanted to separate a unit square region into square regions to show ten-thousandths. How many congruent segments should you make on each side of the square?

Exploration

You know that you can multiply rational numbers with decimal names by thinking about their fraction names. Is there another way?

Consider the product 4.53×0.007

a. $4.53 = \frac{453}{100} = 453 \times \frac{1}{100} = 453 \times \frac{1}{10^2}$

b. $0.007 = \frac{7}{1000} = 7 \times \frac{1}{1000} = 7 \times \frac{1}{10^3}$

c. So, $4.53 \times 0.007 = (453 \times \frac{1}{10^2}) \times (7 \times \frac{1}{10^3})$

d. $= 453 \times (\frac{1}{10^2} \times 7) \times \frac{1}{10^3}$

e. $= 453 \times (7 \times \frac{1}{10^2}) \times \frac{1}{10^3}$

f. $= (453 \times 7) \times (\frac{1}{10^2} \times \frac{1}{10^3})$

g. $= (453 \times 7) \times \dfrac{1 \times 1}{10^2 \times 10^3}$

h. $= 3171 \times \dfrac{1}{10^5}$

i. $= 3171 \times \dfrac{1}{100,000}$

j. $= \dfrac{3171}{100,000} = 0.03171$

1. Explain line a and line b.

2. What property of rational numbers is used in line d ? in line e ? in line f ?

3. What is done in line g ?

4. What law of exponents is used in line h ?

5. How is the 100,000 obtained in line i ?

Lines f and g show that you can find the product of two rational numbers named by decimals by a) multiplying as though they were whole numbers and b) placing a decimal point to indicate the correct place value.

The first step is familiar. How can you tell where the decimal point should be? Look at line k below.

k. $4.53 \times 0.007 = (453 \times \dfrac{1}{10^2}) \times (7 \times \dfrac{1}{10^3})$

$$= (453 \times 7) \times (\frac{1}{10^5})$$

$$= 0.03171.$$

Do you see an easy way to tell what each exponent should be? Is there an easy way to decide how many digits in the product there should be to the right of the decimal point?

6. Write these products as shown in line k.

a. 2.46×3.1 c. 1.68×0.005

b. 0.513×9.2 d. 6.2×1.049

Since you find the product by first multiplying as with whole numbers, it is convenient to arrange your work in vertical form and record your thinking like this:

$$\begin{array}{r} 1.049 \\ \underline{6.2} \\ 2\;098 \\ \underline{62\;94} \\ 6.5\;038 \end{array} \qquad \begin{array}{l} 1049 \times \frac{1}{10^3} \\ 62 \times \frac{1}{10} \\ \\ \\ 65038 \times \frac{1}{10^4} \end{array}$$

Exercise Set 23

Use the vertical form as shown to find r.

1. $r = 3.25 \times 0.04$
2. $r = 6.17 \times 0.29$
3. $r = 0.048 \times 1.46$
4. $r = 3.1 \times 0.307$
5. $r = 58 \times 7.23$
6. $r = 0.96 \times 7.7$
7. $r = 0.18 \times 0.056$
8. $r = 3.68 \times 1.42$
9. $r = 19.03 \times 8.5$

RATIONAL NUMBERS WITH DECIMAL NAMES

You know that the decimal name for a number can be written easily if the fraction name has denominator 10, 10^2, 10^3, etc. What about numbers whose fraction names have other denominators? Can you name the number $\frac{1}{4}$ by a fraction with denominator 10, or 100, or 1000? Since $\frac{1}{4} = \frac{25}{100}$, $\frac{1}{4}$ and 0.25 name the same number.

Exercise Set 24

1. If possible, for these numbers find fraction names with whole number numerators and with denominator 10, 100, 1000, or 10,000.

   a. $\frac{3}{4}$
   b. $\frac{5}{8}$
   c. $\frac{2}{3}$
   d. $\frac{3}{5}$
   e. $\frac{1}{16}$
   f. $\frac{2}{5}$
   g. $\frac{1}{4}$
   h. $\frac{5}{6}$
   i. $\frac{3}{8}$
   j. $\frac{1}{7}$

2. You should have found fraction names with whole number numerators for all but three of the numbers in Exercise 1. Explain why you could not find fraction names with whole number numerators for these three numbers. (Hint: Find the prime factorization of 10, 100, 1000, and 10,000. Find the prime factorization of the denominators of the fractions for the three numbers.)

3. Write decimal names for the other seven numbers.

4. In your answers for Exercise 1, which fractions have the same denominator?

USING THE DISTRIBUTIVE PROPERTY

You have found products of rational numbers by using their fraction names.

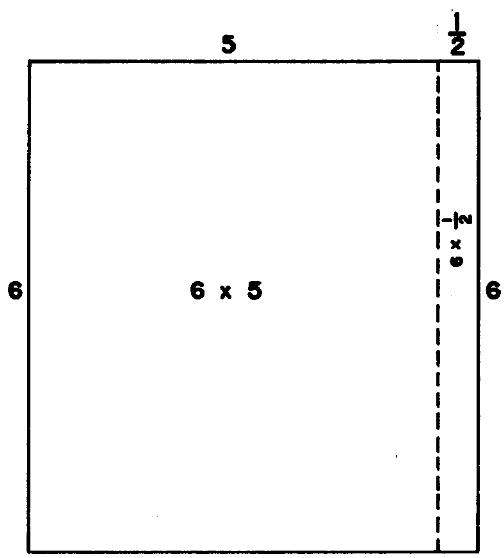
<u>First way</u> $6 \times 5\frac{1}{2} = \frac{6}{1} \times \frac{11}{2} = \frac{66}{2} = 33$

Since $5\frac{1}{2} = 5 + \frac{1}{2}$, you could also use the distributive property.

<u>Second way</u>

$$\begin{aligned} 6 \times 5\tfrac{1}{2} &= 6 \times (5 + \tfrac{1}{2}) \\ &= (6 \times 5) + (6 \times \tfrac{1}{2}) \\ &= 30 + \tfrac{6}{2} \\ &= 30 + 3 \\ 6 \times 5\tfrac{1}{2} &= 33 \end{aligned}$$

You can illustrate the second way by using rectangular regions.



Exercise Set 25

1. Find n by using the distributive property. Write each product in simplest form.

   a. $n = 8 \times 2\frac{3}{4}$

   b. $n = 5\frac{1}{3} \times 7$

   c. $n = 4 \times 16\frac{1}{2}$

   d. $n = 12\frac{1}{8} \times 24$

   e. $n = 6\frac{3}{4} \times 30$

   f. $n = 14 \times 9\frac{3}{7}$

2. Draw a rectangular region and separate it to illustrate one of the products in Exercise 1.

3. Find the products in Exercise 1 by using $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$. Write the simplest name for each product. Which way is easier?

ESTIMATING PRODUCTS

When you are multiplying rational numbers, it is a good idea to estimate the product first.

Consider the product

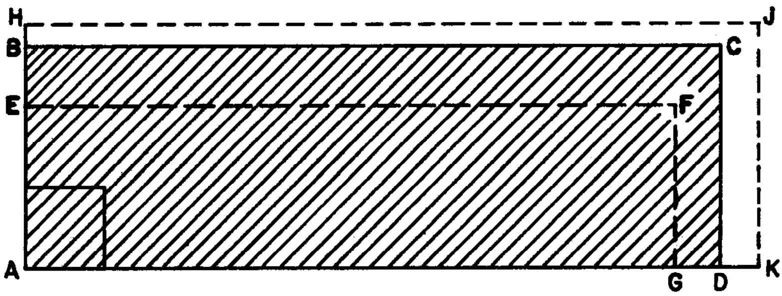
$2\frac{3}{4} \times 8\frac{1}{2}$. How large should the product be?

$2\frac{3}{4} > 2$ $\qquad\qquad$ $8\frac{1}{2} > 8$

so $(2\frac{3}{4} \times 8\frac{1}{2}) > (2 \times 8)$. The product must be greater than 16.

$2\frac{3}{4} < 3$ $\qquad\qquad$ $8\frac{1}{2} < 9$

So $2\frac{3}{4} \times 8\frac{1}{2}$ is between 16 and 27.



In the drawing, the unit square region and the $2\frac{3}{4}$ by $8\frac{1}{2}$ region are in dark lines.

Name the 2 by 8 region.

Name the 3 by 9 region.

Exercise Set 26

Which of the answers below may be right? Which ones must be wrong? Answer by finding two whole numbers between which the product must be.

1. $3\frac{1}{2} \times 5\frac{7}{8} = 20\frac{9}{16}$

2. $12\frac{7}{10} \times 3\frac{5}{8} = 32\frac{41}{80}$

3. $5\frac{3}{4} + 24\frac{7}{8} = 32\frac{5}{8}$

4. $2\frac{3}{4} \times 5280 = 15{,}840$

5. $\frac{7}{8} \times 6\frac{1}{2} = 6\frac{3}{4}$

6. $42\frac{7}{10} + 26\frac{8}{15} = 69\frac{1}{10}$

Between what two whole numbers must each sum or product be?

7. $2\frac{3}{4} + 3\frac{1}{2}$

8. $7\frac{9}{10} + 15\frac{3}{7}$

9. $10\frac{4}{9} + 12\frac{7}{15}$

10. $128\frac{3}{17} + 245\frac{125}{147}$

11. $56.7 + 38.54$

12. $8\frac{3}{4} \times 3\frac{2}{7}$

13. $9\frac{4}{13} \times 11\frac{5}{19}$

14. $6\frac{5}{23} \times 8\frac{46}{47}$

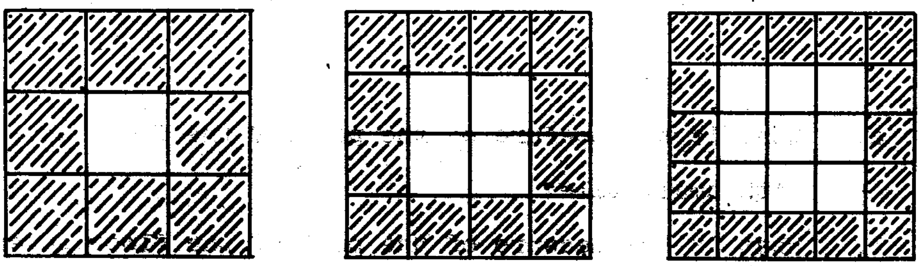
15. $15\frac{1}{2} \times 10\frac{51}{53}$

16. 7.28×0.34

17. Which can you estimate more closely by the method described: the sum of two numbers or the product of the same two numbers?

Exercise Set 27

1. Below are three unit squares. Each is separated into smaller congruent squares, and the border squares are shaded.



A B C

   a. What is the measure of a side of the unshaded square region of each?

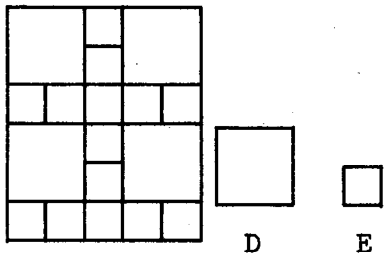
   b. What is the measure of the unshaded square region of each?

   c. What is the measure of the shaded region of each?

2. The sides of the unit squares in Exercise 1 are separated into 3 congruent parts, 4 congruent parts, and 5 congruent parts. Draw another unit square; separate its sides into 6 congruent parts. Separate the unit square region into smaller square regions, and shade the border region.

   Answer questions a, b, and c, from Exercise 1 about your unit square.

3. A tile floor, pictured to the right, is made of tile of the two sizes shown. The measure of tile D is 1 and tile E is $\frac{1}{4}$.

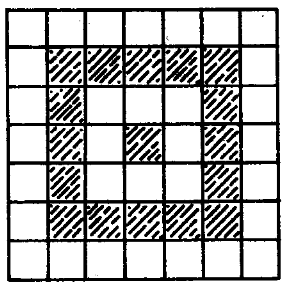


a. What is the measure of the part of the floor covered with small tile?

b. What is the measure of the part of the floor covered with the large tile?

c. What is the measure of the floor?

4. The square pattern to the right was made by fitting together black and white square tile. The pattern has been used by artists for many years.



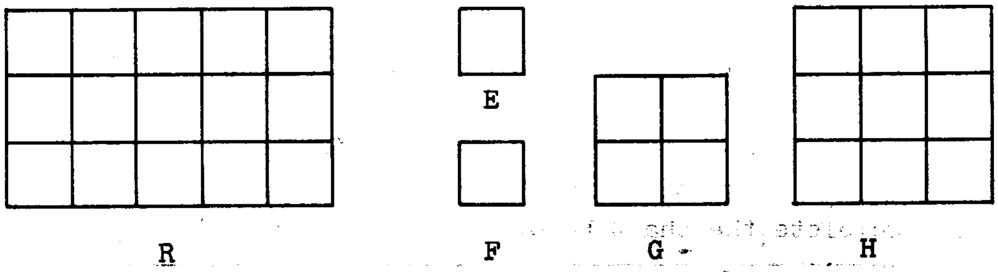
a. If the measure of the whole region is 1, what is the measure of each tile?

b. What is the measure of the white outer border?

c. What is the measure of the white inner border?

d. What is the measure of the black outer border?

5. Below are two arrangements of 15 squares. On the left, they are arranged in a rectangle; on the right, they are arranged in four squares.



a. If the measure of the rectangular region is 1, what is the measure of each square region?

b. If the measure of the square region H is 1, what is the measure of the rectangular region?

c. If the measure of square region G is 1, what is the measure of the rectangular region?

6. Write each of the following in its simplest form:

a. $\frac{4}{3}$	d. $\frac{64}{9}$	g. $\frac{25}{15}$
b. $7\frac{6}{5}$	e. $12\frac{8}{4}$	h. $\frac{399}{1000}$
c. $17\frac{3}{6}$	f. $\frac{37}{100}$	i. $\frac{16}{16}$

<u>Exercise Set 28</u>

1. Supply the missing numerators:

a. $1\frac{2}{3} = \frac{}{3}$

b. $5\frac{1}{8} = \frac{4}{8}$

c. $1 = \frac{}{5}$

d. $\frac{16}{100} = \frac{}{1000}$

e. $17\frac{4}{8} = \frac{}{8}$

f. $\frac{2}{3} = \frac{}{12}$

2. Complete the chart below:

Rectangle	Measurement of Adjacent Sides	Perimeter	Area of Regions
A	$4\frac{1}{3}$ ft. by $6\frac{1}{2}$ ft.		
B	12.75 ft. by 18.18 ft.		
C	9 ft. by 3 ft.		

a. How much greater is the measure of rectangular region A than C ?

b. How much greater is the measure of rectangular region B than C ?

c. How much less is the measure of the perimeter of A than C ?

d. How much less is the measure of the perimeter of C than B ?

3. Arrange from least to greatest:

$\frac{1}{7}$; $\frac{2}{2}$; $\frac{2}{3}$; 0.5; $\frac{3}{4}$; $1\frac{1}{5}$

4. **Arrange from least to greatest:**

$0.5; \quad 0.49; \quad 1.8; \quad 7.09; \quad 0.001; \quad \frac{4}{10}$

5. **Subtract:**

a. $\begin{array}{r} 5\frac{4}{5} \\ \underline{1\frac{2}{5}} \end{array}$

c. $\begin{array}{r} 29\frac{1}{8} \\ \underline{17\frac{3}{8}} \end{array}$

e. $\begin{array}{r} 90\frac{1}{3} \\ \underline{46\frac{1}{2}} \end{array}$

b. $\begin{array}{r} 14 \\ \underline{5\frac{1}{6}} \end{array}$

d. $\begin{array}{r} 57\frac{9}{10} \\ \underline{39\frac{4}{5}} \end{array}$

f. $\begin{array}{r} 74\frac{3}{8} \\ \underline{65\frac{5}{6}} \end{array}$

6. **Find the simplest fraction name for:**

a. $\frac{5}{2} \times \frac{1}{4}$

b. $7 \times 5\frac{2}{3}$

c. $\frac{4}{5} \times \frac{4}{5}$

d. $1\frac{1}{2} \times 2\frac{2}{3}$

e. $\frac{5}{8} \times 10$

f. $4\frac{5}{6} \times 3\frac{1}{2}$

7. **Add:**

a. $\begin{array}{r} 1\frac{1}{5} \\ \underline{2\frac{3}{5}} \end{array}$

c. $\begin{array}{r} 9\frac{5}{6} \\ \underline{8\frac{5}{6}} \end{array}$

e. $\begin{array}{r} 101\frac{1}{2} \\ \underline{47\frac{2}{5}} \end{array}$

b. $\begin{array}{r} 7\frac{3}{8} \\ \underline{6\frac{5}{8}} \end{array}$

d. $\begin{array}{r} 34\frac{2}{3} \\ \underline{15\frac{3}{4}} \end{array}$

f. $\begin{array}{r} 69\frac{5}{6} \\ \underline{82\frac{7}{8}} \end{array}$

Exercise Set 29

1. Name the following numbers by fractions:

   a. 0.7 c. 2.57 e. 0.072

   b. 0.04 d. 3.6 f. 1.25

2. Name the following numbers by decimals:

   a. $\frac{9}{10}$ c. $\frac{125}{100}$ e. $\frac{43}{10}$

   b. $\frac{49}{100}$ d. $\frac{10}{10}$ f. $\frac{417}{1000}$

3. Find n.

   a. $7.29 + 0.7 = n$ c. $0.37 + 0.8973 = n$

   b. $31 + 2.59 = n$ d. $5.235 + 4 + 6.25 = n$

4. Find n.

   a. $0.90 - 0.4 = n$ c. $4.205 - 1.7416 = n$

   b. $6.7 - 4.25 = n$ d. $47 - 0.478 = n$

5. a. $0.3 \times 0.4 = n$ d. $48.8 \times 0.56 = n$

   b. $7.03 \times 0.9 = n$ e. $0.94 \times 6.8 = n$

   c. $0.78 \times 0.5 = n$ f. $3.42 \times 8.6 = n$

<u>Exercise Set 30</u>

1. When the Smith family left on their vacation, the speedometer read 19,628.6 miles. When they returned, it read 22,405.3. How many miles had the Smiths traveled?

2. Mrs. Williams bought four pieces of steak weighing 4.7 pounds, 5.2 pounds, 5.3 pounds, and 3.8 pounds. At 99¢ per pound, how much will the four pieces cost?

3. Jack's mother bought his fall clothes on sale. Shoes originally priced \$7.89 were marked $\frac{1}{3}$ off. A suit originally priced \$15.96 was marked $\frac{1}{4}$ off. A coat originally priced \$19.98 was marked $\frac{1}{2}$ off. How much money did Jack's mother save?

4. When Mark pulled his lobster traps, he had 9 lobsters each weighing $1\frac{1}{4}$ pounds, 13 lobsters each weighing $1\frac{1}{2}$ pounds, and 8 lobsters each weighing $1\frac{3}{4}$ pounds. How many pounds of lobster did he pull?

5. The Ward's house is 42.8 feet by 68.5 feet. Their land is 105.5 feet by 236.2 feet. How many square feet of land do they have surrounding their house?

6. One day, Helen and Rosemary were each given a guinea pig. Helen's guinea pig weighed 0.60 pounds and gained 0.07 pounds each day. Rosemary's guinea pig weighed 0.48 pounds, but ate more, and gained 0.09 pounds each day. Whose guinea pig was the heavier a week later? How much heavier?

7. In a swimming test, Dan stayed under water 2.3 times as long as Charlie. Charlie stayed under water 19.8 seconds. How long did Dan stay under water?

8. Races are sometimes measured in meters. A meter is 1.094 yards. What is the difference in yards between a 50 meter race and a 100 meter race?

9. Paul weighs 40 pounds. Jerry weighs $1\frac{7}{8}$ times as much as Paul. Mike weighs $1\frac{3}{5}$ times as much as Jerry. How much do Jerry and Mike each weigh?

10. Ethel likes to collect colored rocks for her rock garden, but she can carry only 18 pounds of rock in her basket. If she puts in more, the basket will break. She puts six colored rocks in her basket. The first weighs 3.4 pounds, the next three weigh 3.1 pounds apiece, and the last two rocks weigh 2.6 pounds apiece. Will she break her basket? Explain.

CHAPTER 3

SIDE AND ANGLE RELATIONSHIPS OF TRIANGLES

ISOSCELES TRIANGLES

Exploration

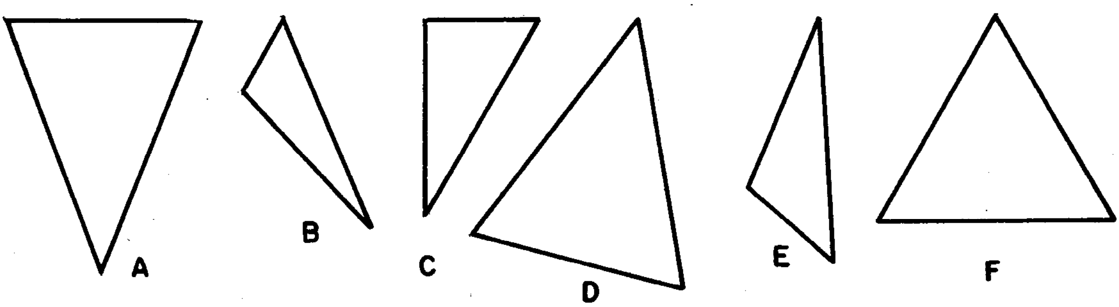
What do we call a triangle which has at least two congruent sides? You have seen many of these triangles and no doubt are able to give them their correct name--isosceles triangles.

1. Make a model of a triangle using the strips and fasteners. For two sides, choose two strips of the same length, the longest ones you have. For the third side, choose a strip about half as long as the others you used.

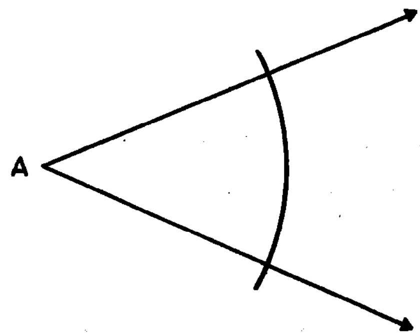
   Recall that a triangle with at least two congruent sides is called an <u>isosceles</u> triangle.

2. Draw an isosceles triangle in which the congruent sides are each two inches long and the third side is three inches long. Do you need to use your compass?

3. Which of these are isosceles triangles? How did you decide?

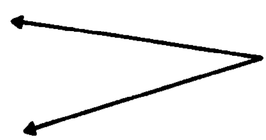


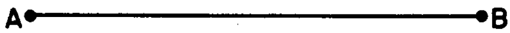
4. Use your compass and straightedge to copy this figure. In the figure, place the letters B and C at the intersection of the arc and rays. What segments are congruent? Does this figure suggest a method for making an isosceles triangle?



Exercise Set 1

1. Use the method of Exploration Exercise 4 to draw an isosceles triangle, starting with a figure like this. How many such triangles could you draw?



2. 

Draw a line segment congruent to $\overline{AB}$. Name your line segment, $\overline{CD}$. How many isosceles triangles can you make on $\overline{CD}$ if it is not one of the congruent sides? Draw several of them. Must all these triangles lie on the same side of $\overline{CD}$?

3. Can you see any examples of isosceles triangles in our room? Could you, by drawing one line on the door, show an isosceles triangle? Do you see any other ways of making isosceles triangles in our room by drawing just one line?

Make $\overline{BA} \cong \overline{BC}$.

4. As you go home tonight look closely at things around you to see if you can find any examples of isosceles triangles. You may find some good examples in your neighborhood, at the dinner table, or even in your car. Most magazines have some good pictures of isosceles triangles in them, too.

BRAINTWISTERS

5. Draw two line segments of different lengths. Name one of them $\overline{EF}$ and the other one $\overline{GH}$. Now draw an isosceles triangle with two sides congruent to $\overline{EF}$ and the third side congruent to $\overline{GH}$. Is it easier if you draw the third side first? Did you have any trouble drawing the isosceles triangle?

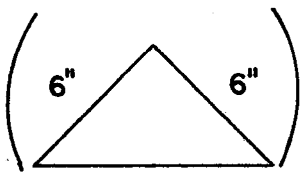
6. See if you can stump your teacher. Ask her to work problem 5 after you have marked $\overline{EF}$ and $\overline{GH}$ for her. Be sure to choose lengths so that she cannot draw the isosceles triangle. How did you do this?

ANGLES OF AN ISOSCELES TRIANGLE

Exploration

You have done many things with isosceles triangles. Let's look at them even more closely.

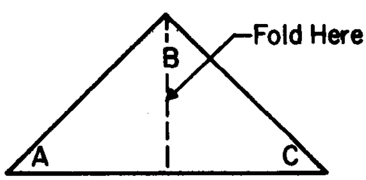
1. Draw an isosceles triangle with the congruent sides 6 inches long. Make the third side any length you choose.

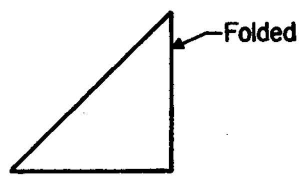


Do you have to be careful of the length you choose? Could you choose 12 inches for the length of the third side? Why? Could you choose a length greater than 12 inches for the length of the third side? Why?

2. Cut out your isosceles triangle with its interior. Label the vertices so that $\overline{AB} \cong \overline{BC}$. Now fold it through point B so that side $\overline{BC}$ fits on side $\overline{BA}$.

3. These sketches show what you did.





Where did vertex C fall? Is $\angle C$ congruent to $\angle A$? If you made the isosceles triangle carefully, $\angle C$ should fit exactly over $\angle A$ so that $\angle C \cong \angle A$.

4. Construct a triangle DEF with $\overline{DE} \cong \overline{FE}$. Trace ΔDEF on a sheet of thin paper and label the vertices. Turn the sheet over and place

vertex D on F,

vertex E on E,

vertex F on D.

Is $\Delta DEF \cong \Delta FED$? If so, which angles are congruent?

5. We call $\angle D$ the angle <u>opposite</u> $\overline{FE}$ since $\overline{FE}$ joins points on the sides of $\angle D$. In a similar manner, we call $\angle E$ the angle opposite $\overline{DF}$, and $\angle F$ the angle opposite $\overline{ED}$.

6. In isosceles ΔDEF, which are the congruent sides? Are the congruent angles opposite the congruent sides?

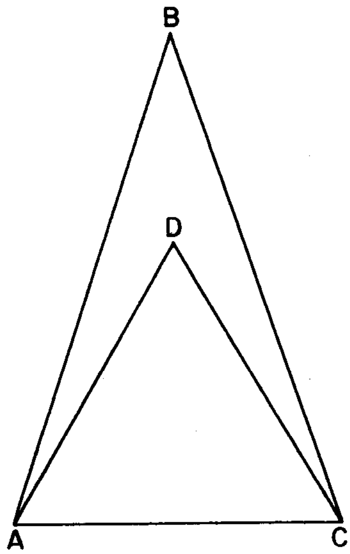
7. Can you finish this sentence?
It seems to be true that if two sides of a triangle are congruent, then the angles

Summary

In each isosceles triangle with which you worked you found at least two congruent angles. Every isosceles triangle has at least two congruent sides and at least two congruent angles, and the congruent angles are opposite the congruent sides.

Exercise Set 2

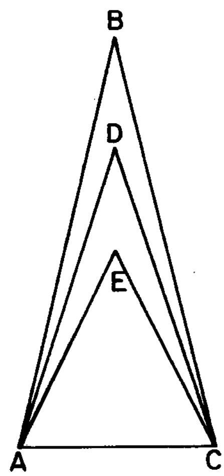
1. In this drawing, $\overline{AB} \cong \overline{CB}$ and $\overline{AD} \cong \overline{CD}$. What kind of triangles are $\triangle ABC$ and $\triangle ADC$?



   Which angle has the greater measure, $\angle BAC$ or $\angle DAC$?

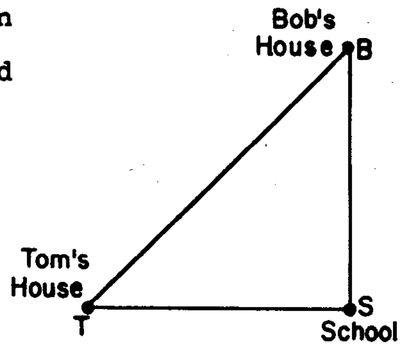
   Is $\angle DAC \cong \angle DCA$?

2. How many triangles can you find in this drawing? If $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$, and $\overline{AE} \cong \overline{CE}$, what kind of triangle are they?



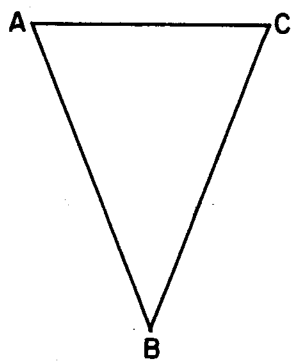
   If $\angle ACE \cong \angle CAE$ and $\angle DCA \cong \angle CAD$, are all four angles congruent?

3. This drawing shows the location of Tom's house, Bob's house and their school. Bob and Tom live the same distance from school. Why is the angle at Tom's house congruent to the one at Bob's house?



Suggestion: If Bob's house is directly north of the school and Tom's house is directly west of the school, what do you know about $\angle TSB$.

4. In ΔABC, $\overline{AB} \cong \overline{CB}$. Choose any way you want to show that $\angle BAC \cong \angle BCA$.



EQUILATERAL TRIANGLES

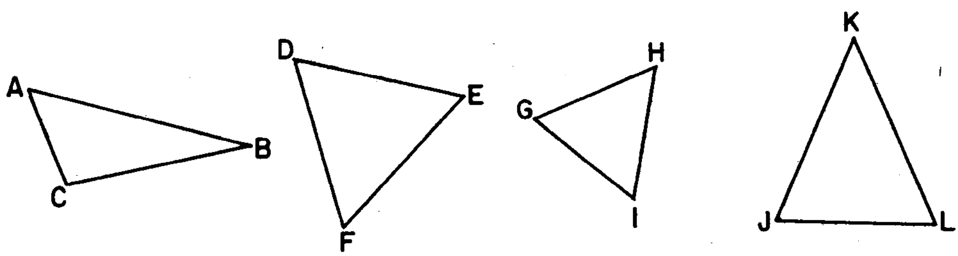
Exploration

A triangle which has three sides congruent to each other is called an <u>equilateral</u> triangle.

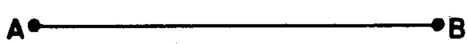
Make an equilateral triangle by using strips. Choose any strips you want, but you must be careful about one thing. What is it?

<u>Exercise Set 3</u>

1. Which of these are pictures of equilateral triangles?

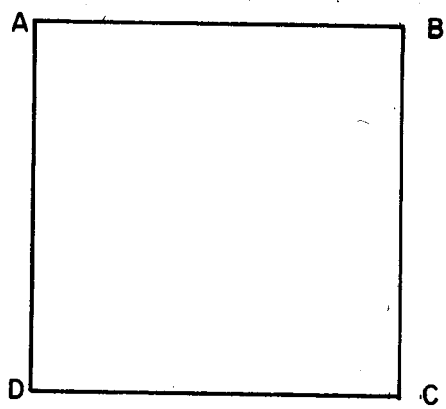


2. Draw an equilateral triangle using your compass and straightedge. Use the length of $\overline{AB}$ as the length of a side.



3. Draw three equilateral triangles. Make the first one with sides $1\frac{1}{2}$ inches in length, the second with sides $2\frac{1}{2}$ inches in length, and the third with sides $3\frac{1}{2}$ inches in length.

4. Figure ABCD is a square.
What do you know about the sides of a square?



Name the congruent segments.

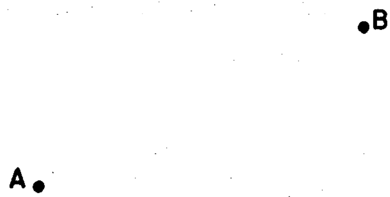
Trace figure ABCD on your paper.

Draw an equilateral triangle, using $\overline{AB}$ as one side. Draw the triangle so that its interior lies in the exterior of ABCD.

Draw equilateral triangles on $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$ so that the interior of each triangle lies in the exterior of ABCD.

Is the triangle with side $\overline{AB}$ congruent to any of the other triangles you have just marked?

5. How many equilateral triangles can you draw which have a vertex at point A and another vertex at point B? Draw them.



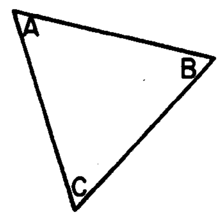
6. How many equilateral triangles with a side 2 inches long can you draw which will have one vertex at point B? How many isosceles triangles can you draw which will have a vertex at point A and a vertex at point B?

ANGLES OF AN EQUILATERAL TRIANGLE

Exploration

You know that an equilateral triangle has three congruent sides. Is there anything else that you think might be true about it?

1. Draw an equilateral triangle which has a side 4 inches long. Name it $\triangle ABC$, putting the letters on the interior like this:



What must be true about $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$?

2. Since $\overline{AB} \cong \overline{BC}$, what can you say about $\angle A$ and $\angle C$?

   Since $\overline{BC} \cong \overline{AC}$, what can you say about $\angle A$ and $\angle B$?

   Now what can you say about $\angle A$, $\angle B$, and $\angle C$?

3. Cut out triangle ABC with its interior. Fold it through point B so that $\overline{BC}$ falls on $\overline{BA}$. Does $\angle C$ fit exactly on $\angle A$? Now open up the figure and fold it through point C so that $\overline{BC}$ falls on $\overline{AC}$. Does $\angle B$ fit exactly on $\angle A$? Open the figure and fold it through A, so $\overline{AC}$ falls on $\overline{AB}$. Does $\angle C$ fit exactly on $\angle B$?

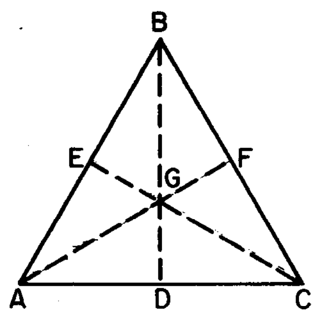
Exercise Set 4

1. Use your strips to make an equilateral triangle. Make another equilateral triangle that is congruent to the first one. Label the first triangle GHI and the second JKL. Can you place ΔJKL exactly on ΔGHI so that vertex J falls on vertex G, K falls on H, and L falls on I? Are there other ways in which ΔJKL will fit exactly on, that is, be congruent to ΔGHI?

2. Draw an equilateral triangle with a side whose length is $1\frac{3}{4}$ inches. Are all three angles congruent?

3. Draw an equilateral triangle with a side whose length is $2\frac{1}{2}$ inches. Are the three angles congruent?

4. When you folded your equilateral triangle, in the Exploration, the folds made lines on it as in the drawing. Notice that we have named the folds $\overline{AF}$, $\overline{BD}$, and $\overline{CE}$. Name all of the different triangles you can find in the figure. Name each triangle only once. Can you find 16 triangles?



BRAINTWISTER

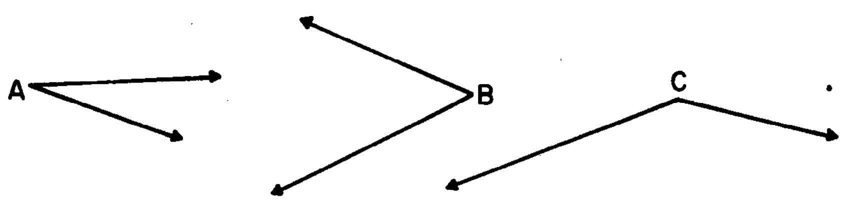
5. Draw an equilateral triangle ABC with a side of length 3 inches. Draw another equilateral triangle DEF with a side 4 inches long and another equilateral triangle with a side 5 inches long. Are these three triangles congruent? Are the angles in all three equilateral triangles congruent?

BRAINTWISTER

6. If you drew an equilateral triangle with sides 8 inches long, would the angles be congruent to those of an equilateral triangles with sides 4 inches long? Would you expect this?

BRAINTWISTER

7. Can you draw an equilateral triangle with its angles congruent to these three angles?



Exercise Set 5

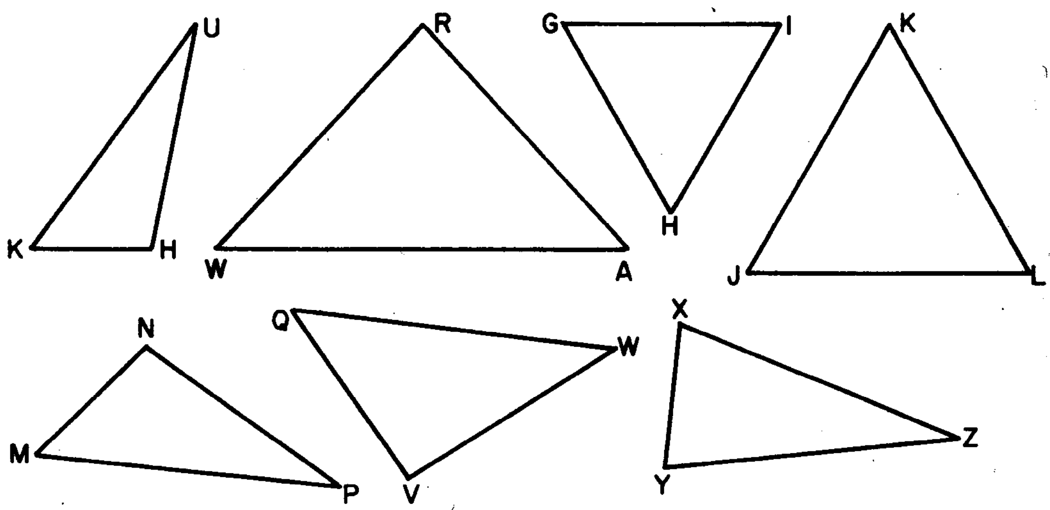
1. Divide the triangles below into four sets:

   Set A - Triangles with exactly two congruent sides

   Set B - Triangles with three congruent sides

   Set C - All triangles not in Set A or Set B

   Set D - All triangles with at least two congruent sides



2. All the triangles in Set A or in Set B are __________ triangles.

   All the triangles in Set B are __________ triangles.

   All the triangles in Set A are __________ triangles.

3. What can you say about the triangles in Set C?

4. Divide the triangles above into four new sets.

Set E - Triangles with exactly two congruent angles. (If you need to, trace the angles on paper to help you decide.)

Set F - Triangles with three congruent angles.

Set G - Triangles with no two angles congruent.

Set H - Triangles with at least two angles congruent.

5. Look at the eight sets you have listed. Which sets have exactly the same members?

Which sets in Exercise 1 are subsets of other sets?

6. What is the intersection of Sets A and B?

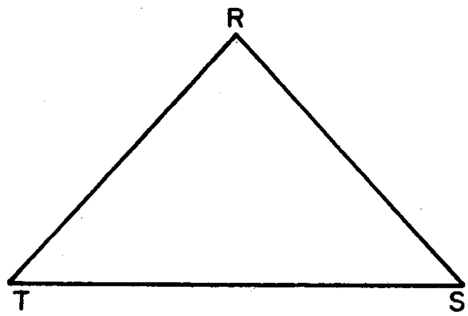
What is the intersection of Sets G and B?

What is the union of Sets H and C?

SCALENE TRIANGLES

Exploration

You know that in a triangle the angles opposite two congruent sides are also congruent. What do you think about angles opposite sides of unequal length?



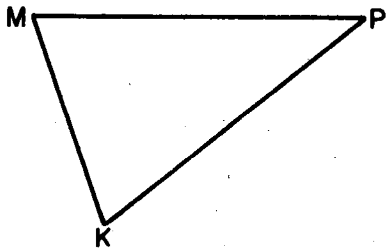
1. In ΔRST, $\overline{RS} \not\cong \overline{RT}$, so which angles must be congruent?

2. Which is longer, $\overline{RS}$ or $\overline{ST}$?

3. What angle is opposite $\overline{RS}$?
   What angle is opposite $\overline{ST}$?

4. **By tracing on thin paper compare $\angle R$ with $\angle T$. Which angle has the greater measure?**

5. Which side is opposite the **greater angle,** $\overline{RS}$ or $\overline{ST}$?



6. In triangle MPK choose a pair of sides which are not congruent.

7. Tell which angle is opposite each of the sides you chose.

8. Compare the sizes of the two angles.

9. Arrange your answers for Exercises 6, 7, 8 like this:

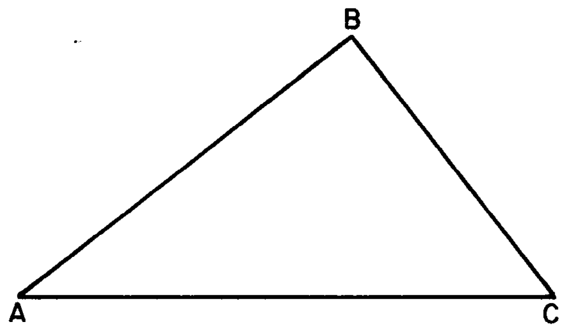
   Longer side: __________ Shorter side: __________

   Angle opposite longer side: __________

   Angle opposite shorter side: __________

   Larger angle: __________ Smaller angle: __________

10. Is the larger angle opposite the longer side or the shorter side?



11. Are any two sides of $\triangle ABC$ congruent? What is this kind of triangle called?

12. Arrange the sides in order of length. Then name the angle opposite each side.

Longest side:	________	Opposite angle:	________
Next longest side:	________	Opposite angle:	________
Shortest side:	________	Opposite angle:	________

13. Compare the sizes of the angles:

    Largest size angle: ________

    Next largest angle: ________

    Smallest angle: ________

14. What do you notice about your answers to Exercises 12 and 13?

Exercise Set 6

1. Draw a scalene triangle. Name it with letters.

2. List the sides in order of size from longest to shortest.

3. List the angles in order of size from largest to smallest.

4. Which length side is opposite the largest angle?

   Which length side is opposite the smallest angle?

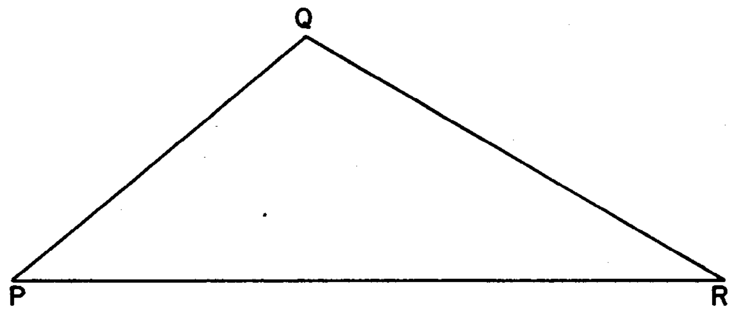
5. Can you make a triangle out of three sticks whose lengths are 5 inches, 7 inches, and 9 inches? If so, where will the largest size angle be?

6. What kind of triangle can you make out of three sticks of length 8 inches, 8 inches, and 6 inches? What should be true about the sizes of the angles of the triangle?

ANGLES OF A TRIANGLE

Exploration

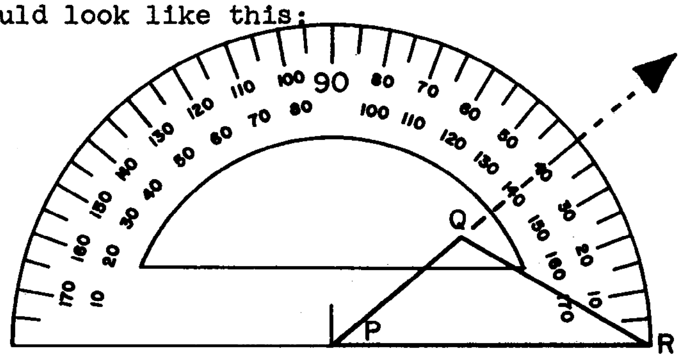
You have learned how to measure an angle with your protractor. Now we will measure angles which are determined by triangles. Suppose we wish to measure the angles of ΔPQR.



1. **Where will you place the point V of your protractor to measure $\angle QPR$? If you place the point V of your protractor on P, along which ray of $\angle P$ may the zero ray of your protractor be placed?**

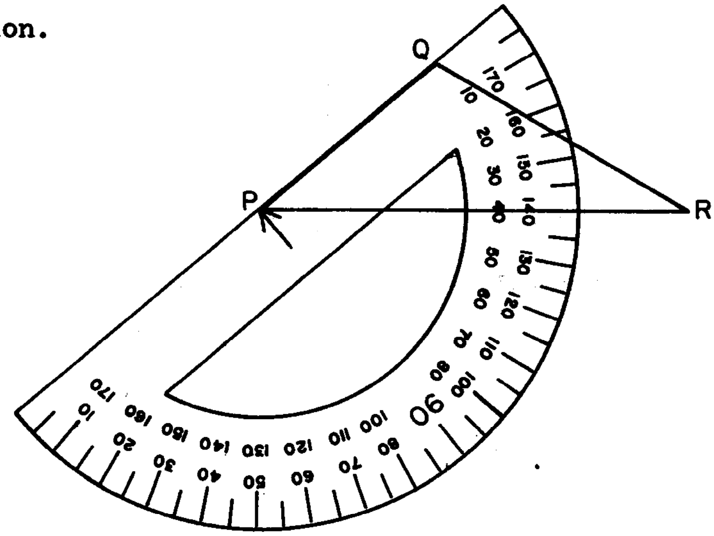
2. **If you place the zero ray along $\overline{PR}$, which side of $\angle P$ should you extend, if necessary?**

   A picture of the protractor placed to measure $\angle P$ would look like this:



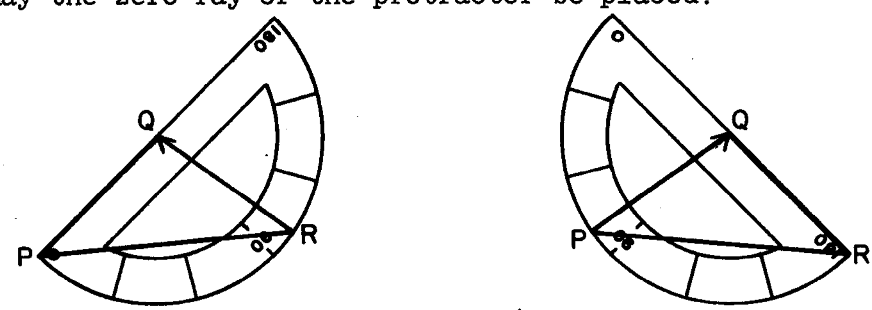
   What is the measure (in degrees) of $\angle P$?

3. **Now place the protractor so the point V of your protractor is on P and the zero ray is on $\overline{PQ}$. This is a picture of the protractor in this position.**



To find the measure of $\angle P$, should you look at $\overline{QR}$ or $\overline{PR}$? Do you get the same reading on the protractor for the $m\angle P$ as before?

4. On which vertex of $\triangle PQR$ must we place point V of the protractor to measure $\angle PQR$? Along which sides of $\triangle PQR$ may the zero ray of the protractor be placed?



Do you get the same measure of $\angle Q$ both times?

5. Measure $\angle R$ by placing the zero ray of the protractor along one of the rays of the angle. Repeat by using the other ray of the angle. Are the two measures the same?

Summary

To measure an angle of a triangle, the point V of the protractor must be placed on the vertex of the angle. The zero ray may be placed along either ray of the angle. The sides of the triangle on which the rays of the angles lie may have to be extended.

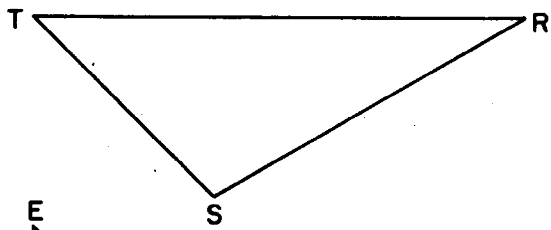
Exercise Set 7

Use your protractor to measure the 3 angles of each triangle. Check your measure (in degrees) in each case by placing the zero-ray of the protractor along the other ray of the angle.

1. $m\angle T =$

   $m\angle S =$

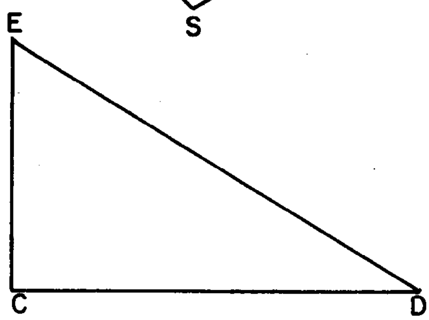
   $m\angle R =$



2. $m\angle C =$

   $m\angle D =$

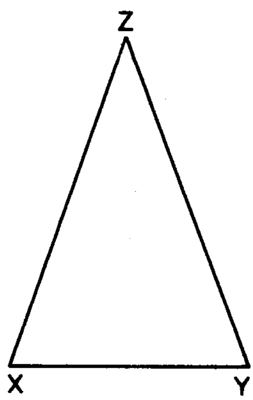
   $m\angle E =$



3. $m\angle X =$

   $m\angle Y =$

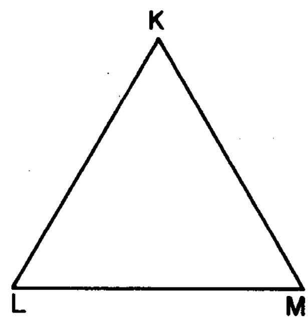
   $m\angle Z =$



4. $m\angle K =$

   $m\angle M =$

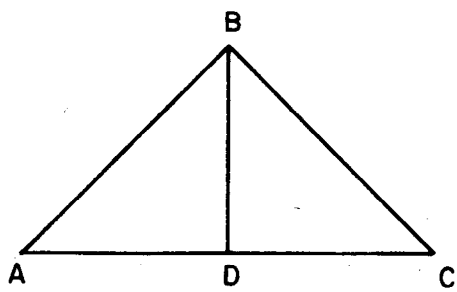
   $m\angle L =$



5. $m\angle BAD =$

   $m\angle ABD =$

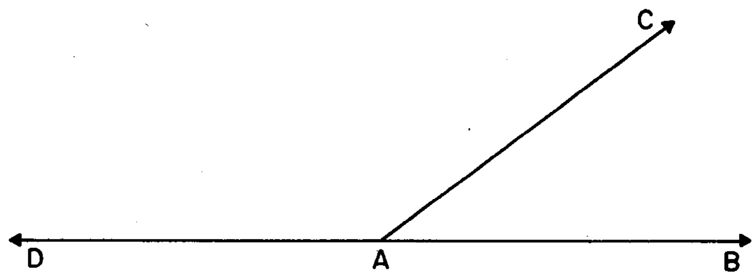
   $m\angle BDA =$



SUM OF MEASURES OF ANGLES OF A TRIANGLE

Exploration

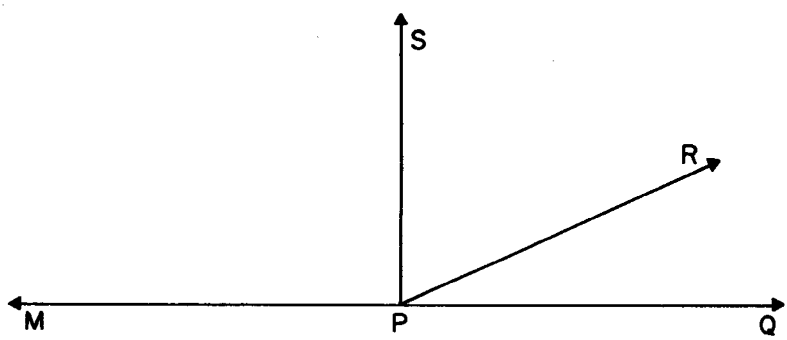
1.



If $\overrightarrow{AD}$ and $\overrightarrow{AB}$ are on a line, then, in degrees, $m\angle DAC + m\angle BAC =$ ____, and, in octons (see Grade 5 text, Chapter 6), $m\angle DAC + m\angle BAC =$ ____

Check with your protractor.

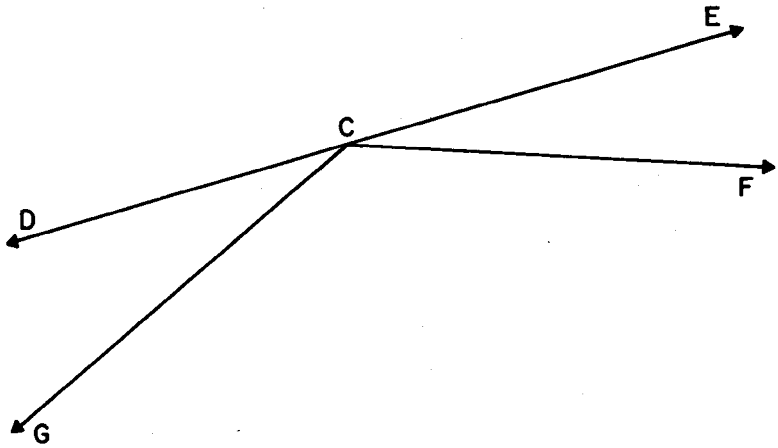
2.



If $\overleftrightarrow{MPQ}$ is a straight line, then, in degrees, $m\angle MPS + m\angle SPR + m\angle RPQ =$ _____ and in octons $m\angle MPS + m\angle SPR + m\angle RPQ =$ _____.

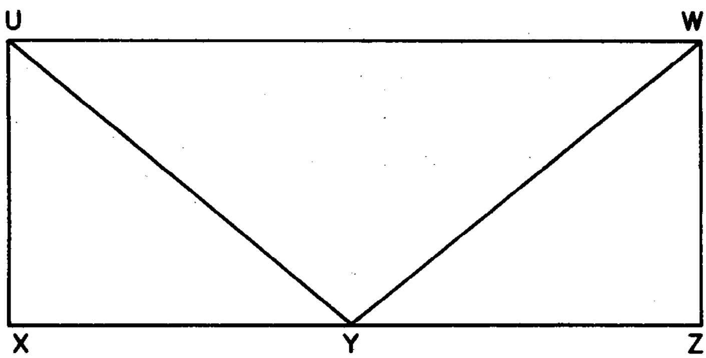
Check with your protractor.

3.



If $\overleftrightarrow{DCE}$ is a straight line, for which three angles is the sum of the measures (in degrees) exactly 180? Check with your protractor.

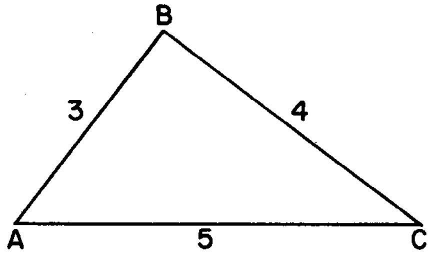
4.



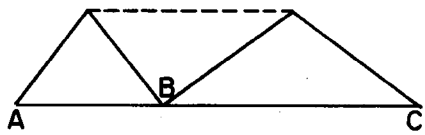
Use your protractor to see if the following statement seems reasonable.

$m\angle UYX + m\angle UYW + m\angle WYZ = 180$

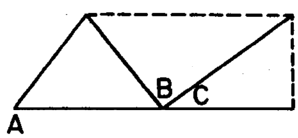
5. Draw a triangle whose sides will have measures, in inches, of 3, 4, and 5. Cut it out with its interior. Hold it like this:



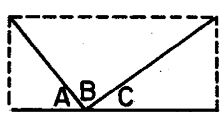
Mark the midpoint of $\overline{AB}$ and the midpoint of $\overline{BC}$. Fold $\triangle ABC$ through the midpoints; B should fall on $\overline{AC}$.



Then fold so that C falls on point B.



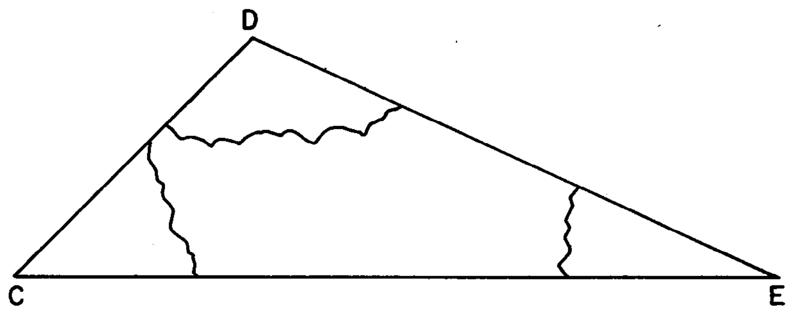
Then finally, A falls on B.



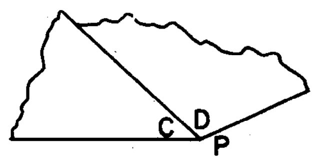
Complete the mathematical sentence

$$m\angle A + m\angle B + m\angle C = \_\_\_\_.$$

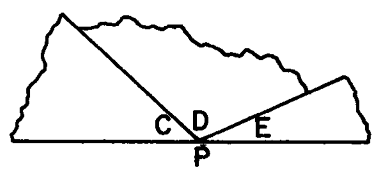
6. Cut out any triangle with its interior. Label each angle of the triangle. Tear the model like this. (Keep all the parts)



Place the models of $\angle C$ and $\angle D$ so that $\angle C$ and $\angle D$ have a common vertex, P, and a common side.



Then place $\angle E$ so that $\angle D$ and $\angle E$ have the same vertex, P, and a common side.



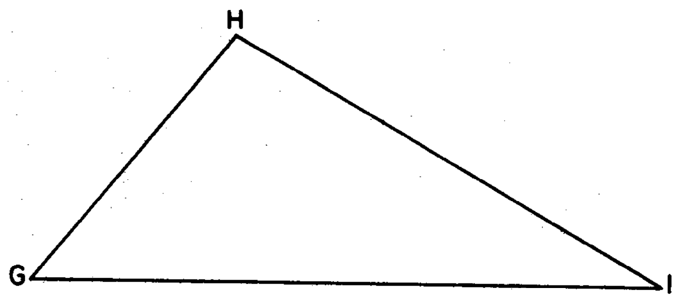
What do you observe?

Complete the sentence

$$m\angle C + m\angle D + m\angle E = \_\_\_\_.$$

7. In Exercise 5, $\angle A$, $\angle B$ and $\angle C$ were the three angles of a triangle. In Exercise 6, $\angle C$, $\angle D$ and $\angle E$ were the three angles of a triangle. In each case what did you find to be the sum of the measures in degrees of the three angles of a triangle?

8.



What do you think is the sum of the measures of the three angles of $\triangle GHI$?

Use your protractor to find the measures of the angles

$m\angle G =$

$m\angle H =$

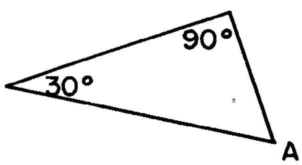
$m\angle I =$

Add the measures. If your sum is not 180, can you account for this?

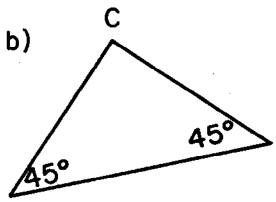
<u>Exercise Set 8</u>

1. In each triangle below the sizes of certain angles are shown. Find the size of each angle whose vertex is named with a letter. Sides marked || are congruent segments.

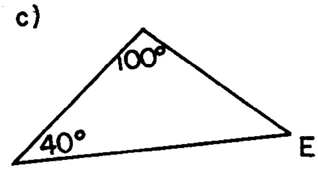
a)



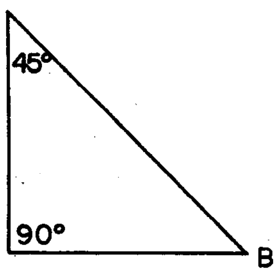
b)



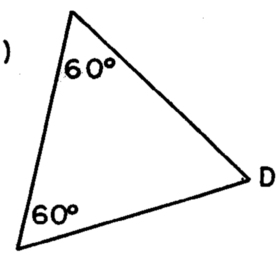
c)



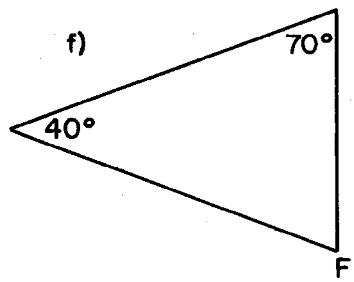
d)



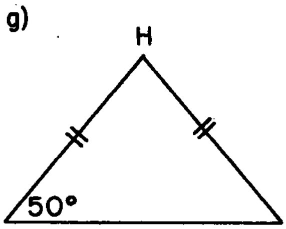
e)



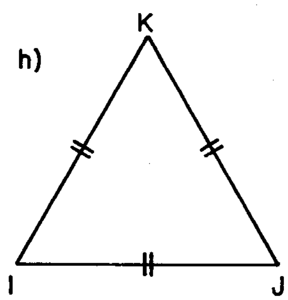
f)



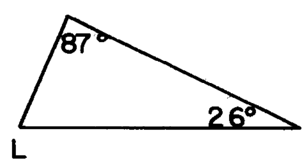
g)



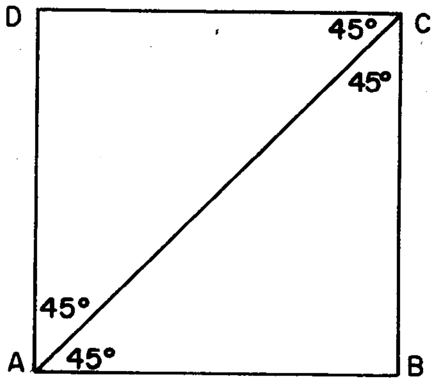
h)



i)



2.



What is the measure (in degrees) of $\angle DAC$?

What is $m\angle CAB$?

What is $m\angle DAB$?

Is $m\angle DAC + m\angle CAB = m\angle DAB$?

What is the measure of $\angle BCA$?

What is $m\angle ACD$?

What is $m\angle BCD$?

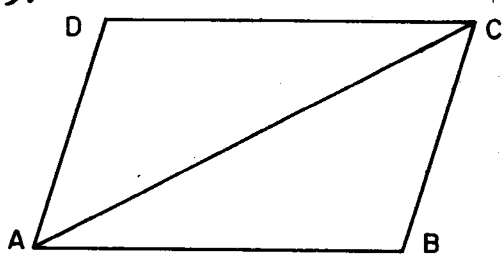
Is $m\angle BCA + m\angle ACD = m\angle BCD$?

What is $m\angle D$? What is $m\angle B$?

Complete the sentence:

$$m\angle DAB + m\angle B + m\angle BCD + m\angle D = \_\_\_\_ .$$

3.



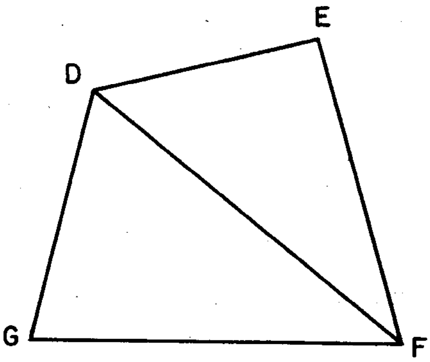
What is the sum of the measures of the three angles of Δ ABC? What is the sum of the measures of the three angles of Δ ADC?

Complete the sentence:

$$m\angle DAB + m\angle B + m\angle BCD + m\angle D = \_\_\_\_ .$$

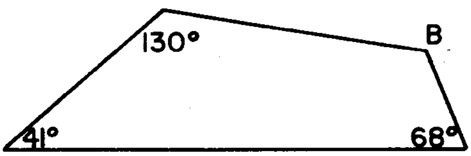
What is the sum of the measures of the 4 angles of the quadrilateral ABCD?

4.



What is the sum of the measures of the angles of Δ GDF? What is the sum of the measures of the angles of Δ DEF? What is the sum of the measures of the angles of quadrilateral DEFG?

5.

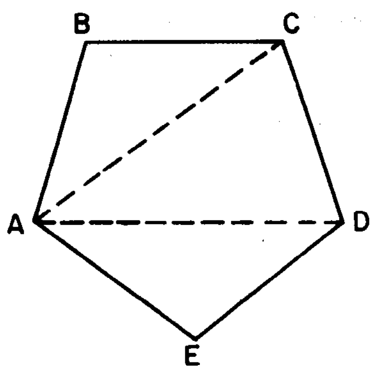


m$\angle$B = ____

How do you know?

BRAINTWISTERS

6.

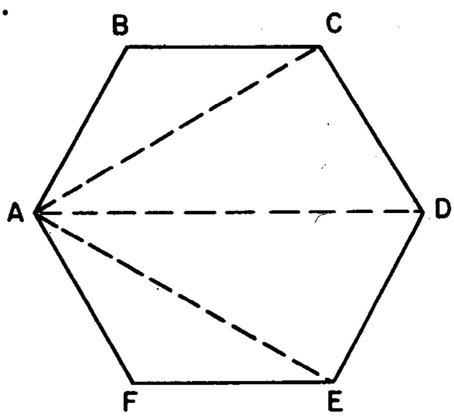


Complete the mathematical sentence:

m$\angle$BAE + m$\angle$B + m$\angle$BCD + m$\angle$CDE + m$\angle$E = ________.

Hint: How many triangles have been made to fit in the interior of the figure ABCDE?

7.



What is the sum of the measures of the angles of the figure ABCDEF?

8. What is the sum of the measures of the angles of a polygon of 7 sides?

9. What is the sum of the measures of the angles of a polygon of 202 sides?

10. Name some numbers which are the sum of the measures (in degrees) of the angles of a polygon.
Name the six smallest such numbers.

11. Draw a polygon with more than 4 sides. Mark a point in its interior. Name this point, C. Draw line segments from C to each vertex of the polygon. In the interior of the polygon draw a small circle using C as a center. From your drawing tell why the sum of the measures (in degrees) of the angles of a polygon is $(n - 2) \times 180$ or $n \times 180 - 360$.

12. Could the sum of the measures (in degrees) of the angles of a polygon be 250? Why?

13. Could the sum of the measures (in degrees) of the angles of a polygon be 160? Why?

MEASURES OF ANGLES OF SPECIAL TRIANGLES

Exploration

You have studied angle relationships in some special triangles.

1. Use your compass to draw an isosceles triangle, with the length of each of the congruent sides the same as the length of $\overline{AB}$. How many such triangles can be drawn?

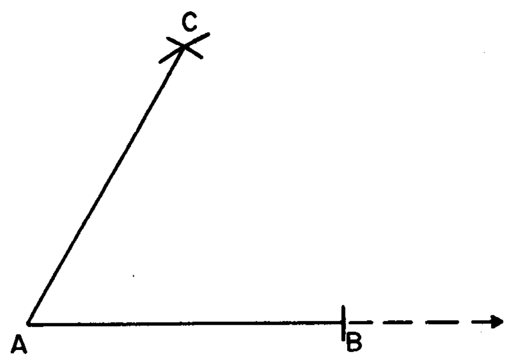
Measure the angles. What seems to be true about the angles of an isosceles triangle?

2. Did each one of your classmates find that the congruent angles of his isosceles triangle had the same measure? If the answer is "no," can you think of the reasons for this?

3. What did you learn about the angles of an equilateral triangle? How should their measures compare? Use your compass to draw an equilateral triangle. Choose any convenient length as the length of a side. Measure the angles. How do the measures compare?

4. How many different equilateral triangles were made in answer to question 3? How did the measures of the angles of your triangle compare with the measures of the angles of the triangles of your classmates?

5. What number is the measure, in degrees, of every angle of an equilateral triangle? Using only compass and straightedge, draw an angle whose measure in degrees is 60. Did you draw an equilateral triangle? Did your drawing look like this?



Was it necessary to draw $\overline{CB}$ to complete the triangle in order to draw an angle whose measure is 60? Check your construction with your protractor.

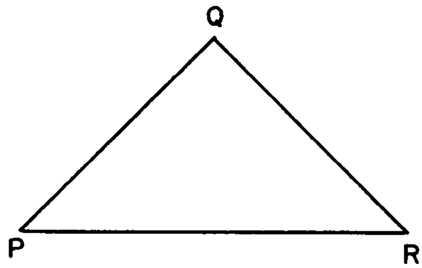
Summary

Angles opposite congruent sides of an isosceles triangle have the same measure.

Sixty is the measure in degrees of each angle of any equilateral triangle.

Exercise Set 9

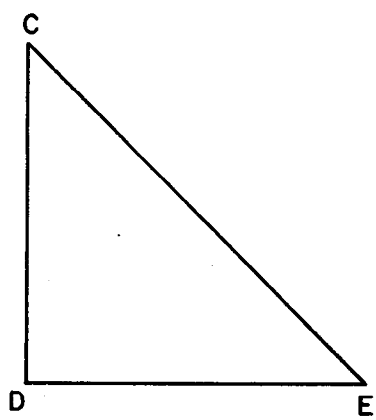
1. In the figure below, ΔPQR is isosceles with $\overline{QP} \cong \overline{QR}$.



Use your protractor to find $m\angle P$. What do you expect for the $m\angle R$? Check your answer by using the protractor.

2. A triangle whose angles have measures (in degrees) of 45, 45, and 90 is of special interest because there are so many examples of it in your surroundings. What examples of right angles can you see in your classroom? Be ready to show how to complete a 45°, 45°, 90° triangle by drawing one segment.

3.



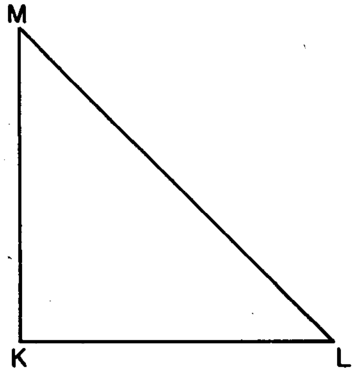
In ΔCDE, $\overline{CD} \cong \overline{DE}$. Which angles have the same measure?

Use your protractor to find the following measures.

$m\angle D =$ _____

$m\angle C =$ _____

$m\angle E =$ _____

4. 

ΔMKL is an isosceles triangle. $\overline{MK} \cong \overline{KL}$. Find

$m\angle K =$ _____

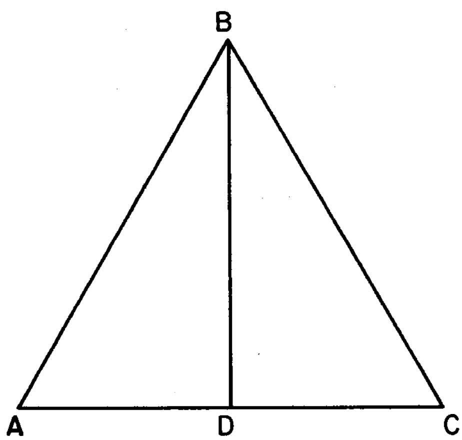
$m\angle M =$ _____

$m\angle L =$ _____

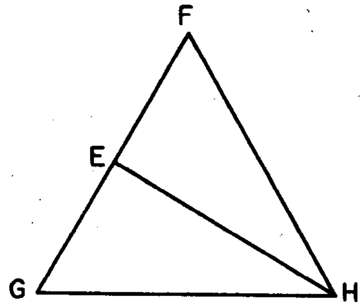
How do these measures compare with those of Exercise 3? Are ΔCDE and ΔMKL congruent?

5. Have you seen other examples of triangles which were not congruent, and whose angles have the same measure? Tell whether this statement is true or false: Two triangles may have three pairs of corresponding angles with the same measures and still not be congruent.

6. How many triangles are there in this figure? One of them is an equilateral triangle. Which one? Name 3 angles whose measure is 60.



7. ΔFGH is an equilateral triangle, and $\overline{EH}$ is drawn so that $\angle HEF$ is a right angle. Look at ΔFEH. For which angles do you know the measures?



8. Look at ΔEGH. List the measures of the 3 angles of ΔEGH.

9. Are all triangles whose angles have measures of 30, 60, and 90, congruent?

A triangle whose angles have measures of 30, 60, and 90 is another triangle of special interest. Look around you for models of right angles. Be ready to show how to draw a 30°, 60°, 90° triangle by drawing one segment.

10. Complete the following statement:

(a) **The measure (in degrees) of any angle of an equilateral triangle is __________.**

(b) **Two triangles of special interest have angle measures (in degrees) of ___, ___, ___ and ___, ___, ___.**

Practice Exercises

Find the number that y represents. Express all answers in simplest form.

1. a) $\frac{1}{2} + \frac{1}{8} = y$
   b) $8\frac{1}{4} + 7\frac{1}{4} = y$
   c) $5\frac{1}{8} + 6\frac{5}{8} = y$
   d) $\frac{1}{2} + \frac{3}{4} = y$
   e) $7\frac{1}{2} + 5\frac{3}{4} = y$
   f) $\frac{2}{3} + \frac{1}{3} = y$
   g) $\frac{3}{4} + \frac{1}{8} = y$
   h) $6\frac{3}{4} + 5\frac{1}{8} = y$

2. a) $36.3 + 23.0 = y$
   b) $5.46 + 1.48 = y$
   c) $269 + 287 = y$
   d) $881 + 482 + 886 = y$
   e) $876 + 867 + 972 + 659 = y$
   f) $993 + 364 + 848 = y$
   g) $8.98 + 4.07 = y$
   h) $698 + 589 = y$

3. a) $.3 \times .3 = y$
   b) $1.1 \times 2 = y$
   c) $64.1 \times 9 = y$
   d) $6.3 \times 8 = y$
   e) $92 \times 7.309 = y$
   f) $5 \times .9 = y$
   g) $10 \times 3.6 = y$
   h) $10 \times 8.54 = y$

4. a) $\frac{2}{3} - y = \frac{7}{15}$
   b) $y - \frac{2}{3} = 8\frac{1}{2}$
   c) $4\frac{1}{3} - 3\frac{1}{4} = y$
   d) $1\frac{3}{4} - y = \frac{11}{12}$
   e) $y - 5\frac{3}{5} = 2\frac{9}{10}$
   f) $2\frac{1}{4} - 1\frac{3}{8} = y$
   g) $\frac{3}{4} - \frac{1}{2} = y$
   h) $y - 2\frac{3}{4} = 3\frac{1}{2}$

5. a) $\frac{1}{2} \times \frac{1}{6} = y$
   b) $\frac{9}{10}$ of $\frac{4}{9} = y$
   c) $5 \times \frac{1}{8} = y$
   d) $\frac{16}{24} \times \frac{3}{8} = y$
   e) $\frac{2}{10^1} \times \frac{6}{10^2} = y$
   f) $2\frac{1}{2} \times 6 = y$
   g) $47 \times 5\frac{1}{5} = y$
   h) $\frac{3}{4}$ of $36 = y$

6. a) $30 \times 478 = y$

b) $256 \times 100 \times 2 = y$

c) $(40 \times 984) + (6 \times 984) = y$

d) $(7{,}000 \times 875) + (200 \times 875) + (5 \times 875) = y$

e) $106 \times 470 = y$

f) $(300 \times 9{,}150) + (80 \times 9{,}150) = y$

g) $400 \times (600 + 52) = y$

h) $209 \times 639 = y$

7. a) $(2{,}200 + 99) \div 11 = y$

b) $69{,}360 \div y = 17$

c) $3{,}332 \div 49 = y$

d) $4{,}984 \div 56 = y$

e) $331{,}705 \div y = 407$

f) $(217{,}200 + 33{,}304) \div 724 = y$

g) $50{,}542 \div 683 = y$

h) $546{,}984 \div 642 = y$

8. a) $8\frac{1}{6} = 7\frac{y}{18}$

b) $7\frac{2}{3} = 6\frac{y}{9}$

c) $\frac{4}{12} = \frac{y}{36}$

d) $\frac{y}{20} = \frac{2}{5}$

e) $1\frac{3}{4} = \frac{y}{8}$

f) $\frac{y}{21} = \frac{2}{7}$

g) $\frac{1}{4} = \frac{12}{y}$

h) $2\frac{1}{5} = \frac{y}{10}$

9. a) $3^2 \times y = 72$

b) $y + 7^2 = 62$

c) $81 \div 3^2 = y$

d) $60 = y + 6^2 + 4^2$

e) $4^3 \times y = 512$

f) $43 = y + 2^4$

g) $y \div 5^2 = 4$

h) $\frac{3}{10^1} \times \frac{7}{y^2} = \frac{21}{10^3}$

10. a) $5\frac{1}{2} = 4\frac{y}{6}$

b) $(56{,}100 \div 187) + (15{,}718 \div 187) = y$

c) $58 \times 2 \times 273 = y$

d) $\frac{14}{10^2} \times \frac{3}{10^2} = y$

e) $5\frac{1}{2} - 4\frac{7}{10} = y$

f) $10 \times 7.936 = y$

g) $875 + 374 + 923 = y$

h) $6\frac{3}{4} + 9\frac{1}{2} = y$

i) $3^4 \div 9 = y$

11. a) $\frac{7}{8} + \frac{3}{4} = y$

b) $16.58 + 8.28 + 787.54 + .56 = y$

c) $10 \times 29.2 = y$

d) $1\frac{5}{6} - 1\frac{1}{4} = y$

e) $\frac{5}{9} \times \frac{3}{5} = y$

f) $3 \times 3 \times 2{,}875 = y$

g) $595{,}161 \div y = 603$

h) $\frac{y}{12} = \frac{1}{4}$

i) $32 \times 2^2 = y$

Braintwisters

1. What four consecutive odd numbers, when added together, will equal 80?

2. Can you find any two prime numbers less than 100 whose sum is an odd number?

Review

SET I

Part A

1. Express in exponent form.

a) $5^4 \times 5^2 =$ ________ e) $16 \div 8 =$ ________

b) $16 \times 2 =$ ________ f) $100 \times 1{,}000 =$ ________

c) $7^3 \div 7^2 =$ ________ g) $4 \times 32 =$ ________

d) $125 \div 5^2 =$ ________ h) $81 \div 3 =$ ________

2. Find the number that n represents. The first one is done for you.

a) $\frac{1}{2} = \frac{n}{20}$ $n = 10$ d) $\frac{n}{5} = \frac{16}{40}$ ________

b) $\frac{1}{3} = \frac{n}{36}$ ________ e) $\frac{12}{18} = \frac{n}{6}$ ________

c) $\frac{3}{n} = \frac{30}{60}$ ________ f) $\frac{3}{5} = \frac{18}{n}$ ________

3. Arrange according to value, putting the smallest first. Example a is done for you.

a) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ $\underline{\frac{1}{4}, \frac{1}{3}, \frac{1}{2}}$ d) $\frac{15}{18}, \frac{5}{9}, \frac{2}{3}$

b) $\frac{1}{3}, \frac{1}{5}, \frac{1}{6}$ e) $\frac{5}{30}, \frac{2}{15}, \frac{1}{10}$

c) $\frac{2}{4}, \frac{2}{5}, \frac{2}{3}$ f) $\frac{3}{10}, \frac{1}{3}, \frac{7}{30}$

4. What exponent does n stand for in $3^n = 9$? You know that $3 \times 3 = 9$. Then $3^2 = 9$ and $n = 2$. Copy the problems and find n for each.

a) $2^n = 8$, n = ___

b) $6^n = 216$, n = ___

c) $3^n = 243$, n = ___

d) $4^n = 1{,}024$, n = ___

e) $7^n = 343$, n = ___

f) $5^n = 625$, n = ___

g) $3^n = 81$, n = ___

h) $4^n = 64$, n = ___

i) $6^n = 1{,}296$, n = ___

j) $10^n = 100{,}000$, n = ___

5. Place parentheses in the following to make the sentences true. Example: $3 + 13 \div 4 = 4$, $(3 + 13) \div 4 = 4$

a) $3 \div 1 \times 3 = 9$

b) $14 - 2 + 5 = 7$

c) $4 \times 5 + 2 = 28$

d) $17 + 3 \times 4 = 80$

e) $6 - 2\frac{2}{3} + 1\frac{1}{3} = 4\frac{2}{3}$

f) $\frac{3}{4} \times 5\frac{5}{6} - 2\frac{1}{8} = 4\frac{1}{4}$

g) $3 + 3\frac{1}{2} - 1\frac{7}{10} = 5.8$

h) $13 - 9\frac{3}{4} + 2\frac{5}{6} = \frac{1}{4}$

6. Estimate the two whole numbers the sum or product must be between. An example is done for you.

Example: $2\frac{1}{2} + 3\frac{1}{6}$, 5 and 7

a) $6\frac{1}{2} + 9\frac{7}{10}$

b) 3.25×2.4

c) $\frac{5}{6} \times 4\frac{1}{2}$

d) $5\frac{1}{2} + 7\frac{2}{5}$

e) 8.9×7.50

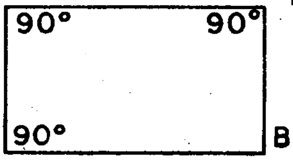
f) $12\frac{2}{3} + 15\frac{3}{6}$

g) $23\frac{1}{7} \times \frac{4}{5}$

h) $85\frac{3}{4} + 92\frac{5}{6}$

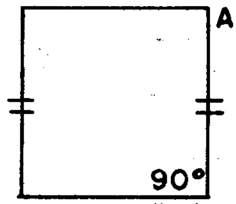
7. In each polygon below the sizes of some of the angles are shown. Write the measure of each angle whose vertex is named by a letter. Tell what each polygon would be called. Sides marked || are congruent. Example a is done for you.

a)

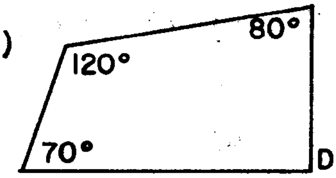


m $\angle B = 90$
Rectangle

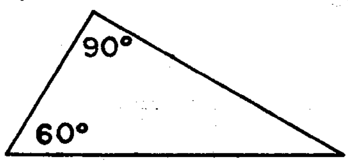
b)



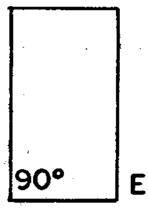
c)



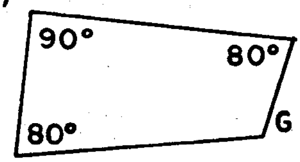
d)



e)



f)



8. Complete the chart below.

Name of Star	Approximate distance in miles, using exponent form	Approximate distance in miles, without using exponent form
Procyon	65×10^{12}	65,000,000,000,000
Betelgeuse	18×10^{14}	______
Regulus	______	410,000,000,000,000
Altair	______	110,000,000,000,000
Vega	16×10^{3}	______
Alpha Centauri	$2^2 \times 3^3 \times 10^{10}$	______
Rigel	______	3,200,000,000,000,000

Part B

Write a mathematical sentence (or two sentences if necessary) and solve. Write an answer sentence.

1. If Mrs. Jones cut $\frac{1}{2}$ of a pie into three equal pieces, each piece is what part of the whole pie?

2. Jim has a triangle design on his garage door. The triangle has one side 8 inches long and the other two sides each 6 inches long. The angle opposite the 8 inch side is 80°. What is the sum of the measures of the other two angles?

3. Ruth lives $\frac{5}{8}$ mile from the library. Linda lives twice as far from the library as Ruth does. How far does Linda live from the library?

4. The length of the playground at the Pine Grove school is 150 ft. The width of this playground is 60 ft. The playground at the Recreation Center has the same area as the playground at the school. The width of the Center's playground is 90 ft. What must be the length of the playground at the Center?

5. Mrs. Brown bought 14 yards of cloth to make curtains. She used $\frac{3}{4}$ of the material for the kitchen and bedroom. How much material is left?

6. A recipe for dessert for six persons calls for $\frac{3}{4}$ cup of sugar. How much sugar will be needed to make the dessert for three persons?

7. In 1950 the population of India was estimated to be $5 \times 2^3 \times 10^7$. Express this as a base ten numeral.

8. A planet moves around the sun. When it is closest, it is 70 million miles from the sun. When it is farthest, it is 90 million miles from the sun. What is its average distance from the sun? Write the average in exponent form.

9. Jack's sister in high school has an average of $2\frac{1}{3}$ hours of home work each school night. How many hours will she spend on homework each school week?

Braintwisters

"Clock Arithmetic"

Is $10 + 6$ always equal to 16 ?

If it is 10 a.m. now, what time will it be 6 hours later?

In counting hours what happens on an ordinary clock after the hour hand gets to 12 ?

The answers to problems in Example a are correct only for "clock arithmetic".

a 1) $3 + 4 = 7$ 2) $7 + 9 = 4$ 3) $8 + 6 = 2$

4) $11 + 7 = 6$

Use "clock arithmetic" to write the sum for these.

b 1) $9 + 6 = n$ 2) $10 + 7 = n$ 3) $11 + 11 = n$ 4) $5 + 4 = n$

Can you find products in "clock arithmetic"?

Review

SET II

Part A

1. Tell which property is illustrated by each of these mathematical sentences. Write the first letter of each work that names the property. For example, write A P M for associative property of multiplication.

   a) $(a + b) + c = a + (b + c)$ ________

   b) $c + d = d + c$ ________

   c) $a \times (d \times c) = (a \times d) \times c$ ________

   d) $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$ ________

   e) $(a + b) \times c = (a \times c) + (b \times c)$ ________

   f) $a \times b = a \times b$ ________

   g) $(b + c) + a = (b + a) + (c + a)$ ________

   h) $a \times c = c \times a$ ________

   i) $\frac{a}{b} + \frac{c}{d} = \frac{a}{b} + \frac{c}{d}$ ________

   j) $(\frac{a}{b} \times \frac{c}{d}) \times \frac{e}{f} = \frac{a}{b} \times (\frac{c}{d} \times \frac{e}{f})$ ________

2. Write the repeated factor form and the numeral for each of the following: Example a is done for you.

   a) $2^4 = 2 \times 2 \times 2 \times 2 = 32$

   b) $4^3 =$ __________ e) $10^6 =$ __________

   c) $6^2 =$ __________ f) $8^3 =$ __________

   d) $4^5 =$ __________ g) $5^4 =$ __________

3. Answer yes or no to each of the following questions.

a) Does $.15 + 2.3 + 7 = \frac{15}{100} + \frac{23}{10} + \frac{70}{10}$? _____

b) Does $4.2 \times 3.5 = \frac{42 \times 35}{10^2}$? _____

c) Does $\frac{5 \times 6}{10^3} = .56$? _____

d) Does $.032 = \frac{4 \times 8}{1000}$? _____

e) Does $4 = \{\frac{4}{1}, \frac{8}{2}, \frac{12}{3}, \frac{16}{4}\}$ _____

f) Does $\frac{8 + 8}{4 + 2} = 8$? _____

g) Does $\frac{21}{15} = \frac{3 \times 7}{5 \times 3} = \frac{3}{5} \times \frac{7}{3}$? _____

h) Does $7 \times \frac{2}{3} \times \frac{3}{6} = \frac{1}{7} \times \frac{2}{3} \times \frac{3}{6}$? _____

i) Does $3\frac{1}{2} \times 5\frac{7}{8} = 20\frac{9}{16} + \frac{2}{16}$? _____

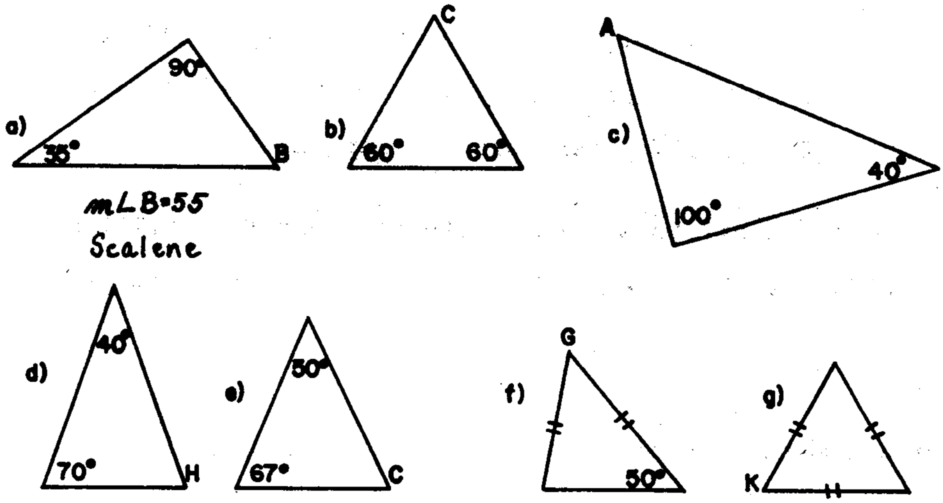
j) Does $\frac{3}{3} \times \frac{4}{5} = \frac{4}{5} \times 1$? _____

4. Write the following as decimals, fractions, and fractions with denominator in exponent form.

Example a is done for you.

a) Two and four tenths	2.4	$\frac{24}{10}$	$\frac{24}{10^1}$
b) Twenty-four hundredths	_____	_____	_____
c) Three and six hundredths	_____	_____	_____
d) Four and thirty hundredths	_____	_____	_____
e) Five and sixteen thousandths	_____	_____	_____
f) Three hundred twenty-four hundredths	_____	_____	_____
g) One hundred sixty-four tenths	_____	_____	_____
h) Thirty-nine and seventy hundredths	_____	_____	_____
i) Two thousand four and one tenth	_____	_____	_____

5. In each triangle below the sizes of some of the angles are shown. Write the measure of each angle whose vertex is named by a letter. Tell what each triangle would be called. Sides marked || are congruent. Example a is done for you.



6. Some countries use the metric system to measure length. It is a decimal system as shown below.

10 millimeters (mm)	=	1 centimeter (cm)
10 centimeters	=	1 decimeter (dm)
10 decimeters	=	1 meter (m)
1000 meters	=	1 kilometer (km)

Fill in the blanks.

a) 1 mm = _____ cm.

b) 1 mm = _____ dm

c) 1 mm = _____ m

d) 1 m. = _____ mm

e) 1 m. = _____ cm

7. An astronomer was trying to find how many miles it is from the center of the earth to the center of the moon. This was his mathematical sentence, $(6 \times 10^1) \times (4 \times 10^3) = n$. How many miles is it?

8. The mass (not weight) of the earth is 6,000 million million million tons. Using exponent form this could be written as 6×10^n. What number does n represent?

9. Some scientists say the earth is about five billion years old. Write the age of the earth four different ways.

Part B

Write a mathematical sentence (or two sentences if necessary) and solve. Write an answer sentence.

1. George bent a piece of wire to form a triangle. He found the size of one angle to be 90° and the size of another angle to be 30°. What will be the size of the third angle?

2. James can run one-fourth mile in $2\frac{1}{4}$ minutes. At this speed how long will it take to run one mile?

3. Eddie rode his bicycle 3.7 miles in fifteen minutes. At this speed how far does Eddie travel in one hour?

4. An airplane is traveling at $2^4 \times 5^2$ miles an hour. Express this as a base ten numeral.

5. In many countries a standard unit of measure is the meter. A meter equals 1.0936 yards. How many yards more than 100 yards is a 100 meter race?

6. Dan bought $\frac{1}{2}$ pound of lunch meat. The first day he ate $\frac{1}{4}$ of the lunch meat. How much lunch meat did he eat?

7. A musical tone is made by vibrations. When struck, string E vibrates 256 times a second. String C vibrates $1\frac{1}{2}$ times as much as string E. How many times a second will string C vibrate?

Group Activities

Relays Using Sequence--The object of the game is to discover the rule and to write the terms of a sequence. The teacher dictates the first three numbers of the sequence to the first player of each team. The first player writes these and then returns to seat. After a brief time, during which class discovers the rule of the sequence, the signal to begin is given. Each player, going up in turn, adds a term to the sequence. The relay continues until the teacher calls time. The winning team is the one with the longest correct sequence.

This game can be adapted for many kinds of interesting practice.

Review

SET III

Part A

1. Arrange in the order of size from least to greatest.

   a) 0.74 , 0.014 , 1.40 , 0.7

   b) 0.65 , 0.8 , 0.07 , 0.10

   c) 1.1 , 1.32 , 1.0 , 1.85

2. Write the pair of equal fractions in each of the following. The first one is worked for you.

   a) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{8}$ ($\frac{1}{2}$, $\frac{4}{8}$)

   b) $\frac{3}{5}$, $\frac{9}{15}$, $\frac{3}{10}$

   c) $\frac{2}{3}$, $\frac{6}{4}$, $\frac{4}{6}$

   d) $\frac{5}{9}$, $\frac{15}{36}$, $\frac{20}{36}$

   e) $\frac{11}{4}$, $\frac{21}{8}$, $\frac{33}{12}$

   f) $\frac{5}{4}$, $\frac{10}{12}$, $\frac{15}{12}$

3. Find the number that the letter represents. Express answers in simplest form.

   a) $\frac{3}{4} + \frac{1}{8} = n$

   b) $\frac{5}{8} \times \frac{1}{3} = p$

   c) $3.6 \times 0.6 = r$

   d) $\frac{4}{3} - \frac{4}{6} = d$

   e) $2.81 \times 0.7 = p$

   f) $\frac{13}{16} - n = \frac{3}{4}$

   g) $16.3 \times 0.09 = b$

   h) $6 \times 5\frac{1}{4} = c$

4. Find the number that n represents. Tell whether it is a prime number or a composite number.

   Examples: $2^3 + 1 = n$, $n = 9$ composite; $2^3 - 1 = n$ $n = 7$ prime

   a) $9^2 - 1 = n$

   b) $2^5 - 1 = n$

   c) $3^3 + 1 = n$

   d) $4^2 + 1 = n$

   e) $6^2 + 1 = n$

   f) $5^2 - 1 = n$

   g) $7^3 + 1 = n$

   h) $4^5 - 1 = n$

5. Express each of these numbers as a product of primes.

Example: $16 = 2 \times 2 \times 2 \times 2$

a) 125 = ______________	f) 68 = ______________
b) 81 = ______________	g) 39 = ______________
c) 27 = ______________	h) 32 = ______________
d) 21 = ______________	i) 48 = ______________
e) 12 = ______________	j) 243 = ______________

Which of the above numbers could be written in exponent form as a power of two? Example: $16 = 2^4$. Write them. Which could be written in exponent form as a power of three? Write them.

6. A ten foot ladder is leaning against a wall. The top of the ladder is 8 feet from the ground. The bottom of the ladder is 6 feet from the wall. What kind of triangle does this suggest? Where will the largest angle of that triangle be located: where the ladder touches the wall or ground, or where the ground and wall touch?

Part B

Write a mathematical sentence (or two sentences if necessary) and solve. Write an answer sentence.

1. Jill is cutting out a pattern. One piece has the shape of an equilateral triangle. One side of the triangle is 34 inches long. How many inches will Jill cut when she cuts out the triangle?

2. There were 36 children in Miss Heyer's class in January. Three-fourths of the class had perfect attendance. How many children had perfect attendance?

3. A snail crawls $\frac{3}{8}$ of an inch in one minute and $\frac{5}{3}$ of an inch the next minute. How far does he crawl in the two minutes?

4. Dan caught a lake trout that weighs $2\frac{1}{2}$ pounds. How many ounces does the trout weigh?

5. An astronaut is traveling around the earth at $5^2 \times 10^3$ miles an hour. He travels 75,000 miles each orbit. How long will it take him to make one orbit?

6. David's father weighs 196.5 pounds. David's weight is $\frac{1}{3}$ of his father's weight. What is David's weight?

7. A dress pattern calls for $3\frac{3}{4}$ yards of material. Mother wishes to make dresses for Mary, Jan, and Denise. How much material does she need?

Braintwisters

1. The following statements are true. What number base other than ten is used in each?

   a) I am $13_{\underline{?}}$ years old. In three years I will be a teen ager.

   b) There are $100_{\underline{?}}$ inches in one yard.

   c) My birthday is in October, the $12_{\underline{?}}$ month of the year.

Chapter 4

INTRODUCING THE INTEGERS

A NEW KIND OF NUMBER

This is a number line. It has a point labeled 0 just like the number line you saw in Chapter 3. But the other points on this one have new labels. These labels are the names of some new numbers we are going to study.

Find the dot numbered 0. As your eye moves to the right, the dots are labeled $^{+}1$, $^{+}2$, $^{+}3$, ... As your eye moves to the left from zero, the dots are numbered $^{-}1$, $^{-}2$, $^{-}3$, ... The dots on the line represent numbers that are called <u>integers</u>. The set of integers is

$$\{\ldots, {}^{-}3, {}^{-}2, {}^{-}1, 0, {}^{+}1, {}^{+}2, {}^{+}3, \ldots\} .$$

The integer " $^{+}2$ " is read "positive two." The integer " $^{-}7$ " is read "negative seven." The 0 is read "zero." It is neither positive nor negative.

Integers on the line can be thought of in pairs. $^{+}6$ is paired with $^{-}6$; $^{-}1$ is paired with $^{+}1$; $^{+}10$ is paired with $^{-}10$; and zero is paired with itself.

These pairs are called <u>opposites</u>. $^{+}12$ is the opposite of $^{-}12$; $^{+}4$ is the opposite of $^{-}4$; $^{-}1$ is the opposite of $^{+}1$; and zero is its own opposite.

A part of a number line is shown above. It may be extended and numbered with integers as far as you wish. Try to imagine how many integers there are.

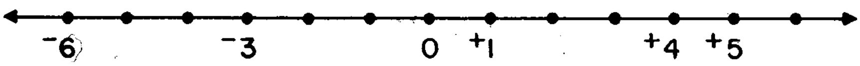
Look at the dot labeled $^+5$. Positive five is greater than $^+4$, or 0, or $^-3$, or $^-8$. In fact, positive five is greater than any integer which labels any dot to the left of the dot labeled by $^+5$. The mathematical sentences ... $^+5 > {}^+4$; $^+5 > {}^+3$; ... $^+5 > 0$; $^+5 > {}^-1$; ... $^+5 > {}^-4$; ... are ways of writing this.

Look again at the dot labeled $^+5$. Positive five is to the left of and less than $^+6$, or $^+7$, or $^+127$. In fact, positive five is less than any integer to its right. The mathematical sentences $^+5 < {}^+6$; $^+5 < {}^+7$; ... $^+5 < {}^+19$; ... $^+5 < {}^+127$; ... are ways of writing this fact.

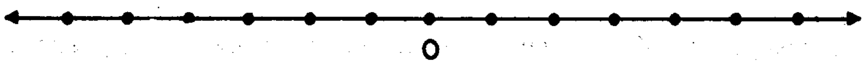
An integer represented by a dot on the number line is greater than any integer represented by a dot to its left; an integer represented by a dot is less than any integer represented by a dot to its right on the number line.

Exercise Set 1

1. Draw a number line like the one below and beneath it write the integers that are missing.



2. On a number line like the one below, locate and write each of these integers: $^{-}1$, $^{+}5$, $^{-}4$, $^{+}2$, $^{-}6$ below the dot which it labels.



3. Match each set with its best description:

a.	E = {1, 2, 3, 4, ...}	Whole numbers
b.	P = {... $^{-}3$, $^{-}2$, $^{-}1$, 0, $^{+}1$, ...}	Fractional numbers
c.	N = {... $\frac{1}{2}$, ... $\frac{7}{8}$, ... $\frac{9}{3}$, ...}	Integers
d.	T = {0, 1, 2, 3, ...}	Counting numbers

4. Copy and complete the following sentences by writing the correct symbol in the blank space, " >," " <," or " =."

a.	$^{+}3$ ___ $^{+}5$	f.	$^{+}479$ ___ $^{+}421$
b.	$^{-}12$ ___ $^{-}4$	g.	$^{+}89$ ___ $^{+}95$
c.	$^{-}8$ ___ $^{+}6$	h.	$^{-}26$ ___ $^{-}26$
d.	$^{+}1$ ___ $^{-}19$	i.	$^{-}3$ ___ $^{-}5$
e.	$^{-}16$ ___ $^{-}32$	j.	0 ___ $^{-}7$

5. Name the integer that is

a.	3 greater than $^{-}12$.	d.	5 less than $^{-}2$.
b.	7 less than 0.	e.	6 greater than 0.
c.	4 greater than $^{+}16$.	f.	2 less than $^{+}9$.

6. Arrange the members of the following sets in the order they would appear on a number line from least to greatest:

$$P = \{{}^{+}4,\ {}^{-}6,\ 0\}$$
$$F = \{{}^{-}19,\ {}^{+}2,\ {}^{+}17,\ {}^{-}36\}$$
$$M = \{{}^{-}1,\ {}^{+}5,\ {}^{+}3,\ {}^{-}7,\ {}^{-}20\}$$
$$R = \{{}^{+}13,\ {}^{-}11,\ {}^{+}1,\ {}^{-}31,\ {}^{-}3\}$$
$$T = \{{}^{-}26,\ {}^{+}4,\ {}^{+}9,\ {}^{+}12,\ {}^{-}2\}$$

7. Compare the numbers a, ${}^{-}4$, b, 0, c, e, ${}^{+}3$, and d which are pictured on the number line below. Copy and fill in the mathematical sentences. Use the symbols ">," "<," or "=."

a. a ___ ${}^{-}4$	d. e ___ ${}^{+}3$	g. 0 ___ a
b. b ___ 0	e. d ___ b	h. c ___ b
c. e ___ e	f. ${}^{-}4$ ___ a	i. ${}^{+}3$ ___ ${}^{-}4$

8. Name the integers which are

a. greater than ${}^{-}2$ and less than ${}^{+}4$.

b. less than ${}^{-}5$ and greater than ${}^{-}10$.

c. greater than ${}^{-}72$ and less than ${}^{-}67$.

d. less than ${}^{+}4$ and greater than ${}^{-}1$.

e. greater than ${}^{+}9$ and less than ${}^{+}10$.

9. Describe these sets of integers as was done in exercise 8 a-e.

a. $\{{}^{-}7,\ {}^{-}6,\ {}^{-}5,\ {}^{-}4\}$

b. $\{{}^{+}11,\ {}^{+}12,\ {}^{+}13\}$

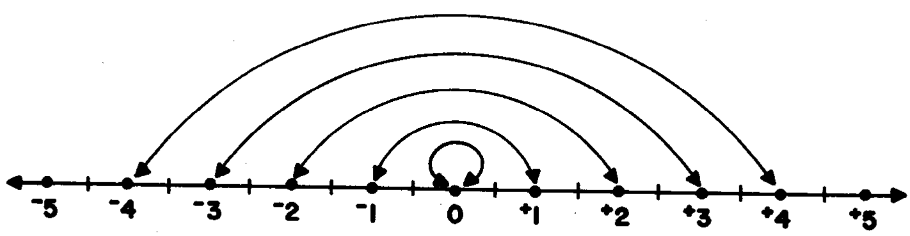
c. $\{{}^{+}1\}$

d. $\{{}^{-}10\}$

INTEGERS AND THEIR OPPOSITES

Expressions like "five degrees below zero," "five hundred feet above sea level," "two points in the hole," and "three yards loss" are very common in our language. We may use integers to express these ideas:

Five degrees below zero	$^-5$ degrees
Five hundred feet above sea level	$^+500$ feet
Two points in the hole	$^-2$ points
Three yard loss	$^-3$ yards



Look at the diagram above. Each integer is one of a pair. $^+4$ and $^-4$ form a pair. Their dots are the same distance from the zero dot. $^+4$ is the <u>opposite</u> of $^-4$; $^-4$ is the opposite of $^+4$. Zero is its own opposite.

<u>Every integer has an opposite</u>.

Exercise Set 2

1. Which of the number pairs below are opposites:

   a. $^-3;\ ^+3$ c. $^+7;\ ^-2$ e. $0;\ 0$

   b. $^-5;\ ^-5$ d. $^-8;\ 0$ f. $^-14;\ ^+14$

2. Choose the greater integer, if possible, from each pair of opposites below.

   a. $^-13;\ ^+13$ c. $^-7;\ ^+7$ e. $^+9;\ ^-9$

   b. $^+6;\ ^-6$ d. $0;\ 0$ f. $^-112;\ ^+112$

3. Write in words the opposite meaning of:

   a. 12° below zero d. 270 feet above sea level

   b. "5 in the hole" e. 15° north of the equator

   c. $10 profit f. 110° east of Prime Meridian

4. Write the opposite for each integer below.

   a. $^+3$ c. 0 e. $^+77$

   b. $^-7$ d. $^-2$ f. $^-256$

5. Use an integer to help describe each of the following.

   a. Twenty degrees above zero _____ degrees

   b. Two hundred feet below sea level _____ feet

   c. Two points in the hole _____ points

   d. Forty degrees south of the equator _____ degrees

6. Write an expression similar to those given in exercise 5 for each of these integers.

   a. $^+17$ c. 0 e. $^-7$

   b. $^-22$ d. $^+126$ f. $^+21$

7. Write numerals for:

   a. negative two thousand twenty-two.

   b. positive five hundred fifty.

   c. negative sixty-one.

   d. positive eighty-nine.

   e. negative one thousand one.

   f. positive ten thousand four hundred ninety.

8. Write words for:

   a. $^{-}317$.

   b. $^{+}203$.

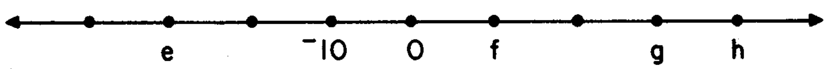
   c. $^{-}6,060$.

   d. $^{+}910$.

9. What integers are represented by the points labeled by letters on these number lines?

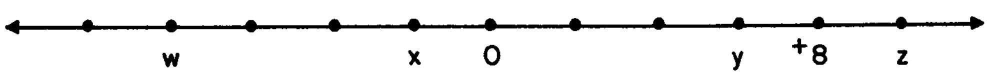
(a)

(b)



(c)

(d)



10. Write these sets of integers.

a. greater than $^{-}1$ and less than $^{+}1$

b. less than $^{-}3$ and greater than $^{-}6$

c. greater than $^{+}5$ and less than $^{-}5$

d. less than 0 and greater than 0

11. Write these numerals.

a. negative three hundred fifty-four

b. negative six thousand eight

c. positive twenty three thousand

d. positive forty seven thousand two hundred

e. negative eight hundred four

f. negative five thousand nine

g. positive two thousand twenty

h. negative twenty-five

12. Write words for:

a. $^{+}29$

b. $^{-}4{,}008$

c. $^{-}8$

d. $^{+}606$

e. $^{+}45$

f. $^{-}370$

g. $^{+}8{,}001$

h. $^{-}2{,}300$

ARROW DIAGRAMS

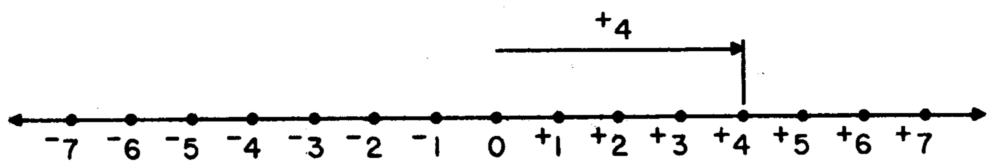
Diagrams may be used to show the result of counting. You can count forward or backward. An <u>arrow</u> can show many things about counting. The arrow in the diagram below shows:

(1) Where you begin to count

(2) The number of spaces you count

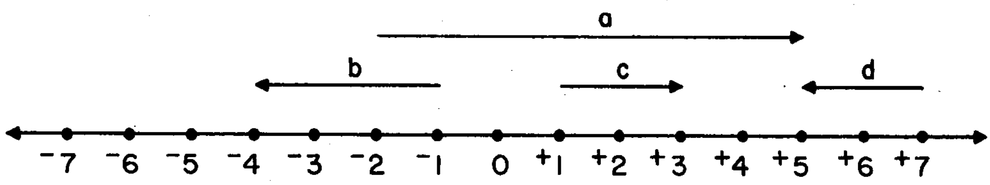
(3) In what direction you count

(4) Where you stop.



In the diagram above, we began at the zero dot and counted 4 in a positive direction to the $^+4$ dot. 4 is called the <u>measure</u> of the arrow. The arrow is named $^+4$ to show the direction of count and the measure. The + shows direction, 4 shows the measure. The tail of the arrow is at zero; the head of the arrow is at $^+4$.

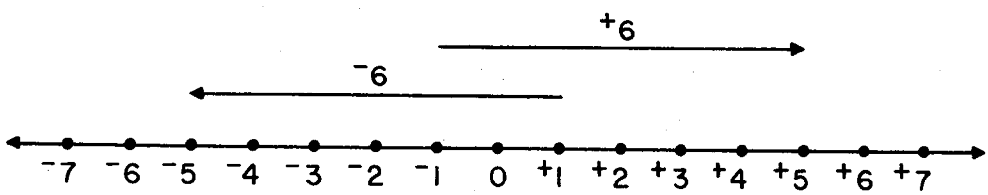
The diagram below shows some other arrows. Name them with integers.



We have agreed that counting in a positive direction means counting to the right from a reference dot on the number line. This may be shown by an arrow diagram and labeled with the symbol " + "; $\xrightarrow{^{+}6}$ means that you start at a reference dot and count six spaces to the right. The <u>measure</u> of the arrow is 6.

Counting in a negative direction means counting to the left from a reference dot on the number line. The symbol " ⁻ " on an arrow diagram shows this; $\xleftarrow{^{-}6}$ means that you count six spaces to the left from a reference dot on the number line. The <u>measure</u> of the arrow is 6.

The figure below shows two arrows with the same measure. One indicates a count in the negative direction; the other a count in the positive direction.

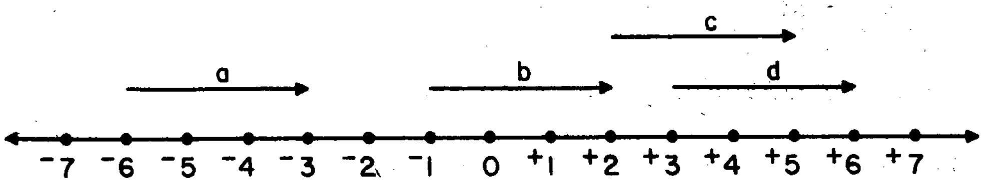


Exercise Set 3

1. Look at the arrows on the number line below.

   a. How are they alike?

   b. How are they different?

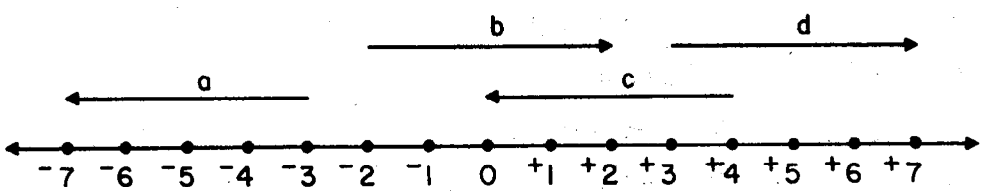


2. Draw a number line. Label it with integers. Draw five arrows that begin at these dots: $^+1$, $^-2$, $^-6$, $^+3$, $^-3$. $^+4$

3. Look at the arrows on the number line below.

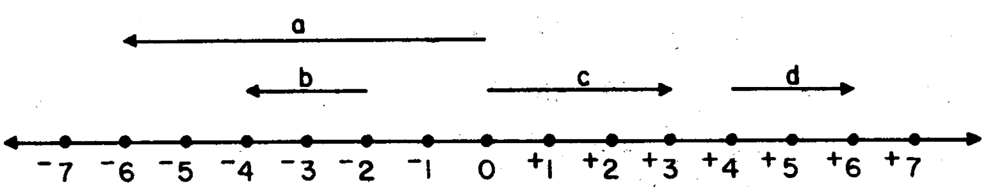
   a. How are they alike?

   b. How are they different?



4. Draw a number line. Label it with integers. Draw four arrows that begin at these dots: $^-3$, $^+3$, $^+6$, $^-1$. $^-3$

5. Write the number name for each arrow below.



6. Draw a number line. Label special points with integers. Draw the arrows described below and label them.

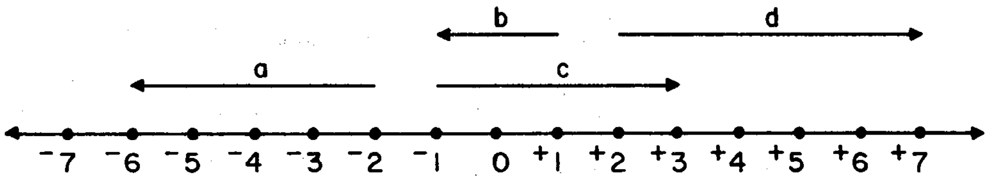
   a. Begin at 0 and end at $^{+}4$.

   b. Begin at $^{+}5$ and end at $^{+}7$.

   c. Begin at 0 and end at $^{-}2$.

   d. Begin at $^{-}2$ and end at $^{-}6$.

7. Write the number name for each arrow below.



8. Draw a number line. Label special points with integers. Draw the arrows described below and label them.

   a. Begin at $^{-}4$ and end at $^{-}7$.

   b. Begin at $^{-}2$ and end at $^{+}4$.

   c. Begin at $^{+}6$ and end at $^{+}7$.

   d. Begin at $^{+}3$ and end at $^{-}1$.

9. Which arrow has the greater measure?

   a. $^{-}2$ to $^{+}6$ or $^{+}2$ to $^{+}6$

   b. $^{+}8$ to $^{+}1$ or $^{+}8$ to $^{-}1$

   c. 0 to $^{+}4$ or $^{-}6$ to 0

   d. $^{+}5$ to $^{-}3$ or $^{-}3$ to $^{-}5$

   e. $^{-}4$ to $^{-}8$ or $^{+}6$ to $^{+}9$

Review

Exercise Set 4

1. At what integer on the number line will you stop, if you
   a. begin at 0 and count 7 to the right?
   b. begin at $^{-}2$ and count 4 to the left?
   c. begin at $^{-}6$ and count 9 to the right?
   d. begin at 0 and count 5 to the left?
   e. begin at $^{+}6$ and count 6 to the left?
   f. begin at $^{+}3$ and count 2 to the right?
   g. begin at $^{-}4$ and count 4 to the right?

2. Draw arrows to represent each answer in exercise 1. Label with an integer.

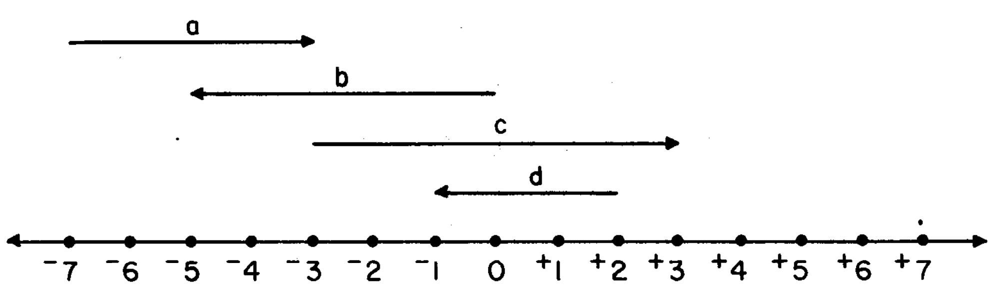
3. Copy and write the opposite for each integer below. Underline the greater integer in each pair.

   a. $^{-}7$ c. $^{+}9$ e. $^{+}6$
   b. 0 d. $^{-}4$ f. $^{-}5$

4. Write in words the names of the following integers.

   a. $^{+}9$ b. 0 c. $^{-}3$

5. Label the arrows below using $^{-}5$, $^{+}4$, $^{-}3$, $^{+}6$.



6. Some of these mathematical sentences are true and some false. Mark them T if true and F if false.

a. $^{+}17 > {}^{-}99$

b. $^{-}17 > {}^{-}99$

c. $^{-}2 < {}^{-}5$

d. $^{+}2 < {}^{+}5$

e. $^{-}6 > 0$

f. $^{-}1 > {}^{-}6$

g. $^{+}14 > {}^{-}14$

h. $^{-}14 < {}^{+}14$

7. Choose the largest integer from each set.

a. $P = \{^{-}29, {}^{+}3, {}^{+}31, {}^{-}50, {}^{-}1\}$

b. $T = \{^{+}5, {}^{+}1, {}^{-}2, {}^{-}1, {}^{-}5\}$

c. $W = \{^{+}23, {}^{-}41, {}^{-}30, {}^{+}29, {}^{+}20\}$

d. $F = \{0, {}^{-}3, {}^{-}7, {}^{-}2, {}^{-}6\}$

e. $G = \{^{-}4, {}^{+}10, {}^{+}15, {}^{-}9, {}^{+}1\}$

8. Choose the smallest integer from each set in Exercise 7.

9. Would the arrow drawn for each of the following be named by a positive or negative integer?

a. from $^{-}3$ to $^{-}7$

b. from $^{+}2$ to $^{-}4$

c. from $^{-}5$ to $^{-}1$

d. from $^{-}4$ to 0

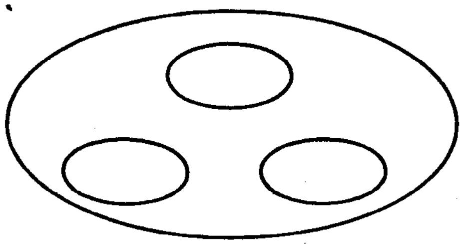
10. Name these sets of numbers. The letter used for each set should help you remember the name of the set.

$$C = \{1, 2, 3, 4, \ldots\}$$

$$R = \{\ldots, \frac{1}{2}, \ldots, \frac{4}{4}, \ldots, \frac{17}{8}, \ldots\}$$

$$I = \{\ldots, {}^{-}2, {}^{-}1, 0, {}^{+}1, {}^{+}2, \ldots\}$$

11. Draw a picture diagram like the one below on your paper. Label the sets given in exercise 10.



12. Write these subsets of the set of integers.

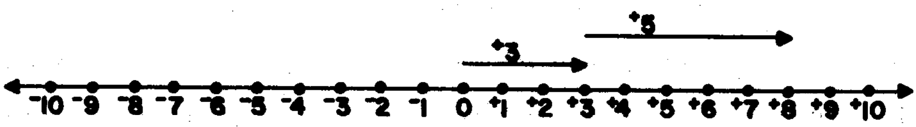
   a. Integers which are positive

   b. Integers which are negative

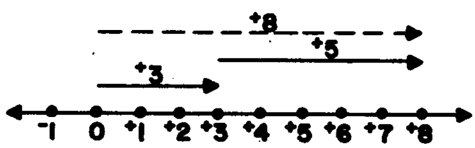
   c. Integers which are neither positive nor negative

Exercise Set 5 (Exploratory)

1. Draw a number line. Label it with integers from $^-8$ to $^+8$.
   a. Begin at zero and count 3 spaces to the right on the number line. Draw an arrow above the number line to show this count. Label it $^+3$.
   b. Begin at a point directly above the head of the $^+3$ arrow and count 5 more spaces to the right. Draw another arrow to show this count. Label it $^+5$. Your diagram should look like this:



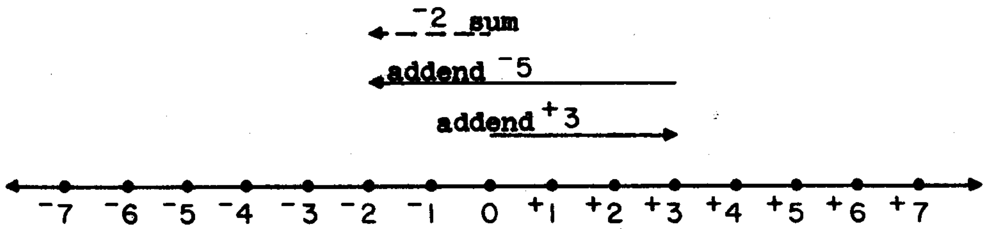
   c. Put a finger on zero where you started counting to make the $^+3$ arrow. Put your pencil on the dot where the $^+5$ arrow ends. What integer does this dot represent? Above the $^+5$ arrow draw a "dotted" arrow, with its tail above the zero, and its head directly above the point where you stopped counting, $^+8$. Label the "dotted" arrow with an integer. Your diagram should look like this:



2. a. Change exercise 1 by beginning at zero and counting 3 to the left then 7 to the right. Show both of these counts with arrows.
   b. Draw a "dotted" arrow which begins at zero and ends at the dot where the counting stops. Label the "dotted" arrow with an integer.

RENAMING INTEGERS

Arrow diagrams may be used to rename integers. The diagram below renames $^{+}3 + {}^{-}5$ as $^{-}2$. This may be shown by the mathematical sentence $^{+}3 + {}^{-}5 = {}^{-}2$.



The diagram is made by following these steps:

(1) Begin at a point directly above zero and draw a solid arrow for the first addend ($^{+}3$). Draw to the right for positive.

(2) Begin at a point directly above the head of the arrow for the first addend and draw a solid arrow for the second addend ($^{-}5$). Draw to the left for negative.

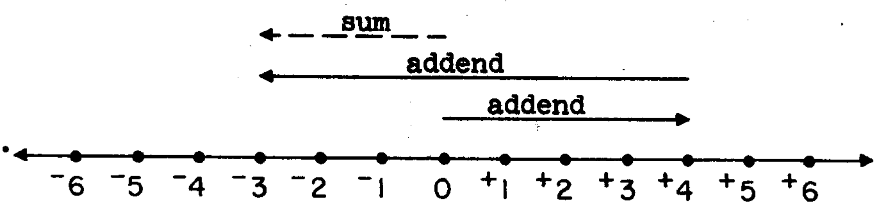
(3) Above this arrow draw a "dotted" arrow from directly above zero to the head of the arrow for the second addend. This arrow ($^{-}2$) renames $^{+}3 + {}^{-}5$. It is the sum of $^{+}3$ and $^{-}5$.

Follow this plan: (1) draw the arrow for the first given addend directly above the number line; its tail should be at 0; (2) draw the arrow for the second addend above the first arrow, starting at the point where the first arrow's head ends; and (3) draw the "dotted" arrow representing the sum above these two arrows. This dotted arrow begins directly above zero and ends at the head of the second arrow. If this plan is followed, we will better understand our diagrams.

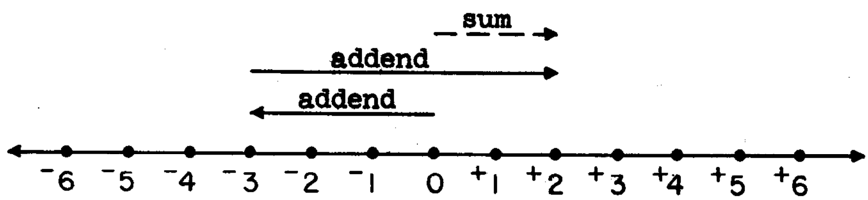
Exercise Set 6

1. Study the arrow diagrams below. Write a mathematical sentence for each:

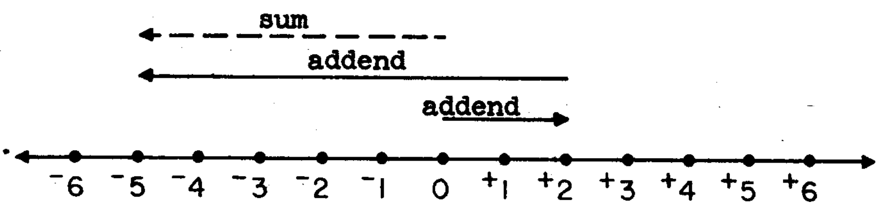
a.



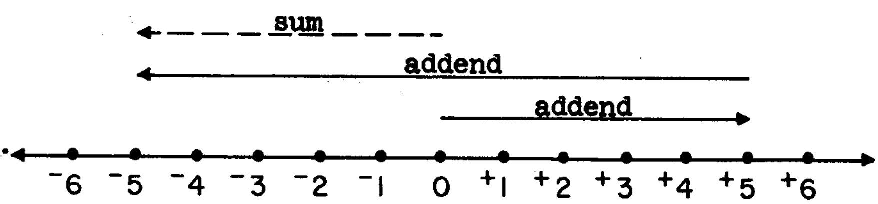
b.



c.



d.



2. Draw a number line and use arrows to illustrate each of these mathematical sentences.

a. ${}^{+}3 + {}^{+}3 = {}^{+}6$

b. ${}^{-}2 + {}^{-}1 = {}^{-}3$

c. ${}^{-}4 + {}^{-}2 = {}^{-}6$

d. ${}^{+}4 + {}^{-}7 = {}^{-}3$

e. ${}^{-}5 + {}^{+}9 = {}^{+}4$

f. ${}^{-}6 + {}^{+}3 = {}^{-}3$

3. Rename each integer in Column A by matching it with another name in Column B. You might be able to do this without drawing arrow diagrams. Look at the first sum in Column B. It is $^{+}13 + {}^{-}2$. Can you "imagine" a number line which has the arrows $^{+}13$ and $^{-}2$ on it? The $^{+}13$ arrow would begin directly above zero and have its head directly above $^{+}13$. The $^{-}2$ arrow would begin directly above the head of the $^{+}13$ arrow and would be drawn 2 spaces to the left. Its head would be directly above $^{+}11$. So the arrow for the sum would begin at zero and have its head directly above $^{+}11$. Thus, $^{+}13 + {}^{-}2 = {}^{+}11$; and d is the answer.

	A	B
a.	$^{+}17$	$(^{+}13 + {}^{-}2)$
b.	$^{-}8$	$(^{-}20 + {}^{-}4)$
c.	$^{+}2$	$(^{+}9 + {}^{+}8)$
d.	$^{+}11$	$(^{-}5 + {}^{+}2)$
e.	$^{-}24$	$(^{-}5 + {}^{-}3)$
f.	$^{-}3$	$(^{+}1 + {}^{+}1)$

4. Complete with an integer.

a. $^{-}4 + {}^{-}3 =$ ___

b. $^{+}2 + {}^{-}4 =$ ___

c. $^{+}3 + {}^{+}5 =$ ___

d. $^{+}4 + {}^{-}9 =$ ___

e. $^{-}7 + {}^{+}14 =$ ___

f. $^{+}8 + {}^{-}3 =$ ___

g. $^{-}11 + {}^{+}10 =$ ___

h. $^{-}9 + {}^{-}7 =$ ___

5. Complete these mathematical sentences.

a. $^{+}8 + \underline{\quad} = {}^{-}15$

b. $\underline{\quad} + {}^{-}9 = {}^{-}15$

c. $^{+}20 + \underline{\quad} = {}^{-}15$

d. $^{+}1 + \underline{\quad} = {}^{-}15$

e. $^{-}2 + \underline{\quad} = {}^{+}6$

f. $\underline{\quad} + {}^{+}15 = {}^{+}6$

g. $\underline{\quad} + {}^{+}3 = {}^{+}6$

h. $^{+}9 + \underline{\quad} = {}^{+}6$

6. Use an integer from Column B to rename a, b, c, and d in Column A.

A	B
a. $^{-}5 + {}^{-}4$	$^{+}9$
b. $^{+}5 + {}^{-}4$	$^{-}9$
c. $^{-}5 + {}^{+}4$	$^{+}1$
d. $^{+}5 + {}^{+}4$	$^{-}1$

7. Mark true or false.

$^{-}2$ is another name for:

a. $^{+}5 + {}^{-}3$

b. $^{-}7 + {}^{-}5$

c. $^{-}8 + {}^{+}6$

d. $^{+}2 + 0$

e. $0 + {}^{-}2$

$^{+}3$ is another name for:

a. $0 + {}^{+}3$

b. $^{+}9 + {}^{-}12$

c. $^{-}4 + {}^{+}7$

d. $^{-}7 + {}^{+}4$

e. $0 + {}^{-}3$

8. $^{+}4$ may be renamed, for example, $^{+}1 + {}^{+}3$ or $^{-}1 + {}^{+}5$. Rename each of the following integers in two different ways.

a. $^{+}3$

b. $^{-}4$

c. $^{+}1$

d. 0

e. $^{-}75$

f. $^{+}18$

g. $^{-}100$

h. $^{+}100$

9. BRAINTWISTER: Rename each of the following numbers with a numeral.

a. $^{+}5 + {}^{-}7 + {}^{-}5 + {}^{+}7$

b. $^{-}3 + {}^{+}14 + {}^{-}4$

c. $^{-}7 + {}^{+}6$

d. $^{-}137 + {}^{+}136$

e. $^{-}8 + {}^{+}8 + {}^{-}5 + {}^{+}5 + {}^{+}1$

f. $^{+}45 + {}^{-}46 + {}^{+}1$

g. $^{-}2 + {}^{-}2 + {}^{-}2 + {}^{-}2$

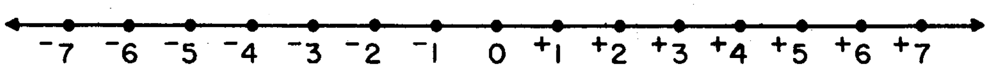
h. $^{+}5 + {}^{+}5 + {}^{-}10$

RENAMING SUMS

When $^{+}5 + {}^{+}2$ is renamed $^{+}7$ from a diagram, when $^{+}5 + {}^{-}3$ is renamed $^{+}2$ from a diagram, and when $^{-}2 + {}^{-}3$ is renamed $^{-}5$ from a diagram, you are finding sums. The sum of $^{+}5 + {}^{+}2$ is $^{+}7$; $^{+}5 + {}^{-}3 = {}^{+}2$; and $^{-}2 + {}^{-}3 = {}^{-}5$.

It is not always necessary to draw a diagram. Some of you can look at a number line and imagine the arrows without making them. Try this.

Find the sum in this sentence: $^{-}4 + {}^{-}2 = s$. Look at the number line below. No drawings, please! Imagine the $^{-}4$ arrow, then the $^{-}2$ arrow. What is the name of the arrow for the sum? It may help to outline the arrows with your eyes or a finger.

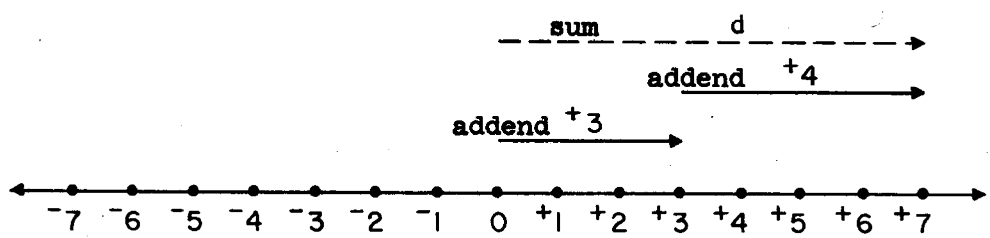


The operation we use when we think of two integers like $^{-}4$ and $^{-}2$ and get $^{-}6$ is called <u>addition</u>. It may be possible for you to add two integers without arrow diagrams or without even looking at a number line. Try it with these: $^{+}3 + {}^{+}5$; $^{-}3 + {}^{+}2$; $^{-}5 + {}^{+}5$.

USING THE NUMBER LINE

The integers and arrow diagrams may be used to solve problems. The diagram below was drawn by a girl to show where a new friend lived. This is the way she explained it:

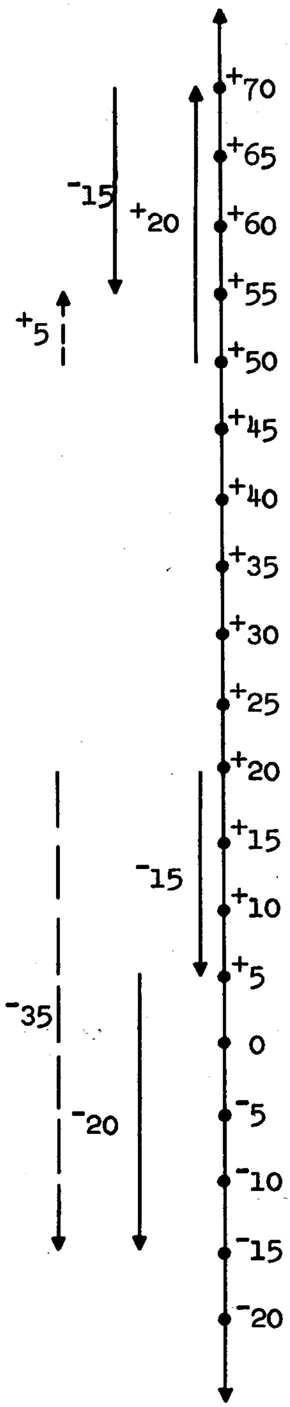
> The line below represents my street and is marked off in blocks. I live at the dot named zero. My friend lives on the same street three blocks east of me. The $^{+}3$ arrow shows this. A new girl has moved in four blocks east of my friend. The $^{+}4$ arrow shows this. The $^{+}7$ arrow shows that the new girl lives 7 blocks east of me.



The diagram above shows the three arrows. The "dotted" arrow shows the sum of the other two arrows. Living seven blocks to the east is the same as living three blocks to the east and then four blocks further east. This is the meaning of the mathematical sentence which shows addition:

$$^{+}3 + {}^{+}4 = d$$

$$^{+}3 + {}^{+}4 = {}^{+}7$$



The change of temperature as shown by a thermometer may be illustrated by arrow diagrams.

Look at the thermometer scale at the left. It is a vertical number line. It is labeled with integers.

There is an arrow diagram at the top of the thermometer scale which shows a rise of 20° in temperature ($^+20$) and then a fall of 15° ($^-15$). The result of these two changes is shown in the diagram by a "dotted" arrow ($^+5^{\circ}$). The mathematical sentence which shows this is $^+20 + {}^-15 = {}^+5$.

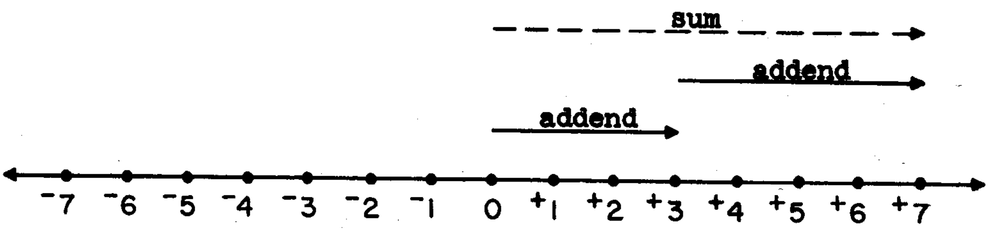
The arrow diagram at the bottom of the thermometer scale shows a fall of 15° ($^-15$) in temperature and another fall of 20° ($^-20$). The "dotted" arrow shows the total change in temperature. The mathematical sentence which shows this is $^-15 + {}^-20 = {}^-35$.

Draw a thermometer scale; sketch in arrows to show two changes. Draw a "dotted" arrow to show the total change shown by the two arrows.

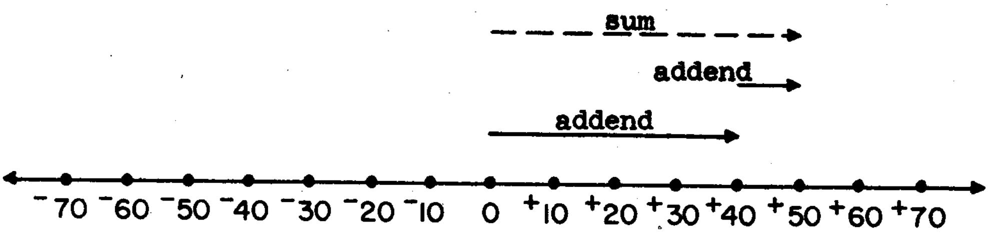
Exercise Set 7

Write a mathematical sentence showing addition for each diagram below.

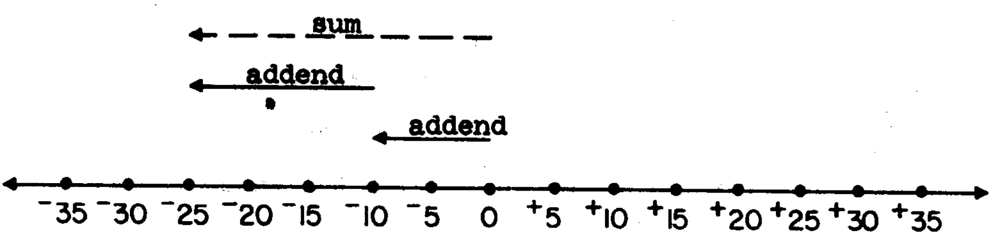
1.



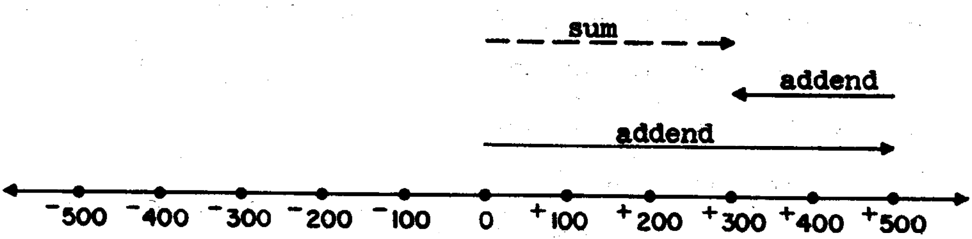
2.



3.



4.



Solve these problems by drawing diagrams if necessary, like those on page 217. Write a mathematical sentence, using integers, for each problem.

5. Charles and John are playing a game. The boy with the greater total score for two games is the winner. Charles has 2 scores. One of them is 5. The other is "3 in the hole." John has 2 scores. One of them is 3. The other is "4 in the hole."

   a. What is Charles' total score?

   b. What is John's total score?

   c. Which boy is the winner?

6. The temperature in a mountain cabin was 15 degrees above zero. During the night the temperature fell 20 degrees. What was the temperature then?

7. A scientist invented a "subcopter." (A helicopter which can also go beneath the surface of the water like a submarine.) The "subcopter" was 30 feet above the water. It dived 40 feet. How far below the surface of the water was it then?

8. Three girls were playing a game. They played it twice. The girl with the smallest total score was the winner. Jane's scores were "5 in the hole" and "8 in the hole." Sandy's scores were "6 in the hole" and 4. Helen's scores were "9 in the hole" and 2.

   a. What was Jane's total score?

   b. What was Sandy's total score?

   c. What was Helen's total score?

   d. Who was the winner?

   e. Which girl was second?

Exercise Set 8

1. Draw an arrow diagram to help rename $^{-}5 + ^{+}5$.

2. Draw an arrow diagram to help rename $^{-}4 + ^{+}4$.

3. Draw an arrow diagram to help rename $^{+}12 + ^{-}12$.

4. What arrow did you draw for the sum in exercise 1? Exercise 2? Exercise 3?

5. How are the answers to exercises 1, 2, and 3 similar?

6. What do we call pairs of integers like $^{-}5$ and $^{+}5$, and $^{-}4$ and $^{+}4$?

7. Choose another pair of opposites and draw an arrow diagram to find its sum.

8. Is the sum in exercise 7 the same as those in exercises 1, 2, and 3?

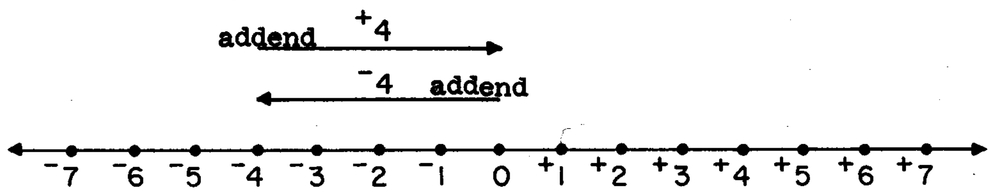
9. What is the sum when opposites are added?

10. Write a sentence in words about adding opposites.

OPPOSITES

You were asked to rename $^-5 + {}^+5$; $^-4 + {}^+4$; and $^+12 + {}^-12$ in Exercise Set 8.

The diagram you drew to rename $^-4 + {}^+4$ was like the one below. The others were similar.



No arrow was drawn for the new name of $^-4 + {}^+4$. You would be drawing an arrow from zero to zero. Counting 4 backward and 4 forward undo each other.

You found: $^-4 + {}^+4 = 0$; $^-5 + {}^+5 = 0$; and $^+12 + {}^-12 = 0$.

$^-4$ and $^+4$ are opposites; $^-5$ and $^+5$ are opposites; $^+12$ and $^-12$ are opposites. We can say:

<u>When opposites are added, the sum is zero</u>.

Exercise Set 9

1. Which of the number pairs below are opposites?

a. $^-3$, $^+3$

b. $^-5$, $^-5$

c. $^+7$, $^-2$

d. $^+2$, $^+2$

e. $^-8$, 0

f. $^+6$, $^-6$

g. $^+1$, $^+1$

h. $^-4$, $^+4$

2. Fill the blanks so the sentences are true.

a. $^+4 + \_\_ = 0$

b. $^-7 + \_\_ = 0$

c. $0 = \_\_ + {}^-5$

d. $\_\_ + {}^-9 = 0$

e. $0 = \_\_ + 0$

f. $^+6 + \_\_ = 0$

3. Which of the following are names for zero?

a. $^+8 + {}^+8$

b. $^-6 + 0$

c. $^-3 + {}^+3$

d. $^-4 + {}^-4$

e. $^+2 + {}^+2$

f. $^-16 + {}^+16$

g. $^-7 + {}^-7$

h. $^+5 + {}^-5$

4. Tell whether each of these is a true mathematical sentence. Write "Yes" or "No."

a. $(^+2 + {}^+4) + (^-3 + {}^-3) = 0$

b. $(^+5 + {}^-3) + (^-5 + {}^+3) = 0$

c. $(^-7 + {}^+6) + (^+6 + {}^-7) = 0$

d. $(^+8 + {}^-6) + (^+4 + {}^-2) = 0$

e. $(^-3 + {}^+4) + (^-4 + {}^+3) = 0$

f. $(^+9 + {}^-12) + (^-2 + {}^-1) = 0$

g. $(^+26 + {}^+5) + (^-18 + {}^-13) = 0$

h. $(^-17 + {}^+3) + (^-2 + {}^+16) = 0$

5. Which statements are true about 0?

   a. It is neither positive nor negative.

   b. It is equal to its opposite.

   c. It is less than any negative integer.

   d. It is the sum of any integer and its opposite.

   e. It is less than any positive integer.

6. Use "positive" or "negative" to complete these sentences.

   a. If an integer is greater than its opposite, the integer is a ______________ integer.

   b. If an integer is less than its opposite, the integer is a ______________ integer.

   c. When you add two negative integers, the sum is a ______________ integer.

   d. When you add two positive integers, the sum is a ______________ integer.

   e. When you add a negative integer and a positive integer, the sum is a ______________ integer if the dot labeled by the positive integer is farther away from 0 than the dot labeled by the negative integer.

   f. When you add a positive integer and a negative integer, the sum is a ______________ integer if the dot labeled by the negative integer is farther away from 0 than the dot labeled by the positive integer.

Exercise Set 10

1. Show the addition of these addends on a number line by drawing arrow diagrams.

   a. $^{-}4 + {}^{-}2$

   b. $^{-}2 + {}^{-}4$

2. Look at your diagrams for exercise 1. Answer these questions.

   a. What is the first pair of addends?

   b. What is the second pair of addends?

   c. How are the two pairs of addends alike?

   d. How are the two pairs of addends different?

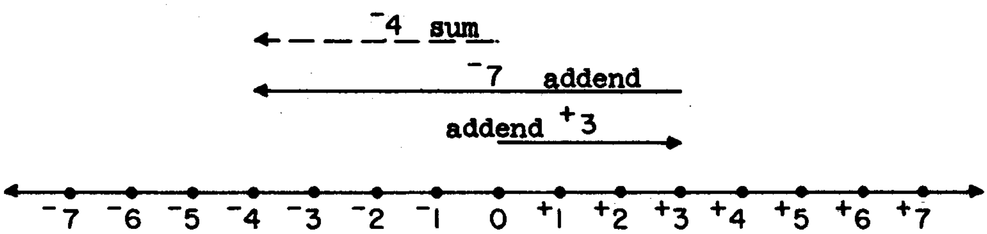
   e. What do you notice about the new names you found for the two pairs?

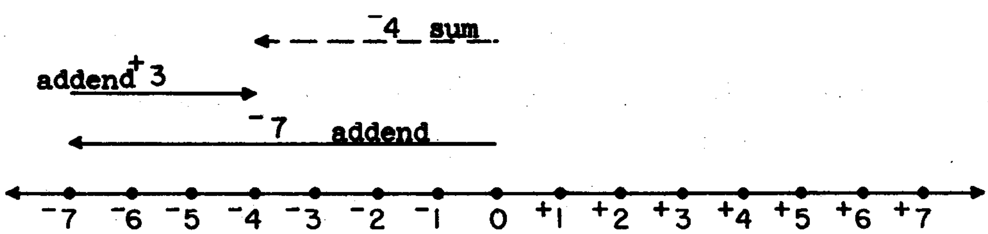
3. Rename $^{-}4 + {}^{+}3$ and $^{+}3 + {}^{-}4$ by drawing arrow diagrams. Answer exercises 2a through 2e for these pairs of addends.

ORDER OF ADDENDS

In Exercise Set 10 you drew diagrams to rename $^{-}4 + {}^{-}2$ and $^{-}2 + {}^{-}4$; and to rename $^{-}4 + {}^{+}3$ and $^{+}3 + {}^{-}4$. You found something very interesting.

The diagrams below are similar to ones you drew. They show the renaming of $^{+}3 + {}^{-}7$ and $^{-}7 + {}^{+}3$.





These diagrams show that $^{+}3 + {}^{-}7$ and $^{-}7 + {}^{+}3$ name the same integer, $^{-}4$.

You found $^{-}4 + {}^{-}2$ and $^{-}2 + {}^{-}4$ each names $^{-}6$; and that $^{-}4 + {}^{+}3$ and $^{+}3 + {}^{-}4$ each names $^{-}1$.

Your work shows that:

> <u>the order of adding two integers may be changed with no change in the sum.</u>
>
> **Addition is commutative in the set of integers.**

Exercise Set 11

1. Fill the blanks so the sentences are true.

   a. $^{-}7 + {}^{+}3 = {}^{+}3 + \_\_\_$
   b. $^{+}14 + {}^{+}8 = \_\_\_ + {}^{+}14$
   c. $\_\_\_ + {}^{-}6 = {}^{-}6 + {}^{-}4$
   d. $^{+}29 + \_\_\_ = {}^{-}12 + {}^{+}29$
   e. $^{+}51 + {}^{-}5 = \_\_\_ + {}^{+}51$
   f. $\_\_\_ + {}^{-}18 = {}^{-}18 + {}^{+}19$
   g. $^{-}6 + \_\_\_ = {}^{-}17 + {}^{-}6$
   h. $\_\_\_ + {}^{+}37 = {}^{+}37 + {}^{-}16$

2. Complete these mathematical sentences. The order of adding two addends may be changed without changing the sum.

   a. $^{+}7 + {}^{-}4 = \_\_\_\_\_\_$
   b. $\_\_\_\_\_\_ = {}^{-}12 + {}^{-}6$
   c. $^{+}3 + {}^{+}11 = \_\_\_\_\_\_$
   d. $\_\_\_\_\_\_ = {}^{-}26 + {}^{-}7$
   e. $\_\_\_\_\_\_ = {}^{+}8 + {}^{-}13$
   f. $^{-}6 + {}^{-}9 = \_\_\_\_\_\_$
   g. $\_\_\_\_\_\_ = {}^{-}5 + {}^{+}10$
   h. $^{+}32 + {}^{-}19 = \_\_\_\_\_\_$

3. Complete the mathematical sentences with " >," " <," or " =."

   a. $^{-}3 + {}^{-}6 \; \_\_\_ \; {}^{-}6 + {}^{-}3$
   b. $^{+}3 + {}^{-}6 \; \_\_\_ \; {}^{-}3 + {}^{+}6$
   c. $^{+}6 + {}^{+}3 \; \_\_\_ \; {}^{-}3 + {}^{-}6$
   d. $^{-}6 + {}^{+}3 \; \_\_\_ \; {}^{+}3 + {}^{-}6$
   e. $^{+}7 + {}^{-}2 \; \_\_\_ \; {}^{+}7 + {}^{+}2$
   f. $^{+}2 + {}^{-}7 \; \_\_\_ \; {}^{-}7 + {}^{+}2$
   g. $^{-}2 + {}^{-}7 \; \_\_\_ \; {}^{+}2 + {}^{+}7$
   h. $^{-}2 + {}^{+}7 \; \_\_\_ \; {}^{-}7 + {}^{+}2$

4. If $^{-}6 + {}^{+}2$ is written in each blank below, will the sentence be true or false?

   a. $\_\_\_\_\_\_ > {}^{-}8 + {}^{+}8$
   b. $\_\_\_\_\_\_ < 0 + {}^{+}5$
   c. $\_\_\_\_\_\_ > {}^{+}6 + {}^{+}2$
   d. $\_\_\_\_\_\_ > {}^{+}6 + {}^{-}2$
   e. $\_\_\_\_\_\_ < {}^{-}4 + {}^{-}2$
   f. $\_\_\_\_\_\_ < 0 + {}^{-}6$
   g. $\_\_\_\_\_\_ > {}^{+}5 + {}^{-}10$
   h. $\_\_\_\_\_\_ < {}^{-}7 + {}^{-}9$

5. Write the set whose members will be:

a. the integers $> {}^{-}4$ and $< {}^{+}2$.

b. the negative integers $> {}^{-}5$.

c. the integers $> {}^{-}3$ and < 0.

d. the integers between ${}^{+}2$ and ${}^{-}2$.

e. the positive integers $< {}^{+}3$.

6. Add the following. Use arrow diagrams only when necessary.

a. ${}^{+}5 + {}^{+}8 =$ ___

b. ${}^{-}7 + {}^{-}4 =$ ___

c. ${}^{+}23 + {}^{+}23 =$ ___

d. ${}^{+}9 + {}^{-}26 =$ ___

e. ${}^{-}34 + {}^{+}11 =$ ___

f. ${}^{-}5 + 0 =$ ___

7. In each pair of statements, only one is true. Write the correct statement.

a. ${}^{+}78 > {}^{-}93$; ${}^{-}78 > {}^{+}93$

b. ${}^{-}15 > {}^{-}2$; ${}^{-}15 < {}^{-}2$

c. ${}^{+}125 < {}^{-}26$; ${}^{-}125 < {}^{+}26$

d. ${}^{+}571 > {}^{-}589$; ${}^{+}571 > {}^{+}589$

e. ${}^{-}2 < {}^{-}35$; ${}^{+}2 < {}^{+}35$

f. ${}^{-}45 > 0$; ${}^{+}45 > 0$

8. An airplane pilot saw that the temperature outside his plane was 23 degrees below zero. A little later, as he was approaching a landing field, he saw that the outside temperature was 40 degrees higher. What was the temperature outside the plane then?

9. The teacher places the end of a pointer on a number line in a sixth grade room. She then moves it along the number line. If it was placed at a point labeled $^{+}8$ and moved 9 spaces to the left, at what point did it stop?

10. These are the scores of three girls on a game.

Betty "6 in the hole."

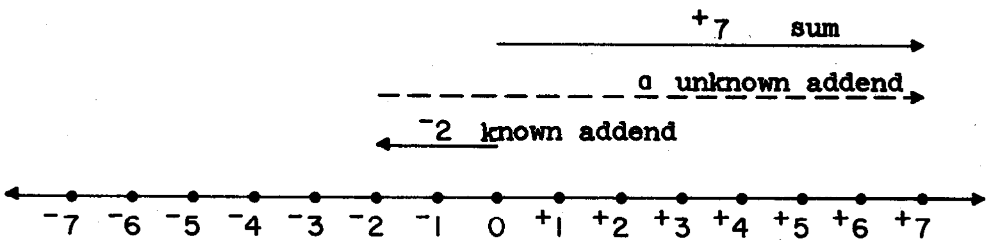
Mable "9 in the hole."

Ruth "8 in the hole."

On the next game, each girl make a score of 12 points. What is each girl's score then?

INTRODUCTION TO UNKNOWN ADDENDS

Diagrams may be used to rename a sum when one addend is unknown. If the sum is $^{+}7$ and one addend is $^{-}2$, the mathematical sentence is $^{+}7 = {}^{-}2 + a$ or $^{-}2 + a = {}^{+}7$. The diagram below renames $^{+}7$ as $^{-}2$ and the number represented by the "dotted" arrow.



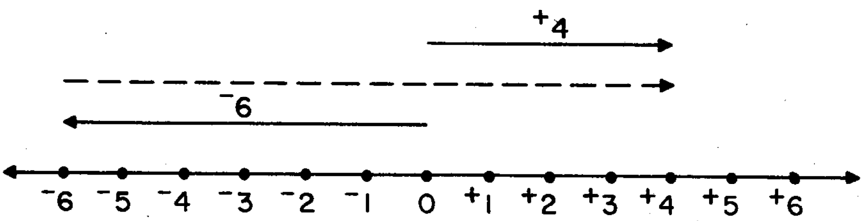
The arrow for the known addend, $^{-}2$, is drawn directly above the number line. It is a solid line because it is a known addend. Notice that the arrow representing the sum, $^{+}7$, is the top arrow. It is a solid line because it is a known sum.

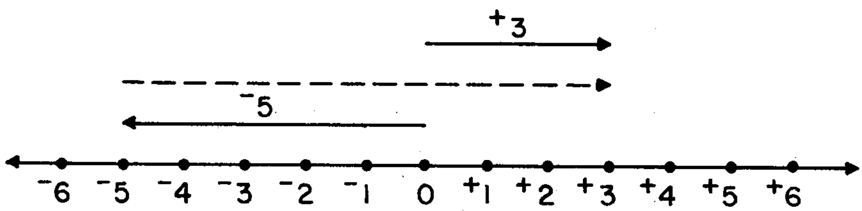
The arrow representing the unknown addend is drawn as a "dotted" line between the other two arrows. It must be drawn so that the sum of it and the arrow representing the known addend is the sum arrow, $^{+}7$.

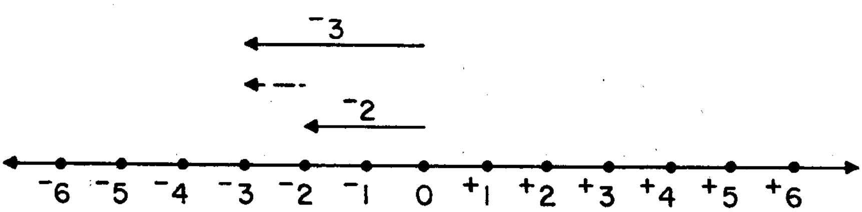
The arrow for the unknown addend is drawn from the head of the arrow representing the known addend to the head of the arrow representing the sum. In this sentence the arrow represents $^{+}9$.

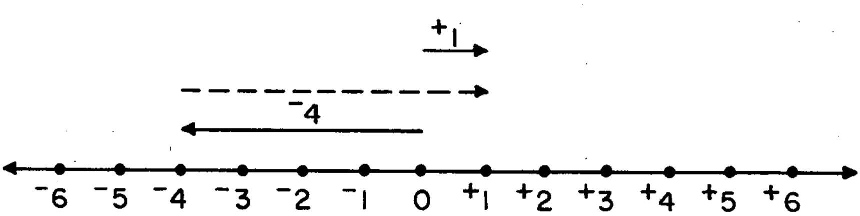
Exercise Set 12

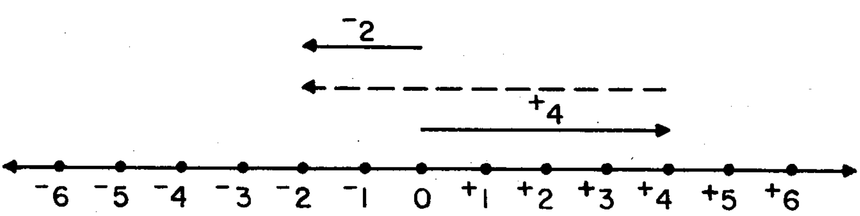
1. Study the arrow diagrams below. Write a mathematical sentence to rename the sum. Find the unknown addend.

a.


b.


c.


d.


e.


2. What integer in Column A may be used to complete each sentence in Column B?

A	B
a. $^{+}9$	a. $\_\_\_ + {}^{-}6 = {}^{-}18$
b. $^{-}12$	b. $^{+}18 + \_\_\_ = {}^{+}11$
c. $^{+}3$	c. $^{-}4 + \_\_\_ = {}^{+}2$
d. $^{-}7$	d. $\_\_\_ + {}^{+}5 = {}^{+}3$
e. $^{+}6$	e. $^{-}9 + \_\_\_ = {}^{-}6$
f. $^{-}2$	f. $^{+}5 + \_\_\_ = {}^{+}14$

3. Complete the following sentences.

a. $^{+}7 + \_\_ = {}^{+}4$

b. $\_\_ + {}^{+}6 = {}^{-}2$

c. $\_\_ + {}^{-}8 = {}^{+}6$

d. $^{-}9 + \_\_ = {}^{+}3$

e. $\_\_ + {}^{+}3 = {}^{-}6$

f. $^{-}4 + \_\_ = {}^{+}3$

g. $^{+}8 + \_\_ = 0$

h. $\_\_ + {}^{+}9 = {}^{-}2$

4. Rename the integers below by completing the mathematical sentences.

a. $^{+}3 + \_\_\_ = {}^{+}9$

b. $\_\_\_ + {}^{-}9 = {}^{+}9$

c. $^{-}15 + \_\_\_ = {}^{+}9$

d. $^{-}1 + \_\_\_ = {}^{+}9$

e. $^{+}4 + \_\_\_ = {}^{-}12$

f. $\_\_\_ + {}^{+}18 = {}^{-}12$

g. $\_\_\_ + {}^{-}6 = {}^{-}12$

h. $^{-}17 + \_\_\_ = {}^{-}12$

5. Diagram each of these mathematical sentences to find the unknown addend.

a. $^{-}5 + m = ^{+}3$ d. $^{+}2 + s = ^{-}3$

b. $n + ^{-}1 = ^{+}6$ e. $^{+}7 + t = ^{+}10$

c. $r + ^{+}4 = ^{-}7$ f. $^{-}2 + p = ^{-}9$

6. Column A represents temperatures at 6:00 a.m. Column B represents temperatures at 4:00 p.m. Find the total change in temperature between 6:00 a.m. and 4:00 p.m. Use an integer to indicate the amount and direction of change.

	A	B
Monday	$^{-}5^{\circ}$	$^{+}2^{\circ}$
Tuesday	$^{+}8^{\circ}$	$^{-}4^{\circ}$
Wednesday	$^{+}3^{\circ}$	0°

7. Below is a list of cities and the location of each. We think of directions north of the equator as positive.

Hilo	20° north
Rio de Janeiro	23° south
San Francisco	38° north
Lima	12° south

Complete:

a. San Francisco is ______ degrees north of Rio de Janeiro.

b. Lima is ______ degrees south of Hilo.

c. Rio de Janeiro is ______ degrees south of Hilo.

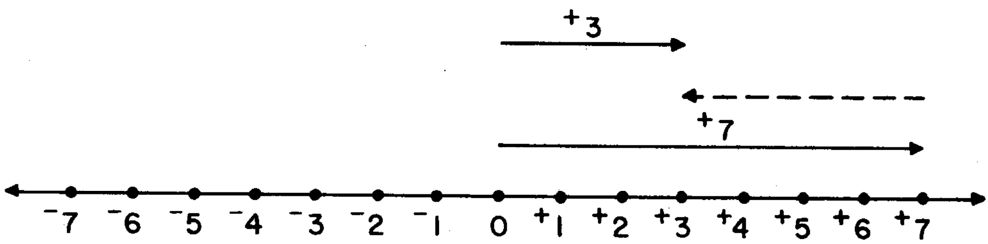
d. Lima is ______ degrees south of San Francisco.

FINDING AN UNKNOWN ADDEND

The process of finding an unknown addend in a sentence like $^-5 + n = {}^-7$ is <u>subtraction</u>.

The diagram on page 228 renamed $^+7$ as $^-2 + {}^+9$. It helped you find an unknown addend which is the result when you subtract $^-2$ from $^+7$.

The sentence $n + {}^+7 = {}^+3$ is diagramed below and the steps outlined.



(1) A solid arrow for the known addend, $^+7$, is drawn.

(2) A solid arrow for the sum, $^+3$, is drawn.

(3) A "dotted" arrow for the unknown addend, n, is drawn from the head of the $^+7$ arrow to the head of the $^+3$ arrow.

(4) The "dotted" arrow is named $^-4$.

The subtraction of integers may be shown by drawing arrow diagrams. To show subtraction by the use of arrow diagrams, you must find an arrow to represent an unknown addend.

Exercise Set 13

1. Find the unknown addends. (Use an arrow diagram.)

a. $^{-}4 + \_\_\_ = {}^{+}3$

b. $\_\_\_ + {}^{-}10 = {}^{-}7$

c. $^{+}16 + \_\_\_ = {}^{+}34$

d. $^{+}12 + \_\_\_ = {}^{+}3$

e. $^{-}7 + \_\_\_ = {}^{-}2$

f. $\_\_\_ + {}^{+}12 = {}^{-}5$

g. $^{-}7 + \_\_\_ = {}^{+}6$

h. $\_\_\_ + {}^{-}15 = {}^{-}10$

2. Use arrow diagrams to find each unknown addend below.

a. $^{-}5 + \_\_ = {}^{+}4$

b. $\_\_ + {}^{-}7 = {}^{-}2$

c. $^{+}2 + \_\_ = {}^{+}6$

d. $\_\_ + {}^{-}5 = 0$

e. $\_\_ + {}^{-}2 = {}^{+}3$

f. $^{+}7 + \_\_ = {}^{-}3$

g. $^{-}5 + \_\_ = {}^{+}7$

h. $\_\_ + {}^{+}3 = {}^{-}4$

3. Diagram on a number line

a. Two trains started from the same station but traveled in opposite directions. Train A traveled north at the rate of 46 miles per hour. Train B traveled south at the rate of 53 miles per hour. How far north of train B would train A be at the end of the first hour?

b. In a game, Jane's score was 56 and Mary's score was "23 in the hole." How many points was Jane ahead of Mary?

The "Short-Cut" Method For Subtracting Integers

Exercise Set 14

1. One addend in each of the following is represented by n. What integer does n represent in each?

   a. $^{+}2 + n = {}^{+}6$
   b. $0 + n = {}^{+}6$
   c. $^{-}2 + n = {}^{+}6$
   d. $^{-}4 + n = {}^{+}6$
   e. $^{-}6 + n = {}^{+}6$
   f. $^{+}7 + n = {}^{-}2$
   g. $^{+}5 + n = {}^{-}2$
   h. $^{+}3 + n = {}^{-}2$
   i. $^{+}1 + n = {}^{-}2$
   j. $^{-}1 + n = {}^{-}2$

2. What addend is represented by n in each sentence?

   a. $n + {}^{-}5 = {}^{-}10$
   b. $n + {}^{-}3 = {}^{-}10$
   c. $n + {}^{-}1 = {}^{-}10$
   d. $n + {}^{+}1 = {}^{-}10$
   e. $n + {}^{+}3 = {}^{-}10$
   f. $n + {}^{+}5 = {}^{-}10$

3. Find the unknown addend.

   a. $^{+}12 = n + {}^{+}4$
   b. $^{-}7 = {}^{-}3 + n$
   c. $^{-}8 = n + {}^{+}6$
   d. $^{+}9 = {}^{+}2 + n$
   e. $^{+}14 = n + {}^{-}2$
   f. $^{-}2 = {}^{-}1 + n$

4. What integer must be added to each of the following to obtain a sum of $^{+}6$?

   a. $^{+}3$
   b. $^{-}3$
   c. 0
   d. $^{+}9$
   e. $^{-}9$
   f. $^{+}6$

5. Write a true mathematical sentence using addition and these integers:

a. $^{-}6$, $^{+}8$, $^{+}14$ Answer: $^{+}8 = {}^{-}6 + {}^{+}14$

b. $^{+}5$, $^{-}3$, $^{-}8$

c. $^{-}4$, $^{+}2$, $^{-}6$

d. $^{+}6$, $^{+}3$, $^{+}9$

e. $^{-}6$, $^{+}3$, $^{-}9$

f. $^{-}6$, $^{+}3$, $^{+}9$

WRITING SUBTRACTION SENTENCES

Subtraction is the operation of finding an unknown addend. Sentences like these have unknown addends.

$$n + {}^{-}5 = {}^{+}8 \qquad {}^{+}6 = n + {}^{+}2$$

$$^{-}2 + n = {}^{-}5 \qquad {}^{+}8 = {}^{+}12 + n$$

To find n, the known addend is subtracted from the sum. To find n in $n + {}^{-}5 = {}^{+}8$, $^{-}5$ is subtracted from $^{+}8$.

To show this in a mathematical sentence you may write

$$n = {}^{+}8 - {}^{-}5$$

$^{-}2 + n = {}^{-}5$ may be written as $n = {}^{-}5 - {}^{-}2$

$^{+}6 = n + {}^{+}2$ may be written as $n = {}^{+}6 - {}^{+}2$

$^{+}8 = {}^{-}12 + n$ may be written as $n = {}^{+}8 - {}^{-}12$

<u>Exercise Set 15</u>

1. Write the following subtraction sentences as addition sentences.

a. $^-4 - {}^+3 = s$

b. $^-9 - {}^+12 = i$

c. $^-7 - {}^-9 = x$

d. $^+27 - {}^+25 = t$

e. $^-35 - {}^+21 = h$

f. $^-18 - {}^-13 = g$

g. $^+19 - {}^+35 = r$

h. $^+45 - {}^-17 = a$

i. $^-45 - {}^+8 = d$

j. $^-12 - {}^+16 = e$

2. Rewrite the following subtraction sentences as addition sentences.

a. $^-4 - {}^-2 = g$

b. $^+7 - {}^-5 = i$

c. $^-3 - {}^-7 = r$

d. $^+2 - {}^-8 = \ell$

e. $^-6 - {}^-13 = c$

f. $^+7 - {}^+9 = a$

g. $^+12 - {}^+5 = t$

h. $^-9 - {}^-1 = s$

3. Choose an integer from Set A to use as an unknown addend in each of these.

a. $^+3 - c = {}^+2$

b. $^-6 - a = {}^+2$

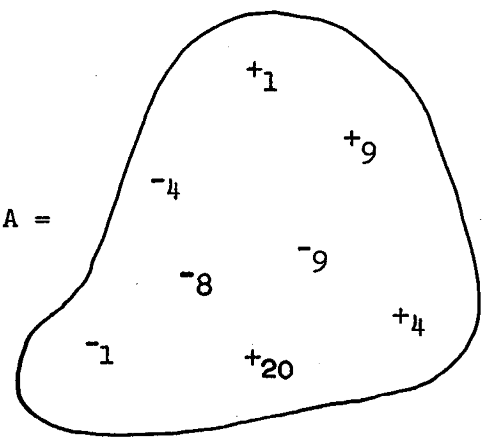
c. $^+16 - n = {}^+7$

d. $^+12 - d = {}^-8$

e. $^-10 - y = {}^-9$

f. $^+5 - z = {}^+14$

A =



4. Rewrite as addition sentences. Then find the unknown addend.

a. $^-17 - ^+5 = n$

b. $^+10 - ^-2 = e$

c. $^-3 - ^+8 = a$

d. $^+2 - ^-6 = c$

e. $^+9 - ^+3 = h$

f. $^-7 - ^-32 = e$

g. $^+2 - ^+19 = r$

h. $^-43 - ^-16 = s$

5. Copy and complete the following sentences by writing the correct sign of operation.

a. $^-5$ ___ $^-7 = ^+2$

b. $^+1$ ___ $^-1 = 0$

c. $^+13$ ___ $^-10 = ^+23$

d. $^-9$ ___ $^+7 = ^-2$

e. $^-1$ ___ $^+6 = ^+5$

f. $^+18$ ___ $^-3 = ^+21$

g. $^-2$ ___ $^-4 = ^+2$

h. $^+26$ ___ $^+11 = ^+15$

6. Mark true or false.

a. $^-2 + ^-3 = ^-3 + ^-2$

b. $^+6 + ^-4 = ^-4 + ^+6$

c. $^-5 - ^-3 = ^-3 - ^-5$

d. $^-7 + ^-5 = ^-5 + ^-7$

e. $^+3 - ^-12 = ^-12 - ^+3$

f. $^-2 + ^+3 = ^+3 - ^-2$

g. $^+16 + ^-18 = ^-18 + ^+16$

h. $^+14 - ^+9 = ^+9 - ^+14$

Exercise Set 16

1. Add these integers. Try to add them without the use of arrow diagrams.

a. $^{+}7 + {}^{+}3$ d. $^{-}4 + {}^{+}4$

b. $^{-}6 + {}^{+}9$ e. $^{-}14 + {}^{+}6$

c. $^{+}10 + {}^{-}5$ f. $^{-}9 + {}^{-}19$

2. Try to find the unknown addend without the use of arrow diagrams.

a. $^{-}7 - {}^{-}3$ d. $^{-}12 - {}^{+}5$

b. $^{+}6 - {}^{+}8$ e. $^{+}18 - {}^{-}2$

c. $^{+}6 - {}^{+}3$ f. $^{+}5 - {}^{-}5$

3. Perform these operations. Try to perform them without arrow diagrams.

a. $^{+}3 + {}^{-}6$ d. $^{-}11 + {}^{-}4$

b. $^{+}5 - {}^{-}7$ e. $^{-}4 - {}^{-}3$

c. $^{-}8 + {}^{+}13$ f. $^{+}8 - {}^{+}10$

4. BRAINTWISTER. Perform the following without the use of arrow diagrams.

a. $^{-}625 - {}^{+}25$ d. $^{+}2{,}300 + {}^{-}300$

b. $^{+}999 - {}^{+}1$ e. $^{-}7{,}225 + {}^{+}125$

c. $^{-}455 + {}^{-}55$ f. $^{-}4{,}376 - {}^{-}4{,}376$

5. On any number line, how many units apart are:

   a. the 9 dot and the 4 dot?

   b. the $^{-}6$ dot and the $^{+}3$ dot?

   c. the $^{-}10$ dot and the $^{+}10$ dot?

6. John has a score of $^{-}8$ points ("8 points in the hole") in a game. How many points would he need to earn to get to a score of $^{+}5$ points ("5 points out of the hole")? We can think of this in this way: "What integer must be added to $^{-}8$ to get a sum of $^{+}5$?"

$$^{-}8 + n = {}^{+}5$$

$$n = {}^{+}13$$

   He would need to earn $^{+}13$ points.

   a. How many points would he need to earn to get to a score of $^{+}8$?

   b. of $^{+}10$?

   c. of $^{+}2$?

   d. of 0?

7. The lowest temperature ever recorded in the United States was 70° below zero at Roger's Pass, Montana. The highest temperature ever recorded in the United States was 134° at Death Valley, California. How many degrees higher was the temperature recorded at Death Valley than the temperature recorded at Roger's Pass?

8. Mt. Everest is 29,028 feet above sea level. The Dead Sea is 1,280 feet below sea level. How much higher is Mt. Everest than the Dead Sea?

9. Complete the addition chart below. Add to each integer given in the left column the integer given in the top row. Use arrow diagrams if you need them.

ADDITION CHART

Addend

Addend +	$^-4$	$^-3$	$^-2$	$^-1$	0	$^+1$	$^+2$	$^+3$	$^+4$
$^+4$									
$^+3$				$^+2$					
$^+2$									
$^+1$			$^-1$						
0									
$^-1$									
$^-2$							0		
$^-3$									
$^-4$		$^-7$							

10. Examine the chart and list as many relationships as you can.

11. Complete the subtraction chart by filling in the unknown addends. Subtract from each integer given in the left column (the sum column), the integer given in the top row (the known addend row).

There are three ways to find integers to complete this chart. Suppose you are subtracting $^-3$ from $^-5$. You could write $^-5 - {^-3} = n$. You can write this as an addition sentence $^-5 = n + {^-3}$.

A. Then use the addition chart. $^-5$ is the sum and $^-3$ is one addend. Find $^-3$ in the left column of the chart. $^-5$ is in the row to the right of $^-3$. The integer in the top row in this column is $^-2$. So $^-2$ is the number that is represented by n. $^-2$ belongs in the subtraction chart in the row to the right of $^-5$ and in the column headed by $^-3$.

B. You can use the counting method to find an unknown addend. You would count from the known addend ($^-3$) to the sum ($^-5$). This count would be to the left for 2 spaces, so n is $^-2$.

C. You can draw an arrow diagram to find the unknown addend. You would draw the arrow for the known addend ($^-3$) and for the sum ($^-5$). The arrow for the unknown addend would start at $^-3$ and have its head at $^-5$. The arrow would be labeled $^-2$.

SUBTRACTION CHART

Known Addend

Sum	$^-4$	$^-3$	$^-2$	$^-1$	0	$^+1$	$^+2$	$^+3$	$^+4$
$^+4$			$^+6$						
$^+3$									
$^+2$								$^-1$	
$^+1$									
0									
$^-1$				0					
$^-2$									
$^-3$							$^-5$		
$^-4$									

12. Examine the chart and list some relationships you noticed as you made the chart.

> When two whole numbers are added, the order of the addends may be changed without changing the sum.

13. What property of addition is stated above?

14. Select at least five pairs of whole numbers. Add them to illustrate this property.

15. Select some pairs of integers and add them. Decide if this property also applies to the addition of integers.

16. If this property of addition of whole numbers also applies to integers, write the statement of the property on your paper.

17. Fill in the chart below:

	Sum	Addend	Addend	Operation
1.	( )	$^{+}2$	$^{-}5$	________
2.	( )	$^{-}3$	$^{+}6$	________
3.	$^{+}7$	$^{+}9$	( )	________
4.	( )	$^{-}6$	$^{+}2$	________
5.	( )	$^{+}4$	$^{-}10$	________
6.	$^{-}7$	$^{+}1$	( )	________
7.	( )	$^{+}8$	$^{-}8$	________
8.	( )	$^{-}9$	$^{-}7$	________
9.	( )	$^{+}7$	$^{-}12$	________
10.	$^{+}3$	( )	$^{-}6$	________
11.	( )	$^{+}12$	$^{+}3$	________
12.	$^{-}11$	( )	$^{-}4$	________
13.	( )	$^{+}24$	$^{+}12$	________
14.	$^{+}18$	$^{-}2$	( )	________
15.	( )	$^{-}4$	$^{+}4$	________

Chapter 5

COORDINATES

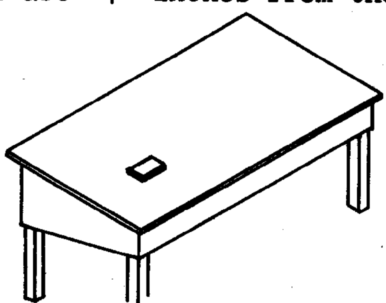
DESCRIBING LOCATIONS

Exploration

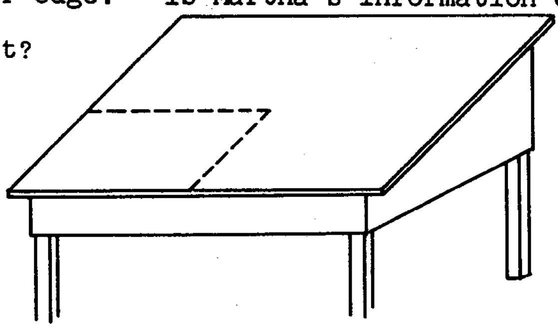
1. How many points are there in a plane? Is each point different from every other point? How can we find a way to identify a particular point?

2. Think of the top of your desk as a part of a plane. Place a small object to represent one point on your desk top. How could you describe its location?

3. Ted said, "The point is 7 inches from the lower left-hand corner of my desk." Does this give you enough information? How many points are 7 inches from the corner of the desk?



4. Martha said, "I didn't measure from the corner. My point is 7 inches from the left-hand edge and 8 inches from the lower edge." Is Martha's information enough to locate the point?



5. Jane said, "I can use Ted's information and some more information to describe a point. My point is 7 inches from the lower left-hand corner and 19 inches from the lower right-hand corner." Does this give you enough information?

How many points are 7 inches from the lower left-hand corner? How many points are 19 inches from the lower right-hand corner? How many points of the desk top are at the correct distance from both corners?

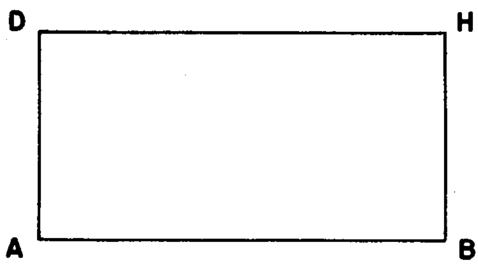
6. Joe said, "I can use Ted's information and some different information. My point is 7 inches from the lower left-hand corner. It is on a ray that makes a 40° angle with the ray from the corner on the lower edge of my desk." How many such rays are there? Does Joe's method work?

7. Are there at least three ways of describing the location of a point on the top of the desk? Can you think of others?

Exercise Set 1

Suppose a rectangular region 6 inches long and 4 inches wide represents a picture and a point C is a particular point of the picture.

1. Use your ruler and protractor to draw a rectangular region to represent the picture. Label it as shown below. $\overline{DH}$ is 6 inches in length. $\overline{AD}$ is 4 inches in length.



2. Suppose C is 5 inches from A and $3\frac{5}{8}$ inches from B.
   a) Use your compass to locate C. Is C exactly one point of the picture?
   b) What property of triangles is illustrated?
   c) What information was used to locate C? What fixed points were used? How far apart are these fixed points?

3. a) Make another copy of the rectangular region.

   Draw $\overrightarrow{AC}$ so that the union of $\overrightarrow{AC}$ and $\overrightarrow{AB}$ is an angle of 37°.

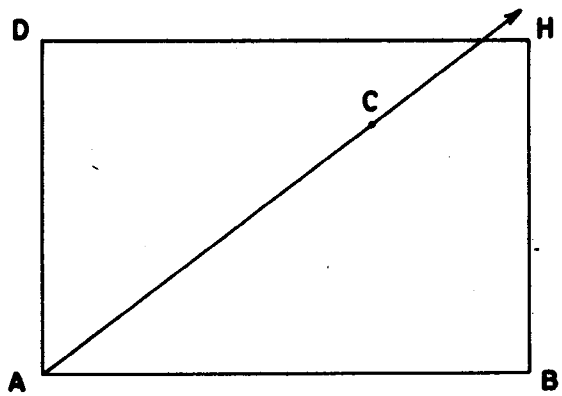
   b) On $\overrightarrow{AC}$ locate point C to make $\overline{AC}$ 5 inches long.

   c) Is C exactly one point of the picture?

   d) What information was used to locate C?

   e) What fixed point and line were used?

   Your drawing should look about like this:



4. Copy the rectangular region again.

   a) Locate a point E on $\overline{AB}$ so that $\overline{AE}$ is 4 inches long.

   b) Draw a ray. Put its endpoint at E. Make it so that it and $\overrightarrow{EA}$ form a right angle.

   c) Locate a point F on $\overline{AD}$ so that $\overline{AF}$ is 3 inches long.

   d) Draw in the rectangular region a ray with endpoint F perpendicular to $\overrightarrow{FA}$.

   e) Does the intersection of the rays you have drawn locate exactly one point C?

f) What information was used to locate C?

g) What fixed lines were used?

5. Look at Exercise 2 in this Exercise Set. Is the method used in it the same as the method used by Jane in Exercise 4 of the Exploration?

6. Look at Exercises 3 and 4 in this Exercise Set.

   a) Which of these exercises uses the same method that Joe used in Exercise 6 of the Exploration?

   b) Which of these exercises uses the same method that Martha used in Exercise 4 of the Exploration?

COORDINATES ON A LINE

Exploration

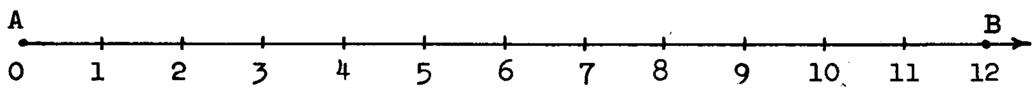
The methods you have considered for locating a point in a plane all involved using:

a) at least one fixed point and at least one line from which measurements were made; and

b) at least two measurements of segments or angles.

1. Think of a situation in which you are given a fixed point A on $\overleftrightarrow{AB}$. Can you describe the position of another point C by just one measurement? Where must C lie if this is possible?

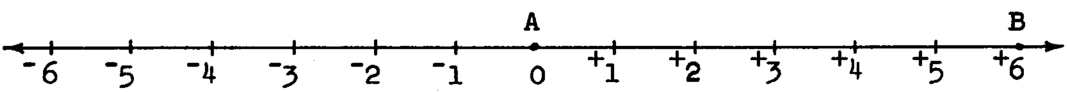
2. Look at the number ray below.



If you know the distance of C from A is 6 units and that C is on $\overrightarrow{AB}$, do you know exactly where C is?

3. Now look at the number line below.

a) What kind of numbers are shown on this number line?



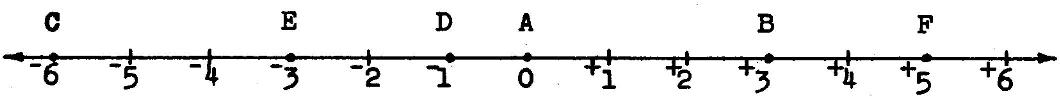
b) If you know the distance of a point C from A, and that C is on $\overleftrightarrow{AB}$, how many different points could be named C?

c) What must you know beside the distance in order to locate exactly one point C ?

4. What kind of numbers tell both the direction and distance of a point from A?

A number that tells both distance and direction of a point on a line from the 0-point is called the <u>coordinate</u> of the point.

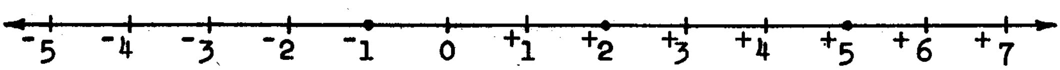
5. On the number line below, what is the coordinate of B? of C? of D?



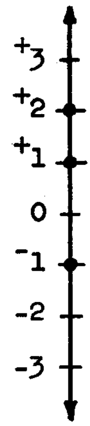
6. What point has the coordinate $^{-}3$? What point has coordinate $^{+}5$?

When you mark on a number line the points which have a certain set of numbers as coordinates we say you are drawing a <u>graph</u> of the set of numbers.

Below is shown the graph of a set of integers. The three heavy dots are the graph of the set $\{^{-}1, ^{+}2, ^{+}5\}$. Is this a different kind of graph from those you have seen before?

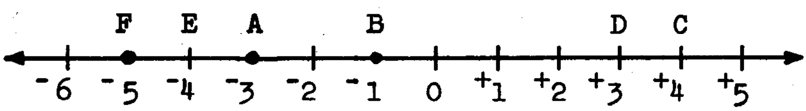


7. Sometimes vertical (up and down) lines are used. What set of integers is graphed on this line?



8. Draw a vertical number line and graph this set of integers: $\{^-4, {}^+4, {}^-2, {}^+2, 0\}$.

9. What is the coordinate of the point half-way between A and B on the number line below? Is the coordinate an integer?



10. What number should be the coordinate of a point half-way between C and D? half-way between E and F? Are these coordinates integers?

Many points have as coordinates numbers which are not integers. In this unit, however, we shall use points whose coordinates are integers.

Exercise Set 2

Draw number lines showing the integers from $^{-}10$ to $^{+}10$ and graph these sets. Use some horizontal and some vertical number lines.

1. $\{^{-}9,\ ^{-}6,\ ^{-}1,\ 0,\ ^{+}8\}$

2. {The positive integers less than $^{+}6$}

3. {The negative integers greater than $^{-}4$}

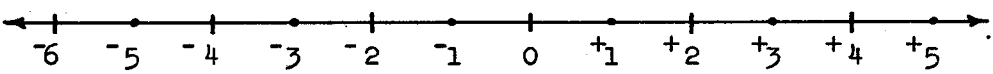
4. {The integers less than $^{-}3$ and greater than $^{-}11$}

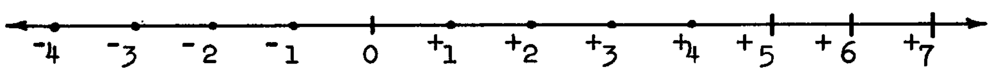
5. {The positive integers less than $^{+}11$ which are divisible by 5, and their opposites}

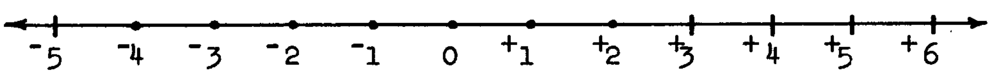
6. {The integers less than $^{+}2$ and greater than $^{-}11$}

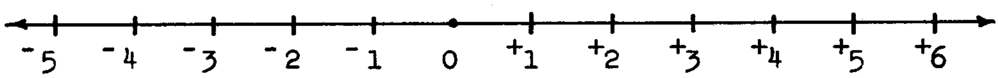
7. {The integer which is 3 greater than $^{-}2$ and the integer which is 3 less than $^{-}2$}

List or describe the sets of integers whose graphs are shown below.

8. 

9. 

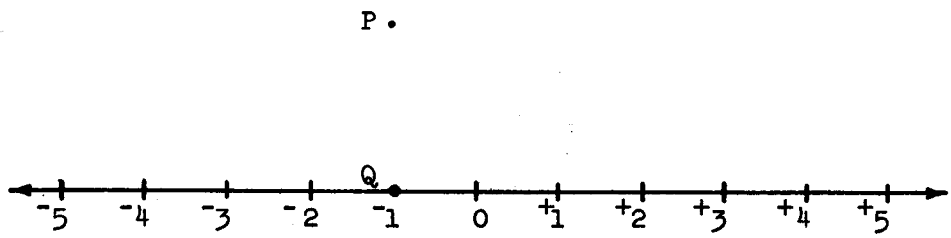
10. 

11. 

COORDINATES IN A PLANE

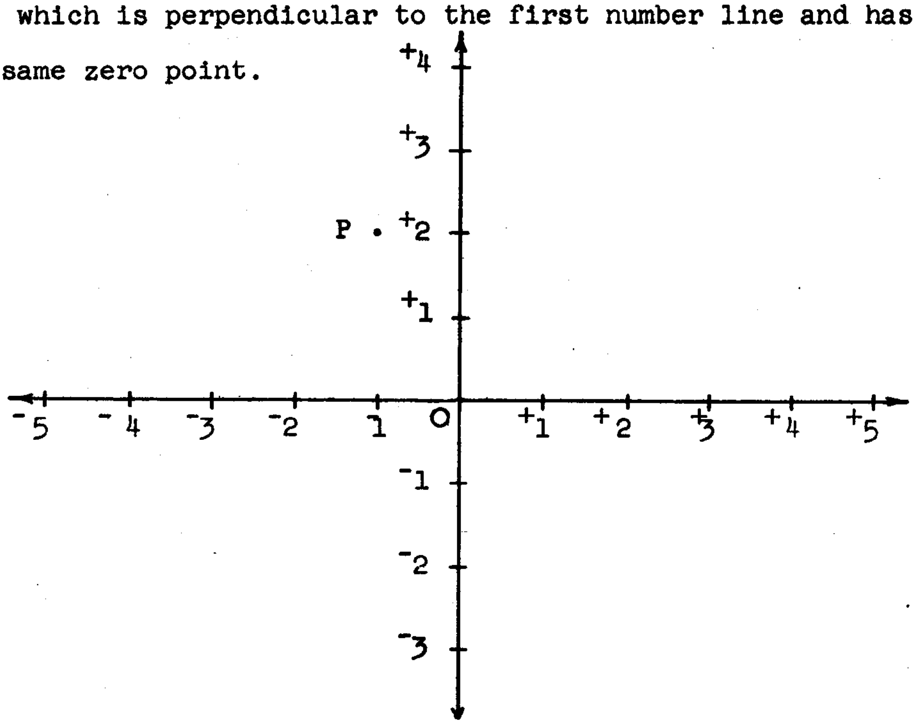
Exploration

Look at the number line and the points P and Q below.



Since Q is on the number line we can state its position by naming its coordinate, $^-1$. On the other hand, since P is not on this number line, we cannot state its position by naming its coordinate. It seems to be directly above $^-1$ on the line, but we need a way to tell how far above the line it is.

We can find a way to do this by using a second number line which is perpendicular to the first number line and has the same zero point.



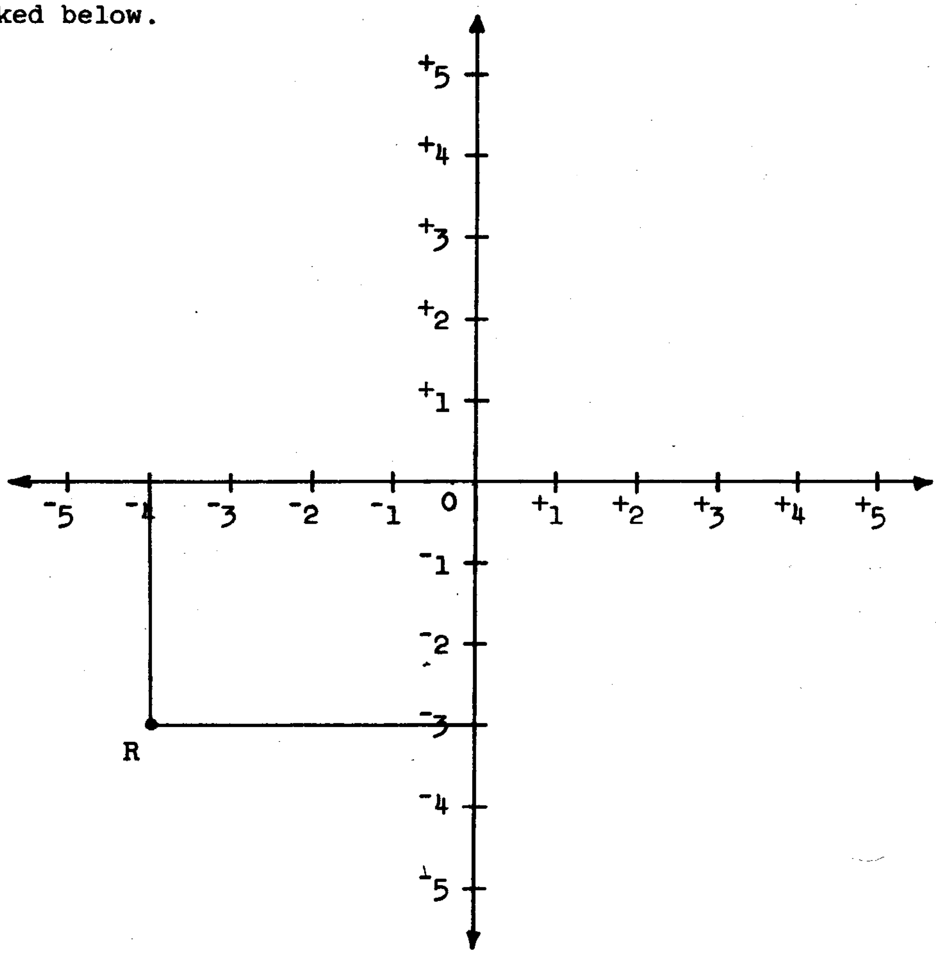
We see now that P is directly to the left of $^+2$ on the vertical number line. We can describe its position by using the two numbers, $^-1$ and $^+2$. Is there any point except P which is exactly above $^-1$ on the horizontal number line and also exactly to the left of $^+2$ on the vertical number line?

What is meant by "exactly above" and "exactly to the left of" in the last two sentences?

The position of P can thus be described not by one number, but by the pair of numbers, $\{^-1, ^+2\}$. The numbers $^-1$ and $^+2$ are called coordinates of the point P.

The first number tells the number exactly below it (in this example) on the horizontal number line. The second number tells the number exactly to the right (in this example) of P on the vertical number line. The order in which the numbers are named is important, so $\{^-1, ^+2\}$ is called an ordered pair.

Let us think more precisely about finding the numbers which describe the position of a point. Look at point R marked below.

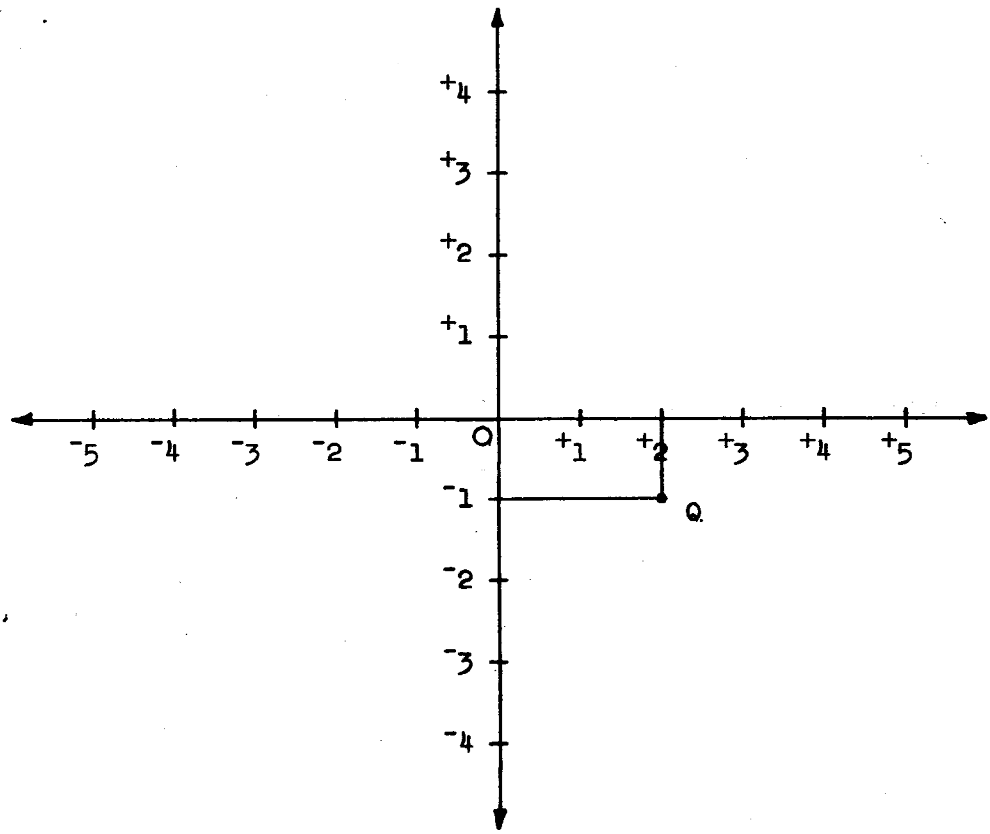


A line segment from R is drawn perpendicular to the horizontal number line. It intersects the number line at $^{-}4$.

A line segment from R is drawn perpendicular to the vertical number line. It intersects the number line at $^{-}3$. The location of R is described by the ordered number pair $(^{-}4, ^{-}3)$.

We can use perpendicular lines to find the position of a point.

Consider the point which is described by the ordered pair $(^{+}2, {}^{-}1)$. Look at the drawing below.



Since the first number is $^{+}2$, find the point for $^{+}2$ on the horizontal number line. A line segment perpendicular to the number line is drawn at this point.

Find the point $^{-}1$ on the vertical number line.

A line segment perpendicular to the number line is drawn at this point.

The two perpendicular line segments intersect at the point labeled Q. Q is the point whose position is described by the ordered pair $(^{+}2, {}^{-}1)$.

Its first coordinate is ___?___.

Its second coordinate is ___?___.

Do the ordered pairs $(^{+}2, {}^{-}1)$ and $(^{-}1, {}^{+}2)$ describe two different points in the plane?

Briefly, we can think:

To locate the point $(^{-}4, {}^{-}3)$, start at $(0, 0)$ count 4 units to the left and then 3 units down.

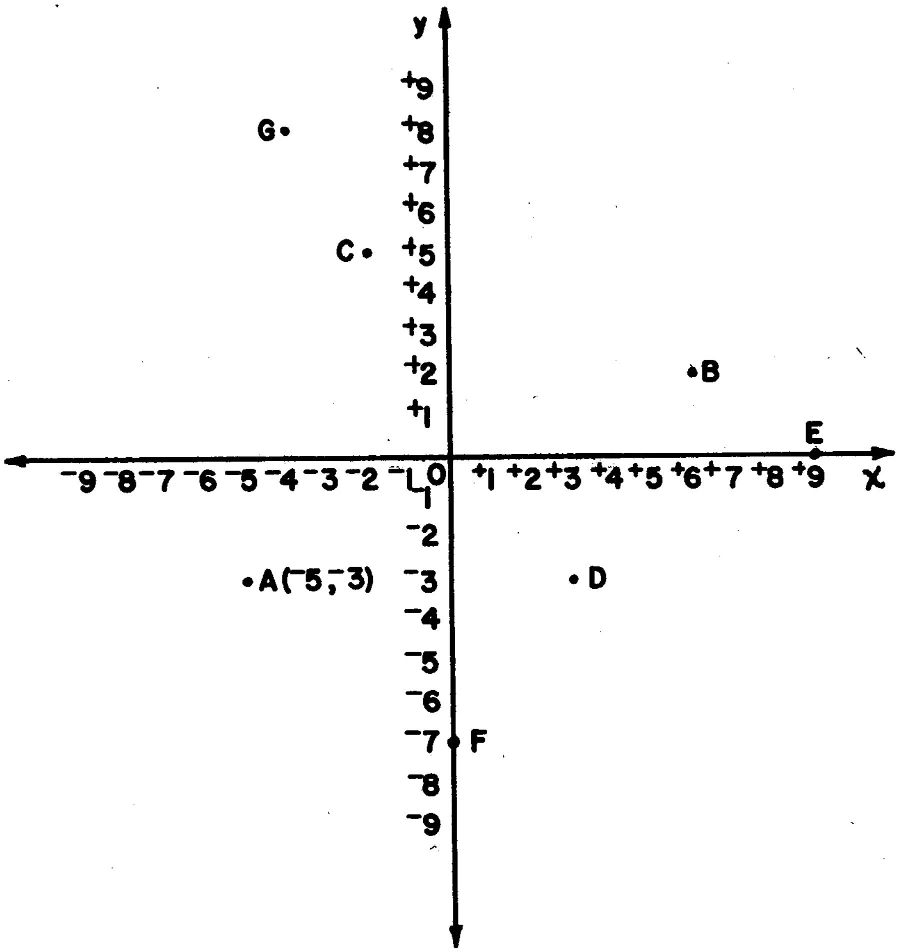
To locate $(^{-}1, {}^{+}2)$, count 1 unit to the ___?___ and then 2 units ___?___ .

To locate $(^{+}2, {}^{-}1)$, count ___?___ units to the ___?___ and then ___?___ unit ___?___ .

To assist in describing accurately the position of points it is customary to use graph paper. On graph paper sets of perpendicular lines are printed forming segments of equal length. Any pair of perpendicular lines may be chosen as the number lines.

The ordered pair ($^-5$, $^-3$) is graphed below and the point it identifies is labeled A. Notice the ordered pair is written in parentheses beside the point.

1. Write the ordered pairs which are the coordinates of points B, C, D, E, F, and G. Write your answer like this: A($^-5$, $^-3$)



2. Use a sheet of graph paper with lines one-half inch apart. Choose two perpendicular lines for number lines and draw heavy lines on them to show the lines you have chosen. Label the number lines from $^{-}6$ to $^{+}6$.

Graph these ordered pairs. Label each with its letter and its coordinates.

$H(^{+}5, {}^{-}4)$

$J(^{-}6, {}^{-}3)$

$K(0, {}^{+}6)$

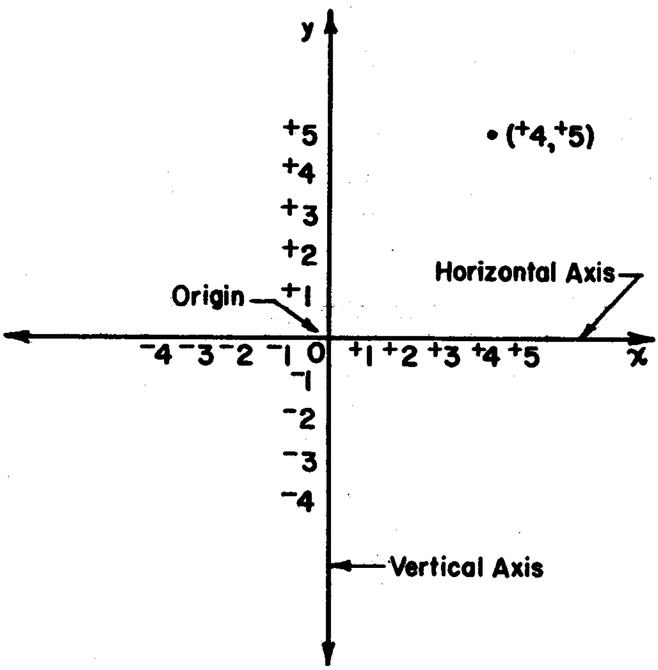
$M(^{-}2, 0)$

$R(^{-}2, {}^{+}5)$

$S(^{+}4, {}^{+}3)$

When number lines are used in this way, we call each number line an <u>axis</u>. The horizontal number line is called the <u>x-axis</u> and the vertical number line is called the <u>y-axis</u>. The point of intersection of the x-axis and the y-axis is called the <u>origin</u>.

When you draw graphs you should always label the axes ("axes" is the plural of axis) in a certain way as shown below.



Write "x" and "y" near the arrows on the rays of the axes which show the positive integers.

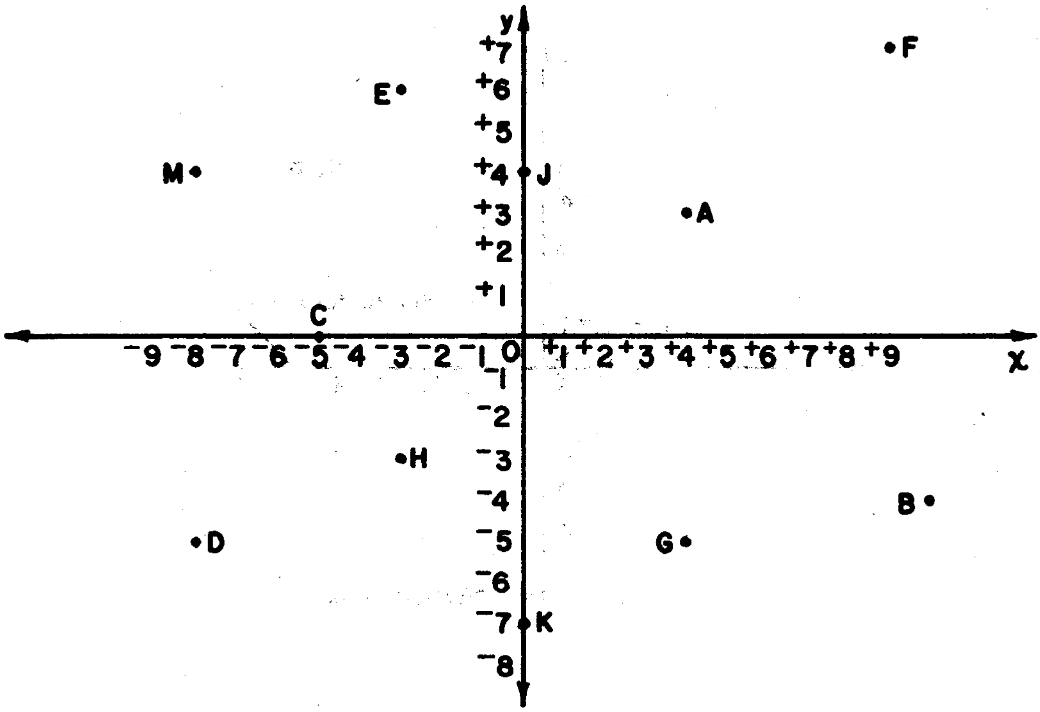
Label the 0-point and several points on each axis.

When you graph an ordered pair, label the point with the ordered pair.

Label the x-axis and the y-axis on your graph paper for Exercise 2.

Exercise Set 3

1. Write as ordered pairs the coordinates of the labeled points.



2. Graph the following ordered pairs. Use graph paper and label the x-axis and the y-axis. Label each point.

A($^-7$, $^+3$)	D(0, $^+10$)
B($^+4$, $^+9$)	E($^-5$, $^-8$)
C($^+6$, $^-5$)	F($^+5$, 0)

3. Can you write an ordered pair of numbers to tell the location of your town on the earth's surface? What would you use for number lines?

Exploration

(X-Coordinates and Y-Coordinates)

You have been using two perpendicular number lines with the same zero point. We called these number lines the x-axis and the y-axis. These lines help you identify the point in a plane which is the graph of an ordered pair of numbers.

We say "ordered pair" because the order in which the two numbers are named is important. The point named by the pair $(^-3, ^+6)$ is a different point from the one named by the pair $(^+6, ^-3)$.

The first number in an ordered pair, which tells how far the point is to the right or left of the y-axis, is called the <u>x-coordinate</u>. The number which tells the distance of the point above or below the x-axis is called the <u>y-coordinate</u>.

1. a) Name the x-coordinate of each point graphed in Exercise 1 of Exercise Set 3.

   b) Name the y-coordinate of each of these points.

2. Write as an ordered pair the coordinates of these points:

A: x-coordinate is $^{+}3$, y-coordinate is $^{-}7$.

B: y-coordinate is $^{+}10$, x-coordinate is $^{-}3$.

C: x-coordinate is 0, y-coordinate is $^{+}4$.

D: x-coordinate is $^{-}5$, y-coordinate is 0.

E: x-coordinate is $^{+}8$, y-coordinate is $^{+}1$.

F: y-coordinate is $^{-}6$, x-coordinate is $^{+}4$.

G: x-coordinate is $^{-}7$, y-coordinate is $^{+}3$.

3. Graph the points whose x- and y-coordinates are given in Exercise 2. Label each with its letter name and its ordered pair.

Exercise Set 4

1. a) Graph the points whose coordinates are given below. Label each point with its letter name and its ordered pair.

   A: $(^{-}1,\ ^{+}7)$

   B: $(^{-}4,\ 0)$

   C: $(^{+}2,\ 0)$

   b) Draw $\overline{AB}$ and $\overline{AC}$. The union of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ is a triangle. What kind of triangle is it?

   c) The base of ____________ triangle ABC is on the ____axis.

2. a) Graph these points. Label each point with its letter name and its ordered pair.

   D: x-coordinate $^{-}3$, y-coordinate $^{+}3$

   E: x-coordinate $^{-}1$, y-coordinate $^{-}3$

   F: x-coordinate $^{+}5$, y-coordinate $^{-}1$

   G: x-coordinate $^{+}3$, y-coordinate $^{+}5$

   b) Draw $\overline{DE}$, $\overline{EF}$, $\overline{FG}$, $\overline{GD}$. Which of these names describe the figure DEFG?

1) Simple closed curve	5) Square
2) Polygon	6) Isosceles triangle
3) Quadrilateral	7) Rectangle
4) Square region	8) Union of four angles

   c) Draw $\overline{DF}$ and $\overline{EG}$. What are the coordinates of their intersection?

3. a) Write three different ordered pairs in which the second number is 0.

   b) Graph these ordered pairs.

   c) Where do the three points lie?

   d) Write three different ordered pairs in which the first number is 0.

   e) Graph these ordered pairs using the same axes you used for b.

   f) Where do these points lie?

4. a) Any point whose x-coordinate is 0 lies on the ___?___.

   b) Any point whose y-coordinate is 0 lies on the ___?___.

5. a) What are the coordinates of the intersection of the x-axis and the y-axis?

   b) What special name is given to the point of intersection of the x-axis and y-axis?

USING COORDINATES TO FIND MEASURES OF SEGMENTS

Exploration

1. a) Write 5 ordered pairs which have the same x-coordinate (do not use 0) and different y-coordinates.

   b) Graph the points for the ordered pairs.

   c) Are all five points on the same line?

2. a) Write 5 ordered pairs which have different x-coordinates and the same y-coordinates (do not use 0).

   b) Graph the points for the ordered pairs.

   c) Are all five points on the same line?

3. What do you notice about the lines suggested in 1 c) and 2 c)?

4. a) Graph the points $R(^{-}2, ^{+}7)$ and $S(^{-}2, ^{+}3)$

   b) What is the measure of $\overline{RS}$ in unit segments?

   c) Could you find the measure of $\overline{RS}$, without counting unit segments, by using the y-coordinates?

5. a) Graph the points $A(^{-}3, ^{-}5)$ and $B(^{+}4, ^{-}5)$

   b) Subtract $^{-}3$ from $^{+}4$. $^{+}4 - {}^{-}3 = ?$

   c) Subtract $^{+}4$ from $^{-}3$. $^{-}3 - {}^{+}4 = ?$

   d) Does your answer to either b) or c) tell you the length of $\overline{AB}$?

6. a) Consider the points $C(^{+}105, ^{+}58)$ and $D(^{+}105, ^{+}69)$. Without graphing C and D, can you find the length of $\overline{CD}$?

   b) What is the length of $\overline{RS}$ if R has coordinates $(^{-}3, ^{-}579)$ and S has coordinates $(^{-}3, ^{-}468)$?

7. From your observations in Exercises 1-6, complete this sentence:

   To find an integer which tells the measure, in units, of the segment between two points:

   a) if the x-coordinates are the same ______ one ___ coordinate from the other.

   b) if the y-coordinates are the same ______ one ___ coordinate from the other.

Exercise Set 5

1. Here are names of points and the coordinates of each:

$$A(^{+}5,\ ^{-}7)$$
$$B(^{-}2,\ ^{+}3)$$
$$C(^{-}2,\ ^{-}7)$$
$$D(^{+}2,\ ^{-}7)$$
$$E(^{+}5,\ ^{+}3)$$
$$F(^{-}8,\ ^{-}1)$$

   a) List the pairs of points with the same x-coordinate. Then find the length of the segment joining each pair.

   b) Find the pairs of points with the same y-coordinate. Then find the length of the segment joining each pair.

   c) Check your answers by graphing the ordered pairs and counting unit segments.

2. a) Graph this set of ordered pairs of integers and label the points of the graph. $\{A(^{+}2, ^{+}9), B(^{+}2, ^{-}2), C(^{+}7, ^{-}2), D(^{+}7, ^{+}9)\}$

   b) Draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$.
   What kind of quadrilateral is ABCD?

   c) Find the lengths of $\overline{AB}$ and $\overline{BC}$. (Don't count, subtract coordinates.)

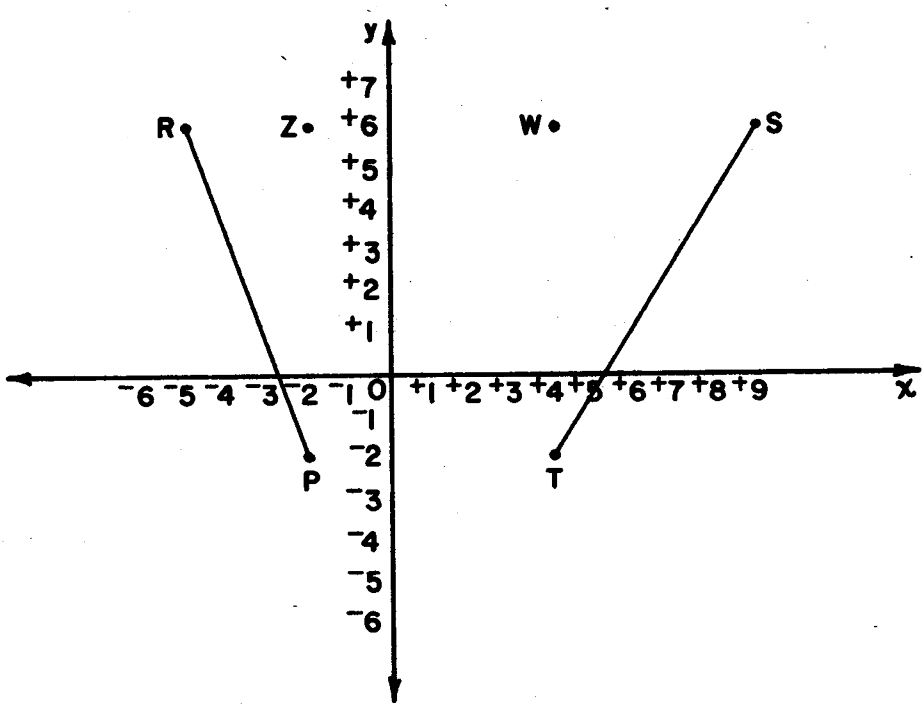
   d) What is the area of region ABCD?

3. a) Graph these points $E(^{+}5, ^{+}8)$, $F(^{-}1, ^{+}8)$, $G(^{-}1, ^{-}4)$.

   b) Draw $\overline{EF}$, $\overline{FG}$, $\overline{EG}$. What segments are the base and height of $\triangle EFG$?

   c) Find the area of $\triangle EFG$.

4. a) Write on your paper the coordinates of each labeled point in the figure below.



b) Find: 1) a set of four labeled points with the same y-coordinate.

2) a set of two labeled points with the same y-coordinate.

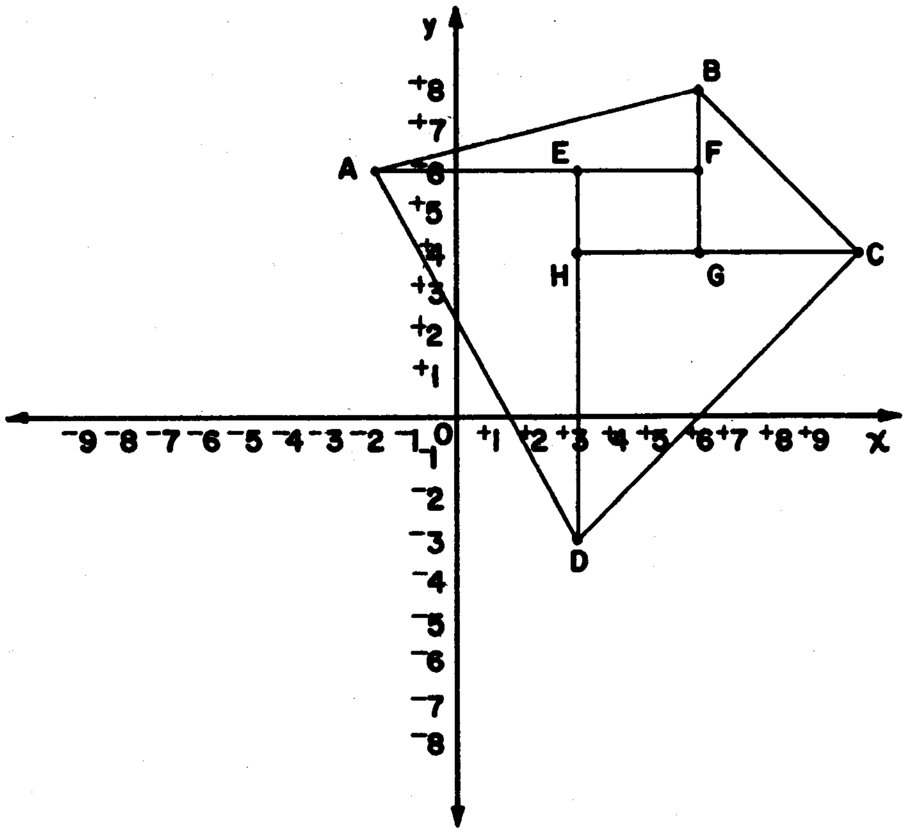
3) two sets of two labeled points with the same x-coordinate.

c) Find the lengths of these line segments: $\overline{RZ}$, $\overline{ZW}$, $\overline{WS}$, $\overline{ZP}$, $\overline{WT}$, $\overline{PT}$.

d) Name two triangles and a rectangle in the figure, and find the area of each region.

e) What is the area of the region bounded by quadrilateral RSTP?

5. a) Write the coordinates of each labeled point in the figure.



b) Figure ABCD is a ______?______.

c) What set of three points have the same x-coordinate? Can you find another set?

d) What three labeled points have the same y-coordinate? Can you find another set?

e) Find the lengths of base and altitude of each right triangle with labeled vertices. (There are four.)

f) Find the area of each triangular region and the area of the rectangular region.

g) Find the area of the region ABCD.

6. Battleship Game 1: Here is a game you might find interesting. It requires two people (or two "sides") to play it. Here are the rules. They are stated for two players. If there are two "sides" with several players on each side then the "sides" play alternately with each player on a side playing in turn. The game can also be played on a piece of paper, rather than on a chalkboard.

a) Draw (on the chalkboard) x- and y-axes and mark each axis with numerals from $^{-}10$ to $^{+}10$.

b) One player marks (with white chalk) 10 points (battleships) each having coordinates which are integers. Do not label the points with their coordinates.

c) The opponent player marks (with colored chalk) 10 new points (battleships) for his side, each point having coordinates which are integers. Do not label the points with their coordinates.

d) The first player calls out an ordered pair of integers. If there is a point (battleship) marked with these coordinates then the marking is erased (battleship sunk). (Sometimes a player makes a mistake and sinks one of his own battleships.) If there is no point marked with these coordinates then the opponent has his turn.

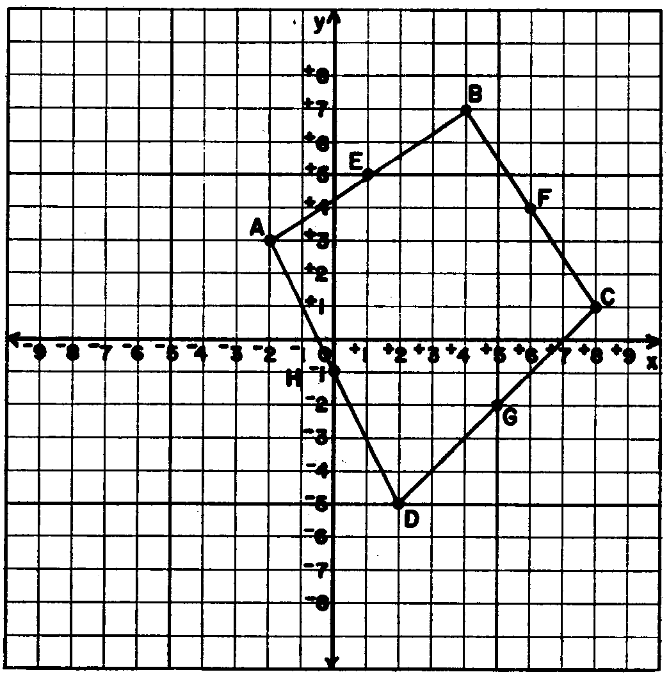
e) The second player now has his turn to carry out Rule d.

f) Players now play alternately.

g) The player whose battleships are all sunk first (after each player has had the same number of turns) loses the game. If all battleships of both sides are sunk, then the game is a tie.

BRAINTWISTER

1. a) Write the coordinates of each labeled point in the figure:



b) Figure ABCD is a ___________.

c) Draw $\overline{EF}$, $\overline{FG}$, $\overline{GH}$, and $\overline{HE}$.

d) Figure EFGH is a ___________.

e) On the graph mark:

(1) A point J whose x-coordinate is the same as the x-coordinate of point A and whose y-coordinate is the same as the y-coordinate of point B.

(2) A point K whose x-coordinate is the same as the x-coordinate of the point C and whose y-coordinate is the same as the y-coordinate of point B.

(3) A point L whose x-coordinate is the same as the x-coordinate of point C and whose y-coordinate is the same as the y-coordinate of point D.

(4) A point M whose x-coordinate is the same as the x-coordinate of point A and whose y-coordinate is the same as the y-coordinate of point D.

f) Figure JKLM is a ___________.

g) Compute the area of the region JKLM.

h) Compute the lengths of the base and altitude of each of the right triangles, ΔAJB, ΔBKC, ΔCLD, and ΔDMA.

i) Compute the areas of each of the triangular regions whose sides are the triangles of Exercise h .

j) What is the area of the polygon region ABCD?

CHANGING COORDINATES

Exploration

1. Suppose that P is the point $({}^{+}1, {}^{+}2)$ and R is the point $({}^{+}3, {}^{+}6)$.

   How many segments are there with P and R as endpoints?

2. Graph the ordered pairs P and R in Exercise 1 and draw $\overline{PR}$.

3. Is $({}^{+}2, {}^{+}4)$ on $\overline{PR}$? If so, mark and label it S.

4. Graph the ordered pair $T({}^{+}5, {}^{+}2)$ and draw $\overline{RT}$.

5. Is the point $({}^{+}4, {}^{+}4)$ on $\overline{RT}$? Mark and label it W.

6. Draw $\overline{SW}$.

7. What letter does the figure look like?

8. The set of ordered pairs you have graphed is:

   $\{P({}^{+}1, {}^{+}2), S({}^{+}2, {}^{+}4), R({}^{+}3, {}^{+}6), W({}^{+}4, {}^{+}4), \text{ and } T({}^{+}5, {}^{+}2)\}$.

   Form a new set of ordered pairs by changing the coordinates of these pairs as follows: Add ${}^{+}7$ to each x-coordinate and, add 0 to each y-coordinate.

   Name the corresponding new points, A, B, C, D, and E.

9. a) Graph and label the set of ordered pairs you found in Exercise 8 and draw the segments AC, CE, and BD.

   b) What does this new figure look like?

   c) How is the new figure related to the old figure?

Exercise Set 6

1. a) Graph and label this set of points:

   $\{A(^+5, ^-1), B(^+5, ^+5), C(^-4, ^+5), D(^-4, ^-1)\}$

   b) Draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$.

   c) What kind of polygon is ABCD?

2. a) Form a new set of ordered pairs by subtracting 0 from each x-coordinate and subtracting $^+8$ from each y-coordinate of the set of pairs of 1a.

   b) Plot this set of points. (Call the corresponding points E, F, G, H.)

   c) Draw $\overline{EF}$, $\overline{FG}$, $\overline{GH}$, $\overline{HE}$.

   d) Is EFGH congruent to ABCD?

   e) How is the new figure we got by changing the y-coordinates related to the old figure?

3. a) Graph this set of ordered pairs. Label the points.

   $\{A(^-3, ^+7), B(^+1, ^+9), C(^+1, ^+4), D(^+3, 0)\}$

   b) Draw $\overline{AC}$, $\overline{BC}$, $\overline{CD}$. What letter does the union of these segments look like?

4. a) Form a new set of ordered pairs {E, F, G, H} from the old set of Exercise 3a as follows: Add $^+3$ to the x-coordinate and subtract $^+6$ from the y-coordinate.

   b) Graph this set of ordered pairs. Draw $\overline{EG}$, $\overline{FG}$ and $\overline{GH}$.

   c) How does the shape and position of this figure compare with the one for Exercise 3?

5. Suppose you wanted to "move" the figure of Exercise 3 five units to the left. What change would you make in the coordinates?

6. What change in the coordinates would "move" the figure three units up?

7. Battleship Game 2: The rules of this game are the same as those for Battleship Game 1 (see end of Exercise Set 5) except that Rule d is replaced by:

   d*) (1) The first player calls out something like "2 units exactly to left of $(^+1, ^+3)$". If there is a point (battleship) marked with the coordinates $(^-1, ^+3)$ then the marking is erased (battleship sunk).

   (2) If the first player said "2 units exactly to the right of $(^+1, ^+3)$" then the marking at $(^+3, ^+3)$ would be erased.

   (3) If the first player said "3 units exactly below $(^+1, ^+3)$," then the marking at $(^+1, 0)$ would be erased.

(4) if the first player said "3 units exactly above ($^+1$, $^+3$)," then the marking at ($^+1$, $^+6$) would be erased.

(5) If there is no point marked at the coordinates described, then the opponent has his turn.

BRAINTWISTER: In the Exploration exercises you graphed ordered pairs and drew segments which formed the letter "A". Draw some other capital letter on a sheet of graph paper. Write on a separate sheet the coordinates of the endpoints of the segments that form the figure. Hand the instructions (but not the graph) to some other student. See if he can follow the instructions to obtain the same letter you have on your graph paper.

GRAPHS OF SPECIAL SETS

Exploration

Suppose we have just the set $\{^+1, ^+2, ^+3, ^+4, ^+5\}$. You wish to write the set of all the ordered pairs of numbers such that both numbers are in this set.

1. Follow a system so you do not omit any pairs.

   First list in the first row all pairs which have $^+1$ as first number. Then list in a second row all which have $^+2$ as first number; and so on.

   Arrange your pairs in a chart as shown below.

$(^+1, ^+1)$	$(^+1, ^+2)$	$(^+1, ?)$	$(^+1, ?)$	$(^+1, ?)$
$(^+2, ^+1)$	$(^+2, ?)$	$(?, ?)$	( )	( )
( )	( )	( )	( )	( )
( )	( )	( )	( )	( )
( )	( )	( )	( )	( )

2) a) Does the chart contain <u>all</u> ordered pairs with both numbers in the set $\{^+1, ^+2, ^+3, ^+4, ^+5\}$?

   b) Where in the chart are all the pairs with $^+1$ as second number? With $^+2$ as second number? With $^+4$ as the first number?

3. a) Find in the chart the ordered pairs in which the x-coordinate and y-coordinate are the same. List the pairs.

   b) Graph these ordered pairs.

4. a) Find in the chart the ordered pairs in which the second number is greater than the first number. List them.

   b) Graph these ordered pairs, using a red crayon. Use the same axes you used for Exercise 3b.

5. a) Find in the chart and list the ordered pairs in which the second number is smaller than the first.

   b) Graph these ordered pairs on the same axes you used for Exercise 3b, using a green crayon.

6. a) Find all the points whose y-coordinate is $^{+}5$. Draw in black the line segment through these points.

   b) Do the same for the points whose x-coordinate is $^{+}5$.

Exercise Set 7

Use the numbers in this set: $\{^-3, ^-2, ^-1, 0, ^+1, ^+2, ^+3\}$.

1. Make a chart of all ordered pairs of numbers with both numbers in the set above. How many should there be?

2. List the set of ordered pairs in the chart in which the two coordinates are the same. Graph this set in red.

3. List the set of ordered pairs in which the second number is 2 greater than the first. (There should be five such pairs.)

   Graph this set in green on the axes you used for Exercise 2.

4. List the set of ordered pairs in which the second number is 3 less than the first. (There should be four such pairs.)

   Graph this set in blue on the same axes.

5. List the set of ordered pairs in which the second number is the opposite of the first. (There should be __?__ pairs.)

   Graph this set in black on the same axes.

6. Does the set of red points suggest a line? the green points? the blue points? the black?

BRAINTWISTERS

1. Graph the set of ordered pairs $\{A(^+1, ^+1)\ B(^+5, ^+1)\ C(^+3, ^+5)\}$ Draw $\overline{AB}$, $\overline{BC}$, $\overline{CA}$. Form new ordered pairs by doubling each number. Graph the ordered pairs and call the points D, E, and F. Draw $\overline{DE}$, $\overline{EF}$, $\overline{FD}$. Is $\triangle ABC$ congruent to $\triangle DEF$? Does it have the same shape? Are corresponding angles congruent?

2. Draw some other figure whose vertices have positive integers as coordinates. Find the coordinates of each vertex. Multiply each coordinate by 3 and draw the corresponding new figure. Are the figures the same shape?

3. Write a sentence that tells what you have observed from Exercises 1 and 2.

Exploration

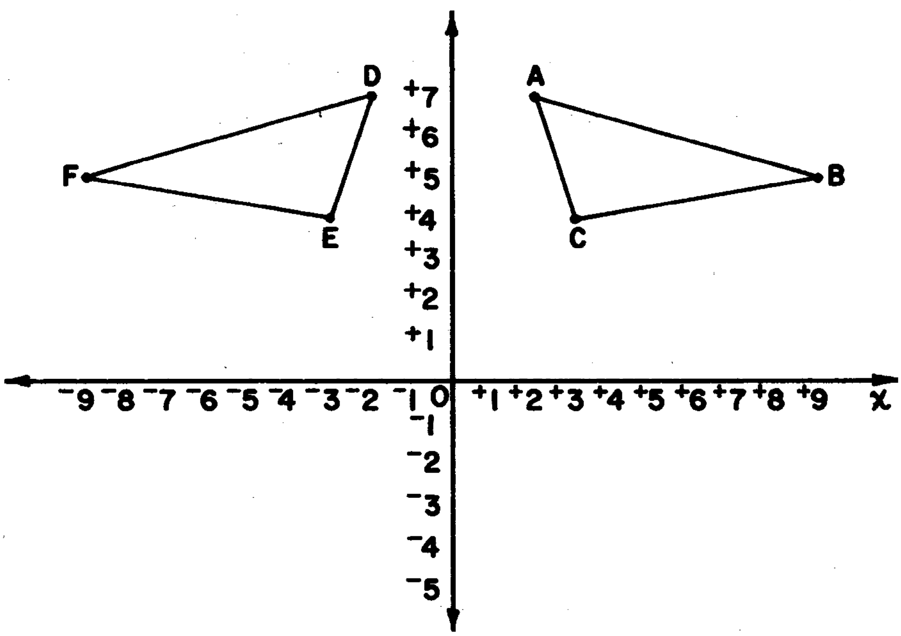
You already know at least one meaning for "reflection." We think of a mirror or a pool of clear water as giving a reflection. Let us see what reflections are in geometry.

1. a) Graph this set of ordered pairs:

$$\{A(^{+}2, ^{+}7), B(^{+}9, ^{+}5), C(^{+}3, ^{+}4)\}$$

Draw segments $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$. The union of these segments is a ___?___.

Does your triangle ABC look like triangle ABC below? This drawing shows also triangle DFE which is a <u>reflection</u> of triangle ABC. Do you see why it is called a reflection?



b) Copy the table below and write the coordinates of points D, E, and F. Fill in the distance of each point from the y-axis also.

Point	Coordinates of point	Distance of point from y-axis
A	$(^+2, ^+7)$	?
D	( )	?
B	$(^+9, ^+5)$	?
F	( )	?
C	$(^+3, ^+4)$	?
E	( )	?

c) What do you observe about the coordinates of points A and D?

d) What do you observe about the distances from A to the y-axis and from D to the y-axis?

e) Are the observations you made for the points A and D similar for B and F? for C and E?

f) Mark and label points D, F, and E on your graph. Draw triangle DFE.

g) Fold your paper along the y-axis and hold your paper up to the light. Does A fall on D? Does B fall on F? C on E?

h) Is $\triangle ABC \cong \triangle DFE$?

Point D is a reflection of point A in the y-axis.

Point F is a reflection of point B in the y-axis.

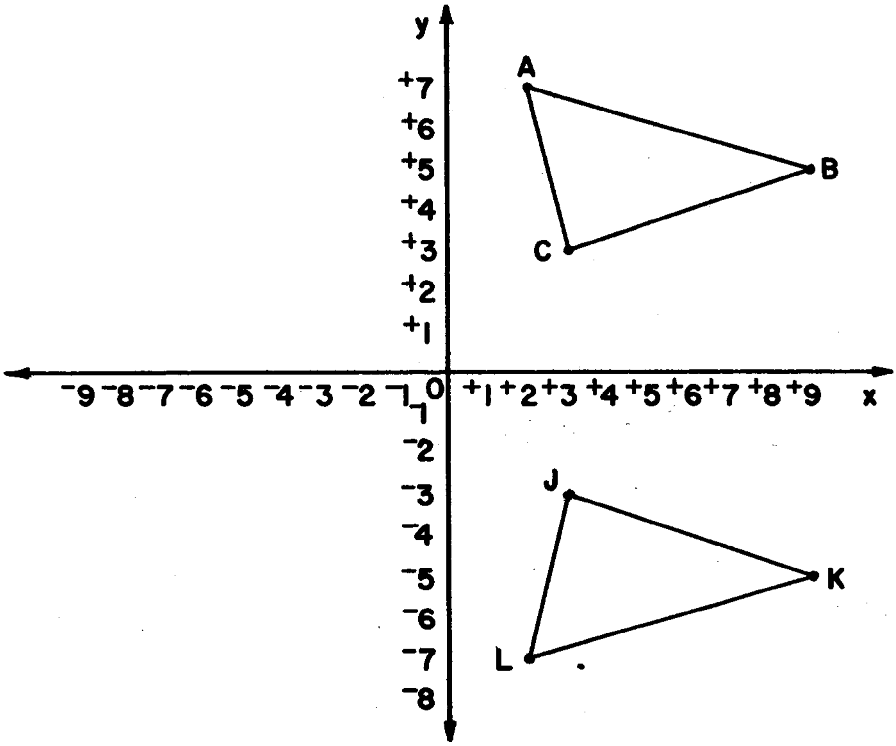
Point E is a reflection of point C in the y-axis.

$\triangle$DFE is a reflection of $\triangle$ABC in the <u>vertical</u> <u>axis</u>.

Note that we get a reflection of a point in the vertical axis when the first coordinate of the point is replaced by its opposite, and second coordinates remain the same.

A figure is a reflection of another in the vertical axis if corresponding points are the same distance from the vertical axis but in opposite directions from it.

2. How can we get a reflection of $\triangle ABC$ in the horizontal axis? In the drawing below, does triangle LKJ look like a reflection of triangle ABC in the horizontal axis?



a) Copy the table below and fill in the missing facts.

Point	Coordinates of point	Distance from x-axis
A	$(^+2, ^+7)$	?
L	( )	?
B	$(^+9, ^+5)$	?
K	( )	?
C	$(^+3, ^+3)$	?
J	( )	?

b) What do you observe about the coordinates of:

(1) A and L ?

(2) B and K ?

(3) C and J ?

c) What do you observe about the distance of:

(1) A and L from the x-axis?

(2) B and K from the x-axis?

(3) C and J from the x-axis?

d) How do these observations compare with those you made from the table in Exercise 1?

e) Along which axis would you fold this drawing so that A, B, and C would fall on the corresponding points L, K, and J?

f) When the second coordinate of each point is replaced by its opposite and the first coordinate remains the same, we get a reflection in the ___?___ axis.

A figure is a reflection of another figure in the horizontal axis if corresponding points are the same distance from the horizontal axis, but in opposite directions from it.

3. a) Graph this set of ordered pairs:

A($^-5$, $^-2$)	D($^-9$, $^-10$)	G($^-7$, $^-6$)
B($^-1$, $^-4$)	E($^-9$, $^-4$)	H($^-7$, $^-10$)
C($^-1$, $^-10$)	F($^-9$, $^-6$)	

Draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EA}$, $\overline{FG}$, $\overline{GH}$.

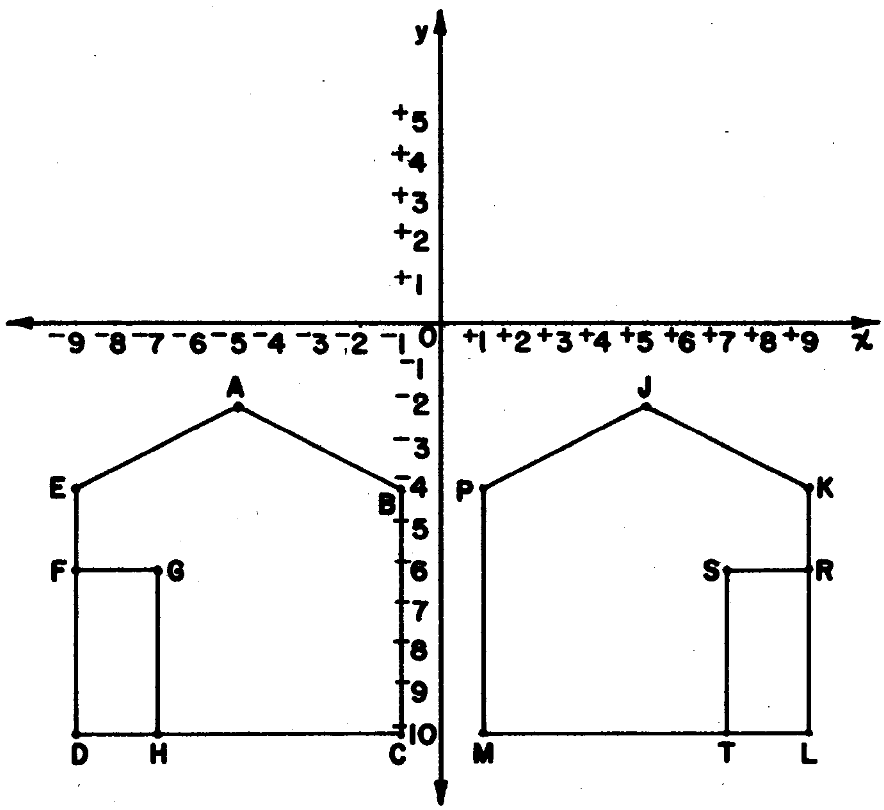
This figure looks like a drawing of a ___?___.

b) Graph the reflection of point A in the y-axis. The reflection of A in the y-axis should have the same ___?___ coordinate as A. Its ___?___ coordinate should be the opposite of A's.

Point A and its reflection in the y-axis should be the same distance from the ___?___ axis.

c) Graph the reflection in the y-axis of the other labeled points. Draw segments to get the reflection of the figure.

d) Do your figures look like the ones in the graph below? Label the points of your reflection figure as shown.



<u>Exercise Set 8</u>

1. a) (1) Graph this set of ordered pairs. Label each point with its letter and coordinates.

$\{A(^+6, ^+4), B(^+8, ^+2), C(^+6, ^+1)\}$

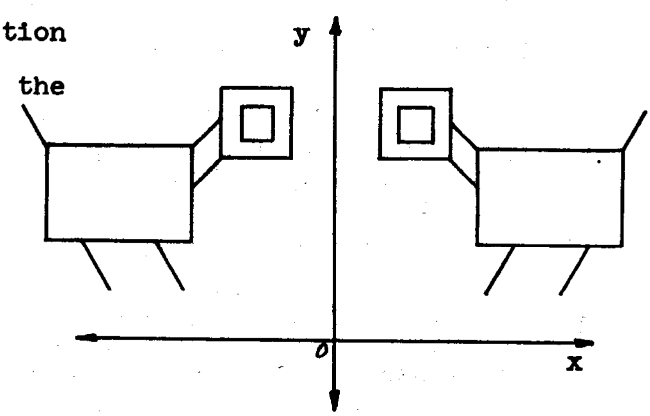
(2) Draw $\overline{AB}$, $\overline{BC}$, $\overline{AC}$.

(3) Is your figure a triangle?

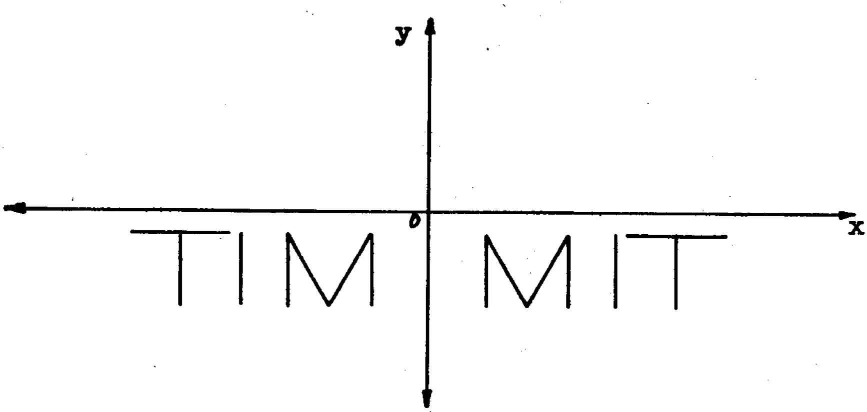
b) Graph the reflection in the vertical axis of ΔABC. Label each vertex of this second triangle with its coordinates.

c) Graph the reflection in the horizontal axis of ΔABC. Label each vertex of this third triangle with its coordinates.

2. The line of reflection in this drawing is the ___?___-axis.

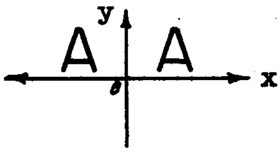


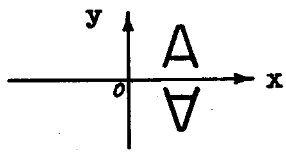
3. The line of reflection in this drawing is the ___?___-axis.



This is a drawing of Chief Pointed Head. Draw his reflection in the vertical axis.

2. The reflection of the letter A in the vertical axis is also a letter A, but the reflection of the letter A in the horizontal axis is not.





Can you think of any other capital letters which would be the same as their reflections in the vertical axes? The horizontal axes? both axes?

3. See the chart for Exploration Problem number 1 on page 280 in your text. Can you find an illustration of reflection of a set of points in a line which is not the x-axis and not the y-axis? If so, write the coordinates of a point and the coordinates of its reflection. What do you observe? Test your observation on three other points.

4. Battleship Game # 3: The rules of this game are the same as those for Battleship Game # 1 (see end of Exercise Set 5) except that Rule d) is replaced by:

d)** (1) The first player calls out something like "reflection of $(^{+}1, ^{+}3)$ in the x-axis". If there is a point (battleship) marked with coordinates $(^{+}1, ^{-}3)$ then the marking there is erased (battleship sunk). If there is also a point marked at $(^{+}1, ^{+}3)$ it is erased

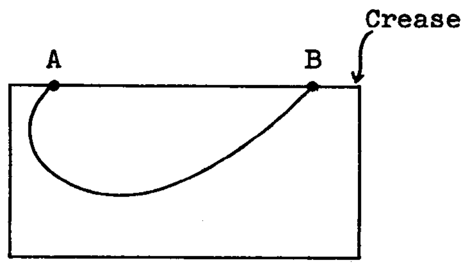
SYMMETRIC FIGURES

Exploration

1. Fold a piece of paper and crease the fold. Mark a point A and a point B on the crease.

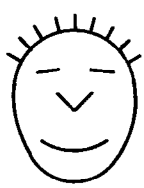
   Start at A and draw a curve which does not intersect itself and ends at B.

   Use scissors to cut along the curve. Be sure to cut through both parts of the sheet of paper. Unfold the part of the paper you cut out.



   The curve is a <u>symmetric</u> figure. The union of the curve and its interior is also a symmetric figure. Either set of points furnishes an example of <u>line symmetry</u> because when the paper is folded along the line suggested by the crease one part of the figure fits exactly on the other. The line represented by the crease is the <u>line of symmetry</u> or <u>axis of symmetry</u>.

2. George has a "crewcut". Is this picture of his head a symmetric figure? If so, lay your ruler along the axis of symmetry.



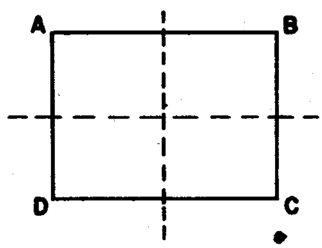
6. Use your compass and straightedge to construct an equilateral triangle. Let the length of each side be three inches.

   Cut out the triangular region.

   a) Can you fold the paper to show an axis of symmetry? Can you show another axis of symmetry? How many axes of symmetry are there?

   b) Can a figure have more than one axis of symmetry? Must it have more than one?

7. Trace this drawing of a rectangle.



   a) Is it a symmetric figure? If so, draw as many axes of symmetry as you can.

   b) How many axes of symmetry does a rectangle have?

   c) How many axes of symmetry does a square have?

8. Construct a circle with a radius of two inches. Cut out the circular region.

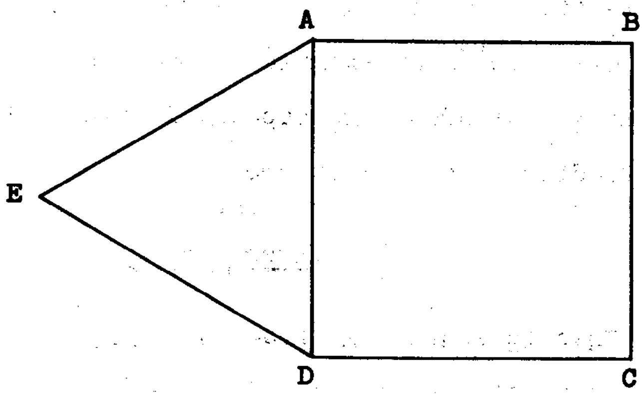
   a) How many axes of symmetry do you think a circle has?

   b) What is the intersection of all the axes of symmetry of a circle?

9. Are there examples of symmetric figures in your classroom? If so, describe the axis of symmetry for each figure.

4.

Polygon ABCD is a square. Triangle ADE is an equilateral triangle.



Trace this figure on your paper.

a) Name three symmetric figures.

b) Draw all of the axes of symmetry for square ABCD.

c) Draw all of the axes of symmetry for equilateral triangle ADE.

d) Is there an axis of symmetry for polygon ABCDE?

e) Look carefully at your drawing and the axes of symmetry you have drawn. If you consider these axes as well as the original figure, do there appear to be even more symmetric figures?

Exploration

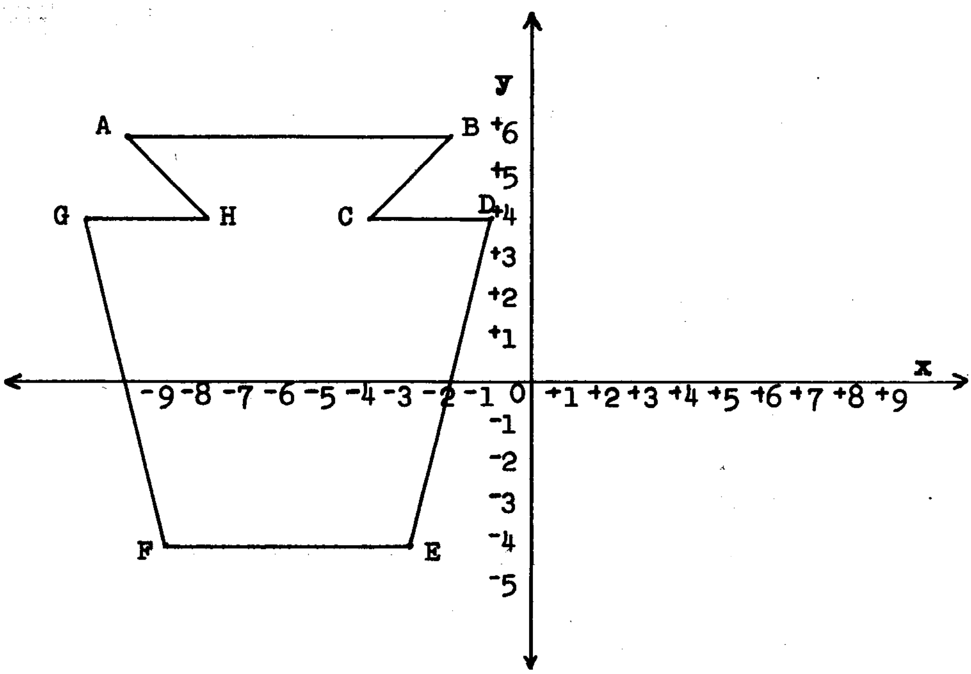
Is an axis of symmetry the same thing as a line of reflection? Think about this question as you work out these exercises.

1. a) Graph this set of ordered pairs.

A($^-10$, $^+6$)	D($^-1$, $^+4$)	G($^-11$, $^+4$)
B($^-2$, $^+6$)	E($^-3$, $^-4$)	H($^-8$, $^+4$)
C($^-4$, $^+4$)	F($^-9$, $^-4$)	

Draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, $\overline{FG}$, and $\overline{HA}$.

Does your drawing look like the one below?



Curve JBCDEK is the reflection of curve JAHGFK.

Curve JAHGFK is the reflection of curve JBCDEK.

The axis of symmetry, $\overleftrightarrow{JK}$, is also the line of reflection.

2. a) Choose a point R in the interior of polygon ABCDEFGH whose coordinates are integers. (Do not choose a point on the axis of symmetry.) What are the coordinates of the point you chose?

   b) Find the point which is the reflection of R in the axis of symmetry, $\overleftrightarrow{JK}$. Call it S. What are the coordinates of S?

   c) Find the point which is the reflection of R in the x-axis. Call it T. What are its coordinates?

   d) Find the point W which is the reflection of R in the y-axis. What are the coordinates of W?

3. a) Graph the set: $R(^+2, ^-3)$, $S(^+4, ^-6)$, $T(^+7, ^+3)$.
Draw triangle RST.

b) Draw the reflection of triangle RST in the y-axis. Call the reflection triangle XYZ.

c) Is the union of triangle RST and triangle XYZ a symmetric set of points? If so, what is the axis of symmetry?

BRAINTWISTERS

1. a) Look at your drawing for Exercise 2, Set 10. Graph the point $(^+1, ^+2)$, and label it E.

b) What point is the reflection of E in the x-axis? in the y-axis? in $\overleftrightarrow{AC}$ (call this reflection point F)? in $\overleftrightarrow{BD}$ (call this reflection point G)?

c) What do you notice about the coordinates of these reflection points.

2. a) Graph this set of ordered pairs:

$A(0, ^+9)$	$D(0, ^+10)$
$B(^+4, ^+9)$	$E(^+3, ^+1)$
$C(^+3, ^+10)$	$F(0, ^+1)$

Draw $\overline{AB}$, $\overline{CD}$, $\overline{CE}$, $\overline{FE}$, $\overline{DF}$.

b) Graph these points using the same axes.

$G(^+2, ^+3)$ $L(^+1, ^+5)$

$H(^+2, ^+2)$ $M(0, ^+6)$

$J(0, ^+2)$ $P(0, ^+5)$

$K(^+1, ^+6)$

Draw $\overline{GH}$, $\overline{HJ}$, $\overline{GJ}$.

Draw $\overline{KL}$, $\overline{KM}$, $\overline{MP}$, $\overline{LP}$.

c) Graph this set of ordered pairs using the same axes.

$R(^+1, ^+8)$ $T(^+1, ^+7)$

$S(^+2, ^+8)$ $V(^+2, ^+7)$

Draw $\overline{RS}$, $\overline{SV}$, $\overline{VT}$, and $\overline{TR}$.

d) Draw the reflection of the figure you now have in the vertical axis.

e) You have a picture of Dandy Dan. Is it an example of a symmetric figure?