TEACHER'S COMMENTARY UNIT NO. 53

MATHEMATICS FOR THE ELEMENTARY SCHOOL

BOOK 1

SCHOOL MATHEMATICS STUDY GROUP

School Mathematics Study Group

Mathematics for the Elementary School

Book 1

Unit 53

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Teacher's Commentary

REVISED EDITION

Prepared under the supervision of the
Panel on Elementary School Mathematics
of the School Mathematics Study Group:

Leslie Beatty	Chula Vista City School District, Chula Vista, California
E. Glenadine Gibb	State College of Iowa
William T. Guy	University of Texas
Stanley B. Jackson	University of Maryland
Irene Sauble	Detroit Public Schools
Marshall H. Stone	University of Chicago
J. Fred Weaver	Boston University
Raymond L. Wilder	University of Michigan

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PREFACE

The increasing contribution of mathematics to the culture of the modern world, as well as its importance as a vital part of scientific and humanistic education, has made it essential that the mathematics in our schools be both well selected and well taught--at all levels, from the kindergarten through the graduate school.

With this in mind, mathematical organizations in the United States cooperated in the formation of the School Mathematics Study Group (SMSG). The general objective of SMSG is the improvement of the teaching of mathematics in grades K-12 in the schools of this country. The National Science Foundation has provided substantial funds for the support of this endeavor.

One of the prerequisites for the improvement of the teaching of mathematics in our schools is an improved curriculum--one which takes account of the increasing use of mathematics in science and technology and in other areas of knowledge, and at the same time one which reflects recent advances in mathematics itself. Among the projects undertaken by SMSG has been that of enlisting a group of outstanding mathematicians, educators, and mathematics teachers to prepare a series of sample textbooks which would illustrate such an improved curriculum. This is one of the publications in that series.

The development of mathematical ideas among young children must be grounded in appropriate experiences with things from the physical world and the immediate environment. The text materials for grades K-3 provide for young children an introduction to the study of mathematics that reflects clearly this point of view, in which growth is from the concrete to the abstract, from the specific to the general. Major emphasis is given to the exploration and progressive refinement of ideas associated with both number and space.

These texts for grades K-3 were developed following the completion of texts for grades 4-6. The dynamic nature of SMSG permitted serious reconsideration of several crucial issues and resulted in some modification of earlier points of view. The texts for grades K-3 include approaches to mathematics which appear to be promising as well as approaches whose efficacy has been demonstrated through classroom use.

It is not intended that this book be regarded as the only definitive way of introducing good mathematics to children at this level. Instead, it should be thought of as a sample of the kind of improved curriculum that we need and as a source of suggestions for the authors of commercial textbooks. It is sincerely hoped that this and other texts prepared by SMSG will lead the way toward inspiring a more meaningful teaching of Mathematics, the Queen and Servant of the Sciences.

Based on the teaching experience of elementary teachers in all parts of the country and the estimates of the authors of the revisions, it is suggested that teaching time be approximately as follows:

BOOK I

Chapter	Time
1	2-3 weeks
2	4-6 weeks
3	1 week
4	2-3 weeks
5	2-3 weeks
6	3-4 weeks
7	6-8 weeks
8	2-3 weeks
9	3-4 weeks
10	3-4 weeks

Teachers are urged to try not to exceed these approximate time allotments so that pupils will not miss the important ideas contained in later chapters.

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Chapter I

SETS AND NUMBERS

Background

Why sets? We know that most of an elementary mathematics program is concerned with arithmetic, and that this begins with counting and addition and subtraction. Why not begin with counting and addition? Why start with the idea of sets? These are questions that you may ask before beginning to teach this chapter, and such questions demand an answer.

One answer to these questions is that a set is what you count. We count 6 cats, or 6 boys, or 6 ice cream cones. Yet the concept of the number 6 does not depend on cats, or boys, or ice cream cones. The thing that is common to all of the collections of objects which we count is that they are sets. The notion of set will recur throughout the entire mathematical program and it is wise to introduce the term early and to use if effectively in building the notion of number. We shall also see that the simplest descriptions of additions and subtraction--indeed the classical descriptions--are in terms of manipulation of sets of physical objects.

The basic notion is that of a set of physical objects. Thus we consider the set of all children in a room, a set (or a bunch) of grapes, or a set of books. A set is completely specified when its members are specified, and the members of a set belong to it. Thus John may belong to the set of pupils in the classroom, and the teacher is a member of the set of people in the room. There need be no relation between the members of a set. We may have a set consisting of two oranges, an apple, and an umbrella.

There is one very special set. Suppose a club is going to have a program and the members are volunteering for the various committees. Then a number of sets are automatically created: the set of volunteers for the refreshments committee, the set of volunteers for the decorations committee, and so on. But what of the set of volunteers for the clean-up committee? We all know this may often be the empty set. There are no members in this set. The empty set has many other descriptions. It is the set of purple cows, or the set of green-haired boys. There is just one empty set. Later we will say that 0 is the number of members of the empty set.

Another curious sort of set is a set which has just one member. We need to consider sets of this kind because we are going to use the number one. If Alexander, Balthazar and Constantine are the members of a set and Constantine and Alexander leave in a huff, the set which remains has just one member. We will eventually relate this situation to the number sentence: $3 - 2 = 1$.

The next task is to prepare for the concept of number. To understand how we do this let us consider a related problem. How do we convey the notion of red? Suppose a child is beginning to learn about color, and we want to explain the color red. We would surely point to, or touch, all of the red objects around, trying to lead him to conceive of the color red as the property which is shared by all of these objects.

What is our conception of the number 7? It is surely the property which is shared by all of the sets having 7 members*. But this raises a difficulty. Can we talk about all of the sets having a certain number of members without having the notion of number? Even simpler: can we decide when two sets have the same number of members without knowing how to count them? Can you discover whether there are the same number of boys and girls in a room without counting? Can you find out whether there are the same

---

* We might go even further and define the number 7 as the collection of all sets having 7 members. Using the ideas which are explained in the next two paragraphs we can state this definition without using the notions of "7 members", and say: 7 is the collection of all sets equivalent to:

number of cups and saucers on a shelf without counting? The solution to our problem is clear. We can pair each boy with a girl, and we can pair each cup with a saucer. In fact, there is a way of deciding whether or not two sets have the same number of members without knowing anything about number. This process is our next concern.

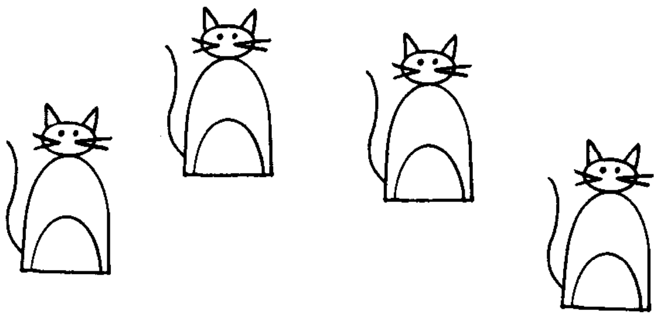
Suppose we are given two sets which we wish to compare. We may pair each member of the first set with a member of the second set as long as possible. If we run out of members of the first set but not of the second we know that there are fewer members in the first set than the second; if both sets are exhausted at the same time we say the first has as many members as the second, and that the second has as many members as the first, or that the sets are equivalent*; and if the second set is used up first we say there are more members in the first set than in the second. We can describe these possibilities in a slightly different way. We try to set up a one-to-one correspondence between the sets. If we succeed, the sets are equivalent.

For example, if we want to compare the set of boys in the classroom with the set of girls, we can form couples as long as possible. If we run out of boys first, there are fewer boys than girls; if we run out of girls first, there are more boys than girls; if everybody has a partner, then there are as many boys as girls, and the set of boys is equivalent to the set of girls. Notice that counting is not necessary.

---

* Equivalent is not synonymous with equal. Two sets are equal if and only if they have the same members; i.e., the sets A and B are equal if and only if every member of A is a member of B and every member of B is a member of A. For example: the set consisting of the President, Vice-President, and the Secretary of State equivalent to the set consisting of the British Prime Minister, the Chancellor of Exchequer, and Minister of War, but these sets are certainly not equal.

These manipulations with sets are intended to establish the concept of number. Children frequently come to the first grade able to "count" to a hundred or more, but they may have failed completely to connect the words for the numbers with sets of the appropriate size. Our concept of number is just this: The number of members of a set, or just the number of a set, is that property common to all sets equivalent to it. To us, four is the property common to all sets equivalent to



or to

We see from the pictures above that the set of cats is equivalent to the set of mice--it is easy to set up a one-to-one correspondence between these sets. However, these two sets are not equal. A set of cats is a very different thing from a set of mice. The number of cass is equal (not equivalent) to the number of mice.

The notion of equivalence thus underlies our concept of number. We will see that the notions of "more than" and "less than" lead us in a natural way to the idea of inequality of number.

After the vocabulary and the necessary notions of set manipulation have been established we begin the process of assigning spoken names to the numbers. (We sometimes call these oral numerals.) This proceeds roughly as follows: We first learn by sheer memory (some rote drill, at last!) the spoken words for 1, 2, ... , 10. We then use this sequence of words--which at first is really a sequence of nonsense syllables--by touching successively the members of a set, saying the words in succession until every member of the set has been touched exactly once. The last word spoken names the number of the set. Later we replace touching by pointing or just looking, and we think of the names rather than saying them. This entire procedure is called counting, and is considerably easier to learn than to explain. The mathematical foundation is simple. For example, 8 is the number of members in the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

We develop skill in counting by counting many sets, and we hope that, at this stage, the children will learn to count sets of at most ten objects with relatively few errors. For the smaller numbers, 0, 1, 2, 3, 4, and 5, we hope for a stronger identification. With enough practice children can perceive, without counting, the number of members in a set having no more than five members. We try to build this perceptive ability by many examples of such sets arranged in different sorts of patterns.

There are two important facts which children should discover in connection with counting. First, the order in which a set is counted is immaterial. Another child counting the same set but starting with a different member and proceeding in a different order will get the same number. Second, if two sets are equivalent, counting will yield the same result. This last fact shows us that the name we assign by counting a set really depends only on the number of the set.

We may notice (but not necessarily point out to the children) that, essentially, we never count beyond ten. Because of our very clever system of numeration we can name the number of members of any set by repeatedly counting sets of ten.

The ordering of sets shows us how to order number. We say that 5 is greater than 3, and that 3 is less than 5 because, if the members of a set of 3 are paired off with the members of a set of 5, there will be members of the latter set left over. Notice that if one set has more members than another, then the number of the first set is greater than the number of the second set; if the first set has fewer members, then the number of the first set is less than that of the second; and if sets are equivalent then they have the same number. The words "more than," "fewer than," and "equivalent" refer to sets. The corresponding words for number are "greater than," "less than," and "equals."

I-1. Set and member

Objective: To introduce the ideas of set and member of set and the use of these terms to describe things we observe around us.

Vocabulary: Set, member.

Background Note:

A set is just a bunch of things. The things are called members of the set and we say that they belong to the set. If there is a pencil, a book and papers on a teacher's desk, then a pencil is a member of the set of objects on the teacher's desk, but a chair is not a member of this set. The members of a set may have nothing to do with each other. Thus, we may have a set whose members are an apple, an orange, and an umbrella.

Materials:

1. A variety of small objects: books, blocks, pencils, crayons, sheets of paper, paint brushes, scissors, game boxes, beads, pegs, balls, clothes pins, ceramic tiles, bottle caps.
2. A variety of flannel board objects: animals, fruit, stars, trees, story-book characters, stick figures, etc. (Exclude geometric shapes at this time.)
3. Magazine pictures of collections: family, automobiles, telephones, clothing, food, toys, planes, trains and any other appropriate illustration.

Suggested Procedure:

Use the terms "set" and "member" informally but systematically with the children before any formal class work on sets.
For example:

This is our set of books.

Is this your set of sea shells? Is this shell a member of your set?

Is John a member of the set of children playing ball?

Children are familiar with sets of dishes, silverware, blocks, tinker toys, etc. Use these examples when first developing meaning of the term set. The following may be helpful as a guide.

> Which children helped to get the table ready for dinner last night?
>
> What do you put on the table? (Plates, glasses, silverware, etc.)
>
> We usually have to be careful with the set of dishes because dishes can break. What do you have in your set of dishes at home, Angie? (plates, cups, saucers, etc.)
>
> Angie's set of dishes is a collection of several objects----plates, cups, etc.

While Angie lists the objects, you may have flannel board materials ready and place them on the flannel board as each is mentioned. This activity should keep the class's attention and provide reinforcement. Show the children a collection and ask for descriptions of other collections. The replies may include collections of stamps, coins, sea shells, butterflies. Refer to each of these as a set as well as a collection. Ask the children to look around the room and describe sets of objects they see. At first the children may hesitate because of the new vocabulary. Help them by describing sets and having them guess what they are. Note the game "I'm Thinking of a Set" under "Further Activities" of this lesson. Some of the following sets may be described: chairs, tables, books, paint brushes, crayons, science-table displays, game boxes. These descriptions of sets should be explicit.

> We have many books in our room. How can we talk about these books? (A set or collection of books in our room.)
>
> Let's look at this set of books in the room. What books do you see? (Set of reading books, library books, etc.)

There is little difficulty in introducing the term "member". The children are members of the class; they belong to it. Some children may belong to community recreation groups. Soldiers are members of the Army. Each child is a member of his family. The family is a collection of people; each person in the family is a member of his family.

The ideas of set and member should be used throughout the day in classroom activities. The vocabulary can be used as opportunities arise.

Further Activities:

1. Art activity: Have children draw pictures of sets and orally name their members. Suggestions: child's own family, story-book family, toys, things that grow in the yard.

2. Oral reading of a story dealing with a family or set of some sort.

3. A class project of creating a science-table display with collections of rocks, sea shells, leaves, and butterflies.

4. Game: (This is time-consuming.) The teacher says, "I'm thinking of a set of - - - ?" If the answer is correct, the child becomes the leader and thinks of a set. If he is incorrect, further description is given until the set is identified.

5. Captions posted around the room:
   Here is a set of things that grow.
   This is a set of shells.
   Here is a set of rocks.
   This is a picture of the members of Sue's family.

6. The daily calendar can be used in conjunction with the vocabulary introduced. (The set of days in the week, the set of days we go to school.)

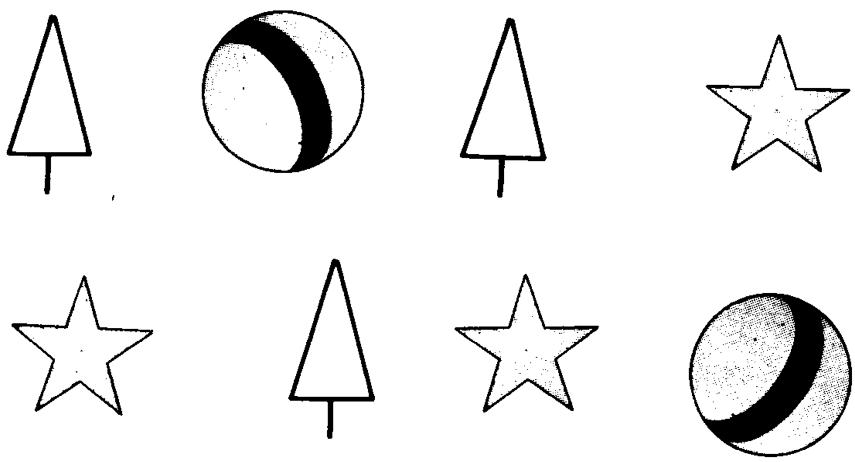
7. Four or five children are assigned a show-box or cigar box as "homework". They are to bring a set which can be used for the classwork the following day. Some discussion may be needed concerning size, quantity, etc. o of items which could be brought for this purpose. This activity is helpful in providing a variety of materials used and serves in a small way to familiarize parents with the child's work.

The following lesson is suggested in conjunction with "the family" presented in most children's readers. Use illustrations and make references to this family to clarify the discussion.

We read a story about a family today. The family is a collection of people. Is that a set? (Yes.) Who were the members of this family? (Use flannel board material or pictures to suggest mother, father, etc.) Each one in the family is an important member of it. Just as each person in the family is a member of that set, so is each thing in any set called a member (e.g., children are members of our class.) Let's think of some of the sets we found in our room and name their members. (Review the sets previously discussed and decide what their members are. First describe the set; then name its members.)

Point out that some things are <u>not</u> members of some sets. For instance: The books are not members of the set of paint brushes; Johnny is not a member of the set of girls. Children enjoy answering such questions as: "Is the flag a member of the set of books?"

Be sure to emphasize that the members of a set need not be the same kind of objects. You might use a flannel board illustration of the following sort.



Be sure that the classroom activities includes considerable manipulation by the children of sets of objects. Worksheets may be used (you might ask the children to draw a ring around the set of stars, or the set of trees, on a worksheet) but physical manipulation of objects is essential. Key relationships that children need to understand are extremely difficult to convey in pictures because the pictures do not permit movement of the objects shown. Furthermore, the actions of the children contribute to learning.

I-2. The empty set

Objective: To develop the idea of the set with no members.

Vocabulary: Empty set.

Background Note:

Zero is the number of members of the empty set.

Materials:
1. Small objects: disks, paper clips, keys or like material.
2. Four containers (covered boxes, bags) in which to place collections of materials.

Suggested Procedure:

A concept of the set with no members can be developed through an activity such as the following:

Place collections of materials in three of the four boxes. Begin the lesson by observing the four boxes and saying that we are looking for a special set in one of the boxes. Have one child at a time select the box that he thinks is the one that they are trying to identify. Ask him to open the box and describe the set of objects in the box and also to name the members of the set. (Sets may consist of members which are very dissimilar, they may be dissimilar only in color, e.g., red, yellow, and blue pencils, or the members may be similar.) The "special" box will have a set which has no members. After all boxes are open, observe the sets in the four boxes.

> What box contained a set with no members? (Indicate the box.)
>
> When there are no members in a set, it is called the empty set.

Then ask the children to describe other sets in the room which have no members, for example, the set of grandfathers, the set of watermelons, the set of tigers. You may wish to use children's pockets instead of boxes or bags, to repeat the kind of activity and again observe the set with no members.

I-3. Pairing and equivalence

Objectives: To introduce the idea of pairing members of sets and the idea of equivalence of sets.

Vocabulary: Pair, equivalent, as many as.

Background Note:

Two sets are equivalent if we can pair each member of the first set with just one member of the second set in such a way that every member of the second set is used. The notion of equivalent sets is fundamental in building number concept, and the most important fact about counting is that if two sets have the same number of members then they are equivalent.

Materials: 9" x 12" paper, crayons, name cards, materials for the flannel board, small objects for individual use.

Suggested Procedure:

The following lesson concerns pairing the members of a set of children with the members of a set of name cards. You may prefer to use a set of boys and a set of girls, or a set of cups and a set of saucers, or a set of children and a set of chairs, or some other one of the many pairing situations which occur in the classroom.

Have name cards for only those children present so that sets of cards and children are equivalent.

> I had a set of name cards for the children in our class. I have removed the set of cards of the children who are absent. What cards do I have now? (The set of name cards of the children here.)

Ask the children to suggest ways that the cards might be passed out.

> As I hold up your card, please come, get the card and put it on your desk.

We say that each member of the set of cards is paired with a member of the set of children.

Is each name card paired with a child? (Yes.)
Is each child paired with a name card? (Yes.)

When members of one set are paired with members of another set and there are no members left over, we say that there are just as many members in the first set as there are in the second set. We call these equivalent sets.

At the top of the flannel board put a set of objects. This set might consist of two bananas, two apples, an orange and a lemon; also, another set of geometric shapes consisting of two circles, two triangles and two squares. Have the two sets described.

Let's see if we have equivalent sets of fruit and geometric shapes. What can we do to find out? (Pair the fruit with the shapes.)

Alice, will you please start the pairing using one member of each set? Choose any geometric shape and any kind of fruit that you would like.

Continue this process until the sets are paired. Stress the fact that here it makes no difference which member of one set you choose to pair with a member of the other set.

Are the sets equivalent? (Yes.)

On the flannel board, place a set of five trees and directly below a set of seven circles so that the first five members of each set are obviously paired. Have the sets described.

Is the set of trees equivalent to the set of circles? (No.)

What can you do to make the set of trees equivalent to the set of circles? (Supply more trees to pair with the circles, or remove some of the circles.)

• * Each child will need six or seven set objects on his desk. You will need an equal number of objects for the flannel board.

> Let's see if you can pair the members of a set on your desk with the members of a set I put on the flannel board.

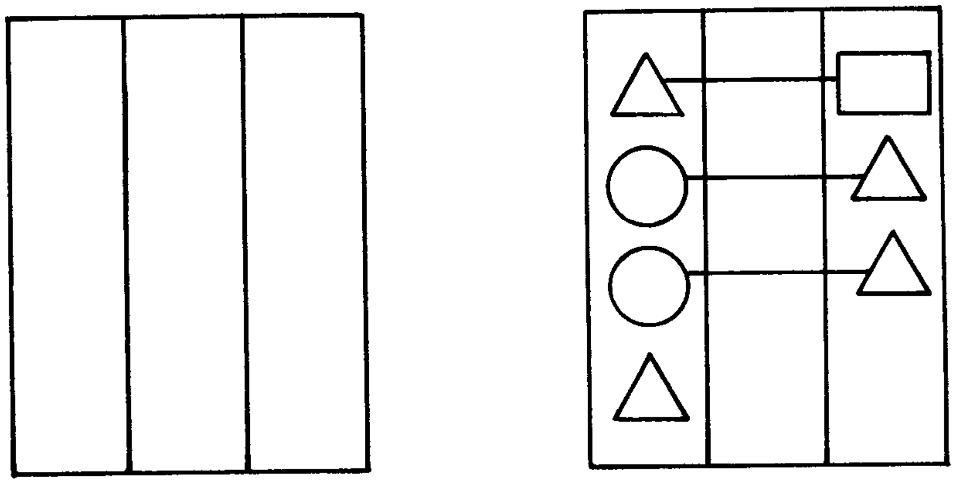
Then you should place one object (rabbit) on the flannel board. Ask the children to put their equivalent sets in the middle of their desks so that you can quickly see them.

> I will put another object with my set. Now I have a new set. Is your set equivalent to my new set? What will you need to do to your set?

Remove the objects and start again with a set of two rabbits. This time remove a member of your set and continue as before. Use no more than two or three members in your set so that the child can see it without counting.

• Distribute 9" X 12" papers that have been folded into thirds. Have each child make a picture of a set on the left third of his paper and fold that third so that his work does not show. The paper is then passed to another child who makes a set on the right third without looking at the other set.

The illustrations can then be collected and used for group work in finding sets which are equivalent. If desirable, they could be marked.



* This notation is used to denote a variation in either the activity used or the idea being developed.

<u>Pupil's</u> <u>book,</u> <u>pages</u> <u>1</u>, <u>2</u>, <u>3</u>: Pair members of the sets by drawing lines. Decide whether or not the sets are equivalent.

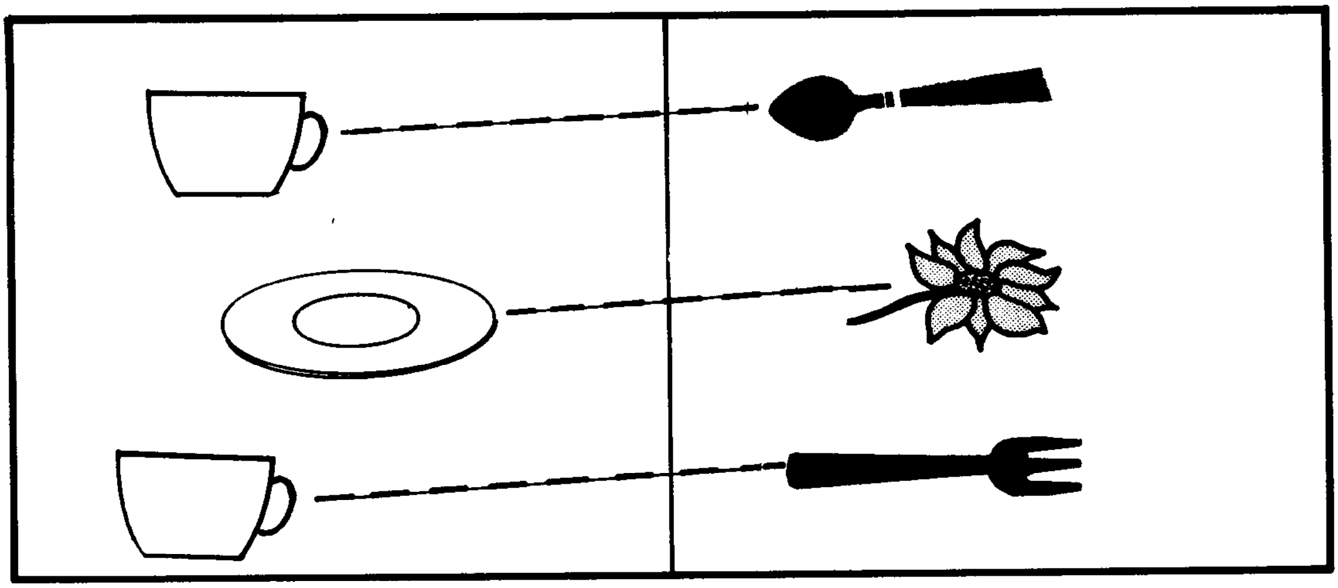
<u>Pupil's</u> <u>book,</u> <u>page</u> <u>4</u>: Show a set of things on the right which is equivalent to the set on the left. Children may use X's for members of the set.

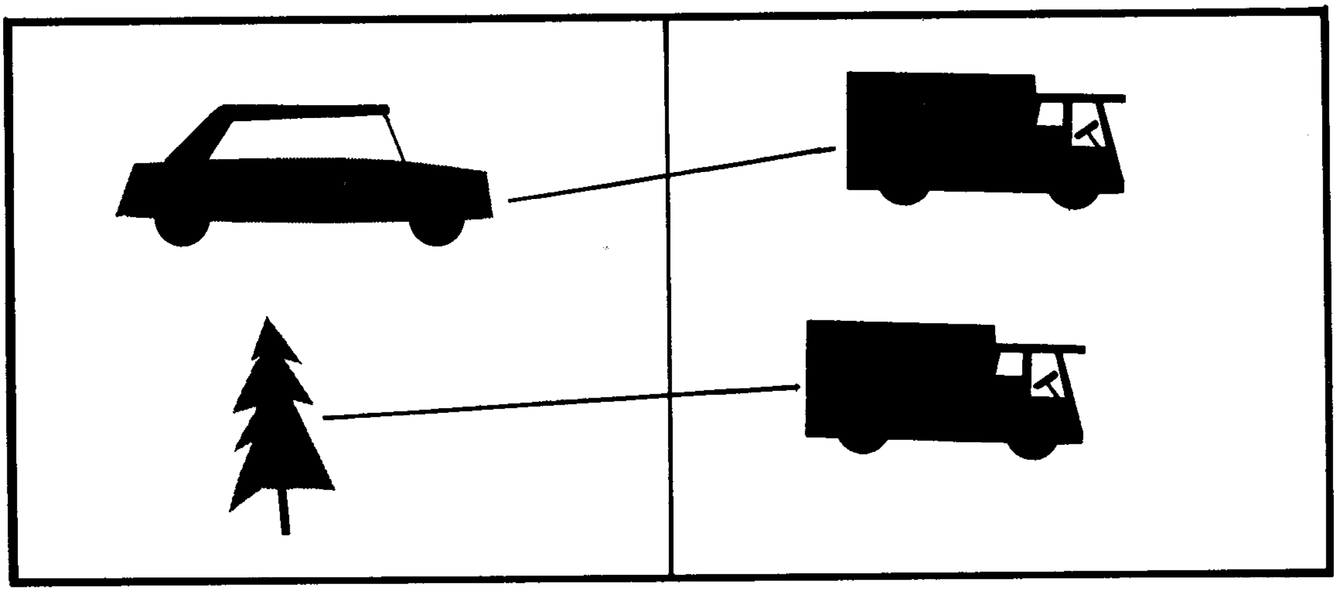
<u>Further</u> <u>Activities</u>:

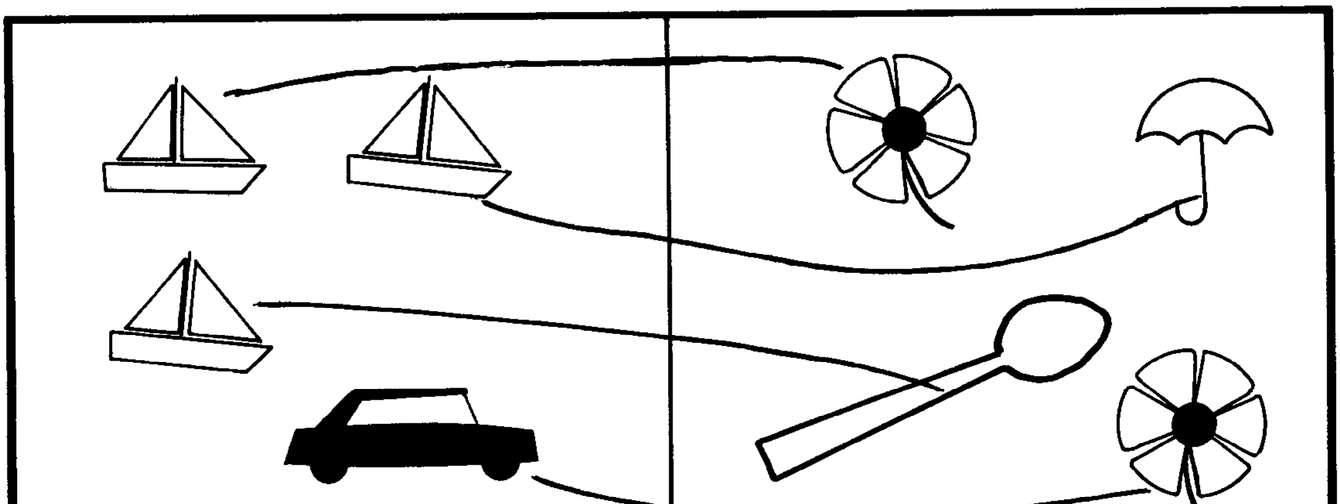
Many classroom situations offer examples of pairing: the pairing of chairs and children, the pairing of books and children in the reading circle, the pairing of children for singing games and for relays.

<u>NOTES</u>

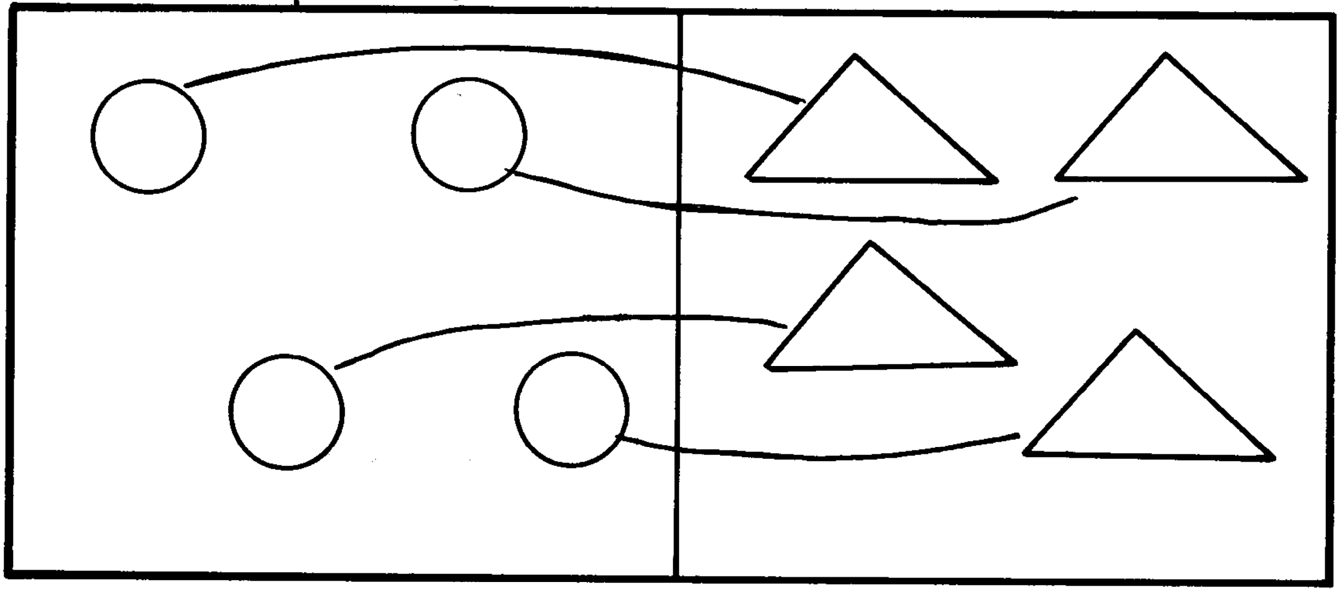
Are the sets equivalent?

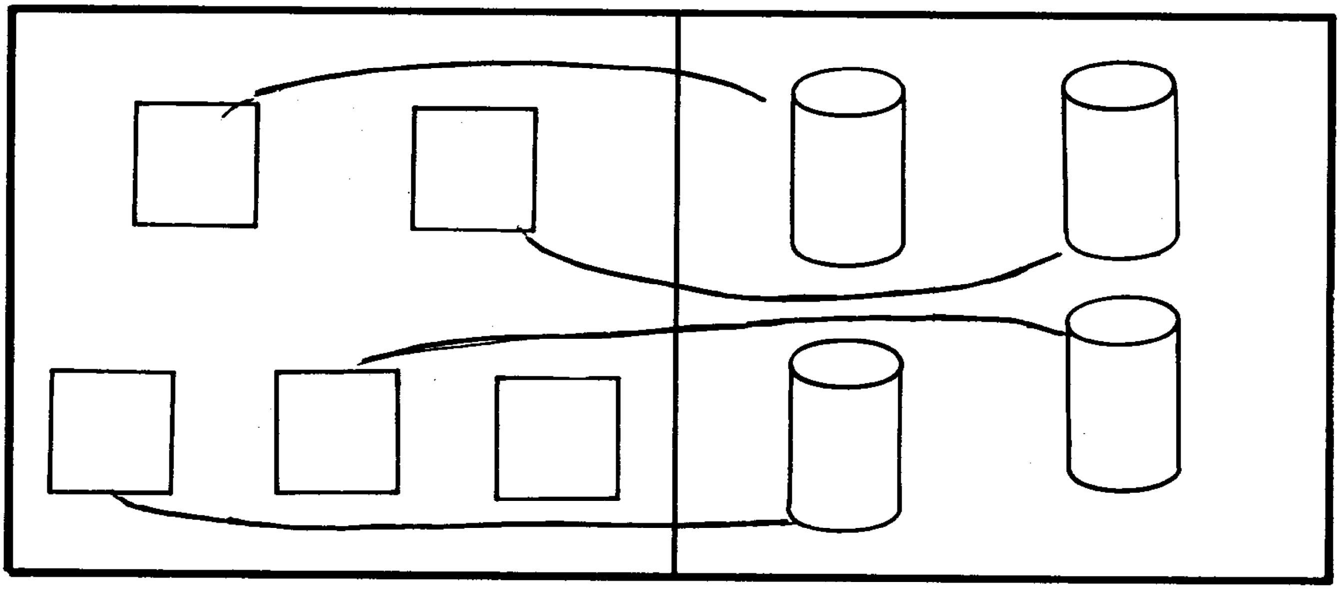


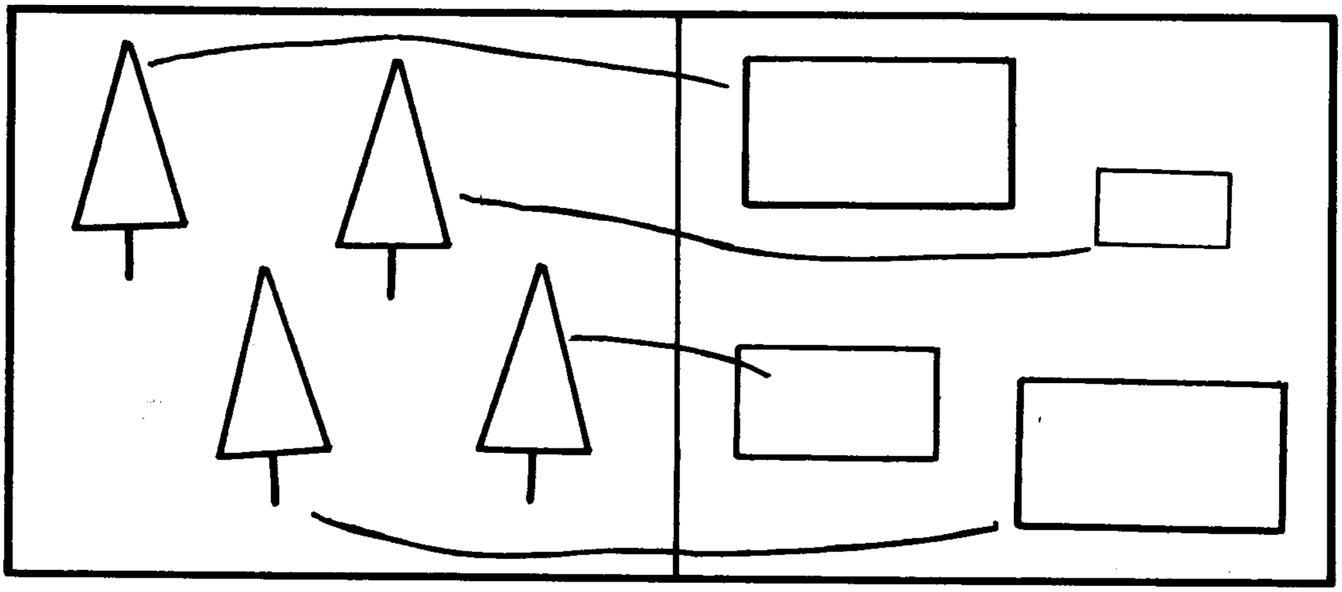




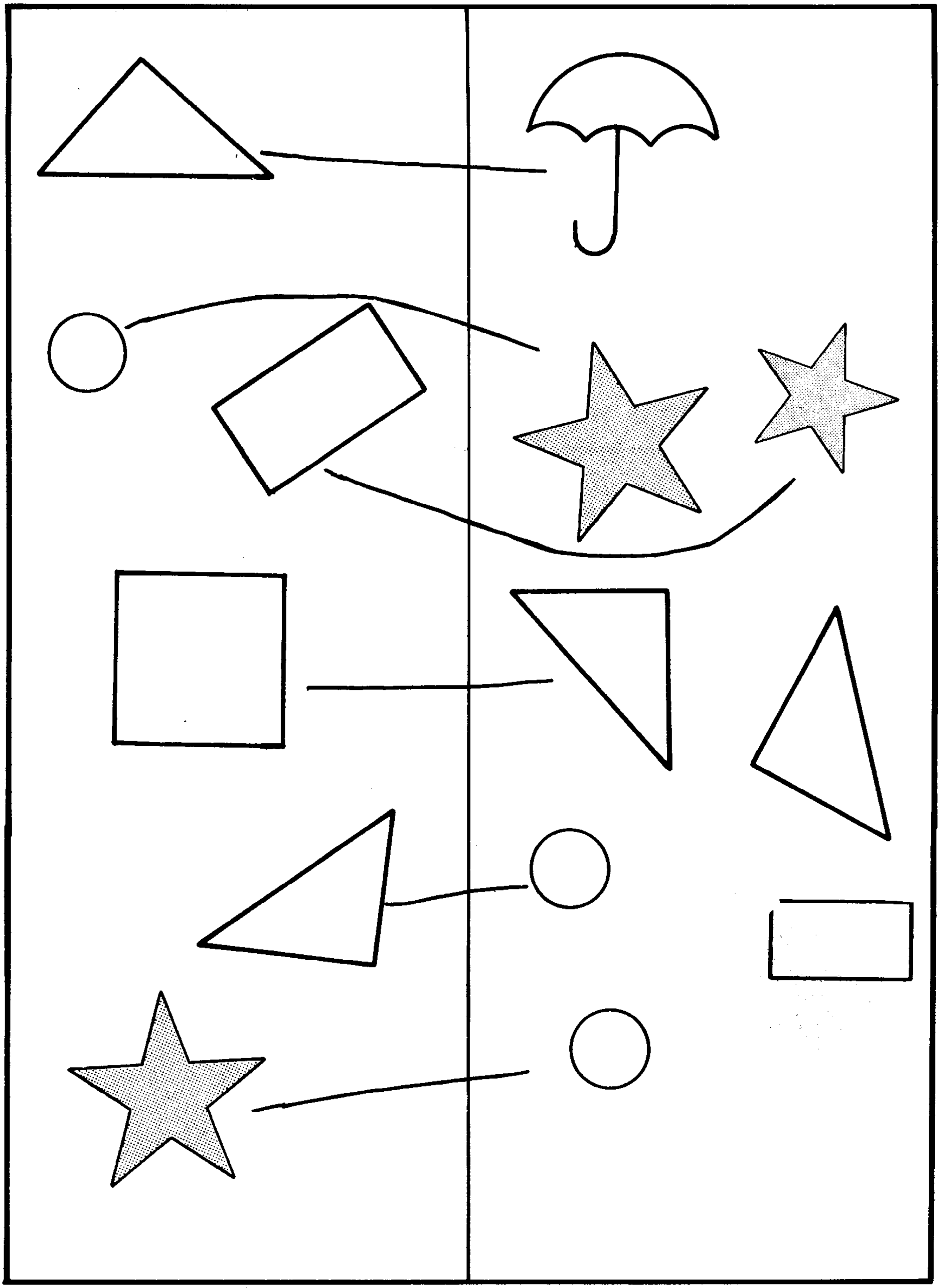
Are the sets equivalent?



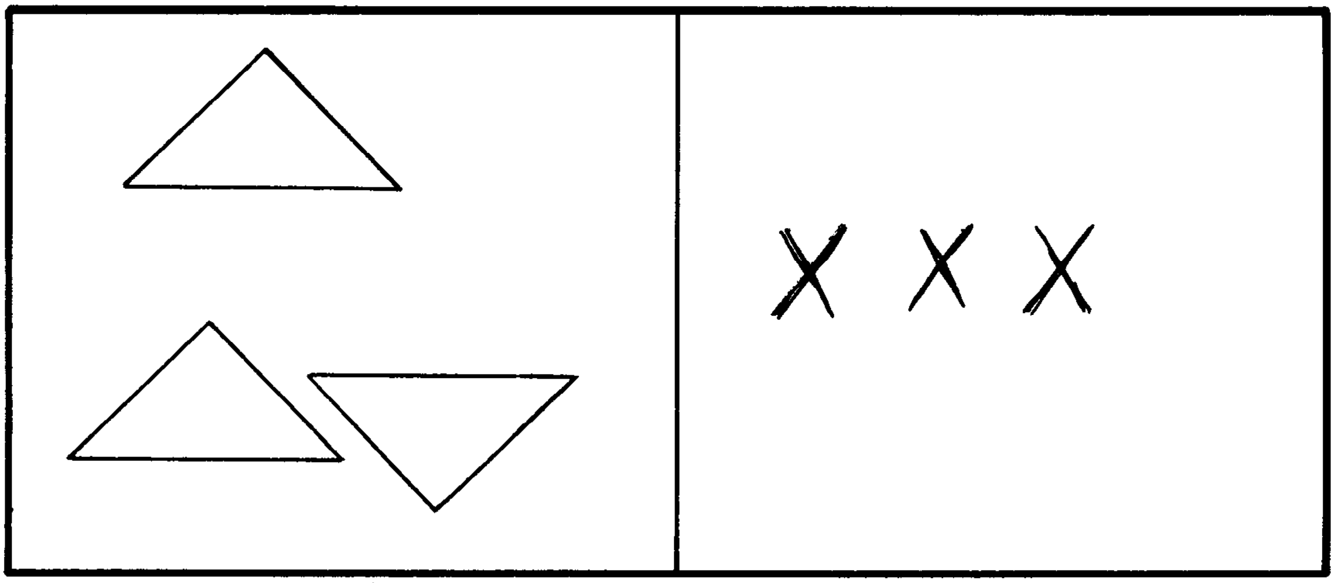


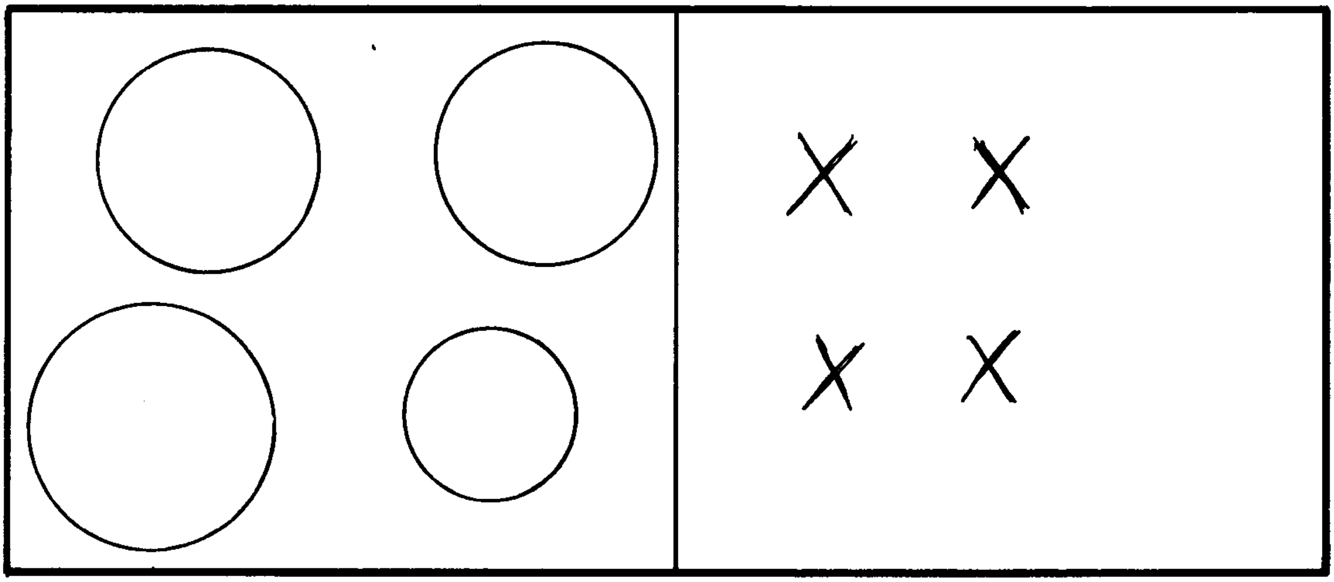


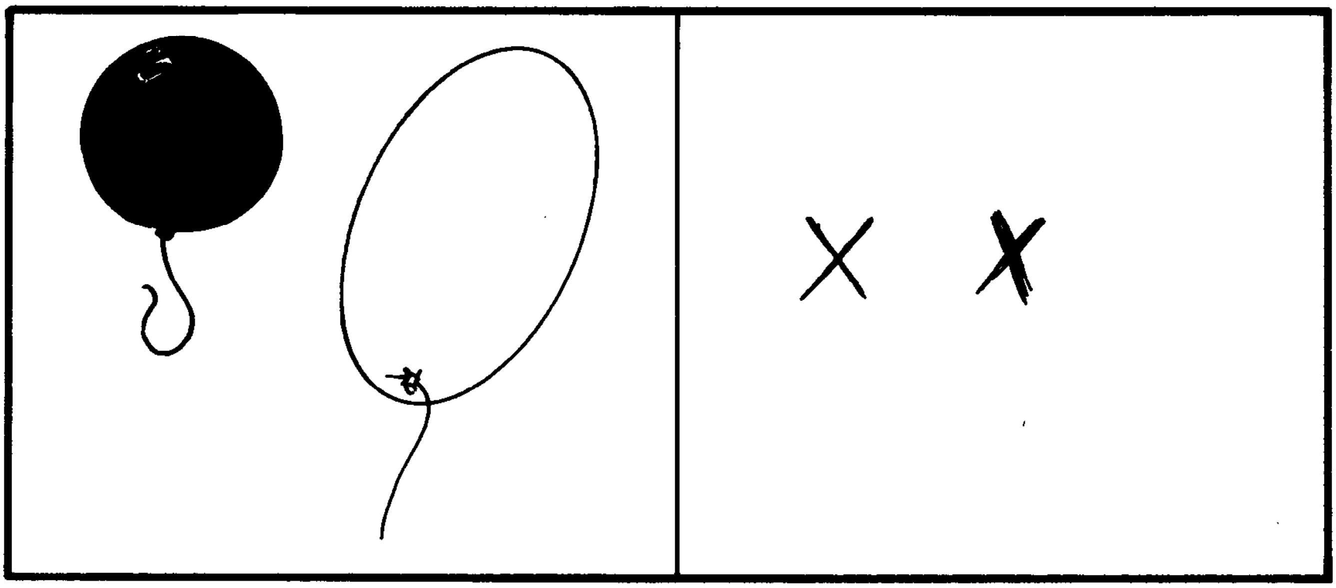
Are the sets equivalent?



Show equivalent sets.







I-4. Comparison of sets

Objective: To introduce the idea that a set may have more or fewer members than another set.

Vocabulary: More than, fewer than, not as many as.

Background Note:

This lesson continues the work on pairing members of sets. If sets are not equivalent, we find that one set has more members than another, and that the latter has not as many members as, or fewer members than, the former. This is preparation for the concept of inequality of numbers. We shall later say that a number is greater than another if a set corresponding to the former has more members than a set corresponding to the latter. We shall say that a number is less than another if a set corresponding to the former has fewer members than a set corresponding to the latter. We use the terms "greater than" and "less than" for numbers, and the terms "more than" and "fewer than" for sets. These distinctions need not be emphasized with the pupils, but you should be reasonably consistent in your use of these words in this way.

Materials: Buttons, clothes pins, or other small objects, pictures of four story-book characters (Goldilocks and the Three Bears):

Suggested Procedure:

You may want to introduce the concept of "more than" first, waiting a day or two before considering "fewer than".

Give each child a handful of buttons and clothes pins, or have these already placed in the individual boxes or envelopes. Include a greater number of buttons. Have the children describe the sets and ask that the members of the sets be paired on their desks. Offer no help except when the pairing idea is misunderstood.

Were you able to pair all the members of your set of buttons with all the members of your set of clothes pins? (No, there were some buttons left over.) Yes, we say that your set of buttons has more members than your set of clothes pins. Your set of clothes pins does not have as many members as your set of buttons. Which set has more members, your set of buttons or your set of clothes pins? Which set does not have as many members as the other set? We say there are fewer buttons. Let's look around our room. Can you see a set that has more members than another set? Fewer members than another set?

Pupil's book, pages 5, 6: In each box on the right, show a picture of a set that has more members than the set in the box on the left.

Pupil's book, pages 7, 8: In each box on the right, show a picture of a set with fewer members than the set in the box on the left.

Further Activities:

1. Ask three children to stand by the flannel board. Ask another child to give a story character picture to each of the three children. (There should be four pictures.)

Is there a child for each character in the story? (No.)

Are there more children than there are story characters? (No.)

Are there as many children as there are story characters? (No.)

Are there fewer children than story characters? (Yes.)

Ask the three children to put the pictures back on the flannel board and have another child give the pictures to the three children. Encourage this child to pair the pictures and the children in a different order.

Do we now have as many children as story book characters? (No.)

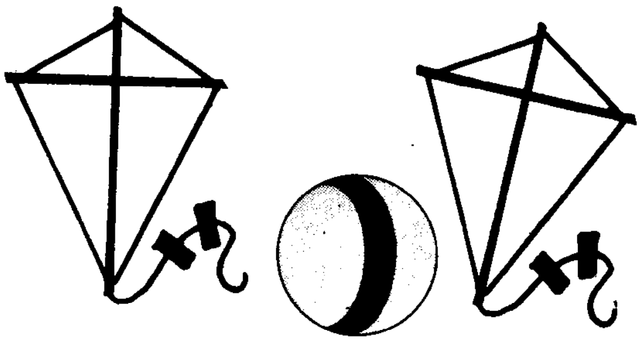
Are there fewer children than story characters? (Yes.)

How many more story characters than children do we have? (One.)

2. Choose some set of objects in the room and ask the children to look around for sets that are equivalent, for sets that have more members, and for sets that have fewer members.

3. Ask each child to fold a piece of paper, and to draw a set on one half of the paper. Then ask each to pass his paper to another pupil. That pupil is asked to draw a set with more (or fewer) members.

Show a set with more members.



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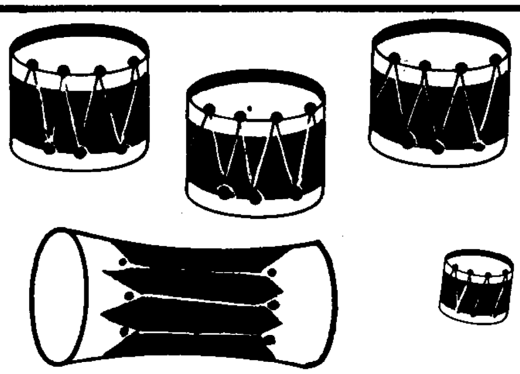

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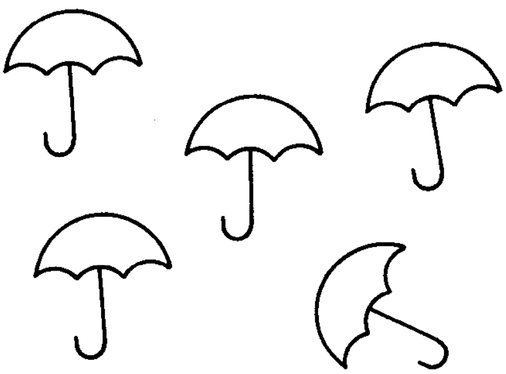

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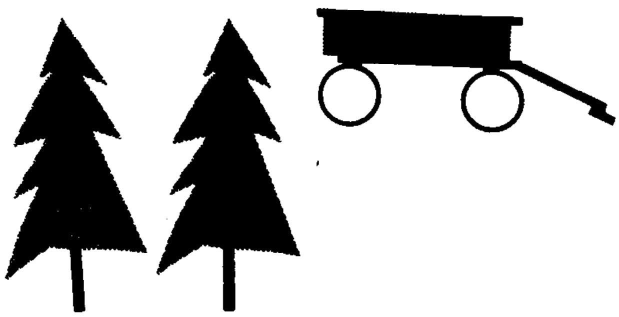
Show a set with more members.



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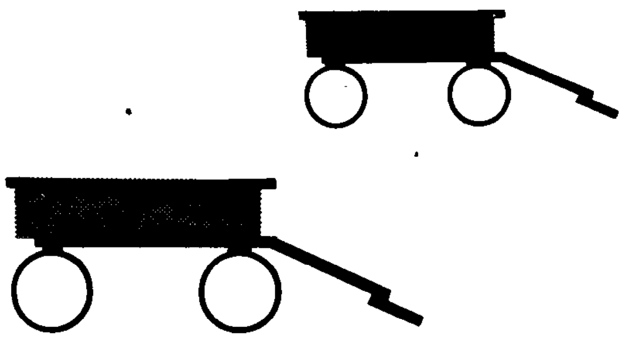

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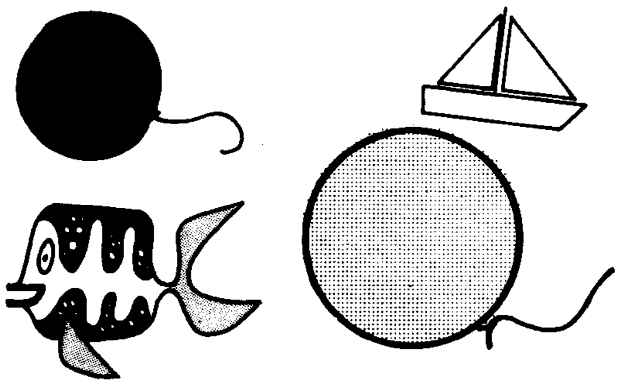

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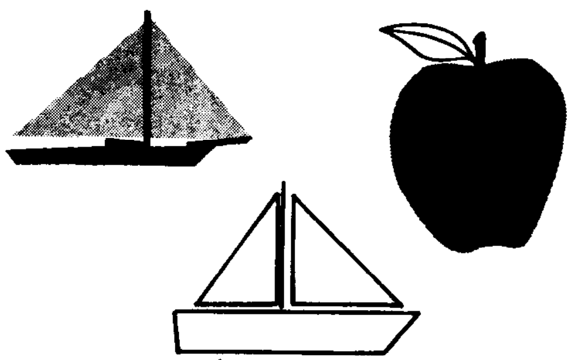

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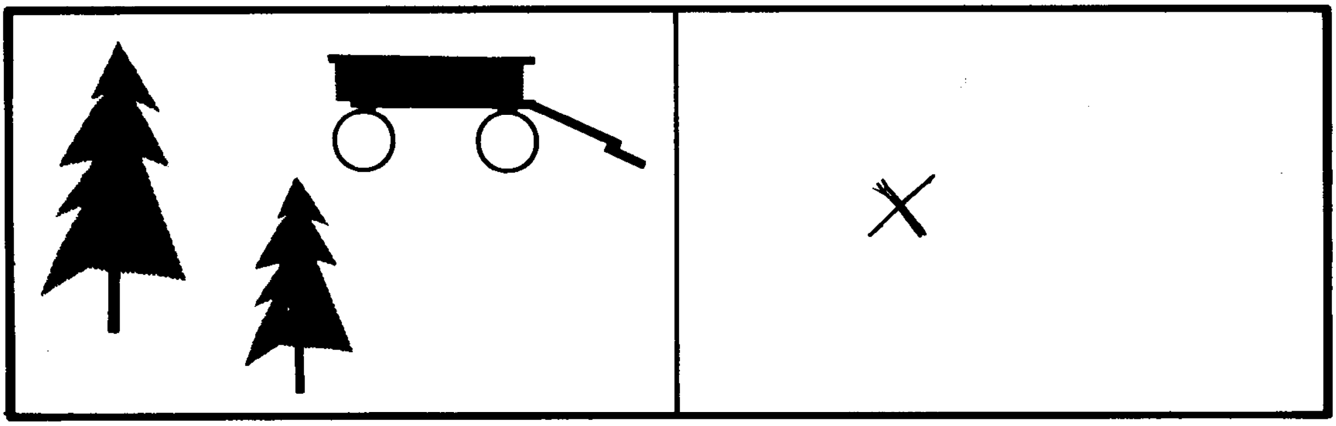
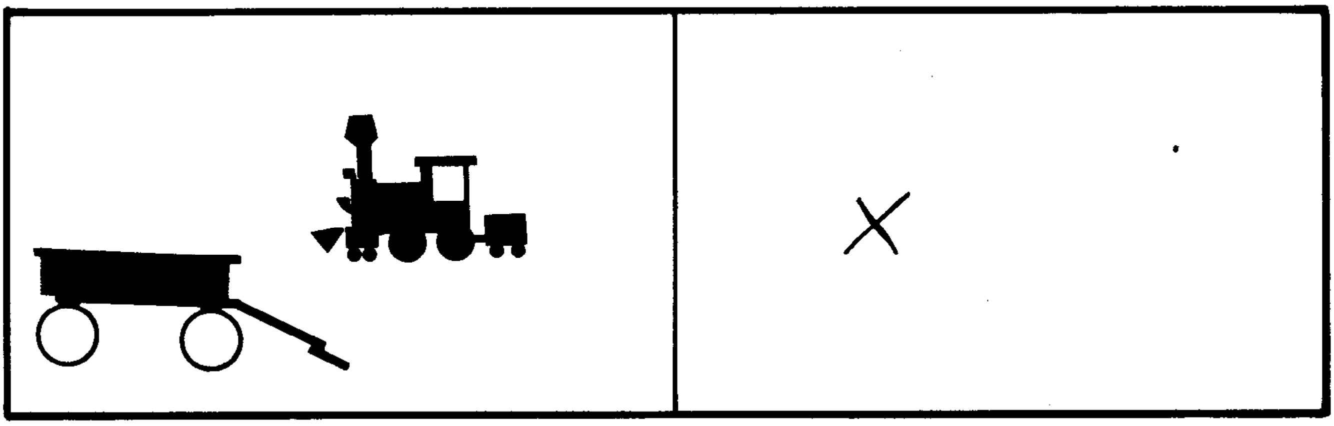
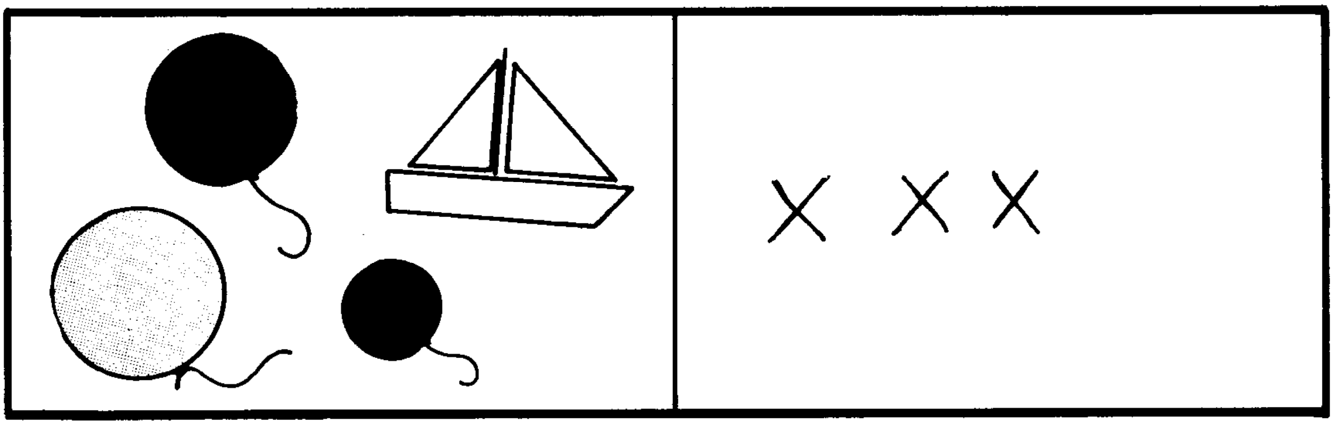
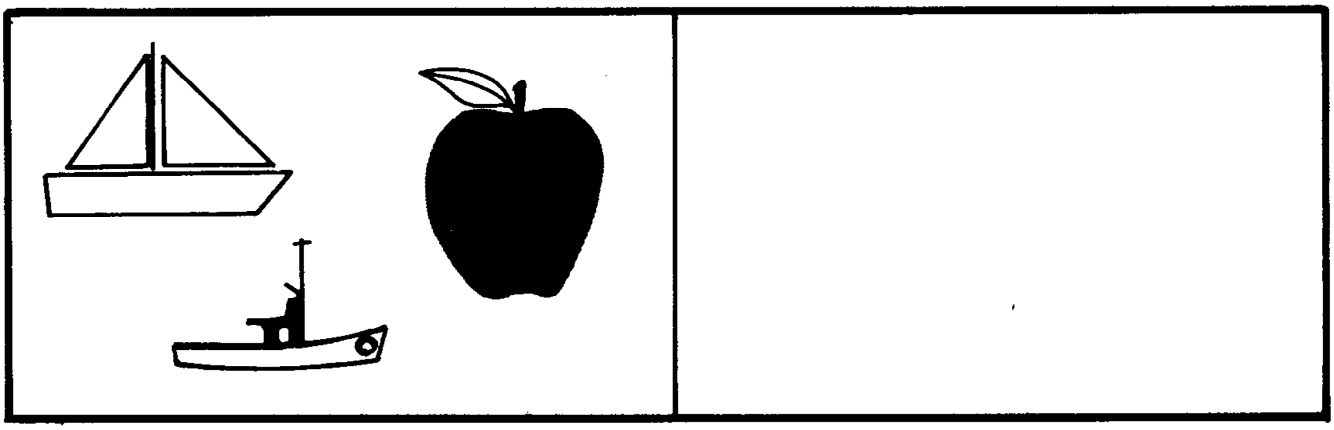
Showa set with fewer members.

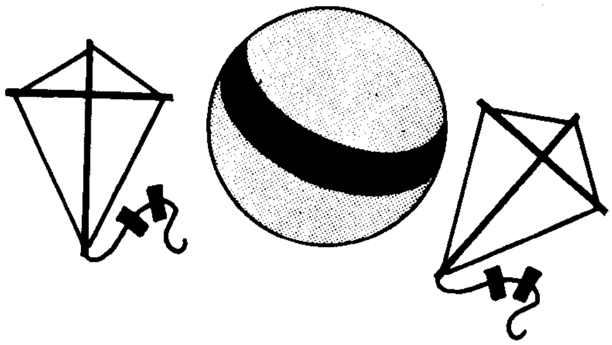








Show a set with fewer members.



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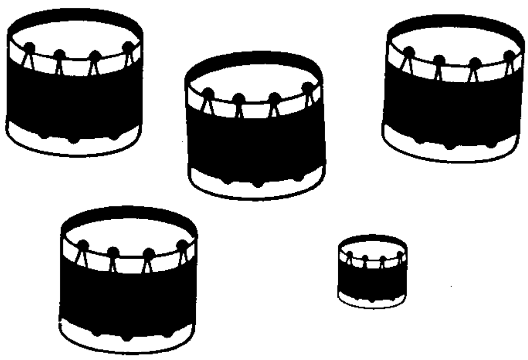

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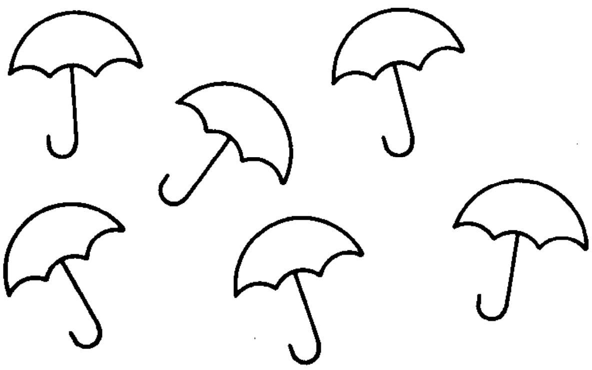

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I-5. Set with one member

Objective: To introduce the idea of a set with one member.

Vocabulary: (No new words.)

Materials: Small objects, chalkboard illustrations, materials to be used on the flannel board.

Suggested Procedure:

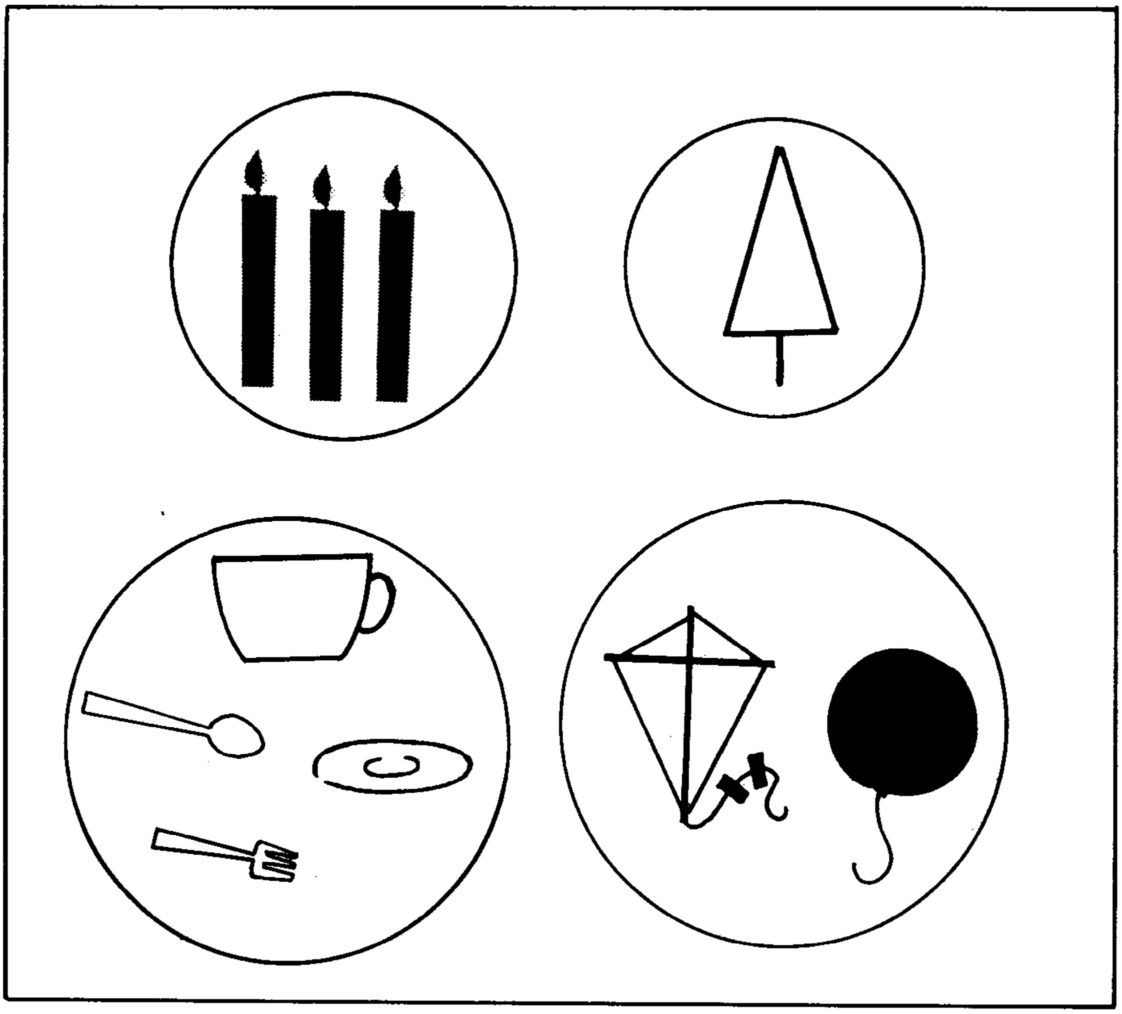
The concept of a one-member set can be easily introduced if it has not been considered in previous discussions. The following activity may be helpful.

> Today, let's have all the girls wearing red stand. (The teacher should now be seated on a chair.) Now, will the set of boys stand? set of girls stand? set of teachers stand? (Reactions at this point will appear from the class.) Let's all be seated.
>
> Yes, I am the only member of the set of teachers in this room (assuming that there is not more than one teacher). Your sets have many members but my set has just one member. Can you think of any other sets in our room with just one member? (The set of pianos, the set of teacher's desks, etc.) Sets can have many members such as the set of the children in our school; or they can have just a few members or just one member or no members at all. Is the set of elephants in our room a set with one member? (No! There aren't any elephants in here. It's the empty set!)

The following activities may be used for further development of the objective.

Further Activities:

Such chalkboard illustrations as those below can help to stress concept.



1-6. Review

Objectives: To review the concepts of pairing, equivalent, more than and fewer than.

Vocabulary: (No new words.)

Materials: Pencils, scissors, or small objects.

Suggested Procedure:

Your children do not need this lesson if they understand clearly that they can always decide whether one set is equivalent to another, and whether one set has more or fewer members than another, by pairing members. If the children compare sets by counting, accept their answers but continue until they are sure that the comparison can be made without counting.

On a desk by the door, place as many pencils as there are boys in the class and as many scissors as there are girls in the class. As the children enter the room, ask each boy to take a pencil and put it on his desk and each girl to take scissors and put them on her desk.

> We discovered the other day when we paired our set of name cards with the set of children in our room that we had just as many members in one set as we had in the other set. Do you remember what we said about these sets? (Yes: equivalent.) We also discovered that two sets aren't always equivalent. We know that a set may have more members than another set or fewer members than that set. Let's check today to see if our set of boys is equivalent to the set of girls. If the two sets are not equivalent, then we can say that the set of boys has more or fewer members than the set of girls.

Have the children pair the members of the two sets (pencils and scissors) by asking a boy and a girl to go to the teacher's desk together and place their materials side by side. Continue until all children have gone to the desk. In case there are more boys or more girls, of course, they will not be able to go in pairs.

> We have said that sets are equivalent when there are just as many members in one set as there are in the other. Do we have equivalent sets of boys and girls?
>
> Is the set of pencils equivalent to the set of boys?
>
> Is the set of scissors equivalent to the set of girls?
>
> Is the set of pencils equivalent to the set of scissors?

If the answer is "No," continue with these questions.

> Can we say that the set of pencils has more members than the set of scissors?
>
> Which set has more members?
>
> How can you tell?
>
> Can we say that one set has fewer members than the other set?
>
> Which set has fewer members?
>
> How can you tell?

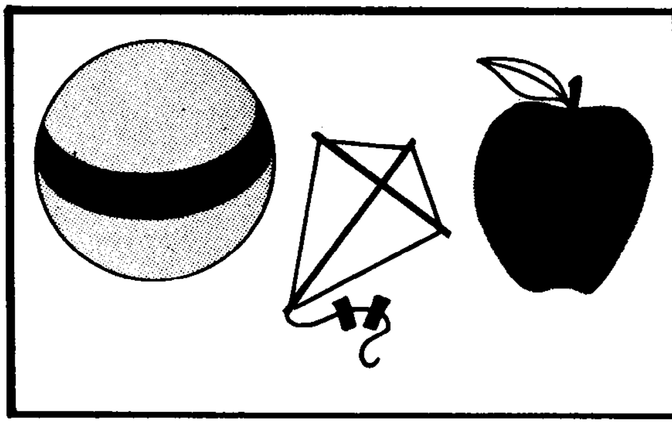
<u>Pupil's</u> <u>book,</u> <u>page</u> <u>9</u>: Show a set on the right equivalent to the set on the left.

<u>Pupil's</u> <u>book,</u> <u>page</u> <u>10</u>: Mark the set with more members.

<u>Pupil's</u> <u>book,</u> <u>page</u> <u>11</u>: Mark the set with more members.

Give each child a large handful of beans and a handful of toothpicks (or peas, pebbles, or acorns) and ask them to compare the sets.

Show an equivalent set.



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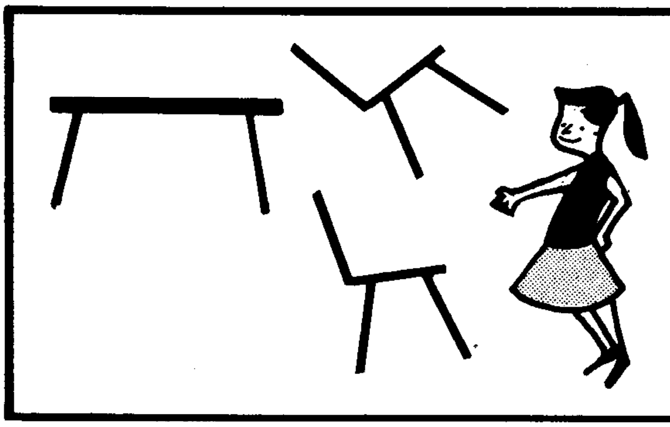

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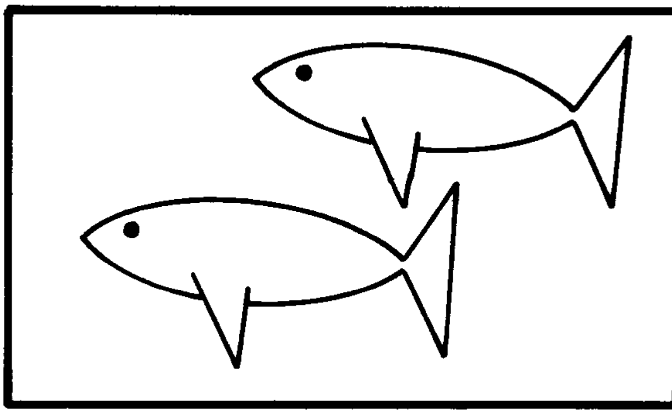
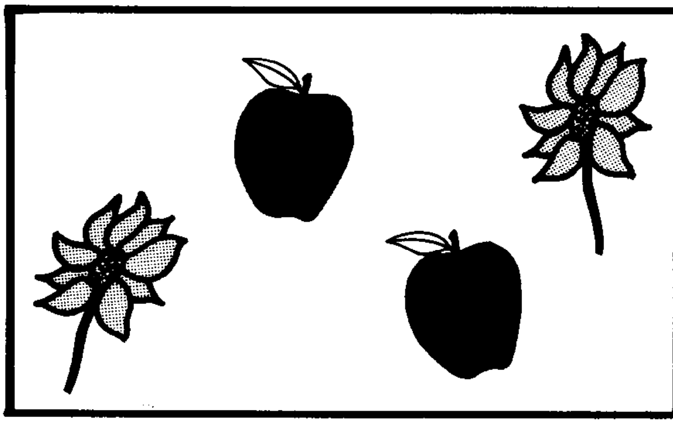


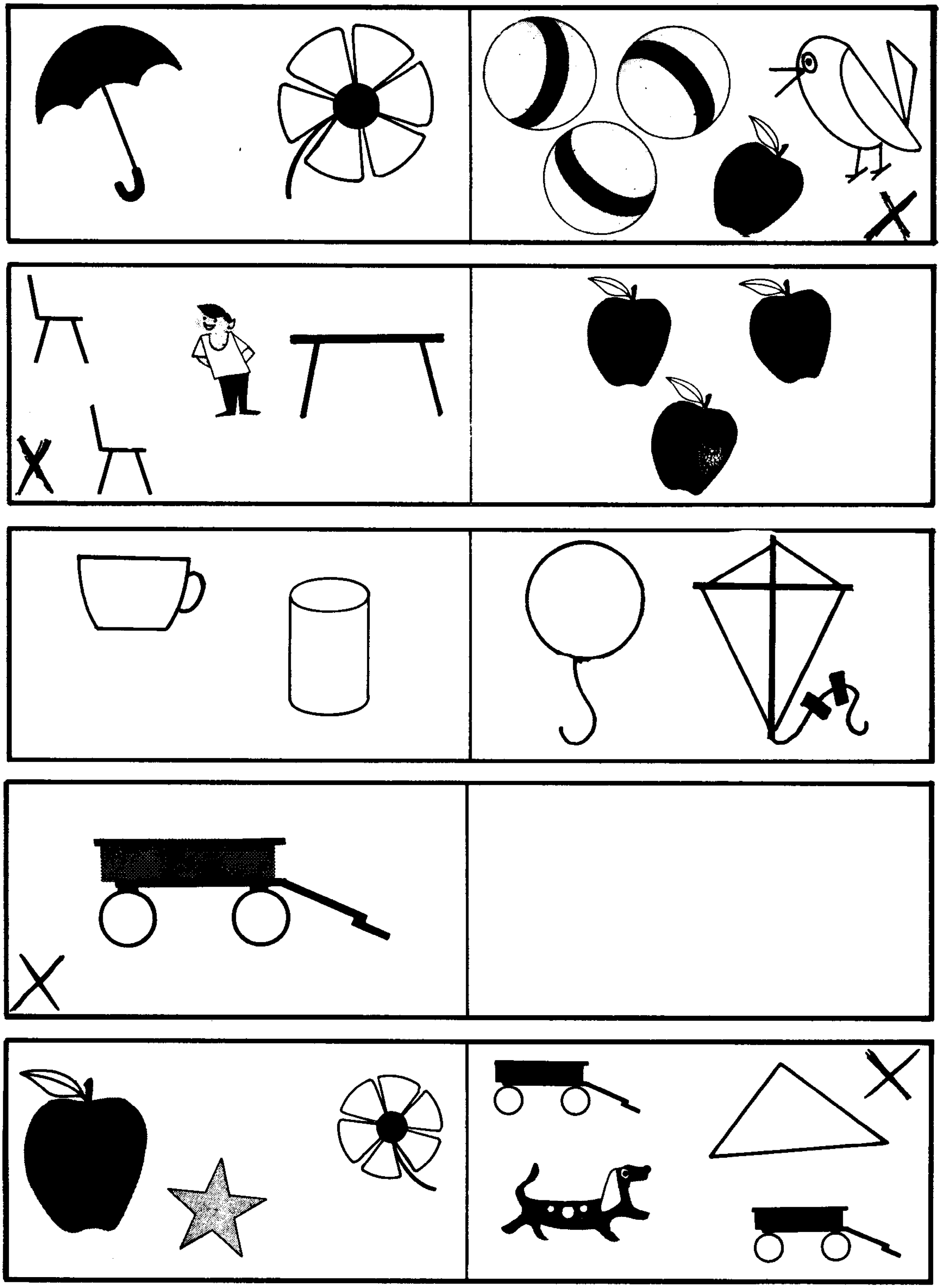
X X



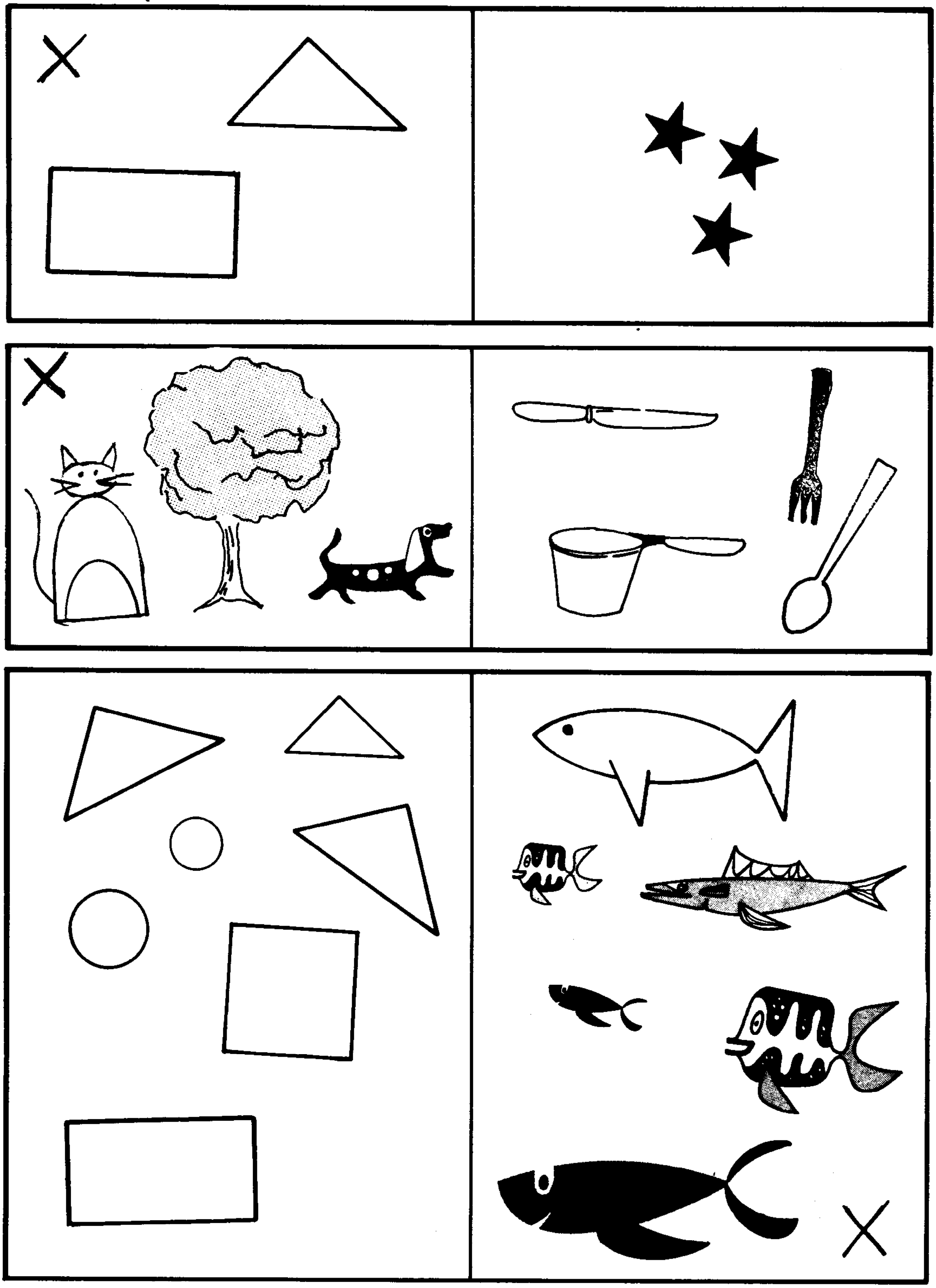
X X

X X

Mark the set with more members.



Mark the set with fewer members.



I-7. Counting sets

Objective: To help children learn to count sets of as many as ten members.

Background Note:

This lesson begins the process of naming the numbers. The counting process developed in this lesson enables us to find the name for the number of members of any set of at most ten members. Care should be taken that this is more than a rote process. It is necessary to take particular pains in this and the following lessons to bring out the underlying concepts. The number of members of a set is the property common to all sets equivalent to it. Thus 3 is the property common to all sets equivalent to

and "three" is the name of this property. Counting is essentially a way of remembering the name of the number of members of a set. The important facts about number, that equivalent sets have the same number of members and that sets having the same number of members are equivalent, must be continually emphasized.

For many classes this lesson is entirely unnecessary. In any case, this is the appropriate time to check children's ability to count sets of at least five, and preferably ten, objects and to help those children who need further practice. The lesson below is given in abbreviated form because many classes will need little or no practice of this kind.

Vocabulary: (No new terms)

Materials: Sets of small objects.

Suggested Procedure:

An important step in developing ability to count is to know, in order, the spoken names of the numbers from one to ten. There are many ways of aiding the memorization process. Such nursery rhymes and songs as "One little, two little, three little Indians," "One, two, three, four, five, I caught a hare alive" and "One, two, buckle my shoe" are very useful. At any rate, the child must learn to say the words in sequence. For many children, the rhymes are primarily nonsense rhymes at the early stage; even after attaining perfection in saying the words a child may have little conception of the meaning of, say, eight.

In beginning to count sets of objects a child must touch each member of a set in sequence, saying in sequence the names of the numbers. (The child is actually pairing the members of the set with members of a set of numbers.) The name spoken when the last member of a set is touched designates the number of members in the set. Later comes the "point and say" stage, next the stage of pointing and thinking and finally, the "look and think" stage.

During counting practice the child should be led to realize that the arrangement of the set being counted does not matter. That is, if he counts the set, then rearranges it and counts in another way, the same number is obtained. The relationship between number and equivalence should be constantly emphasized: If two sets are equivalent then they have the same number of members, and if two sets have the same number of members then they are equivalent. Thus, if a group of boys is counted and the same number of girls is counted, we are sure that we can form couples with no one left over.

This entire development is accomplished by a great deal of practice which should not be confined to the period set aside for the mathematics lesson. Children should count sets of chairs, sets of children, sets of toys, sets of desks, and so on as part of their everyday classroom activities.

I-8. The number zero

Objective: To understand that zero is the number of members of the empty set.

Vocabulary: Zero.

Materials: A set of five objects; e.g., pencil, crayons eraser.

Suggested Procedure:

Place the set of objects on a demonstration table. Have children grouped around where all can see. Question children concerning the number of the set (five). Ask a child to come up and take an object to his chair. Repeat, each time asking for the number of the set that remains.

When the last object is removed, ask about the set remaining on the table.

> What set do we have now on the table? (The empty set.)
> Do you know a number that tells how many things there are in the empty set? (Zero.)
> Zero is the number that tells us how many things there are in the empty set.

Continue with illustrative questions, e.g.:

> What is the number of giraffes in our room today? (Zero.)
>
> What is the number of boys in this room who have green hair? (Zero.)

- Count a set of objects and put them in a grocery bag. Remove one object at a time and ask a child to tell the number of objects still in the bag. Continue until there are no objects in the bag. Reinforce the idea that zero is a number.

I-9. Number perception, without counting

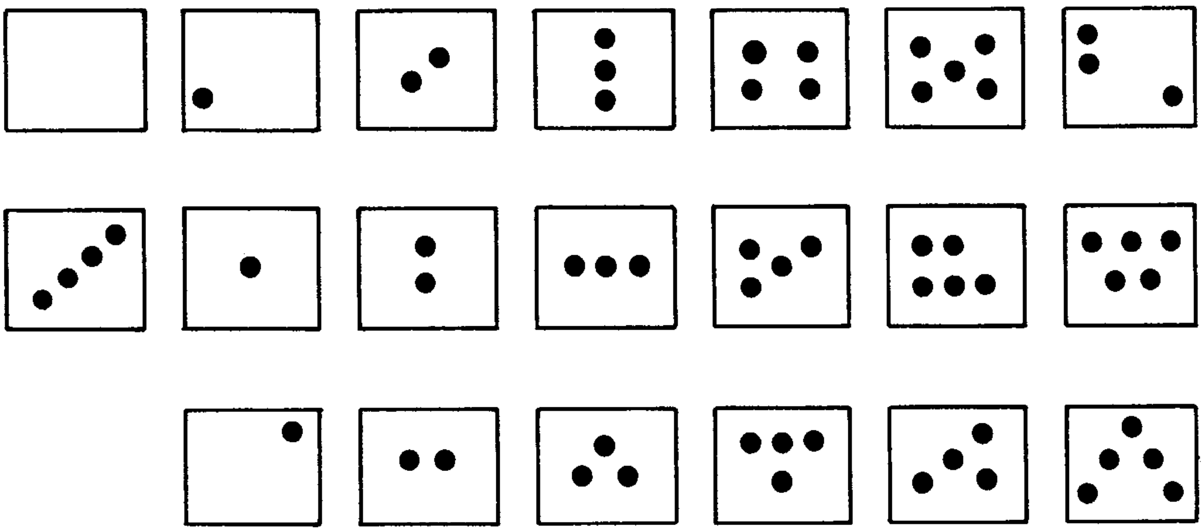
Objective: To help children perceive, without counting, the number of objects in a set of no more than five members.

Vocabulary: (No new words.)

Materials: Perception cards, in various patterns, sets zero through five; flannel board materials.

Suggested Procedure:

Use perception cards (without numerals) of sets of from zero through five members. Perception cards should have different arrangements of the members of the sets:



Give each child five objects. Show a card and have the child put on his desk a set equivalent to that displayed on the card.

Put a set of objects on the flannel board. Have children show equivalent sets at their desks.

Display a perception card very quickly; then conceal it, and have children show equivalent sets at their desks.

Display a card, cover it, and have a child give orally the number of the set that was shown.

Further Activities:

There are many opportunities to point out equivalent sets in the classroom and to encourage children to perceive the number of the set without counting. For instance, if there are two sides of an easel, two children can paint. If there are four balls for the recess period, four children may take out balls. If there are three fish in a fish bowl, put three books about fish on the table beside them and encourage children to see "three," rather than "one, two, three."

NOTES

I-10. Numbers and equivalence

Objective: To use simple story problems to emphasize the fact that equivalent sets have the same number of members, and that sets having the same number of members are equivalent.

Vocabulary: (No new words.)

Materials: Set of small objects for the children.

Suggested Procedure:

The problems given are samples. There will be no difficulty in devising other similar problems. Small objects and dramatization can be used to involve all of the children. The problems are to be read or told to the children.

1. There were 6 cups on the table, and there was a saucer under each cup. How many saucers were on the table? (Six.)

2. Five children were going to draw pictures and each child wanted a pencil. How many pencils did the children need? (Five.)

3. There were seven children at a party. Each child had a hat, and there was a feather on each hat. How many feathers were there? (Seven.)

4. Three little girls had a tea party. The set of girls was equivalent to the set of dolls. How many dolls were there? (Three.)

5. Once there were four dogs. Each dog had a cat, each cat had a rat, and each rat had a mouse. How many mice were there? (Four.) Was the set of mice equivalent to the set of cats? (Yes.)

6. John had three guinea pigs, seven marbles, and two tops. David had as many mice as John had marbles. How many mice did David have? (Seven.)

7. On the table are a set of bottles and an equivalent set of straws. There are 8 bottles. How many straws are there? (Eight.) Is there a straw for each bottle? (Yes.)

8. Six toy soldiers are on the table. Six toy guns are on the floor. Is the set of guns equivalent to the set of soldiers? (Yes.)

9. How many balloons are in a set of balloons which is equivalent to a set of 7 children? (Seven.)

10. How many bows are in a set of bows if there is only one bow for each girl and there are 5 girls? (Five.) Is the set of bows equivalent to the set of girls? (Yes.)

11. Six boys are playing with boats. The set of boats has ten members. Is the set of boats equivalent to the set of boys? (No.)

12. Sue has 4 bracelets. Three girls came to play with Sue. Is the set of bracelets equivalent to the set of girls? (Yes.)

13. Each child has at least one crayon, and some children have two crayons. Is the set of crayons equivalent to the set of children? (No.)

14. Jim's class has this set of animals: a turtle, a guinea pig, a hamster and a rabbit. The class has 5 cages. Is the set of cages equivalent to the set of animals? (No.)

Further Activities:

Ask the children to make up their own story problems about numbers and equivalence, and dramatize these.

I-11. Comparison of numbers

Objective: To introduce the concepts of greater than, less than.

Vocabulary: Greater than, less than.

Background Note:

To compare two numbers, say 6 and 3, we choose a set of 6 members and a set of 3 members and by pairing find that the first set has more members than the second. We therefore say that 6 is greater than 3 and that 3 is less than 6. We use the terms "more than" and "fewer than" in connection with sets, and "greater than" and "less than" in connection with numbers. You should not over-emphasize linguistic precision with the children, but should be precise in your own use of these words.

Materials: Sets of small objects for the children.

Suggested Procedure:

Have the children build sets with 2 and 3 members. Ask the children to put the sets at the front of their desks.

> Which set has more members? (The set of 3 members.) Why do we say the set of 3 members has more members? (If we pair members of this set with those of the set of 2, we have a member left over.) Since a set of 3 things has more members than a set of 2 things, we say that the number 3 is greater than the number 2.

Next, notice that the set of 2 has fewer members than the set of 3.

> Why do we say the set of 2 members has fewer members? (If we pair members of this set with those of a set of 3 things, we run out of members before we have finished pairing.) When a set has fewer

members than another, we say the number for that set is less than the other number. Is 2 less than 3? (Yes.)

Repeat with several other examples.

- Ask the children to close their eyes. Then ask them to think of numbers greater than 2. Do not limit them to the number 9 but insist that each child explain why his number is greater. (It is the number of a set with more members than 2.)

  Ask for a number less than 9, and why it is less. Repeat with other numbers.

- Have the children use small objects to dramatize the solving of word problems. (These are supposed to be both vocabulary drill and mathematics.) For example:

  1. Johnny has 3 marbles and Jim has 5 marbles. Which boy has the greater number of marbles? (Jim.) If we paired John's marbles with Jim's marbles, which boy would have some marbles left over? (Jim.)

  2. Sue has four dolls. Sharon has fewer dolls than Sue. How many dolls does Sharon have? (Three, or two, or one, or zero.)

  3. Max has 4 marbles and Frank has 6 marbles. Jim has fewer marbles than Frank, but Jim has a greater number of marbles than Max. How many marbles does Jim have? (Five.)

  4. Matilda has one bracelet. The number of bracelets Jane has is less than the number of bracelets Matilda has. How many bracelets does Jane have? (Zero.)

5. Joe has 3 tops. Bill has fewer tops than Joe. The set of Bill's tops is equivalent to the set of John's tops. How many tops does John have? (Two, or one, or zero.)

Chapter II

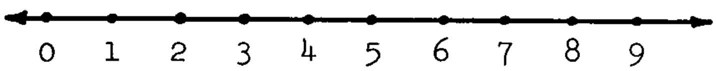
NUMERALS AND THE NUMBER LINE

Background

This chapter has several objectives. First, we wish to teach children to recognize and to write the numerals: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Each of these symbols must be connected with the correct spoken word and with those sets having the correct number of members. It is worth noticing that children in the first and second grades have to establish the association of four different sorts of objects with each number. For example, they must learn to connect the number 8 with sets having eight members, with the symbol "8", with the written word "eight", and with the spoken word "ate". At the same time children are learning other names for 8, such as $4 + 4$, $9 - 1$, and so on. Each of these associations must be established by many different experiences.

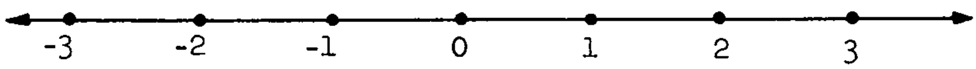
Second, we wish to reinforce the connection between comparison of sets and comparison of numbers. We recall that one set has fewer members than another if, when the members of the first set are paired with the members of the second, there are members of the latter left over. This relationship between sets leads to a relationship between numbers. We say that 5 is greater than 3, and that 3 is less than 5 because if the members of a set of 3 are paired with the members of a set of 5, there will be members of the latter set left over. Recall that if one set has more members than another, then the number of members of the first set is greater than the number of members of the second set; if the first set has fewer members, then the number of members of the first set is less than that of the second; and if sets are equivalent then they have the same number of members. The words "more than", "fewer than", and "equivalent" refer to sets. The words "greater than", "less than", and "equal" refer to numbers.

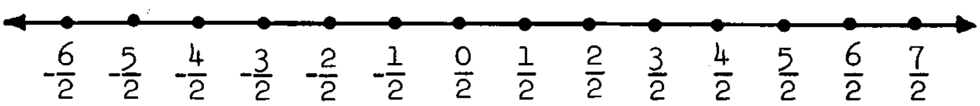
Lastly, this chapter begins the development of the concept of the number line. At this stage we are trying to help children think of the numerals 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 as equally spaced labels on a line. We put arrows on the ends of the line we draw to help children understand that a line extends indefinitely in both directions, but the picture we want to give now is the following:

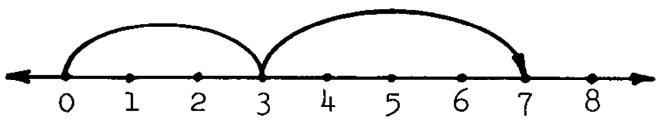


Be sure that they understand that the numerals are associated with the number of steps from 0. The "starting point" is 0, and each number is the number of "steps from the starting point".

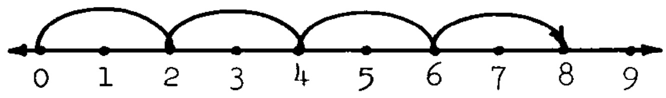
Later, pictures such as these will be used:







$3 + 4 = 7$



$4 \times 2 = 8$

Without going into details, these pictures indicate that the number line will be used to motivate the introduction of negative numbers, to motivate the introduction of rational numbers, to emphasize that one number may have many names, to illustrate the notion of addition and its properties, and to give an interpretation of multiplication. We use the number line in this chapter primarily to reinforce understanding of the natural ordering of the numbers, but there are many other pedagogical uses for the concept. There are also profound mathematical reasons for the early introduction of the number line. We are associating an algebraic object (a number) with a geometric object (a point), and this foreshadows the close connection between algebra and geometry which was discovered by Descartes and called <u>analytic geometry</u>.

> IMPORTANT NOTE: This chapter does not require that the children be able to write numerals, but they will be required to write numerals in Chapter III. You should teach the writing of the numerals in the writing periods but not until children have the understandings and abilities developed by the end of Section II-2.

II-1. Arranging sets in succession

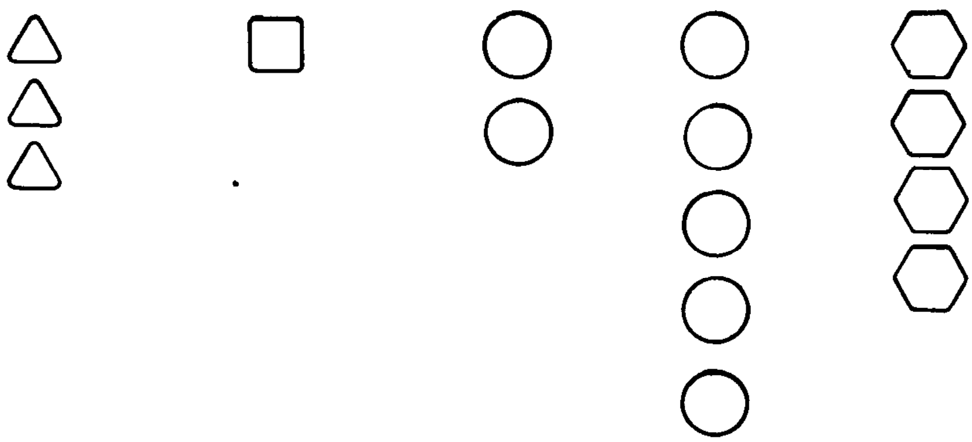
Objective: To put sets with 0 through 10 members in order so that each non-empty set has one more member than the set which precedes it.

Vocabulary: (Review) order, more than, fewer than.

Materials: Materials to be used on flannel board, including numerals and pairing symbols or pieces of string (or similar materials for the bulletin board or the magnetic board); sets of objects for children; pennies (real or play money).

Suggested Procedure:

Place five sets of objects ranging from one to five members on the flannel board in some such arrangment as sets of three, one, two, five, and four members. Use a left to right arrangement.



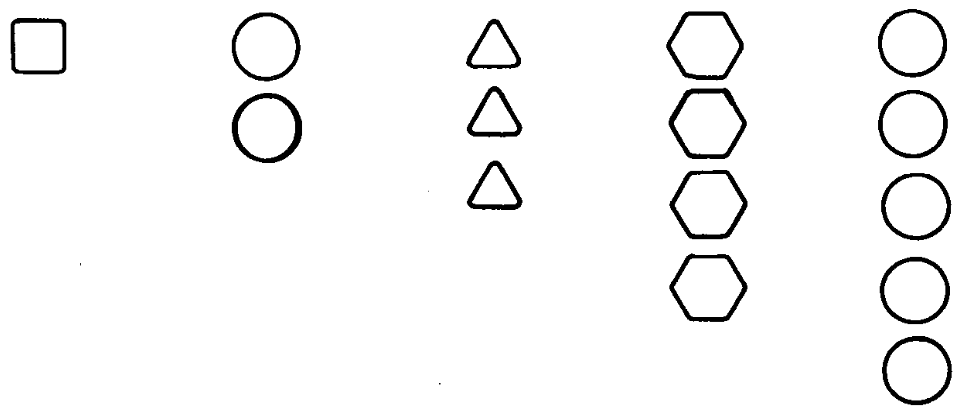
Ask which set has fewest members. Agree that the set with one member can be at the left.

Next point to the set with three members.

> Does this set (the set with three members) have more members than the set with one member? (Yes.) Let's place it at the right of the set with one member.
>
> Is there a set which has more members than this set (indicating the set with one member) and fewer members than this set (indicating the set with three members)?

Ask where the set with two members belongs. Bring out by pairing that it has one more member than the set with one member, and one fewer member than the set with three members. Place the set with two members between that with one member and that with three members.

Complete the arrangement as shown, using the techniques suggested above. You may want to look for the set with the most members next, and then fit in the remaining sets. Use a flannel pairing symbol or a piece of yarn to show the pairing between members of the set with one member and the set with two members. Cover the set with one member and the pairing symbol, and ask about the set of two and the set of three. Proceed in the same way until the display is complete.

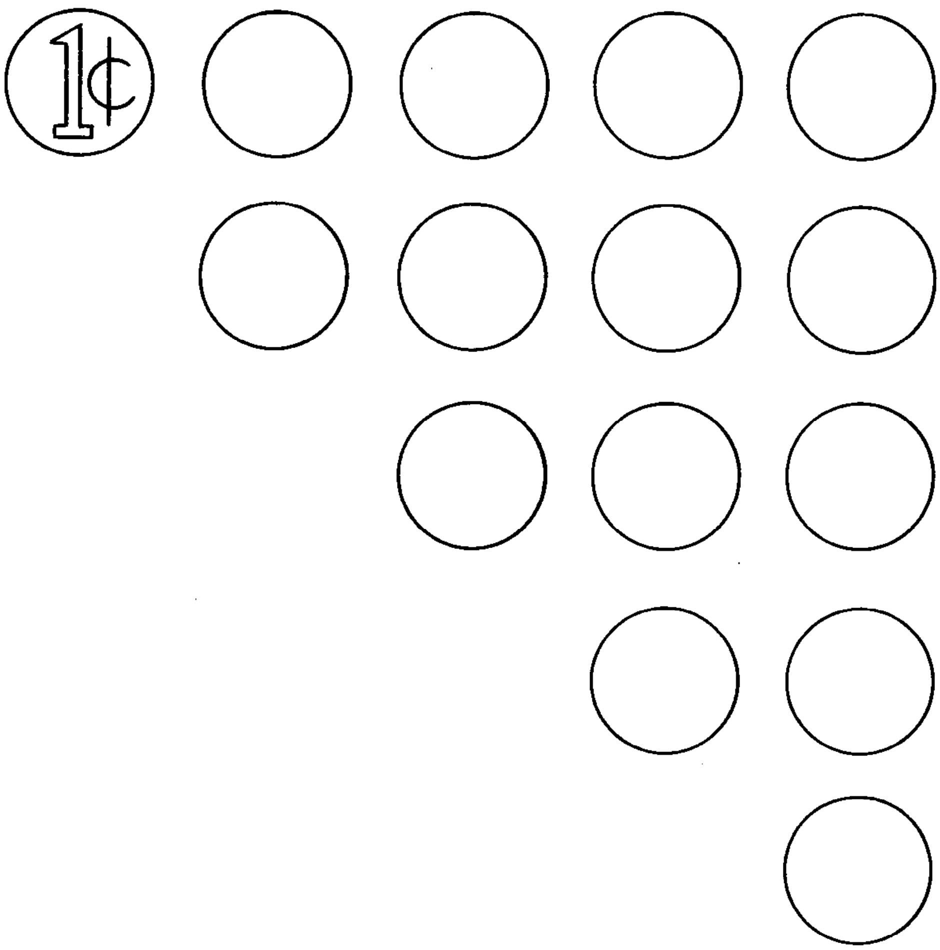


Ask if there is any set with fewer members than the set with one member. Try to bring out that the empty set belongs on the left-hand side. (No picture, of course.)

Pupil's book, page 12: Missing sets are to be drawn.

Further Activities:

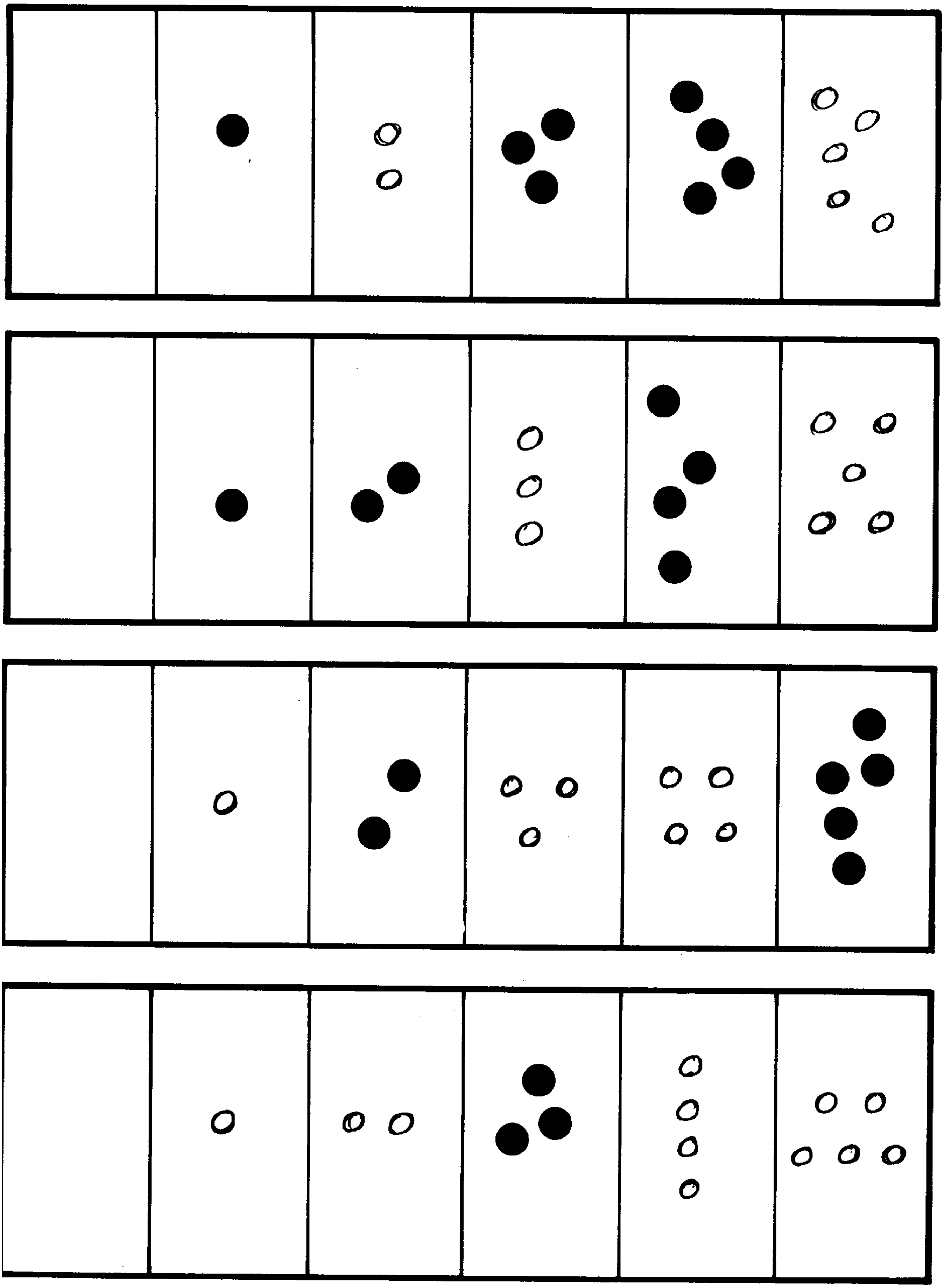
Give each child fifteen pennies (real or play money). Ask him to make a set of one, and then make, to the right of this set, a set of three. Ask if there is a set missing between the set of one and the set of three. Continue working until each child has arranged sets of pennies as follows:



Ask the children if they know a coin that is worth five cents. Bring out that five cents is worth just as much as one nickel.

CAUTION: Do not say that five cents equals one nickel. We use "equal" only in sense of logical identity.

Draw the missing sets.

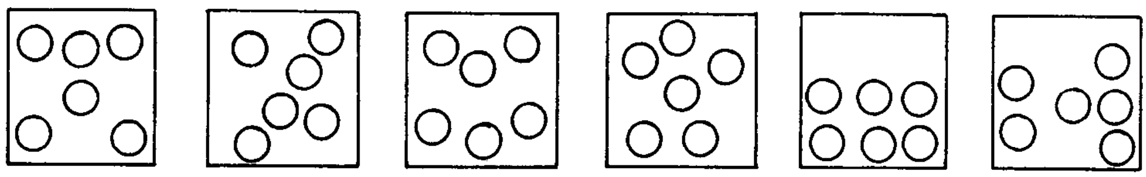


- Now extend the ordering of sets to include those with 6, 7, 8, 9 and 10 members. Use a procedure similar to the one you followed for sets with not more than 5 members.

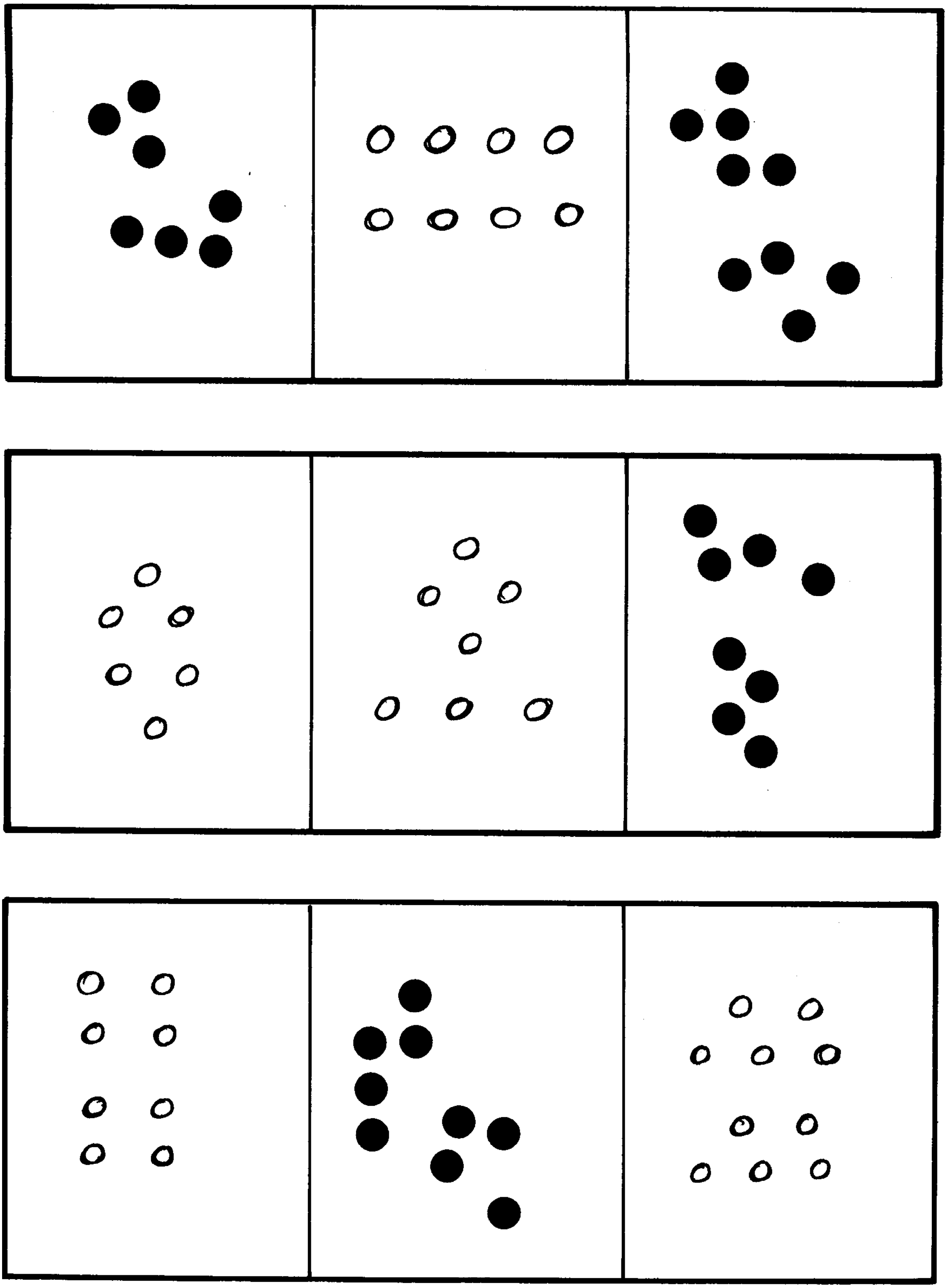
  Pupil's book, page 13: In each row there is a set missing Draw a set that fits.

- As a further activity, scatter a set of objects with 6, 7, 8, 9, or 10 members on a small rug or towel. Ask a child to pick up a subset which he can name immediately and begin his counting from that number. Each child should be encouraged to pick up the subset he identifies first rather than to look for a particular subset. Some children might pick up the first set and then recognize the remainder set without counting and then add the two numbers. Some children will also determine how many members in a set by recognizing three subsets and adding these three numbers.

- The teacher can make cards that show sets of 6, 7, 8, 9, and 10 in a variety of arrangements. Each set should have a minimum of six arrangements so that children do not just learn a particular pattern. Sets should be constructed on the same color of card, preferably with the objects drawn alike to prevent association of a given set with a given object shape or card color.



Draw a set that fits.



Display the cards in the same way that the flannel board objects were shown in the earlier lesson. At times when this work is being done ask a child to "think out loud" while he is finding out how many. Children may be encouraged to use these cards in an activity period on their own time and develop some of their own games that will help them learn to recognize these sets.

II-2. The numerals 0 through 9

Objective: To associate the written numerals 0 through 9 with their corresponding numbers, and to order these numerals.

Vocabulary: Numeral.

Background Note:

This section has been divided into three parts: the numerals 0 through 5, the numerals 6 through 9, and the words for numbers from zero through ten. You may wish to introduce the complete set of numerals, 0 through 9, at one time rather than in two parts. Feel free to make this modification if appropriate for your class.

Materials: Peg board, flannel board, numeral cards (0 - 9), blocks, books, counters, brushes.

Suggested Procedure:

Part I- The numerals 0 through 5

Tell the children that many kinds of marks can be used to tell how many. Make a set of marks (/////) and tell a child to bring a set of books with that many members to the front of the room. Ask another child to bring a set of blocks whose number of members is the same as this set of marks ($o^o o^o o$). Ask another child to bring ☆ ☆ ☆ ☆ ☆ counters. Ask another child to get △ △ △ △ △ brushes. Identify the number in each set as it is brought to you. (5.)

Ask each child to place on his desk a set of objects which has as many members as the one displayed. Introduce a numeral card for 5. Explain that the figure is called a numeral, that it is a special mark for the number five, and that it is read "five".

- Show a word card with Betty (or the story character from your reading series) on it.

Is this word really Betty? Does the word have curly yellow hair and a smiling face? (No.) It is just Betty's _name_.

A numeral is the name of a number. This (pointing at the numeral card) is a name for five. "Five" (written on the chalkboard) is another name for five.

- Have ordered sets from 1 to 5 on the peg board. Leave some space at the left. Have children locate a set of 4 spools on the board. Have the numeral placed below the set of 4. Continue with the introduction of other sets and the numeral cards 1 to 5. Ask where the numeral for the empty set belongs. Place it to the left of the numeral 1 on the peg board. The flannel board and chalkboard can be used to vary the procedure. Order the sets from left to right. Use peg board or spool board displays to relate sets and numerals.

Pupil's book, page 14: Ask the children to draw a line from each set to the numeral that names the number of the set.

Pupil's book, page 15: Ask the children to draw a line from the numeral 5 to each set of 5.

Pupil's book, page 16: Ask the children to ring the numeral that names the number of the set.

Pupil's book, page 17: Ask the children to ring the correct numeral.

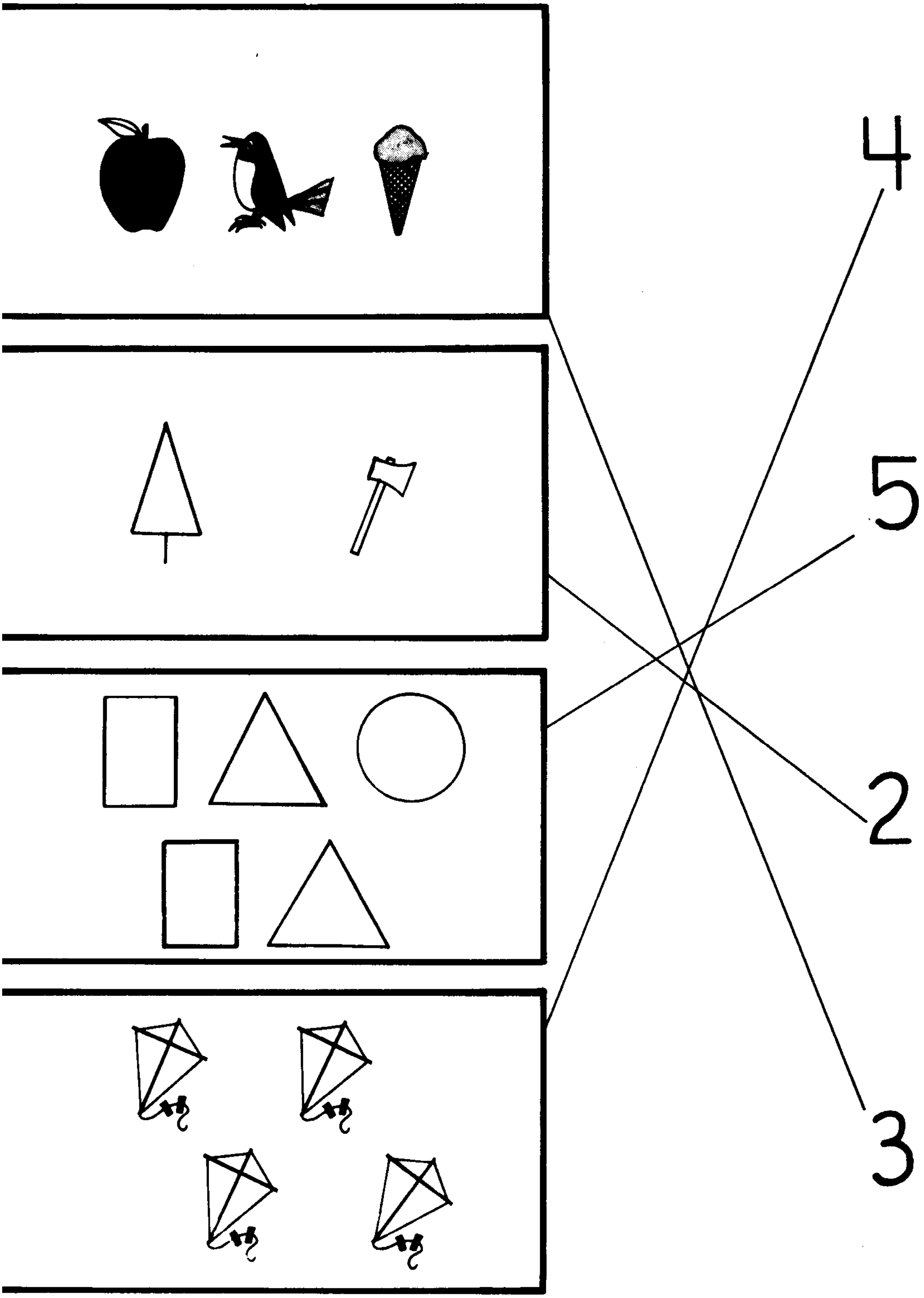
Pupil's book, pages 18 and 19: Ask the children to mark the numerals for numbers greater than that named in the left-hand box.

Pupil's book, pages 20 and 21: As for 18 and 19, but "less than".

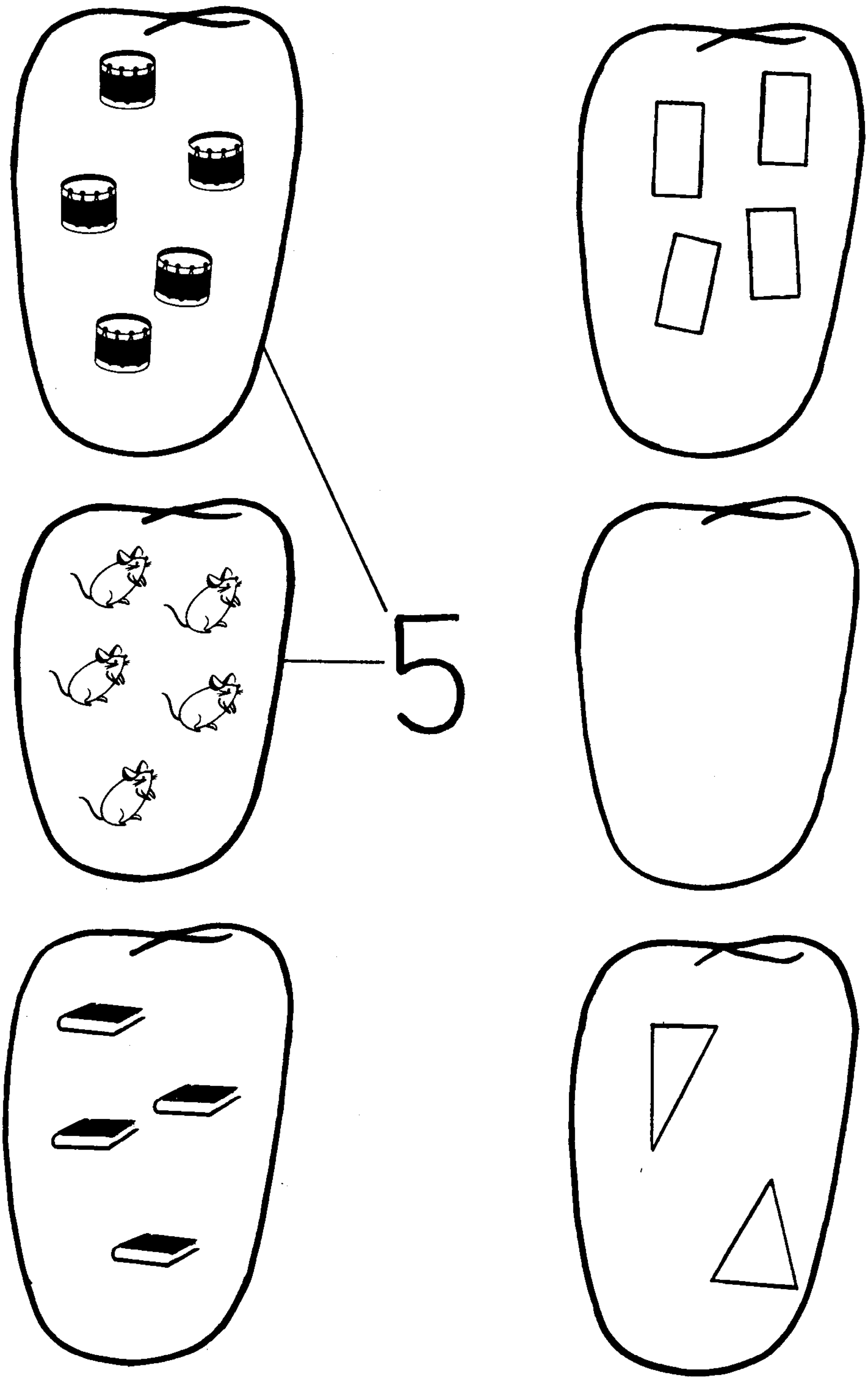
Pupil's book, page 22: As for 18 and 19.

Pupil's book, page 23: As for 20 and 21.

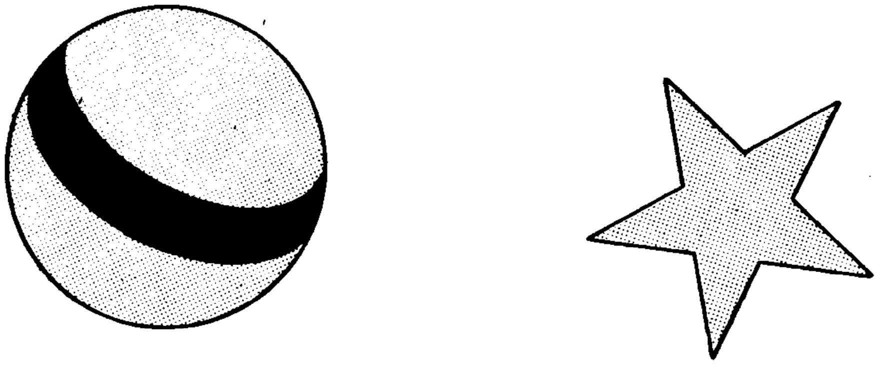
Which sets go with which numbers?



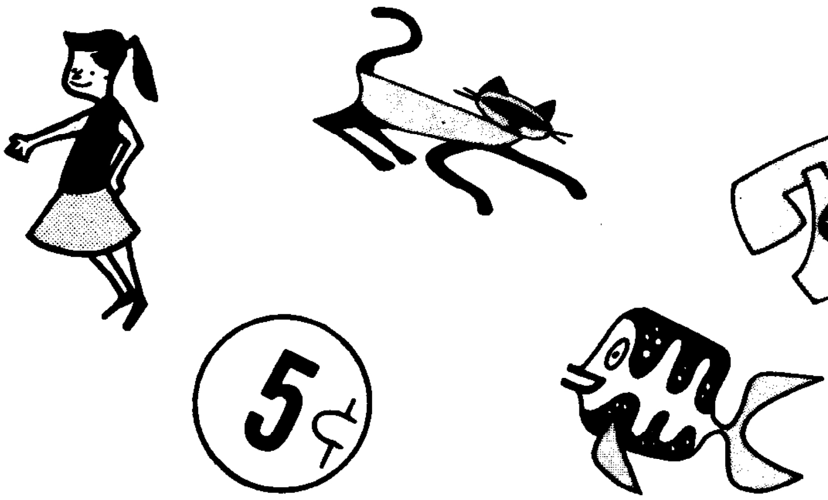
Which sets go with 5?



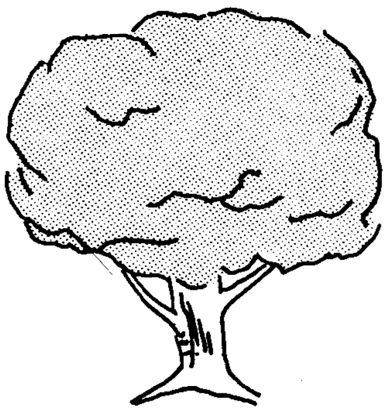
Ring the correct numeral.



2
4
0

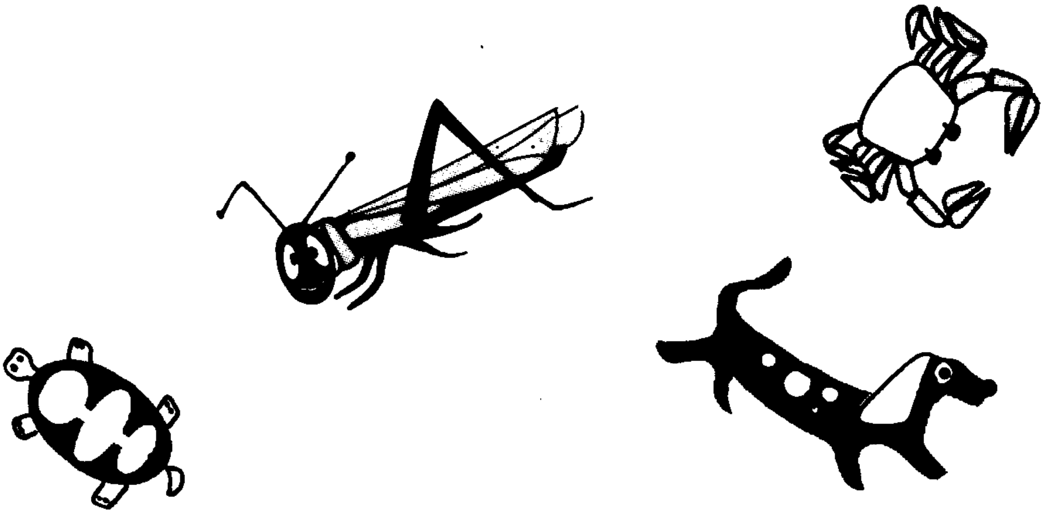


5
3
4



3
1
0

Ring the correct numeral.



3

4

5

0

1

3

2

1

0

Which numbers are greater?

3	2	~~5~~	~~4~~
4	0	~~5~~	1
1	~~2~~	~~4~~	0
5	4	0	1
0	~~3~~	~~5~~	~~2~~
2	0	~~4~~	1

Which numbers are greater?

3	2	3	1	~~4~~
	0	~~5~~	2	0

5	4	2	3	1
	5	0	2	4

2	~~5~~	1	0	2
	~~3~~	1	~~5~~	~~4~~

Which numbers are less?

3	~~2~~	5	4
4	~~0~~	5	~~1~~
1	2	4	~~0~~
5	~~4~~	~~0~~	~~1~~
0	3	5	2
2	~~0~~	4	~~1~~

Which numbers are less?

3	~~2~~	3	~~1~~	4
	~~0~~	5	~~2~~	~~0~~

5	~~4~~	~~2~~	~~3~~	~~1~~
	5	~~0~~	~~2~~	~~4~~

2	5	~~1~~	~~0~~	2
	3	~~1~~	5	4

Which numbers are greater?

4	4	2	1	0
	3	~~5~~	2	1

1	~~2~~	0	~~4~~	~~3~~
	~~5~~	1	~~3~~	0

0	~~1~~	~~4~~	0	~~2~~
	~~3~~	~~5~~	~~1~~	~~3~~

Which numbers are less?

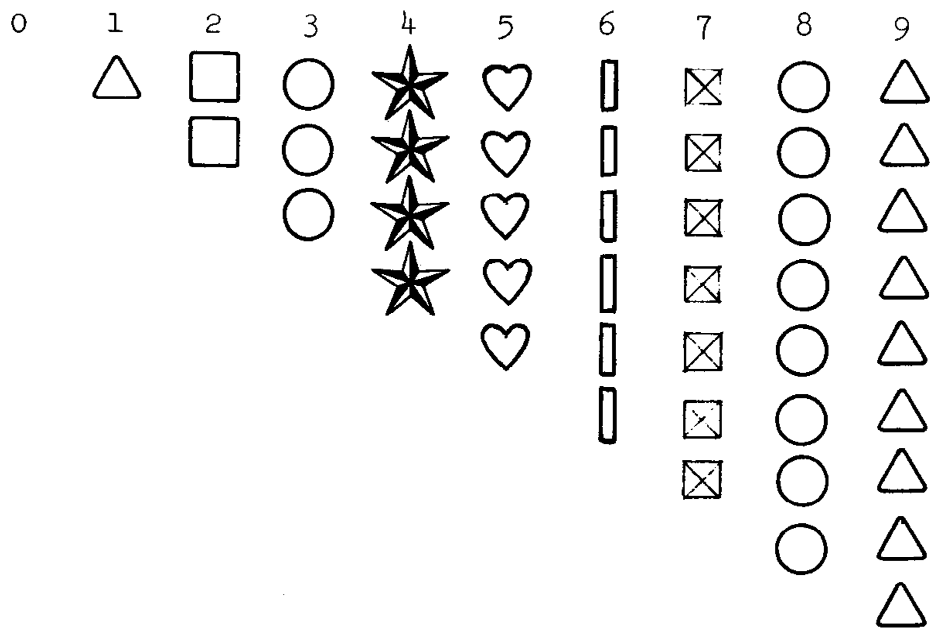
4	4	~~2~~	~~1~~	~~0~~
	~~3~~	5	~~2~~	~~1~~

1	2	~~0~~	4	3
	5	1	3	~~0~~

0	1	4	0	2
	3	5	1	3

Part II - The numerals 6 through 9

If you did not introduce the numerals for 6, 7, 8 and 9 at the beginning of the lesson you should now repeat the same sort of procedure you used there. In either case, you should keep a display on the chalkboard, flannel board, or bulletin board of the following sort.



For this lesson it is essential for children to be able to count on from a given number so that when a set is recognized the child can count the additional members of the set without starting again at one.

Distribute 7 to 10 objects to each child. Have child arrange the set on his desk so that members of the set are not touching. Without moving any of the objects, ask children to frame 3 members of the set then count the rest of the set. Frame 3, 4, or 2 members of the set on the desk. Increase or decrease the number of objects with which a child is working and repeat the work. Continue the work by asking a child to frame the number of members which he can name without counting. Then ask the child to push those objects aside and count the rest of the set. Some children may push aside the first set and recognize the number of members still on the desk and in effect, add the numbers.

<u>Pupil's</u> <u>book</u>, <u>pages</u> <u>24</u> <u>and</u> <u>25</u>: Ask the child to ring a subset within each set whose number he can recognize without counting. Then, he identifies the number of the set and writes the numeral for that number in the space provided. (You can expect a child to recognize no more than 4 objects without counting.)

<u>Pupil's</u> <u>book</u>, <u>page</u> <u>26</u>: Instruct the children to connect each set with the appropriate numeral.

<u>Pupil's</u> <u>book</u>, <u>pages</u> <u>27</u>, <u>28</u>, <u>29</u>, <u>30</u>, <u>31</u>: Have the children ring the correct numeral.

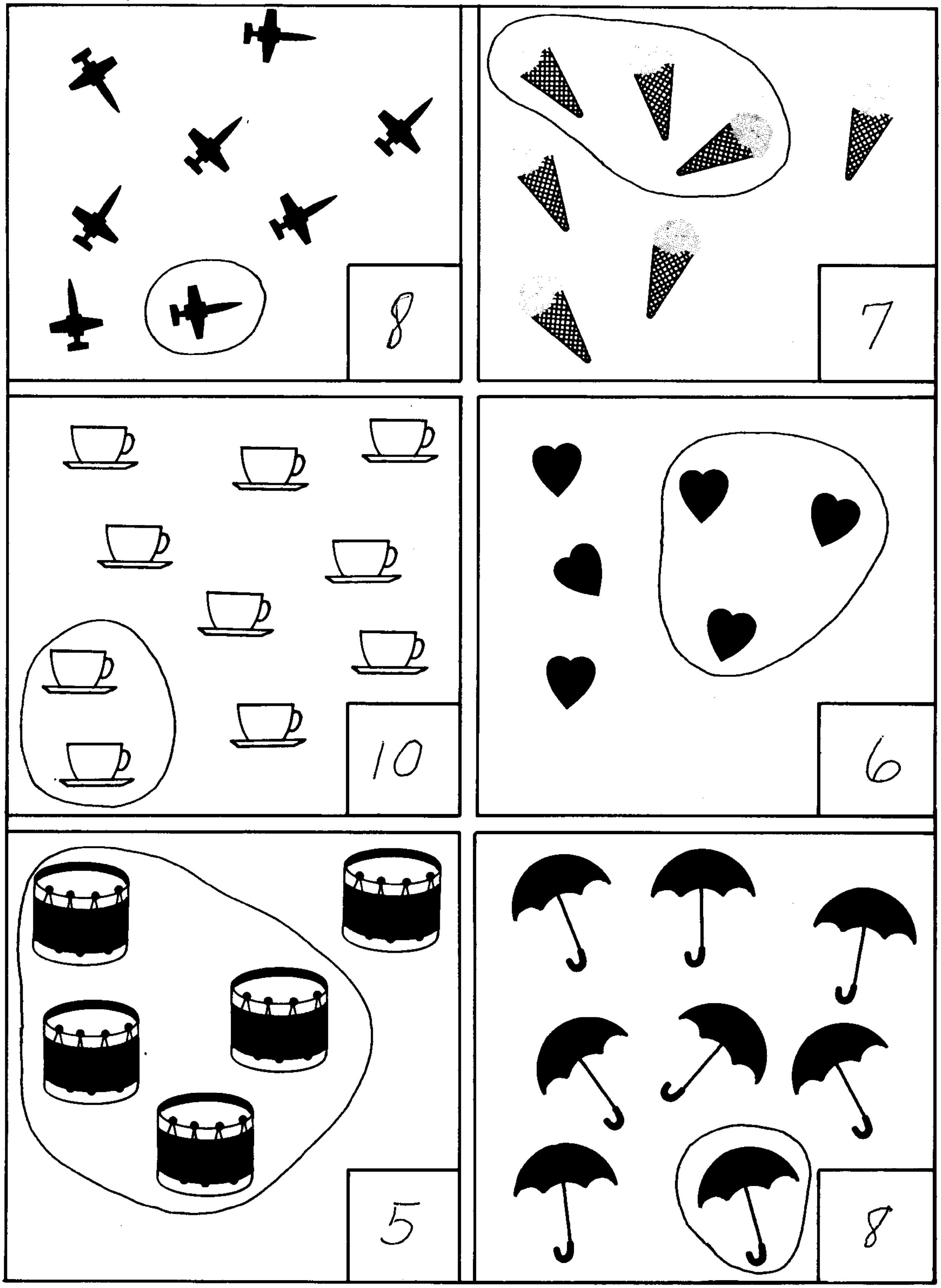
<u>Pupil's</u> <u>book</u>, <u>page</u> <u>32</u>: In each box of two numerals, ring the one which names the greater number.

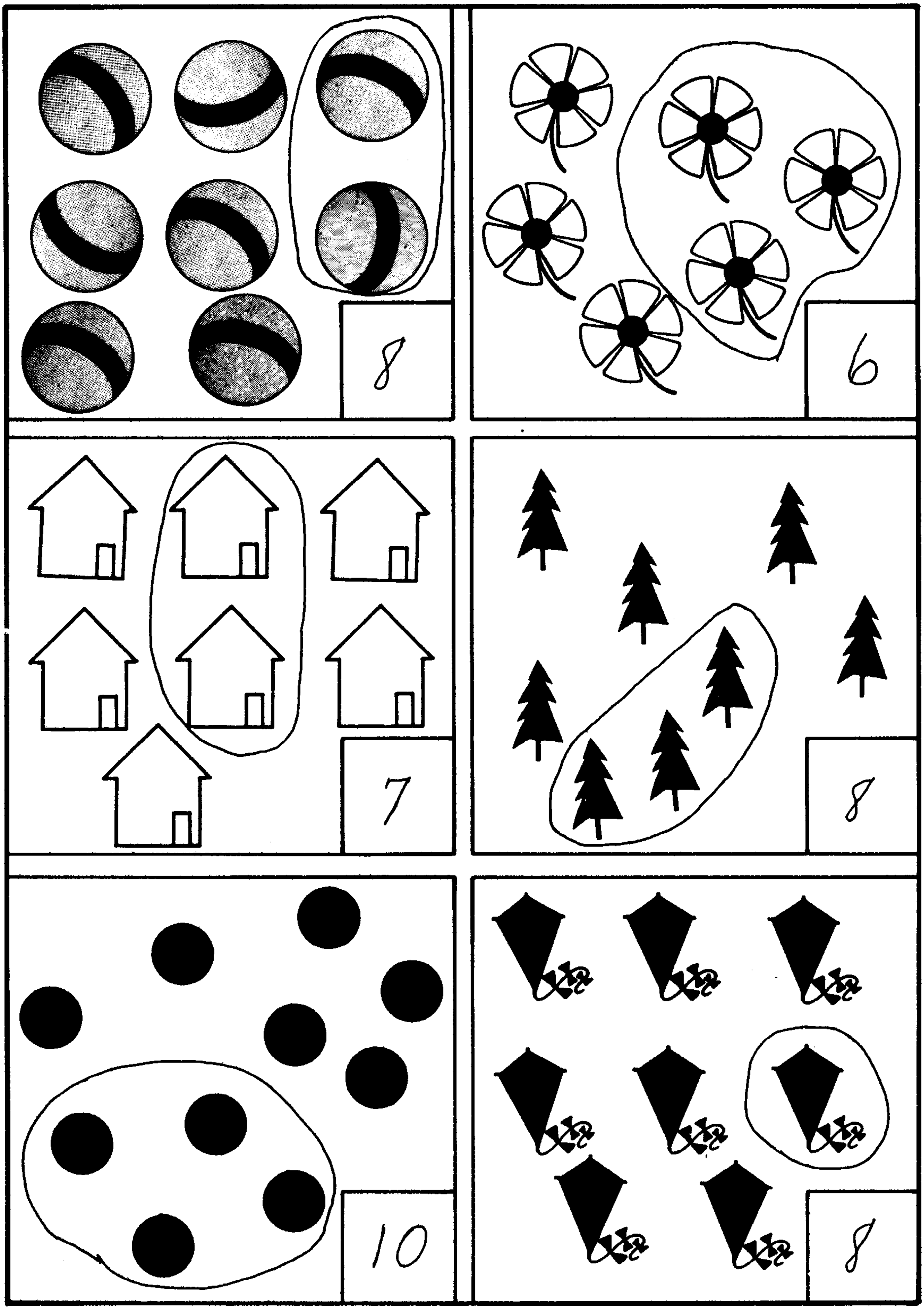
<u>Pupil's</u> <u>book</u>, <u>page</u> <u>33</u>: In each box of two numerals, ring the one which names the lesser number.

<u>Pupil's</u> <u>book</u>, <u>page</u> <u>34</u>: Read the numeral at the center of the doughnut. Mark each numeral which names a greater number.

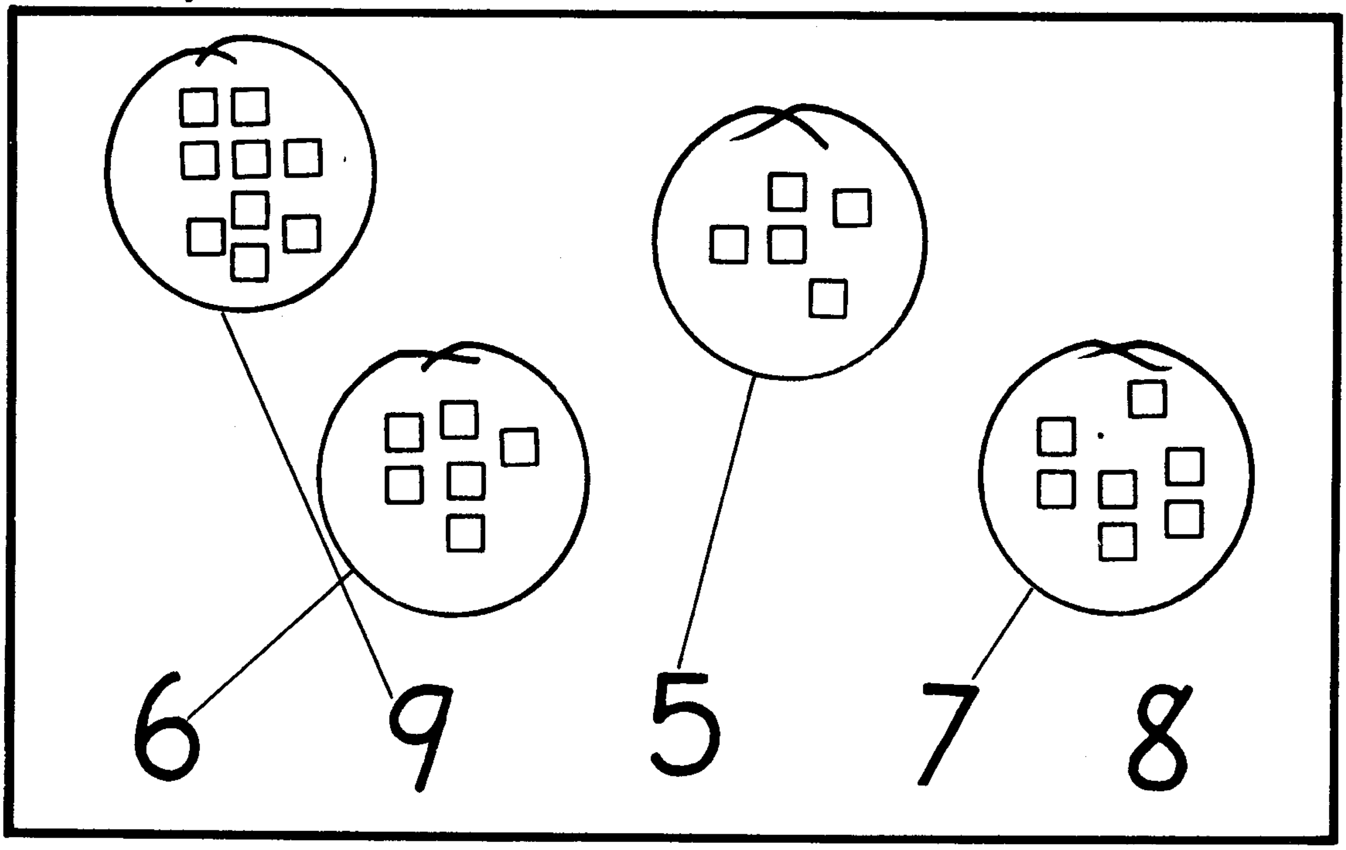
<u>Pupil's</u> <u>book</u>, <u>page</u> <u>35</u>: Read the numeral at the center of the doughnut. Mark each numeral which names a lesser number.

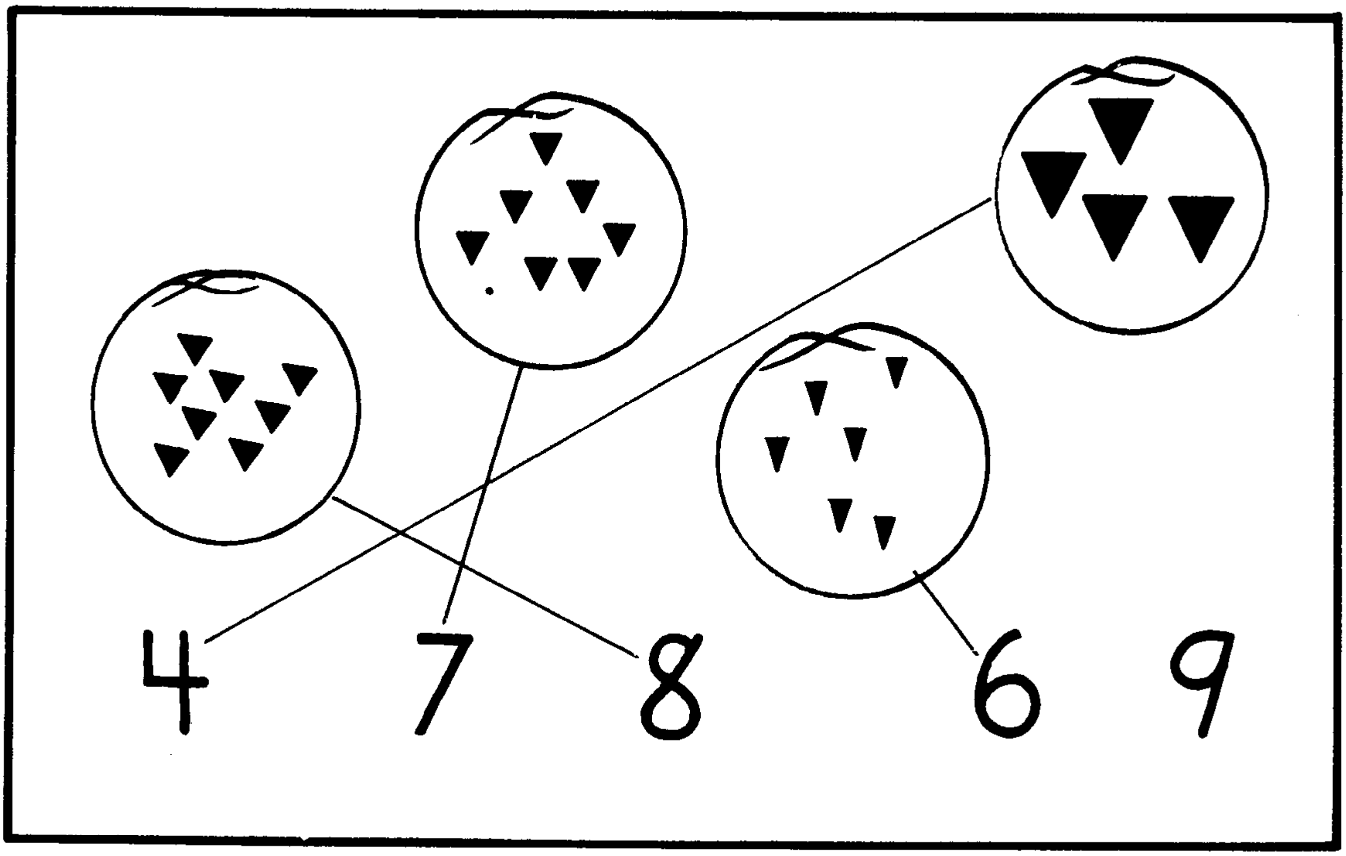
How many?



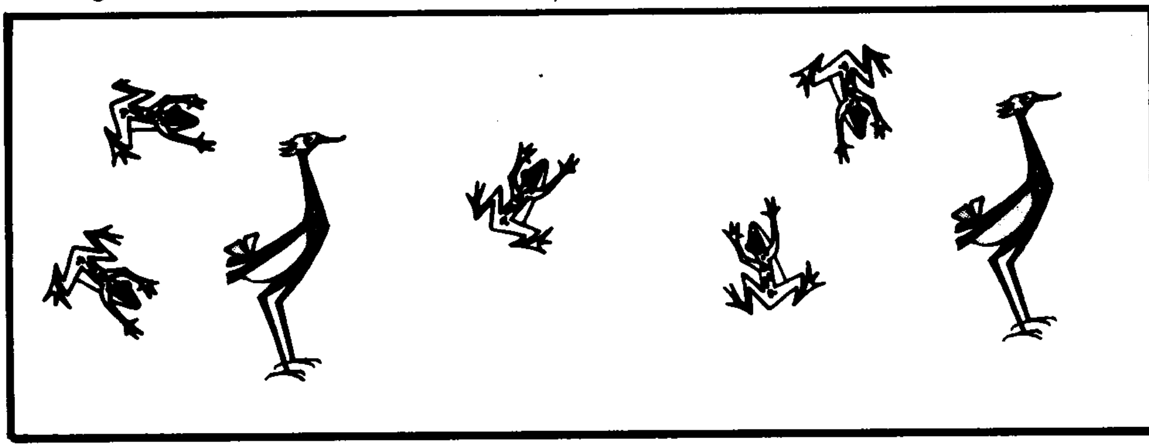

8
6
7
8
10
8

How many?

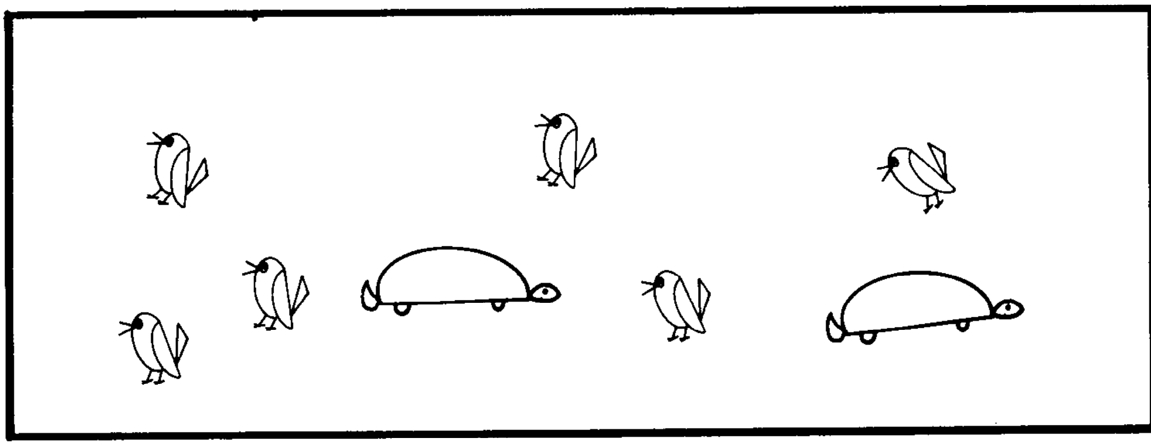




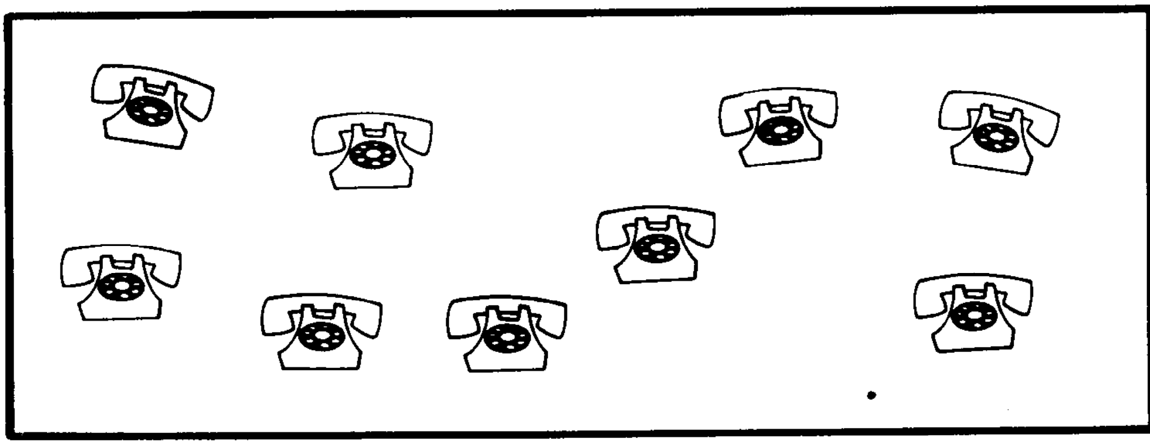
Ring the correct numeral.



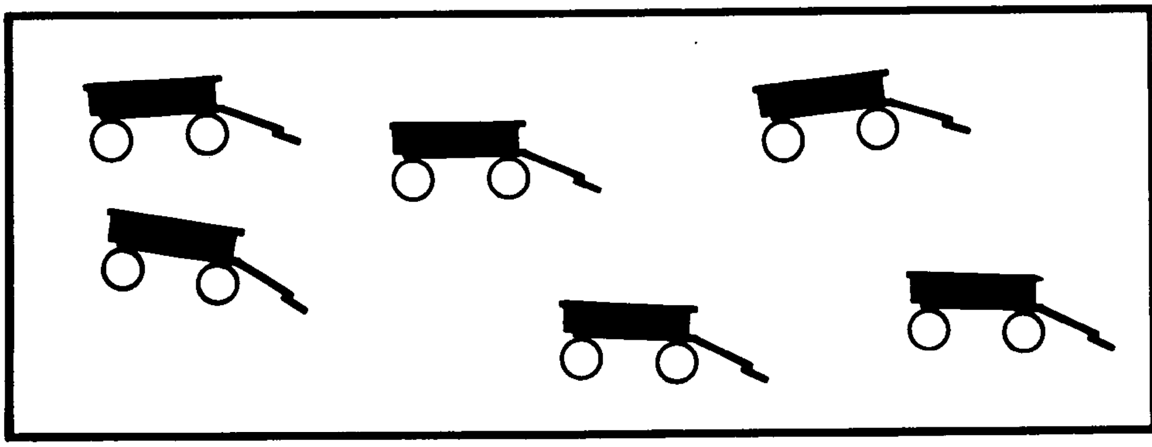
2
7
4



8
6
4

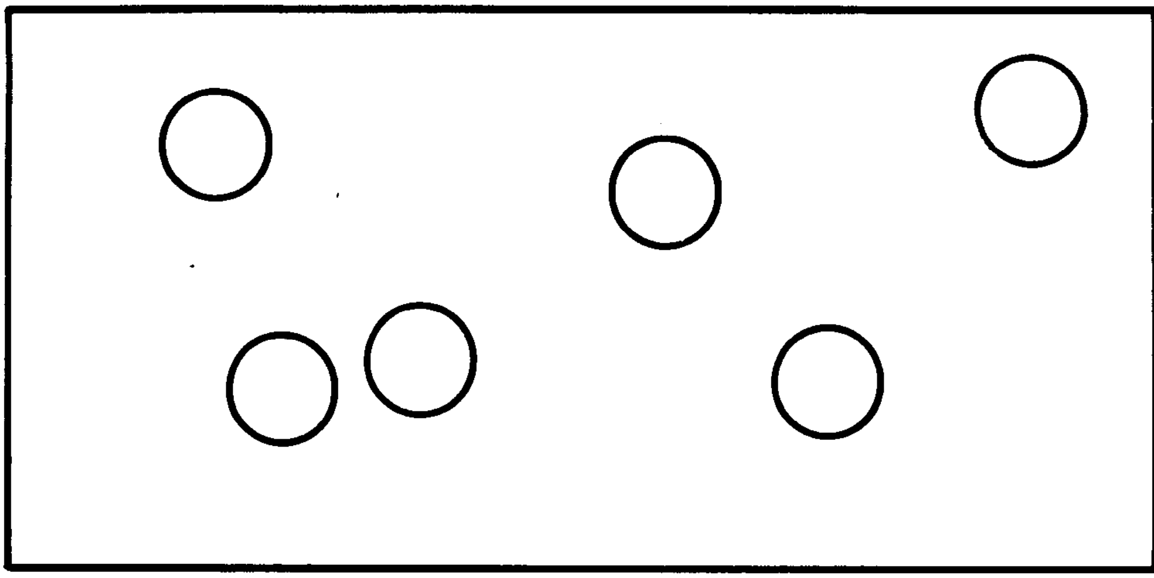


3
9
0

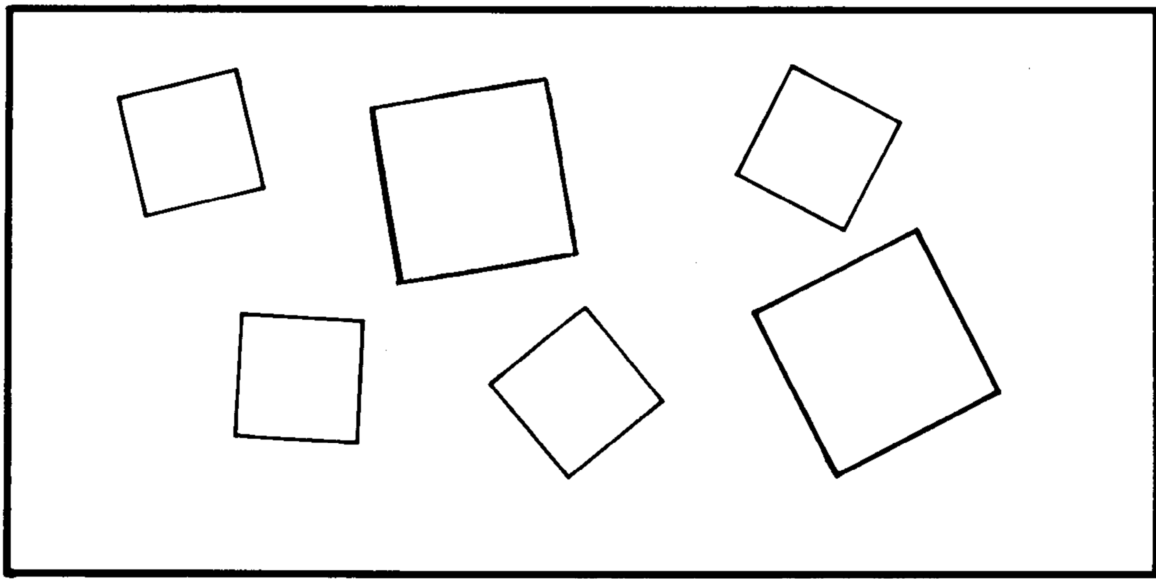


5
6
1

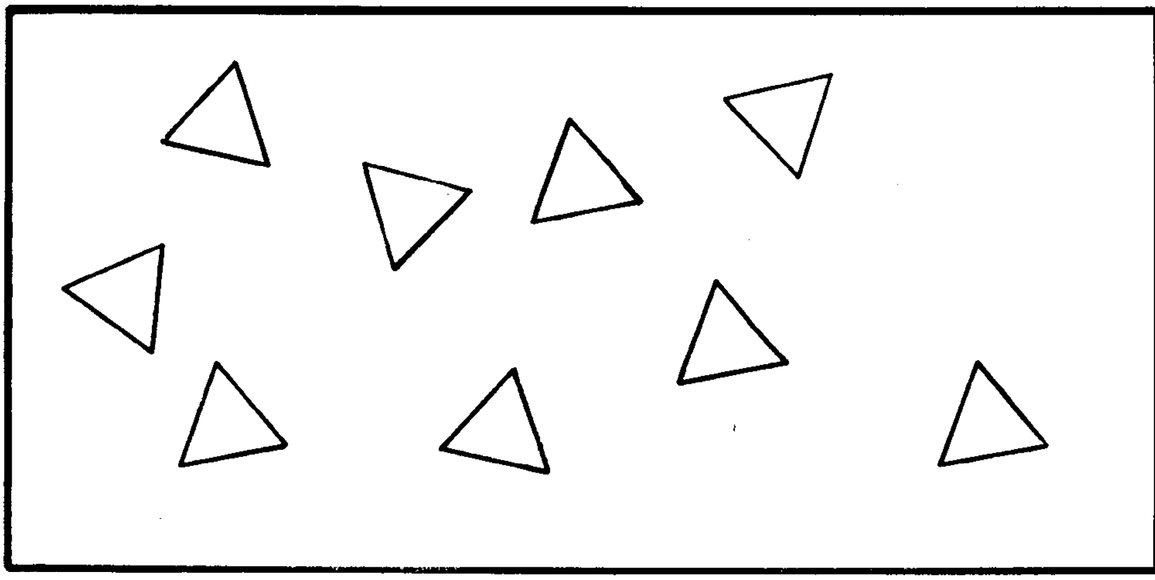
Ring the correct numeral.



6 7 8

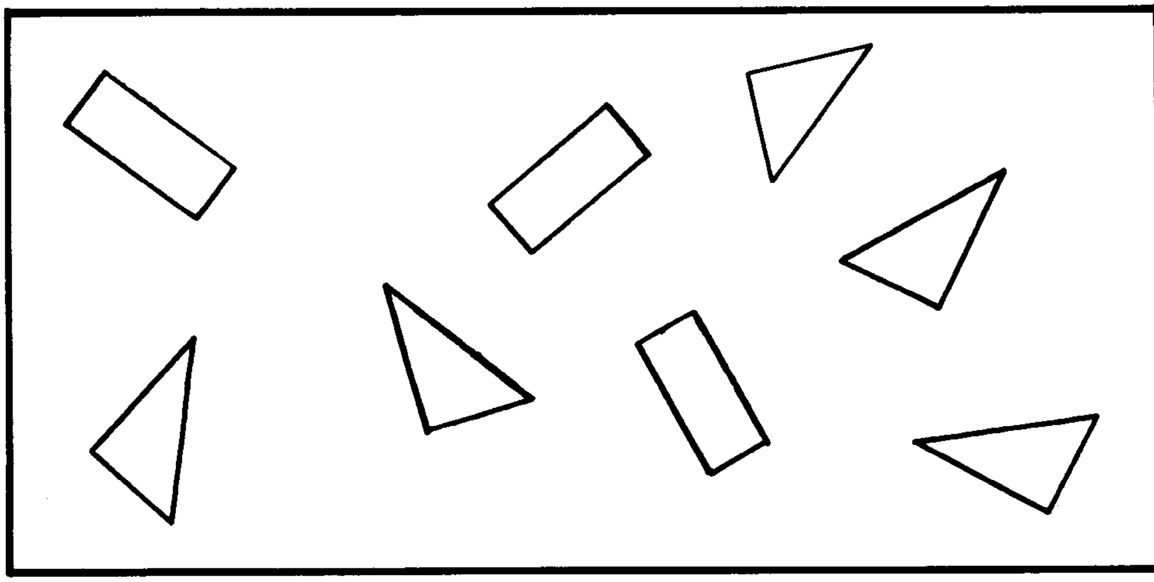


6 7 8



7 8 9

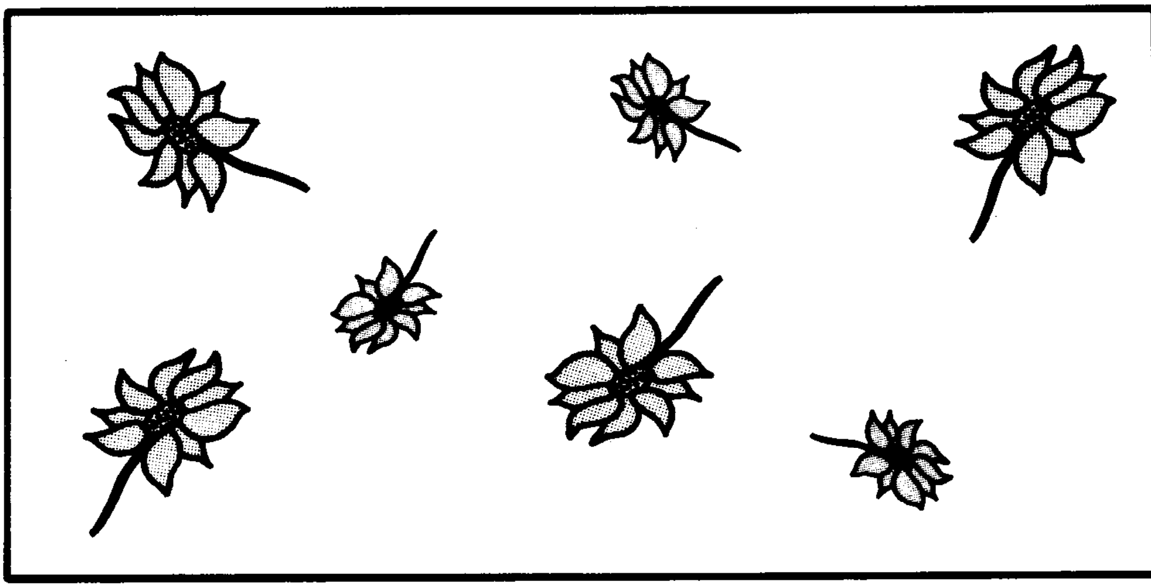
Ring the correct numeral.



6

7

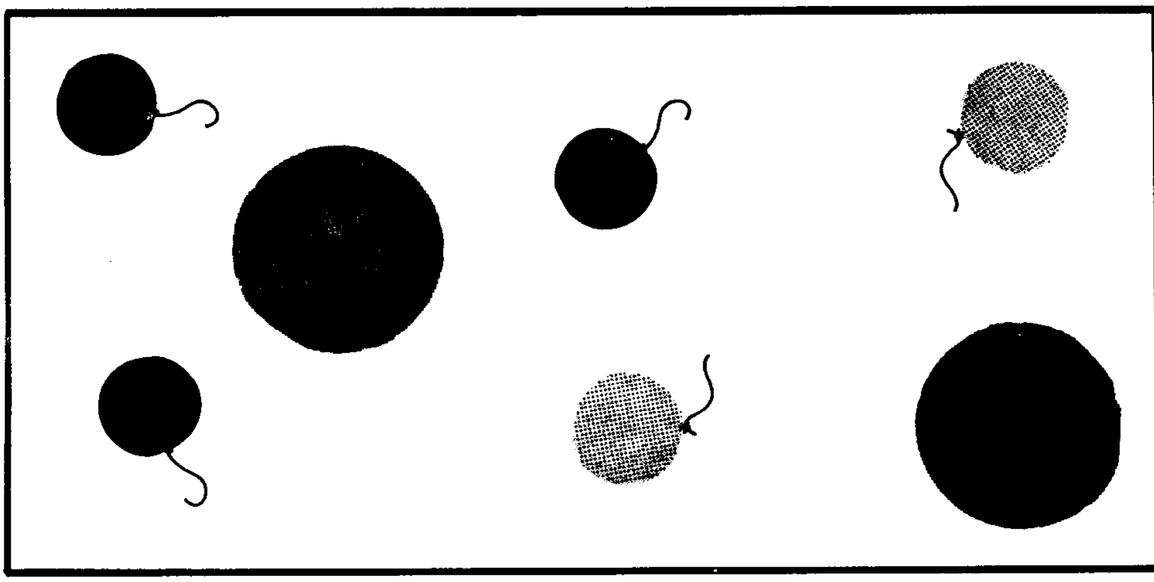
8



6

7

8

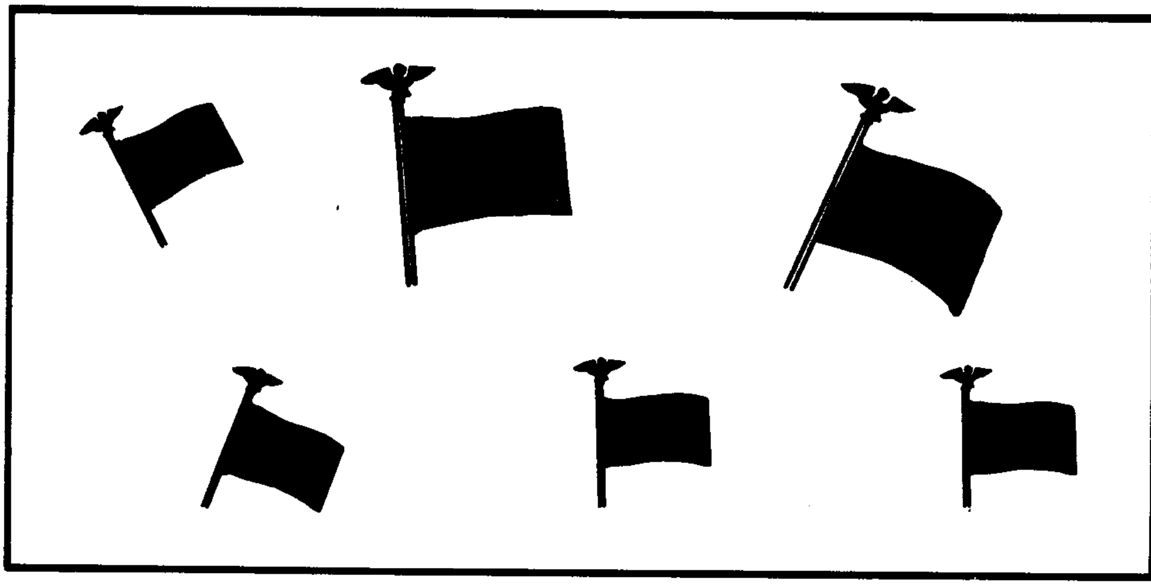


6

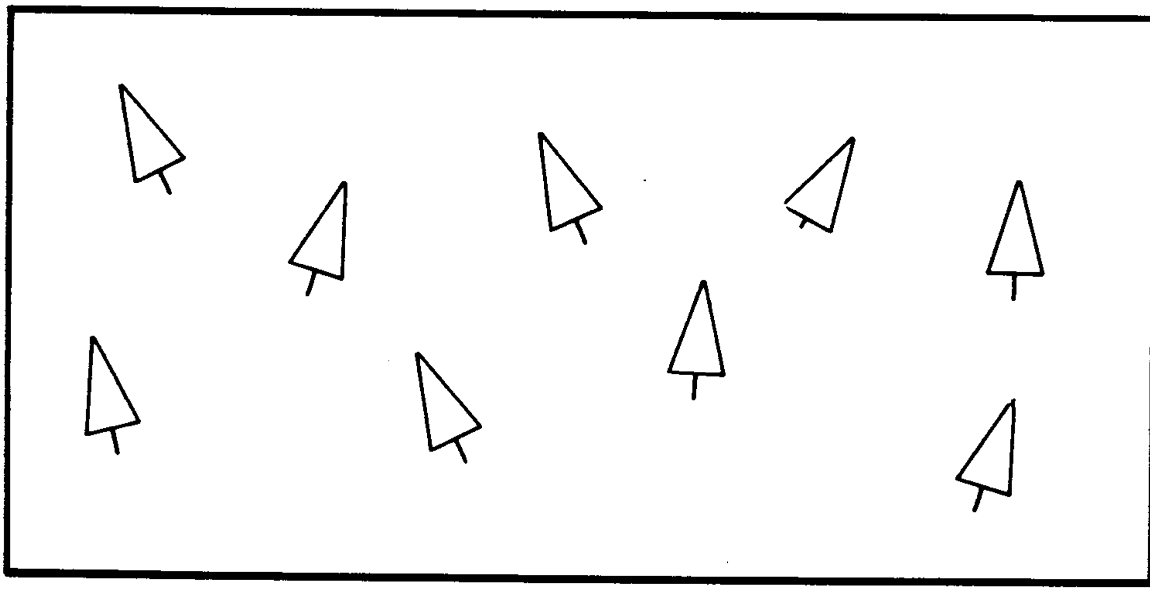
7

8

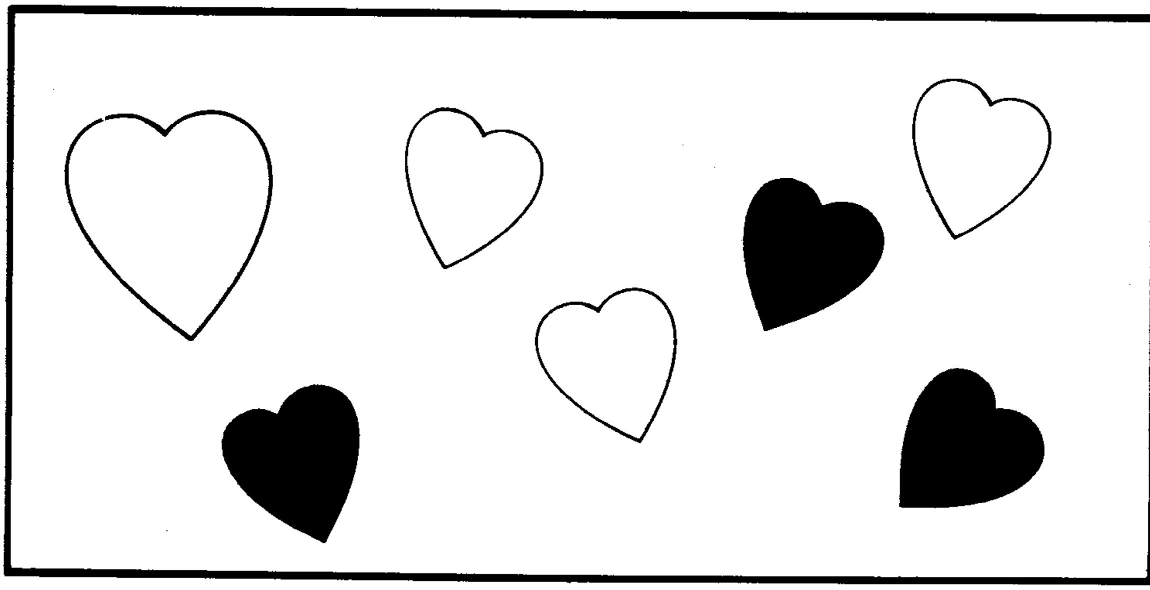
Ring the correct numeral.



6
7
8



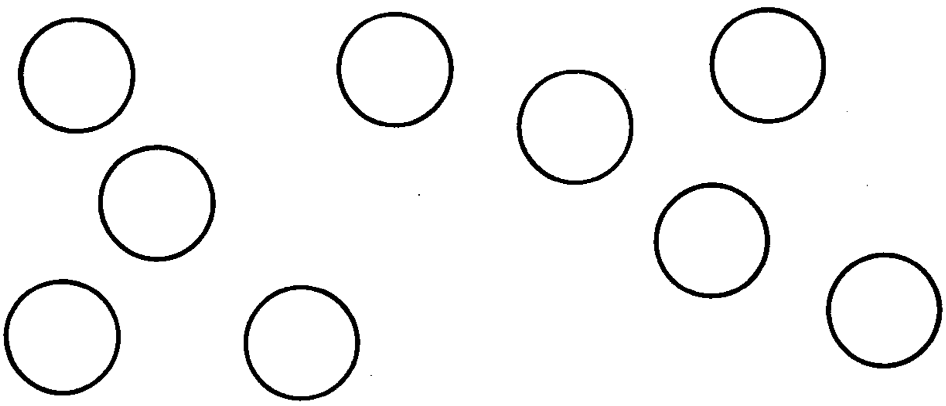
7
8
9



6
7
8

Ring the correct numeral.

6
7
8



7
8
9

6
7
8

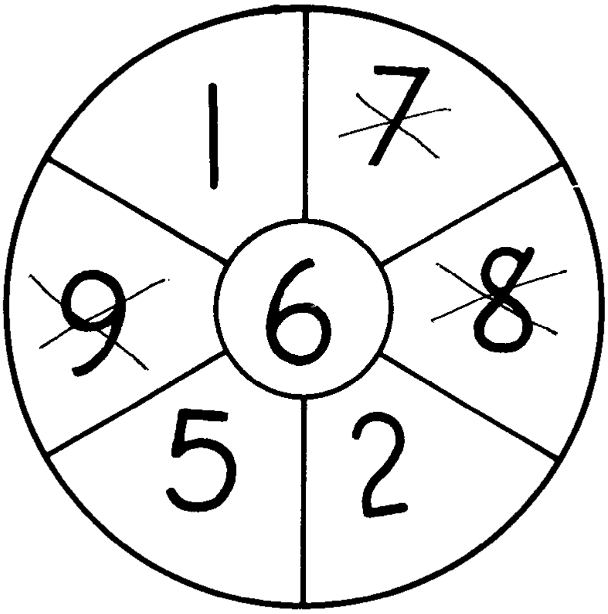
Which number is greater?

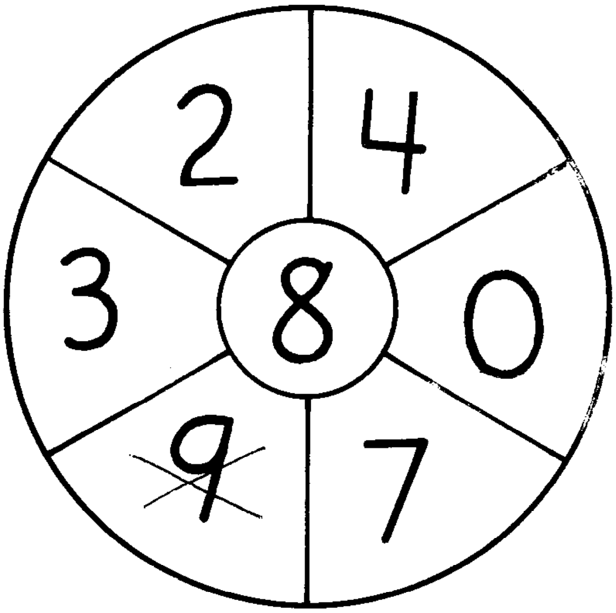
2 / 4	3 / 1	5 / 9
2 / 0	7 / 6	2 / 3
0 / 9	5 / 7	9 / 2

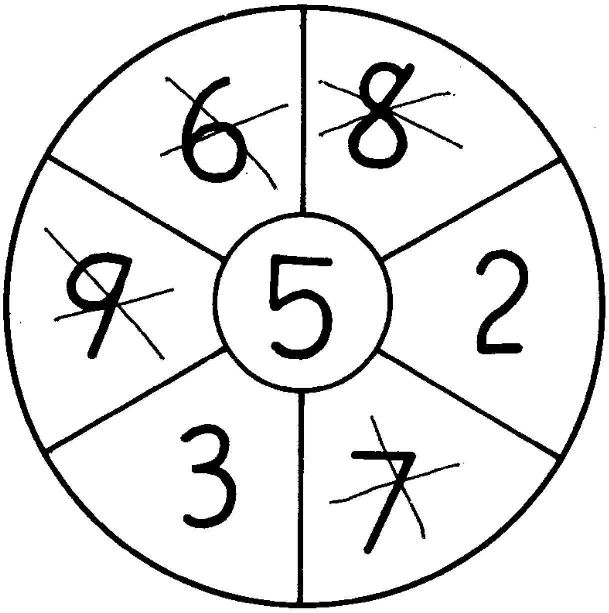
Which number is less?

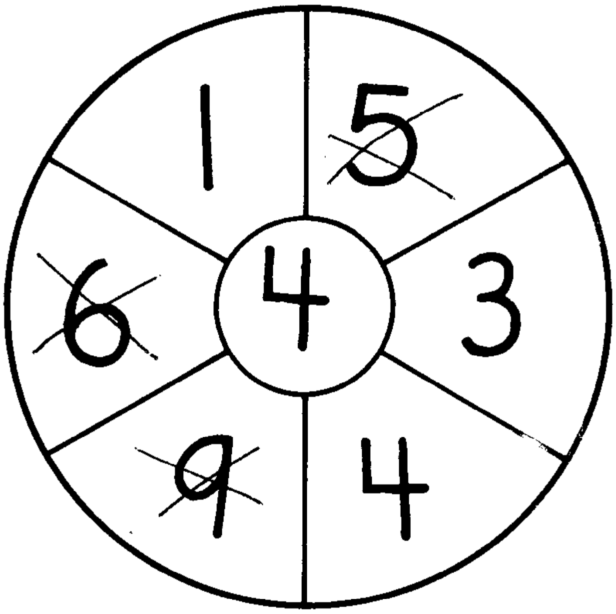
8 / 3	9 / 8	3 / 5
4 / 3	7 / 5	1 / 9
6 / 2	8 / 0	4 / 6

Which numbers are greater ?

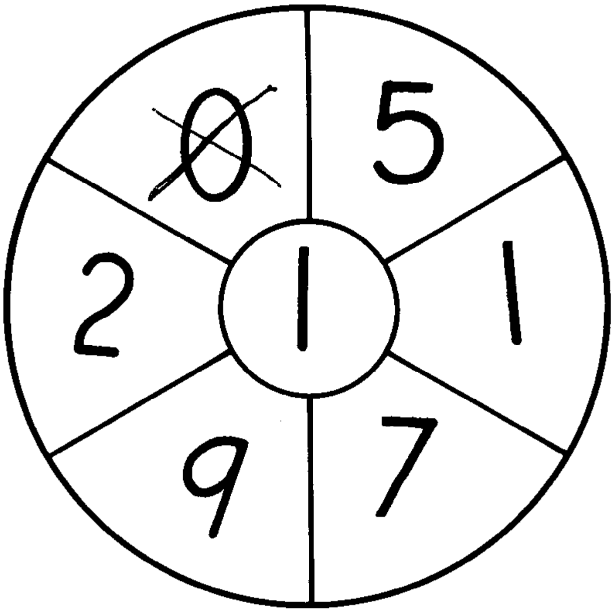


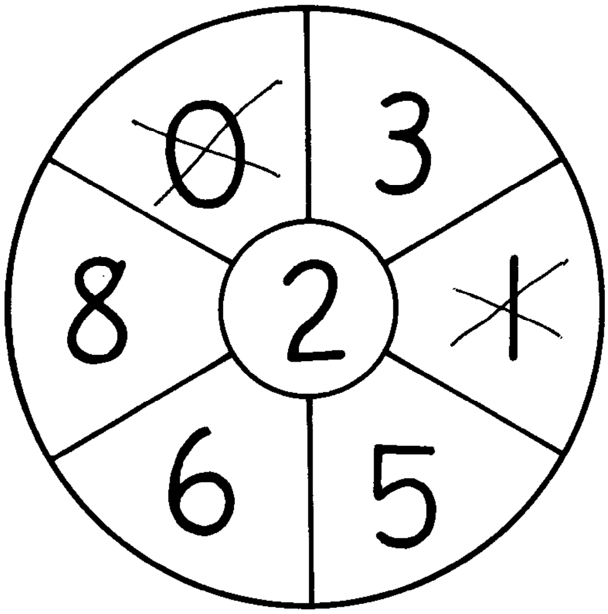


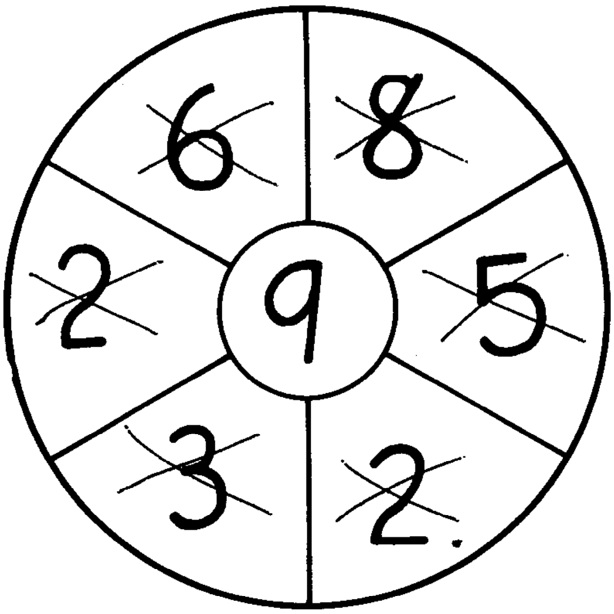


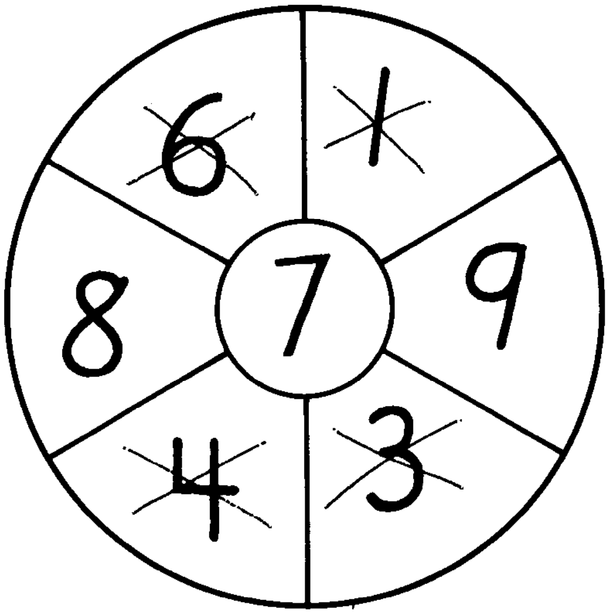


Which numbers are less?









Part III-The words "zero, one, two, ..., ten"
Children should learn to recognize the words, zero, one, two, three, four, ..., ten as names for numbers just as they recognize the numerals 0, 1, 2, 3, 4, etc. Some children who are capable of understanding the concepts which are presented in this chapter and can learn the numerals without difficulty may not yet be ready to recognize the words. It is not intended that mastrry of the words precedes continuation of the book but that a start be made so that when they are used there will be some familiarity with the words. You should expect a level of mastery which seems appropriate for the ability of your children.

Since the teaching of these words is closely related to reading, you should use techniques and activities which you use in your reading program to teach these words. You will want the children to associate the written word with the spoken word, with the set, and with the numeral. Where the words are used on pupil pages you will find that they are used with the numeral. Cards with numerals on one side and the words on the back, and cards with pictures of a set of objects on one side and the word on the back provide for individual or small group practice.

Further Activities:

1. The following game may be made for use by two children as drill:

   Make 2 sets of cards 2" × 3" which have the numerals 0 to 9 on one side; one set will have X on the reverse side and one set will have 0 on the reverse side. Make a tick tack toe chart approximately 9" × 12". Fill in the boxes with nine of the words zero to nine. One word must be left off on each card but the numeral should be included in the cards.

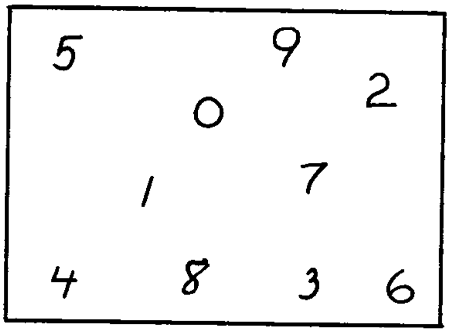
four	six	two
one	zero	five
eight	three	seven

   One child uses the X cards, another child the 0 cards. Children take turns as in tick tack toe, placing a card over the corresponding box of the chart. Place cards down with X or 0 showing. If a word has already been used the child gets another turn. A score is made when one child has 3 X's or 3 0's in a row as in tick tack toe.

2. Give each child a set of numeral cards. Show sets of objects on the flannel board or on the chalk board, describe sets in the room or show pictures of sets, and have each child show the correct numeral card.

3. Pass out sheets of paper, each with a numeral written on it. Ask the children to draw a set with that many members.

4. Give each child a piece of newsprint or construction paper. Tell the children to make each numeral 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 on the page. The numerals should be scattered all over the page, and each child's page will probably be different. Display a set of objects on the flannel board. Ask the children to find the numeral which tells the number of members in the set. They "show" the numeral by framing it with their hands.



II-3. The number line

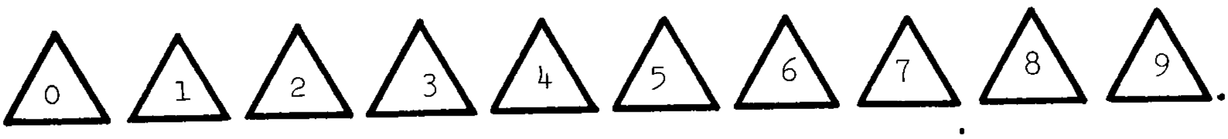
Objective: To develop the idea of the number line.

Vocabulary: Point, line.

Background Note:

This lesson introduces the number line. Eventually we shall use the number line for addition, for subtraction, and to introduce rational numbers. In this lesson, the principal use of the number line is to reinforce the children's concept of inequality.

Materials: Strong string, paper clips, a printed number line and two sets of numeral cards shaped as shown:



Suggested Procedure:

Have one set of numeral cards in random order on the chalk ledge. Ask a child to come to the front of the room. Tell children, "Today we are going to jump and count the jumps." Mark, on the floor, a starting point.

> Susie, you stand at our starting point.
> (To Susie:) How many jumps have you taken from the starting point? (None.)
> What number tells how many jumps you have taken? (0.)

Place the zero card by the child's feet with the point toward her feet. Tell her to take a jump and stop.

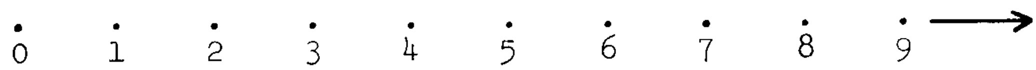
How many jumps from the starting point have you taken now? (1.)

Have a child choose the numeral card and place it at Susie's feet. Continue with the same procedure until 9 jumps have been taken. Ask the child to try to take the same sized jumps. Keep emphasizing that the numeral card tells how many jumps from the starting point.

Would we have to stop with 9 if we had more cards? (No. We could go on and on.)

Remove the numeral cards from the floor and tell the children to imagine that jumps had been taken on the chalkboard. Have a child indicate the starting point and nine jumps. Make a dot to show these points. Ask what numeral to write for the point that shows the starting point. (0.)

Continue to number the points through 5. Have children look at unnumbered points, and ask one to come up and put the pointer on the point where he thinks the next jump would be. Ask if points could continue to be made in this direction. Draw an arrow to show in what direction we would go if nine jumps are taken. (We begin with dots and an arrow and gradually build the picture of the number line.)



Try to establish for the children, using further experiences if necessary, the picture of equally spaced dots extending indefinitely to the right, with each dot associated with a number. Repeat that the numeral names the number of jumps from the starting point.

- Now help the children to understand the relation between inequality of numbers and order on the number line.

Is 3 less than 5? (Yes.)

Which side of the "5" is the "3" in our picture?

("3" is to the left of "5".)

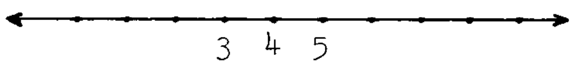
Continue, bringing out that "less than" corresponds to "to the left of" and "greater than" corresponds to "to the right of".

- A day or two later, have two children, one at each end, pull the piece of string tightly. Ask children to pretend to watch a frog jumping across a tight rope. Suggest a spot on the string (to the left as the class sees it) where it begins, and ask what numeral to use at this point. Clip the zero card to the string (using a paper clip at the top of the card).

Have a child come up and stand by the string at that point, take a single jump and "stop" there. Place numerals after children tell you which one for each jump.

Ask children to imagine a long, long tight rope on which many jumps can be taken. Suggest that it be thought of as a line of numbered jumps, all of the same length, or a "number line".

Dispense with the string and draw a line on the board. Mark points and label as shown.



Instead of using a tight rope for our number line, let's use a line on the board.

How is this number line different from the one we made on the tight rope? (It doesn't have all the numerals. It doesn't show the first jump.)

Ask a child to use the pointer to show another point, to tell what numeral to write there, and to tell why he decided on that numeral. (Names the number of jumps from the starting point.) Have children notice that although not all the numerals are written, they know where others must go because of the ones already written.

Have the other points named and write the numerals (not beyond 9 however). Emphasize that 0 is the starting point, and that each numeral tells how many jumps from the starting point.

Use the line for children to point to the numeral *before* "5", the numeral *after* "1", *before* "1", etc.

The numeral just before "5" names the number that is one less than 5; the numeral just after 5 names the number that is one greater than 5. It should be emphasized that all numerals before "5" name numbers *less than* 5 and that all numerals after "5" name numbers *greater than* 5. Ask children to use the number line to find:

A number greater than 7.

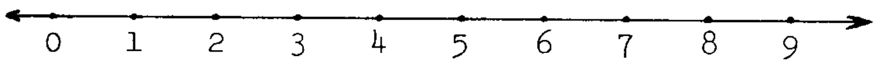
A number greater than 2.

A number less than 4.

A number less than 6.

Use riddle games. For example: "I am thinking of a number that is two less than seven;" "I am thinking of a number that is one greater than six."

Mount above the chalkboard, or elsewhere, a large printed number line:

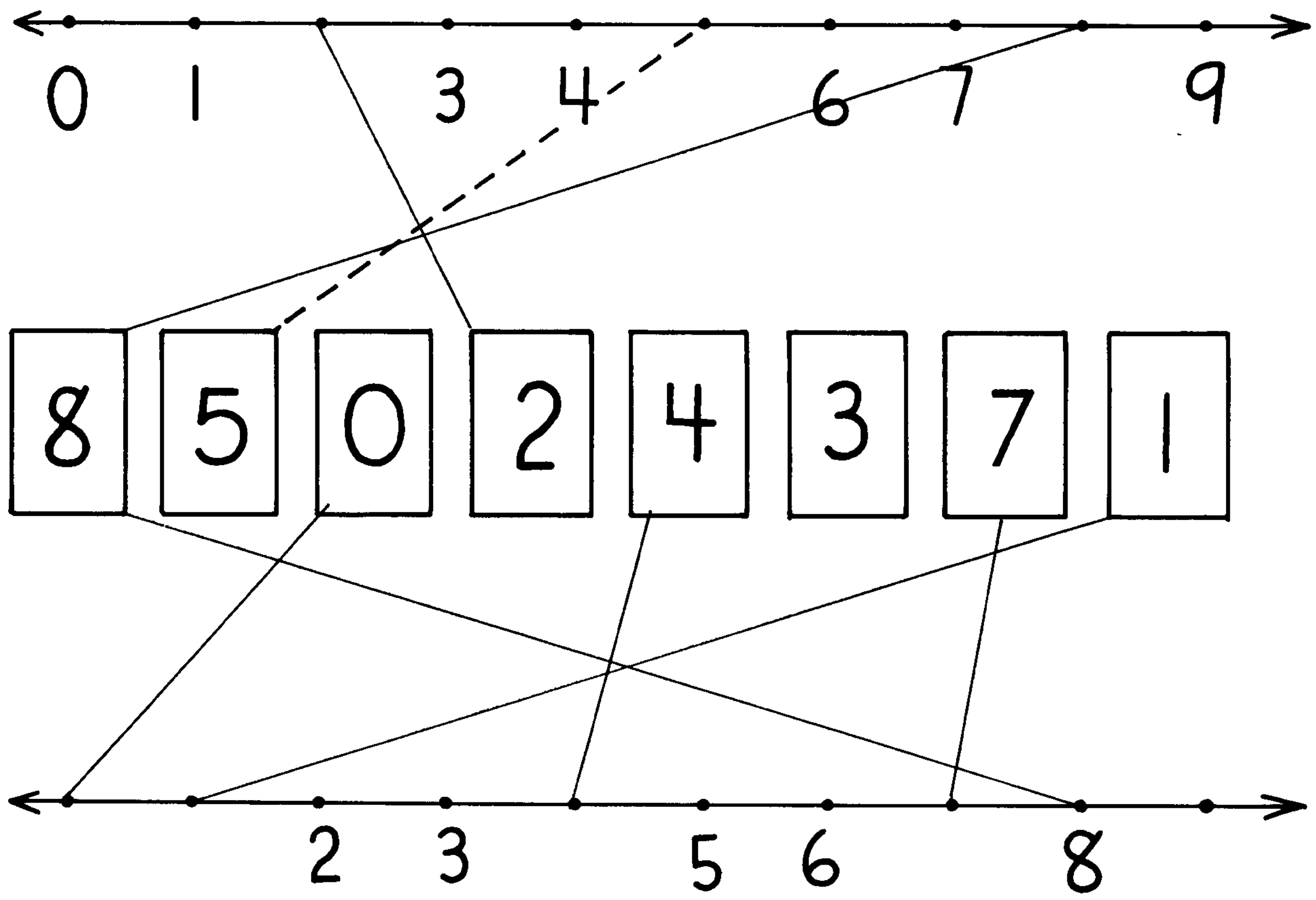


CAUTION: If the material you have has the "0" to the right of "9" cut off the "0" and tape it to the left of the "1".

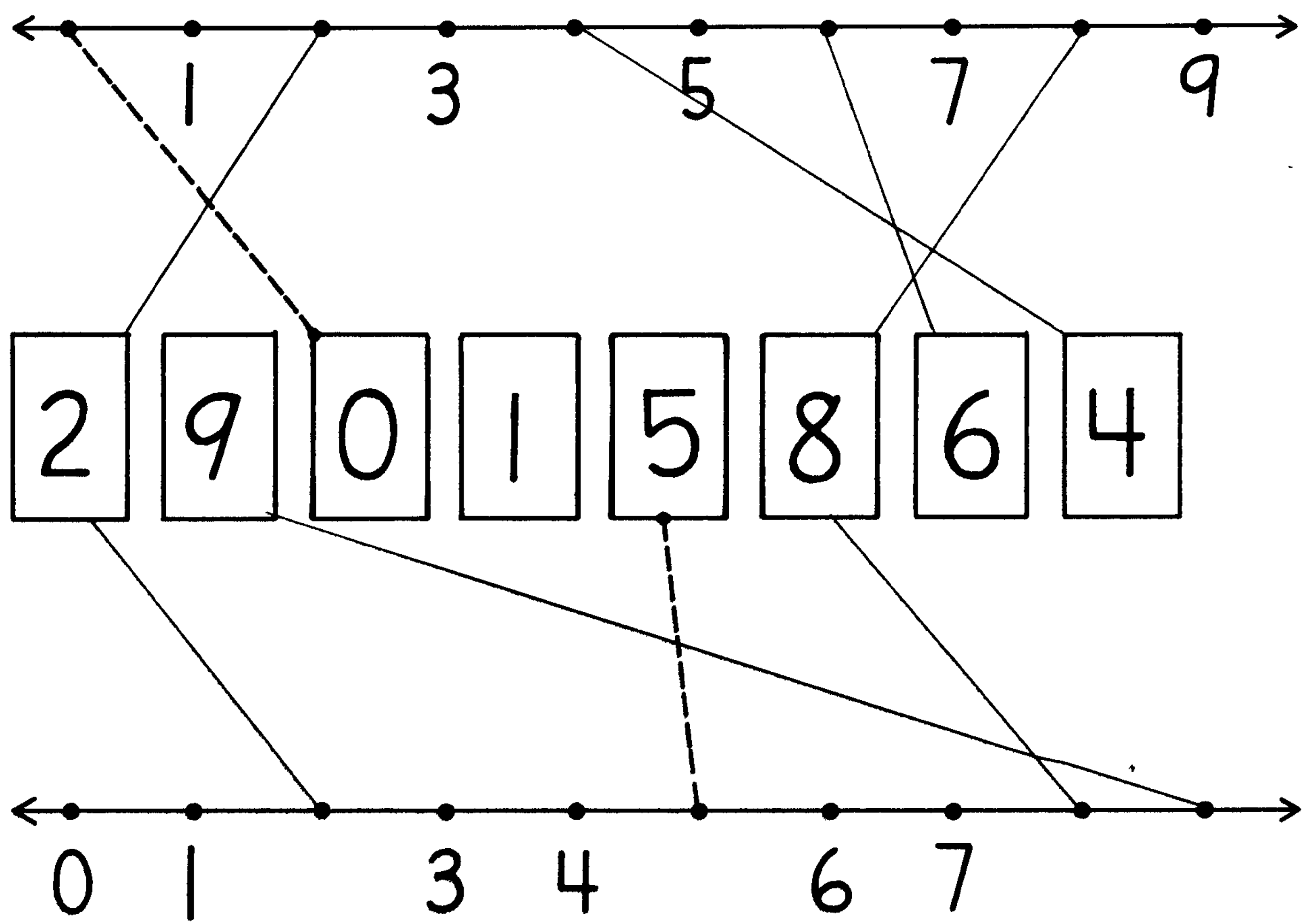
Pupil's book, pages 36 and 37: Ask the children to draw lines to show where the numerals in the boxes belong on the number lines.

<u>Pupil's book, page 38</u>: Ask the children to look at the picture of the number line. Using the number line, they are to mark all of the numerals in the first box that name numbers less than 4; in the second box, greater than 7; and in the third box, less than 0.

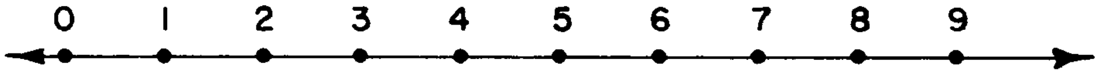
Where do the numerals belong?



Where do the numerals belong?



Use the number line.



Which numbers are less than 4?

9	~~3~~	5	~~2~~	~~1~~	6

Which numbers are greater than 7?

5	7	~~9~~	3	6	~~8~~

Which numbers are less than 0?

5	7	3	4	1	0

Chapter III

SETS OF TEN

Background

We have been concerned so far with sets of single objects--that is, with bunches of things. In this chapter we reach a slightly higher level of sophistication: we consider sets whose members are themselves sets. We count sets of objects by partitioning them into sets of ten and then counting the sets of ten. We extend our system of numeration by agreeing that, for example, a set which can be partitioned into 3 sets of ten has thirty members.

This short chapter is devoted entirely to sets of ten and it is primarily preparation for the study of place value. In a later chapter we shall partition a set into equivalent subsets which are not necessarily sets of ten. We then connect the fact that, for example, a set of thirty can be partitioned into 5 sets of 6, with the facts: $5 \times 6 = 30$ and $30 \div 5 = 6$.

III-1. Sets of ten

Objective: To introduce counting sets of ten members each.

Vocabulary: (No new words.)

Materials: For teacher--a box with sets of ten disks and other cutouts for use at the flannel board; large sheet of tagboard or cardboard; counting sticks, rubber bands.

For pupils--sets of materials in multiples of ten (10 to 100 of each); e.g., blocks, kindergarten beads, buttons, lima beans, spools, sticks, theater tickets, pegs, and paper clips.

Teaching Note:

The activities described here may be carried on for several days, the time depending on the ability and previous experiences of the children. It should be remembered, however, that the intent is to "open up" an idea rather than to develop full understanding. No worksheets are needed.

Suggested Procedure:

Show the children a box containing thirty flannel cutouts (ten disks and ten each of two other shapes), bunched so that it is necessary to count and sort them. Dump them on a table.

I wonder how many sets of ten are in this box?

Let's see how we can find out.

Pick out one disk and hold it up.

How many disks have I? (One.)

Place that disk on the flannel board. Repeat the procedure, making a row of disks on the flannel board. (As you work, a child may be asked to help pick out the cutouts with which you are working and hand them to you, one at a time.)

> How many disks are there in the set on the flannel board?

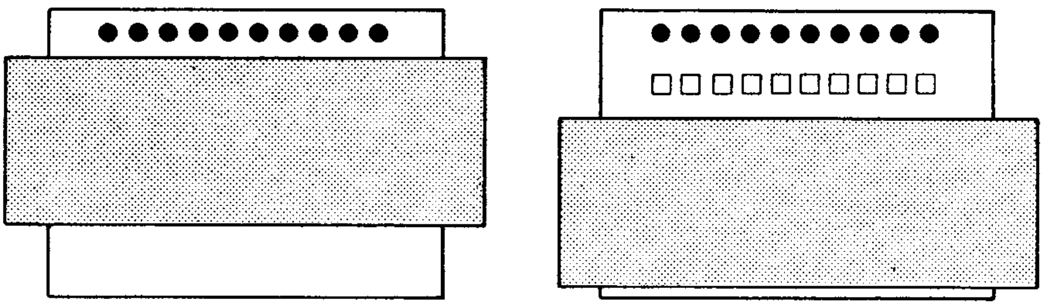
Point to each disk as children count to ten.

> This set of disks on the flannel board has how many members? (Ten.)
>
> Do we have one set with ten members? (Yes.)
>
> Let's see if we can use the set of ten disks to find how many trees we have.

Place ten trees in a row beneath the disks so that the one-to-one correspondence is obvious.

> Is the set of trees equivalent to the set of disks? (Yes.)
>
> What is the number of the set of disks? (Ten.) Of the set of trees? (Ten.)
>
> How many sets of ten cutouts do we have on the flannel board? (Two.)

Hold tagboard to cover the sets, exposing one row, then two, while children count with you: one set of ten, two sets of ten.



Place another set of ten cutouts beneath the trees.

How many sets of ten cutouts have we now? (Three.)

Again cover and expose rows while children count: one set of ten, two sets of ten, three sets of ten. Tell the children that we can say "one ten", "two tens", "three tens", and not say as many words when we count. Let them repeat the counting: "one ten", and so on.

How many sets of ten cutouts did we have in the box? (Three.)

Have children count ten sticks as you bundle them together. Ask how many are in the bundle; put a rubber band around it. Continue in like manner with ten sets (bundles) of ten sticks, asking questions similar to those related to the sets of cutouts on the flannel board.

Have sets of ten blocks counted and stacked in "tens", questioning the children as the work proceeds as to how many tens there are after each stack is completed.

- Other experiences with sets of ten:

A strip of ten theater tickets that has been torn apart into ten single tickets can be put back together as a set of ten. Paper clips may be strung together as sets of ten to lead to the idea that ten ones and one ten are names for the same number. Sets with a variety of members as well as with similar objects should be grouped into sets of ten, e.g.

A chart may be made on the chalkboard with blanks in which children can record the number of sets of ten that have been counted.

tickets _____ tens

clips _____ tens

- The preceding experiences have had considerable guidance by the teacher. The next step, and an important one, is to give each child a set of materials to count into sets of ten. If your children work well in small groups, you may prefer to have several children work together. Let each child (or group) report the number of sets of ten (tens) that was counted. Make a list on the chalkboard, letting children write the numerals and their own names.

I counted	_____	tens.	Les
	_____	tens.	Chris
	_____	tens.	Alicia

Further Activities:

1. How many fingers are in the set of fingers on both of your hands? Then let ten children stand in front of the class, while the other children count the ten sets of ten fingers. Have one child come to the front and count the sets of ten by starting at one end of the row of children. Have another child count by starting at the other end of the row. Did both count ten sets of ten? Did both say the counting names in the same order?

2. Let children work as partners. Have each child in turn draw around the other's outspread hands palms down. Let children color and cut out their "hands". Then paste the hands, in pairs, to a strip of wrapping paper to use in practicing counting by tens.

3. Have children count sounds as you tap the desk or a triangle. After a set of 10 taps, each child holds up one finger; after the next set of 10 taps he holds up another finger, and so on. Stop after every multiple of ten, and have children tell you how many "tens" of taps there have been. Or, each child may record the sets of ten by marking a tally mark for each of the sets.

III-2. Naming multiples of ten

Objective: To emphasize that the order of counting "tens" is the same as the order of counting "ones"; to teach the names for multiples of ten.

Vocabulary: Row, column; ten, twenty, thirty...one hundred.

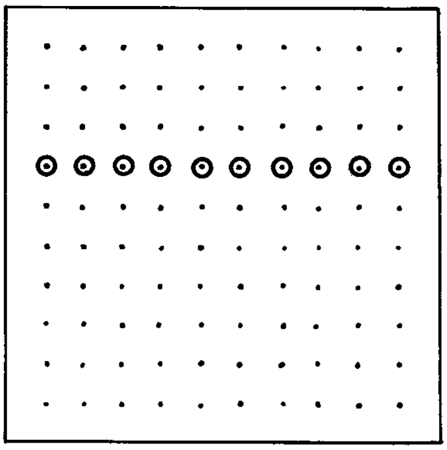
Materials: Spool board with 100 spools or pegboard with 100 pegs, large piece of plain tagboard or cardboard to cover rows for counting by tens; 1" × 1" cutouts for flannel board and ten strips of ten of these made by affixing to masking tape; objects for children to use in forming sets of ten, e. g., bundles of sticks, lima beans or other small objects in plastic bags, strips of tickets, the strips of 1" × 1" cutouts, buttons on cards.

Suggested Procedure:

Call attention to the rows and columns of the spool board or peg board.

> How many places for spools (pegs) are there in one row? (Indicate a row.) In one column ? (Indicate a column.)
>
> How many spools will each row of the board hold? (Ten.)
>
> How many spools will each column of the board hold? (Ten.)
>
> How do you know?

Ask a child to choose a row and fill it with spools. (If he selects a column, help him change to a row without belaboring the point. The distinction between row and column is introduced at this time. Clarification will come with use.)



How many spools are in the set on the board? (Ten.)

How do you know? (Answers will vary.)

Do we have one set of ten spools? (Yes.)

Place another row of spools on the board.

How many sets of ten spools are on the board now? (Two.)

Continue placing rows of spools on the board letting the children count them: 1 set of ten, 2 sets of ten, etc. to 10 sets of ten. Have the children count to ten so that the order will be recalled easily. Then have the rows of spools counted as you move a sheet of tagboard to expose them: 1 ten, 2 tens, 3 tens, etc.

Can we count tens in the same way we count ones? (Yes.)

Cover all but the two top rows of spools with the tagboard.

How many sets of ten spools do you see? (Two.)

Who will tell us another name for 2 tens? (Twenty.)

Continue in similar fashion, introducing any names that are not known by a child. Then let the children count the rows of spools again with you: ten, twenty, thirty,...one hundred.

Give each child sets of materials that have been grouped into sets of ten. Let each child count the sets he has, using both names, for example: 2 tens, twenty; 3 tens, thirty. Make a chart on the chalkboard. Let children write the numerals and their own names.

I counted __4__ tens		Sue, Bob, Anita
I counted __8__ tens		Phil, Helen

Follow up by asking such questions as:

> Who has 7 tens? What is another name for 7 tens? (Seventy.)
>
> Who has 4 tens? What is another name for 4 tens? (Forty.)

- Write numerals on the chalkboard, e.g., 9 tens, 1 ten, 10 tens. Ask the children who have the corresponding number of sets of 10 to display them to the class.

Further Activities:

1. Place sets of objects on the table or shelf reserved for activities for children to choose for individual work. With them place duplicated sheets that children may complete.

Name ________

I counted

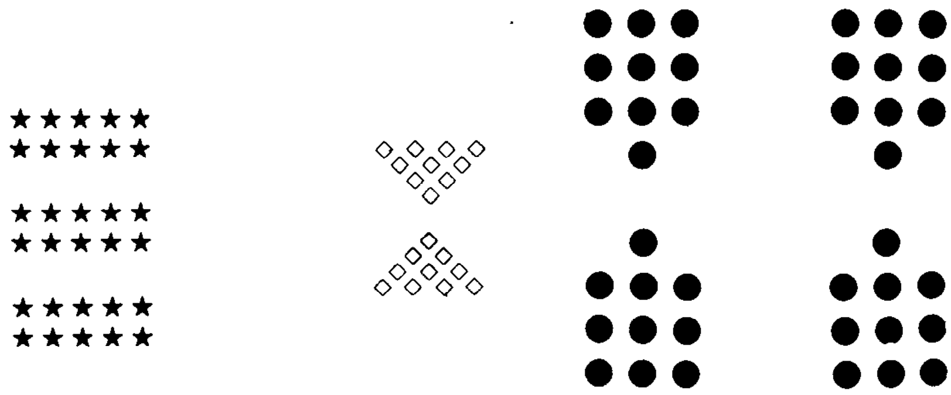
_____ set of ten

_____ sets of ten

_____ sets of ten

_____ sets of ten

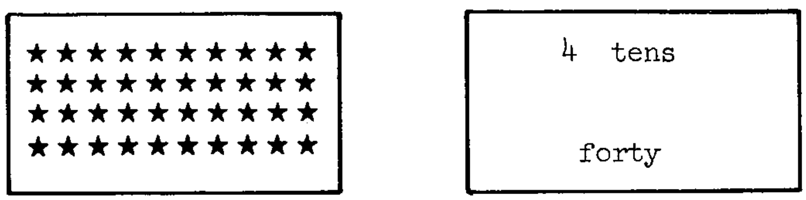
2. Use gummed stickers to make perception cards for sets of ten. Arrange the sets of ten in various patterns, but keep them as easily distinguishable units on the cards. For example,



3. The teacher may serve as leader to introduce the activity; later a child may be the leader.

   This card shows forty stars. Forty is how many sets of ten? (Four.)

   Then briefly expose the card to the children. Show the card again so that a check can be made. Following several experiences with the cards, children may use them as an independent activity. After these names have been learned, they can be written on the reverse sides.



(Front of card) (Back of card)

III-3. Application: Money

Objectives: To introduce the relative values of ten cents and one dime.

To reinforce understanding of "ten" as "ten ones" or "one ten".

Vocabulary: Dime, worth, value.

Materials: 100 pennies, 10 dimes, sheet of construction paper.

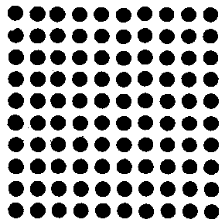
Suggested Procedure:

Have the children gather around a table where all may see. Let them discuss briefly their experiences in using money. Show them a dime and a cent.

> If you could choose just one of these coins, which one would you choose? Why?

Note whether or not any children use the words cent, penny, dime, value or worth spontaneously. If not, introduce the words into the discussion when appropriate.

Put 100 pennies on a table and ask a child to count a set of ten pennies. Ask if anyone knows how this set of ten could be used to help in finding the number of the set of all the pennies on the table. A child may suggest arranging the rest of the pennies into sets of ten that match (are equivalent to) the set of ten that was counted. If the suggestion does not come from a child, introduce the idea and let a child arrange the pennies as shown. Hold a sheet of construction paper to cover all but the first row, and uncover rows as children count: one ten, ten cents; two tens, twenty cents; three tens, thirty cents; and so on.



Ask questions to bring out the idea that there are ten rows of pennies with ten pennies in each row; the number of pennies in the set on the table is one hundred; there are one hundred cents altogether.

Ask how many pennies there are in the first column of pennies. Use a procedure like that used with the rows and let a child count the columns by tens.

Have a child place a dime at the end of each row of pennies, slightly apart from the row. Note that there is a set of ten pennies for each dime, and a dime for each set of ten pennies; the number of the set of dimes on the table is ten.

> If you need a dime to buy something, how many pennies will it take to buy the same thing?
>
> Do ten pennies have the same value as one dime?
>
> If someone wanted to give you nine cents for one dime, what would you do? Why?
>
> If you have two dimes, do you have as much money as someone who has twenty pennies? Do you have more coins or fewer coins?

Remove the pennies from the table. Ask how much one dime is worth. Tell the children that a dime is sometimes called a "ten-cent piece"; when money is counted, dimes are counted as tens. Let a child play "banker" and count the dimes: "one dime, ten cents; two dimes, twenty cents;...ten dimes, one hundred cents". Give ten pennies or more to each of several children, letting them count ten cents and give it to the "banker" in exchange for one dime.

> Does anyone know the name of a bill you get in exchange for ten dimes?

Further Activities:

1. Make a chart showing the values of the coins cent, nickel and dime, and use it for discussion and review. Real coins may be fastened to tagboard with transparent tape. If you wish, some space may be left for coins of larger denomination.

2. Use opportunities that arise naturally in the classroom for counting money and discussing the values of coins.

Pupil's book, page 39:

Discuss with the children the pictures of cent and dime, and give help in reading the names beneath them. Have them find the picture of the toy car, and the dime that will buy the car. Ask how many pennies it would take to buy the car. Let the children mark an X on enough pennies to buy the car.

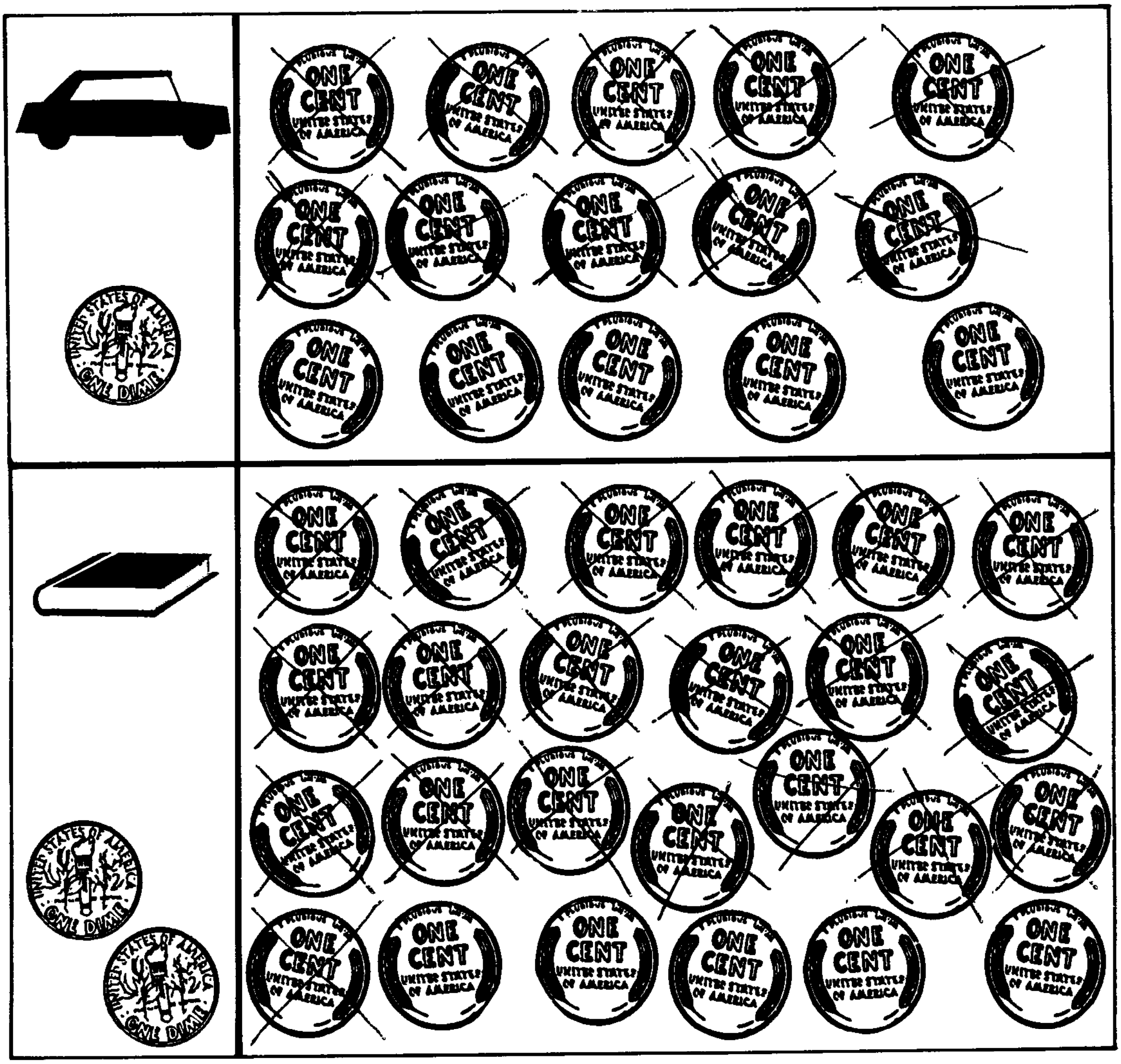
Ask the children what they think they should do in the next exercise. After discussion, let them do it independently. (Mark enough pennies to buy the book.)

Dimes and cents

one cent

one dime
ten cents

Mark enough pennies to buy each toy.



III-4. Problem solving with sets of ten

Objective: To extend understanding of "tens" through using sets of ten in problem solving; to develop ability to solve problems involving inequality, one more ten and one less ten.

Vocabulary: (No new words.)

Materials: Sets of ten tickets, sticks, toothpicks, tongue depressors, clothes pins, plastic spoons, held together by rubber bands or tape.

Suggested Procedure:

The intent is that these problems be read or told to the children, and that no written equations be used in connection with them. The sequence is from relatively simple to more complex problems. Selections should be appropriate for the children in the class.

1. Suppose that you could choose either a set of ten sticks of candy or a set of twenty sticks of candy? Which set would you choose? Why?

Let several children choose and explain their answers. Some may choose the set with more members and some may select the set with fewer members. The essential point is the comparison: a set of ten members (1 ten) has fewer members than a set of twenty members (2 tens); a set of twenty members (2 tens) has more members than a set of ten members (1 ten). Sticks or toothpicks may be used to represent sticks of candy if physical representation is needed.

Many similar problems can be posed, involving different numbers and different choices. Choices can be given between things that children tend to either like very much or dislike very much.

Answers may be written on the chalkboard; for example:

I choose twenty sticks of candy. Fran

I want ten sticks of candy. Twenty is too much. Mill

You write the sentence; the child writes his name.

2. Bill has twenty marbles. John has thirty marbles. Which of the two boys has more marbles?

Do three sets of ten have more or fewer members than two sets of ten? (More.)

Is thirty greater than or less than twenty? (Greater than.)

After the answer is given, write the sentence on the chalkboard.

Jim has more marbles than Bill.

Other similar problems can be formulated and discussed in this way. Children enjoy the use of their own names in such problems.

3. Billy's class is going on a picnic. Billy brought ten spoons. The other children brought thirty spoons. How many spoons did they have?

What is the question? What do we want to find out? How many spoons did Billy bring? Who will put that number of spoons on the table? (Let a child select from the materials you have provided to use in solving the problems.)

> Thirty is how many sets of ten? Who will put that number of spoons on the table? How many spoons are there on the table now? Three sets of ten spoons and one set of ten spoons are how many sets of ten spoons? How many spoons are on the table? How many spoons did the class have? (They had forty spoons.)

This problem also may be used.

4. Alice counted her money and found that she had exactly forty pennies. Mary has ten pennies more than Alice. How many pennies does Mary have?

> How many sets of ten are there in forty? How many sets of ten pennies does Alice have? How many sets of ten pennies does Mary have if she has ten pennies more than Alice?

If the children have had experiences with problems that cannot be solved because there is not sufficient information, problems like the one that follows may be presented without preliminary comment.

5. Sue has thirty buttons. She buys ten buttons. How many blue buttons does she have?

If a child answers <u>4</u> <u>tens</u> or <u>forty</u>, ask questions such as:

> Are you sure? How do you know that Sue has forty <u>blue</u> buttons? Listen while I tell the story again.

In working with the situation, handle it so that no child is embarrassed by having given an answer without sufficient information on which to base it.

A variation in working with "story problems" is to give the children information about numbers and let them formulate the story.

6. We know that two tens and one more ten is three tens. Who wants to tell a "story problem" that these numbers suggest.

With children early in first grade it may be necessary to give considerable help in formulating the story. Some children may be helped by being able to use some of the manipulative materials that are available in the classroom. Such experiences are worthwhile, despite the "struggling", for their contribution to reading and interpreting problems that will be encountered later.

Further Activities:

Let children make pictures of "story problems" and write, or dictate for you to write, a question about the picture. Such "problems" may be shared with the class.

Children who are writing with ease may enjoy writing story problems. These may be stapled together between sheets of construction paper to make booklets. These can become sources of problems to use with the class.

Chapter IV

INTRODUCTION TO ADDITION AND SUBTRACTION

Background

We describe the operation of addition in terms of joining sets. One set joined to another is the set consisting of all objects which belong to either set. In your classroom, if the set of boys is joined to the set of girls, the result is the set of all children in your classroom. We may represent two sets on the flannel board, and think of joining the first set to the second, or of joining the second set to the first. Of course the same set results so we may say that joining is _commutative_. This fact serves later to show us that addition is commutative.

The idea of joining sets underlies the arithmetical _idea of addition_. In a classroom, if the set of all boys is joined to the set of all girls, the result is the set of all children. The _number_ of girls _plus_ the _number_ of boys is equal to the _number_ of children. Thus $2 + 3$ is by definition the number of members in the set obtained by joining a set of 3 to a set of 2. There is one complication (which you may or may not want to point out to your pupils). The sets which we join must have no members in common. We call such sets _disjoint_. Thus the set consisting of John and Mary has 2 members, the set consisting of Mary, Sue and Jane has 3 members, but the set obtained by joining these two sets, which consists of John, Mary, Sue, and Jane, does not have $2 + 3$ members. The trouble is that Mary is a member of both sets, and the sets are not disjoint.

Joining a set of 2 to a set of 3 always gives the same set as joing the set of 3 to the set of 2. Consequently $2 + 3 = 3 + 2$. Similarly, $4 + 1 = 1 + 4$, $5 + 7 = 7 + 5$, and so on. We say that the operation of addition is commutative.

We wish to make very clear the sense in which equality and the equals sign, (=), are used in mathematics. We write, for example, $3 + 4 = 7$, or $7 = VII$, because "3 + 4", "7" and "VII" are all names for the same thing, the number seven. Any statement of equality means that the symbols to the left of the equals symbol and the symbols to the right of the equals symbol are names for the same thing. A number, like a person, may have many names. All of the following

$$7, \quad 3 + 4, \quad 5 + 2, \quad 8 - 1, \quad 49 \div 7, \quad \frac{14}{2} \quad \text{VII}$$

are names for the number seven. The reason we adopt this meaning for equality is that we always want to be able to substitute equals for equals. Thus, for example, if we know that $6 = 3 + 3$, we can infer that the sum of 7 and 6 is the sum of 7 and $3 + 3$.

Addition, which we denote +, is an operation. If we apply the operation + to a pair of numbers, say 3 and 4, the result is another number, $3 + 4$ or 7. We write $3 + 4 = 7$, because "3 + 4" and "7" are names for the same number. It is also true that $3 + 4 = 6 + 1$, and indeed, if we ask a child "What is $3 + 4$ equal to?" then all of the following are correct answers: 7, $6 + 1$, $3 + 4$, $4 + 3$. The procedure which we sometimes call addition really amounts to finding the common name, a name not involving +, for a number which we have named in a more complicated way--for example, "3 + 4".

We say that one set is a subset of another if each member of the first is a member of the second. The set of girls in a classroom is a subset of the set of children, and the set of all tricycles is a subset of the set of all toys. The set consisting of John and Mary is not a subset of the set of all girls because John does not belong to the set of girls.

Let us list all possible subsets of the set consisting of an apple and an orange. It is clear that there are two subsets which have just one member each: the set consisting of the apple and the set consisting of the orange. Are there other subsets? What of the empty set? Is it true that every member of the empty set is a member of the set consisting of an apple and an orange? If not, then some member of the empty set must fail to belong to the apple-and-orange set, and this is impossible since the empty set has no member. We must therefore agree that the empty set is a subset of the set consisting of an apple and an orange, and in fact that the empty set is a subset of every set. (Later we will relate this to statements like $5 - 0 = 5$.) Finally, we may ask if this apple-and-orange set is a subset of itself. Is it true that every member of the set consisting of an apple and an orange belongs to the set consisting of an apple and an orange? This is obviously true, so we agree that the apple-and-orange set is a subset of itself. In fact, every set is a subset of itself. (We will relate this fact to statements like $7 - 7 = 0$.) Mathematicians have expressed their general feeling that it's somewhat improper for a set to be a subset of itself by agreeing that a proper subset of a set is to be a subset which is different from the whole set. [A proper subset of a set always has fewer members than the set.]

Suppose we are given a set and a subset of it and we remove the subset. The remaining set consists of those objects which belong to the original set but not to the subset. If we remove the set of boys from the set of children in the classroom, the remaining set is the set of girls.

We describe subtraction in terms of removing a subset of a set: $5 - 3$ is the number of members in the remaining set if a set of 3 is removed from a set of 5. Thus, if John has 5 marbles and his older brother takes away 3 marbles, then the remaining set has $5 - 3$ members.

There is a close relation between addition and subtraction. If we join a set to another set and then remove it, the remaining set is just the original or "starting" set. If we remove a subset from a set and then join it to the remaining set, then we again have the original set. We sometimes say that joining a set, and removing the same set, are inverse operations, in the sense that doing these in succession to any set always gives back the original set. This fact about manipulation of sets shows us something about addition and subtraction. If we add 2 to a number and then subtract 2, we have the original number. If we subtract 2 and then add 2, we again have the original number. Thus, adding 2 and subtracting 2 are inverse operations, in that doing these in succession to any number always gives the original number. Of course, adding 4 and subtracting 4 are also inverse operations, and so on.

There is one matter of notation which needs to be made clear. In some of the equations in this chapter we have left boxes for the children to write in (for example, $2 + \square = 5$). These boxes are nothing more than places for the children to write; they are not interpreted as placeholders or variables, as is the case in some of the other mathematics programs. We introduce variable notation in the second grade, where we write, for example, $2 + n = 5$.

IV-1. Joining

Objective: To introduce the operation of joining and its commutative property.

Vocabulary: Join.

Background Note:

If one set is joined to another, the result is the set whose members are those things which belong to either of the sets. This is preparation for the concept of addition.

Materials: Materials for flannel board or magnetic board demonstration; beans, bottle caps, buttons, blocks and spools.

Suggested Procedure:

After the children are familiar with the ideas of set and member, they should be ready to understand the concept of join. Some will understand the word as they have heard it used in other situations.

You might start with a demonstration at your desk, at a low table, or on a flannel or magnetic board where all can see. On one side of the table, place a set of ten or fifteen various sized buttons and on the other side a set of blocks, another kind of material, or another set of buttons which can be distinguished from the first set. Each set should contain too many members to be counted quickly as no counting is wanted at this time.

After the sets have been described by the children, proceed somewhat as follows:

We will move this set of buttons over to join it to the set of blocks.

When we join these sets, we have a new set.
What are the members of this set? (Buttons and blocks.)

Touch a member of the set formed by joining the set of buttons to the set of blocks. (A button.)

Is this button a member of the set of buttons? (Yes.)

Is this button a member of the set of blocks? (No.)

Is it a member of the set of buttons and blocks? (Yes.)

Is each member of the set of buttons a member of the set of buttons and blocks? (Yes.)

Is each member of the set of buttons and blocks a member of the set of buttons? (No.)

Discuss several other members of the set in a similar manner. Through this discussion the children should be helped to understand that each member of the new set (buttons and blocks) was a member of either the set of buttons or the set of blocks and that each member of the set of buttons and the set of blocks is a member of the new set.

Move the set of buttons back to its original position.

Now let's move the set of blocks over to join the set of buttons.

When we join the set of blocks to the set of buttons, we have a new set.

What are the members of this set? (Buttons and blocks.)

How is this set like the set we had when we joined the set of buttons to the set of blocks? (It has the same members.)

- You should follow the same procedures using a variety of materials. Emphasize the ideas that one set is joined to the other to form a new set; each member of the new set is a member of one or both of the sets joined; the order of joining the sets does not change the new set.

We have our set of books about animals here at the library table. On my desk is the set of new animal books that I got at the library today. John, will you bring the set of new books and join it to the set here at the table? How would the new set be different if we took the set of animal books from the library table back to join the set of books on my desk? (No difference.)

- See that each child has a set of spools and a set of beans or other sets of two different kinds of material on his desk, one set on each side. Again have too many members in each set to be counted quickly.

Put your right hand on the set of beans on your desk. Now move this set over to join the set of spools.

What is the new set on your desk? (A set of beans and spools.)

Ask the children to pick up a member of the new set which was not a member of the set of beans. (A spool.) Pick up a member of the new set which was not a member of the set of spools. (A bean.) Ask the children to pick up a member of the new set that was not a member of the set of beans or the set of spools. (None.)

Move the set of beans and spools back where they were when the lesson started. This time move the set of beans over to join the set of spools.

What set do we have when we join a set of beans to a set of spools? (A set of beans and spools.)

How is this set like the set we had when we joined the set of spools to the set of beans. (The members of the new set are the same as before.)

Emphasis is made here on the idea that the order of joining sets does not affect the new set.

Further Activities:

Describe two sets in the clasroom; a set of chalkboard erasers, a set of staplers. Ask a child to show the set of staplers joined to the set of chalkboard erasers on a demonstration table or desk.

IV-2. Joining sets and counting

Objective: To prepare for the operation of addition by introducing the joining of set in association with the spoken names of numbers 0 through 9.

Vocabulary: (Review) Joining, written numerals for numbers 0 - 9, member, set, empty set, set with one member.

Materials: Materials for flannel board, sets of small objects; perception cards (see instructions under Further Activities), a box of small objects for each child.

Suggested Procedure:

As a pre-addition activity you will find it useful to establish an understanding of joining sets and counting, counting both the sets with which they started, and the resulting set. An introduction by means of a flannel board demonstration follows. (The same procedure, with slight modification, is applicable to chalkboard illustrations.)

Place a set of objects on the flannel board and have the set described. (Example: set of apples, set of red apples, set of 5 apples.)

Identify the number of the set and place the set to one side of the board. On the other side of the board place another set. (Example: 3 oranges.) Identify the number of this set as you did the first.

> We now have a set of 5 apples on our board and a set of 3 oranges. Let's put the oranges with the apples. What have we done to the two sets? (Joined the set of oranges to the set of apples.) We began with a set of 5 apples and a set of 3 oranges.
> How many members do we have in the new set that we just made by joining the two sets? (8.)

By moving the set on the flannel board, the children are able to see the joining of the separate sets and formation of the new set.

- After the children are familiar with the verbal presentation, cards with numerals written on them, one numeral per card, should be placed on the flannel board or table during the discussion and the appropriate card displayed after the number of members of a given set is identified.

  What is the number of members in the set of apples? (5.)

  What is the number of members of the set of oranges? (3.)

  What is the number of members in the set of apples and oranges? (8.)

Place the oranges on the flannel board. Then join the set of apples to the set of oranges and ask that this set be described.

  When we join the set of apples to the set of oranges, what is the new set? (A set of oranges and apples.)

  What is the number of members of the set of apples? (5.) Display the numeral card with 5 written on it.

  What is the number of members of the set of oranges? (3.) Display the numeral card with 3 written on it.

  What is the number of members of the new set? (8.) Display the card with 8 on it.

  How is the set we formed by joining the set of apples to the set of oranges like the set formed by joining the set of oranges to the set of apples? (They have the same number of members. They have the same members. We changed the order in which we joined the sets but this does not change the set which is formed.)

- Place 4 rabbits on the flannel board. Ask a child to name the number of members in the set. Place a card with this numeral on the flannel board. Place 3 kittens on the flannel board. Ask a child to name the number of members in the set of kittens. Place a card with this numeral on the flannel board. Ask a child to join the set of kittens to the set of rabbits.

  What are the members of the new set? (Kittens and rabbits.)

  What is the number of members in the new set? Place a card with this numeral on it on the flannel board.

A discussion such as the following may serve to evaluate how well children remember which set had a given number of members.

Remove the set of kittens and rabbits from the flannel board and point to the card with the numeral 3 on it.

  What set had this number of members? (The set of kittens.)

Point to the card with the numeral 4 on it.

  What set had this number of members? (The set of rabbits.)

Point to the card with the numeral 7 on it.

  What set had this number of members? (The set of rabbits and kittens.)

If we had joined the set of rabbits to the set of kittens would we have the same number of members in the set of rabbits and kittens as in the set we had earlier? (Yes.)

Further Activities:

1. Perception cards made by the teacher help to visualize the joining of sets. Several cards should be made for each number so that children do not merely learn to recognize a pattern. Magazine pictures pasted on heavy paper are also useful. Some advertisements have excellent pictures and are fun for children to find. They also enjoy making these cards themselves. Hold up one card and have the children identify the number of members. (Identification may be made either by recognition or counting, or a combination of both.) Place this card on a stand and hold up another card using the same procedure. Join the two sets and identify the number of the resulting set.

2. Sets of objects such as books, writing equipment, art supplies, and blocks may be placed on a demonstration table. First, identify two sets and join one to the other to form a new set. Remember to identify the number of members in each set as well as the number of members in the new set.

   Manipulative materials may be used at the pupils' desks to be joined to form new sets. Proceed as before, being certain to use orally the number of both the set with which you started and the resulting set.

3. Use sets of class members like ball monitors, errand helpers, etc. Join one set to another and name the number of the resulting set. (Example: ball monitors 2, errand helper 1, resulting set 3.)

4. An overhead projector, if available, is useful in developing this lesson. It provides an excellent means of viewing the sets, writing names of numbers associated with these sets, and showing the result of joining one set to the other.

IV-3. Joining sets and adding numbers

Objectives: To develop an understanding of addition.

To use the terms plus and equals and the symbols + and = to write equations.

Vocabulary: Add, plus, equals, number, equation.

Materials: Set of books, set of blocks, large sheets of paper divided into three columns for a chart.

Suggested Procedure:

Ask a child to pick up a set of 2 books from the shelf. Have the child place the books on the table, saying that this is our first set. Explain to children that we will keep a record of our work and record this number in the first column of the chart.

2		

Ask a child to pick up a set of 3 books. We want to join this set to the first set by putting these books with the books on the table. You should then explain that you will record this number of books (set being joined) in the second column of the chart.

As the chart is developed, continue to point out to the children that a numeral in the left hand column shows the number of objects in the set we started with and a numeral on the middle column shows the number of objects in the set joined to the first set. Join the set of 3 books to the set of 2 books.

How many members are in the new set of books? (5.)

Record 5 in the right-hand column of the chart. Explain to the children that this numeral (5) shows the number of members in the new set when the second set is joined to the first set. Continue with other examples, recording the numbers for each example on the chart.

Pupil's book, page 40: Record in the left column of the chart the number of members in the set on the left, record on the center column the number of members in the set on the right, and record in the right column the number of members you would have if you joined these sets.

Page 40 in the pupil's book may be used for an independent activity.

- Continue with the information on the chart.

    Look at the record of our work with the sets of books. We can use what is written to make an equation. In the equation we say, "two plus three equals five".

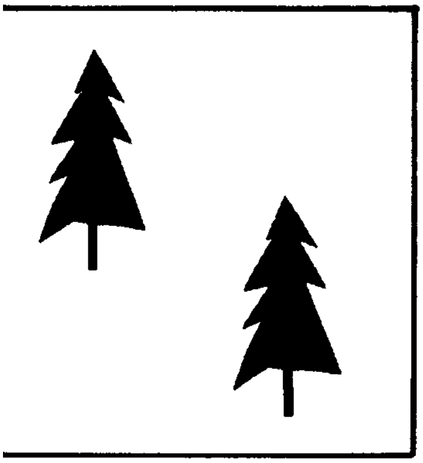
    To the number 2 we are adding the number 3. The result is the number 5.

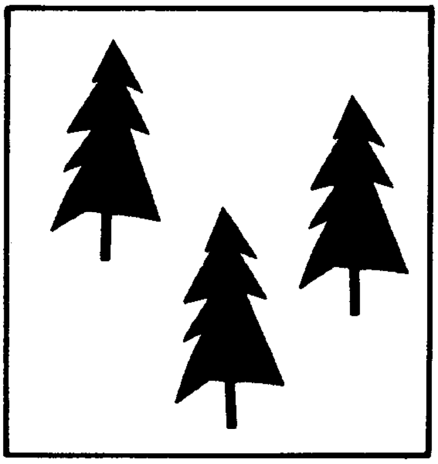
Extend the chart and write the equation, 2 + 3 = 5 on the chart.

    This is how we write the equation.

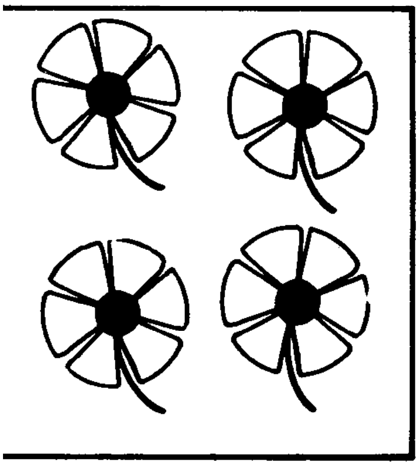
Continue to extend other rows of the chart in a similar manner.

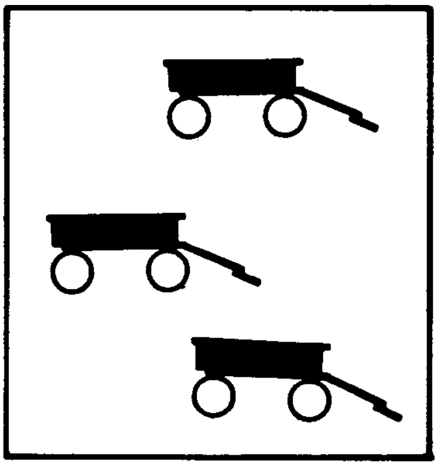
How many?



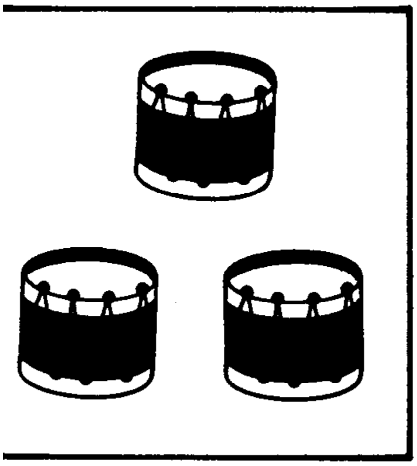


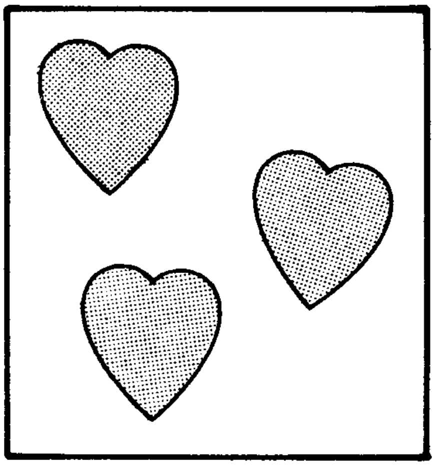
2	3	5





4	3	7





3	3	6

4. Four mice were eating.

   Three mice came to eat.

   Then, how many mice were eating?

Distribute sets of small objects. Display the numeral 5. Ask children to "show" that many objects on their desk. Display the numeral 3. Ask children to join a set with that many members to the set on their desk.

> How many members are in the new set? (8.)

Display the numeral 8.

> Can we use an equation to tell about joining these sets? (5 plus 3 equals 8.)

Put the + and = symbols in the correct places in the equation.

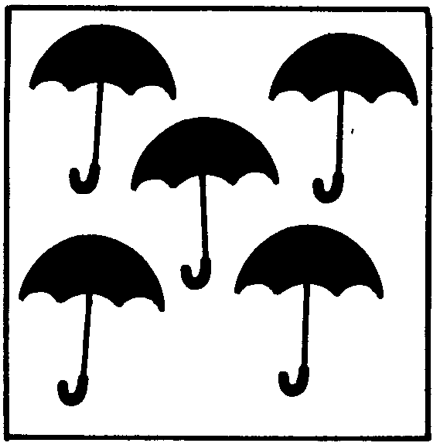
Write an equation, $7 + 2 = \square$. Help children use materials to complete this equation, as well as other equations such as $6 + \square = 7$, $\square + 3 = 8$.

<u>Pupil's Book, page 41</u>: Record the numerals in the chart as on page 40. Then write the equation.

<u>Pupil's book, page 42</u>: Direct the children to complete these equations. If they need to use objects to do this, they should be allowed to do so.

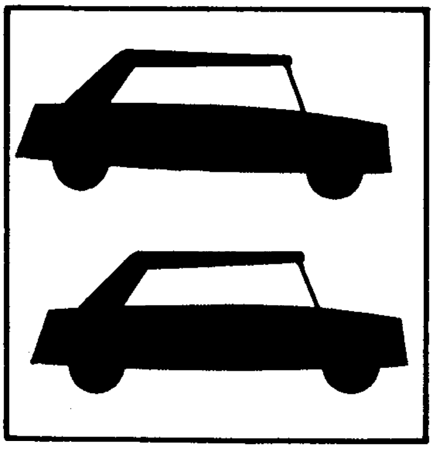
<u>Pupil's book, page 43</u> : Record the numbers of members in each set. Complete the equations.

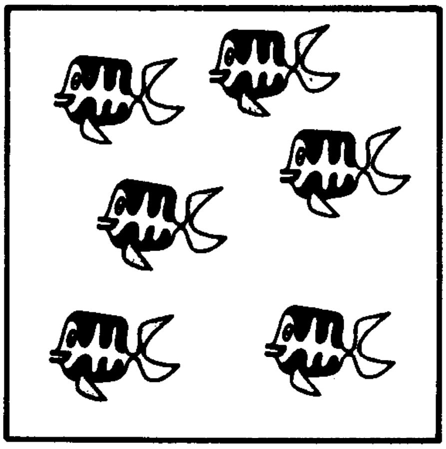
How many? Write the equation.



5	1	6

$\underline{5} + \underline{1} = \underline{6}$





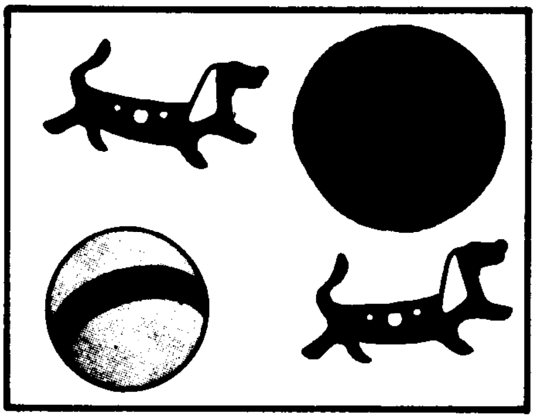
2	6	8

$\underline{2} + \underline{6} = \underline{8}$

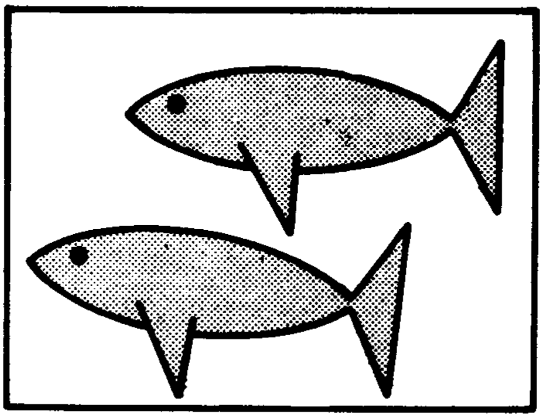
Equations

$4 + 1 = \boxed{5}$	$2 + 3 = \boxed{5}$
$3 + 0 = \boxed{3}$	$0 + 6 = \boxed{6}$
$2 + 2 = \boxed{4}$	$1 + 3 = \boxed{4}$
$1 + 5 = \boxed{6}$	$2 + 4 = \boxed{6}$

Pairs of Equations



How many? 4

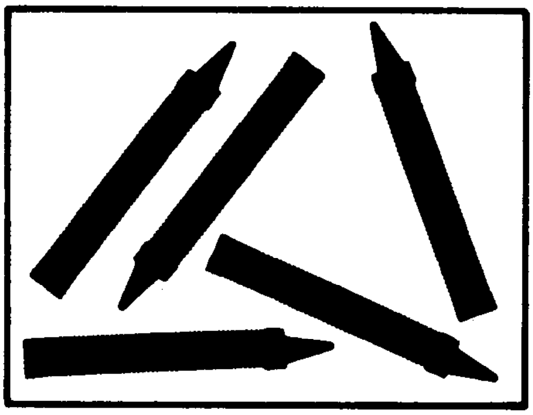


How many? 2

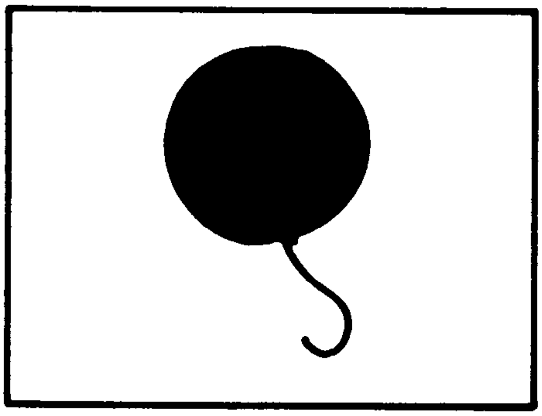
Write two equations.

$$4 + \boxed{2} = \boxed{6}$$

$$2 + \boxed{4} = \boxed{6}$$



How many? 5



How many? 1

Write two equations.

$$5 + \boxed{1} = \boxed{6}$$

$$1 + \boxed{5} = \boxed{6}$$

IV-4. Subsets

Objective: To introduce the idea of subset of a set.

Vocabulary: Subset; (review) set, collection, member.

Background Note:

One set is a subset of a second set if each member of the first set is a member of the second. Thus the set of all dogs is a subset of the set of all animals because each dog is an animal, but the set of all animals is not a subset of the set of all dogs because there are animals which aren't dogs. This lesson is preparation for the concept of subtraction, which we will describe in terms of removing a subset of a set and identifying the number of members in the remaining set.

Materials: Sets of objects, materials for flannel board displays, colored construction paper (several sheets of different colors), jars of multi-colored beads, rhythm instruments, set of play dishes.

Suggested Procedure:

The concept of a subset of a set may be developed as follows:

Will each member of the set of children in our room raise one hand?

Do all the boys have one hand up? (Yes.)

Do all the girls have one hand up? (Yes.)

Everyone put his hand down.

Now, let's have the girls stand. Is Jane a member of the set of girls? (Yes.) Repeat this question about several girls; then ask the question inserting a boy's name.

Is Michael a member of the set of girls? (No.)

Michael is not a member of the set of girls. Of what set is he a member? (The boys.) Will the boys stand? (Repeat as with the girls.) We have seen that there is a set of children in our room. We also know that there is a set of boys and a set of girls. We say that the set of girls is a subset of the children because every member of the set of girls is a member of the set of children. The set of boys is a subset of the set of children because every member of the set of boys is a member of the set of children.

(Ask the boys to stand again. Then ask three children to raise their right hands.) Tom, Billy, and Johnny have each raised their right hands. Are these three boys members of our set of boys? (Yes.) However, they have their hands raised. We call the set of boys in our class with their hands raised a subset of the set of boys in our class.

Repeat the above process with the girls: set of girls and set of girls holding books; or, the set of girls and girls wearing white shoes.

This exercise can be developed further using the entire set of children and describing subsets of this set.

- At this point you may wish to use a chalkboard or flannel board to develop further the idea of subset of a set.

Place a set of "fruit" on the flannel board. Talk about the set of apples as being a subset of the set of fruit, the set of pears as a subset, etc.

Caution: It is necessary to identify the set first before talking about a subset of that given set. We cannot start with a subset and then think of a set.

It is important to have experiences naming subsets whose members are not selected on the basis of size, color, or use, or children develop the misconception that a subset is a subset because the members belong together for some of these reasons.

- Place a set of materials on the flannel board.

  How can we describe this set?
  (A set with seven members. A set of flannel objects. A set of a tree, a star, a cup, a cone, a kite, a moon and a ball.)

  Is the set of a star and a cup a subset of the set of flannel board objects? (Yes.)

  Is the set, a ball, a subset of the set of flannel board objects? (Yes.)

  Is the set of a swing and a seesaw a subset of the set described? (No.)

Emphasize that each member of the subset must be a member of the given set.

  Is the set of a ball, a tree and a watermelon a subset of the set of flannel board materials? (No.)

Then proceed with these questions.

  Is there any member of the set "cup, cone, kite" that is not a member of the set of flannel board objects? (No.)

  Is the set "cup, cone, kite" a subset of the set of flannel board objects? (Yes.)

  Is there any member of the empty set that is not a member of the set of flannel board objects? (No.)

  Is the empty set a subset of the set of flannel board objects? (Yes.)

- Make a list of things used during art class. (Substitute if this list is not appropriate.)

clay	crayon
glue	chalk
brush	scissors
paper	ink
pencil	string
paint	cloth

Use this list as the basis for questions about subsets to reinforce the ideas previously dealt with in this lesson.

Exhibit a set of objects on the flannel board. Identify the number of members of the set. Place this numeral on the flannel board. Place a piece of yarn around a subset of the set of flannel board objects. Ask a child to name the number of members in the subset. Place the numeral for this number on the flannel board. Include the subset which consists of all of the members of the set and the empty set.

After this experience children should be ready for the following independent work:

- Distribute sets of small objects, nine or less per child. Give each child a piece of yarn. Ask the children to use the yarn to make a ring around a subset of the set of materials on the desk. Give each child a half sheet of paper which has been labeled:

Set	Subset

Tell the children to record the number of members in the set of his desk under set, make the ring to show a subset of the set and then record the number of members in the subset. Repeat activity asking children to show other subsets of this set.

Further Activity:

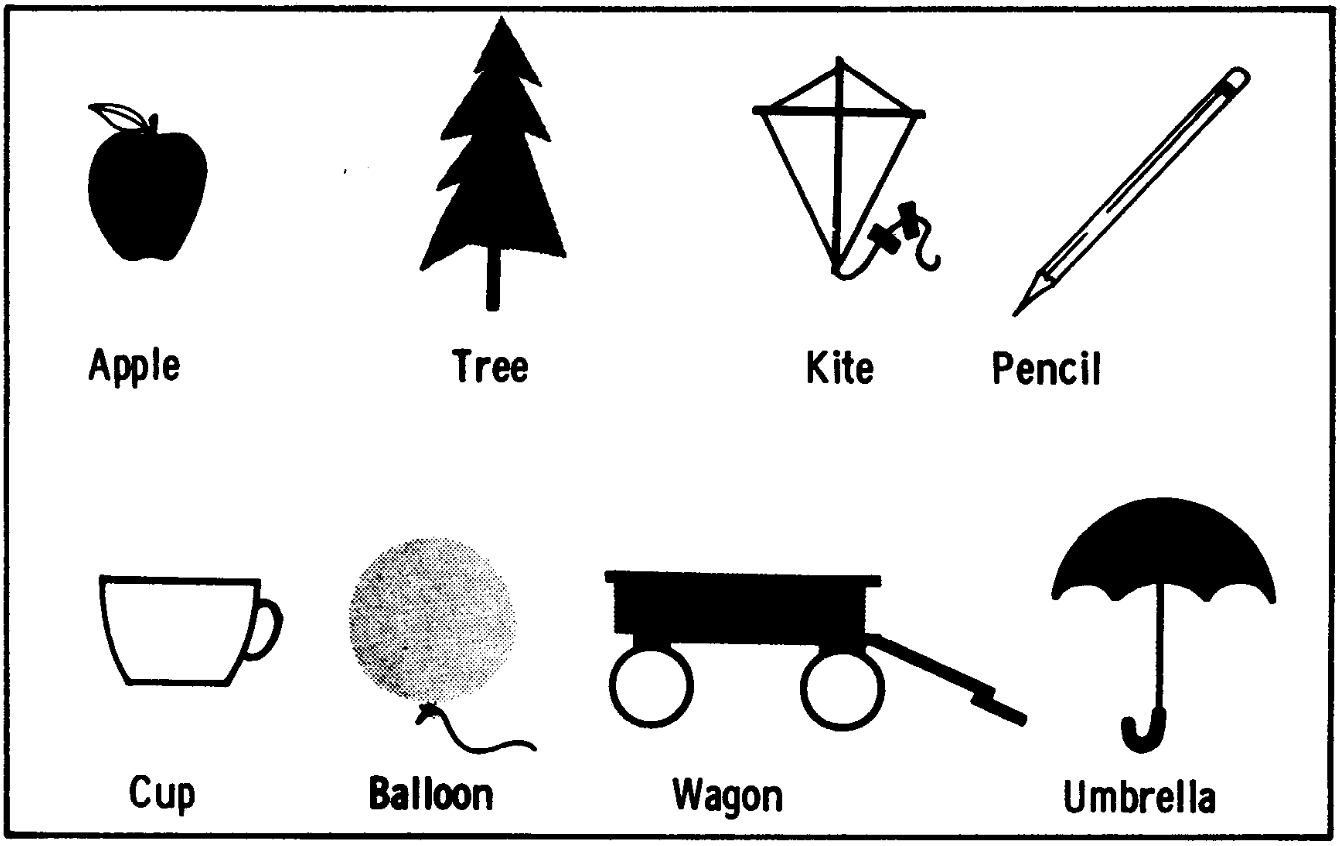
If children wear costumes on Halloween, the following activity could be used: Ask children to stand in a large ring. Touch four or five children and ask them to stand inside the ring. (2 witches, 1 bunny, 1 ghost, 1 pirate.)

Ask a child to name the members of the set in the center of the ring.

> Does every member of the set of children in the center of the ring belong to the set of children in our room? (Yes.)
>
> Is the set of children in the center of the ring a subset of the set of children in our room? (Yes.)
>
> Is a skeleton a member of the subset? (No.)
>
> Are all of the children in our room members of the subset? (No.)

★ Pupil's book, page 44: Make a ring around the letter which names a subset of the set in the frame at the top of the page.

★ A Set and Some of its Subsets



A. apple  kite  and cup 

B. tree  and barn 

C. wagon 

D. apple  tree  and cup 

E. gun 

F. umbrella  and apple 

IV-5. Removing sets and the remaining set

Objective: To introduce the ideas of removing sets, the remaining set and the use of these ideas.

Vocabulary: Remove, remaining set.

Background Note:

This is further preparation for the concept of subtraction.

Materials: Materials for flannel board such as apples, bananas, oranges, pineapple, cherries, small objects as beans, sticks and buttons.

Suggested Procedure:

On a table place a set of beans with more members than can easily be counted. Tell a child to remove a subset of the set. When we remove a set, the set that is left is the remaining set.

Point to the remaining set.

These observations should be made in relation to the removing of a set and the remaining set:

> Each member of the set removed was a member of the set with which we started.
>
> Each member of the remaining set is a member of the set with which we started.
>
> Each member of the starting set is a member of either the set removed or the remaining set.

- On the flannel board place a set of fruit containing six or seven members. Ask a child to describe the set. It may be a set of fruit, such as apples, oranges, pineapple, cherries, banana.

> Let's pretend that we are going on a picnic today. We will each take a sandwich for ourselves and something to share with the whole group. On the flannel board is the set of fruit which Dick brought. How many members are in the set? (7.) Jerry wanted some of the fruit to eat with his sandwich. Jerry will take some of the fruit from the set. (Ask a child to take some fruit from the set displayed.) This is the set of fruit Jerry wanted to eat. Is it a subset of the set of fruit? (Yes.) When Jerry took the subset of fruit from the set of fruit, he removed it from the set. What is the number of members in the set removed? (2.)

Ask a child to describe the set remaining on the flannel board. (Cherry, orange, apple, pineapple, and banana.) The remaining set is the set that is left after we have removed a subset from the starting set.

> Is the remaining set a subset of the starting set? (Yes, because all members of the remaining set must have been members of the starting set.) How many members are in the remaining set? (5.)

Call attention to the many situations in the school day experience which could illustrate a set removed and the remaining set, where the starting set is the children in your classroom; for example, the set of children who went home for lunch today (the set removed) and the set of children who ate lunch at school today (the set remaining).

IV-6. Removing sets and subtracting numbers

Objectives: To review the idea of removing a subset.

To use number in relation to this set experience.

Vocabulary: Minus, subtract, - .

Background Note:

Subtraction is described in terms of removing a subset: 7 - 3 is the number of members in the remaining set if a subset of 3 members is removed from a set of 7 members. We shall later consider other descriptions (which include, the missing addend description).

Materials: Set of flannel cutouts, numeral cards 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and chart to display numeral cards.

Suggested Procedure:

Ask a child to form a set with 5 objects on the flannel board. Display the set of numerals and ask if a child can find the card for the number of members in the set. Place the card on the left at the bottom of the flannel board. Ask a child to describe a subset of the set. Remove the subset and place it on the flannel board apart from the set with which you started.

> How many members are in the set that was removed? (2.)

Ask the child to find the numeral card which tells this number and place it on the flannel board at the right of the first card.

> What do we call the set that is here (indicate the part that was not moved)? (The remaining set.)
>
> How many members are in the remaining set? (3.)

Ask a child to find the numeral card for the number of members in the remaining set and place it on the flannel board at the right of the other cards.

> The numerals at the bottom of the flannel board can be used to make an equation. We are subtracting the number 2 from 5. The result is the number 3.
>
> We say 5 minus 2 equals 3.
>
> We write the equation, $5 - 2 = 3$.

As a review you might ask children the following questions:

> Which set has 5 members? (The set we started with.)
>
> Which set has 2 members? (The set we removed.)
>
> Which set has 3 members? (The remaining set.)
>
> What is the equation? (5 minus 2 equals 3.)

- In further development of this lesson the numerals should be recorded on a chart similar to that used in Section 1. The column on the left is for the number of members in the starting set; the column in the middle is for the number of members in the set that was removed; and the column on the right is for the number of members in the remaining set. Write the equation which corresponds to each experience. The completed chart might look like this:

Set	Set removed	Remaining set	Equation
5	2	3	$5 - 2 = 3$
4	1	3	$4 - 1 = 3$
2	2	0	$2 - 2 = 0$
3	1	2	$3 - 1 = 2$

Have each child work at his desk with sets of small objects, removing subsets from sets of objects and recording the numbers of each of the three sets. The paper which each child uses to record the work should have these headings:

Set	Subset removed	Remaining set	Equation

- <u>Solving Problems</u>

Use sets of objects and dramatization in order to solve these problems.

> Five rabbits were in my garden.
> Two rabbits hopped away.
> Then how many rabbits were in my garden?
>
> What is it that we want to find out?
> (How many rabbits are in the garden after some go away.)
>
> Yes, the question is, "<u>How many rabbits were left</u> (remained) <u>in my garden</u> ?" after some hopped away.
>
> Describe the first set you heard about when I read the story. (A set of 5 rabbits in a garden.)

Have a child use materials on flannel board to show a set that matches the set of five rabbits.

> What happened to this set of rabbits?
> (Two rabbits hopped away.)

Ask a child to show what happened using the objects on the flannel board. Discuss the set of three members that remains after the subset with 2 members is removed.

> Do you know the answer to the problem? What is it? (Some children may answer, "Three".)

Ask if the answer is 3 ____, naming whatever materials were used to represent the rabbits. Bring out again that we are answering the question, "How many rabbits remained in the garden?" We use these materials to help us find the answer, but the answer is, "There are still three rabbits in my garden."

What equation can be written about this story?
$(5 - 2 = 3)$

Other problems to be developed in the same way:

1. Seven birds were sitting in a tree.
   Four of these birds flew away.
   How many birds were still sitting in the tree?

2. Mark had 5 cents.
   He gave 4 cents to Father.
   Then how many cents did Mark have?

3. Mother baked eight gingerbread men.
   She gave three of them to Susan.
   How many did Mother have then?

4. Judy had 9 pencils.
   She lost 2 of these pencils.
   Then how many pencils does Judy have?

<u>Further</u> <u>Activities</u>:

1. Distribute sets of small objects. Display the numeral 7. Ask each child to "show" a set with that many members on his desk. Display the numeral 4. Ask children to remove a subset with that many members from the sets on their desk.

   How many members are in the remaining set?
   (3.)

Display the numeral 3.

What is an equation that describes removing a subset with 4 members from a set with 7 members?

(7 - 4 = 3.) (7 minus 4 equals 3.)

Place the symbols - and = in the correct places with the numerals to form the equation.

Have the children use manipulative materials to find the numbers which complete these equations: $7 - \square = 2$; $\square - 1 = 4$; and $6 - 5 = \square$.

2. To implement work with removing a set, have the children imagine a birthday cake with 6 to 10 candles. For illustrative purposes 8 will be used in the discussion here. Place numeral cards 0 through 8 in a box. (cards 0 - 6 if six, candles, etc.). Draw a card from the box and show it to the children. Tell them this is the number of candles which you were able to blow out with one try.

   Identify the number of candles which would still be burning. (If necessary, a row of candles, with detachable flames might be placed on the flannel board. When the number indicates how many are blown out is shown, that number of flames could be removed.) Each experience should be followed by writing an equation.

A chart can be made for these experience to show the different equation which were formed. Then when a child draws a card which has been drawn earlier he can be asked to find the equation which represents this. Only the different equations would be added to the chart.

Pupil's book, page 45:

In each box the members of the starting set and the number of members in the subset to be removed are given. The children are to ring and shade the subset which they are to imagine has been removed from the set. They are then to record into the space provided the number of members in the starting set and in the remaining set.

Pupil's book, page 46:

Direct the children to identify the number of members of the set. They are to indicate a subset to be removed by making a ring around it. They are to write the numerals which would be used to make a record.

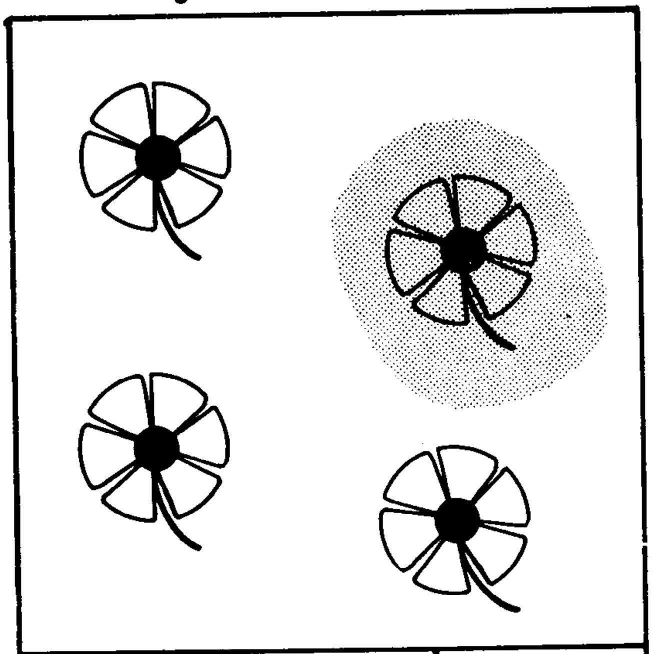
Pupil's book, page 47 and 48:

Direct children to complete the chart which is shown and write the corresponding equation. On page 48, they will need to make the ring around the subset.

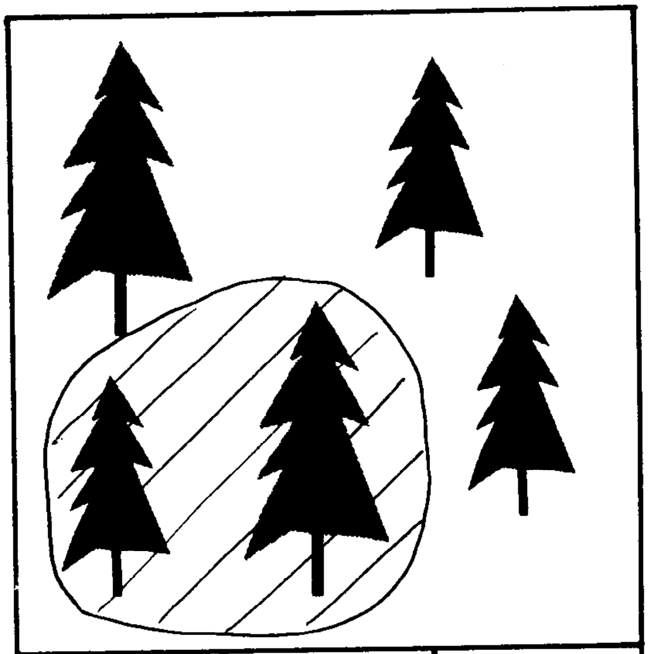
Pupil's book, page 49:

Direct the children to complete these equations. If they need to use manipulative materials, they should be allowed to do so.

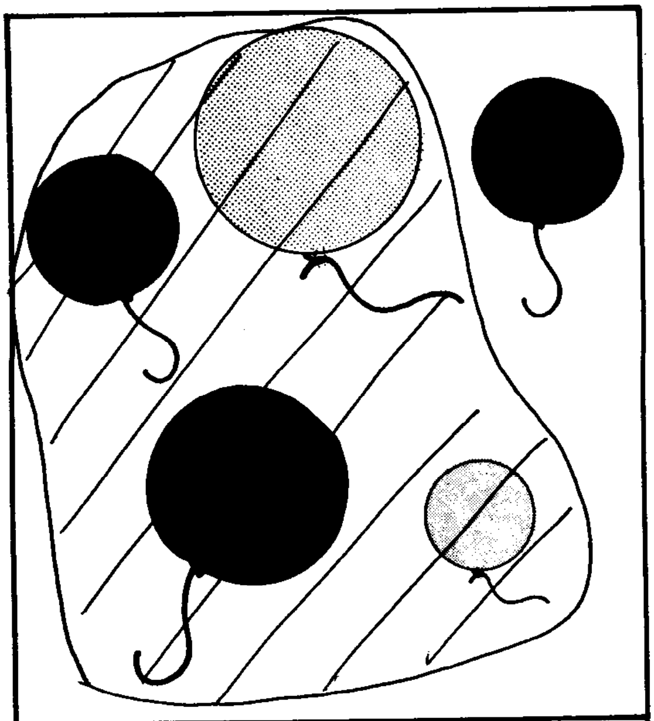
Removing Subsets



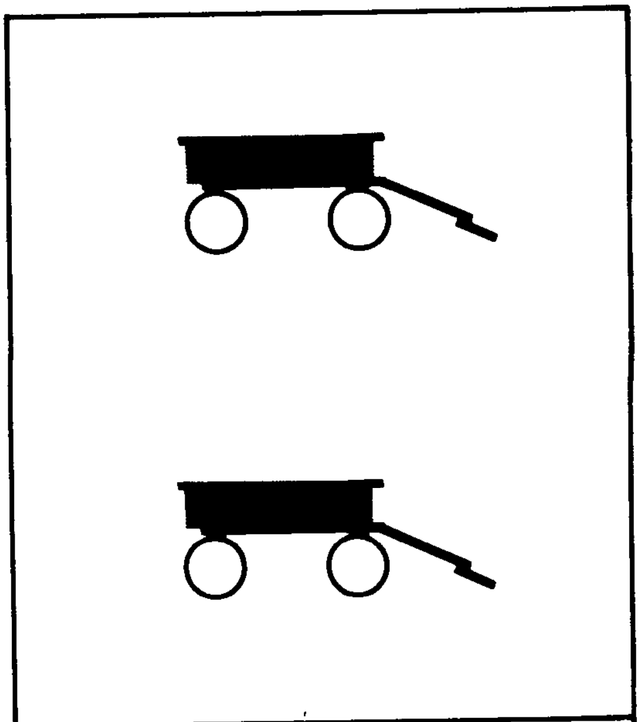
4	1	3



5	2	3

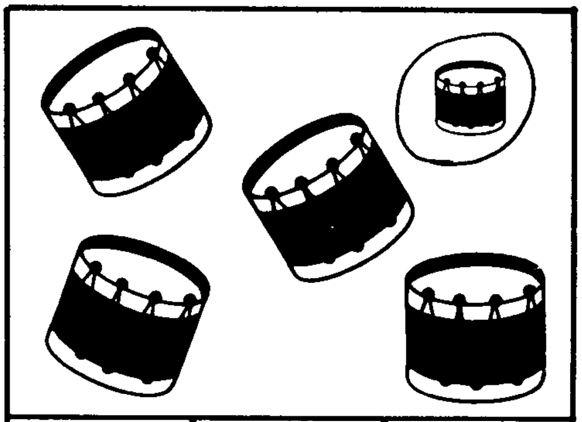


5	4	1

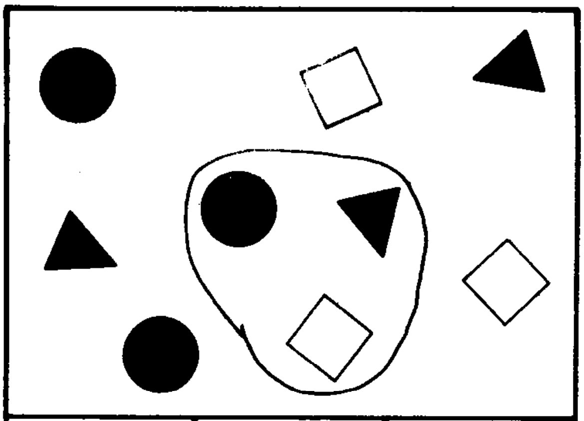


2	0	2

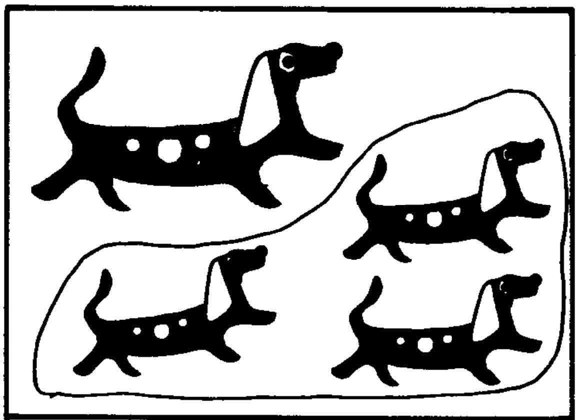
Remove a subset. How many?



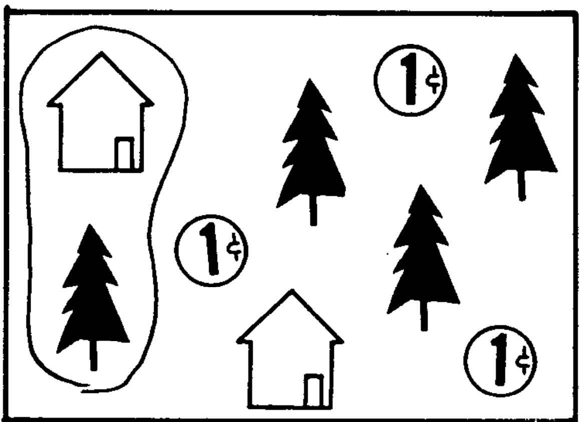
5	1	4



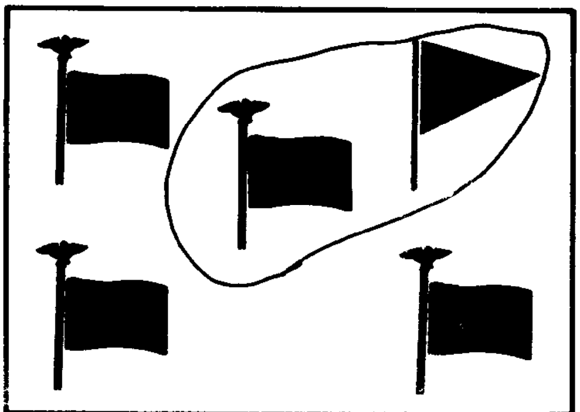
9	3	6



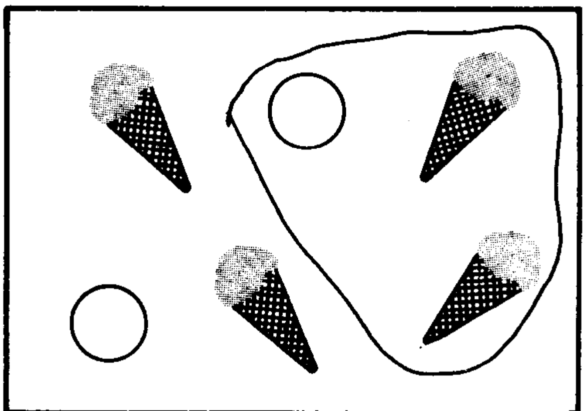
4	3	1



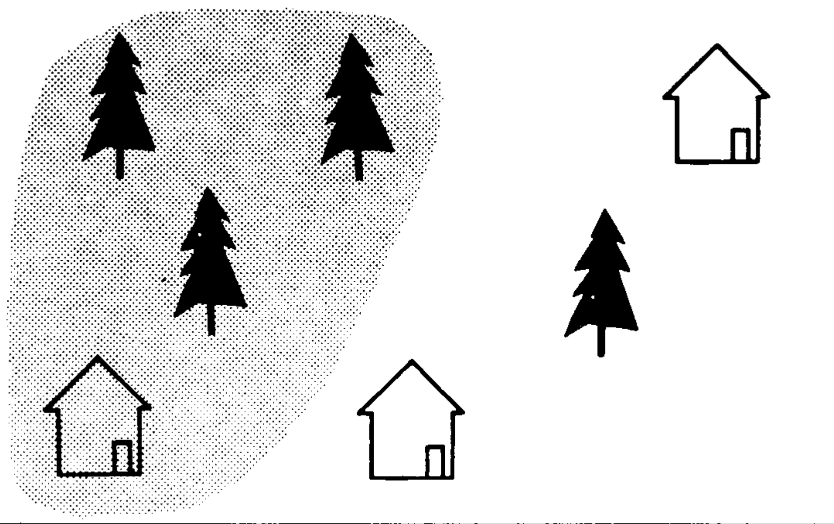
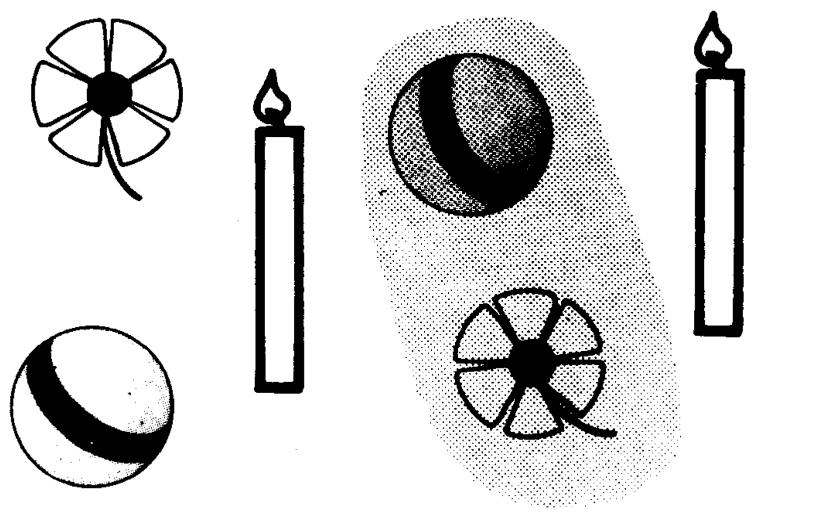
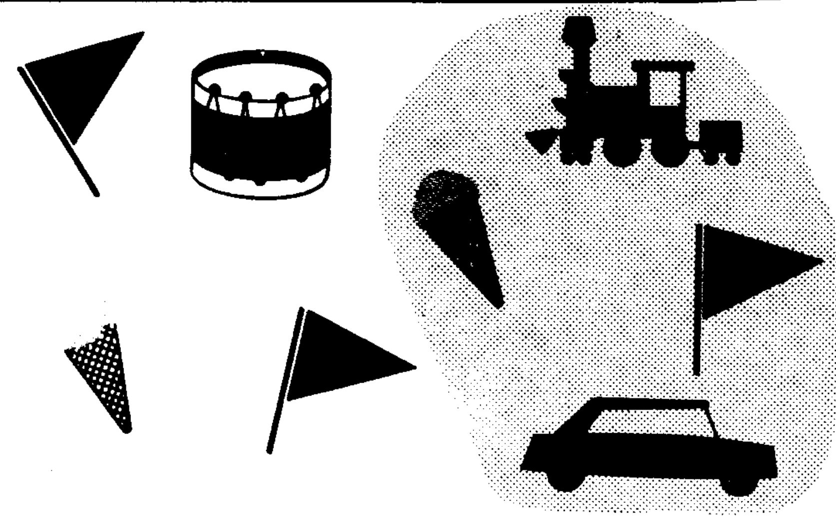
9	2	7

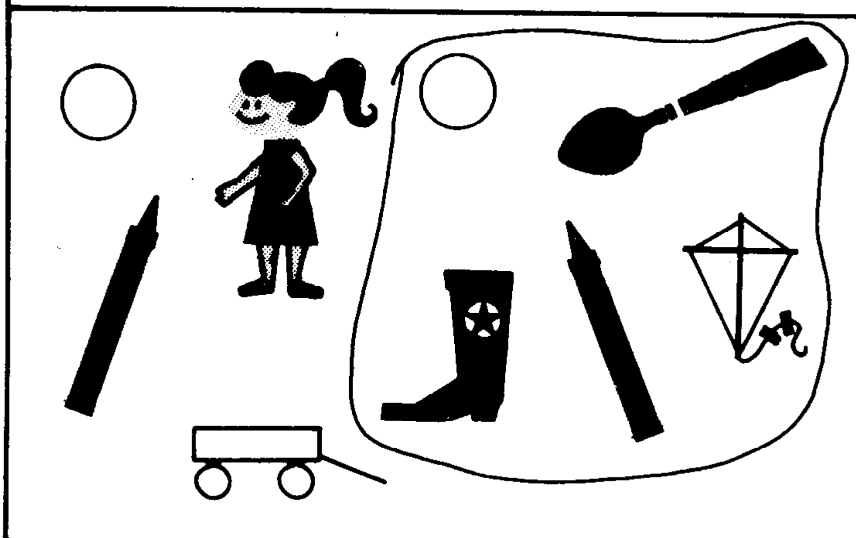
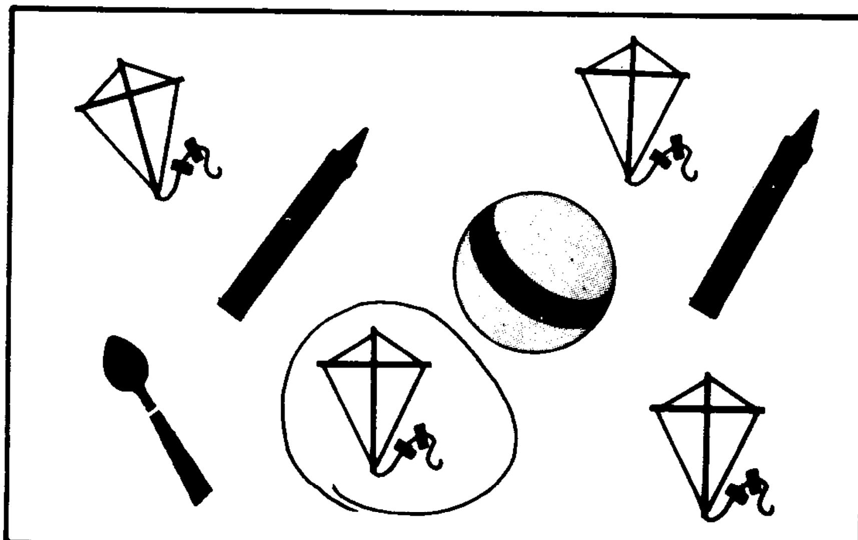
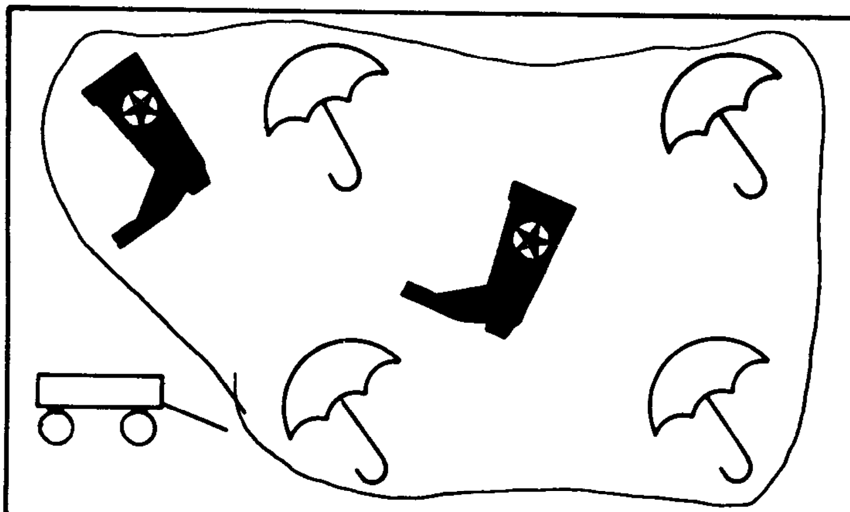


5	2	3



6	3	3

Remove a subset.	Equation
	7 | 4 | 3 $7 - 4 = \boxed{3}$
	6 | 2 | 4 $6 - \boxed{2} = \boxed{4}$
	8 | 4 | 4 $8 - \boxed{4} = \boxed{4}$

Remove a subset.	Equation
	9 | 5 | 4 $9 - 5 = 4$
	8 | 1 | 7 $8 - 1 = 7$
	7 | 6 | 1 $7 - 6 = 1$

Write the equations.

$5 - 1 = \boxed{4}$

$1 - 1 = \boxed{0}$

$3 - 1 = \boxed{2}$

$4 - 3 = \boxed{1}$

$6 - 2 = \boxed{4}$

$3 - 2 = \boxed{1}$

$4 - 2 = \boxed{2}$

$2 - 1 = \boxed{1}$

$5 - 2 = \boxed{3}$

$7 - 5 = \boxed{2}$

IV-7. Doing and undoing

Objective: To introduce the notion that adding a number and subtracting the same number are inverse operations.

Vocabulary: Doing, undoing.

Background Note:

Adding 3 to a number and then subtracting 3 from the result always gives the original number. We may also interchange the order of the operations. Subtracting 3 from a number and then adding 3 to the result always gives the original number. We say that the operation of adding 3 and the operation of subtracting 3 are inverses. (We do not use the term "inverse" with the children.) This relation between addition and subtraction is based on the corresponding relation between joining and removing. Joining a set to a second set and then removing it leaves the second set unchanged; removing a set which is a subset of a set, and then joining it to the remaining set leaves the starting set unchanged.

Suggested Procedure:

Place a set of toy cars on the table. Ask a child to describe the set. Identify this as the set with which we started. Put a set of toy animals on the table. It is not necessary to know the number of members in either set. Use sets which have too many members to count.

Join the set of toy cars to the set of animals.

> What are the members of this set?
> (Cars and animals.)
>
> When we joined the set of cars to the set of animals we formed a new set.

Remove the subset of toys which was joined to the set of cars.

What are the members that are remaining? (The set of animals.) This is the set with which we started.

- On the flannel board place a set of ducks with three members. Have the class describe the set, name the number of members of this set and refer to it as the starting set. Place a set of two dogs on the right side and identify the number of members in the set.

We have 2 sets on our flannel board. Let's join the set with 2 dogs to the set with 3 ducks. (Show by moving the set of dogs to the set of ducks.)

We now have one set with how many members? (5.)

What is the equation which tells we have added 2 to 3?

$(3 + 2 = 5.)$

Continue the discussion using this new set formed by joining the set of dogs to the set of ducks.

How many members have we in this set? (5.)

Ask a child to remove the subset of dogs from the set of ducks and dogs.

What is the number of members in the subset of dogs which was removed? (2.)

Name the members of the remaining set.

(The set of ducks, the set of 3 ducks, the set of ducks with which we started.)

What is the equation which shows that we have subtracted 2 from 5? $(5 - 2 = 3.)$

Is three the number of ducks with which we started? (Yes.)

Is the set the same set as the one with which we started? (Yes.)

Emphasize with the children that if a set which is joined to a set is removed from the new set which was formed, then we have the set with which we started.

If a number which is added to a second number is subtracted from the sum of the two numbers, then we have the number with which we started. The equations which we have developed while working with these sets are repeated here for comparison.

$$3 + 2 = 5$$

$$5 - 2 = 3$$

- Place a set of spools on the table.
  Ask a child to remove a subset of the spools.

  What is on the table? (A remaining set.)

  Join the set removed to the remaining set.

  What is on the table?
  (The set with which we started. The set of spools.)

  Was each member of this set in the set with which we started? (Yes.)

- Place a set of balls on a table. Describe the members of the set.

  What is the number of members? (7.)

  Ask a child to remove a subset of the balls.

  What is the number of members in the subset which was removed? (3.)

  What is the number of members in the remaining set? (4.)

  What equation describes removing a set of 3 from the set of 7? (7 - 3 = 4.)

Ask the children to think of the remaining set as a starting set.

How many objects are in this set? (4.)

Join the set of 3 balls which was the set removed to the set of 4 balls.

What is the number of members in the new set? (7.)

This is the set which we had before we removed the subset of three balls.

What is the equation to show that we have joined a set of 3 balls to the set of 4 balls? (4 + 3 = 7.)

Solving Problems

1. Bobby had five balloons.
   During the night some of the air went out of two balloons.
   How many were still blown up? (3.)
   How many balloons were full of air? (3.)
   Bobby's mother blew up the balloons which had lost some air.
   How many had lost some air? (2.)
   How many are full of air again? (5.)

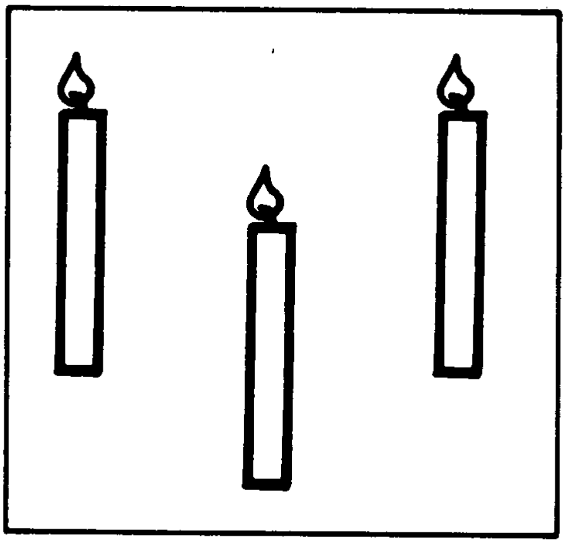
2. Tom had 2 marbles in his pocket.
   He had 4 jacks in his pocket.
   How many objects were in his pocket? (6.)

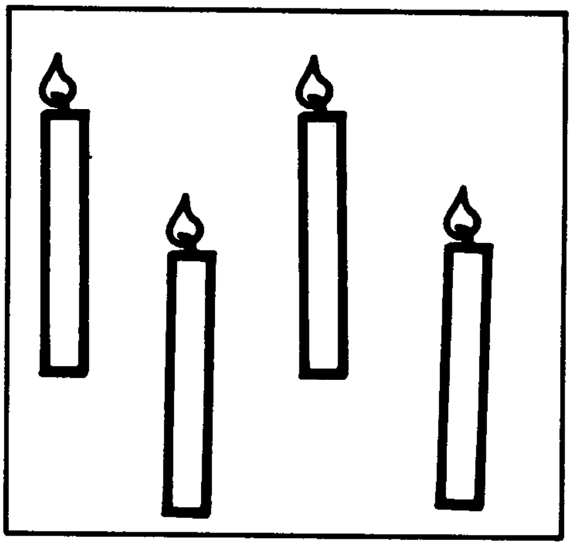
   Tom gave the jacks to his brother to play with. Then how many jacks did Tom have? (0.)
   What was left in Tom's pocket? (The set of marbles.)
   How many objects were left in Tom's pocket? (2.)

Pupil's book, page 50: Tell children to fill blanks and to record addition and then subtraction.

Pupil's book, page 51: Tell children to fill blanks and to record subtraction and then addition.

Addition and Subtraction

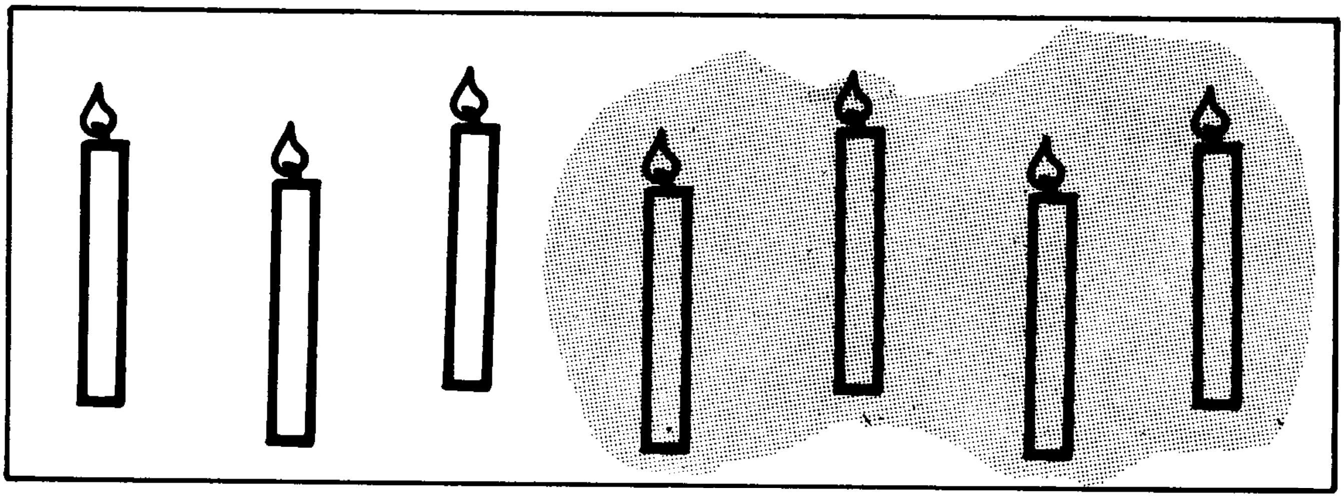




How many? 3 How many? 4

How many all together? 7

Equation: $3 + 4 = 7$

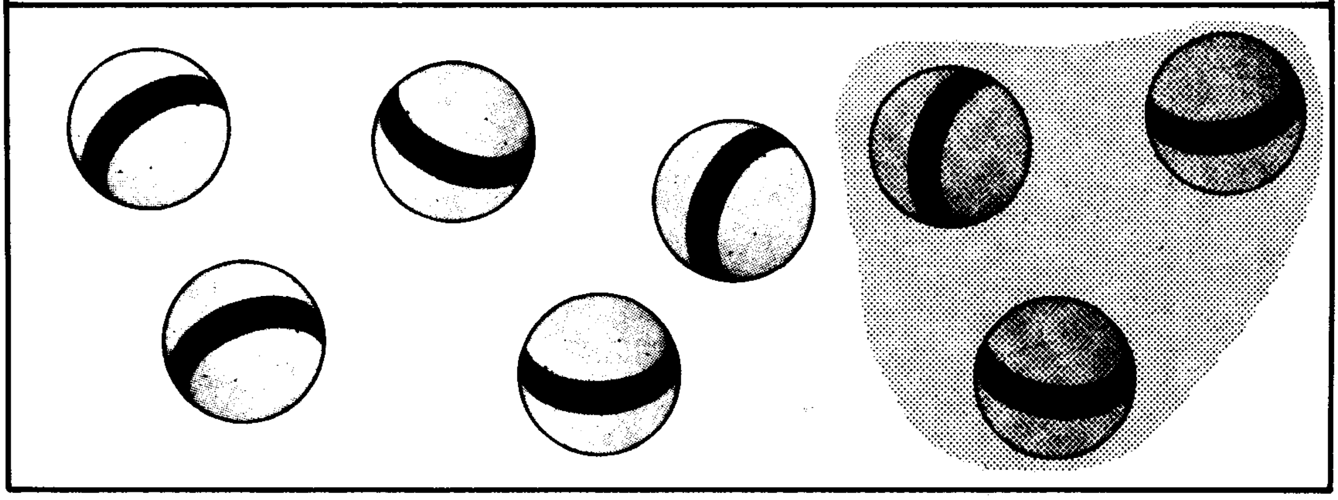


How many all together? 7 How many removed? 4

How many in remaining set? 3

Equation: $7 - 4 = 3$

Subtraction and Addition

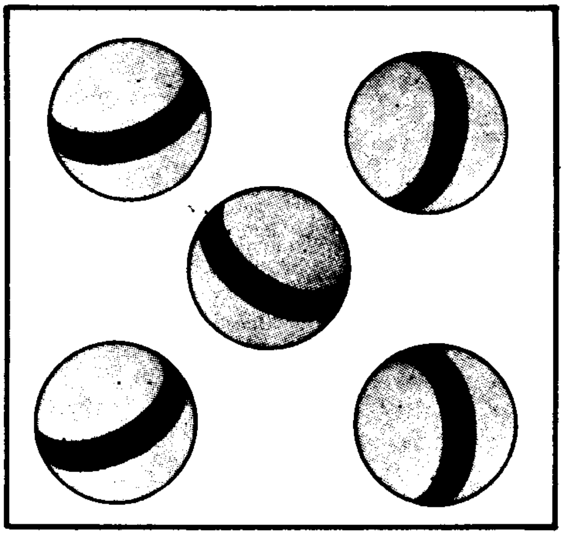


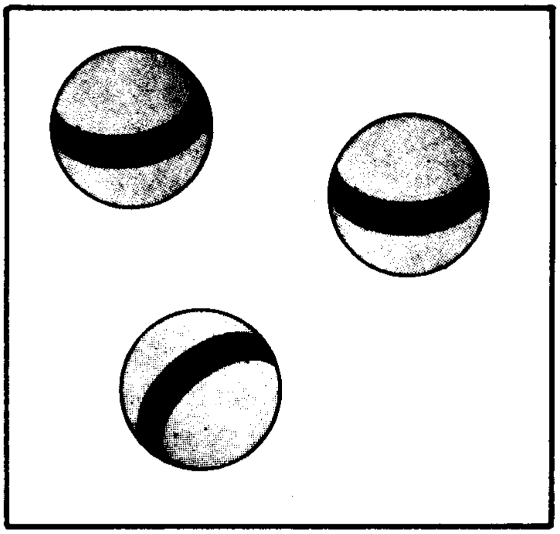
How many all together? 8

How many removed? 3

How many in the remaining set? 5

Equation: $8 - 3 = 5$





How many? 5

How many? 3

How many all together? 8

Equation: $5 + 3 = 8$

IV-8. Problem solving

Objective: To develop skill in solving story problems.

Vocabulary: Solve (to be used informally, not emphasized).

Materials: Play money (cents); numeral cards 0-9 and additional symbol cards for +, -, = (large size set for demonstration and additional sets for the children); "balloons" for flannel board; story problems printed on 12" × 24" tagboard; small objects for use by children when needed.

Suggested Procedure:

Display the following story written on tagboard.

> Mary had ☐ cents.
>
> She gave ☐ cents to Tom.
>
> How many cents does Mary still have?

I will read this story to you. In the first two sentences the "box" shows where something is missing. I will pause there each time.

After you have read the problem to the children, ask questions such as these:

Does the story ask a question? (Yes.)

What is the question it asks? (How many cents does Mary still have.)

What does the story tell you? (Mary had some money and she gave some away.)

Can you tell the answer to the question? (No.)

Why not? (There are no numbers to tell how many.)

Let's use some numbers in the story. Then we will see if we can answer the question.

Ask a child to show with play money how many cents Mary had at first. Write the numeral in the first box and read the sentence aloud. (e.g., Mary had 7 cents.)

Can you answer the question now? (No, because we don't know how many cents she gave away.)

Have the child who is holding the money decide how many cents Mary gave to Tom. Point out that this could not be more than 7 cents. Complete the second sentence by writing the numeral in its proper position. (e.g., 3) and then read the sentence. (She gave 3 cents to Tom.)

Encourage the children to answer the question asked by the third sentence in the story.

How can we show that Mary has 4 cents left?

The child who has the money can show the result using the set of 7 cents with which he started, and removing a subset of 3 cents.

Ask a child to come and choose numeral cards and additional symbol cards to show the equation.

$7 - 3 = 4$

Copy the equation on the chalkboard where everyone can see it.

Is this equation the answer to the question in our story? (No.)

Through discussion bring out that the equation is not the answer to the question in our problem, "How many cents does Mary have now"?

Who can tell me the answer to the question in our problem?

"Now Mary has 4 cents," is the answer to the question in our problem. We can say we have solved the problem.

- Display and read to the children the following problem:

Five dogs were playing.

Two dogs ran away.

How many dogs were there then?

Discuss the problem from the standpoint of ways in which it is different from and also like the first problem. Emphasize that although the story problems tell about different things, both problems involve removing a subset from a set and finding the number of members in the set that remains.

Ask the children to show with a numeral card on their desks the number of members in the set of dogs that remained. (3.) (Some children may need to manipulate representative objects. Permit them to do so.) Then have a child write on the chalkboard an equation that relates the numbers 5, 2 and 3. (5 - 2 = 3)

Ask another child to state the answer to the question in the problem.

Three dogs were left.

- Display this story problem written on tagboard:

John had ☐ balloons.

Father gave ☐ balloons to John.

How many balloons did John have then?

Remind the children to listen and think while you read the story problem to them, and to remember that each box is there to show that something is missing. Pause at each box as you read. Then discuss this problem in the same way in which you discussed the first problem, about Mary and Tom, i.e., emphasize the need for specific numbers to indicate how many balloons John had at first and how many balloons Father gave to John.

Ask one child to be John and choose enough flannel board balloons to show how many balloons John had before Father gave more to him. Write the numeral in the box in the first sentence, (e.g, 4). Read the sentence to the children.

Can we solve the problem now? (No, because we don't know how many balloons Father gave to John.)

If Father gave John at least one balloon, how many balloons would John have then? (At least 5.)

Ask another child to be Father, to choose the number of balloons that Father gave to John (e.g., 3), and to display that many flannel board balloons.

Write the numeral to show how many balloons Father gave to John (e.g., 3) and read the second sentence aloud.

How can we find the number we need to solve the problem? (We can join the set of balloons John had and the set Father gave to him.) How many balloons did John have then? (7.)

Have a child use the large demonstration cards to show the equation that relates the numbers 4, 3 and 7. ($4 + 3 = 7$.)

Stress the fact that $4 + 3 = 7$ is the equation associated with the problem but that the answer to the question asked in the story is, "Then John had 7 balloons."

Further Activities:

1. Give the children large sheets of newsprint and suggest that they think about a problem of their own and draw a picture about it. Encourage them to share their story with the group. Write some of the story problems on sheets of oaktag for use in this connection.

2. Give each child a card with a pair of numerals on it, e.g., 3, 2. Using large sheets of paper have the child draw a picture of a story problem which would fit the numbers named on his card. Discussion in presenting the activity could bring out the different possibilities for using the same pair of numbers. For example, 3, 2 might be used as 3 and 2 more, as 2 and 3 more, or as a subset of 2 removed from a set of 3.

3. Give each child a sheet of 12 × 18 newsprint and a few gummed dots of each of two colors. Direct the child to stick dots on the paper wherever he may wish to place them. He may imagine the dots are anything he wishes them to be and he is to draw a picture around them. For example, the dots might be eggs in a nest, or a bed of flowers, or balls, or different kinds of food. When the child finishes his picture he may tell the class a story problem that goes with his picture.

4. Story problems such as these may be used for further experience with problem solving work.

    Some cars were in front of my house. Two cars came. Then there were four cars. How many cars were there at first?

    Susan spent 3 cents for candy and 2 cents for gum. How many cents did she spend?

    Jack had 4 balloons. He gave some to Jimmy. Then he had 2 balloons. How many did he give to Jimmy?

    This morning David read 1 page in his book. This afternoon he read 5 pages. How many pages did he read?

    Mother had 5 sticks of candy. She gave 4 sticks to the children. How many sticks did she have then?

    Bill borrowed some paper from Bob. He returned 2 sheets and he still owes Bill 3 sheets. How many sheets did he borrow?

    Mark had some toy boats. He gave 2 boats to Albert. Then Mark had 5 boats. How many boats did Mark have in the beginning?

    Mary had 3 dolls. She got more dolls for her birthday. Then she had 6 dolls. How many dolls did she get for her birthday?

Chapter V

RECOGNIZING GEOMETRIC FIGURES

Background

Introduction

This chapter is devoted to geometry.

The subject is introduced to the children by means of familiar three-dimensional shapes. This part of the discussion is very informal and the classification crude: objects are differentiated according to whether they are "round", "flat", "square", and so on.

The rest of the chapter, as well as the ensuing geometric material through the next several books, deals with plane geometry only. For convenience of reference we now outline the main ideas (even though many of them will not be encountered until later).

We shall study what may be called physical geometry--that is, the geometry of the world around us. The study involves a certain amount of abstraction, for the fundamental objects we shall deal with are not things we can pick up or feel or see. We shall think of a *point*, for example, as an exact location in space. A point, then, has no size or shape or color; it has no physical attributes at all except its location. We indicate a point by making a pencil dot or a chalk dot; but every child will agree that such a dot does not mark an *exact* location, and he will enjoy imagining the unseeable points.

We may remark that the geometry studied in college courses is of a higher degree of abstraction still. There the fundamental geometric objects like *point* and *line* are not defined at all, and the study proceeds deductively from certain formally stated assumptions about them (called axioms).

Our purpose here is to help the pupil observe and describe fundamental geometric relationships. The discussion is intuitive. In the primary grades we are not particularly concerned with formal deductions.

Point

By a point we mean an exact location--for example, the exact spot at the corner of a room where two walls and the ceiling meet. We indicate points by drawing dots; but we realize that a pencil dot, no matter how small, gives only an approximate location, not an exact one. (In fact, it is clear that a pencil dot on a sheet of paper covers infinitely many points--that is, more than can be counted.) Nevertheless, in order to keep the language simple, we refer to the dots themselves as the actual points.

It is customary to denote points by capital letters.

A point is a fixed location: points do not move. The point at the corner of the ceiling remains even if the whole building falls down. Nevertheless, it must be remembered that fixing a location is a meaningful notion only with respect to some particular frame of reference. Frames of reference in common usage are: the sun, the earth, a car, a person, a ruler. A point that is fixed with respect to one frame of reference need not be fixed with respect to a different one. For example, when a ruler is carried across the room, a point on the ruler remains fixed with respect to the ruler but does not remain fixed with respect to the earth.

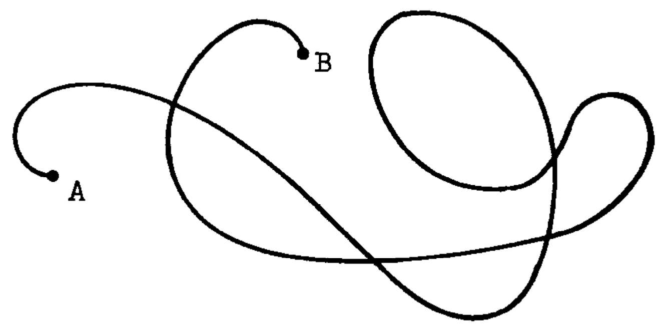
A geometric figure is any set of points.

Congruence

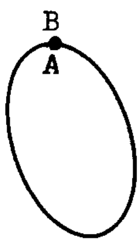
The idea of congruence in geometry is basic. Two geometric figures are said to be congruent provided that they have the same size and shape. A test is whether one will fit exactly on the other. In practice, the objects may not be conveniently movable; then one tests for congruence by making a movable copy of one and checking it against the other. Of course, all such tests, since they involve actual physical objects, often including the human eye, are only approximate. Nevertheless, in order to keep the language simple, we shall say, "the segments $\overline{AB}$ and $\overline{CD}$ *are* congruent" (rather than *seem to be*)--just as people say, "Johnny and Jimmy are exactly as tall as each other" (rather than *seem to be*).

Curve

By a *curve* we mean any set of points followed in passing from a given point A to a given point B. Inherent in this definition is the intuitive notion of continuity; this is a curve:



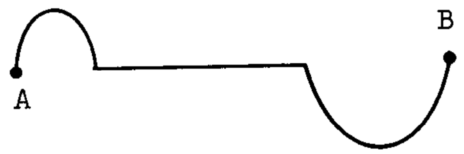
and so is this:



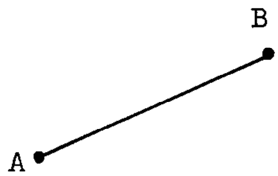
while this is *not* a curve:

(However, it is a union of three curves.) We agree that a single point is not a curve.

It is also noteworthy that, according to the definition, a curve can be straight (in contrast with everyday usage). This is a curve:



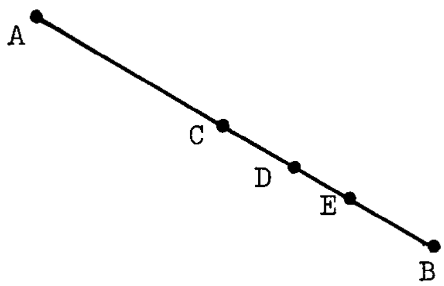
and so is this:



Line Segment

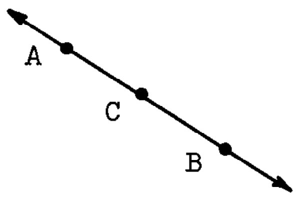
The last picture is an example of a line segment, that is, a straight curve. The endpoints are marked A and B; the line segment is denoted, accordingly, by either $\overline{AB}$ or $\overline{BA}$. Again, we agree that a single point is not a line segment.

Observe that a line segment can always be expressed in many different ways as a union of other line segments. For example, the line segment $\overline{AB}$ shown here is the union of the line segments $\overline{AC}$ and $\overline{CB}$, the union of the line segments $\overline{AD}$, $\overline{AE}$, and $\overline{CB}$, etc.



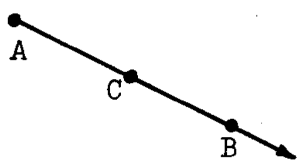
Line

When a line segment is extended infinitely far in both directions, we get a line. Such extensions are only conceptual, of course, not practical. A line has no endpoints. No matter how far out we go in either direction along a line, still more of the line will lie ahead. The infinite extent is indicated by arrows. The line containing points A and B is denoted by $\overleftrightarrow{AB}$. The line shown contains points A, B, and C; some names for this line are, therefore, $\overleftrightarrow{AB}$, $\overleftrightarrow{BA}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BC}$, etc.

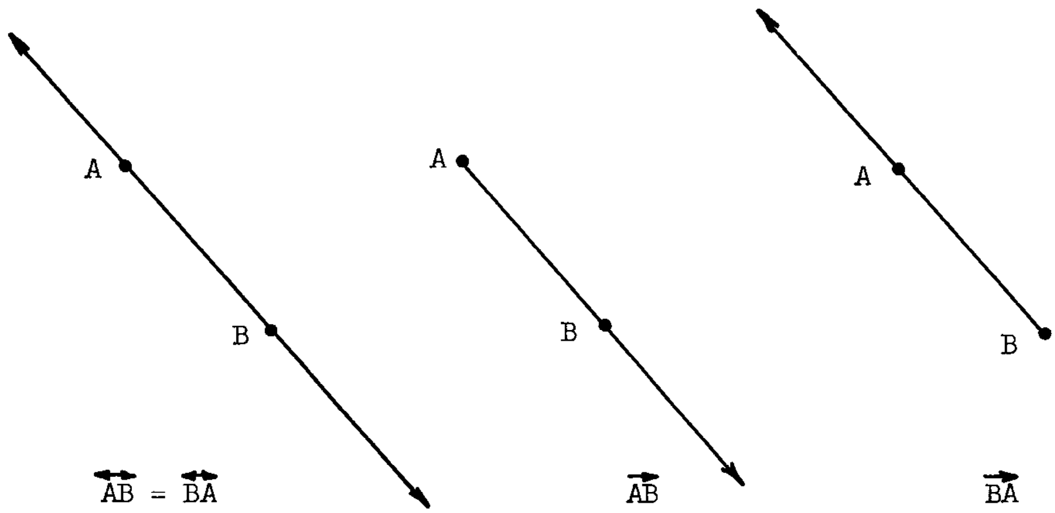


Note that, although $\overline{AB}$ and $\overline{AC}$ are different line segments, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are the same line.

Just as a line is the infinite extension of a line segment in both directions a ray is the infinite extension of a line segment in one direction. A ray therefore has a single endpoint. The infinite extent of a ray is indicated by an arrow. The ray with endpoint A and containing another point B is denoted by $\overrightarrow{AB}$. The ray shown has endpoint A and contains points B and C; some names for this ray are, therefore, $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

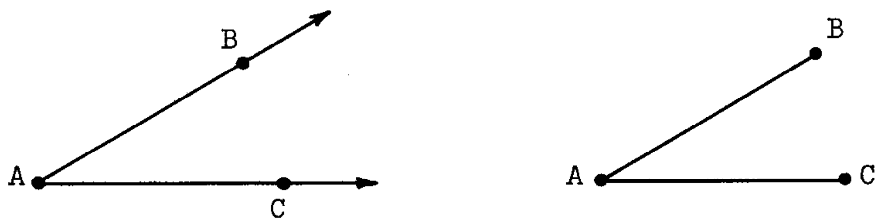


Note that, although $\overline{AB}$ and $\overline{BA}$ are the same line, $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays.



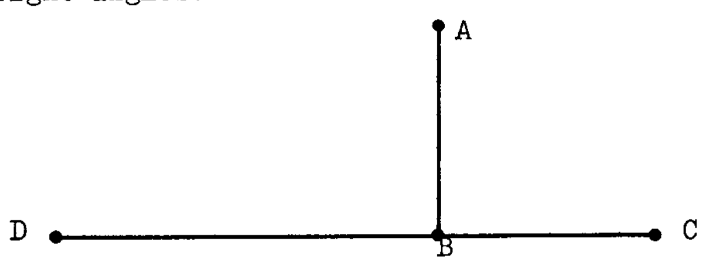
Angle

By an angle we mean the union of two rays having the same endpoint. (We exclude the case in which the two rays are part of the same line.) The common endpoint is called the vertex of the angle. The plural of "vertex" is "vertices". The angle formed by rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is denoted by $\angle BAC$ or $\angle CAB$. Two segments with a common endpoint determine an angle: segments $\overline{AB}$ and $\overline{AC}$ with common endpoint A determine the angle $\angle BAC$ with vertex A:

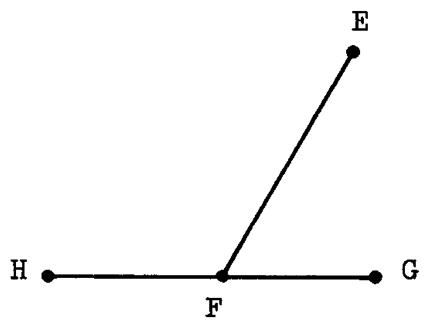


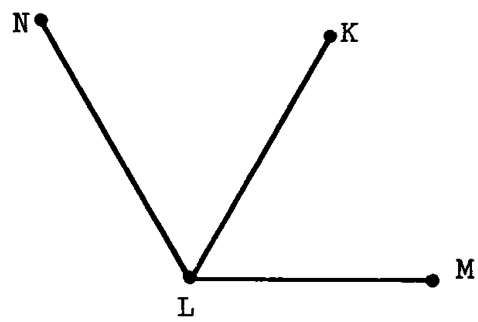
Right Angle

An angle is called a right angle if "two of them can fit together to form a line". In the diagram, $\angle ABC$ is congruent with $\angle ABD$, and the three points C, B, and D lie on a line; therefore, $\angle ABC$ and $\angle ABD$ are right angles.



Note that there are two parts to the definition: the part concerning congruence, and the part concerning the line. In the next diagram, ∠ EFG and ∠ EFH form a line but are not congruent, while ∠ KLM and ∠ KLN are congruent but do not form a line.



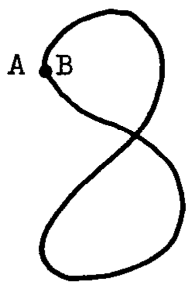


Plane

When a flat surface such as a table top, wall, or sheet of glass, or even this sheet of paper, is extended infinitely in all directions, we get a plane. Notice that if two points of a line lie in a given plane, then the entire line is contained in the plane. Two intersecting lines determine a plane. In the teaching material, the infinite extent of the plane is not stressed.

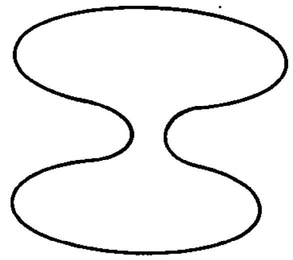
Closed Curve, Simple Closed Curve

We have called a curve any set of points followed in passing from a given point A to a given point B. When the points A and B coincide, the curve is said to be closed.



A closed curve

A closed curve that lies in a plane and does not cross itself is <u>simple</u>.

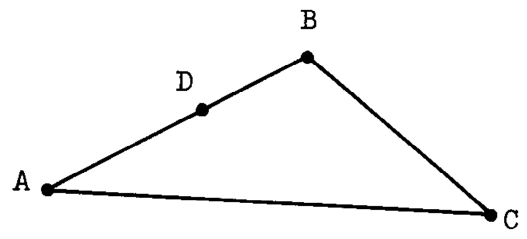


A simple closed curve

A simple closed curve has the interesting property of separating the rest of the plane into two subsets, an inside or <u>interior</u> (the subset of the plane enclosed by the curve) and an outside or <u>exterior</u>. Any curve connecting a point of the interior with a point of the exterior necessarily intersects the simple closed curve. (It may be of interest that this seemingly obvious fact is actually quite hard to prove.)

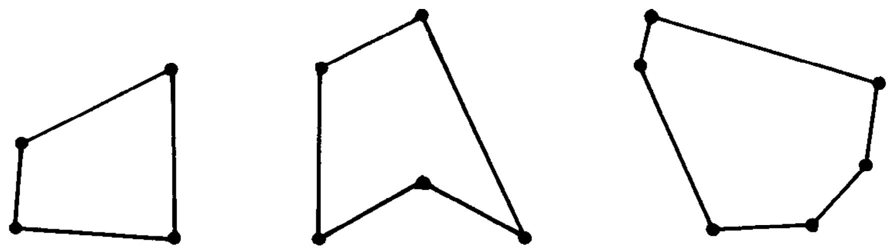
<u>Polygon</u>

An important class of simple closed curves is the class of polygons. A <u>polygon</u> is a simple closed curve that is a union of line segments. Recall that a line segment can always be expressed in many different ways as a union of line segments. Hence a polygon, too, can be expressed in different ways as a union of line segments.



The union of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ =
the union of $\overline{AD}$, $\overline{DB}$, $\overline{BC}$, and $\overline{CA}$.

If we look at the various line segments in a polygon, we notice that they are of two kinds: those that are contained in other line segments, and those that are not contained in other line segments. For example, in the picture above, $\overline{AD}$ is of the first kind, since it is contained in the line segment $\overline{AB}$. On the other hand, $\overline{AB}$ is of the second kind, since it is not contained in any line segment except itself. Line segments of this second kind are called sides: a line segment in a polygon is called a side if it is not contained in any other line segment in the polygon. The polygon shown has three sides: $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$. A polygon of three sides is called a triangle. A polygon of four sides is a quadilateral; of five sides, a pentagon; of six, a hexagon. (The last two names are not used in the teaching material.)



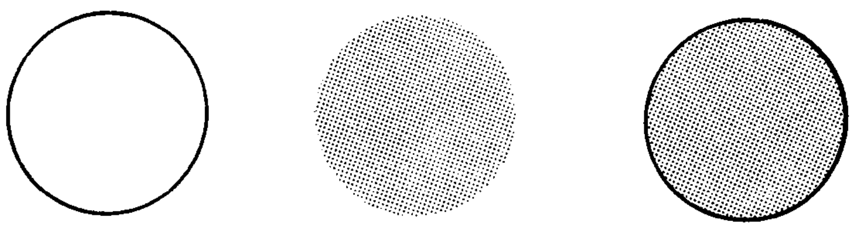
quadrilateral pentagon hexagon

It may be observed that two consecutive sides of a polygon--that is, two sides with an endpoint in common--never lie on the same line. The endpoints of the sides are the vertices (singular: vertex) of the polygon. The vertices of the triangle shown above are A, B, and C.

Rectangles are special kinds of quadrilaterals. Squares are special kinds of rectangles.

Region

The union of a simple closed curve and its interior is called a region. We refer to a triangular region, rectangular region, or circular region, etc., indicating that the simple closed curve is a triangle, rectangle, or circle, etc. For example, an ordinary sheet of paper is a rectangular region; the edges of the paper form a rectangle.



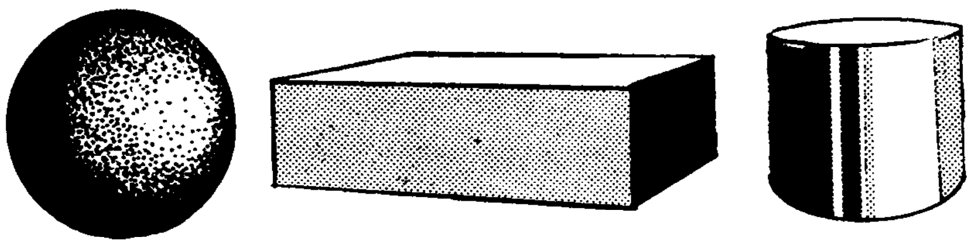
circle interior circular region

V-1. Familiar three-dimensional shapes

Objective: To lead children to observe distinguishing features of spheres, rectangular prisms, and cylinders.

Vocabulary: Shape, round, face, edge, corner, surface.

Materials: A table on which there are familiar objects (at least 15): balls, boxes, blocks, plastic containers, and the like. These should be restricted to objects that can serve as models of spheres, rectangular prisms, and cylinders. A set of commercial models is highly recommended.



Sphere   Rectangular prism   Cylinder

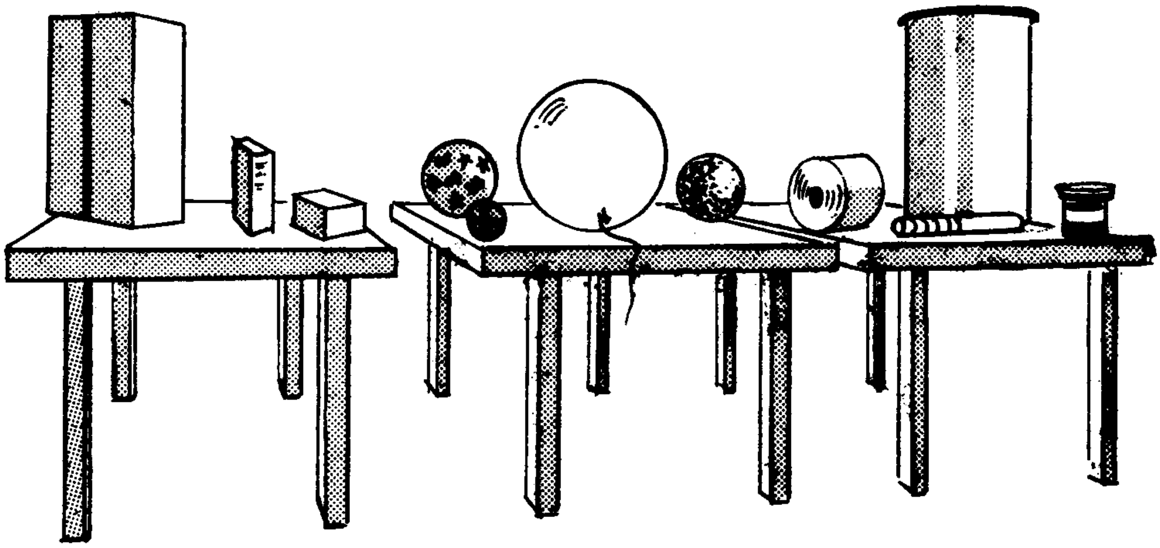
Suggested Procedure:

This exploratory lesson directs attention to the geometry of spheres, rectangular prisms, and cylinders. There should be a sufficient number of objects (varied in color and shape) so that all children have an opportunity to handle and to discuss the objects. They should run their hands over surfaces, along edges, etc. As the lesson proceeds, use the words object, item, and thing interchangeably until the children have learned the word object.

You may begin this lesson by designating desks on which children are to place objects that have some kind of likeness to each other. Begin by asking a child to place an object (item or thing) in one of the places. Ask another child to select a second object. If he does not think it should be placed with the first object, he may place it on another desk and explain in what way these objects are different. The classification has been established at this point.

When the other children place objects in the various sets, they should use this same classification. You may find that the first sorting is done according to color or size or use of the object or material from which it is made, etc. Let the children continue the classification by using six or more objects. As each object is placed with a set, discuss with the children whether or not it belongs with the other objects in the set.

Start again with all objects in one set and tell the children to think of other ways to sort them. Let the children develop several classifications. If shape has not been used as a basis for sorting, introduce it. First place a ball on one desk, a box on the next desk, and a can on the third. Then select another object and ask the children why it should be placed on a particular table. If a response is made that it has a shape like a ball, agree, and comment that it is a figure shaped like a ball. The activity should result in some such arrangement as that pictured below.



After the sorting is completed, the children should identify what the objects in each set have in common. Their description of the sets may be: objects like boxes, objects like balls, objects like spools. Help develop the awareness of these shapes by describing the boxes as having edges, flat sides (faces), and corners; the cans as having edges (rims) but no corners; and the balls as having neither edges nor corners.

Pupil's book, pages 53-54: Recognizing the Shapes of Figures

Ideas

Objects are shaped in different ways. (Balls, cans, boxes.)

Page 53.

Call attention to some of the pictures of objects on the page. Ask the children to look at the first row. Note that a row goes across the page, not up and down. Ask what the first object in the first row is. (Ball) Trace the mark on the ball.

What are the names of the other pictures in the row? (Crayon, golfball.)

Which picture has the same shape as the baseball? (Golfball.)

Mark the golfball in the same way the baseball is marked.

Then ask the children to mark the first picture in the other rows, and one other picture shaped like the first one in the same row.

Page 54.

This page has more choices for marking in each row. Ask the children to look at the first row and mark the two objects that have the same shape. Check the accuracy of their markings, then give instructions to complete the page.

Further Activities:

1. Ask a child to put his hands behind his back. Then place in his hands an object shaped like one of the three kinds in this lesson. (It would be advisable to include objects which had not been used in the earlier sorting.) Ask the child to identify its shape. Continue with other children and other objects. In each case, ask why the object is classified as it is. Chalk, dominoes, and cylindrical pinboxes would be helpful.

2. Have children identify other objects in the room that could be placed in one of the three categories. Children may wish to bring from home various objects to add to the collection. Flashlight batteries, balls, blocks, pencils, chalk, or simple toys can be classified as they are brought in.

3. Pictures of objects can also be brought in and classified. Have the children tell why each object can

be put in that particular classification. This procedure not only helps to identify the geometric figures but also provides the association of the picture with the object and with the geometric figure they represent. The pictures may be arranged on a bulletin board, in a scrapbook, etc.

4. If children ask the geometric names of the objects that they handle, supply these names whenever possible. Although introduction of such names as "rectangular prism", "cylinder", and "sphere" is not the purpose of this chapter, some children are interested in new words and will take pleasure in hearing them.

5. Have several small packing boxes (more than necessary) available in which to pack the objects. Ask the children how the objects might best be packed to save room. Some suggestions might be

   Rectangular figures can fit together because of their edges.

   Large round figures have waste space in which smaller objects can be packed.

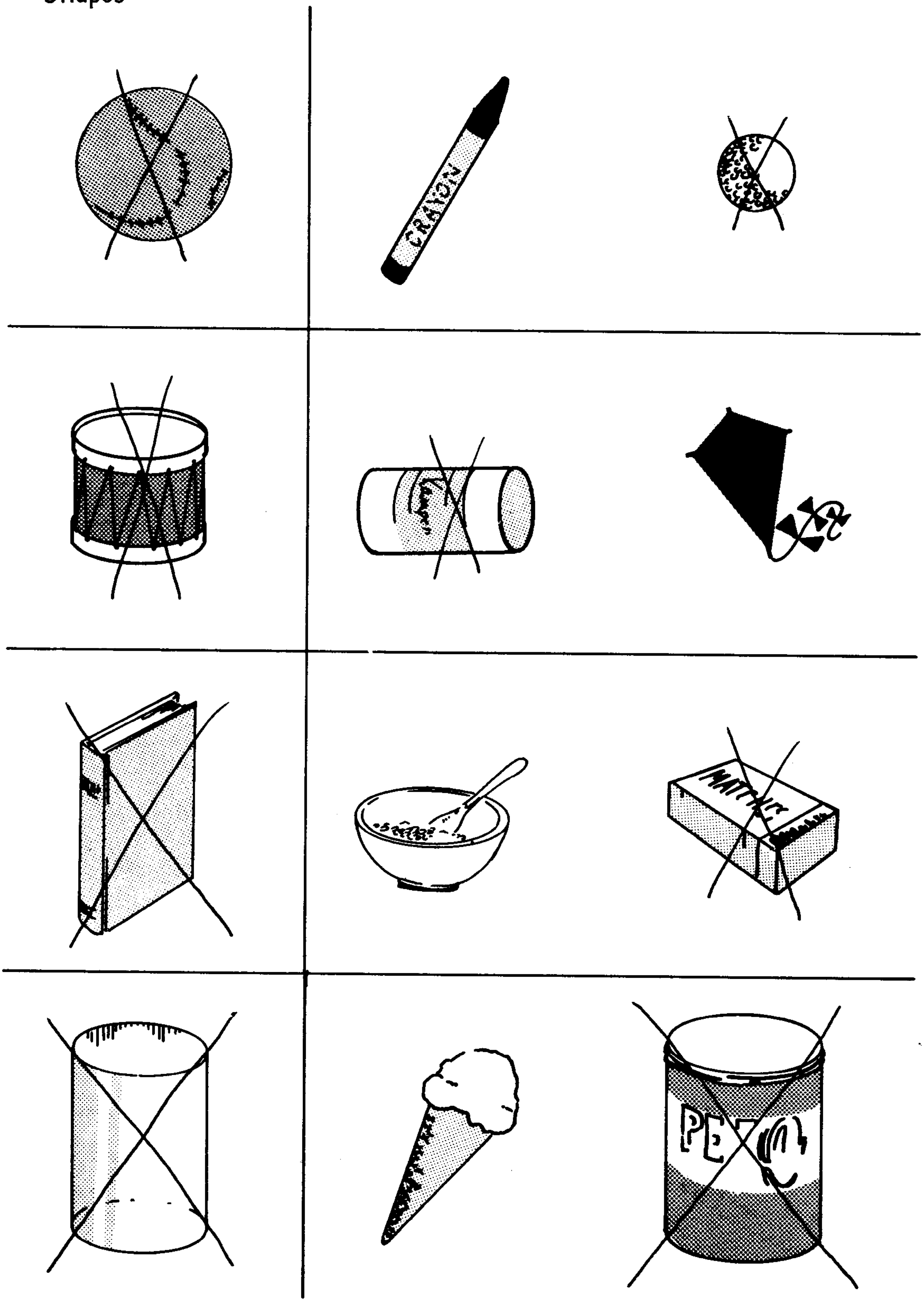
   Balloons could be deflated.

   Then have the children experiment with ways of packing. Compare the ease or difficulty of packing with that of a box of dominoes or checkers.

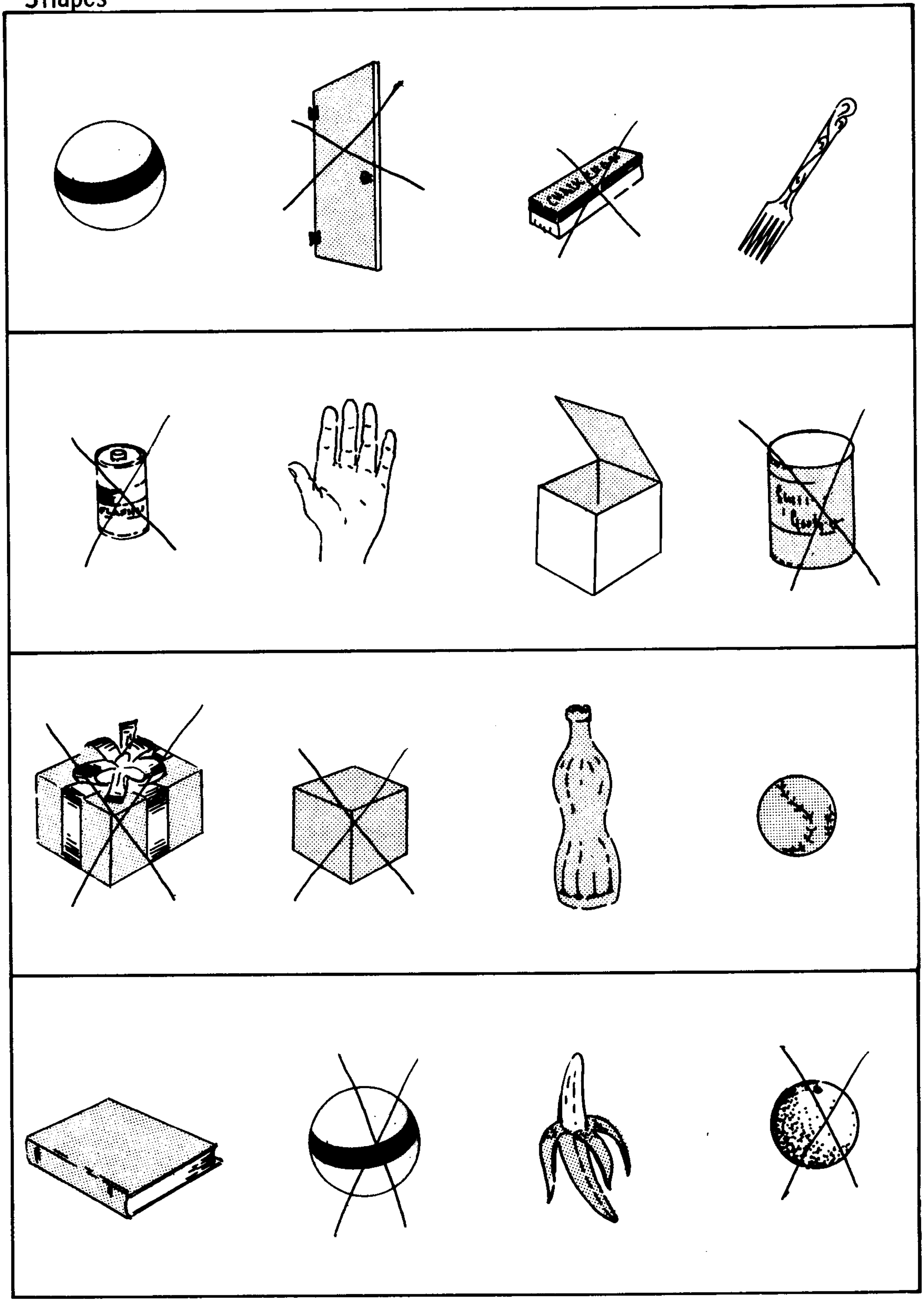
6. Read to the children such books as:

   Berdney, A Kiss is Round
   Kuskin, Square as a House
   Roberts, The Dot
   Schlein, Shapes
   Wolff, Let's Imagine Thinking of Things

Shapes



Shapes



V-2. Simple closed curves

Objective: A preliminary classification of some simple closed curves.

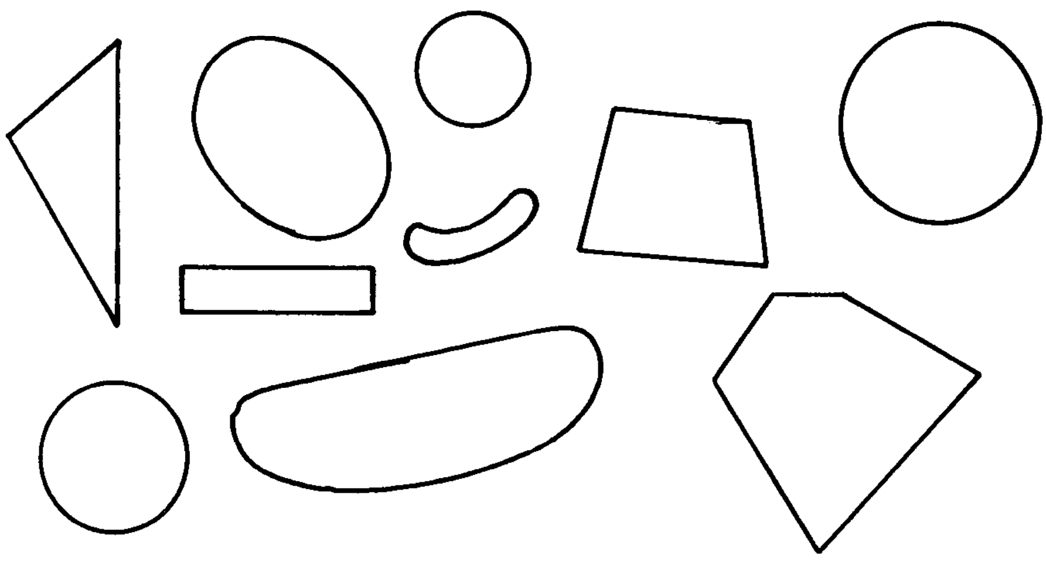
Vocabulary: Straight, rounded, circle.

Materials: Balls, boxes, and cans as in the preceding section; models of circles, triangles, rectangles, and other curved or polygonal figures, such as triangles from rhythm instruments, rectangular picture frames, circular embroidery hoops, rubber bands, stretched around pegs on a pegboard (or nails in a piece of ceiling tile), models made from wire or starched string (Do not use cardboard sheets as they suggest the regions rather than the curves themselves.); chalk and string for drawing circles on the chalkboard.

Suggested Procedure:

Distinguishing between "straight" and "rounded"

Before the lesson draw several polygons and other simple closed curves on the chalkboard. Include at least three circles.



Point out that some of the figures are rounded, while others have straight sides. Discuss and classify each figure in turn.

Display and discuss the triangles, frames, and hoops, and the pegboard and wire models.

Display the balls, boxes, and cans. Show the circular seam of a ball. (Do not use a baseball; its seam is not a plane curve.) Indicate the rounded rims of the cans. Point out the straight edges of the boxes.

Have the children look for objects about the room whose shapes they can classify: the rounded rim of the waste-basket or clock, the straight edges of the desk or window, etc.

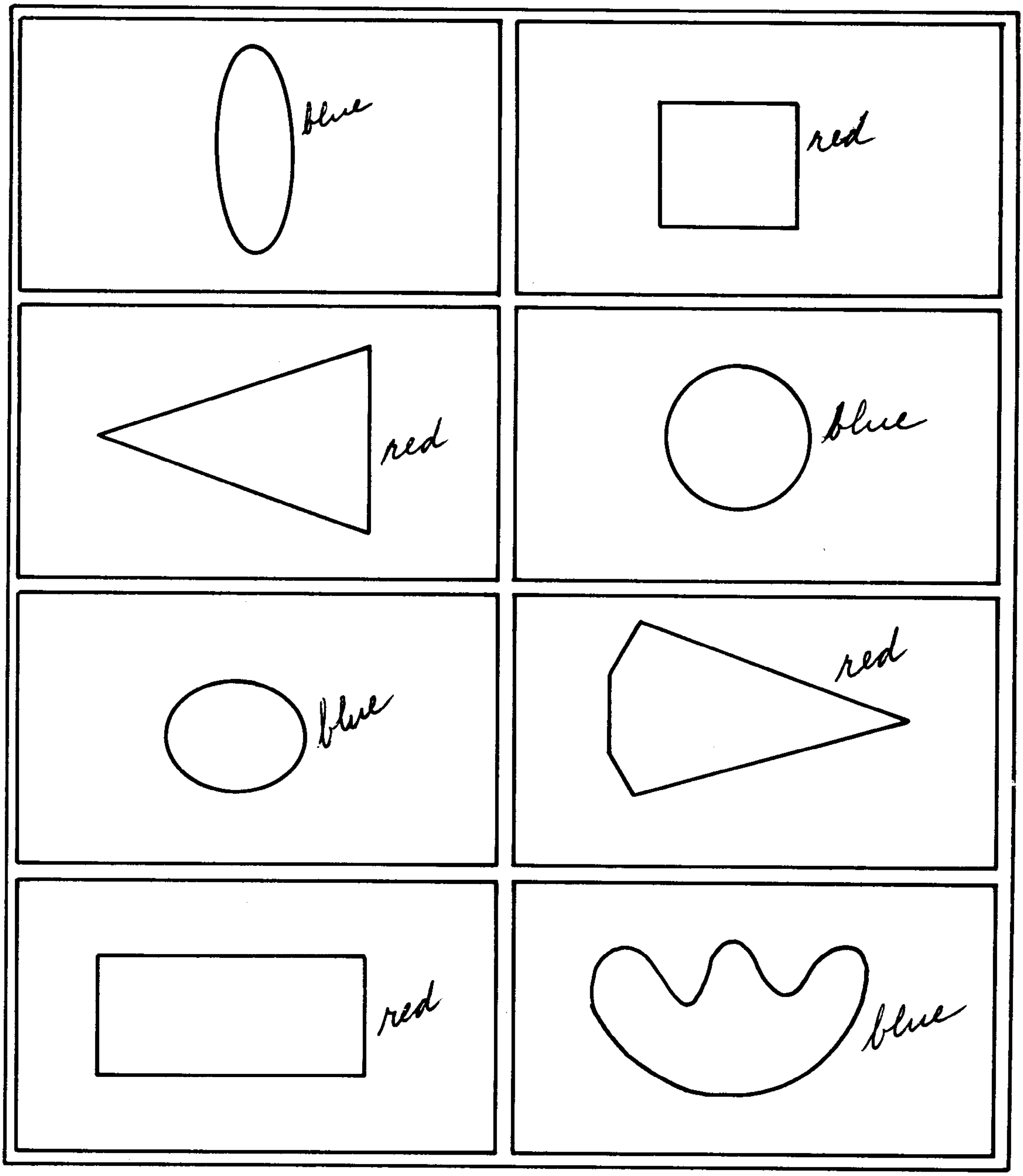
Pupil's book, page 55: Rounded or Straight

Read the instructions to the children. The child is to make a mark somewhere on the figure.

Rounded or Straight

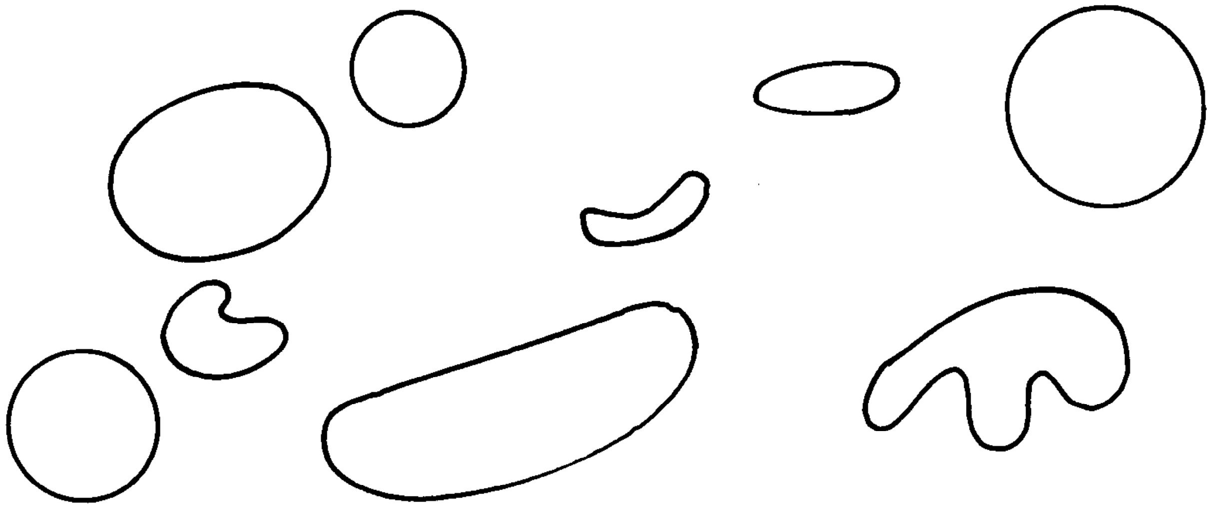
Mark each rounded figure blue.

Mark each figure with straight sides red.



Distinguishing circles from other rounded shapes

Direct children's attention again to the figures on the chalkboard. Tell the children that you are going to erase all the figures (or pictures) with straight sides. Have them pick out the figures for you. When all the polygons have been erased, replace them with curved figures that you can draw freehand.



Introduce the word circle. Consider the figures one by one, picking out the circles. Have the children tell why the circle is special. ("It looks the same from every direction", etc.)

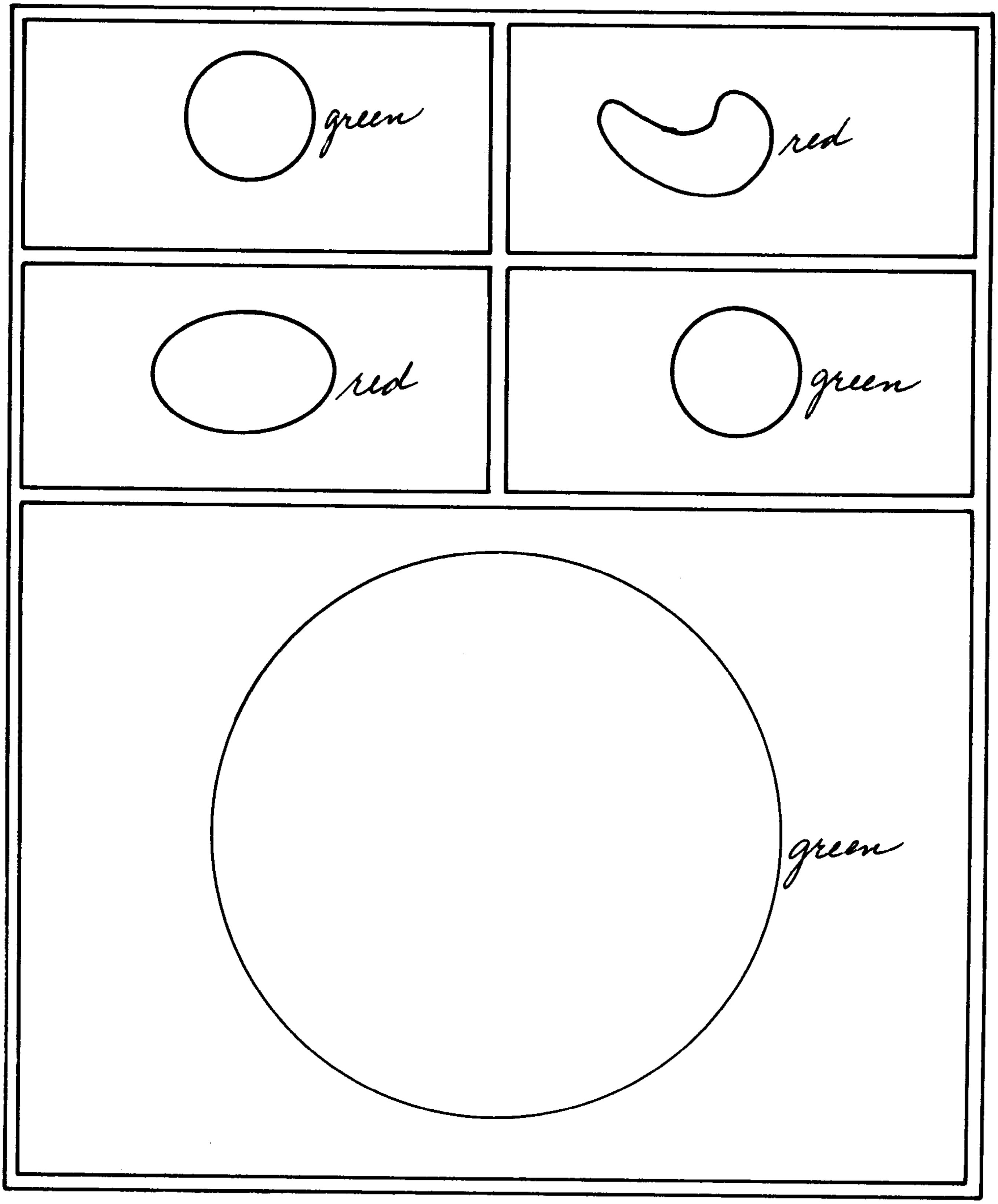
Pupil's book, page 56: Circles

Read the instructions to the children. The pupil is to make a mark somewhere on the figure.

Circles

Mark each circle green.

Mark each other figure red.



V-3. Polygons

Objective: A preliminary classification of some polygons.

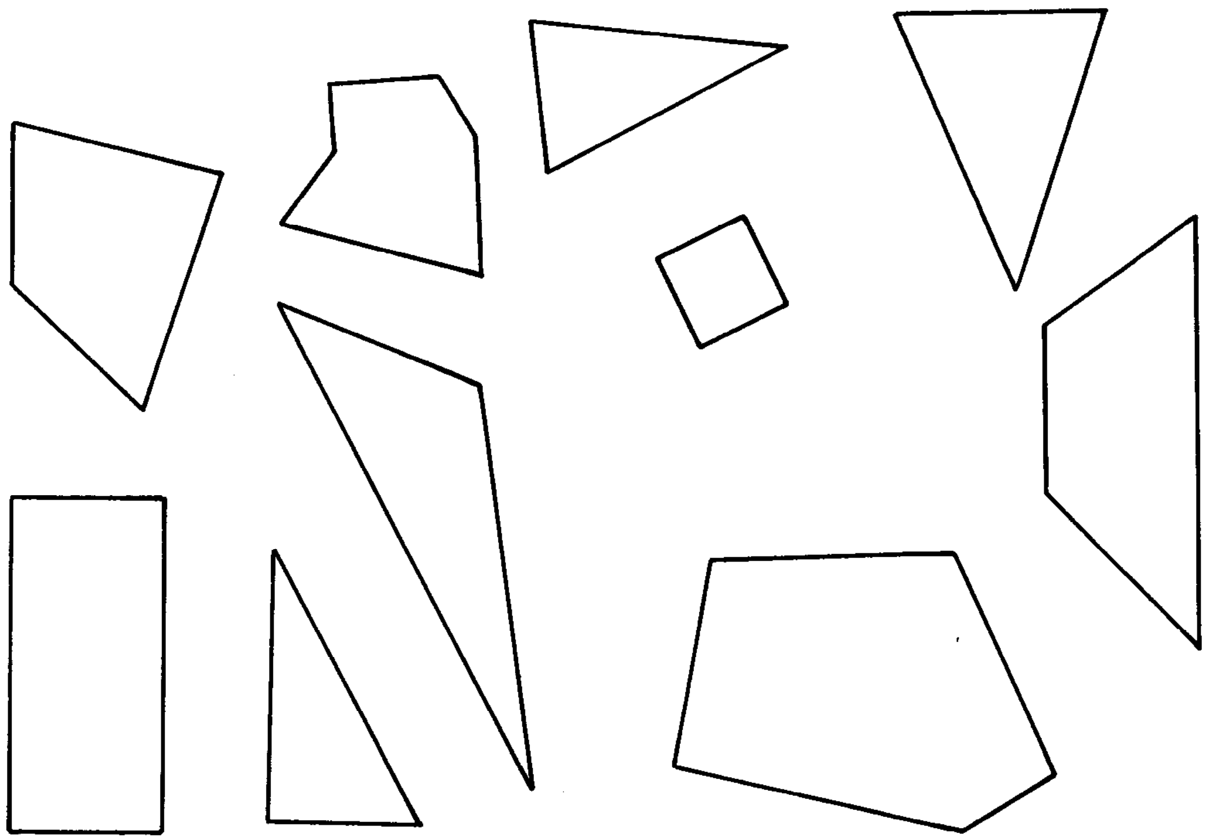
Vocabulary: Triangle, rectangle, square.

Materials: Boxes, models of triangles, rectangles, and other polygons, such as triangles from rhythem instruments, rectangular picture frames, rubber bands stretched around pegs on a pegboard (or nails in a piece of ceiling tile), models made from wire or starched string; sticks of various lengths.

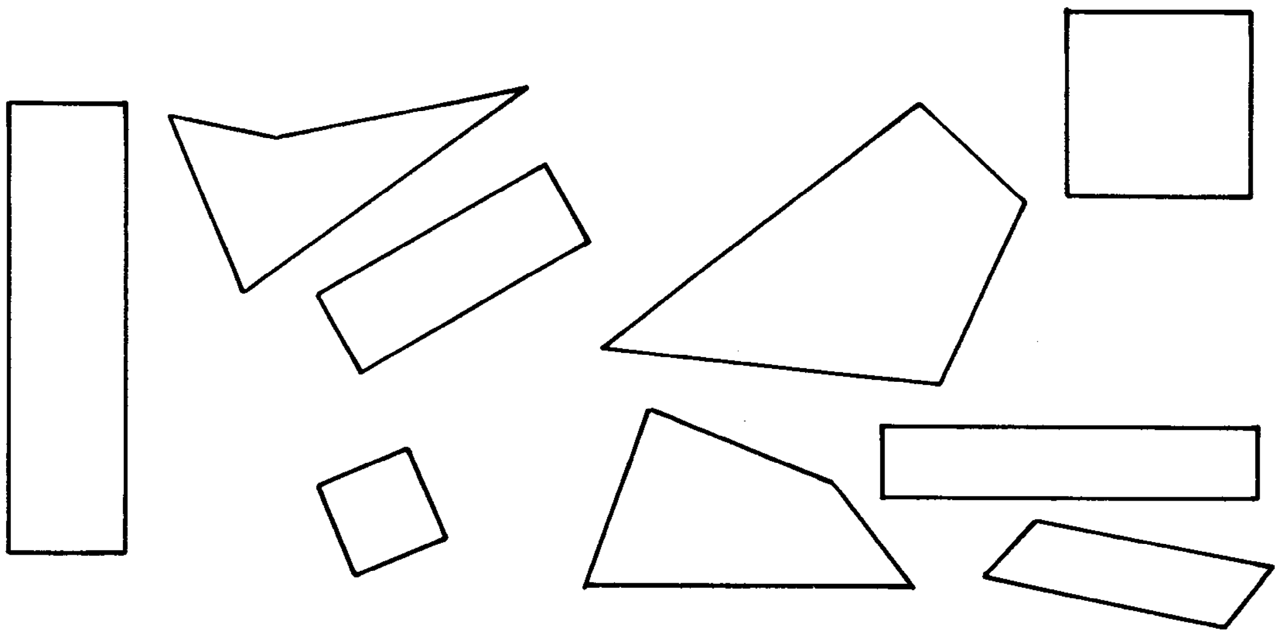
Suggested Procedure:

This lesson requires some preparation of the chalkboard.

On the left side of the chalkboard, draw several polygons. Include at least three triangles and three quadrilaterals, and a few polygons with five or more sides.



On the right side of the chalkboard, draw several quadrilaterals. Include at least five rectangles, two of which are squares; at least two of the rectangles, including one of the squares, should be "tilted". Keep this section covered from view until needed.



Classifying polygons according to the number of sides

Ask the class how the set of figures (or pictures) drawn here differs from those discussed last time. (All of these have straight sides.) Pick out a triangle and show that it has three sides; write "3" inside the triangle. Pick out a quadrilateral and show that it has four sides; and write "4" inside. Then consider the remaining figures in turn, getting the children to agree on the number of sides, and recording the number inside the figure.

See if children know the name, triangle, for polygons having exactly three sides. Suggest the name if necessary. Consider the figures once more, picking out the triangles. The word "quadrilateral" is not introduced at this stage, but should be given if a child asks for the name of a polygon of four sides. For five or more sides, it is enough to tell the children that special names do exist. (Possible exception: some children will know the word "pentagon.")

Display the metal triangles, the picture frames, and the pegboard and wire models of polygons. Have the children classify their shapes.

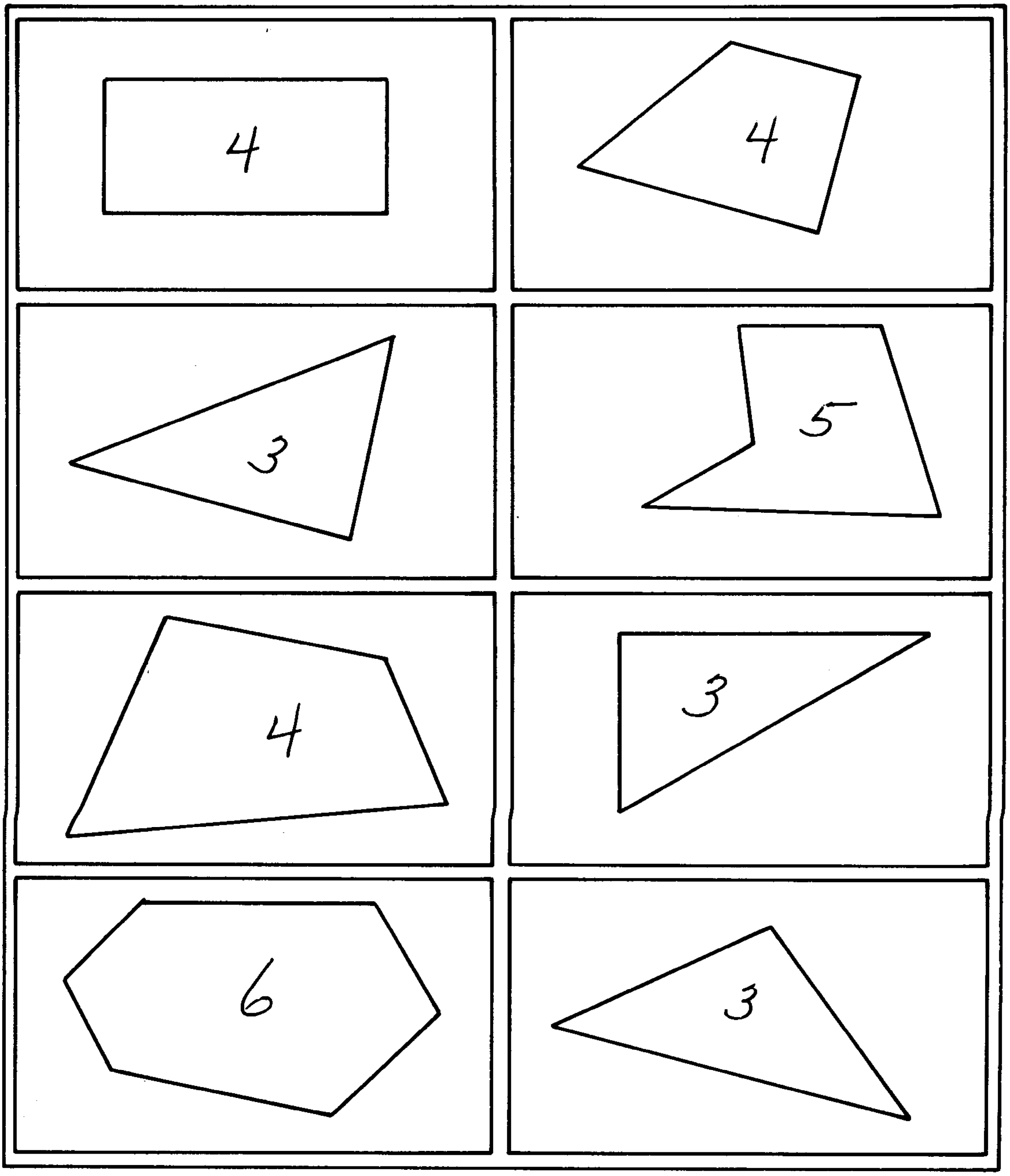
Supply sticks of various lengths for the children to form into triangles. Make sure that the two shortest sticks have a combined length greater than the longest; then, no matter which three the child picks out, he will always be able to construct a triangle.

<u>Pupil's book, page 57:</u> <u>Number of Sides</u>

Read the instructions to the children.

Number of Sides

Write the number of sides inside each figure.



Distinguishing rectangles and squares from other quadrilaterals

Disclose the figures on the right side of the chalkboard. Ask the class how the set of figures (or pictures) drawn here differs from the set discussed earlier in this section. (Each of these has exactly four sides.) Tell the class you are all going to look for some special figures in the set. Ask whether some child sees a figure that is special in any way. Point to the rectangular picture frame and the rectangular window frame as examples of the special shapes we are looking for. If necessary, ask explicitly about the corners. Try to lead the children to the idea that in a rectangle, all four corners "look alike". Introduce the words "rectangle" and "square". Some children may object to calling the square a rectangle; point out that it is a special kind of rectangle, just as a lollipop is a special kind of candy. You may even refer to a square from the beginning as a "square rectangle".

Have the children make rectangles by bordering a sheet of paper with a crayon.

Display several boxes and point out how their edges form rectangles or squares. Have the children look for rectangles in the room as boundaries of desks, the chalkboard, and so on.

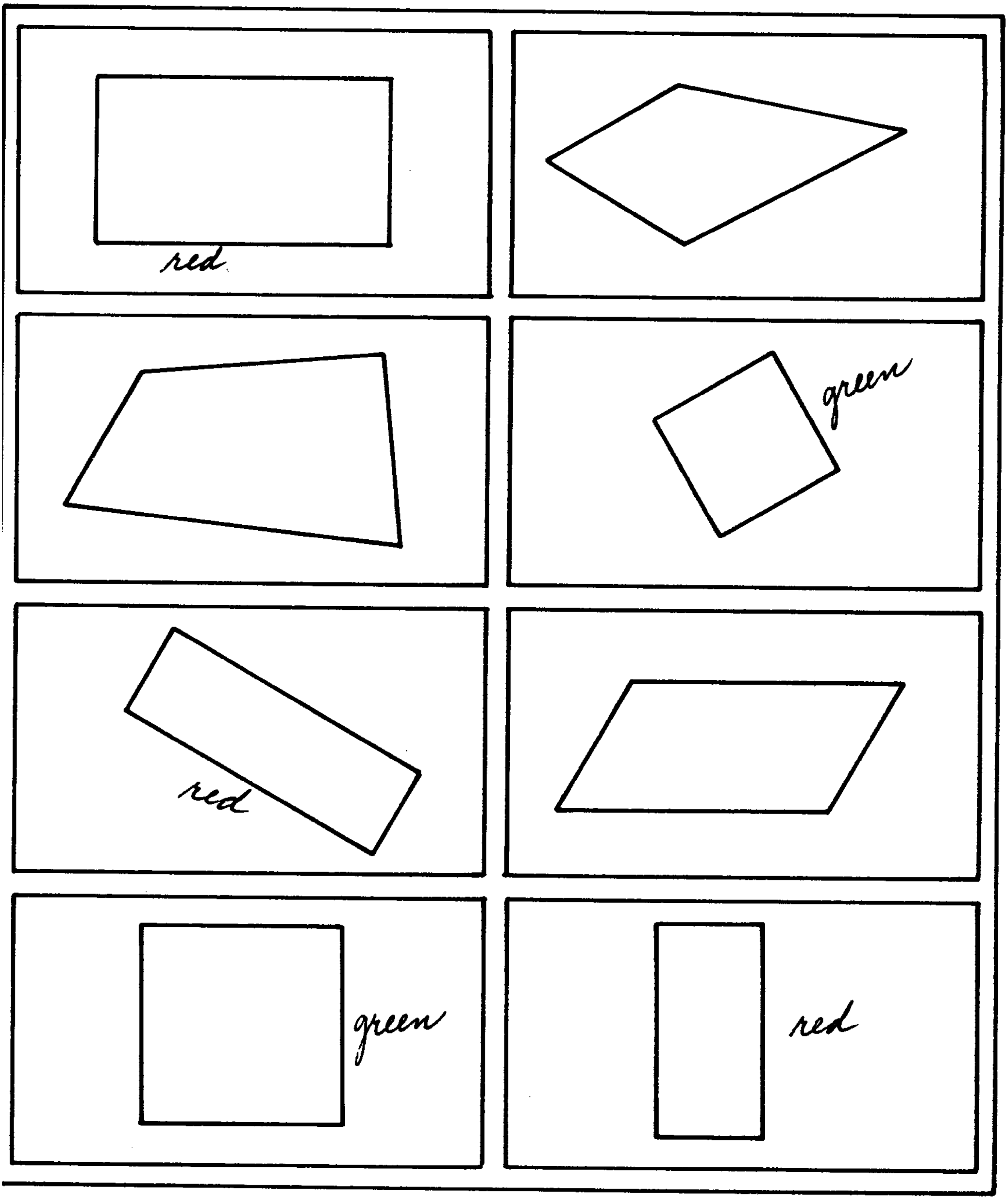
Pupil's book, page 58: Rectangles and squares

Read the instructions to the children. The child is to make a mark somewhere on the figure.

Rectangles and Squares

Mark each square green.

Mark each other rectangle red.



V-4. <u>Classifying regions</u>

<u>Objective</u>: To recognize that a circular region, rectangular region, etc., consists of the curve itself plus its interior.

To identify circular, rectangular, triangular, and square regions.

<u>Vocabulary</u>: Circular region, rectangular region, triangular region, square region, inside, outside, on.

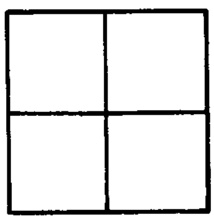
<u>Materials</u>: Wire models of circles, rectangles, squares, triangles; flannel regions of the same shapes.

<u>Suggested Procedure</u>:

In preparation for study of regions, review the ideas of <u>inside</u>, <u>outside</u>, and <u>on</u>. In the SMSG Kindergarten book there are many activities that call attention to these ideas.

The use of playground circle games can reinforce the idea of a circle through their references to the above terms. Such games include: "Froggie in the Middle", "The Farmer in the Dell", "Bow Belinda", "In and Out the Window", "Looby Loo", "The Old Brass Wagon", and "Hokey, Pokey". Step on the circle to show where the curve is.

The playground outlines for "Four Square" can be used to find several squares.



The outlines of the volleyball or basketball court are examples of rectangles, though these may tend to be too large for delineation at this time.

- On the flannel board place an assortment of regions of the types above. Compare these with models of circles, rectangles, triangles, and squares. Ask how a circular figure is like a circle and how it is different. (Alike in shape; the edge of the felt figure is like the wire circle; the inside of the felt figure is "full"; and so on.)

Tell the children that any object like the felt cutout has a longer name. It is called a circular region. Its edge is a circle.

Continue with the other figures. Refer to their straight edges as sides. Use the terms triangular region, rectangular region, and square region.

Place the wire models on a table in separate classifications. Ask a child to go to the flannel board, remove a region, compare it with a wire model, name the region, and place it in the proper classification. Continue until all the figures have been removed and classified.

Pupil's book, pages 59 - 63: Regions

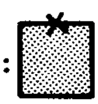
Ideas

A circular region, rectangular region, etc., consists of the curve itself plus its interior.

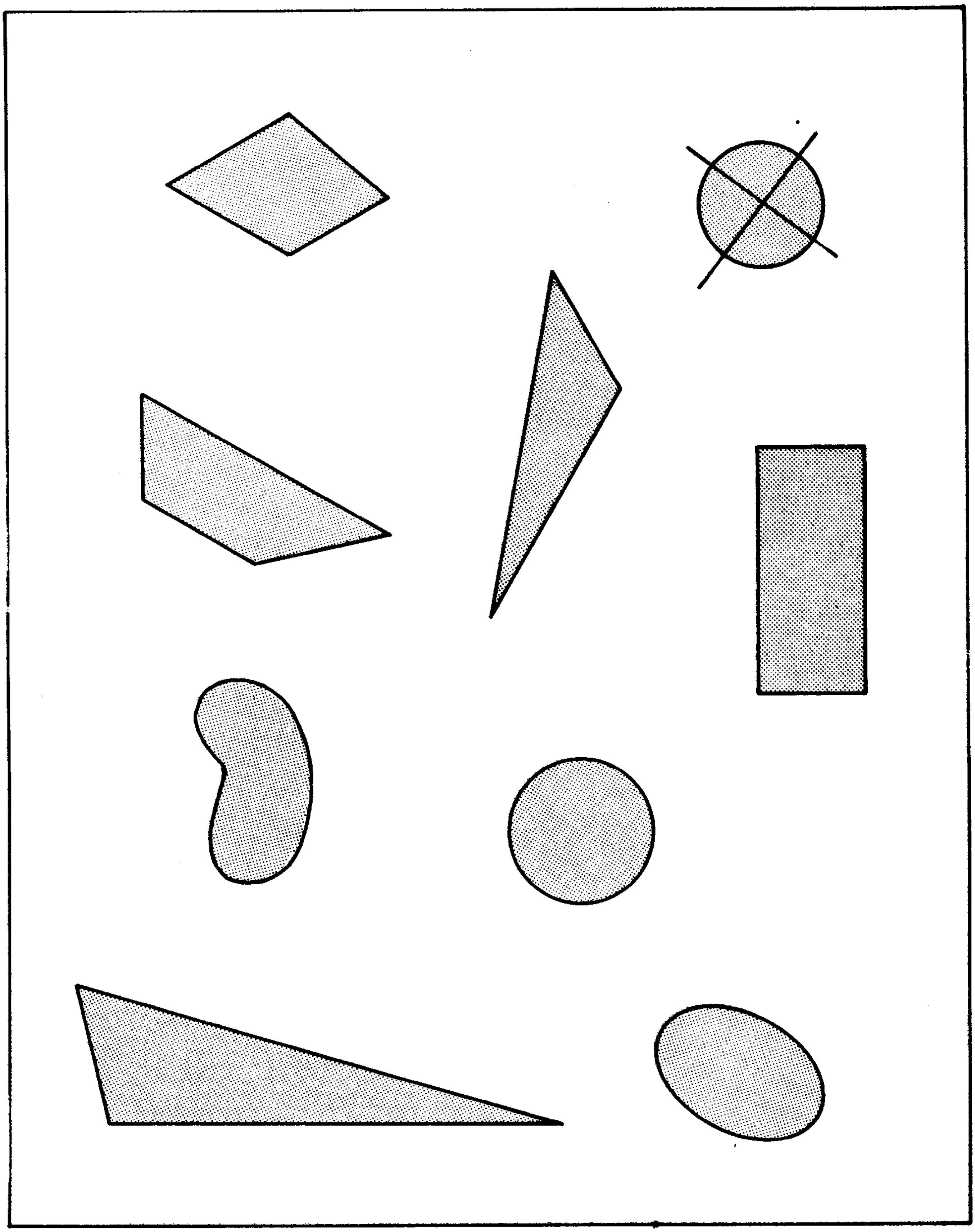
Pages 59 to 62.

Each page includes a different type of region to classify. The instructions should be read and the sample answer noted on each of the first two pages.

Page 63.

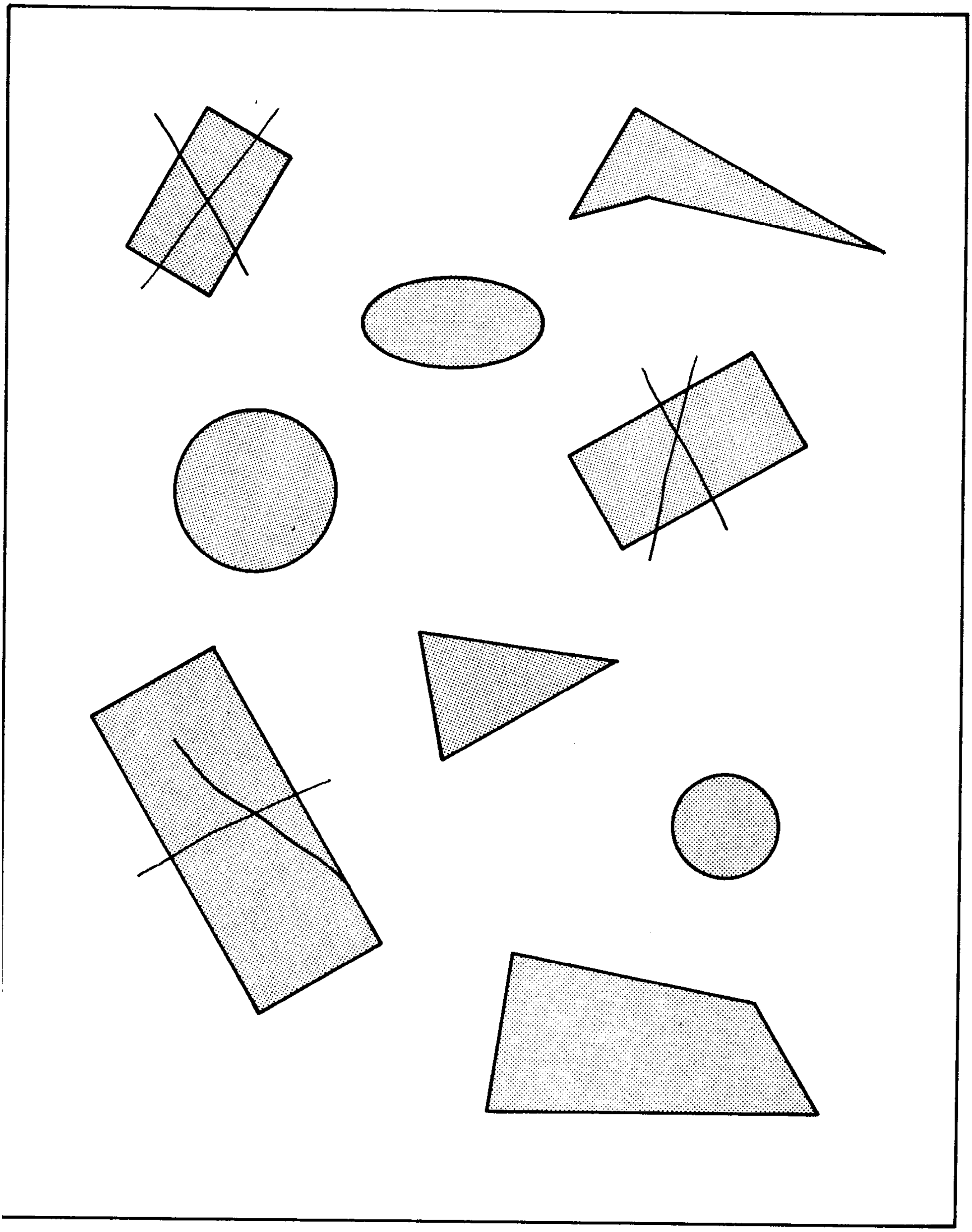
Here the children need to mark the curve itself: 

Regions
Mark each circular region.



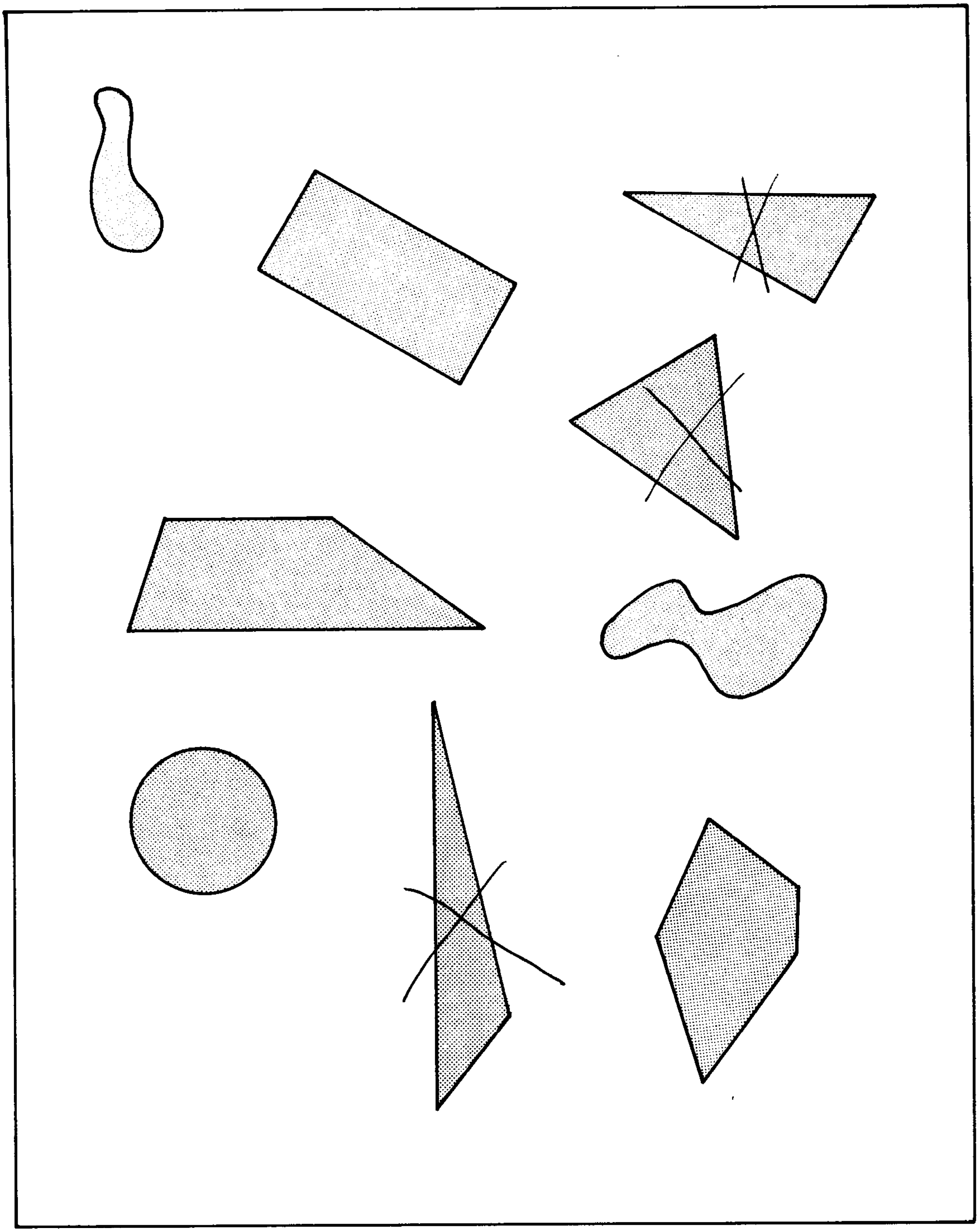
Regions

Mark each rectangular region.



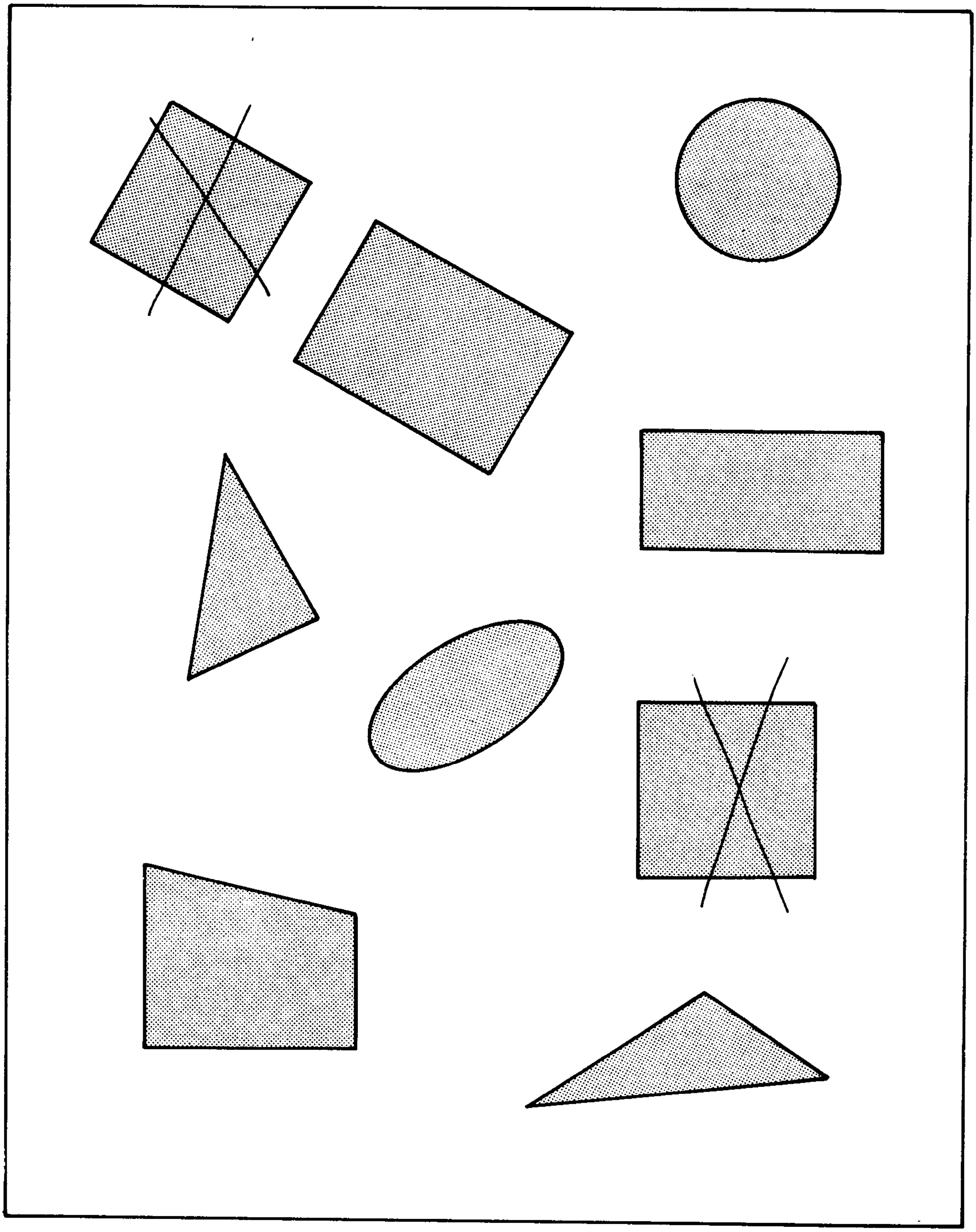
Regions

Mark the triangular regions.



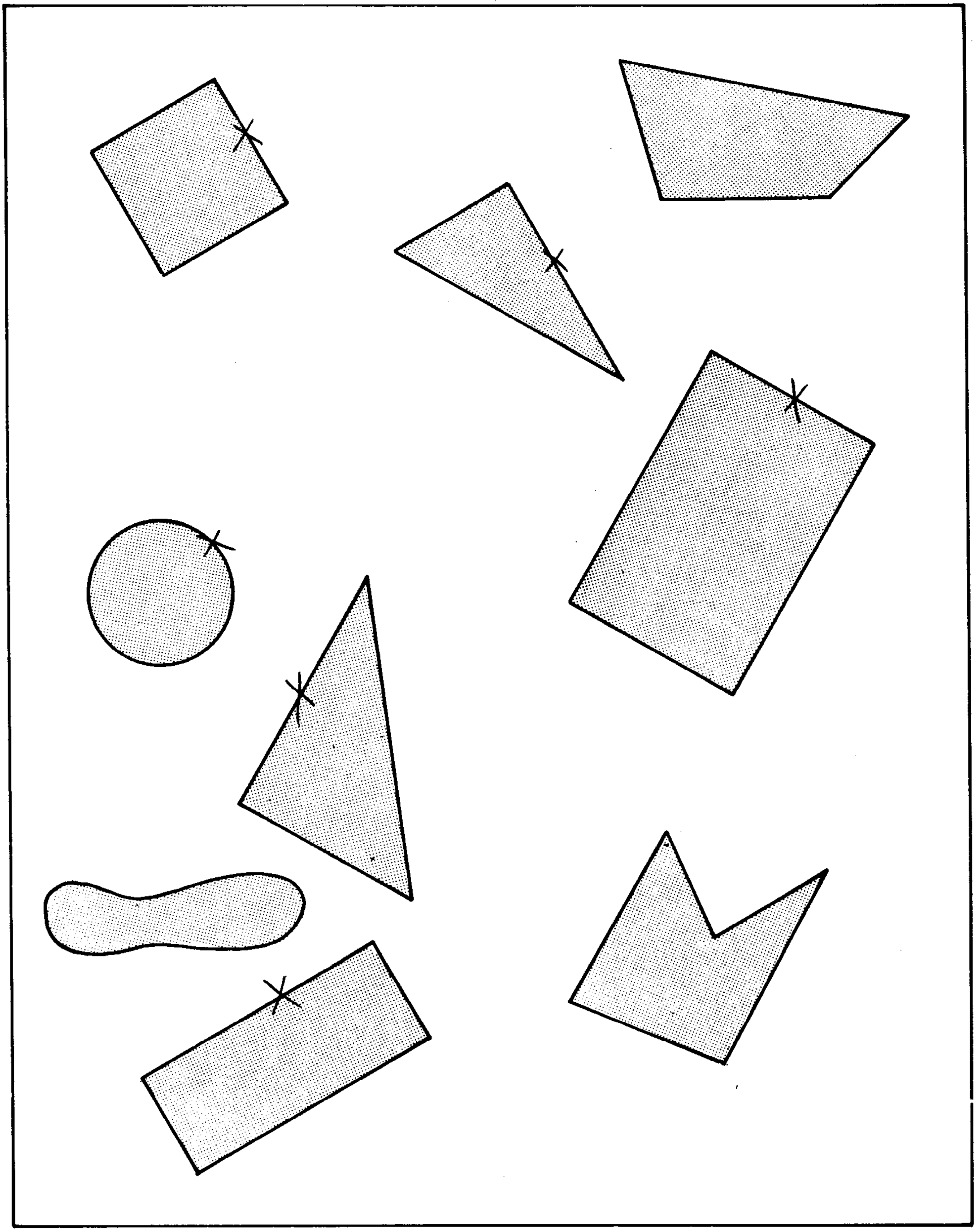
Regions

Mark each square region.



Regions

Mark an X on each of the rectangles, squares, circles, and triangles.



Further Activities:

1. Place parquetry blocks in a bag for a game of identifying figures. If blocks are not available, figures cut from tagboard or cardboard may be used. Children take turns. Each reaches into the bag without looking and identifies the shape of a block by feeling it. He may say, for example, "The block is shaped like a triangle." Then he brings out the block. If the other players agree that he is correct, he places the block in front of him. Otherwise he returns it to the bag. At the end of the game, the child having the most blocks is the winner. It is necessary, of course, to establish the rule that each child must have the same number of turns. Children can make tally marks to keep track of their turns.

2. Start Our Big Book of Shapes with a page for each of the figures--rectangular region, triangular region, and circular region. Paste a model cut from construction paper at the top of each page. Children may cut pictures from magazines and paste them on appropriate pages. Do not hastily reject a child's selection as incorrect; inquire. Some aspect or detail that escapes your attention may have been seen by the child.

3. Give children geometric regions cut from colored construction paper. They may assemble the shapes into "pictures" of animals, people, boats, buildings, tree forms, and so on.

4. Provide parquetry blocks and design blocks for children to use in making designs, pictures, etc. Further intuitive understanding among geometric figures can be developed by such experiences. The intent of this chapter is to introduce concepts and vocabulary rather than to have children "master" the content.

V-5. Fitting regions

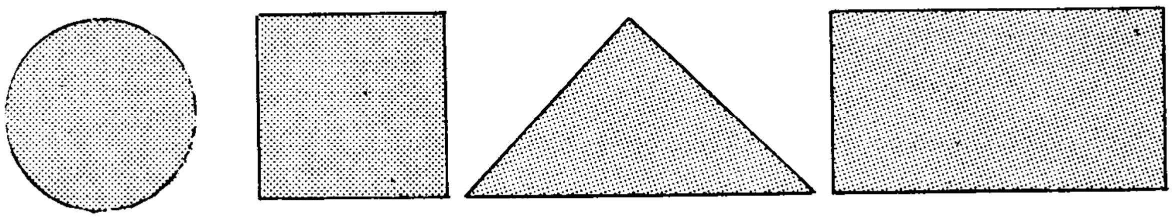
Objective: To distinguish different regions by seeking to fit them on each other.

Vocabulary: Match, fit.

Materials: Flannel board regions of different sizes and shapes; there should be two sets of congruent figures of contrasting colors (red and green, for instance); also, one square clearly larger than the congruent figures; a few sets of construction paper regions in two colors, as above.

Suggested Procedure:

Place on the flannel board some of the red figures as shown:



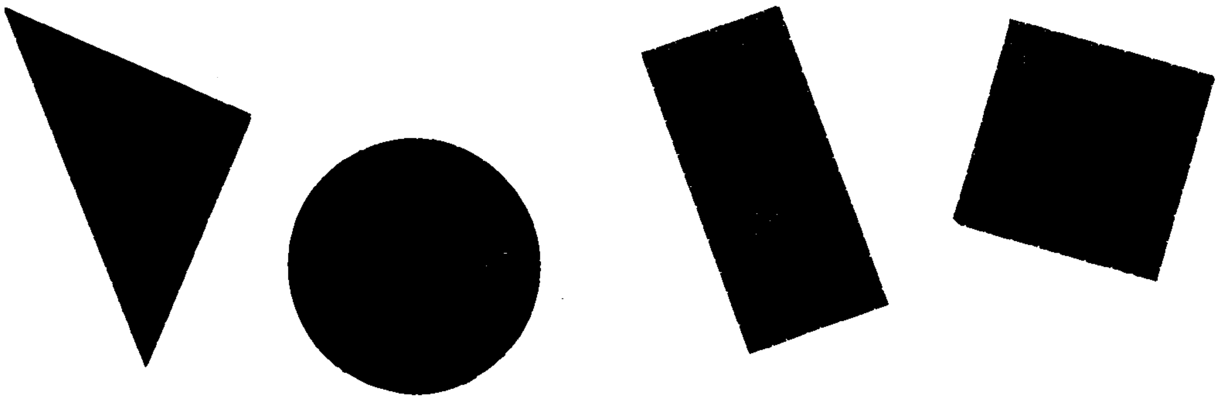
Talk about what it means to fit exactly, or to match exactly. Show how edges of coins of like denomination match exactly; discuss the way the edges of slices of bread often fit exactly in a loaf. Pages in a book match, and one end of an unsharpened pencil may fit exactly against the end of another, with nothing left over of either pencil.

Hold up a green rectangular region which will fit exactly one of the red ones on the flannel board. Hold it with its sides parallel to the sides of the one on the board. Have it described as a rectangular region. Ask whether this region will exactly fit any of those on the flannel board. Have a child do the matching, and show that all sides match, or fit.

Remove the two figures, place the green one on the flannel board, and ask whether the red one can be matched to it. Remove the green figure and ask whether it would fit on any of the other regions on the flannel board. Have a child try to fit it, and show clearly that there are some parts not covered up either on the red or on the green figure.

Match the other green regions to the appropriate red regions in the same way, having them described each time as a ____ region.

Without letting the class see what you are doing, arrange the green regions on the flannel board in different positions.



Hold up the red triangular region in the same position it was in when it was first matched. Ask with what kind of region it might be matched. Turn the flannel board so that the children can see the different shapes; then ask the children whether the red region can be matched to one of those on the board. Caution the children to be careful, for even this simple arrangement can cause difficulty for children who expect to see regions in positions with one side parallel to the floor. Continue fitting the other figures.

Hold up the square region of larger size and ask whether it could be matched to any of those on the board. Discuss the fact that a region must not only be the same shape but also the same size in order to fit exactly.

Pupil's book, pages 64 - 66: Regions that fit

Ideas

Regions of the same size and shape can be fitted one on the other.

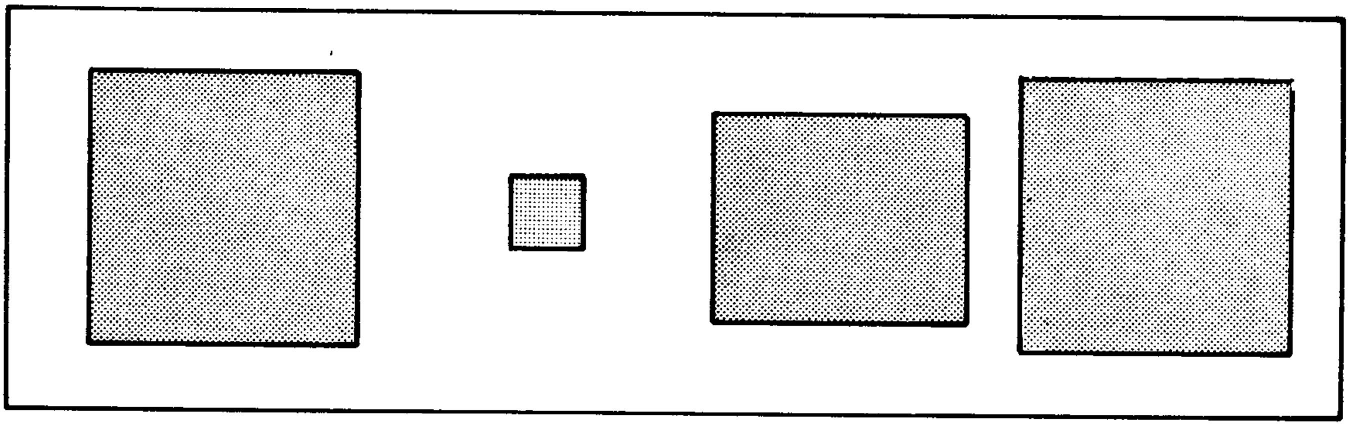
A region can be rotated for possible fitting.

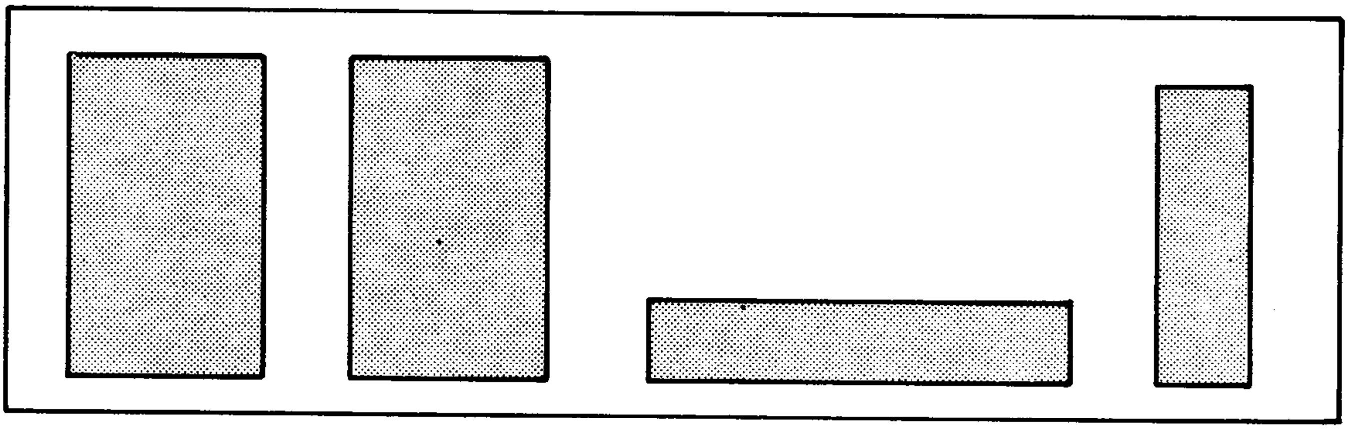
Pages 64 - 66.

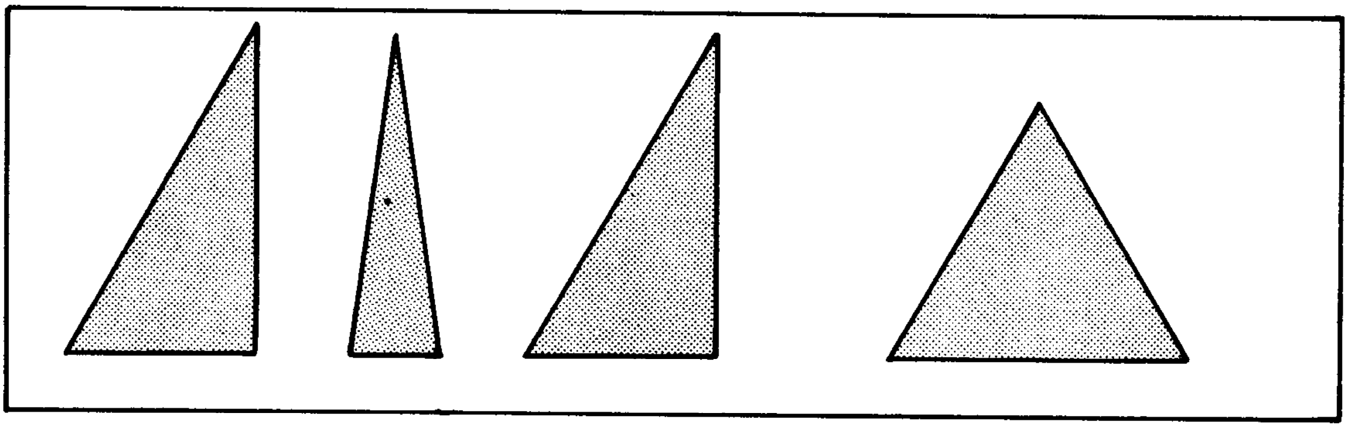
The first page shows regions in the so-called horizontal position. It represents no problem for identification. The second and third pages will need more careful scrutinizing since rotating will be necessary in most instances to get the figures to fit.

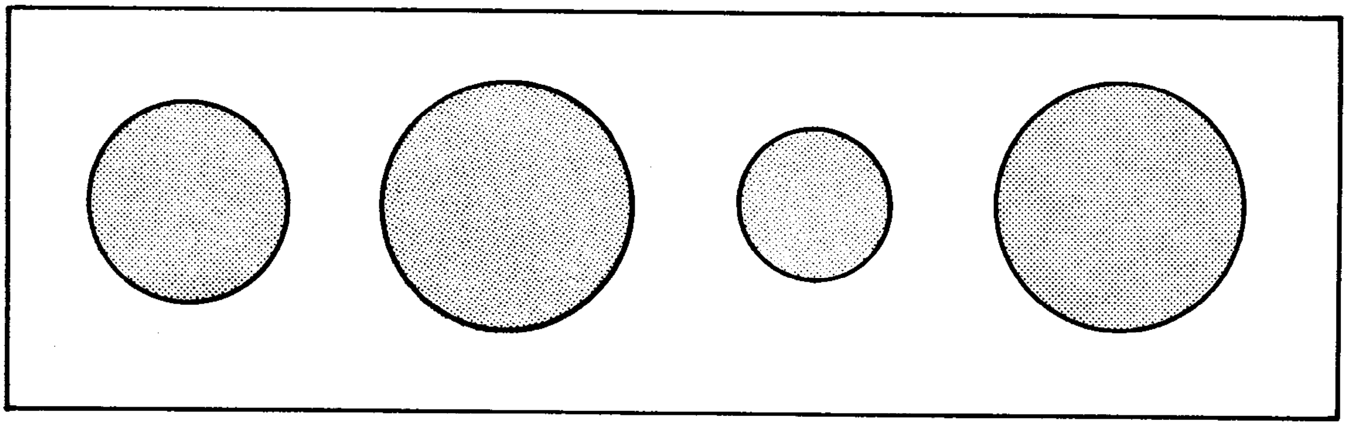
Regions that Fit

Mark the regions that fit.



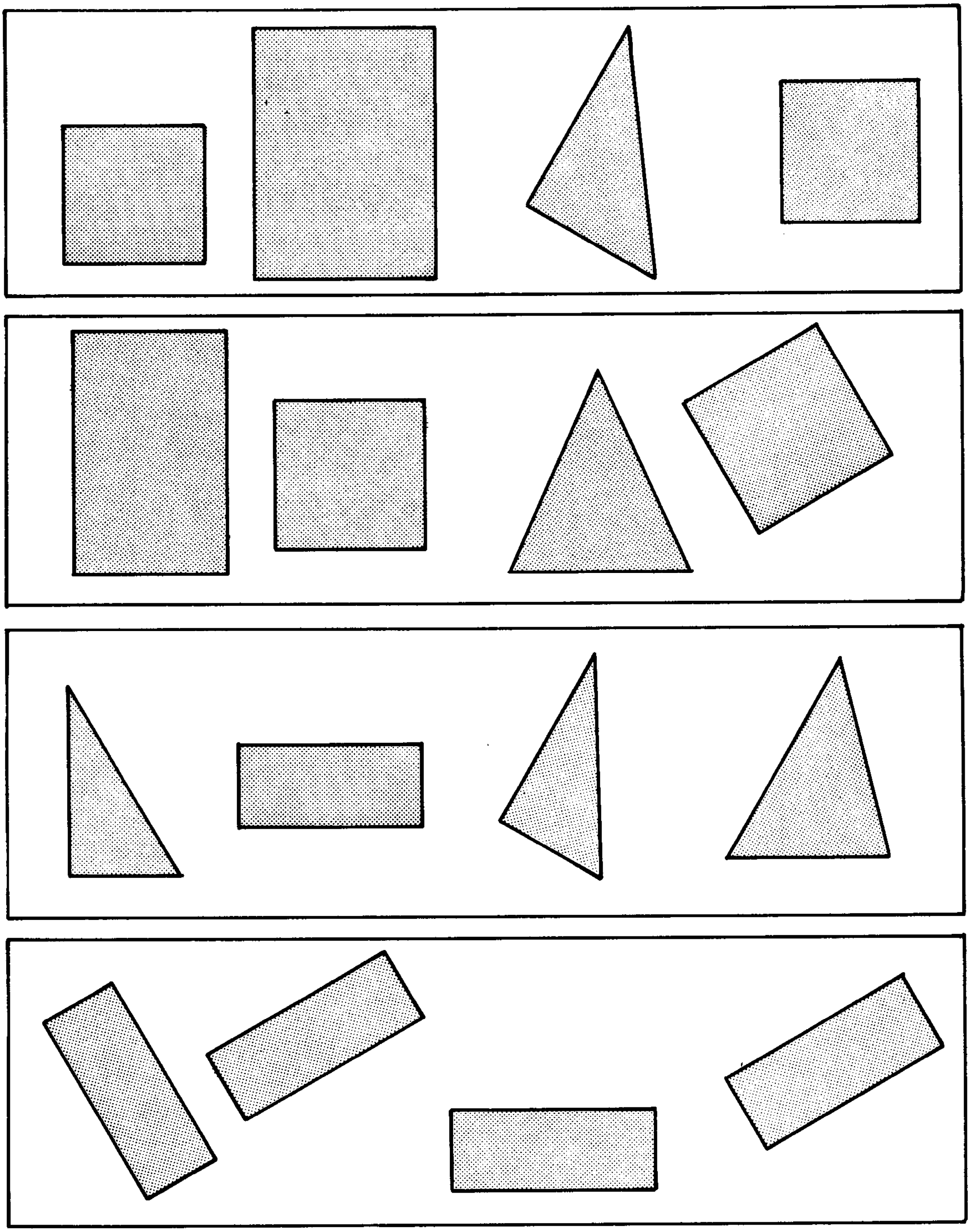






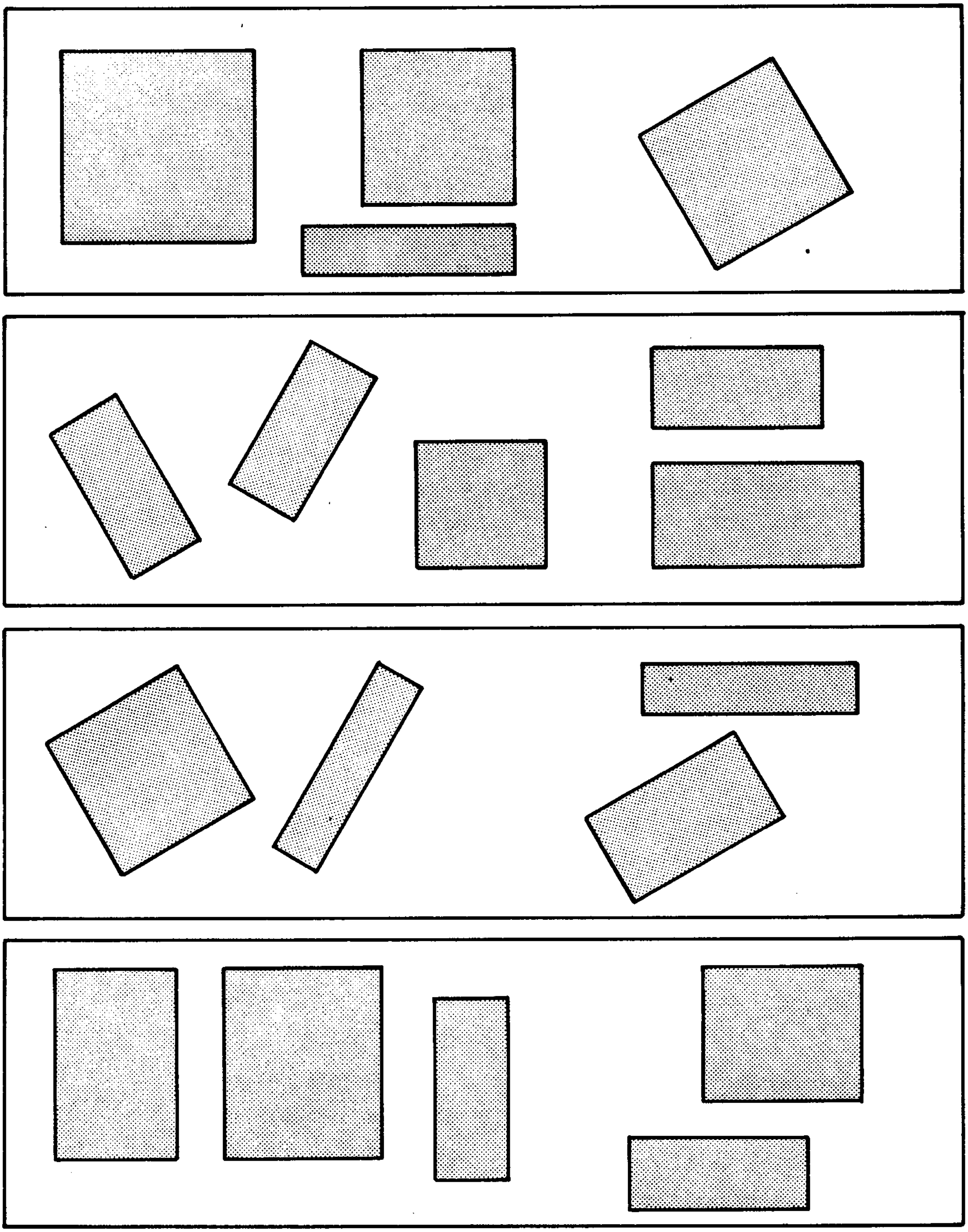
Regions that Fit

Mark the regions that fit.



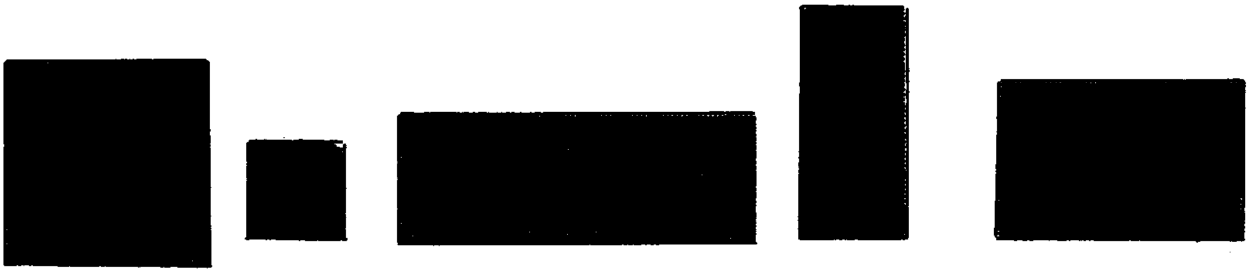
Regions that Fit

Mark the regions that fit.

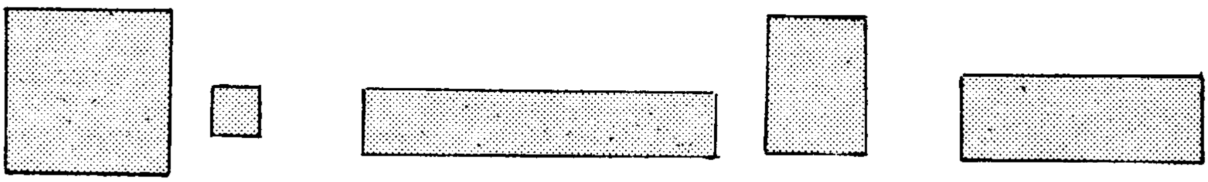


- An additional series of lessons can be developed to refine comparisons by fitting. A long thin rectangular region can be included with one that is nearly square.

  All the red regions might be rectangular regions (including some square ones) such as two different sized square regions and three or more rectangular regions of differing dimensions and proportions.



The green regions should include all of these shapes as well as other rectangular (and square) regions of different proportions, including some like the following for which one pair of sides fits somewhere above but the other does not.



This time the pupil will need to recognize that, for the fitting, the lengths of opposite sides must be the same. Choose some of the green regions quite similar to the red ones, but not actually the same; good practice can then be developed in estimating relative lengths. In most cases it would be profitable, before any attempted fitting is made, to discuss whether a given green region will fit and what would be reasonable places to try it.

Another time some of the red regions on the board should be turned in different positions. It is important to plan specifically for such a lesson.

Geometric insights of these additional lessons would include:

1. An awareness of the impossibility of matching

a long, thin rectangle with one which is nearly a square;

2. An awareness of the possibility of comparing visually the sides of a region to see whether they are likely to fit;

3. An awareness of the possibility of rotating a region to make it fit another;

4. The recognition of a rectangular or triangular region which does not have a side parallel to the floor.

Chapter VI

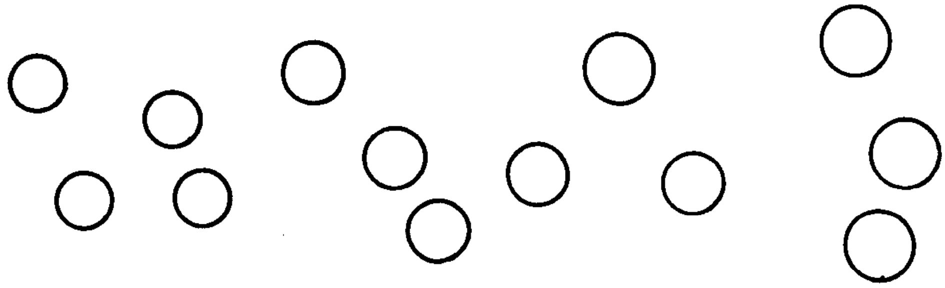
PLACE VALUE AND NUMERATION

Background

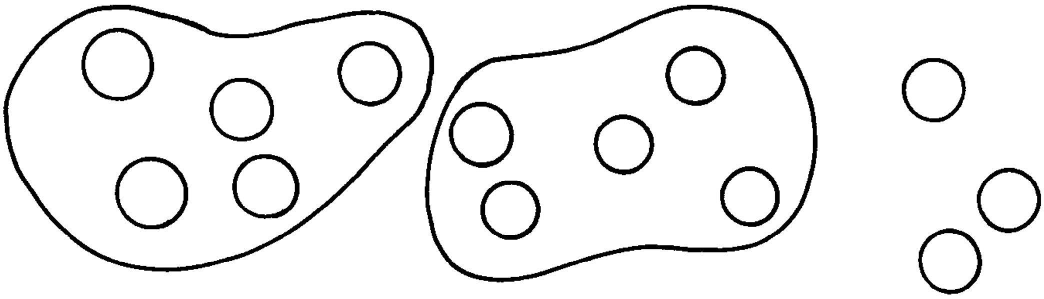
The fundamental purpose of this chapter is to learn assigned names of numbers greater than nine. We have named the first few numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and ten, but the procedure of assigning a new name to each successive number is clearly impractical. Some sort of system of naming numbers is necessary. This chapter is devoted to the Hindu-Arabic system of numeration, our decimal system of numeration. It is interesting to notice that this is a relatively modern system--quite unknown to the Greeks and Romans. Indeed, mathematicians have conjectured that the rather feeble accomplishment of the Greeks in algebra was due to their lack of a reasonable notational system. The system which we now use is only about a thousand years old; it was carried to Europe, along with spices and sandalwood, by Arab traders.

The simplest numeration systems are very closely related to tallying. For instance, the Romans used I, II, III, and IIII for the first four numbers. Of course, this sort of notation is completely impractical for large sets, and people soon found ways of simplifying the naming system. The first step was to count by groups of some agreed-upon size, so that, for example, we might refer to seven dozen eggs, or a gross of pencils.

Let us state in mathematical terminology just what this sort of "grouping" amounts to. Suppose we are trying to describe a set which has a great many members. We select a subset of some standard number of members (like a dozen, or a gross) and <u>partition</u> (split up) the set into as many equivalent subsets as possible. There may or may not be a <u>remainder</u> (that is, members left over). Thus if 5 is the standard number, we may partition the set



into the sets



and describe the original set as consisting of 2 fives and 3 ones. The number of members of the standard subset is more or less arbitrary. Thus we describe the number of members of the set pictured above in any of the following ways:

Fives	Ones
2	3

Threes	Ones
4	1

Tens	Ones
1	3

We customarily group by tens--presumably because we have ten fingers. Computing machines customarily group by twos, and the barefoot Mayans grouped by twenties.

This system of counting by groups has been used by most civilizations. But as greater and greater numbers of objects were considered, new names for greater and greater standard numbers became necessary. Thus the Romans used I, V, L, C, D, and M for one, five, fifty, one hundred, five hundred, and one thousand. At each stage, as names for greater numbers were needed, a new symbol was needed. But the Hindu system circumvents this difficulty by assigning meaning to the <u>place</u> a digit occupies, and manages to create numerals for every number from the ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

This is a truly remarkable achievement.

The idea of grouping, together with place value, is enough to permit us to assign numerals to the first hundred numbers. The step from the pattern:

Tens	Ones
4	7

to the numeral 47

is a simple one, and it should be clear that this number is to be assigned to a set which consists of 4 tens and 7 ones. The number 10 is described in precisely the same way: this is the number which is to be assigned to a set of 1 ten and 0 ones. We say that the right hand digit is in the ones' place, and that its neighbor on the left is in the <u>tens' place</u>.

There is a further step in our system of numeration. Suppose that a set consists of 23 tens and 4 ones. In counting the 23 tens we would normally group these in tens, so that our record keeping might look like either of the following:

Tens	Ones
23	4

Tens of tens	Tens	Ones
2	3	4

In either case, naming this number is 234 is completely natural. We say that the right hand digit is in the ones' place, its left hand neighbor in the tens' place, and the next left hand digit is in the <u>hundreds'</u> place. We call tens of tens "hundreds", and we call tens of tens of tens "thousands". But the practice of naming these greater numbers eventually becomes impractical and we fall back on the numerals. Thus

234,468,789,345,863,456,998,567,452,345,765,989

names a certain number in a perfectly well-defined way, but it is doubtful if many of us remember the ordinary names beyond quadrillion.

In order to be sure that you understand our numeration system, you might want to imagine the following whimsical situation:

On Mars there are several forms of life, and the dominant form is called the dozer because of its habit of taking cat naps. The dozer has neither hands nor feet, but it manages manipulations very nicely using its twelve tentacles. The dozers are mathematically accomplished (all little dozers doze through calculus in the first grade) and they have invented a system of numeration which is quite similar to ours. But of course they count by dozens, for anatomical reasons. They use the numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 just as we do, but they use N for ten and L for eleven. It is curious that they have not found it necessary to invent a digit for twelve (or is it?). Naturally the counting is done by dozens and every little dozer understands that N7 is ten dozens and 7 ones, whereas 7N is 7 dozens and ten ones. They also count by dozens of dozens (which they call gross), so that L5N means eleven dozens of dozens, 5 dozens, and N ones. Now the problems:

(a) How do the dozers write twelve? (b) What would we call the number which they label 100? (c) 7 + L = ? (Of course the answer must be written so that dozers can read it.) (d) A little dozer undulated down to the store with 2N shekels to buy L shekels worth of licorice. How much change did he bring his parents?

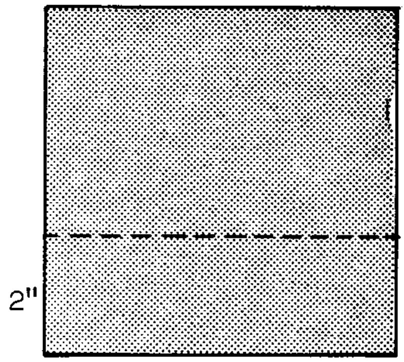
Note:

The sequence of topics in this chapter requires a little explanation. We begin by partitioning a set into as many sets of ten as possible. We then record the number of sets of ten (number of tens) and the number in the remaining set (the number of ones). We then begin to name these numbers. The twenties, thirties, and so on, are discussed first because the pattern of naming is simple and is like the pattern of the numerals. The names for the numbers between ten and twenty are delayed because the naming pattern is so much more complex. Eleven and twelve have very special names, but the names of the "teens" reverse the usual pattern as the word, "thirteen", gives the number of ones first, then the number of tens. *On the other hand, "twenty-seven" states the number of tens* and then the number of ones.

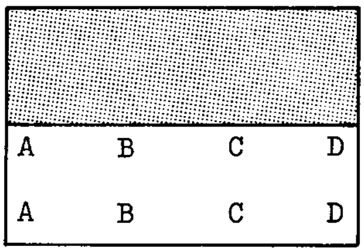
Again, note that the numeral "10" was delayed until we could assign it the natural meaning: one ten and zero ones.

HOW TO MAKE "SHOW-ME" CARDS

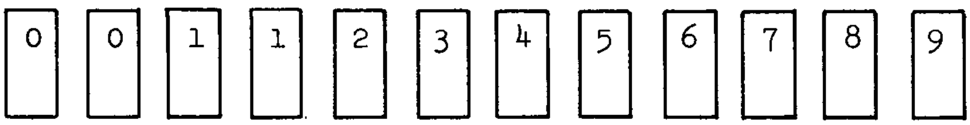
1. Use a piece of tagboard 6" × 6". Fold up 2" from the bottom.



2. Staple at A, B, C, and D to make 3 pockets, each almost 2" wide.



3. Cut a strip of tagboard 18" × 4" into 12 strips $1\frac{1}{2}$" × 4". With felt pen, write numerals as follows:



4. Children should be taught early to lay out numeral cards in order on their desks and to replace them <u>in order</u>.

5. In the game, the children figure out the solution to a problem you give orally or on the chalkboard. They then place the numerals for the answer in the pockets, hold the cards against their chests with the answers concealed until you say, "Show-me!" Then all turn answers toward you, while you make a quick survey to see who is right.

VI-1. Counting by tens and ones

Objective: To help children learn to count sets with many members by counting sets of ten.

Vocabulary: (No new terms.)

Materials: Flannel board squares and strips of ten, similar flannel board material, other types of counting material.

Background Note:

A set of objects may be partitioned into subsets of ten members each and a set of not more than 9 objects. (We do not use the term "partition" with the children.) In this lesson the children learn to do this partitioning into subsets of ten and to name the number of members in the set; e. g., 3 tens and 7 ones, or (orally) thirty and seven.

Teaching Note:

The lack of pages in the pupil's book for use with this lesson is not an oversight. Teachers have found that actual manipulation of sets of objects is much more effective than working with pictures of sets. Such pictures necessarily either group the members of the set artifically, or else present an impossibly cluttered appearance.

Suggested Procedure:

Place the material to be counted in a box. Ask a child to remove ten from the set and place the objects in a row on the flannel board. Have children count to determine how many. Be sure they understand that the name that tells how many is "ten" or "ten ones". Have a child place on the flannel board a set which matches the set already there. This can be done without counting. Have children note that there are now two tens. Do the same for a third set of ten.

Show the remaining 4 objects.

Do we have enough to make another row of ten? (No.)

How many sets of ten do we have? (3.)

What is another name for three sets of ten? (Thirty.)

What number tells how many objects are not in sets of ten? (4.)

These are the ones.

How many ones are there? (4.)

How many objects were in the box? (Many answers should be given, such as 3 tens and 4 ones, thirty plus four, thirty-four.)

Repeat the experience with a set in which the number of members is 40.

Now how many sets of ten do we have? (4.)

We have separated all our material into sets of tens.

Do we have a set of ones? (No.) (There are no members in the set of ones.)

What is the number that we use to tell that a set has no members? (0.)

How many sets of ten do we have, and how many ones? (4 tens and 0 ones.)

Use other types of material to develop understanding of counting by tens and ones.

● Let children use sets of small objects at their desks to count sets of tens and ones. Keep a record of their results on the chalkbaord.

	Tens	Ones	oral
Dick	2	5	twenty-five
Harry	6	1	sixty-one
Tom	2	8	twenty-eight

As review, read each item on the chart in both ways; e.g., 2 tens 5 ones, twenty-five. Discuss which child had the most objects. (Harry , with 6 tens 1 one.)

- Put sets of small objects for counting into boxes or envelopes. Put a letter of the alphabet on each box or envelope. Give each child a paper which is marked:

A	_____	tens	and	_____	ones
B	_____	tens	and	_____	ones
C	_____	tens	and	_____	ones
D	_____	tens	and	_____	ones
E	_____	tens	and	_____	ones

Children may work as teams or alone to count contents of envelope and record the number of sets of tens and ones. After a child has completed one envelope, he replaces its contents and exchanges it for another envelope.

One child of a team may serve as the recorder or each may want to keep his own chart. A class chart can be used in order to verify the independent charts. Children can help in setting up materials of this kind. The number of objects in the envelopes can be changed and the activity repeated.

VI-2. Spoken names of the numbers: 21 through 99

Objective: To help children understand how to count sets of more than twenty members using the spoken names for whole numbers.

Vocabulary: The spoken names of numbers from 21 through 99.

Materials: Different kinds of objects in groups of 20 or more.

Background Note:

This lesson makes the transition from "tens and ones" to the spoken names of the numbers. We avoid the names for the numbers between ten and twenty at this stage.

Teaching Note:

Work sheets for pupils are not recommended here.

Suggested Procedure:

Place 2 tens and 5 ones on the flannel board.

> Who can tell how many squares are on the flannel board?
> (Two tens and 5 ones, or possibly twenty and five.)

Cover the 5 ones with tagboard.

> How can we name the 2 tens in a shorter way?
> (Ten, twenty.)
>
> Then let's go on from twenty: twenty-one, twenty-two, twenty-three, twenty-four, twenty-five.

Repeat with different sets.

- Give each child a set of small objects. Ask the children to separate the set of objects into subsets of ten, to make as many subsets with ten members as possible, and then to be ready to report to the class how many tens and how many ones there are in the set partitioned. Ask several children to tell how many things they counted by reporting tens and ones. Record the number of tens and ones on a chart or the chalkboard. When they have told how many tens and ones, point to the chart and read, 6 tens and 5 ones, sixty and five, sixty-five.

Further Activities:

Distribute sets of small objects for counting. On a chart write:

	Tens	Ones
Martha	2	4
Sarah	4	7
George	5	3

Have each child display a set of small objects on his desk which corresponds to the set listed by his name on the chart. Then ask each child for other names for the number of objects he has shown. Emphasize names such as twenty-four, forty-seven, fifty-three, etc.

VI-3. The written numerals: 20 through 99

Objective: To help children associate the correct written numerals, as well as spoken names, with the numbers 20 to 99.

Vocabulary: (No new words.)

Materials: Different kinds of objects for counting: blocks, sticks, pegs, flannel board materials, etc. Show-Me cards for further activities.

Suggested Procedure:

This activity should follow many experiences with counting and naming sets of ten and single objects. The children should be able, for example, to name a set of 4 tens and 5 ones as forty and five, and as forty-five.

Place three stacks of ten blocks and a stack of two blocks on the chalk tray. Have children name, in several ways, the number of members in the set. (3 tens and 2 ones; thirty and two; thirty-two.)

Begin a tabulation on a chart showing:

Tens	Ones
3	2

Place 5 sets of ten small objects on the flannel board. Have children tell, in two ways, how many there are. Make 2 bundles of ten sticks each and put them with 6 sticks on a table. Ask a child to tell the number of tens and ones. Continue to develop the chart as each of these sets is counted. Show 4 sets of ten and 7 ones. Have children tell where you should write the numeral for the tens and the numeral for the ones for each set of objects.

The completed chart might look like this.

Tens	Ones
3	2
5	0
2	6
4	7

Each of these numbers can be named in several different ways. When we see the numerals on the chart we read 3 tens and 2 ones. We may say thirty and two or thirty plus two. We also say thirty-two. We write: 32.

Continue to rewrite the numerals from the chart.

A completed chart might look like this:

Tens	Ones	Numerals
3	2	32
5	0	50
2	6	26
4	7	47

Pupil's book, page 67:

In each case the child is to show the other way to name the number.

Pupil's book, page 68:

Make a ring around a set of objects whose number is indicated at the right.

Pupil's book, page 69:

Write the numeral which names the number of objects in the set.

Pupil's book, page 70:

Join the points in succession, beginning with 10 and counting by tens.

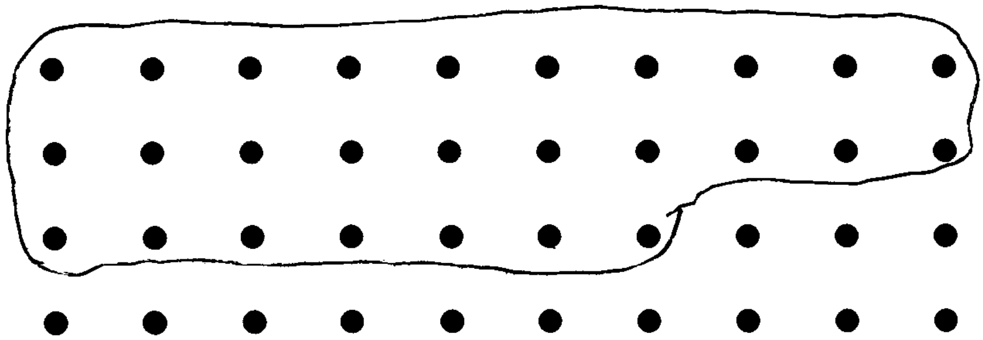
Two Names for a Number

Write another name.

Tens	Ones		
2	3	is	23
4	7	is	47
3	0	is	30
7	2	is	72
5	5	is	55
8	7	is	87
9	6	is	96

How many?

Draw a ring around a set.



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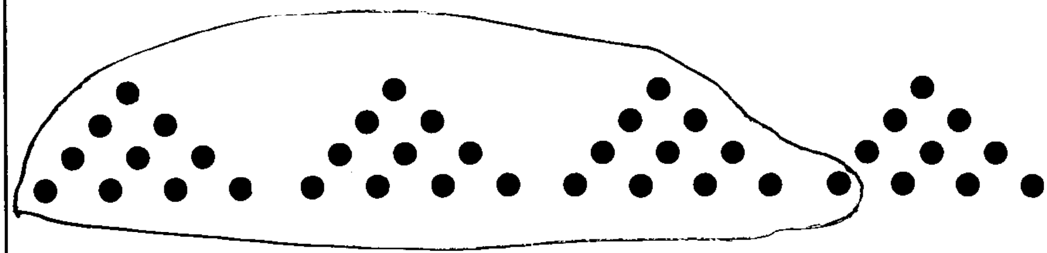

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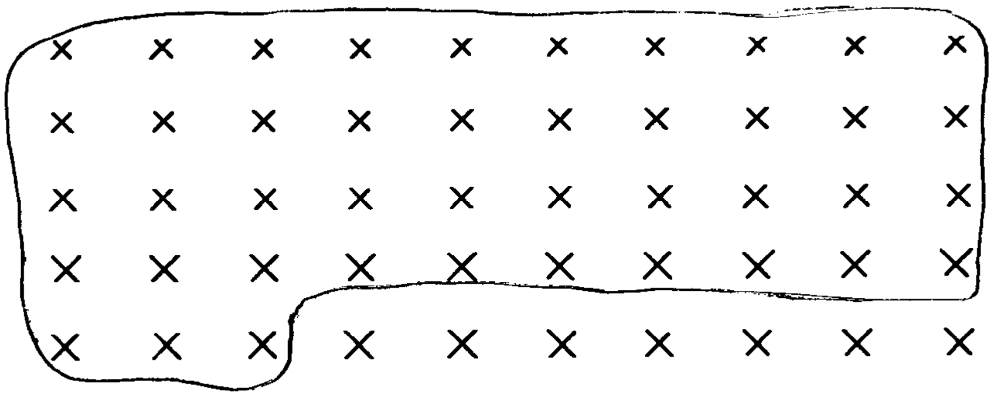

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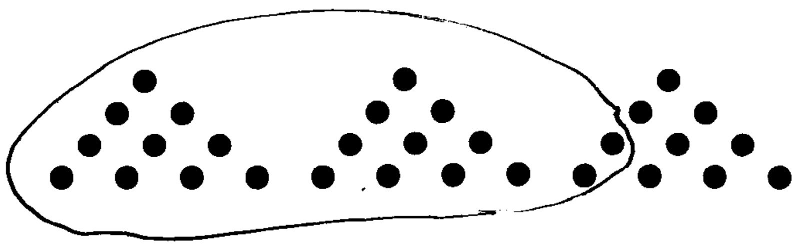

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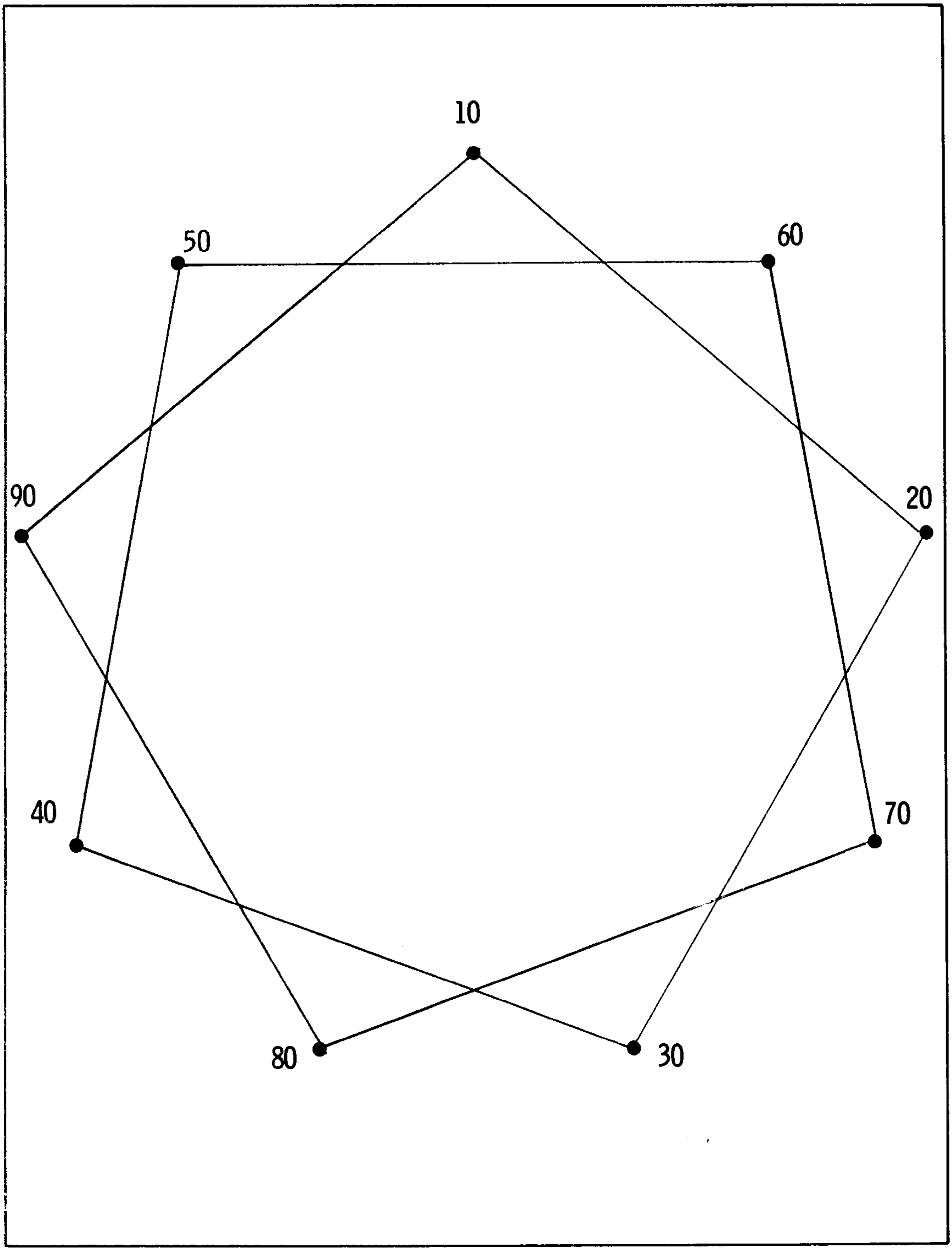
How Many?

XXXXXXXXXX XXXXXXXXXX XXXXXXXXXX XXXXXXXXXX **40**	0000000000 0000000000 0000000000 0000000000 0000000 **47**
FFFFFFFFFF FFFFFFFFFF FFFFFFFFFF FFFFFFFFFF FFFFFFFFFF FF **52**	PPPPPPPPPP PPPPPPPPPP PPPPPPPPPP PPPPPPPPPP PPPPPPPPPP **50**
* **35**	¢¢¢¢¢¢¢¢¢¢ ¢¢¢¢¢¢¢¢ **18**
+ **21**	&&&&&&&&&& &&&&&&&&&& &&&&&&&&&& &&&&&&&&&& &&&&&&&& **48**

What is it?



Further Activities:

Distribute individual set materials. Give each child one numeral card which has on it a numeral from 20-99. The child will display on his desk a set with that number of objects. It should be displayed to show sets of tens and sets of ones. You can then check to see if the child has the correct set. If it is right give him a different numeral card. The child then repeats the activity for another number. Be alert to see that the work is not too difficult for any child. Such an activity can be varied to consider individual differences.

Give each child a set of "Show-Me" cards. Say, "Four tens and six ones." Children should insert numeral cards in the correct places in their cards. Vary the procedure by saying, "Thirty-two," etc. Place sets of objects on the flannel board or hold up bundles of sticks and single sticks and have children show you the numeral for the number.

VI-4. Eleven, twelve, and the teens

Objective: To present spoken names and written numerals for the numbers 11 through 19.

Vocabulary: Eleven, twelve, thirteen, ..., nineteen.

Materials: Flannel board materials, blocks, sticks, theater tickets, etc. (One ten and ten ones of each.)

Suggested Procedure:

Place ten objects on the flannel board and have children name the number two ways, as one ten or ten ones.

Underneath, at the left, place another object.

> How many do we have now? (One ten and one one, or ten and one more.)

Ask children to go back to the beginning and count by ones. If they hesitate, supply the word "eleven".

Using the same procedure, place another object and develop the idea of twelve as 10 and 2 more, or twelve ones. Continue with thirteen. Be sure children know they are saying thir<u>teen</u>, not thir<u>ty</u>. It would be good to write the word <u>thirteen</u> on the chalkboard. (Notice that if the names for sets with one ten and some ones followed the same pattern as the other number names pupils have been studying, we would say something like "onety-one, onety-two, etc." It would not be quite so hard if we said, "Teen-one, teen-two, teen-three, teen-four", but that just isn't the way it's done in the English language!) Use other sets of materials and emphasize the oral names, and the idea of one ten and so many ones. Use the tabulation form, then the written numeral. With some classes, it probably will be wise to stop at "thirteen" the first day. Take the rest of the teens another day (or two!). A thorough understanding of the teens will save much difficulty later on.

Pupil's book, page 71:

Have children write the conventional numeral opposite the tens and ones tabulated, or write the tens and ones opposite numeral.

Pupil's book, pages 72 - 73:

For each set, make a ring around the numeral that names the number of members in the set.

Pupil's book, page 74:

Have children write numerals for numbers that are 1 less than and 1 greater than, or 10 less than and 10 greater than, the numbers that are named.

Further Activities:

Give many opportunities for separating sets fewer than 20 into tens and ones, recording and saying the result as tens and ones and as the teen number.

1. Have children build sets from directions given using written numeral, "Show a ten and six ones", and "Show a set of fifteen", or "Show a set of 12". Be sure to use "one ten and zero ones" for 10.

2. Write numerals on chalkboard and have the children draw a ring around the digit that names the tens or the digit that names the ones.

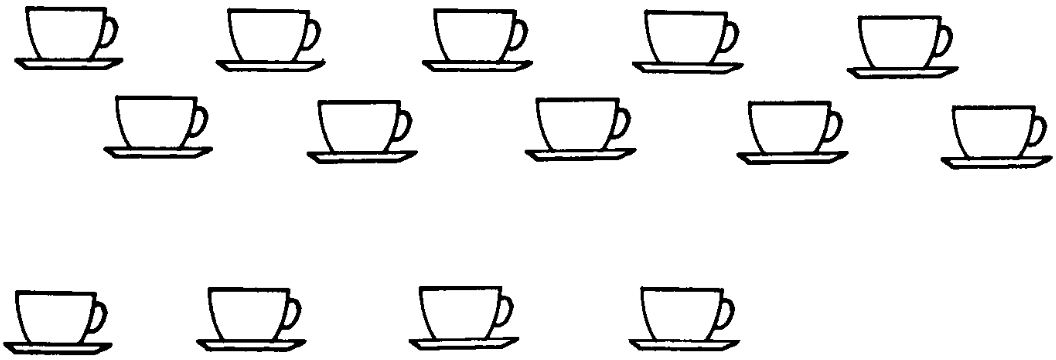
3. With "SHOW-ME" cards have children show the numeral from ten-and-ones instructions and from the spoken word.

4. When you are sure children have oral and written names for teen numbers firmly established, reintroduce numbers greater than twenty, and check to make sure they do not confuse 12 with 21, 15 with 51, and so on.

Two Ways to Name Numbers

Number		Tens	Ones
21	is	2	1
15	is	1	5
12	is	1	2
50	is	5	0
40	is	4	0
14	is	1	4

How Many?



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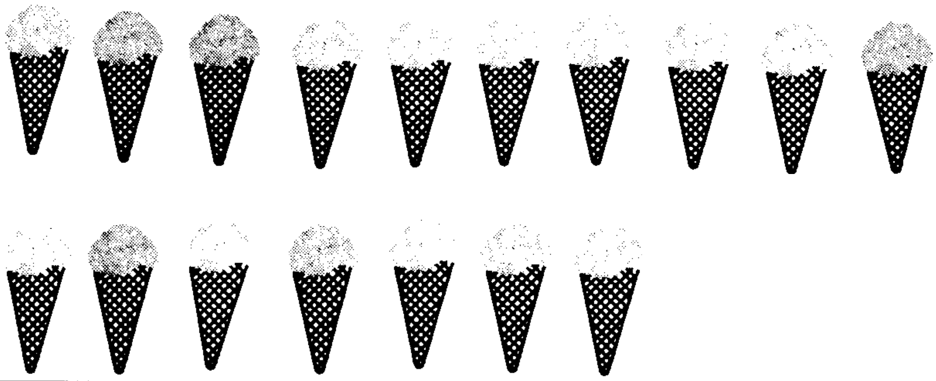
11

17



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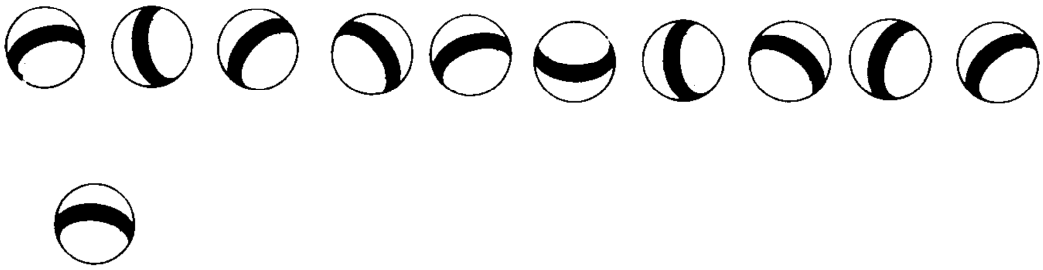

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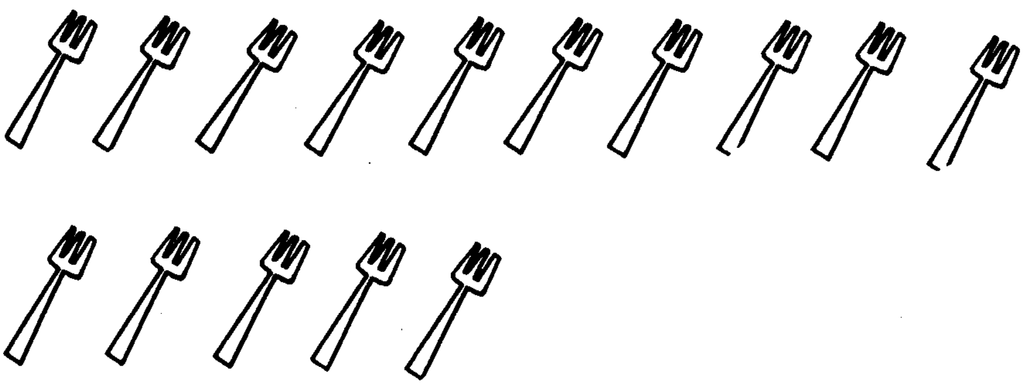
11

16



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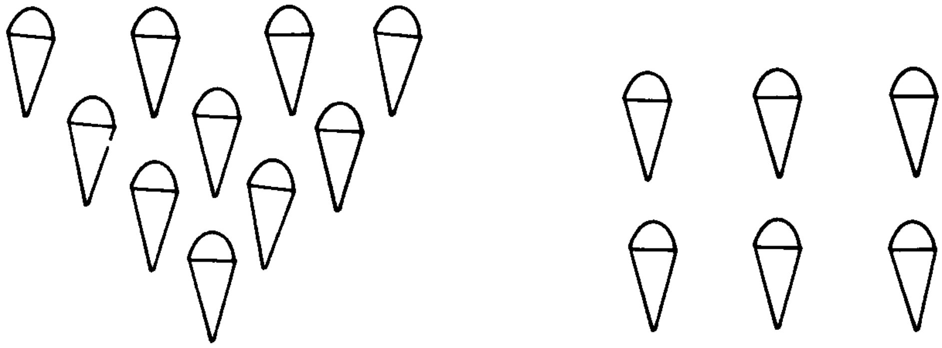
19

15

How Many?



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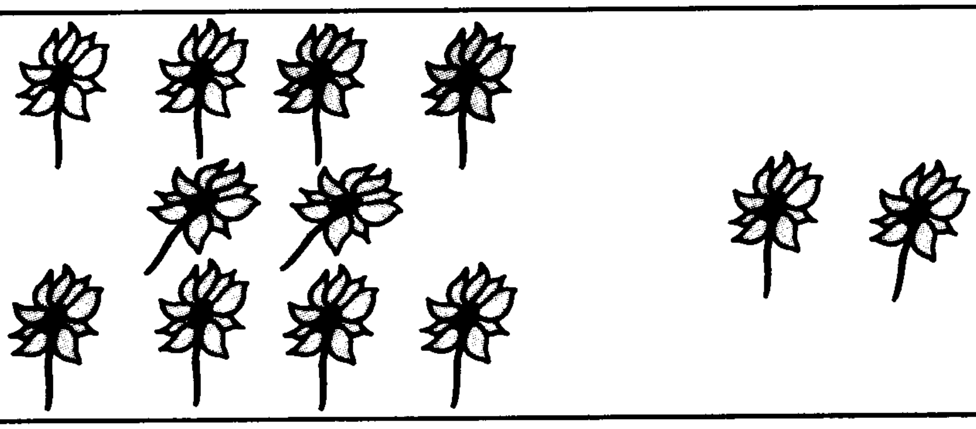

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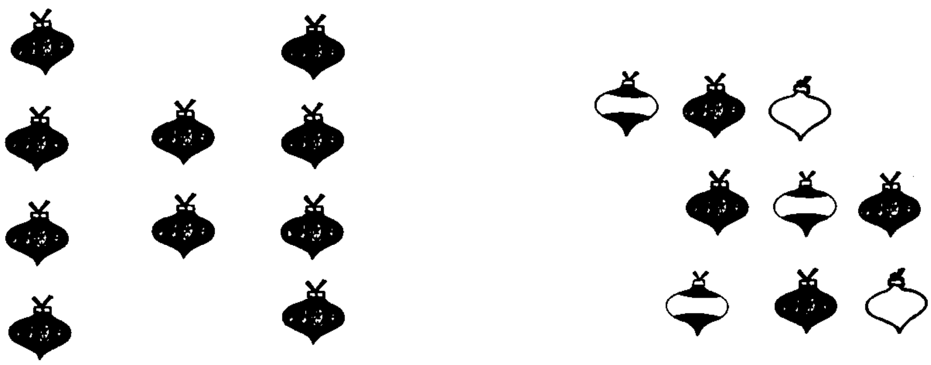
15

17



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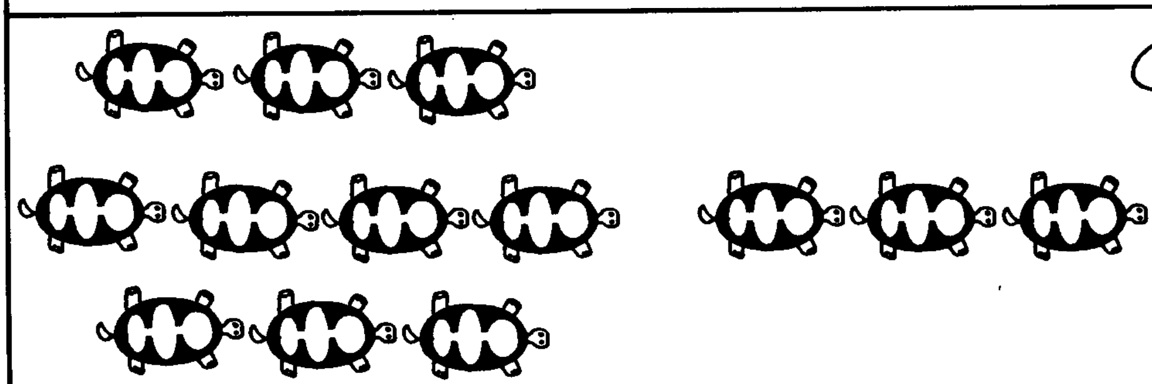
19



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17

10

Name the Number

1 less than		1 greater than
21	22	23
35	36	37
83	84	85
60	61	62

10 less than		10 greater than
12	22	32
26	36	46
74	84	94
51	61	71

VI-5. Order relations for numbers 0 through 99

Objective: To extend the ideas of "greater than" and "less than" to include the numbers 11 through 99.

Vocabulary: (Review) greater than, less than.

Materials: One container of 28 sticks or other small counting materials (theater tickets) that can be bundled into easily recognizable tens, and one container of 31 of the same kind of materials.

Suggested Procedure:

Empty two containers, one containing a set of (say) 28 objects and the other a set of 31 objects on a table or desk. Ask the children which set has more members. Have a child determine this by using a one-to-one correspondence. Explain that this is a way to find out which set has more members. Then ask, "If we know the number of members in each set, we can decide which set has more members without making a one-to-one correspondence." Ask which number is greater, 8 or 5. Continue by asking whether 28 or 25 is greater, 58 or 55, and so on.

Help children to generalize that if there are the same number of tens in two numbers, then we need only compare the number of ones.

Write the numerals 16 and 6 on the chalkboard and ask a child to tell which names the greater number. Ask him how he can tell. Ask whether all numbers in the teens are greater than the numbers which contain 0 tens. Ask whether every number in the fifties is greater than numbers in the thirties, and so on. Continue with specific examples:

> Which is greater 25 or 17?
> Is two tens and five ones greater than one ten and seven ones?
> Which is greater, 37 or 40? And so on.

In each case, restate the problem in terms of tens and ones.

Use the sets of 28 and 31 counting objects again. Have a child separate each of the sets into tens and ones. Bundle the tens so that you can show that: "This ten (of 28) corresponds to that ten (of 31); and this ten corresponds to that ten. But here (in the set of 31) there is one more ten than in this set (of 28).

> If we have numbers between 10 and 99, which digits of the numerals should we look at first to help us decide which is the greater number? (The tens digit.)
> If the numerals have the same tens digit, how do we decide which number is greater? (The ones digit.)

Write numerals on the chalkboard and have children draw a ring around the numeral for the greater number. Have them tell you, if they hesitate, what the parts of the numerals show.

<u>Pupil's</u> <u>book</u>, <u>page</u> <u>75</u>:

> Top--Draw a ring around the numeral for the greater of the two numbers in each set.
> Bottom--Draw a ring around the numeral for the greatest number in each set.

<u>Pupil's</u> <u>book</u>, <u>page</u> <u>76</u>:

> This is like the preceeding page but with "less than" and "least".

<u>Pupil's</u> <u>book</u>, <u>page</u> <u>77</u>:

> Write the numerals in each box in order, beginning with the least.

Which is greater?

36	27	17	24	30	13
79	81	77	96	65	32
21	19	91	58	28	23
10	6	45	50	88	95
53	97	61	59	56	49

Which is greatest?

36	27	40	11	9	10
2	55	26	20	19	21
31	18	23	46	38	29
75	89	64	82	91	43
49	51	25	14	32	73

Which is less?

21	19	95	88	27	36
91	58	97	53	79	81
28	23	61	59	17	13
10	6	56	49	30	13
45	50	32	61	65	32

Which is least?

72	14	32	31	18	23
82	91	43	20	27	21
25	51	49	2	55	26
75	89	64	11	9	10
29	46	38	40	27	36

Order of Numbers

Write in order from the least to the greatest.

19 31 25 19 25 31	23 12 18 12 18 23
86 45 29 29 45 86	77 68 92 68 77 92
62 66 85 62 66 85	53 36 16 16 36 53
56 84 99 56 84 99	42 74 37 37 42 74
63 48 72 48 63 72	15 34 83 15 34 83
94 28 54 28 54 94	59 39 54 39 54 59

Further Activities:

1. Write a numeral on the chalkboard and have children name the number that is one greater or one less than the one whose name you have written. Ask them to name the number that is ten greater or less.

2. Let children use hundreds-square paper (10 rows of 10 squares) and write numerals from 0 through 99.

3. Numerals may be written either vertically or horizontally. To check understanding of greater than and less than, write on the chalkboard:

   12, 21
   45, 49
   62, 57
   38, 25
   70, 39
   98, 89

Ask children to copy the pairs of numerals and draw a ring around the one in each pair that names the greater number. (They will ring 21, 49, 62, 38, etc.) The activity may be varied by asking them to draw a ring around the numeral for the lesser number in each pair. On other days you may wish to write 3 numerals in each group and ask children to draw a red ring around the numeral for the greatest number and a blue ring around the numeral for the least number.

4. Give each child three numeral cards and a piece of lined writing paper. The child is to arrange the three numeral cards in order from least to greatest number.

   $\boxed{23}$ $\boxed{43}$ $\boxed{68}$

   The child then copies the three numerals on his paper. When the first set is finished the child gets a new set of cards and repeats the activity for another set of cards. It is possible to modify this work by giving some children only two cards and other children as many as five cards.

5. Game for three children. Each child starts with 20 numeral cards, any of the set 0 through 99; no duplicates. The cards are in a stack, face down. Each child turns one card face up. The children compare the three numbers named, and the child whose numeral card names the greatest number takes all three cards and puts them on the bottom of his stack. The game is finished when one child has all of the cards in his stack.

VI-6. The hundreds (Optional)*

Objective: To develop the idea of the hundreds place in written notation.

Vocabulary: Hundred(s).

Materials: Small counting materials, all of one kind--beans, corn, pegs, etc.--more than 300 of them--in a container, preferably glass or clear plastic.

Teaching Note:

This section can be omitted if you feel it is too difficult, or that you do not have time in your program for this extension at this time. An introduction to the hundreds place will be given in Book 2.

Suggested Procedure:

Ask children to guess how many beans are in the container. Suggest that they find out by counting, and that everyone help. Give each child a handful of beans and ask him to count the beans in sets of ten. When all have finished, put all the remainder sets (ones) together, and have them counted into sets of ten.

Ask a child to count the sets of ten. Using chart below, record the number of tens. Ask another child to count the ones. Record it.

Tens	Ones
32	7

How many beans do we have? (We have 32 tens and 7 ones.)

Ask them if that is the way we expected to name the number of beans.

---

* The sections, 6 and 7, may be delayed until end of year or used with only some children.

Ask the children to count the tens; ten, twenty, thirty, etc. When they get to one hundred, stop, and emphasize the pronunciation of the word. Print it on the board. Ask a child to count how many tens it took to make one hundred.

Make another tabulation form on the chalkboard and ask where you would put hundreds on this chart.

Tens	Ones
10	0

Show that the hundreds is really ten tens, and that the numeral 10 could be written in the tens column, but that this would not suggest that we read it "one hundred".

Draw still another chart, this time writing hundreds, tens, ones.

Hundreds	Tens	Ones

Request that the children count the ten tens with you again: "Ten, twenty, thirty, ... one hundred".

> How many hundreds did we count? (One.)

Enter a 1 in the chart.

> If this set (point to the ten tens) were all we were counting, would we have any more tens to put in our chart? (No.)
> Would we have any ones? (No.)

Enter zeros in the tens and ones columns. Ask the children if they can tell you how to write the numeral for one hundred. Write it to the right of the chart. (100.)

(Continue counting beyond one hundred.) Show the children some more sets of ten and again count by tens - ten, twenty, etc. Stop them at fifty. Ask them how they would write the numeral for the number they have now counted. If anyone says 15 tens, enter it on the tens and ones chart and ask if there is another way to write it, so that you will know that you have already said "one hundred". (1 in hundreds column, 5 in tens column, 0 in ones column). Write the numeral to the right of the chart as before, and ask someone to read it. Point out that 15 tens and 150 name the same number. Go back, now, to the sets of ten, and have children again count by tens. When they get to one hundred, continue, "one hundred ten, one hundred twenty, one hundred thirty, etc." to two hundred. Emphasize the two, "Two hundred ten, etc." If the children find it difficult to count by tens when they say "one hundred" or "two hundred" first, go back and count again, this time just saying, when you reach the number, "<u>one</u> hundred, one hundred ten, one hundred twenty, ... <u>two</u> hundred, two hundred ten ... <u>three</u> hundred, three hundred ten". When they arrive at the total number of objects counted, repeat the number clearly and ask the children how to enter it on the chart of hundreds, tens, and ones.

Point out that the first chart, showing 32 tens and 7 ones looks very much like the present one showing 3 hundreds, 2 tens, and 7 ones, and means exactly the same number of objects. Ask a child how to write the numeral as we would read it. (327.)

Write three-place numerals on the chalkboard. Have children tell what each digit of the numeral means, and read the numeral. Be sure to include numbers like 401, 210, etc., with zero ones or tens.

<u>Further Activities</u>:

1. Children should have opportunities to count large collections of things and tell you how to write the number of each set. This can be done in small groups or individually, and the length of time needed will depend on the maturity and ability (among other things) of your class. Theater tickets are easy to count into strips of ten, with the strips bundled to make hundreds. Hundreds-squared paper may be cut into strips of ten and counted in the same way. In either case, however, you will need to point out that you are counting tickets or squares, <u>not</u> strips, to determine the number of objects,

and that a strip containing ten squares is one ten, not just one thing.

2. Give children cards on which 3-digit numerals are written. Ask each child to read his numeral and tell you how many hundreds, tens, and ones it means. Write it on the board after his name. When five or six have answered, ask which numeral names the greatest number. Ask which one names the least number. Enter greatest and least in a special place on the chalkboard, perhaps with the names of the children who read them. Repeat with other groups of children. When all have had a chance to report, and the numerals for the greatest and least numbers for each group have been recorded, identify which was the greatest number of objects counted and which was the least.

3. When children have understanding of the concepts developed here, let those who wish make booklets, using hundreds-square paper, and write the numerals from 1 to 1000. Page 1 would be 1 to 100, page 2 from 101 to 200, etc. While not emphasizing the point, you may tell the children who finish the project (and many won't !) that the number after 999 is 1000, which means, as they can see from the 10 pages they have done, ten hundreds, or one thousand, zero hundreds, zero tens, and zero ones.

4. Use "SHOW-ME" cards, giving names orally, as "three hundred twenty-one", as "three hundreds, two tens, and one one", and as "thirty-two tens and one one", and asking children to display the numeral on their card.

VI-7. Hand numerals (Optional)

Objective: To extend understanding of place value by using base five.

Vocabulary: (No new words.)

Materials: Thirty-five objects for counting; sets of objects for the children.

Suggested Procedure:

Write the numerals 0, 1, 2, 3, 4 on the chalkboard.

> We are going to count a set of objects today and use only these five numerals. We will have to find a way to count more than four objects. When we have counted four and are ready to count one more we will say that we have one hand. This will be our only new name. We will record what we are counting on a chart which shows the number of hands and the number of ones.

hands	ones

Now count several sets by grouping into hands (fives) and ones, and record the results on the chart. Have the children partition sets of objects into hands and ones, and record the numbers. Then (in much the same way as for the decimal system) begin a third column. Explain to the children that to show we are not using the ordinary numerals, we make an "H" to the right of the numerals. Your chart might look as follows.

Hands	Ones	Hand numeral
3	1	31 H
2	3	23 H
4	0	40 H

Caution: Do not read 31 H as thirty-one. Read as "three-hand-one" (or "three-handy-one" if you prefer).

Pupil's book, page 78:

Children are to record the number of hands and the number of ones in each set and then write the H numeral.

Display an H numeral such as 23 and ask the children to construct a set with that many members. (They should show 2 fives (hands) and 3 ones.)

Pupil's book, page 79:

The children are to draw a set of the indicated number of members.

Ask the children to count, using the hand numerals. Record on a chart as below (you will probably need two columns on the chalkboard). Begin: one, two, three, four, one hand zero, ... until you reach four hand four.

Hands	Ones	Hand numeral
	1	1
	2	2
	3	3
	4	4
1	0	10 H
1	1	11 H
4	3	43 H
4	4	44 H

Ask what the next number would be. Probably someone will suggest "five hand zero". If so explain that in naming numbers with hands we only use 0, 1, 2, 3, and 4. Bring out that five is one hand zero, and that we write this as 10 H. The entry in the chart will then read;

Hands	Ones	Hand numeral
10 H	0	

Try to get one of the children to suggest the correct hand numeral: 100 H. Go on to say that we could call a hand of hands a fist, and that we could make our chart as follows:

Fists	Hands	Ones	Hand numeral
1	0	0	100 H
1	0	1	101 H

This may be about as far as you wish to carry this discussion at this time. Be sure to point out the similarities with the decimal system:

One ten and zero ones is 10.
One hand and zero ones is 10 H.
One ten of tens (hundred) is 100.
One hand of hands (fist) is 100 H.
234 is 2 tens of tens (hundreds), 3 tens, and 4.
234 H is 2 hands of hands (fists), 3 hands, and 4.

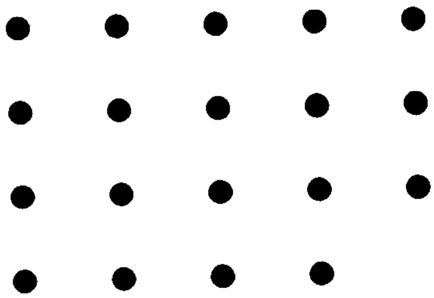
Ask a child to place two-hand-zero on the chart. Ask another child to place three-hand-two on the chart.

Which has more members two-hand zero or three hand-two? (Three-hand-two.)
What number is greater by one than two-hand-one? (Two-hand-two.)
What number is one less than four-hand-zero? (Three-hand-four.)

How Many?

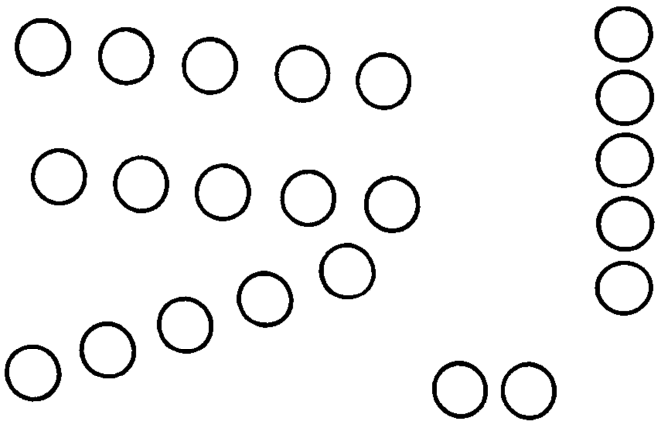
Hands	Ones
2	3

23 H



Hands	Ones
3	4

34 H



Hands	Ones
4	2

42 H

Draw a set of 13 H.

XXXXX
XXX

Draw a set of 24 H.

XXXXX
XXXXX
XXXX

Draw a set of 40 H.

XXXXX
XXXXX
XXXXX
XXXXX

VI-8. Application: Money

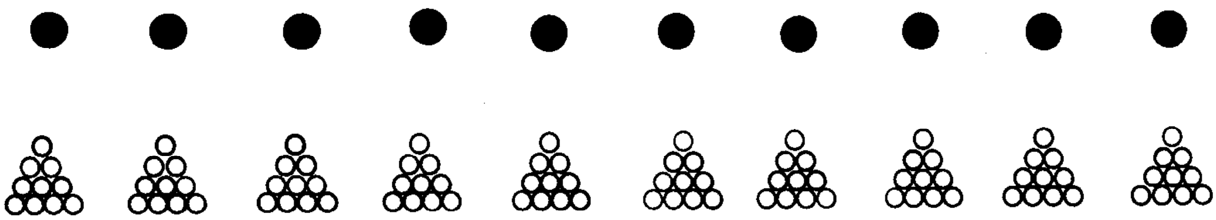
Objective: Review value of pennies and dimes.
Introduce value of dollar.

Vocabulary: Dollar, nickel.

Materials: 100 pennies, 20 nickels, 10 dimes, 1 dollar.

Suggested Procedure:

Place 100 pennies on a table. Tell one child to separate (partition hasn't been introduced yet) the set of pennies into sets of 10. As he does, arrange the piles of pennies in a row, and place a dime beside each pile.



Note that there is a set of pennies for each dime, and a dime for each set of ten pennies.

> Does this help us find out something about how many pennies a dime is worth? If you need a dime to buy something, how many pennies will it take to buy the same thing?
>
> If it takes ten cents to buy something, how many dimes will you need to buy the same thing?

Tell the children that ten dimes can be traded for one dollar. The dollar is worth ten dimes.

> What else could be traded for one dollar? (100 pennies.)
> One dollar is worth how many pennies? (100.)
> One hundred pennies have the value of how many dollars? (1.)
> How many dimes are worth one dollar? (10.)
> This is like something we have done with numbers.

At this point it is hoped that children will have noticed that the 100 pennies, 10 dimes, 1 dollar pattern is like the place-value ideas they have recently learned. Make a tablulation form on the chalkboard.

dollars	dimes	cents

Give a child 37 pennies. Ask what pieces of money he could trade **them** for so that he had the same value but fewer pieces of money.

Give a child 5 dimes and 2 pennies. Ask how this could be traded so that value of the money was the same but the number of pieces of money was far greater.

Give children amounts of money with which they must trade 10 dimes for a dollar to get the fewest number of pieces of money. Do not introduce notation \$1. or \$1.35 at this time. Call the amount read from the chart.

dollars	dimes	cents
1	3	5

one dollar, thirty five cents and relate to one hundred thirty-five.

Ask a child to pretend he is going to the store to buy a toy car that costs 49¢. Have him count out the money, all in pennies, and hold it in his hand. Have another child count out 49¢ using as few coins as possible.

Which is easier to carry?

Point out the fact that 9 pennies are still rather awkward and that it would be easier to manage if there were some coin between a dime and a penny in value, some coin worth more than a penny but less than a dime.

Does anyone know whether we have a coin like that?
What is it called?

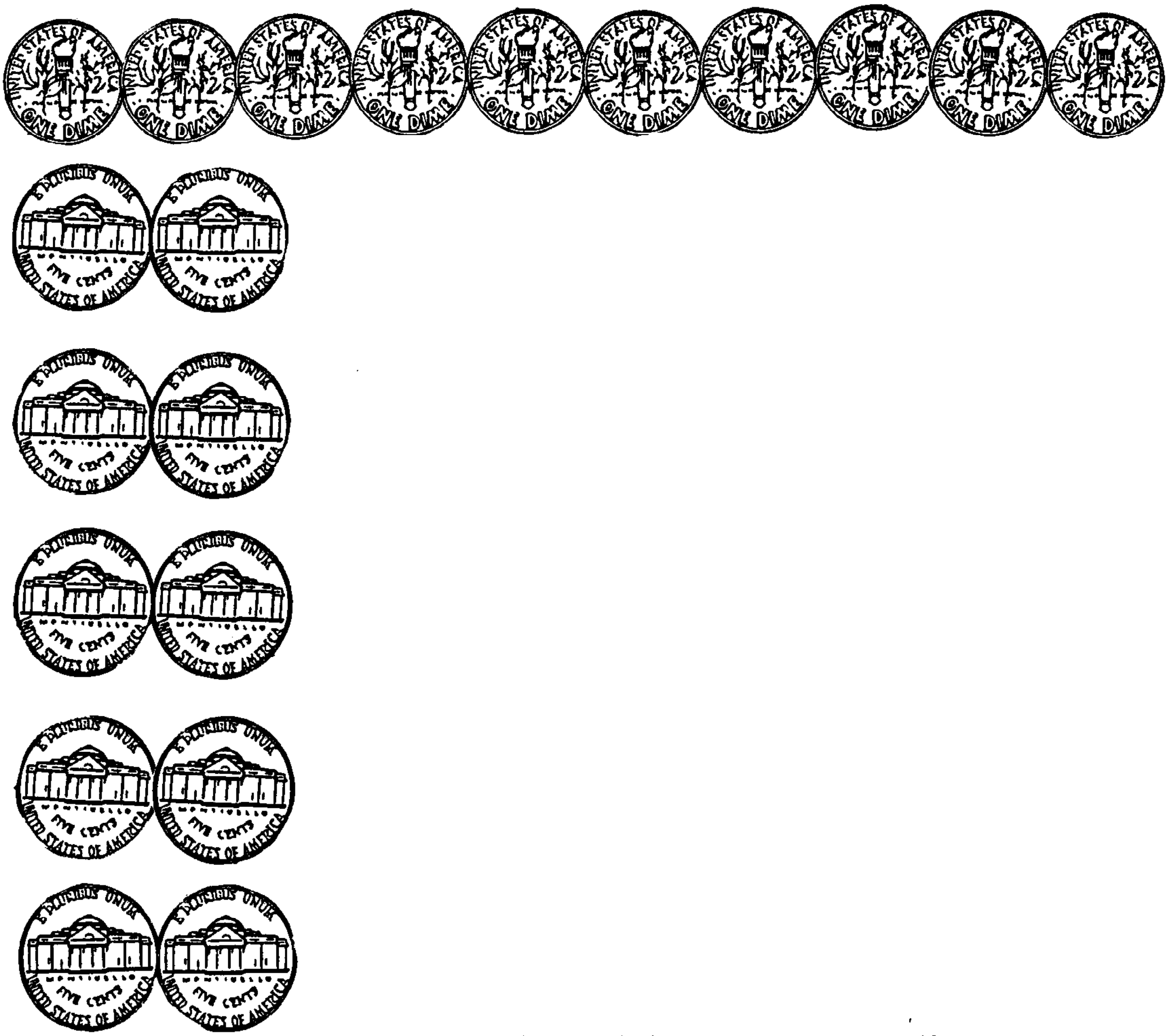
Hold up the envelope of nickels and ask what children think may be in it. Have them guess how many nickels are in it. Tell them it is a dollar's worth of nickels, and have someone count the nickels to find how many there are.

Does anyone know how many nickels one dime is worth?

Remind children that one dollar's worth of pennies and one dollar's worth of dimes could be counted so that there were 10 pennies for every dime. If Jimmy says 5 nickels are worth one dime say

Let's see if we can count out 5 nickels for every dime and have enough to go around.

Use the following arrangement on a table or desk.



Let a child do the counting, putting 5 nickels on the table beside each dime. When he has finished, ask what happened.

> Are there 5 nickels for each of the 10 dimes on the table? Let's try something else. Do you think there are more than 5 or fewer than 5 nickels for every dime?

Try to get children to think, not guess. When the idea of 2 nickels for each dime appears to them to be the most reasonable suggestion, have a child count out 2 nickels beside each dime on the table.

> If ten pennies are worth as much as a dime and a dime is worth as much as two nickels, how many pennies would be worth as much as two nickels?

● After the idea that ten pennies have the value of two nickels seems firmly established, ask how many pennies one nickel is worth. Even though some children at this age know the answer, it may be worthwhile to ask them to count out a set of ten pennies into two sets of five pennies each and to put each set of five pennies next to a nickel.

> When we count the number of cents in ten dimes we may count to one hundred by tens.

Remind children that two nickels have the value of one dime. Add a nickel to each heap of coins and show the children how to count it as "ten, twenty, thirty," (moving two nickels at once) "forty."

> When we count the number of cents in twenty nickels we count five, ten, fifteen, twenty... We could count the sets of five pennies in the same way.
>
> Let's go back to that toy car Jimmy wanted to buy. Think how heavy 49 pennies are. We found that it was easier to carry 4 dimes and 9 pennies. Now if a nickel is worth as much as 5 pennies, can we use a nickel instead of 5 of those 9 pennies? Let's see what happens.

Make the exchange of coins, and count the money for the children.

> Ten, twenty, thirty, forty cents (counting dimes) and then we have five cents because that is what a nickel is worth. Forty-five cents, forty-six cents, forty-seven cents, forty-eight cents, forty-nine cents. We still have forty-nine cents.

Give children many experiences mixing dimes, nickels and pennies and ask children to count the number of cents.

<u>Further</u> <u>Activities</u>:

Place 3 dimes, 2 nickles, and 7 pennies on a table. Display another set of coins which is 2 dimes, 7 nickels, and 3 pennies.

These are intentionally the same number of pieces. Ask children to determine which set is worth more.

Tell several children to pick up 7 coins. On a chart record the different numbers of cents which a set of seven coins may be:

7	dimes					are	70¢
5	dimes	2	nickels			are	60¢
7	pennies					are	7¢
2	dimes	2	nickels	3	pennies	are	33¢

Tell a child to pick up any number of coins which are 36 cents. Ask another child to find sets of coins which are worth 36 cents. A chart might be made of these. Each child might choose an amount of money and try to list 5 different sets of money which make that amount.

36 cents

3 dimes		6 pennies
3 dimes	1 nickel	1 penny
		36 pennies
2 dimes	3 nickels	1 penny
	6 nickels	6 pennies

Pupil's book, page 80:

Write numeral in box at right for the amount shown.

Pupil's book, page 81:

Mark coins needed to make amount shown at left.

Pupil's book, page 82:

Write numeral in box at right for the amount shown.

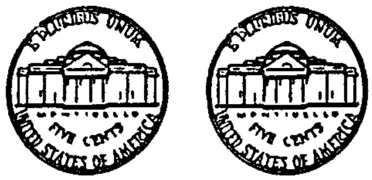
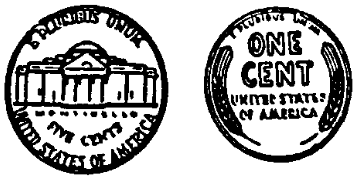
Pupil's book, page 83:

Mark coins needed to make amount shown at left.

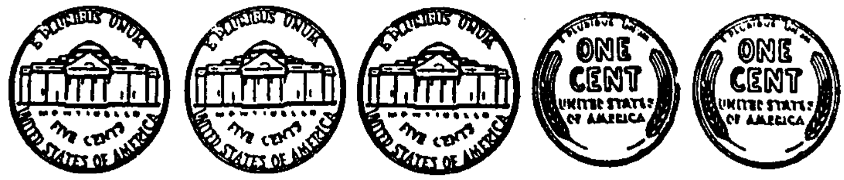
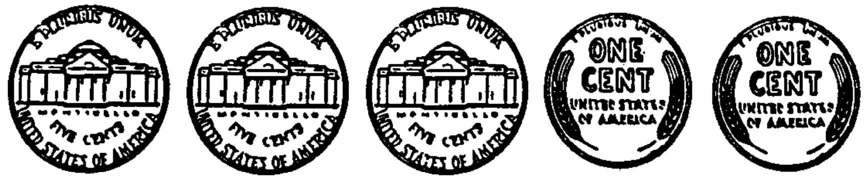
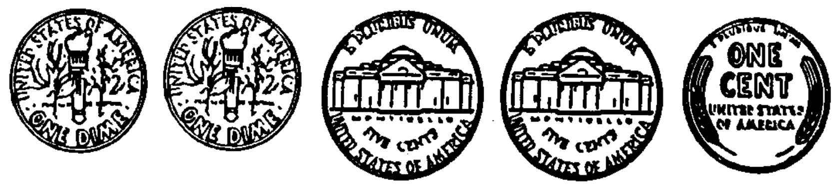
Pupil's book, pages 84 - 85:

Mark number of coins designated to make amount of money designated.

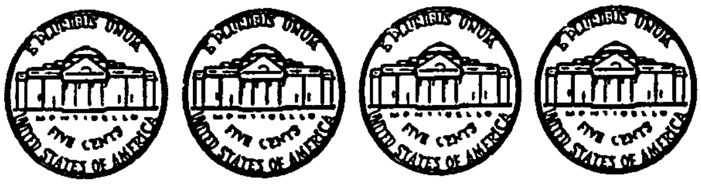
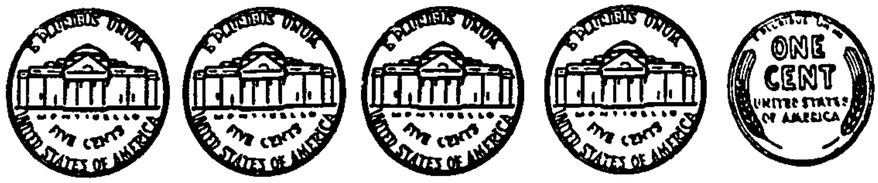
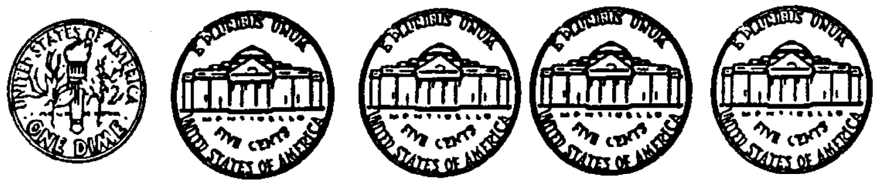
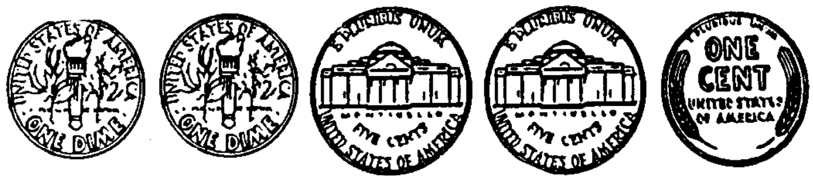
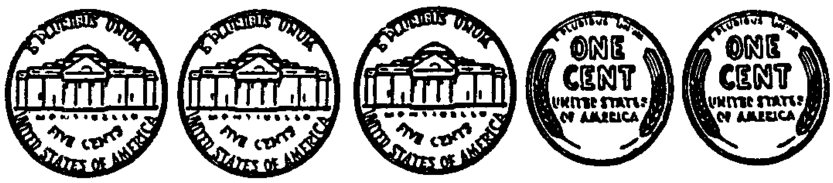
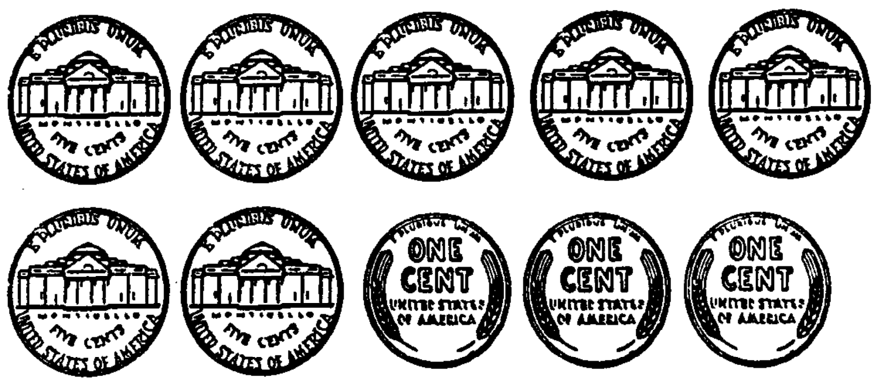
Money

	10¢
	12¢
	10¢
	6¢
	6¢
	9¢
	35¢
	36¢

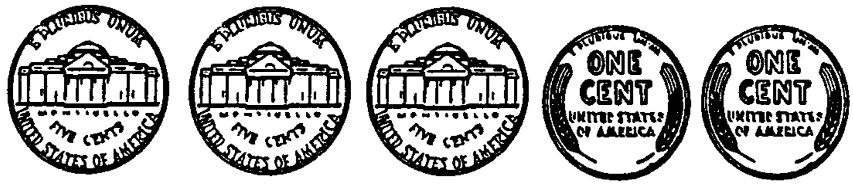
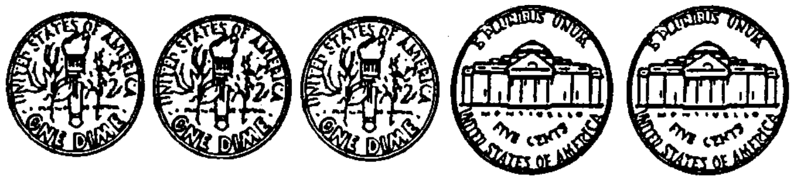
Money

7 ¢	
11 ¢	
25 ¢	
36 ¢	
10 ¢	
8 ¢	
49 ¢	

Money

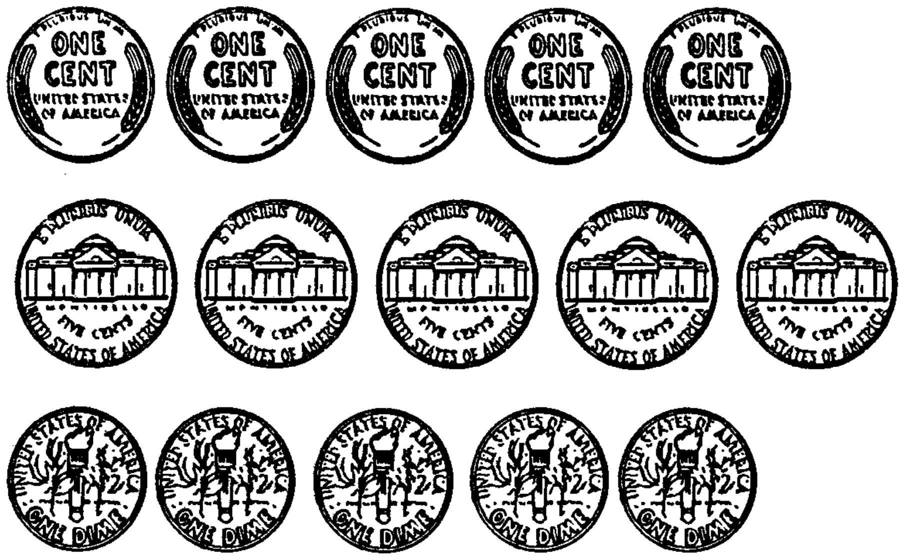
	20¢
	21¢
	30¢
	26¢
	31¢
	17¢
	38¢

Money

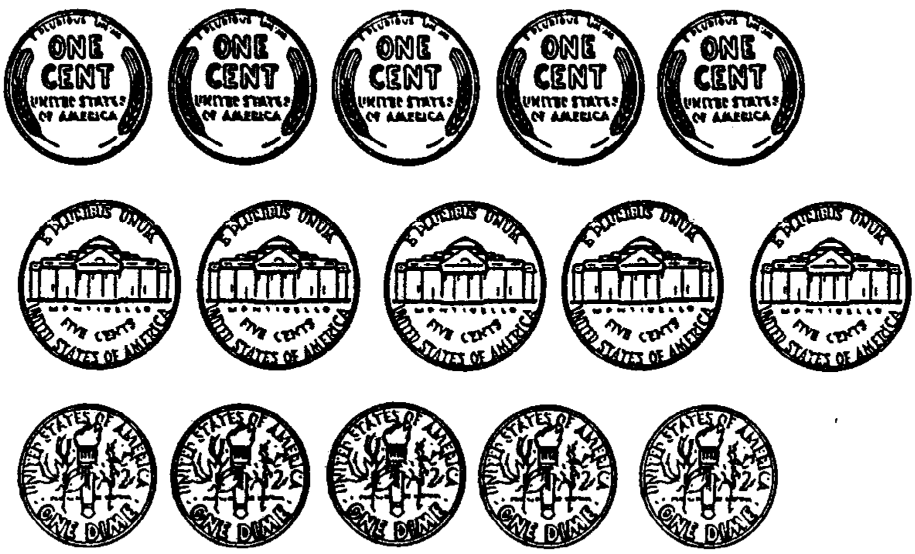
10¢	
16¢	
25¢	
32¢	
69¢	

Money

Mark 9 coins that make 48¢.

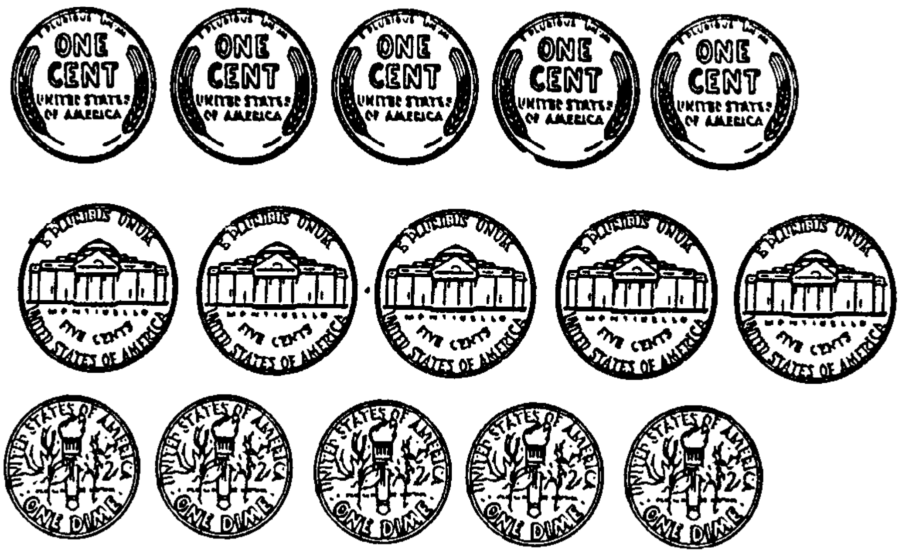


Mark 7 coins that make 41¢.



Money

Mark 11 coins that make 60¢.



Mark 6 coins that make 31¢.

Chapter VII

ADDITION AND SUBTRACTION

Background

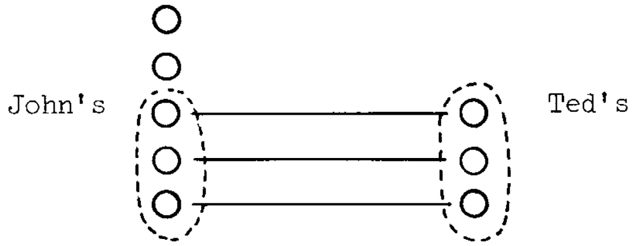
You may want to review the background for Chapter 4, where the fundamental definitions concerning addition and subtraction were made. We recall, for example, that $3 + 5$ is the number of members in the set obtained by joining a set of 5 to a set of 3; that $5 - 3$ is the number of members remaining if a subset consisting of 3 members is removed from a set of 5; and that adding 3 and subtracting 3 are inverse operations in the sense that adding 3 to a number and then subtracting 3 from the result always gives the original number, and subtracting 3 and adding 3 also gives the original.

We introduce the idea of partitioning sets in this chapter. This idea is used here primarily for reinforcement of various number relationships, but we shall later use partitioning into equivalent sets in the discussion of place-value and division. Partitioning a set is just separating it into two disjoint subsets. For example, we may partition the set consisting of Mildred, Jean, Stan, and Mary into the set consisting of Mildred, Jean, and Stan, and the set consisting of Mary. (We shall later partition a set into more than two subsets.)

Partitioning is related to both joining and removing. For example, if we join the set consisting of Mildred, Jean, and Stan to the set consisting of Mary, we have the original set consisting of Mildred, Jean, Stan, and Mary. Because of the relation between joining and addition, we see that, in general, the number of members in the original set is equal to the sum of the number of members in the two sets of the partition (in this case, $4 = 1 + 3$ or $4 = 3 + 1$).

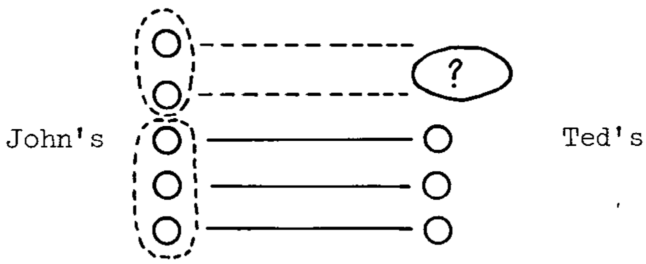
Then, $4 - 3 = 1$, and in general, the number of members of the original set minus the number of members of one of the sets of the partition is the number of members of the other set of the partition.

There are also other problems which lead to subtraction equations. For example: If John has 5 marbles and Ted has 3 marbles, how many more marbles has John? We may think of pairing Ted's marbles with John's, as shown below,



removing from the set of John's marbles a set which is equivalent to Ted's, and identifying the number of the remaining set. Since the number of Ted's marbles is equal to the number of the equivalent subset of John's marbles, we see that John has $5 - 3$ more marbles than Ted.

The following is a closely related problem: if John has 5 marbles and Ted has 3 marbles, how many marbles must we give Ted so that he has as many as John? Schematically, we can pose the question as follows:

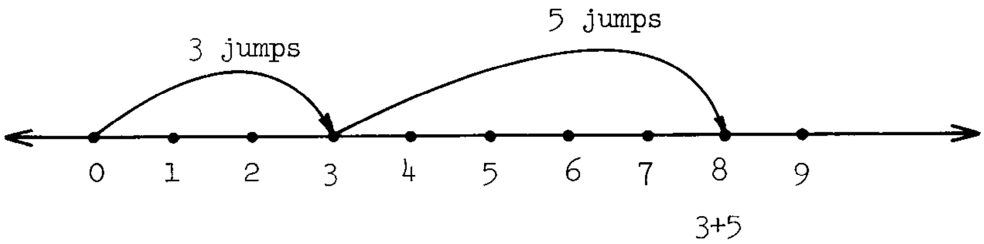


If we remove from the set of John's marbles a subset which is equivalent to Ted's set, then the remaining set is equivalent to the unknown set. We conclude that we must give Ted $5 - 3$ marbles.

This last description of subtraction in terms of sets leads to a formulation which does not depend on sets, but only on the idea of addition which we have already introduced. (Of course, the definition of addition does depend on manipulation of sets.)

Suppose again that John has 5 marbles and Ted has 3 marbles, and we wish to know how many marbles to give Ted so the boys will have the same number of marbles. The union of the set we give Ted and the set that Ted has must be equivalent to the set of John's marbles. Hence, the number of marbles we must give, which is $5 - 3$, is the answer to the following question: $3 + ? = 5$. In the same way, $4 - 2$ is the answer to the question, $2 + ? = 4$, and so on. This is sometimes called the missing addend description of subtraction. It is important that children work with this description as well as with the descriptions in terms of set manipulation since this will be the fundamental notion underlying the subtraction of numbers in later grades. In general, we try to give the children experience with several of the ways that subtraction problems arise.

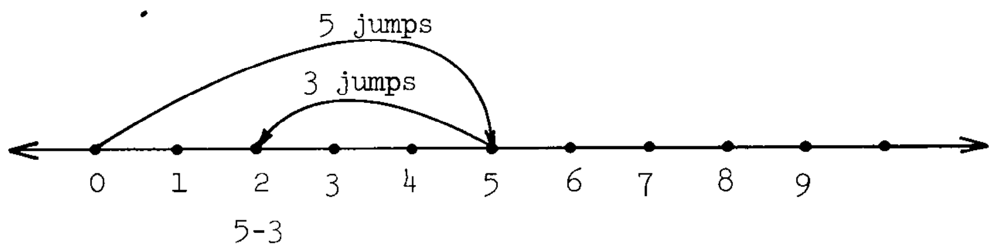
The number line is useful in learning about the operations of addition and subtraction. If we think of 0 as the starting point, then each numeral indicates the number of "jumps" required to get from the starting point to the point marked by the numeral. We may find the sum of 3 and 5 by taking 3 jumps, and then 5 jumps, and then reading the numeral (which indicates the number of jumps taken from the starting point).



$3 + 5 = 8$

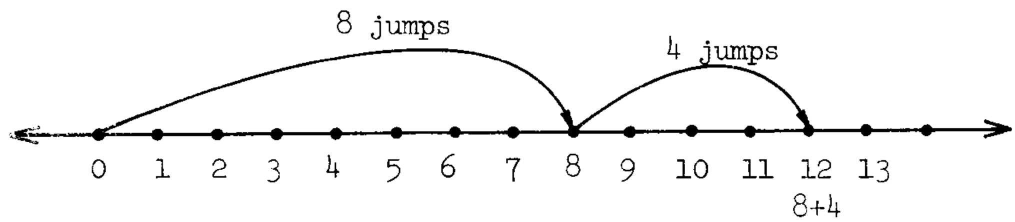
Notice that we do not have to count out the 3 jumps: the numeral "3" shows where counting 3 jumps would have gotten us.

The number line can also be used for subtraction: 5 - 3 is the number of jumps from the starting point which results from taking 5 jumps forward and then 3 jumps backward.

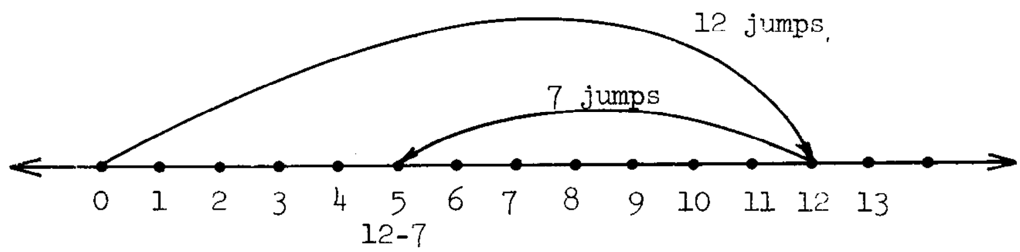


$5 - 3 = 2$

Finally, we note that the number line, which we use here primarily for reinforcement and variety, can be used in a very important way in introducing such problems as $8 + \square = 12$ and $12 - \square = 5$.



$8 + 4 = 12$



$12 - 7 = 5$

VII-1. Partitions and addition

Objective: To reinforce and extend the child's understanding of addition by using partitions of sets.

Vocabulary: Partition.

Materials: Flannel board shapes, colored paper shapes or sets of small objects, sheets of white 9" x 12" paper, yarn.

Background Note: Partitioning a set into two subsets simply means dividing it into two parts. Each member of the set with which you started then belongs to just one of the two subsets. The union of the two subsets is the set with which you started and, because of the relation between adding and joining, each partition gives us information on addition. Thus, the fact that a set of 5 can be partitioned into a set of 2 and a set of 3 shows us that $5 = 2 + 3$ and $5 = 3 + 2$ (since joining and adding are both commutative).

Suggested Procedure:

Place a set of objects on the flannel board. Observe how many objects are in the set. Separate this set into two subsets, using yarn or any other suitable item. Observe how many objects are in each of the subsets. Repeat the procedure, partitioning the same set in different ways.

Have each child place a 9" x 12" sheet of paper on his desk and place four circular shapes on the paper.

> Lay a piece of yarn on the paper so that you have some members of the set on one side of the yarn and some on the other.

What could you call the part of the set on each side of the yarn? (A subset.)

Fred, how many members are in your whole set? (4.)
How many members are in each of your two subsets? (Answers will vary.)

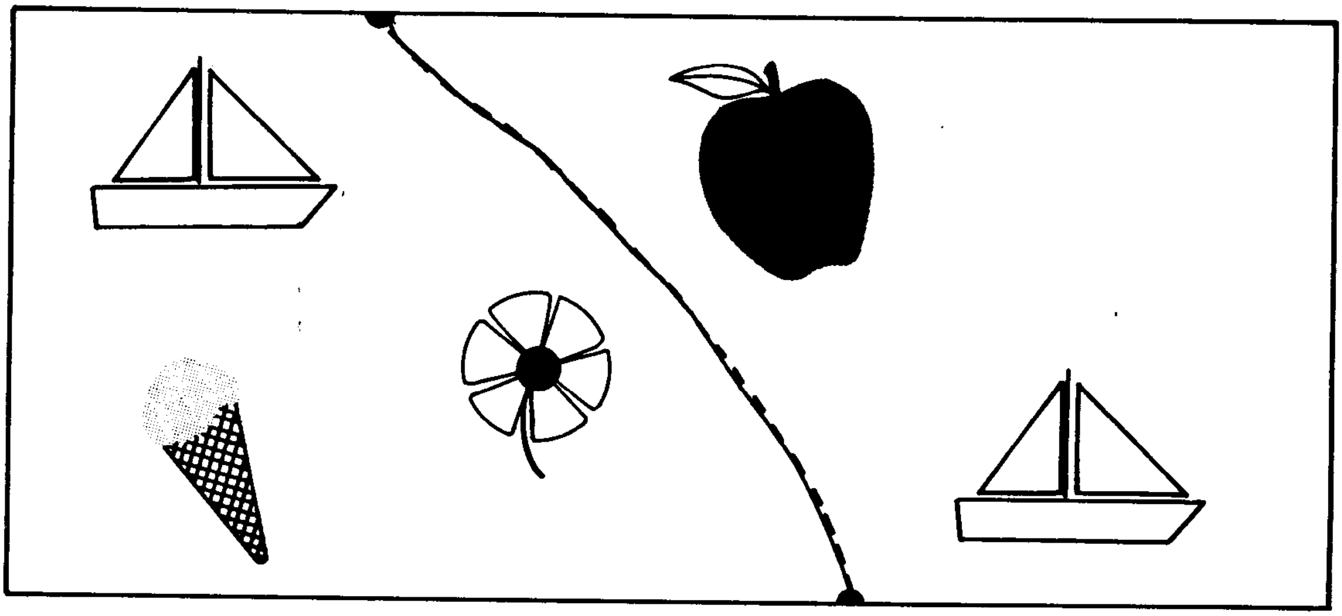
Glenn, how many members are in your whole set? (4.)
How many members are in each of your two subsets? (Answers will vary.)

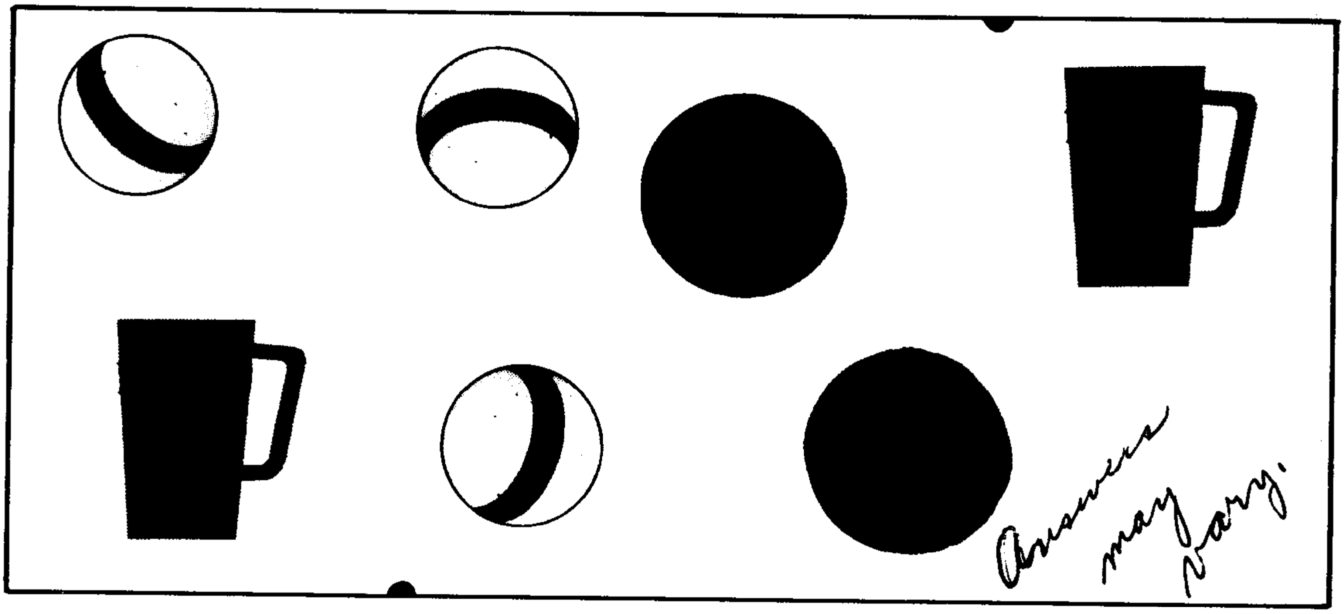
Continue until all possible partitions of a set of four have been found.

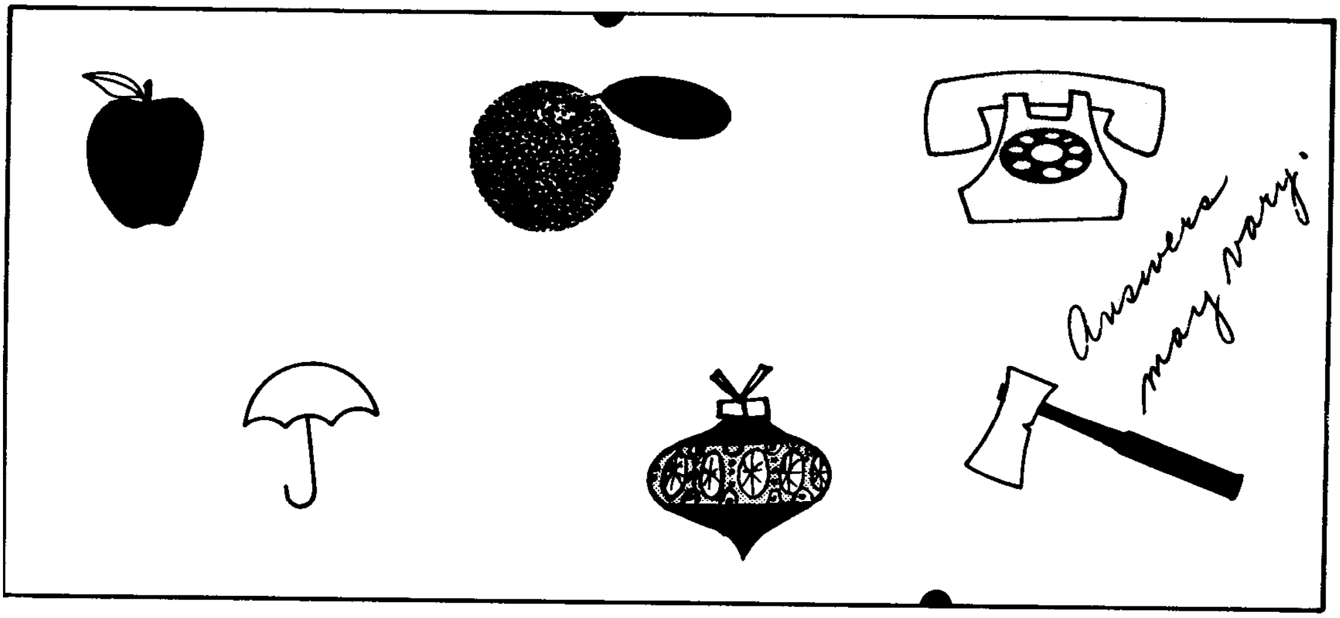
Pupil's book, page 86:

Have the pupils imagine that each rectangle is a "fence" around a set of things. The two dots represent "fence posts". Each set is to be partitioned by drawing a "line" from one fence post to the other, as indicated by the dashed line in the first example. Have children trace over the dashed line in the first example, and then draw lines between the fence posts in each of the other two examples. Discuss each partitioning, having pupils indicate the number of members in the whole set and the number of members in each subset.

Partition the sets.







- Place a set of 8 objects on the flannel board.

  What is the number of members of the set? (8.)

Place a piece of yarn across the flannel board to show a partition of the set.

  What is the number of members in one subset? (3.)
  What is the number of members in the other subset? (5.)
  We can add the number of members of the two subsets.
  What is the equation? ($5 + 3 = 8$.)

Place a set with nine members on the flannel board; place a piece of yarn to show four and five members in the subsets. Tell children that it is easy to find the number of members in the set if we add the numbers in the two subsets.

  How many members are in the subsets? (5, 4.)

  When we add 5 to 4 this tells the number of members in the set. We write the equation $5 + 4 = 9$. We could also write $4 + 5 = 9$.

Display other sets with 6 to 9 members. Encourage children to use partitions and addition to determine the number of members in the sets.

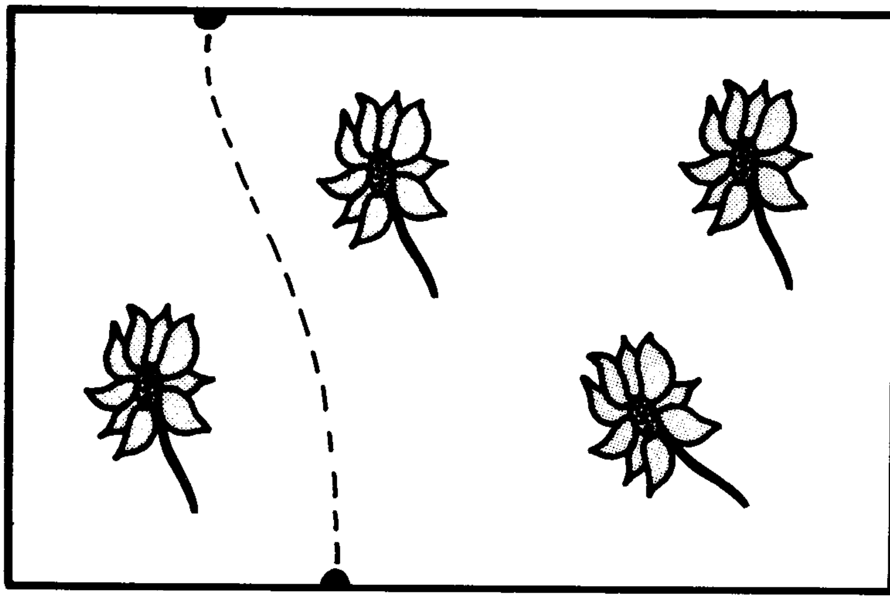
Pupil's book, pages 87 - 90:

These pages show partitions of various sets. Ask the children to write equations for each partition.

Pupil's book, pages 91 - 94:

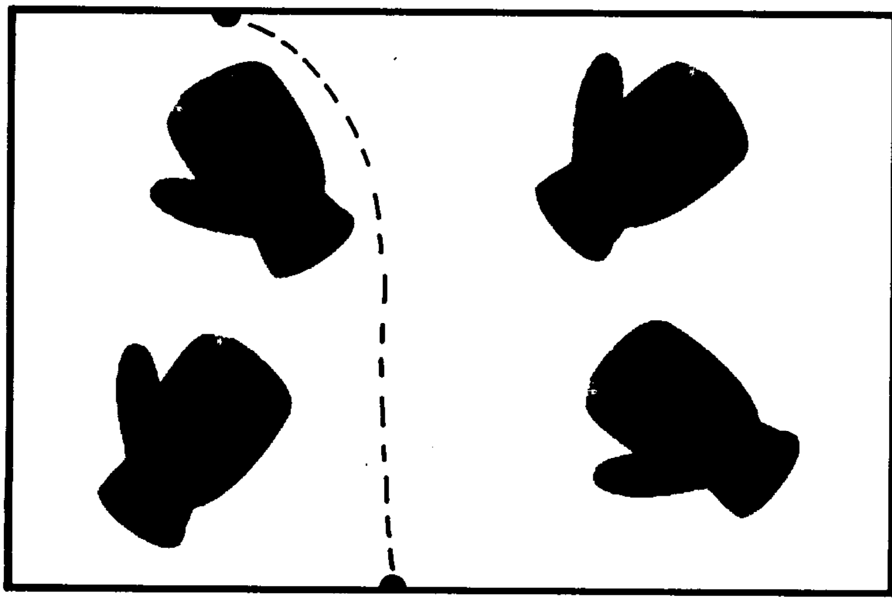
Ask the children to partition the set pictured at the top of each page by placing some object across the box. Record the equations in the space provided at the bottom of the page.

Partitions

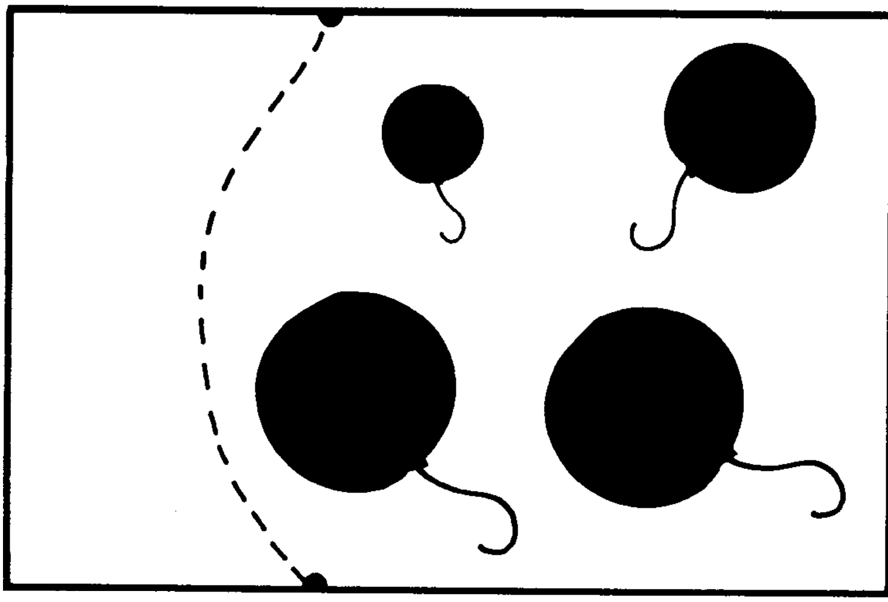


1 + 3 = 4

3 + 1 = 4



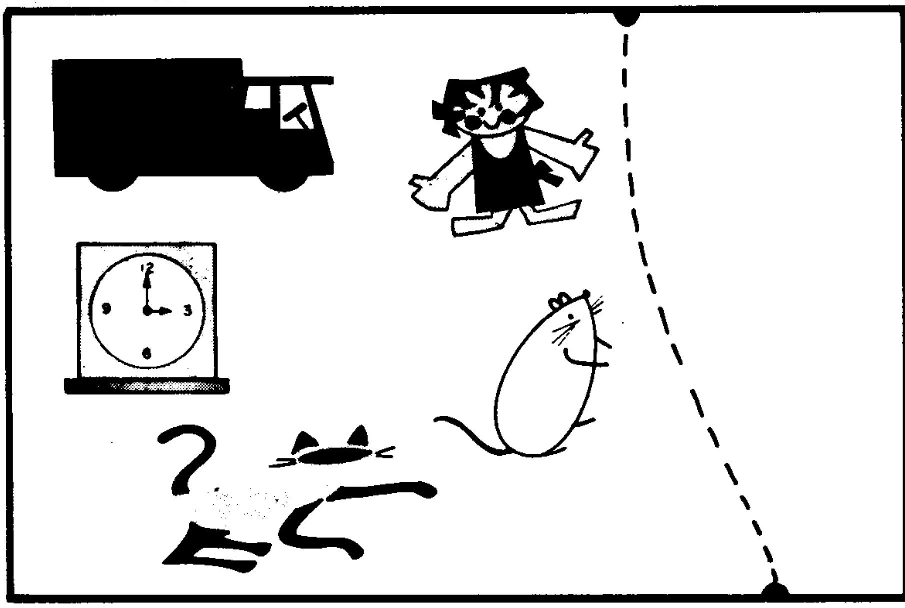
2 + 2 = 4



0 + 4 = 4

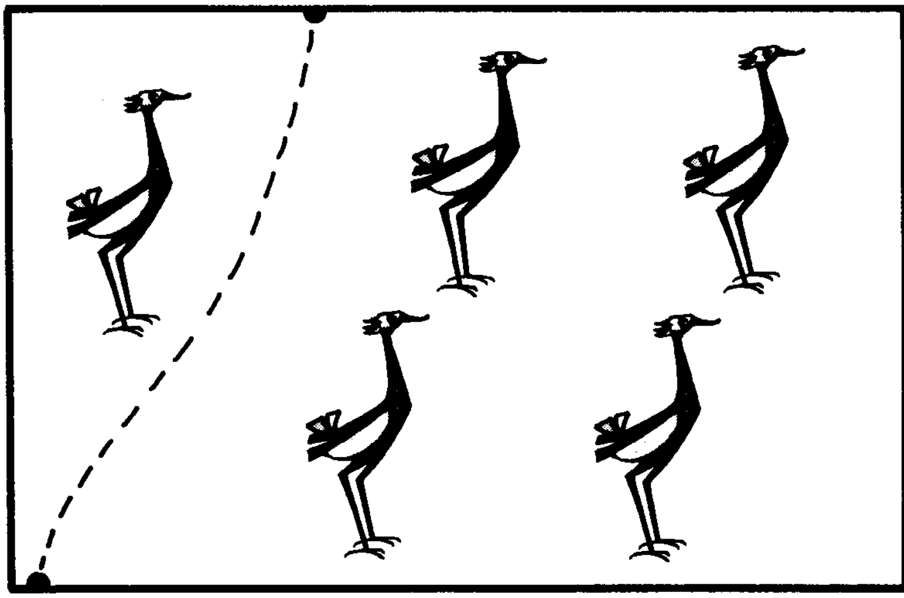
4 + 0 = 4

Partitions



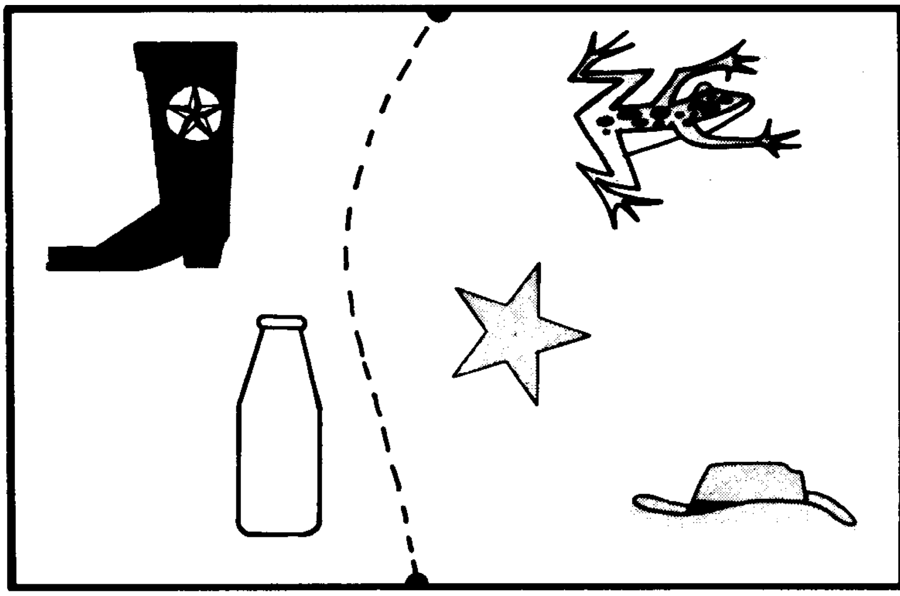
$5 + 0 = 5$

$0 + 5 = 5$



$1 + 4 = 5$

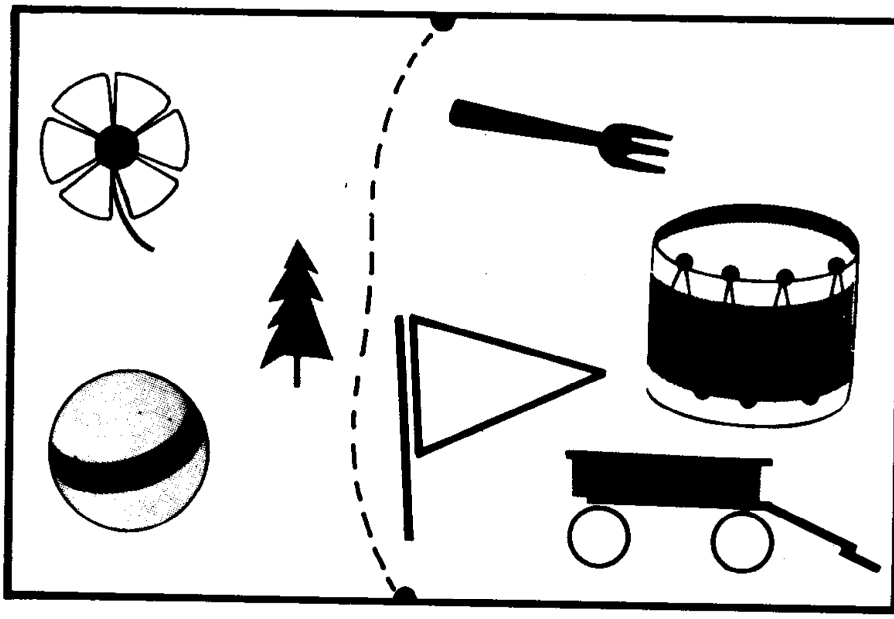
$4 + 1 = 5$



$2 + 3 = 5$

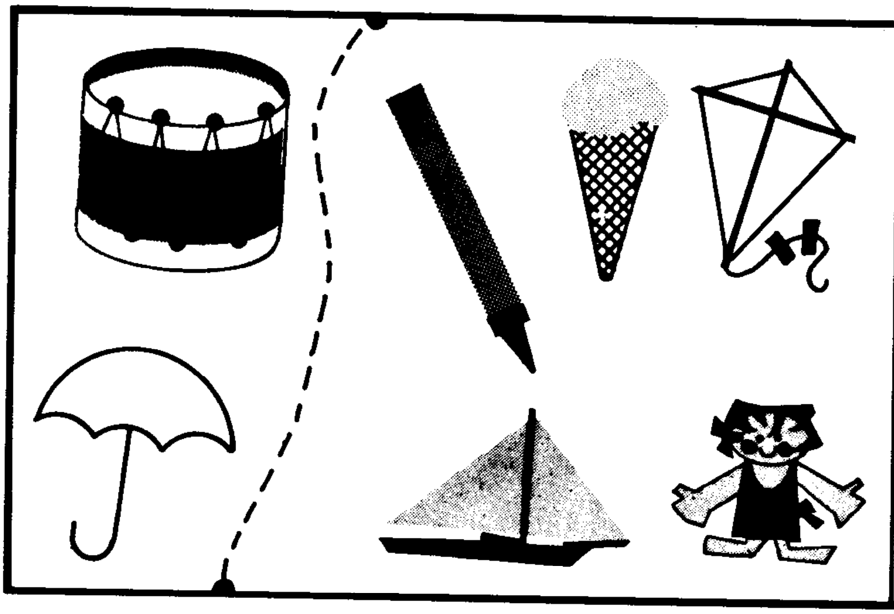
$3 + 2 = 5$

Partitions



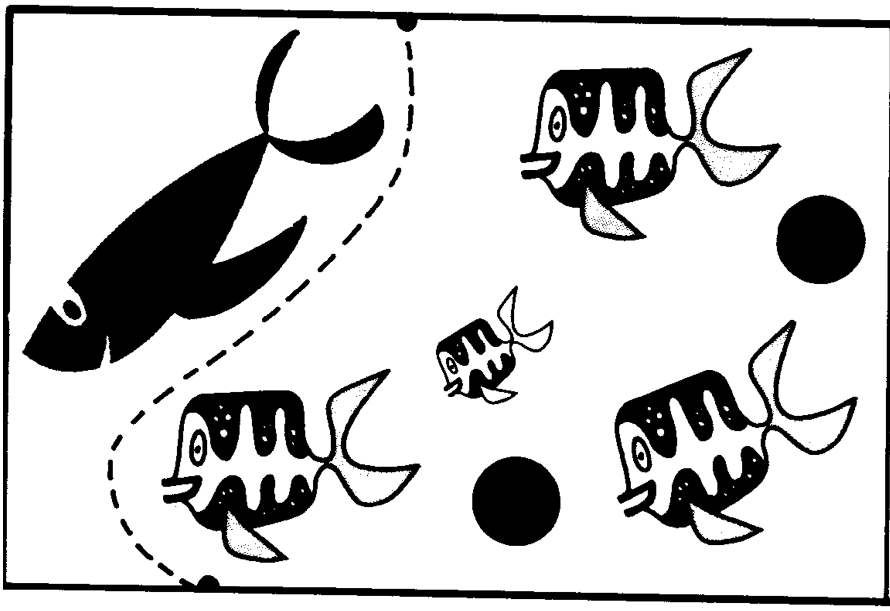
$\underline{3} + \underline{4} = \underline{7}$

$\underline{4} + \underline{3} = \underline{7}$



$\underline{2} + \underline{5} = \underline{7}$

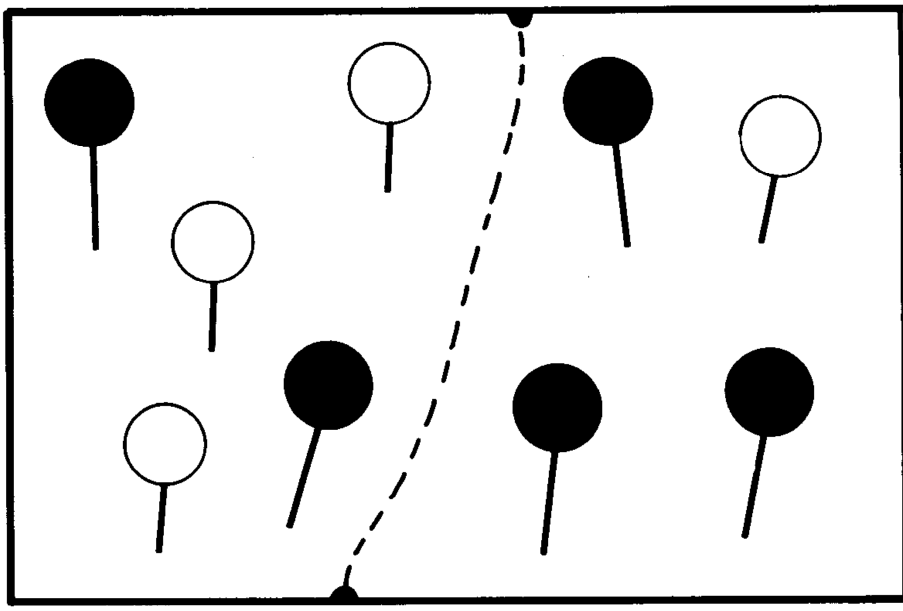
$\underline{5} + \underline{2} = \underline{7}$



$\underline{1} + \underline{6} = \underline{7}$

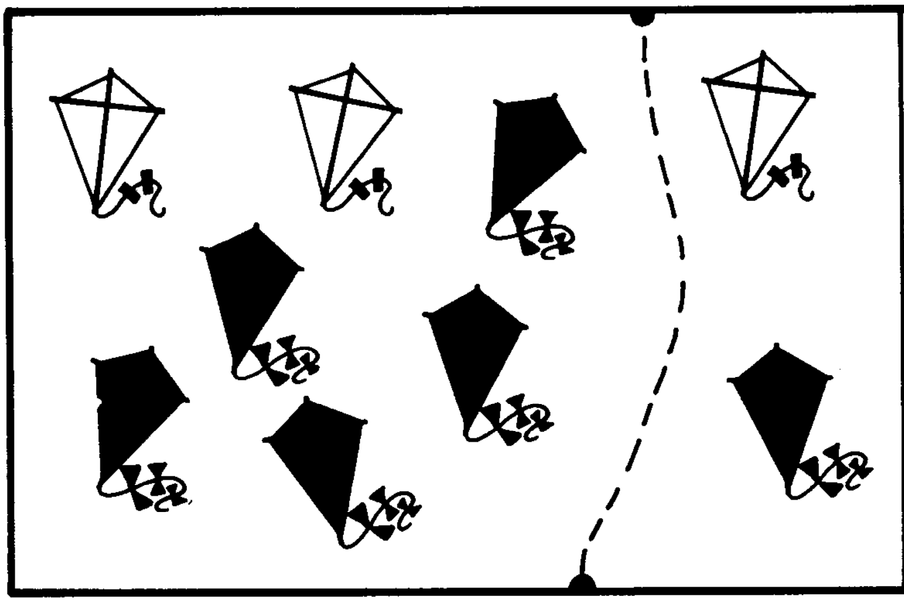
$\underline{6} + \underline{1} = \underline{7}$

Partitions



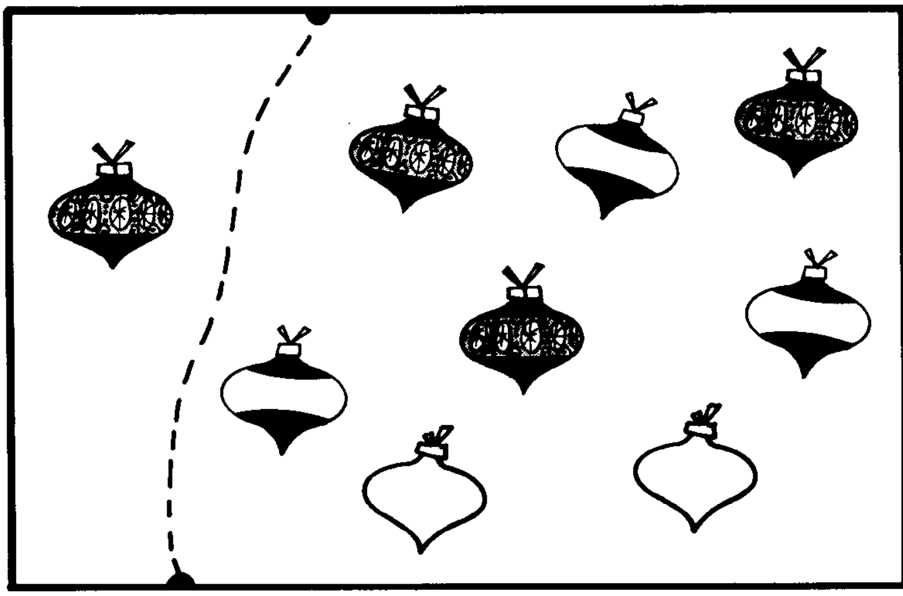
$5 + 4 = 9$

$4 + 5 = 9$



$7 + 2 = 9$

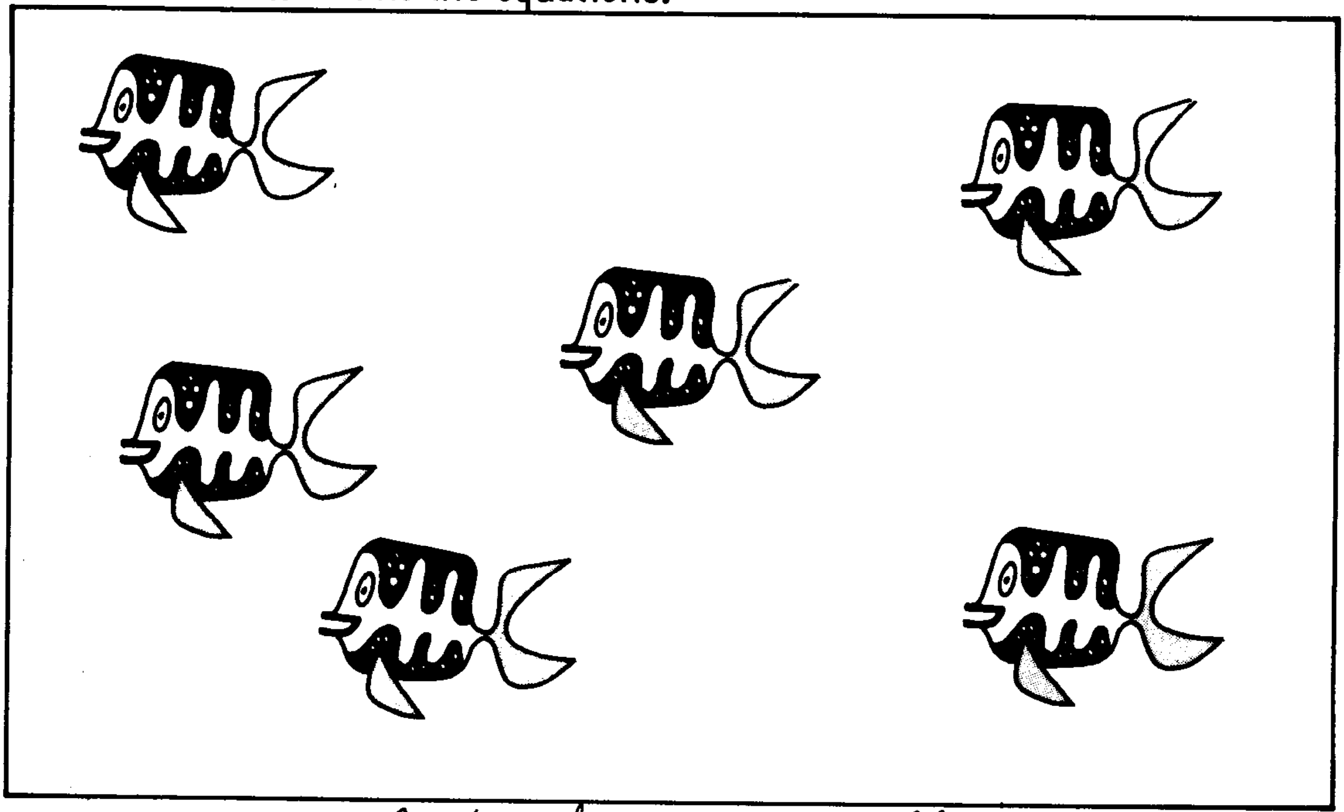
$2 + 7 = 9$



$1 + 8 = 9$

$8 + 1 = 9$

Partitions of 6. Write the equations.



Order of answers will vary.

6 = 6 + 0 or 0 + 6

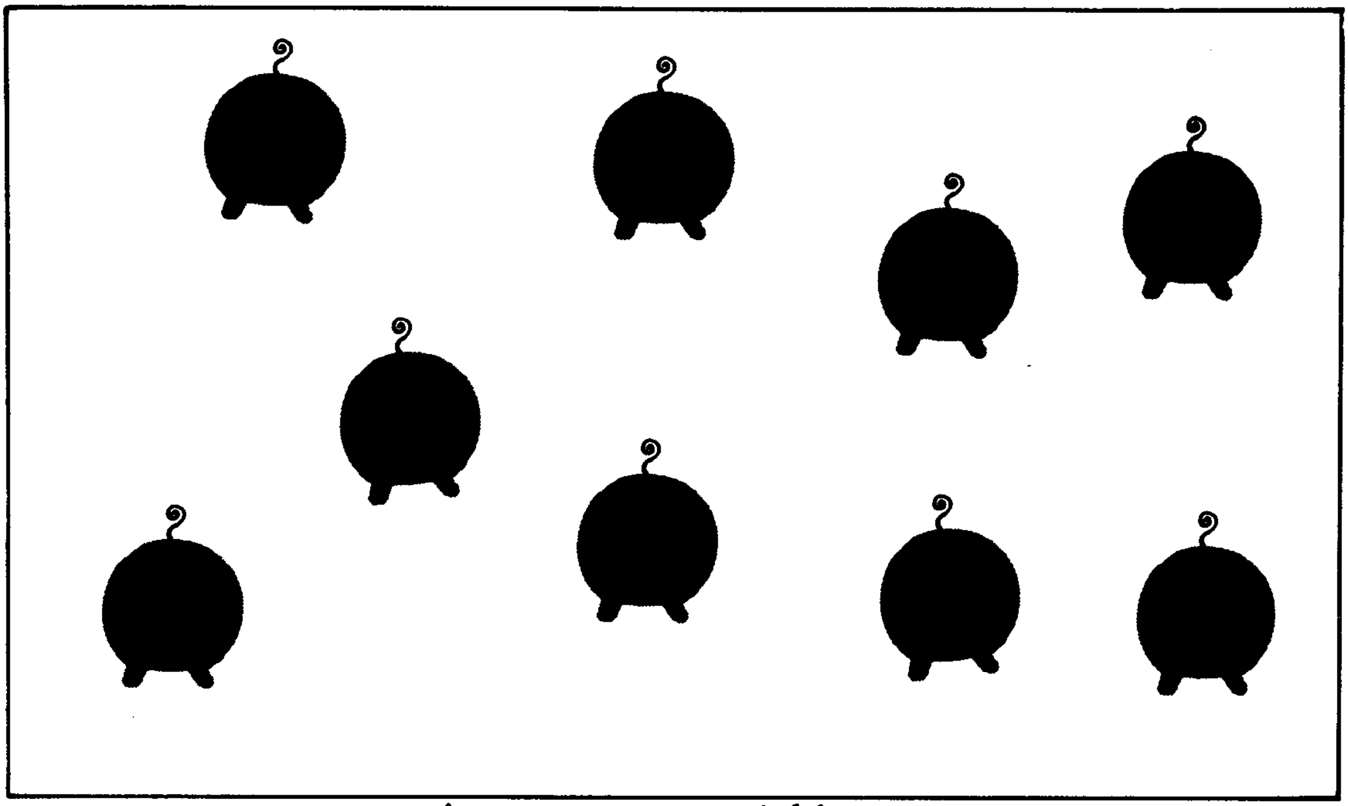
6 = 5 + 1 or 1 + 5

6 = 4 + 2 or 2 + 4

6 = 3 + 3

6 = ______ + ______

Partitions of 9. Write the equations.



Order of answers will vary

9 = 0 + 9 or 9+0

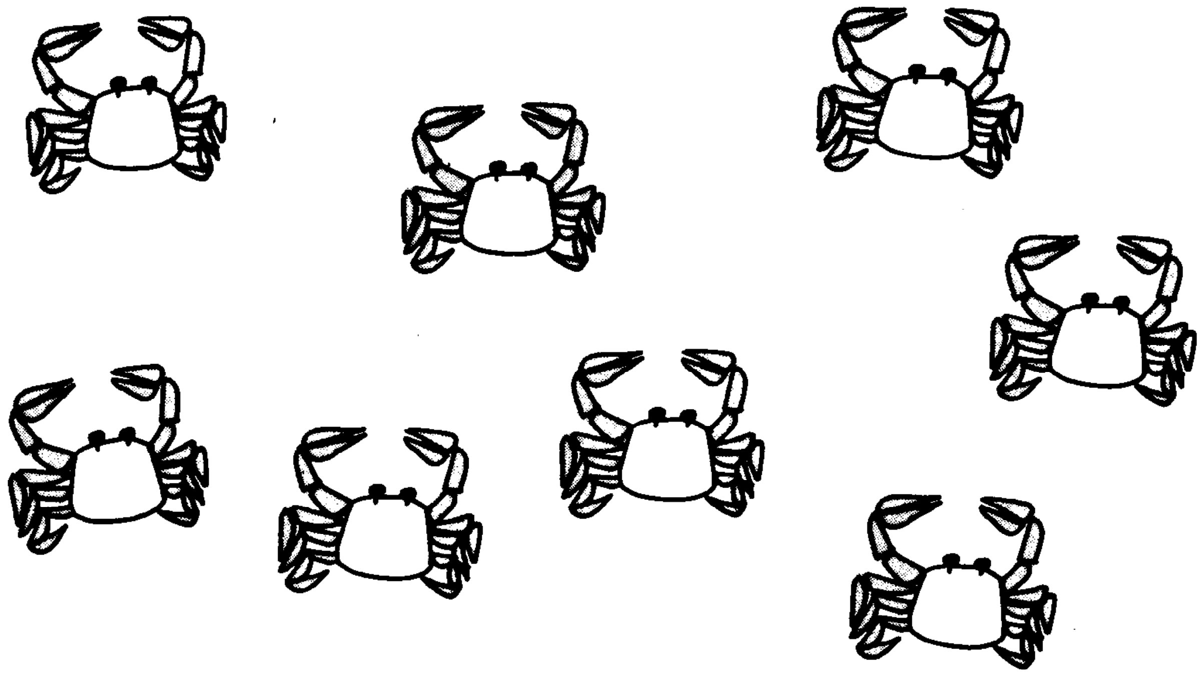
9 = 1 + 8 or 8+1

9 = 2 + 7 or 7+2

9 = 3 + 6 or 6+3

9 = 4 + 5 or 5+4

Partitions of 8. Write the equations.



Order of answers will vary.

8 = 0 + 8 or 8 + 0

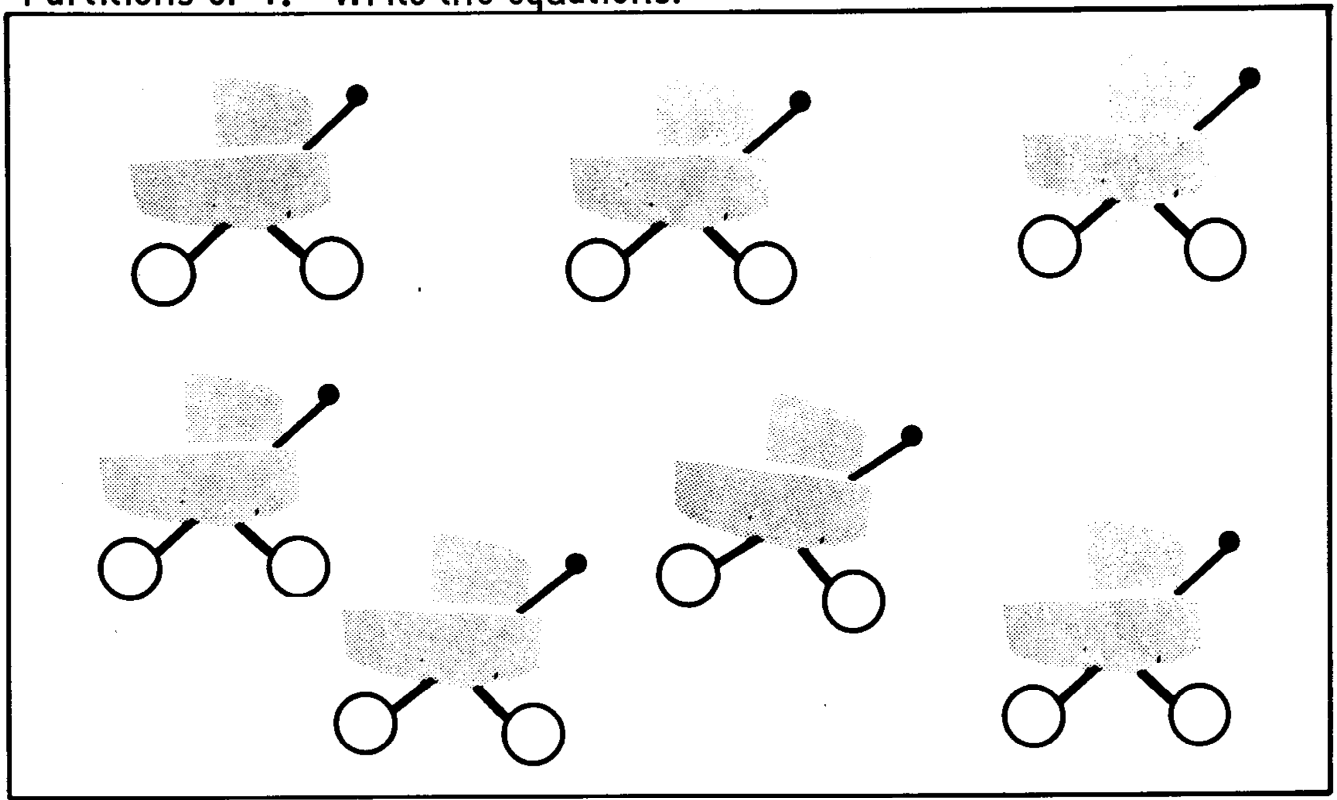
8 = 1 + 7 or 7 + 1

8 = 2 + 6 or 6 + 2

8 = 3 + 5 or 5 + 3

8 = 4 + 4

Partitions of 7. Write the equations.



Order of answers will vary.

7 = 0 + 7 or 7 + 0

7 = 1 + 6 or 6 + 1

7 = 2 + 5 or 5 + 2

7 = 3 + 4 or 4 + 3

7 = ____ + ____

Further Activities :

1. Duplicate sheets with sets of geometric figures. Have each child partition each set as he wishes. Then have children work in pairs, exchanging papers and writing the equations for the partitions.

2. Tell the children that you are thinking of a set with 5 members--that your set is partitioned into two subsets--and that there are 3 members in one of the subsets. Ask how many members are in the other subset. (If necessary, demonstrate on the flannel board how children could use a set of 5 objects to answer your question.) Repeat with other partitions of sets with no more than 5 or 6 members.

VII-2. Partitions and subtraction

Objective: To reinforce and extend the child's understanding of subtraction and addition by using partitions of sets.

Vocabulary: (No new terms.)

Materials: Small objects.

Background Note: If a set of 5 is partitioned into a set of 3 and a set of 2, and if the set of 2 is removed, then the set of 3 is the remaining set. We therefore see that $5 - 2 = 3$, because of the relation between subtraction and removing. By removing the set of 3, we see, in similar fashion, that $5 - 3 = 2$. Each partition thus leads to two subtraction equations. We have then 4 equations (two addition and two subtraction) for each partition into two non-equivalent subsets.

Throughout the following lesson you should encourage children to use sets of small objects to find sums and differences which they have not yet mastered.

Suggested Procedure:

Give each child 6 to 10 objects. Ask him to partition the set on his desk into any two subsets. (There will be many different partitions.) Tell the children to pick up the objects in one of the subsets, thus removing it from the set.

> What set do you see now on the desk? (The other subset.)

Ask each child to join to the set still on his desk the subset that he had removed.

> What set is on your desk? (The set I started with.)

Ask children to make the same partition again. This time ask them to remove the other subset.

> What set is on your desk? (The subset that we removed last time.)

Ask each to join the set he removed to the set that is on his desk.

> Do you have the set you started with? (Yes.)

Discuss that removing either of the subsets of a partition leaves the other subset as the remaining set, and that joining the subset which had been removed to the remaining set results in the set with which they started.

- Give each child seven objects. Ask the children to partition the set of seven so there are three members in one of the sets.

> How many are in the other set? (4.)
> What equation can we make about this partition? ($7 = 3 + 4$ or $7 = 4 + 3$.)
>
> If you remove the set of 3, how many members are in the remaining set? (4.)
> What equation suggests that you are removing a set of 3 from a set of 7? (hopefully, $7 - 3 = 4$.)

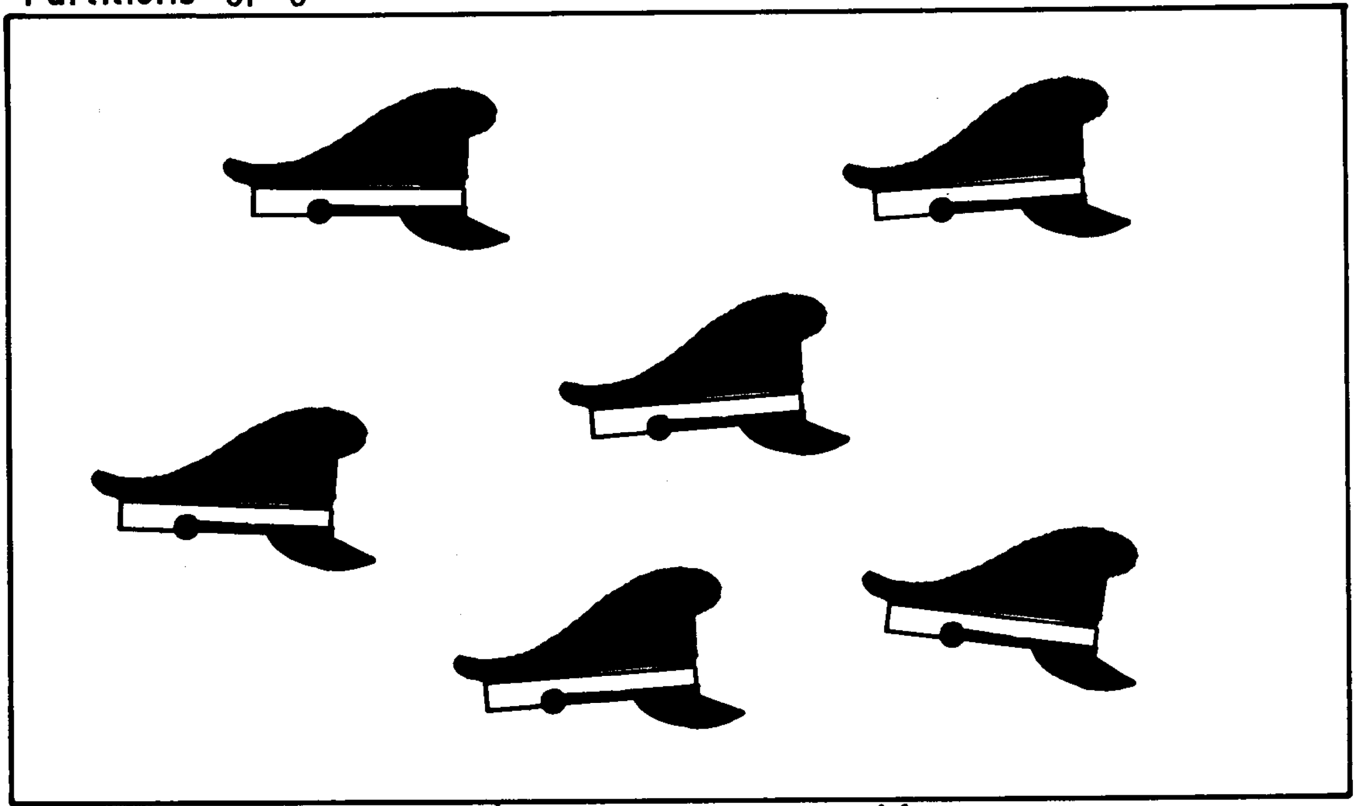
Say that there are several equations that are suggested by this partition. Try to get the children to state them: $3 + 4 = 7$, $4 + 3 = 7$, $7 - 3 = 4$, $7 - 4 = 3$.

Continue working with partitions of 7 and identifying the 4 equations associated with each partition.

<u>Pupil's book, pages 95 - 99</u>:

Ask the children to partition the set pictured at the top of the page by placing string or yarn across the box. Think of removing one subset which is formed and write the appropriate equation. Think of removing another subset and write the equation.

Partitions of 6



Order of answers will vary.

6 — 1 = 5 $\quad$ 6 - 0 = 6

6 - 6 = 0

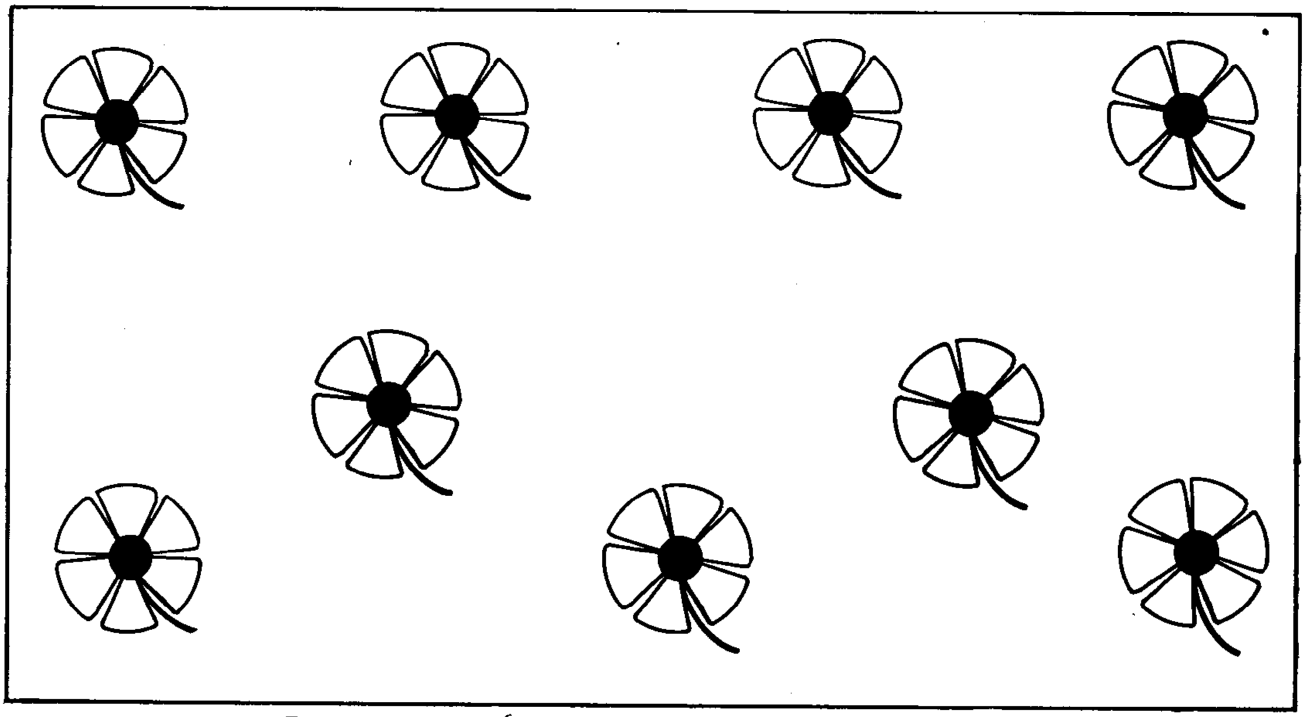
6 — 2 = 4

6 — 3 = 3

6 — 4 = 2

6 — 5 = 1

Partitions of 9



Order of answers will vary.

9 — 9 = 0 or 9-0=9

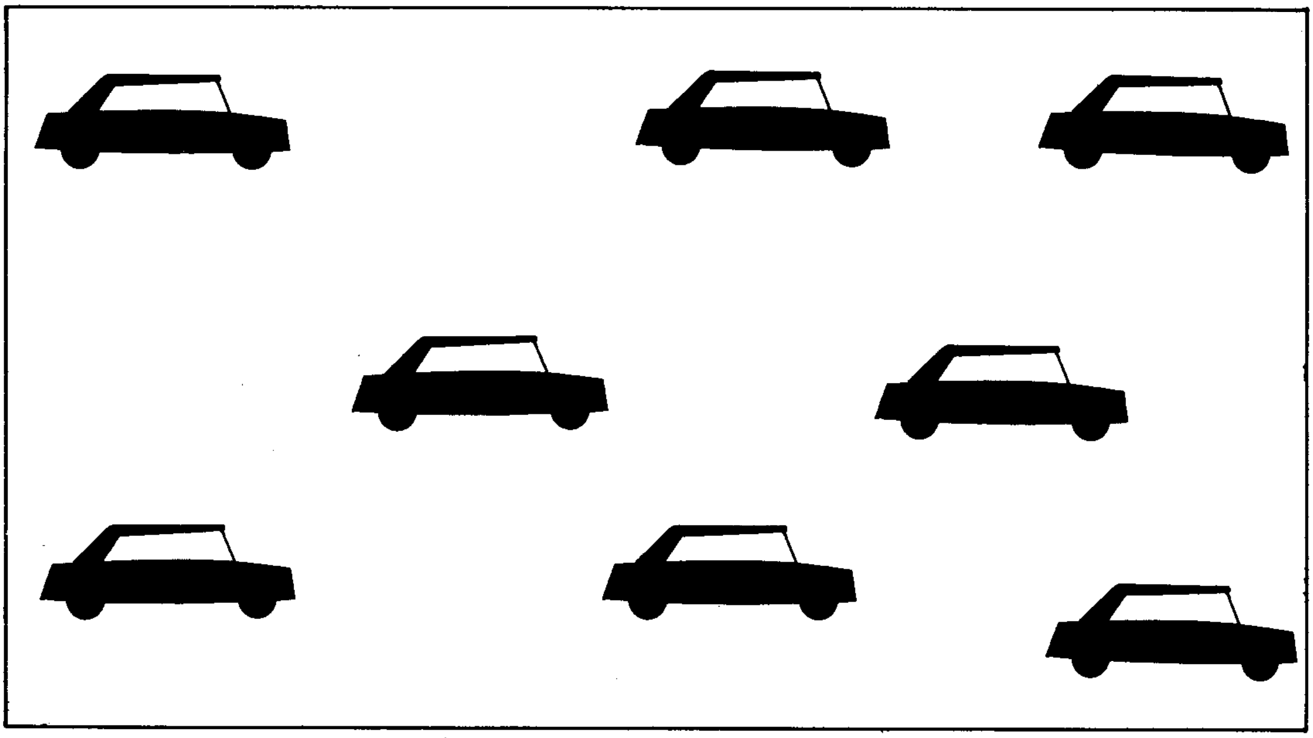
9 — 8 = 1 or 9-1=8

9 — 7 = 2 or 9-2=7

9 — 6 = 3 or 9-3=6

9 — 5 = 4 or 9-4=5

Partitions of 8



Order of answers will vary.

8 — 8 = 0 or 8 - 0 = 8

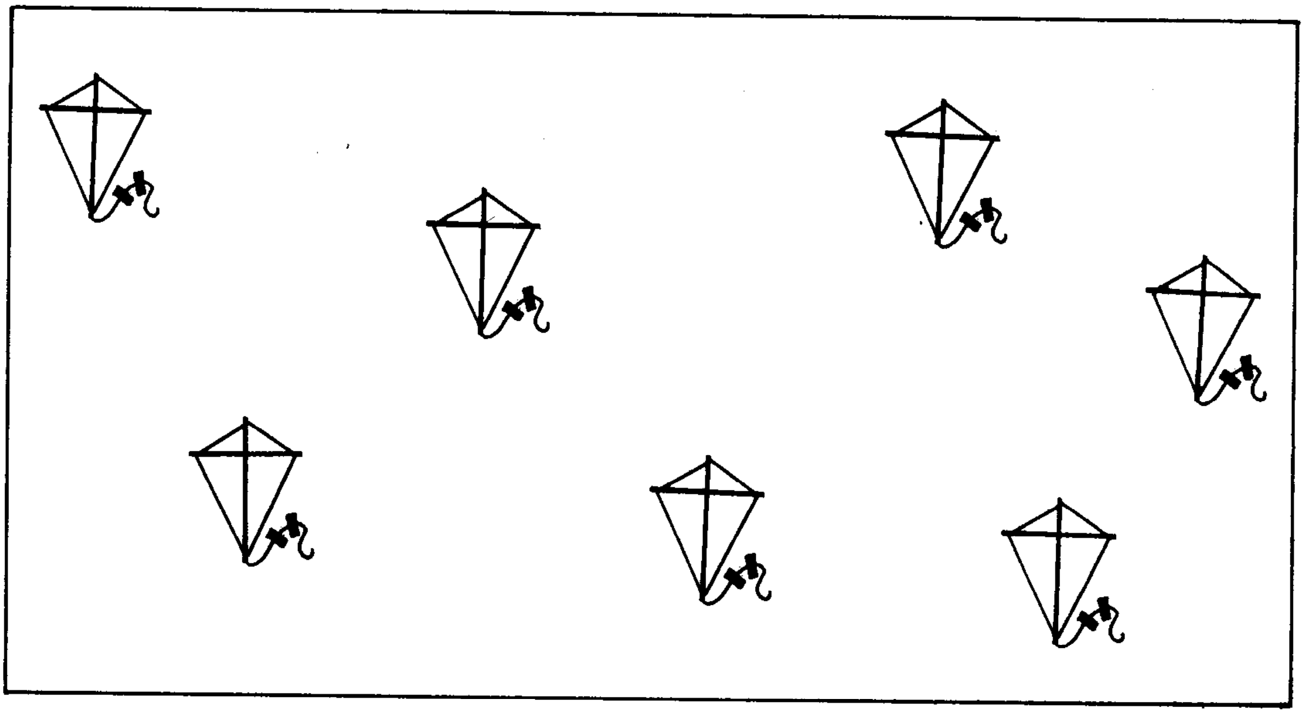
8 — 7 = 1 or 8 - 1 = 7

8 — 6 = 2 or 8 - 2 = 6

8 — 5 = 3 or 8 - 3 = 5

8 — 4 = 4

Partitions of 7



Order of answers will vary.

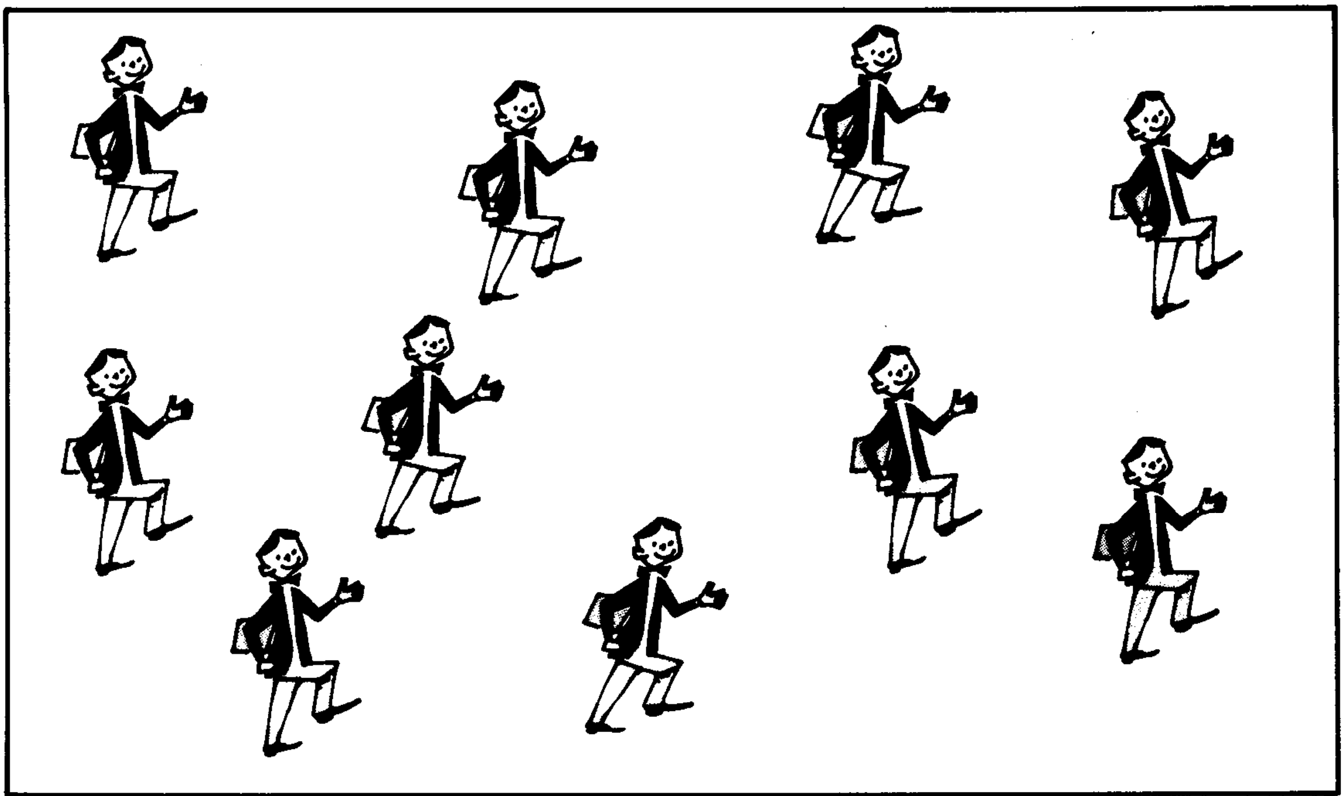
7 − 7 = 0 or 7 − 0 = 7

7 − 6 = 1 or 7 − 1 = 6

7 − 5 = 2 or 7 − 2 = 5

7 − 4 = 3 or 7 − 3 = 4

Partitions of 10



Order of answers will vary.

10 − 1 = 9 or 10 − 9 = 1

10 − 2 = 8 or 10 − 8 = 2

10 − 3 = 7 or 10 − 7 = 3

10 − 4 = 6 or 10 − 6 = 4

10 − 5 = 5

- Play "Acting out Stories". Ask a group of six children to come to the front of the class to act out stories. For example: To act out $6 = 2 + 4$, the children separate into a group of 2 and a group of 4, and then the groups come together. $(2 + 4 = 6$ or $4 + 2 = 6)$ To act out $6 - 2 = 4$, they begin in one set, and then a set of 2 children move away from the set of 6. To act out a partition of 6 into a set of 2 and a set of 4, they begin in a set of 6, then a set of 2 moves one way and a set of 4 moves the other way. (There are four equations for this play!) Ask the other children to hold up their hands as soon as they think of an equation that tells about the play, or later, ask them to write the equation on paper.

Dramatize problems such as the following. Children may use manipulative materials to represent the objects in each story problem.

> Four girls and three boys were playing kickball.
> How many children were playing kickball?
> Can you make an equation about the story?
>
> Billy had 8 marbles. He shared his marbles with John. Billy kept 5 marbles. How many marbles did John get?
> Can you make an equation about the marbles?
>
> Sally was helping her mother set the table. She carried 4 plates to the table. Then she went back to the kitchen to get 3 more plates. How many plates did Sally put on the table?
> Can you make an equation about the plates?
>
> Ethel and Jim were counting cars. Jim counted white cars, and Ethel counted red cars and blue cars. Ethel counted 6 red cars and 3 blue cars, and Jim counted 8 white cars. Who counted the most cars?

Can you make an equation about the cars Ethel counted?

Polly had nine crayons. She kept some of them in a shoe box, but she kept 3 in her pocket. How many crayons did Polly keep in the shoe box?
Can you make an equation about Polly's crayons?

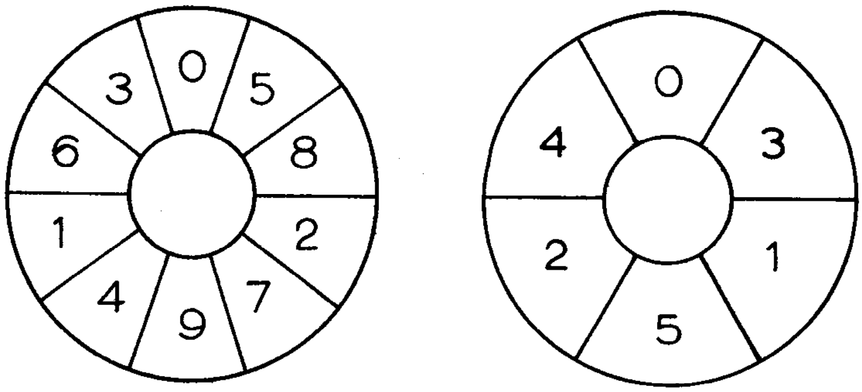
Use Drill Doughnuts to reinforce the learning of addition and subtraction. 2 + , 1 + , 3 + , 4 + , 5 - , and 6 - might now be appropriate. (Instructions for making Doughnuts are on page 283.)
Pupil's book, pages 100 - 102:

Each box shows a set which has been partitioned. Write two addition and two subtraction equations suggested by the partition.

Drill Doughnuts

"Drill Doughnuts" may be used to advantage for additional practice in adding and subtracting, both for the basic facts and for encouraging mental computation later, e.g., 35 + 4, 49 - 6, etc.

For each child in the class, prepare a Doughnut. Cut from cardboard, tagboard, or other heavy stock. Circle is $4\frac{1}{4}$ inches in diameter, center hole is 1 inch in diameter. (Use red felt pen to write the numerals on one side and blue felt pen to write the numerals on the other side.)



Blue side Red side

Give each child a Doughnut and a sheet of newsprint, 9 x 12. Tell children to fold paper in half (to yield two sections on each side of paper, each 9 x 6). Have Doughnut placed on paper so that zero is at the top and there is room on each section of paper to write numerals around the edge of the Doughnut. Tell children to hold Doughnut still, not to trace around it, but to write on paper through the hole in the middle "4 + ". Beyond the circumference of the Doughnut, on the newsprint, they should write sums of 4 and the numbers indicated on each section of the Doughnut.

Next move Doughnut to another section of the paper. Give the operation sign and the number to be written: For example, "6 -" might be written in the center of the Doughnut. Children again write answers around the Doughnut on the paper. This can be repeated when the paper is turned over.

When all four sections of paper are used, 40 problems have been done if blue side is used, 24 if red side is used. Only the center entry and the answers appear for each section and are necessary in making a key for checking papers.

Pupil's book, pages 103 - 104:

Ask the children to fill in the blank spaces in the tables. You should fill in a similar addition table on the chalkboard before asking the children to do these sheets.

Pupil's book, pages 105 - 106:

Ask the children to fill in the blank spaces in the subtraction table.

Pupil's book, page 107:

Ask the children to look at the sums in the left hand column and to circle each which names 8. Those which name ten in the right hand column are to be circled.

Pupil's book, page 108:

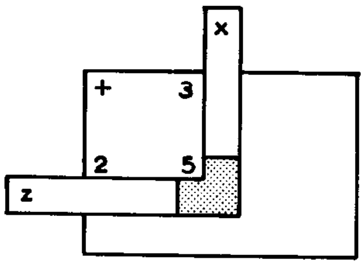
On this page the pupil is to ring the sum or difference which names the number listed at the top of the page.

Pupil's book, page 109:

Mark the box after 7 with red, after 8 with blue, after 9 with yellow; we use these as keys. Ring in red each name for 7, in blue each name for 8, in yellow each name for 9.

★ Further Activity:

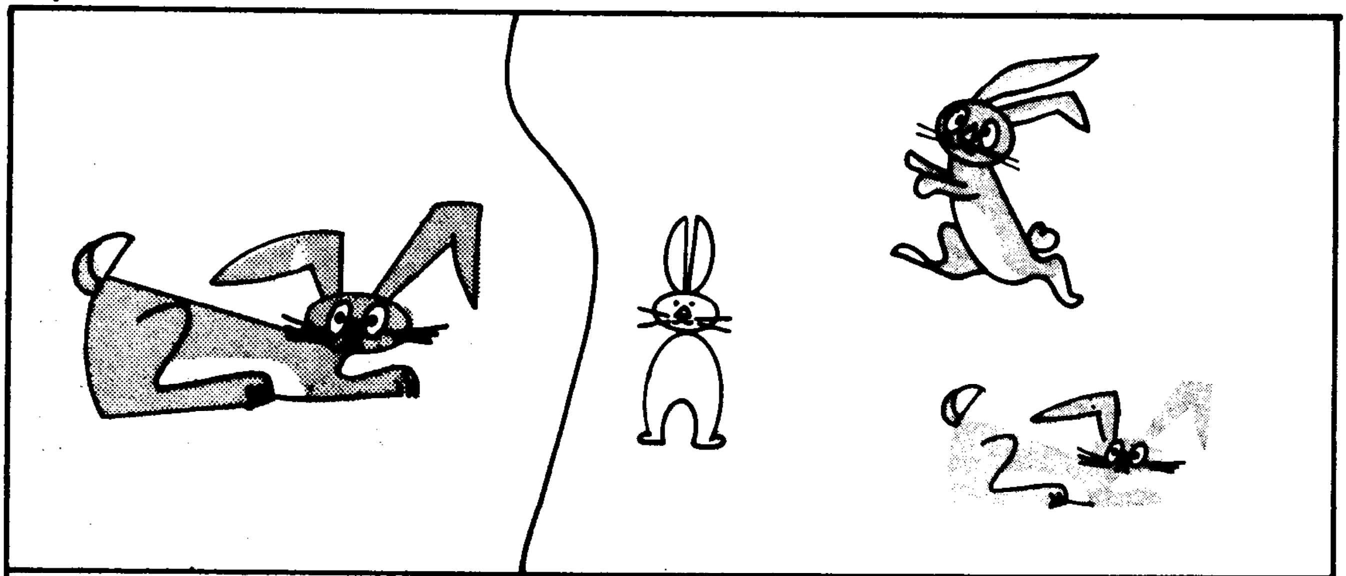
The addition chart can be used as a review of addition facts. It can be used to solve an equation. If a number on the top is added to a number on the left, place a (which may be cut from plain white paper) so that the numerals nearest the shaded edges (left of side x and above side z) are the numbers which you wish to add. The answer is the number at the point where the two sides of the join (darker shading).



★ Pupil's book, page 110:

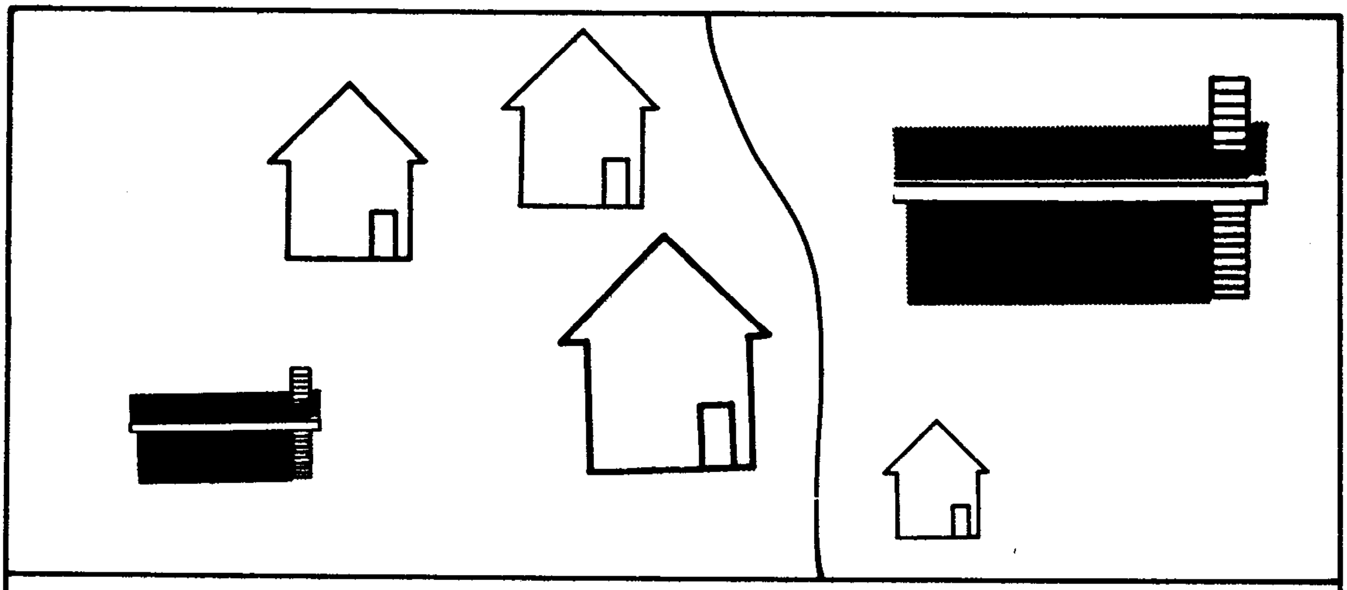
Direct children to write the equation which was completed when A was the answer. Continue with B, C, D, etc.

Equations



1 + 3 = 4 and 3 + 1 = 4

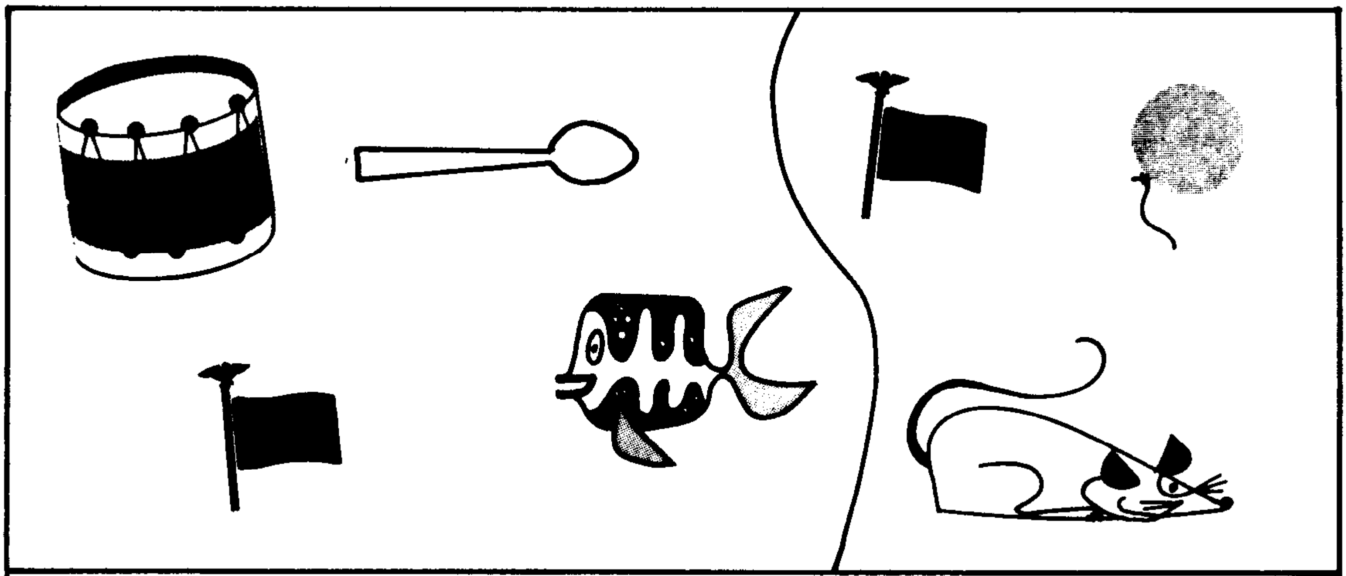
4 - 3 = 1 and 4 - 1 = 3



4 + 2 = 6 and 2 + 4 = 6

6 - 2 = 4 and 6 - 4 = 2

Equations



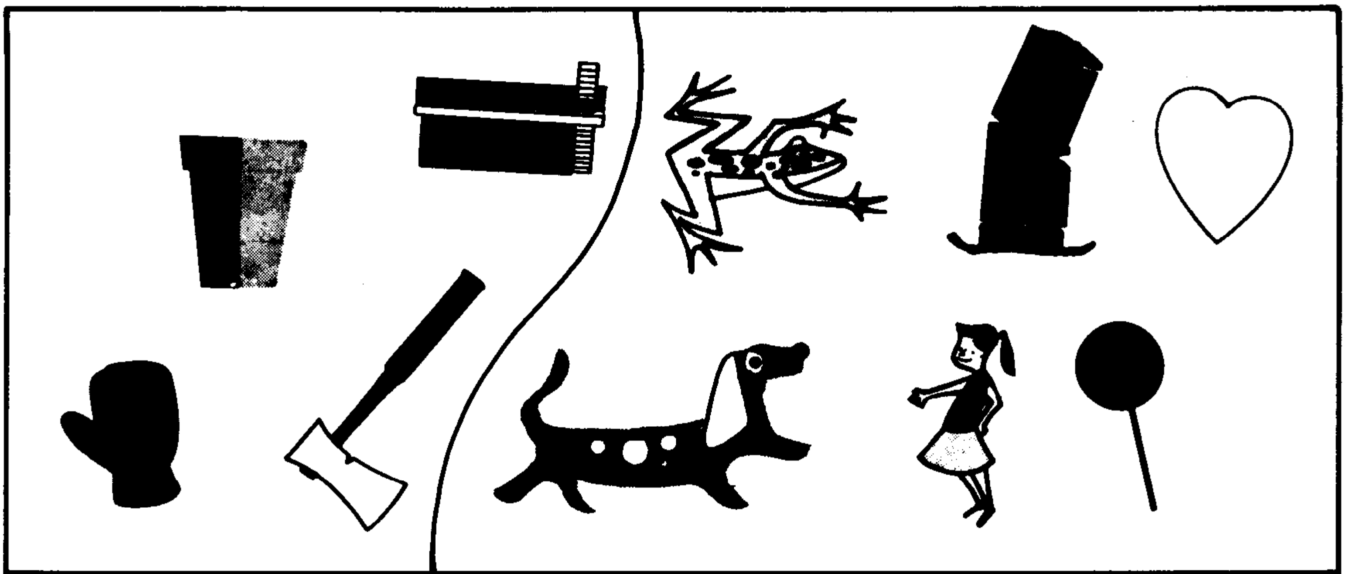
4 + 3 = 7 and 3 + 4 = 7

7 - 3 = 4 and 7 - 4 = 3

3 + 5 = 8 and 5 + 3 = 8

8 - 5 = 3 and 8 - 3 = 5

Equations



4 + 6 = 10 and 6 + 4 = 10

10 - 6 = 4 and 10 - 4 = 6

6 + 3 = 9 and 3 + 6 = 9

9 - 3 = 6 and 9 - 6 = 3

Addition

Fill in the tables.

	+5	+3
2	7	5
4	9	7

	+2	+6
3	5	9
4	6	10

Addition

Fill in the tables.

	+2	+0
7	9	7
6	8	6

	+3	+2
5	8	7
2	5	4

Subtraction

Fill in the tables.

	-3	-4
8	5	4
6	3	2

	-2	-4
5	3	1
7	5	3

Subtraction

Fill in the tables.

	-5	-4
9	4	5
8	3	4

	-4	-3
7	3	4
6	2	3

Which are equal to 8?	Which are equal to 10?
7 + 1	6 + 2
5 + 2	5 + 5
6 + 2	4 + 5
3 + 5	2 + 8
4 + 3	7 + 4
5 + 4	3 + 7
1 + 6	0 + 9
4 + 4	4 + 6
8 + 0	1 + 9

Which are equal to 7?

3 + 4

4 + 4

8 + 1

2 + 7

5 + 3

6 + 1

9 + 3

1 + 8

2 + 5

Which are equal to 9?

8 + 1

8 + 2

5 + 4

2 + 6

3 + 6

10 + 1

1 + 8

7 + 5

9 + 0

★ Names for Numbers

7 $\square$ red	8 $\square$ blue	9 $\square$ yellow
3 + 4 red	2 + 5 red	4 + 4 blue
0 + 9 yellow	8 + 0 blue	5 + 1
6 + 1 red	3 + 8	0 + 8 blue
6 + 2 blue	3 + 3	3 + 1
6 + 0	1 + 8 yellow	3 + 6 yellow
5 + 4 yellow	2 + 7 yellow	1 + 6 red
7 + 2 yellow	7 + 4	2 + 6 blue
7 + 1 blue	4 + 3 red	8 + 1 yellow

★ Addition Table

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	B 5	6	7	8	9	
2	2	3	4	5	6	7	8	F 9		
3	3	A 4	5	6	7	C 8	9			
4	4	5	6	D 7	8	9				
5	5	6	7	8	9					
6	6	7	E 8	9						
7	7	8	9							
8	G 8	9								
9	9									

Write the equations:

A $3 + 1 = 4$

B $1 + 4 = 5$

C $3 + 5 = 8$

D $4 + 3 = 7$

E $6 + 2 = 8$

F $2 + 7 = 9$

G $8 + 0 = 8$

VII-3. Addition and subtraction on the number line

Objective: To extend the children's understanding of addition and subtraction by using the number line.

Vocabulary: (No new words.)

Materials: (Possibly, construction paper for number line.)

Suggested Procedure:

The number line (which was introduced in Chapter 2, Section 3) may be used as an aid in addition and subtraction.

Review carefully the fact that the numeral on the line shows the number of jumps from the starting point, 0.

> Find the point on the number line which shows that you have taken 8 jumps from the starting point, 0.
>
> Find the point which shows that you have taken 3 jumps, from the starting point, 0.
>
> Find the point on the number line which shows that you have taken 0 jumps from the starting point 0.
>
> Find the point which shows that you have taken 3 jumps and then 2 jumps, from the starting point 0.

If this is difficult, ask a child to point to the place on the number line which shows that he has taken 3 jumps from the starting point. Use a pointer or pencil to indicate the point on the line. Ask him not to go back to the starting point but to take two more jumps. (It may be helpful to think of this as a "stop for a rest". A number line drawn on the floor can be used if necessary to dramatize this idea.)

> Now that you have taken 3 jumps and 2 jumps from the starting point, where are you on the number line? (5.)
>
> This tells us that three plus two equals five.

Write the equation, $3 + 2 = 5$.

> If we have a story problem, we can solve it on the number line if we know the numbers which we want to add.

Jerry had 5 books.
He bought 2 books.
How many books does he have?

What is the number of books which Jerry had to begin with? (5.)

Ask a child to take the same number of jumps from the starting point on a number line as the books Jerry had.

How many books did Jerry buy?

(2.)

Ask the child to take that many more jumps on the number line, this time starting at 5.

What is the number of jumps that you have taken? (7.)

We wanted to add 5 and 2. When we take those numbers of jumps on the number line we find that we have taken 7 jumps. The set of 5 books and 2 books if joined would be the same number of books. Thus, $5 + 2 = 7$.

- <u>Subtraction</u>

Draw a number line on the chalkboard.

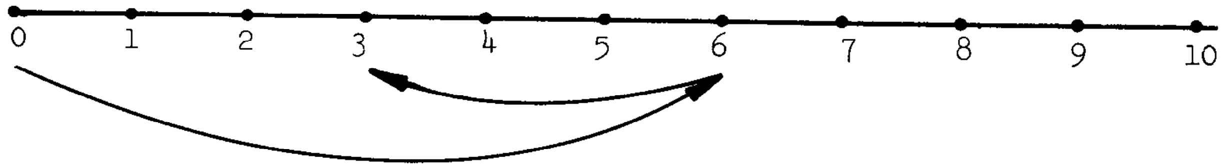
Write the equation, $6 - 3 =$ ____.

To complete this equation we must first take six jumps on the number line. Start at 0.

Ask a child to do this and either to mark the point where he stops or to draw the curve which shows the number of jumps.

To solve the equation we must subtract 3 from six. On the number line this means we will have to go back three jumps.

Ask a child to touch the point which shows where you would be if you went back three jumps from 6. Draw the curve which shows you have gone back 3 jumps.



Where did we stop this time? (3.)
When we went 6 steps forward and back 3 steps we stopped at 3. This is one way of showing that 6 minus 3 equals 3.

It may be necessary to use a line which is drawn on the floor for the first work with subtraction on the number line. This would enable children to take jumps forward and then to come back on the number line.

Some children have difficulty using the number line to complete equations because they are confused by the numerals at which they are looking while they count the second set of jumps.

Two ways to use the number line are given here. You may want to develop one to a great extent and exclude the other or try to use both of the ideas suggested.

1. Make a number line on a piece of cardboard or oaktag 12" x 36". This should be large enough for all children to see. Fold the top of the paper to cover the numerals.

(Figure 1) Place a pencil on the starting point. (0) Take jumps on the number line which correspond to the set to which another set is joined in the problem. (Some children will make one jump at a time (Figure 2) while other children will go to five in one jump (Figure 3).)

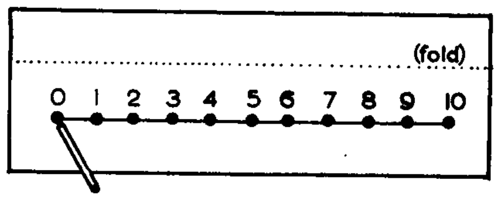


Figure 1 (Open)

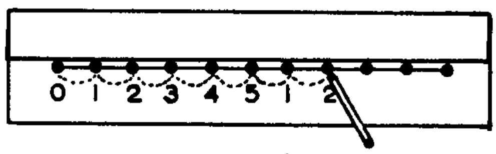


Figure 2 (Closed)

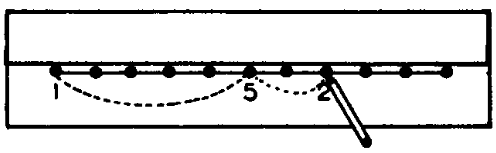


Figure 3 (Closed)

Move the pencil to that point. Some children may need to move the pencil as they take each jump. Keep the pencil on the last point and unfold the page. The pencil will be on the point which tells the total number of jumps. (Figure 4)

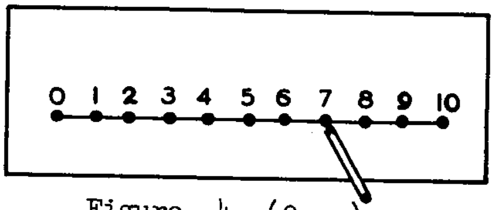
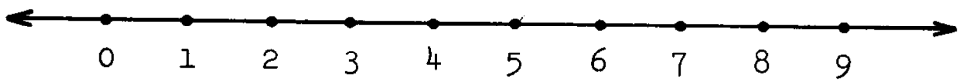


Figure 4 (Open)

Pupil's book, pages 111 - 114: have been designed to use in this way.

Some children may leave the page unfolded to find the point which shows the number of the first set, then fold the page and take jumps which correspond to the second set and open the page to find how many jumps in all. Children who are not confused by the numerals and can use the page without needing to fold it should be encouraged to do so. Subtraction is developed by finding the number of members in the set described and relating the removing of a subset to "backing up" on the number line. This page may be helpful in solving other story problems or incomplete equations such as $3 + 2 = \_\_\_\_$. Pages in the pupil's book should not be used unless children have had enough experience with this number line to use it independently and without difficulty.

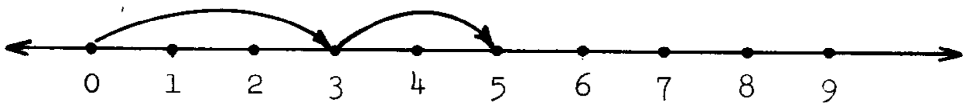
2. Draw a number line on the chalkboard.



Write an equation, $3 + 2 = \_\_\_$

Place the chalk on the starting point. Ask a child to put a finger on the point which tells the number of jumps corresponding to the number of the first set. (The child should touch 3.) Draw a curve from the starting point to the point the child is touching.

Now, <u>using</u> 3 <u>as the starting point</u> ask where 2 more jumps would take us. (Child should now touch 5.) Draw a curve from the 3 to the 5. This represents the second set of jumps.

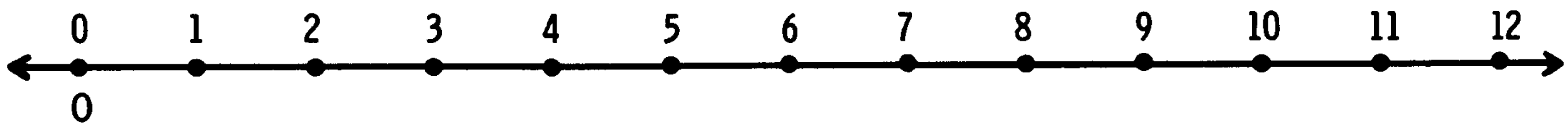


The point where the last curve ends tells the number of jumps and enables the children to complete the equation.

<u>Pupil's book, pages 115 - 117</u>: are designed to be used in this way.

Heavy plastic taped over cardboard on which a number line has been drawn can be useful as an aid to independent work. The child can mark with a crayon the curve which shows the jumps he has taken and then remove the mark with a paper towel and use the same number line again.

Fold until edge covers numbers



$2 + 1 = \boxed{3}$

$5 + 1 = \boxed{6}$

$1 + 5 = \boxed{6}$

$1 + 2 = \boxed{3}$

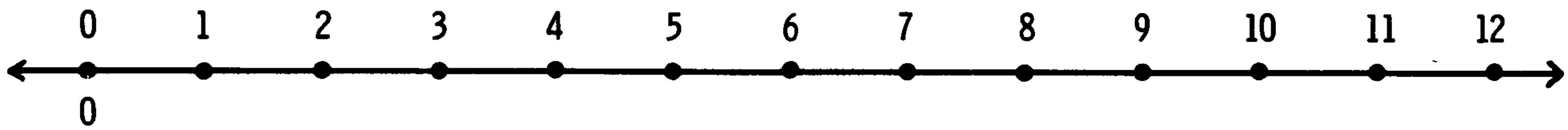
$5 + 3 = \boxed{8}$

$2 + 4 = \boxed{6}$

$1 + 6 = \boxed{7}$

$3 + 4 = \boxed{7}$

Fold until edge covers numbers



$3 + 2 = \boxed{5}$

$4 + 5 = \boxed{9}$

$4 + 2 = \boxed{6}$

$2 + 5 = \boxed{7}$

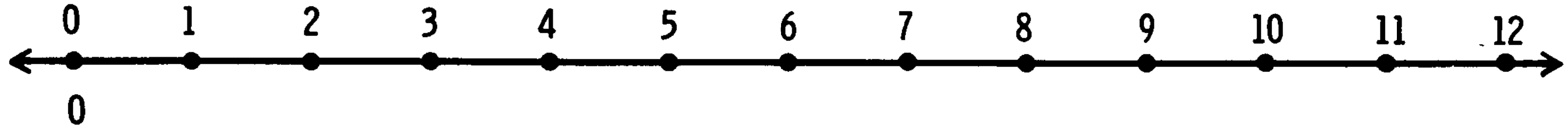
$1 + 7 = \boxed{8}$

$6 + 2 = \boxed{8}$

$3 + 5 = \boxed{8}$

$4 + 3 = \boxed{7}$

Fold until edge covers numerals



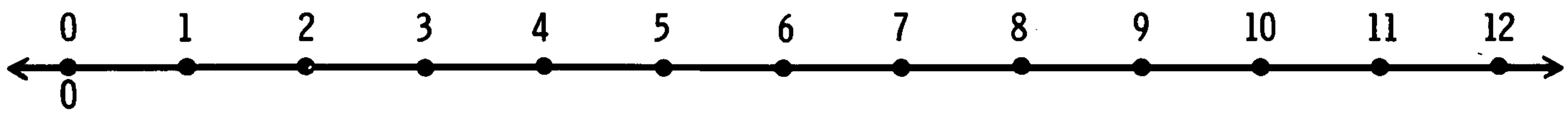
$5 - 3 = \boxed{2}$ $\quad$ $9 - 7 = \boxed{2}$

$8 - 5 = \boxed{3}$ $\quad$ $6 - 1 = \boxed{5}$

$7 - 2 = \boxed{5}$ $\quad$ $5 - 2 = \boxed{3}$

$8 - 2 = \boxed{6}$ $\quad$ $5 - 1 = \boxed{4}$

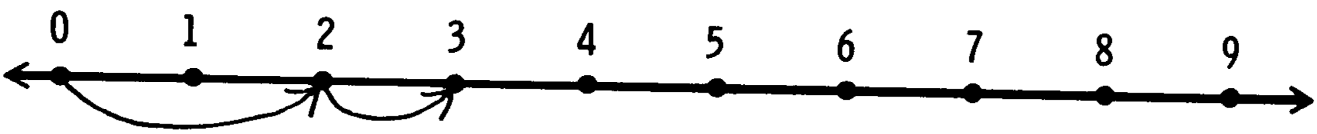
Fold until edge covers numerals



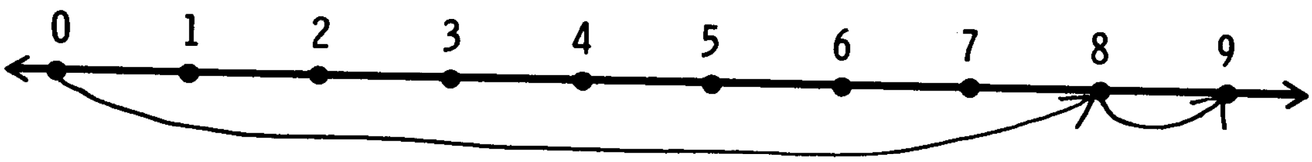
$8 - 1$	$\boxed{7}$	$2 + 7$	$\boxed{9}$
$3 + 6$	$\boxed{9}$	$9 - 5$	$\boxed{4}$
$10 - 3$	$\boxed{7}$	$1 + 4$	$\boxed{5}$
$7 - 6$	$\boxed{1}$	$2 + 6$	$\boxed{8}$

Use the number line.

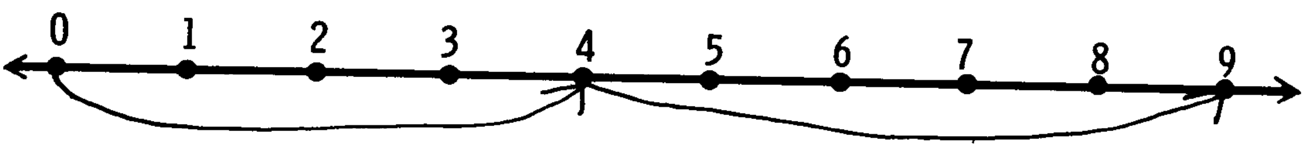
$1 + 3 = \boxed{4}$



$2 + 1 = \boxed{3}$

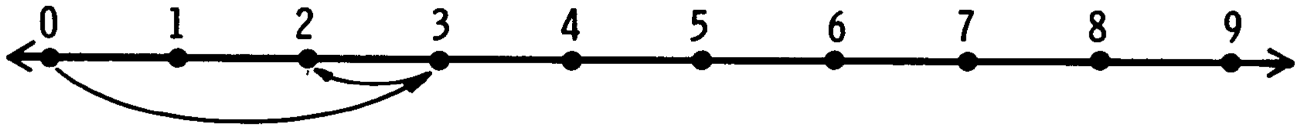


$8 + 1 = \boxed{9}$

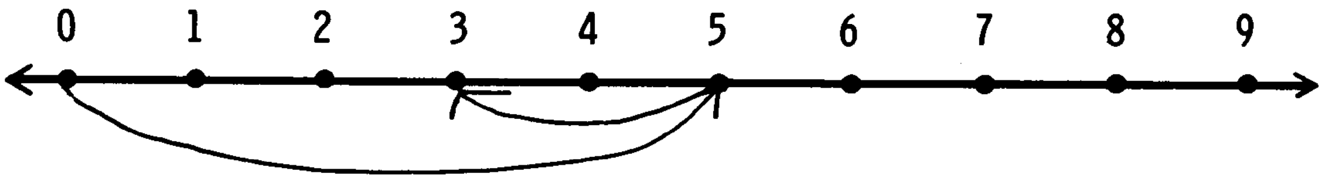


$4 + 5 = \boxed{9}$

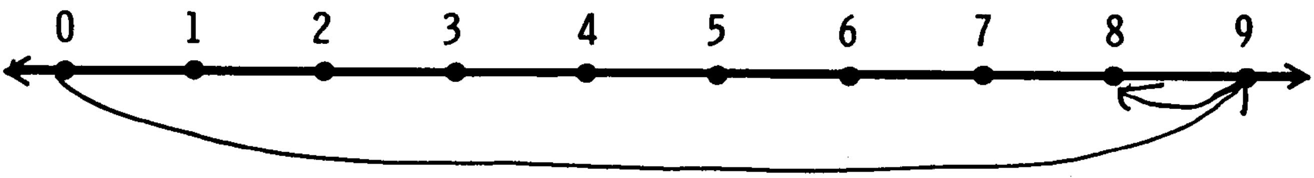
Use the number line.



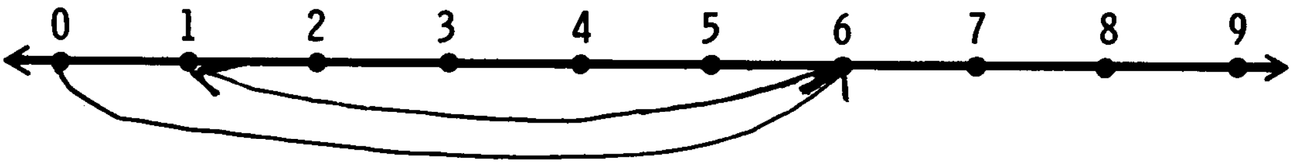
$$3 - 1 = \boxed{2}$$



$$5 - 2 = \boxed{3}$$

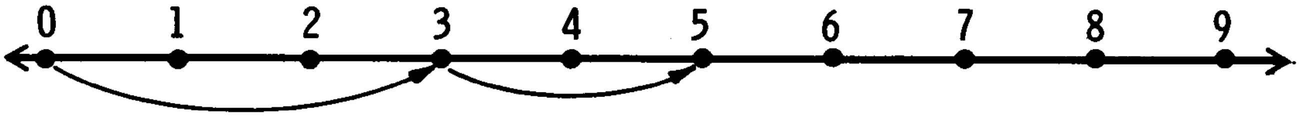


$$9 - 1 = \boxed{8}$$

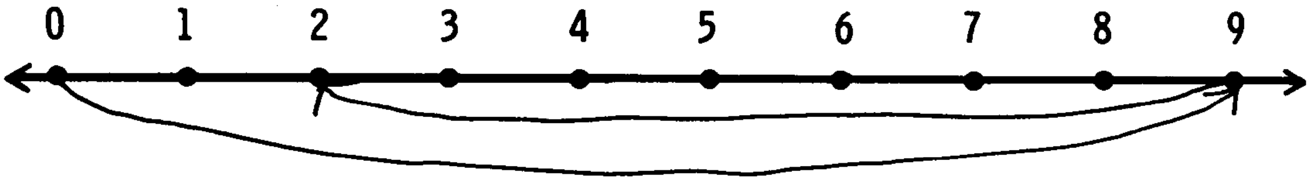


$$6 - 5 = \boxed{1}$$

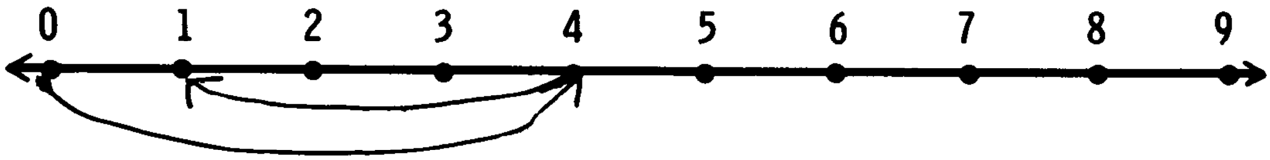
Use the number line.



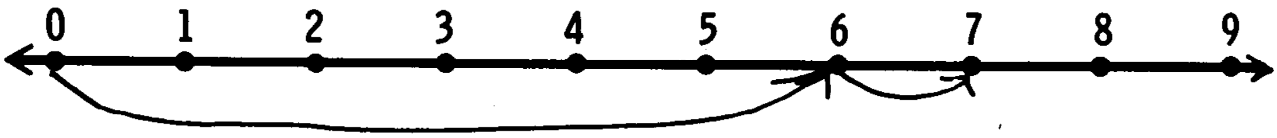
$$3 + 2 = \boxed{5}$$



$$9 - 7 = \boxed{2}$$



$$4 - 3 = \boxed{1}$$



$$6 + 1 = \boxed{7}$$

VII-4. How many more?

Objective: To find how many more members there are in one set than in another set.

Vocabulary: (No new words.)

Materials: Material, such as apples and lemons for flannel board display, flannel pairing symbols, two sets of numeral cards, (Sandpaper or flannel strips on the back of the card will make it stay on the flannel board).

Background Note:

Recall that one set has more members than a second set if, when members of the first are paired with the members of the second, there are members of the first left over. In this lesson we continue development of this idea by using subtraction to find how many more members there are in the first set. We say that there are 2 more members in a set of 5 than in a set of 3 because, if we pair members of the set of 5 with members of the set of 3, there are 2 members left over. We also say that there are 2 fewer members in the set of 3.

Suggested Procedure:

Ask a child to place a set of 3 apples on the flannel board. Ask another child to place a set of 5 lemons on the board.

Ask whether there are more lemons, or more apples, and ask whether we can decide which set has more members than the other without counting. Bring out that we can accomplish this by pairing the lemons with the apples. Have one of the children show the pairing with pairing symbols or with yarn. Ask how many more lemons there are than apples. (2.) Explain that we say that there are 2 more because, if we remove the set of lemons that are paired to apples, the remaining set has 2 members. Display the numeral cards for the set of lemons, the set of apples, and ask the children if they can make an equation about these sets by observing that there are $(5 - 3)$ or 2 more lemons than apples, and that $5 - 3 = 2$ is the equation which tells us how many more lemons. Also, ask how the equation $3 + 2 = 5$ is suggested by this problem.

Repeat this procedure with other sets, in each case displaying the pairing and expressing the equation.

Give each child some small objects. Ask each to place a set of 8 members on one side of the desk, and a set of 5 members on the other side. Ask them how many members are in the first set. Ask what equation describes the problem, and display the equations which are suggested.

$$5 + 3 = 8 \qquad\qquad 8 - 5 = 3$$

Repeat, sometimes with the first set having more members and sometimes with the second set having more members.

<u>Pupil's book, pages 118 - 124</u>:

Ask the children to first find out how many members are in each set. Then ask them to pair the members of the set on the left with the members of the set on the right. They are then to decide which set has more members, and how many more.

(You may want to follow up the work on the pages by asking how children compared the sets. This would also be helpful to develop an awareness of the ways in which children find out how many more.

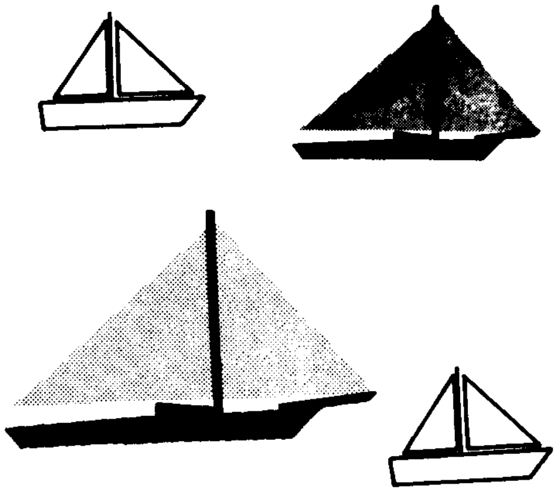
Some children will have paired the sets by touching or marking the same number in each set. Some children will have subtracted the numbers; for this you will want to show the equation $6 - 2 = 4$. Some children will have thought of the additional number of members needed in order to have equivalent sets; for this you will want to write the equation $2 + \underline{\quad} = 5$.)

- Problem Solving:

Give each child some small objects to work with at this desk. Ask the children to select sets of objects equivalent to objects in the story problems. Notice that the problems differ in the kind of set operation--joining, removing, partitioning. In these problems, children should be expected to find the number to be used in answering the question and then answer the question. An equation is not expected.

> Tom has 6 marbles. Joe has only 4 marbles. How many more marbles has Tom than Joe?
>
> Jane had 3 crayons. Then Mary gave her 5 crayons. Now how many crayons does Jane have?
>
> Bill had 9 cars and Sue took 3 of them. How many cars does Bill have?
>
> Sharon had 5 apricots. John had 8 apricots How many more apricots did John have than Sharon?
>
> Sara has 3 more dolls than Susan. Susan has 4 dolls. How many dolls does Sara have?

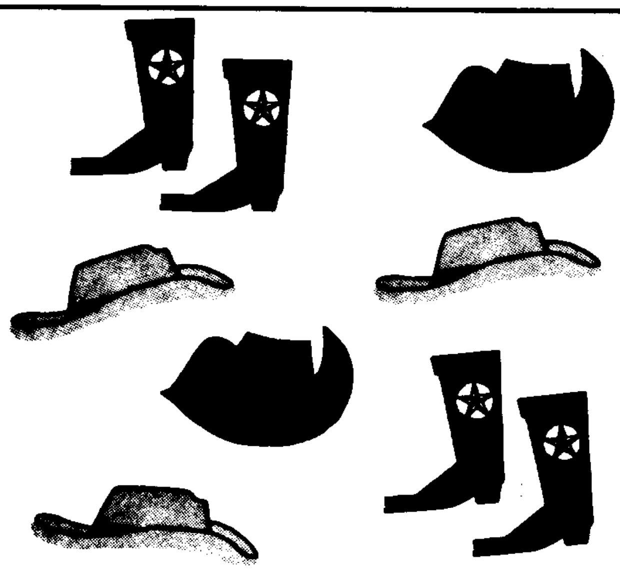
Comparing Sets

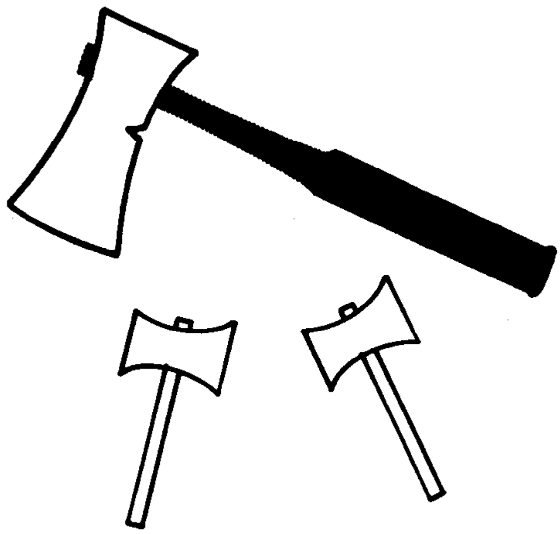


How many? 6

How many? 4

How many more? 2



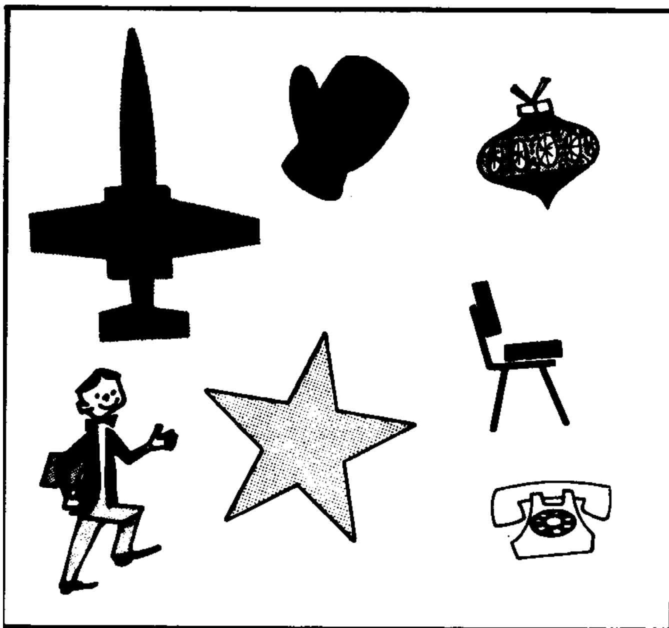


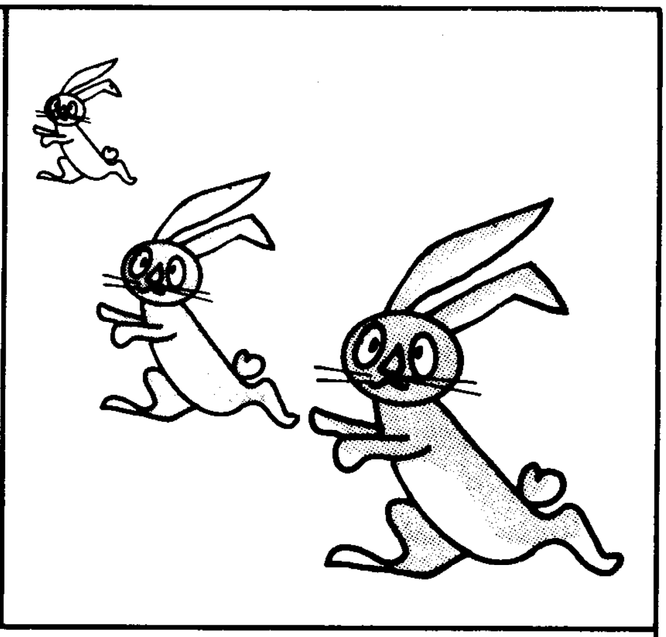
How many? 9

How many? 3

How many more? 6

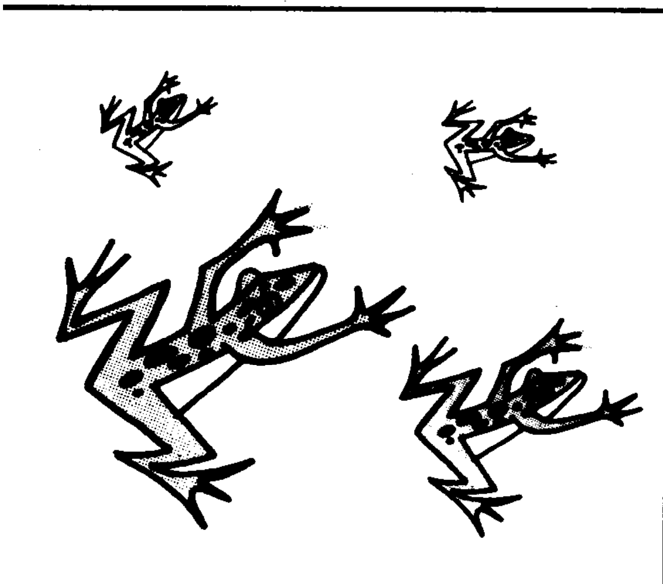
Comparing Sets

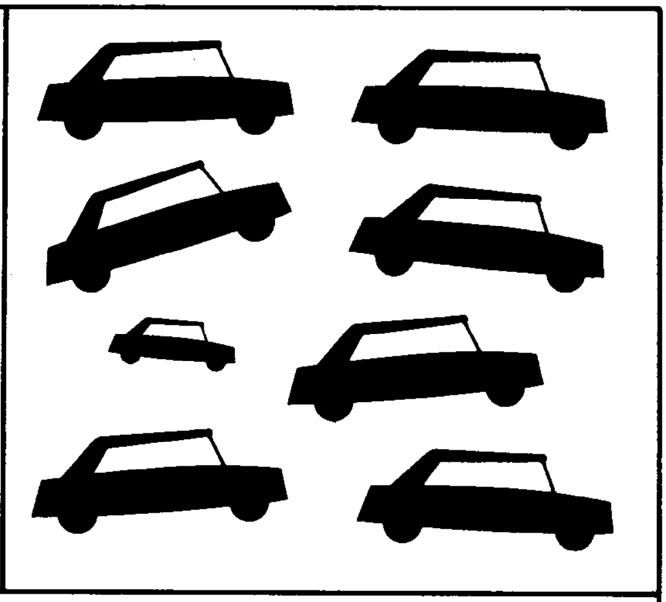




How many? 7 How many? 3

How many more? 4

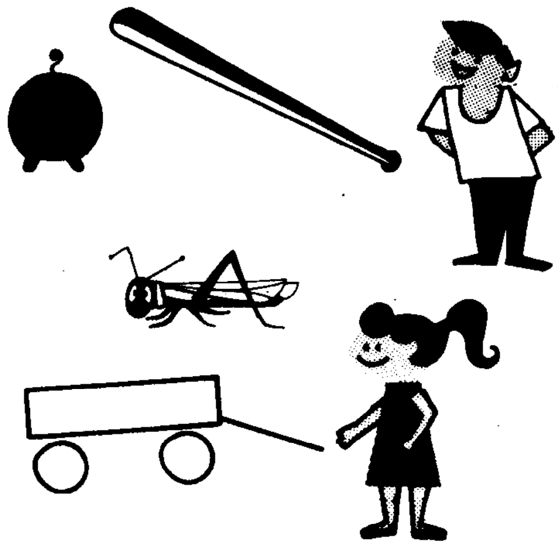


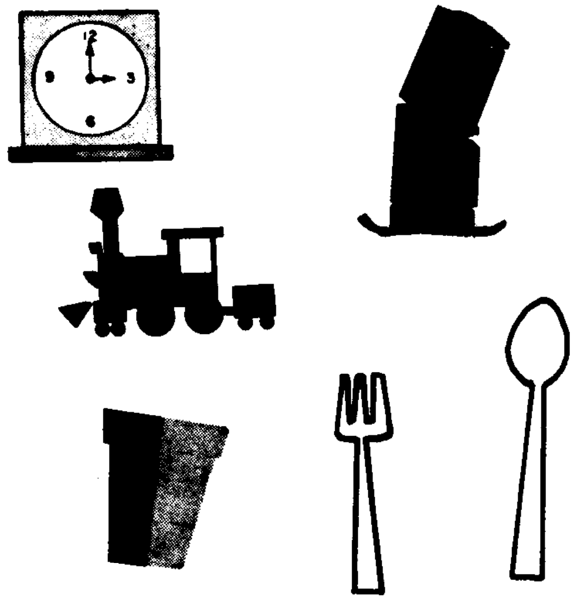


How many? 4 How many? 8

How many more? 4

Comparing Sets

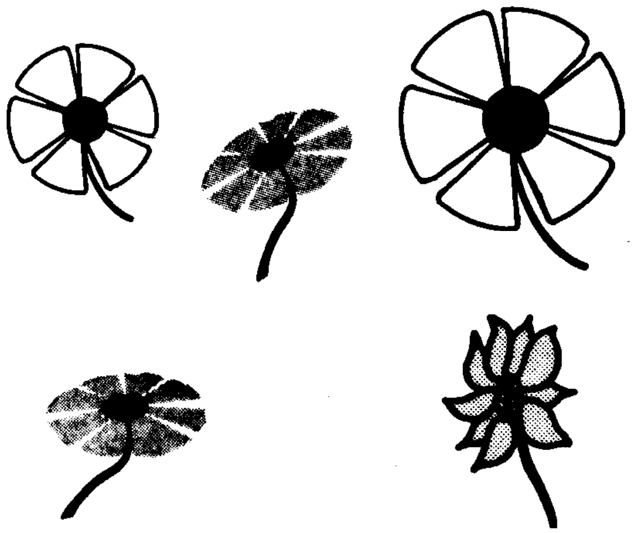




How many? 6

How many? 6

How many more? 0

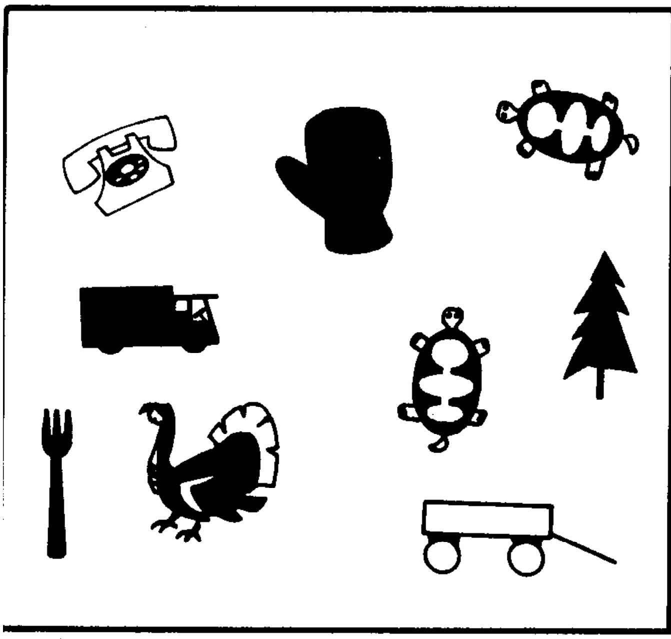


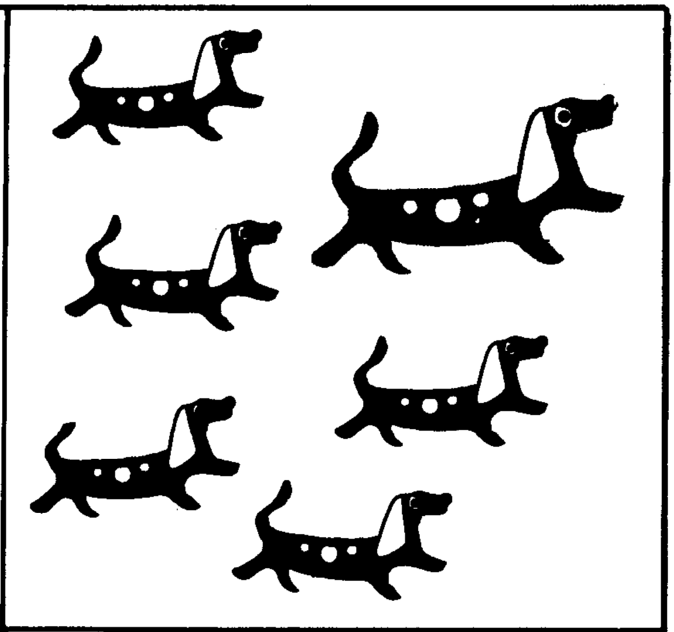
How many? 5

How many? 10

How many more? 5

Comparing Sets

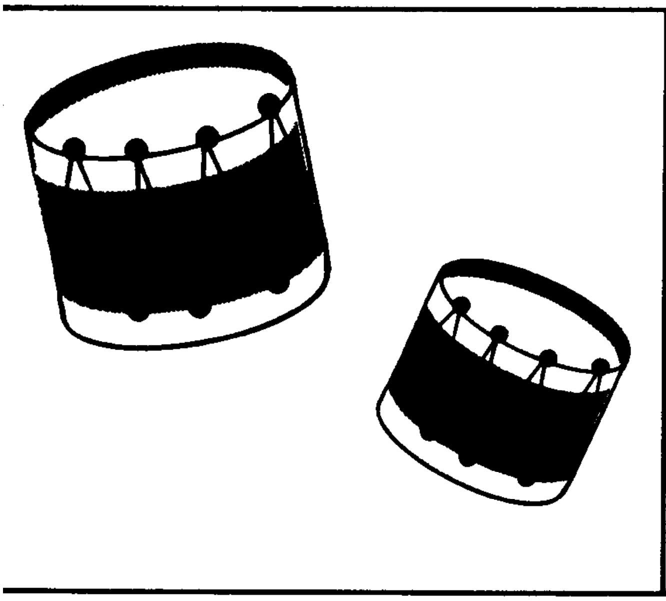


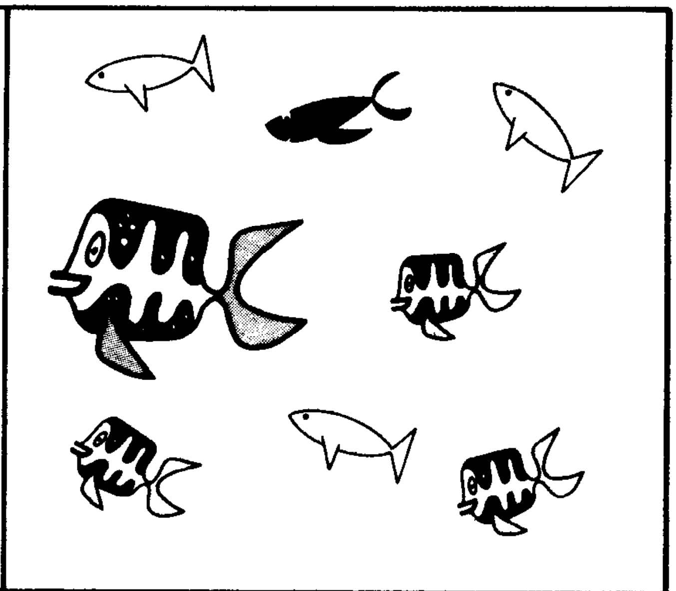


How many? 9

How many? 6

How many more? 3



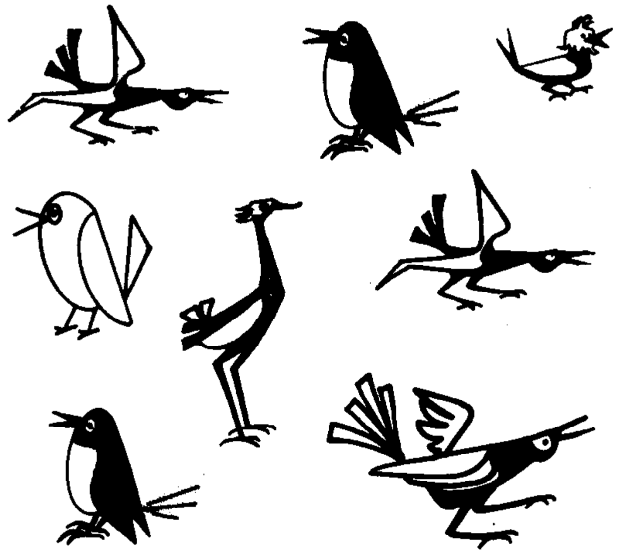


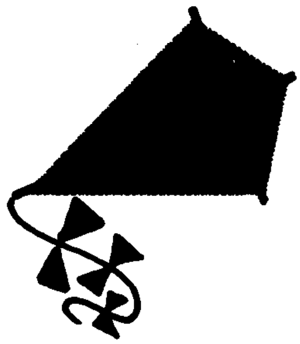
How many? 2

How many? 8

How many more? 6

Comparing Sets

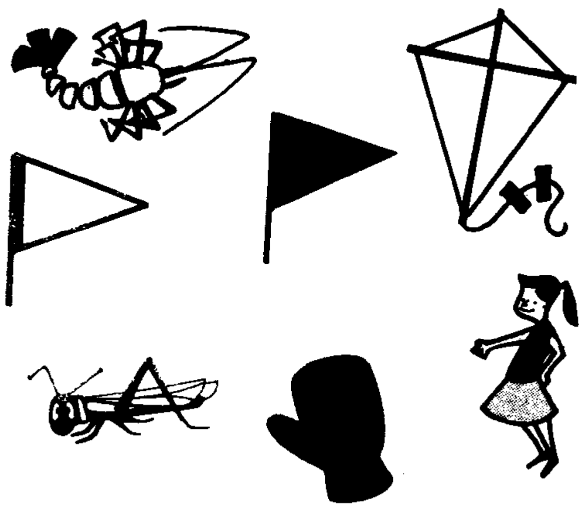


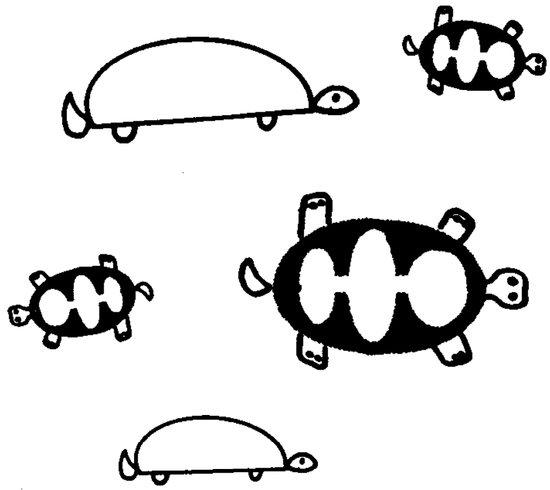


How many? 8

How many? 1

How many more? 7



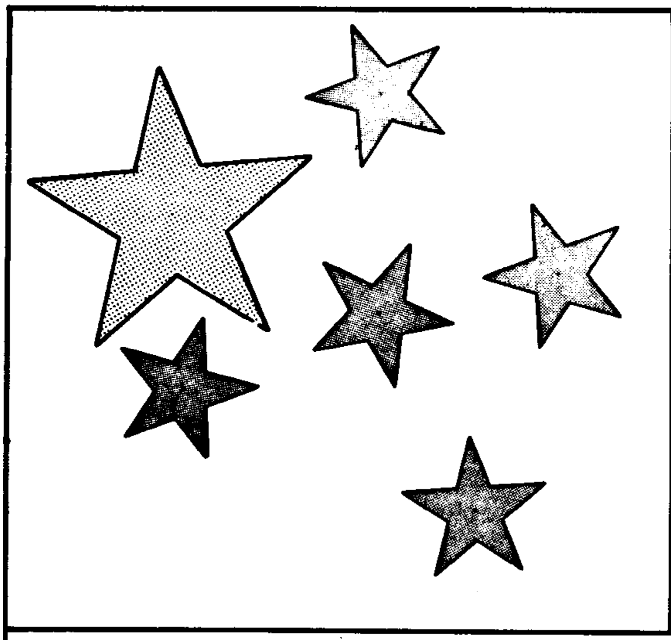


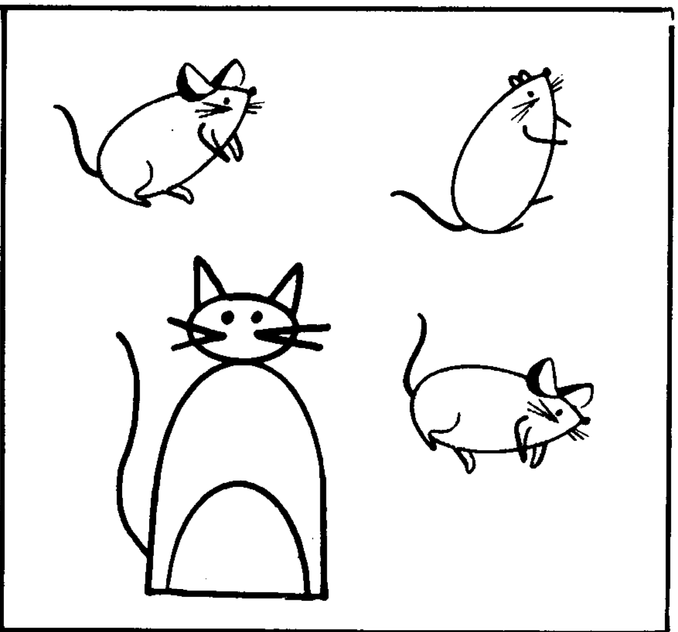
How many? 7

How many? 5

How many more? 2

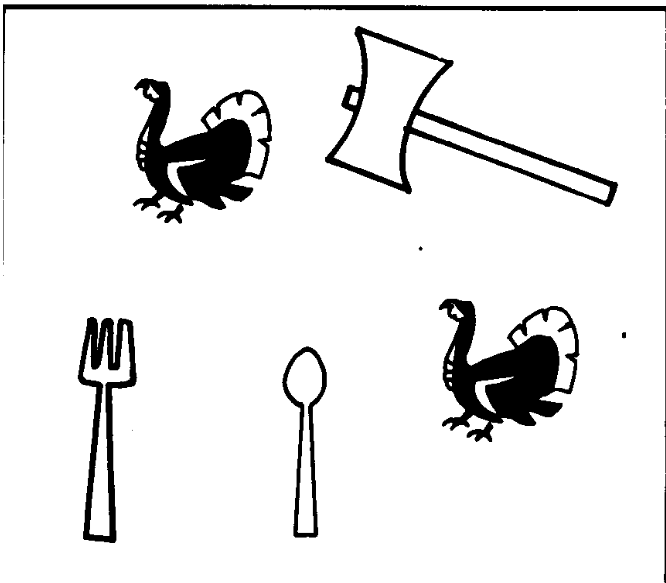
Comparing Sets





How many? 6 How many? 4

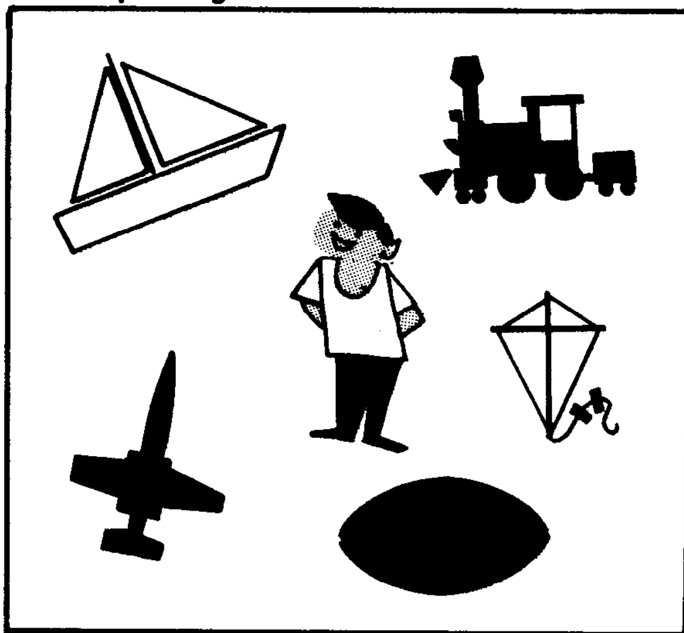
How many more? 2

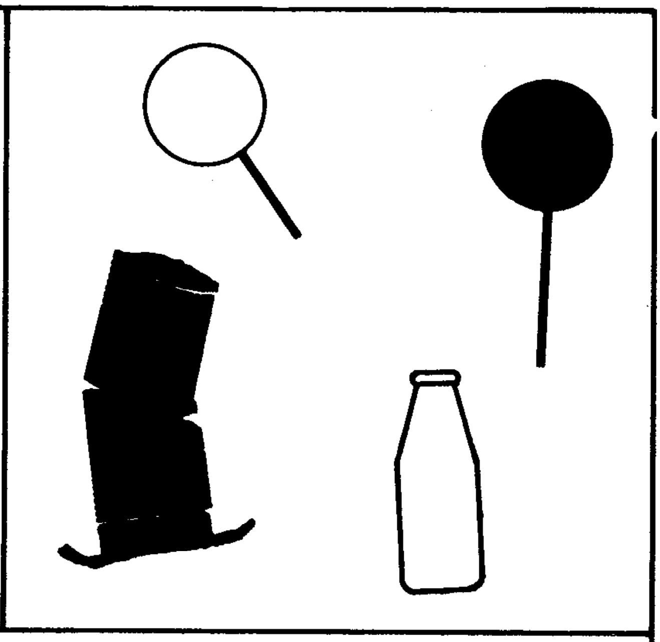


How many? 5 How many? 0

How many more? 5

Comparing Sets

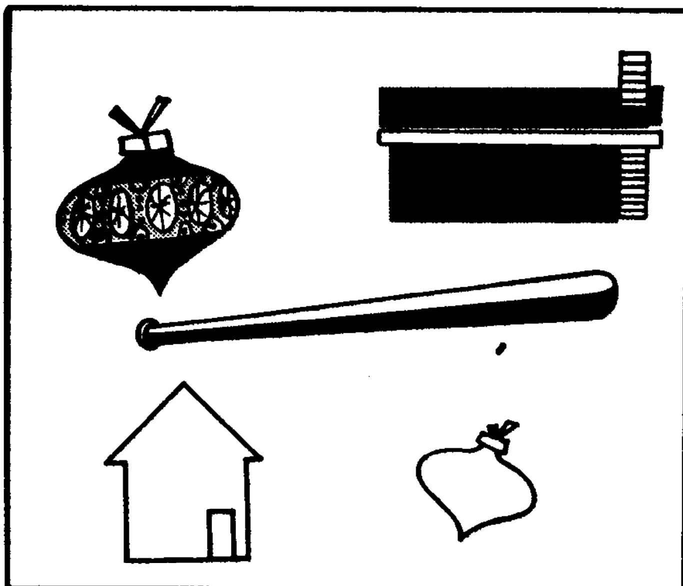


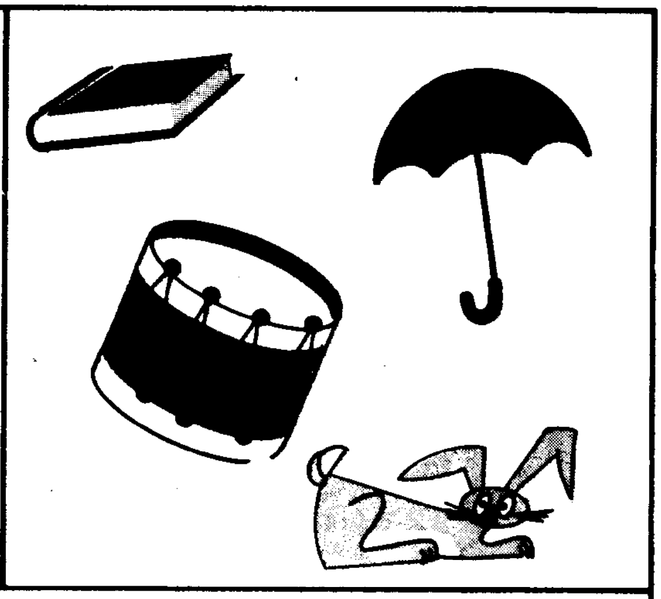


How many? 6

How many? 4

How many more? 2





How many? 5

How many? 4

How many more? 1

VII-5. Problem solving and equations

Objective: To use addition and subtraction to solve simple story problems, and to find missing numbers in equations involving addition and subtraction.

Vocabulary: (No new words.)

Materials: Sets of small objects.

Background note: There are several different sorts of problems which lead to addition and subtraction equations. For example:

One set has 4 members and a second set has 9 members. How many members must I join to the first to get a set equivalent to the second? ($4 + \square = 9$ or $9 - 4 = \square$)

One set is joined to a second set. The second set had 4 members. If the set obtained by joining the two sets had 6 members, how many members did the first set have? ($\square + 4 = 6$ or $6 - 4 = \square$)

A set has 4 members. How many members must be joined to the set to get a set of 6 members? ($4 + \square = 6$ or $6 - 4 = \square$)

This lesson is devoted to problems of this kind, and to the equations that these lead to. You should be careful not to classify problems by type, and to encourage each child to use his own method of thinking and to be as imaginative as possible in approaching the problems.

Suggested procedure:

Begin with an equation such as $6 + \square = 9$. Explain that this asks us to name the number which added to 6 will give a sum of 9. Try to get the children to visualize the equation by using sets of objects. Continue with equations like $3 + 4 = \square$ and $\square + 4 = 8$. Do not point out different types of equations, but try to get the children to examine and visualize each equation question as it comes.

<u>Pupil's book, pages 125 - 128</u>:

Ask the children to complete the equations. Suggest they use sets of objects if they have difficulty.

The next step is to present story problems, to find equations which describe the problems, and to name the missing number. Continue to have the children use sets of objects to answer the questions.

The following are examples of story problems:

> There are 9 saucers and 4 cups on the table. How many more cups do we need if we want to put a cup on each saucer? ($4 + \square = 9$, or $9 - 4 = \square$)
>
> John has 4 cents and Sue has 7 cents. How much must John save to have as many cents as Sue? ($4 + \square = 7$ or $7 - 4 = \square$)
>
> Mary had two ribbons.
> Her mother gave her some ribbons.
> Now she has seven.
> How many did her mother give her? ($2 + \square = 7$ or $7 - 2 = \square$)
>
> Tom had six boats.
> He gave some to Bill.
> Now Tom has five boats.
> How many did he give to Bill? ($6 - \square = 5$ or $5 + \square = 6$)
>
> Gordy has 5 guns.
> Eddie has 7 guns.
> How many more guns does Gordy have? (No equation.)
>
> Ben has 8 marbles.
> Three are red.
> The rest are green.
> How many are green? ($3 + \square = 8$ or $8 - 3 = \square$)

Mark had three cookies.
He ate some on the way to school.
Now he has only one for lunch.
How many did he eat on the way to school?
($3 - \square = 1$ or $\square + 1 = 3$)

Karen had three caps.
Her friend gave her five caps.
How many caps does Karen have? ($3 + 5 = \square$)

Bryan bought some ice cream cones.
Four were vanilla and the rest were strawberry.
How many did he buy in all? (No equation)

Jack's father gave him three model rockets.
His grandfather gave him five.
Jack gave one of them to his friend Douglas.
How many rockets does Jack have? ($3 + 5 = \square$ and $8 - 1 = \square$)

Display an equation on the chalkboard or flannel board, for example, $7 + \square = 9$. Ask the children to make up a story that goes with the equation.

Ask the children to make up story problems, discuss these, and find equations (if there are any) that go with the stories.

Write the following story on the chalkboard:

> Mary had 3 cookies.
> She gave 1 cookie to Bill.
> Find how many cookies Mary had then.

Read the story to the children or have it read by an able child. Encourage the children to recall some of the ideas presented previously; (Chapter 4) namely:

(1) Story problems ask you to find something. You may be asked to find the answer to a question.

(2) Some information is given to help you solve the problem. Sometimes not enough information is given to solve the problem.

(3) Thinking about sets, equivalent to the sets in the story, helps us solve story problems. (One child might demonstrate with set materials to show Mary's cookies.)

(4) We can write an equation to show us what number we will need to use in the answer but the equation is not the answer we are asked to find. (The equation $3 - 1 = \_\_\_$ might be written on the chalkboard at a spot removed from the story itself. Remind the children that the 3 in the equation says nothing about cookies, it only names the number "3". For this reason we must always tell what it is that was found. In this case we might say, "Mary had 2 cookies then.")

(5) We can use what we know about solving one story problem to help us solve other story problems.

Erase the equation from its place on the board and write it directly under the story problem. Write the following story on the chalkboard:

> Jack had some toy cars.
> Mother gave Jack 2 toy boats.
> Then Jack had 7 cars and boats.
> How many toy cars and boats did Jack
> have to begin with?

Ask a child to show sets equivalent to the sets of toys. Discuss what set operation could be used to find the answer to the question. Ask what equation describes the set operations and have it written on the board, directly under the story. Continue by telling the children the following stories. In each case have the children tell what equation they can use to help them solve the problem. Would it be like the first story problem, a subset removed from a set, using an equation that shows one number minus another number; or would it be like the second story problem, two sets joined, and use an equation that shows one number plus another number? Have the equation written and the answer to the problem given.

1. Father had 3 rakes and 2 shovels.
   How many rakes and shovels did Father have?

   (It is like the second equation. $3 + 2 = \underline{5}$)
   (Father had 5 rakes and shovels.)

2. Beth had 4 new dresses.
   Mother bought some more new dresses for Beth.
   Now Beth has 6 new dresses.
   Find how many new dresses Mother bought for Beth.

   (It is like the second equation. $4 + \underline{\ 2\ } = 6$)
   (Mother bought 2 more dresses.)

3. Jack had some red apples.
   Mary took 5 of them for Mother.
   Then Jack had 2 apples.
   How many red apples did Jack have to begin with?

   (It is like the first equation. $\underline{\ 7\ } - 5 = 2$)
   (Jack had 7 apples in the beginning.)

4. Mary had some sticks of gum.
   She gave 3 sticks to Joe.
   Find how many sticks she has now.

   (You can't solve it. You must either know how many sticks Mary had to begin with or how many she has now.)

<u>Pupil's book, pages</u> <u>129</u> - <u>133</u>:

Read each of the stories aloud to the children. In some cases it may be well to ask a less able child to reread the story after it has been read once. The pupils should then be directed to complete the page by drawing more objects as needed or crossing off the subset which is removed. The equation which may be used to help solve the problem is to be written on the line and the answer sentence completed by filling in the blank. Let those children who are able complete the page independently. Give individual help to those children who need it.

Pupil's book, pages 129 and 130 are to be used in this way.

Pupil's book, pages 131 - 133: children should be directed to use ○'s to show sets equivalent to the sets of objects described in the story. Be sure to read the stories aloud before the children begin to work independently. Give individual help whenever needed.

Complete the equations.

$\boxed{2} + 5 = 7$	$8 - 5 = \boxed{3}$
$10 - \boxed{2} = 8$	$6 + \boxed{2} = 8$
$7 - 0 = \boxed{7}$	$\boxed{9} - 3 = 6$
$1 + 9 = \boxed{10}$	$6 + 2 = \boxed{8}$
$0 + 8 = \boxed{8}$	$4 - 1 = \boxed{3}$
$\boxed{4} - 1 = 3$	$5 - \boxed{1} = 4$
$9 - \boxed{3} = 6$	$0 + \boxed{7} = 7$

Complete the equations.

$\boxed{8} - 2 = 6$

$3 - \boxed{2} = 1$

$7 - 2 = \boxed{5}$

$8 - 3 = \boxed{5}$

$4 - 0 = \boxed{4}$

$\boxed{7} - 6 = 1$

$10 - 2 = \boxed{8}$

$9 - \boxed{5} = 4$

$\boxed{10} - 4 = 6$

$7 - 4 = \boxed{3}$

$8 - 1 = \boxed{7}$

$10 - \boxed{5} = 5$

Complete the equations.

$3 + \boxed{4} = 7$

$0 + \boxed{6} = 6$

$4 + \boxed{5} = 9$

$5 + 4 = \boxed{9}$

$\boxed{2} + 8 = 10$

$\boxed{5} + 2 = 7$

$\boxed{8} + 1 = 9$

$\boxed{4} + 4 = 8$

$7 + 0 = \boxed{7}$

$\boxed{4} + 6 = 10$

$6 + \boxed{2} = 8$

$2 + \boxed{7} = 9$

$\boxed{2} + 4 = 6$

$7 + 3 = \boxed{10}$

Equations
Write + or − .

$2 + 1 = 3$

$5 - 3 = 2$

$7 - 1 = 6$

$4 + 4 = 8$

$1 + 9 = 10$

$4 - 3 = 1$

$5 + 2 = 7$

$8 - 5 = 3$

$9 - 8 = 1$

$9 - 9 = 0$

$3 + 4 = 7$

$1 + 8 = 9$

$6 - 3 = 3$

$5 + 4 = 9$

$7 \; \overset{+}{\underset{-}{\text{or}}} \; 0 = 7$

$3 + 6 = 9$

$6 - 4 = 2$

$1 + 5 = 6$

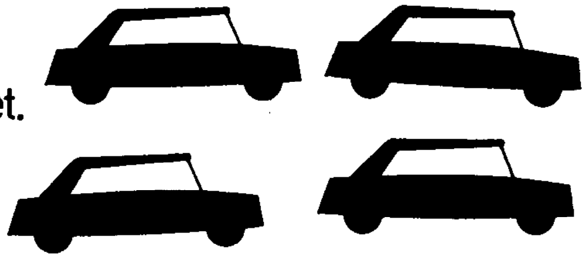
Solving Problems

1. Sam wants 6 toy cars.

   He has 4 toy cars.

   Find how many toy cars he must get.

   Problem Picture



   $4+2=6$ or $6-4=2$

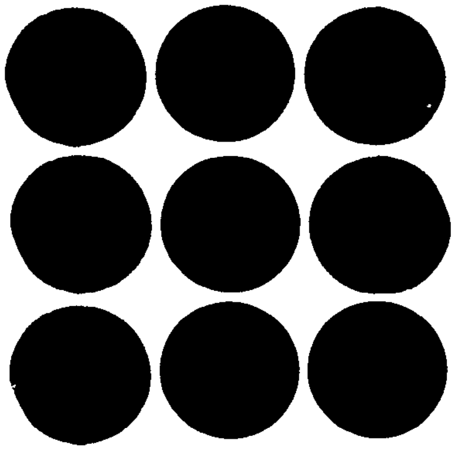
   Sam must get 2 toy cars.

2. Pat had 9 marbles.

   He gave 5 marbles to Dick.

   Find how many marbles Pat had then.

   Problem Picture



   $9-5=4$

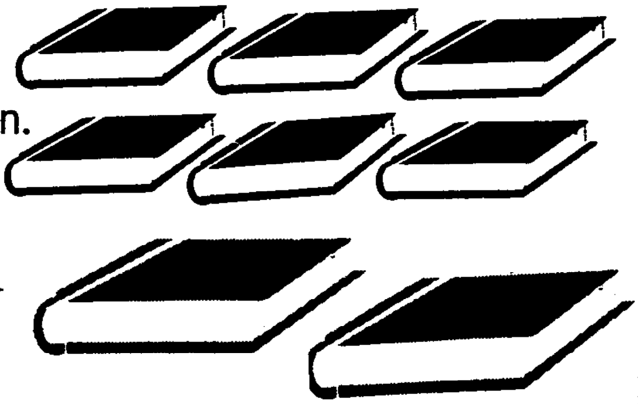
   Pat had 4 marbles then.

3. Nan had 6 books.

   She got 2 new books.

   Find how many books Nan had then.

   Problem Picture



   $6+2=8$

   Nan had 8 books then.

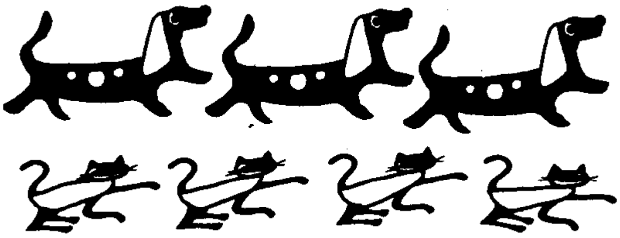
Solving Problems

1. Bob's dog had 3 puppies.
His cat had 4 kittens.
Find how many baby animals Bob had.

$3 + 4 = 7$

Bob had 7 baby animals.

Problem Picture

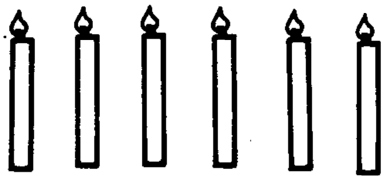


2. Mother needs 10 candles.
She has 6 candles.
How many candles must she get?

$6 + 4 = 10$ or $10 - 6 = 4$

She must get 4 candles.

Problem Picture

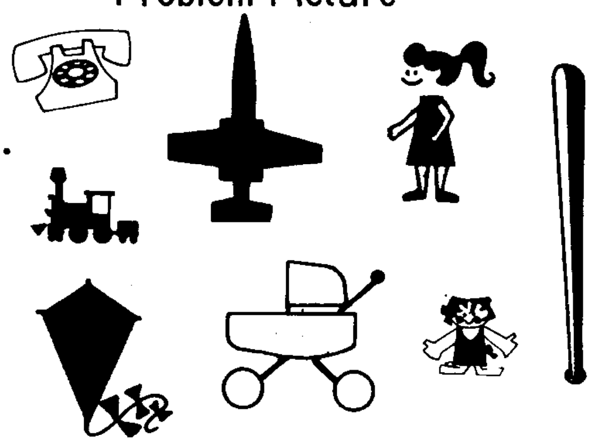


3. Mary had 8 toys.
Tom took 3 of the toys.
Find how many toys Mary had then.

$8 - 3 = 5$

Mary had 5 toys then.

Problem Picture



Solving Problems

1. David had 6 toy cars.

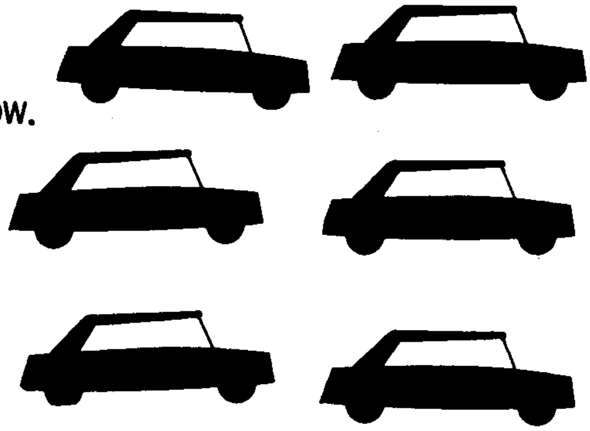
   He gave 1 car to Jim.

   Find how many cars David has now.

   6 - 1 = 5

   David has 5 toy cars now.

Problem Picture



2. Sally has 2 crayons.

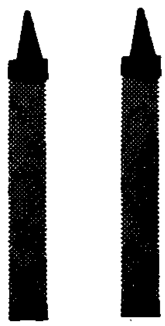
   She needs 6 crayons.

   How many crayons must she get?

   2 + 4 = 6 or 6 - 2 = 4

   Sally must get 4 crayons.

Problem Picture



3. Joan had 2 cookies.

   Mother gave 2 cookies to Joan.

   How many cookies did Joan have then?

   2 + 2 = 4

   Joan had 4 cookies then.

Problem Picture

Solving Problems

1. Jane wants 4 dolls.
   She has 3 dolls.
   How many dolls must she get?

   $3+1=4$ or $4-3=1$

   Jane must get 1 doll.

Problem Picture

2. Susan had 6 cookies.
   Spot ate 2 cookies.
   Find how many cookies Susan had then.

   $6-2=4$

   Susan had 4 cookies then.

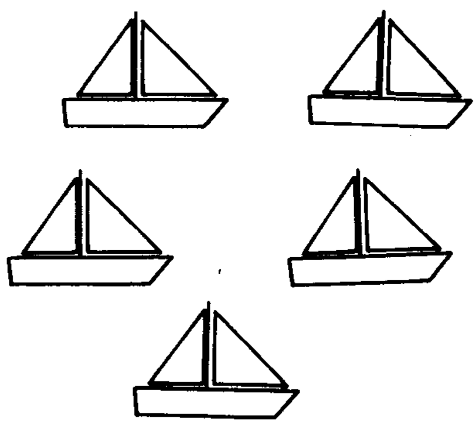
Problem Picture

3. Jack had 5 boats.
   He made 2 more boats.
   How many boats did Jack have then?

   $5+2=7$

   Jack had 7 boats then.

Problem Picture



Solving Problems

1. Ann made 10 cookies.

   She gave 3 cookies to Bill.

   How many cookies did Ann have then?

   10 - 3 = 7

   Then Ann had 7 cookies.

2. Mrs. Lee had 3 hats.

   She got 2 new hats.

   Find how many hats she has now.

   3 + 2 = 5

   Mrs. Lee has 5 hats now.

3. Mother baked some cakes.

   She gave 4 cakes to the church.

   Then she had 2 cakes.

   How many cakes did Mother bake?

   6 - 4 = 2 or 4 + 2 = 6

   Mother baked 6 cakes.

VII-6. Addition and Subtraction: numbers greater than ten

Objective: To begin the study of addition and subtraction of numbers greater than ten.

Vocabulary: (No new words.)

Materials: Sets of small objects for counting.

Teaching Note:

This lesson contains a sample presentation of addition and subtraction of 2 digit numbers. The teacher may pursue this as far as seems appropriate with her class.

Suggested Procedure:

Billy's family is going on a picnic.
His mother put ten cookies in the basket.
Billy put in thirty cookies.
How many cookies are in the basket?

We can add the number of cookies that Billy put in the basket to the number his mother put in the basket to find the number of cookies.
We would write the equation $10 + 30 = \_\_\_\_$.
To finish the equation we need to add the numbers. It may help us to think of the number of tens in 10 and in 30 and add the tens.

How many tens are in ten? (1.)
How many tens in thirty? (3.)
If we add 1 ten and 3 tens, how many tens will we have? (4.)
What do we call 4 tens? (Forty.)
When we add 30 to 10 we have 40, we finish the equation $10 + 30 = 40$.

If necessary, bundles of ten objects each should be available to use as an aid in solving these problems.

Develop the following problems in the same way·

Jerry found forty rocks in his back yard.
He put them in the fish bowl in his room.
On the way to the store he picked up ten rocks.
When he got home he put them in the fish bowl.
How many rocks does he have in the bowl?
(Jerry has fifty rocks in the fish bowl.
40 + 10 = 50.)

Alice had twenty pennies in her purse. Her mother gave her twenty pennies. She put them in her purse, too. How many pennies are in Alice's purse? (There are forty pennies in Alice's purse. 20 + 20 = 40.)

Carl has eighty plastic army men. He has twenty plastic army trucks. He has thirty plastic army jeeps. He took his army trucks and jeeps to Bob's house to play. How many toys did he take? (Carl took fifty toys to Bob's house. 20 + 30 = 50.)

These problems involve removing a set:

Steve had fifty marbles. He traded twenty marbles for a kite. How many marbles did he keep? (Steve kept thirty marbles. 50 - 20 = 30.)

Frank found forty sea shells while he was at the beach. He gave twenty to his friend David when he got home. How many sea shells did Frank keep? (Frank kept twenty sea shells. 40 - 20 = 20.)

Shirley has twenty paper dolls. She has forty paper doll dresses. She left ten dresses at Mary's house. How many dresses does she have to play with at home? (Shirley has thirty dresses to play with. 40 - 10 = 30.)

John has forty pencils. He gave ten to his brother Jerry. How many pencils does his brother, Tim, have? (Cannot be solved with information available.)

Place two sets of small objects on a table. Do not tell the number of objects in each set.

> How can we find how many objects we would have if we joined the sets?
> (Pupils may suggest that they could join the sets and then count all the objects. This procedure should then be followed.)

Place two new sets of objects on the table. Now tell the children the number of members in each set as you place them on the table. (Be sure that joining the set of ones to the set of ones will result in a set of not more than 9 ones.

> This set has 22 members. We will join to it a set with 36 members. How many objects are in our new set? Can we find the number of members in our new set without counting each member? Remember, you know the number of members in each of the sets.

Pursue suggestions that children may give. If not suggested, ask a child to arrange the set with 22 members as sets of ten and a set of ones. Another child should arrange the set of 36 as tens and ones.

> How many sets of ten are in the set with 22 members? (2.)
> How many sets of ten are in the set with 36 members? (3.)

Join the set of 3 tens to the set of 2 tens.

> How many tens in all? (5.)
> How many ones in the set with 22 members? (2.)
> How many ones in the set with 36 members? (6.)

Join the set of 6 ones to the set of 2 ones.

> There are how many ones in all? (8.)
> We have 5 tens and 8 ones.
> How many are in the set? (58.)

Show children an envelope. Tell them that inside is a set of sticks. There are 45 sticks in all. Show them another envelope. Tell them that this one has 23 sticks in it. Tell the children we will use the numbers to help find out the number of sticks we would have if we joined the set of 23 sticks to the set of 45 sticks. We will do this by adding 23 to 45.

> How many tens are in 45? (4.)
> How many tens are in 23? (2.)
> If we add 2 tens to 4 tens, what is the number of tens? (6.)

On the chalkboard, write 4 tens + 2 tens = 6 tens.

> How many ones are in 45? (5.)
> How many ones are in 23? (3.)
> If we add 3 ones to 5 ones, what is the number of ones? (8.)

On the chalkboard, write 5 ones + 3 ones = 8 ones.

> We have 6 tens and 8 ones in all.
> How many sticks are in the envelope? (68.)

Join the sets, group into tens and ones, count the sets of tens and sets of ones to check this work.

Place a set of objects on the table. Do not tell the number of members.

> If I remove 21 members of the set, what will be the number remaining? (We will need to count the members of the remaining set in order to find out.)

Place another set on the table. Arrange the set as 4 tens and 8 ones.

> This set has 48 members. Now I will remove a subset of 21 objects. How many objects are in the set that remains?
> (Remove 2 tens from 4 tens.) What is the number of tens remaining? (2 tens.)
> (Remove 1 one from 8 ones.) What is the number of ones remaining? (7 ones.)

> The remaining set has 2 tens and 7 ones. What is the number of members in the remaining set? (27.)

Show the children a box. Tell them that inside the box are 57 beads.

> If I remove 34 beads, how many beads will be left in the box?

Tell the children we can use numbers to find the number of members in the remaining set. We do this by subtracting 34 from 57.

> How many tens are in 57? (5.)
> How many tens are in 34? (3.)
> If we subtract 3 tens from 5 tens, what is the number of tens? (2.)

On the chalkboard, write 5 tens - 3 tens = 2 tens.

> How many ones are in 57? (7.)
> How many ones are in 34? (4.)
> If we subtract 4 ones from 7 ones, what is the number of ones? (3.)

On the chalkboard, write 7 ones - 4 ones = 3 ones.

> We have 2 tens and 3 ones in the remaining set if we have subtracted the numbers correctly. How many are in the remaining set? (23.)

Remove the set of 34 beads from the set of 57 beads. Count the number of members in the remaining set in order to check this work.

Chapter VIII

ARRAYS AND MULTIPLICATION

Background

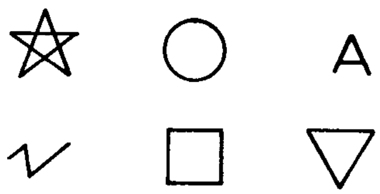
Section VIII-1 introduces the idea of array. An array is a rectangular arrangement of objects into rows, each row containing the same number of objects. Shown below is an array of 3 rows, each row containing 5 x's.

x x x x x

x x x x x

x x x x x

The objects in an array are called its members. These need not be all alike. Here, for instance, is an array of 2 rows of 3 members each, all different:



Arrays may also consist of rectangular arrangements of flannel board objects, or blocks on the floor, or drawers in a cabinet, or panes in a window, or compartments in a carton, etc., etc. If an arrangement of objects in rows does not have the same number of members in each row, then it is not called an array.

Arrays are used in defining multiplication. When we multiply 5 by 3, for instance, the result is defined as the number of members in an array of 3 rows of 5 members each. In counting the number of members in an array of, say, 3 rows of 5 members each, children are led to count by rows (5, 10, 15) and to say, "Three fives are fifteen." There is then an easy transition, in Section VIII-2, to the statement "Three times five is fifteen." The further transition to the equation

$$3 \times 5 = 15,$$

using the multiplication sign, can easily be made if the teacher feels her class is ready for this. It should be noted here that no mastery of multiplication facts is expected in this grade.

Section VIII-3 points out two simple properties of multiplication. The first of these is that multiplying numbers in either order always gives the same result. For instance,

$$4 \times 5 = 5 \times 4.$$

Arrays make this very easy to see: when we turn up on end an array of 4 rows of 5 members each, we get an array of 5 rows of 4 members each.

By considering an array of just 1 row of, say, 3 members, we see that

$$1 \times 3 = 3.$$

Similarly, 1 times any whole number is that number. Also, by considering an array of 3 rows of just 1 member each, we see that

$$3 \times 1 = 3,$$

and similarly, that any whole number times 1 is that number.

VIII-1. Arrays

Objective: To introduce the idea of array, and to find the number of members in an array by counting by rows.

Vocabulary: Array, row.

Materials: Counting disks, buttons, beans, or other small objects; felt cut-outs for flannel board; hundreds-square paper.

Suggested Procedure:

Give each child a set of 20 counting disks or other small objects. Ask 6 children to select 2 of their disks to put into a box in which you will collect them. When you have collected the disks, discuss with the class the fact that you have joined 6 sets, and that the sets were equivalent to each other, each having 2 members. Have a child count the disks in the box to see how many disks are in the union.

```
0   0
0   0
0   0
0   0
0   0
0   0
```

Suggest that the disks be arranged in rows, with 2 in each row, and show that in this arrangement it is possible to count by twos. Explain that this kind of arrangement is called an array. It has 6 rows with 2 members in each row. There are 6 sets of 2, and hence 6 twos, or 12 members in the array. Explain that an array is an arrangement of things in rows in which each row has the same number of members. Point to arrays of window panes, bulletin board pictures, etc., or draw on the chalkboard pictures of arrays of different kinds: 5 rows of 4 members each, 3 rows of 8 members each, 6 rows of 5 members each, etc., and have children observe how they know they are arrays. Ask children to give other examples. Point out that unless all rows have the same number of members the arrangement is not called an array. Give examples of arrangements that are not arrays.

Return the disks to the children and ask them to make an array (demonstrate again, if necessary) of 4 rows with 2 members in each row. Ask how many members there are in the whole array. Continue, in the same way, having other arrays made with 2 members in each row.

Have children make arrays of 6 rows with 2 members in each row, again, and then rearrange the disks to show 2 rows of 6 members each. Children should be aware that these are different arrays, but that they have the same number of members.

Repeat, using an array with 4 rows of 2 members each, and rearranging to form an array with 2 rows of 4 members each.

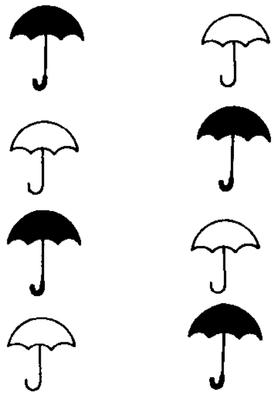
Have children form an array of 3 rows with 5 members in each row. Ask how they might count to find the number of members in the array. (5, 10, 15.) Draw arrays with 5 or 10 members in each row and have children count by rows to find the number of members in the array. Distribute hundreds-square paper. Show children how to write X's in the boxes to form various arrays of 2, 3, 5, or 10 members in each row, as you direct them. For instance, use red crayon to show an array of 8 rows with 2 members in each row. Use blue crayon to show ... Under each array they should write the number of members in the array.

```
x  x        x  x  x  x  x
x  x        x  x  x  x  x
x  x        x  x  x  x  x
x  x        x  x  x  x  x
x  x              20
x  x
x  x
x  x
 16
```

<u>Pupil's</u> <u>book</u>, <u>pages</u> 135 - 137:

Children are to write the number of rows, the number of members in each row, and the number of members in the array.

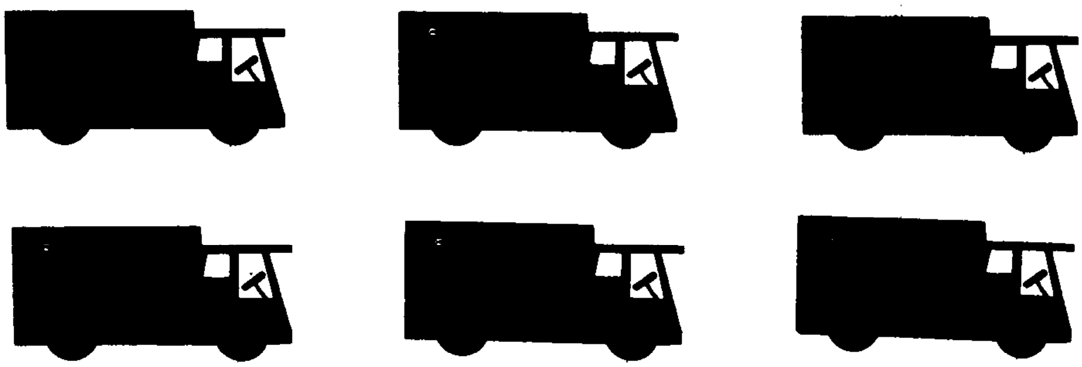
Arrays



How many rows? 4

How many in each row? 2

How many in the array? 8

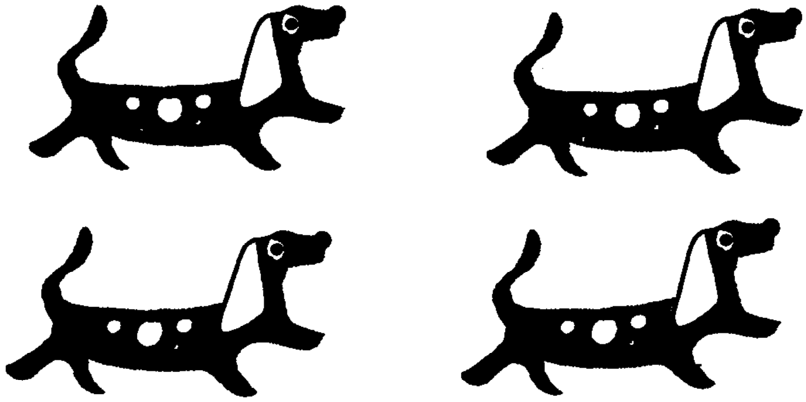


How many rows? 2

How many in each row? 3

How many in the array? 6

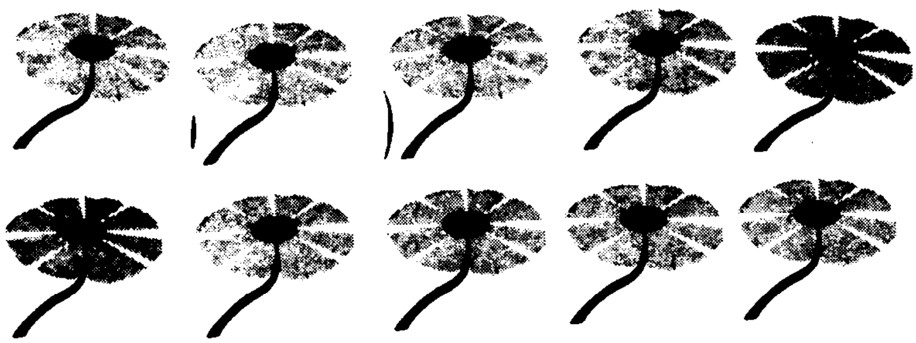
Arrays



How many rows? 2

How many in each row? 2

How many in the array? 4

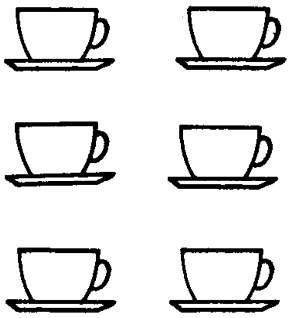


How many rows? 2

How many in each row? 5

How many in the array? 10

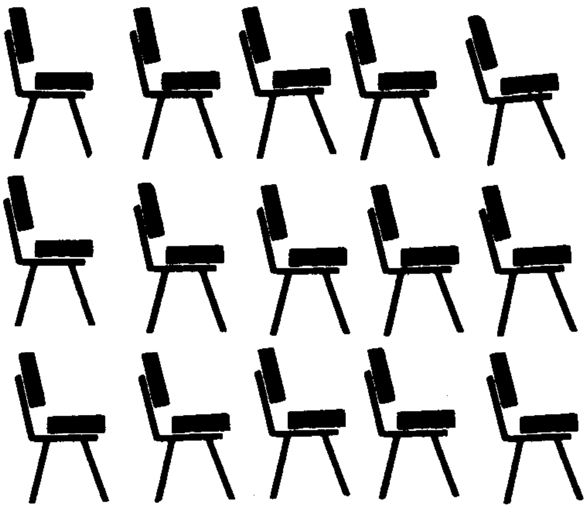
Arrays



How many rows? 3

How many in each row? 2

How many in the array? 6



How many rows? 3

How many in each row? 5

How many in the array? 15

How many rows? 4

How many in each row? 5

How many in the array? 20

- Show on the chalkboard an array of 5 rows with 2 members in each row. Have children tell how many rows there are and how many members there are in each row. Point to each row in turn.

Is this two? (Yes.)

Then ask children how many twos there are in the array. (5 twos.) Have them count by twos to find how many members there are in the array. Write:

5 twos are 10.

Show other arrays with rows of 2 members and repeat, having children tell how many rows, and how many twos the array has. Have them count by twos. Write:

___ twos are ___.

Include an array with 10 rows with 2 members in each row. When children have said, "Ten twos are 20", show an array with 2 rows of 10 members in each row, and observe that 2 tens are 20.

Have children make arrays with rows of 5 members in each row and ask how many fives there are, etc. Then write:

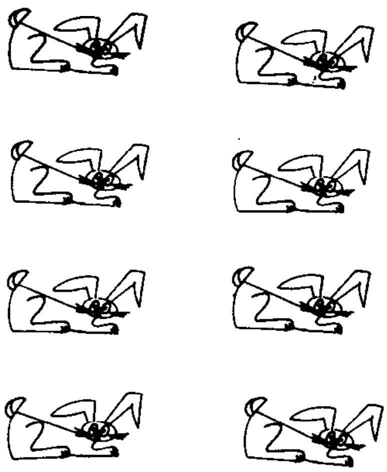
___ fives are ___.

Do the same for rows of 3 members each.

<u>Pupil's</u> <u>book</u>, <u>pages</u> 138 - 141:

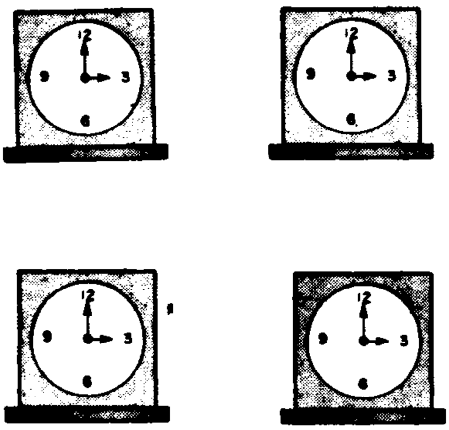
Children first count the rows, then fill the blanks.

Arrays



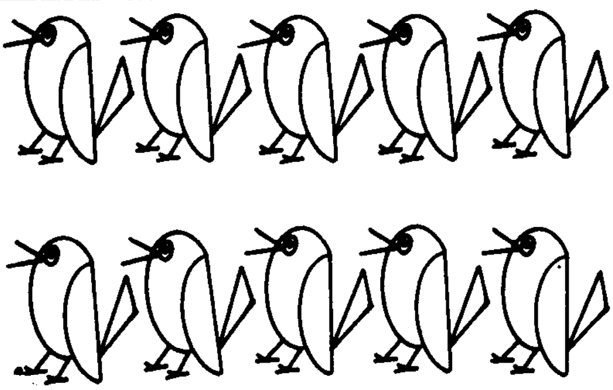
How many twos? 4

4 twos are 8.



How many twos? 2

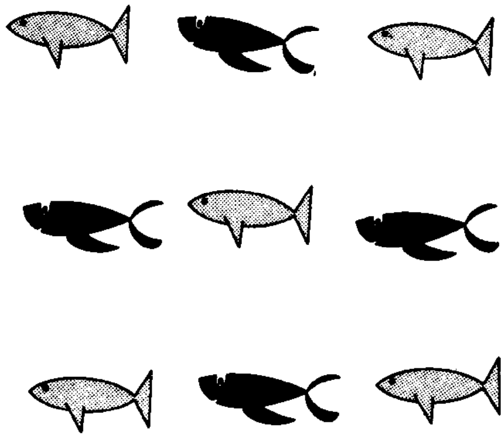
2 twos are 4.



How many fives? 2

2 fives are 10.

Arrays



How many threes? 3

3 threes are 9 .

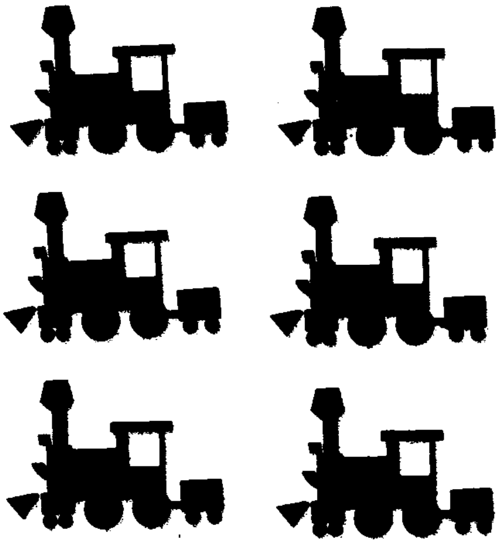
How many tens? 3

3 tens are 30 .

How many fours? 2

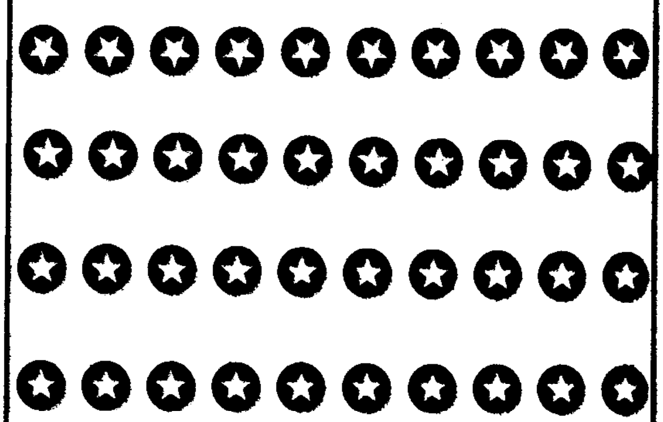
2 fours are 8 .

Arrays



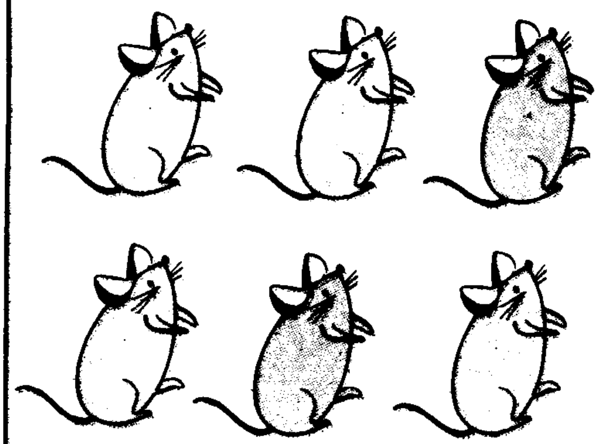
How many twos? 3

3 twos are 6.



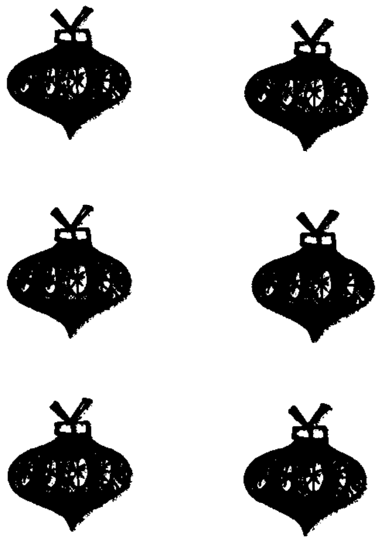
How many tens? 4

4 tens are 10.



How many threes? 2

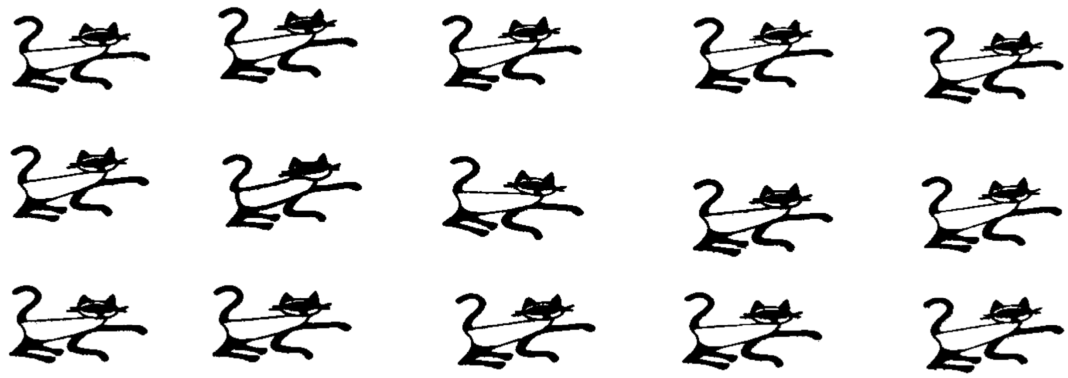
2 threes are 6.



How many twos? 3

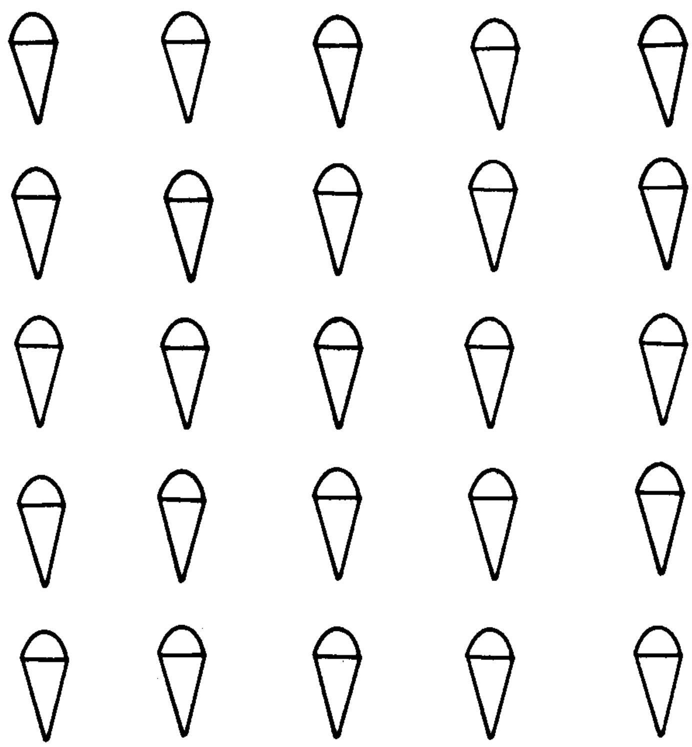
3 twos are 6.

Arrays



How many fives? 3

3 fives are 15.



How many fives? 5

5 fives are 25.

VIII-2. Multiplication

Objective: To introduce the idea of multiplication, using arrays.

Vocabulary: Multiply, multiplication, times.

Materials: Felt cut-outs for flannel board; sets of counting disks or other small objects, paper bag.

Suggested Procedure:

Tell the following story.

> Mrs. Brown was getting ready for a picnic. Eight people were going, and she wanted to take two cookies for each person. She put the cookies into a bag, two at a time. I'll pretend to be Mrs. Brown, and put in enough cookies for the eight people.

Put disks in the paper bag, counting as you do so: "One person, two people, three people, etc." When you have finished, ask

> How many times did I put 2 cookies into the bag? (8.) We can say there are 8 twos in the bag, or we can say there are 8 times 2 in the bag. Eight twos are...? (16.) What is 8 times 2? (16.)

Write:

8 times 2 is 16.

You may or may not wish to introduce the symbol $\times$ and write equations, such as $8 \times 2 = 16$.

> Suppose you earned 5 pennies each school day this week. That would be Monday, Tuesday, Wednesday, Thursday, and Friday. How many times would you have earned 5 pennies? (5.) What is 5 times 5? (25.)

Have children use disks or other objects to form an array and find out. Ask children how they might use an array to solve the following problem:

> A man has 5 ponies, and they all need new shoes. He wants to know how many shoes will have to be put on.
> How many shoes does each pony need? (4.)
> How many ponies are there? (5.)
> We can say that together all the ponies need 5 times 4 shoes.

Have the children use their counting disks. They should put 4 disks in a row to show how many shoes the first pony needs, 4 disks in another row for the next pony, etc., to learn that 5 times 4 is 20. Therefore 20 shoes are needed.

> When we say 5 times 4 is 20, we are <u>multiplying</u>. In <u>multiplication</u>, we multiply one number by another.

Go directly to problems of the form: what is 3 times 5? Restate the problem in several ways: Three fives are...? If an array has 3 rows, and each row has 5 members, how many members are in the array? Have children use disks or other objects to answer these questions. Some may choose to draw arrays on paper.

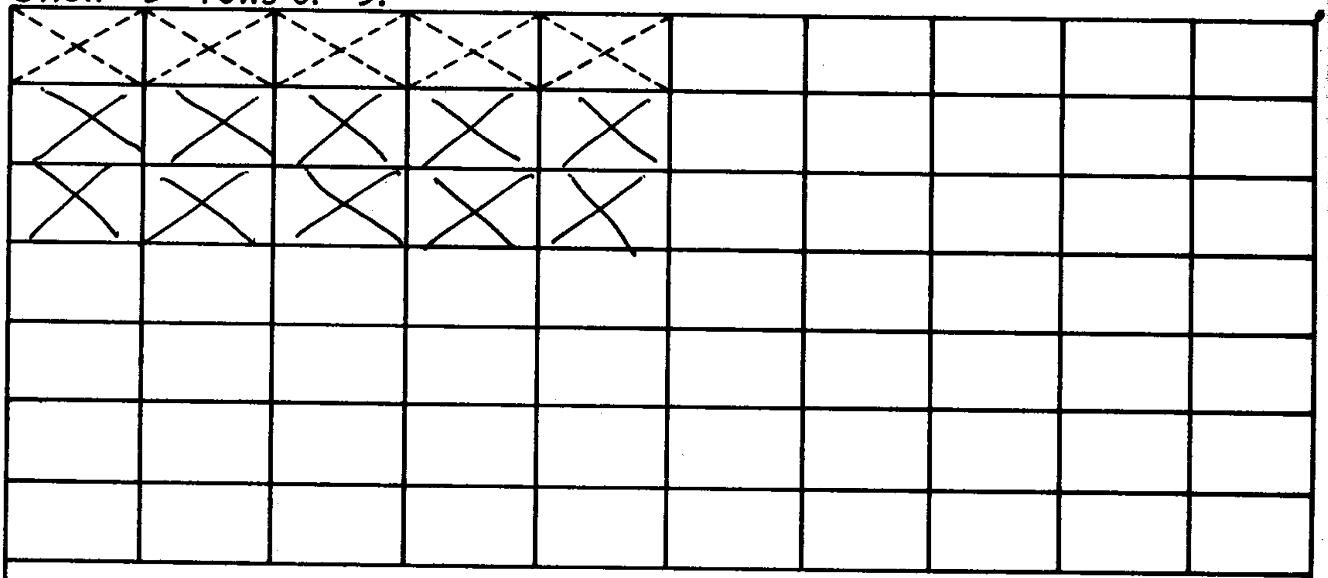
<u>Pupil's book, pages</u> 142 - 144:

> Children are to write X's in the boxes to form arrays, as indicated. They will write in the blank the number of members in the array.

Multiplication

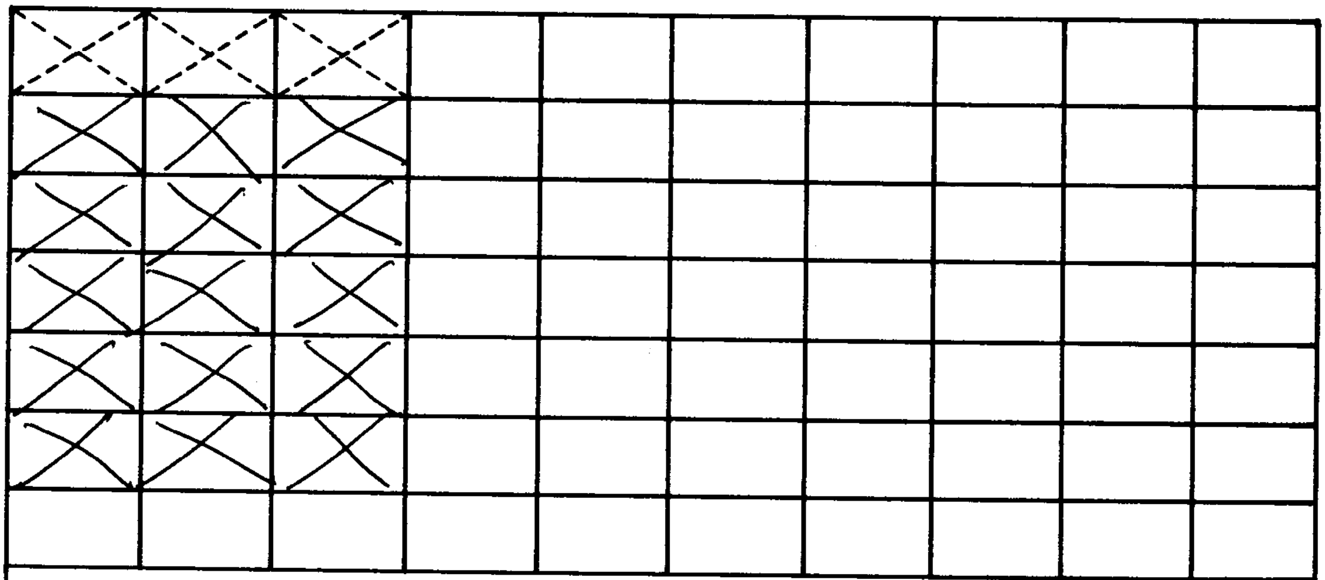
Draw arrays. Fill the blanks.

Show 3 rows of 5.



3 times 5 is 15.

Show 6 rows of 3.

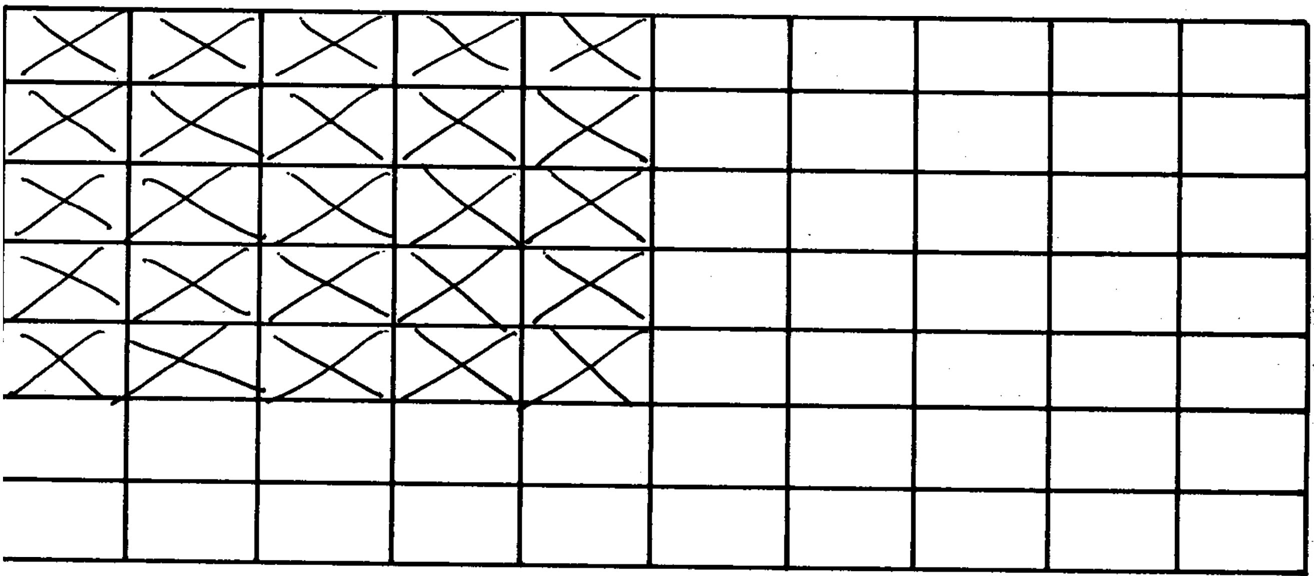


6 times 3 is 18.

Multiplication

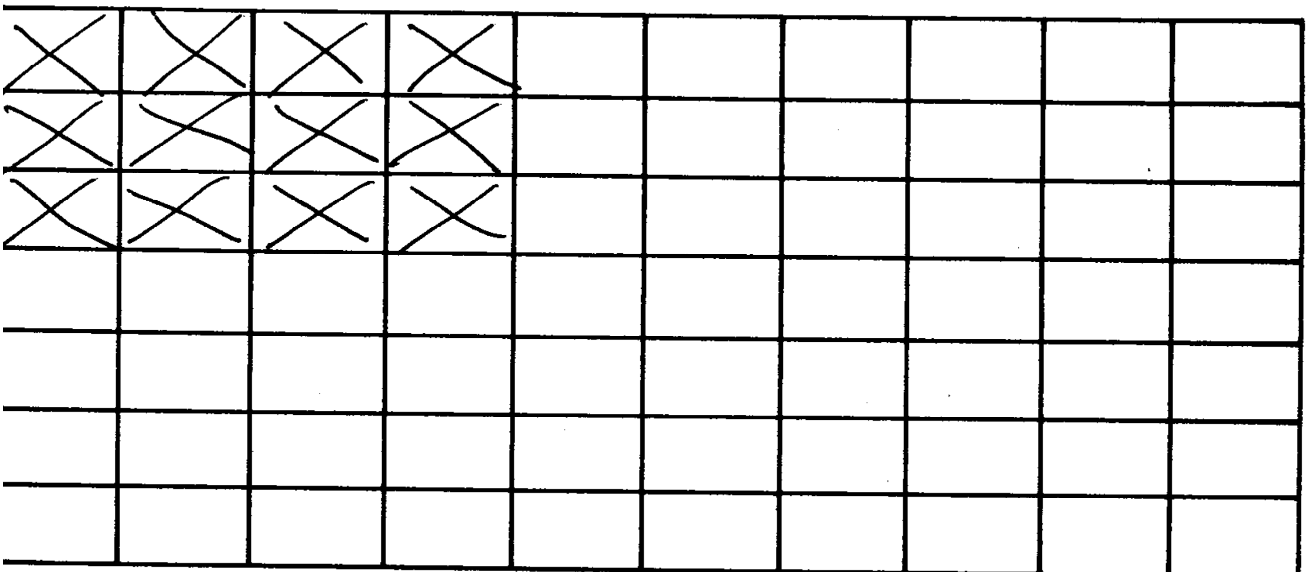
Draw arrays. Fill the blanks.

Show 5 rows of 5.



5 times 5 is 25 .

Show 3 rows of 4.



3 times 4 is 12 .

Multiplication

Draw arrays. Fill the blanks.

Show 5 rows of 10.

X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X

5 times 10 is 50.

Show 6 rows of 2.

X	X								
X	X								
X	X								
X	X								
X	X								
X	X								

6 times 2 is 12.

Further Activities:

Use story problems such as the following to deepen understanding of multiplication. Read each story to the children. Encourage them to use manipulative materials or to draw arrays to solve the problem. The problem should also be stated, and written on the chalkboard, in mathematical terms, using, at your option, either of the following forms: 3 times 6 is ____ or $3 \times 6 =$ ____. "Show Me" cards are very useful for observing the response of each child to the problem situation.

1. Mother washed 5 pairs of stockings.
   How many stockings did Mother wash?

2. On Mary's street there were 3 houses.
   In each of the houses lived 3 children.
   All together how many children lived in the 3 houses?

3. Joe ate 3 apples each day for 4 days.
   In 4 days how many apples did Joe eat?

4. Judy was weaving a doll blanket.
   She wove 2 squares every day for 5 days.
   How many squares did she weave in the 5 days?

5. Tom hit a home run each game for 3 games.
   How many home runs did he hit all together?

6. Beth ate 2 cookies each day for 2 days.
   How many cookies did Beth eat?

7. David put 1 butterfly in each of 5 jars.
   Find how many butterflies were in the 5 jars.

8. Each of the girls had 4 dolls.
   Find how many dolls 2 of the girls would have.

9. Mark got 3 new books each week for 2 weeks.
   Find how many new books Mark got in the 2 weeks.

10. Jim polished 4 pairs of his father's shoes.
    How many of his father's shoes did Jim polish?

11. Bob had several boxes of toys.
    Each box had 2 toys in it.
    How many toys were there in 3 of the boxes?

12. Each backyard in Jane's block had 1 tree.
    How many trees were there in 4 backyards?

13. Each house in Jan's block had 1 chimney.
    All together how many chimneys were there on 2 houses?

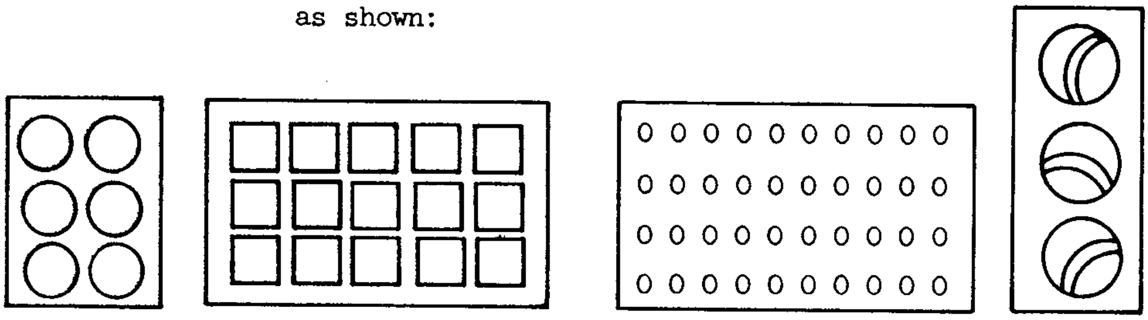
14. Each of the backyards had 3 flowering plants.
    How many flowering plants were there in 4 backyards?

VIII-3. Simple properties of multiplication

Objective: To use arrays to show the commutative property of multiplication and the multiplication property of 1.

Vocabulary: (Optional) Commutative.

Materials: Manipulative objects for children, pictures of arrays on tagboard or construction paper, as shown:



A B C D

Chart on newsprint, as shown:

4	times	5	is	20	10	times	8	is	___
8	times	2	is	16	1	times	41	is	___
3	times	2	is	6	5	times	2	is	___
2	times	5	is	10	2	times	3	is	___
8	times	10	is	80	5	times	4	is	___
41	times	1	is	41	2	times	8	is	___

Suggested Procedure:

Show picture A to the class. Have the array described. (3 rows with 2 members in each row.) Ask children to county by 2's to find what 3 times 2 is, and write:

3 times 2 is 6.

Turn the picture to show 2 rows of 3 members each. Have the array described and ask children to count by rows. (3, 6.) Write:

2 times 3 is 6.

Show pictures B and C in the same way. Have children use objects and make arrays of 4 rows of 3, 3 rows of 4, etc., so that they see that exactly the same number of objects may be used to make the arrays. Help children to generalize, (using the term "commutative property" if it has been used in lessons on addition) that one number times a second number gives the same result as the second number times the first.

Show picture D. Have the array described (3 rows with 1 member in each row.) Ask how children will count by rows. (By ones.) Write:

3 times 1 is 3.

Turn the picture to show the array as 1 row of 3 members. Have the array described, and ask children what they will say if they count by rows. (3.) Draw other arrays either with 1 row or with one member in each row, and help children to generalize: any number times 1 is that number itself, and 1 times any number is that number.

Show the chart and have children read the sentences at the left. Read the first sentence at the right. Ask which sentence to the left gives the information needed to finish, "10 times 8 is ____. (8 times 10 is 80.) Use crayon or felt pen to write 80 and to draw a line between the sentences that show commutativity. Complete the chart in this way.

<u>Pupil's book, pages</u> 145 - 146:

Children are to fill in the number of members in each row and the number of members in the array. They should observe that each pair of boxes shows commutativity.

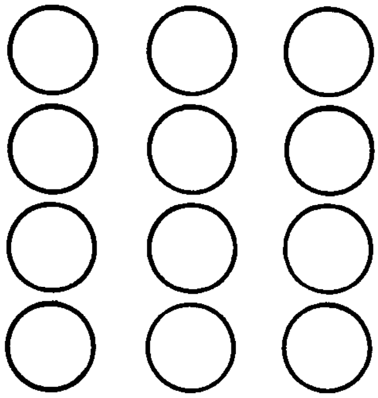
<u>Pupil's book, page</u> 147:

Children fill in the number of members in each array.

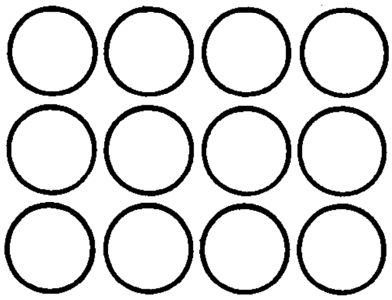
Pupil's book, page 148:

Children should be able to use their understanding of commutativity and of multiplying with 1 as a factor. If necessary, allow them to draw arrays to solve some of the problems at the bottom of the page.

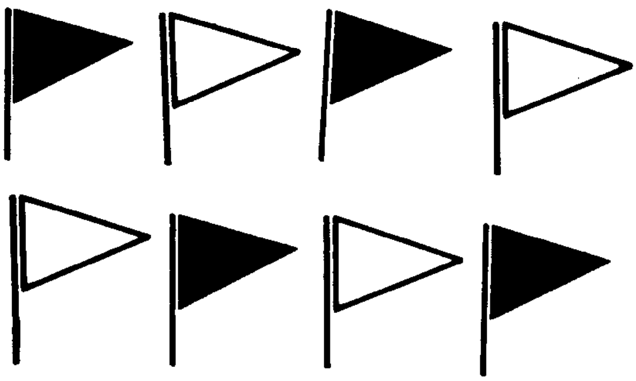
Multiplication



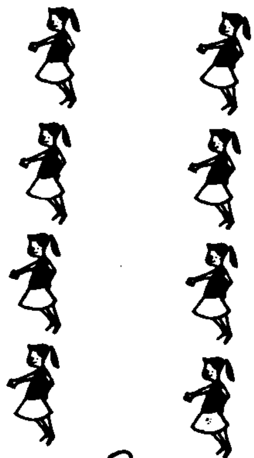
4 times 3 is 12



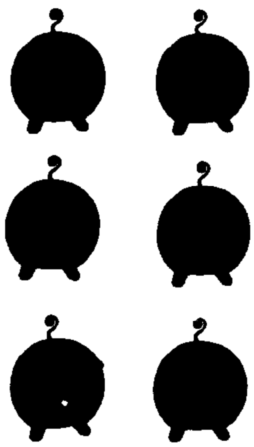
3 times 4 is 12



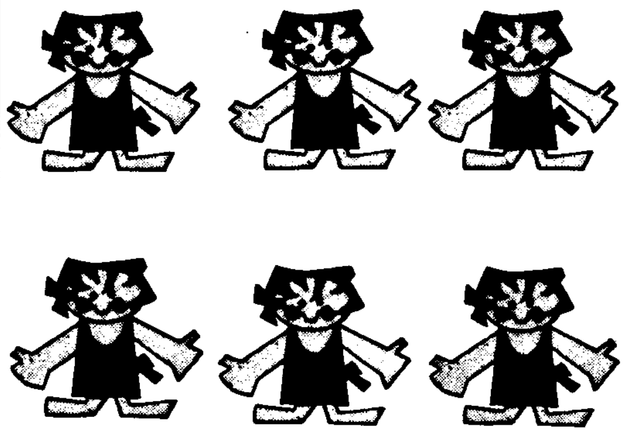
2 times 4 is 8



4 times 2 is 8

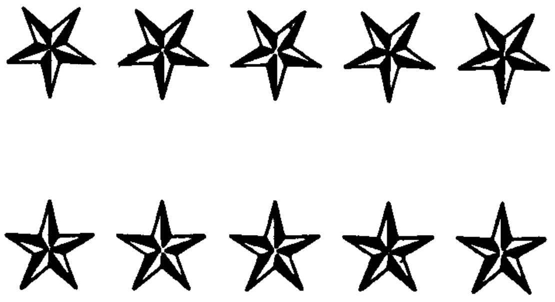


3 times 2 is 6

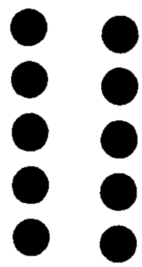


2 times 3 is 6

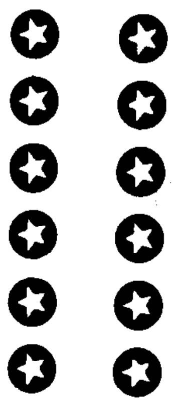
Multiplication



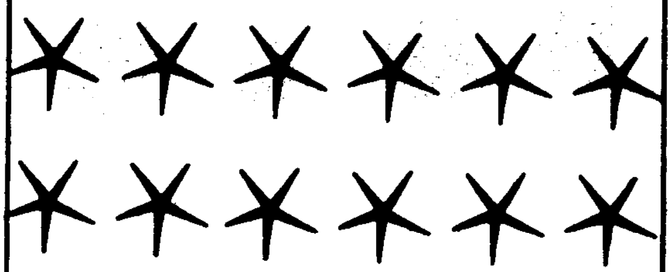
2 times 5 is 10



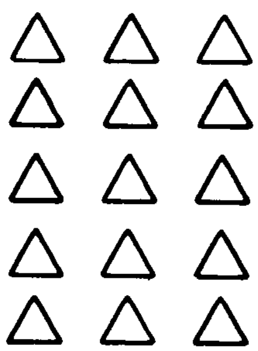
5 times 2 is 10



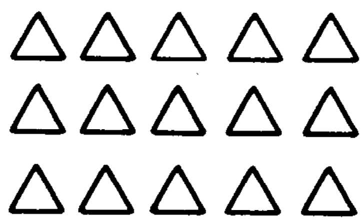
6 times 2 is 12



2 times 6 is 12



5 times 3 is 15

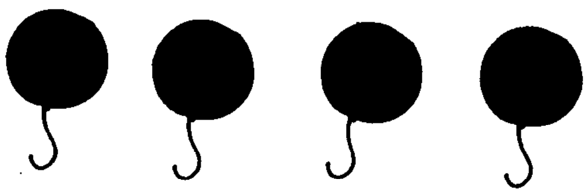


3 times 5 is 15

Multiplication

6 times 1 is 6

1 times 6 is 6



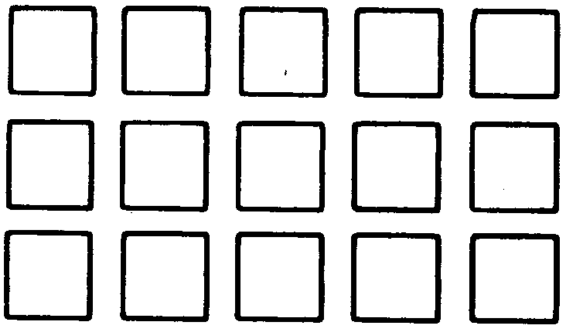
1 times 4 is 4

4 times 1 is 4

1 times 7 is 7

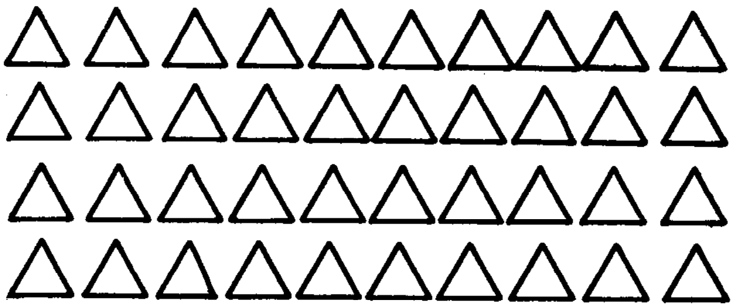
7 times 1 is 7

Multiplication



3 times 5 is 15.

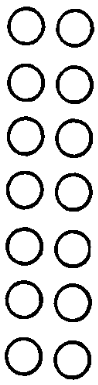
So, 5 times 3 is 15



4 times 10 is 40

So,

10 times 4 is 40



7 times 2 is 14

So, 2 times 7 is 14

5 times 2 is 10	1 times 68 is 68
2 times 4 is 8	3 times 2 is 6
3 times 5 is 15	89 times 1 is 89

CHAPTER IX
PARTITIONS AND RATIONAL NUMBERS

Background

In Section IX-1 we partition a set into subsets of a given number of members, and then count the number of subsets obtained. For instance, we may partition a set of 20 members into subsets of 5 members each and find that we get 4 such subsets. We may think of these as forming 4 rows of 5 members each in an array. The point is to lead children to see that 4 answers the question "How many 5's are there in 20?" With children this is approached concretely: If 20 players are divided up into teams of 5, how many such teams will there be?

Section IX-2 considers a slightly different partition situation. Here we partition sets into a given number of equivalent subsets and then count the number of members in such a subset. For instance, when we partition a set of 20 into 5 equivalent subsets, we find that there are 4 members in each such subset. We may think of these 5 subsets as forming the rows in an array of 5 rows of 4 members each. The kind of concrete question answered here is the following: If 20 cookies are distributed fairly to 5 children, how many cookies will each child get?

Section IX-3 deals with very simple problems of this last sort: If 10 cookies are distributed fairly to just <u>2</u> children, how many cookies will each get? In more abstract language, we are partitioning a set of 10 into 2 equivalent subsets and asking how many members each will have. We are of course leading up to the statement

$$\frac{1}{2} \text{ of } 10 \text{ is } 5.$$

Notice that here we speak of $\frac{1}{2}$ <u>of</u> something. In later grades it will be recognized that the statement

$$\frac{1}{2} \text{ of } 10 \text{ is } 5$$

is a reading of the multiplication equation

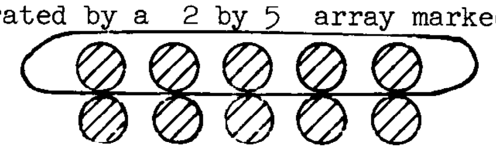
$$\frac{1}{2} \times 10 = 5.$$

In this equation $\frac{1}{2}$ clearly appears as a number (a rational number we call it, because it is the ratio of the whole numbers 1 and 2), and the "of" is associated with the "times" of multiplication. But we can't do all this at once. In this chapter we content ourselves with telling children that $\frac{1}{2}$ is a new kind of number and that the symbol "$\frac{1}{2}$" is an example of a special way of writing a number that we call a fraction. That is, a fraction consists of a bar with a numeral above it and a numeral below it.

The statement

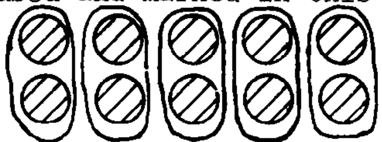
$$\frac{1}{2} \text{ of } 10 \text{ is } 5$$

is illustrated by a 2 by 5 array marked in this way:



where we are interested in only one of the two rows and its relation to the entire array.

In Section IX-4, instead of asking "What is $\frac{1}{2}$ of 10?", we ask "How many 2's are there in 10?" The answer is of course again 5, and a 2 by 5 array may be used and marked in this way.



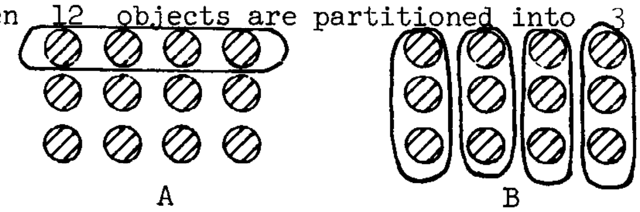
but attention is placed on the twos.

Section IX-5 introduces $\frac{1}{3}$ in much the same way

Section IX-3 introduces $\frac{1}{3}$. We ask: If 12 cookies are distributed fairly to 3 children, how many cookies will each get? We arrive at the statement

$$\frac{1}{3} \text{ of } 12 \text{ is } 4,$$

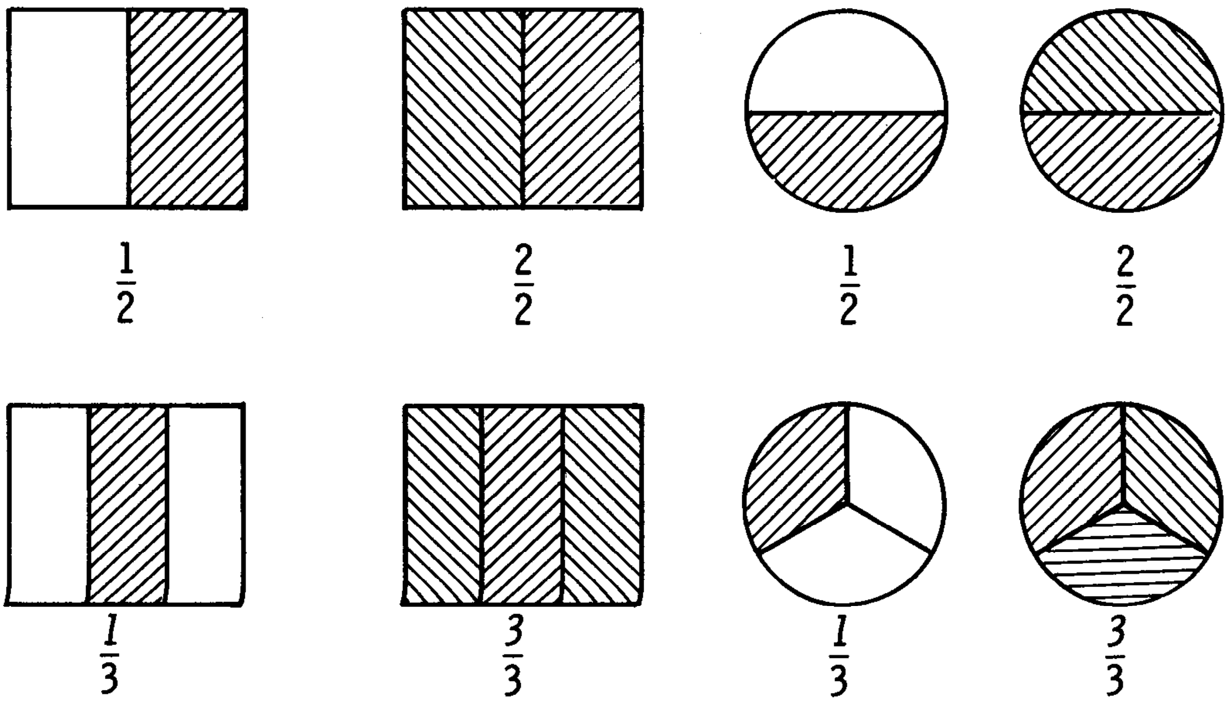
and then use arrays to see that $\frac{1}{3}$ of 12 is the same as the number of members in each equivalent set when 12 objects are partitioned into 3 rows.



In A, $\frac{1}{3}$ is the number associated with the ringed regions in the array if the array is regarded as 1 set.

In B, 4 is the number of 3's in the 3 by 4 array.

Section IX-6 uses the familiar device of cutting up regions into congruent parts (i.e., parts of the same size and shape) to make vivid $\frac{1}{2}$, $\frac{2}{2}$, $\frac{1}{3}$, and $\frac{3}{3}$:



IX-1. Partitioning sets into equivalent subsets

Objectives: To partition sets into equivalent subsets of a given number of members and to use these to form arrays.

Vocabulary: (Review) partition, equivalent.

Materials: Sets of small objects, materials for flannel board.

Suggested Procedure:

Ask the children to select a set with 12 members.

Now count 4 members of your set and pull them just a little to one side.

What do we call this part of the set? (Subset.)

Have them keep on counting out subsets with 4 members each until they have made as many subsets as they can.

How many subsets did you have when you finished? (3.)

Did each subset have 4 members? (Yes.)

Could we say we have 3 fours in our original set? (Yes.)

We have partitioned the original set into 3 sets with 4 members each.

Have children put all subsets into one set again and then partition into subsets with 3 members each, using above procedure. When 4 sets of 3 have been obtained and discussed, ask children to partition the set of 12 members into sets of 5 members each.

How many subsets do you have? (3.)

Does each subset have 5 members? (No, all but one.)

How many subsets have 5 members? (2.)
Can we partition 12 into sets of 5? (No.)

Use the same procedure and partition the set material into subsets with 2 members. Say that 12 is 6 twos. Partition the set into subsets with 6 members. Say that 12 is 2 sixes.

Further Activities:

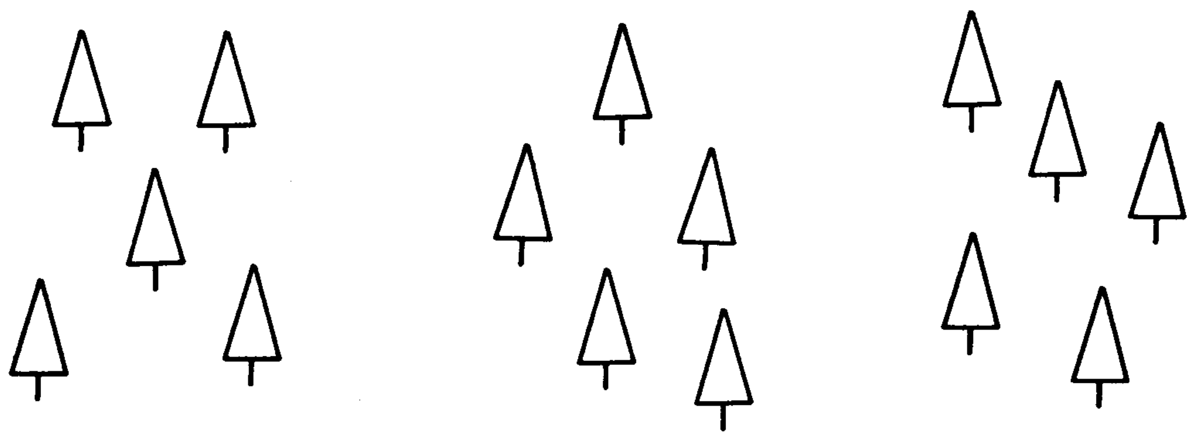
1. Have children choose 8 objects from set boxes and let individual children suggest ways of partitioning. ("Bill, how many things shall we use for our first subset?") Make sure that sometime during the period it is noticed that 8 can be partitioned into subsets of 2 and 4.

   What would happen if we used only one member in the first subset? (Then we would get 8 sets of 1 member each.)
   What would happen if we used 8 members in our first subset? (Then we would get just 1 set of 8 members.)

2. Draw sets on chalkboard (balls, trees, kites, etc.) and ask children to draw rings around equivalent subsets of various numbers of members.

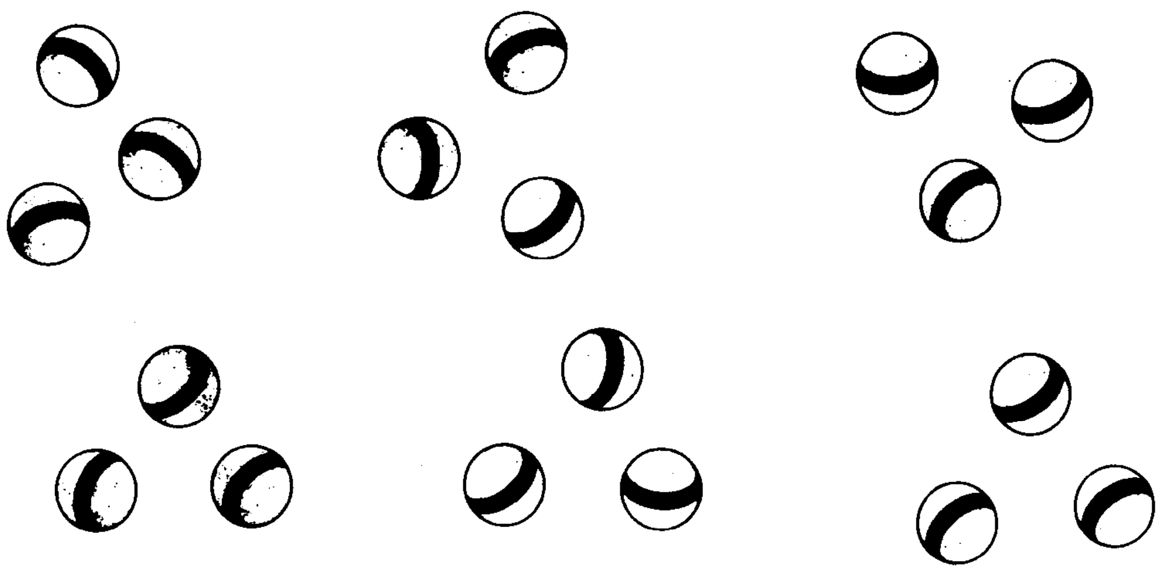
Pupil's book, pages 149 - 150: Children should count the number of objects in each box in order to fill the first blank. They should then ring sets, as indicated, and count the number of equivalent sets.

Partitioning



15 in all.

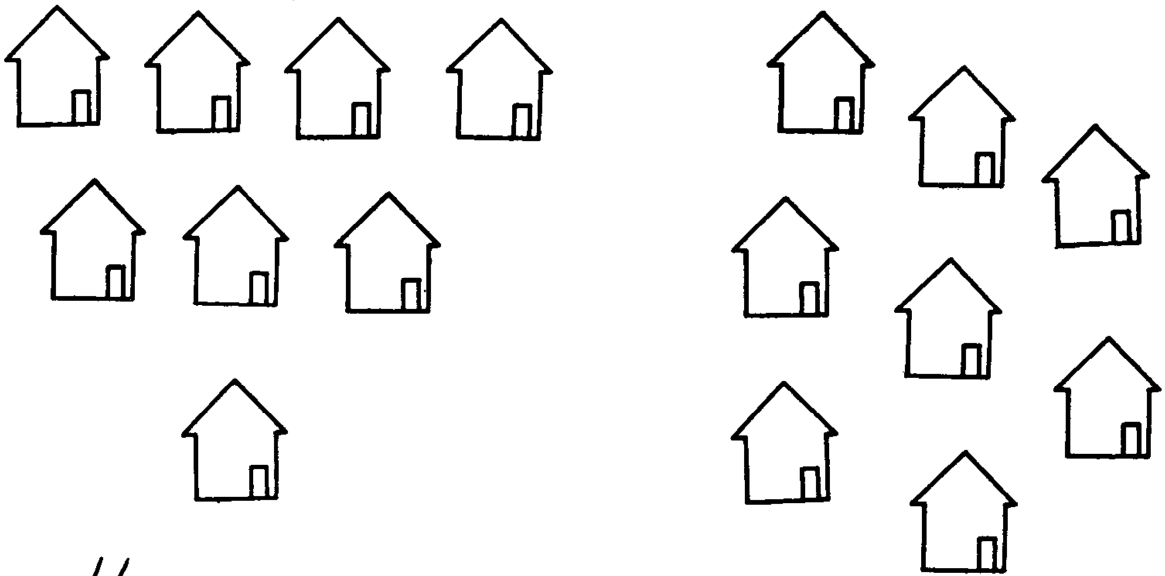
Ring sets of 5. 15 is 3 fives.



18 in all.

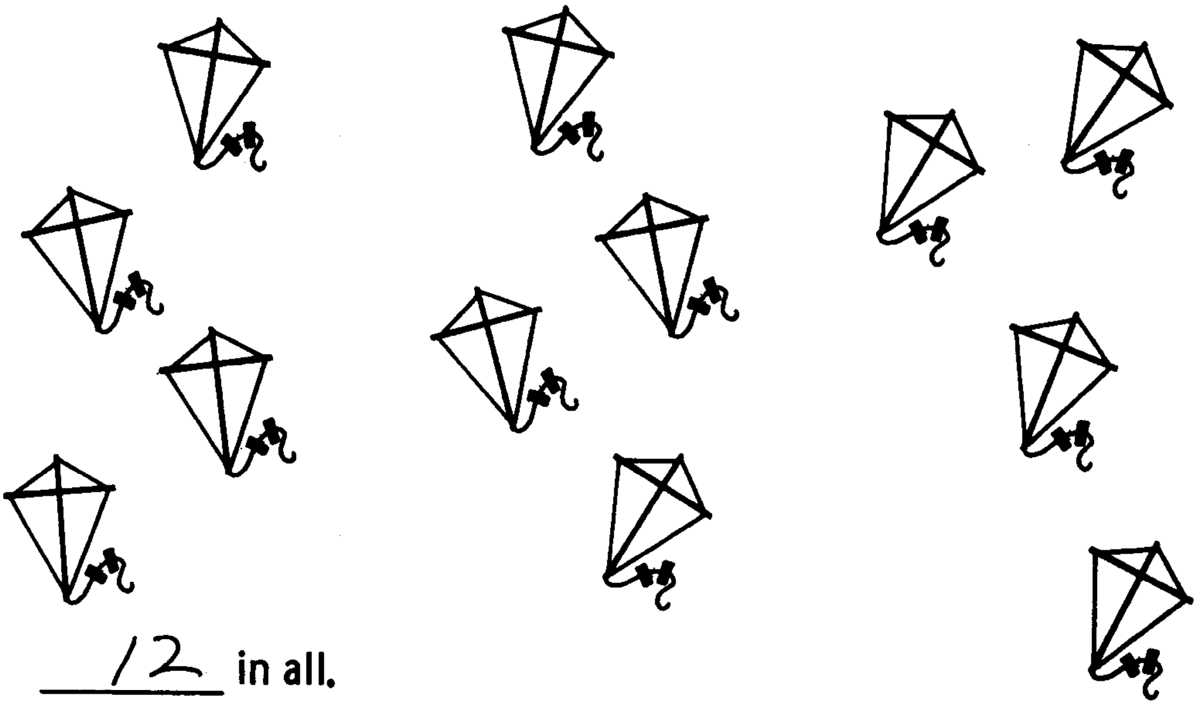
Ring sets of 3. 18 is 6 threes.

Partitioning



16 in all.

Ring sets of 8. 16 is 2 eights.



12 in all.

Ring sets of 4. 12 is 3 fours.

- Place a set with 16 members on the flannel board. Ask a child to count the number of members of the set. Then ask a child to partition the set into sets of four and observe that there are 4 fours in a set of sixteen.

    We know there are four members in each of these subsets because someone counted them as he partitioned the set of 16. How could we arrange the objects so it would be easier to see that there are 4 in each set?

Make an array.

    We know that if there are four in the first <u>row</u> and the other rows are the same, then each row has four members. We can make a set of 16 into an array with 4 rows, with 4 members in each row. We know just by looking that a set of 16 can be partitioned into 4 sets of 4. How many fours are there in 16?. We see that 16 is 4 fours just by looking.

Repeat this procedure, partitioning a set of 16 into sets of 2 by arranging it in an array so that each row has 2 members.

    How many rows do we have? (8.)

Have the set of 16 arranged in rows of 8 members each.

    How many rows do we have? (2.)

Have the set of 16 arranged in rows with 1 member in each row, etc.

Distribute counters (disks, buttons, etc.) Ask children how they show an array to find out how many sets of 3 there are in a set of 9.

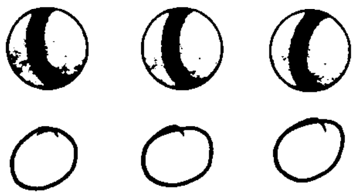
Continue in the same way, having children find how many rows of 7 members each will be in an array of 14 members, how many rows of 5 members each will be in an array of 10 members, etc.

Pupil's book, page 151 - 152: Children draw apples or write X's to find the number of 2's, 3's, etc.

Partitioning

Show 6 balls.

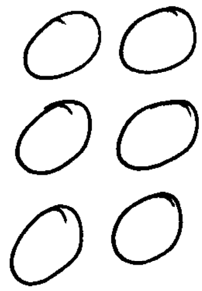
Have 3 balls in each row.



6 is ___2___ threes.

Show 6 balls.

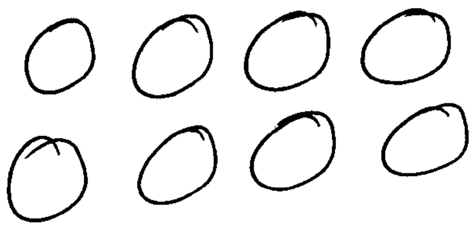
Have 2 balls in each row.



6 is ___3___ twos.

Show 8 balls.

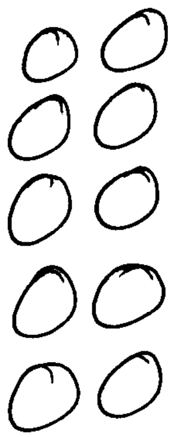
Have 4 balls in each row.



8 is ___2___ fours.

Show 10 balls.

Have 2 balls in each row.



10 is ___5___ twos.

Partitioning

Show 8 X's. Have 2 X's in each row. X X 8 is 4 twos.	Show 12 X's. Have 3 X's in each row. 12 is 4 threes.
Show 15 X's. Have 5 X's in each row. 15 is 3 fives.	Show 12 X's. Have 4 X's in each row. 12 is 3 fours.

IX-2. Partitioning into a given number of equivalent sets

Objective: To partition a set into a given number of equivalent sets.

Vocabulary: (No new words.)

Materials: Sets of small objects, materials for flannel board

Suggested Procedure:

Present the following problem:

> Mary, Sue, and Betty have 18 cookies. They want to share the cookies fairly, so that each girl will have the same number of cookies. How can they do this?

Children will probably suggest giving one to each girl in turn until all cookies have been distributed, "dealing them out" until all are gone. Explain that each girl's cookies should then be counted to be sure everyone has a fair share.

Suggest using 18 felt objects on the flannel board to find out how many cookies each girl should get. Place 3 objects one below the other, on the flannel board, and then put another in each row, etc., until all 18 have been used.

Children should observe that an array has been made, and that it is easy to see that each row has 6 members.

18 is three ________'s.

Discuss the fact that making an array has shown that the missing word is sixes.

Have children use manipulative objects to find out how many members there would be in each set if a set of 16 were partitioned into 4 equivalent subsets. Show that they are solving the problem:

16 is four ________'s.

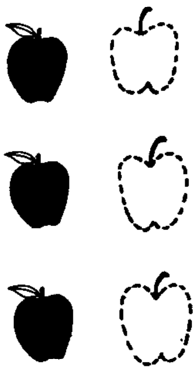
Continue with several other problems of this sort: 12 as 4 sets of how many members, 14 as 2 sets of how many members, etc.

Have children use objects to solve similar problems.

<u>Pupil's</u> <u>book,</u> <u>pages</u> 153 - 155: Have children draw more objects to make an array with the number of members indicated and the rows started. They are to find the number of members in each row.

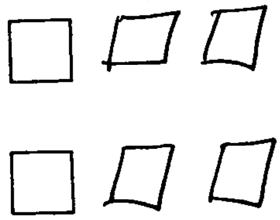
Partitioning

Show 6 in all.



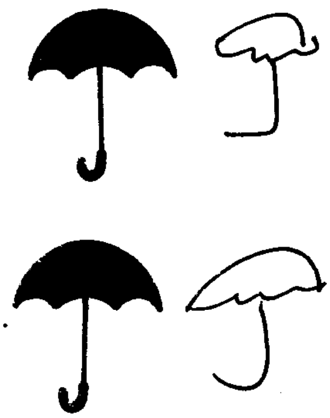
6 is three __2__'s.

Show 6 in all.



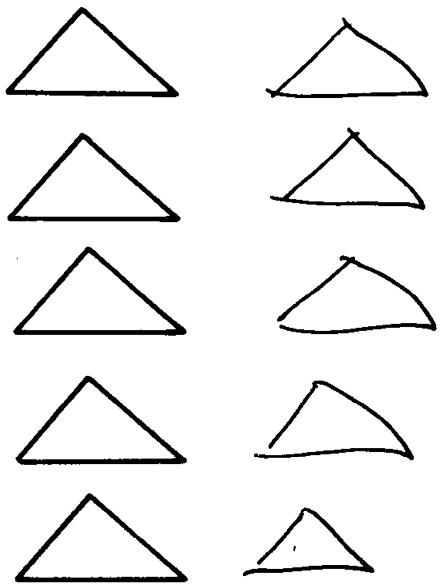
6 is two __3__'s.

Show 4 in all.



4 is two __2__'s.

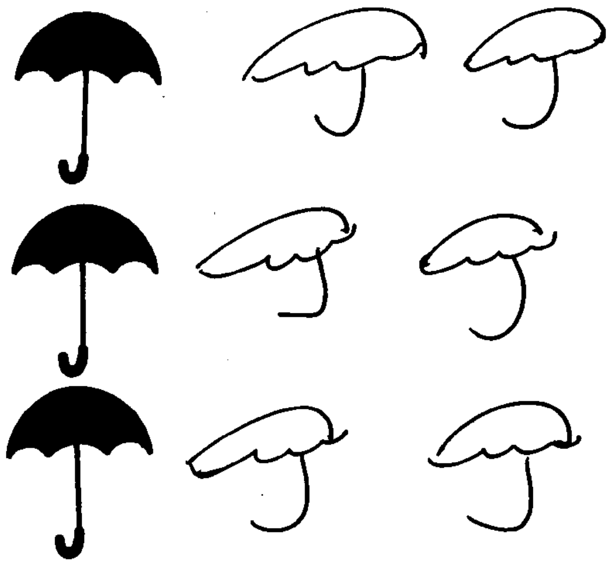
Show 10 in all.



10 is five __2__'s.

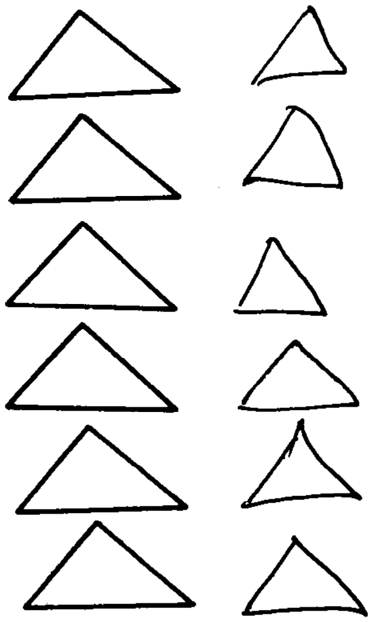
Partitioning

Show 9 in all.



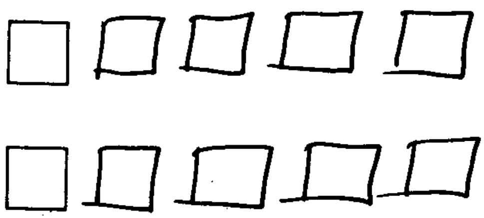
9 is three 3 's.

Show 12 in all.



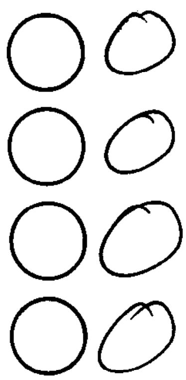
12 is six 2 's.

Show 10 in all.



10 is two 5 's.

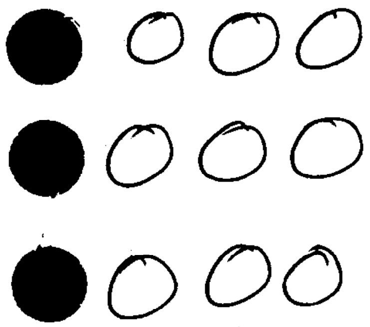
Show 8 in all.



8 is four 2 's.

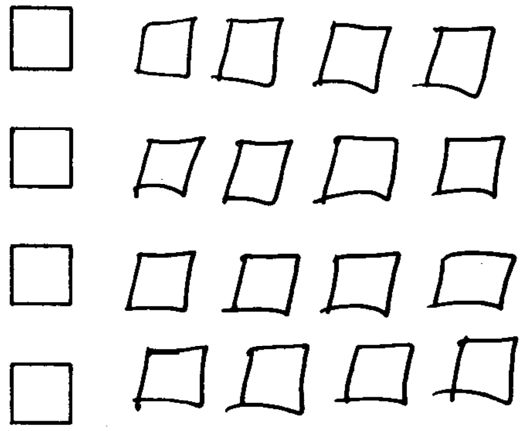
Partitioning

Show 12 in all.



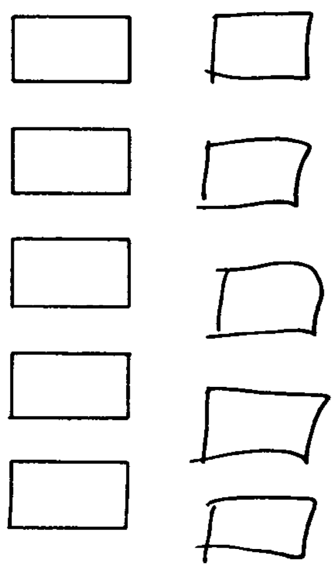
12 is three 4 's.

Show 20 in all.



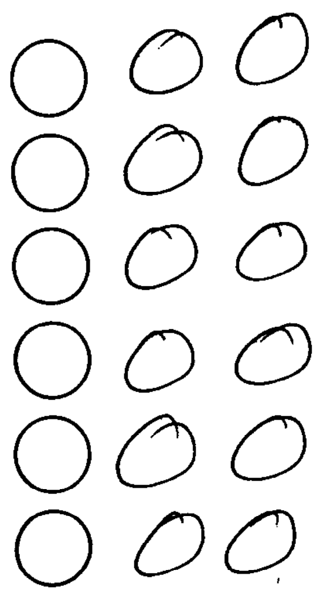
20 is four 5 's.

Show 10 in all.



10 is five 2 's.

Show 18 in all.



18 is six 3 's.

IX-3. One-half

Objective: To introduce the idea of one-half and the written symbol, "$\frac{1}{2}$".

Vocabulary: One-half, part.

Materials: Materials for flannel board, sets of small objects.

Suggested Procedure:

Place 6 flannel cut-outs (apples on the flannel board in no particular arrangement. Ask children how many apples are on the flannel board.

Then say--

> Now, suppose we let someone in the class have one-half of the apples in this set. Who can show us how many apples he would take? (Let a child who thinks he can show us remove the apples that we will give away.) How many apples did he take? (3) What part of the set of apples did he take? ($\frac{1}{2}$)
>
> Now suppose we want to give away the other apples. How many apples can we give someone else? (3.) (You may wish to name a child who is to receive these.) What part of the set of apples did he get? ($\frac{1}{2}$.) How many one-halves of a set of apples are there? (2.) How many children received apples? (2.) How many apples did each child get? (3.) $\frac{1}{2}$ of 6 objects is how many? (3.)

Because many children have only the idea that one-half means a part of something, or less than all of a set, it is necessary to emphasize the fact that finding one-half of a set requires partitioning the set into 2 equivalent subsets.

Provide experiences for showing one-half of many different sets-- 4, 8, 10, 12. It is important that children learn to think of the set of objects as one set and that to find $\frac{1}{2}$ of the set, they partition (or separate) the set into two subsets so that there are just as many objects in one subset as in the other.

- After they understand how they can find one-half of a set of objects by partitioning the set into two equivalent subsets, display on the chalkboard how we can describe what we have done using the names of the numbers. For example, $\frac{1}{2}$ of 8 is 4. Explain that $\frac{1}{2}$ is a new kind of number. Point out that the numeral "$\frac{1}{2}$" is made by using names for 1 and for 2. You may wish to ask if there are other numerals which are made by using the numerals for one and two. (12 and 21) Emphasize that we write these names in a different way. We write "1", put a bar under it, and write "2" under the bar: $\frac{1}{2}$. This is a name for the number <u>one</u>-<u>half</u>.

- Repeat some of the previous experiences again and this time write the sentences that can be associated with finding $\frac{1}{2}$ of a set of objects.

  Write on the chalkboard:

  $\frac{1}{2}$ of 14 is ________.

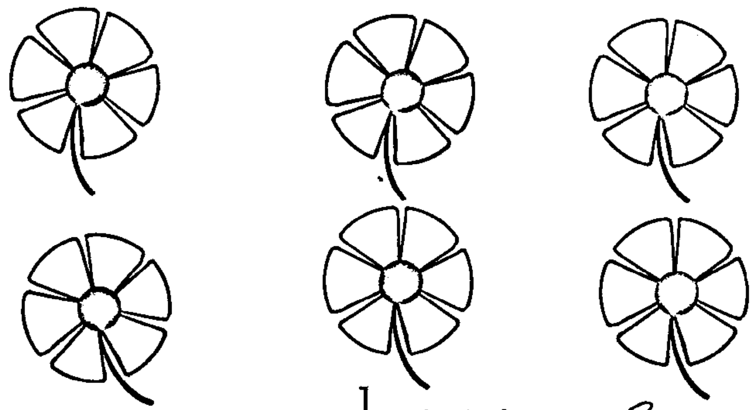
  Ask how many objects are to be in the set to be partitioned. Ask how children can find the number so that they can complete the sentence. Have children use sets of small objects and partition of a set of 14 into 2 equivalent subsets to determine the answer.

  Write other sentences and have children use their sets of objects to complete the sentences.

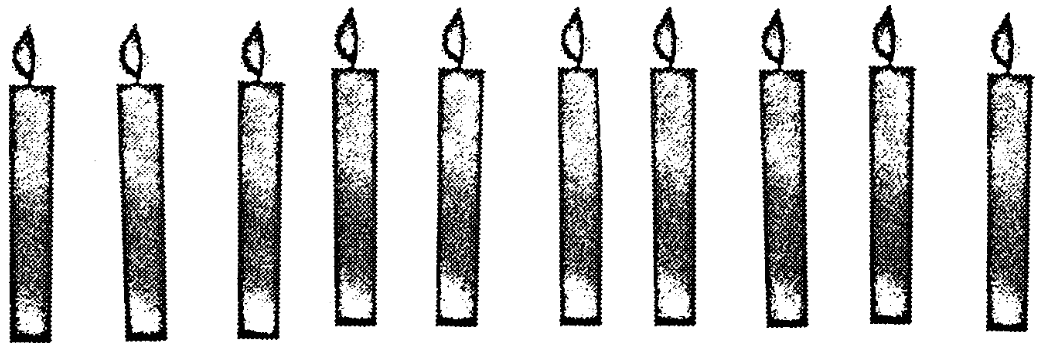
<u>Pupil's book, pages</u> 156 - 158   Children are to ring $\frac{1}{2}$ of each set.

One Half

Color $\frac{1}{2}$ of each set blue. Color the other half red. Fill the blanks.



$\frac{1}{2}$ of 6 is 3.

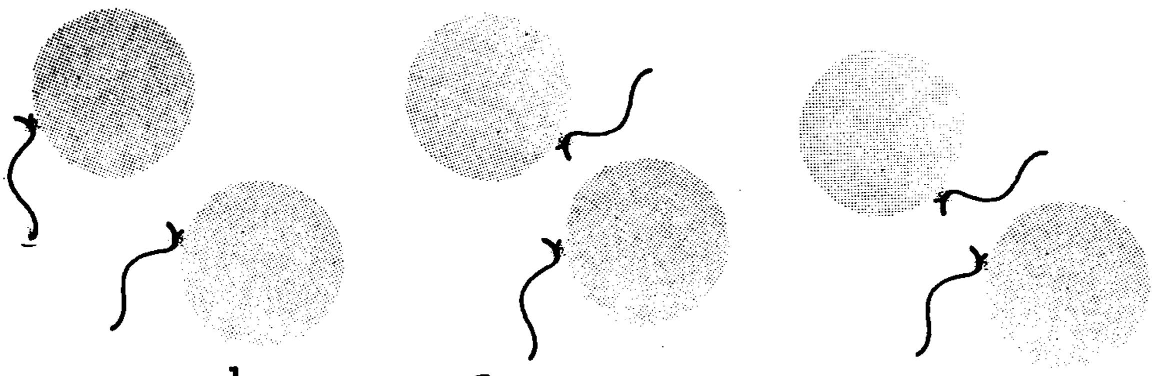


$\frac{1}{2}$ of 10 is 5.

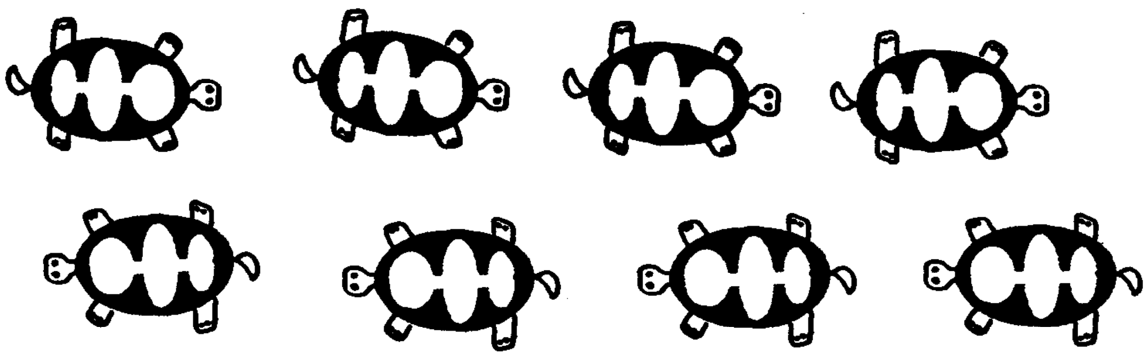
$\frac{1}{2}$ of 4 is 2.

One Half

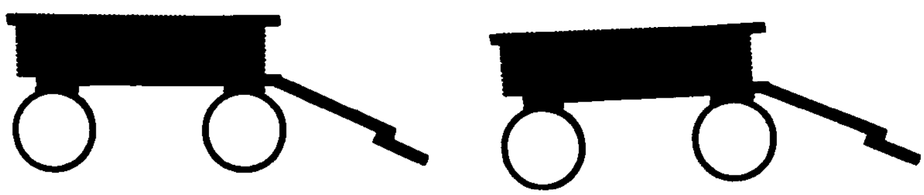
Color $\frac{1}{2}$ of each set blue. Color the other half red. Fill the blanks.



$\frac{1}{2}$ of 6 is 3.



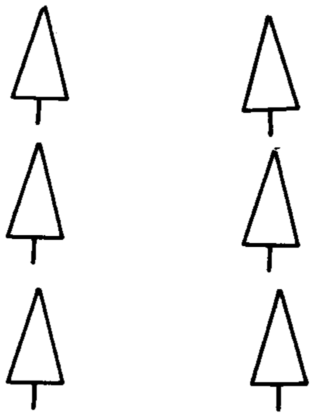
$\frac{1}{2}$ of 8 is 4.



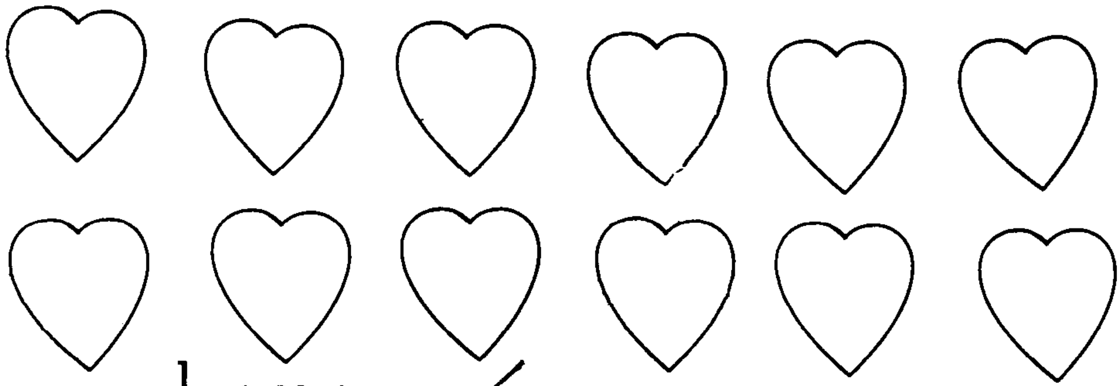
$\frac{1}{2}$ of 2 is 1.

One Half

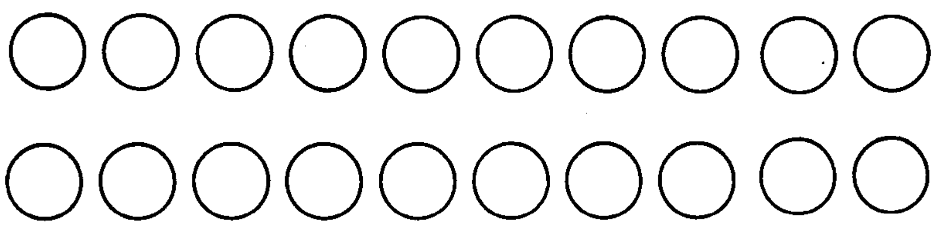
Color $\frac{1}{2}$ of each set blue. Color the other half red. Fill the blanks.



$\frac{1}{2}$ of 6 is ___3___.



$\frac{1}{2}$ of 12 is ___6___.



$\frac{1}{2}$ of 20 is ___10___.

<u>Further Activities</u>:

Read the following story problems to the children.
Let them use manipulative materials to find the answers.

1. 6 boys were playing ball.
   One-half of the boys went home.
   How many boys went home? (Three boys went home.)
   How many boys were still playing ball? (Three boys were still playing ball.)
   What part of the group was still playing ball?
   (Half of the group was still playing ball.)

2. Mother had 8 sticks of gum.
   She kept $\frac{1}{2}$ of the gum for herself.
   How many sticks of gum did mother keep for herself? (She kept 4 sticks.)

3. Father had 2 golf balls.
   He lost $\frac{1}{2}$ of the balls.
   Find how many balls he lost. (He lost 1 golf ball.)
   What part of the set of golf balls did Father still have? (He had $\frac{1}{2}$ of the balls.)

4. Mother had 4 cookies.
   She gave 2 cookies to Sarah.
   What part of the set of cookies did Sarah get?
   (Sarah got $\frac{1}{2}$ of the cookies.)
   What part of the set of cookies did Mother still have? (Mother still had $\frac{1}{2}$ of the cookies.)

5. Father had 12 nails.
   He used $\frac{1}{2}$ of the nails to make a bird house.
   How many nails did he use? (He used 6 nails.)
   What part of the set of nails did he still have?
   (He had $\frac{1}{2}$ of the nails.)

IX-4. Halves and two times

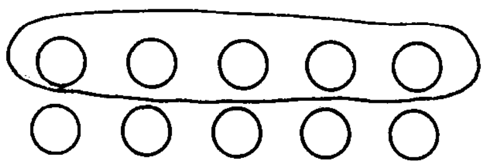
Objective: To lead children to see that 2 times one half of a number is the same as that number.

Vocabulary: (No new words.)

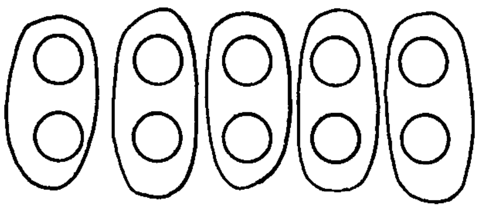
Materials: Materials for flannel board, yarn, manipulative objects.

Suggested Procedure:

Begin with a display of ten oranges on the flannel board and explain that we want to find one half of ten by partitioning this set into two equivalent subsets. Ask if there is any nice arrangement which we could make to show the two sets of the partition and display the fact that they are equivalent. Lead up to arranging the set in an array with two rows, and five members in each row. Decide that 5 is one half of ten. Ring one of the rows with yarn.



Then ring sets of two to show that the same number, 5, indicates how many twos there are in 10:

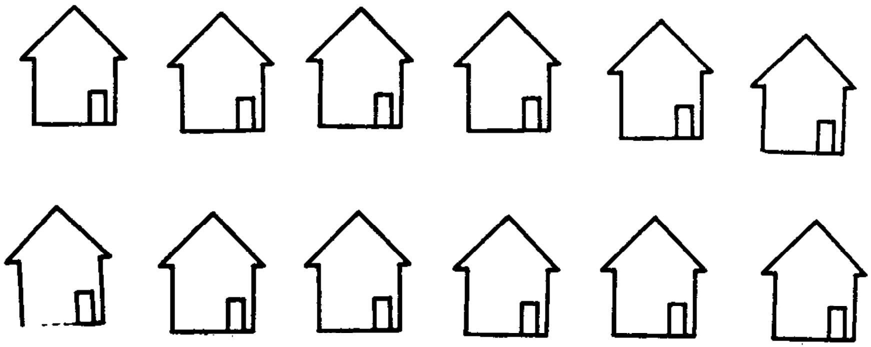


Now use 18 objects on the flannel board. Construct an array with two rows, and decide that nine is one-half of 18, and that there are two nines in 18.
Repeat with other sets.

<u>Pupil's book, pages</u> 159 - 161:

Children first record the number of members in the array. They will ring or color $\frac{1}{2}$ of the set.

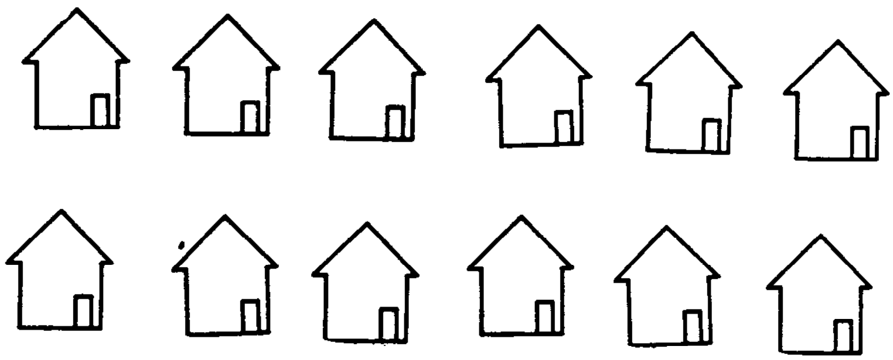
Partitions



How many? 12

Color $\frac{1}{2}$ of the set.

$\frac{1}{2}$ of 12 is 6.

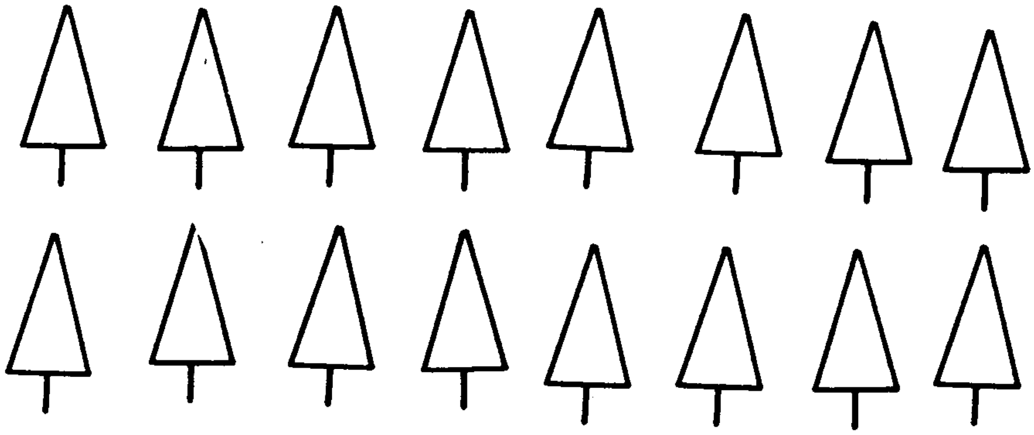


How many? 12

Ring subsets with 2 members.

6 twos in 12.

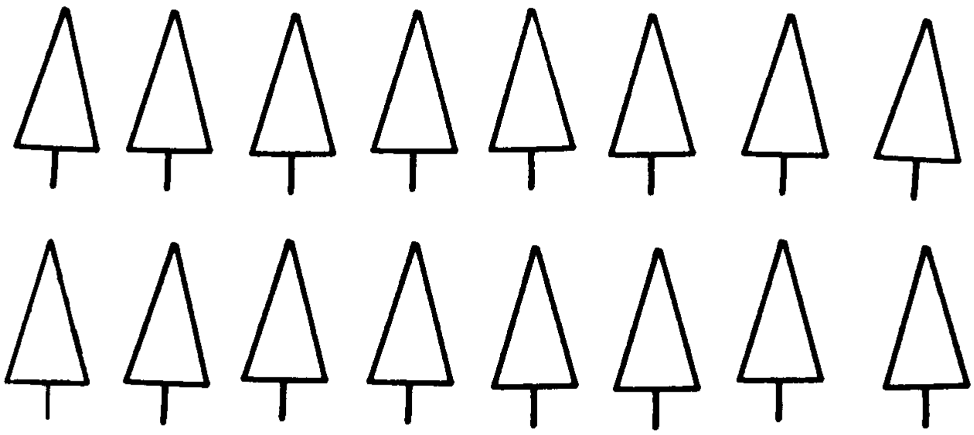
Partitions



How many? 16

Color $\frac{1}{2}$ of the set.

$\frac{1}{2}$ of 16 is 8.

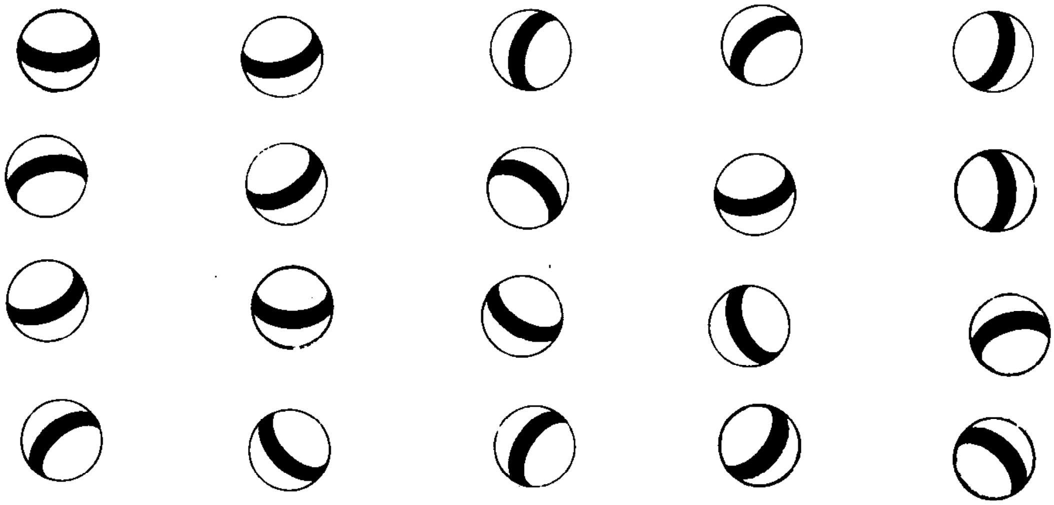


How many? 16

Ring subsets with 2 members.

8 twos in 16.

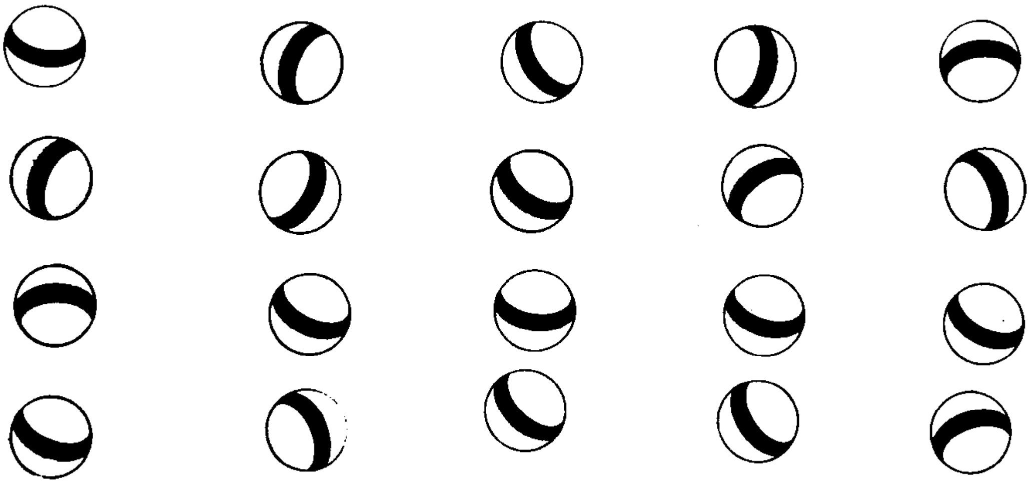
Partitions



How many? 20

Color $\frac{1}{2}$ of the set.

$\frac{1}{2}$ of 20 is 10.



How many? 20

Ring subsets with 2 members.

10 twos in 20.

IX-5. One third

Objective: To introduce the ideas of one third, written symbols "$\frac{1}{3}$".

Vocabulary: One third.

Materials: Objects for flannel board, manipulative materials.

Suggested Procedure:

Make a flannel board display of 12 disks. Ask the children to imagine that the disks are cookies and explain that three children are to share these so that each child gets the same number of cookies. Ask three of the children to come to the board and decide how to do this. They may each take one in turn, till all cookies are gone. Ask the first child to place his share of the cookies in a row, the next child to place his in a row below that of the first child's, and so on. Say that each child has 4 cookies, that each child has one-third of the 12 cookies, and that one-third of 12 is 4.

Write "$\frac{1}{3}$ of 12 is 4" on the chalkboard and, as you did in the case of $\frac{1}{2}$, explain that this is a new number and so on. Repeat with other sets until the idea that to find $\frac{1}{3}$ of a set, we partition the set into 3 equivalent sets is understood.

Have the children, working individually with sets of objects, find one-third of 15, of 21, and so on.
Use "Show-me cards" to check the results.

After understanding of the concept of one-third has been developed, we help children to see that one-third of a number can be associated with: 3 x ______ = given number.

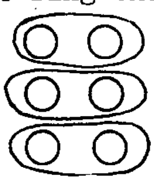
We do this with flannel board demonstrations, just as in the preceding section, when we learned that, for example

$\frac{1}{2}$ of 8 = 4 is related to 2 x 4 = 8.

Begin with a display of 6 objects on the flannel board.

0 0
0 0
0 0

Ask how to show one third of the set with yarn.
(Ring the 2 members of 1 row.)
Ask whether anything else may be seen from the array.
Try to lead children to see that there are 3 twos. in 6. Use yarn to ring the three twos.



Write: $\frac{1}{3}$ of 6 is 2.

There are 3 twos in 6.

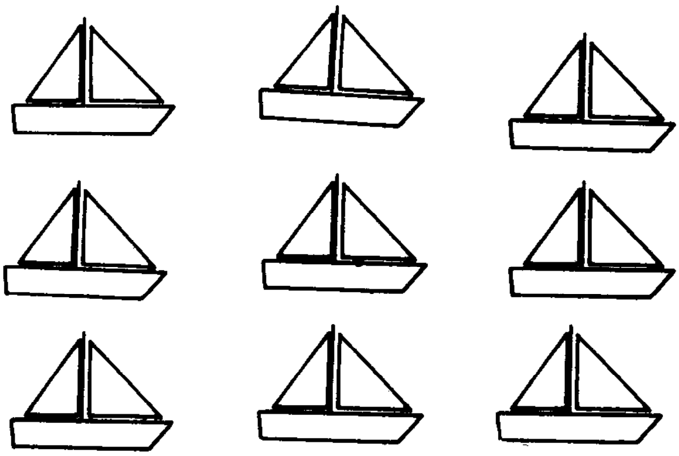
Repeat with other sets.

Pupil's book, pages 162 - 163: Children are to mark $\frac{1}{3}$ of the objects in each box, and fill the blanks, as indicated.

Pupil's book, page 164: Children should observe the relationship between $\frac{1}{3}$ and $\frac{1}{2}$.

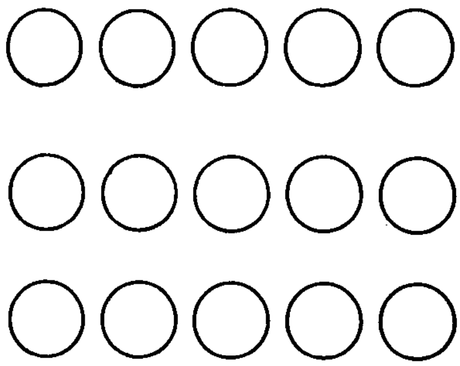
One Third

Color $\frac{1}{3}$ of each set. Fill the blanks.



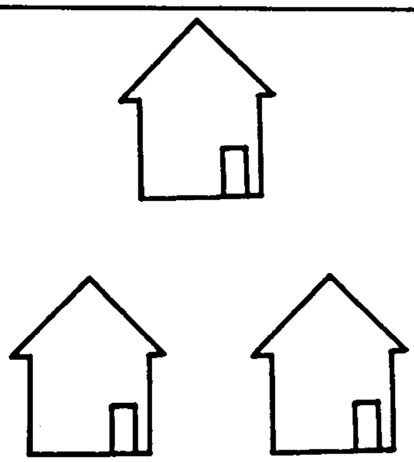
$\frac{1}{3}$ of 9 is 3.

9 is 3 threes.



$\frac{1}{3}$ of 15 is 5.

15 is 5 threes.

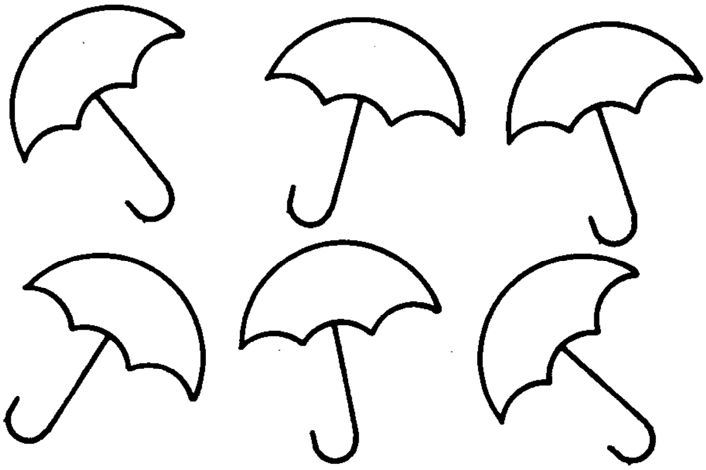


$\frac{1}{3}$ of 3 is 1.

3 is 1 three.

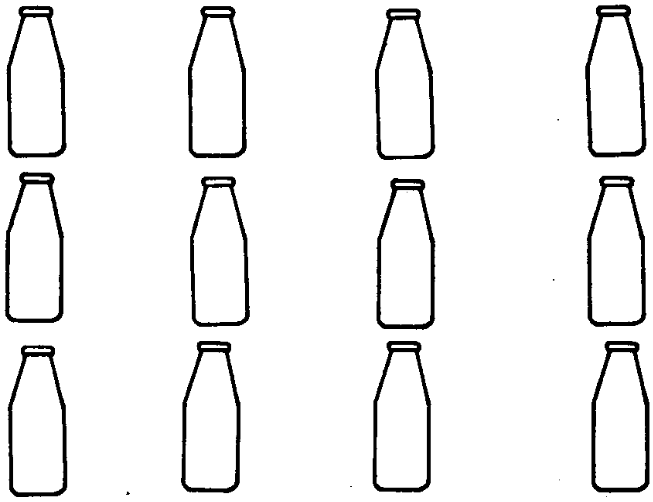
One Third

Color $\frac{1}{3}$ of each set. Fill the blanks.



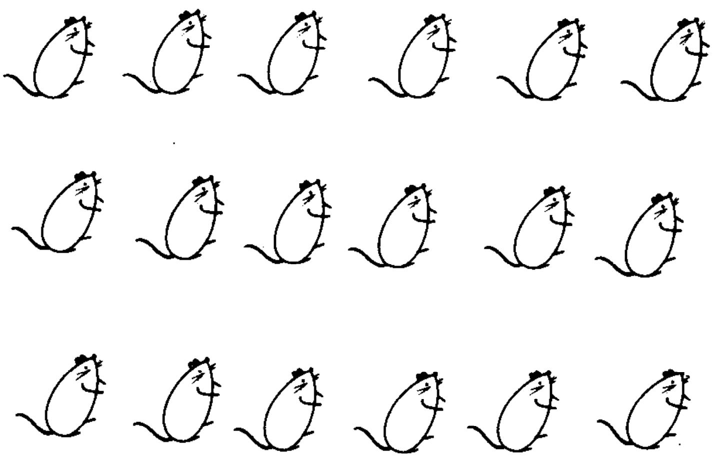
$\frac{1}{3}$ of 6 is 2.

6 is 2 threes.



$\frac{1}{3}$ of 12, is 4.

12 is 4 threes.

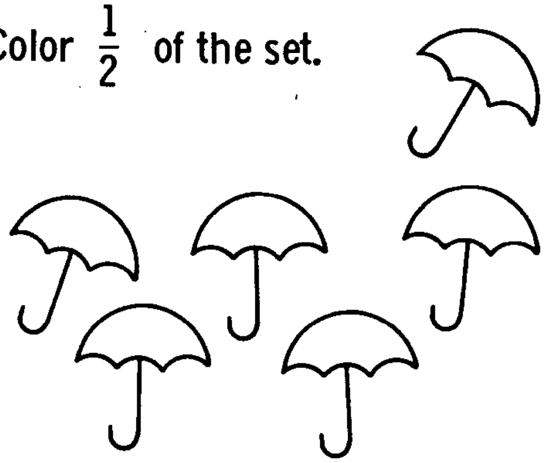


$\frac{1}{3}$ of 18 is 6.

18 is 6 threes.

One Half and One Third

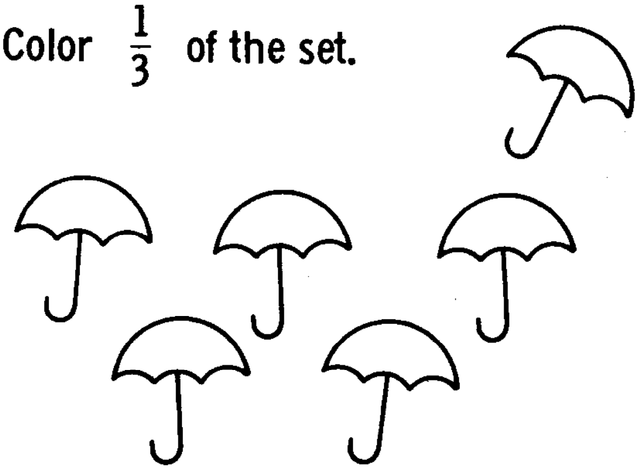
Color $\frac{1}{2}$ of the set.



$\frac{1}{2}$ of 6 is 3.

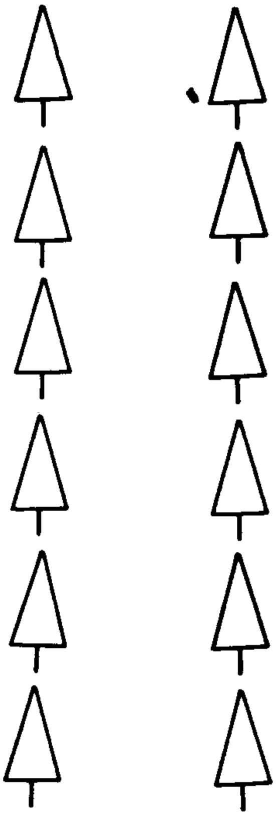
6 is 3 twos.

Color $\frac{1}{3}$ of the set.



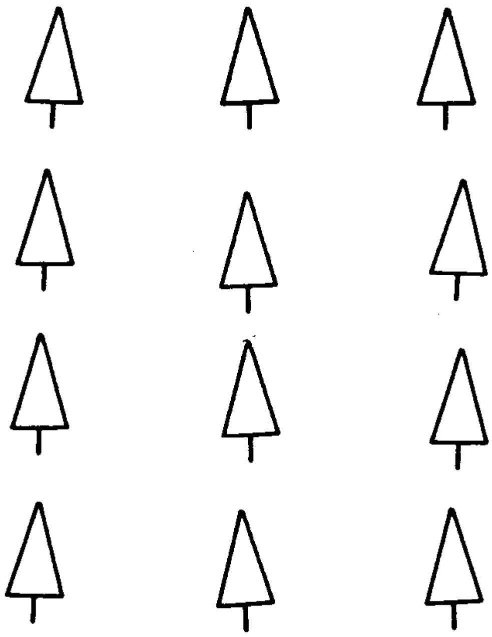
$\frac{1}{3}$ of 6 is 2.

6 is 2 threes.



$\frac{1}{2}$ of 12 is 6.

12 is 6 twos.



$\frac{1}{3}$ of 12 is 4.

12 is 4 threes.

★ Further Activities: (Optional)

At a later time, when children show understanding of the ideas of one third and one half, you may wish to introduce the ideas of two thirds and three thirds.

Begin with a flannel board display with 3 rows with 4 members in each row. Agree that one third of 12 is 4 and record this result on the chalkboard. Then ask what two thirds of 12 should be, and agree that it should be two fours or 8.
Record in the form:

$\frac{1}{3}$ of 12 is 4

$\frac{2}{3}$ of 12 is 8

$\frac{3}{3}$ of 12 is ______.

Agree that $\frac{3}{3}$ of 12 should be one twelve, and explain that "$\frac{3}{3}$" is just another name for one. You may want to discuss "$\frac{2}{2}$", which is also another name for one, at this time, or you may wish to postpone this until later. Have the children, working individually with sets of objects, find one third and two thirds of 15, of 21, and so on. Use "Show-me" cards to check the results.

IX-6. Parts of regions

Objective: To relate halves and thirds to regions and to physical objects.

Vocabulary: (No new words.)

Materials: Rectangular and circular regions of construction paper, 1 yellow and 1 blue of each. Numeral cards $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{2}$, $\frac{3}{3}$.

Suggested Procedure:

Jimmy and Tim each want a piece of yellow paper for a picture. I have only one piece of yellow paper. What shall we do to the paper? (Cut it into two pieces.)

There are many ways in which we could cut the paper. If we cut it in a way so that Jimmy and Tim each have a piece of the same size and shape then we say that each boy has one half of the sheet of paper. If one piece is larger than the other, then the pieces can not be called halves.

If we cut the paper into halves, what number of pieces will we have? (2.)

The piece that Jimmy gets will be what part of the paper? The piece that Tim gets will be what part of the paper? Show how this is written. ($\frac{1}{2}$.)

What is the number of halves in the piece of paper? (Two halves.)

This is written $\frac{2}{2}$.

Two halves is another name for 1.

What would we have done if three boys had wanted to use this blue paper? (Cut it into three pieces.)

If we cut it so that there are three pieces of paper and all three pieces are the same size and shape, then each is called one third.

This is written $\frac{1}{3}$.

What is the number of thirds in the whole piece of paper? (3.)

Place the papers on the flannel board. Place them up in such a way that the sides of the parts are touching and look like the whole piece of paper. It may be advisable to mark the lines which were cut with black crayon to indicate the pieces which were made. Have some numeral cards with $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{2}$, $\frac{3}{3}$. Ask children to touch or pick up various pieces which are named or that match the numeral card which is displayed.

Emphasis needs to be given again to the idea that the numbers $\frac{2}{2}$ and $\frac{3}{3}$ are each equal to 1.

Use various rectangular regions as well as circular regions to show halves, and thirds.

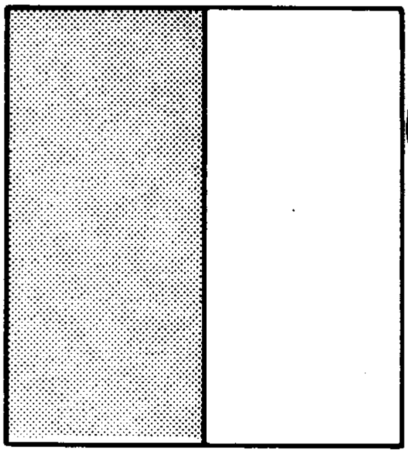
Use objects such as a cookie, candy stick, candy bar. Demonstrate and discuss how these must be cut into pieces of the same size and shape. Call the parts one half or one third of the object.

Pupil's book, page 165: Children are to ring the numeral which shows what part of the region is shaded.

Pupil's book, page 166: Children are to color parts of regions as indicated.

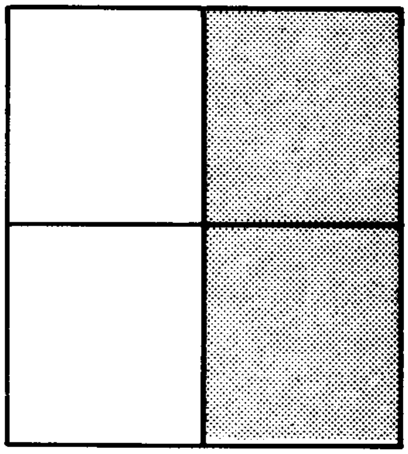
Regions

Ring the correct numeral.



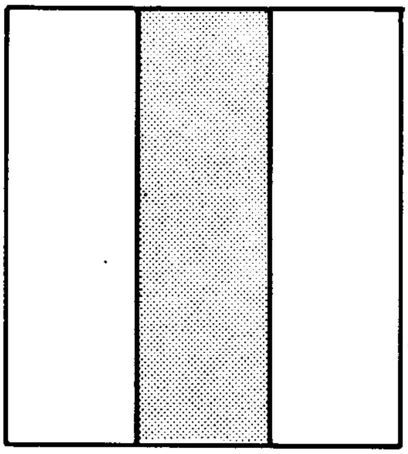
$\frac{1}{2}$

$\frac{1}{3}$



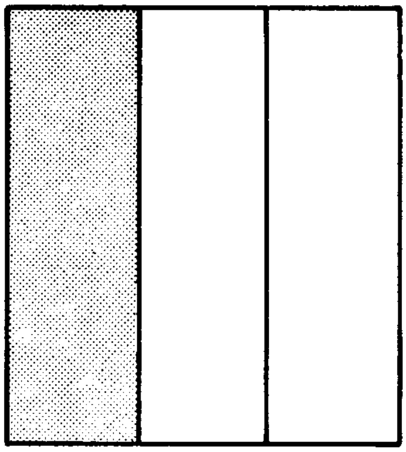
$\frac{1}{3}$

$\frac{1}{2}$



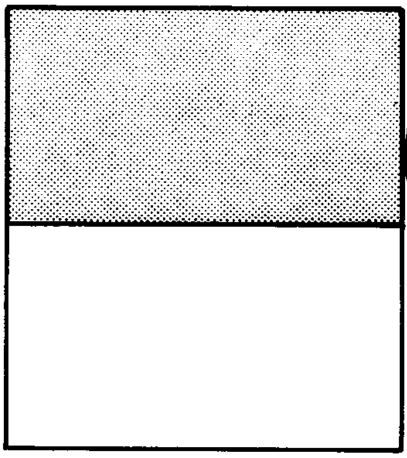
$\frac{3}{3}$

$\frac{1}{3}$



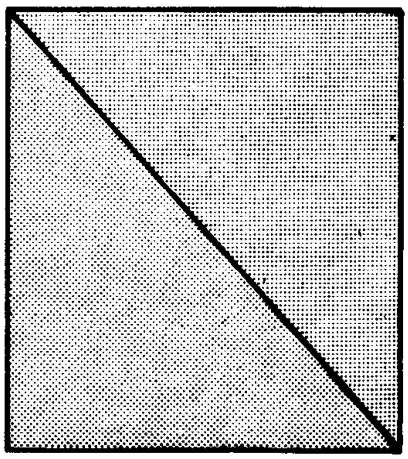
$\frac{1}{3}$

$\frac{3}{3}$



$\frac{1}{2}$

$\frac{1}{3}$

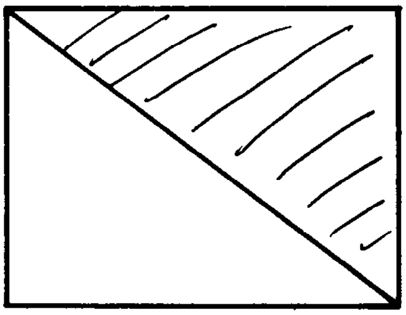


$\frac{2}{2}$

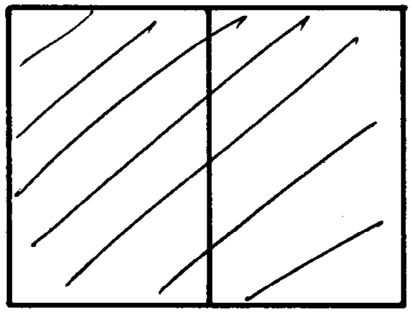
$\frac{1}{2}$

Regions

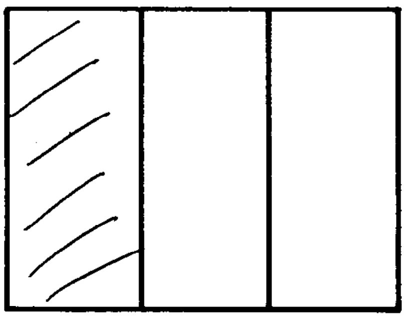
Color $\frac{1}{2}$ red.



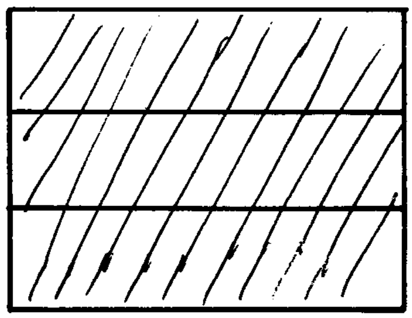
Color $\frac{2}{2}$ blue.



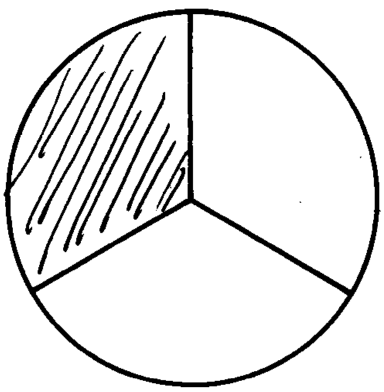
Color $\frac{1}{3}$ yellow.



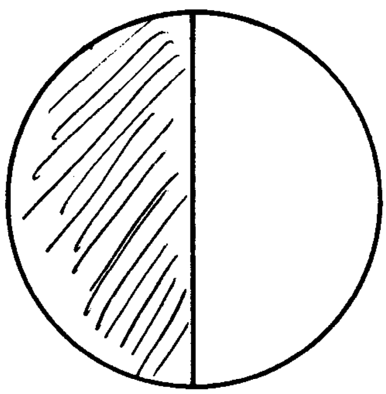
Color $\frac{3}{3}$ green.



Color $\frac{1}{3}$ red.



Color $\frac{1}{2}$ blue.



Chapter X

LINEAR MEASUREMENT

Background

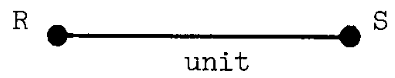
In this chapter we discuss the measurement of line segments. Recall that a line segment is the set of points followed in passing along a straight path from a given point A to a given point B. Two line segments are congruent provided that they have the same size, so that one will fit exactly on the other.

Long before the child comes to school he has experience in comparisons of order: his father is taller than he is; his sister is younger than he is; the new house is bigger than the old house; he woke up today before his mother did; this pail is heavier than that pail. He has also had experience with the notion of measure; he understands and makes such statements as, "My dad is 6 feet tall," "We get 3 quarts a day," "It takes me 15 minutes to get to school." Here we wish to extend the child's knowledge of linear measure and to deepen his intuitive understanding.

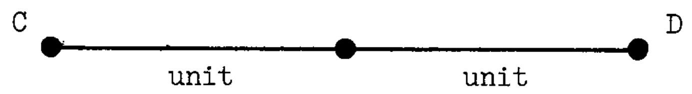
Our development parallels the historical one. The counting of separate objects (say, sheep) was a technique not applicable to measuring a region or curve (like a field and its boundary). Nevertheless, one could often make comparisons: this field is larger than that; this boundary is longer than that. Later, when fields bordered more closely on each other, actual measurement became necessary. When a unit of measure (e.g., that part of a rope between two knots) was agreed upon, it was possible to designate a piece of property as having a length of "50 units of rope" and having a width of "30 units of rope". With the increase in travel and communication it became obvious that "50 units of rope" did not always represent the same length. Hence, standard units were adopted. For convenience in measuring, rules or scales marked in these standard units were introduced.

Measure, Length, Units

In measuring line segments, we first select a particular line segment, say $\overline{RS}$, to serve as a unit.



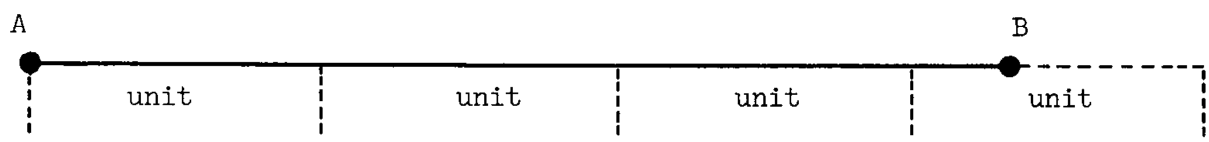
The length of $\overline{RS}$ itself is then 1 unit. To measure any given line segment $\overline{CD}$, we lay off the unit $\overline{RS}$ along it.



If the unit can be laid off exactly twice, as in the picture, we say that the measure of $\overline{CD}$ is 2, and that the length of $\overline{CD}$ is 2 units. If the unit could be laid off exactly three times, we would say that the measure of $\overline{CD}$ is 3, and that the length of $\overline{CD}$ is 3 units. The measure of a line segment is a number: the number of times the unit can be laid off on the line segment. When naming a length, we use both the measure and the unit.

Length to the nearest unit

More often than not, the unit will not fit exactly some number of times, but there will be a part of a unit left over. In the picture, the unit can be laid off along the segment $\overline{AB}$ 3 times, with a part of a unit left over, but it does not fit 4 times.



The length of $\overline{AB}$ is then greater than 3 units but less than 4 units. Moreover, in our examples the length of $\overline{AB}$ is visibly nearer to 3 units than to 4 units. In this case, we say that the length of $\overline{AB}$ to the nearest unit is 3 units. This approximation is the best we can give without introducing fractional parts of a unit or shifting to a smaller unit. In this chapter we will not introduce the phrase, to the nearest unit, but will note that the length of $\overline{AB}$ above is between 3 and 4 units.

A word about terminology: We do not add inches, any more than we add apples. All we add are numbers. If we have 3 apples and 2 apples, we have 5 apples altogether, because

$$3 + 2 = 5.$$

Likewise, if we have 3 yards of ribbon and 2 more yards of ribbon, we have 5 yards of ribbon altogether, again because

$$3 + 2 = 5.$$

Standard Units and Systems of Measures

The acceptance of a standard unit for purposes of communication is soon followed by an appreciation of the convenience of having a variety of standard units. An inch is a suitable standard unit for measuring the edge of a sheet of paper, but hardly satisfactory for finding the length of the school corridor. While a yard is a satisfactory standard for measuring the school corridor, it would not be a sensible unit for finding the distance between Chicago and Philadelphia.

The last section includes work with the idea of time. The placement of the material here is to extend physical measurement in which a linear scale is used.

X-1. Line segment, straightedge

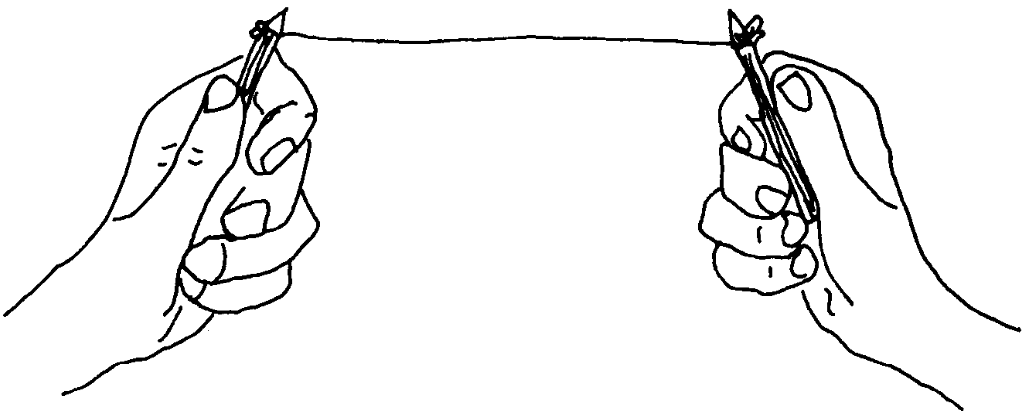
Objective: To introduce the concept of line segment and the use of the straightedge.

Vocabulary: Straightedge, line segment.

Materials: Jump rope, yarn, string, thread; various models of line segments; unmarked strips of cardboard (at least 10 inches in length).

Suggested Procedure:

Show a loosely held string between two pencils. Pull the string tightly to demonstrate the idea of a straight path.

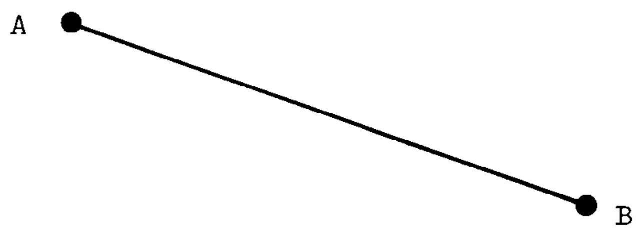


Ask the children to identify objects that display straight edges: the edge of a desk, a sheet of paper, etc.

Explain that these are all examples of line segments; a straightedge from one point to another. Call attention to the physical things that suggest the endpoints. For example, if the edge of a block is mentioned as a line segment, then the corners of the block represent the endpoints.

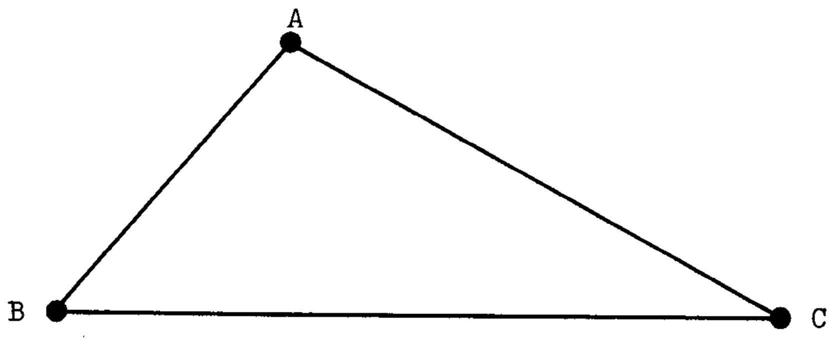
On the chalkboard show two points. Draw a line segment between them. Use an unmarked cardboard straightedge, since one will be used later by the children.

Explain that it is often helpful to give names to the end points. Label them A and B as shown.



Explain further that the names of the endpoints may be used to name the line segment as either line segment $\overline{AB}$ or line segment $\overline{BA}$. Illustrate and name several other line segments.

Uncover on the chalkboard a picture of a triangle. Ask the children if there is a way in which they can use line segments and letters to describe the triangle. Then label the triangle.



Help the children to visualize that this triangle can be described as being made up of line segment $\overline{AB}$, line segment $\overline{BC}$, and line segment, $\overline{CA}$.

Show two more points on the board. Demonstrate a technique for using straightedge and chalk. Show that if a piece of chalk is placed on one point, the straightedge lined up slightly below the other, then the line segment drawn will include both points. Also, discuss the importance of holding the straightedge at the center rather than at an end.

Ask several children to come to the board for a demonstration of the need to hold a straightedge firmly. Ask what will happen if fingers overlap the edge on which a segment is to be drawn.

Distribute a cardboard strip to each child.

Pupil's book, pages 167 - 170: Line segments

Ideas

A line segment connects two points.

A straightedge can be used to draw a line segment.

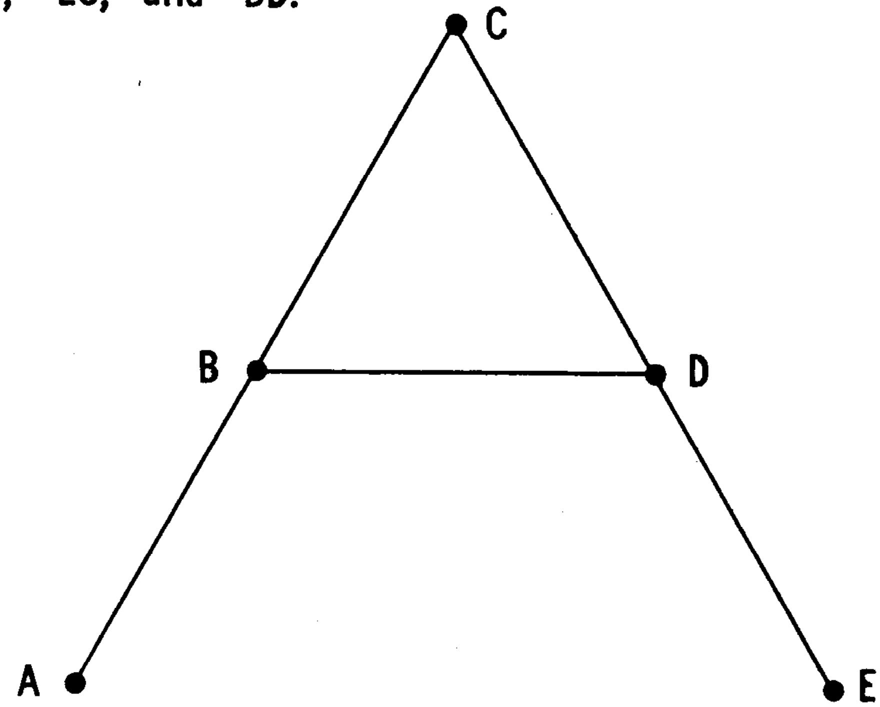
Page 167:

Give oral directions to draw line segments $\overline{AC}$, $\overline{EC}$, and $\overline{BD}$. Tell the children to place their pencils on point A, line up the straightedge with point C, hold it in the center, then draw $\overline{AC}$.

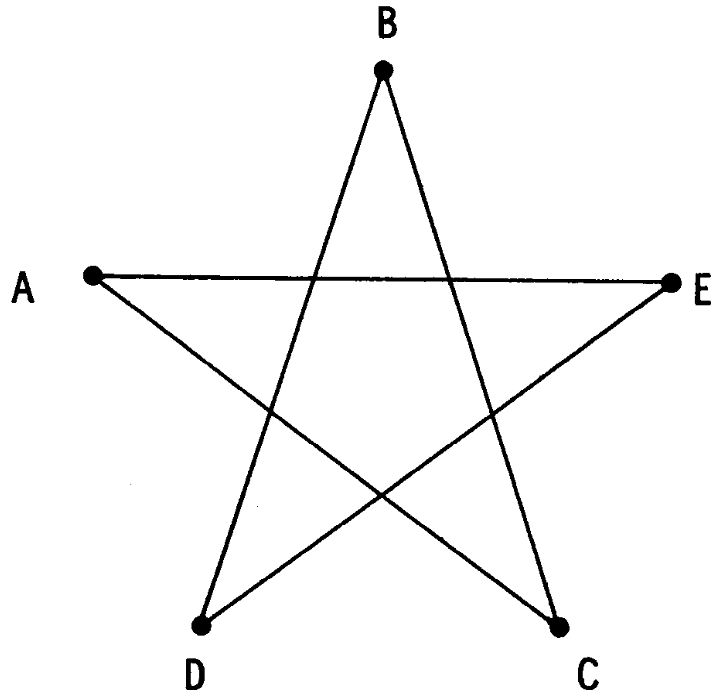
Encourage the children to guess what the lower figure will be. Tell them to draw the line segments as shown.

Line Segments

Draw $\overline{AC}$, $\overline{EC}$, and $\overline{BD}$.



Draw $\overline{AE}$, $\overline{AC}$, and $\overline{DE}$.



Page 168:

Read instructions and give help where needed. Some children may not think to count $\overline{AD}$ and $\overline{BC}$ as line segments. In discussion help them see that the two shorter line segments are part of the longer line segment. Note also that two small triangular regions such as $\triangle ABE$ and $\triangle ACE$ are part of the larger triangular region $\triangle ABC$.

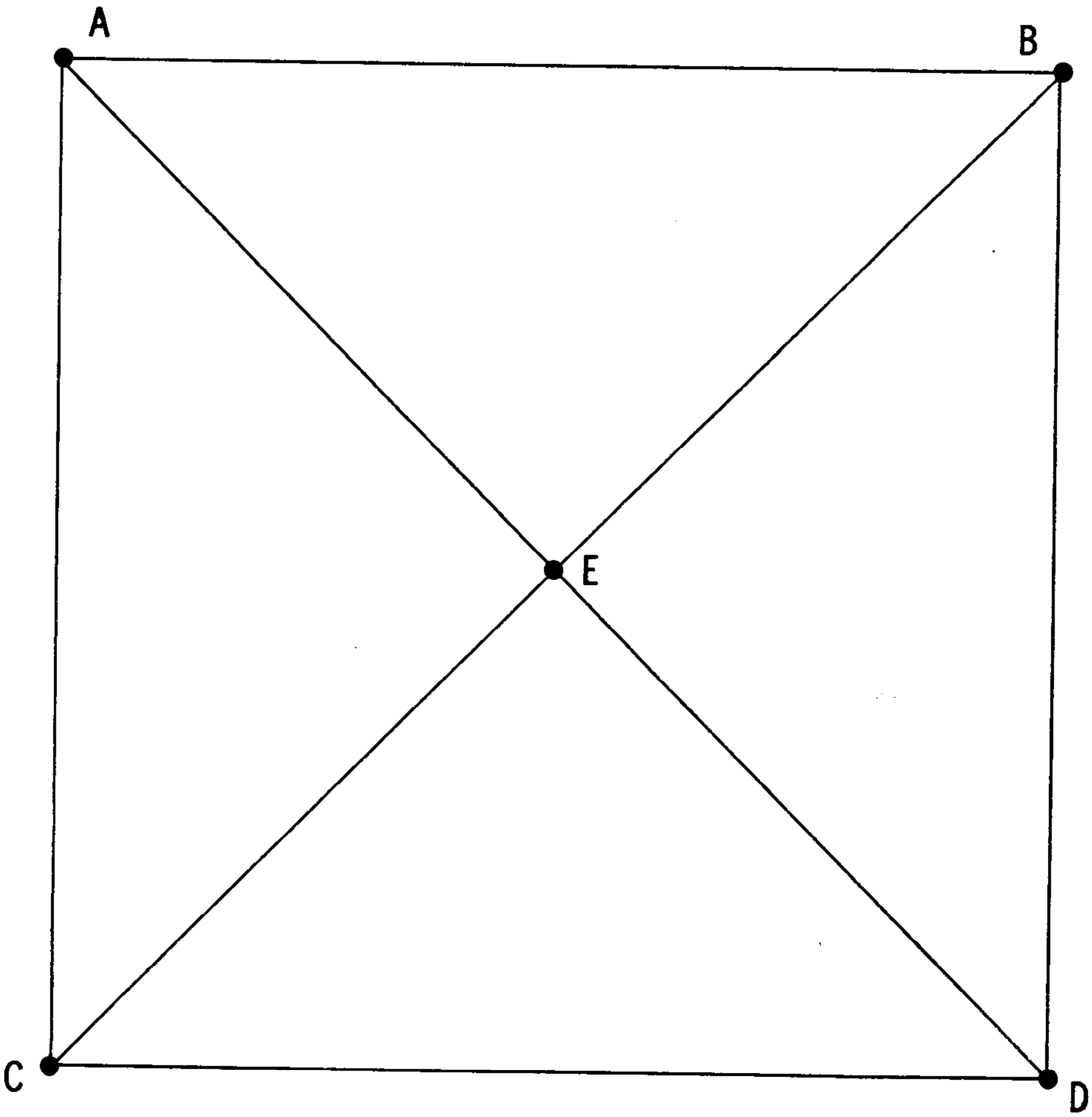
Page 169 - 170:

Read instructions for both pages, then let children work independently. When page 170 is completed, ask the children to compare the two examples (what happens when point D is inside, outside, the triangle).

Line Segments

Draw $\overline{AB}$, $\overline{BD}$, $\overline{DC}$ and $\overline{CA}$.

Connect point E with the other points.



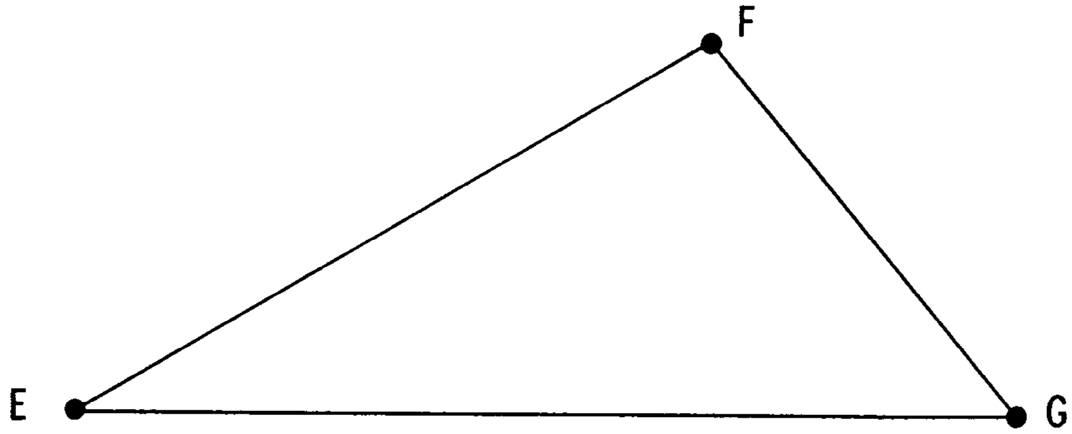
How many line segments can you count? 10

Color a square region red.

Color one triangular region blue.

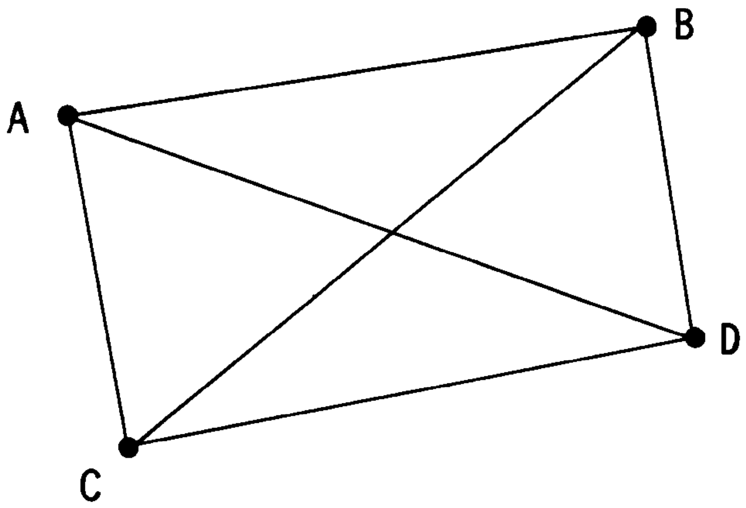
Line Segments

Connect each point by a line segment to each of the other points.



Do any line segments cross?

Mark <u>Yes</u> or <u>No</u>. Yes No

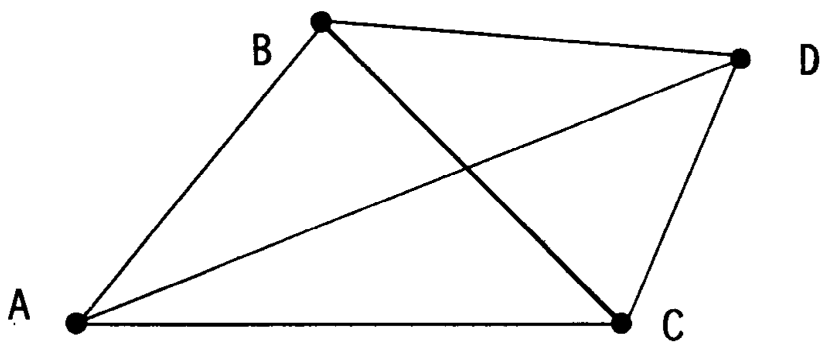


How many line segments cross? 2

Line Segments

Draw $\overline{AB}$ and $\overline{AC}$.

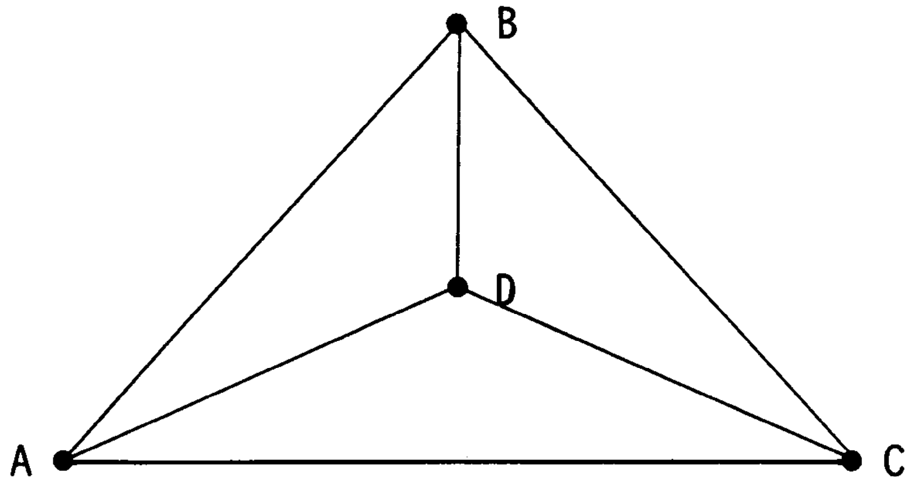
Now connect point D with the other points.



How many line segments cross? 2

Draw $\overline{BA}$ and $\overline{BC}$.

Now connect point D with the other points.



Do any line segments cross? Yes No

X-2. Comparing Line Segments

Objective: To introduce the ideas of longer than, longest, shorter than, shortest, same length as.

To compare line segments by using an intermediate model.

Vocabulary: Compare, longer than, longest, shorter than, shortest, same length as.

Materials: One long easel brush and one short paint brush for each child, several tagboard or chipboard sheets of varied lengths, flannel board, three strips of cloth of different lengths, individual pieces of string, each 8 inches long, and as needed, pencils, pipe cleaners, pick-up sticks, book, straws.

Suggested Procedure:

Comparing Lengths of Objects

Give each child one short paint brush and one long easel brush. There should be a distinct difference in length between the brushes. If brushes are not available in quantity, use straws.

Ask the children to put the brushes on end on their desks. Find out how the brushes are alike. (Both are brushes, wood, etc.) Find out how they are different. (This brush is longer than the paint brush.)

Suggest to a child that he observe the brushes of the child next to him. Ask him to find a brush the same length as one of his, and to display the two. Continue with another child finding a brush longer than (shorter than) his.

Select the children at one table for demonstration. Give an easel brush to one child. Ask him to compare the brush with the two he has. Seek the response that the new brush is the same length as his easel brush, and longer than his paint brush. Repeat with different children, alternating with a short and a long brush.

Pupil's book, pages 171 - 173: Comparing Lengths

Ideas

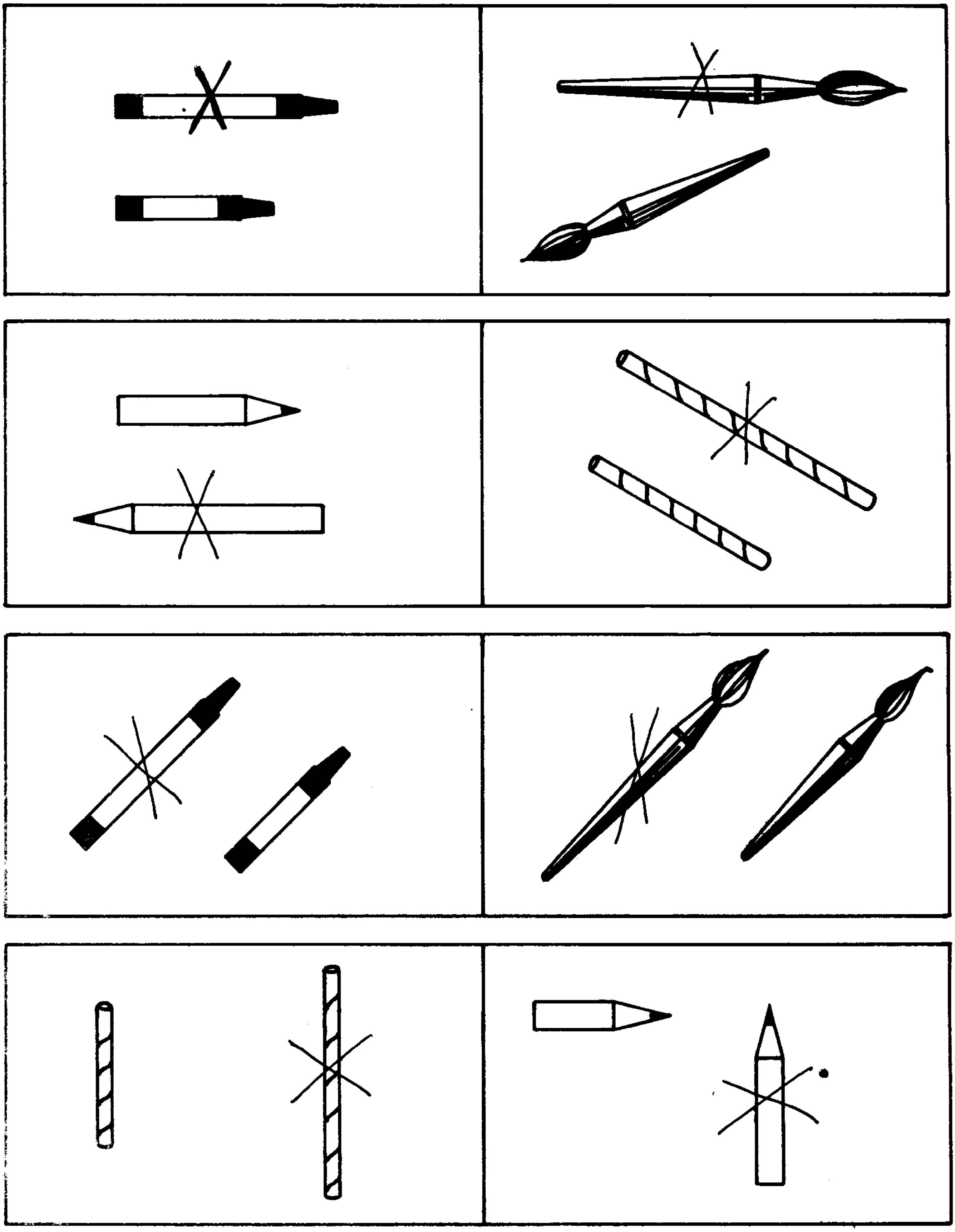
An object can be longer than, shorter than, or the same length as another object.

Pages 171 - 173:

Each page presents one of the ideas of this section for visual comparison. Read the instructions with the children. Make sure that they agree that the marking of the first example on each page is correct.

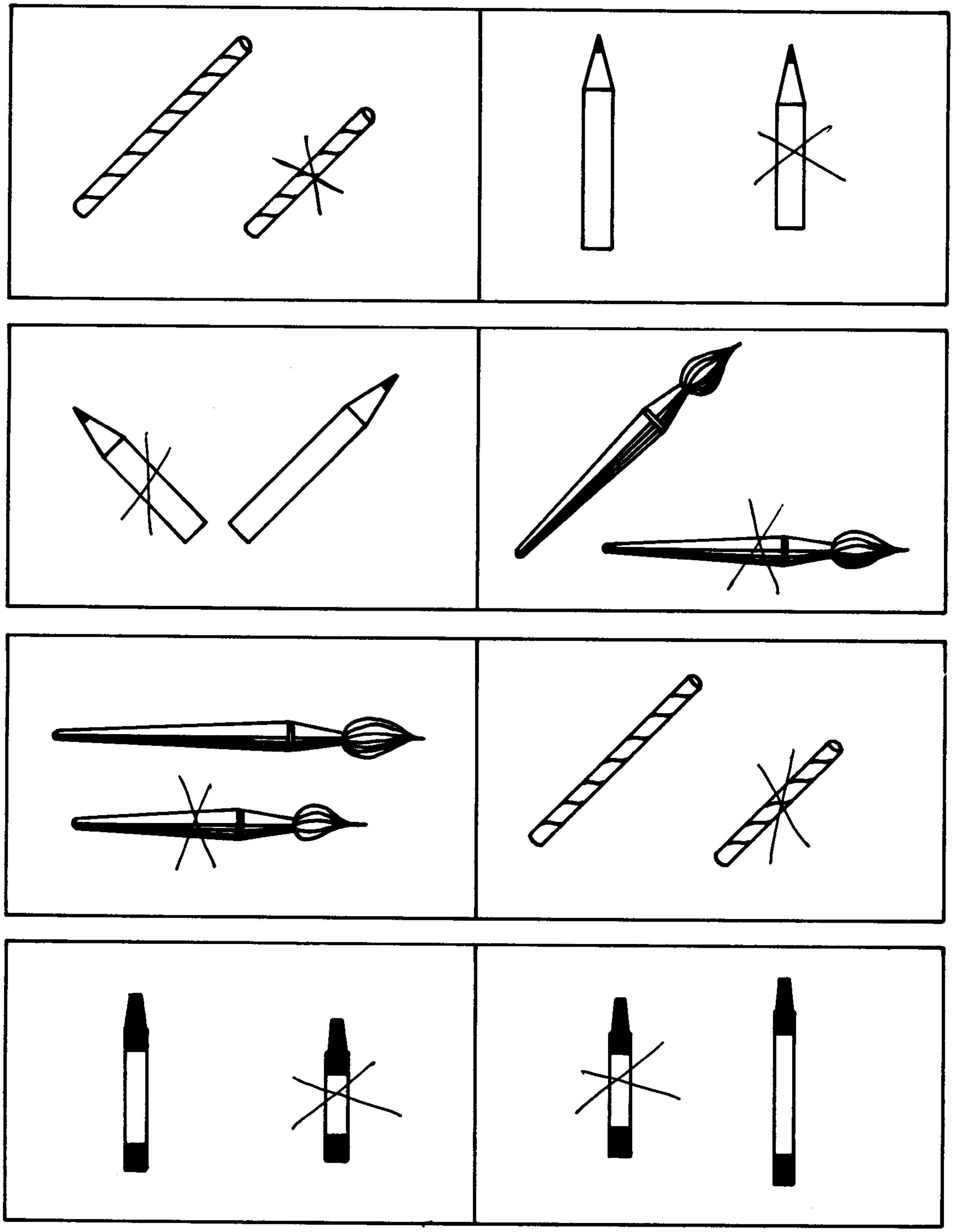
Comparing Lengths

Mark the one that is longer than the other.



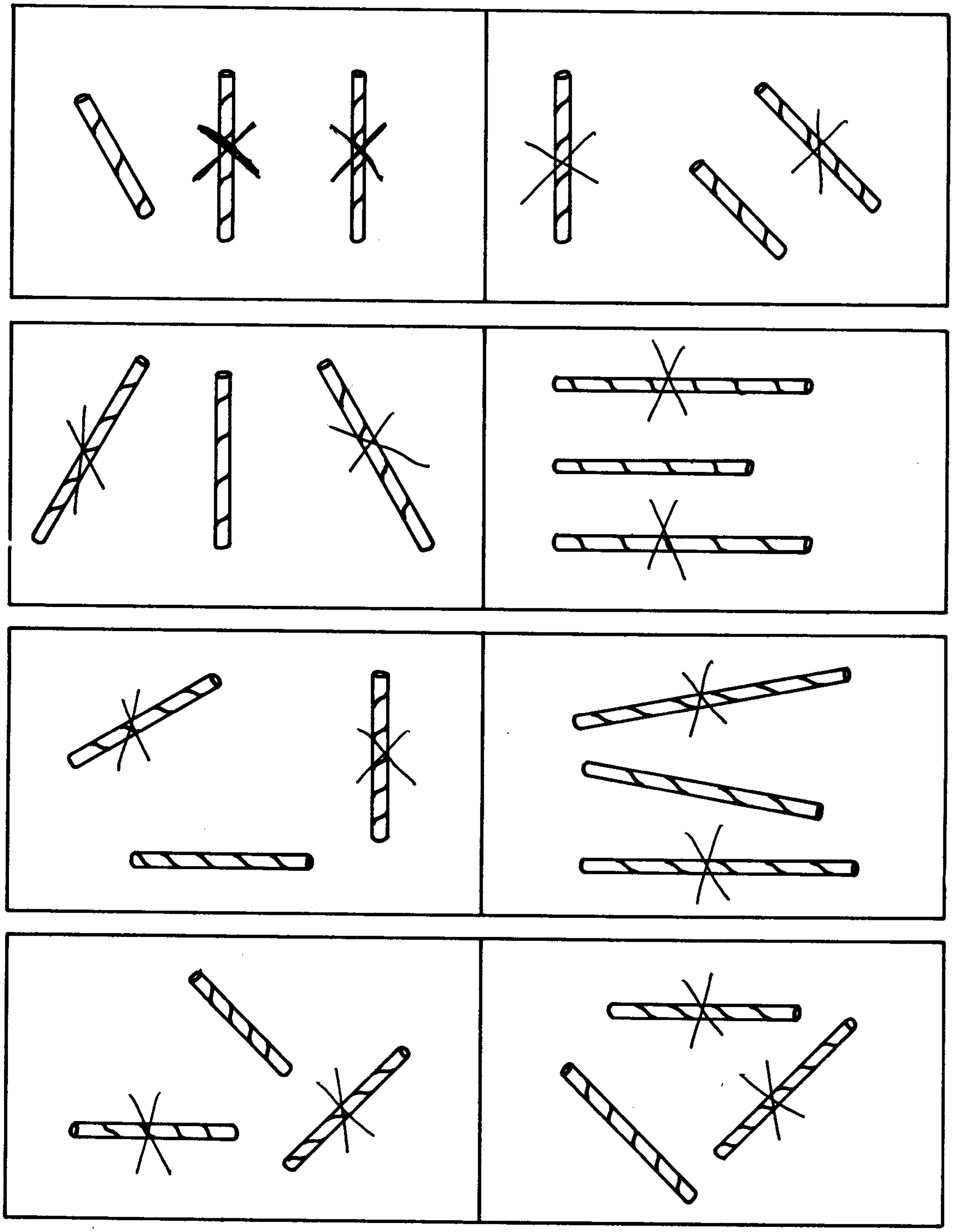
Comparing Lengths

Mark the one that is shorter than the other.



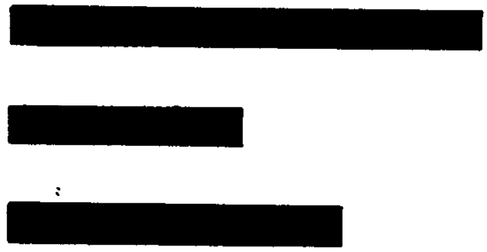
Comparing Lengths

Mark the drawings that are the same length.



Comparing line segments

Direct the children's attention to the flannel board where three strips of colored cloth of distinctly different lengths are displayed. These strips should be placed horizontally and have a common beginning position.



Discuss which strips are longer, then ask which is longest. (The one that is longer than any of the others). Repeat with shorter and shortest. Test for length by moving one edge against another.

In two parts of the room place two objects (fairly narrow) that are obviously not the same length. Compare them at a distance, then bring the objects together for comparison of their edges. Then place two objects that are the same length and repeat the comparison.

Introduce two narrow tagboard or chipboard sheets, one only slightly longer than the other. When the comparison is made, point out the advantage of being able to bring the objects together to check the lengths of their edges.

Call attention to two different edges of the flannel board (one edge should be shorter). Ask how these line segments could be compared.

Accept any of the following ideas:

1. Holding one's hands at the ends of one line segment and using this to transfer to the other line segment. (The end points are marked by the hands. Keep in mind that this method is quite imprecise.)

2. Laying a piece of string beside one line segment, and then grasping it carefully at the end points of the line segment and carrying it over to the other line segment. (Clarify that the string represents the line segment, and the places where it is held the end points. The method is imprecise because the string may stretch if tension in it is increased.)

3. Using a long unmarked stick or piece of paper by placing one end of the stick or paper at one end of the line segment, marking a point on the object at the other end of the line segment, and then comparing the marked object with the other line segment. (Indicate that the edge of the stick from one end to the mark represents the line segment.)

Clarify that in each case above, in one way or another, a model has been made of one line segment. This model has been superimposed on the other segment for comparison.

Use string to show how the edges of the flannelboard can be compared.

<u>Pupil's book, pages</u> 174 - 177: <u>Comparing line segments</u>

Ideas

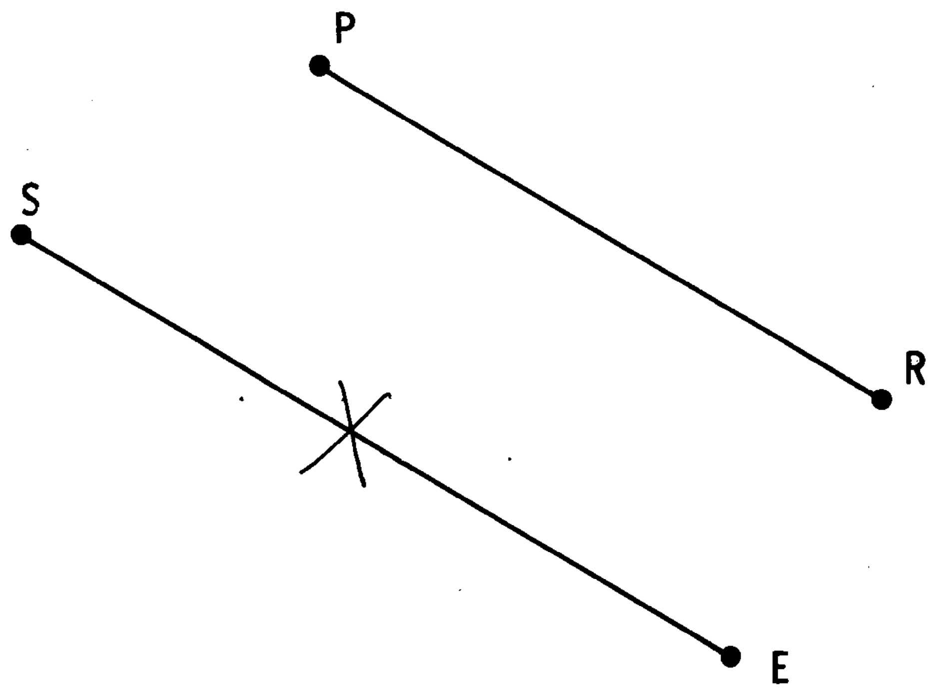
Two line segments can be compared by using a model of one and placing it on the other.

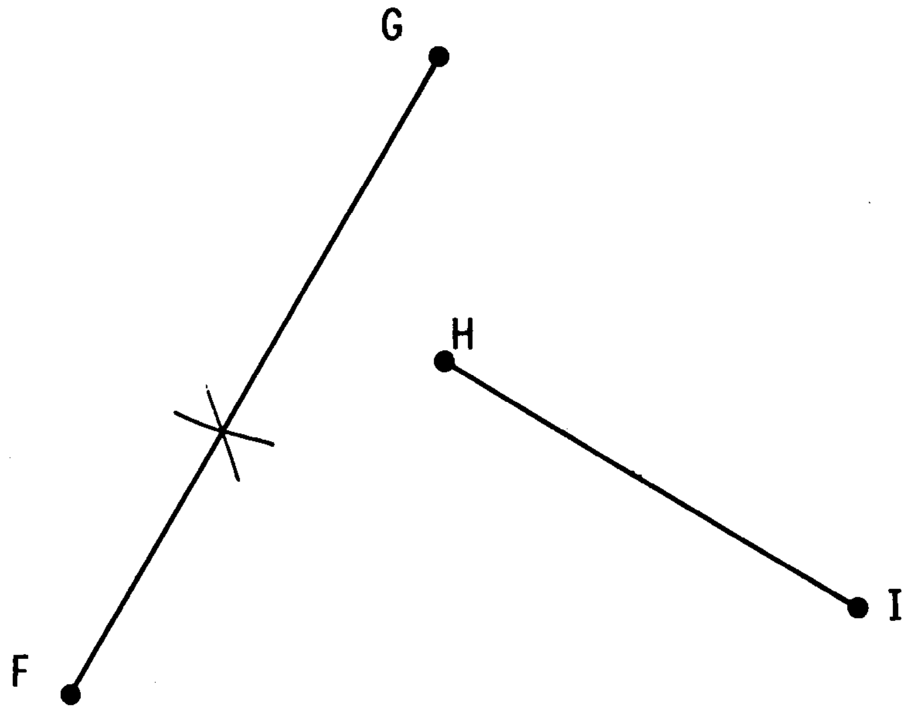
Pages 174 - 177:

Pass out string to the class. Read the instructions and tell the children they are to use the string to compare the line segments in each set. Give no more instructions, but move around and ask leading questions to those who are obviously copying or are not able to get started.

Comparing Line Segments

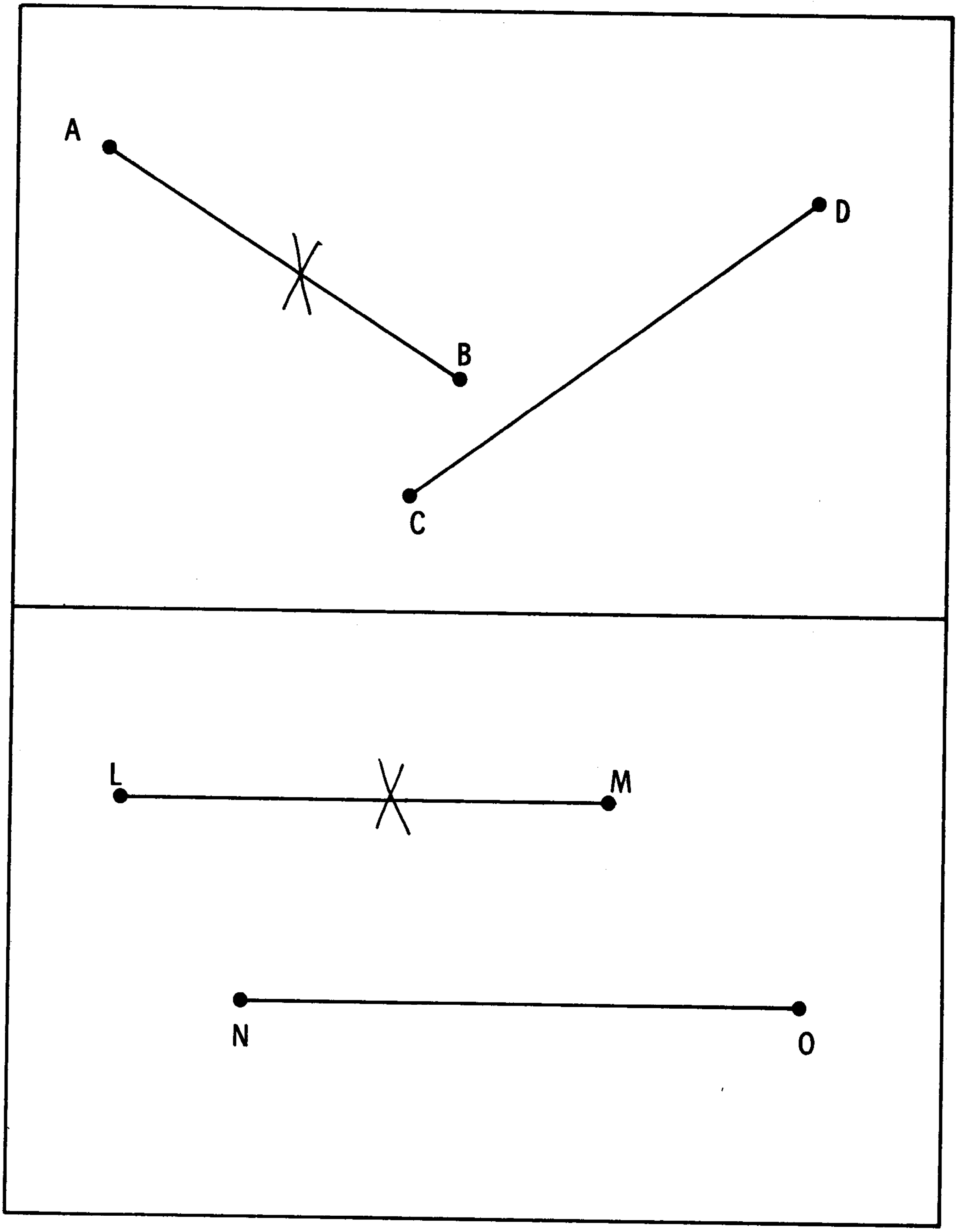
Mark the line segment that is longer than the other one.





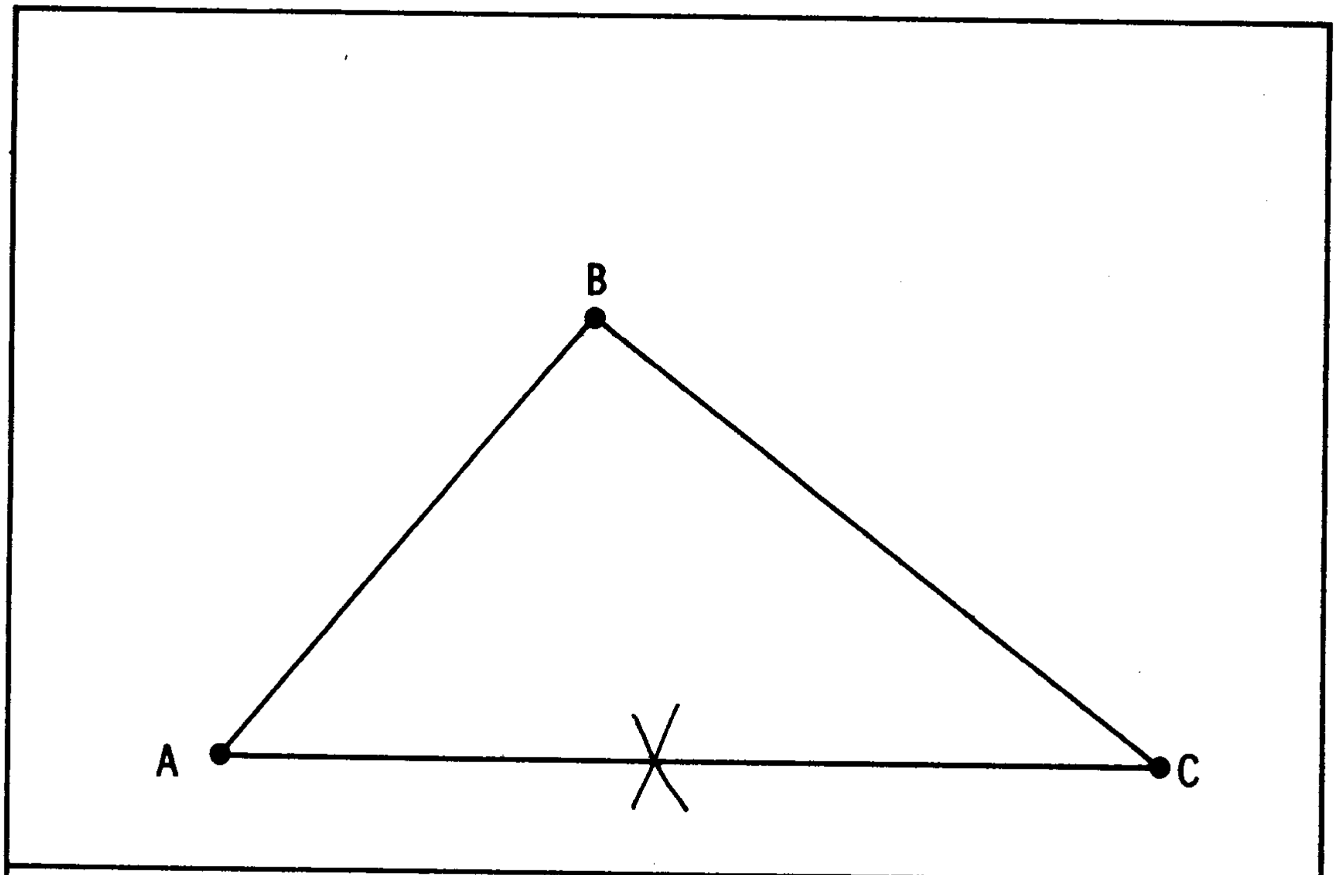
Comparing Line Segments

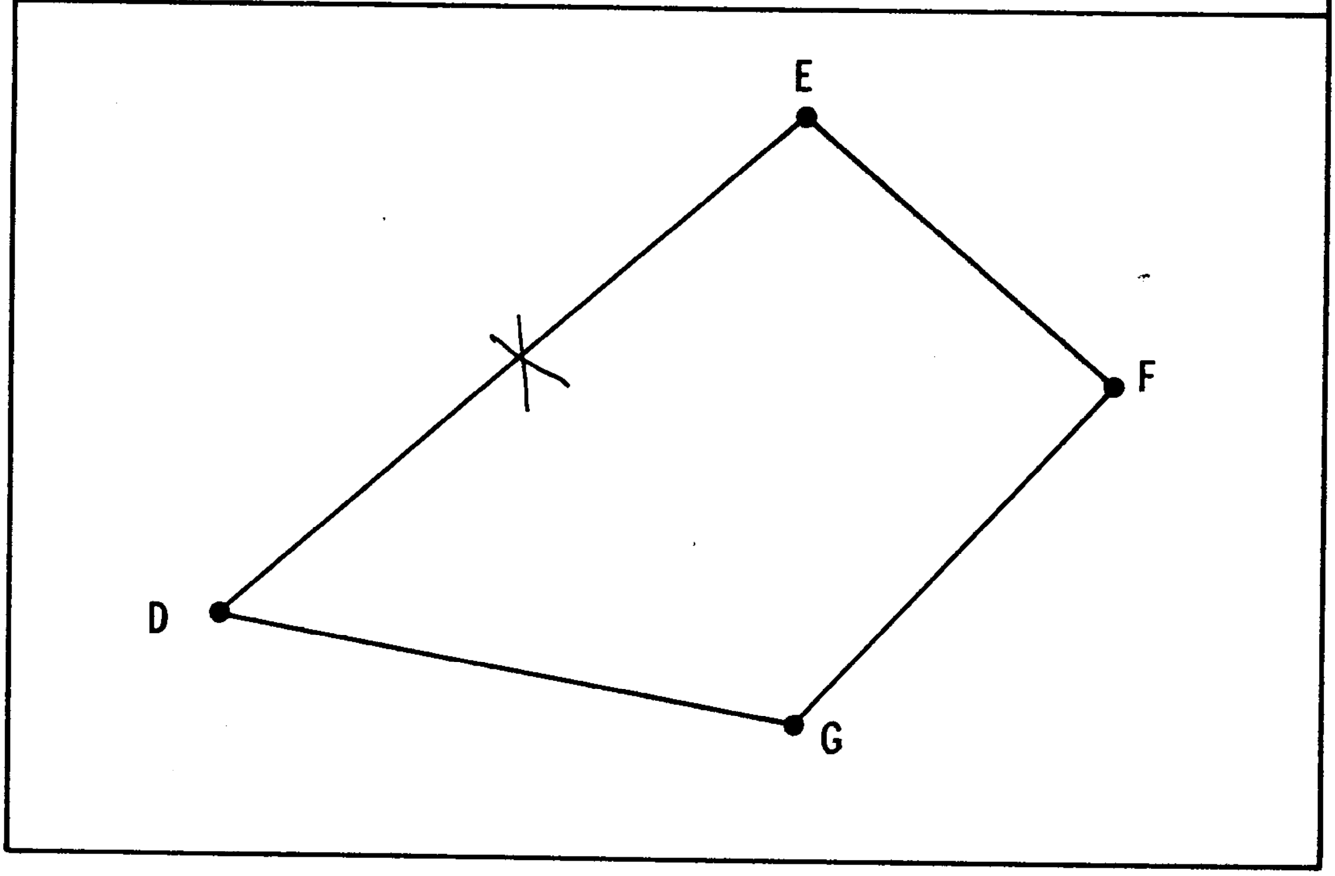
Mark the line segment that is shorter than the other one.



Comparing Line Segments

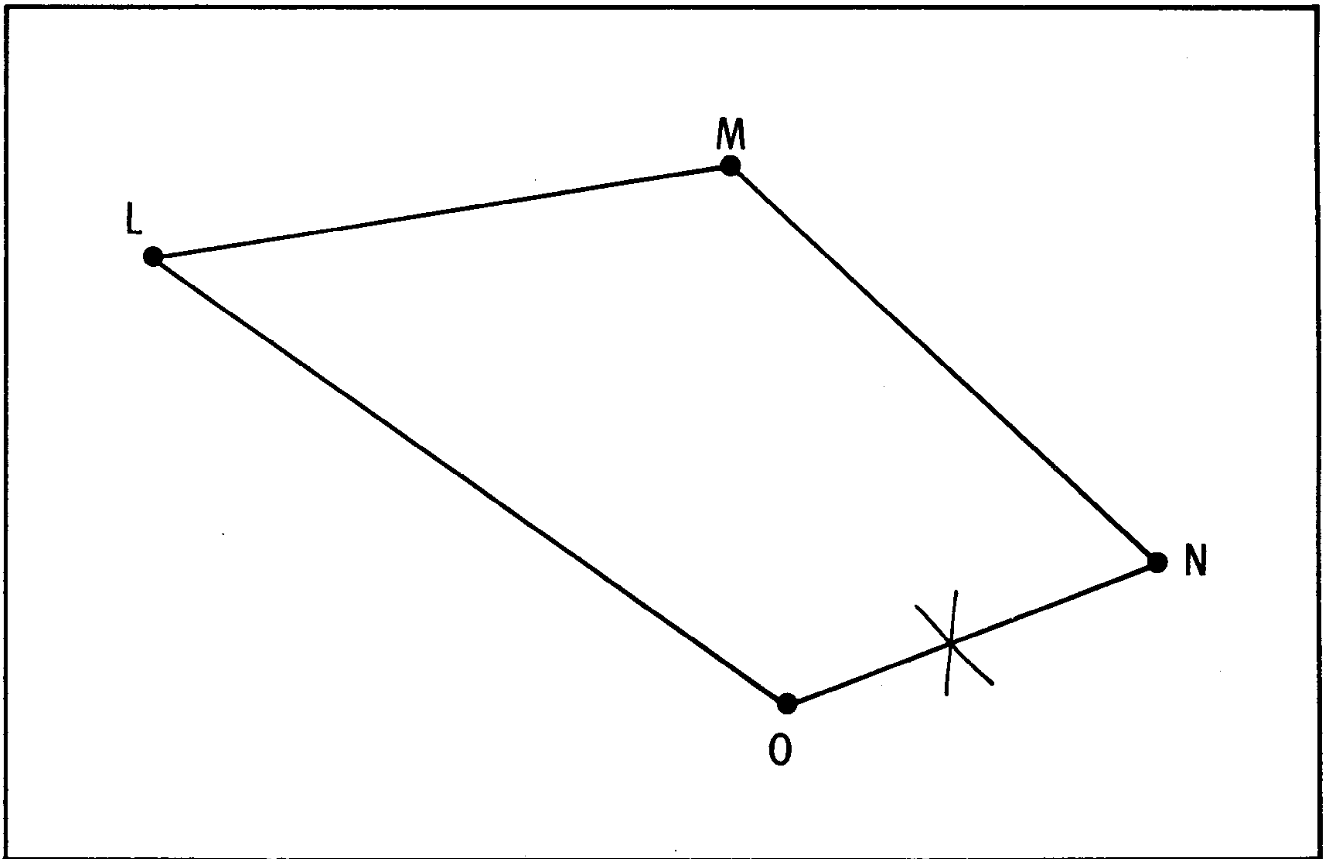
Mark the longest line segment.

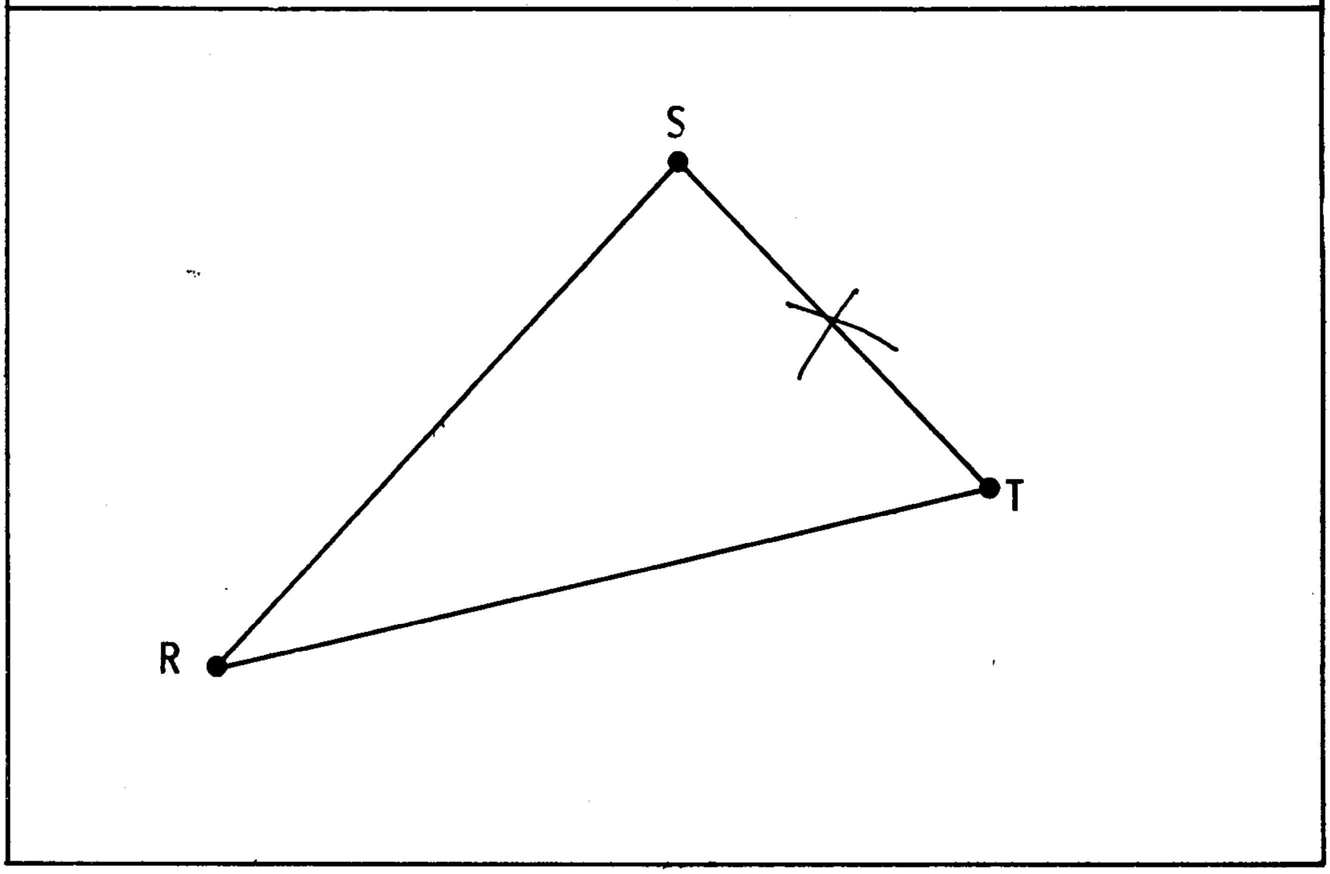




Comparing Line Segments

Mark the line segment that is shortest.





X-3. Measurement of line segments

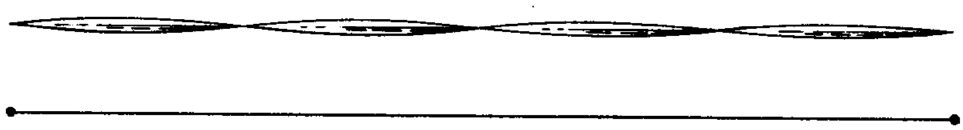
Objective: To introduce the idea of measurement of a segment as the number of unit segments necessary to cover it.

Vocabulary: Unit segment, units, (review) length.

Materials: Toothpicks, pieces of drinking straws, line segments drawn on paper.

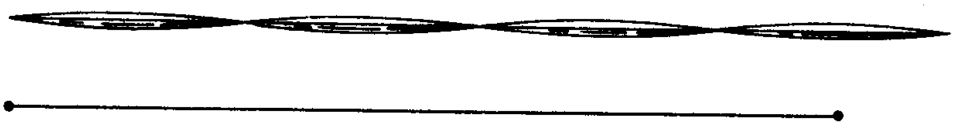
Suggested Procedure:

Provide each child with a number of toothpicks of the same length. Make provision for a number of line segments to be measured. The endpoints should be clearly indicated. The exercise is to see how many of these toothpicks can be laid end to end along each line segment. Indicate that the toothpick is but one of many objects that we might use to measure line segments. We call the toothpick a unit segment. The length of the toothpick is one unit. The length of the line segment is 4 units. Have the children write the number 4 on their paper. Then continue with several other segments where the length is at least approximately the same as several toothpicks that are lined up.



The length is 4 units.

The next set of examples should be those where the unit segment does not fit exactly, as shown below. Have the children count the toothpicks and discover that the segment is between 3 and 4 toothpicks in length. Read the sentence and have the children write the numbers 3 and 4 where blanks are noted.



The length is between 3 units and 4 units.

Ask the children to check the examples again, this time using just one toothpick. Demonstrate how the toothpick is to be laid off and a mark made at the end each time so that the next measurement can be done carefully.

Pupil's book, pages 178-179: Measuring line segments

Ideas

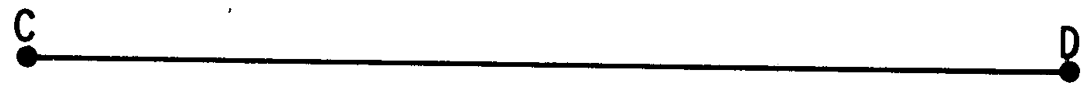
A line segment may be measured by repeatedly using a unit segment.

Pages 178 - 179:

Have the children lay the toothpick repeatedly along the segments. Ask them to count the number of times the unit is used and to write the correct numbers where shown. Some of the examples may result in the last mark falling on the end of the line segment. In these cases explain that the number is not "between", but is the count of the unit segments.

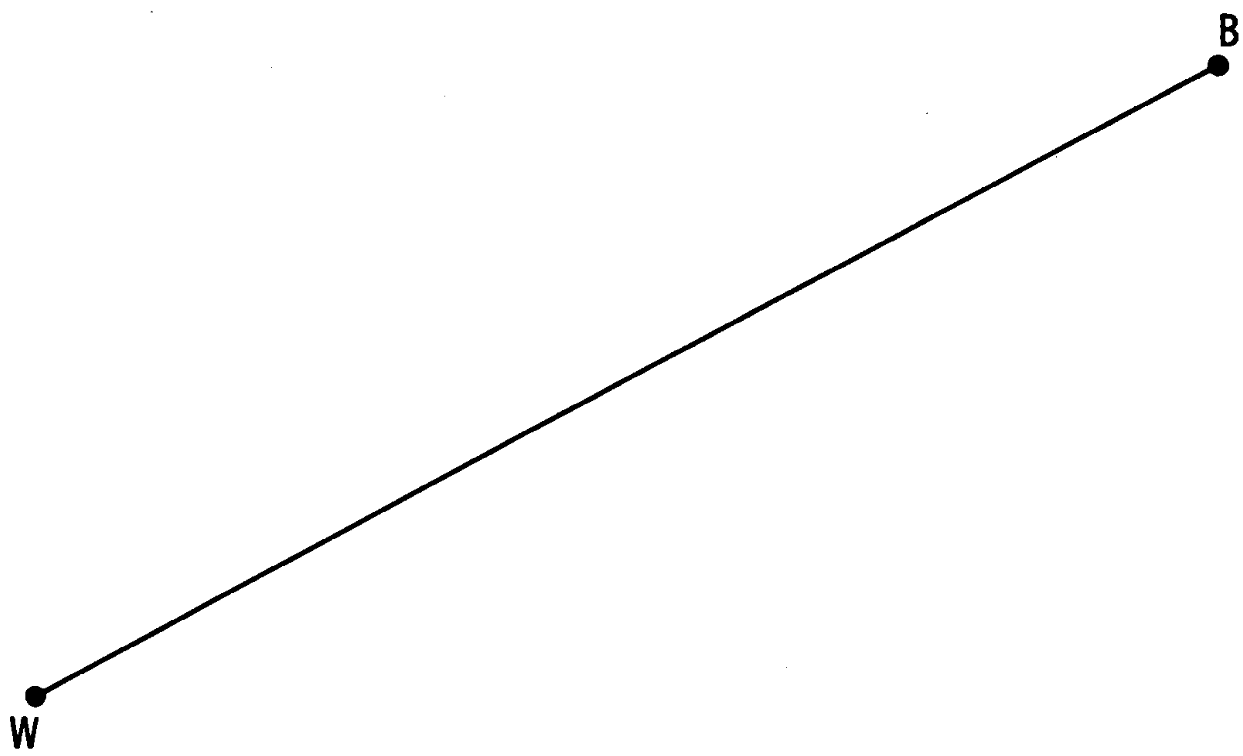
Measuring Line Segments

Use a unit segment to find each length. *Answers depend on the unit used.*



The length of $\overline{CD}$ is between ______ and ______ units.

The length of $\overline{SR}$ is between ______ and ______ units.



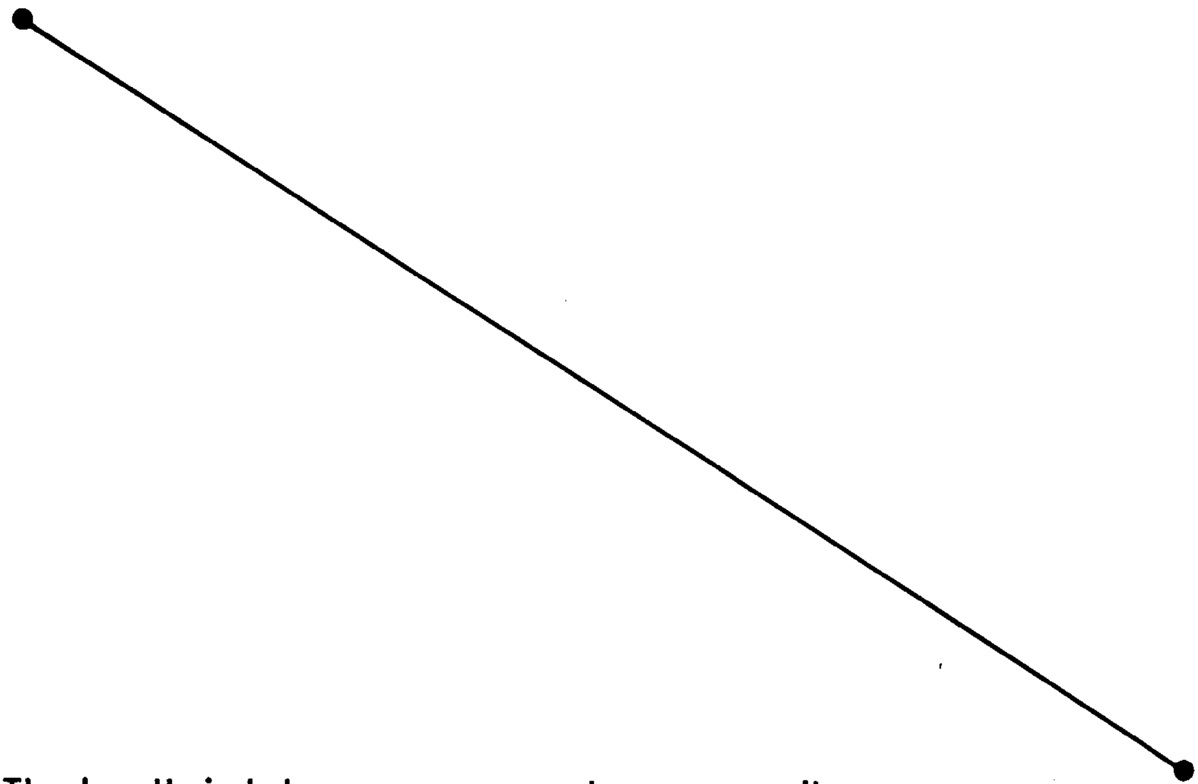
The length of $\overline{WB}$ is between ______ and ______ units.

Measuring Line Segments

Use a unit segment to find each length. *Answers depend on the unit used.*

The length is between ______ and ______ units.

The length is between ______ and ______ units.



The length is between ______ and ______ units.

Further Activities:

1. Provide each child with several different units (say pieces of drinking straws) and have him measure the same line segments with each unit. If straws are used, for example, there should be some designation attached to the different ones such as "long straw", "medium straw", and "short straw" so the pupil can describe his results as so many "short straws", etc. An alternative would be to use different objects as unit segments, such as pencil, chalk, etc.

2. Have different pupils measure the same line segment with different units. For example, have two children (with different sized feet) see how many of their foot lengths it takes to cover a crack in the schoolroom floor.

3. Have the pupils invent their own units and use them. For example, how many of some child's hand-spans is it across the edge of the bookshelf?

In all exercises, try to make sure that the pupils keep clearly in mind that the unit is a line segment. It is easy to have this idea obscured.

The exercises themselves will make clear the possible variety of units. Class discussion should crystallize the idea that for different units, a measurement has different numbers. To tell a length you need to tell not only the number of units but also to tell what unit is used. It should also be possible to develop the understanding that the smaller the unit, the greater the number needed for any particular measurement.

X-4. Construction of a ruler

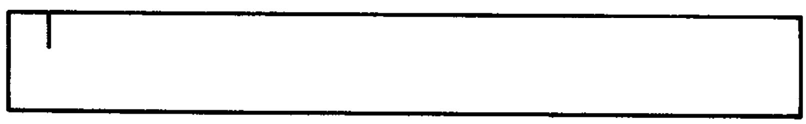
Objective: To introduce the idea of a scale as a measuring device.

Vocabulary: (No new words.)

Materials: Light cardboard straightedge (unmarked) perhaps a foot long, one for each pupil, some convenient unit segment (toothpicks or pieces of drinking straws), one for each pupil. To be convenient for handling, the units chosen should be around two inches or a little less.

Suggested Procedure:

Distribute straightedges and ask each pupil to make a mark not far from the end. (This point is to be the zero point of the ruler. Note that the zero point is not at the end of the straightedge. In addition to being easier to identify, it avoids the problem that corners are always getting bent and dog-eared.

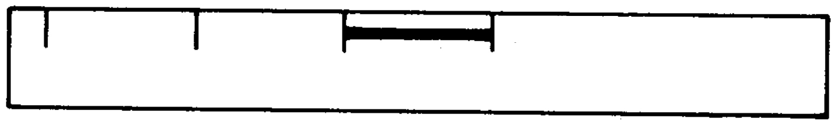


Now ask each child to put his unit segment on the straightedge with one end on the initial mark and to mark the other end.

The piece of the straightedge is now a line segment one unit long.

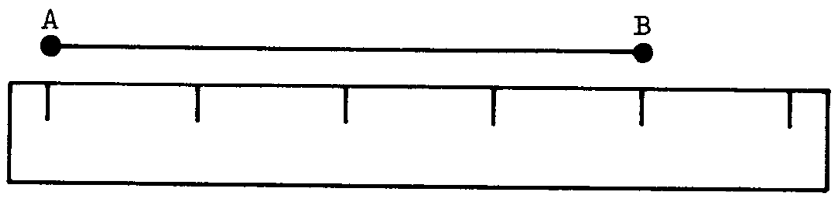
Now the unit segment can be laid down again,

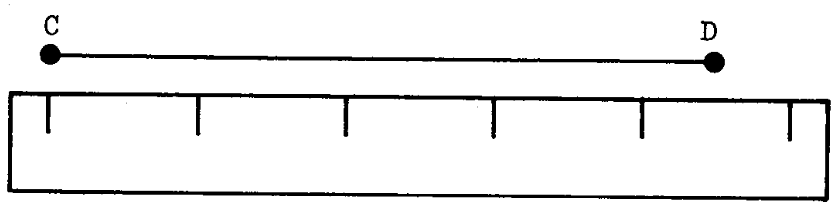
and again,



as often as the length of the straightedge allows. It is now easy to see that the marked straightedge shows line segments 1 unit long, or 2 units long, etc.

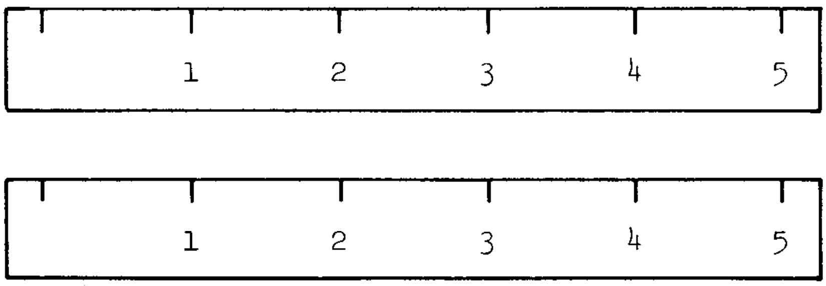
The straightedge in its present form can now be used for measuring line segments as shown below, where it is seen that the length of line segment $\overline{AB}$ is 4 units and the length of line segment $\overline{CD}$ is between 4 and 5 units.



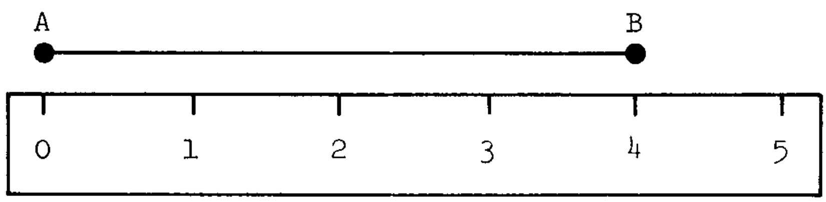


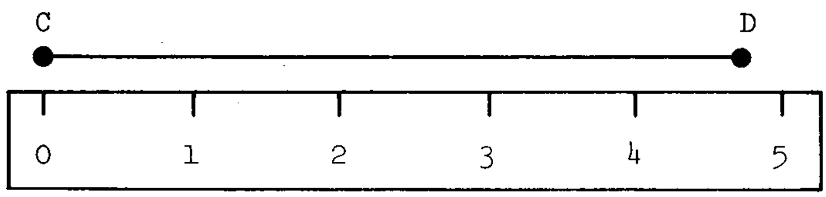
These numbers are found by counting the number of unit segments. The placing of the straightedge will need to be emphasized, i.e., the placing of the original mark at one end point of the line segment.

The next stage is to encourage the children to label the marks on the straightedge. The idea is to put a 1 beside the mark that was made the first time the unit was used, a 2 beside the mark that was made the second time the unit was used, and so on. The instrument then looks like this.



Discussion should produce the suggestion that the original mark be labeled 0. The instrument is now complete and may properly be called a ruler. Indicate that the ruler shows part of a number line. In using it to measure line segments $\overline{AB}$ and $\overline{CD}$ as before, the numbering of the points produces a simplification.





The fact that in measuring line segment $\overline{AB}$ the point B is opposite the 4 mark shows that there were 4 copies of the unit segment between A and B. Thus instead of looking back and counting the segments as was done before, the length of 4 units can be read directly from the ruler. Similarly, on line segment

$\overline{CD}$, the point D is between the 4 and 5 marks; this shows that the length of line segment $\overline{CD}$ is between 4 and 5 units.

Practice should be given in measuring with this device, but it need not be pushed too hard as the mechanics of using the ruler will be developed when standard units are introduced in a later book.

X-5. Telling Time

Objective: To teach telling time, emphasis on hour, half hour.

Vocabulary: Time, hour, minute, o'clock.

Materials: Real or educational clock, duplicated clock faces to be glued on paper plates.

Suggested Procedure:

Hour hand - Children will be familiar with expressions of time which are a part of their daily program. Lead the children to discuss reasons for measuring, recording, or knowing exact times within a day. (When to get up, come to school, have lunch, keep a dental appointment, watch television, have a music lesson, catch a train, airplane, or bus, etc.) If the room clock has a second hand, have the children notice that its movement can easily be seen. Ask them to look carefully at the position of the other hands and see which one of the hands will be in a different position when you ask to check the time. Wait two minutes (if possible) and ask whether the long hand or the short hand has moved. Tell the children that the short hand is the hour hand and that it goes all the way around the clock twice a day. Display a clock with only an hour hand. Say that this clock has no minute hand but that if it were necessary, one could show the time fairly well anyway.

Point the hour hand exactly at the 4 and explain that when the hour hand points exactly to that spot, it is four o'clock.

Move the hand half-way between the 4 and the 5 and ask how far between the two it is. Explain that we would say it is half-past four.

Repeat with other numerals on the clock face, and lead children to decide whether the hour hand would show "o'clock" or "half-past".

Have children make their own clock faces. These should have only an hour hand. The teacher should ditto a clock face on which numerals are written and the minutes marked. This face is to be cut out and pasted on a paper plate. As the teacher names times such as two o'clock, half-past five, the children place the hour hand in the right place.

<u>Pupil's</u> <u>book</u>, <u>pages</u> 180 - 181: <u>Telling</u> <u>Time</u>

Ideas

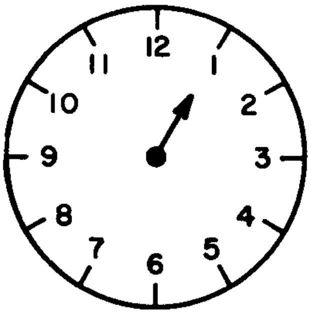
Time can be told by the hour hand alone, and can be read as "o'clock" or "half past".

Pages 180 - 181:

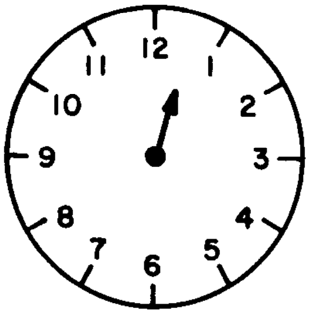
Ask why 1 o'clock is the correct answer in the sample. Ask the children to enter the number which tells the nearest hour or half-hour.

Telling Time

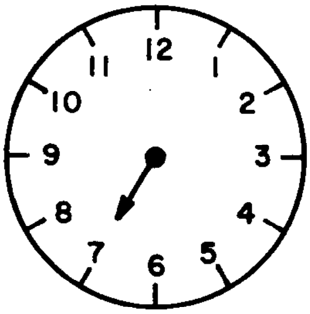
Write the number that tells the time.



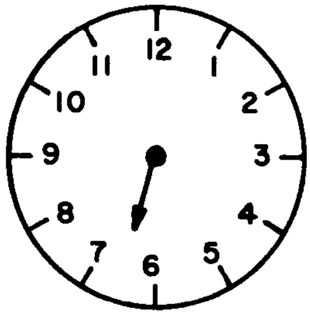
1 o' clock



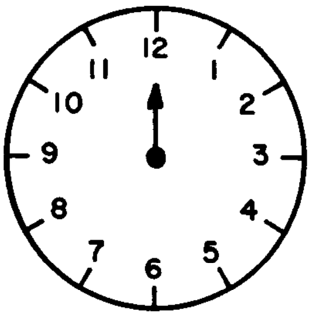
half past 12



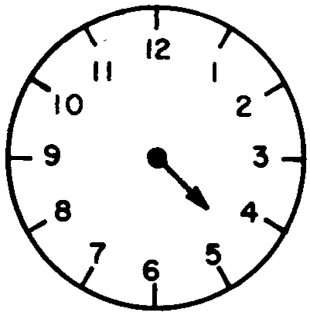
7 o' clock



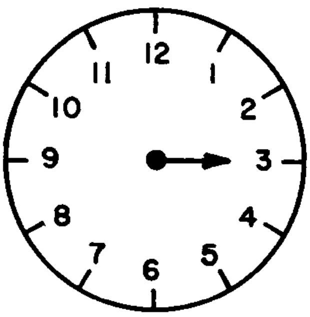
half past 6



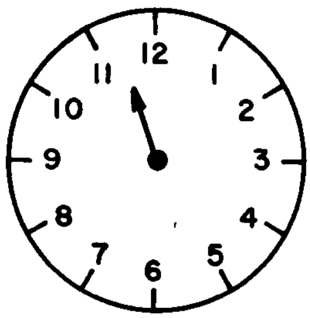
12 o' clock



half past 4



3 o' clock



half past 11

Minute hand

Display a clock with the minute hand only. Explain that the long hand of a clock goes completely around every hour and that it travels from one little mark to another in one minute, from one numeral to the next in five minutes.

Point the minute hand to the 12 and tell the children that whenever the long hand points exactly to the 12 it will be "something o'clock". Use a piece of paper to cover the 6 to 12 section of the clock and ask how much of the clock can be seen (half). Move the minute hand slowly to the 6 and show that it will have gone half way around, so that it will be half past the time it showed when it was at 12. Have children say "something o'clock" or "half-past something" as you move the hand to 12 or 6. (If you wish to go on to the quarter hours, use yarn fastened with masking tape to the clock face and proceed as above, moving the hand to a quarter past, half past, a quarter till, etc.)

Show that in order to tell time accurately, both clock hands are needed. Point the hour hand of one clock to the 2 and the minute hand of the other to the 12. Ask children tq tell what time it is. Move the hour hand half-way to the 3, and the minute hand to 6 and ask what time it is. Continue until the children seem to have no difficulty. Then proceed to use the clock which has both hands. Do not go beyond the "o'clock" and "half-past" times at first. Put the minute hand on the clocks which were made to use with the hour hand lesson.

Children should learn to write both 3 o'clock and 3:00 as names for the same time, and half-past 1 and 1:30 as names for the same time.

Pupil's book, pages 182-185: Telling Time

Ideas

In telling time more accurately both the hour and minute hand are used.

Page 182:

Children are to fill the blanks.

Page 183:

Children should write the time expressions as 2:00, 2:30, etc., on the blanks.

Pages 184-185:

Explain that the minute hand should almost touch the outer edge of the clock, and that the hour hand should either touch the numeral or be half-way between two numerals, but not too near the outer edge.

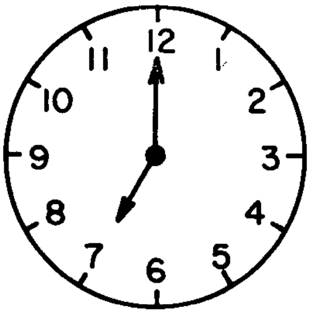
Further Activities:

1. Some children will be able to tell time by the five minutes. Count the number of minutes which the minute hand takes to go around. Point out that the time between any two numerals is 5 minutes. The time needed for the minute hand to go around the clock face, starting at any numeral and returning to that numeral is one hour.

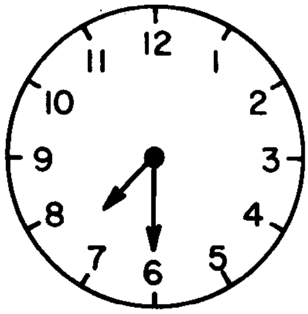
2. Set one clock at 3:00 and another at 5:30. In one hour check the clocks and note that an hour means not only from 3:00 to 4:00 but also from 5:30 to 6:30.

Telling Time

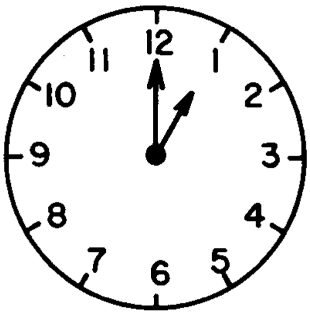
Write the number that tells the time.



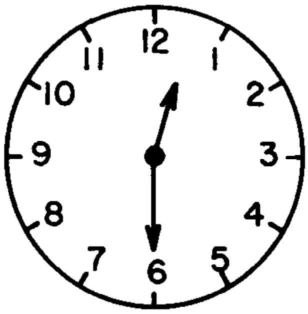
7 o' clock



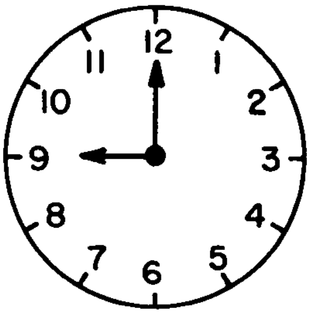
half past 7



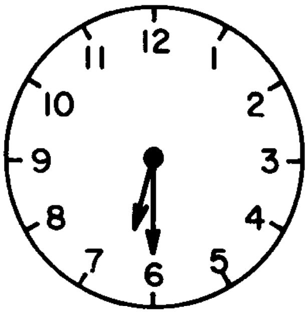
1 o' clock



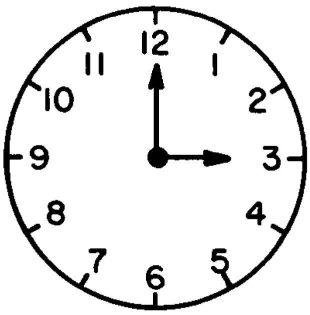
half past 12



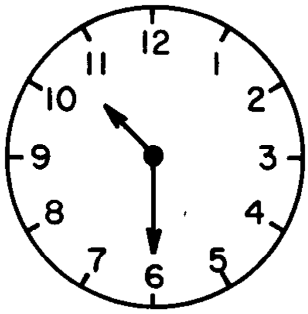
9 o' clock



half past 6



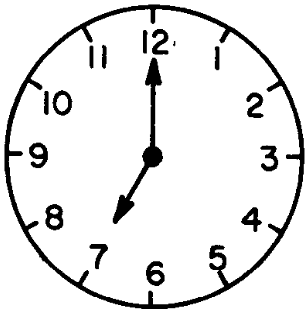
3 o' clock



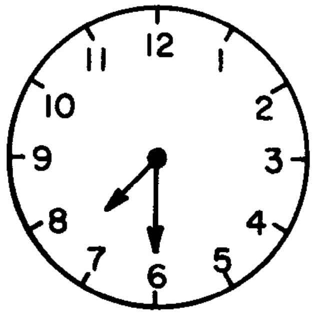
half past 10

Telling Time

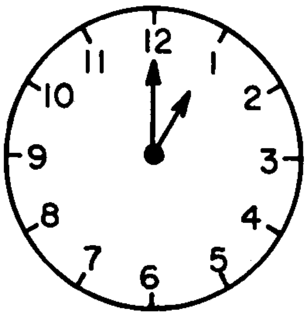
Write the number that tells the time.



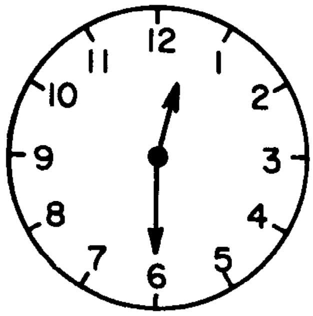
7:00



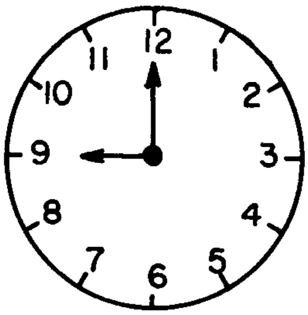
7:30



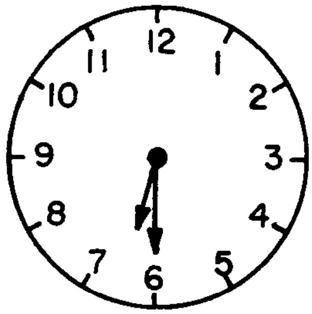
1:00



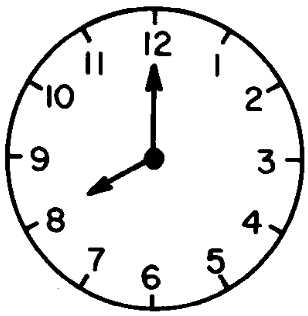
12:30



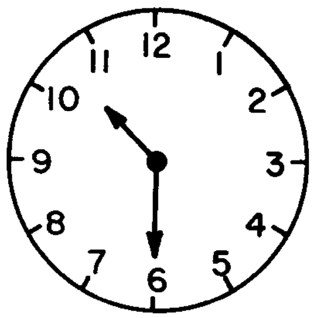
9:00



6:30



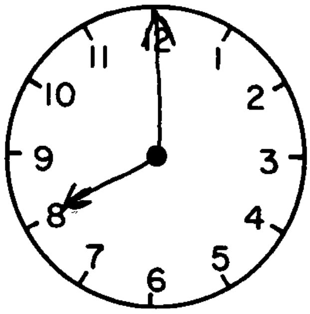
8:00



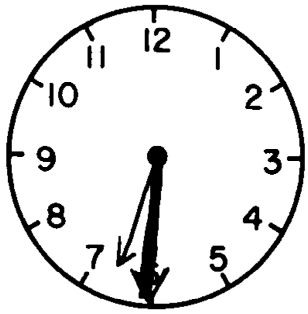
10:30

Telling Time

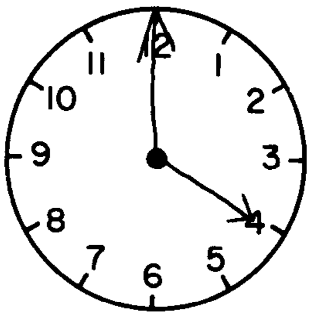
Put the clock hands in the correct places.



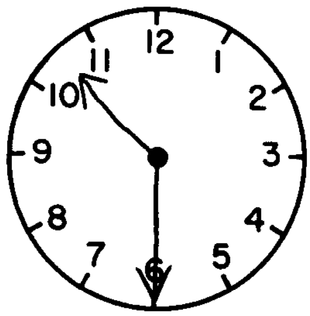
8 o' clock



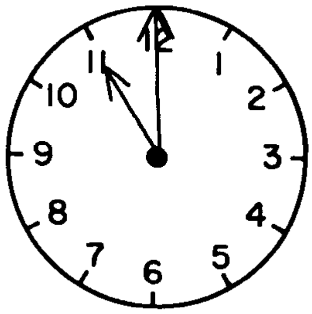
half past 6



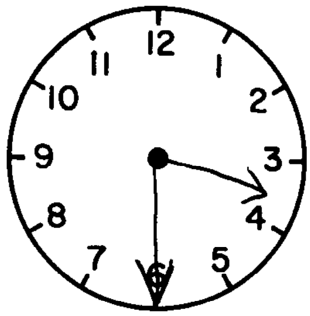
4 o' clock



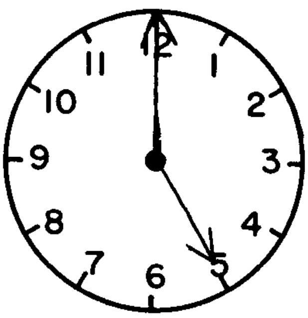
half past 10



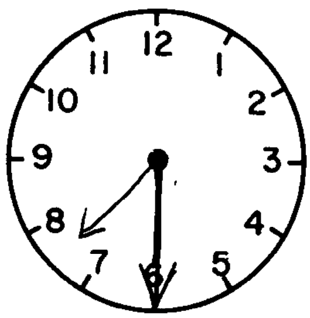
11 o' clock



half past 3



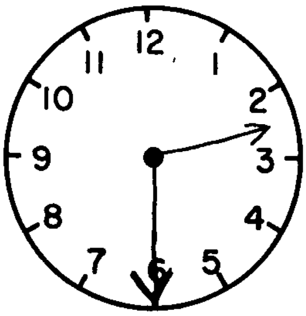
5 o' clock



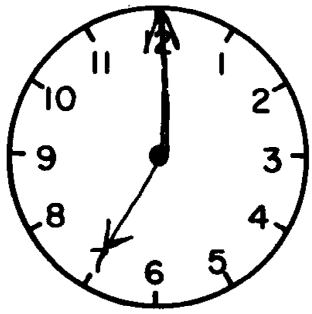
half past 7

Telling Time

Put the clock hands in the correct places.



2:30



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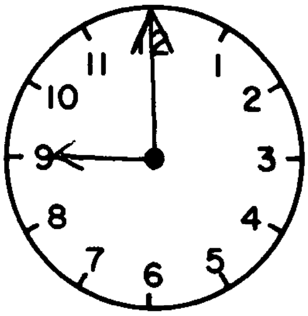

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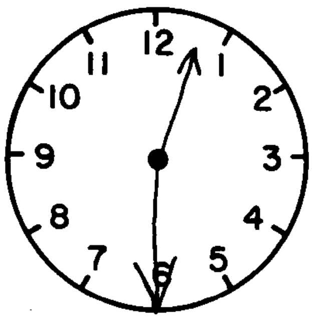

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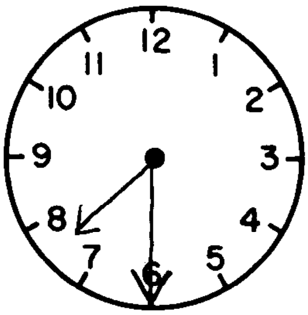

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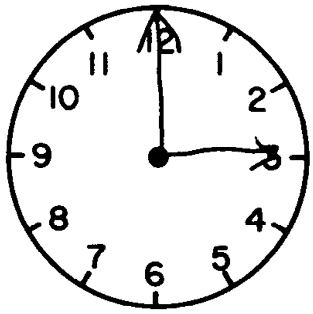

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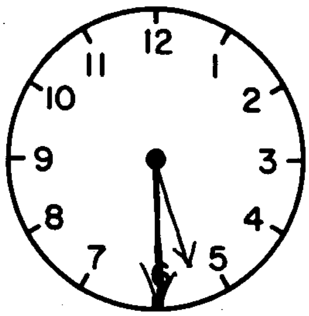

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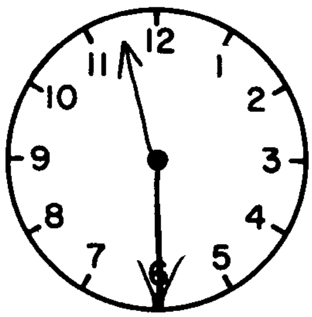

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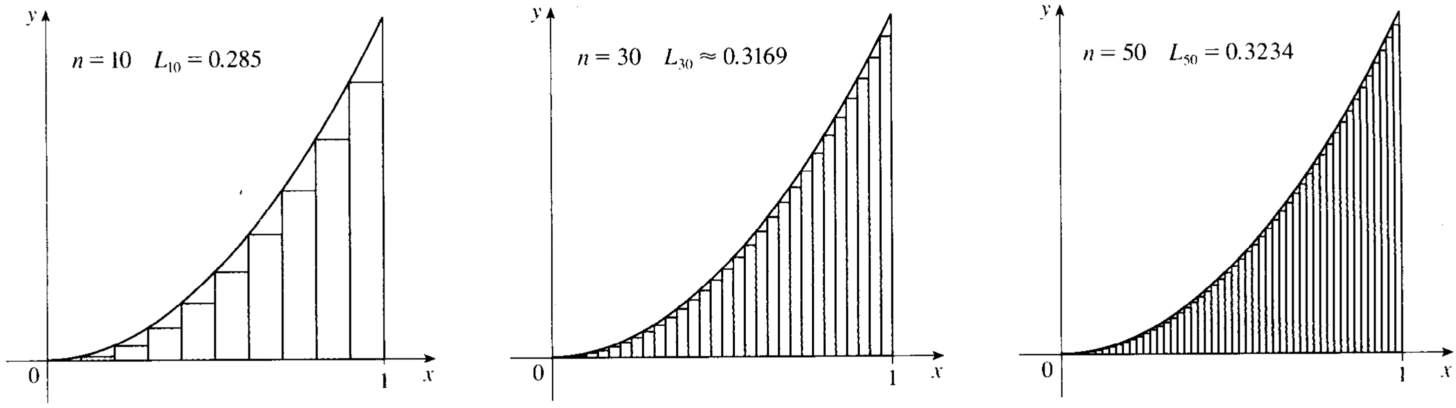


FIGURE 9
The area is the number that is smaller than all upper sums and larger than all lower sums

sums of the areas of the approximating rectangles, that is,

$$A = \lim_{n \to \infty} R_n = \lim_{n \to \infty} L_n = \tfrac{1}{3}$$

Let's apply the idea of Examples 1 and 2 to the more general region S of Figure 1. We start by subdividing S into n strips $S_1, S_2, \ldots, S_n$ of equal width as in Figure 10. The width of the interval $[a, b]$ is $b - a$, so the width of each of the n strips is

$$\Delta x = \frac{b - a}{n}$$

These strips divide the interval $[a, b]$ into n subintervals

$$[x_0, x_1], \quad [x_1, x_2], \quad [x_2, x_3], \quad \ldots, \quad [x_{n-1}, x_n]$$

where $x_0 = a$ and $x_n = b$. The right endpoints of the subintervals are

$$x_1 = a + \Delta x, \quad x_2 = a + 2\,\Delta x, \quad x_3 = a + 3\,\Delta x, \quad \ldots$$

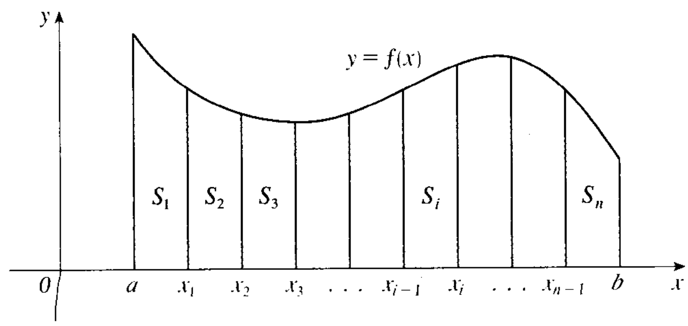


FIGURE 10

Let's approximate the ith strip S_i by a rectangle with width Δx and height $f(x_i)$, which is the value of f at the right endpoint (see Figure 11). Then the area of the ith rectangle

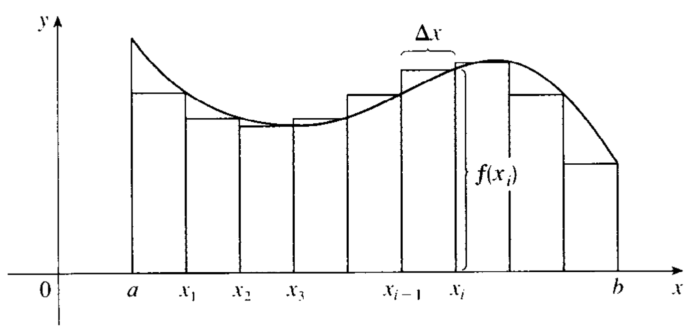
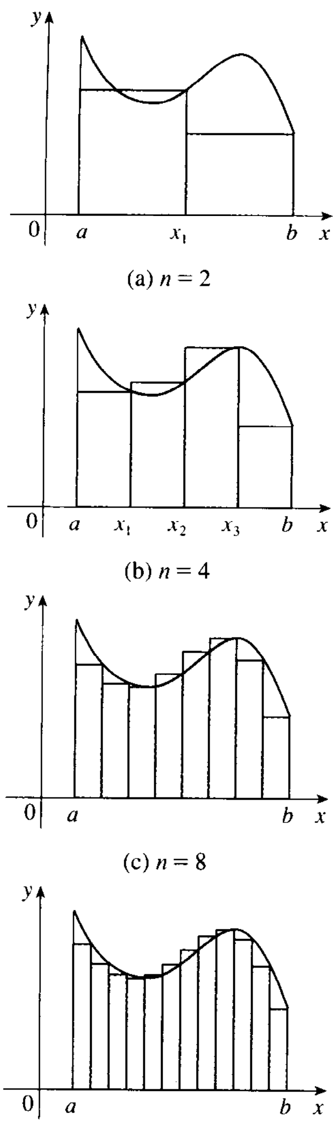


FIGURE 11

is $f(x_i)\,\Delta x$. What we think of intuitively as the area of S is approximated by the sum of the areas of these rectangles, which is

$$R_n = f(x_1)\,\Delta x + f(x_2)\,\Delta x + \cdots + f(x_n)\,\Delta x$$

Figure 12 shows this approximation for $n = 2, 4, 8,$ and 12. Notice that this approximation appears to become better and better as the number of strips increases, that is, as $n \to \infty$. Therefore, we define the area A of the region S in the following way.



(a) $n = 2$

(b) $n = 4$

(c) $n = 8$

(d) $n = 12$

FIGURE 12

2 Definition The **area** A of the region S that lies under the graph of the continuous function f is the limit of the sum of the areas of approximating rectangles:

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} [f(x_1)\,\Delta x + f(x_2)\,\Delta x + \cdots + f(x_n)\,\Delta x]$$

It can be proved that the limit in Definition 2 always exists, since we are assuming that f is continuous. It can also be shown that we get the same value if we use left endpoints:

3 $$A = \lim_{n\to\infty} L_n = \lim_{n\to\infty} [f(x_0)\,\Delta x + f(x_1)\,\Delta x + \cdots + f(x_{n-1})\,\Delta x]$$

In fact, instead of using left endpoints or right endpoints, we could take the height of the ith rectangle to be the value of f at *any* number x_i^* in the ith subinterval $[x_{i-1}, x_i]$. We call the numbers $x_1^*, x_2^*, \ldots, x_n^*$ the **sample points**. Figure 13 shows approximating rectangles when the sample points are not chosen to be endpoints. So a more general expression for the area of S is

4 $$A = \lim_{n\to\infty} [f(x_1^*)\,\Delta x + f(x_2^*)\,\Delta x + \cdots + f(x_n^*)\,\Delta x]$$

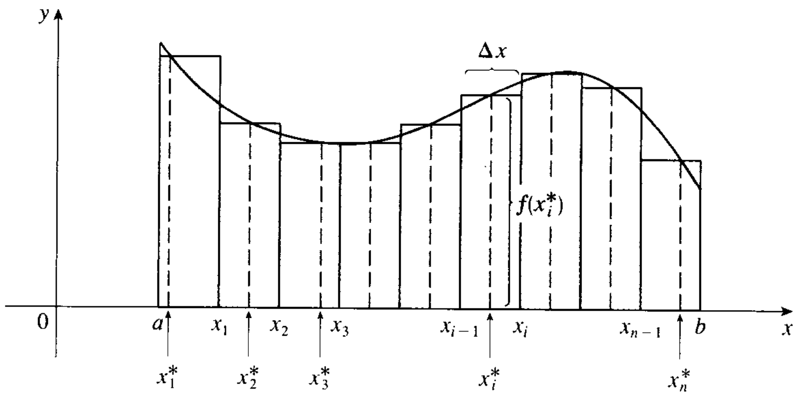


FIGURE 13

This tells us to end with $i = n$.

This tells us to add. $\displaystyle\sum_{i=m}^{n} f(x_i)\,\Delta x$

This tells us to start with $i = m$.

We often use **sigma notation** to write sums with many terms more compactly. For instance,

$$\sum_{i=1}^{n} f(x_i)\,\Delta x = f(x_1)\,\Delta x + f(x_2)\,\Delta x + \cdots + f(x_n)\,\Delta x$$

So the expressions for area in Equations 2, 3, and 4 can be written as follows:

IIII If you need practice with sigma notation, look at the examples and try some of the exercises in Appendix E.

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_{i-1})\,\Delta x$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

We could also rewrite Formula 1 in the following way:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

EXAMPLE 3 Let A be the area of the region that lies under the graph of $f(x) = \cos x$ between $x = 0$ and $x = b$, where $0 \leqslant b \leqslant \pi/2$.
(a) Using right endpoints, find an expression for A as a limit. Do not evaluate the limit.
(b) Estimate the area for the case $b = \pi/2$ by taking the sample points to be midpoints and using four subintervals.

SOLUTION
(a) Since $a = 0$, the width of a subinterval is

$$\Delta x = \frac{b-0}{n} = \frac{b}{n}$$

So $x_1 = b/n$, $x_2 = 2b/n$, $x_3 = 3b/n$, $x_i = ib/n$, and $x_n = nb/n$. The sum of the areas of the approximating rectangles is

$$\begin{aligned} R_n &= f(x_1)\,\Delta x + f(x_2)\,\Delta x + \cdots + f(x_n)\,\Delta x \\ &= (\cos x_1)\,\Delta x + (\cos x_2)\,\Delta x + \cdots + (\cos x_n)\,\Delta x \\ &= \left(\cos\frac{b}{n}\right)\frac{b}{n} + \left(\cos\frac{2b}{n}\right)\frac{b}{n} + \cdots + \left(\cos\frac{nb}{n}\right)\frac{b}{n} \end{aligned}$$

According to Definition 2, the area is

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{b}{n}\left(\cos\frac{b}{n} + \cos\frac{2b}{n} + \cos\frac{3b}{n} + \cdots + \cos\frac{nb}{n}\right)$$

Using sigma notation we could write

$$A = \lim_{n\to\infty} \frac{b}{n} \sum_{i=1}^{n} \cos\frac{ib}{n}$$

It is very difficult to evaluate this limit directly by hand, but with the aid of a computer algebra system it isn't hard (see Exercise 25). In Section 5.3 we will be able to find A more easily using a different method.