OL
EMATICS
Y GROUP

Report of

A CONFERENCE ON RESPONSIBILITIES FOR SCHOOL MATHEMATICS IN THE 70's

October, 1970

A CONFERENCE ON RESPONSIBILITIES FOR SCHOOL MATHEMATICS IN THE 70's

October, 1970

Financial support for the School Mathematics Study Group has been provided by the National Science Foundation.

Contents

Preface

The Status Quo -- And What to do About It?
Philip S. Jones, University of Michigan 1

Problems of Curriculum Development for the 70's
Gail S. Young, University of Rochester 15

Research and Evaluation in Mathematics Education
E. G. Begle, Stanford University 27

Federal Support of School Mathematics in the 70's
John M. Mays, Executive Office of the President 35

Goals and Objectives
Karl Kalman 51

Teacher Education
Joseph Payne 57

Research
Jeremy Kilpatrick 61

Curriculum: A Position Paper Based on the San Francisco Conference
H. O. Pollak 63

Evaluation in Mathematics Education
Donovan A. Johnson 67

Communications
E. G. Begle 73

Exploitation or Effective Utilization of the Programs of the 60's in Attacking the Problems of the 70's
B. H. Colvin 77

Proposal for a New Organization for Mathematics Education 81

Conference Participants 83

Appendix
Some Considerations on the Role of Probability and Statistics in the School Mathematics Programs of the 1970's
Frederick Mosteller 87

PREFACE

The purpose of this Conference was to provide the Advisory Board of the School Mathematics Study Group with suggestions as to what it ought to do next. This Study Group, which came into existence in 1958, devoted its major efforts to development of new mathematics curriculum materials and also to a careful evaluation of the effects on students of different kinds of curricula. To reach its objectives, SMSG developed a specific kind of organization and specific kinds of operating procedures.

As a result of SMSG's activities, and the activities of others, substantial improvements came about in school mathematics programs in the United States. Certainly, the situation in 1970 was far better than it was in 1958.

Standing at the beginning of a new decade, it was therefore appropriate to ask whether the organization and operating procedures which were developed for the 60's were still appropriate for the 70's.

A broad cross section of the entire mathematical community was brought together, October 23 and 24, 1970, to provide suggestions to the SMSG Advisory Board.

The Conference began with four presentations addressed to some of the particular problems to be faced during the 70's. The participants were then divided into four groups, each of which was asked to make specific recommendations as to what needed to be done during the 70's for a particular topic. These topics were: curriculum construction, dissemination of new curricula, research and evaluation, and teacher training.

Reports from these four committees were presented in a plenary session at the beginning of the second day of the Conference. After discussion of these recommendations, the participants were again divided into four small groups; this time each group was asked to prepare recommendations and suggestions as to how these recommendations could be carried out, who might be expected to do the work, what organizations would be needed, and who should assume responsibility.

Reports from these four separate committees were presented and discussed at a final, closing plenary session.

The SMSG Advisory Board met the day after the conclusion of the Conference to review the suggestions and recommendations that had been put forth. However, because of the large number of these, the Board requested its Chairman to appoint an ad hoc committee to meet later to review all these suggestions and

recommendations and to organize them in a way which would make discussion easier.

This ad hoc committee (E. G. Begle, Burton Colvin, Donovan Johnson, Karl Kalman, Jeremy Kilpatrick, Joseph Payne, and Henry Pollak) met December 11 and 12. Detailed accounts of the discussions and group reports, prepared by the SMSG staff, were made available to this ad hoc committee in advance. With these as a starting point, the committee was able to organize the conclusions, suggestions and recommendations from the Conference into summary reports for seven broad areas - objectives, teacher training, research, curriculum, evaluation, communication, and exploiting the work of the past decade in the next decade. The committee also prepared a proposal for a new organization for mathematics education for consideration by the Advisory Board.

These summaries and the draft proposal were sent to the Advisory Board in advance of its meeting January 20, 1971. At that meeting the summaries and the draft proposal were reviewed in detail. Some minor editorial changes were made, after which the proposal was approved unanimously by the Advisory Board.

The Conference Report, which follows, consists of the texts of the four opening addresses, the summary reports prepared by the ad hoc committee as revised by the Advisory Board, the proposal for a new organization for mathematics education as approved by the Advisory Board, a list of participants in the Conference, and, as an appendix, an article written and submitted during the Conference.

THE STATUS QUO - AND WHAT TO DO ABOUT IT?

Phillip S. Jones
The University of Michigan

The task set for me was to present a perspective view of the present situation in mathematics education, together with a list of the most urgent problems as seen by one person. The modification, acceptance or rejection of this cataloguing, and all the solutions for the problems are the task for the rest of the group in the discussions of the next two days.

When I asked myself what are the areas of our greatest current concerns and difficulties, I found that in this day of strong and sometimes illogical views, kidnappings, and guerilla warfare, I was thinking first in terms of the forces impinging upon us rather than in terms of the issues, the open questions, which might seem the more logical beginning. For mathematics education I see three groups of forces: (1) Those from the mathematicians themselves. Here I see (i) a continuing concern for introducing more mathematics earlier. This means a more varied mixture of mathematical concepts, perhaps an "integrated curriculum", as well as a translation downward of the topics of the existing curriculum. I believe some of this can and should be done, but that there are serious dangers and overly simplistic analyses in some proposals for doing it. (ii) I think that I also see in the mathematical community a slight movement toward less formalism and perhaps less stress on rigor and precision in the early years of mathematical training. (iii) I believe that there is also a growing concern for displaying the connections between mathematics and the rest of the world we live with. This concern is not merely to motivate students by displaying the undoubted utility of our subject, nor merely to give them practice in solving story problems, but also to give them an understanding of mathematical models, their nature, uses, and construction. Concerns for pushing content down in the curriculum, for teaching insight into good modern mathematics, and for a proper regard for the utility of our subject have all functioned as issues in the teaching of mathematics for literally hundreds of years, but I have tried to imply that there are new views of them and new forces bringing them to the fore as currently important issues.

(2) The second group of forces operating strongly in mathematics education today are those stemming from the allied fields of educational philosophy and psychology. We seem to be in a period of revivalism. (i) Skinnerism, programmed materials, and some computer-assisted instructional materials seem

to represent a return to a stimulus-response psychology, while (ii) the pronouncements of Bruner stir recollections of both mental discipline and gestalt theories, and (iii) Gagne's stress on the importance of hierarchies in learning recalls Thorndike's theory of bonds. Here, too, the newer views are not identical with those of the past.

There also seems to be a similar revival in educational philosophies. We hear much of open schools, of free schools, of students "doing their thing" and of need for relevance. In a recent panel discussion of problems in the schools on the Today television program, the "interlocutor" repeatedly asked the participants if this were not a return to the progressive education of thirty or more years ago. I did not hear a good reply to this.

This leads to my third (3) category of forces impinging upon mathematics education. These are the societal forces. They are embodied in both the deep problems of affluence, colonialism, war, race, etc., and also in more overt public attitudes and demands. A recent Gallup survey(1) of the current public views of the major educational problems listed in order: 1. Discipline; 2. Integration-segregation; 3. Finance; 4. Teachers; 5. Facilities; 6. Drugs; 7. Curriculum.

Several of these may appear to have little to do with mathematics education. For example, I feel sure that when discipline was listed first, the persons interviewed visualized protests, trashings, riots, and fights. Although this may seem to have little to do with mathematics education, if the causes of these breaches of discipline include lack of "relevance" in the schools, failure to recognize and deal with students as individuals, overly formalized and routine instruction as well as administrative organization, and just plain failure to teach effectively, then we must ask if either our mathematical content or methods may not be significant contributors to the problem of discipline.

But turning back to our list of major problems, and skipping for the moment integration, we arrive at finance and teachers. Here the name of the game is accountability. In the Gallup Poll, 75% of the adults favored national educational tests, 67% believed that teachers and administrators should be held "accountable", 58% expressed the opinion that teachers should be paid in accordance with the quality of work which they did, and 53% registered themselves as opposed to teacher tenure as it is known today. If some computation of the cost-effectiveness of education as a whole and of individual teachers is to become a basis for determining salaries and tenure, as well as being tied in with the employment of private educational

contractors, and the purchase of expensive equipment and materials, then it behooves the educational community as a whole and the mathematics educational community in particular to study critically the tests to be used and the educational objectives upon which test development must be based.

It will not be fair to schools or to teachers or to the students if the expectations of the community are not phrased and measured in terms of both the carefully defined goals of mathematics education and also in terms of the nature of the educational setting. If teachers are to be judged, possibly even paid, on the basis of some measure of their educational effectiveness, then this effectiveness must not be measured solely on the basis of a performance of students on any old tests. All of our objectives, whatever they are, must be measured and the effectiveness of teachers must not be measured merely by the raw scores of our students. Effectiveness must be measured in terms of the relationship between test scores and such other factors as the size of the class, the backgrounds and native abilities of the students, and the materials available for use in instruction. A teacher who is able to make smaller changes in his students but against greater odds may actually be more effective than one who with a small class and able students can produce high scores on the college board exams. This new stress on assessment, national, state, and local, also has its revivalistic overtones, recalling the testing movement of the early 20th century and its role in the attacks upon mathematics prior to World War II. At that time the poor showing of students on computational exams supported accusations that we were doing a bad job of teaching skills many of which were rather useless. This could happen again.

We must become quite specific about the goals for our instruction. We should not ignore, even if they would let us, those who insist on a behavioral formulation for our objectives and testing in terms of these behavioral objectives. However, mathematicians and mathematics educators must decide whether or not they do believe in such goals as training in reasoning, problem solving, perception of mathematical structure, understanding of the model-building process, appreciation of the nature and role of mathematics, the discovery of fun and aesthetic satisfaction in mathematics. If we really do believe that we can and do teach for any of these goals, then we must come to grips with the problem of defining them more clearly and testing them in some reasonable and reliable manner. If understanding is important, we must try to develop testing devices which will measure different levels of understanding as well as the well-known problems which measure manipulative facility.

We should also look carefully and realistically at our own estimates of the levels we seek to achieve. The National Assessment Testing Program attempted to classify its questions into three levels: those which were expected to be answered by 90%, 50%, and 10% of the tested population. After this classification had been completed by experts, the 90% exercises were tested out on a sample group of students. Of the 54 exercises tested, 43 scored at the 60% level or below, 42 scored at below the 50% level, and no one of the questions was answered correctly by more than 80% of the students. Either our teaching is poor or our goals are unrealistically set. Our problem is of course to raise the level of our students, but it is also essential that we be realistic with reference to these levels of achievement.

Let us now turn away from this analysis of the forces for change being brought to bear today by mathematicians, educational philosophers, and the general public. Let us try another, a semi-historical approach to setting the stage for the discussions to come.

At a conference on this same general topic and under the same sponsorship ten years ago, the opening, rather extensive, historical survey ended with the warning that the mathematics education community could not expect the level of financial support and popular interest in mathematics education to continue indefinitely, that mathematics educators themselves must guard against the onset of lethargy and a diversion of interest to other fields and other problems, and finally that continuing regular and frequent assessment of the curriculum materials projects then in their hey-day was necessary.

In a look at the future for mathematics education, Marshall Stone pointed out at that conference that there would be a continuing need to push more advanced mathematical materials down in the curriculum, but that there may be some things that you can not do with children at too early an age. Hence, he felt that there was a great deal of need for additional data from psychology. He also stressed the need to continue educational reform down into the elementary school, to seek and utilize better coordination with the other subjects of the school, that attention must be given to the high school mathematics desirable for non-college-bound students, and that there must be a closer relationship between the mathematics we teach and the applications of mathematics. In the following decade attacks were mounted on all those problems, but many remain largely unsolved, probably because they are difficult, perhaps because they are unsolvable. Let us hurriedly list some of these attacks.

SMSG continued its development of a program for the elementary school, introducing numeration, geometric concepts, negative numbers, and algebra-related structural properties, terminology, and symbolism. Simpler versions of some secondary school materials were developed and successfully taught to slow learners. High school courses in analytic geometry and calculus were developed. These activities might be gathered together under the heading of the development of <u>materials</u>.

SMSG and some others, especially the Minnemast project, have developed courses and units built around relationships between mathematics and science. These courses and the computer-related materials developed by SMSG, the NCTM, a current project in Colorado, and others might be grouped together as having a primary bearing upon <u>motivation</u> although they also embody some direct and immediate intrinsic utility and partially bring out ideas related to the process of formulation of mathematical models.

Other groups developing curriculum materials during this period have included the Columbia University centered Secondary School Mathematics Curriculum Improvement Study and the Comprehensive School Mathematics Project at Carbondale, Illinois. Both of these have tended to focus on materials for superior, or at least college preparatory, students with the former planning to write a complete secondary school program stressing an integrated and modern approach. None of the projects of the later sixties seemed to have the combination of drive, excitement, range, and extended involvement and interest which were featured in SMSG and UICSM earlier.

The latter program has turned, in part at least, to a concern for materials for culturally disadvantaged. This group of non-college bound or non-college-motivated students have been the recent concern of an Oakland County, Michigan, project, an NCTM project, and a Texas-centered project which, under Glenadine Gibb has been concerned with materials and methods usable with Mexican Americans and other students with a culturally different home environment and background.

It is not always clear whether some projects should be classified as material preparation or as classroom <u>management</u> programs. I would list William Johntz' S.E.E.D. project, the University of Illinois Arithmetic project under David Page, Robert Davis' Madison and Webster projects under a classroom <u>management</u> heading. These projects have all stressed open-ended, exploratory, discovery procedures, but of an intellectual type and with substantial teacher participation and leadership. SMSG itself experimented with programmed materials, another contributor to classroom management and

individualization programs. There is a growing interest in materials - centered and manipulative laboratory activities patterned after some English elementary school models. The problems of adapting to individual interests and capabilities at all levels are substantial and increasing. Experimentation seems to have shown that students learn better and develop better attitudes when they see their objectives clearly, and when they have some freedom to self-select projects and materials of especial interest to themselves. However, sharply defined conclusions on classroom management which are easily passed on to young teachers are hard to come by.

In addition to new curriculum materials and classroom management experimentation, the past decade has seen several major measurement projects, S M S G's NLSMA and the International Study of Achievement in Mathematics testify to how costly and difficult measurement is. The rapid current growth of national and local assessment point out how urgent it is to work on this problem.

Probability and statistics has probably been the one area which could be classified under the heading of motivation (although it also involves curriculum materials) which has had the most vocal and insistent proponents. However, materials in this field have been in short supply, and little used. Those experiments in teaching probability and statistics with which I am familiar have been disappointing in their effect on both students' knowledge and attitudes. It seems that a great deal more needs to be done here.

This past decade has also seen the development of five new journals, several of them stressing research in mathematics education, as well as a research-oriented organization and a first international congress. Organization seems to flourish; major definitive advances in motivation, management, materials, and measurement are harder to come by.

How can we assess the actual effect of the so-called "revolution" and of the activity of the past ten years. An interesting study was published by Milton Beckman in School Science and Mathematics for April, 1969. Beckman has long been interested in the "basic competencies" which were defined by Post-War Plans of the National Council of Teachers of Mathematics as being the essence of what they called mathematical literacy. Their work was done and published prior to the development of so-called "modern mathematics", and their recommendations focused on the over-all needs of the school population as a whole. In 1950-51, Beckman gave a test based on these competencies to 42 schools and found that the mean score in the fall of the year on these tests was 45.7 and that it was 54.3 in the spring. He gave the same tests

in virtually the same schools in 1965-66. The means were 54.9 in the fall and 61.1 in the spring. His conclusion, then, was that students who had been studying so-called modern mathematics and modern algebra seemed to be doing better with respect to those old objectives than students who studied purely traditional algebra.

We should not be too delighted with such a result because the change of course is not tremendous, and one can always raise questions or objections with reference to the validity of the entire process used. Furthermore, a variety of studies have tended to show that students taught via our new stress on structure and meaning and understanding with some added insistence on technical terminology and precision have tended to do less well than more classically instructed people, particularly with reference to computational and manipulative skills. There seems to have been in many of these studies some gain under the new mathematics in problem solving and perception or understanding of basic mathematical ideas. One must always raise the question as to whether or not tests in terms of older goals and terminology can be validly administered or interpreted with respect to students taught under a different regime.

At the college level, a number of studies have shown improvement in the background of college freshmen, which a person teaching in the college hardly needs to document. All of us have seen freshman courses omitting college algebra, trigonometry, and even a substantial amount of the time put in on analytic geometry as evidence of the fact that content is being pushed down from the upper reaches to the lower levels. We have seen students continuing to perform and even performing better than in the past in courses which are not only more advanced in content but taught at a higher level of rigor and abstraction. We are now seeing linear algebra incorporated into the freshman-sophomore program either incidentally to the calculus or in courses companion to the calculus. Similarly, units and even shorter courses in probability and statistics are being developed for engineers as a part of their required beginning mathematics program and we're on our way towards the requirement of some acquaintanceship with the computer for all engineers and mathematics majors, perhaps even for all college mathematics students. If we needed data on the changed preparation of the average college freshman, we have a study (2) which showed that at Western Michigan University there has been a substantial decrease in the number of students entering without mathematics or with algebra only and a substantial increase in the students entering with algebra, geometry, and trigonometry incorporated into a year or longer high

school mathematics program. These changes apply to freshmen of both sexes. More recently, Irene Williams of the Educational Testing Service has analyzed data gathered from students taking College Entrance Examination Board Tests (3). She sought to determine whether or not their high school programs reflected the recommendations of the Commission on Mathematics. I shall not use our time here to detail those recommendations; however, 85% of her sample had studied the field properties and 3/4 of them had met them before their junior year in high school, 1/5 before grade 9. Inequalities had been studied by 90% of her population, and 2/3 of them had done this before grade 11. The concepts of inverse function were familiar to 70% of her sample, and 50% recorded that they had met exponential functions. 50% also reported that their high school geometry had included coordinate geometry, and $33\frac{1}{3}$% rated their geometry course as being both plane and solid geometry. However, only 20% of her student population had studied probability and statistics for as long as at least one month.

Insofar as teachers are concerned, we have the evidence of the tremendous growth of the National Council of Teachers of Mathematics from a membership in 1950-54 of 8,000 to a membership of 76,000 in 1965-69. However, the rate of growth of the NCTM has been declining in recent years, the interest of elementary teachers and supervisors has been turning away from mathematics if one can judge on the basis of subscriptions to The Arithmetic Teacher and attendance of elementary teachers at meetings. Studies of the effect of the CUPM teacher training recommendations for elementary teachers show a tremendous growth in one-semester courses in mathematics for prospective elementary teachers, a moderate growth in two-semester courses, but far from any general acceptance of their recommended four semesters of mathematics courses for elementary teachers. It is my impression in fact that the CUPM is going to withdraw a little from this advanced stand with reference to required mathematics for elementary teachers in the forthcoming revised teacher training recommendations.

Another trend in the past decade which we should not ignore has been the change in the objectives of our students. As early as 1964, a study of the career decisions of very able students showed that "Talented male students appear to have shifted their interests from physical science and engineering to the humanities and the social sciences. Exceptions to this general trend are losses in the fields of education and music and the gain in mathematics." (4) This was a study of high school graduates who performed well on National Merit Tests. Enrollments in engineering and science continue to decline while students are flocking into the social sciences.

Apparently associated with this trend are the data drawn together in an editorial by Philip Ableson in the May, 1967, issue of Science (5). He pointed out that recent campaigns to increase interest in science and engineering have not been very successful and raised the question as to whether or not this shift was chargeable to more stringent secondary school curricula? He felt that there was a growing concern that too much is being asked of the young in our secondary schools, especially with reference, of course, to the college-bound students. In 1967, Carol King, professor of chemistry at Northwestern, stated in a speech that secondary school students are being asked to do "too much, too fast, too soon". Here we see an echo of Marshall Stone's concern of ten years ago. Ableson ends his editorial with the statement, "Evaluation, looking toward proper changes, is in order", and this, you see, is again an echo of my earlier expressed concern for testing and evaluation, though admittedly expressed in a different situation and probably envisaging a little bit different type of evaluation. Let me now turn from this semi-historical survey of the past decade to a summary of those forces which seem most active on the current scene and those issues in mathematics education which seem most to the fore in today's schools.

Both current educational panaceas involving "assessment" and "accountability" and long range educational research and development demand a careful review of the goals of mathematics education parallel with the development of new and imaginative testing procedures. The question of whether or not all the valid goals of mathematics education have been or can be expressed behaviorally must be wrestled with. If accountability is to be stressed and based upon some set of more or less objective measures, then these measures must also include parameters associated with the size of classes, the background and motivation of the students, the support received by students and faculty from parents, the assistance provided to teachers by the school system in terms of materials, time, and other forms of supervisory support. Further, if schools and teachers are to be measured in these ways, they must be involved, along with mathematics educators, not only in the formulation of the tests and in the specification of additional parameters used to determine educational return per dollar investment, but they must also be heard with reference to the realism with which the goals are formulated and the tests are weighted. Measures of mathematical understanding, insight, problem solving, and perception of the structures should be added to testing programs, but the nature and level of achievement in these areas and the stress upon them in the measuring instruments must be realistically adapted to the facts that children develop their capacities for higher levels of abstraction and

generalization over many years. It is not consistent with either good psychology or good experience to lay the same stress on these goals in either teaching or testing at all levels of instruction and with all groups of students.

Another currently popular social outcry, that for relevance, should be associated with a longtime debate amongst mathematicians themselves. I believe that it is important for learners to see both aspects of mathematics: its internal structure, beauty and abstractness, and also its relationship to the physical world which frequently has been the source of problems and initial ideas, and the concrete embodiment of theorems, as well as the place to which mathematical theories have returned to aid in applications. I believe that a stress on interrelationships both within mathematics and with the rest of our environment, mental and physical, is actually a part of teaching children what mathematics is about. Such teaching contributes to their appreciations and insights. This is my belief from the viewpoint of mathematics. From the viewpoint of psychology, I believe the same goals, teaching applications and modeling, are important aspects of motivations. How to define, teach, and measure for such outcomes is the problem.

There is much that is not new about my major points: reconsideration of goals has been called for repeatedly ever since instruction began. The basing of testing on well-defined objectives has been explicit and implicit ever since man began to test students. The importance of motivation and the recognition of utility as one aspect or type of motivation has nothing new about it. However, I have tried to show that there is a modern flavor and a modern urgency about all of these items which say there is something new and different about the situation today.

A somewhat similar statement may be made with reference to today's demands upon classroom management. We are witnessing what appears at first glance to be a return to the progressive education of the thirties. Some of the terminology is new. We hear of "free schools" and "open schools", and everyone today associates some vision with the phrase "doing your thing". Here again the philosophers and psychologists who call for these types of classroom management must not be ignored. Their educational goals are broadly desirable, and unmotivated, disinterested, unwilling students are at their very best only poor learners. On the other hand, a significant part of the essence of mathematics is structure, deduction, and organization. Although there is no unique sequence for the organization of the learning and teaching of mathematics, it is difficult to support mathematics instruction which is totally disorganized

and incidental. We must provide a basis upon which classroom teachers who are responsible for classroom management can resist undue, improper, and unprofitable pressures for dependence upon incidental learning and insistence upon a narrow utility as an only and essential motive. We must at the same time encourage and help them to recognize the values of incidental learning and of pointing out interrelationships between mathematics and its environment. We must not lose sight of the existence of intrinsic interest and motivation, but we should not regard it as the sole motivation to be presented. Similarly we should continue to build problems and exercises that provide incidental maintenance of past learning and skills, but experience has tended to show that this is not enough. Adequate recurring practice must be planned for. In the thirties and forties mathematics educators had to fight the postponement and disorganization that came from undue stress upon incidental learning and felt need as curriculum determiners. We replaced felt need motivations by an insistence upon the existence of implicit interest and motivation within the framework of mathematics itself. We insisted that the utility of mathematics lay in its very abstractness and generalization, in the capacity for making one mathematical structure fit many real world situations. All of this was fine, correct, and I personally believe in it. However, I also believe in the existence, reality, and importance of <u>both</u> sequenced learning and incidental learning, of motivation from intellectual curiosity and the internal structure and beauty and logical constructs of mathematics and also motivation from perceptions of the physical world, of social-economic problems which may be formulated in mathematical terms and use the abstract structures as a part of the model-building process which helps to solve problems and predict changes. In the past ten years we have tended to swing from one extreme to another, to focus on a new and changed perception of mathematics and its goals and thereby lose a valid view which should have been retained from the past. To incorporate all of these factors into teaching and teachers is a tremendous challenge which must be done not only to <u>improve</u> the instruction in mathematics in the next ten years but even to <u>retain</u> for mathematics the public view of its role and importance which has developed in the past decade.

Teachers also need help in evaluating and using the opportunities presented by new forms of classroom management. Do mathematics educators know enough about the role of computer-assisted instruction, of modular scheduling and team teaching to prepare teachers to reject or accept them, or to function effectively in schools using them? Is it possible to design relatively independent mathematics models which will allow different branching and content

adapted to individual needs and interests within the same grade, possibly even within the same class? Can teachers be trained and motivated even to create their own "incidents" while at the same time organizing and planning a coherent program?

The role of the teacher needs analysis as never before. As I see it, today's varied concatenations of computers, tests, para-professionals, teams, and modules call, as never before, for well prepared flexible teachers with background and self confidence. The teacher still plays a major role in the selection, sequencing, and assigning of priorities to the content specified by a text or curriculum and this role becomes even more important today. Further, the teacher is responsible for developing the informal and intuitive approaches which lead to desired understandings. It is largely the trained teacher, not a machine or a para-professional, who will communicate to students enthusiasm, perceived relevance, and perceived goals. Training such teachers is an increasing challenge.

In a somewhat impassioned speech given in 1894 the mathematician and logician Charles Sanders Peirce (6) stated that the intellectual powers needed by a mathematician are concentration, imagination, generalization. Then, pausing for effect, he asked, "Did I hear someone say demonstration? Why demonstration is but the pavement on which the chariot of the mathematician rolls!" Let us keep that chariot rolling!

Notes

(1) George Gallup, "Second Annual Survey of the Public's Attitudes Toward Schools". The Phi Delta Kappan, Vol. 52 (Oct. 1970), p. 99 ff. Arthur Coxford called my attention to this article.

(2) Haym Kruglak, "The High School Background of College Freshmen after Sputnik". The Mathematics Teacher, Vol. 63 (April, 1970) pp. 339-341.

(3) S. Irene Williams, A Survey of the Teaching of Mathematics in Secondary Schools. Princeton: Educational Testing Service, 1969.

(4) Robert C. Nichols, "Career Decisions of Very Able Students". Science Vol. 144 (June 12, 1964) pp. 1315-1319.

(5) Philip H. Abelson, "Excessive Educational Pressure". Science, Vol. 156 (May 12, 1967).

(6) Raymond Clare Archibald, "A Semi-Centennial History of the American Mathematical Society, 1888-1938." Semicentennial Publications, Volume I. New York: American Mathematical Society, 1938. p. 7.

PROBLEMS OF CURRICULUM DEVELOPMENT FOR THE '70s

Gail S. Young
University of Rochester

I will begin by reminding the reader how short a time we have been working systematically and on a large scale on the problems of the school mathematics curriculum. A few dates will help: the first National Science Foundation Summer Institute in Mathematics was in 1954; the first grant was awarded to SMSG in 1958; and the report of the Commission on Mathematics of the College Entrance Examination Board was published in 1959. I first met set theory 32 years ago in my first graduate course with R. L. Moore, and I contrast that with a little ten-page book made by the six-year-old son of one of my department secretaries, entitled Sets, on the kth page of which there appears a picture of a set of k elements. We have been working on the problems of the mathematics curriculum in a systematic way only a very short time, and we have come a very long way. Mathematics itself has changed enormously, in methods, in approach, and in terminology, in the 28 years since I received my Ph. D., and these changes will certainly be reflected soon in the school curriculum. At the same time, the electronic computer has brought in an almost inconceivable change in the speed and cost of numerical computation and has created a whole set of new concepts that are already showing up in the curriculum. Combining all these, you will see why I feel distinctly hesitant in making predictions now about the changes in the curriculum to occur by 1980. I am uneasily aware that as I write there is quite possibly some young assistant professor in Boston trying out cohomology theory on a fourth grade class in a ghetto school, and finding that it is wonderfully successful. I would have more concern that such developments would invalidate my predictions if it were not for the comforting fact that there are about a million elementary school teachers, and there is no way on earth of teaching many of them cohomology theory in the next ten years.

Our work has made great improvement in the mathematics education of the rather talented youngster bound for four-year colleges. One indication of this had been the sharp increase in level of freshman mathematics in the four-year institutions. This change in level is thoroughly documented in Volume I of the report of the CBMS Survey Committee,[1] summarizing data gathered in

[1] Aspects of Undergraduate Training in the Mathematical Sciences, Conference Board of Mathematical Sciences, Washington, D. C., 1967.

1965. Additionally, the Survey questionnaire had a specific question as to the impact of the new curriculum materials.[2] About 70% of the responders said that the change in level was due to the new materials, and only 26% said no.

We have also changed the elementary school program from rote drill in number facts to an understanding of mathematical concepts. And we have wiped out from the junior high school a vast amount of material which, as Professor Begle once remarked, could only be justified under the assumption that, on completion, the students would marry, get a job, and start raising a family. I believe that we can now say that the child in an upper middle class suburban school system receives a very good mathematical training indeed.

But for the rest of the population, we have been much less successful. The Survey Committee asked the same questions about the impact of the new curricula in the two-year colleges. The returns here were half yes, and half no. When I examine the enrollment figures for various courses in the two-year colleges, I can only interpret this coin-tossing data as meaning that the two-year colleges saw no change in their students from the new curricula.[3] A re-survey to be made this fall (1970) will, I think, show no great change in the enrollment pattern in two-year college mathematics. What that will mean to me is that for somewhere between a half and a quarter of the students going into post-secondary education, all our work on the curriculum has meant little. I believe that this is unacceptable. Even if those of us in this room can live with it, our society cannot.

The technological revolution resulting from the computer has made irreversible changes in society, and the rate of change is accelerating. It is not a question of whether we can enrich our students' emotional and intellectual life through the contemplation of mathematical truth; it is rather, can we teach him enough mathematics to keep him off welfare? I think it is that question, in its various ramifications, that will dominate curriculum work in the next ten years. For the next few paragraphs, I am going to abandon my role of prophet and say what it is I think we must do. We must, first of all, take a quite different approach to the elementary curriculum. A common

[2]The exact question is, "It is desired to know the impact, if any, of the new curriculum materials (such as those of the School Mathematics Study Group, The University of Illinois Curriculum Study in Mathematics, the Minnesota National Laboratory, etc.) on the entering college student. In your judgment, have such materials had an impact (e.g., extent of preparation of students, level of preparation, etc.)?"

[3]Approximately half the enrollment in mathematics courses in the junior colleges, freshmen and sophomores alike, in the fall of 1966, was in courses below the 12th grade.

characteristic of all the new elementary curricula is that the development of mathematics has been logical rather than psychological, adult, so to speak, rather than childish. By this, I mean that if one had an educated adult who inexplicably had failed to learn any arithmetic, the way one would approach his education would be exactly in the spirit with which we approach the six-year-old. One could, for example, take Landau's Grundlagen[4] and proceed through this. This is exactly the line of development of some commercial series, developing set theory, the whole numbers, the integers, the rational numbers, and finally the real numbers. Or one could take something of an axiomatic approach and begin with a simplification of the definition of the real numbers as a complete ordered field. This is approximately the line of development urged by the Cambridge Conference[5] which advocates the introduction of the number line in the first grade. If any of these approaches worked universally, we would have no problems, and it is easy to find external reasons which could explain why these approaches have not been universally successful. One can give many explanations and there is something to be said for each of these. One could talk about the realities of teacher training in the United States. One can talk about the conditions under which many of our children live, or any number of other factors which are outside the control of groups like ours. But the fact remains that, because of our ignorance, we have now no real choice but to teach in a logical order. We know almost nothing about the psychology of mathematical learning in the child - or indeed of any other kind of learning. We have experience, we have intuition, we have introspection, but little knowledge. We know more about the mental processes of schizophrenics than we do about normal healthy third-graders.

What, it seems to me, learning theory attempts is the isolation of some part of the psyche that controls "learning", while ignoring the psycho-sexual development of the child. Anna Freud has, I think, much more to say to us about the teaching of rational numbers than does - well, I won't select a victim.

I am beginning to encroach on the territory of Professor Begle's paper, but the encroachment seems to me unavoidable. A major part of our effort must go into finding out how the child learns mathematics, and teaching him in the order of topics best suited his psychology, with less regard to the logical order.

[4]E. Landau, Grundlagen der Analysis, Chelsea, New York, 1965.

[5]Goals for School Mathematics, Cambridge Conference on School Mathematics, Houghton Mifflin, Boston, 1964. Also see their Goals for Mathematical Education of Elementary School Teachers, ibid, 1967.

Let me say more concretely what I mean by this psychological order. A friend remarked that his child was fascinated by the concept of infinity at age 10, but at age 12 had lost interest and was then fascinated by finite combinatorial ideas. Logically, one proceeds from the finite to the infinite, and actually we shield the innocent child from concepts of infinity for quite a long time. If my friend's observation had general validity, we should take up finite sets in the 4th grade, whether or not we had prepared a logical base. By grade x, $4 \leq x \leq 20$, the whole process of instruction should presumably have covered the logical structure necessary for adult understanding, and the student should so understand it. But it is not necessary before grade x to have always been logical, certainly not if we can teach more.

I will remark parenthetically that most research in mathematical education has been the accumulation of facts, of the sort that in the physical sciences is recorded in handbooks. That is, we learn such things as that 100 children taught fractions by method A do significantly better on standard tests than 100 children taught by method B. This is a fact, and if the experiment is reliable, we adopt method A over method B. But there is nothing in this sort of research that gives us any powers of prediction. We cannot say, for example, what the results of using method C would be.

Let me elaborate that a little. A fact of physics, in the handbooks, is that copper is a better conductor of electricity than iron. That one can discover by experiment, with no understanding of why it is true, and no basis for saying what the answer is for lead. The line of scientific explanation I once knew was concerned with atomic structure, and was rather deep. It is a level of theory out of our reach in teaching, perhaps forever. But consider the conductivity of heat. Here again, copper is a better conductor than iron, and here again there is a deep explanation. But there is also a scientific explanation, at a much shallower level, that makes prediction possible. Iron "holds more heat" than copper, and so lets less through - to sound more impressive, iron has a higher specific heat than copper. We can tell the heat conductivity of lead by determining its specific heat, a simpler thing.

Besides these problems of research and psychology, it seems to me that we have not really thought through the aims of our curriculum. I first became aware of this myself when I went two years ago with Henry Pollak to Africa to study the effectiveness of Entebbe mathematics.[6] We found ourselves for the first time in societies where, for the indefinite future, a large fraction of

[6] "Entebbe" refers to primary and secondary school mathematics programs developed by the African Education Program of Education Services, Inc., Newton, Mass. These have been published by Science Research Associates.

the children would not get beyond the fourth grade. This fact raised in our minds many novel questions about the elementary curriculum, questions which, we soon realized, we had not thought about in connection with our own country.

To illustrate, many of the children who left school this early would be farmers in cooperatives. There are decisions to be made by such cooperatives, such as, "With this year's profits, do we buy another pump or buy a tractor?" The analogous problems in a major U.S. corporation would be approached mathematically. In Africa, in the first four grades, should some attempt be made at least to make the child aware that such problems can be studied analytically? That is a question about goals. We still have the difficult, if not impossible, task of implementing that goal, if adapted.

In this country, we have no such dramatic problems. But nevertheless I cannot say what are the goals of our curriculum. I think it is not too much of an exaggeration to say that the bulk of our curriculum work in the schools has been aimed to maximize the number of children who will be prepared for calculus. Is that really the goal we should have? My asking the question does not mean that I am convinced the answer is no. But we must think through what it is we should be doing.

A major part of curriculum development in the next decade will be the development of variant curricula for the various minority groups in our culture, the black, the rural poor, the Mexican-Americans, to name three. For a variety of reasons, we are failing very badly with these groups. To deal with them successfully, however, requires more than mere curriculum changes. The remarkable work of William Johntz and his SEED project in California has much to teach us, but can be misinterpreted. To return to my African experience, Pollak and I decided that what we saw there shot down many of the usual explanations for the difficulties minority children have in our schools. In one country we saw elementary school children living in what could be regarded as appalling proverty, going home to houses without electricity, without books or magazines, with illiterate parents, but doing Entebbe mathematics with the enthusiasm and apparent success of Chicago suburban children. Their teachers were, by our standards, inadequately trained, usually the products of two-year normal schools, yet clearly very successful. But in the next country, we saw nothing like that much success. How could one account for the difference? The two countries were alike ethnologically, culturally, and economically. The difference, we felt, was that in the first country the schools were firmly in control of the Africans, and in the other country the schools were still controlled by expatriate teachers. In the second country, it seemed quite

clear that no one expected the African children to perform at anything like a European level, and the children didn't. Now that, I believe, is the situation in our ghetto schools. No one believes that the students can do well, and so they don't. The success of Johntz's work, to me, rests not so much on what he has been teaching, as on the firm belief he and his co-workers have had that these children can do wonderful things.

Let me give another concrete example, an embarrassing and painful one to present publicly. I was finally convinced that I had eradicated all vestiges of prejudice in me long before I went to Africa. In Dodomo, Tanzania, 200 miles from the coast, I taught for an hour the most responsive, brightest class of my life. I began with a puzzle question, which I had tried on classes of every level in this country. The class response was better than most first-year graduate classes. I was amazed by these students. Later, I realized that until I taught these students, I had never really believed that a class of black students could do as well as a class of whites, though I would never have admitted that to myself. This was a totally unconscious vestigial prejudice.

There is evidence besides my own limited experience to confirm me, but, if I am correct, our major problem in the ghetto schools is the eradication of racism. This will be a long painful process, and, like most psychological disorders, it will not vanish until we have become much more conscious of it than most of us are. I emphasize this point because, if it is valid, it should affect the nature of special curricular materials for minority groups, and also affect teacher training for these groups. There are objective differences in culture that must be taken account of. But such materials will not be successful if they are based on our unconscious presupposition that the children cannot do well.

I would like now to address myself to specific problems of the curriculum. Let me summarize quickly the mathematics curriculum of the '40s and early '50s. Grades 1 thru 6 studied "arithmetic", the rote manipulation of whole numbers, fractions, finite decimals, percentage and proposition, with some allegedly practical material, such as the preparing of grocery bills. Grades 7 and 8 applied this arithmetic to the problems of interest, stock broker's commission, and other such matters of vital concern to the age group. Grades 9 thru 12 followed a curriculum which ultimately had its origin in British school mathematics of the 1850s, and which had changed primarily by successive dilutions as the secondary school itself changed to mass education. I have said earlier that the changes since then have been amazing, but they have been

motivated primarily by the desire to give the student the same content in a form that can be understood. This clarification has been more successful in those projects that have included professional research mathematicians than in those that have not. There have been astonishing distortions of what I regard as the spirit of mathematics in some of the others. I have in mind, for example, a commercial 8th grade book in which the definition of multiplication of rationals is given in terms of operations on equivalence classes of ordered pairs of equivalence classes of ordered pairs of equivalence classes of ordered pairs. I hope in fairness I have the right number of repetitions. Or of another 8th grade text in which it is explained that rational numbers were introduced to make it possible to solve equations of the form $ax = b$, a and b integers, and in which, if I may attempt to get at the meaning, a rational number is defined as an equivalence class of such equations. How was it possible for such distortions to occur? I suppose that fundamentally it had its origin in the very small number of mathematicians the country had 15 years ago. There simply were not enough well trained mathematicians to do the necessary tasks. Another factor, arising from the first, has been the neglect of teacher training in the stronger mathematical centers, a problem still not satisfactorily resolved. Still a third reason was the emphasis in the first NSF summer institutes, where the college and university mathematician, faced with a group of students of poor background, almost unanimously selected logic and the foundations of the real numbers as something they could teach to their classes. These distortions must be corrected, and I hope that one result of the several new journals in mathematical education will be thorough-going critical discussion of such matters, bringing all these things out into open air.

But we have more to do than the elimination of these misconceptions. One task is the incorporation of the effects of the computer in the school. Let me indulge in a little speculation. In 1975, it has been estimated that we will have 100,000 computers in operation. It does not seem unreasonable to me, and I have tried this figure out on various computer experts, that, on the average, for each such computer there will be or should be about 10 people with some direct involvement in the computer having at least some mathematics and computer science training at the undergraduate level. This includes people in managerial positions who must make decisions based on an understanding of what the computer can and cannot do, advanced programmers, and other technical personnel, but does not include, for example, engineers using the computers for purposes of computation. That is a million people who must have a specialized technical training, out of a total working

population of around 90 million. Would it be too wild to say that 5 times that number will find themselves using the computer? I find myself unconcerned about the accuracy of these particular predictions because nothing that is being done at this meeting will affect the employment of anyone in 1975. Whatever the outcome of this meeting, it will not be until the '80s that any significant number of students will be affected.

One of the major tasks of the '70s will be determining the proper place of the computer in the school program. It is already clear that much more will happen than a senior course in programming for the college-bound. Such concepts as flow diagrams have proved themselves valuable in the actual exposition of mathematical ideas, and will certainly work their way down further and further in the grades. But I believe that more than that will occur. I wish I were technically competent to map out the changes. But I cannot even guess as to whether, for example, sixth graders will have direct access to computers by 1980. In an earlier paper,[7] I said at great length that nobody knows what the implication of the computer is for calculus. To say it more briefly, a very great deal of the calculus turns out to be in there for the purpose of avoiding hand computation by altering answers to forms findable in tables, or in setting up and justifying methods which, at that level, have their justification as numerical methods. In discussing what topics should remain, I said in the earlier paper that I feel like an old flint-worker, aware that the Bronze Age has arrived, discussing what techniques of flint making should be taught to all the young braves. All I know for sure is that we must give most serious attention to the implications of the computer for the school.[8]

Let me give one small concrete example. I am sure that many of my readers have seen George Forsythe's CUPM paper, "How to Solve Quadratic Equations."[9] When we come to the quadratic formula, should we now stop and talk about the computational questions Forsythe raises?

The advent of the computer has meant a great increase in the mechanization of society. It is, I believe, the ultimate cause of the tremendous acceleration of technological change.

---

[7] "The Computer and the Calculus", Educational Studies in Mathematics, Vol. 1, 1968, pp. 105-110.

[8] For a discussion of computers in the colleges that has many implications for the schools, see Computers in Higher Education, Report of the President's Science Advisory Committee, Government Printing Office, 1967. This is the report of the "Pierce Committee".

[9] A CUPM monograph, 1967.

I mean by this that the computer makes possible the non-experimental solution of problems of design, replaces much testing by computer simulation, and controls directly processes beyond human skill. The time scale for development has been altered. It seems to me that one implication of this is that there must be a greater increase in the student's understanding of mathematics as a tool for solving real problems. At the least, the student being prepared for the '80s and the '90s must be aware that there are whole classes of problems which yield to mathematical methods coupled with the computer. It is beyond my powers to estimate how far this will have to spread. Should every truck dispatcher, chief shipping clerk, service manager, shop foreman, airline passenger manager, have some understanding of linear programming? We cannot afford to wait until the '80s and '90s to find the answers to such questions.

Once one turns away from the classical applied mathematics based on physical laws to the newer areas of application, one finds probability theory becoming important. I myself managed to take mathematics for 19 years without ever running into probability, and I suspect that this still occurs, but that is also something that must stop.

I have been describing up to now a grimly practical approach to the curriculum, an approach designed to keep the students off welfare, to teach them all about computers, to teach them applied mathematics and probability. I would now like to switch abruptly to another side of the curriculum. It may appear that I am trying to make sure that when 1980 comes I will be able to say that I predicted successfully the direction of the curriculum, whatever that direction has been. But suppose that I now were given a class of 9th graders, told that I could teach them for 4 years, and that I could rest assured that every one of those students would go into scientific work in the physical sciences, engineering, or mathematics. What would I do with them? First of all, I suspect, I would start teaching them group theory. Additionally, that first year I would teach them geometry from the standpoint of transformation groups and begin the study of morphisms. By the time they graduated, they would have had a good deal of abstract algebra, linear algebra, and a very thorough course in calculus. All through the 4 years I had them, there would be great emphasis on the working of difficult, interesting problems, most of them purely mathematical, and simultaneously, great emphasis on structure. If I were properly successful, I don't know of a college in the country I could send them to. It is clear that when I think of students like that, I have entirely different curricular ideas. How do I resolve this conflict in my own mind? At this point of writing, I don't. There is a real difficulty here. To take the case of physics alone, it is clear that the highest and

most recent abstractions are finding employment there. I have seen the reports of a summer research seminar on relativity theory which included a chapter on cohomology theory of locally compact spaces. Lie groups and group representations are becoming commonplace. In my own university, the physics department offers a course called the Algebraic Methods of Theoretical Physics, parallel to the traditional Courant-Hilbert course, that includes linear algebra, vector spaces, and the theory of finite and continuous groups. Special attention is given to the symmetric group, the 3-dimensional rotation group, and the unitary groups. This is very different from the traditional concept of "useful mathematics" and was pure mathematics 30 years ago.[10] A very few children in each school will need such concepts professionally. It is not an easy question as to how to meet that need. In the preparation of students in the schools for scientific research, the European curricula seem to me to have it all over our own, or over the Russian. This superiority is gained at a price. It seems to me that both in Russia and the United States, countries with amazingly similar problems, much better attention is paid to the mathematical needs of everyone.

My own belief is that there is a genuine conflict here, that we cannot use the same basic curriculum, altered for different levels of ability. "Tracks" are valuable, but run along the ground. This small group of students I am now considering needs to be airborne. Is this socially feasible? I do not know. There is, however, experimental work with such a highly mathematical curriculum going on, for example, the work of Howard Fehr and his group, and work of this sort must be continued.

Finally, I would like to talk about some manpower questions. At the beginning of the "new mathematics" movement, there was a great involvement of university research mathematicians, some of whom switched permanently to work in mathematics education. I think that nothing like this is now occurring. For one thing, we have succeeded in alleviating the frustrations of the university freshman teacher by providing him with students that are often better trained than he knows. Yet none of the developments that I have described can possibly take place unless we have the cooperation of mathematicians at the highest level of research training. We must regain the attention of the research community.

---

[10] For a further discussion of this point, see parts of my note, "Comments on a 'Commentary'", School Science and Mathematics, 1964 pp. 545-549.

A vastly larger manpower problem, so far as numbers are concerned, is that of teachers. My readers will all be aware of the recommendations of the CUPM Teacher Training Panel for 12 hours of special mathematics for the elementary school teacher, and will be aware of how far we are from this modest goal. I myself am ready to say that we will have to switch to mathematics specialists in the early grades in order to get faculty competent enough to teach the curricula that the children can demonstrably handle and demonstrably need. The CUPM recommendations have been much more successful at the secondary level, but we are still far from producing secondary teachers really well trained to begin teaching in such experimental curricula as I have outlined.

A necessary part of the solution to these manpower problems is money. Money is scarce. I do not want to blame Washington, or the Viet Nam war, or other such highly visible targets for this shortage. Society itself has not seen the need. Curriculum development is really not expensive. To use my favorite figure, one that must strike some deep chord of moral revulsion in me, the extension of the Massachusetts Turnpike into downtown Boston cost thirty million dollars a mile. I think that the sum total of all expenditures on curriculum development in mathematics since Sputnik has not come near that.

This paper has presented a bewildering number of tasks. If that is the impression the reader has, then I think my exposition has been successful, because that is exactly what we have. But success in all these tasks is necessary - though not sufficient - for the preservation of our society.

RESEARCH AND EVALUATION IN MATHEMATICS EDUCATION

E. G. Begle
Stanford University

In mathematics education we have been attacking two problems: the problem of teaching better mathematics, and the problem of teaching mathematics better. I submit that we have done very well on the first problem and very poorly on the second. In the last decade or two, as Professor Young pointed out, we have made it possible for children in our schools to learn much better mathematics than we were exposed to when we were in school. While we may not have made as much progress as we had originally hoped for, we certainly have made substantial improvement in the quality of the mathematics in our school programs; and we have substantial and powerful evidence that students can and do learn this better mathematics.

Furthermore, we have learned over the past decade how we can continue to make sure that students are provided with better mathematics. We have learned how to arrange the needed cooperation between classroom teachers and research mathematicians, how to write new text materials, and how to evaluate them. In short, the problem of teaching better mathematics is under control.

The problem of teaching mathematics better is not. Let me list some of the attempts that have been made during the past dozen years. In the late 50's numerous efforts were made to teach by means of movies or television. Next came teaching machines and programmed learning with the promise that these would make it possible for all students to learn, although perhaps at different rates. Team teaching once commanded, and to some extent still does command, considerable attention. The discovery method of teaching has been held out as the answer to our problem of teaching mathematics and other subjects better. More recently, individually prescribed instruction has been proposed, as has been computer assisted instruction. We have been offered modular scheduling and flexible scheduling. Quite recently our attention has been called to mathematics laboratories.

However, the actual state of affairs is that for each of these panaceas either there is very little empirical evidence of any kind, or else there is a great deal of empirical evidence which demonstrates that the new way of teaching is no better, though often no worse, than our old-fashioned ways.

Recently I tried to find out something about the effectiveness of individually prescribed instruction and discovered that empirical information is quite scarce. Also, at the request of the SMSG Advisory Board, I tried to

obtain empirical information about the effectiveness of mathematics laboratories. All I could find was one report of a study done in England about ten years ago, which indicated that a kind of mathematics laboratory in use there at that time was slightly less effective than traditional methods as far as student achievement in mathematics went.

On the other hand, it turns out that there is quite a bit of information about the effectiveness of teaching by television. A large number of studies have been carried out using a variety of subject matter areas and comparing television teaching with standard face-to-face procedures. The net result is a stand-off. For each case where television teaching shows an advantage, there is another case where it is at a disadvantage, and in all cases the differences are small.

The same thing is true for teaching machines and programmed learning. A vast amount of experimentation has been carried out, and a vast number of comparisons between programmed learning and more conventional teaching procedures have been made. The distribution of differences in such comparisons seems to have a mean of 0.

The same is true for discovery teaching. A vast number of comparisons with more conventional teaching procedures have been made and again the distribution of differences seems to have a mean of 0.

To sum up, our attempts to teach mathematics better have either failed to demonstrate any improvement or have failed as yet to provide any evidence one way or the other. The problem of teaching mathematics better is <u>not</u> under control.

In reviewing this list I was reminded of two books I read shortly after the end of World War II. One was a report on the development of the atomic bomb; the other included a report on the development of the proximity fuse. This latter was a small radio set which could measure the distance between a anti-aircraft shell and an enemy aircraft and explode the shell when the distance became small enough. This radio set had to be strong enough to withstand the shock when the shell was fired out of the anti-aircraft gun.

The developers built a radio set of the proper size, and then threw it out the window onto the pavement of the next door parking lot. They then retrieved the set, opened it up, and looked to see what had broken. The broken pieces were replaced with stronger ones, and the set was then tossed out of the second-story window, and so on until they finally had a set all parts of which were strong enough.

I have the impression that in education we have been trying to construct an atomic bomb rather than a proximity fuse, and that we might very well have been better off right now if we had tried to make small step-by-step improvements rather than spending all our time looking for major break-throughs.

If we <u>are</u> to undertake a research program with these more modest aims, then the question arises as to where we start. Before trying to provide any kind of answer to this question, let me turn to another matter. I provided in advance of the Conference a copy to each of the participants of a paper* which I prepared a year ago. I did this in order to call to the attention of the participants two very basic laws about mathematics education, and probably education in general.

The first of these states:

> The validity of an idea about mathematics education and the plausibility of that idea are uncorrelated.

The subject of teacher effectiveness provides a number of confirming examples. Many of our most plausible ideas about teachers have turned out, on the basis of empirical evidence, to be wrong. I can now add a footnote to the discussion of this in the above-mentioned paper. In reviewing our data we discovered that a number of the fourth-grade teachers involved in the SMSG longitudinal study were in the Study the following year teaching fifth grade. Similarly, a number of teachers at the seventh-grade level in the first year of the Study were again teaching at the eighth-grade level in the second year. For these teachers we computed effectiveness scores for the second year of the Study and then calculated the correlations between first-year effectiveness and second-year effectiveness. These correlations were not very high. Teachers who are effective one year may be less effective the next year. What this implies for teacher training I am not yet prepared to say.

I can supply a few more illustrations of this law. When I first became a member of the faculty at Yale University many years ago, I remember being very impressed with the Dean of Yale College, because he knew perfectly well that it was all right to teach English literature, or political science, or freshman chemistry, or practically anything except mathematics, by means of large lectures. When it came to mathematics, however, teaching had to be done in small discussion groups. I was very pleased that this obvious fact was so clearly understood by the Dean.

---

*<u>The Role of Research in the Improvement of Mathematics Education</u>. Educational Studies in Mathematics, Vol. 2 (1969) pp. 232-244.

Only recently did I come across evidence indicating that both the Dean and I were wrong.

It turns out that there have been a large number of studies, at the college level, comparing different procedures which ranged from large lectures through discussion classes to independent study. When these studies are examined together, it becomes clear that no one procedure has any advantage over any other, and in particular that small discussion classes are no more effective than large lectures on the one hand, or individual study on the other. The plausible (in fact, the obvious) just was not true.

The same organization which carried out this compilation of studies on class size was also the one which compared TV with face-to-face teaching, as mentioned above, in which no significant advantage for either procedure could be found. However it was pointed out that, unlike face-to-face teaching, TV teaching provided no possibility for student feedback or questioning. Consequently some studies were carried out in which students had access to a microphone and could question the lecturer. This plausible suggestion, however, turned out to be a mistake. Feedback and questions from the students resulted in significantly less student achievement than without.

My second law about mathematics education reads as follows:

> Mathematics education is much more complicated than you expected even though you expected it to be more complicated than you expected.

Professor Higgin's study of the effects on student attitudes of a junior high school science-mathematics unit is a good illustration of this law. Another illustration is provided by an analysis of the effects in grades four, five, and six of certain conventional and certain modern textbook series. Over a three-year period students were administered a large number of different mathematics tests. The patterns of achievement on these various tests within the three modern textbook groups were very complicated, as was also the case for the three conventional textbook groups. Finally, contrasts between the modern textbook groups and the conventional textbook groups were equally complicated. The simplistic answers which many of us had originally expected to obtain from this study did not appear.

I try to keep these two laws in mind when I am planning research in mathematics education. In the first place, I do not choose the most plausible alternative before me and invest all of my time and resources in it. I have no expectation of developing a major breakthrough.

No matter what I do decide to investigate, I expect the results to be quite complicated and therefore I am not satisfied with simplistic measures. I prefer to measure the values of many different independent and dependent variables.

One source of problems worth empirical investigation is suggested by the question of objectives of mathematics education. A large number of objectives, each specific enough to be measured, have been suggested at one time or another. These seem to fall into three classes. First, a topic may be recommended for inclusion in the mathematics curriculum because it is intrinsically valuable. For example, these days it is often stated that every well educated citizen should have some understanding of what a computer can do and what it cannot do. A statement of this kind is, of course, a value judgment and if there are differences of opinion about such a statement, there are no rational ways of adjudicating these differences.

However, there are not too many objectives in this class--most are in one or the other of the following two classes. The second class consists of statements of the form: this topic belongs in the curriculum because, when mastered, it permits students to solve that particular class of problems. For example, many topics in arithmetic are in the curriculum because we feel that practically every adult will often have to use these topics in solving everyday problems.

A third class of objectives are of the form: this topic should be in the curriculum because it is a prerequisite for another topic. An example would be the statement that the concept of a derivative is a prerequisite for the concept of marginal cost.

Most objectives for mathematics education belong to the second or the third class. Now it is important to note that an objective in either of these two classes can be tested empirically. If it is claimed that a particular arithmetic topic should be in the curriculum because it provides an essential tool for solving a certain class of problems, then by testing suitable children in suitable numbers, both on the arithmetic topic and on the problem, the actual relationship can be ascertained. Similarly, teaching "marginal cost" to students, some of whom understand the notion of a derivative and others not, will tell us if the derivative is indeed a prerequisite for the understanding of marginal cost.

During our curriculum development work over the past decade we have built many things into the curriculum because we felt intuitively that they were useful, without checking in advance to see whether these objectives could be empirically substantiated.

Let me cite two examples: one rather global and one much more narrow. It seems to have been an article of faith for SMSG, from its very beginning, that stressing understanding of mathematical ideas over rote learning of mathematical techniques led to easier learning, greater retention, and greater facility in problem solving. A decade ago there was only a modicum of evidence in favor of this point of view, and many of us were unaware of even that.

Today there is a considerable body of evidence, some of it resulting from the SMSG longitudinal study, but much from other smaller studies, indicating that our faith was well placed, with respect to this particular aspect of the curriculum.

But now let us look at a much smaller piece of the curriculum. We felt that elementary school students should understand why the standard algorithm for long multiplication works before they were drilled on the use of this algorithm. Since the algorithm depends very much on our decimal place value system, we felt that better understanding would be reached if they saw an example of a different numeration system. Base k $(k \neq 10)$ was the standard suggestion, and was implemented not only in the SMSG program but in many of the others developed during the 60's.

Here the empirical evidence indicates that our intuition was not very good. It now seems quite clear that the study of non-decimal numeration systems does not contribute nearly as much to the understanding of arithmetic algorithms as we had originally hoped.

Another set of potentially fruitful research projects is suggested by the fact that there are various ways of providing the first introduction to a new mathematical concept, just as there are various forms of many mathematical algorithms.

Most of us have very strong feelings on these. Most of us, I imagine, would feel that physical manipulation of concrete objects would be a most effective way of introducing mathematical concepts to elementary school children. There is, however, at least one study which indicates that passive observation of the teacher's manipulation of objects is equally effective.

A much more important, but also much more difficult, area for research is aimed at the development of a theory of the learning of mathematics. Our colleagues in psychology have nothing to offer us. Mathematics educators have put forth a few suggestions, but these have been based on very scanty evidence and possess very little empirical justification. Until we possess a theory of mathematics learning that has some validity, it is difficult to ascertain in which direction we should be aiming our research efforts.

In much of the research which has been done to date the effects of two different teaching procedures are compared. Almost invariably, these procedures differ on a number of variables. Consequently, if the two procedures have different effects, it is not possible in general to separate out those variables which were responsible. For this reason, we are working at SMSG Headquarters to prepare some teaching units which are so clearly structured that it will be possible to manipulate one variable at a time. With these as tools, it should be possible to get a much better understanding of the variables that are important for the learning of mathematics.

A still more important, and still more difficult, area for research is that of problem solving. We know even less about problem solving than we do about mathematics learning; and I, myself, would not know where to start, in any research effort in this area.

Finally, let me say a few words about evaluation of mathematics programs and the uses to which it can be put in the decade we have just entered. Evaluation, of course, ties in closely with research in education because they both use the same measuring instruments, namely, tests. On this we are much better off than we were a decade ago. We have available a much larger array of tests, and we know what each one is good for. Consequently, we are now equipped to undertake much more searching evaluations of mathematics programs than we were in the past; and whether we like it or not, evaluation is becoming a very important part of our educational scene.

One example of this is the National Assessment of Education. A booklet has been prepared describing the procedures followed in constructing the test items for the assessment of mathematical knowledge. The mathematical community had very little say in the construction of these tests. However, they happen to be fairly good, which is fortunate, because some important educational decisions may be made for us as a result of this assessment.

Another practice which involves evaluation is not yet very widespread, but it is under consideration in many school districts. This is the practice of contracting with private industry to teach certain subjects to certain of the students in the district. Often the payments are based on student achievement as measured by certain specified standard tests. An instance which has received considerable publicity recently is that of the Texarkana school system which contracted with a private agency to have the agency teach reading to certain students in the Texarkana School District. Unfortunately it was recently discovered that some of the teachers working for the agency were teaching the tests to the students. This is an example of one of the problems of evaluation.

Texarkana is only one of many such cases. As far as I can find out, little attention has been paid in these cases to the quality of the evaluation instruments, and most of them seem to me to be not very imaginative. We would be doing a great service if we could educate school boards to the fact that more powerful and more useful tests have been developed recently.

Another notion has been put forward recently which also involves evaluation. This is the notion of accountability. Schools, and even individual teachers, are being held accountable for the progress of their students. An interesting example is a recent action by the District of Columbia School Board. A recommendation was made last year to that Board that this year (1970-71) be very heavily devoted to increasing the reading ability of the students. Along with this recommendation was another one to the effect that teachers in the District of Columbia school system be paid on the basis of the gains in reading scores made by their students. This would, of course, mean that there would no longer be a uniform salary schedule.

The teachers, of course, were most unhappy with being held accountable in this fashion, but the School Board nevertheless accepted the recommendation. Whether this is just an isolated case, or whether it is the beginning of a trend toward accountability, it is too early to tell. If the latter, however, then evaluation will play a still larger role in the near future.

To summarize very briefly, there is more than enough research on mathematics education that ought to be carried out during the 70's than our resources and available manpower will be able to handle. It seems likely that evaluation will play a more prominent role during the 70's than it has in the past. We will need to not only continue the production of more refined measuring instruments, but also to educate school administrators, school boards, and concerned parents to the existence of more useful tests and more penetrating evaluation procedures.

FEDERAL SUPPORT OF SCHOOL MATHEMATICS IN THE 70'S

John M. Mays
Office of Science and Technology
Executive Office of the President

I shall start with the President's March 3 Message on Education which is significantly entitled Message on Education Reform and whose opening sentences are:

> "American education is in urgent need of reform.
>
> "A nation justly proud of the dedicated efforts of its millions of teachers and educators must join them in a searching re-examination of our entire approach to learning."

A little later, perhaps anticipating this conference, he goes on to say:

> "... the decade of the 1970s calls for thoughtful redirection to improve our ability to make up for environmental deficiencies among the poor; for long-range provisions for financial support of schools; for more efficient use of the dollars spent on education; for structural reforms to accommodate new discoveries; and for the enhancement of learning before and beyond the school."

and then makes, among others, the following proposal:

> "... I propose that the Congress create a National Institute of Education as a focus for educational research and experimentation in the United States. When fully developed, the Institute would be an important element in the nation's educational system, overseeing the annual expenditure of as much as a quarter of a billion dollars."

Having established the tone of the present Administration's concern for educational reform I should like to discuss for a while pertinent programs of the National Science Foundation, the Office of Education, and the Office of Economic Opportunity. I shall then return to a more detailed discussion of The National Institute of Education.

National Science Foundation

The National Science Foundation has, of course, been the major Federal source of support of new approaches to school mathematics. Major projects and the amount of support they have received through FY 1970 are:

School Mathematics Study Group (SMSG) E. G. Begle, Stanford Univ., Director Complete curricula for grades K-12 with alternative courses; teacher training materials; longitudinal study of mathematics achievement.	$14.2 million
Minnesota School Mathematics and Science Teaching Project (MINNEMAST) James H. Werntz, Jr., Director This project has produced close to 30 coordinated math-science units for grades 1-3.	5.0
University of Illinois Committee on School Mathematics (UICSM) Max Beberman, Director Emphasis in recent years has been on the underachiever and in the last year or two has been exploring the British integrated day approach to mathematics and science in elementary school.	4.4
Computer-Based Mathematics Education Project Patrick Suppes, Stanford Univ., Director Included are drill and practice for grades 1-6 and a "tutorial" curriculum in logic and algebra for bright students in grades 4-8.	2.4
University of Illinois Arithmetic Project David A. Page, Education Development Center, Director Films and written materials for elementary school teachers.	1.5
Madison Mathematics Project Robert B. Davis, Webster College and Syracuse Univ., Director Davis has a particular interest in influencing what goes on in the classroom.	1.0
	28.5

Other interesting projects now in progress include:

Development, under the direction of Howard Fehr at Columbia Teachers College, of a grade 7-12 curriculum for the college-bound which eliminates the traditional separation into arithmetic, algebra, geometry, and analysis, organizing instead about the concepts of set, relation, mapping, and operation and structures - group ring, field, and vector space. This project was supported in its earlier stages by the Office of Education.

Development, under the direction of Earle Lomon at Education Development Center, of "problem" pamphlets which involve the student in learning both mathematics and science in a somewhat self-directed fashion. Accompanying will be a comprehensive teachers manual - a compendium of information, source materials, techniques, and data designed to help the teacher handle this more difficult open-ended type of activity. This project grew out of the Cambridge Conference on the Correlation of Science and Mathematics in the Schools held in August 1967.

Development of instructional models, under the direction of Glenadine Gibb at the Southwest Educational Development Laboratory in Austin, designed to improve teaching of mathematics to minority groups including Mexican and Afro-Americans.

The themes discernible here are, in addition to continuing concern with mathematics for the college-bound:

mathematics for the disadvantaged or minority student
integration of mathematics and science, often in an open-ended style
encouraging student pursuit of special interests
use of computers in education.

Support of course content improvement in mathematics in FY 69 and 70 has been in the neighborhood of $1.5 million a year and is expected to be about the same in the present FY 71. Levels for future years are unpredictable but NSF remains open to proposals for imaginative projects in school mathematics. Areas of particular emphasis at this time are integration of mathematics and science and research into the learning process as it relates to mathematics.

NSF support for teacher training in mathematics amounted to roughly $15 million in FY 1970 in the following categories:

Summer Institutes	$8.3 million
Academic Year Institutes	3.7
In-Service Institutes	1.9
Cooperative College-School Prog.	1.3

A separate program aimed at developing model preservice training programs at individual institutions was started last year and has $2 million available this year. Mathematics is involved in about half of the projects.

Office of Education

The U.S. Office of Education (USOE) supports efforts directed at improvement of school mathematics through several programs. Those supported under the program of Regional Educational Laboratories and Research and Development Centers are perhaps particularly germane to the subject of this conference.

Probably the best known of these is the Individually Prescribed Instruction project developed at the Learning Research and Development Center at the University of Pittsburgh and taken up for field testing by Research for Better Schools, the Regional Laboratory in Philadelphia. The project has produced a complete mathematics curriculum for grades K-6 designed to allow students to proceed at their own pace. Detailed specifications for what is to be learned were prepared in the form of so-called behavior objectives and instructional materials including texts, filmstrips, records, audiotapes and manipulative devices were prepared to achieve these objectives. Through placement tests and pre- and post-tests associated with each unit teachers prescribe next steps in the instruction. Through FY 70 the Pittsburgh Center has spent $1.4 million in the development of this mathematics program and the Philadelphia Laboratory has spent $2.35 million in the field testing and disseminating of the program.

Whereas the creators of IPI have not been concerned with innovation in curriculum content, the Central Midwest Regional Educational Laboratory in St. Louis is engaged in development of an individualized K-12 curriculum in the spirit of the Cambridge Conference. While the management scheme resembles that of IPI, the approach is not on the behavioral objectives model. A number of university mathematicians are associated with the project. As with IPI, the materials have been found to work well with classes of inner city children. Support has totaled $2.5 million through FY 1970.

The Southwest Educational Development Laboratory in Austin has been working on improving mathematics instruction for black, Mexican-American, and migrant students. At this time they are working on a modification of IPI style which will make use of contemporary content and give greater emphasis to development of concepts through activities similar to those being developed at the Central Midwest Laboratory (CEMREL). They are also exploring the usefulness of a bilingual approach to mathematics for Spanish speaking students. Funds expended to date (including NSF contributions) total $700,000.

The Wisconsin R and D Center for Cognitive Learning has completed development, at a cost of roughly $800,000, of Patterns in Arithmetic, a course in mathematics for grades 1-6 involving 336 15 minute videotapes and associated student workbooks and teacher's manuals. It is now working on a K-6 sequence providing for small group work in arithmetic, geometry, algebra, and probability and statistics, with provision for students who learn well from the printed work after beginning with concrete manipulative materials and those who profit by continuing in the concrete style. Expenditures to date for this and some exploratory work in computer management of instruction are $530,000.

In addition to these and some other smaller projects relating to mathematics in the Regional Laboratories and R and D Centers, roughly $200,000 a year has been devoted to individual projects in mathematics. As will be seen later, present plans are that the research activities of the Office of Education will be transferred to the National Institute of Education.

The OE Bureau of Education Professions Development deals with mathematics education as part of its focus on areas such as education of the disadvantaged. Roughly $800,000 was devoted to mathematics in FY 70.

Early Childhood Education

Though this conference is explicitly concerned with school mathematics, it may wish to give some consideration to early childhood education. Experimental programs in this area, which often spill over into the school years, include in the Office of Education the National Laboratory on Early Childhood Education, with six university centers and Head Start Follow Through which is systematically studying a number of approaches to early education for disadvantaged children. Elsewhere in HEW the new Office of Child Development, headed by Dr. Edward Zigler on leave from Yale where he is professor of psychology, manages the Head Start program which includes an experimental component and has a modest research program which is expected to grow.

Office of Economic Opportunity

The Office of Economic Opportunity, in a search for better means of helping break the cycle of poverty through education, have embarked on two experimental programs - one in so-called performance contracting; the other in use of education vouchers.

The performance contracting experiment will focus on educational results rather than the means for achieving them. The experiment will attempt to determine whether it is possible, in the words of OEO, to:

hold accountable those providing the instruction--individual teachers, a teachers' union, or private firms--for the success of their students in such basic areas as reading and mathematics.

guarantee poor children the same results from classroom effort that now is achieved by students from nonpoor homes; i.e., equality of results.

Contracts have been let to 18 school districts, which in turn have subcontracted with six private firms to provide instruction in their schools. In these cases, the private firms are not being reimbursed, even for costs, if the students they instruct do not improve by at least one grade level in reading and mathematics. The firms will not begin to make a profit until their students improve by 1.6 grade levels--three to four times the improvement now achieved in the average classroom of poor children. Students were carefully selected to avoid "creaming;" the vast majority are at least two grade levels below norm.

The second experiment is in an earlier stage of development as is indicated by the title of an OEO pamphlet dated this month: A Proposed Experiment in Education Vouchers. Following are quotations from that pamphelt:

"For many years it has been argued that the quality and relevance of education would be improved if parents and students were given greater choice in the selection of the type of education they received. It is argued that the necessity of providing common education in neighborhood schools, combined with the monolithic decisionmaking structure inherent in any large school system, often results in education that cannot be responsive to the needs of many citizens of diverse backgrounds and interests. One means of developing a more responsive educational system is a vouchers plan, which gives the consumer (the parent) control over the education marketplace.

"Several types of vouchers systems are being considered; for example, vouchers could be provided to particular segments of the community, such as the poor, or vouchers could be provided for limited services, such as compensatory or special education. The most fully developed concept, would provide vouchers to the entire community for all education services. While the Office of Economic Opportunity is not an advocate for adoption of any type of vouchers system, it does believe the concept merits consideration.

"Such a program envisions providing funds equivalent to the current expenditures in the public school system to each child to be expended in a school of his choice. Such a plan, of course, requires many safeguards and regulations. It is quite probable that an unregulated system would worsen

the quality of education available to the vast majority of our citizens. Much of the analysis and planning to date has been devoted to considering regulations that shall be applied.

"Proponents of the vouchers system believe that these benefits can be expected from its implementation:

(1) Individuals would have a greater freedom within the public education system because they would not be required to accept standardized programs offered in assigned public schools. Middle income and poor parents will have the same freedom to choose schools that wealthy parents can exercise by moving to an area where the public schools appeal to them or by enrolling their children in private schools.

(2) Parents would be able to assume a more significant role in shaping their child's education, thus renewing the family's role in education and resulting in the concomitant desirable impact upon attitudes of both the parent and child.

(3) A range of choices in schools would become available. Small new schools of all types will come into operation--Montessorri, Summerhill, open classroom, and traditional style schools.

(4) School administrators and teachers could arrange their curriculum to appeal to a particular group or to reflect a particular school of thought on educational methods. Schools could emphasize music, arts, science, discipline, or basic skills. Parents not pleased with the emphasis of one school could choose another. Thus, public school administrators and teachers would be freed from the necessity of trying to please everyone in their attendance area, a practice that often results in a policy that really pleases no one.

(5) Resources could be more accurately channeled directly to the target group, the poor, since funds will follow the child holding the voucher."

National Institute of Education

Returning now to the National Institute of Education, I will first quote again from the President's message on Education Reform:

"As the first step toward reform, we need a coherent approach to research and experimentation. Local schools need an objective national body to evaluate new departures in teaching that are being conducted here and abroad and a means of disseminating information about projects that show promise.

"The National Institute of Education would be located in the Department of Health, Education, and Welfare under the Assistant Secretary for Education, with a permanent staff of outstanding scholars from such disciplines as psychology, biology and the social sciences, as well as education.

"While it would conduct basic and applied educational research itself, the National Institute of Education would conduct a major portion of its research by contract with universities, non-profit institutions and other organizations. Ultimately, related research activities of the Office of Education would be transferred to the Institute.

"It would have a National Advisory Council of distinguished scientists, educators and laymen to ensure that educational research in the institute achieves a high level of sophistication, rigor and efficiency.

"It would develop criteria and measures for enabling localities to assess educational achievement and for evaluating particular educational programs, and would provide technical assistance to State and local agencies seeking to evaluate their own programs.

"It would also link the educational research and experimentation of other Federal agencies - the Office of Economic Opportunity, the Department of Labor, the Department of Defense, the National Science Foundation and others - to the attainment of particular national educational goals."

He then goes on to mention a "few of the areas the National Institute of Education might explore".

> "Compensatory Education. The most glaring shortcoming in American education today continues to be the lag in essential learning skills in large numbers of children of poor families...
>
> The first order of business of the National Institute of Education would be to determine what is needed - inside and outside of school - to make our compensatory education effort successful."
>
> Reading - development of new curricula, wider and better application of what we know and additional research.
>
> Television and other technology - to stimulate the desire to learn and to help in teaching.
>
> Experimental schools trying out new ways of organizing education.
>
> New measures of educational achievement - to achieve "fundamental" reform it will be necessary to develop broader and more sensitive measurements of learning than we now have.

> The National Institute of Education would take the lead in developing these new measurements of educational output. In doing so it should pay as much heed to what are called the "immeasurables" of schooling (largely because no one has yet learned to measure them) such as responsibility, wit and humanity as it does to verbal and mathematical achievement."

It is expected that Congressional hearings on the bills* providing for establishment of the Institute will begin when Congress reconvenes. During the last few months there has been in progress a planning effort, under the direction of Dr. Roger Levien of the RAND Corp, exploring in more detail questions relating to the objectives, program, and organization of the NIE. A number of working meetings have been held involving persons from education, the natural and social sciences including mathematics, and the arts and humanities.

In these meetings there has been rather general agreement on several ideas:

1. The basic objective should be improving the education of Americans through R and D - with all the implications this has for delivery as well as discovery.
2. The need for concentrating considerable resources at any time on a relatively small number of important and promising problems or opportunities. Some have emphasized problems close to the firing line such as improving education of the disadvantaged, increasing the effectiveness of use of educational resources, and improving the quality of education in specific ways. Others have emphasized more basic problems such as language acquisition, social interactions and learning, and the biology of learning. Most have agreed that both sorts of work should be supported.
3. The need for a broad range of activities from individual basic research projects through major field-sited model development and special concern for making it easy to adopt developmental products.
4. The inadequacy of present models for educational R and D and of present means of bringing together the wide range of talents - school people, social and natural scientists including educational researchers, engineers, artists, humanists, and persons from other professions - that can contribute to the planning and execution of educational R and D.

*S. 3531 H. R. 16262

5. The importance of special attention to educational measurement, as called for in the President's message.

6. The advisability of concentrating first on the extramural program, with a slower buildup of the intramural program. Initial efforts for the intramural program might be concentrated on:

   (a) clarifying and defining the problems and opportunities in educational R and D in continuing collaboration with first rate people throughout the country.

   (b) evaluation and measurement including both development of new measures and actual evaluation of results of large scale Federal education programs.

A possible organization of the extramural program management staff would be on a matrix model. One assistant director would have responsibility and funds for attacks on problems such as education of the disadvantaged while another would be responsible for support of more discipline-oriented or basic projects. Individual NIE extramural staff members would work in both areas, thus fostering interconnections between more basic work and attacks on pressing educational problems. A third assistant director might head an analysis and planning unit, with a considerable rotating complement of first rate people, which would help in the definition of problems and planning of attacks on them involving the whole range of disciplines.

The Experimental Schools Program, which will be an important element in the NIE has already been authorized by Congress. A Federal program of experimental schools has been proposed by several Presidential advisory groups in the last decade and is only coming into being now at a time when a remarkable number of school districts are responding to dissatisfaction with conventional schooling and trying out new forms of education, almost always with inadequate funds for development and evaluation. The basic purpose of the program is to go well beyond experimentation with this or that aspect of conventional schools into experimentation with fundamentally new ways of organizing education along lines that informed intuition and earlier small scale trial suggest are likely to be fruitful. The resources in each case would be large enough so that the idea can be properly tested and shown conclusively to work or not under carefully specified conditions, and if successful to be put in shape for wider adoption. The hope is thereby to get off the treadmill that has affected fundamental experimentation with educational organization in the past. Numerous experiments in school organization have been tried in American communities, but little cumulative advance has been achieved because no one

knows for sure what happened. The experimental schools would not be like some laboratory schools where, typically, classes of children from the university community are available for small scale experiments. Rather, the student body would have a composition like that of an ordinary school, appropriate to the community in which it was imbedded, and indeed would normally be a part of the local school system. The experiment would be a whole-school (or whole-school-system) experiment, though there might be some exploration of alternative versions of the basic scheme from class to class. The possibility that the scheme being tested might require considerable modification or that it might prove unworkable would be understood from the start. Though the Hawthorne effect should insure that students would not suffer relative to students in ordinary schools, adequate precautions would be taken in any case. Schools that produced obvious and significant improvement over ordinary schools would be studied further to determine, as far as possible, the conditions that were critical to the improvement, so that we would both have firm data on what can be accomplished by specified means and also be able to make the model available, stripped to essentials, to other school systems with assurance that it will work. It is envisioned that the work carried out by a mixed group including school people, educational researchers, social and behavioral scientists, and persons from science, the arts, and the humanities with advice from parents, business and industry - not to mention the students themselves. I have already mentioned three experiments along these lines being supported under other Federal programs - performance contracting and the voucher system under the Office of Economic Opportunity and Individually Prescribed Instruction at the Pittsburgh Learning R and D Center and the regional Laboratory in Philadelphia supported by the Office of Education. Candidates for development and study in the new program might include:

> The British infant school model characterized by a rich variety of equipment, specimens, books and other objects in the classroom and considerable freedom for the student to determine not only his schedule but to some extent the nature of his studies. As I hardly need tell you this model is attracting great interest in this country.
>
> A school of the sort proposed by Ralph Tyler* making much greater use of "work and other areas of life as a laboratory in which youths

---

*R. W. Tyler in Agenda for the Nation, Kermit Gordon, editor, The Brookings Institution, Washington, 1968, pp. 207-236.

find real problems and difficulties that require learning and in which they can use and sharpen what they are learning".

A school where parents or students or members of the community or all of them actively participate in planning the school and in all its activities.

A school putting prime emphasis on linguistic development in the early grades.

A school combining features of the British infant school and the highly structured individually prescribed instruction.

At this point I should like to make some comments and raise some questions about school mathematics as it relates to the Federal programs I have described. I am pleased that almost every point I wish to make has already been touched on by the previous speakers, first because I feel the appropriate role for Federal officials in educational R and D is principally to see that the important questions are being addressed rather than to give answers to those questions and second because I believe the existence of rather general agreement on what needs to be done means that things are likely to move. I shall give my comments in the form of questions.

1. Are existing mathematics curricula well designed to meet the range of likely mathematical needs of adults with various occupations and interests and do they take account of what is known about the residue, in adults, of earlier study of mathematics?

   It has been my experience that mathematics curriculum development projects involving mathematics come rather rapidly to agreement on the general goals of school mathematics and that the basic theme, implicitly or explicitly, is likely to be, as Gail Young has pointed out, preparation for calculus. What I am suggesting is that it would be useful if mathematicians and mathematics educators gave more careful attention to an analysis of the mathematical needs of various classes of persons in the light of information on what is retained by adults. I am not suggesting some simplistic basing of curricula designed to prepare for the future on minimal needs as measured now but rather the desirability of having as background knowledge for such development a much better idea of what these needs are and the extent to which mathematics once studied is actually retained and used where it would be useful. I would like to see mathematicians applying to their consideration of the proper goals of school mathematics

standards of rigor and evidence closer to those they would apply in their regular professional work. As we move toward greater opportunity for diversity and more control by the individual student and his parents over his educational program, there is going to be greater need and demand for more precise answers than we are able to give now to questions of what is essential for various objectives the individual may have, and what is necessary in order to preserve various options, and what can be left to personal taste. It would be nice to be able to give better answers to these questions than general statements about the obvious need for mathematics in our technological society and the beauty of the great edifice of the calculus.

2. To what extent can the effectiveness of mathematics curricula be improved by greater attention to individual differences in students?

   I would have to say that I believe SMSG has given little attention to this problem beyond the provision of some alternative texts for "slow learners". In contrast, the Individually Prescribed Instruction project and others aiming at mastery by all students of a set of "behavioral objectives" have generally derived their objectives from very conventional mathematics curricula. Surely there is a need for more of the synthesis of these concerns as, for example, in the project, described earlier, at the Regional Laboratory in St. Louis.

3. Are paper, pencil, and blackboard methods of teaching mathematics ultimately as effective in leading students to <u>use</u> mathematics as are other methods such as those involving projects requiring the use of mathematics?

   Presumably a major objective of school mathematics is to lead students to use mathematics in their everyday lives and their work. Yet everyone has observed that students often fail to make the connection between the mathematics they have learned in the classroom and situations where that mathematics would be useful to them. Almost all mathematics achievement tests measure the ability of the student to perform some mathematics under circumstances where it is made clear to him that mathematics is what is called for. The achievement thus measured is, of course, a valuable preparation for future mathematics courses and ultimately for careers explicitly requiring mathematics. For many students, perhaps for most, however, one has an uneasy feeling that 9 - 12 years may have been spent in teaching mathematics much of which will never be used even in situations where

it would be very useful. This suggests the need for both new ways of teaching mathematics so that it will become a part of the student's way of thinking and also ways of measuring whether this aim has been accomplished. Many people, I among them, believe that instruction that includes projects in which mathematics must be used is more likely to produce users of mathematics than is paper and pencil instruction alone. More experimentation along this line seems very worthwhile. Rather than demanding that this produce greater achievement on standard tests, I would be willing to settle for equal or even somewhat poorer performance there, if it could be demonstrated that the aim of producing better users had been achieved.

4. To what extent can mathematical ideas and activities be used to engage the interest of inner city children?

   I have in mind here not only finding ways of teaching mathematics to disadvantaged children but also of using mathematical acitivites, perhaps for example, in the form of games, as a way of interesting such children in school and in learning. There are many reports by individual teachers of success in engaging students deeply in mathematical activities, and several projects are providing a variety of manipulative materials, but it might be worth concentrating some resources on systematic search for mathematically significant activities that are particularly attractive to children who are not normally enthusiastic students.

5. Can instructional material in mathematics be designed to contribute significantly to improving skill in reading?

   This question is prompted by the tendency of many projects aimed at improving reading to treat reading as a mechanical skill unrelated to the content of what is to be read or to the uses to which what is being read might be put. This approach passes up a source of motivation for reading and opportunities for sharpening skill in reading. An obvious remedy is to integrate instruction in reading with instruction in other areas. In mathematics this might take such forms as making reading a part of some pleasurable mathematical game or systematic development of precision in reading and verbal expression through relating written descriptions to numerical and geometrical situations and concepts.

I would like now to summarize very briefly what I see as the main outlines of Federal programs as they relate to school mathematics.

First, the likely creation of a National Institute of Education with a broad charter for research, development, and dissemination of results in education, drawing very broadly from the various intellectual communities and seeking to create new syntheses of their contributions. The Institute would be concerned both with education outside regular schools and with exploration of fundamentally new ways of organizing schools and other educational institutions. Though it would ultimately take over much of the present responsibility of the present OE National Center for Educational R and D it would not be developed as an expanded version of that organization.

Second, continuing support of educational innovation in the sciences and mathematics by the National Science Foundation, with increased interest in associated research on learning.

Third, support by the Office of Economic Opportunity of experimentation with new ways of arranging the relationship between producers and consumers of educational services, with mathematics education an important element.

Finally, increasing interest in and support of early childhood education by the Office of Child Development and the National Institute of Education.

There is clearly an important role to be played by mathematicians and mathematics educators in all these programs. I hope they will increase their involvement and will come into these enterprises with an open mind and willingness to explore new ways of combining their talents with those of persons from other disciplines for the good not only of mathematics education but of American education generally.

GOALS AND OBJECTIVES

Karl Kalman

I have attached some enclosures from the discussions at the Conference, enclosures A through E, and I have underlined significant passages.

Selected from these and from the discussions of the Ad Hoc Committee are the following:

1. Goals and objectives ought to be more than behavioral objectives* which are concerned with "measurable objectives." Goals such as problem solving, appreciation of mathematics, and recognition of the significance of mathematics in society are important. Some objectives may not, at first, seem measurable but may turn out to be measurable in time.

2. Some textbooks for grades K to 8 are written in terms of behavioral objectives.

3. Statement of goals and objectives ought to be broad--general enough to be concerned with <u>all</u> pupils of our school society regardless of abilities, background, race, inner-city or not, etc., but specific enough to be useful. Also, it should be couched in language understood by laymen.

4. The scope of the objectives ought to be the 3 year old nursery through grade 14 and recognize the needs of college-bound, employment-bound, technical-oriented, the average "guy", the dropout.

5. Someone suggested a "minimum" core. E.G. Begle reported that some such list will be written for the new junior high school program.

6. It would be useful to include sample test items in a list of behavioral objectives.

7. It is useless to set up goals without the support and assistance of competent teachers at all levels, and representation from administration, principals, and communities--perhaps the students also, if only for political reasons. A contest might be designed in which students compete by submitting papers on "why mathematics?" for a spot on the panel.

---

*During discussions at the conference, it became clear that few of the mathematicians present were familiar with the phrase behavioral objectives." For an explanation of the way in which this phrase is currently used in discussions of educational goals, see the article "Objectives and Instruction" by W. James Popham in <u>Instructional Objectives</u>, AERA manograph series on Curriculum Evaluation, 1969, Rand McNally and Co.

8. Some work has been done in this area and this should be looked at. IPI* has written behavioral objectives and RBS is putting this on tape.

9. Procedures and committees might be set up to consider priorities and, perhaps, a rationale for the inclusion of each objective.

10. It is not entirely clear, nor did the discussion indicate, who should do this work. It appears, though, that a committee should be established under the aegis of a coordinating agency or commission to consider this activity and make steps to provide proper mechanisms.

(A)

The discussion centered on the explanation and implications of behavioral objectives. It was clear that many of the participants were not familiar with the term "behavioral objectives." Some participants found the term offensive. One participant felt that the areas discussed by the research and evaluation group were not areas that money should be spent in. Another participant attempted to define behavioral objectives as behaviors we can measure. He stated that behavioral objectives were an attempt to pin down the outcomes of instruction. Consequently, any desired goal of mathematics instruction that can be broken down in terms of specific behaviors, may be used as a behavioral objective. For example, if one can specify the behaviors involved in appreciating the beauties of mathematics, he can use this as a behavioral objective.

It is very natural for natural scientists to find the term behaviorial objectives an offensive one. The term is often thought of as jargon that schools of education have invented. However, you find that they are talking about the behavior in the sense of the actual performance which you are requiring of the people in the classroom. You reflect upon your own performance in the classroom and you have ambitions of communicating a great deal beyond what you can actually measure. When an instructor gives a test he is doing nothing more than specifying behavioral objectives for his course. Unfortunately, most instructors probably never thought about this in the first place. In fact, they probably taught before deciding on the objectives. They should ask the question, "What do I want the students to learn?" before teaching.

*IPI : Individually Prescribed Instruction. This refers to any program, of which there are now several, in which each student proceeds at his own pace. One of the better known of these is being evaluated and implemented by Research for Better Schools (RBS), Philadelphia, of the Regional Education Laboratories funded by the U.S. Office of Education.

Many texts are written in such a way that the author merely stops writing when he tires of the subject. Authors should decide what information they wish to communicate before writing. So in reality, behavioral objectives are an outline of what you wish to convey to the student. They may include goals which we do not know how to measure. However, they include many objectives that can be measured and one assumes these measures are correlated with things we cannot measure. For example, we have no guarantee that a student who makes 100 percent on a test understands the subject, but we have faith that the test reflects the student's understanding.

It is rather surprising that the mathematical community has had no communication with the wide group of people who have been discussing behavioral objectives for years. Some publishers have written texts for grades K-8 in terms of behavioral objectives. The books are written so that adoption committees can look only at the behavioral objectives and decide whether the scope of the content of the text is acceptable. Actually, further study would be needed to see if the text books can support the claims for the objectives. Under these conditions it is very arrogant of us, and, in fact, a great mistake for us not to look at behavioral objectives. Since students will be taught in terms of behavioral objectives, we should be involved in setting these objectives. Otherwise, non-mathematicians will be setting the goals for mathematics courses. The mathematical community should decide whether or not behavioral objectives are appropriate and, if they are, should be involved in setting these objectives. The mathematical community should try to extend behavioral objectives beyond computational skills.

One can think of many misuses of behavioral objectives. However, the State of California has passed legislation that requires formulation of behavioral objectives. Schools will be judged according to the level to which their students display achievements at the end of instruction. We are concerned about the political side of this development. Also behavioral objectives present a massive assessment problem. However, if the mathematical community ignores these issues, they will not be involved in the crucial decisions that will determine mathematics courses in the future.

(B)

Let me begin with WHAT since this occupied a significant portion of our time and is probably the priority question. What is it that we disseminate? The single suggestion that generated the most heat in our discussions was that a sort of "Bureau of Standards" be established for mathematics curricula. It would be the function of such a body (which would be a collection of experts of some kind) to place a sort of "Good Housekeeping Seal of Approval" on

selected curricula. Those that receive the seal would merit dissemination and implementation. This suggestion led almost immediately to a number of alternative proposals!

First it was suggested that professional organizations of some kind establish a carefully prepared set of minimal objectives which could serve as a bench mark for the evaluation of curricula. Some people felt that a minimal set of objectives was not sufficient. They suggested each developer should, in addition to demonstrating that he meets the minimal objectives, specify very clearly those objectives that his program attempts to meet beyond the minimum, so that the consumer (the buyer of a curriculum) can make choices among curricula on what is included over and above the minimal objectives. It was suggested that these statements of objectives should include more than content objectives; they should also include methodological objectives and objectives in other areas. Another suggestion along the same line was that the professional organizations again establish a systematic review service that would focus on qualitative judgments as well as on information. There have been attempts, I understand, in the past by NCTM in the Mathematics Teacher and by other groups to provide such review services. It was felt that we need to enlarge upon these attempts. We need people who will stick their necks out and make evaluative judgments about respective curricula and textbooks. The NCTM analysis of new programs at one time did this in a more or less informative way without passing very much of a value judgment. They said "These programs have some strong points." Something a little more than this is needed. As an alternative to the "Good Housekeeping Seal of Approval," the suggestion is more in the nature of a consumer research service which would report to the consumer on what a program will do.

There was concern that if the mathematics education community does not formulate objectives, there is a real danger that the consumer will use those of the National Assessment as guidelines for selecting curricula. Several members of the group felt that these guidelines would not be sufficient. It was also suggested that, at the very least, the target audience needs to be made aware of the range of materials that are available and even an objective unevaluative listing of curricula resources would be of value. We do have a beginning in that direction in the ERIC* center for mathematics education which is just getting off the ground; it is precisely this kind of service that ERIC intends to fulfill at some time.

---

*ERIC : Educational Research Information Center. This is an information system sponsored by the U.S. Office of Education.

(C) Publish Objectives.

Job for SMSG. Put together a collection of "behavioral objectives" for math (elementary and secondary--? and JC?) and, parallel to these, a set of thematic or mathematical objectives. The latter are harder to determine, but ought to be rational justification for the inclusion of certain mathematical topics, stated in forms which could be understood by an educated school administrator or curriculum supervisor. They should not be based solely upon utilitarianism. Several examples:

e.g., model and modelling

examples: graphing as device for displaying relationships, flow diagram for displaying relationships, syntax analysis, etc.,

e.g., equivalence classes, relation

instances: stereotypes, nouns, simplification of a complex situation or structure,

e.g., concepts of proof

(hard to do) disproof vs. not proved, proof by authority, etc.,

e.g., probabilistic thinking

or dealing with uncertainty (not techniques but rather the concepts themselves).

(D)

There was some concern, I guess, for who establishes or selects objectives, and who would make some decision as to priorities. But certainly the group in their discussion, considered broader objectives than the skills and concepts that we usually think about. I think that it was very evident that the group was highly concerned about attitude. The idea that learning mathematics should be a joyful experience, and as a result of this we could break down some of the hostilities. And I think that there was a common desire for greater interest to be shown in applications of mathematics and the relation of mathematics to other disciplines, perhaps, even the idea of interdisciplinary approaches.

(E)

There was some discussion about the relation of setting goals and developing behavioral objectives or simply trying to state objectives behaviorally. It was pointed out that behaviorist have probably overstated and over-simplified the matter but that we shouldn't over-react to them but try to improve our ability to state objectives as clearly as possible. Developing process objectives is a difficult matter and too often they are at a low level, however, they do have value in judging the overall quality of a teaching program, a curriculum or a test.

The question was raised who should do this job. There are probably a number of objectives tacitly assumed within groups in the mathematics community, but no organization of this community exists to focus on these objectives.

TEACHER EDUCATION

Joseph Payne

It was clear from all the discussion on teacher education that we should give serious study to the preparation of teachers for all levels in the decade of the 70's. NSF Institutes were of great help in the 60's. Groups such as CUPM and AAAS, both active in the 60's, continue to be active in the 70's. CUPM has revised recommendations aimed primarily at content preparation of mathematics teachers. The AAAS Commission on Science Education has at this time a Preliminary Report on "Guidelines for the Preparation of Secondary School Teachers of Science and Mathematics." In the science area, AAAS has developed guidelines, standards, and recommendations for research and development for "Preservice Science Education of Elementary School Teachers."

Serious study on teacher education will continue to be given by universities, schools, and professional organizations. As a result of the San Francisco Conference some of the problems these groups face were identified and suggestions were given for direction of teacher education.

The education of mathematics teachers should be viewed as a life-long career development. With such a view, it should be possible to analyze the needs of the teacher at various stages and to make available the kind of education needed.

At the undergraduate level, primary focus probably should be given to the tasks faced by a teacher during the first one or two years of teaching. Mathematical content, as now recommended by a variety of organizations, would be a strong component. The study of curriculum materials and methods of teaching would go as far as a beginner could take, deliberately excluding those parts that require considerable experience in teaching for comprehension. The goal is to make a bridge from being a student to being a teacher. Perhaps the bridge is best made through a longer training program, including an internship that gradually gives the prospective teacher greater responsibility. There were strong suggestions that the internship start earlier in college to help young people identify with teaching and to ascertain the strength of their interest in teaching. The additional training time and thought might also make a transition to full-time teaching easier.

If the teacher survives the initial shock of teaching (we don't know how many nor the reasons) he should be eager, willing at least, to increase his sophistication in mathematics and to examine curriculum and methodology more critically.

Most teachers are ready to respond to suggestions about opening up the curriculum and trying new methods after they have gotten some experience under their belts and have mastered the routines of classroom management. Considerable work is needed on the kinds of courses, mathematical and pedagogical, that best suit the needs of teachers at this level. Perhaps special courses such as problem solving à la Polya would be helpful. NSF Summer Institutes and regular summer school and evening classes have provided a great help for many teachers at this stage. Can we find out which of these have been genuinely helpful to the teacher and design a more coherent and more productive program?

The long-time career person needs things that keep him growing intellectually and professionally. Again, institutes and special courses have helped. However, what may be of even greater value is some joint experimental effort involving school, universities, and national projects. We saw the excitement and genuine rejuvination that occurred with the SMSG Experimental Centers when SMSG first started. Can we plan similar experimental work on a continuing basis and try to assess the effect on the teacher and his students?

There are some special problems in teacher training that need careful study:

1. Performance contracting and accountability seem certain to have an effect on the teacher. The teacher will need to know more about learning and how to produce it, how to motivate students to learn, how to teach students to take tests and how to manage students in a classroom setting more effectively. This certainly will influence training programs and also will force more careful study of curriculum, objectives, and assessment.

2. There is a lack of information on selection, success, and durability of the mathematics teachers we train. What happens to the mathematics teachers we graduate? How many of them stay in teaching and for how long? For what tasks that they met were they prepared or not prepared? There is a host of questions on which we need reliable data so that we will know better where we are. Perhaps a wider data collection could be augmented by a several year longitudinal study of students from selected institutions.

3. Differentiated training probably is needed for the wide variety of jobs in mathematics education, e.g., junior high school teacher, senior high school teacher, department chairman, supervisor. Can we give some guidance on the differentiated training? Can we assess whether or not differential training is needed for a given cultural setting, e.g., inner-city-suburban?

4. Overall, we have done poorly in helping teachers see a wide variety of reasons for teaching and learning mathematics. For a great many students, mathematics is not a "now" subject. How can we help teachers assess honestly the values of mathematical study and communicate them to their students? Can this be a more obvious part of instructional materials?

5. Mathematicians, mathematics educators, and school mathematics teachers all have a stake and a responsibility for the preparation of teachers. What are some ways that cooperative effort can be achieved?

The sheer number of elementary teachers and their rapid turnover make teacher training, both pre-service and in-service, overwhelming. Some of the problems related to training elementary school teachers were identified as follows:

1. Do we need special mathematics teachers in elementary schools, particularly upper elementary? Do they produce better results with pupils?

2. Can we design better mathematics courses for elementary teachers, taught in the spirit with which we want elementary school mathematics taught? There are many people who feel that many courses now are too formal, too axiomatic and too unrelated to the content an elementary school teacher teaches.

3. How do we prepare elementary school teachers to operate in the variety of organizations and with the variety of instructional materials one finds in the elementary schools?

With the complex problem of teacher education, there is needed a coordinated effort of many people and several institutions with the goal of developing some model teacher training program for elementary school and secondary school teachers. With impetus coming from an organization like SMSG, there could be a consortium of universities with each working on a different facet of the problem. Planning conferences would be held first. Universities, in

instruments are also needed to measure attitudes, aptitudes, and personality traits related to the learning of mathematics.

If one surveys the field of research in mathematics education, one is likely to be discouraged by the view. Studies are limited in scope and rarely build upon previous work. Results of exploratory studies are not widely disseminated. Most of the research is done by doctoral students; few researchers have published more than one research study; almost no one devotes more than a fraction of his time to research. This situation is particularly depressing in the light of an observation made by E. G. Begle in his paper at Lyon; namely, that the complexity of the phenomena we are studying--the mind of the child, the children in a classroom--demands that we make much more careful observations on larger and more diverse samples of students and teachers than has been customary in the past.

It does seems to be true, however, that the number of mathematics educators trained in empirical research has grown enough in the last decade or so to enable us jointly to attack some selected research problems, if these problems can be identified. At the San Francisco conference, the group charged with making recommendations on research called for an organization to coordinate research activities in mathematics education. The structure and the precise functions of this organization were left unspecified, but it might identify problems needing research, set priorities, contract with groups and individuals to carry out investigations, and perhaps conduct research studies of its own. It might also undertake reviews of the research literature in selected areas, lay down criteria for evaluating research proposals and completed studies, and provide samples of designs that could be used for various kinds of studies.

Although legitimate concerns were expressed that no group should be given exclusive power to control the direction of research in mathematics education, some coordination seems to be essential if future research is to have an impact. At present, collaborative research is almost nonexistent across universities and seldom occurs on a sustained basis between universities and school systems. Too many researchers are isolated; they need the stimulus of like-minded colleagues from other institutions. At the same time, teachers can profit from participation in research studies; they can be challenged to reflect on their teaching and their instructional goals. An organization that would coordinate research could stimulate collaborative studies in a concerted, sustained attack on research problems in mathematics education.

CURRICULUM: A POSITION PAPER BASED ON THE SAN FRANCISCO CONFERENCE

H. O. Pollak

The San Francisco conference seemed to be fairly well agreed that totally new curriculum development is for the most part relatively low in the hierarchy of priorities for mathematics in the 1970's. [There was high priority for curriculum studies with the goal of teaching mathematics better.] Nevertheless, a large number of suggestions related to curriculum matters may be found throughout the conference. This may perhaps be a form of cozy, comfortable conservatism as in many organisms when growing old; it may be a feeling that after all curriculum development is one of the things we know best how to do; there may be some darned important items on the list; it may be the only subject some of us could talk about--but anyway there it is. The list consists of two kinds of items with a separation in degree rather than in kind--curriculum problems in the "old" style of the last ten years, that is, items where we would probably know how to proceed if we decided to undertake them, and problems particularly caused by changes in or new looks at the world around us where we probably do <u>not</u> know how to proceed. Before giving you a list of each of these, there are some other remarks at the conference about the people and aims of curriculum work that should be mentioned.

We observe that the second round of SMSG, a new look at grades 7 to 9, has a structure which is strongly motivated by putting that mathematics first which is most important to the population as a whole. It is thus an example of a curriculum development closely tied to social objectives and one from which therefore a useful set of objectives might be and is being abstracted. This is related to the question of why kids should take math, or particular parts of math, anyway. Is there always an answer to the question of a topic's relevance?

It was remarked that the time allotted to mathematics, particularly in the elementary school, was likely to come under attack, and that we will have to be ready to defend it. It was also remarked that in future curriculum work, social scientists, scientists and perhaps also secondary school dropouts should participate.

Coming now to specific suggestions for curriculum development, let us first mention areas in which the method of attack is probably "traditional."

People were interested in the combination of mathematics, science and social science, especially in the elementary school. USMES* is doing something in this line, but not on a specific problem which has plagued us for years and was mentioned again, and that is the problem of being clear to ourselves what measurement is all about. There was interest in a collection of real problems showing applications of all kinds of school mathematics to all kinds of human activities. These might also be useful in talking to lay people, and in integrating mathematics into the currently popular form of instruction built around contemporary issues. Descriptive courses unfairly but best described as mathematics or computer appreciation courses were again suggested. It was felt that the form and role of a number of controversial or rapidly developing areas in the curriculum were not yet settled. These topics include the computer, geometry, statistics, and logic. The computer in particular has not as yet really come up against the calculus many of whose topics are after all there for computer purposes! Gail Young raised an interesting "traditional" kind of question: If I had a class of 9th graders known to be going into science, what would I teach them? His answer: Group theory, transformation geometry, and other topics in the algebraic methods of mathematical physics. As a final item in the present category, there was reference to the big themes of mathematics, themes that occur also in many areas outside of mathematics, themes around which units might be organized at many grade levels. Examples: the organization and display of information, modeling, equivalence, proof versus non-proof versus disproof, invariance, randomness, extensions of systems with their gains and losses, partial ordering.

In the category of candidates for curriculum development where the possible procedures are more fuzzy, we begin with a major crises in our society, in the inner city schools. Closely related to this (in fact one of our difficulties is that we do not know what the relation is) are the future dropouts, the 25% who will physically be elsewhere, and the x% who will mentally be elsewhere. What do we do for them? Are multiple approaches available to the teacher for the same subject an answer? What is the basic minimum of mathematical knowledge for everybody? What happens after this core? What is the polylithic (as opposed to monolithic) structure that might follow? Is there a smorgasbord of models, and how can we help the student and teacher to set priorities? What about mathematics for everyone at his own pace? What

*The "Unified Science and Mathematics for the Elementary Schools" project is being carried out under the auspices of Educational Development Center, Newton, Massachusetts.

mathematics should be taught in community colleges, technical institutes and junior colleges? If we organize the curriculum in its psychological rather than its logical order--and neither of these is unique--what would it look like?

It is perhaps fitting to close this section with one more question: Why do curricula work? It is clear from the preceding that there are situations which are not felt to be covered, students for whom appropriate materials are not available, potential participants in curriculum thinking who have not yet been tapped. In this last category one important remark needs to be reported: the overall mathematics education effort needs the participation, among many other people, of the research mathematician. He can do many things, but we need to get his attention. Curriculum development is perhaps the best means for this.

Who should do these things? SMSG or other curriculum groups might well handle the first category. If we knew who could do the second they would be half done.

EVALUATION IN MATHEMATICS EDUCATION

Donovan A. Johnson

In the Conference, it was clear that appropriate decisions regarding the accelerating changes currently taking place require evaluation of the appropriateness of mathematics programs, the total achievement of students, the effectiveness of instruction, and the role of instructional material. To evaluate these different factors requires measuring instruments which make comparisons possible, measuring instruments that are precise enough to detect small differences. These measuring instruments must be usable for the students, teachers, materials, or programs involved and in the school situation for which they are designed. Then the measures obtained must be analyzed and evaluated. This evaluation may require standards, norms or the subjective judgement of the evaluators.

The changes in modern society which make evaluation such a critical need today include the following:

1. the criticism of mathematics programs because of the level of computational skill of students as measured by standardized tests.
2. the role of commercial enterprise in contracting for the education of groups of students.
3. the variety of new programs promoting individualized instruction.
4. the increasing role of the computer as a tutor and as an instructional tool.
5. the freedom of choice of students in schools with modular scheduling and mathematics laboratories.
6. the increased role of the computer in record keeping, test administration, and decision making.
7. the increased sophistication of and potential for research in mathematics education to be performed by experts with computer assistance.
8. the innovations in teacher education which need to be evaluated before they are accepted.
9. the variety of instructional materials produced by commercial enterprises.

10. the crises in the classroom, especially in the schools in the inner city.

11. the demand for accountability of an educational program and the instruction.

12. the growing interest in performance contracting.

Evaluation of Achievement in Mathematics

There is a critical need for new instruments for measuring achievement in mathematics. These instruments should be designed to meet the following criteria:

1. Tests should measure the attainment of all types of objectives. If objectives are not available, the first step would be the statement or collection of appropriate objectives. If these objectives are stated as behavioral objectives, the writing of test items is facilitated.

2. In order to measure broad cognitive objectives and objectives in the affective domain, test items should include measures of achievement in the following categories:

understanding of computational algorithms
logic of a proof used in a unique setting
solving of original problems
attitudes such as appreciation, curiosity, loyalty
applications of concepts to new situations
discovery of generalizations
creation of a mathematical system
learning independently

This requires new test situations that are currently not available. These are the type of tests which are desperately needed at this time.

3. The test items should measure different levels of mastery of a given objective.

4. The test items should measure the residue of achievement sometime after instruction has taken place.

5. Some test items should be designed for different settings, for example with a text available, or a laboratory device, or a computational device.

Tests which are constructed should be used experimentally to establish the reliability, validity, and discriminatory power of the test. The tests might collect information from stratified samples to provide bench marks for comparative studies.

Test items and complete tests should be examples which could be used by teachers and publishers. The attached proposal suggests one way to develop new tests. Hopefully, these would be developed in such a way that they will be understood and interpreted properly by the public. When used for accountability of a project, they must be used in terms of the objectives measured.

Evaluation of Mathematics Programs

A mathematics curriculum needs to be evaluated before it is accepted as appropriate for a given school. It would be the purpose of this project to establish guidelines and standards for a mathematics program. In establishing these guidelines the following aspects should be considered:

1. Philosophical: Does this program have acceptable objectives?
Is it designed to meet the needs of society and the needs of students?
Is it relevant in today's world?

2. Psychological: Will it be of interest to students?
Does it have appropriate difficulty level for the students involved?
Does it make provision for individual differences?

3. Mathematical: Is the mathematics correct?
Is the mathematics significant?
Is the sequence appropriate?

4. Pedagogical: Is it teachable by the teachers available?
Is it teachable in the time available?
Are adequate materials available?

5. Evaluative: Is there a means of evaluating students' achievement?
Is there a means of comparing achievement with that of another program?

Evaluation of Instruction

At the present time there is no valid or reliable device for measuring the quality of a teacher's performance in the classroom. The checklists, interaction analyses, or attitude inventories now used are notoriously inadequate. It is obvious that the main criteria of a teacher's performance is the learning of the students. However, what students learn depends on the student's ability, the student's prior educational experience, the environment in which the student lives and other factors, all of which are outside the

control of the teacher. Thus, it does not seem probable that a major effort at this time would be productive in finding a way to measure instructional effectiveness of a teacher. Hence, at this time, it does not seem reasonable for teacher effectiveness to be evaluated by student achievement or current rating devices, although both these methods are frequently used.

Evaluation of Instructional Materials

There are a variety of instructional devices, audio-visual aids, and published material available from commercial companies. The teacher who must select those items which he can use to improve his instruction needs help. The purpose of this project would be to establish guidelines for selecting instructional material. It would provide standards of quality which could be used for decision by curriculum consultants and state departments of education.

To implement these proposals for the evaluation of programs or instructional materials, it might be desirable to establish a "Bureau of Standards" for mathematics education materials. If this were to be done, the Bureau should be an independent organization and not a part of the organization suggested in this proposal.

A TEST DEVELOPMENT PROJECT

Jeremy Kilpatrick

(A Supplement to the Report in Evaluation in Mathematics Education)

One of the themes running through the conference was a concern for behavioral objectives, what they are, and whether, if the mathematics community abdicates responsibility, the task of specifying objectives will be taken up by others. A related theme concerned "performance contracting" projects and the notion that, in the absence of anything better, narrowly-based tests are being used in these projects to evaluate students' performance.

A perusal of the National Assessment Project's "Mathematics Objectives" and a close look at some of the standardized mathematics achievement tests now on the market convince me that a major effort should be made to develop tests to measure some of the things that we consider important, but that are not touched by existing tests. We can waste a lot of time talking about behavioral objectives, but unless we spell out what we mean by devising actual test items, trying them out, and putting them into usable form--complete with norms, etc.,--the test publishers, like the textbook publishers, will not be moved.

The development of tests to go after some of the ideas tossed around at the conference--problem-solving ability, appreciation of the beauties of mathematics, attitudes toward mathematics--seems, at first glance, a utopian goal. But if we don't make a start on this, who will? It seems to me that the one place in which the professional mathematician can and should make a substantial, immediate contribution to research in mathematics education is in the development of new testing instruments. Needless to say, the practical value of such tests would also be considerable. SMSG has made a start toward the development of new mathematics tests in the National Longitudinal Study of Mathematical Abilities, but much more would have to be done to produce "sample tests" for classroom use.

Accordingly, I propose that a small study group be established--as an offshoot of whatever mechanism is devised to deal with problems of mathematics education in the next decade--to undertake the development of mathematics achievement and attitude tests. The study group would convass the mathematics community for test ideas and sample items--drawing on the expertise of the sizable group of mathematicians who have worked for such enterprises as the College Boards. Then the study group would contract with researchers in

mathematics education to undertake the tryout studies that would be needed to get the tests into shape. Since the study group would have a national constituency, norming studies could be conducted in schools across the country--an almost impossible task for a researcher working alone.

Like SMSG, the new study group would not compete with commerical publishers. Instead, it would provide samples of the sorts of tests that mathematicians and mathematics educators consider appropriate for measuring the outcomes of modern curriculum programs.

COMMUNICATIONS

E. G. Begle

Two general observations need to be stated. First, communication must be thought of as a two-way flow of ideas. Much of our present communication system is designed for one-way dissemination and needs to be supplemented with better feedback mechanisms.

Second, different communication channels are usually needed for different messages.

Here are some examples of different important messages:

1. Objectives. A message might be a list of mathematical topics asserted to be important for all students, or a list of topics important for any student planning to attend a four-year college, etc. The source of such a message would typically be a committee or panel of experts. The targets of such a message would be many, among them being curriculum developers, textbook publishers, school administrators, and parents. Each of these targets should be able to provide feedback to the source, and it (the source) would also wish to be the target for other messages from such sources as educational researchers or scientists and other consumers of mathematics.

Typical channels for these messages would be journal articles, lectures at local, state, and national conventions. More useful meetings bringing together representatives of the source and of the targets.

A mechanism for noting the need for and then organizing such meetings is needed.

2. Standards. A message here might be that for high ability students this textbook results in excellent achievement, for middle ability students rather poor achievement, and for low ability students no achievement. Another message might be that this test does not discriminate between high and low achieving students for topics A, B, and C, but does discriminate for topics D and E. The source of such messages could be a "National Bureau of Standards" or a "Consumers' Union" for mathematics education.

Since any message of this kind will have financial implications, great care will need to be exercised in setting up an agency to make these evaluations. However, the need for it is great.

It is not necessary to list all of the targets for such messages or the channels through which they would flow. It is clear, though, that there would be feedback with respect to which texts, tests, etc., should be evaluated.

3. Research. Here a message might be that there is an interaction between IQ and verbosity of a presentation of probability concepts. The source of such a message would be an individual research worker or a research project, such as an R and D Laboratory. The targets would be other research workers, school administrators, teachers, textbook writers, parents, etc. The channels would be, usually, research reports or journal articles and oral reports at various kinds of conventions.

Here it must be noted that the wording and format of these messages will depend on the particular targets. To explain to a parent the meaning and implications of the above message requires a different wording from that appropriate for a fellow research worker.

Present channels between researchers are reasonably satisfactory, but other channels need either improvement or construction from scratch. It would be helpful, for example, if NCTM could arrange for an annual review, aimed explicitly at classroom teachers, of educational research, and if MAA could do the same for research on post-secondary mathematics education. How to convey research results to parents and other laymen is an important question with no easy answers.

Finally, it should be noted that one target of research messages should be an organization which is concerned with quality control in research. At the moment, the SMSG Panel on Research through its Journal of Abstracts serves this function.

4. New Curriculum Materials. A message here might be a new textbook. The source might be a curriculum development project or it might be an individual author. The target would be the usual combination of school administrators, teachers, parents, and laymen (and also the Bureau of Educational Standards). The usual channels of communication, journal articles and lectures (and advertisements) are not very efficient.

The system of tryout centers, developed by SMSG, seems to have been quite efficient. For each new text, a dozen or so centers were established, with a wide geographical spread. Each center had enough teachers included so that

none of them felt lonely or isolated and so that there were several others to call on in case of trouble. A mathematician provided inservice instruction.

These centers made it possible for a large number of other teachers in the area to see the new text in use. The participating teachers were asked to speak at local meetings. Direct information about the new curriculum unit was easily available at that locality. Some of the participating teachers, especially those who were also inovlved in the writing sessions, were invited to testify before text selection committees, and spread information even wider.

A study of the geographical distribution of orders for SMSG texts early in SMSG history showed that adoptions clearly radiated out from these tryout centers, and that these were much more influential than articles, lectures, or advertisements.

Agencies willing to provide financial support for the preparation of new curriculum material should demonstrate their faith in what they are supporting by budgeting also for a number of information spreading tryout centers.

5. Information. At this moment (December, 1970) it is not clear whether there is an oversupply or an undersupply of high school mathematics teachers. This is just one example of an information gap. We could plan improvements in our teacher-training programs much more effectively if we had a better estimate of the current supply of teachers.

CBMS has already demonstrated that it can collect and disseminate useful information, and it should be encouraged to continue.

Also, we urge CBMS, in its work on a National Information System, to give a high priority to the needs of mathematics education.

EXPLOITATION
OR
EFFECTIVE UTILIZATION OF THE PROGRAMS OF THE 60's IN ATTACKING THE PROBLEMS OF THE 70's

B. H. Colvin

The decade of the 1960's has seen an unprecedented surge of progress in curriculum development in the U.S. and in many other countries. New programs in mathematics, in astronomy, in physics, in chemistry, in the biological sciences, and in the earth sciences, have been brought to the schools. At the elementary level, programs in the processes of science and in pre-science topics have been developed, in addition to a variety of new mathematical programs and curriculum materials. At the secondary level, some more advanced programs in engineering concepts, in computing and in computing application are available.

In fields outside the physical sciences and mathematics, comparable strides have been made in the language arts, in social studies and in the social sciences and, indeed, in almost every spectrum of the curriculum.

A variety of approaches have been tried in developing and in introducing these programs. Moreover, a number of different approaches to improvements in teacher training, both in-service and pre-service, have been explored.

During the period of these developments the evolution of our society has brought new problems into critical focus. Thus the decade of the 1970's presents new problems, some of different character and different scope from those which led to the curriculum development activities of the 60's. Typically, we identify:

1. A general societal and educational concern for interrelationships between and among subject matter field and a grand concern for relevance of all education to "real" world activities - science and society; mathematics and society, etc.

2. An articulated national concern for improved education for certain subgroups in our society, e.g., inner-city children, for rural area children, and, quite broadly, for "dropouts" everywhere.

3. The recognition of problems of a new magnitude in preparing for the vocational and technical education of millions of students at the high school and early college level. This is one component of vast new educational concern for the programs preparatory to and appropriate for junior college, community college, and two-year or four-year technical colleges.

4. The recognition that individual educational patterns are rapidly changing from one of continuous sequential school attendance to one of interrupted periods of education where a lock-step sequence becomes difficult if not impossible, and certainly not optimal.

The decade of the 70's thus adds a new spectrum of problems to those attacked in the 60's. New visions, new goals, new patterns of organization and fresh ideas will be needed.

Nevertheless, in our planning it is important not to sacrifice any possibilities for exploiting the substantial achievements of the past in tackling the problems of the present and of the future. One proposal for school mathematics activities of the 1970's must be to explore all possibilities for exploiting the curriculum improvement, teacher training, and course-content improvement accomplishments of the 1960's in seeking solutions for the problems of the 1970's.

As possible examples of such exploratory activities, we identify the following questions for study.

1. Can the course content programs in physical sciences and mathematics be used as the basis for developing an "interdisciplinary" curriculum of broad interest and relevance?

2. Can the existing programs in the social sciences, social studies, language arts, etc., be "integrated" in some reasonable way with mathematics and physical sciences topics, to achieve a similar advance in relevance, interest and teaching effectiveness?

3. Can "modules," some disciplinary in character - some interdisciplinary in character - be developed from existing curriculum programs to meet the need for flexible blocks in a revolutionary new type of smorgasbord, selective access, multi-stage type of instruction? (cf. item 13)

4. What more effective use can be made of present programs, or of these freshly-baked casserole courses, in improving the teacher training programs?

5. How can such unifying, interdisciplinary, socially relevant recombinations of newly developed course content programs be used in the development and refinement of goals, objectives and evaluation criteria? Can these studies contribute to developing a rationale for such objectives?

6. How can these exploratory studies be used to achieve a greater efficiency in communication and in understanding with teacher certification groups, state curriculum officials, parents, teachers and school boards?

7. As an example of special significance, can the various materials for learning the operation and application of computers be further developed into helpful series of computer-oriented modules for the physical sciences, social science activities, business and industrial applications, medical and hospital applications, etc.?

8. Can the exploration of such integrating studies be tied to an effort to develop mathematical models and to develop an expanding source of relevant, related, understandable, exciting applications in mathematics and computing?

9. How can the existing programs in the various areas of mathematics, physical sciences, social sciences, language arts, etc., be used to provide special materials for special use in urban schools, poor rural schools, newly integrated schools, etc.? Such packages would in general serve as valuable enrichment packages for the average classroom.

10. How can the developments at grades 11, 12, 13, 14 be utilized as a possible mechanism for developing curricula at black colleges and black community colleges and for two-year colleges in general?

11. How can the activities, the developments and the data accumulated in the course content improvement activities, etc., of the 60's be effectively made available to suggest research topics or to refine the choice of research topics or to help coordinate the development of research projects in the 1970's? This would include possible research in learning, in teacher training, in teaching approaches, in school organization, in "modules" use, in studying the effectiveness of learning games, mathematical materials gadgets and laboratories, ...

12. Are there programs or approaches developed or now developing in other countries which offer the possibility for exploitation in the U.S.?

13. Can the development of new course materials--e.g., a minimal core mathematics program, an interdisciplinary mathematics, science, social science, language arts program, a flexible module series in some area--be utilized in a radically different type of teacher training geared to the teaching in just such courses? Could we develop the background training in conjunction with the actual teaching of the course? (This is really second-order exploitation, although a first order exploitation might use an existing course sequence.)

14. How can we exploit the discipline training of some mathematicians, physicists, chemists and engineers in retraining them for teaching opportunities? Does this offer a special opportunity to help a group of citizens with unique educational qualifications to contribute to educational progress?

Characteristically, most of these suggested studies involve greatly increased collaboration between mathematics and other disciplines and with numerous sectors of the education and government communities. By the nature of the problems confronting us in the 70's, this seems inevitable and should be recognized in planning, in proposals and in organization for the work. Especially, to achieve major alterations in the educational patterns associated with mathematics it will be necessary to develop more realistic working relationships with a number of organizations--e.g., the Education Commission of the States, NASDTEC, ...

PROPOSAL FOR A NEW ORGANIZATION FOR MATHEMATICS EDUCATION

On October 23 and 24, 1970, SMSG sponsored a Conference on Responsibilities for Mathematics Education for the 70's. The extensive proceedings of this conference have been examined by an ad hoc committee of the SMSG Advisory Board.

At its working session in Washington on December 11 and 12, the ad hoc committee organized ideas suggested by the conference into seven major categories--objectives, teacher training, research, curriculum, evaluation, communication, and exploiting the work of the past decade in the next decade. Each of the attached summaries points up the major problems in the given category with indications of what might and should be undertaken in the 70's. For some of the most urgent problems, specific projects are suggested for action.

The ad hoc committee thought of who might be encouraged to take the action. At least one-half of the suggestions would easily fit into SMSG as presently constituted. One or two could be incorporated into the present activities of CBMS. Several are new kinds of activities and new ideas, e.g., coordinated research efforts and the consortium on teacher training, requiring a different kind of organization.

Overall, there is a recurring feeling--implicit and explicit--that the nature and size of the problems identified and the actions suggested require the participation of a wide variety of people in the mathematics community. Many of the problems are too big to be undertaken by a single university, school, or other existing organization. Marshalling the efforts of the mathematics community at large requires some SMSG-type organization that can cut across the various specialties needed to work on the problems.

We believe that the organization set up by SMSG was appropriate to deal with the problems of the past decade and that a number of current problems could be attacked by the present organization. However, we feel that a fruitful attack on the problems of the 70's ideally requires new people, fresh ideas, and new organizations.

We believe that one new organization is needed to plan, to stimulate, and to coordinate work on the problems identified. The organization itself should initiate action. Action is needed if the organization is to have vitality and by its vitality attract competent people with needed expertise. Furthermore, productive action results in the confidence and acceptance necessary to attract and keep widespread support of the academic community as well as financial

support from government or foundation funds. However, the organization should be free to enlist the cooperation of schools, universities, and other groups in its various activities.

The organization should consist of a Director, some permanent staff, and a working Board of Directors of from five to seven members. Board members should meet three or four times a year for sessions of three or four days so that they can be aware in depth of the activities of the organization and can provide thoughtful leadership. The Board should be representative of the various constituencies in the mathematics community. Since the effectiveness of the Board and the Director will depend very much on the quality of the people, special effort should be made to ensure the appointment only of individuals of sound judgment and with a wide understanding of mathematics education.

The Conference Board of the Mathematical Sciences seems a natural parent for such an organization because CBMS does represent all organizations. Procedures for election of the Director and the Board would have to be worked out with CBMS.

We recommend that the SMSG Advisory Board go on record as supporting the formation of an organization as described herein. The problems identified in our recent conferences would provide an initial focus for the organization. It would, of course, be encouraged to identify other problems, initiate planning, stimulate the mathematics community, and move to some course of action.

We also recommend that action on this recommendation take place as soon as feasible so that the organization will be functioning at the time SMSG activities are completed. The problems in mathematics education are crucial and serious. They deserve forthright action by the best talent of the country.

Ad Hoc Committee

E. G. Begle
Burton Colvin
Donovan Johnson
Karl Kalman
Jeremy Kilpatrick
Joseph Payne
Henry Pollak

January 5, 1971

CONFERENCE PARTICIPANTS

Frank B. Allen	Elmhurst College
Edward G. Begle	Stanford University
Ralph P. Boas	Northwestern University
Truman A. Botts	Conference Board of the Mathematical Sciences
R. Creighton Buck	University of Wisconsin
L. Ray Carry	The University of Texas
Peter Christiansen	Madison Public Schools
William G. Chinn	City College of San Francisco
Dan Christie	Bowdoin College
Leon W. Cohen	University of Maryland
Burton H. Colvin	Boeing Scientific Research Laboratories Seattle, Washington
F. Joe Crosswhite	Ohio State University
Robert P. Dilworth	California Institute of Technology
Edwin C. Douglas	The Taft School Watertown, Connecticut
Marion G. Epstein	Educational Testing Service
Trevor Evans	Emory University
W. Eugene Ferguson	Newton High School Newtonville, Massachusetts
F. A. Ficken	New York University
Daniel Finkbeiner	Kenyon College
Jack E. Forbes	Purdue University
E. Glenadine Gibb	The University of Texas
Leonard Gillman	The University of Texas
A. M. Gleason	Harvard University
John W. Green	University of California at Los Angeles
Herbert J. Greenberg	University of Denver
Clarence E. Hardgrove	Northern Illinois University
John G. Harvey	University of Wisconsin
Julius H. Hlavaty	New Rochelle, New York
David C. Johnson	University of Minnesota
Donovan Johnson	University of Minnesota
Phillip S. Jones	University of Michigan
Karl S. Kalman	Philadelphia School District
Mildred Keiffer	Cincinnati Board of Education
Jeremy Kilpatrick	Columbia University
Peter A. Lappan	Michigan State University

John F. LeBlanc	Indiana University
Saunders MacLane	University of Chicago
Dexter A. Magers	U.S. Office of Education
John M. Mays	Office of Science and Technology Executive Office of the President
Vincent O. McBrien	College of the Holy Cross
Robert H. McDowell	Washington University
Lyn McLane	Lexington School System
Bruce E. Meserve	University of Vermont
H. Stewart Moredock	Sacramento State College
Frederick Mosteller	Harvard University
Sheldon S. Myers	Educational Testing Service
Elliott I. Organick	University of Houston
Joseph N. Payne	University of Michigan
George B. Pedrick	Committee on the Undergraduate Program in Mathematics
Len Pikaart	University of Georgia
Henry O. Pollak	Bell Telephone Laboratories, Murray Hill, New Jersey
Thomas A. Romberg	University of Wisconsin
Arnold E. Ross	Ohio State University
Isabelle P. Rucker	Virginia State, Department of Education
Glyn H. Sharpe	Jefferson County Public Schools Denver, Colorado
Edwin H. Spanier	University of California at Berkeley
Zella Stewart	Seattle, Washington
J. Fred Weaver	University of Wisconsin
Paul A. White	University of Southern California
Alfred B. Willcox	Mathematical Association of America Washington, D. C.
James W. Wilson	University of Georgia
Frank L. Wolf	Carleton College
Gail S. Young	University of Rochester
Marilyn Zweng	University of Iowa

Observers

Mary Kohlerman	National Science Foundation
John Downes	Emory University

SMSG Staff

Terry Chay
William Geeslin
Joseph W. Green
Helen McCullough
Gordon McLeod
Bertrand Morin
Barbara Pence
Stanley Puryear
Luanne Thodey
Sally Thomas
Norman Webb
Robert Wise

SOME CONSIDERATIONS ON THE ROLE OF PROBABILITY AND STATISTICS IN THE SCHOOL MATHEMATICS PROGRAMS OF THE 1970'S

Frederick Mosteller

In plans for the future of probability and statistics in school mathematics, the same empirical investigative attitude encouraged by Begle for other issues about content, objectives, methods, and equipment continue to be appropriate. At the same time, some information about current trends in statistics may have value, even though they inevitably are colored by the beliefs of the author. It should also be noted that these remarks are being written during a conference and therefore mercifully cannot be very complete or detailed.

The American Statistical Association and the National Council of Teachers of Mathematics have a joint committee on the curriculum in statistics and probability.* The Committee has two immediate projects for which they hope to complete manuscripts by June 30, 1971.

Examples Volume

This volume develops statistical teaching almost entirely through the analysis of real life statistical problems. Generally speaking, an opening problem is solved and then the student is offered a set of exercises. The material is in "example sets" which correspond to sections in a text and may build up to a complicated point. Example sets run from 1 to 8 examples and from about 2 to 25 pages. We have about 40 sets. Committee members and secondary school teachers criticize the examples which are supplied by statisticians, Committee members, and secondary school teachers. Then the authors revise in the light of the critiques.

We do not think material comparable to this is at all available (in school or college); on the other hand, that does not prove that we have a good way to teach statistics. Our notion is that if we can make examples of genuine statistical thinking available, at a voltage level consistent with elementary work, others can consider how to adapt the ideas for the curriculum. We are not preparing what we regard as a textbook. We are offering topics treated in this manner. Coherent units of various sizes can be constructed, but there are intellectual gaps between the units.

*The work of the Committee has been facilitated by a grant from the Sloan Foundation.

Some examples are argumentative and require considerable care because of the complications of real life problems. Others require us to take a second attack on a problem after a seemingly successful first one. Some do not have just one answer, or just two either. Some deal with the art of data analysis, some with modelling. (I emphasize this variety because the Conference has been discussing the desirability of such problems. A major drawback can be that the beginning teacher will find some of the material substantially different from both his experience and his preconceptions of statistics.)

Some illustrations of the types of examples treated but not the format of the treatment appear in a recent Mathematics Teacher (Vol. 63, March, 1970, pp. 199-208), a recent American Statistician (June, 1970, pp. 8-12), and a recent issue of the Journal of the American Statistical Association ("Collegiate Football Scores, U.S.A.", Vol. 65, March, 1970, pp. 35-48) all by the author. Additional material by William Kruskal, "Statistical Examples For Use in High Schools" in a Proceedings of An International Conference on the Teaching of Probability and Statistics at the Pre-College Level held at Carbondale, edited by Lennart Rade and to be published by Almqvist and Wiksell, Stockholm.

Essays on Applications

This volume consists of about 40 essays on uses of statistics in problems of significance to everyday life, science, government, etc. These do not "teach" how to do things but show successful uses, for instance: measuring unemployment, consumer price index, postponing death, safety of anesthetics, smoking, anti-aircraft fire, baseball, introducing a new product, epidemics, etc. Except for the essay on epidemics the mathematical exposition is minimal, nearly zero, though graphs, figures, and tables are sometimes used. It is not intended that one exercise explicit mathematical skills to read it. The volume is intended to familiarize the reader with the breadth, variety, and importance of the applications of statistics and to communicate some basic notions.

Data analysis. What is new in statistics just now is a revival of interest in exploratory data analysis partly because of the availability of computers and partly because of a natural resource called J. W. Tukey. Semi-systematic approaches to exploring data are being codified and tried out in various colleges and universities. Several substantial distinct research projects will be contributing their findings to the common pool. Some work in interactive mode, some batch, is available on computers. At several places data sets having considerable variety of subject matter are available.

This data analytic development is refreshing because it moves strongly away from the simple and often artificial problems of "confirmatory data analysis" into the complicated world of the structure of the data. In this part of the work the probabilistic attitude toward the material may be largely neglected. Graphics may play a great role. Curved lines may be drawn in by hand and further calculations done.

New courses in this subject obviously can be developed in a variety of ways. I think that the use of the display tube will be too expensive except for a few demonstration installations until the very late '70s. Meanwhile, I think there are some promising ideas for its development. First, by associating it with either batch mode with fast turnaround or interactive mode we can get a close relation with the computer and make the computer work pay off by decreasing the drudgery and by increasing the variety of parameters that can be adjusted in the analysis.

Second, I think that there are some sophisticated ideas where the computer will be used to produce material which can be passed out to a class, and a great deal about data analysis can be learned from these materials which would cost too much for each class member to program or produce himself. Thus the computer will be used in a small way by the specific student but its product can become familiar, and discussions about what the next step in the analysis should be can still be quite satisfactory.

Therefore there are three main ways I see at this moment to introduce this course. Paper and pencil based, computer material based, with modest direct computer support, and direct computer based, with or without displays.

Interdisciplinary with the social sciences. Most of the social sciences are becoming strongly behaviorally based, which means quantitative. At Dartmouth a data bank and related programs called Project IMPRESS is illustrative. The freshman sociologist or economist is able to study social problems directly by attending to such data sources as the one-in-1000 sample from the census. He has many variables at his disposal and can ask for a variety of breakdowns and percentages, and he can also ask for various statistical devices to be applied, with reminders from the machine as to what they mean and warnings about how the particular statistic may be a silly one to compute, but the student can have it. There is also material on a large number of companies over a substantial period. This project is a fascinating one, and we would do well to go study it, talk to the students, and see whether it has some hints for relating social science work in schools to statistics, and mathematics. The data are genuine and extensive, and so ideas of a factual

nature of a non-experimental sort--that is, the one-shot observational study sort like a census--can be made available.

Naturally, this sort of relationship between statistics and social sciences does not have to be computer based. There are other matters, such as simple crosstabs, how samples are actually taken, and so on, that can be treated. The Essays on Applications section would be useful. The mathematical content can be rather small, but the interest of some students may be high. I do not want to say all or even most students would be delighted with such work. But I might emphasize that it moves in the same direction that SMSG is going. That is, it offers empirical research as a substitute for armchair thinking about social problems and the properties of society. (Let me say though that many vigorous changes in society have been done from the armchair, but others have been in response to rather simple but overwhelming data.) There is here the opportunity to see data so massive that it can be regarded as population information and to see other data that is most queasy in both its validity and its reliability. Under the latter circumstances first-class discussion is a must. Under the former, the issue often is whether the data is adequate for policy and asks what policy should be developed.

One could have inferential statistics here without necessarily developing the mathematical basis for it.

Interdisciplinary with the physical sciences. I have seen units prepared for the analysis of data from experimental work at the school level by physical scientists, for example to see whether two methods produce observations with means alike, or whether the difference observed agrees with physical theory. I believe that it would be worthwhile for the mathematical community to ask itself whether it should have a role in discussing or preparing such material, or whether it wishes to have the physicist, chemists, and biologist take the leadership and responsibility for such teaching.

Probability (and statistics) more generally. As mathematics comes to be taught in a more integrated way, it seems to me that there will be many places for short sections on probabilistic and statistical methods, where the statistics and probability illustrate the active use of a technique in a way that may go well beyond the simplicity of the original mathematical idea. (The binomial distribution is now the basis of many sections.) Discovering places where this approach is useful and valuable is something the ASA-NCTM considered as a possibility for its own work and rejected. We felt that this required a direct team attack on the whole curriculum and needed many minds with curriculum ideas before them. It seemed more appropriate for summer

work for a group than for isolated people working on a committee. The committee thus sees a need for this to be done, but does not at this moment have have plans or funds of its own to do it. I believe that it would encourage some other group to do so, and that it would be happy to give advice or discussion. Some of the materials mentioned in the Examples Volume section that it has gathered and edited are, of course, appropriate for such spots.

This approach rather considers the opportunities for the use of statistical and probabilistic techniques in the exposition of other mathematical techniques. Another related project is this: "What P and S ideas and methods should be presented to various sorts of students in a well-rounded program?" Again this is a worthy piece of research and could go along the lines the Conference has discussed for other sorts of mathematics, except I think that we are a little better prepared elsewhere in mathematics to recognize a constellation of ideas needed than we are here. I believe this is a project that will require the help of research statisticians from the applied fields as well as those from the more mathematical side to cooperate with secondary teachers and curriculum developers. Again, I am confident that the ASA-NCTM Committee would encourage having this work done. Whether they would be a resource for the effort or not I could not say at this time.

Timing. From our experience in teaching students in college, most of us have found that there are topics which present no basic intellectual difficulty but which do not seem to be teachable to students who have not had certain experiences. It is not that they are so hard, but that the student can't take an interest in them. The difficulty seems to be that he has had no experience that makes the discussion reasonable to him. An example is experimental design, which is fascinating in many ways, both mathematical and practical, and calls for our best thought and ingenuity. Nevertheless, a student who has never been involved in a genuine research situation finds it hard to get engaged intellectually, partly I suspect because he cannot believe that anyone would go to the amount of trouble implied by the work. The same student two or three years later will be back wondering why he was not taught this material. We have therefore a rather clear anecdotal experience about the "level" at which some things can be taught. I think this particular one is culture-bound in the sense that it depends upon the order in which different students have certain quantitative experiences. A freshman in college who has spent a summer doing a chemical apprenticeship is just as ready as the second year graduate student who has embarked upon a thesis topic and is desperate to find the effect of the variables. The relevance becomes more obvious with the task in hand.

I am not trying to say what probability and statistics should be taught in what order but rather accepting the point that just because something can be taught and might even be sensible to have taught at a given level, this does not mean that students will pick it up efficiently if they take it then. There may be room for electives.

Still another point that I think worth mentioning is the deterministic world problem. Many students have been brought up on the theory that the world is a very deterministic place, and so it sometimes is when you can control many of the conditions. But statistics and probability makes some of these students uneasy. Again, it is not that the material is so intellectually difficult, but this feeling that if there are many different answers and if one can't tell how things come out then the world is a threatening place. Consequently one will have troubled restless students who are resisting the material not from difficulty with the mathematical features, but from the implications which they vaguely see it having for the world. And of course it raises questions about the ease of instant improvement of unsatisfactory situations in the world. We are fond, too, of saying that we don't know about this, and we don't know about that. Unfortunately, sometimes when we do know about this and that, the mathematics tells us we can't change things to suit ourselves because of excessive cost or lack of control of variables. All these ideas can be hard for an idealistic young man or even a hardnosed old one to accept. I must not overemphasize this syndrome, but I have seen many a good student very troubled. Except for gambling, uncertainty is an unpopular product.

I do not expect all students to like any particular part of P and S any more than they are all going to like proof in geometry or graphing conics. But I think a valuable project would be to discover the better orders in which to present things and the better subject matter vehicles for interesting the student.

On the one hand we can make the material very relevant, but we all have seen the bored student, whether bright or dull, take off about a problem that has no relevance to anything except that he suddenly wants to know. Very often relevant problems are terribly complicated and specific and have so many troubles that their simplification can be downright misleading. Anyway, while relevance can be offered in these subjects, I find that the birthday problem, the secretary problem, and the 1 and 3-engine plane problem are attractive and fun in their own right even if they do have some genuine application in more sophisticated forms that the student is unlikely to know about.

Future of ASA-NCTM Committee. Partly stimulated by questions raised and discussion at this SMSG Conference, and partly by its own timetable, since the manuscripts for the items Examples Volume and Essays on Applications should be finished next year, the ASA-NCTM Committee is convening a small conference itself, December 13, 1970. The Conference will discuss the further needs and future tasks in elementary and secondary school work in probability and statistics. With the aid of discussions from that Conference, the Committee hopes to make further recommendations to the parent societies.

FOOTNOTE. Confirmatory data analysis includes methods of testing significance or other methods of inference which many people associate with statistical methods for research workers. In many of these methods the idea lurking in the background is that an experiment has been executed with a specific variable or distinction to be "tested". Provided we do not challenge any assumptions, this work can be neat and tidy. The mathematics can be "closed" with specific answers. More of this work is done in elementary statistics courses today than exploratory data analysis. The latter offers ways of tackling large bodies of data and trying to make some sense of them. Both areas are important, and they in turn are somewhat different from sample survey methods and experimental design, and still another area we might call methodology dealing with "validity", "bias", "defining variables", and "non-sampling errors".