CSMP Mathematics for the Intermediate Grades

A Supplement for Entry Classes

4TH GRADE ENTRY TABLE OF CONTENTS

NOTES TO THE TEACHER

SUGGESTED SCHEDULE

EN LESSONS

EL LESSONS

EW LESSONS

The Fourth Grade Entry Program

A Supplement for Fourth Grade Entry Classes is designed for fourth grade classes using *CSMP* for the first time. The "veteran" fourth grade program is contained in two volumes, *CSMP Mathematics for the Intermediate Grades, Part I* and *Part II* (*IG-I* and *IG-II*). This supplement contains fifteen introductory lessons, drawn from earlier grades. These lessons acquaint students with the Minicomputer, the languages of strings and arrows, and negative numbers, thus providing necessary background for future lessons in the fourth grade program. A modified first semester schedule is included.

Notes to the Teacher

How to Use the Fourth Grade Entry Schedule

As a teacher of a fourth grade entry class, you will be teaching lessons from both this supplement and from the *IG-I* teacher's guide. The schedule contained in this booklet shows you how they integrate. Follow this schedule rather than the one in the *IG-I* teacher's guide. Pay close attention to the letters preceding a lesson number. Lesson numbers beginning with **E** can be found in this booklet. For example, Lesson **EN1** is the first supplement lesson to the World of Numbers (**N**) strand; its description begins on page EN-1 of this booklet.

strand abbreviation (in this case, World of Numbers strand)

lesson number within the strand

supplement lesson
for entry classes

EN1

If there is not an **E** before a lesson number, then the description of that lesson can be found in the

Materials

All the materials you will need can be found in your classroom set, as well as a few you will not need because the lessons requiring those materials are not scheduled for entry classes.

The worksheets that your class will need for lessons in the fourth grade entry program can be found in blackline form following the lessons. Parent letters and home activities can be found in the Blackline section of the *IG-I* teacher's guide.

Suggested Schedule

Master Schedule for CSMP Mathematics for Fourth Grade Entry Classes

Week	World of Numbers	Languages of Strings & Arrows / Probability & Statistics	World of Numbers	Workbooks	Geometry and Measurement
1	EN1 Minicomputer Introduction #1 EN-1	EL1 Introduction to Arrows EL-1	EN2 Minicomputer Introduction #2 EN-13	EW1 Detective Story #1/ Eli's Magic Peanuts #1	Adjustment Day
2	EN3 Minicomputer Introduction #3 EN-21	EL2 Set Membership EL-7	EN4 Minicomputer Introduction #4 EN-31	EW2 Which Road? EW-9	G1 Length #1 G-5
3	EN5 Doubling and Halving on the Minicomputer EN-43	EL3 String Game with A-Blocks EL-13	EN6 Assorted Arrow Problems EN-47	EW3 Detective Story #2/ Eli's Magic Peanuts #2	G2 Length #2 G-9
4	EN7 Parentheses EN-51	EL4 Multiples EL-17	N2 Composition of Functions #1 N-15	EW4 Detective Story #3/ Eli's Magic Peanuts #3	N3 Road Map of Illinois N-21
5	N4 Subtraction #1 N-25	L1 String Game with A-Blocks #1 L-5	N5 Guess My Rule #1 N-31	W1 Collection of Problems #1 (Lesson One) W-5	G3 Area and Perimeter #1 G-13
6	N6 Minicomputer Golf #1 N-35	L2 Composition Games #1 L-11	N7 Tag the Arrows #1 N-41	W2 Collection of Problems #1 (Lesson Two) W-7	G4 Square Regions G-17
7	N8 Number Line #1 N-47	L3 String Game with A-Blocks #2 L-17	N9 The Function 10x #1 N-53	W4 Collection of Problems #2 (Lesson One) W-23	N10 Subtraction #2 N-59
8	N11 Composition of Functions #2 N-65	P1 The Island of Tam-Tam #1	N12 Decimals #1 N-71	W5 Collection of Problems #2 (Lesson Two) W-25	G5 Parallelism #1 G-25
9	N13 Minicomputer Golf #2 N-75	P2 The Island of Tam-Tam #2	N14 Calculator Relations #1 N-79	W6 Halloween Puzzles (Lesson One) W-35	G6 Parallelism #2 G-31

Week	World of Numbers	Languages of Strings & Arrows / Probability & Statistics	World of Numbers	Workbooks	Geometry and Measurement
10	N15 Multiplication #1 N-85	L5 Composition Games #2 L-25	N16 Multiples and Divisors #1 N-91	W7 Halloween Puzzles (Lesson Two) W-39	P3 Baseball Statistics P-15
11	N17 Sasquatch #1 N-95	L6 String Game with Numbers #1 L-29	N18 Assorted Problems #1 N-101	W8 Collection of Problems #3 (Lesson One) W-47	N19 Number Line #2 N-105
12	N20 Division #1 N-111	L7 How Many Disguises? L-33	N21 Guess My Rule #2 N-117	W9 Collection of Problems #3 (Lesson Two) W-49	G8 Parallelism #3 G-41
13	N22 Decimals #2 N-123	L9 String Game with Numbers #2 L-41	N23 Multiples and Divisors #2 N-127	W10 Collection of Problems #4 (Lesson One) W-59	G9 Parallelism #4 G-45
14	N24 Sasquatch #2 N-131	P4 Same or Different #1 P-19	N25 Calculator Relations #2 N-137	W11 Collection of Problems #4 (Lesson Two) W-61	N26 Multiplication #2 N-141
15	N27 The Function 10x #2 N-147	P5 Same or Different #2 P-23	N28 Division #2 N-153	W12 A Strange Country (Lesson One) W-71	G10 Area and Perimeter #2 G-49
16	N29 Tag the Arrows #2 N-161	P6 Same or Different #3 P-29	N30 Fractions #1 N-165	W13 A Strange Country (Lesson Two) W-77	G11 Area ofRectangles and Triangles G-55
17	N32 Assorted Problems #2 N-177	P7 Averages P-35	N33 Multiplication #3 N-181	W14 Collection of Problems #5 (Lesson One) W-83	G13 Hand Covers G-63
18	N34 Fractions #2 N-187	P8 Climate of U.S. Cities L-39	N35 Decimals #3 N-193	W15 Collection of Problems #5 (Lesson Two) W-85	G14 Basketball Hands G-65

EN Lessons

EN1 MINICOMPUTER INTRODUCTION #1

Capsule Lesson Summary

Introduce the values of the squares on the Minicomputer. Use the Minicomputer to represent numbers less than 20. Sometimes represent the same number in different ways. Model some trades and represent numbers less than 100. Introduce the hundreds and thousands board and extend trades and representations of numbers to include hundreds and thousands.

Materials

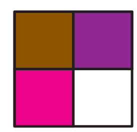
Teacher	• Minicomputer set† • Base-10 blocks or other place-value manipulative	**Student**	• Worksheets EN1*, **, ***, and ****

Advance Preparation: Use the blacklines following this lesson to make copies of student worksheets.

Description of Lesson

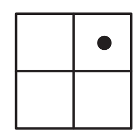
Exercise 1

Display a Minicomputer board.



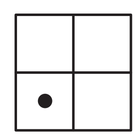
T: ***This is the Minicomputer. We put numbers on the Minicomputer using checkers.***

Put one checker on the purple square and hold up four fingers as you say,



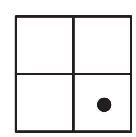
T: ***This is the number 4 on the Minicomputer.***

Move the checker to the red square.



T: ***This is the number 2 on the Minicomputer.***

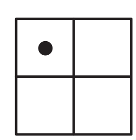
Move the checker to the white square.



T: ***This is the number 1 on the Minicomputer.***

Review the configurations for 4, 2, 1 and then again for 1, 2, 4 on the Minicomputer. Do this a couple times letting students tell you the numbers.

Move the checker to the brown square.



T: ***What number do you think this is?***

Some students may guess 3 or 5.

T: ***No, it is not 3 (or 5).***

†A teacher's Minicomputer set consists of four demonstration Minicomputer boards and a sufficient number of demonstration Minicomputer checkers.

Pause. Review 1, 2, 4 and then pause as you put the checker on the brown square. After a moment a student may suggest 8. If not, simply say,

T: ***This is 8.***

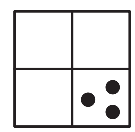
Move the checker quickly from one square to another and ask the class to call out each number. In doing this, follow the doubling pattern: 1, 2, 4, 8. Visually suggest the doubling pattern by putting two checkers on the white square and saying, “1 + 1 = 2.” Then take off the two checkers and put one checker on the red square. Repeat for “2 + 2 = 4” and “4 + 4 = 8.”

Remove the checkers from the Minicomputer.

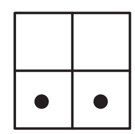
T: ***Who can put 3 on the Minicomputer?***

Ask volunteers to tell you first how many checkers they will need. (Two and three are both correct answers.) Let a student put 3 on the Minicomputer.

A student might put three checkers on the white square.



If a student places the checkers this way, lift the checkers one by one and say, “1 + 1 + 1 = 3.” Remove the checkers from the Minicomputer. Ask if someone can put 3 on the Minicomputer in another way.



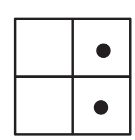
By now someone should be able to put this configuration for 3 on the Minicomputer. After a student places the checkers in this way, lift each checker as you mention its value and then replace it quickly on the Minicomputer.

T: ***What number is on the red square?*** (2) ***What number is on the white square?*** (1)
2 + 1 = 3. 1 + 2 = 3.

Remove the checkers from the Minicomputer.

T: ***Who can put 5 on the Minicomputer?***

Ask volunteers to tell you how many checkers they will need. (Two, three, four, and five are all correct answers.) Ask a volunteer to place the checkers on the Minicomputer. Add the numbers on the various squares out loud as you did for 3. Whenever more than two checkers are used, ask for another way until you get the standard configuration.

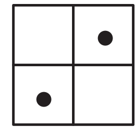


When the checkers are in this position, lift each checker as you mention its value and replace it quickly on the Minicomputer.

T: ***What number is on the purple square?*** (4)
… on the white square? (1)
4 + 1 = 5. 1 + 4 = 5.

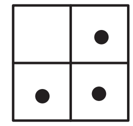
Remove the checkers from the Minicomputer.

T: ***Can anyone put 6 on the Minicomputer?***



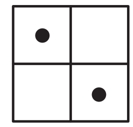
Accept all correct answers. If more than two checkers are used, ask for another way until you get the standard configuration.

T: ***Can anyone put 7 on the Minicomputer?***

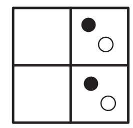


Accept all correct answers. If more than three checkers are used, ask for another way until you get the standard configuration.

T: ***Can anyone put 9 on the Minicomputer?***



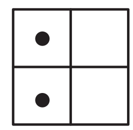
Accept all correct answers. If more than two checkers are used, ask for another way until you get the standard configuration.



Put on 5 and ask what number it is. Then put on 5 again and ask what number 5 + 5 is. (10)

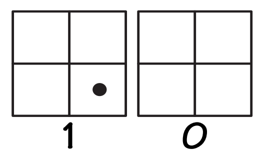
T: ***Can anyone put 10 on the Minicomputer in another way?***

At some moment someone will probably suggest this configuration.



T: ***There is a way to put 10 on the Minicomputer using only one checker, but we will need another board.***

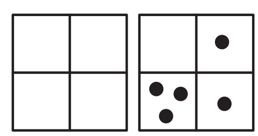
Display a second board (the tens board) to the left of the first board and place one checker on the white square of the tens board. As you do this say, “8 + 2 = 10.” Write 1 below (or above) the tens board and 0 below the ones board.



At this point you may like to use base-10 blocks or some other place-value manipulative to model a trade of 10 ones for 1 ten and 0 ones.

Exercise 2

Put some checkers on the ones board very gradually. For example:

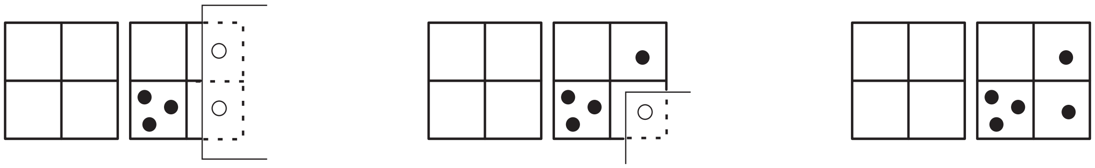


- place three checkers on the red square, pause;
- then one checker on the purple square, pause again;
- then one checker on the white square.

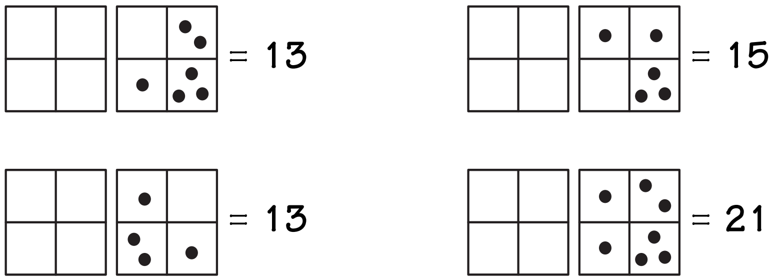
This gives your students a chance to calculate mentally.

T: ***What number is on the Minicomputer?*** (11)

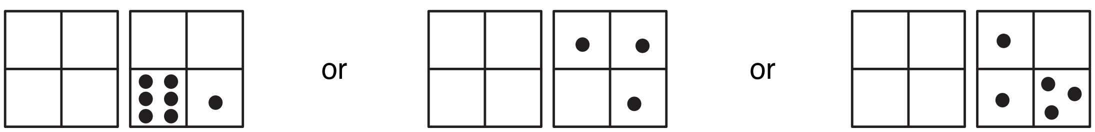
No explanation is necessary if everybody gets the right number. If someone gives the wrong number, you might use this procedure: Cover part of the board with a piece of paper to focus the class's attention on certain checkers, and gradually uncover the full set of checkers.



Repeat this activity with other examples; some possibilities are suggested below.



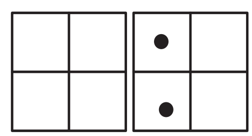
Note: It is interesting to present the same number several times, each time with a different number of checkers. For instance, 13 can be represented as



and so on.

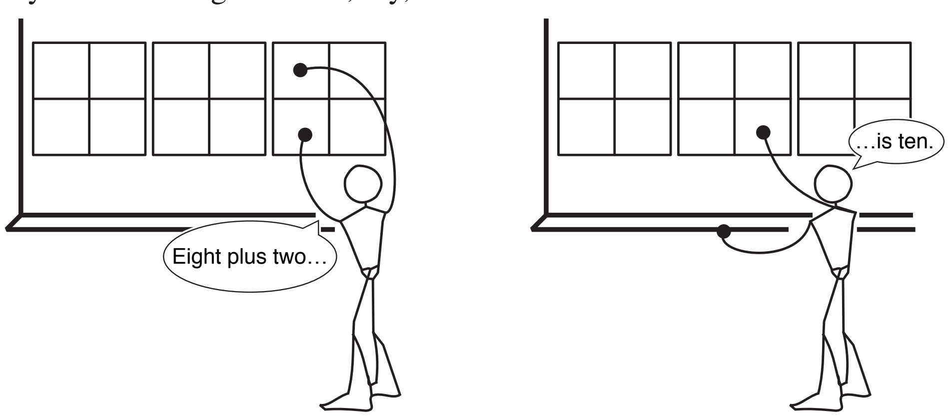
Exercise 3

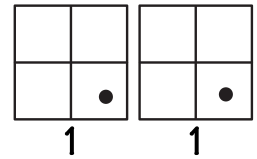
Put one checker on the 8-square and one on the 2-square.



T: ***What number is on the Minicomputer?*** (10)

Pick up the checker on the brown square with one hand and the checker on the red square with the other. Then put one of these checkers on the 10-square and take the other checker away (put it in the chalk tray). As you are making the trade, say, "8 + 2 = 10."





T: ***This is a way to put 10 on the Minicomputer with just one checker. With one more checker, can you put 11 on the Minicomputer?***

Ask students to remove the checker from the ones board.

T: ***With one more checker, can you put on 12?*** (Yes)
Can you put on 14? (Yes)
Can you put on 17? (No)
How many more checkers do you need? (At least three more)
Can you put on 18? (Yes) ***19?*** (No)

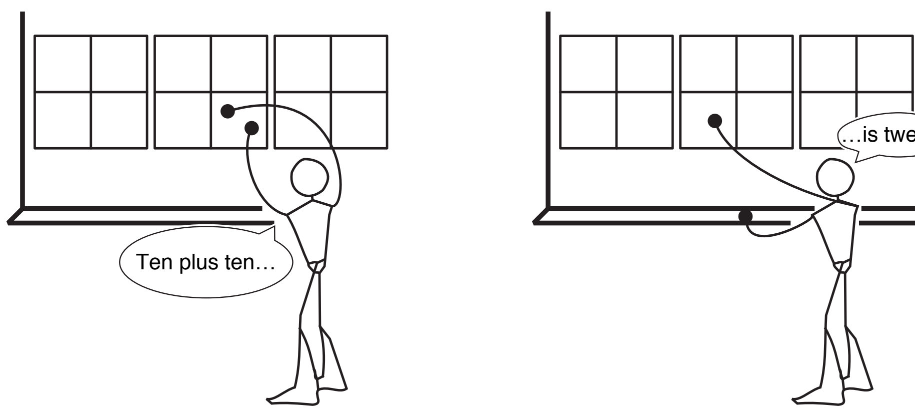
Note: The question, "How many checkers do you need for 17?" has many answers. For the standard configuration you need four checkers, but it is possible to put 17 on the Minicomputer with three checkers: 8 + 8 + 1 = 17. In this case, however, one checker is given on the 10-square.

In some cases, you may request a student to model a number as well with base-10 blocks or another place-value manipulative; for example, model 14 as 1 ten and 4 ones.

Ask someone to put 20 on the Minicomputer using two checkers. If no one volunteers, put two checkers on the 10-square.

T: ***There is a way to put 20 on the Minicomputer with one checker.***

Make the trade and say, "10 + 10 = 20."

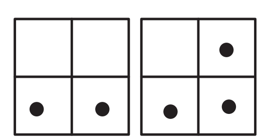


Remove the checkers and continue with other numbers.

T: ***Who can put 25 on the Minicomputer? 27? 30?***

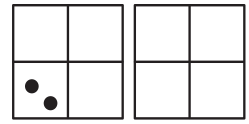
Again you may invite students to model 25 as 2 tens and 5 ones with base-10 blocks or another place-value manipulative.

Put this configuration on the Minicomputer.



T: ***What number is this?*** (37)

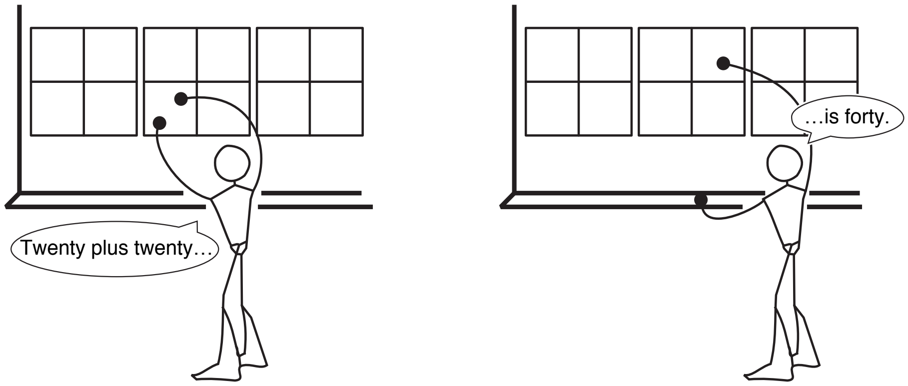
When you receive the correct answer, write 3 below the tens boards and 7 below the ones board.



Put two checkers on the 20-square.

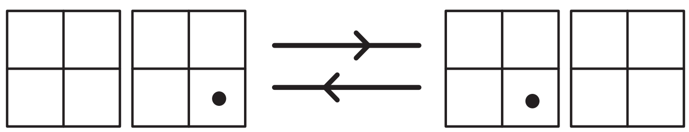
T: ***What number is this?*** (40)

Make the trade yourself and say, “20 + 20 = 40.”



Remove the checker from the 40-square.

Move a checker back and forth very quickly from the 1-square to the 10-square. Each time you move the checker, ask the class which number is on the Minicomputer: one—ten; one—ten; and so on.

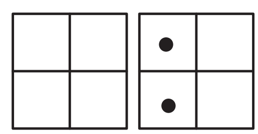


Repeat this with 2 and 20; 4 and 40; 8 and 80. You may use this as an introduction to the 80-square.

T: ***Who can put 47 on the Minicomputer? 53? 96? 32?***

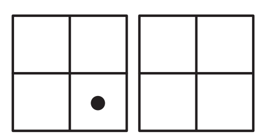
Exercise 4

Put 10 on the Minicomputer as 8 + 2.

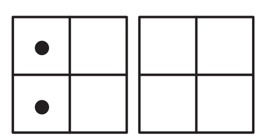


T: ***What number is this?*** (10)

Who can put 10 on the Minicomputer with one checker?



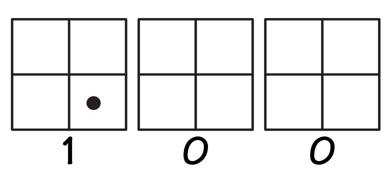
Put 100 on the Minicomputer as 80 + 20.



T: ***What number is this?*** (100)
How do you know? (80 + 20 = 100)

There is a way to put 100 on the Minicomputer with just one checker, but we need another board.

Display the hundreds board and make the trade yourself as you say, “80 + 20 = 100.” Write 100 below the Minicomputer.

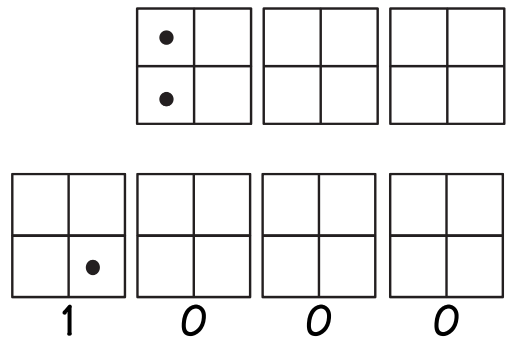


You may want to model a trade of 10 tens for 1 hundred with base-10 blocks or another place-value manipulative.

T: ***Who can put 105 on the Minicomputer? 126? 157?***

Occasionally, you may wish to ask a student to write a numeral below the Minicomputer while other students model the number with base-10 blocks or another place-value manipulative.

Put 1000 on the Minicomputer as 800 + 200.



T: ***What number is this?*** (1 000)
How do you know? (800 + 200 = 1 000)

Display the thousands board and make the trade yourself as you say, “800 + 200 = 1 000.” Write 1 000 below the Minicomputer.

You may want to model a trade of 10 hundreds for 1 thousand with base-10 blocks or another place-value manipulative.

T: ***Who can put 1 008 on the Minicomputer? 1 016? 1 253?***

Occasionally, ask students to write a numeral below the Minicomptuer.

Move checkers quickly from one board to another to show the following:

1; 10; 100; 1000
2; 20; 200; 2000
4; 40; 400; 4000

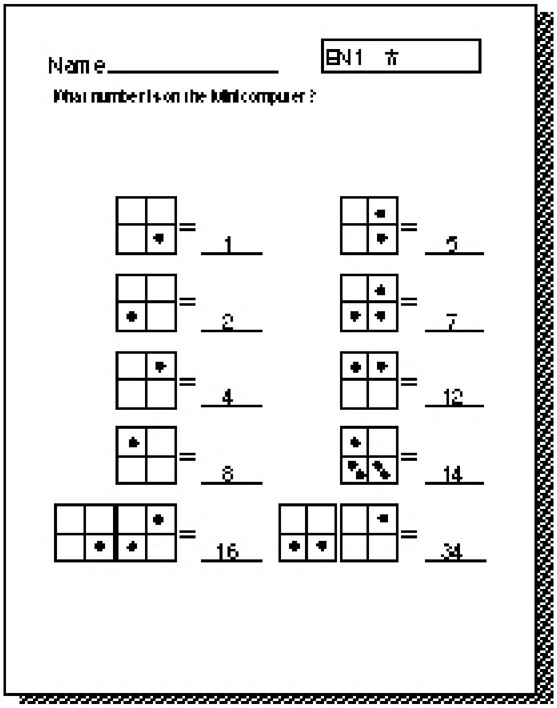
8; 80; 800; 8000
5; 50; 500; 5000
3; 30; 300; 3000

T: ***Who can put 2 425 on the Minicomputer? 5 048? 9 703?***

Worksheets EN1*, **, ***, and **** are available for individual work.

Center Activity

Put Minicomputer boards and checkers in a center. Let pairs of students practice putting on numbers and reading each other’s numbers on the Minicomputer. Task cards made with a number on one side and its standard Minicomputer configuration on the other can be used by individual students.



Name ____________ EN1 ☆

What number is on the Minicomputer?

= 1

= 5

= 2

= 7

= 4

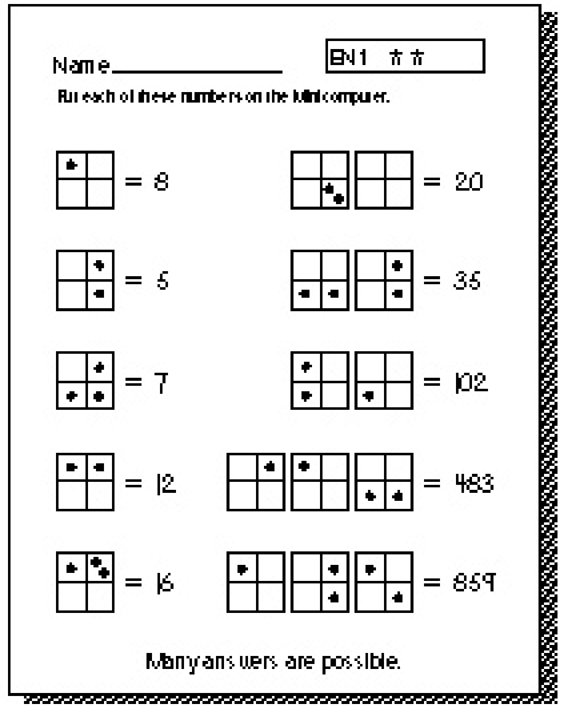
= 12

= 8

= 14

= 16

= 34



Name ____________ EN1 ☆☆

Put each of these numbers on the Minicomputer.

= 8

= 20

= 5

= 35

= 7

= 102

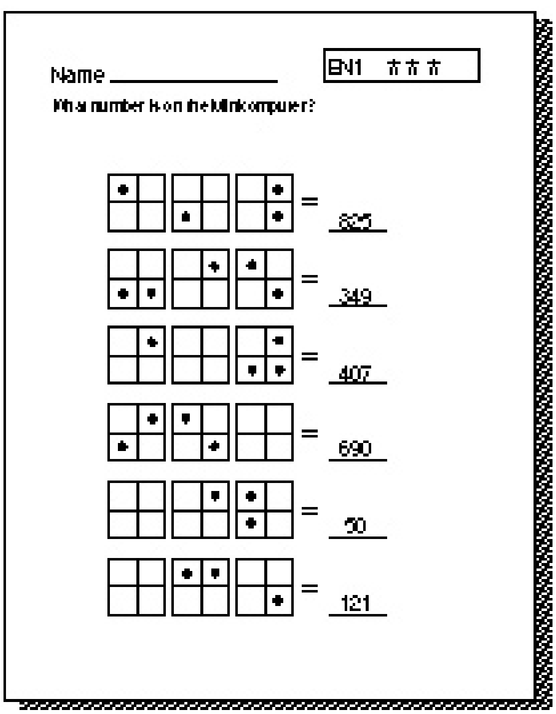
= 12

= 483

= 16

= 859

Many answers are possible.



Name ____________ EN1 ☆☆☆

What number is on the Minicomputer?

= 625

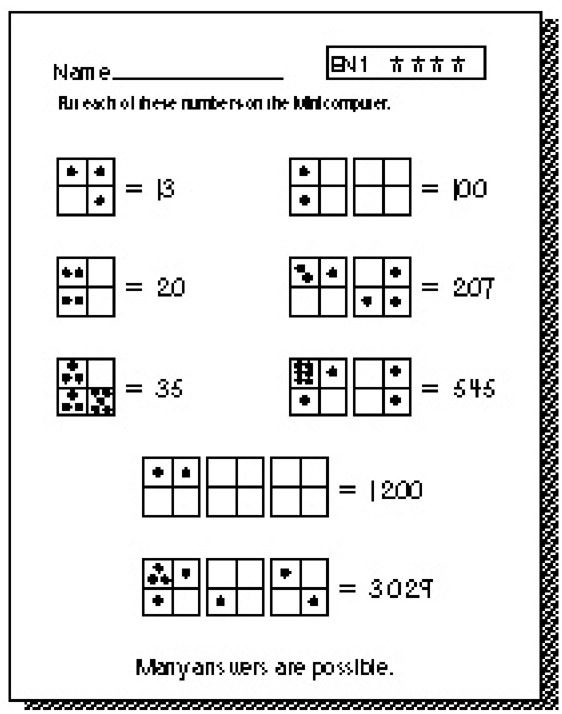
= 349

= 407

= 690

= 50

= 121



Name ____________ EN1 ☆☆☆☆

Put each of these numbers on the Minicomputer.

= 13

= 100

= 20

= 207

= 35

= 645

= 1200

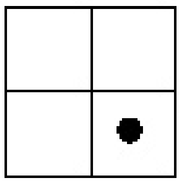
= 3029

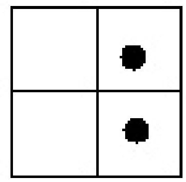
Many answers are possible.

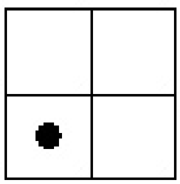
Name ____________________

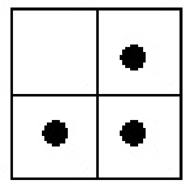
EN1 *

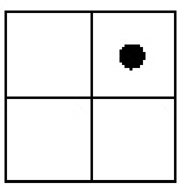
What number is on the Minicomputer?

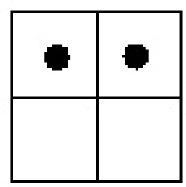
 = ______

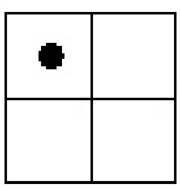
 = ______

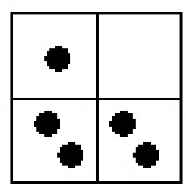
 = ______

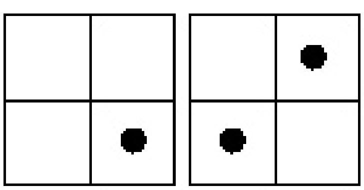
 = ______

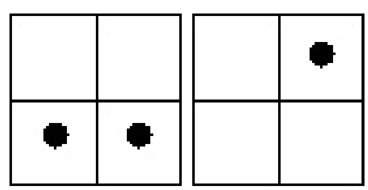
 = ______

 = ______

 = ______

 = ______

 = ______

 = ______

Name ____________________ EN1 **

Put each of these numbers on the Minicomputer.

= 8

= 20

= 5

= 35

= 7

= 102

= 12

= 483

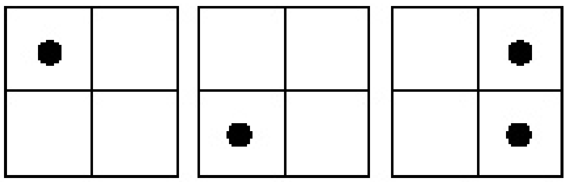
= 16

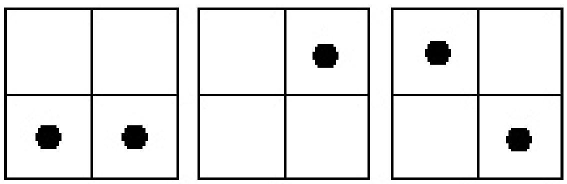
= 859

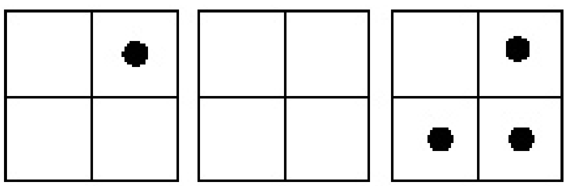
Name ____________________

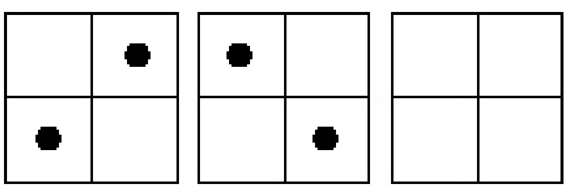
EN1 ★★★

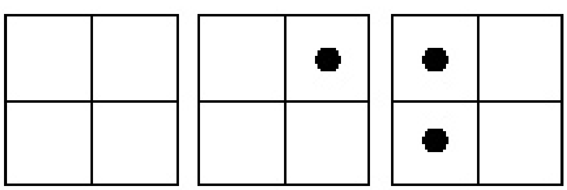
What number is on the Minicomputer?

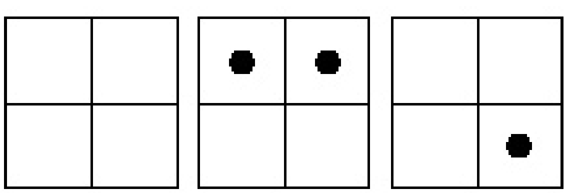
 = ______

 = ______

 = ______

 = ______

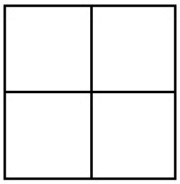
 = ______

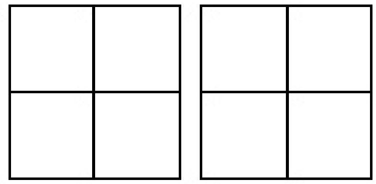
 = ______

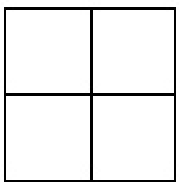
Name ____________________

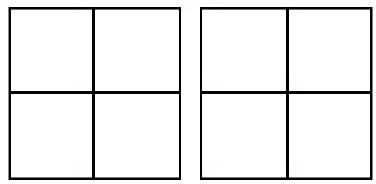
EN1 ★★★★

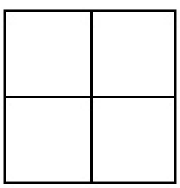
Put each of these numbers on the Minicomputer.

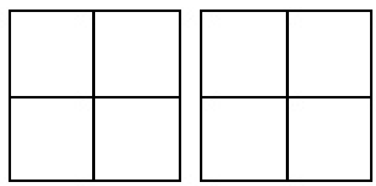
 = 13

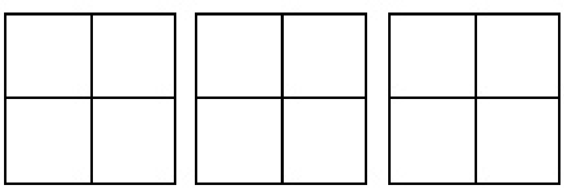
 = 100

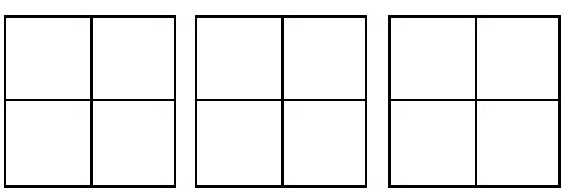
 = 20

 = 207

 = 35

 = 545

 = 1200

 = 3029

EN2 MINICOMPUTER INTRODUCTION #2

EN2

Capsule Lesson Summary

Review the value of the squares on the Minicomputer. Use the Minicomputer to represent a variety of numbers from 1 to 8 000. Introduce a game called *Minicomputer Tug-of-War*.

Materials

Teacher
- Minicomputer set†

Students
- Paper
- Minicomputer set†

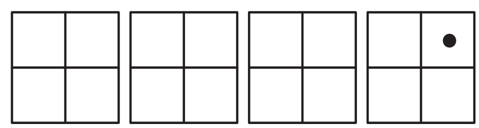
Note: Paper for students can be scratchpads, notebooks, slates, or whatever serves your classroom management and record-keeping purposes.

Description of Lesson

The first four exercises are for review and should move at a quick pace. Target students who need extra help. You may want to let students work with a partner; team those who are less proficient with the Minicomputer with "experts."

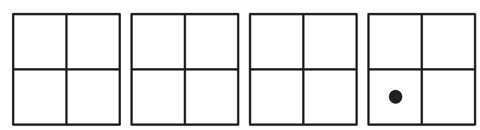
Exercise 1

Display four Minicomputer boards.

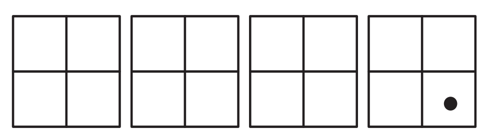


T: ***This is the number 4 on the Minicomputer.***

To prevent confusion, always remove the checkers from the Minicomputer before asking the next question.

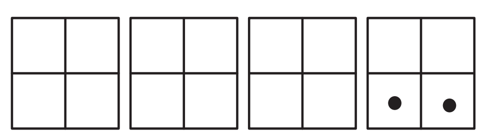


T: ***Who can put 2 on the Minicomputer?***



Who can put 1 on the Minicomputer?

Very quickly, review placement of the checker for 1, 2, 4, and 8.



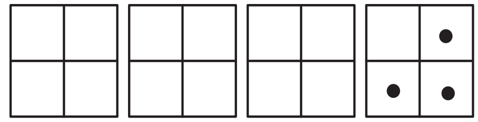
T: ***I'll give you two checkers.***
Can you put 3 on the Minicomputer?

Can you put 5 on the Minicomputer using exactly two checkers?
… 6 using exactly two checkers?
… 9 using exactly two checkers?
How many regular checkers do you need to put 7 on the Minicomputer? (At least three)

†A teacher's Minicomputer set consists of four demonstration Minicomputer boards and a sufficient number of demonstration Minicomputer checkers. A student's Minicomputer set consists of two sheets of Minicomputer boards (two boards per sheet) and cardboard checkers.

EN2

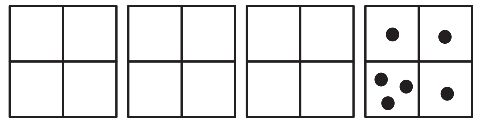
Ask someone to put 7 on the Minicomputer using exactly three checkers.



Exercise 2

Gradually put this configuration on the Minicomputer, pausing after each step.

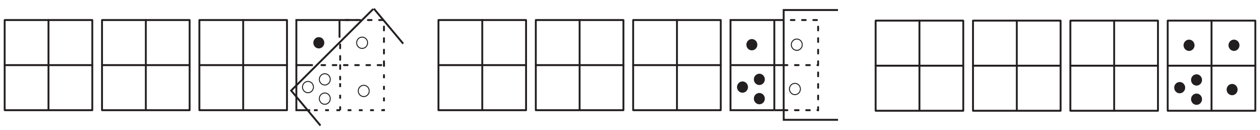
- Put one checker on the 8-square.
- Put three checkers on the 2-square.
- Put one checker on the 4-square and one checker on the 1-square.



This gives your students a chance to do a mental calculation of the number.

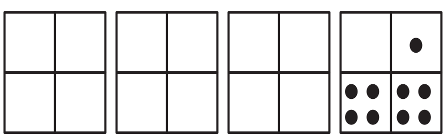
T: ***What number is on the Minicomputer?*** (19)

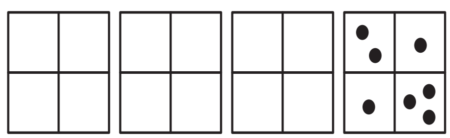
No explanation is necessary if everybody gets the right number. If someone is wrong, you might use this procedure: Cover part of the board with a piece of paper to focus the class's attention on certain checkers, and gradually uncover the full set of checkers.



T: ***What number is this?*** (8) ***And 6 more?*** (14) ***And 5 more?*** (19)

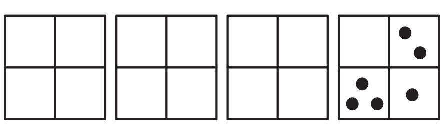
Repeat this activity with other configurations; some possibilities are suggested below. Ask the students to write their answers on scratch paper or whisper them to their partners; then quickly get class agreement.

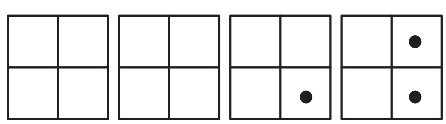
 = 16

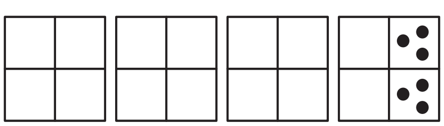
 = 25

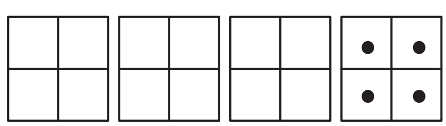
T: ***Who can put 15 on the Minicomputer?***

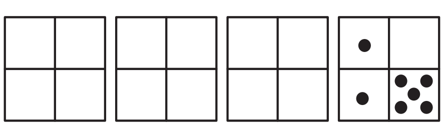
Let students suggest a variety of ways of putting 15 on the Minicomputer. Some of the many possibilities are shown below.

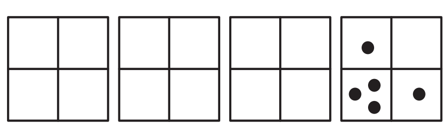
 = 15

 = 15

 = 15

 = 15

 = 15

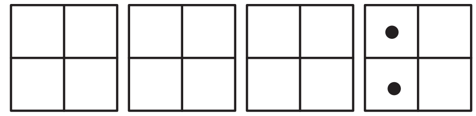
 = 15

Exercise 3

Quickly review trades on the ones board:

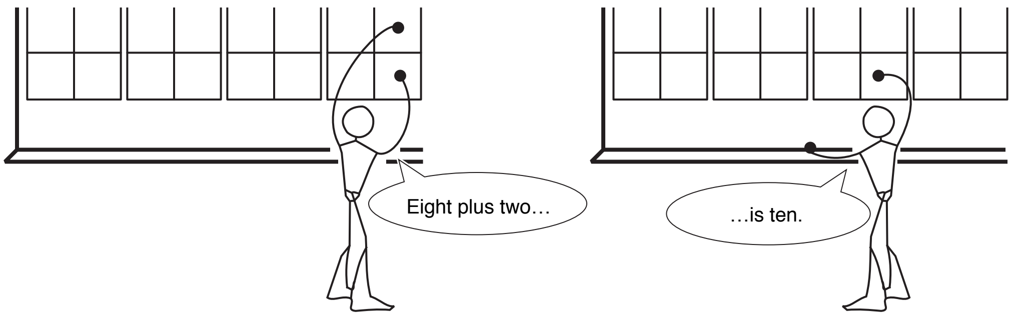
$1 + 1 = 2$ and $2 = 1 + 1$
$2 + 2 = 4$ and $4 = 2 + 2$
$4 + 4 = 8$ and $8 = 4 + 4$

Put one checker on the 8-square and one on the 2-square.

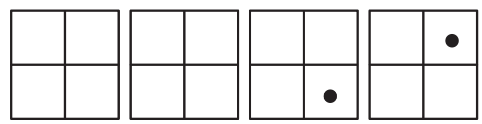


T: ***What number is on the Minicomputer?*** (10)

Pick up the checker on the brown square with one hand and the checker on the red square with the other. Then put one of these checkers on the 10-square and take the other checker away (put it in the chalk tray). As you are making the trade, say, "8 + 2 = 10."



T: ***This is a way to put 10 on the Minicomputer using just one checker. I'll give you one more checker. Can you put 14 on the Minicomputer?***



Remove the checker from the ones board (4-square).

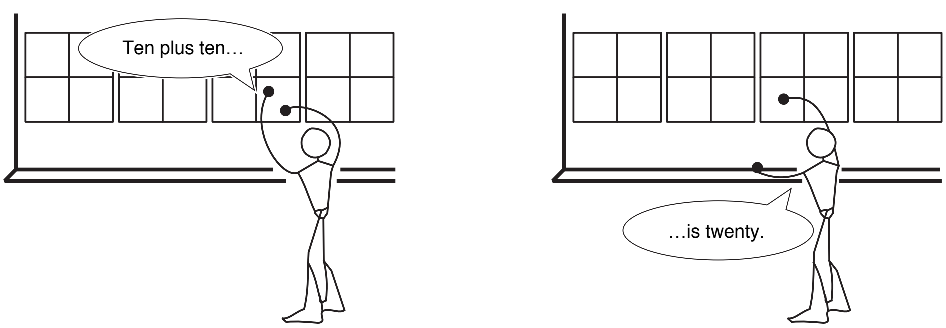
T: ***Who would like to put a secret number on the Minicomputer?***

Whisper a whole number less than 20 to a volunteer. When that number has been put on the Minicomputer, call on another student to say which number it is. Repeat this activity several times.

Ask someone to put 20 on the Minicomputer using two checkers. If no one volunteers, put two checkers on the 10-square.

T: ***There is a way to put 20 on the Minicomputer using just one checker.***

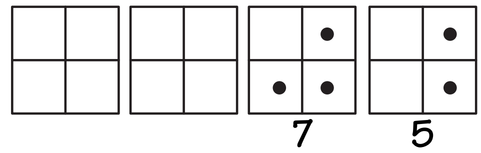
Make the trade and say, "10 + 10 = 20."



Remove the checkers and continue with other numbers.

T: ***Who can put 23 on the Minicomputer? 25? 30?***

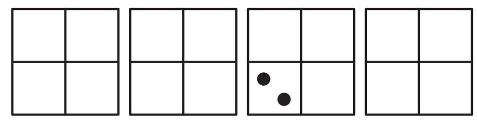
Put this configuration on the Minicomputer.



T: ***What number is this?*** (75)

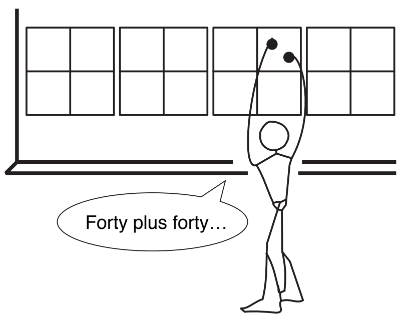
Write 7 below the tens board and 5 below the ones board.

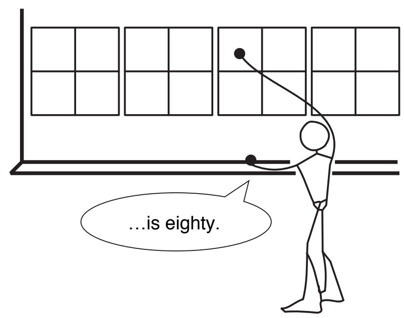
Put two checkers on the 40-square.



T: ***What number is this?*** (80)

Make the trade yourself and say, “40 + 40 = 80.”





Remove the checker from the 80-square.

Move a checker back and forth very quickly from the 1-square to the 10-square. Each time you move the checker, ask the class which number is on the Minicomputer, one—ten, one—ten, and so on.

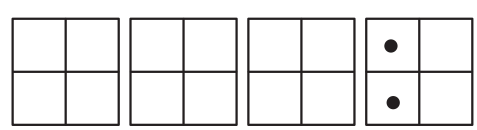
Repeat this activity with 2 and 20; 4 and 40; and 8 and 80.

T: ***Who would like to put a secret number on the Minicomputer?***

Whisper a whole number less than 100 to a volunteer. When that number has been put on the Minicomputer call on another student to say what the number is. Repeat this activity several times.

Exercise 4

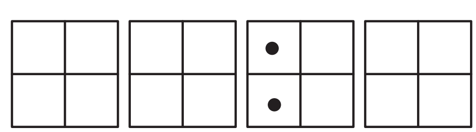
Put this configuration on the Minicomputer.



T: ***What number is this?*** (10)
Who can put 10 on the Minicomputer with only one checker?

Encourage the volunteer to say, “8 + 2 = 10,” as the trade is made.

Put this configuration on the Minicomputer.



T: ***What number is this?*** (100)
Who can put 100 on the Minicomputer with only one checker?

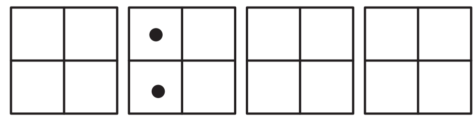
Encourage the volunteer to say, "80 + 20 = 100," as the trade is made.

Move a checker from the 1-square to the 10-square to the 100-square. Each time you move the checker, ask the class what number is on the Minicomputer: 1, 10, and 100. Repeat this activity with 2, 20, and 200; 4, 40, and 400; and 8, 80, and 800.

T: ***Who would like to put a secret number on the Minicomputer?***

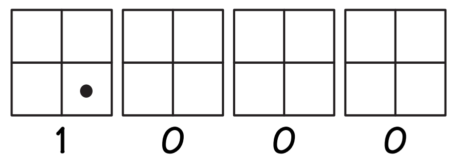
Whisper a whole number between 100 and 200 to a volunteer. When that number has been put on the Minicomputer, call on another student to say which number it is. Repeat this activity several times.

Put this configuration on the Minicomputer.



T: ***What number is this?*** (1000)
Who can put 1 000 on the Minicomputer with only one checker?

Encourage the volunteer to say, "800 + 200 = 1 000," as the trade is made.



T: ***Who can write 1 000 below the Minicomputer?***

Move a checker from the 1-square to the 10-square to the 100-square to the 1000-square. Each time you move the checker, ask the class what number is on the Minicomputer: 1, 10, 100, and 1000. Repeat this activity with 2, 20, 200, and 2000; 4, 40, 400 and 4000; 8, 80, 800, and 8000.

Exercise 5

Display three Minicomputer boards. Position four red and four blue checkers on the Minicomputer as in the illustration on the next page.† Divide the class into two teams (the **RED** team and the **BLUE** team).

T: ***We are going to play a game called Minicomputer Tug-of-War. The BLUE team will play with the blue checkers and the RED team will play with the red checkers.***

The BLUE team starts with the number on the Minicomputer shown with blue checkers. What is the starting number of the BLUE team?

S: ***15.***

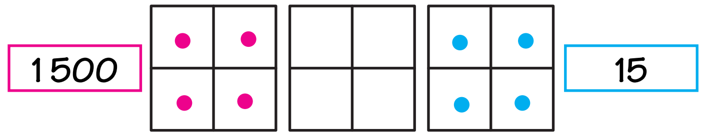
Draw a blue rectangle to the right of the Minicomputer and record 15 inside it.

T: ***The RED team starts with the number on the Minicomputer shown with red checkers. What is the starting number of the RED team?***

S: ***1 500.***

†You m□
However, four checkers give more options in the game.

Draw a red rectangle to the left of the Minicomputer and record 1 500 inside it.



T: ***Teams alternate turns. A player moves one checker on a turn from the square it is in to another square. Players on the* BLUE *team can move blue checkers only and must increase their number. Players on the* RED *team can move red checkers only and must decrease their number. If the* RED *team always decreases its number and the* BLUE *team always increases its number, eventually the red and the blue numbers will tie or pass each other. If the* RED *team makes the red number the same as or less than the blue number, the* RED *team loses. If the* BLUE *team makes the blue number the same as or more than the red number, the* BLUE *team loses.***

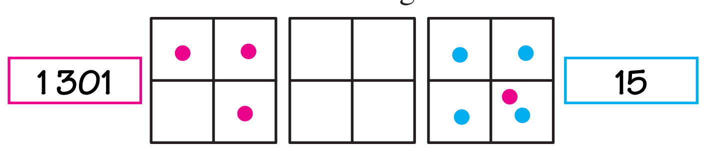
You may like to discuss how this game is like *Tug-of-War*. Then begin playing the game.

Note: We suggest that during this first time playing the *Minicomputer Tug-of-War* game you call on volunteers from each team rather than asking students to play in some order. This will speed up the game and allow students who are unsure of the rules to become more familiar with the game before they take a turn.

A description of a possible game is given here.

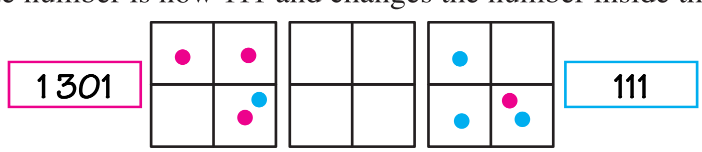
T: ***The* RED *team plays first. You must decrease the red number without tying or passing the blue number.***

A student from the **RED** team moves a checker from the 200-square to the 1-square. The student determines that the red number is now 1 301 and changes the number inside the red box to 1 301.



T: ***The* BLUE *team plays next. You must increase the blue number without tying or passing the red number.***

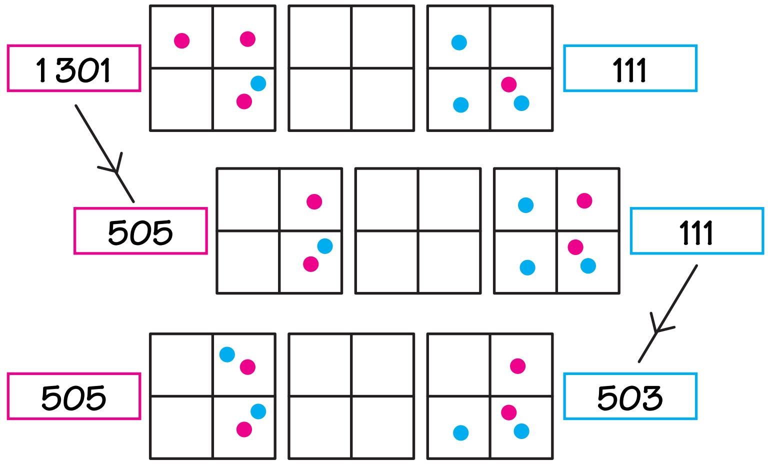
A student from the **BLUE** team moves a checker from the 4-square to the 100-square. The student determines that the blue number is now 111 and changes the number inside the blue box to 111.



T: ***111 is less than 1 301, so the blue number is still less than the red number.***

Note: Do not expect your students to make such big moves on the first few turns. Often students are very cautious at first as they are learning how the game is played.

The game continues as shown below.



It is the **RED** team's turn to play, but no one volunteers and some students claim it is impossible for the **RED** team to play without tying or passing the blue number.

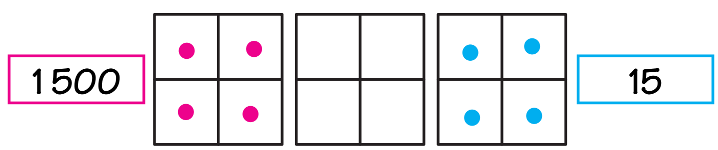
T: ***The* RED *team has to move one of its checkers and decrease the red number without tying or passing the blue number. Is this possible?***

S: ***We can't move the red checker on the 1-square, because there is no square less than 1.***

S: ***We can't move the checker on the 100-square or the 400-square, because we have to keep the 500 (any move with one of these checkers would put us below 500).***

S: ***We can't move the checker on the 4-square either, because the least backward move we can make would be from 4 to 2, and then the red number would be 503.***

Conclude that the **RED** team cannot move without losing, and congratulate the **BLUE** team. Return the checkers to their original positions, change the numbers in the rectangles, and play the game again.



Note: When you play the game with students playing in order, avoid forcing a student to make a losing play. Whenever a student declines to play because he or she cannot find a move that will prevent his or her team from losing, ask if any other member of the team is able to play. If none of the team's members is able to play, conclude that the team has lost the game.

Center Activity

Put desk Minicomputer sets in a center so that students can play *Minicomputer Tug-of-War* in pairs, or groups of four or six.

EN3 MINICOMPUTER INTRODUCTION #3

Capsule Lesson Summary

Review trades on the Minicomputer and the value of the squares. Explore the effect of moving various checkers in a configuration on the Minicomputer. In each case, does the move increase, decrease, or leave the same number on the Minicomputer? Put numbers on the Minicomputer with many checkers, estimate, and make trades to make them easier to read. Find many ways to represent a given number on the Minicomputer.

Materials

Teacher
- Minicomputer set

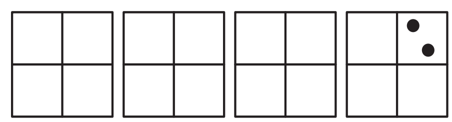
Student
- Paper
- Minicomputer set
- Worksheets EN3*, **, ***, and ****

Advance Preparation: Use the blacklines following this lesson to make copies of student worksheets.

Description of Lesson

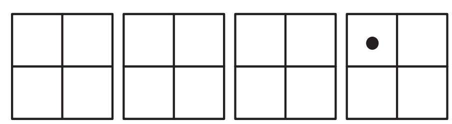
Exercise 1: Minicomputer Review

Put this configuration on the Minicomputer.



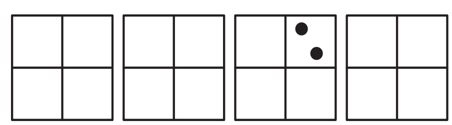
T: ***What number is on the Minicomputer?*** (8)
Could we put on 8 with one checker?

Invite a student make the 4 + 4 = 8 trade.



Demonstrate the 8 = 4 + 4 and 4 + 4 = 8 trades again yourself.

Put this configuration on the Minicomputer.



T: ***What number is on the Minicomputer?*** (80)

Ask a volunteer to make a trade so that 80 will be on the Minicomputer with only one checker. As the trade is made, the student should say, "40 + 40 = 80."

Repeat this activity with the following trades:

400 + 400 = 800	20 + 20 = 40	8 + 2 = 10
4 000 + 4 000 = 8 000	200 + 200 = 400	80 + 20 = 100
	2 000 + 2 000 = 4 000	800 + 200 = 1 000

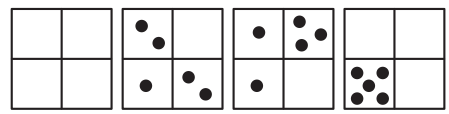
Move a checker from one square to another very quickly to review the value of the squares. Each time you move the checker, ask the class which number is on the Minicomputer.

Review the standard configurations of the following numbers. Move the checkers very quickly from one board to another, each time asking which number is on the Minicomputer.

5; 50; 500; 5 000	9; 90; 900; 9 000
3; 30; 300; 3 000	7; 70; 700; 7 000
6; 60; 600; 6 000	12; 120; 1 200; 12 000

Exercise 2: Transforming a Number

Put this configuration on the Minicomputer.



T: ***I put a number on the Minicomputer.***
It's not easy to read, but let's try to estimate.
Is this number more than 100? How do you know?

S: ***Yes. There are several checkers on the hundreds board.***

T: ***Is this number more than 1 000?*** (Yes)

S: ***The two checkers on the 800-square make 800 + 800 = 1 600 and that is more than 1 000.***

T: ***We know this number is more than 1 000. Is it more than 2 000?*** (Yes)
How do you know?

S: ***Yes, this pair of checkers (800 + 200) is 1 000 and these three checkers (800 + 100 + 100) equal 1 000, so the number is more than 2 000.***

T: ***We know this number is more than 2 000. Is it more than 3 000?***

A student should explain to the class why this number is less than 3 000. Your class may estimate the number to be more than 2 000 and less than 2 500, or perhaps closer to 2 200.

T: ***We do not know exactly what number is on the Minicomputer, but we can still compare it to other numbers.***

Write these words on the board.

More
Less
Same

T: ***I am going to move, remove, or add some checkers.***
Each time, tell me if the number on the Minicomputer is more than, less than, or the same as this number.

Move a checker from the 800-square to the 400-square. Point to each of the three words in turn as you ask,

T: ***Who thinks this number is more than the number we had?***
Who thinks this number is less than the number we had?
Who thinks this number is the same?

Decide on a method for students to show whether they believe the new configuration is for a number more, same, or less than before. For example, they can write three words on an index card and then hold up the card, pinching it on their choice.

Repeat the move very obviously if many students do not know that the number now on the Minicomputer is less than the previous one.

Return the checker to its original position.

Continue this activity with the following moves or similar ones. After each move, return the checkers to their original positions. When appropriate, ask students exactly how much more or less a number is than the previous one.

- Move a checker from the 80-square to the 200-square. (120 more)
- Replace any checker with a checker of another color. (Same)
- Make a 2 + 2 = 4 trade. (Same)
- Move a checker from the 20-square to the 1-square. (19 less)
- Make an 80 + 20 = 100 trade. (Same)
- Move two checkers from the 40-square to the 10-square. (60 less)
- Make an 800 = 400 + 400 trade. (Same)
- Move a checker from the 2-square to the 1 000 square. (998 more)

After you make several moves yourself, invite students to transform the number on the Minicomputer.

At the end of this activity, leave the checkers on the Minicomputer for Exercise 3.

Exercise 3: Estimation

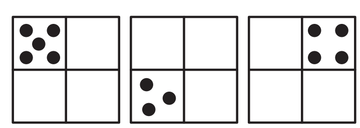
Remind the class that they previously estimated this number to be between 2 000 and 2 500. Invite students to make several trades, and then guide the class to make a closer estimate. You may allow students to guess what the number is and to record some guesses on the board.

Continue making trades until standard configuration is obtained. Invite a student to write the number below (above) the Minicomputer and to determine which guess was the closest. Erase the board and remove the checkers from the Minicomputer.

Display three Minicomputer boards. Give a student twelve checkers as you say,

T: ***Put these checkers on the Minicomputer wherever you want.***

Suppose the student chooses this configuration.



T: ***Is this number more than 100? How do you know?***
Is it more than 1 000? How do you know?
Is it more than 2 000? How do you know?
Is it more than 3 000? How do you know?

Allow the students to guess what number is on the Minicomputer, and record their guesses on the board.

T: ***How can we make the number easier to read?***

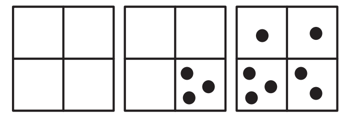
S: ***Make some trades.***

Invite students to make trades and encourage them to announce the trades they make. You may need to initiate backward trades the first few times they are needed. Display a fourth Minicomputer board if the configuration is for a number greater than 999.

When standard configuration for the number is obtained, ask a student to record the number below the Minicomputer. Determine which guess was the closest.

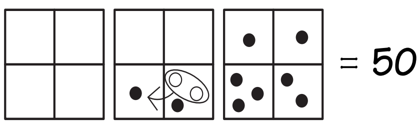
Exercise 4

Put the following configuration of checkers on the Minicomputer gradually, allowing students to calculate the number mentally.

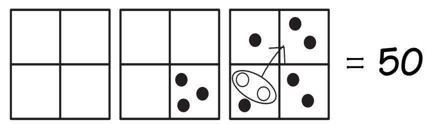


T: ***What number is on the Minicomputer?*** (50)
How many checkers did I use to put 50 on the Minicomputer? (Ten)
Can you put 50 on the Minicomputer using one less checker (that is, nine checkers)?

If the students do not respond, suggest that they make a trade. Any forward trade will solve the problem. For example:



or



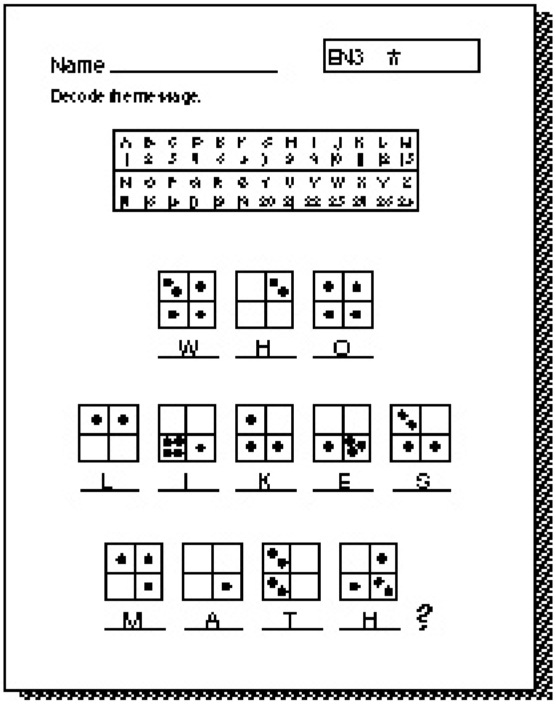
Other solutions involve making a 1 + 1 = 2 trade or a 8 + 2 = 10 trade.

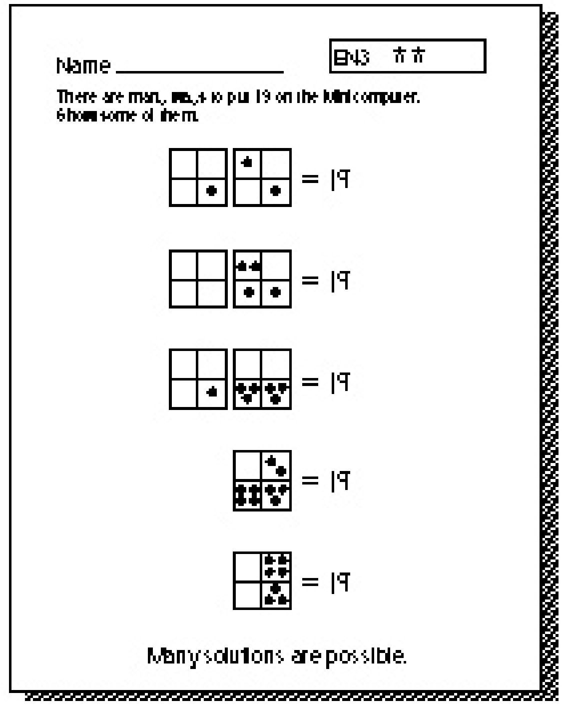
T: ***This is 50 with nine checkers.***
Can you put 50 on the Minicomputer with one less checker (that is, eight checkers)?

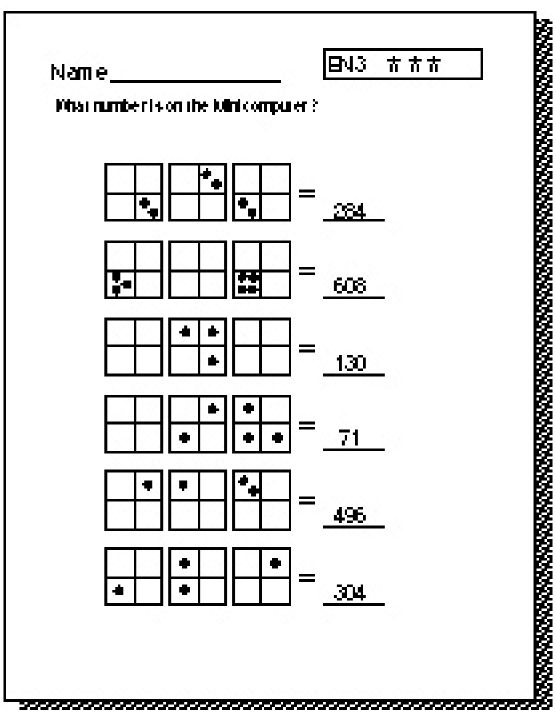
Continue by representing the number 50 on the Minicomputer with seven and then six checkers.

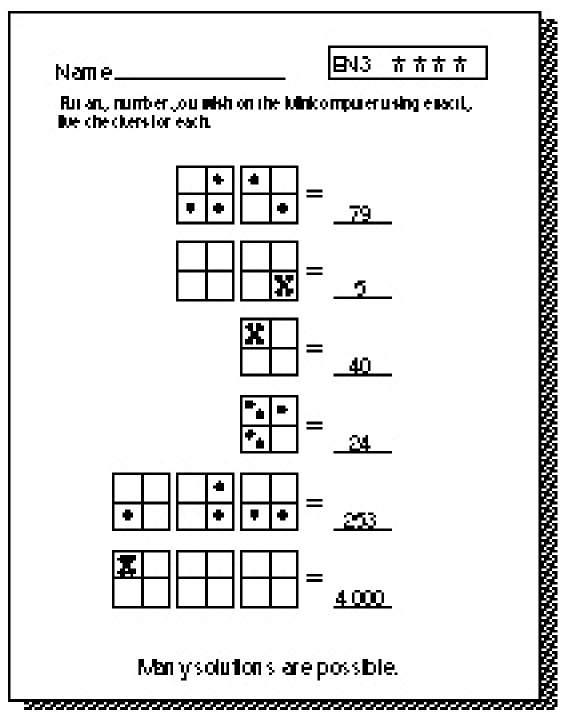
Provide student pairs with desk Minicomputer sets and ten checkers. Instruct them to put a number on with ten checkers, calculate the number, and then proceed to put the same number on with nine, eight, seven, six checkers, and so on (whatever is possible).

Worksheets EN3*,**, ***, and **** are available for individual work.


Name
EN3 ☆
Decode the message.
A B C D E F G H I J K L M
1 2 3 4 5 6 7 8 9 10 11 12 13
N O P Q R S T U V W X Y Z
14 15 16 17 18 19 20 21 22 23 24 25 26
W H O
L I K E S
M A T H ?


Name
EN3 ☆☆
There are many ways to put 19 on the Minicomputer.
Show some of them.
= 19
= 19
= 19
= 19
= 19
Many solutions are possible.


Name
EN3 ☆☆☆
What number is on the Minicomputer?
= 284
= 608
= 130
= 71
= 496
= 304

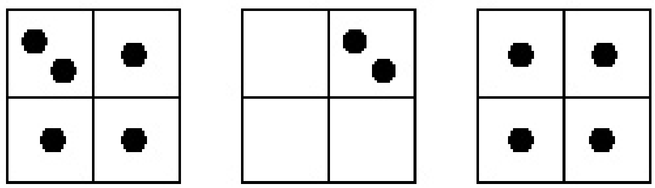

Name
EN3 ☆☆☆☆
Put any number you wish on the Minicomputer using exactly five checkers for each.
= 79
= 5
= 40
= 24
= 253
= 4 000
Many solutions are possible.

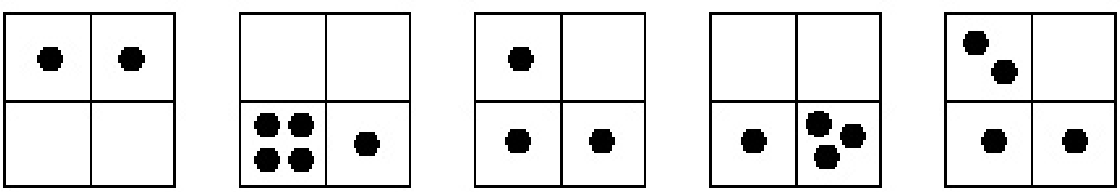
Name ____________________

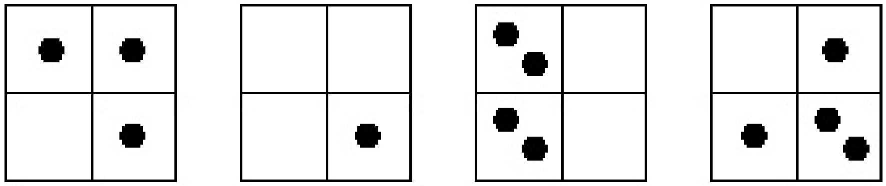
EN3 *

Decode the message.

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26





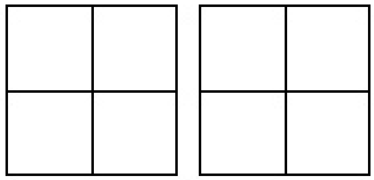


?

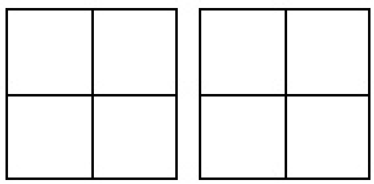
Name ________________

EN3 ★★

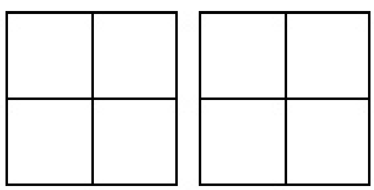
There are many ways to put 19 on the Minicomputer.
Show some of them.



= 19



= 19



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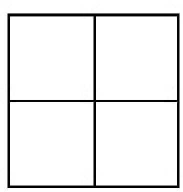

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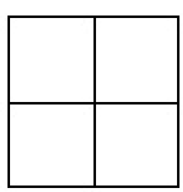
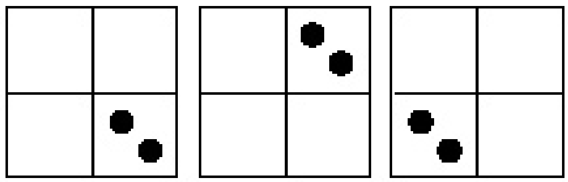


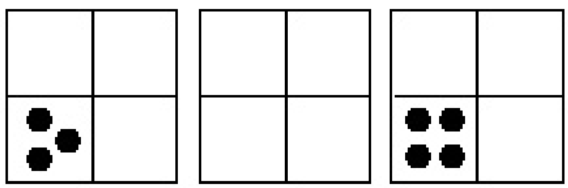
= 19

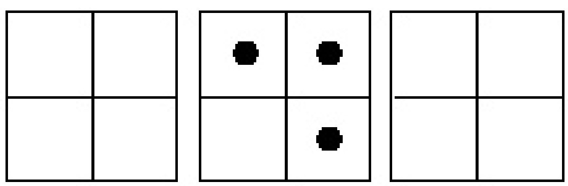
Name ____________________

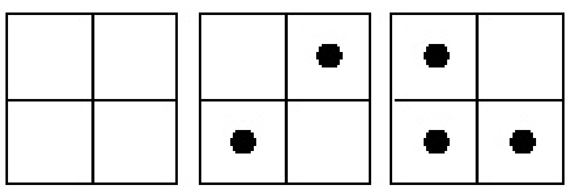
EN3 ★★★

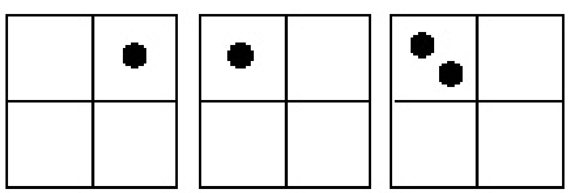
What number is on the Minicomputer?

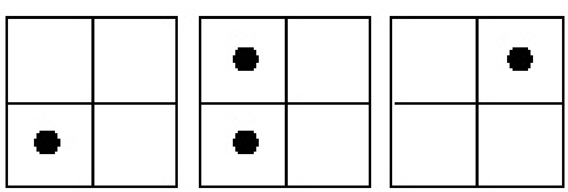
 = ______

 = ______

 = ______

 = ______

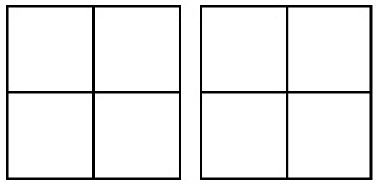
 = ______

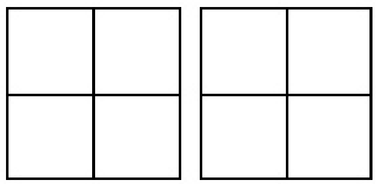
 = ______

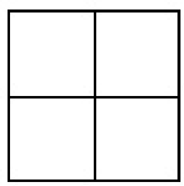
Name ____________________

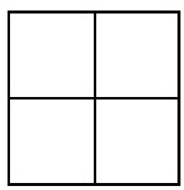
EN3 ★★★★

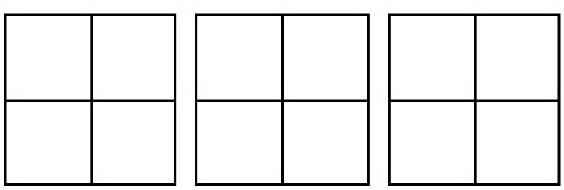
Put any number you wish on the Minicomputer using exactly five checkers for each.

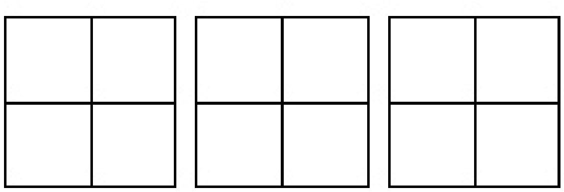
 = ______

 = ______

 = ______

 = ______

 = ______

 = ______

EN4 MINICOMPUTER INTRODUCTION #4

Capsule Lesson Summary

Use the Minicomputer to solve addition, subtraction, and multiplication problems. Multiply by 10 on the Minicomputer, and introduce the ⑩-checkers together with a big trade: ten checkers on a square is the same as one checker on the same color square one board to the left.

Materials

Teacher
- Minicomputer set
- ⑩-checkers

Student
- Paper
- Minicomputer set
- Worksheets EN4*, **, and ***

Advance Preparation: Use the blacklines following this lesson to make copies of student worksheets.

Description of Lesson

Exercise 1: Addition Problems

Write this problem on the board.

280 × 160 =

T: ***Can you think of a story problem for which we need to do this calculation?***

Allow students to compose (write) a story problem with a math partner. Accept a couple example story problems, but don't allow anyone to give an answer to the calculation at this moment.

When students respond to the following estimation questions, ask them to explain their answers.

Note: Some students may know 280 + 160 = 440. Accept this knowledge but discuss why you may want an estimate rather than an exact answer. Estimates are sometimes what we use to compare numbers or to check that our results are reasonable.

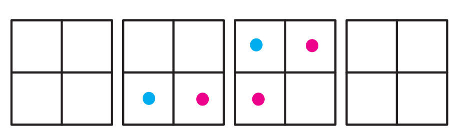
T: ***Do you think 280 + 160 is more or less than 200?*** (More)
More or less than 300? (More) ***More or less than 500?*** (Less)
We know that 280 + 160 is between 300 and 500. Would it be more or less than 400?

If the students disagree, say, "We don't know if it is more or less than 400, but we do know it is between 300 and 500." If the class is certain that 280 + 160 is more than 400, say, "Now we know it is between 400 and 500."

T: ***What number is 280 + 160?***

Accept several guesses and list them on the board. Insist that guesses be between 300 and 500 (or between 400 and 500 if this has been determined).

T: ***Who would like to put 280 on the Minicomputer using blue checkers?***
Who would like to put 160 on the Minicomputer using red checkers?



Invite students to make trades so that the number will be easier to read. When all the trades have been made, call on a student to write 440 below the Minicomputer and conclude that 280 + 160 = 440.

400
420 < 440
450 > 440
460

Point to the list of estimates on the board. Acknowledge anyone who correctly predicted the answer. If no one guessed correctly, determine which estimate was the best. Indicate whether certain estimates are more or less than the sum. For example, if you had 420 and 450 as estimates, you would indicate that 420 is less than 440, and that 450 is more than 440. However, 450 is the best estimate because it is only 10 more than 440.

Note: Use the symbols > and < throughout the year whenever you have made a list of estimates and wish to decide which is the best estimate. These symbols are introduced in the *CSMP* first grade program. Most entry students will have been introduced to these symbols in other math programs.

Repeat this exercise with 256 + 485 = 741.

Exercise 2: Subtraction Problems

First pose a story problem for subtraction. For example:

T: ***One day I opened a package from a friend and found 37 bright orange rocks. I gave 14 of them away. How many did I have to keep? What do you think I should do with them?***

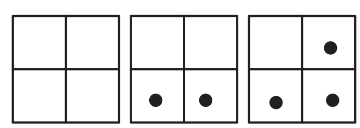
37 – 14 =

Write this problem on the board.

When students respond to these estimation questions, ask them to explain their answers.

T: ***Do you think 37 – 14 is more or less than 30?*** (Less) ***More or less than 20?*** (More)
We know that 37 – 14 is between 20 and 30. What number do you think it is?

If you get an answer that is not between 20 and 30, remind the class that they already know 37 – 14 is between 20 and 30.

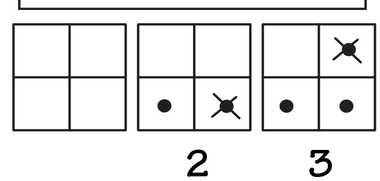


T: ***Who can put 37 on the Minicomputer?***
What should we do next?

S: ***Take away 14.***

Note: When students are asked to put numbers on the Minicomputer, they may or may not use standard configurations for the numbers. The lesson description assumes that standard configurations of numbers are put on the Minicomputer. Adjust the lesson depending upon which configurations your students display.

An "X" through a checker indicates that the checker has been removed from the Minicomputer.



A student should remove a checker from the 10-square and a checker from the 4-square.

Ask a student to write 23 below the Minicomputer, and conclude that 37 – 14 = 23.

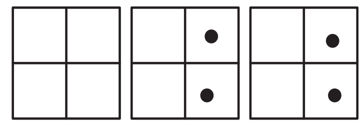
Repeat this exercise with 53 – 42 = 11 and 76 – 34 = 42. As in Exercise 1, give math partners a few minutes to compose (write) story problems for which you need to do the calculations. Let students share some of their stories.

Exercise 1: Multiplication on the Minicomputer

Put the pictured configurations on the Minicomputer as you tell this or another multiplication story.

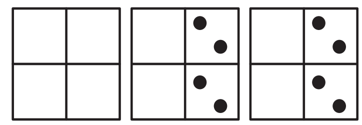
T: ***Lyle is in charge of cleaning potatoes for a big Thanksgiving dinner at the community center. He puts the number of potatoes from one bag on the Minicomputer.***

How many potatoes in one bag? (55)



When he finishes cleaning the potatoes from one bag, he counts the potatoes in a second bag and puts the number on the Minicomputer.

How many potatoes in two bags?
What number is this?



Suggest students write their responses on paper or whisper them to a neighbor. Then call on several students to explain how to calculate the number on the Minicomputer. Include mention of 2 x 55 = 110.

T: ***What number is on the ones board?***

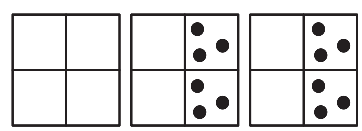
S: ***10; 2*** x ***5 = 10.***

T: ***What number is on the tens board?***

S: ***100; 2*** x ***50 = 100.***

T: ***2*** x ***55 = 110.***
Now Lyle puts the number of potatoes from a third bag on the Minicomputer.
How many potatoes in three bags?
What number is this?

$$\begin{array}{r} 2 \times 5 = 10 \\ 2 \times 50 = 100 \\ \hline 2 \times 55 = 110 \end{array} \qquad \begin{array}{r} 55 \\ \times\ 2 \\ \hline 10 \\ 100 \\ \hline 110 \end{array}$$



Again, suggest students write their responses on paper or whisper them to a neighbor. Then call on several students to explain how to calculate the number on the Minicomputer. Include mention of 3 x 55 = 165.

T (pointing to the ones board)**: *What number is 3*** x ***5?***

S: ***15.***

T (pointing to the tens board)**: *What number is 3*** x ***50?***

S: ***150.***

T: ***What number is 15 + 150?***

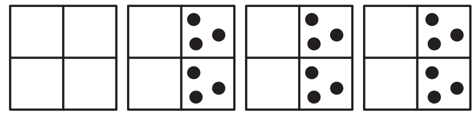
S: ***165.***

T: ***3*** x ***55 = 165***

$$\begin{array}{r} 3 \times 5 = 15 \\ 3 \times 50 = 150 \\ \hline 3 \times 55 = 165 \end{array} \qquad \begin{array}{r} 55 \\ \times\ 3 \\ \hline 15 \\ 150 \\ \hline 165 \end{array}$$

Continue by adding three 500s to the configuration on the Minicomputer. Ask students to do this calculation on their papers and then get explanations.

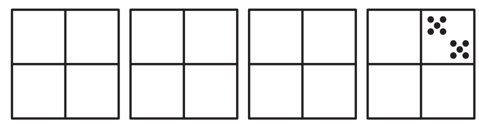
$$\begin{array}{l}3 \times 5 = 15\\ 3 \times 50 = 150\\ \underline{3 \times 500 = 1\,500}\\ 3 \times 555 = 1\,665\end{array} \qquad \begin{array}{r}555\\ \times\ 3\\ \hline 15\\ 150\\ 1\,500\\ \hline 1\,665\end{array}$$



$3 \times 555 = 1\,665$

Exercise 2: Multiplication by 10

Put this configuration on the Minicomputer.



T: ***What multiplication calculation is shown on the Minicomputer?***

S: ***10* x *4.***

Write the calculation on the board.

10×4

Note: If a student says 4 x 10, remark that although 4 x 10 = 10 x 4, four checkers on the 10-square of the Minicomputer would be a better way to show 4 x 10.

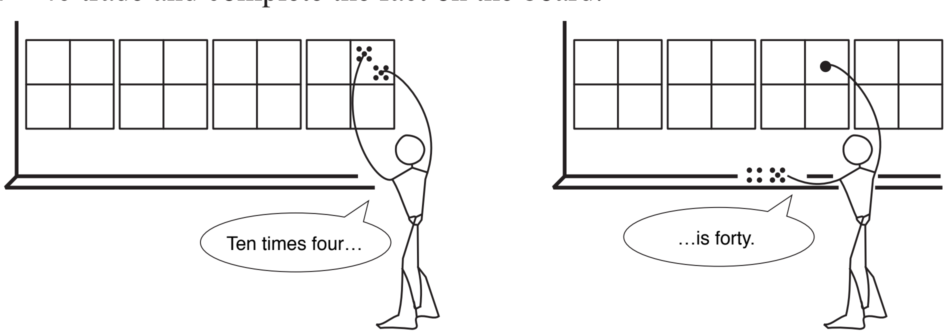
T: ***What number is this? Write it on your paper.***

Check several responses before asking a student to answer aloud.

S: ***40.***

T: ***10* x *4 = 40. Ten checkers on this purple square (4-square) is the same as one checker on this purple square (40-square).***

Make a 10 x 4 = 40 trade and complete the fact on the board.



Repeat this activity for 10 x 40 and 10 x 400.
Write each number sentence below the previous one.

$$\begin{array}{r}10 \times 4 = 40\\ 10 \times 40 = 400\\ 10 \times 400 = 4\,000\end{array}$$

T: ***Do you see a pattern?***

Encourage different responses. If a student suggests that you simply write another 0 on the right when you multiply a number by 10, accept this idea but do not emphasize it yourself.

Note: Putting 0 to the right of the numeral works only when an integer is multiplied by 10. It does not work for non-integer decimals; for example, 10 x 0.3 is not 0.30.

Extend the list on the board another step with 10 x 4 000.

T: ***What number is 10 x 4 000? How do you know?***

S: ***40 000. 40 has one zero, 400 has two zeros, 4 000 has three zeros, and 40 000 has four zeros.***

Ask students to complete the number sentences on the board. Point out that a comma may be inserted between the thousands place and the hundreds place to help us read large numbers. The digits to the left of the comma tell us how many thousands there are.

$$10 \times 4 = 40$$
$$10 \times 40 = 400$$
$$10 \times 400 = 4\,000$$
$$10 \times 4000 = 40\,000$$

Erase the board and remove the checkers from the Minicomputer.

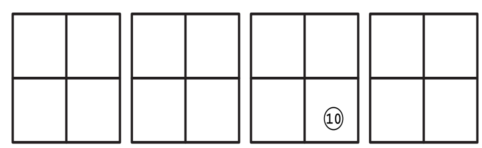
T (holding up a ⑩-checker)**:** ***Let's use this ⑩-checker to put ten checkers on the same square.***

Put a ⑩-checker on the 10-square.

T: ***What number is this?***

S: ***100.***

T: ***10 x 10 = 100***

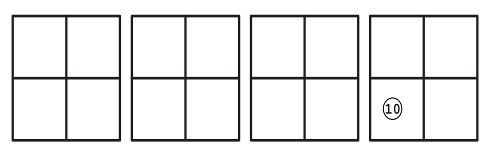


Move the ⑩-checker to the 2-square.

T: ***What number is this?***

S: ***20.***

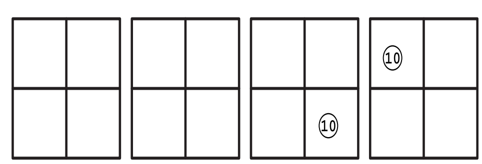
T: ***10 x 2 = 20.***



Put this configuration on the Minicomputer.

T: ***What calculation do you see on the Minicomputer?***

S: ***10 x 18.***



Write the calculation on the board.

T: ***What number is this? Write it on your paper.***

$$10 \times 18 =$$

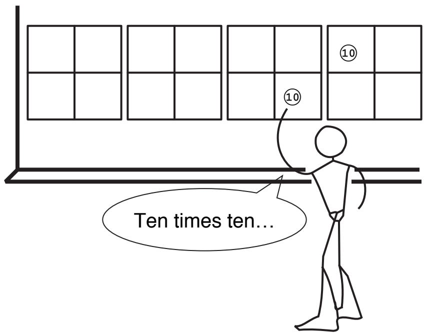
Check several responses before asking a student to answer aloud.

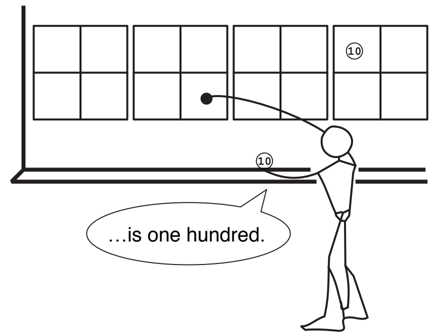
S: ***180.***

EN4

T: ***10 x 18 = 180. Ten checkers on this white square (10-square) is the same as one checker on this white square (100-square).***

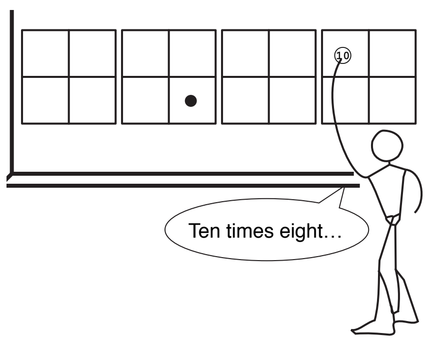
Make a 10 x 10 = 100 trade.

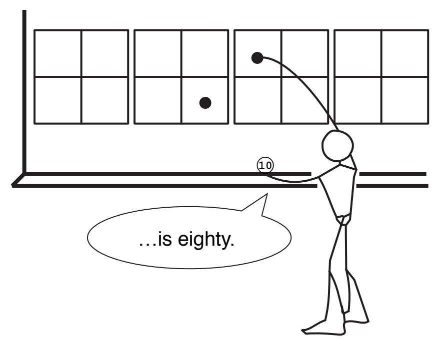




T: ***Ten checkers on this brown square (8-square) is the same as one checker on this brown square (80-square).***

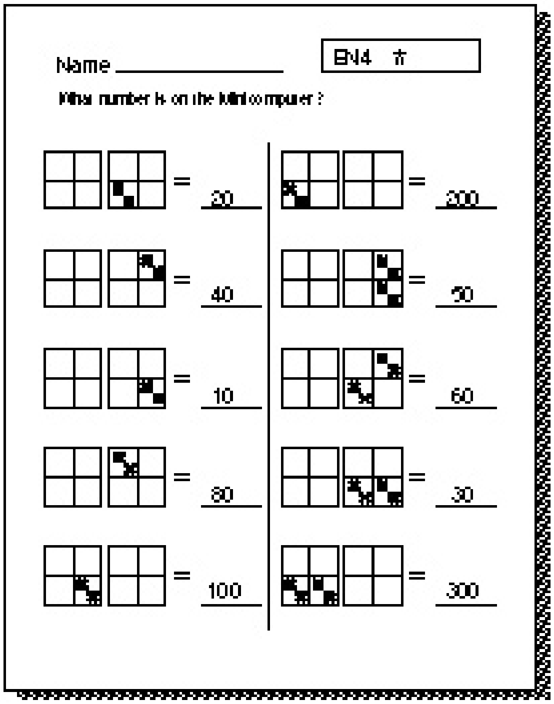
Make a 10 x 8 = 80 trade and complete the calculation on the board.





Repeat this activity with 10 x 180, 10 x 25, and 10 x 250.

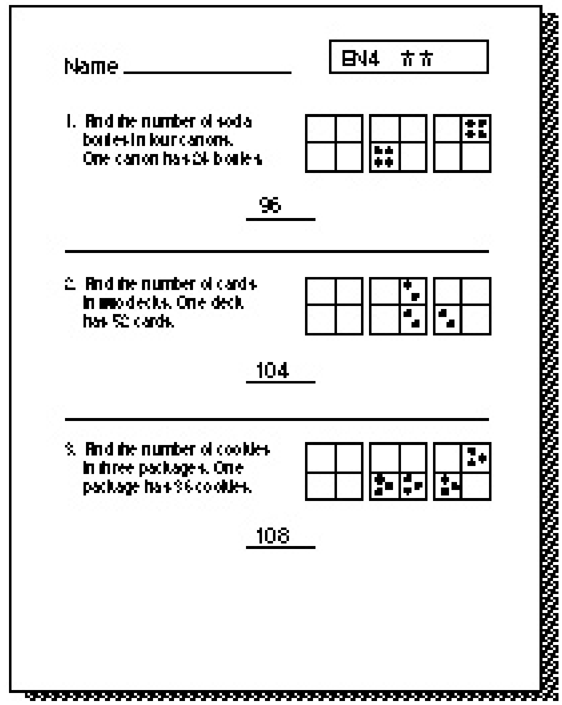
Worksheets EN4*, **, and *** are available for independent work. Provide individual Minicomputers to students who wish to use them.



Name ____________ EN4 ☆

What number is on the Minicomputer?

= 20 | = 200
= 40 | = 50
= 10 | = 60
= 80 | = 30
= 100 | = 300



Name ____________ EN4 ☆☆

1. Find the number of soda bottles in four cartons. One carton has 24 bottles.

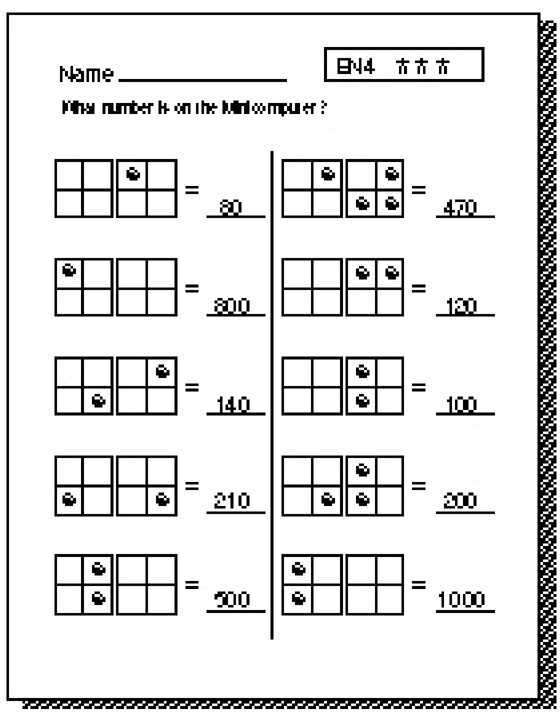
96

2. Find the number of cards in two decks. One deck has 52 cards.

104

3. Find the number of cookies in three packages. One package has 36 cookies.

108



Name ____________ EN4 ☆☆☆

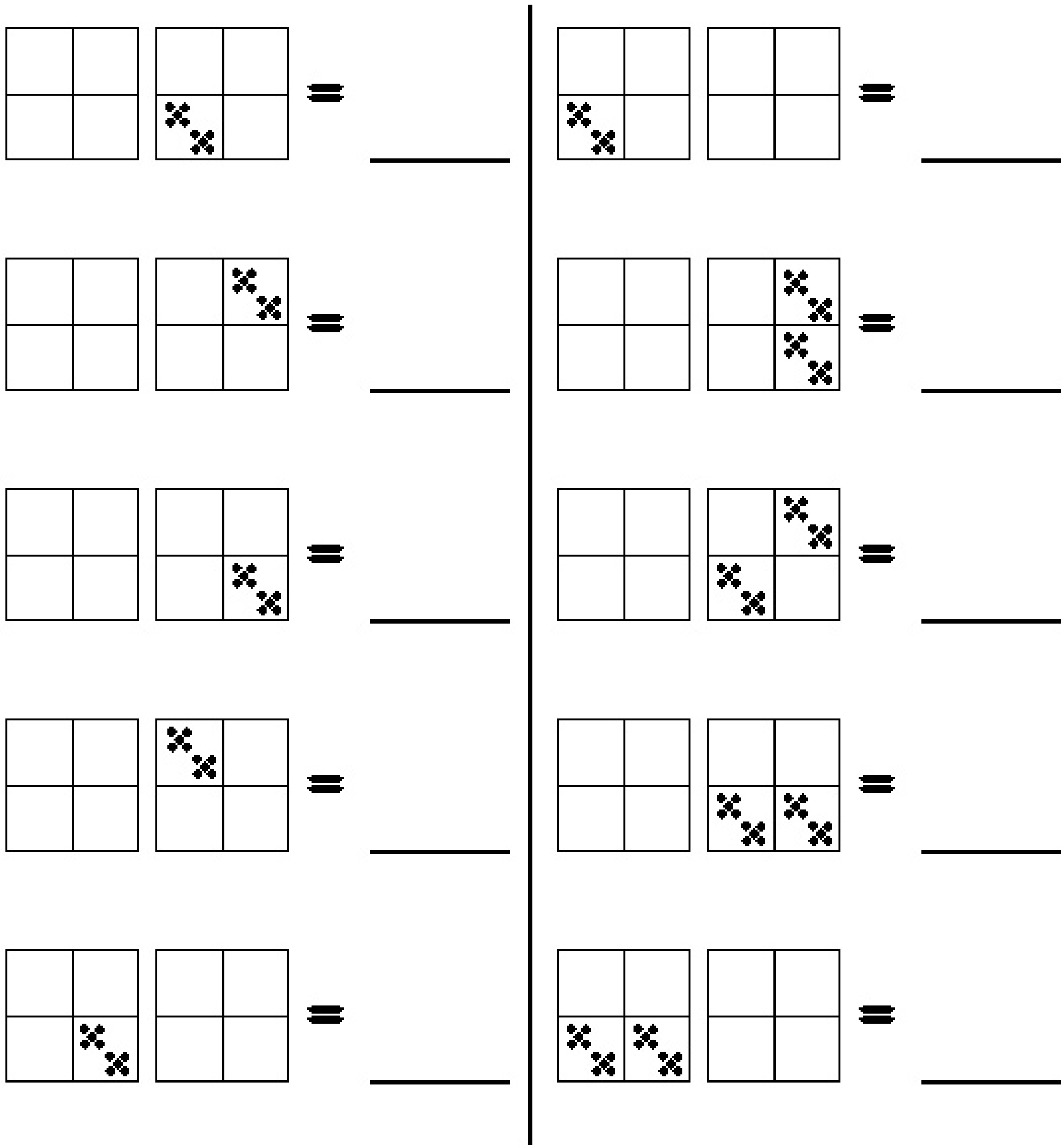
What number is on the Minicomputer?

= 80 | = 470
= 800 | = 120
= 140 | = 100
= 210 | = 200
= 500 | = 1000

Name ____________________

EN4 *

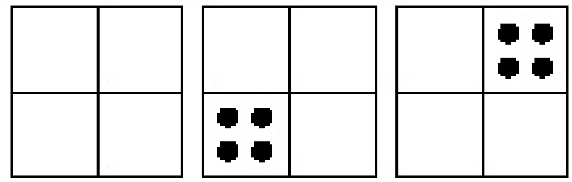
What number is on the Minicomputer?



Name ____________________

EN4 **

1. Find the number of soda bottles in four cartons. One carton has 24 bottles.



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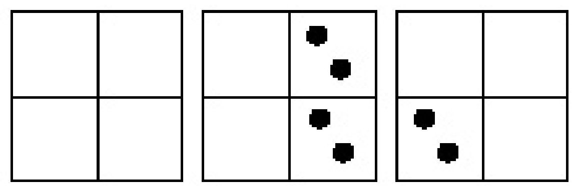
2. Find the number of cards in two decks. One deck has 52 cards.



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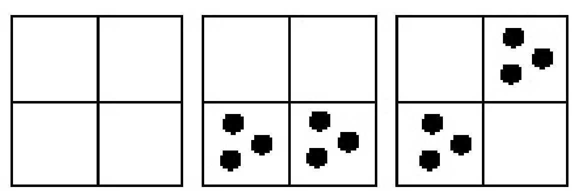
3. Find the number of cookies in three packages. One package has 36 cookies.



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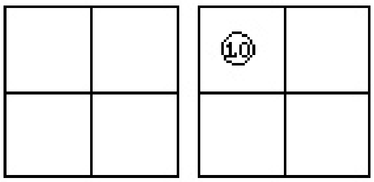
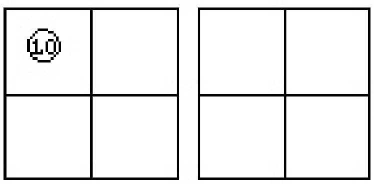
Name ____________________

EN4 ★★★

What number is on the Minicomputer?


= ______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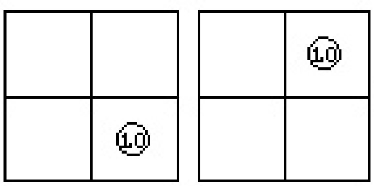
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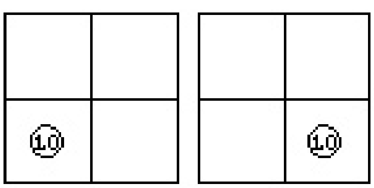
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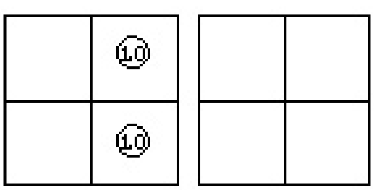
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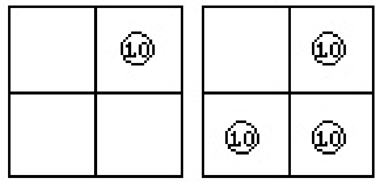
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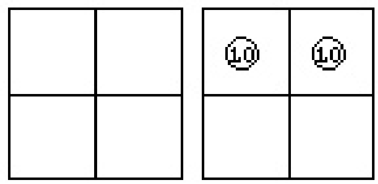
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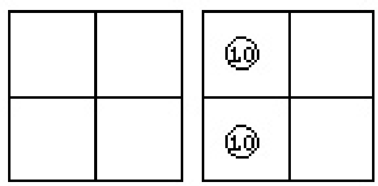
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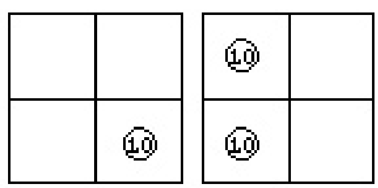
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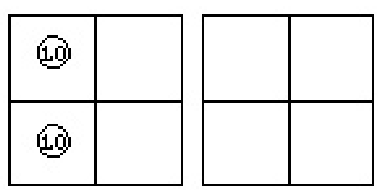
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EN5 DOUBLING AND HALVING ON THE

Capsule Lesson Summary

Find a configuration of checkers on the Minicomputer. Do some mental arithmetic involving doubling and halving. Consider a configuration on the Minicomputer in which the checkers are grouped in pairs on the same square. First recognize what one-half of the number is and then find the number. Record the situation in an arrow picture. Use the Minicomputer to calculate $\frac{1}{2}$ x 1 994.

Materials

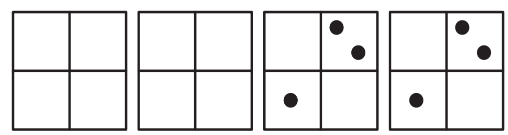
Teacher
- Minicomputer set
- Colored chalk

Student
- Paper

Description of Lesson

Exercise 1

Put this configuration on the Minicomputer.



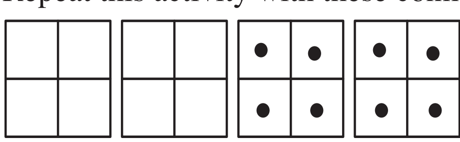
T: ***What number is on the Minicomputer? Write it on your paper (or whisper it to your neighbor).***

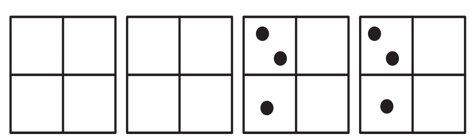
Look at (or listen to) many of the students' answers.

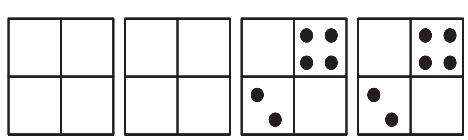
T: ***What number is on the ones board?*** (10)
What number is on the tens board? (100)
What number is this? (110) ***How do you know?***

Let students announce which calculations they did. If many are uncertain that this number is 110, ask a volunteer to make trades on the Minicomputer until you have the standard configuration for 110.

Repeat this activity with these configurations.

 = 165

 = 198

 = 220

Exercise 2

Conduct a brief mental arithmetic exercise involving doubling and halving. A sequence of problems is suggested below; answers are in the boxes. Try to involve all the students by varying the difficulty of the calculations. Occasionally ask students to explain how they calculated an answer.

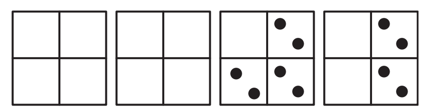
$2 \times 20 = \boxed{40}$	$2 \times 30 = \boxed{60}$	$2 \times 50 = \boxed{100}$	$2 \times 36 = \boxed{72}$
$\frac{1}{2} \times 40 = \boxed{20}$ †	$\frac{1}{2} \times 60 = \boxed{30}$	$\frac{1}{2} \times 100 = \boxed{50}$	$\frac{1}{2} \times 72 = \boxed{36}$
$2 \times 25 = \boxed{50}$	$2 \times 45 = \boxed{90}$	$2 \times 51 = \boxed{102}$	$2 \times 49 = \boxed{98}$
$\frac{1}{2} \times 50 = \boxed{25}$	$\frac{1}{2} \times 90 = \boxed{45}$	$\frac{1}{2} \times 102 = \boxed{51}$	$\frac{1}{2} \times 98 = \boxed{49}$

Exercise 3

T: ***I am going to put a number on the Minicomputer. As I put on checkers, try to calculate, and when you think you know the number, write it on your paper.***

Gradually put this configuration on the Minicomputer, pausing after each step:

- start with two checkers on the 40-square;
- then put two checkers on the 10-square;
- then put two checkers on the 20-square;
- then put checkers on the 4-square and the 1-square (5), and then 5 again.



Look at many of the students' answers before asking someone to announce the number.

S: ***150.***

T: ***What calculations did you do to decide that the number is 150?***

Let several students tell which calculations they did; a few possibilities are given here.

S: ***40 + 40 = 80; 80 + 20 = 100; and 100 + 20 + 20 = 140; and 140 + 5 + 5 = 150.***

S: ***The checkers on the 40-square and on the 10-square together make 100. 100 + 40 + 10 = 150.***

S: ***The number on the tens board is 140 because 2 x 70 = 140, and the number on the ones board is 10 because 2 x 5 = 10. 140 + 10 = 150.***

T: ***Look at the Minicomputer. What number is $\frac{1}{2} \times 150$?***

S: ***75.***

T: ***Can you tell that $\frac{1}{2} \times 150 = 75$ just by looking at these checkers on the Minicomputer?***

S: ***The number on the Minicomputer is 150. The checkers are in pairs, so you can just look at one checker in each pair.***

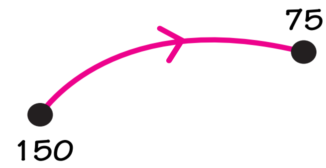
Record this number sentence on the board.

$$\frac{1}{2} \times 150 = 75$$

T: ***Let's show this in an arrow picture.***

† Read $\frac{1}{2} \times 40$ as "one-half of forty."

Draw this arrow picture on the board.



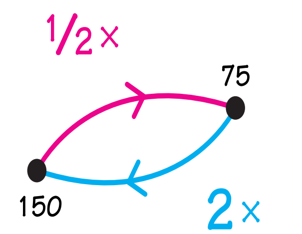
T: ***What could the red arrow be for?***

S: *$\frac{1}{2}$x.*

Note: The arrow could also be for –75 or "is more than" or several other relations, but in this case we are interested in $\frac{1}{2}$x.

Write $\frac{1}{2}$x in red near the arrow picture.

T (tracing the $\frac{1}{2}$x arrow)**:** ***$\frac{1}{2}$ x 150 = 75. If I drew an arrow here*** (trace an arrow from 75 to 150)***, what could it be for?***



S: *2x.*

Draw a blue arrow from 75 to 150 and label it 2x.

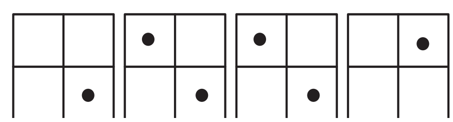
T: ***What number sentence is told by this blue arrow?***

S: ***2 x 75 = 150.***

Record this number sentence on the board.

$$\frac{1}{2} \times 150 = 75$$
$$2 \times 75 = 150$$

Erase the board and then put this configuration on the Minicomputer.



T: ***What number is on the Minicomputer? Write it on your paper.***

Look at several students' answers. Call on a student to write this number (1 994) on the board and to read it aloud.

S: ***One thousand, nine hundred, ninety-four.***

T: ***What number is $\frac{1}{2}$ x 1 994. Is $\frac{1}{2}$ x 1 994 more than 500? How do you know?***

S: ***Yes, because $\frac{1}{2}$ x 1 000 = 500 and 1 994 is more than 1 000.***

T: ***Is $\frac{1}{2}$ x 1 994 more than 1 000? How do you know?***

S: ***No, because $\frac{1}{2}$ x 2 000 = 1 000 and 1 994 is less than 2 000.***

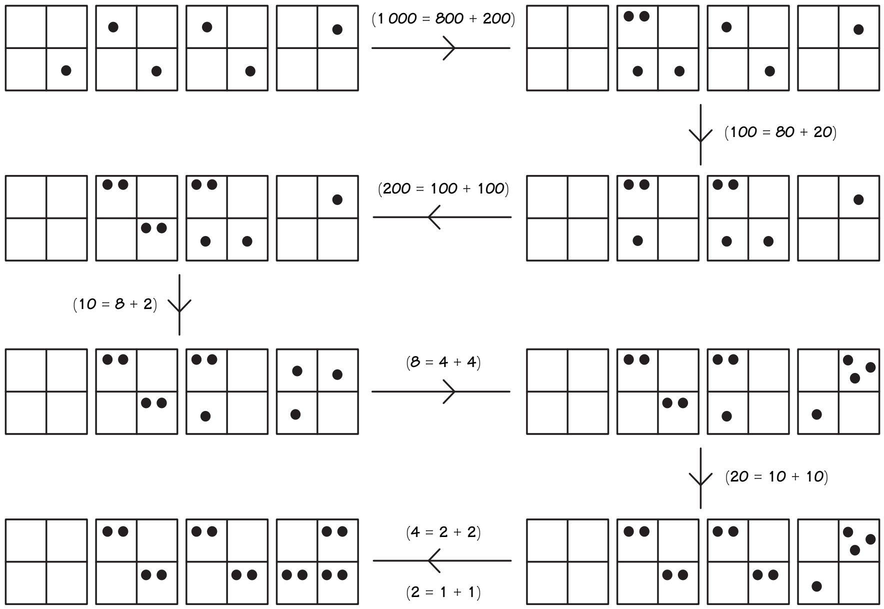
T: ***So we know that $\frac{1}{2}$ x 1 994 is between 500 and 1 000.***

If your students are able to give good explanations, ask if $\frac{1}{2}$ x 1 994 is more than 800 or even more than 900. Then solicit an estimate for $\frac{1}{2}$ x 1 994 and record some estimates on the board, but only record those between 500 and 1 000.

T: ***How can we find exactly what number $\frac{1}{2}$ x 1 994 is?***

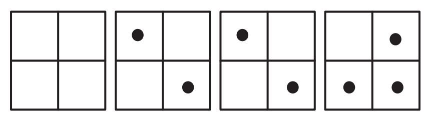
S: ***We could make backward trades and get checkers in pairs (the two checkers in any pair being on the same square).***

Invite students to make trades. Any initial backward trade will result in a pair of checkers on one of the squares. Continue this activity until all the checkers are in pairs. Discourage a student who wants to make a trade with checkers that are already paired. One possible sequence of trades is shown here.



When all the checkers are paired, ask a student to remove checkers so that one-half of the number will be on the Minicomputer. The student should remove one checker from each pair of checkers.

Perhaps you will have this configuration on the Minicomputer.



T: ***What number is*** $^{1}/_{2}$ **x** ***1 994?***

S: ***997.***

Write $^{1}/_{2}$ x 1 994 = 997 to one side of the Minicomputer.

T: ***How could we check that 997 is*** $^{1}/_{2}$ **x** ***1 994?***

S: ***We could calculate 997 + 997.***

T: ***If*** $^{1}/_{2}$ **x** ***1 994 = 997, then 2*** **x** ***997 = 1 994. Calculate 997 + 997 on your paper.***

Conclude that 2 x 997 = 1 994 and $^{1}/_{2}$ x 1 994 = 997. With the class, decide which estimate was closest to 997.

Give student pairs a desk Minicomputer set. Call out several numbers for students to find one-half of ($^{1}/_{2}$x) on their Minicomputers.

Center Activity

Write a large number on a paper near a Minicomputer set in a center. Instruct students who visit the center to estimate what one-half of that number is, write their estimate, and then use the Minicomputer to calculate one-half of the number. Change the number periodically.

Writing Activity

Instruct students to write a story in which they need to find one-half of a large number.

Capsule Lesson Summary

Consider an arrow picture with +6 and +5 arrows. Locate the greatest and least numbers in this picture. Locate six numbers in an arrow picture with +10 and –3 arrows where there are two pieces to the picture. Consider arrow roads using 2x and –3 arrows, and try to find roads with nine arrows that start at 5 and end at specified numbers.

Materials

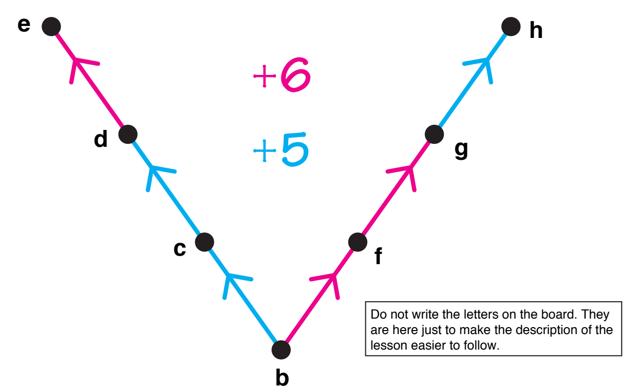
Teacher
- Minicomputer set
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

Draw this arrow picture on the board.



T: ***You don't know yet what numbers are in this picture, but you still can answer some questions about them.***

Is this number (at* c*) more or less than this number (at* b*)? (More)
How much more? (5 more)
Is this number (at* b*) more or less than this number (at* d*)? (Less)
How much less? (10 less)
How do you know? (5 + 5 = 10)

Ask similar questions for other pairs of numbers; for example:

- the numbers at **g** and **b** (The number at **g** is 12 more than the number at **b**.)
- the numbers at **b** and **h** (The number at **b** is 17 less than the number at **h**.)
- the numbers at **e** and **b** (The number at **e** is 16 more than the number at **b**.)

T: ***Where is the least number in the picture? Where is the greatest number?***

S (point to **b**)**:** ***This number is least because if you start here, you add to get all of the other numbers.***

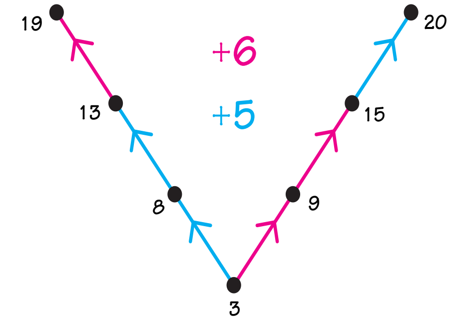
S (point to **h**)**:** ***This number is greatest because it is 17 more than the least number (at* b*). The number here (at* e*) is only 16 more than the least number (at* b*).***

T: ***20 is the greatest number in this arrow picture.***

Put 20 at **h** and then collectively label one or two other dots.

T: ***Copy the arrow picture and finish labeling the dots.***

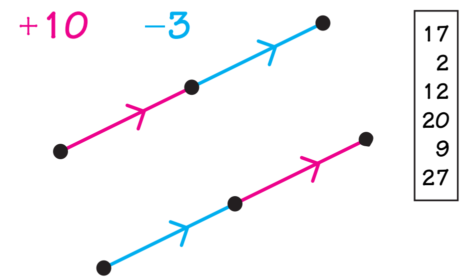
After a while, invite students to the board to label the dots.



Exercise 2

T: ***Copy this arrow picture as I draw it on the board.***

The six dots in this arrow picture are for six numbers: 17, 2, 12, 20, 9, and 27. (Point to one of the red arrows.) ***Which two of these numbers could be joined by a red arrow?***

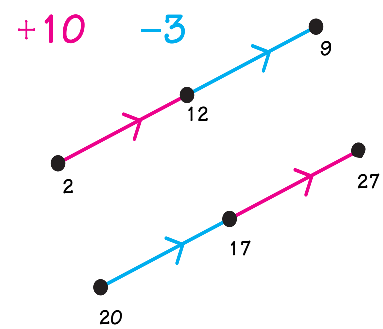


S: ***2 and 12, because 2 + 10 = 12.***

S: ***17 and 27, because 17 + 10 = 27.***

T: ***With this clue, try to place these six numbers in the arrow picture.***

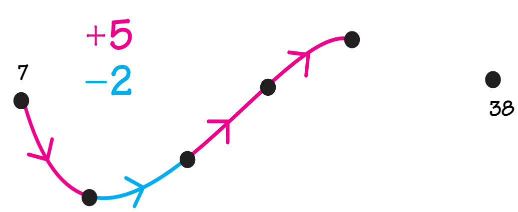
As you observe the individual work, remind students of the above clue as necessary. When most of the students have finished, let one student label the dots in the picture on the board.



Exercise 3

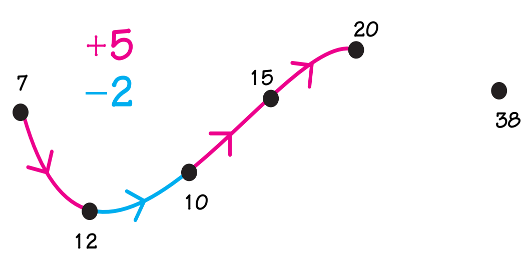
Draw this arrow picture on the board.

T: ***Our goal is to build an arrow road from 7 to 38 using +5 and –2 arrows. The first few arrows of a road have already been drawn. Who can label the dots?***

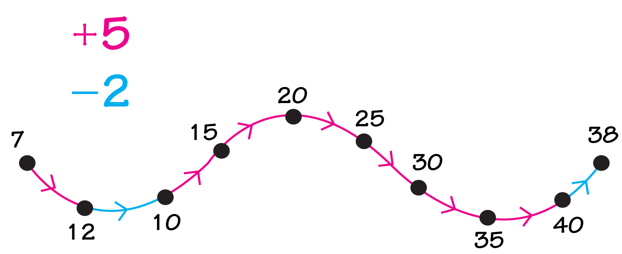


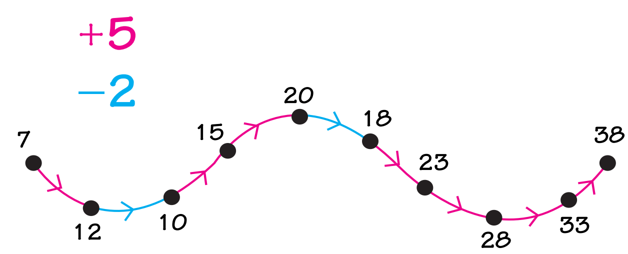
Invite students to complete this part of the picture.

T: ***Copy this picture onto your paper. Then use +5 arrows and –2 arrows to complete the road by going from 20 to 38. Many solutions are possible. Try to find one of them.***



After a while, let students draw different solutions on the board. Two correct arrow roads are shown below.

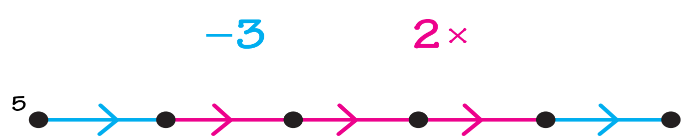




Many solutions are possible. Ask students if they notice any similarities among the various roads from 7 to 38. They are likely to notice that most roads discovered by their classmates have nine arrows: seven +5 arrows and two –2 arrows. Actually, many other longer roads are possible, for example, sixteen arrows with nine +5 arrows and seven –2 arrows. You may like to ask if anyone found an arrow road from 7 to 38 with more than nine arrows.

Exercise 4

Draw this picture on the board.



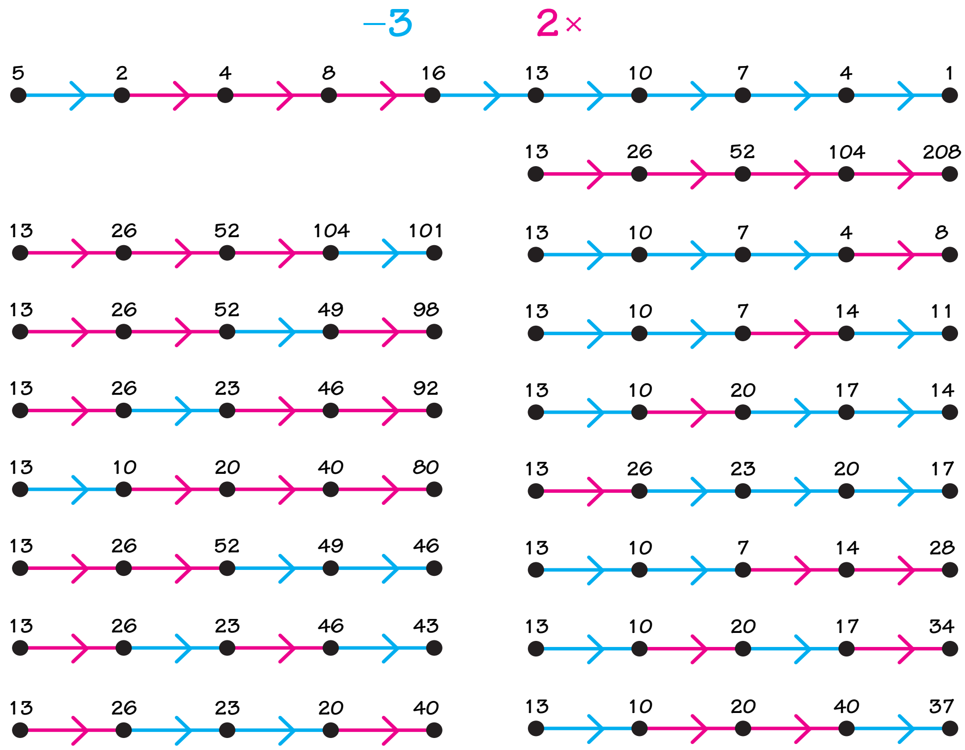
T: ***The blue arrows are for –3 and the red arrows are for 2*x*. Copy the picture and label the dots. Then draw a road starting at 5, using exactly nine arrows. The first five arrows are drawn for you, so complete the road by adding four more arrows. You may use red arrows or blue arrows or both kinds of arrows.***

As you observe students, ask some of the following questions. Choose more challenging problems for students who need extra challenge.

- Can you make the arrow road end at 11 (or 46 or 92)?
- What is the least (greatest) possible ending number for such a road?
- Are there many possible ending numbers? How many?

EN6

There are 16 possible ending numbers for roads in this problem. All the possibilities are shown below (all but one are shown only from 13 to their completions).



Capsule Lesson Summary

Discover that parentheses must be added to some expressions to determine which numbers they name; for example, (2 x 5) + 3 = 13 and 2 x (5 + 3) = 16. Solve a detective story about a secret number in which the clues involve (i) finding all of the numbers that can be named using the symbols 3, 5, 4, +, x, (,), each symbol exactly once; (ii) locating the secret number in a two-string picture where the strings are for multiples of 4 and 5; and (iii) observing that the secret number is on the same +5 arrow road as $\widehat{6}$.

Materials

Teacher		Student	
	• Colored chalk		• Paper • Colored pencils, pens, or crayons • Worksheet EN7* and **

Advance Preparation: Use the blacklines following this lesson to make copies of student worksheets. For use in Exercise 2, write a large 19 on a piece of paper, and then fold the paper so that 19 is hidden.

Description of Lesson

Exercise 1

Write this expression on the board.

$$2 \times 5 \times 3$$

T: ***What number is this?***

S: ***It could be 13, because 2 x 5 = 10 and 10 + 3 = 13.***

S: ***It could be 16, because 5 + 3 = 8 and 2 x 8 = 16.***

If no student suggests 16, suggest it yourself as in the following dialogue.

T: ***You think the number is 13. I think it could be 16. Why do I say it could be 16?***

S: ***Because 5 + 3 = 8 and 2 x 8 = 16.***

T: ***Is the number 13 or 16?***

S: ***We are not sure, it could be either.***

Add parentheses to the expression on the board.

$$2 \times (5 \times 3)$$

T: ***What number is this?***

S: ***16, because you calculate 5 + 3 first.***

Explain that the parentheses indicate which operation should be done first. Write this expression on the board.

$$(2 \times 5) \times 3$$

T: ***What number is this?***

S: ***13, because you calculate 2 x 5 first.***

Write this expression on the board. $5 \times 2 \times 8$

T: ***Who can add parentheses to make this number as great as possible?***

Let a student add the parentheses and calculate the number. $5 \times (2 \times 8) = 50$

T: ***What other number could this be?***

S: ***18, but 18 is smaller than 50.***

Invite a student to add parentheses to show 18. $(5 \times 2) \times 8 = 18$

Repeat the exercise with the following expression. $4 \times 7 - 1$

This number could be (4 x 7) – 1 = 27 or 4 x (7 – 1) = 24.

Exercise 2

Display the piece of paper on which the secret number, 19, is hidden.

T: ***The name of a secret number is written on this paper. I'll give you some clues, and we'll see how good you are at being detectives.***

Clue 1

Write these symbols on the board.

$$3 \quad 5 \quad 4$$
$$\times \quad \times \quad (\quad)$$

T: ***The secret number can be written using each of these symbols exactly once. Who can find one number that could be the secret number?***

Invite a student to write an expression, using each symbol exactly once, and then to calculate the number. For example: $3 \times (4 \times 5) = 23$

Ask students to find other possibilities and to write them on a piece of paper. As you observe students' work, announce some of the possibilities.

T: ***Nicole found a number that the secret number could be. Nicole used the symbols in this way. What number is this?*** $(3 \times 4) \times 5$

S: ***35, because 3 + 4 = 7 and 7 x 5 = 35.***

Complete the number sentence on the board. $(3 \times 4) \times 5 = 35$

Let students continue to work individually. Periodically announce a student's solution and ask the rest of the class which number the expression names. Continue until all six possibilities have been found and are listed on the board.

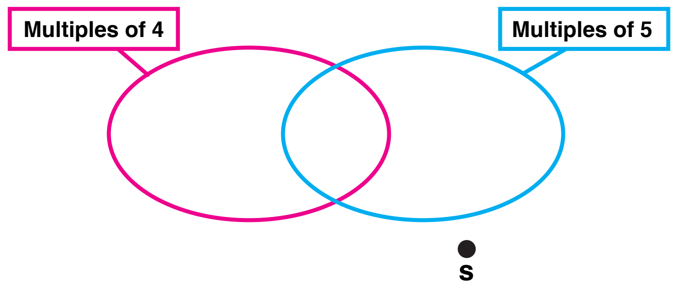
$3 \times (4 \times 5) = 23$	$3 \times (4 \times 5) = 27$
$(3 \times 4) \times 5 = 35$	$(3 \times 5) \times 4 = 32$
$(3 \times 4) \times 5 = 17$	$(3 \times 5) \times 4 = 19$

Students may think they have found another combination and suspect it will give a new possible number. For example, if a student suggests (4 + 3) x 5 when (3 + 4) x 5 = 35 has already been given, use the opportunity to comment on commutativity (4 + 3 = 3 + 4). You need not use this word with the students.

T: ***The secret number could be 23, 35, 17, 27, 32, or 19.***

Clue 2

Draw this string picture on the board.



T: ***The secret number is here*** (point to **s**)***.***
What does this picture tell us about the secret number?

S: ***The secret number is not a multiple of 4 and not a multiple of 5.***

T: ***Which numbers from our list of possibilities could be the secret number?***

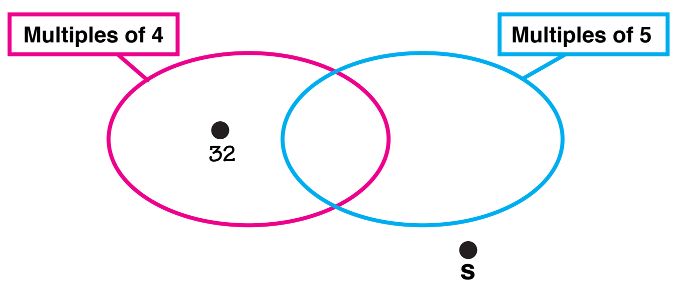
S: ***23 (17, 27, or 19).***

For each suggestion, ask students to check that it is not a multiple of 4 and not a multiple of 5.

A student might suggest a number that cannot be the secret number, and very likely many students will disagree immediately. Suppose someone suggests 32, for example.

T: ***Where does 32 belong in this picture?***

Invite a student to locate 32 in the string picture.



T (tracing the red string)**:** ***How do you know that 32 is a multiple of 4?***

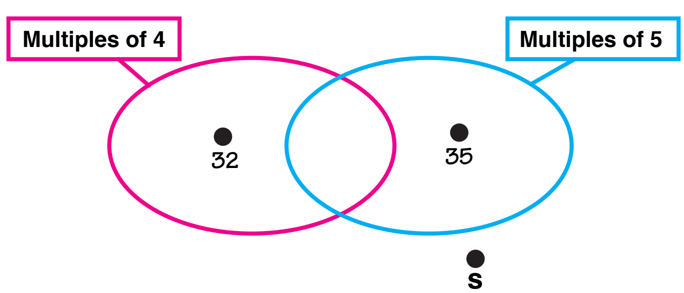
S: ***8*** x ***4 = 32.***

T (tracing the blue string)**:** ***How do you know that 32 is not a multiple of 5?***

S: ***32 does not end in 0 or in 5.***

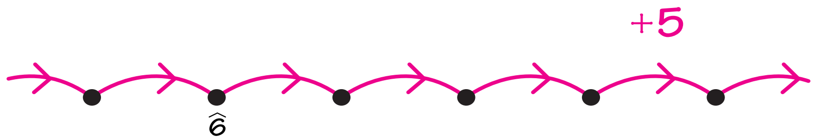
Continue until the class concludes that the secret number could be 19, 17, 27, or 23.

23
~~35~~
17
27
~~32~~
19



Clue 3

Draw this arrow picture on the board.



T: ***The secret number is on the +5 arrow road that meets the number $\widehat{6}$. I only drew part of the road because it goes on and on. Which of the numbers 23, 17, 27, or 19 could be the secret number? Write it on your paper.***

As you observe students' papers, acknowledge correct answers.

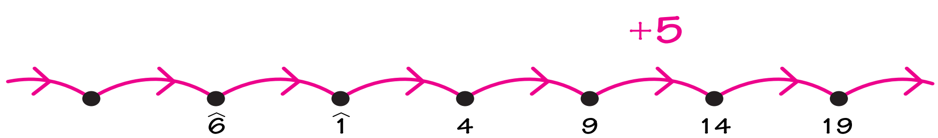
T: ***Who can convince us that one of these numbers (23, 17, 27, 19) is on this arrow road?***

Allow students to label the dots to the right of $\widehat{6}$ to show where 19 occurs. Use the magic peanut model, if necessary, to show that $\widehat{6} + 5 = \widehat{1}$ and $\widehat{1} + 5 = 4$.

Continue until the four dots to the right of $\widehat{6}$ have been labeled.

S: ***The next number (to the right) is 19.***

Finish drawing the arrow and label its ending dot 19.



T: ***How can we be sure that 17, 23, or 27 could not be the secret number?***

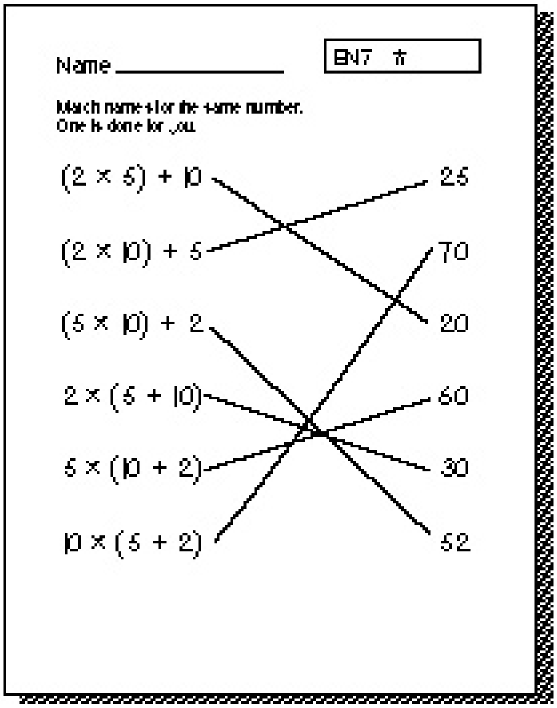
S: ***We already skipped 17 on the arrow road.***

S: ***The next number we meet is 24, because 19 + 5 = 24. We skip 23.***

S: ***The next number after 24 we meet is 29, because 24 + 5 = 29. We skip 27.***

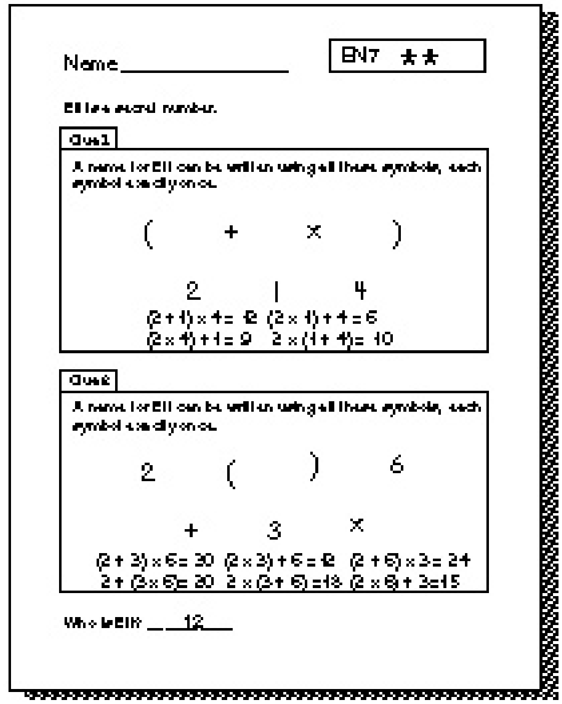
Reveal that the secret number, 19, is written on your piece of paper.

Worksheets EN7* and ** are available for individual work.


Name ______________ EN7 ☆

Match names for the same number.
One is done for you.

(2 × 5) + 10	25
(2 × 10) + 5	70
(5 × 10) + 2	20
2 × (5 + 10)	60
5 × (10 + 2)	30
10 × (5 + 2)	52


Name ______________ EN7 ★★

Bill is a secret number.

Clue 1

A name for Bill can be written using all these symbols, each symbol exactly once.

(+ ×)

2 1 4

(2 + 1) x 4 = 12 (2 x 1) + 4 = 6
(2 x 4) + 1 = 9 2 x (1 + 4) = 10

Clue 2

A name for Bill can be written using all these symbols, each symbol exactly once.

2 ( ) 6

\+ 3 ×

(2 + 3) x 6 = 30 (2 x 3) + 6 = 12 (2 + 6) x 3 = 24
2 + (3 x 6) = 20 2 x (3 + 6) = 18 (2 x 6) + 3 = 15

Who is Bill? 12

Name ____________________

EN7 *

Match names for the same number.
One is done for you.

$(2 \times 5) + 10$	25
$(2 \times 10) + 5$	70
$(5 \times 10) + 2$	20
$2 \times (5 + 10)$	60
$5 \times (10 + 2)$	30
$10 \times (5 + 2)$	52

Name ___________________

EN7 ★★

Elf is a secret number.

Clue 1

A name for Elf can be written using all these symbols, each symbol exactly once.

(+ ×)

2 1 4

Clue 2

A name for Elf can be written using all these symbols, each symbol exactly once.

2 ( ) 6

+ 3 ×

Who is Elf? __________

EL Lessons

EL1 INTRODUCTION TO ARROWS

Capsule Lesson Summary

Explore ways to put a number on the calculator when the keys for that number cannot be used. Teach a calculator to count by ones and by threes. Label the dots and draw return (opposite) arrows in several arrow pictures. Relate number sentences such as 7 + 3 = 10 and 10 – 3 = 7.

Materials

Teacher
- Overhead calculator
- Colored chalk
- Number line

Student
- Calculator
- Paper
- Colored pencils, pens, or crayons

Description of Lesson

Display an overhead calculator (if available) and distribute calculators to individual students. Depending on your students' experience with calculators, review the parts and some features of the calculator. In particular, students should be able to turn on the calculator, enter numbers, read the display, clear the calculator, use the calculator for simple addition and subtraction problems, and use the [=] key to display the result.

Exercise 1

Tell the students you want them to pretend that a particular number key on the calculator is broken. Then ask them to display that number without using the broken key. For example:

T: ***Suppose the [8] key is broken. Try to display 8 without using the [8] key.***

S: ***Press [5] [+] [3] [=].***

S: ***Press [9] [–] [1] [=].***

S: ***Press [2] [×] [4] [=].***

Allow students to find several solutions and demonstrate their solutions on the overhead calculator.

Repeat this exercise a couple times with other broken keys.

T: ***Suppose both the [2] and the [5] keys are broken. Try to display 25.***

S: ***Press [1] [4] [+] [1] [1] [=].***

S: ***Press [9] [+] [8] [+] [7] [+] [1] [=].***

S: ***Press [3] [1] [–] [6] [=].***

S: ***Press [1] [0] [0] [÷] [4] [=].***

Again, allow students to find several solutions and demonstrate their solutions on the overhead calculator. You might not expect the same variety given here, but be open to many different solutions and encourage students' experimentation.

With students, review (or introduce) how to teach a calculator to count. You may want to let students describe the counting process first. Demonstrate with the overhead calculator while students use their calculators.

T: ***We teach a calculator to count by ones by***
(1) putting on the starting number;
(2) pressing $\boxed{+}$ $\boxed{1}$***; and then***
(3) pressing $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ***and so on.***

Let students spend a few minutes making their calculators count up to "big" numbers.

T: ***How do you suppose we can teach a calculator to count by threes?***

S: ***Put on the starting number.***
Press $\boxed{+}$ $\boxed{3}$.
Then press $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ***and so on.***

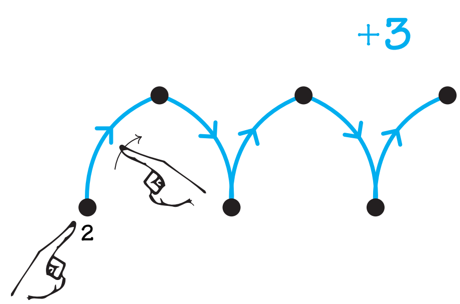
Spend a few minutes exploring the counting calculator. Some students may want to make their calculators count by other numbers as well.

Exercise 2

Draw the arrow picture below on the board. Put your left forefinger on the dot for 2.

T: ***This dot is for the number 2. The blue arrow is for +3.***

Trace the blue arrow, starting at 2, with your right forefinger in the direction of the arrowhead as you say,



T: ***2 + 3. What number is this?*** (5)

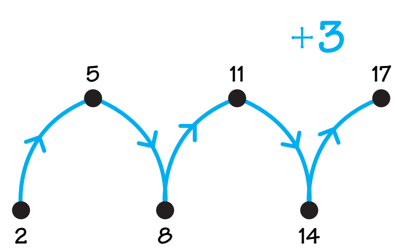
Label the second dot 5. Put your left forefinger on the dot for 5 and trace the blue arrow with your right forefinger as you say,

T: ***5 + 3. What number is this?*** (8)

Label the third dot 8. Point to the unlabeled dots.

T: ***What are these numbers?***

Invite students to point to the dots as they announce the numbers. Continue until all the dots are labeled. Occasionally, you may want to write a number sentence corresponding to an arrow on the board; for example, 8 + 3 = 11.



T: ***Could we go on with more +3 arrows?*** (Yes)

Do not draw more arrows. You can trace more arrows if you like.

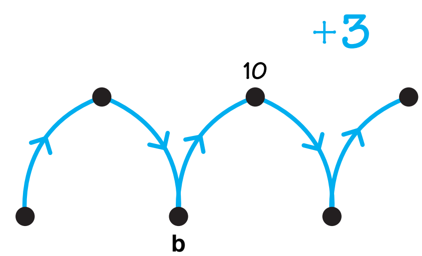
T: ***If I keep drawing arrows, do you think we will ever meet the number 30?*** (No)

If several students respond correctly but many are uncertain, trace imaginary arrows and ask students which numbers come next in the picture until you pass 30. Ask if you will ever meet the number 35. (Yes) How about 50? (No) Encourage students to explain why they think yes or no to such questions.

At this time you may like to let students use their calculators, counting by threes, to follow the arrow picture and to help answer such questions. Students may also use the number line to explain answers to questions about an extended arrow picture. That is, the arrow picture starts at 2 and makes “jumps” of 3 (+3 arrows). It lands on 35 but jumps over 30 and 50.

Erase the numerals but not the dots or arrows.
Label the fourth dot from the left 10.

Point to the dot labeled **b** in the illustration.

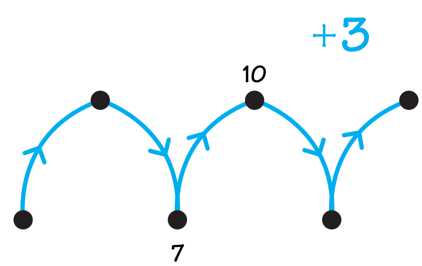


Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: ***What number is here?***

Invite students to whisper their answers to you or to write on the paper for you to check.† Then ask a student to answer aloud.

Label the dot.



T: ***How do you know this is 7?***

S: ***7 + 3 = 10.***

Write the number sentence on the board.

T: ***Does someone have another way to see that this is 7?***

S: ***10 – 3 = 7.***

It may be necessary to give the 10 – 3 = 7 observation yourself.
Write the number sentence under 7 + 3 = 10 on the board.

7 × 3 = 10
10 – 3 = 7

T: ***We also know that this number is 7 because 10 – 3 = 7. We could draw a red arrow for –3.***

Put your left forefinger on the dot for 10. Trace an arrow (starting at 10 and ending at 7) with your right forefinger as you say, “10 – 3 = 7.”

Draw the return arrow in red from 10 to 7, and write –3 in red near the arrow picture.

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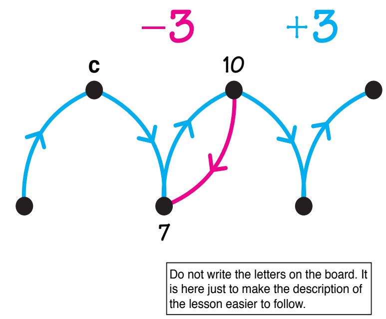
†Often in☐
respond. If☐
or not an answer is correct because you will want students to continue to think about the situation.

Point to the dot labeled **c** in this illustration.



T: ***What number is here?***

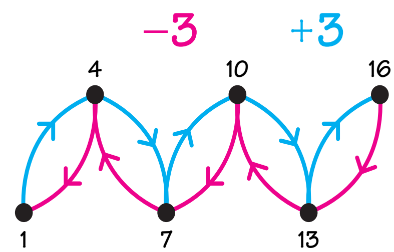
S: ***4, because 7 – 3 = 4 (or 4 + 3 = 7).***

T: ***What are the other numbers in the picture?*** (1, 13, 16)

Quickly label the remaining dots.

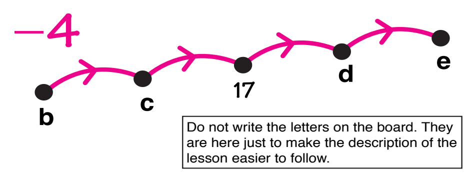
T: ***Could we draw more –3 arrows in this picture?*** (Yes)

Ask students to draw the red arrows. After each red arrow is drawn, point to its starting number and trace the arrow to its ending number while you say, for example, "13 – 3 = 10." Continue until the arrow picture is complete.



Exercise 3

Erase the board and draw this arrow picture.



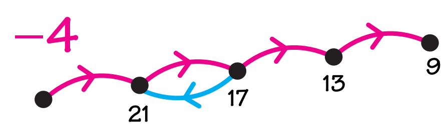
T: ***How should we label the other dots?***

S: ***13 is here (at* d*), since 17 – 4 = 13.***
The next dot (*e*) is 9, since 13 – 4 = 9.

S: ***21 is here (at* c*) because 17 + 4 = 21 and 21 – 4 = 17.***

T: ***Yes, maybe a return (opposite) arrow would help show why 21 is here (at* c*).***

Draw a blue arrow from 17 to **c**.

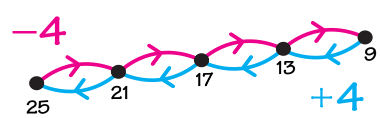


T: ***If the red arrow is for –4, what could the blue arrow be for?***

S: ***+4.***

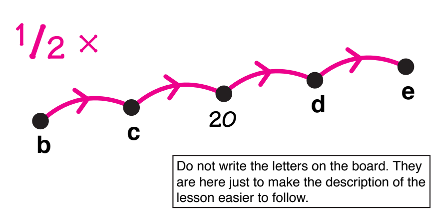
T: ***Can we draw some more blue arrows for +4?***

Write +4 in blue on the board and invite students to draw blue arrows. Also call on a student to label the first dot in the picture.



Exercise 4

Erase the board and draw this arrow picture.



T: ***The red arrow is for*** $^1/_2$x (read as "one-half times" or "one-half of"). ***Who can label a dot?***

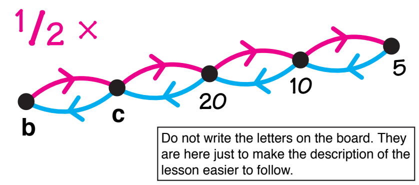
S: $^1/_2$ x ***20 = 10. That number (at* d*) is 10.***

S: ***And*** $^1/_2$ x ***10 = 5. That number (at* e*) is 5.***

T: ***What number is here (at* c*)?***

S: ***40, because one-half of 40 is 20.***

If some students seem confused, suggest the use of return (opposite) arrows. Add the following blue arrows to the picture.



T: ***What could the blue arrows be for?***

S: ***2x, because 2 x 5 = 10 and 2 x 10 = 20.***

Some students are likely to suggest +5 or +10 for the blue arrows if they are looking at only a part of the arrow picture. If this occurs, tell them that one relation is needed for all of the blue arrows.

Write 2x in blue near the picture.

T: ***Is this number (at c) 40?***

S: ***Yes, 2 x 20 = 40 and*** $^{1}/_{2}$ ***x 40 = 20.***

S: ***The other number (at b) is 80 since 2 x 40 = 80 and*** $^{1}/_{2}$ ***x 80 = 40.***

T: ***The red arrows are for*** $^{1}/_{2}$***x. What is another name for the red arrows?***

S: ***÷2. 80 ÷ 2 = 40, 40 ÷ 2 = 20, and so on.***

T: ***Yes,*** $^{1}/_{2}$***x and ÷2 are different names for the same relation.***

Suggest that students draw arrow pictures of their own, similar to those in Exercises 2, 3, and 4, but with other functions (such as +5, –9, 2x, ÷3) and their opposites. Working with a partner, each student can put one number in his or her arrow picture, and then exchange. Partners can label the rest of the dots.

Capsule Lesson Summary

Locate dots for students in a three-string picture and identify which students a certain dot could be for. Determine where a person belongs in a string picture by asking appropriate questions about the person. Explore the idea of an empty set. Place A-blocks correctly into a two-string or three-string picture.

Materials

Teacher
- Colored chalk
- A-Block String Game kit
- Small box

Student
- Paper

Advance Preparation: Before this lesson, you will need to prepare some materials from the A-Block String Game kit. These materials will be used again in future lessons on *The String Game*, so keep them together in the envelope with the kit.

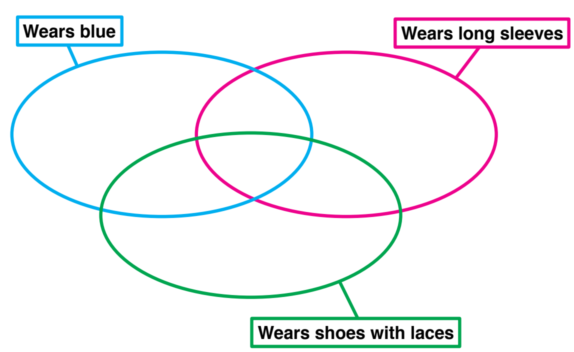
1. Punch out one set of shapes (24). If you have a magnetic board, magnetize each A-block by sticking a small piece of the magnetic material to the back. If there is no magnetic board available, you can still use a regular chalkboard. In this case, have loops of masking tape ready to stick to the back of each A-block so that they can be stuck to the board. Masking tape loses its stickiness quickly, so be prepared to reinforce the A-blocks regularly with new loops of tape.
2. Prepare the string cards in the same manner as the A-blocks.
3. Prepare a team board as pictured in Exercise 4. This board should be metallic if you have magnetized A-blocks. Post a list of attributes (posters found in the A-Block String Game kit) above the team board. You may want to laminate the A-Block pieces and cards to make them more durable.

Description of Lesson

Exercise 1

Before starting this exercise, look over the attire of the students in your class today. Try to determine labels for three overlapping strings so that there is at least one student represented in each of the eight regions (see the illustration).

Draw and label three different-colored strings. You may want to discuss the string labels so students understand what it means for a person to be inside a string.



T: ***Who would like to show us where they are in this picture?*** (Laura volunteers.) ***Is Laura wearing some blue?***

S: ***Yes, blue stripes.***

T: ***Is Laura wearing long sleeves?***

S: ***No.***

T: ***Is Laura wearing shoes with laces?***

S: ***No.***

T: ***Laura, draw a dot for yourself in our picture.***

EL2

If the student needs help locating a dot for herself, use large sweeping motions with your open hand to indicate the interior of the strings.

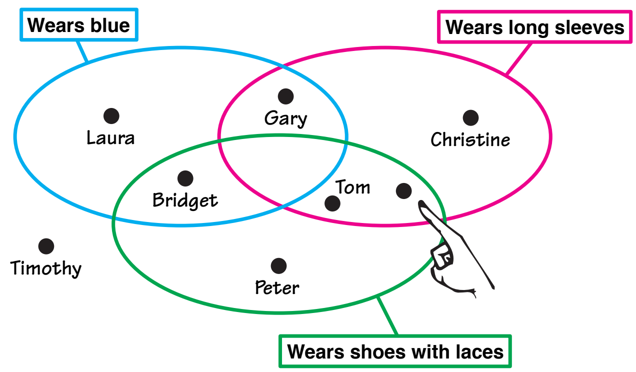
T: ***Laura is wearing blue, so is she inside or outside the blue string?*** (Inside) ***Laura is not wearing long sleeves, so is she inside or outside of the red string?*** (Outside) ***Laura is not wearing shoes with laces, so is she inside or outside of the green string?*** (Outside)

Help the student draw a dot for herself in the correct place. (See the illustration in Exercise 2.)

T: ***Let's write Laura's name by her dot so we can remember that this dot is for Laura.***

Call on several other students to locate themselves in the picture. If necessary, ask the three important questions, and then ask the student to draw his or her dot. Try to select students whose dots go in different regions, including the region outside of all three strings.

Exercise 2



Leave all the dots drawn thus far in the picture. Draw a new dot, making sure the class knows which dot you have just drawn.

T: ***I am thinking of somebody in this class and the dot for that person is here.***

Who could this be?

S: ***Kenneth.***

Note: If students answer with a name already in the picture, for example, Tom, remind them that you have a dot for Tom and this dot should be for someone else.

T: ***Stand up Kenneth. Is Kenneth wearing some blue?***

S: ***No.***

T: ***So a dot for Kenneth should be outside the blue string, and this dot is. Is Kenneth wearing long sleeves?***

S: ***Yes.***

T: ***Then a dot for Kenneth should be inside the red string, and this dot is. Is Kenneth wearing shoes with laces?***

S: ***No.***

T: ***Then a dot for Kenneth should be outside the green string. This dot is inside the green string.***

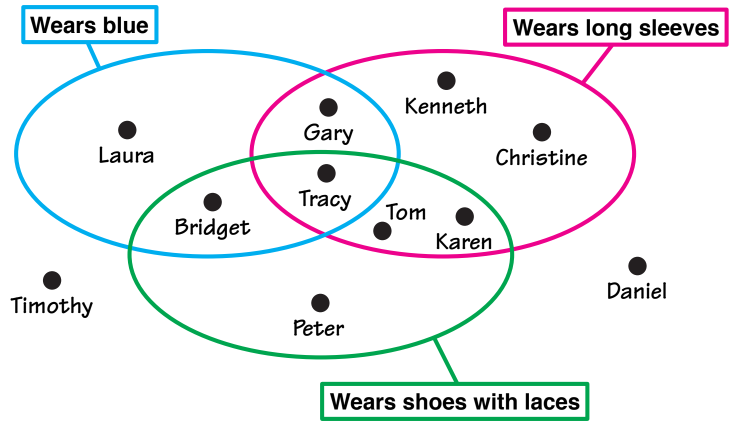
S: ***It can't be for Kenneth.***

T: ***Does someone else have an idea?***

S: ***Karen.***

Ask the students to check with you to see if Karen is not wearing blue, is wearing long sleeves, and is wearing shoes with laces.

Repeat this exercise, drawing a dot inside all three strings (Tracy's dot in the picture below) and a dot outside all three strings (Daniel's dot).



Students may suggest that a dot outside all three strings could be for things other than people or students in your classroom.

S: ***My nose.***

S: ***An elephant.***

T: ***Yes, those things are not people and they do not wear clothes or shoes.***

Exercise 3

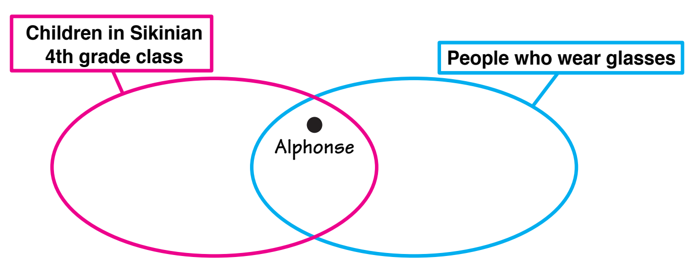
Erase the picture from Exercise 2 and draw a new string picture as in the next illustration.

T: ***Now I'm going to tell you about a school in a country called Sikinia. In this school, there is just one fourth-grade class and all of the children in that class wear glasses.***

Let students discuss this situation for a few minutes if they wish.

T: ***The red string is for all of the fourth graders in the school in Sikinia, and the blue string is for all of the people who wear glasses. There is a fourth-grader in the Sikinian school named Alphonse. Who can put a dot for Alphonse in our picture?***

Let a student do this.



T: ***Why must Alphonse be in the middle?***

S: ***He is a fourth grader, and all fourth graders in that school wear glasses.***

T: ***I have a friend who lives next door to me and who wears a beautiful pair of silver-rimmed glasses. Who can put a dot for my friend in our picture?***

Let a student do this and ask for class agreement.

T: ***Why does the dot for my friend belong inside the blue string but outside the red string?***

S: ***Because your friend wears glasses, but your friend is not a fourth grader.***

T: ***How do you know that? In fact, my friend is a fourth grade girl named Julie.***

S: ***But your friend does not live in Sikinia.***

T: ***Yes. Each of you who wears glasses would also belong in the blue string but outside the red string, just like Julie. How about Dan*** (choose a boy in your class who does not wear glasses)***? Where does he go?***

S: ***Outside both strings, because he does not wear glasses and he is not from Sikinia.***

T: ***That fourth grade Sikinian class is very small. It has only ten students in it altogether. Where are the dots for the other nine students?***

S: ***They are all in the middle.***

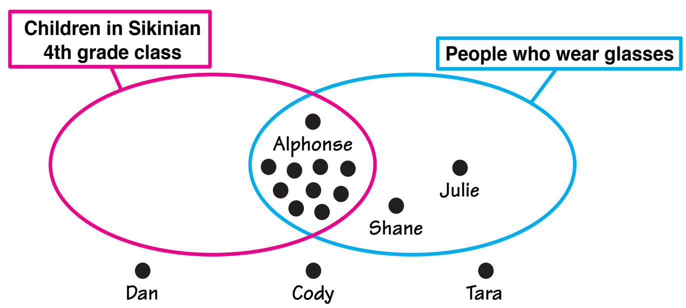
T: ***Why?***

S: ***Because they are all in the Sikinian fourth grade and all fourth graders in Sikinia wear glasses.***

Invite a student to put dots for those nine children in the string picture.

Choose several students to locate dots for themselves.

Point to the region inside the red string and outside the blue string.



T: ***Who could be here?***

S: ***Nobody.***

T: ***Why?***

S: ***Because all fourth-graders in Sikinia wear glasses. There are only ten of them and their dots are in the middle.***

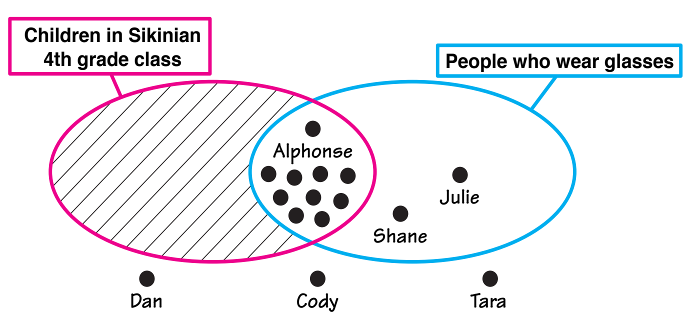
T: ***You're right. What should we do to show that no one is here?***

Many suggestions are possible. Let the class discuss the situation.

T: ***Your ideas are interesting, but in order that we all do it the same way, we will do what other people who study sets do. We will hatch this part of the picture to show that nothing is there.***

Hatch the appropriate region.

T: ***We say this region is empty.***



Exercise 4

Put the 24 A-block pieces in a box (a greeting card box is a good size).

T: ***In this box I have some pieces called A-blocks with different shapes, colors, and sizes. There are three different shapes. What shapes do you think the pieces are?***

As an attribute of the A-blocks is mentioned, show the class a piece having that attribute.

S: ***Squares, circles, and triangles.***

T: ***There are four different colors. What colors do you think the pieces have?***

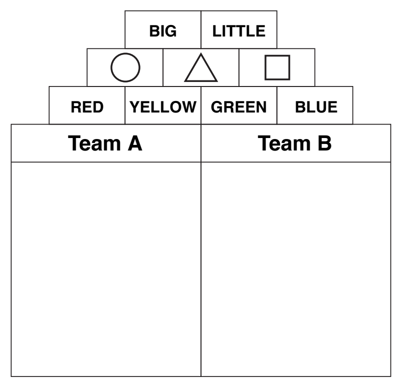
S: ***Red, blue, green, and yellow.***

T: ***There are two sizes. What should we call them?***

S: ***Big and little.***

T: ***Each A-block piece is described by its shape, color, and size. I have one and only one piece in the box for every combination of a shape, color, and size. How many A-block pieces are there in the box?***

Let students make guesses, perhaps listing a few guesses on the board. Then count the pieces by looking at just one shape (or one color). There are eight of each of three shapes (or six of each of four colors) so there are 3 x 8 (or 4 x 6) or 24 pieces altogether.



Tape a copy of the poster of nine possible string labels (A-Block String Game Poster, Version A) to the board and prepare a team board as shown here.

Note: If you prefer, a game can be played with more than two teams and the team members can choose names for their teams (rather than **A** or **B**). For example, you might use three or four cooperative groups as teams. Prepare the team board accordingly.

T: ***Your first task will be to get all the pieces out of the box and onto this board*** (point to the team board)***. Teams will take turns; someone on Team A will tell me a piece to put on Team A's side of the board, and then someone from Team B will tell me a piece to put on Team B's side. Remember, you must describe a piece that is still in the box, and I need to know exactly which piece you want me to put on your side of the board.***

Alternating teams, call on students to describe A-block pieces. Insist on complete descriptions as in the following discussion.

S: ***A big circle.***

T (looking in the box)**:** ***I have several big circles. Which one do you want?***

S: ***The red one.***

T: ***I have a big red circle*** (holding up this piece)***. Is this the piece you want on your side of the team board?***

Continue in this way, keeping a brisk pace, until all the A-block pieces are on the board. If a student describes a piece that is already on the team board, point to it and ask the student to choose a different piece, one that is still in the box. You may want to select students to call on early in the exercise when there are many choices open to them. Encourage all students to be thinking about a piece they will ask for when you call on them.

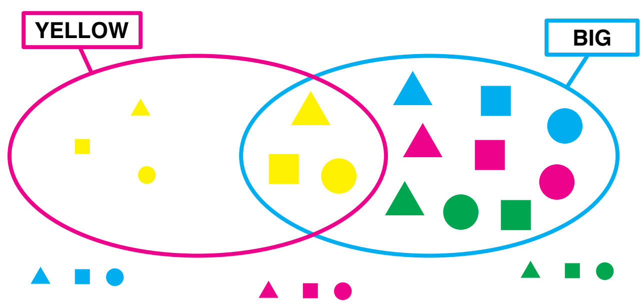
Exercise 5

Note: If this exercise seems too easy for your class, use the alternative Exercise 5.

Draw two different-colored, overlapping strings on the board. Label them as in the next illustration.

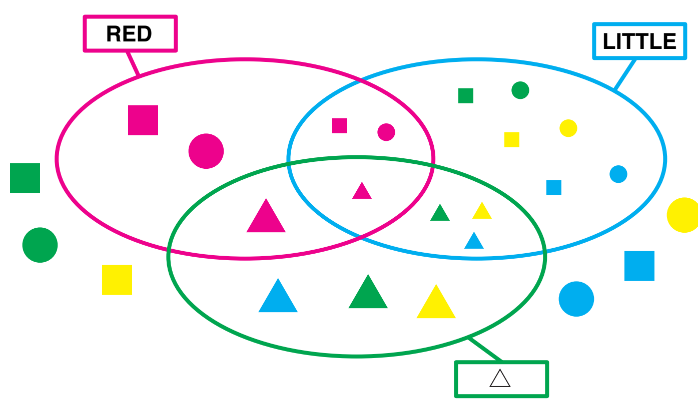
T: ***Now the teams will take turns. Each of you will try to place a piece from your team's side of the board in this string picture. If you put it in its correct place, I will say yes and it stays there. If you do not, I will say no and you must return it to your team's side of the board. The first team to get all of its pieces in the picture is the winner.***

Play the game, alternating teams, and alternating turns among the members of each team. When all the pieces are in the picture, your board will look like this.



Exercise 5 (alternative)

Draw three different-colored, overlapping strings on the board. Label the strings as in the next illustration. Give directions as in Exercise 5. When all the pieces are in the picture, the board will look like this.



Capsule Lesson Summary

Introduce and play *The String Game* with two strings and nine possible string labels. Discuss not-cards as string labels, and place the A-blocks in a two-string picture where one string is labeled with a not-card. Play *The String Game* allowing that the strings could have any of sixteen possible labels.

Materials

Teacher
- A-Block String Game kit
- Box
- Colored chalk

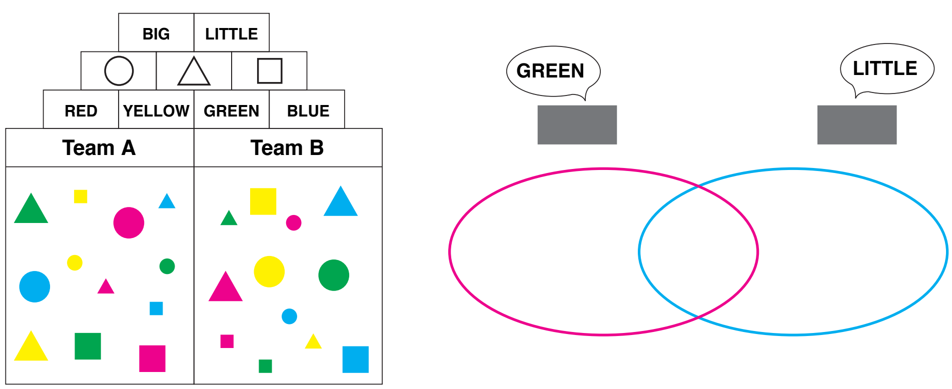
Student
- None

Description of Lesson

Exercise 1

Prepare to play *The String Game* with A-blocks by setting up a team board and taping a list of the nine possible string labels above it. Divide the class into two or more teams and distribute the game pieces on the team board. Draw overlapping red and blue strings on the chalkboard near the team board. Label the strings with facedown cards as shown below.

Note: If you prefer, the game can be played with more than two teams and the team members can choose names for their teams (rather than **A** or **B**). For example, you might use three or four cooperative groups as teams. Prepare the team board accordingly.

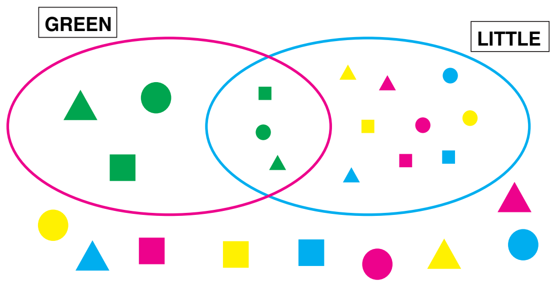


Explain to the class how *The String Game* is played: Teams take turns trying to place their pieces (from the team board) correctly in the string picture. The object is to identify the string labels. Before starting the game, allow each team to select one piece for you to place correctly in the picture. These correctly placed A-blocks serve as starting clues.

The rules of the game call for you to judge the positioning of the pieces. If a piece is correctly placed, say yes, and immediately invite the player to try to place a second piece (bonus turn). No player should have more than two consecutive turns. If a piece is incorrectly placed, say no, and ask the player to return the piece to his or her team's side of the team board.

For this game, the player who correctly places the last piece from a team's choices may then attempt to identify both string cards. If the player gives an incorrect label for one of the strings, the next player on the opposing team has a chance to either place pieces from that team's unplayed pieces or to identify the strings if all their pieces have been placed correctly already. A team wins by being first to get all of its pieces in the string picture and to identify the strings.

The following picture shows correct placement for all the A-block pieces. You may use this picture as a crib sheet during play of the game.



Exercise 2

Clear the board from the last exercise and draw one string on the board. Have the not-cards ready for discussion.

T: ***I am going to show you some other possibilities for string labels.***

Show the class the string card **NOT BLUE** and place it next to the string on the board.

T: ***If this string were for* NOT BLUE*, what pieces would go inside the string?***

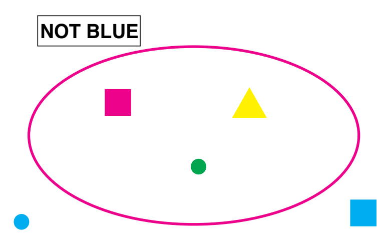
S: ***The little green circle.***

S: ***The big red square.***

Let students place several pieces that would be inside the string and discuss their placement.

T: ***Which pieces would go outside the string?***

S: ***The big blue square.***



Take the A-blocks off the board and erase the string.

T: ***What other new string labels do you think we have?***

Show the class each not-card when it is mentioned.

S: **NOT RED*, NOT GREEN*, *and* NOT YELLOW**

T: ***I also have a card* NOT ○*.*** (Show the card to the class.) ***Can you describe a piece that is not a circle?***

S: ***A small blue triangle.***

Continue in this manner considering **NOT △** and **NOT □**.

T: *Do I need a* **NOT LITTLE** *or* **NOT BIG** *card?*

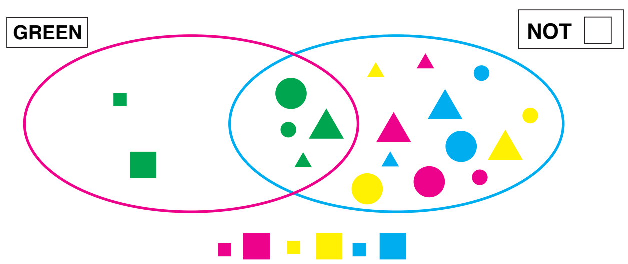
S: *No,* **BIG** *is the same as* **NOT LITTLE.**

S: *No,* **LITTLE** *is the same as* **NOT BIG.**

Draw two overlapping strings on the board using two different colors. Label them as shown below (with faceup cards).

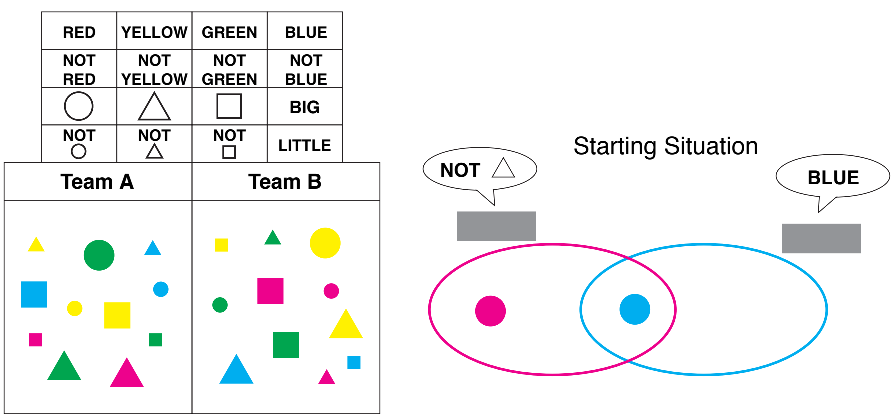
Let students place various pieces of their choice (or yours) in the picture. After about eight to ten pieces have been put on the board, the class should be better acquainted with the new labels.

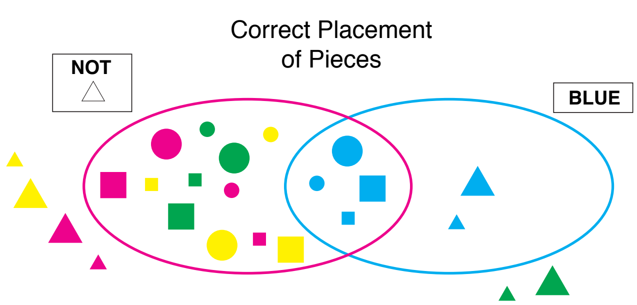
This picture shows the correct placement of all the A-block pieces.



Exercise 3

Prepare your board to play a two-string version of *The String Game* in the usual way with facedown string cards. This time tape the poster list of 16 possible string labels above the team board. Distribute A-block pieces for Team A and Team B on the team board. The next illustration shows the starting situation for a game, followed by the correct placement of all the A-blocks.





Center Activity

Place materials from The A-Blocks String Game kit in a center for students to conduct their own games.

Capsule Lesson Summary

Use a calculator to generate multiples of 3 and place the multiples of 3 in a string picture. Generate multiples of 4 on the calculator and extend the string picture to include a string for multiples of 4. Discuss the various regions of the string picture using multiples of 3 and multiples of 4 for labels.

Materials

Teacher		Student	
	• Calculator • Colored chalk		• Calculator • Unlined paper • Colored pencils, pens, or crayons

Description of Lesson

You may like to let students work in pairs during the lesson. Each pair of students should have a calculator with an automatic constant feature (see "Role and Use of Calculators" in Section One, Notes to the Teacher of the *IG-1* Teacher's Guide). Allow a few minutes for students to experiment freely with their calculators.

Exercise 1

Direct students to clear their calculators and be ready to follow your directions.

T: ***Slowly press* $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ *and continue pressing* $\boxed{=}$*. What numbers appear on the display?***

S: ***3, 6, 9, 12, 15, ….***

T: ***What is the calculator doing?***

S: ***It's counting by threes.***

T: ***Now push the button* $\boxed{C}$ *to clear the display. What number is on the display now?***

S: ***0.***

Direct students to check that 0 is on the display and then to cover the display with a finger. They will need to be careful not to cover the light panel on light powered calculators.

T: ***Press* $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$*. What number do you think is on the display?***

S: ***9.***

T: ***Check the calculators.*** (Students who do not have 9 may need to start again, i.e., press $\boxed{C}$ $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$). ***Hide the display again. Now press* $\boxed{=}$ *four more times. What number do you think is on the display?***

S: ***21.***

T: ***Check the calculators. What numbers between 9 and 21 were on the display that we did not see?***

S: ***12, 15, and 18.***

T: ***Hide the display again. Press* $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$*. What number do you think is on the display?***

S: ***36.***

T: ***Check the calculators.***

T: ***Hide the display on the calculators. This time press $\boxed{=}$ ten times. What number do you think is on the display?***

S: ***66.***

T: ***Why do you think it is 66?***

S: ***I counted by threes.***

S: ***I added 30 (ten 3s), 36 + 30 = 66.***

S: ***Four 3s is 12. Four more 3s is another 12; that makes 24. Two more 3s is 6—so 30 more altogether.***

T: ***Check the calculators.***

S: ***66.***

T: ***How many times would we need to press $\boxed{=}$ to go from 66 to 78?***

Suggest students whisper their answers to their partners or write answers on paper. Ask one student to respond aloud. (Four times)

T: ***Press $\boxed{=}$ four times and tell us the number that appears.***

S: ***78.***

T: ***How do you think we could go back from 78 to 0 in such a way that all the same numbers would reappear but in reverse order?***

S: ***Press $\boxed{-}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on.***

T: ***Hide the display. Press $\boxed{-}$ $\boxed{3}$ $\boxed{=}$. What number is on the display?***

S: ***75.***

T: ***Check your calculator. Keep pressing $\boxed{=}$ until 0 appears. Now 0 is on the display. What number will appear if we press $\boxed{=}$ once again?***

S: ***$^{-}3$.[†]***

T: ***…and if we press $\boxed{=}$ again?***

S: ***$^{-}6$.***

T: ***Will $^{-}13$ ever appear?***

This question may result in disagreement.

T: ***What are some numbers that appear if we kept pressing $\boxed{=}$?***

S: ***$^{-}9$.***

S: ***$^{-}12$ and $^{-}15$. We missed $^{-}13$.***

Instruct students to turn the calculators off and put them away for a moment.

[†]Read $^{-}3$ just like $\overset{\frown}{3}$: negative three. This would be a good time to comment on different notations for negative numbers.

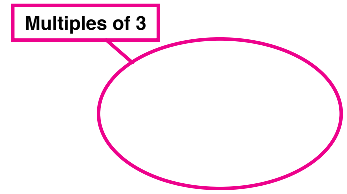
T: ***What did the calculator do when we pressed*** $\boxed{+}\ \boxed{3}\ \boxed{=}\ \boxed{=}\ \boxed{=}$ ***and so on?***

S: ***Counted by threes.***

T: ***Do you know a name for the numbers that appeared on the display?***

S: ***Multiples of 3.***

If necessary, give this name to the numbers yourself. Then draw a string on the board and label it **Multiples of 3**.



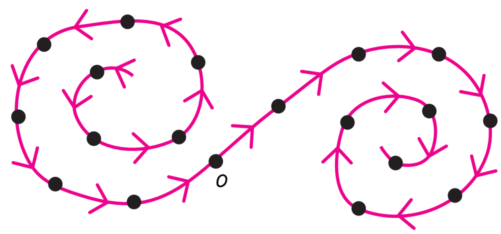
T: ***This string is for the multiples of 3. Name some multiples of 3.***

Let seven or eight students give multiples of 3. If someone suggests a very large number, for example, 2 096, say, "I'm not sure. You'll have to convince me." (In this case 2 096 is not a multiple of 3.)

Exercise 2

Draw a double spiral arrow picture on the board and label the middle dot 0.

T: ***What do you think about this picture?***

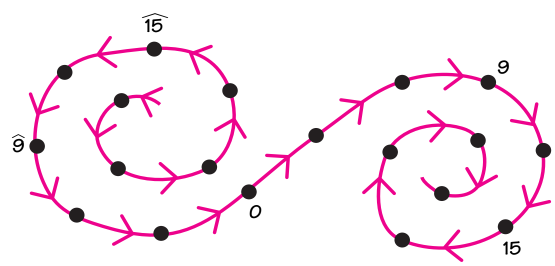


Let the class react. Perhaps they will mention its symmetry, that 0 is in the center, that it could go on and on in both directions, and so on. Allow a minute or two for such observations before continuing, but do not insist that all of these things are mentioned.

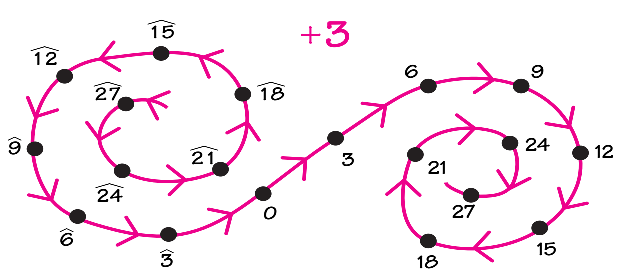
T: ***Let's put some more numbers in the picture.***

Label dots for 9, $\widehat{9}$, 15, and $\widehat{15}$.

T: ***What could the red arrows be for?***



Allow students to experiment with some possibilities. For example, if a student suggests +9, observe that in that case 9 would be at the end of the arrow starting at 0, not where it is. Perhaps a student will suggest "is less than"; if so, observe that this is a possibility, but ask for another possibility. When a student suggests +3, check to find that indeed the red arrows could be for +3, and write the key near the picture. If no one has suggested +3, put in some more numbers, for example, 6 and $\widehat{6}$, and ask the question again. After it is established that the red arrows are for +3, invite students to label the other dots.



T: ***What do you notice about the numbers in this picture?***

S: ***They are alike on both sides; 3 and $\hat{3}$, 6 and $\hat{6}$, 9 and $\hat{9}$, and so on.***

S: ***They are the numbers we say when we count by threes, forward and backward.***

S: ***They are the numbers we saw on the calculator when we made it count by threes (forward or backward) starting at 0.***

S: ***They are the numbers inside the string, the multiples of 3.***

If this last observation is not made, call the class's attention back to the string picture and ask specifically which numbers in the string picture would be in the arrow picture. (The numbers inside the string)

Continue by asking students to predict whether certain numbers are multiples of 3; that is, whether they would be in the double spiral for +3. Include some large numbers such as 300, 333, 900, 906, and 3 000. You may again like to use the calculator to follow the arrow picture and to check students' predictions of multiples of 3.

At this point, if there are no negative numbers in the string picture, direct the class to observe that all the numbers in the arrow picture should be inside the string, including the negative numbers.

Exercise 3

Erase the board, leaving a string for multiples of 3. Refer the class back to the calculator.

T: ***How do you think we could get the multiples of 4 to appear on the calculator?***

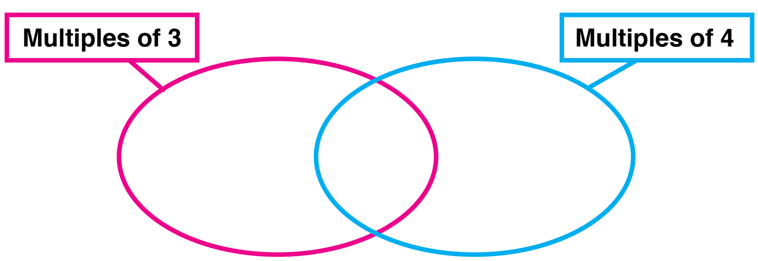
S: ***Do the same thing we did to get the multiples of 3—only press*** $\boxed{+}$ $\boxed{4}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$****....***

T: ***Let's try that. Turn the calculators on and press*** $\boxed{+}$ $\boxed{4}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ***and so on. What numbers appear on the display?***

S: ***4, 8, 12, 16, 20, ...***

T: ***Who can draw a string on the board for the multiples of 4?***

Let a student draw the string in a different color than the first string.



If a student draws the string so that it does not overlap the multiples of 3 string, ask where 12 would be in the picture.

T: ***Give me some numbers that are multiples of 4.***

S: ***20.***

T: ***Clear your calculators. Press*** $\boxed{+}$ $\boxed{4}$***. How many times do you need to press*** $\boxed{=}$ ***to get 20?***

S: ***Five times.***

T: ***Check that on your calculator. Give me another multiple of 4.***

S: ***44.***

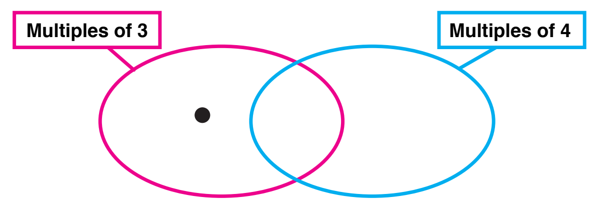
T: ***Does anyone know how many fours is 44?***

S: ***11.***

Continue in this way, letting several students suggest numbers and explain why they are multiples of 4. At the end of the exercise, ask the students to turn off their calculators.

Exercise 4

Direct students to copy this picture on their papers and to follow along with the class to locate several numbers in the picture. Draw a dot in one of the regions in the string picture on the board.



T: ***What number could be here*** (point to the dot)***?***

S: ***9.***

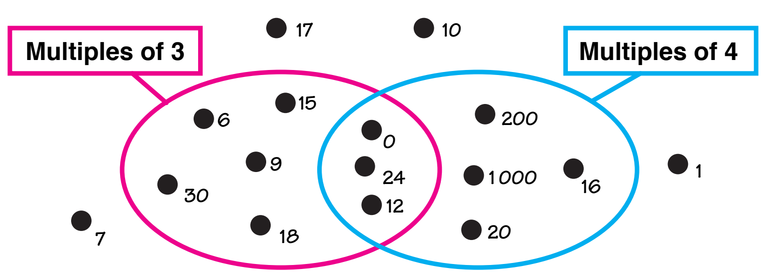
T: ***Is 9 a multiple of 3?*** (Yes)
Is 9 a multiple of 4? (No)

Label the dot in the picture. Draw a dot in another region. Continue in this manner until all regions have been considered. If someone gives a number that does not belong in a particular region, ask if the number is a multiple of 3 and then if it is a multiple of 4. Lead the student through these questions to discover where that number should be, and then let the student draw and label a dot in the correct region.

Present some numbers, one at a time, and ask students to locate them in their pictures as well as on the board. Use these or other numbers depending on your students' previous choices.

10	24	18
0	17	1
200	1 000	30

Use calculators to check the placement of a number when necessary. The next picture shows the correct location of the given numbers.



If the lesson is moving quickly, you can ask where the following numbers would be in the picture. Encourage students to locate them without first doing the calculations.

3 x 7	3 x 80
4 x 7	4 x 39

EW Lessons

EW1 DETECTIVE STORY #1/ELI'S MAGIC PEANUTS #1

Capsule Lesson Summary

Solve a detective story in which the secret number is an odd number, and the other clues involve an arrow picture and the Minicomputer. Tell the story of Eli the Elephant who found magic peanuts. When a magic peanut meets a regular peanut, both disappear, much to Eli's confusion.

Materials

Teacher
- Colored chalk
- Calculator
- Minicomputer set

Student
- Paper
- Colored pencils, pens, or crayons
- Calculator
- Minicomputer set
- Worksheets EW1* and **

Advance Preparation: Use the blacklines following this lesson to make copies of student worksheets.

Description of Lesson

Exercise 1: Detective Story

Before the lesson, write a large 21 on a slip of paper and fold the paper so that 21 is hidden.

Ask the class what a detective does. Guide the discussion to include the idea that a detective tries to uncover secrets by following clues. Tell the students that they are going to be detectives today, and they will be able to discover a secret number if they follow the clues.

Note: You may like to make the secret number something real for students to figure out such as "How old is my brother?" or "How many miles do I drive to school?" or "How many spots on my dog?"

Clue 1

T: ***The name of a secret number is written on this paper. Listen carefully to my clues and you can discover what it is. The first clue is that the secret number will appear on the calculator display when we teach the calculator to count by twos starting at 1.***

Remind or show students how to teach the calculator to count by twos.

1. Put on the starting number (1).
2. Press $\boxed{+}$ $\boxed{2}$.
3. Then press $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on.

Observe which numbers appear and suggest that students write ten or more of these numbers on their papers in the order they appear. Encourage students to make observations and, if not mentioned, tell the class that the secret number is an odd number. Begin listing the positive odd numbers in order on the board with the students' assistance. Start with 1 and continue to at least 25. Mention that you could continue pressing $\boxed{=}$ on the calculator or writing odd numbers all day, but instead you will put three dots at the end of your list meaning "and so on."

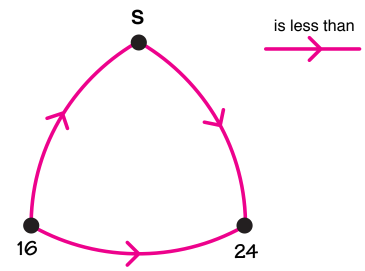
1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, ...

T: ***Now we know that the secret number is one of these numbers, a positive odd number, but we do not know which one. What do we need?***

S: ***Another clue.***

Clue 2

Draw this arrow picture on the board and ask students to copy it on their papers.



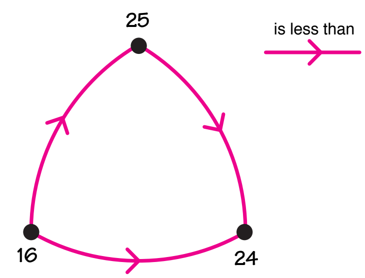
T: ***The secret number is here*** (point to **s**) ***in this arrow picture. What are the red arrows for?***

S: ***Is less than.***

Point to the dot labeled **s** and remind the class that the secret number is an odd number. Direct students to check possible secret numbers at **s** in their pictures. Choose an odd number and ask if it could be the secret number. If the class disagrees on whether a number could be the secret number or not, put that number in the arrow picture and ask a student to read the number sentence told by each arrow and its starting and ending numbers.

For example, suppose the class is not sure whether **s** could be 25 or not.

Trace the arrow from 16 to 25.



S: ***16 is less than 25.***

T: ***Is that true?***

S: ***Yes.***

Trace the arrow from 16 to 24.

S: ***16 is less than 24.***

T: ***Is that true?***

S: ***Yes.***

Trace the arrow from 25 to 24.

S: ***25 is less than 24.***

T: ***Is that true?***

S: ***No.***

T: ***So our secret number cannot be 25.***

Erase 25 from the arrow picture and cross it out of the list of odd numbers.

As the class discovers which numbers in the list of odd numbers cannot be the secret number, cross them out. Continue until every odd number in your list has been considered. Ask if there are any other odd numbers that could be the secret number. Commend a student who concludes that none of the numbers indicated by the three dots at the end of the list could be the secret number because they are all more than 24. If necessary, supply this information yourself and cross out the three dots at the end of the list. Erase everything that has been crossed out.

~~1~~, ~~3~~, ~~5~~, ~~7~~, ~~9~~, ~~11~~, ~~13~~, ~~15~~, 17, 19, 21, 23, ~~25~~, ~~27~~, ~~...~~

Clue 3

Display two Minicomputer boards.

T: ***Now we know that the secret number is one of these numbers: 17, 19, 21, or 23. Listen carefully because I will give you only one more clue. The secret number can be put on the Minicomputer with exactly two checkers.***

Ask the students to put the secret number on their desk Minicomputers and check some of their responses.

T: ***Do you know the secret number?***

S: ***21.***

Invite a student to put 21 on the demonstration Minicomputer with two checkers. Ask how many checkers would be needed to put each of the numbers 17, 19, and 23 on the Minicomputer. Show the class that 21 is written on your paper.

Exercise 2: Eli's Magic Peanuts

T: ***There is an elephant named Eli. Eli is always very hungry. What do you think is Eli's favorite food?***

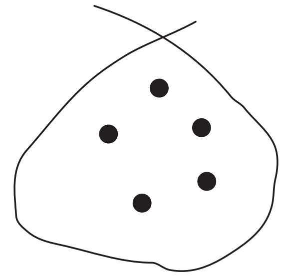
Accept some answers from the class and then tell them that Eli's favorite food is peanuts.

T: ***Eli likes peanuts so much that he always keeps a little bag of peanuts with him wherever he goes. One day as Eli was out walking, he found a plant that looked a lot like a peanut plant, but a little different than a regular peanut plant. Eli pulled up this strange plant and found some peanuts. Eli didn't know it, but the peanuts from this plant were magic peanuts! Eli gathered some of the magic peanuts and put them in his bag with the other peanuts. What do you suppose is so special about magic peanuts?***

Allow students to discuss this briefly.

T: ***Let me show you what happens when Eli puts both regular peanuts and magic peanuts in his bag.***

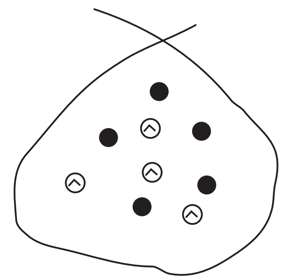
Draw this picture on the board. If you prefer, use one color of magnetic checkers for the regular peanuts and ^ checkers for magic peanuts.



T: ***This is Eli's bag of peanuts with five regular peanuts in it. Now Eli also put some magic peanuts into his bag; here are the magic peanuts.***

How many magic peanuts did Eli put into his bag?

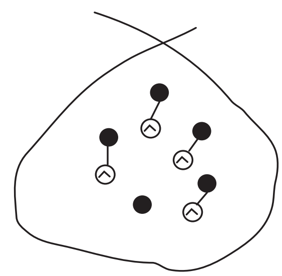
S: ***Four.***



T: ***When Eli returned home, he was hungry from walking all day. He decided to eat some peanuts. When he opened his bag, he was very surprised. There was just one regular peanut in the bag. What do you think happened when Eli put both the regular and the magic peanuts in the bag?***

Let students make suggestions and lead the discussion to the idea that when a regular and a magic peanut come together, they both disappear. Model this idea in the picture by pairing a regular peanut with a magic peanut and then removing them both from the picture.

If you use connecting lines to pair magic peanuts with regular ones, leave the picture on the board. Write an appropriate number sentence next to the picture.



$5 + \hat{4} = 1$

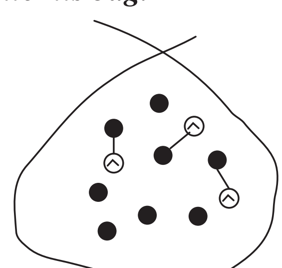
T: ***Five regular peanuts plus four magic peanuts is …?***

S: ***One regular peanut.***

S: ***1.***

T: ***Poor Eli was puzzled. He didn't know the secret of the magic peanuts and he couldn't imagine where his peanuts had gone! He was still hungry, so he went looking for more peanuts. This time, he found eight regular peanuts and put them into his bag.***

Erase your previous picture and draw a new one.



T: ***Eli also found three magic peanuts and put them into his bag.***

Add these three magic peanuts to your drawing.

T: ***When Eli returned home, what do you suppose he found when he opened his bag?***

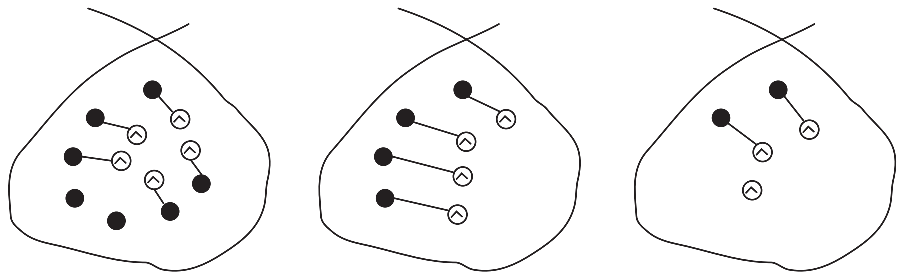
Ask several students to explain their answers; then choose volunteers to pair up regular peanuts with the magic ones. Write the number sentence next to the picture.

$8 \times \hat{3} = 5$

T: ***Eight regular peanuts plus three magic peanuts is…?***

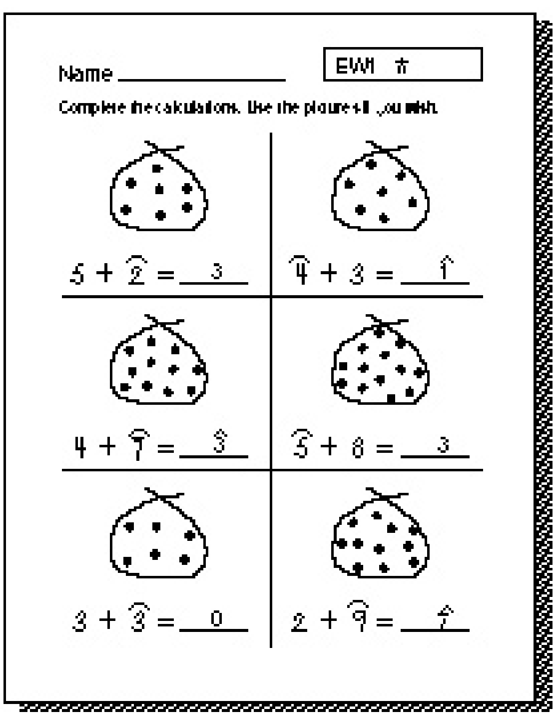
S: ***Five regular peanuts.***

Do a couple more examples such as the ones shown here. Include one example in which Eli finds magic peanuts when he opens the bag.



7 + b5 = 2   4 + b4 = 0   2 + b3 = b1

Worksheets EW1* and ** are available for independent work. Students who finish quickly can write their own stories about Eli's peanuts.



Name ______________ EW1 *

Complete the calculations. Use the pictures if you wish.

$5 + \hat{2} =$ 3

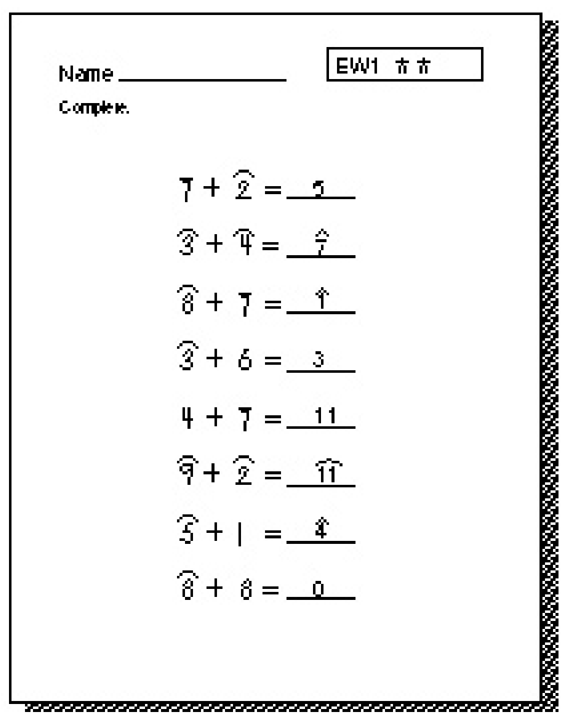
$\hat{4} + 3 =$ $\hat{1}$

$4 + \hat{7} =$ $\hat{3}$

$\hat{5} + 8 =$ 3

$3 + \hat{3} =$ 0

$2 + \hat{9} =$ $\hat{7}$



Name ______________ EW1 **

Complete.

$7 + \hat{2} =$ 5

$\hat{3} + \hat{4} =$ $\hat{7}$

$\hat{8} + 7 =$ $\hat{1}$

$\hat{3} + 6 =$ 3

$4 + 7 =$ 11

$\hat{9} + \hat{2} =$ $\hat{11}$

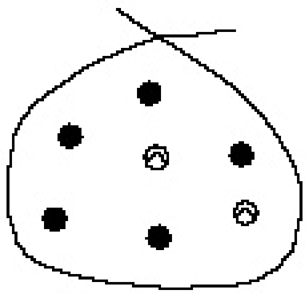
$\hat{5} + 1 =$ $\hat{4}$

$\hat{8} + 8 =$ 0

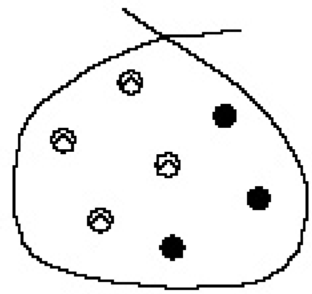
Name ____________________

EW1 *

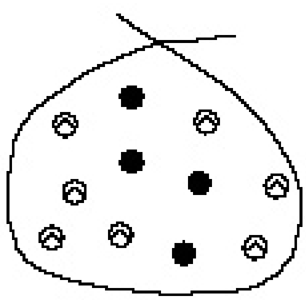
Complete the calculations. Use the pictures if you wish.



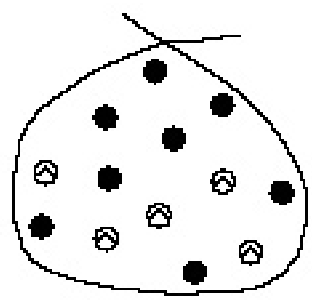
5 + 2 = ______



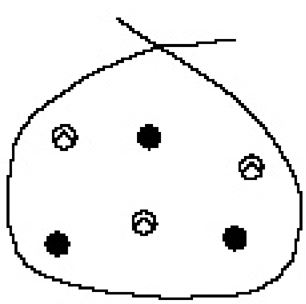
4 + 3 = ______



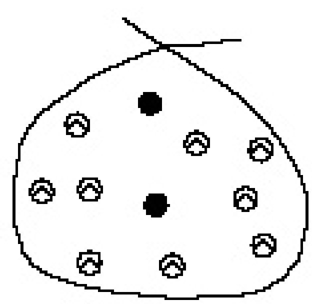
4 + 7 = ______



5 + 8 = ______



3 + 3 = ______



2 + 9 = ______

Name ____________________

EW1 **

Complete.

$7 + \widehat{2} =$ ______

$\widehat{3} + \widehat{4} =$ ______

$\widehat{8} + 7 =$ ______

$\widehat{3} + 6 =$ ______

$4 + 7 =$ ______

$\widehat{9} + \widehat{2} =$ ______

$\widehat{5} + 1 =$ ______

$\widehat{8} + 8 =$ ______

Capsule Lesson Summary

Investigate which of five functions could be used to build a road between two given numbers when all arrows in the road are the same kind. Look for shortest and longest roads. Solve similar problems in the *Which Road?* Workbook.

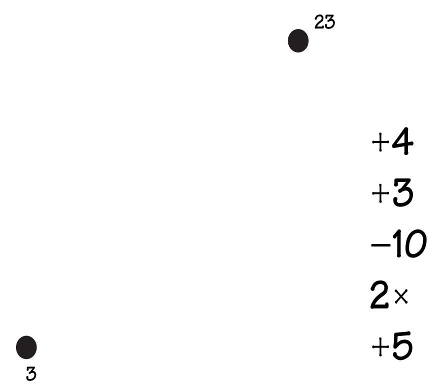
Materials

Teacher		Student	
	• Colored chalk		• Paper • *Which Road?* Workbook • Colored pencils, pens, or crayons

Description of Lesson

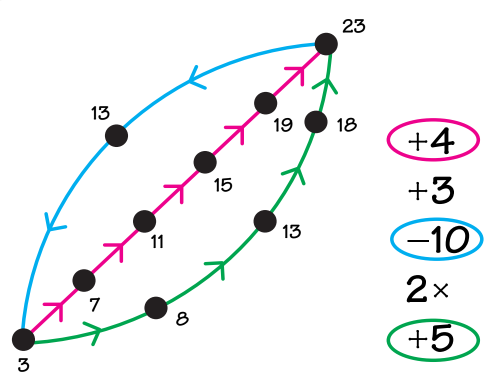
During this warm-up activity, encourage students to experiment on their papers before volunteering to put solutions on the board. You may even want to check work before selecting volunteers.

Write this information on the board.



T: ***Suppose that we want to build an arrow road between 3 and 23 using only one kind of arrow, but using it as many times as we wish. These are our choices for arrows*** (point to the list of functions)***. Can we build such a road? The road can start at 3 and go to 23, or it can start at 23 and go to 3.***

Call on volunteers to circle in color their choices of functions and then to build roads in the corresponding colors. In this case there are three possibilities. You may like students to put these three roads in three separate pictures on the board.



T: ***Which of the roads is the shortest (has the least number of arrows)?***

S: ***The –10 arrow road has only two arrows.***

T: ***Which is the longest?***

S: ***The +4 arrow road has five arrows.***

Your class should discover that a road cannot be built with either +3 or 2x arrows alone.

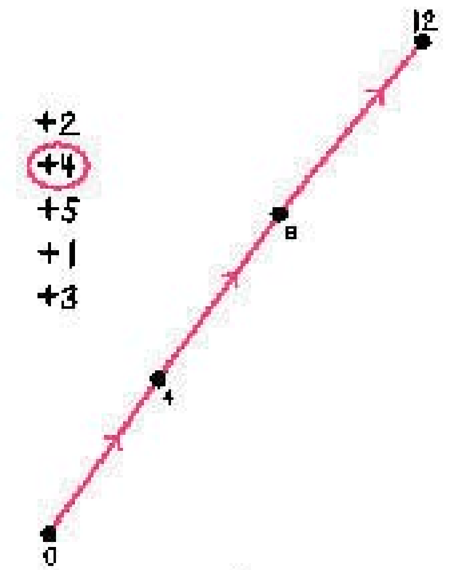
EW2

Distribute copies of the *Which Road?* Workbook. Emphasize that on each page of the workbook, the students are to build a road between the given numbers. They may only use the functions listed on that page, and each road must have only one kind of arrow.

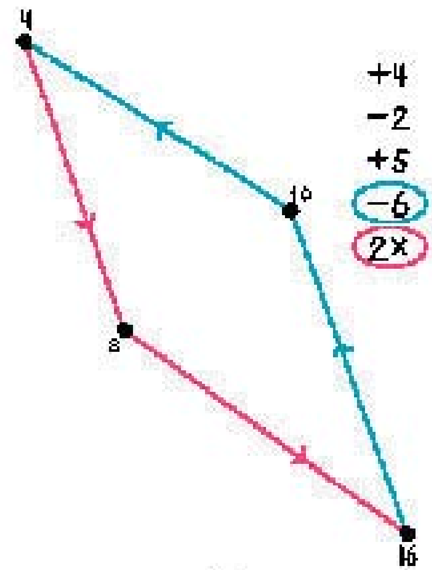
Explain that as soon as they have built a road between the two labeled dots, they have solved the problem, but that you would like them to try to build the shortest possible road. Make finding the shortest road a challenge by occasionally suggesting to students that you think there may be a shorter road than the one they have drawn.

Note: On each page of the workbook there are several functions that could be used successfully to build the road and at least one function that will not work. Of those functions that do work, some will give shorter roads than others.

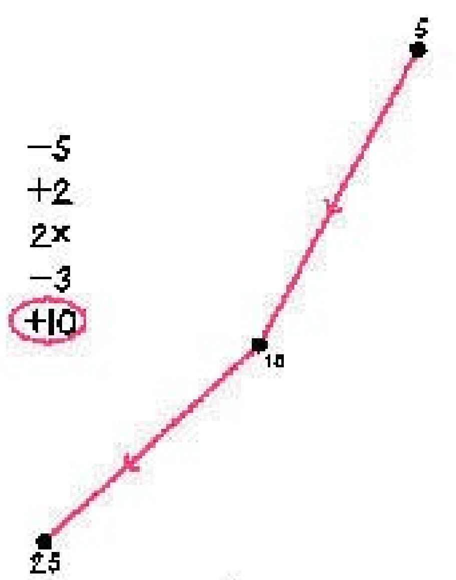
Allow approximately 30–45 minutes for the students to work individually on the *Which Road?* Workbook. Students who are able to complete the workbook quickly, finding shortest roads on each page, can be challenged to find all the possible roads on each page. Alternatively, invite such students to make up similar problems to challenge a comparable classmate or you. Collect the workbooks for your review.



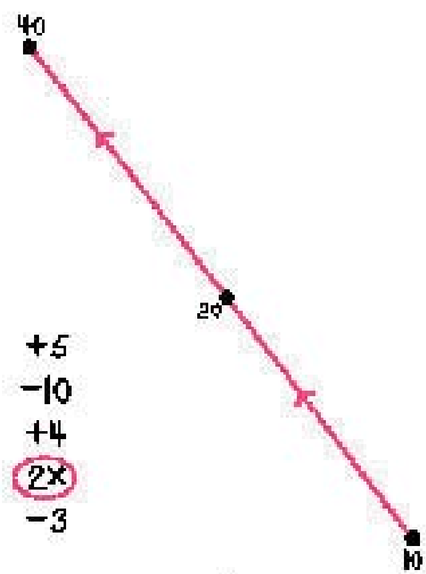
Other solutions are possible using +2, +1, +3.



Other solutions are possible using +4 or −2.

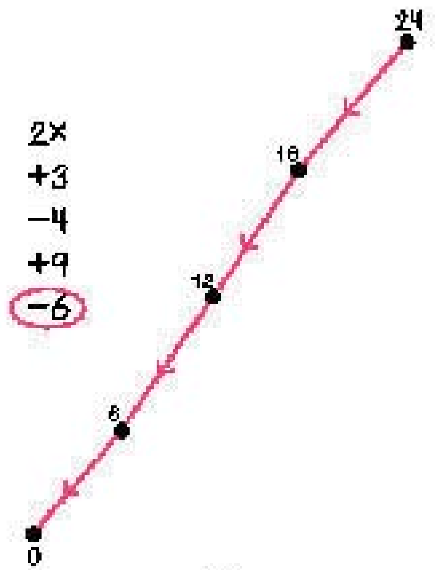


Other solutions are possible using −5 or +2.

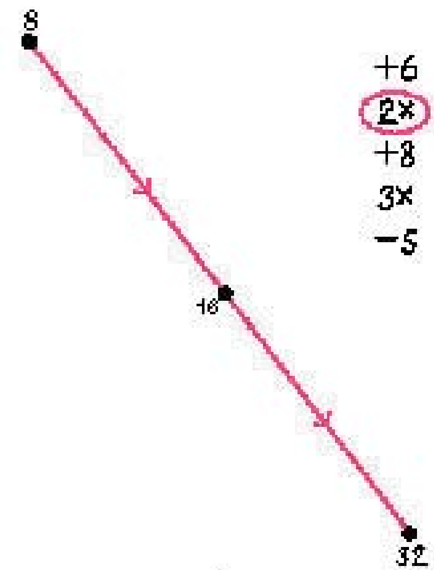


Other solutions are possible using +5, −10, or −3.

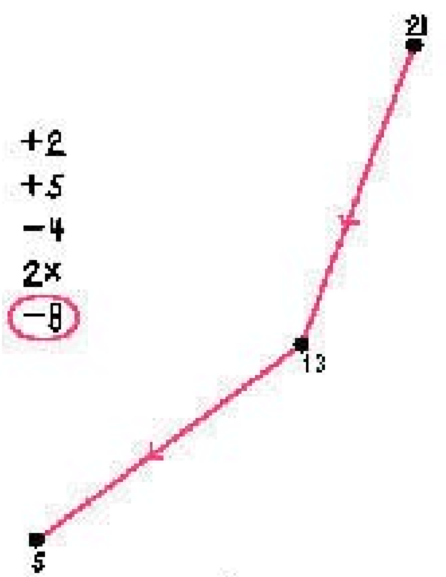
EW2



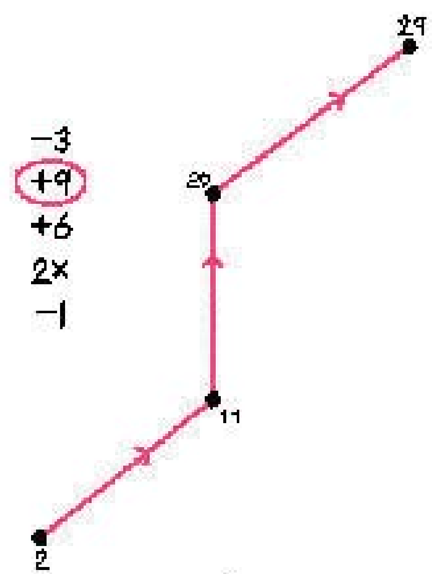
Other solutions are possible using −4 or +3.



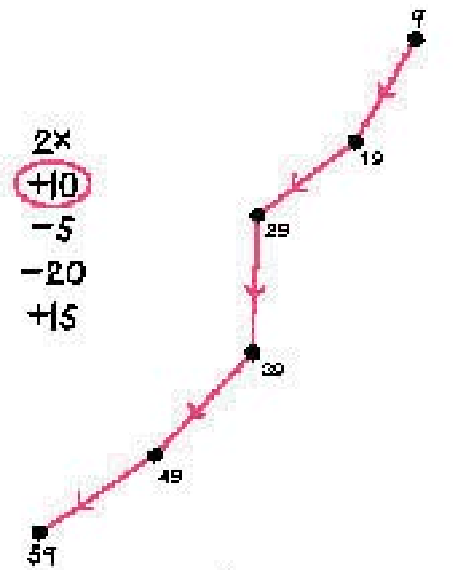
Other solutions are possible using +6 or +8.



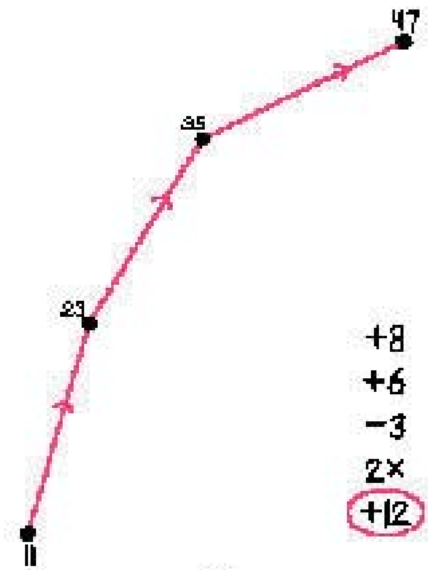
Other solutions are possible using −4 or +2.



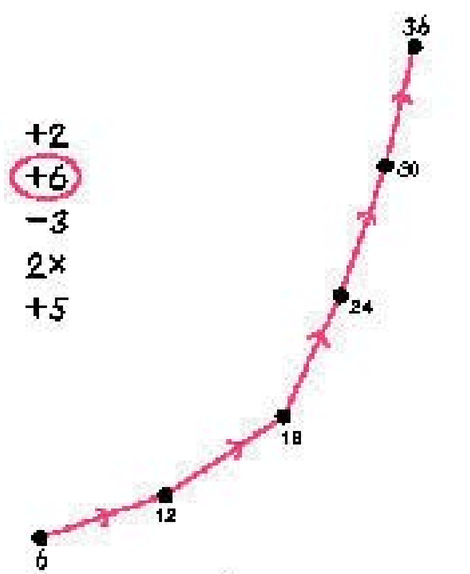
Other solutions are possible using −3 or −1.



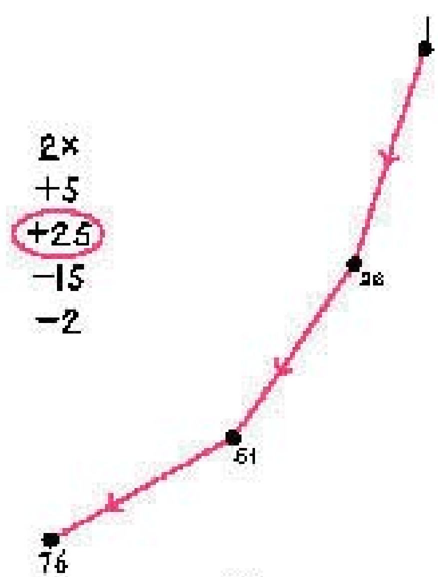
Another solution is possible using −5.



Other solutions are possible using +6 or −3.

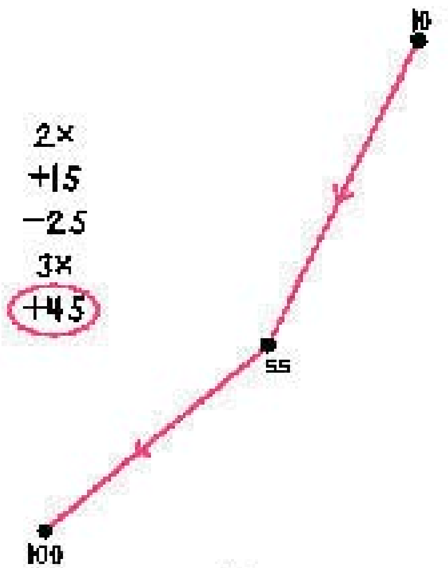


Other solutions are possible using +2, −3, or +5.

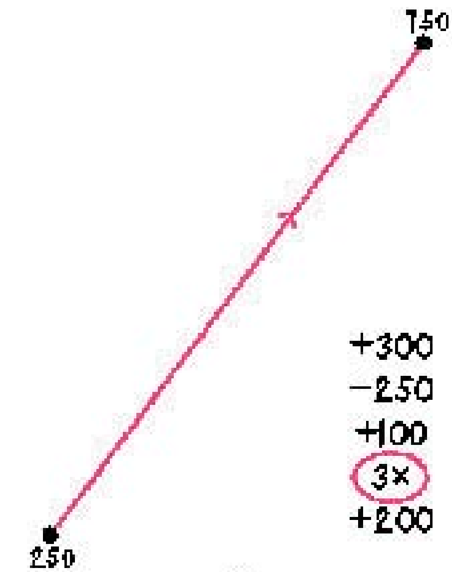


Other solutions are possible using +5 or −15.

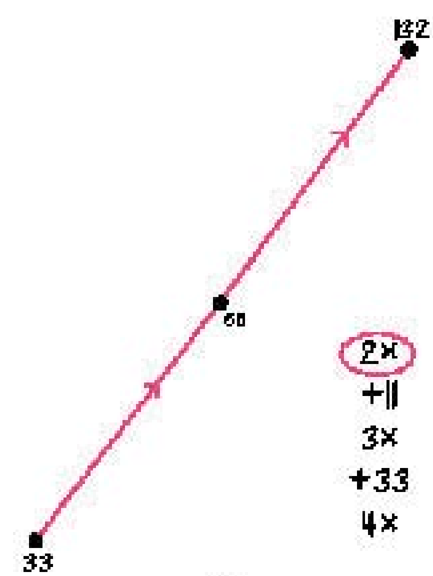
EW2



Other solutions are possible using +5 or +15.



Other solutions are possible using −250 or +100.



Other solutions are possible using +11 or +33.

EW3 DETECTIVE STORY #2/ELI'S MAGIC PEANUTS

Capsule Lesson Summary

Solve a detective story with clues involving the calculator, a string picture, and the Minicomputer. Introduce negative checkers to represent the number of magic peanuts in Eli's bag.

Materials

Teacher
- Minicomputer set
- Colored chalk

Student
- Paper
- Minicomputer set

Description of Lesson

Exercise 1: Detective Story

Allow students to work with a partner during this exercise.

Write a large 32 on a slip of paper and fold it so that 32 is hidden.

Clue 1

Distribute calculators to student pairs. If you have an overhead calculator, you may prefer to use it to introduce this clue and to let students help with pressing keys.

T: ***Today you must discover a secret number. The calculator will help us with the first clue. We will teach the calculator to count by fives starting at 2. The secret number will appear on the calculator's display.***

Instruct students to turn on their calculators and check that 0 is on the display. (They may need to press C a couple of times to get 0 on the display.) Then ask them to follow your instructions exactly.

T: ***Start with 0 on the display. Press 2. What number is on the display?***

S: ***2.***

Record 2 on the board.

T: ***Press + 5 =. What number is on the display?***

S: ***7.***

T: ***What did the calculator do?***

S: ***It added 5; 2 + 5 = 7.***

Record 7 on the board.

T: ***Press = again. What number is on the display?***

S: ***12.***

T: ***What did the calculator do?***

S: ***Added 5 again; 7 + 5 = 12.***

Record 12 on the board.

T: ***The calculator will continue to count by fives (add fives) as we press* $\boxed{=}$. *Press* $\boxed{=}$ *one more time. What number is on the display?***

S: ***17.***

Continue your list on the board.

2, 7, 12, 17

T: ***Think about pressing* $\boxed{=}$ *three more times. What are the next three numbers we will get on the calculator?***

Allow the class to predict these numbers. Then record them on the board as the students press $\boxed{=}$ three more times and note them on the display. As soon as most of your students are able to predict the sequence quickly, you can abandon the calculator. Continue counting and recording until the class realizes this sequence goes on and on. Your list on the board should have several numbers greater than 50. You may like to ask the students to suggest a number that would be in the list that is greater than 100, or between 150 and 160.

2, 7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, 62, 67, 72, ...

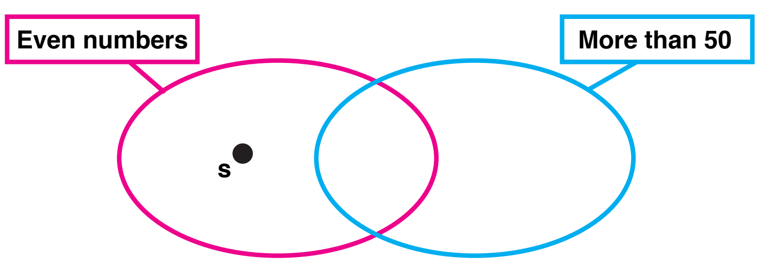
Explain that the three dots mean that the list goes on and on.

T: ***Our secret number is one of the numbers in this list. Do we know which one? What do we need?***

S: ***Another clue.***

Clue 2

Draw this picture on the board and ask students to copy it on their papers.



T: ***The second clue is that the secret number is here*** (point to **s**). ***Is it an even number?*** (Yes) ***Is it more than 50?*** (No) ***Which of these numbers could be the secret number?***

Suggest students put numbers from the list into their string pictures and find possibilities for the secret number. Each time a number is suggested, label the dot with that number and ask the class if the number is placed correctly. Suppose, for example, a student suggests 72.

T: ***Could this be 72?*** (Disagreement) ***Is 72 even?*** (Yes) ***Then it must be inside the red string. Is 72 more than 50?*** (Yes) ***Then it must be inside the blue string. Where should 72 be?***

Ask a student to point to the correct region (the middle).

T: ***72 cannot be the secret number.***

Put 72 correctly in the string picture and cross 72 off the list on the board.

If a correct number is suggested, elicit class agreement and circle it on your list. Continue until many of the numbers in the original list are either circled or crossed off.

Point to the three dots.

T: ***Could the secret number be any of the numbers we didn't write on the board?*** (No) ***How do you know?***

S: ***Because the secret number is not more than 50.***

Cross off the three dots.

Point to each unchecked number and guide your class in deciding if it could be the secret number. When all the numbers have been checked, erase those that have been crossed off.

T: ***Now we know the secret number is one of these*** (point to the five remaining numbers)**.**

2, 12, 22, 32, 42

Clue 3

Display two Minicomputer boards.

T: ***This will be the final clue. When you know which of these numbers is the secret number, write it on your paper. You cannot put the secret number on the Minicomputer with only one or two checkers. At least three (regular) checkers are needed to put the secret number on the Minicomputer. Which is the secret number?***

Look at (or listen to) the responses of several student partners. After most students realize that 32 is the secret number, ask a student to say the secret number aloud. Check the other numbers by letting students demonstrate that each of them can be put on the Minicomputer with one or two checkers. Reveal that 32 is written on your paper.

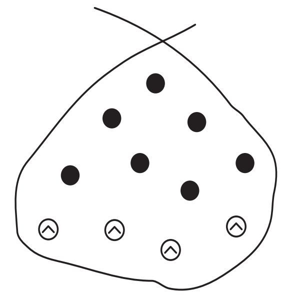
Exercise 2: Eli the Elephant

T: ***Who remembers Eli the Elephant?***

Allow students to recall whatever they remember. Be sure to mention that Eli collects peanuts; he sometimes finds regular peanuts and sometimes magic peanuts. Review what happens when a magic peanut and a regular peanut meet.

T: ***Today Eli went walking in the park and found seven regular peanuts and four magic peanuts. He put all the peanuts in his bag. He plans to eat the peanuts tonight while he watches television. How many peanuts will he find when he opens his bag?***

Draw a picture of the bag with regular and magic peanuts in it.
Write a number sentence about the situation on the board.



$7 + \hat{4} = ?$

T: ***Seven regular peanuts plus four magic peanuts is…?***

S: ***Three regular peanuts.***

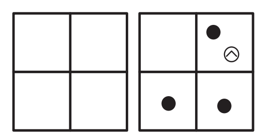
Complete the number sentence. Display two Minicomputer boards.

T: ***Let's show this story about Eli's peanuts on the Minicomputer. How many regular peanuts did Eli find?***

S: ***Seven.***

Invite someone to put 7 on the Minicomputer. Show the class the negative checkers, .

T: ***We can use negative, or magic, checkers to show how many magic peanuts Eli found in the park. How many magic peanuts did he find?***



S: ***Four.***

T: ***Show this on the Minicomputer using negative checkers.***

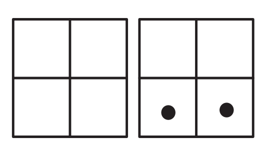
Do not allow students to remove any checkers from the Minicomputer for the moment.

T: ***What will happen?***

S: ***The regular checker and the magic checker on the purple (4-) square will disappear.***

Explain that four magic peanuts and four regular peanuts disappear, and ask a volunteer to show the class what happens on the Minicomputer. The student should remove both checkers from the purple square.

T: ***Seven regular peanuts plus four magic peanuts equals…?***



S: ***Three regular peanuts.***

Erase the board and remove the checkers from the Minicomputer.

T: ***Last week Eli found a lot of peanuts. One day he found 39 magic peanuts and 25 regular peanuts. When Eli opened his bag that night, what did he find?***

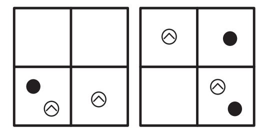
S: ***Magic peanuts.***

Write a number sentence about this situation on the board.

$$\widehat{39} \times 25 = ?$$

If several students know the answer, ask them to write the result on their paper or whisper the number to you. Encourage students to estimate how many magic peanuts are in the bag.

T: ***Who can put $\widehat{39}$ on the Minicomputer with negative checkers?***
Who can put 25 on the Minicomputer with regular checkers?



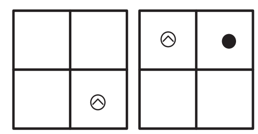
T: ***What happens?***

S: ***The checkers on the 1-square disappear, and the checkers on the 20-square disappear.***

Invite students, one at a time, to remove the checkers from the 1-square and then from the 20-square. Encourage students to explain that one regular and one magic peanut disappear, and then twenty regular and twenty magic peanuts disappear.

T: ***What could we do to make the number easier to read?***

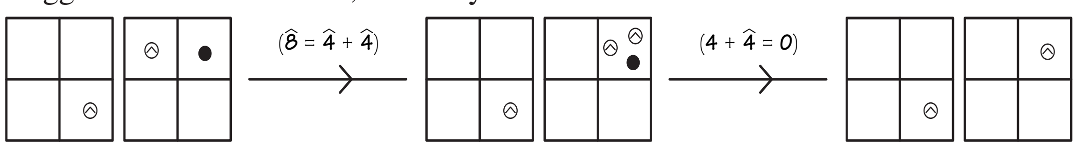
S: ***Make a backward trade.***



Do not expect the verbalization of this trade to be exact. If a student says, "8 = 4 + 4," accept it, but rephrase it by saying that eight magic peanuts is the same as four magic peanuts plus four magic peanuts.

If a student suggests that checkers not on the same square will disappear, for example, $\widehat{8}$ and 4, remind the class that those checkers are for eight magic peanuts and four regular peanuts.

If no one suggests a backward trade, make it yourself.



Exercise 3

T: ***Today Eli the Elephant went to visit his grandmother. On the way to her house, he took a detour through a field where he had seen many peanut plants. Eli's grandmother loves peanuts, so Eli planned to bring her a bag full of peanuts as a surprise.***

Eli spent a long time collecting peanuts. He was thinking how pleased his grandmother would be when he gave her the bag of peanuts.

Eli's grandmother met him at the gate and gave Eli a big hug. Eli held out the bag and said, "Grandmother, I've brought you a bag of peanuts." Eli's grandmother was very happy. She gave Eli a big elephant kiss and went to get a bowl for the peanuts. She shook the bag and out fell—one peanut! She searched inside the bag and then Eli searched inside the bag, but there were no more peanuts. Eli felt confused.

What do you think happened to the peanuts Eli picked?

Allow the students to comment. Very likely someone will suggest that the other peanuts disappeared. If a student suggests that Eli ate the other peanuts, or that there was a hole in the bag, or any other reasonable answer, accept it as a possibility.

T: ***Why would the peanuts disappear?***

Guide this discussion until it is clear that a number of regular peanuts and the same number of magic peanuts would disappear.

T: ***Do we know how many peanuts Eli put in his bag?***

S: ***No.***

If students begin to give you suggestions of how many regular and magic peanuts Eli picked, record them on the board. If no one suggests any specific numbers, ask,

T: ***How many regular and how many magic peanuts could Eli have collected?***

Each suggestion should be considered to see if all but one peanut would disappear. Write the corresponding number sentence for each suggestion. Here are some possibilities:

$$10 \times \widehat{9} = 1$$

$$\widehat{10} \times 9 = \widehat{1}$$

$$59 \times \widehat{58} = 1$$

$$58 \times \widehat{59} = \widehat{1}$$

$$\widehat{1000} \times 1001 = 1$$

$$5 \times 7 \times \widehat{3} \times \widehat{4} \times 4 \times \widehat{2} \times \widehat{6} = 1$$

T: ***Poor Eli, he still doesn't know about magic peanuts. We will never know exactly how many peanuts Eli gathered that day for his grandmother.***

EW4 DETECTIVE STORY #3/ELI'S MAGIC PEANUTS

Capsule Lesson Summary

Discover a secret number from three clues. The first involves putting numbers on the Minicomputer using one positive checker and one negative checker; the second involves building roads with exactly two 2x arrows and two +3 arrows; and the third gives the secret number's place in a three-string picture. Introduce Clarence the Crafty Crocodile who plays a trick on Eli the Elephant. Observe that the trick could have involved adding magic peanuts or taking peanuts out of Eli's bag.

Materials

Teacher
- Minicomputer kit
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons
- Minicomputer set

Description of Lesson

Exercise 1

Before the lesson begins, write a large 36 on a slip of paper and fold the paper over so that 36 is hidden.

Begin the lesson with a brief discussion of detectives. Then refer to the paper.

T: ***I wrote a secret number on this paper. You are going to be detectives. I will give you some clues to help you find out the secret number. If you are good detectives, you can discover the secret number without just guessing.***

Clue 1

Display two Minicomputer boards. Provide students with one Minicomputer sheet and two checkers, one positive and one negative. You may like to let students work in pairs or cooperative groups.

T: ***The secret number can be put on these Minicomputer boards using exactly two checkers. It can be put on using one regular checker on the tens board*** (display a regular checker by the tens board) ***and one negative checker on the ones board*** (display a negative checker by the ones board)***.***

Instruct students to find all possibilities for the secret number. Encourage them to make a list on their papers. When you observe that many students (groups) have completed their work, direct the class's attention to the demonstration Minicomputer.

T: ***What are some numbers that could be my secret number?***

For each suggestion, ask the student to show the number on the Minicomputer using one regular checker on the tens board and one negative checker on the ones board. If correct, record it in a list of possibilities on the chalkboard.

EW4

Try to organize the list so that when all the possibilities are found your list delineates some patterns. In fact, the patterns here may help students find some of the solutions.

2	6	8	9
12	16	18	19
32	36	38	39
72	76	78	79

Take away the Minicomputer boards, but leave the list of numbers on the chalkboard.

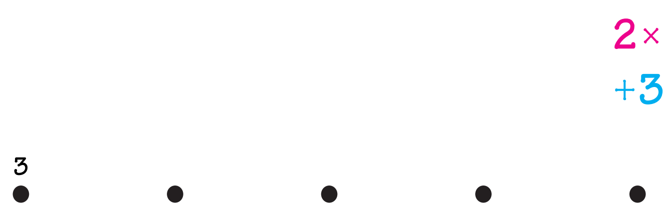
T: ***My secret number is one of these*** (point to the list)***. Do you know my secret number? What do you need?***

S: ***Another clue.***

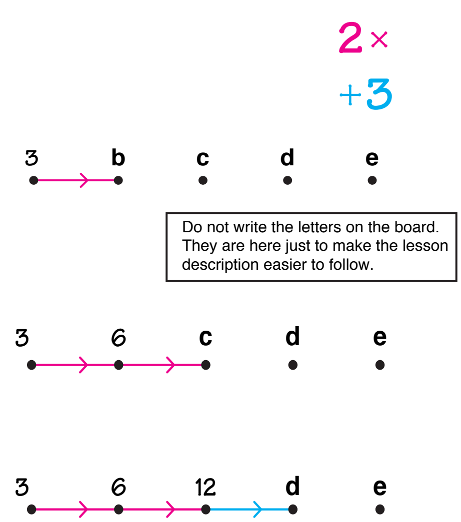
Clue 2

Draw the picture below on the board.

T: ***Here is a second clue. My secret number is the ending number of an arrow road that starts at 3*** (point to the dot for 3), ***and has exactly two red (2x) arrows and two blue (+3) arrows.***



Invite students to suggest arrow roads. There are six possibilities; one is described in the following sample dialogue.



T: ***What color is the first arrow?***

S: ***Red.***

T: ***What number is here (at* b*)?***

S: ***6.***

T: ***What color is the second arrow?***

S: ***Red.***

T: ***What number is here (at* c*)?*** (12)
What color is the next arrow?

S: ***Blue.***

T: ***What number is here (at* d*)?*** (15)
What color is the last arrow?

S: ***Blue.***

T: ***What number is here (at* e*)?***

S: ***18.***

T: ***Does this road have two red (2x) and two blue (+3) arrows?***

S: ***Yes.***

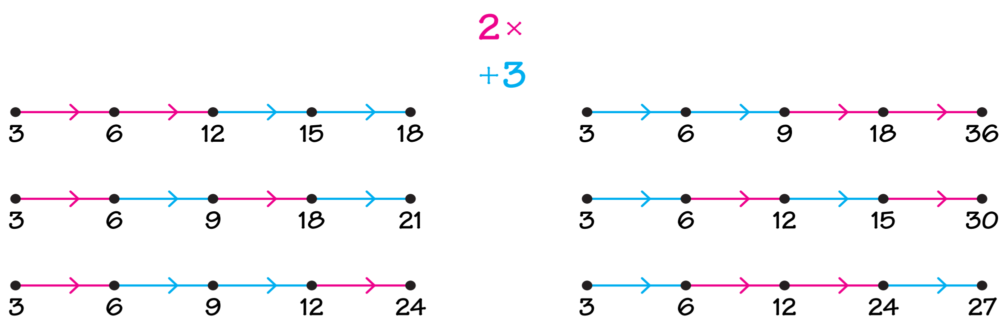
T: ***This road ends at 18. Is 18 on our list?***

S: ***Yes.***

T: ***So my secret number could be 18. I will circle it in the list.***

Instruct students to work with a partner or in groups to find all the different roads with exactly two red (2x) arrows and two blue (+3) arrows. Ask students to draw the roads on their papers as you did on the board, and to remember all the roads start at 3.

As students are working, encourage them to be thinking about how many different roads there are. After a while, invite students to put the six different roads on the board.



With all six roads drawn on the board, draw the class's attention to the list of numbers from the first clue, and consider whether the ending number of each road could be the secret number. Circle any entry in the list that is also the ending number of one of the arrow roads. For example, suppose the road ending at 30 is being discussed.

T: ***This road is blue-red-blue-red and ends at 30. Could 30 be my secret number?***

S: ***No, 30 is not in the list.***

Note: If there is confusion as to whether a number in the arrow pictures other than an ending number could be the secret number, erase the numbers from all but the starting and ending dots.

After comparing all the ending numbers of the arrow roads with the numbers in the list from the first clue, you should have only 18 and 36 circled. Erase all others.

18

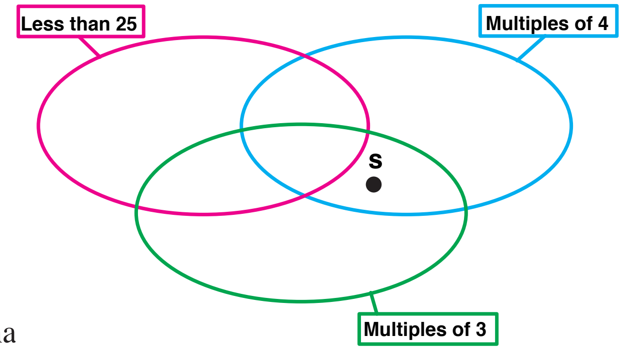
36

T: ***What do you need to help decide which of these numbers (18 or 36) is my secret number?***

S: ***Another clue.***

Clue 3

Erase the arrow roads from the board and draw this string picture. You may need to remind students about multiples of 3 and 4.



T: ***My secret number is here*** (point to **s**). ***Decide whether 18 or 36 is my secret number and write it on your paper.***

Check papers and when most students have found tha the secret number is 36, show the class the slip of paper.

T: ***Who can explain why 36 is here*** (point to **s**)***?***

S: ***36 is a multiple of 3 and a multiple of 4.***

T: ***Is 36 less than 25?***

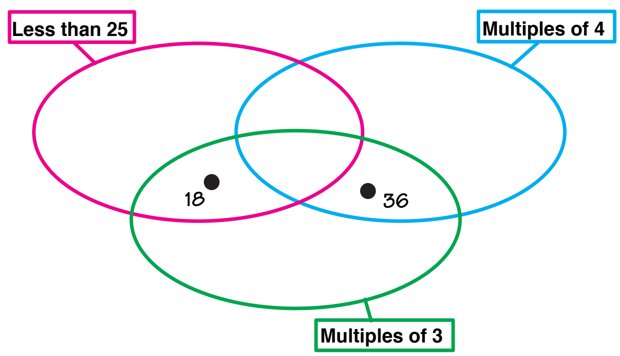
S: ***No.***

T (tracing the red string)**:** ***So 36 must be outside the red string.***

Erase the **s** and label the dot 36.

T: ***Who can show us where 18 belongs in this picture?***

When a student indicates the correct region, draw a dot in that region and label it 18.

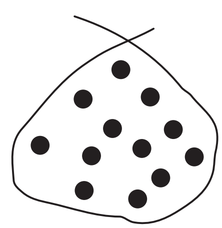


T (tracing the appropriate strings in turn)**:**
Is 18 less than 25? (Yes)
Is 18 a multiple of 3? (Yes)
Is 18 a multiple of 4? (No)

Exercise 2: Eli the Elephant

T: ***One day Eli was walking through the jungle. There were 12 regular peanuts in his bag.***

Draw this bag of peanuts on the board.



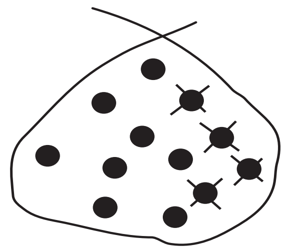
T: ***Eli walked a long time; he was very tired and decided to take a nap. As soon as Eli fell asleep, along came Clarence the Crafty Crocodile. Clarence saw that Eli was asleep and decided that it would be fun to play a trick on Eli. Clarence opened Eli's bag of peanuts and did something. Then he closed Eli's bag quietly and went away.***

Eli felt hungry when he awoke, so he opened his peanut bag. Eli counted his peanuts and cried, "Oh no! There are only eight peanuts. Four peanuts are missing." What do you think Clarence the Crafty Crocodile did?

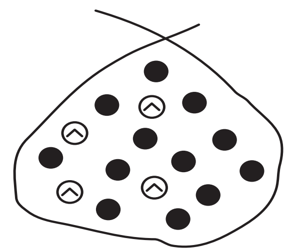
Allow the class to discuss this problem. Two possibilities should emerge: Clarence could have removed four peanuts from Eli's bag, or Clarence could have put four magic peanuts into the bag.

Note: There are other more complex possibilities which might be suggested. For example, Clarence could have added two magic peanuts and removed two regular peanuts. Accept such suggestions as correct, but for the purpose of this lesson, focus primarily on these two possibilities: Clarence removed four regular peanuts, or Clarence added four magic peanuts.

Illustrate each of these situations as it is suggested. Write the appropriate number sentence under each bag of peanuts. Arrange the board so that the two pictures are side by side.



$12 - 4 = 8$



$12 + \widehat{4} = 8$

Emphasize that you do not know what Clarence did when he opened Eli's bag, because removing four regular peanuts and adding four magic peanuts have exactly the same effect. Write this as a number sentence on the board.

$$12 + 4 = 12 - 4$$

T: ***Another day when Eli was walking in the jungle he gathered 55 regular peanuts. He became very tired and decided to take a nap. Can you guess what happened while Eli was sleeping?***

S: ***Clarence the Crafty Crocodile came by and played another trick on Eli.***

T: ***Eli felt hungry when he awoke, so he opened his peanut bag. Eli counted his peanuts and found that 29 peanuts were missing. What do you think Clarence the Crafty Crocodile had done?***

S: ***Clarence could have removed 29 peanuts from Eli's bag.***

S: ***Clarence could have put 29 magic peanuts into Eli's bag.***

As each possibility is suggested, write the appropriate number sentence on the board.

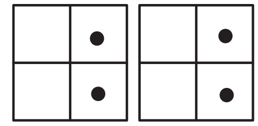
$$55 - 29 = ?$$
$$55 + \widehat{29} = ?$$

T: ***How are we going to find out how many peanuts Eli has left in his bag?***

If someone suggests drawing a picture of Eli's bag, accept this as a good but not very practical method, because you would have to draw so many peanuts. If no one suggests using the Minicomputer, suggest it yourself. Point to the number sentences on the board.

T: ***Which of these problems do you want to do first on the Minicomputer?***
Suppose the students want to calculate 55 – 29 first.

Invite someone to put 55 on the Minicomputer.



T: ***Can someone take 29 off the Minicomputer?***

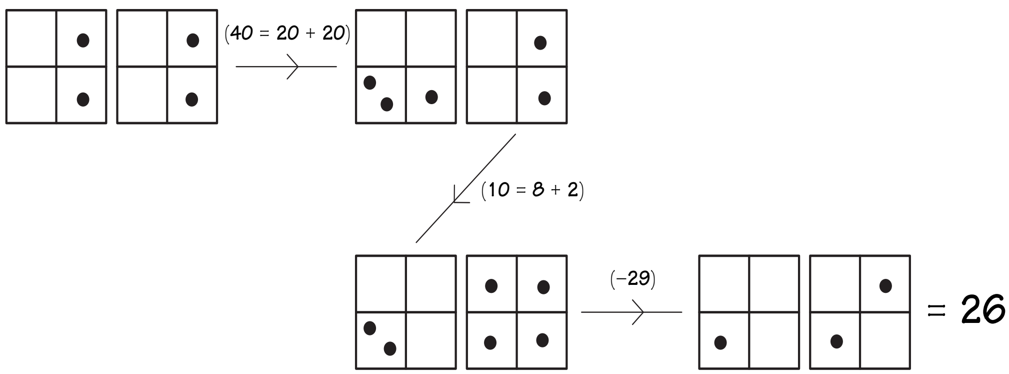
S: ***We do not have checkers in position for 29.***

Let someone point to where the checkers for 29 would be.

T: ***Who can make a backward trade that will help us to calculate 55 – 29?***

Ask a student to identify a trade before moving checkers. This will help to discourage trades that are not useful. Whenever a trade is made that puts a checker in a position for the subtraction, mention this to the class and indicate that they are getting closer to the goal.

Let the students make trades until 29 can be taken off the Minicomputer. A possible sequence of trades is shown below.

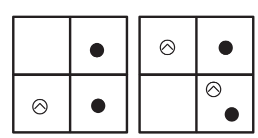


Complete the appropriate number sentence on the board and remove the checkers from the Minicomputer.

$$55 - 29 = 26$$

T: ***Now let's suppose Clarence put 29 magic peanuts into Eli's bag.***

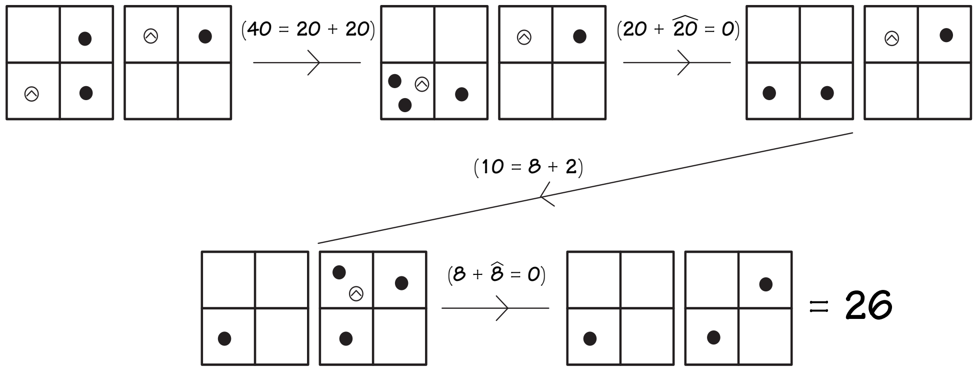
Invite a student to put 55 on the Minicomputer and another to put $\widehat{29}$ on the Minicomputer.



T: ***We want to get a regular and a negative checker on the same square so that we can remove them. Who can make a trade?***

Let students make trades until the checkers on the Minicomputer are all of the same kind. In this case, they will all be regular.

One sequence of trades is illustrated below. (**Note:** $1 + \widehat{1} = 0$ has already been made.)



Complete the number sentence on the board.

$$55 - 29 = 26$$
$$55 \times \widehat{29} = 26$$

Emphasize that we do not know what Clarence did when he opened Eli's bag, because removing 29 regular peanuts and adding 29 magic peanuts have exactly the same effect. Write this as a number sentence.

$$55 - 29 = 26$$
$$55 \times \widehat{29} = 26$$
$$55 - 29 = 55 \times \widehat{29}$$

T: ***Which calculation did you find easier on the Minicomputer?***

Accept either response, letting students comment on why they thought one calculation was easier than the other.

Continue this activity with a similar situation in which Eli has 83 regular peanuts and later discovers that 57 peanuts are missing. Let student partners choose which calculation they will do, either 83 – 57 or 83 + $\widehat{57}$, on their desk Minicomputers. It is not necessary to do both calculations.

Writing Activity

Ask students, possibly working with their partners, to write and illustrate a story about Eli and his friend Clarence. The story should include an example of what happens with Eli's peanut bag. Stories can be used to explain this episode of the Eli story to parents/guardians.