Mathematics for the Elementary School
School Mathematics Study Group

Mathematics for the Elementary School

Book K

Unit 51
Mathematics for the Elementary School
Book K

Teacher's Commentary

REVISED EDITION

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PREFACE

The increasing contribution of mathematics to the culture of the modern world, as well as its importance as a vital part of scientific and humanistic education, has made it essential that the mathematics in our schools be both well selected and well taught—at all levels, from the kindergarten through the graduate school.

With this in mind, mathematical organizations in the United States cooperated in the formation of the School Mathematics Study Group (SMSG). The general objective of SMSG is the improvement of the teaching of mathematics in grades K - 12 in the schools of this country. The National Science Foundation has provided substantial funds for the support of this endeavor.

One of the prerequisites for the improvement of the teaching of mathematics in our schools is an improved curriculum—one which takes account of the increasing use of mathematics in science and technology and in other areas of knowledge, and at the same time one which reflects recent advances in mathematics itself. Among the projects undertaken by SMSG was that of enlisting a group of outstanding mathematicians, educators, and mathematics teachers to prepare a series of sample textbooks which would illustrate such an improved curriculum. This is one of the publications in that series.

The development of mathematical ideas among young children must be grounded in appropriate experiences with things from the physical world and the immediate environment. The text materials for grades K - 3 provide for young children an introduction to the study of mathematics that reflects clearly this point of view, in which growth is from the concrete to the
abstract, from the specific to the general. Major emphasis is given to the exploration and progressive refinement of ideas associated with both number and space.

These texts for grades K - 3 were developed following the completion of the texts for grades 4 - 6. The dynamic nature of SMSG permitted serious reconsideration of several crucial issues and resulted in some modification of earlier points of view. The texts for grades K - 3 include approaches to mathematics which appear to be promising as well as approaches whose efficacy has been demonstrated through classroom use.

It is not intended that this book be regarded as the only definitive way of introducing good mathematics to children at this level. Instead, it should be thought of as a sample of the kind of improved curriculum that we need and as a source of suggestions for the authors of commercial textbooks. It is sincerely hoped that this and other texts prepared by SMSG will lead the way toward inspiring a more meaningful teaching of Mathematics, the Queen and Servant of the Sciences.
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A mathematics program begins in the kindergarten.

The development of mathematical ideas in Books K - 3 is based for the most part upon appropriate experiences with sets of physical objects. This development has several distinct but related aspects.

1. Sets of physical objects are observed, described, and manipulated. Sets of objects are compared with each other and are ordered. Sets of objects are joined to other sets of objects. Sets of objects are removed from other sets of objects. Sets of objects are partitioned. Oral descriptions accompany all such manipulations.

2. Whole numbers are associated with sets and set manipulations. Each whole number reflects a property common to a particular class of sets. Operations on whole numbers--addition, subtraction, multiplication, and division--are associated with particular set manipulations and representations. Properties of operations on whole numbers are developed from properties of sets.

3. Geometric ideas are developed in relation to sets of points. Familiar geometric figures--rectangles, squares, triangles, circles--are interpreted as particular sets of points. Whole numbers are used to describe specific points on a line, leading to the "number line", and to describe points in a plane.

4. The set of whole numbers is reinterpreted as a subset of the set of rational numbers. For example, the whole number 2 is reinterpreted as the rational number that may be named by fractions such as $\frac{2}{1}$, $\frac{4}{2}$, $\frac{6}{3}$, $\frac{8}{4}$, etc.
An informal, intuitive foundation for the development of these mathematical ideas begins at "ground level", in the kindergarten wherever one exists within the school organization, or in the first grade if a kindergarten does not exist.

We have good reason to believe that children will be interested in, and will profit from, these foundation experiences which emphasize activities with physical objects from the child's immediate environment. Your guidance in these experiences will make a significant contribution to the development of mathematical ideas among the children in your class.

**All these new words in the kindergarten?**

Most kindergarten children are rapidly and eagerly adding new words to their vocabulary. As their teachers, we are on the alert to see that these words are introduced in a meaningful situation, yet as naturally and normally as possible. Many children come to school using the word "littler" and we try to capitalize on the situations whereby we can substitute the words, "smaller" or "smallest". They can get "in back of" someone or something but "behind" must be taught to many children. They begin to enjoy the various meanings one word may have as it is used in different situations. One little girl proudly announced that the dentist had put a "cap" on one tooth. There was an immediate response of "Laurie, you wear a cap on your head." "A cap is what you put on the top of a pop bottle." "A cap is what you shoot in a cap-gun."

Mathematical vocabulary can be taught in the same calm, matter of fact manner. Just as we are alert to make or use situations to develop other vocabulary, we can be alert to make or use situations to develop mathematical vocabulary. Instead of always saying something like, "Will Sue, Paul, and Ruth come and sit with the people at this table", remember often to say, "Will Sue, Paul, and Ruth **join** the people at this table." "Yes, Jack, you may **join** the children playing on the big blocks." instead of, "You may play with the children on the big blocks." You might ask a child to **remove** the
scissors from the table instead of asking her to put away the scissors, and the children remaining at the tables might be asked to get their resting rugs.

Mathematical concepts are of prime importance. Special vocabulary carefully used to express these ideas becomes a part of the children’s vocabulary as they hear you use it frequently and correctly, and as you encourage them to make it a part of their speech. However, the ideas are much more important than the unique vocabulary, and if some children are unable to use this vocabulary, let them express the ideas in words that can have some meaning for them.

No books for pupils!

"Where are the pupil pages?" "Why don't you include examples of busy work for duplicating?" "Isn't this material too easy for the kindergarten?"

These questions may be among those you may ask. No workbooks or duplicating pages have been included in this material. Mathematical ideas develop slowly and can be developed and understood most easily through activities that call for thoughtful manipulation of concrete objects and those activities that make children aware of mathematics in their everyday lives.

By meaningful manipulation of concrete objects, children become active participants and not merely passive watchers or listeners. Their attention is aroused by the use of materials near and dear to them. The game approach catches the interest and enthusiasm of all the children regardless of their intellectual level. It seems to be of special aid to the more immature child and a challenge to the more able child in discovering other relationships new to him. Playing is the serious work of the child at this age!
The experiences provided by workbooks and/or worksheets too often make little contribution to effective learning experiences. Too many children become preoccupied with the mechanics of finding and marking the right picture or drawings when the hand and eye coordination necessary for this type of activity is not yet adequately developed for them. Furthermore, the concepts the children need to understand are difficult to convey in pictures because the pictures do not permit the movement of the objects shown. Action is a keyword for children. The active manipulation of objects makes a more lasting impression and contributes to a deeper understanding of the ideas under study.

When do we start? How do we proceed?

Children, their activities, interests, and the possessions which they bring to school provide an ideal starting point for a gradual development of the mathematical concepts and mathematical vocabulary in the kindergarten program. Early in the year the incidental daily conversations and activities that are closely connected with mathematics help children to become aware of mathematical ideas and vocabulary. By the second semester, or maybe earlier for your children, a definite time (two or three times a week) can be used when the focus is on mathematics. It is at this time that the specific suggestions made in this book are intended for your use. You must decide for yourself when to begin using this material, since you are the only one who really knows your children and their levels of maturity. These suggestions should be considered only as a starting point. You will have many excellent ideas of your own that will have meaning and interest to your own children in their particular situation.

When you feel that your children are secure and comfortable in the school situation, when their attention
span has lengthened, and when they are ready for "new worlds to conquer", it is then time to set aside a period two or three days a week when the focus of attention is on some mathematical concept. It will be most advantageous to have an over-all view of all the material that is to be presented so that you may be alert to use it as opportunities arise during the day-by-day kindergarten program.

You may find that a week's work might go as follows:

Monday: Speak about the "sets" and the "members of the set" the children bring for "sharing time".

Tuesday: Use the first activity in Section 1. (Focus entirely on the mathematical concept.)

Wednesday: The second activity in the chapter on the "member of a set" which uses a story might be used at the story time and for a few minutes after.

Thursday: No particular attention to mathematics.

Friday: Third activity in Section 1 which uses the sets of toys in the room. (Focus on mathematical concepts.)

However, these materials do not need to follow each other idea by idea. For example, on Tuesday you may use an activity from Section 5 to further the concept of joining; on Wednesday it may rain and you may wish to use the activity on comparison of lengths of umbrellas as suggested in Section 6; and on Thursday or Friday a geometry activity from Section 2 or 8 may seem most meaningful for the focus of attention. However, when working with each concept it will be necessary to progress in an orderly fashion so that the new ideas can be built upon concepts already understood by the children. (The child must have a good understanding of set and member of the set before the idea of subset and the removal of a subset can be introduced.)
Later in the year, it will be possible to use some of the suggestions given in Section 8, Using Geometric Figures for Directions and Games, in connection with other activities. Then these suggestions need not be the center of focus but can be used to add interest and variety to the activity while at the same time strengthening the geometrical concept: for example, the first activity in the section whereby geometric figures are used to designate the seating arrangement when preparing for an activity other than mathematics.

As you read the material in the Commentary, you will be aware that daily you can find or make opportunities to introduce easily and normally new ideas and new vocabulary. Those which already have been introduced are reinforced by using them in different situations.

On "Sharing Day" children come in with sets: sets of little cars, books, dolls, bow and arrows, etc. It can be very easy to comment on "the set" that Bill brought and to ask questions about the various "members" of the set without particularly dwelling on the vocabulary.

During rhythm time, two leaders may choose members for their trains and we then discover that one train has more members than the other, or fewer members than the other, or as many members as the other.

Daily there are many opportunities to use mathematics during the work period. "Are there as many playing with the big blocks as can be playing there at one time?" "Is there a space for me at the easel?" "Is there one more saw that Tom can use?" "How many colors of finger paint may I use on my paper?" These and many similar questions arise constantly.

Checking on a one-to-one correspondence is a very real situation during lunch time every day, as it always is when anything is distributed to the class. Is there a glass of milk for every child? Is there a child for each glass of milk?
We can ask another group to join our class to see a puppet show or enjoy a flute concert given by one of the kindergarten mothers. We might join the other classes as they go to the auditorium to see a program.

Developing children's imagination

Helping a child to develop his imagination is a daily, on-going activity in the kindergarten. You use it to add interest, fun, and zest to his daily life and to help him enlarge upon his vocabulary so that he has understanding of the words he hears and takes this understanding to his formal reading in the first grade. This imagination also plays an important part in the child's ability to understand mathematical concepts and ideas. As we want him to think of the number three as that number common to all sets having three members, he must be able to imagine many, many sets containing three members each, and many different types of sets (three pieces of candy; three boys; a boy, a dog, and a fishing pole; a father, a mother, and a child; etc.) in many places and under many circumstances. As he later works with a set of points, he will need imagination to "see" these sets of points and he will need it to be able to "see" a line going on and on.

Enlarging and developing this imagination can be a source of great pleasure to both the children and their teacher and can do much to add warmth and real group feeling to the class situation. We all need to remain alert to take advantage of situations (or make these situations) that will provide this growth in imagination.

Objects children might bring from home for "Sharing Time"

A young child is eager to explore and to learn more about his environment. The things he brings to school to "share" with the others often point out some of his special interests.
Objects that are brought in by the various children range from dolls, balls, bats, puzzles, books, stuffed animals, ceramic animals, paper dolls, purses, rockets, trains, trucks, nurses' kits, doctors' kits, guns, catchers' mitts, maps, rings, plastic cowboys, Indians, and dinosaurs to rusty nails and old tin cans. Something of interest can be and must be found in each thing a child brings. The rusty nail can lead to an interesting discussion on the likenesses and differences between that nail and a new nail of similar size brought from the workbench in the room. The old tin can be discussed as to size, shape, and use, and might lead to a discussion and an experiment to see whether or not it will hold as much, more than, or less than another container that is being used in the room.

Since these "sharing time" objects are often of much interest to all the children, they can sometimes be used to provide the center of focus for mathematics for the day. A new coat might well lead to a discussion of the heights of the children. "Is there another child in the room who is as tall as Jim so that he, too, might be able to wear the new coat?" "Can you name any children who would be too tall to wear the coat comfortably?" "Are there any who are not tall enough to wear it?"

"Larry has brought a set of horses and Lynn a set of cowboys. Is there a horse for each cowboy? What can we do to find the answer? Yes, let's have Larry and Lynn pair the members of their sets so that we can all see and know the answer."

Sometimes your planned activity for the day will need to be postponed in order to make immediate use of these items that have some special interest to the children. Whenever they relate to the concepts we are trying to develop, use them and wait to use your prepared material another day. In working with the mathematics program, as in every other phase of the kindergarten work, we must be alert to take advantage of every learning situation as it arises.
Mathematical Background

Sets. A set is simply a collection of things. The things belonging to a set are called its members. A set may be defined by some property common to its members (the set of all books on this shelf; the set of all children in this room). Sets may also consist of quite unrelated objects and may be defined by simply listing their members, as in the set whose members are a certain boy, a certain book, and the moon.

It is possible for a set to have only one member. For example, the set whose only member is the teacher of the class, or the clock on the wall, or the classroom piano. There is also one special set called the empty set, which by definition has no members at all. For example, the set of all elephants in the classroom is the empty set.

One set is a subset of another if every member of the first set is also a member of the second. Thus, the set of girls in a classroom is a subset of the set of children in that classroom; and the set of all tricycles is a subset of the set of all toys. There are two tricky special cases of the definition of subset. First, any given set is a subset of itself (because it fits the definition of subset: every member of a given set is also a member of that set!). Second, the empty set is a subset of every set, because all the members of the empty set (There aren't any!) belong to that given set.

Comparison of Sets; Numbers. We can compare two sets by pairing the members of one set with those of the other. Example:
In this example there are members of the first set left unpaired with members of the second set, so the first set has more members than the second (and hence the second has fewer members than the first).

When such a pairing process "comes out even", so that there are no members left over in either of the two sets, we say that the two sets have been placed in one-to-one correspondence and that they are equivalent.

When two sets are equivalent we say that they have the same number of elements. The number of elements in a set is the property shared by all sets equivalent to it. Thus, four is the property of all sets equivalent to
We see from the pictures above that the set of cats is equivalent to the set of turtles—it is easy to set up a one-to-one correspondence between these sets. However, these two sets are not equal. A set of cats is a very different thing from a set of turtles. The number of cats is equal (not equivalent) to the number of turtles.

When one set has more members than another, we say that the number of members in the first set is greater than the number of members in the second (and hence, of course, that the number of members in the second set is less than the number of members in the first). Thus the ideas of more than and fewer than for sets lead to the ideas of greater than and less than for numbers. The relation of fewer than (or more than) permits us to rank or order several sets of different numbers of members, the set with the fewest members coming first, the set with the next-fewest coming next, and so on, the set with the most members coming last in order.

Joining and removing sets. To take a set and join to it a second set means to form a new set whose members are all those of the first set together with all those of the second. This new set is called the union of the two given sets.

Example:

| First set: | a, b, c |
| Second set: | p, q |
| Union | a, b, c, p, q |
In this example, the first set and the second set happen to be disjoint: that is, these two sets have no members in common. When this happens, the number of members in the first set plus the number of members in the second set is the number of members in their union, so, we get the addition fact

\[ 3 + 2 = 5. \]

Thus, the joining of disjoint sets serves as an approach to the adding of numbers.

In the same way, removing from a given set one of its subsets serves as an approach to subtraction. For example:

Given set: \( a, b, c, p, q \)
Subset removed: \( p, q \)
Remaining set: \( a, b, c \)

The number of members in the given set minus the number of members in the subset removed is the number of members in the set that remains.

In this illustration, the corresponding subtraction fact is

\[ 5 - 2 = 3. \]

You are reminded that sets are joined while numbers are added; a subset of a set may be removed while numbers are subtracted. The members of sets are never added nor subtracted. The operations addition and subtraction apply only to numbers, not to sets.

Geometric Figures. To lead children to recognize very simple geometric figures, show them some cut-out models like these:
Point out that each model, which has been cut out of cardboard, has been outlined with a heavy black mark. This outline, referred to as the edge, border, or boundary, determines the shape and is called a geometric curve. The more common geometric shapes used here are triangular, rectangular, and circular.

Only the edge, border, or boundary represents the triangle, the rectangle, or the circle. A simple, yet satisfactory model of each may be constructed from wire, a pipe-cleaner, a starched string, or similar material. Such a model should keep children from including the inside when talking about a triangle, a rectangle, or a circle. These represent models made out of wire.

Models such as these help children to see that a circle is round or smoothly curved, while a triangle has 3 straight sides and 3 corners, and a rectangle has 4 sides and 4 corners.

Another distinction we try to convey is that of size: when it is that one circle is larger than or smaller than another, when one side of a triangle or rectangle is longer than or shorter than another, when one side of a triangle or rectangle is longer than or shorter than another, etc. Recognizing objects as being the same in size or in length demands closer discrimination and comes later.
It is convenient at times to locate points according to whether they are inside, outside, or on the triangle, the rectangle, or the circle. The use of the words inside and outside instead of interior and exterior has been found helpful in teaching children this idea.

Look again at the triangular shaped model.

When we think of the triangle together with the inside, we have a triangular region. More generally, a boundary together with its interior forms a region. Here is shown a rectangular region and a circular region.
Chapter 1

SETS

Objectives: To develop the concept of set and the use of set terminology, including the idea that a set may have only one member or even no members at all.

Set concepts can be effectively developed when the vocabulary is used incidentally (but reasonably naturally!) throughout the school day. Informal situations occur where reference can be made to particular sets of objects and their members. For example, "Billy, please put your set of blocks away. Children, will those of you who have blocks that belong to Billy's set, put them back." Michael "lines up" with the set of boys because he belongs there or is a member of that group. As the school year progresses, set descriptions should become more explicit. Instead of referring to the set of blocks in general, you may want to refer to the set of small blocks on the bottom shelf. The need for more exact descriptions may be better understood from the following example: If a child is asked to bring you the set of red pencils on your desk, a specific set of pencils is identified. If asked to bring simply a set of red pencils, he may return with any set of red pencils which he collects in the room.

As sets are introduced, the terms, member and members of a set may come naturally into the conversation; e.g., "I have a new set of zoo animals for the room. Jeanne, can you name all the members of this set?" "Bruce, you brought an interesting set of animals today. As you talk about it, will you hold up the members of your set one at a time so that we may see each animal as you tell us about it?" "You have already made many new friends in the set of children in our room. Bill, can you name all the members in our set?"
After set and member have been used informally for a period of time, children may be led naturally to the idea that a set may have only one member or even no member at all. A familiar example of a one member set is the set of pianos in the room (or the set of teachers of the class, or the set of clocks on the wall, or the set of classroom American flags). As for sets with no members, make reference to such "amusing" sets as the set of elephants in the room or the set of lions at the store. Also, you may refer to sets such as the set of crayons in a crayon container when there are no crayons in the box.

Deliberately, we have used the word "set" almost to the exclusion of other words, such as group, bunch, collection, etc. This has been done to illustrate the wide variety of situations to which the term can be applied. However, do not conclude that it was our intention to suggest that words such as groups, bunches, etc., not be used. They should be used when it is natural to do so.

Also, we encourage you to bring in the use of terms naturally. Just because we suggest, "Will the set of boys in our class stand?", don't think you should always say this. Sometimes it is best to say, "Will the boys stand?" In brief, you can try so hard to use these terms in so many activities that they may become too artificial.

Vocabulary: Set, collection, member, set with one member, set with no members (empty set).

Materials: A variety of materials, such as: books, cars, trucks, blocks, pencils, crayons, sheets of paper, paint brushes, scissors, game boxes, puzzles, beads, paper clips, pegs, balls, play house furnishings, etc.
A variety of materials to be used on magnetic or flannel boards: stars, trees, storybook characters, models of circular, triangular, and rectangular regions.

Magazine pictures: family, automobiles, telephones, clothing, food, toys, planes, trains, and any other appropriate illustrations of collections of objects.

Story book about a family and others for making set references.

Activities to help develop concepts:

Set. Since the children are familiar with the playhouse in the classroom, materials from it may be used for early discussion of sets of objects. In particular, the set of dishes may be most meaningful. Place the playhouse table and chairs in front of the children. Have a set of dishes, a set of silverware, and a set of glasses or cups available. These questions can be used as a guide:

- Which of you have helped to get the table ready for dinner?
- What do you put on the table? (Plates, glasses, silverware, etc.)
- Let's pretend that we're going to have dinner and need to get our table ready. Jim, will you put the plates on our table?
- Is this plate (indicate plate) a member of our set of dishes on the table? (Yes.)
- John, what would you like to put on the table for us? (Possibly the silverware.)

Ask if a particular knife or fork or spoon is a member of the silverware.
Julie, what set do we still need to put on the table? (Glasses.) Will you please put a set of glasses on the table?

Now, is our table ready for dinner? (Yes.)

Let's have some people come to eat the dinner. Can we call this a set too? (Yes.)

Name four children to be those who go to eat at the table. Ask if the other children are members of the set named to eat.

Finish the discussion by identification of other sets in the playhouse, at home, etc.

**Members of a set.** Select a story about a family and read this to the class. Some suggestions are listed below:

- Flack, Marjorie, *The Story of Ping*
- McCloskey, Robert, *A Day in Maine*
  - Make Way for Ducklings
  - Blueberries for Sal
- Ward, Lynd, *The Biggest Bear*
- Zion, Gene, *The Plant Sitter*
- (Traditional), *Goldilocks and the Three Bears*
  - Little Red Riding Hood
  - Cinderella

After reading the story, have the children identify various sets described in the story. Further attention can be focused upon the things that are members of each set. The sets may range from the set of buildings on a particular street to the set of people or animals about whom the story is written. Draw the discussion to a particular group of people or animals in the story. **Make reference to this group with the following questions or statements:**
Today we read a story about a family. The family was a group of people (animals, etc.). Is that a set? (Yes.) Who were the members of this set? (Members of the set should be listed.) Yes, all of these are members of the set of people in our story. We say that each person or thing that belongs in the set is a member of the set. You children are members of our class. You belong to the set of children in our room. You are also members of the set of children who come to our school.

We have many sets of toys in our room. Let's look at the toys and name the sets we can see from where we sit. (Wheeled toys, boats, dolls, blocks, puzzles, Lincoln logs, etc.)

Michael, will you show us your favorite set of toys? (Boats.) The boats are fun aren't they? Will you put all the members of the set of boats on one of the tables?

Martha, what is your favorite set of toys? (Dolls.) Will you put all the members of the set of dolls on another table?

Did Michael find all the members of his set? (Depends on what he did.) Did Martha find all the members of her set? (Depends on what she did.)

Talk further about the sets previously discussed, identifying the members of these sets (e.g., the set of dishes in the playhouse; each plate, each cup, and each saucer, etc., is a member of this set). Then ask if the table is a member of the set of dishes.
Point out that some things are not members of some sets. For instance, the books are not members of the set of paint brushes. Steve is not a member of the set of girls; the trucks are not members of the set of blocks, etc.

Set with a single member.

Today let's have all the girls wearing red dresses stand. (Continue with a variety of set descriptions. The teacher should be seated.)
Now will the set of boys stand? (Be seated.)
Set of girls stand? (Be seated.) Set of teachers stand? (Reactions at this point should appear from the class.)
Yes, I am the only member of the set of teachers in this room (assuming that there is not more than one teacher).
Can you think of any other sets in our room with just one member? (The set of pianos, the set of teacher's desks, the set of clocks, etc.)

Sets can have more than one member, such as the set of children in our room. Sets can also have just one member. We call this a set with a single member. Is the set of record players in our room a set with a single member? (Yes.)

Discuss other sets which have a single member, such as: the set of swimming pools at a particular city park; the set of principals at your school.
Sets with no members: the empty set. On the day of this discussion the teacher might wear a piece of clothing containing at least one pocket. It should be empty. (The children will search this pocket for a set of things.) Before the activity, select as "secret helpers" at least three children who have clothing with pockets. Place some small objects in a pocket of each child's clothing. Tell them that they will be asked to empty their pockets when the game begins. After the class is together, call the "helpers" up, one at a time. Inform the children that the pockets contain surprise sets and ask them to describe the set as it is placed on the table. The following may be helpful:

Bruce, will you please empty your pocket for us?
Let's look at the things Bruce had in his pocket. What are the members of this set? (Paper clips, jacks, beads, etc.)

Repeat the above for each child previously chosen. Make it amusing and fun for the children by using the game idea. After each child has emptied his pocket, select one child to come up and empty the teacher's pocket.

What are the members of the set of things in my pocket? (No members.)
You mean my pocket is empty! There is nothing in it. This helps us to think about the set with no members. We call it the empty set.

See if children can suggest other examples of the set with no members, but do not be concerned at this time if the idea needs further development.
The following activity may be used as a variation of the previous one. Whereas the first procedure actually involved the removal of the sets from the pockets, the following is a game of "pretend".

Before the class period, prepare four boxes instead of pockets. Select three children and tell them that each is to pretend that the things in the box given him were things that came out of his pocket last night. The fourth box is empty, representing the teacher's pocket.

We are going to pretend that each of these boxes contains the things that Mary, Michael, Kathy, and I removed from our pockets last night.

Mary, show the class the set of things you had in your pocket. (e.g., doll shoe, penny, bobby pin, and button.) What are the members of Mary's set? (Class describes.)

Things found in the other children's boxes are described in a similar way.

Now, let's see the set of things I had in my pocket last night.
Mark, describe the members of this set. (Child finds teacher's box empty.)
You mean my pocket was empty! I guess I had nothing in my pocket last night.

Observe sets of objects in the room and describe these using the terms: set, member, etc. (e.g., sets of chairs, tables, paint brushes, puzzles, etc.).
Daily directions are given for moving children from one activity to another, for their participation in some special activity, and for dismissal. From time to time, refer to the group under consideration, e.g.,

Will the set of girls wearing ribbons in their hair please get their hands washed for lunch?
Will the set of boys wearing brown shoes be the first to get ready to go to the library?
Will all the girls who are five years old please get ready to go home?

The set of days in the week, the set of days we go to school, the set of the weeks in the month, etc., may be mentioned from time to time as the calendar is used.

Initiate a class project of creating a science table display with collections of rocks, sea shells, leaves, butterflies, etc.

Preparation for work period: When selecting or assigning work activities, separate the children according to sets. (e.g., set of children who wish to paint, etc.) Note: At a later date this experience may be used for "pairing" children with easels, paper, etc.
Prepare "surprise boxes" which contain different objects. The children can sort these objects into sets, and describe the sets. These boxes may be used to develop other concepts in the future; e.g., boxes of buttons may contain several different colored buttons. The children may sort these into sets of each color, sets of large buttons, set of small buttons, etc.

Make perception cards using pictures from magazines. Show these to the class and have children describe the set of objects and name the members of each set.

Children may enjoy making their own cards.

Display wooden stand-up figures in the classroom and have children classify the objects; e.g., sets of farm animals, wild animals, community helpers, etc.

Read stories about people, animals, things, etc., and describe the sets and the members of the sets. Also have children name objects which are not in these sets.

Some children may wish to paint or draw pictures of sets and organize their own "Set Pictures" or "Set Scrapbook".
"What's Missing?" Place several sets of objects on a table—books, blocks, cars, and scissors. Ask a child to describe each set.

Lori, can you describe these sets for me? Yes, a set of books, a set of blocks, etc.

Now close your eyes while we remove one of the sets. (Remove the set of books.) Look at the table again, Lori. Can you tell us which set is gone?

This game can be varied by selecting different objects for each game, such as ceramic figures, geometric figures. The class may also enjoy preparing the sets and directing the activity.

Dramatize stories: set of characters, members in the set; e.g., *Three Billy Goats Gruff*, *Three Bears*, *Blueberries for Sal*. 
Chapter 2
RECOGNIZING GEOMETRIC FIGURES

Objective: To help children to recognize geometric figures (circle, rectangle, triangle, and square).

Vocabulary: Circle, rectangle, triangle, square, figure, inside, outside, on, side, edge.

Materials: Geometric figures (curves and regions) made from wire, starched string, sandpaper, construction paper, and felt or flannel; oatmeal box tops, rectangular and square box tops; triangle from rhythm instruments; rings; macaroni; fruit jar rings; plain, round bracelets; rope; yarn.

Children learn to recognize the shapes of objects about them, identifying those objects shaped like triangles, rectangles, circles, etc. They may become acquainted with these shapes by moving their finger around the edge of such objects as dominoes, checkers, coins, and cut-outs from cardboard and plywood. Through such activities, they begin to distinguish between the objects which have a feeling of roundness and those which have corners. The objects which have corners also have sides. Children will be interested in how many sides and how many corners these objects have. They discover that an object having sides will always have more than two, and it will have as many sides as it has corners. Children begin to associate the idea of triangle with a 3-sided object, circle with an object that is round like a coin, and rectangle when the object has 4 sides and corners like a domino.

Activities are designed to help children to learn to distinguish between a region, its boundary, its inside, and its outside.
Activities to help develop concepts of circle, rectangle, triangle, and square.

You can provide many informal opportunities in which children can distinguish the relationships among geometric curves, regions, and solids. In general, begin with familiar objects in the classroom, such as balls, blocks, boxes, cans, etc., and discuss their shapes. Gradually introduce the idea that the terms circle, rectangle, and triangle refer to the edges of these solids.

When a child looks for one of these curves, say, a rectangle, he is most likely to find examples such as the base of a block, or a cupboard door. Each shows not only a rectangle but also its interior. Again, with each region emphasize that it is the edge that is a rectangle.

Use of regions. Before school, place around the room several construction paper models of regions. When the children are seated, hold up a circular figure.

I want you to see how quickly you can find two figures that are shaped like the one I'm holding. They may be the same size or they may be larger or smaller than this one. They may or may not be made of the same colored paper.

Do not mention that you are holding a circular figure. Some children may call this circular region a circle. At this point, do not make an issue of this. However, merely refer to it as a circular shape, or a cut-out shaped like a circle.
Repeat the game using triangular and rectangular regions. Have many types of triangular regions represented.

Give the children seven colored paper regions of the same size to arrange in designs. Some will be delighted to find that if six are arranged in a ring, there will be room to fit the seventh in the center. Some children will enjoy pasting colored disks of various sizes on sheets of manila paper or newsprint to form designs.

Using geometric regions, start various patterns on the flannel board, and have pupils continue them, e.g.,

Form similar patterns using kindergarten beads, macaroni, straws, etc.

Cut various figures from colored construction paper and have children paste them on a larger sheet to create a pattern design. For variety, these can be done on long rectangular pieces of construction paper.
**Curves.** On the magnetic or flannel board, place at random models of circular, triangular and rectangular (including squares) regions. (Do not crowd these models.) Include different sizes and colors.

Watch what I choose from the board. When I stop, I will point to someone to come and take off the one he thinks I should take next.

Without another word, take a circle, another circle of a different color, and another circle of a different size. Then point to a child to remove the next shape. If he doesn't pick another circle, say,

No, that is not what I would have taken next.

Jim, show us what you would have chosen.
Put the cut-outs back. Again remove those of different size and color and point to another child. At no time call these figures circles. After a few such experiences, explain or let a child explain what is happening to any child who has not yet figured it out for himself. If the figures are called circles, agree but do not emphasize the word. The child doing the explaining may say that "they are all round".

Then let one of the children pick out three models of circles and point to a child to remove the model he thinks would be chosen next.

At another time, use the same cut-outs and remove the triangular models.

At another time, use the same cut-outs and remove some members of the set of rectangular models. The set you remove may look like this:

![Diagram of rectangular models]

From the set of rectangular models, remove the set of squares. Ask how this subset is alike and different from the members of the set of rectangular models. (Intuitively, children learn that squares are a special kind of rectangle.)
Display models of circles. Ask if anyone knows the name of the figure. Supply the name if necessary. Have children run their fingers around the edges of the models. Ask the children if they can find objects in the room that are "shaped like a circle". Ask if they can tell about objects outside the classroom that are shaped like a circle.

Similar activities can be planned using models for rectangles including special rectangles we call squares, and triangles. Models for rectangles can be made from tagboard picture frames, wire, or from starched string, when you wish to use these instead of regions. There will be many objects in the room that are "shaped like rectangles". Whenever possible, reinforce the idea that the term rectangle refers to the edges of the shape about which we are talking.

Triangles can also be made from wire or from starched string. Also, triangles from the rhythm instruments and the triangular shaped blocks from the set of blocks can be used.

On the magnetic or flannel board, place many models of rectangles including squares. Repeat the activity used on page whereby the models are removed from the board. Introduce the square as "a special kind of rectangle that we call a square".
Use of solids. From a set of rectangular shaped boxes, have a child or children pick out all the square boxes.

Call special attention to the squares that make up the tiled floor. (If your floor is tiled.)

The geometric figures cut out of sandpaper help reinforce the "feel" of the shape for some children.

In a bag, place some solid geometric figures (square, rectangular, triangular blocks and a wooden wheel). Hold a newspaper between a child's eyes and hands. Ask him to reach in, pick out one object, feel it, and tell what shape it is. This same procedure may be changed by asking the child to feel for and pick out a designated shape.

Use of regions and solids. Before the children come in, put different sized circular, rectangular, and triangular figures (blocks, boxes, and circular, triangular, and rectangular felt regions, oaktag cut-outs, etc.) around the room. When the children are seated, hold up a circular figure. Have the children identify it. Give them three minutes to see how many circular objects they can find. Hold up another geometric shape and proceed as before. Continue until all three shapes are used.
From where they are seated, have the children visually check to see that the right shapes have been found each time.

Put an object into a bag and describe it as to shape and use, (e.g., It is circular in shape. It is hard, smooth, and rather flat. Food is put on it.). After a child guesses what is in the bag, he may feel the object to determine whether or not he still thinks he has made a good guess. If he is not right, let another child guess and then feel the object. Some objects might be: a small block, a plate from a dollhouse, a small drum, a paper cup, a cracker, etc. Some children may be able to describe to the other children the object or model that is in the bag. The models could include some of the smaller tagboard and construction paper cut-outs, music triangle, etc.

"I'm Thinking Of . . ."

I'm thinking of something in this room. It is shaped like a triangle and is made of wood. What is it? (A triangular shaped block.)

I'm thinking of something else that has three sides and three corners but this one is made of metal. What is it? (The triangle from our rhythm instruments.)

I'm thinking of something that is shaped like a rectangle. It is made of metal. What is it? (The tray of an easel.)
To reinforce the concept that a rectangle is a rectangle regardless of the position in which it is seen, use many models of rectangular regions of various sizes out of oaktag or colored construction paper and place in different positions on the rug, floor, or bulletin board, e.g.,

One by one, ask various children to find a rectangle and place it on the rug in front of him.

The figures that you chose were put on the rug in just any position.
Did it make any difference how they were placed or were they still all rectangles?
Pick up your rectangle and put it down in a different position. Do you still have a rectangle?
Now pick up your rectangle and turn it over. Is it still a rectangle?

The above procedure might be used for triangles. Here again, it is important that many sizes be used and that they be placed in various positions.
Reserve one section of the board for placing these models.

Activities to develop concepts of inside, outside, and on. Many activities must be provided whereby children learn to distinguish and describe points or locations according to whether they are inside, outside, or on a given circle, triangle, or rectangle.

Make a rope circle on the floor and have the children stand around it. Ask one or two who you are sure understand the meanings to help you demonstrate your directions for inside, outside, or on.

Stand inside the circle.
Stand on the circle.
Stand outside the circle.

After they are back in the group, tell all the children that you are going to give some quick directions and they are not to let you fool them.

Boys stand inside the circle.
Boys stand on the circle.
Boys stand outside the circle.
Girls stand inside the circle.
Girls stand outside the circle.
Girls stand on the circle.
Everyone stand on the circle.
Jim, Jack, Susan, and Mildred stand inside the circle. Etc.

Later this could be used as a "Simon Says" game.

Large geometric figures might be formed on the floor by pulling yarn taut and using masking tape to hold the corners in place. Inside, outside, and on games and directions can be used with these easily seen figures.
Using string or yarn and chalk, make a circle on the floor. Each child will be given a marble as he is seated around the circle. From where he sits, or kneels, he gently rolls his marble toward the circle. As his marble stops, he calls out its position with respect to the circle: "inside", "outside", or "on". After each child has had his turn, have all the marbles "outside" put into one set, those "on" into another set and those "inside" into the third set. (The "on" set may be empty!)

Later in the year: After a study of comparison and order, this activity can be repeated and the children can decide which set has the most members and which the fewest. These sets can then be ordered with the set with the fewest members on the left, the middle set next, and the set with the most members on the right. Pairing can be used to decide the order if there is any doubt.

Ask the children to stand around a circle made of yarn, clothesline or a hoop and take turns tossing a beanbag into the circle telling whether the bag lands inside, outside, or on the circle.

This game could also be used as an outside activity using a tire, hoop, or rope for the circle.
The concepts of inside, outside and on can be extended by "Let's Pretend" games using children and objects. With a hula hoop or large model of a circle made of rope, pretend the class is at a rodeo. Place the hoop or rope on the floor or table. Have enough of the small rubber or plastic cowboys, women, children, and men in other occupations in a bag so that each child may reach in and quickly pull out an object. He will then play the part of that figure. The cowboys can go inside the circle with the horses to be ridden, some of the children can sit on the circle fence, and the others can stand where they wish outside the circle while the rodeo is in progress.

In another "Let's Pretend" game, the children might go to the circus. Small plastic or rubber circus animals could be put in a bag and each child would act the part of the animal he pulled from the bag. Each animal would go inside the circus ring as it was his turn to perform. The children who were not playing the part of an animal would watch from outside the ring.

Give each child a 9 x 12 sheet of paper on which is drawn a circle, a rectangle, and a triangle. Also give each a small object (small plastic car, animal from farm set, etc.). Ask each to describe his toy to the others.

Give directions such as:

Place your set inside the circle.
Place your set on the triangle.
Place the set of cars outside the rectangle.
Place the set of pigs on the circle.

If possible, have some of the children give directions for placing the objects.
Many singing games call for a circle. Call attention to the group forming the circle, the child who is inside the circle, and those who are outside the circle.

Some of the games are: "Froggie in the Middle", "The Farmer in the Dell", "Bow Belinda", "In and Out the Window", "Looby Loo", "The Old Brass Wagon", and "Hokey, Pokey".

Circle games can be used in the same manner; e.g., "Duck, Duck, Goose" and "Drop the Handkerchief".

Using Geometric Figures for Directions and Games. Many opportunities may be provided whereby children will see geometric figures and objects shaped like geometric figures in different positions and under varying circumstances. A few ideas for activities of this type are suggested here. Added suggestions for more mature children can be found in Chapter 7.

Have colored disks cut into two parts. e.g,

Place one of each of the parts on a table and keep the other. Each child picks up one part of a disk as he is ready to enter the group situation and puts it in front of him on the rug. His turn to "share" with the class will come when the teacher holds up the part of the disk that will fit with the one he has in front of him. He may merely match the pieces and sit down if he has nothing to say to the group.
This same type of activity may be used with models of rectangular and triangular regions. The triangle sections have been found to be the hardest for the children to visualize and might well be introduced toward the end of the year.

These cut-outs can be used as a method of finding a partner when going for a walk, to the library, to the auditorium, for a singing game, etc.

"What is Missing?" Game

Put four figures in a row on the magnetic board. Have the children name the shapes in order from left to right. As they close their eyes, the teacher removes one of the figure. One child must tell which one is missing. This may be continued for a few minutes using different arrangements of the materials.
Chapter 3

COMPARISON OF SETS

Objective: To compare sets by pairing their members and to describe the result using the terms as many as, more than, and fewer than.

Vocabulary: Pair, as many as, more than, fewer than.

Materials: Paint brushes, paint jars, paper, easel, crayons, tools, name cards, objects for magnetic or flannel board, paper clips, buttons, bottle caps, clocks.

Note: As the concept of "pairing" is introduced, it is very necessary that the child get the feeling of doing, of bringing a member from one set and pairing or matching it to a member from another set. Avoid using "a pair of shoes" or a "pair of mittens", etc., as this is not the concept we are trying to develop.

Activities to help develop these concepts

As Many As. Use daily activities to illustrate pairing the members of one set with members of another set. (Be sure to have children identify the sets before beginning the activity.) For example, pair the pupils with sheets of paper for art activities, or with easels, rhythm instruments, toys, etc., asking:

Is there a sheet of paper for each child?
Is there a child for each sheet of paper?
Is there a child for each instrument (or toy)?
Is there an instrument (or toy) for each child?
Class parties offer a wealth of opportunities for pairing; favors with refreshments, refreshments with place mats, place mats with children, etc.

Select a child to place a set of objects on the left side of the magnetic or flannel board. (See Picture 1.) Limit the objects to be used since the members of two sets will have to be paired. Have the class describe the set. On the right side of the board, place another set with as many members as the first set. Have this set described too.

![Picture 1](image1.png)  ![Picture 2](image2.png)

We now have two sets on our board. (Indicate) (See Picture 2.) Let's see if there are as many members in one set as there are in the other. Yes, we can pair the members of one set with the members of the other.

John, will you pair the triangle with a member of the set of fruit? Place the members side by side in the center of the board.

Repeat this process until all the members are paired.
Picture 3

Is there a geometric figure for each piece of fruit? (Yes.)
Is there a piece of fruit for each geometric figure? (Yes.)
Yes, there were as many members in one set as in the other.

Have other equivalent sets ready to use.

Let's ask Bill, Kathy, and John to play the game.
Bill, you can choose a set of things to put on the left side of the board.
Kathy, you choose a set to put on the right.
Now, John, will you pair the members of the two sets?

Repeat this game until each child has had a turn or as long as interest is maintained. Each time emphasize that there are as many members in the one set as there are in the other.

I have put a set of flags on the left side on the magnetic board and a set of ducks on the right. Glenn, will you pair these two sets as we did the other day?
Is there a flag for each duck? (Yes.)
Is there a duck for each flag? (Yes.)
Yes, there are as many members in one set as in the other.
Today we will pair the members of some sets that I have put in these boxes.

Give each child a box or bag containing two types of set materials: buttons, small blocks, felt cut-outs, small toys, etc. Seat the children so that you can readily see them as they work.

Now, let's see what's inside the boxes. What different kinds of objects did you find? (Have them named.)

Yes, we can say there's a set of (buttons) and a set of (blocks).

Take out the blocks and put them on the right side of the box.

Now, put the buttons on the left side of the box.

Now pair members of your set of blocks with your set of buttons, just as we paired the members of the set of geometric figures and the set of fruit.

Is there a button for each block? (Yes.)
Is there a block for each button? (Yes.)
Are there as many members in one set as there are in the other? (Yes.)

The above activity may be used during free time. Children might enjoy selecting sets of their own. Simple sets can be formed from: shapes of various colors of construction paper, pegs from peg boards, beads, pieces of yarn, etc.
Place five chairs in a row. Select five children and ask them to be seated in the chairs. Point out that the children are paired with the chairs and that there are as many children as there are chairs and there are as many chairs as there are children.

Ask children to show that they have as many fingers on one hand as on the other; stress that they cannot count the fingers but must show you. If needed, suggest that they pair their fingers by putting the finger tips from opposite hands together. Thus, each finger is paired and there are no fingers not paired. The set of fingers on the left hand has as many members as the set of fingers on the right hand.

Talk about the story of the three bears.

Was there a bowl for each bear?
Was there a chair for each bear?
Was there a bed for each bear?
Were there as many people in the story as there were bears?

As the children are seated, give each a set containing several members. (Have a different number of members in the sets for children sitting side by side.) Along the length of the rug, scatter small set objects so that each child may find enough members to form a set that is equivalent to his.

Today you each have a set. On the rug you see many objects that can be used to make other sets.
Let's see how quickly you can pick up a set of the objects near you and pair **them with the members** of your set. You must make a new set that has **as many members as** the set I gave you.

**More Than.** Place yourself so that the boys in your class are on one side of you and the girls on the other. Then, pair a member of one set (the set of boys) with a member of the other set (the set of girls). This procedure might be used to determine the partners for a walk.

Suppose Jack and Jane are partners. You may ask,

Jack, name the member from the set of girls who is your partner.
Jane, name the member from the set of boys who is your partner.

When all **members** have been paired **in the two sets,** ask,

Does every member of the set of girls have a member from the set of boys as a partner? Does every member of the set of boys have a member from the set of girls as a partner?

Have a set of cars and a set of pets ready for children to use on magnetic board. You will need more cars.

Sam, will you please put this set of cars on the left side of the magnetic board? Charles, please put this set of pets on the right side of the board.
Julie, will you pair the members of these two sets?
Was there a car for each pet? (Yes.)
Was there a pet for each car? (No.)
Did one set have more members than the other? (Yes, there were more cars than pets.)

Use the same type of procedure as was used for as many as on page 41. Give each child two sets of objects that cannot be put into one-to-one correspondence. Ask each child to tell which of his sets had more members than the other.

As the children come into the room, ask each girl to pick up a cut-out shaped like a rectangle from the pile of geometric figures and put it on the magnetic board. Each boy is asked to take a cut-out shaped like a triangle and pair it with a rectangular model.

Is there a rectangle for each triangle? (No.)
Is there a triangle for each rectangle? (Yes.)
Do we have more members in one set than we do in the other? (Yes, we have more triangles.)
Are there more boys present today? (Yes.)

Have a box of set materials ready to distribute to the children. Without counting, quickly pick out a handful for each child.
Today you will pair the members of your set with those of the set held by the person sitting to your right. We'll start with Susan. Susan, who is sitting to your right? (Robert.) Yes, you will pair the members of your set with those of Robert's set. We'll skip over Robert to John. John, you will pair the members of your set with whom? (Jim.) Yes.

Continue until every other child is paired with the child to his right. (Make some special arrangement for the child who may be "left over".)

When you have finished pairing the members of your sets, let's have each child stand whose set has more members than that of his partner.

Put a set of bottle caps with many (20 to 30) members on the floor where all can see them. Place another set with a few (2 to 4) members on the floor near it. Ask the children if one set has more members than the other. If the children choose the correct set, ask why they chose it. If a child responds, "It is bigger", substitute some very large blocks for the set with the few members and ask again which set has more members.

For some groups, there may be a need to include more examples of comparing sets which contain objects of different sizes (large blocks with pick-up sticks; larger play trucks with small plastic cars or trucks) to help children concentrate on the members of the set rather than the size of the objects in the set.
Comparing Imagined Sets:

Each of us sees many pictures in his mind every day. Today I'm going to tell you what I want you to see.

First, think of a robin's nest with eggs. Now think of an apple tree full of apples. Which set has more members, the set of robin's eggs or the set of apples?

Next, think of a little yellow chick looking for a bug. Now think of a spider walking along the ground. Which has more legs, the chick or the spider?

Think of the leaves on a tree. Now think of the leaves on a tulip. Which has more leaves, the tulip or the tree?

Many other comparisons which will be familiar to the children may be used. Some other suggestions are:

1. The lights on a Christmas tree and the lights on a floor lamp.

2. The wheels on a truck and the wheel on a wheelbarrow.

3. The boots belonging to all the children in our room and the boots belonging to the children in your family.

4. The candles on your birthday cake and the candles on your father's birthday cake.

Play "Musical Chairs". Ask that each child place a chair in position for the game. Remind them that in playing the game one chair is removed each time the music stops so that the set of children will always contain one member more than the set of chairs. Therefore, when the music stops there will be one child without a chair.
Fewer than. Note: The idea of "fewer than" and the words "fewer than" seem difficult for some children. You may find that you can help clarify the meaning by also using, "not as many as". As the opportunities arise for such comparison, the children will gradually learn to use "fewer than" when talking about the members of a set and "less than" when talking about numbers.

Place a set of spools and a set of buttons, each with nearly the same number of members (8 to 10) on the floor. Ask the children which set has fewer members. There may be some disagreement. Ask a child to pair the members of one set of spools with those of the other to find the answer. Emphasize that the set with the fewer members is the one which does not have as many members as the other set.

As the children enter the room, ask that each girl pick up a white heart from the pile of heart and that each boy take a red heart. As each child's name is called for roll call, he places his heart on the flannel board where all can see--the white ones on the left and the red ones on the right. Ask a child to pair the red hearts with the white hearts in order to answer the question:

Which set has the fewer members?

Place a pair of bags containing familiar toys or equipment in various spots around the room. (More groupings allow more participation for more children in a shorter space of time.) In each pair of bags you might put sets such as: set of 8 pieces of doll clothing and 4 paint
brushes; a box of sand and a set of 4 blocks; a
dozzen pieces of paper and 1 pair of scissors; a drum
and a set of 10 small plastic cowboys; etc. Have
a small disk to give to each child.

When we lift a bag, can we always tell
how many things are in it by feeling how
heavy it is? (No.)

Today I have placed around the room several
bags filled with sets of familiar objects.
As you can see, these bags have been paired
and they must stay together.
I will tell you which bags you will use.
Lift both bags in the pair and quickly
guess which bag contains the fewer members
in the set. When you have decided, put a
small disk by that bag and then come back
to where you were sitting. Remember I
said "lift". Please don't shake the bags.

After the children are back in the group, have two
children bring one pair back to the group where the
bags can be opened and the sets compared. Also compare
the disks that were by each bag noting each time which
set contained the fewer members. The members of
most sets will not need to be paired, but if there
is doubt, ask that a child pair the members to see
which had fewer. Continue until all sets are compared.
"You made some good guesses but some of the bags were
"foolers", weren't they?"

Comparing imagined sets:

Let's see more pictures in our mind today.
I'll tell you what I want you to see.

Think of the wheels of a tricycle. Now
think of the wheels of a bicycle. Which
has fewer members, the set of wheels of the
tricycle or the set of wheels of the bicycle?
Think of the wheels of a wagon. Think of the wheels of a tricycle. Which has fewer members, the set of wheels of the wagon or the set of wheels of the tricycle?

Think of a wagon. Now think of a wheelbarrow. Which has fewer members, the wheels of the wheelbarrow or the wheels of a wagon?

Many other comparisons which are familiar to the children may be used. Some suggestions are:

1. the stars in the sky and the stars in our flag
2. the stars in the sky and the moon
3. the jars containing paint and the easels
4. the books in our room and the books in the library.

Using an activity similar to that on page , have each child pair the members of his set with the members of those held by other children. This time ask all children to stand whose sets have the fewer members.

Prepare for games, dances, etc., which require teams with the same number of members on each team. Pair the members of the team to see if each team has as many members as the other or if one team has more members than or fewer members than the other.

Provide further experiences in pairing members of sets. On some occasions be certain that there are more or fewer members in one set than in the other. On other occasions there should be just as many members in one set as in the other.
Chapter 4

SUBSET OF A SET

Objective: To introduce the idea of subset of a set.

Vocabulary: Set, subset (set within a set).

Materials: Variety of cut-outs to use with magnetic and flannel board, construction paper (several sheets each a different color), jar of multi-colored beads, rhythm instruments, set of play dishes, real fruit, American flag, toys, teacher's "Treasure Box".

Note: The expression, "set within a set", may be used to bring meaning to subset as the idea of subset is being introduced to the children. When identifying subsets of a set, make clear that a subset may but need not consist of "like" members. You may find that it will be effective to do more "showing" than "telling". (Start with a set that includes two cups, a ball, a toy truck, and a doll. The two cups form a subset of this set; the ball and the truck form another subset.)

It is necessary to identify the set first before talking about a subset of that given set. We do not start with a subset and then think of a set.

Activities to help develop this concept:

Have a set of objects on the magnetic or flannel board; e.g., apples, stars, ducks, trees. Have the set identified. Move together a duck, a star, and a tree.
Do all these objects belong to the set that is on the board? (Yes.)
Put those objects back in place and group together an apple, two stars, and a tree.
Are all these objects members of the set on the board? (Yes.)
Yes, we can say that they are a set within our set.
Move all the ducks so they are together within the set.
Are these ducks a set within the set that is on the board? (Yes.)
Yes, we can say that the ducks are a subset of the set on the board.

Place the small dollhouse furniture on a table or on the rug. Ask a child to identify the set.

Jane, will you touch the members of a subset that could be used in the kitchen?
Leonard, will you touch the members of the subset that could be used in the living room?

Continue until all the furniture has been touched.

Is there someone who would choose a different subset for the living room?
A different subset for the dining room?

Assemble a set of toys that show the "Ways We Can Travel" --cars, trucks, wagons, boats, planes, etc. After the set has been identified, ask a child to touch (without moving) a subset he would like to work and play with during the day. Ask several other children to quickly touch subsets of their choice.
There is a little yellow truck on my desk. Is it a subset of this set of toys? (No. It does not belong to this set of toys.)

This same type of activity might be carried on using objects from the teacher's "treasure box". As the children sit in a circle on the floor, open the box and quickly take out things such as 3 "teddy" bears; soft, yellow chicks; unclaimed rings; small party-favor Japanese umbrellas; small ceramic animals, etc. Ask individual children to touch subsets of their choice, hoping they will take a mixed group.

Jim, in the lower right drawer of my desk there is a set of animal puppets. Are the members of that set a subset of the set of things in the treasure box? (No. They don't belong to the set of things in the treasure box.)

If classroom "helpers" are used, all of the helpers can be considered as a set of helpers. Identify the subsets of this set. (The helpers who water the plants in the room, etc.)

Refer to an imaginary family and create subsets of mother, father, and children--mother and father each being one-member sets.
Have as your set for the day, real fruit consisting of an orange, a grapefruit, an apple, a banana, and a tangerine.

Today as we look at each member of our set, let's see how many clues we can find that would help us to identify the subsets of our set if we could feel but not see them.

After the clues are identified, have the fruit passed around the circle so that each child can feel each piece of fruit. Then after either blindfolding a child or holding a newspaper between his eyes and his hands, place a piece of fruit in his hands.

Feel the fruit and tell us what subset you have in your hands.

Continue until many or all have had an opportunity to so identify a subset of the set of fruit.

A carrot or potato might be put into the hands of a more alert child to see if he would recognize the difference and know that he was not holding a subset of the set he had talked about as the activity began. There might be many children in the class who would enjoy participating in this little "joke".

On the flannel board, place a set that may include fruit, geometric figures, trees, flags, and animals. Ask a child to indicate a subset of fruit (which might include a red apple). Ask another to indicate a subset of all red objects (which might include the same red apple). Ask another to indicate a subset of fruit that grows on a tree (and the apple might again be included).
Place several colored blocks (or crayons, sheets of construction paper, etc.) on a table. Have the set described. Invite the children to form various subsets (red blocks, yellow blocks, some red and some yellow blocks, etc.) Ask if the red blocks are members of the set of blocks, if the yellow blocks are members of the set of blocks, etc., to focus attention back on the set with which we started.

Refer to the buildings in a town as the set of buildings. Identify the subsets. (For example, houses, offices, hotels, stores, schools, libraries, etc.)

In addition to separating the class into subsets of boys and girls, you may separate into subsets of children with curly hair, boys wearing red, girls wearing green, children with tie shoes, those who want to work in certain work groups, etc.

Identify various subsets within a set of multi-colored beads or parquetry blocks.

On the flannel board, arrange a set of felt or construction paper farm and zoo animals. After the set has been identified, ask one or two children to find specific sets.

John, will you make a subset of the animals we might find on a farm near here?
Mildred, please show us a subset of all the baby animals.
Next, ask individual children to form subsets and to tell what subsets they are forming. For example: a subset of animals that can go into the water (seal, turtle); a subset of animals that can climb a tree (a squirrel and a monkey); a subset of animals that like honey (bears).
Chapter 5
JOINING AND REMOVING

Objectives: To introduce the set operation of joining and its commutative property. To introduce vocabulary and meaning for removing a subset from a set and the meaning of remaining set.

Vocabulary: Join, remove, remaining.

Materials: Cows and sheep from small, wooden farm animals, small zoo animals, blocks, "wheeled" toys, dolls, doll clothes, name cards, materials for magnetic or flannel board.

Activities to help develop these concepts:
Have the sheep and cows identified as the sets to be used today.

You have had fun using these small wooden farm animals and the blocks. Today, let's pretend these blocks which I'll put on the table will form a fence around a pasture. We'll put the sheep in the pasture. Mr. Brown, the farmer, has milked his cows this morning and is now taking them to the pasture to join the sheep.

Mike, will you please open the gate so that the cows can join the sheep?

When we join one set with another, we form a new set. What are the members of this new set? (Each member of the set of sheep and each member of the set of cows.)
During the daily activities, there will be many opportunities to have one set joined to another. For example:

1. one set of picture books joined to another set of picture books.
2. one set of toys joined to another set of toys.
3. one set of blocks joined to another set of blocks.
4. a set of sand shovels joined to a set of sand pails.
5. a set of brooms joined to a set of mops.
6. a set of new games joined to the set of old ones in the cupboard.

On the rug or floor, make "a circus ring" using yarn, string, or rope. Have rubber or wooden zoo animals at hand (or mounted pictures of the animals if the objects are not available).

Let's go to the circus today. The lions will be the first animals to perform. Wade, will you be the lion-tamer and put your set of lions inside the ring?

Now that we've enjoyed their act, let's ask Glenn to put the tigers inside the ring to join the lions in a new act.

In our unusual circus, the elephants perform an act with the lions and tigers. Sharon is the elephant trainer who will join her animals to the others in the ring.

Now we have a new act using a set of ____ ? (lions, tigers, and elephants)

Does this set contain a subset of lions? (Yes.)
(Yes.) a subset of elephants? (Yes.)
(Yes.) a subset of tigers? (Yes.)
Ask children to join other sets to the set of animals in the ring and identify the new set. After two or three sets are joined in the ring, ask to have them removed so that a new act may start.

Place on the magnetic or flannel board a set consisting of trees, apples, flags, ducks, and rabbits. Have the set identified.

Leonard, will you remove the subset of apples?
Bill, are there any apples remaining on the flannel board? (No.)
Mildred, please remove a subset containing a flag, a duck, and a tree.
Mary, are there any flags remaining on the board? (Yes.) Will you remove the flags that have not yet been taken away?
Sharon, remove the remaining ducks and trees. Now, what objects still remain on the board? (The rabbits.)

We call this the remaining set. The remaining set is the subset which is left when we remove a subset or subsets from our starting set.

As the children are sitting in the circle on the rug or floor, provide each with a set containing three or four members (plastic cars, small plastic or wooden animals, marbles, small dolls, doll clothes, blocks, etc.).
You each have a set of toys that you like. Let's pretend you're going to a friend's home to play and are taking your set with you. Ruth, will you choose someone with whom you would like to play, join your set to hers, and tell us what new set you and your friend now have?

Continue until all children are paired or only one child remains without a partner. If the latter happens, let him join his set with the sets of the two other children. Name the new set.

Today I want us to see more pictures in our mind. We will be joining one set to another.

Think of a set of three hungry boys. Now join a set of three big ice cream cones to these boys. Can you see the new sets of boys and ice cream cones?

Think of a set of two girls. Now join to them a set of two new bicycles. Can you see the new set of girls and bicycles?

Think of a set of Halloween witches. Now join a set of black cats to the set of witches. What do we have in the new set we just made? (Witches and black cats.)

Think of a set of white kittens and red balls. Now remove the set of balls. Can you see just the set of kittens?

Think of a set of red kites and green balloons. Remove the set of balloons. Which set can you still see?
After giving several other examples, let the children suggest their own imagined sets. Ask them to think of joining one set to another or of removing a subset from a set.

Have name cards for children placed face down on a table. Ask a child to pick out one card and tell and show the group the name on it.

Sharon's name was picked out. She may make her choice of activity now and invite three others to join her in this activity. The set will now include Sharon and (Ruth, Mark, and Lisa).

Have another child pick out a name. Continue until children and activities are paired. Emphasize the word "join" when asking the child to choose three others to join him in the activity and have him name the new set of children.

Put the wheeled toys together in a group and ask to have the set described. Show the set of cars without moving it from the larger set.

Is this set of cars a subset of these wheeled toys? (Yes.)
Can we remove these cars from the set of toys? (Yes.)

Move the cars slightly away from the rest of the toys.

What set do we have remaining here on the rug? (The other wheeled toys.) If you know what we call the set that is remaining on the rug, come up one at a time and whisper the answer into my ear. (The remaining set.)
Using flannel board materials, tell a story about a pond in the woods where the animals come to drink.

A set of ducks comes to the pond and a few minutes later is joined by a set of rabbits. (We now have a set of ducks and rabbits.) Soon a shy fawn quietly joins the ducks and the rabbits. (a set of ducks, rabbits, and fawn). Some cardinals come flying down to join the set that is already there (a set of ducks, rabbits, deer, and cardinals).

Unknown to them, a hungry fox is lying in the deep grass at the water's edge. He greedily watches to see which member he can catch for his dinner, but before he can spring, the fawn smells him and bounds away, removing herself from the danger. (Remove fawn.)

Her movement frightens the rabbits who quickly hop away to their homes, thus removing any chance the fox has for a rabbit dinner. (Remove rabbits.) The cardinals see the fox and fly away calling, "Fox, fox!" in time for the ducks to fly away with them. (Remove birds and ducks.)

Sadly the hungry fox looks to see what animal remains for his dinner. What set does remain? (The empty set.)
Chapter 6
COMPARISON OF SIZES AND SHAPES

Objectives: To become aware of the vast differences in sizes of objects and to learn the meaning and use of comparison words: large, larger, largest, etc.

Vocabulary: Small, smaller, smallest; large, larger, largest; long, longer, longest; short, shorter, shortest; thick, thicker, thickest; wide, wider, widest; narrow, narrower, narrowest; big, bigger, biggest; close, closer, closest.

Materials: "Nest" of boxes or barrels, clothesline, yarn, balloons, paper plates, balls, umbrellas.

Activites to help develop these concepts:

A good comparison of the size of objects is found in the "nest" of boxes or barrels whereby the child opens a box and finds a smaller one inside, opens that and finds another smaller one, etc., until he at last finds the smallest box.

After the children have seen the boxes opened and noted that each is smaller than the one seen before, "scramble" them in a larger box or bag and have as many children as there are boxes pick out one without seeing the size. Allow a certain length of time for this "team" to put the boxes back together correctly. A three-minute egg timer might be used to designate the time allotment.
Tie two different lengths of clothesline rope or heavy twine to the legs of chairs and coil the remaining rope by the chair. Have children estimate which is the longer rope by the size of the coil. Then ask two children to pick up the ends of the rope and walk in a designated path as far as the rope will permit them. Was the estimate right? Recoil the ropes.

Bring out a third such coiled rope to see if the longest can be determined by looking at the size of the coil. Again check by having children walk with the ropes as far as they can go.

Repeat the activity using different materials (different lengths of rope, twine, yarn) in each coil to have children note the difference in the thickness of the material used. See if they will take this difference into consideration when estimating which coil will be the longest and which will be the shortest.

A similar activity can be carried out using white yarn and white envelopes. Cut the yarn in different lengths having some as long as other lengths, some shorter than and some longer than the others. Scotch tape one end of each piece of the yarn in separate long business envelopes. Loop the yarn back and forth in the envelope, leaving the free end hanging slightly out.
Again designate the path the yarn is to be carried, to see who has the shortest or the longest pieces of yarn and which pieces are the same length. A child may be used to hold an envelope while another walks with the yarn. If you have a rug, pin the yarn to it when the length is fully out of the envelope so the differences in lengths can be seen easily by all.

This same type of activity may be continued at another time by using a bright or dark yarn in a thin envelope so that a clue is obtained by being able to slightly see the number of loops or how closely the loops are arranged in the envelope.

Many opportunities for the understanding of the concepts of narrow and wide can be found in comparing the widths of: rickrack on dresses, the widths of the stripes in boys' T-shirts and girls' dresses, the widths of the turned up cuffs on the boys' jeans, the width of bracelets worn by the girls or the teacher, etc.

Unless you have a child in the class who has been made quite sensitive to his long or short feet, comparing the lengths of hands, feet, shoes, boots, and mittens brings meaning to the words short, shorter than, shortest, long, longer than, and longest.

If there are several pairs of books that look much alike and do not have distinguishing marks on them, bring them to the group.
I want you all to sit with your legs out in front of you. Let's look at the length of this pair of boots and then at the length of all the shoes in our room. See if you can decide who might be able to comfortably wear these boots.

Mildred, will you please take these boots to one person who you think they might fit? Good. Is there another person in our group who might wear these boots?

Jerry, can you find someone who has shoes that are shorter than these boots?

Sara, can you find someone who has shoes that are longer than these boots?

A similar activity might be carried on using mittens.

"I'm Thinking Of . . ."

I'm thinking of something that is bigger than I am. I'm thinking of a horse. Can you tell me something that is bigger than you are?

Now I'm thinking of something that is larger than the horse. I'm thinking of an elephant. Can you think of something that is larger than the thing you named before?

I'm thinking of something that is smaller than I am. I'm thinking of a kitten. Can you tell me something that is smaller than you are?
I'm thinking of something that is smaller than the kitten. I'm thinking of a piece of pie.
Can you think of something that is smaller than the thing you named before?

This type of thinking about relationships as to size can be used with wide and narrow; tall, taller than and tallest.

Turn 5 or 6 paper plates of a graduated size upside down and arrange on a tray so that only the largest shows.

What do you see on the tray? (A white paper plate.)
I'll remove the plate. Now what do you see? (Another plate.)
I'll remove it. Now what do you see? (Another plate!)

Continue until all plates have been removed.

How was I able to arrange these plates so that you saw only one plate at a time?

Many children will be able to tell you that each plate was just a little smaller than the one they had seen before. To help some see more clearly, you may want to ask a child to demonstrate by holding his hands just far enough apart for you to slip the smallest plate between them.

Show me with your hands what John will have to do before I can slip this plate that is just a little larger between his hands.

This plate is still larger than the last one. Show me how your hands will have to move now.
Continue until all plates have been used.

Scatter the plates around in a small space and have a child see how quickly he can rearrange them as they were.

Then have them scattered and ask a child to turn them over and arrange them with the largest on the bottom.

Paper plates of graduated sizes, "nests" of boxes or barrels, graduated sized beads, etc., might be made available for children to arrange and rearrange during the work period.

I have a balloon for each of you today. Let's stretch them before we start blowing so that we can blow them up more easily. You may blow them as large as you wish after we finish our little game, but for now I will want you to stop blowing when I say, "Stop!". Blow.

Have children blow until a balloon is about 3" in diameter.

Stop! Is your balloon larger than it was when we started? (Yes.)

Blow. Stop!
Is your balloon still larger than it was? (Yes.)
Blow. Stop!
Is your balloon the largest it has been since you started blowing? (Yes.)
Now you may blow your balloon as large as you wish. When you have it that large, you can fasten it with a rubber band you'll find on the workbench. I'll give you help if you need it.

Before the children come into the room or while they are busy elsewhere, place five or six umbrellas on a small table with the longest umbrella to the front and the rest arranged in graduated size to the smallest umbrella. (e.g., man's umbrella, woman's smaller umbrella, child's umbrella, a Japanese toy sun umbrella, and a small party-favor umbrella.)

What do you see? (An umbrella.)
Show me how far apart you think you'd need to stretch your hands to be able to reach from the point of the umbrella to its handle. Chris, will you come and see if your hands are stretched out enough to reach that far?

As he comes to the table, whisper to him to say nothing about what he sees behind the big umbrella but to pick up the umbrella and stand in front of the others.

Chris's hands were just about the right distance apart, weren't they? Chris, you may put the umbrella on the chair.
What do you see now?
Yes, you see another umbrella.

Continue with the remaining umbrellas in the same manner stressing that each is smaller than the one they saw before and that their hands are getting closer and closer together.
At another time, the same umbrellas might be used to demonstrate width. Open all the umbrellas and line them up according to graduated size with the handles facing away from the children.

Is each umbrella just as wide as the ones next to it? (No.)
June, will you please go and sit behind the widest umbrella. Did she choose the right one?
Can you see her? (No.)
Jack, will you sit behind the smallest umbrella.
Did he choose the right one? Can you see him? (Yes.)
Carol, is there another one that is wide enough for you to sit behind so that we can't see you. (Yes.) Can you see her? (No.)

Close the umbrellas and place them at random on the floor.

Tom, if we held the umbrellas up straight with the points on the floor, which one would be the shortest? Please show us. Sara, which one do you think would be next in size? Stand it up by Tom's and see if it is taller.

Continue in this manner until all are used. Speak of the umbrellas as being taller than and tallest.

With the children's help, arrange the class according to height from the shortest to the tallest. Using a roll of wrapping paper tacked to a bulletin board (if the space is narrow unroll just enough for use with
one or two children at a time) measure each child, mark his height, and write his name under the mark. After the children have had time to compare the heights as they wish, roll the paper and put it away until the last week or two of school. Then again mark the heights stressing how much taller each child is than he was at the time of the first measurement.

| June | Jean | Jim | Terry | Sue | Sally | Mike |

Another way to use the sense of feel to help establish the concept of larger than or smaller than is with various sizes of balls, going from a small jacks ball on up to a large beach ball. (Jacks ball, golf ball, a slightly larger rubber ball, tennis ball, etc.)

Hold this jacks ball tightly in your hand. (Palm downward.) Can you see the ball? (No.)

Hold the golf ball in the same way. Can you see the ball? (Yes, just a little.) Can you still curl your fingers around the ball and hold it comfortably in one hand? (Yes.) Let's use this rubber ball next. Can you hold it in one hand with your fingers curled around it? (Yes.) Has your hand spread farther out as you hold this ball? (Yes.) Is this ball larger than the others? (Yes.)
Continue until the child has handled all the balls. When he comes to the first ball he can't hold in one hand (palm downward so that the ball isn't just balanced on the hand), ask him to put the ball between his hands. Help him to get the feel of largeness by the feel of stretching as he handles larger and larger balls. Immediately after he has handled the largest, give him the smallest so that he may feel the sharp contrast.

The classroom equipment and the materials brought into the room by the children offer excellent opportunities for the comparison of length, width, and size.

My jumping rope is longer than yours.
I have the biggest ball.
You aren't as tall as I am.
I want a piece of paper the same size as Tim's.

These and many other statements or questions can lead naturally into a situation calling for an on-the-spot comparison.
Chapter 7

ORDERING

Objective: To introduce ordering of sets by using ideas of more than and fewer than.

Vocabulary: More than, fewer than.

Materials: Small objects, paper bags, perception cards, chart holder.

Activities to help develop these concepts:

Place two sets that are not equivalent on the magnetic board. Let one set consist of fruit and the other of birds.

Let the children assist you in pairing the members of the set of fruit with those of the set of birds to determine which set has the fewer members.

Which set has the fewer members? (The set of birds.)
Which set has more members? (The set of fruit.)
Let us arrange the set of birds here in this portion of the magnetic board and the set of fruit here. (See drawing for the arrangement.)
Select a set of flowers which has more members than the set of birds and fewer members than the set of fruit. Select a child to demonstrate by pairing members of the sets whether the set of flowers has more or fewer members than the set of birds.

Does the set of flowers have more members than the set of birds? (Yes.)
Does the set of birds have fewer members than the set of flowers? (Yes.)

Select another child to demonstrate whether the set of flowers has more or fewer members than the set of fruit.

Does the set of flowers have more members than the set of fruit? (No.)
Does the set of flowers have fewer members than the set of fruit? (Yes.)
Does the set of fruit have more members than the set of flowers? (Yes.)

Since the set of flowers has more members than the set of birds, but fewer members than the set of fruit, we shall place our set of flowers between the set of birds and the set of fruit like this.
Give children materials for three sets (buttons, shells, pick-up sticks), each containing a different number of members. Ask that they put one set in front of them and choose another and pair the members. If there are fewer members than in the first, the set is to be placed to its left. If more, the set will go to the right. (Above for fewer than, and below for more than, may be substituted for right and left.) Now they pair the third set with the first. If there were fewer put that set to the left and if more put it to the right.

Put small, familiar objects (from 1 through 5) in paper bags. Place the bag at random on the floor so that neither you nor the children know which bag holds a particular set. With all children facing a low table, or other area easily seen, pick a bag and place its contents in the center of the table. Ask a child to choose another bag, remove the set, and determine whether or not it has more or fewer members than yours. If his set has more members, have him put it on the right of the set on the table. You may wish to put these objects on colored paper or cards to help children identify the members of each set. If it has fewer members, place it to the left. Be sure that all children are sitting so that the directions, left and right, are correct from their viewpoint. Put the objects back in the bags and repeat the procedure several times. Do not count the sets at this time. If a child cannot determine whether the set has more members than or fewer than the set on the table, ask him to check by pairing the sets. Then repeat activity by continuing to arrange sets in order of numbers of members. For
example, after the first set selected is placed to the right or to the left of the starting set, have another child select another. It may go between the first two or to the left of one or to the right of the other.

Prepare perception cards from 1 through 5. Place face down in a box. Pick one card at random to put into the center of the chart holder. Ask a child to pick out another card and determine whether the set on his card has more or fewer members than are in your set. If his set has fewer members, have it placed to the left of your card. If it has more members, ask that it be placed to the right. Again, check to see that all children are sitting so that the directions, left and right, are correct from their viewpoint.

Using the perception cards, the teacher will pick a card from the box for the beginning of the game. A child will then pick another card and without looking at it place it either to the right or left of the first. This child will then ask another child if he has placed it correctly. If he did not, the second child will tell him why it is not correct and move the card to its right position. The second child then removes all sets and starts the game with a new card picked at random. (When the second child does not know if the set has more than or fewer members, ask him to check by pairing the members of the sets.) Again, arrangements can be made by continuing to draw cards without replacements as was suggested for the objects in paper bags.
Chapter 8

USING GEOMETRIC FIGURES FOR DIRECTIONS AND GAMES

Objective: To provide added opportunities for the children to see geometric figures in different positions and under varying circumstances.

Vocabulary: Between.

Materials: Colored construction paper cut into geometric figures, wire models, various shaped boxes, string, yarn, objects for "houses" and "people", large kindergarten beads, chairs, tables, collages.

Activities to help achieve desired results:

If there are times when you want children to sit in designated places around the rug, their names can be put on cut-outs from colored paper of various geometric regions. This affords another opportunity of seeing these shapes in different sizes and in different positions, e.g.,

![Diagram of triangular figures](image)

Use the rectangular and circular regions in the same way. These figures may also be varied as to size and color.
Have some objects close at hand that can be used in conjunction with your directions to the children. To add extra suspense and humor to the game, intersperse some funny directions that might be given to the children who know these place words but are still eager to be included in the game.

Jim, put this car between John and Tim.
Tom, put this paper in the lower, right-hand drawer.
Julie, bring us a book from the top shelf.
Sara, hold the book above Carol's head.
Now put it on her head.

Have a child (Jerry) stand several feet away from you and bounce a ball to him. Put another child (Joe) between you and the first child.

Can I bounce the ball to Jerry now?
(No, Joe is between you and Jerry.)

"Where is . . .?"

Where is your nose? (between my eyes and my mouth.)
Where is your mouth? (between my nose and my chin.)
Where is your neck? (between my head and my shoulders.)
Comparison with children

As the group is seated, ask one child to name two children of different heights to stand before the group. Choose another to quickly name a third child who he thinks will be between the other two in height. (Tall, taller, tallest.)

Put cut-outs of regions on magnetic or flannel board. Ask a child to arrange a piece of yarn on the board and describe its position.

The yarn is between the circle and the square.
It is above the rectangle.
It is around the circle and the triangle.

Follow the Leader

Directions can be used during the rhythm period to vary a marching activity. The "leader" must be alert to hear and respond to the direction and those behind him must follow the leader. Change leaders often.

Walk between the tables.
Walk behind the green chair.
Jump over the block.
Seat four children at a table, e.g.,

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Jean</td>
<td>Sara</td>
</tr>
<tr>
<td>Mike</td>
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If I serve Mike first, can I serve Jean without going past another child? (Yes.)
If I went the other way, who would be between Mike and Jean? (Jack and Sara.)

Start reading a story to the children and then hold a paper between them and the story but go on reading.

Why do you complain that you can't see the pictures? (The paper is in the way.)
Is the paper between you and the book? (Yes.)

Put the paper behind the book.

Can you see when the paper is in this position? (Yes, it isn't in between.)

Paint a simple closed curve (a road) on a large piece of wrapping paper. Make, or encourage a few children to make, pictures of various things along the road (house, trees, bridge, lake, barn, etc.). Have a plastic car to travel on the road. As a child rides in the car, ask questions such as:
This is Jean and this is her home. Can Jean go home without passing Ruth? Show me how she could do it. If she wanted to see Ruth before she went home, how could she go?

Let the string or yarn lie at random on the rug, forming a path. Place "Jean" at one end of the path and her home and a tree at the other. Somewhere between put another "house" for "Ruth".

Jean is taking this path to get to her home which is over by the tree. Ruth lives in this new house. Can Jean follow the path to her home without going past Ruth's home? Show us.

You may use one or more of several ways to enable children to "see" the playground; looking at it from the window, seeing it in the mind's eye, using objects in the room to represent the permanent playground equipment, or using the materials of the magnetic board to show the placement of the equipment.
When Jim goes down the slide, does he have to go past the sandbox to get to the seesaw? (Yes.) Is the sandbox between the slide and the seesaw? (Yes.)

When Joan goes down the slide, does she have to go past the garden to get to the tile. (No.) Is the garden between the slide and the tile? (No.) Is the tile between the slide and the garden? (Yes.)

Create collages using your cut-outs of regions.

Some questions about the collage might be:

What figure has the string around it?
The string goes between what figures?
Use perception cards, materials for the magnetic or flannel board, and construction paper to make visual aids such as the following:

Display only a few such figures at a time. Many questions could be asked, such as:

In which one do we see a triangle inside a square?
In which do we see two figures of the same shape but of different sizes?

Make cut-out felt or paper figures and cut out different figures from the inside. These can be used to fit into each other. Directions similar to these may be used.

Find the triangle that fits inside the rectangle.
Which square fits around the green circle?
Which circle will fit into the triangle?
Eye Puzzlers

Using the same figures as above, make two items to fit the inside or to fit around each shape, one larger or smaller and one that will just fit. The children will need to be more discriminating as they choose which blue triangle will fit around the circle or which yellow circle will fit inside a rectangle.

This will give more challenge to the brighter child who is ready for a little more difficult task.

With the above figures, the children might be directed to:

Place the largest triangle on top of the largest circle.
Put the smallest triangle inside the largest square.
Arrange the largest triangle between the smallest circle and the smallest square.
Chapter 9

USING NUMBERS WITH SETS

Objective: To help the child identify the number of members in certain sets. (These sets will be limited first to no more than 5 members. Some groups and some children will be ready to think about sets with more than 5 members. Yet, we ask these be limited to no more than 10 members.)

The systematic development of number ideas is undertaken in grade one of this program. However, kindergarten children already have some familiarity with whole numbers such as 1, 2, 3, 4, and 5. The suggestions in this section of the Commentary serve to provide meaningful number associations as background for future systematic development of number ideas.

Vocabulary: (No new words.)

Materials: Set objects, materials for magnetic or flannel board, perception cards.

Activities to help develop these concepts:

There are many opportunities in the daily program for the child to recognize, with or without counting, the number of members in small sets. (Although the following activities use only sets with few members, you may extend these activities to include sets with more members if you feel your children are ready.)
In some kindergartens, certain work areas are limited to 4 children at a time. A child soon is able to see at a glance whether or not there is room for him on the big blocks, in the dollhouse, at the workbench, etc.

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Using a card holder, put in pictures showing the various work centers. As each child chooses his work for the day, he will put his small name card to the right of the pictured activity. The children can see at a glance whether or not there is room for more at that activity. This provides another opportunity for seeing "twoness", "threeness", etc.

Singing in duets, trios, and quartets helps him to see 2, 3, or 4 in another situation.

Six children may sit at each table at lunch time, three on a side.
Arrange in various ways on the rug, 3 large blocks, 3 marbles, 3 balls, 3 paint brushes, and 3 small toys. Help the child to see that the objects, the arrangement, or the size did not change the fact that each set had 3 members. Similar arrangements may be made with materials on the magnetic board.

Use this same type of activity with sets containing 2, 4, and 5 objects.

There may be 4 pictures on a bulletin board which he might see as 2 and 2, or 4.

Perception cards might be used, e.g.,

When the set of girls with curly hair stands, there may be 3 in one place and 1 in another which the children can see as 4.
Name cards may be used to designate where each child is to sit as he enters the group. The number of children absent for the day will be the same as the number of the name cards that are not paired with children.

Cut-outs of different shaped regions might be used in the same manner.

Pairing with an imaginary set

Before the children come in, place many small set materials around the room. Have materials that you can use to arrange in sets of 2, 3, 4, or 5. It may be best to use a variety of objects in the arrangements to reinforce the idea that sets may be made of dissimilar members.

I have an imaginary set of (a car, a cowboy, and a bear).
I would like to have you find a set that has as many members as my set has.
I have a set containing (a book, a ball, a top, and a doll). Find a set that has the same number of members in it.

I'm thinking of a set whose members are (an airplane and a horse). Find a set to match mine.
Sometimes we talk about things that come in twos. We call the two shoes we wear a pair of shoes.
Can you think of other pairs?
Talk about clothing that comes in pairs; mittens, shoes, boots, etc.

Create a game or activity using the parts of the body that are paired; eyes, ears, lips, shoulders, arms, hands, hips, legs, knees, ankles, and feet.

Simon says, "Put your hands on your hips."
Simon says, "Put your hands on your ears."
"Put your hands on your feet." etc.

The singing game or chant "My Toes, My Knees, My Shoulders, My Head" might be fun and point up the twoness of the other parts mentioned because there is only one head.

Using materials for the magnetic board, perception cards, or felt pen on newsprint, arrange a set of three objects and at least three other sets so that one matches the first. Ask a child to find the set that matches the first set. For example, sets of objects might appear as illustrated above where sets (2) and (4) are shown to be equivalent to set A.
Use other sets of objects in the same way with fewer and with more members.

Ask children to find sets in the room that will match the model set. Have them pair the members of their set with the members of yours to see that each has as many members as the other. Be sure that children do not get the idea that the members of a set have to be like objects or that they must be arranged in the same way.

Rhythms and counting beats to the music offer many opportunities to create sets that are intangible. This will be a change from the normal perceptual sets. The music transmits feelings of movement for the body such as skipping, jumping, walking, etc. The faster tempos are counted quickly and often suggest running. Slower tempos of 1, 2, suggest walking. By responding to the counting we create the music or patterns of movement such as walking to the "1, 2; 1, 2; 1, 2" of the "Amaryllis". Marching develops from the "1, 2, 3, 4" of marches like "Yankee Doodle". For variety, play something of $\frac{3}{4}$ time such as the "Skater's Waltz". Ask if the children could march to this music. (No.) Tap out the beat possibly using the rhythm sticks in a swinging motion. Tap the floor each time "one" is counted. Gallop music and lullabies such as "Rock-a-bye-baby" make use of the $\frac{6}{8}$ time.

Individual rhythm instruments may be used to create such sets. The beats of a drum, triangle, etc., can be listened to and children may clap their hands to form a set equivalent to the one they hear.
Give each child a set of 5 objects.

I am going to put a set under this box while you have your eyes closed. When I ask you to open your eyes, put a set in front of you that you think may have as many members as I have in my set. Then I will show you what's under the box.

Open your eyes. If your set has as many members as mine, please stand.

Repeat the game a few times and then ask various children to be "it".

A Rainy Day Game

Have the children scattered around the center of the room. At the sound of a bell or a chord of the piano, they are to form into sets with 1, 2, 3, or 4 members. Ask one child in each group to call out his set number.

There are some singing games and many songs that use number. "Ten Little Indians", "This Old Man", "Six Little Mice", "Two Little Birds", "Six Little Chickadees", etc.

Place ten sets containing 4 small objects each, two sets with 3, and one set of 2 on a table or countertop. Show a perception card with the numeral 4 or show a set of 4 and ask that two children go to
the table, get a set with as many members as you have indicated, bring it back and arrange it for all to see. Ask that this be done by the time you slowly count to five.

When the sets of 4 have been used and the children can no longer find such a set, do not let them take members from another set but send them to a special box where they can get as many more members as they need to complete their set of 4.

This may be repeated using sets with more or fewer members.

Problem Solving

Give each child a set containing 5 small objects and a piece of 6" × 8" construction paper.

Today I'm going to tell you some stories. You will use the set materials to answer the questions I'll ask. After I ask the question, put your set answers under the paper so they won't be seen until it is time.

After you have given the story problem and the children have put the answering set under the paper, show the story using the magnetic board and then have the children check their answers and stand if the set is equivalent to the one on the board.

Mildred had two dolls and mother gave her one new doll.
How many dolls does Mildred have?

Jane has a book, a doll, and a top.
May I see how many toys Jane has?

Bill started to school with a ball and found one on the way.
How many balls did Bill bring to school?
Use perception cards illustrated in the list of **Materials** at the back of the book for set description of various types:

1. Recognizing sets with 1, 2, and 3 members
2. Using sets of 2 and 3 members to describe larger sets (a set of 6 members is made up of two sets of three members each or three sets of 2 members each, etc.).
3. Identification of subsets within sets.
4. Pairing members of two sets and discovering that one has as many members as the other or more than or fewer than the other.
5. Joining sets of few members and creating a new set; removing a subset to find the remaining set.
6. Recognize that a set made up of three members arranged in a large space represents the same number as a set of three members within a small space.

7. Identify two sets which have the same arrangement of members, as well as the same number of members.
MATERIALS

The following list of materials contains suggestions for visual aids helpful in the mathematics program. It is not exhaustive by any means and the teacher may find other materials to use. Also, not all of those listed are necessary.

MATERIALS FOR TEACHER DEMONSTRATION AND PUPIL USE

General classroom supplies: paper clips, rubber bands, furniture, paint brushes and paint, crayons, etc.

Flannel or magnetic board
Flannel board objects: geometric shapes, animals, etc.
Toys: dolls, cars, dishes, plastic animals, stand-up figures, doll clothes, etc.
Balls: a variety of sizes ranging from golf to beach
Rhythm instruments
Blocks of all shapes and sizes
Lincoln Logs
Seasonal materials: Indian corn, pumpkins, Christmas decorations, etc.
Games: dominoes, object lotto, puzzles, etc.
Paper bags
Boxes of various sizes and shapes
Perception cards
Empty cans of various sizes
Oatmeal box
Embroidery hoop
Plain ring; Hula Hoop if available
Cookie or donut cutter
Wire models of geometric figures
Sandpaper cut-outs of geometric figures
Balloons
Books
String, yarn, clothesline rope
Umbrellas of various sizes
Old magazines
(Note the list of "Individual Set Materials" and use as applicable.)

INDIVIDUAL SET MATERIALS

Bottle caps
Buttons
Pegs
Beads
Geometric figures made from construction paper
Crayons
Macaroni shells
Beans
Small toys: animals, cars, cowboys, surprises from cereal boxes, etc.
Tongue depressors
Corks
Straws
String, yarn
Marbles
Spools
Paper and wooden disks
Rocks
Shells
Plastic spoons and forks
One-inch-square wooden cubes
"Food for thought and stomach": marshmallows, cookies, crackers, etc.
General room supplies
Set containers: boxes, cottage cheese cartons, paper bags, envelopes, etc.

MISCELLANEOUS IDEAS

**Individual set boxes:** These may be made from any materials available. They have been useful in set manipulation done by the pupil. The following ideas have been suggested:

1. Boxes of various sizes which are convenient to store. If one wishes to, felt or flannel may be glued in the inside lid and thus create individual flannel boards for each pupil. Place the objects for use inside the box and children may manipulate these such as directed or during free-time activities.

2. Cottage cheese cartons are easily obtained and very good for storage. Use small individual set materials such as: beads, pegs, paper discs, bottle caps, etc. These may be stored inside the cartons as they are stacked.
3. Paper sacks of various sizes to accommodate almost any individual set material desired. These can be used with an element of surprise: grab bag.

**Perception cards:** These aids showing sets of various members are excellent sources for reinforcing concepts and vocabulary. The cards may be made by the teacher for demonstration purposes and also by the children for class work. The latter individual cards are particularly valuable when working with "pairing, more than, larger than, same as, etc." By keeping a variety of cards, the children will not tend to memorize the answers sought. At the same time interest and intrigue may result from the element of surprise. Some suggestions are listed below:

1. Pictures of sets from magazine advertisements, story and picture books. Mount these on heavy tagboard for demonstration purposes.

2. Geometric figures (individual and combinations) such as circle, triangles (three types), rectangle and square.

3. Various objects, such as geometric figures with an "X" placed in various positions to show "inside, outside, over, between, etc." For example: make a square and mark and "X" inside it. Ask the children where the "X" is located.
4. Smaller and larger figures to show comparison of size; individual or a complex group.

![Diagram of different size figures]

5. Sets of dots, geometric figures and other objects to compare set sizes (include a blank card depicting the empty set.)

![Diagram of various set configurations]

6. Sets which can easily be partitioned into subsets of like members, one member sets, etc.

![Diagram of partitioning sets]
BIBLIOGRAPHY

Children's Books Related to Mathematics


__________, *One Fish, Two Fish, Red Fish, Blue Fish*. New York: Random House, 1960.


## KINDERGARTEN VOCABULARY

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