The MANS Tests:

Mathematics Applied to Novel Situations

Developed at CEMREL, Inc.

Mathematics Research and Evaluation Studies

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INTRODUCTION

The MANS Tests (Mathematics Applied to Novel Situations) are a collection of short tests that assess how well students in grades 2 to 6 can use mathematical thinking and skills to solve problems that are new or unfamiliar to them.

Both the content and the format of the MANS tests are markedly different from tests in mathematics now available. The emphasis of most current tests is on mathematical content such as computation, terminology, or word problems. Students demonstrate that they have learned this content by answering questions presented in formats they have seen many times before.

The MANS Tests broaden the traditional content of tests in mathematics by emphasizing abilities such as estimation, mental arithmetic, production of many correct answers to a given problem, and recognition of patterns and relationships, in addition to computation and word problems. Students demonstrate that they have acquired these abilities by solving exercises presented as a series of interesting, almost game-like, problems.

Poor problem solving skills in mathematics have become a matter of national concern (National Assessment for Educational Progress [1], The National Council of Teachers of Mathematics [2]). Within the next several years mathematics curricula and instructional strategies will be developed that emphasize mathematical problem solving. For educators who will be using these new programs, or who wish to assess the present status of students' problem solving abilities, the MANS Tests can be used to assist them in curricular and instructional planning.

More than seven years of research with more than 20,000 students has already been done on the MANS Tests. The tests are sensitive to the effects of mathematics programs that emphasize problem solving. The MANS Tests are practical to administer; they are technically sound; and, unlike conventional tests in mathematics, most students like to take them.

1 Changes in Mathematical Achievement, 1973-78. Report No. 09-MA-01. Denver, Colorado: NAEP, August 1979.

2 An Agenda for Action: Recommendations for School Mathematics of the 1980's. Virginia: NCTM, 1980.

ADVANTAGES OF THE MANS TESTS

- The MANS Tests measure and report separate scores for a broad range of mathematical processes that reflect goals identified by the National Council of Teachers of Mathematics. Current standardized achievement tests give only two or three scores, typically in traditional areas such as computation, concepts and applications.

- The MANS Tests challenge students to apply their mathematical skills to unfamiliar, novel problems. Current tests emphasize knowledge and computational skills using item formats that students practice frequently.

- The MANS Tests require little or no reading; when reading is required, sentences are short and words are easy. Current tests require reading comprehension skills that are beyond what many students possess.

- The MANS Tests are a series of short tests; each test has a few items dealing with a single kind of problem, has specific directions targeted to that problem, and has flexible time limits (except where estimation skills are tested). Current tests contain as many as 50 items, with general directions that must fit all items, and timed as a single unit.

- The MANS Tests often require students to generate answers for themselves and to record them directly in the student test booklet; sometimes students are given credit for as many correct answers as they can give to the same problem. Current tests require students to select one correct answer from four or five given alternatives.

- The MANS Tests compare class performance to norms based on the ability level of the class; ability is measured by a special vocabulary test that is part of the MANS. Current tests compare individual students to general norms for the whole population, rather than to students of similar ability.

- The MANS Tests have been used in research to show curricular and instructional differences in several kinds of problem solving. The problem solving section of current tests are dominated by one-step word problems with heavy computational demands.

- Students enjoy taking the MANS Tests because of the novelty, variety, and intrinsic mathematical content of the tests.

ADMINISTERING THE MANS TESTS

The MANS Tests are contained in two student booklets at each grade level. These booklets are consummable and are printed on inexpensive newsprint.

The booklets contain several tests. Each test has its own directions from the Tester Directions Manual, which the tester follows in explaining the task and the sample items. Students then complete the test items on their own. A flexible time limit, usually about 5 or 6 minutes, allows almost all students to finish.

Each MANS Test takes up one or two pages in a booklet so that diagrams and illustrations are large, words are easy to read and there is ample space for students to do scratch work. For most tests, students produce their own answers instead of selecting one of several given alternatives. Answers are to be written in the booklet and can be erased or crossed out; no special pencil is required.

Most tests have two parallel forms, one for each half of the class. The two forms deal with the same problem solving task, with common tester directions and sample items. Only the actual test items are different and the forms look the same at first glance. The score for the class on a test is then based on both forms of the test. Thus the time required for testing is much less than if every student did every item.

In addition to the Tester Directions Manual there is a Coordinators Manual, which local test coordinators can use to train MANS testers. Both manuals are important to the MANS testing process; the directions for each scale have been developed very carefully so that the nature of the task, but not the means to solve it, can be presented efficiently to students.

On the next pages, two sample tests are shown. For each test, the relevant pages from the Tester Directions Manual and the student booklet are shown in reduced form.

Sample test from third grade.

Tester Directions

Look at the number line at the top of the page. (Demonstrate.)

The numbers on that line are 2, 6, 10, then a box, then 18, 22, 26, and 30. You have to figure out what number would go in the box.

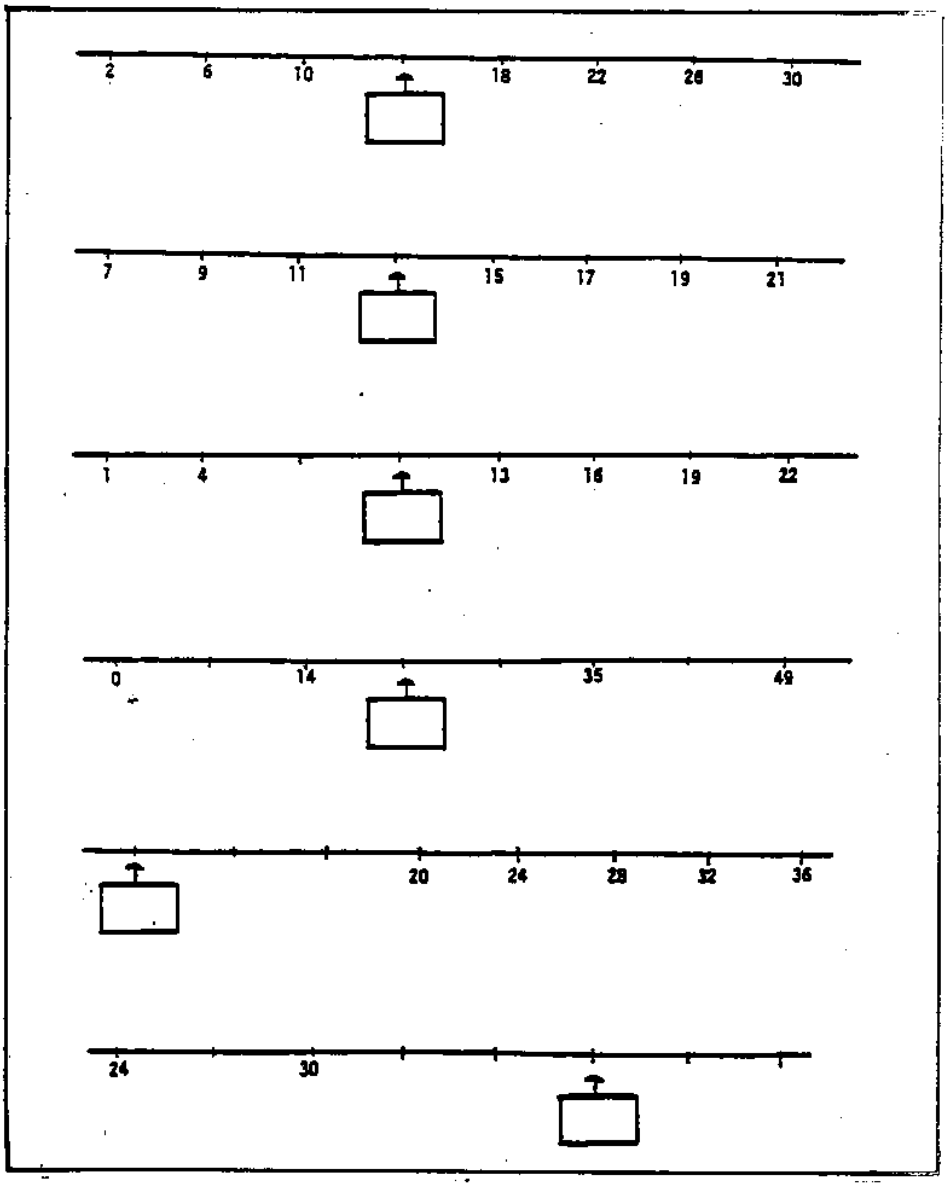
Let's find out. (Pause.) The marks are going up by 4 each time, and 4 more than 10 is 14, so you write 14 in the box and then put your pencils down. (Pause.)

In the problem we just did, the number line went up by 4 each time. But the rest of the problems may be different and you'll have to figure out for yourself how much <u>they</u> are going up by. Some of these are hard, and if you get stuck just go on to the next one.

Some of the marks don't have a number under them. You can write in the number if it will help you, but you don't have to.

Alright, go ahead and do the rest of the problems.

Student Page



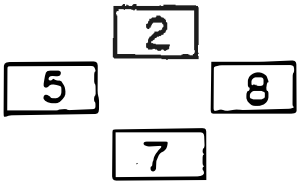
Sample from fourth grade

Tester Directions

On this page we have to make up numbers using the digits at the top of the page: 5, 7, 2 and 8. There are two rules we always have to obey. First it says, "Use only these digits." That means you can't use any other digits. Then it says, "Use each digit only once." You can't use the same digit twice. Look at example 1. It says, "What is the largest three digit number?" and nine hundred ninety-nine is crossed out. It's wrong because it has nines in it and you can only use 5, 7, 2 and 8. (Pause.) Eight hundred eighty-eight is wrong because they used 8 three times and you can't use a digit more than once. If you follow the rules, 875 is the largest 3 digit number you can make.

Now look at example 2. It says, "What is the smallest two digit number between 70 and 86?" So you couldn't say "25" because it's too small; it has to be between 70 and 86. You couldn't say "71" because "1" isn't a digit you can use --the answer must be "72". Remember, you can only use the digits 5, 7, 2 and 8 and you can't use the same digit twice in an answer. Now you do the rest of the problems on this page.

Student Page



Rules: Use only these digits.

Use each digit only once.

Example: 1. What is the largest three digit number? ~~9 9 9~~ ~~8 8 8~~ 8 7 5

Example: 2. What is the smallest two digit number between 70 and 86? ~~2 5~~ ~~7 1~~ 7 2

What is the smallest two digit number? ___

What is the largest two digit number between 74 and 86? ___

What is the second largest three digit number? ___

What is the smallest three digit number between 730 and 850? ___

What is the largest four digit number? ____

What four digit number between 2,000 and 3,000 is closest to 2,800? ____

CONTENT

MANS Test booklets provide an interesting and varied sequence of tasks for students. For analyzing and reporting purposes, however, tests are grouped into categories of similar mathematical processes.

Seven categories comprise the "core" group of MANS Tests; there are tests in each of these categories at each grade level. These categories are described below with samples from tests at various grade levels.

Computation

Straightforward calculation with basic facts and algorithms.

Examples from a third grade test.

SUBTRACTION

$$\begin{array}{r} 11 \\ -\,5 \\ \hline \end{array} \qquad \begin{array}{r} 73 \\ -\,5 \\ \hline \end{array} \qquad \begin{array}{r} 64 \\ -\,28 \\ \hline \end{array}$$

MULTIPLICATION

$5 \times 8 =$

$$\begin{array}{r} 31 \\ \times\,2 \\ \hline \end{array}$$

Estimation

Rapid calculation of approximate answers. For this category only, fixed time limits are prescribed.

Examples from a sixth grade test. Short time limit.

		CHECK ONE		
		Less than 1	Exactly 1	More than 1
Sample	$\frac{3}{4} + \frac{3}{4}$			✓
	$\frac{7}{12} + \frac{5}{12}$			
	$2\frac{1}{2} \div 3$			

Mental Arithmetic

Solution of numerical problems that emphasize an understanding of numbers and operations, but do not require great mental computational facility.

Examples from a sixth grade test. No "scratch work" is allowed.

7,001 − 6,999 = ☐

☐ − 250 = 150

98,001 − ☐ = 98,000 − 5,000

12 x 500 = ☐

101 x 43 = ☐

7 x 43 = 301

14 x 43 = ☐

Number Representations

Recognition, or prodution of different ways of representing numbers, including place value, number lines, measurement.

Example from a second grade test For each of the first group of items (A through F), the tester says aloud a number for students to write in the blank.

E. _____

F. _____

What number is 1 more than 356? _____

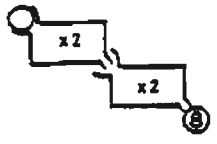
What number is 10 more than 402? _____

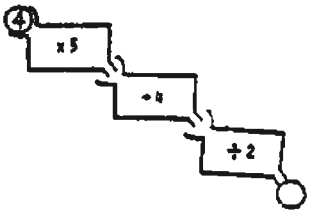
What number is 100 more than 601? _____

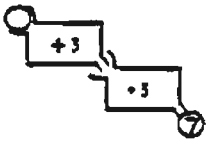
Relationships and Number Patterns

Recognition or application of given patterns, orders, or relationships in sets of numbers.

Examples from a fourth grade test. Three sample items, explaining how the "machines" work, are done previously.



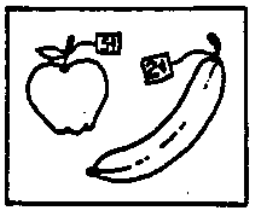




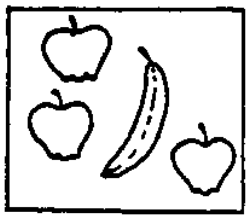
Word Problems

Solution of word problems requiring low levels of reading comprehension and computation and classified according to type of problem.

Examples from a second grade test. Tester reads the items aloud, frame-by-frame.



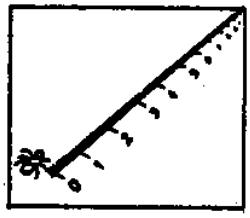
Apples cost 5¢ each and bananas cost 2¢ each.



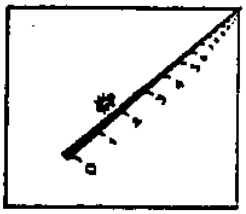
Sally buys 3 apples and 1 banana.

How much does it cost altogether?

_____



A fantastic ant is starting a trip.



After one day the ant has gone 2 miles.

At that same speed, how far will he be after 5 days?

_____

Elucidation of Mutiple Answers

Production of many correct answers to a given problem.

Examples from a sixth grade test.

Rules: Take out three balls.

Add to get a total score. ~~50~~, ~~100~~

Give all the possible scores. 52, _____

_____

Rules: The numbers must be between 500 and 940. ~~499~~

Two of the digits must be 9. ~~900~~

Give all the correct answers. 909, _____

_____

Special Topics

In the upper elementary grades there are also tests in one or more of the special topic areas of algebra, geometry, probability and organizing/interpreting data.

DEVELOPMENT

The MANS Tests were developed to evaluate the Comprehensive School Mathematics Program (CSMP), an innovative mathematics curriculum now being used by more than 50,000 students throughout the country.

CSMP is a curriculum that develops mathematical problem solving skills. It is a spiral curriculum based on generalized rather than behavioral goals. CSMP uses representational "languages" to develop key mathematical concepts through a spiral approach of carefully developed lessons.

The MANS Tests evolved from a need to develop tests which could be used to compare the progress of CSMP and Non-CSMP students. The tests had to reflect the emphasis CSMP places on generalized thinking skills and problem solving. However, to be fair to non-CSMP students, they could not contain any of the representational languages or activities associated with CSMP.

To meet the need for tests that could be used to evaluate the effectiveness of CSMP as a program that develops mathematical thinking and problem solving skills, an extensive process of test development and revision was undertaken. The development occurred sequentially, one grade level at a time. At each grade level, the Mathematics Research and Evaluation Studies (MRES) staff first developed prototype tests. Sometimes the ideas for the tests were adapted from ideas in previous research in mathematics education; most times the ideas were original. The Advisory Panel (whose members are listed on the front cover) independently reviewed all of the test prototypes. Occasionally, teachers, math supervisors and researchers also reviewed the tests. If the tests survived these reviews, they were pilot tested in a few local classes. On the basis of results from these pilot classes, tests were revised or in some cases eliminated.

The original version of the MANS Tests resulted from this continuing process of development, review, testing, and revision. The original version of the MANS Test was used in the first CSMP evaluation study involving 15 to 20 classes. After further refinement of the tests, they were used in the final evaluation of CSMP at that grade level. This evaluation study involved from 40 to 60 classes.

To simplify administration of the tests, and to provide coherent organization and year-to-year continuity of all of the tests that comprise the MANS, one final revision of the whole set of MANS Tests, grades two through six, was completed and pilot tested at the end of the entire sequence of evaluation studies.

At each stage in this process of development, review, testing, and revision, the work was guided by the Advisory Panel. Some of the important considerations in the review and revisions of these tests, were the following:

- o Intrinsic Merit: importance of the mathematical skill required; curricular fairness; student interest in the novel problem context.
- o Administration: clarity and brevity of directions; student understanding of the task; low reading level; attractive format; unspeeded.
- o Technical: item analysis including range of difficulty levels, error analysis, discrimination coefficients; test analysis including ceiling and floor effects, ability level differences, reliability (KR 20 studies of internal consistency), analysis of class means and evidence of construct validity.

SCORING AND REPORTING

To satisfy previous research needs, the MANS Tests have been scored by a combination of hand scoring and direct key entry, so that detailed item analysis and patterns of errors could be studied.

The next proposed step in making the tests usable to schools at competitive cost is the adaptation of the tests for local processing of data via a microcomputer scoring diskette. Research has indicated that the data from a classroom set of MANS Tests could be entered in approximately 40 minutes, resulting in an immediate computer generated report including lists of scores, tables and graphs. Such a diskette could easily be adapted to each of the widely used microcomputers. This optional scoring feature could be selected by school districts or individual teachers with access to this commonly available equipment: a centralized scoring service would also be available.

Scores of individual students will be reported for each MANS category in one of the usual norm-referenced ways. The major innovation in reporting, however, is the emphasis on class mean scores, which are also reported for each MANS category. At each grade level, one of the built-in tests is a vocabulary test. The class mean score on this test provides a measure of the general ability level of that class and plays an important role in subsequent reporting.

The reporting of class mean scores for the various MANS categories is done in a normative way. But instead of showing the standing or rank of a given fourth grade class compared to all other fourth grade classes (through standard scores, grade equivalents or percentile ranks), tables and graphs are used to show the performance of that class compared to other fourth grade classes of similar ability.

The graph below shows class means (actual data) for the second grade MANS. Each dot on the graph represent one class. The ability level of the class is measured by vocabulary score; classes farther to the right are higher in ability and, consequently, tend to have higher MANS scores (the vertical axis).

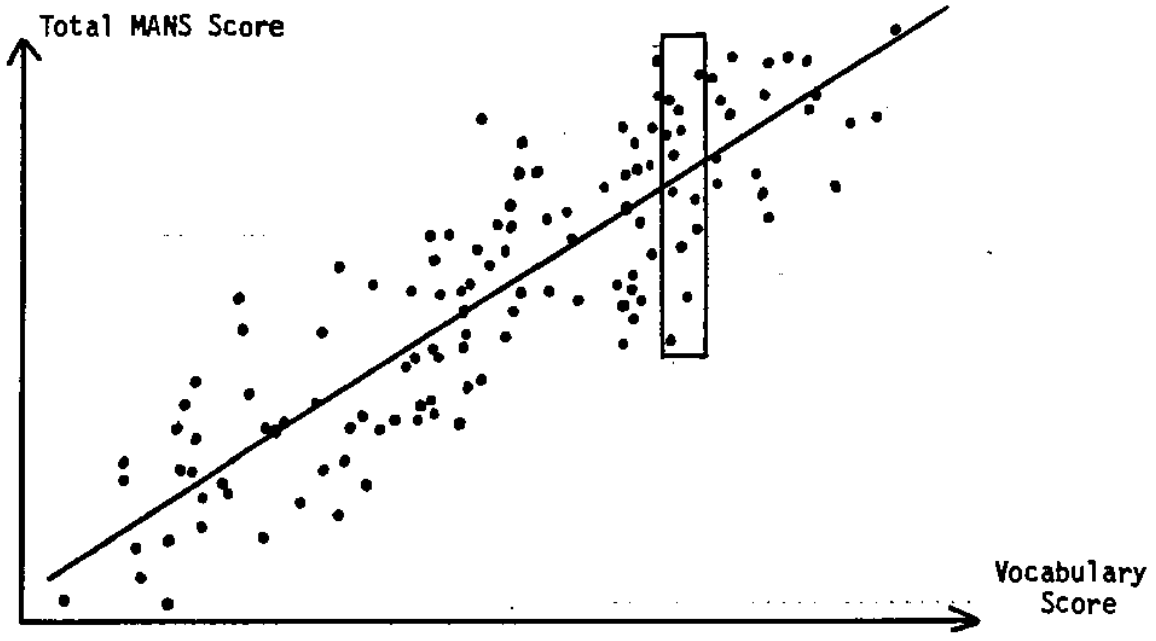


Figure 1, Second Grade Class Means, Total MANS

The line drawn through Figure 1 represents the best predictor of class MANS score for given vocabulary scores. One can easily focus on any single class and determine how well the class is doing given the level of ability (vocabulary score) of the class. Classes represented above the line have higher than expected scores, classes below the line have lower scores. In this way, attention is drawn to the performance of the class compared to what should reasonably be expected of the class, instead of being compared to other classes who may be much higher (or much lower) in ability.

Figure 2 shows a group of 18 classes with approximately the same vocabulary score. This figure is the inset from Figure 1, where it can be seen that the group is above average in ability.

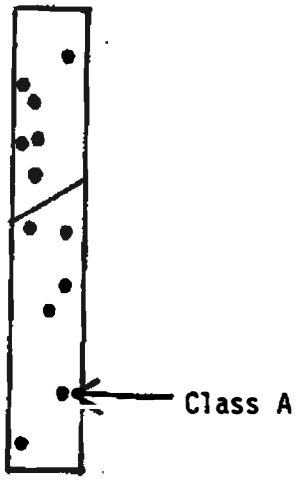


Figure 2, Inset From Figure 1 showing
MANS scores for classes of similar ability

For illustrative purposes, consider Class B. From Figure 1, it appears that Class B has an average MANS score; compared to the entire group of 126 classes, its percentile rank on the Total MANS is between 45 and 50. But from Figure 2 it is clear that Class B is not doing average work when compared to classes of similar ability; in fact, it is well below average.

Similarly, if a group of low ability classes had been examined, the converse would occur. There would be classes with MANS scores that would be below average compared to the whole group, but who would be doing very well for their ability level.

A table will be developed, showing for each range of vocabulary scores, the expected Total MANS Score and the upper and lower limits of MANS Scores that would include 2/3 of the classes at that vocabulary range.

In addition to a graphical and tabular presentation of results for the Total MANS, similar data will be provided for each of the MANS categories. In this way, a teacher or administrator can evaluate the strengths and weaknesses of a given class on various kinds of problem solving.

MANS Scale Descriptions

Comprehensive List

Process Categories:

C: Computation
E: Estimation
M: Mental Arithmetic
N: Number Representations
R: Relations & Number Patterns
U: Elucidation
W: Word Problems

Special Topic Categories:

A: Algebra
G: Geometry
L: Logic
O: Organization of Data
P: Probability

Category C: Computation

C1 Whole Number Computation

Abstract: Given straightforward computation problems involving whole numbers, produce exact answers (by calculating on paper if necessary). The items do not have the multiple choice response format but are similar in range and difficulty to those found in the standardized achievement tests of the appropriate grade level.

Grade Levels: 2, 3, 4, 5, 6

Examples (from Grade 4):

$$\begin{array}{r} 352 \\ +683 \\ \hline \end{array} \qquad \begin{array}{r} 675 \\ -469 \\ \hline \end{array} \qquad \begin{array}{r} 143 \\ \times\ 5 \\ \hline \end{array} \qquad 6\overline{)492}$$

C2 Fraction Computation

Abstract: Given straightforward computation items involving simple fractions, produce exact answers (by calculating on paper if necessary). Though the items do not have the multiple choice response format, they are similar in range and difficulty to those found in the standardized achievement tests of the appropriate grade level.

Grade Levels: 4, 5, 6

Examples (from Grade 5):

$\frac{3}{5} - \frac{1}{5} = \square$ $\quad \frac{1}{2} + \square = 1$ $\quad \frac{1}{2} \times \frac{1}{2} = \square$

C3 Decimal Computation

Abstract: Given straightforward computation items involving one and two place decimals, produce exact answers (by calculating on paper if necessary). Though the items do not have the multiple choice response format, they are similar in range and difficulty to those found in the standardized achievement tests of the appropriate grade level.

Grade Level: 6

Examples: $0.5 + 0.25 = \square$ $\quad .5 - 1.5 = \square$ $\quad 0.5 \times 0.5 = \square$

Category E: Estimation

E1 2 or 5 or 10 Times

Abstract: Given two numbers, quickly estimate whether the first is about 2 or 5 or 10 times as large as the second. A sample is worked collectively.

Grade Levels: 3, 4

Examples (from Grade 3): 65 is about _____ times as large as 12

98 is about _____ times as large as 51

E2 Estimating Intervals: Addition

Abstract: Given a computation problem involving whole number addition, and 5 fixed intervals (0-10, 10-50, 50-100, 100-500, 500-1000), determine which interval contains the answer to the problem, and put an x in the interval. By instruction, format and short time limits, students are discouraged from computing exact answers. Two or three sample items are done collectively.

Grade Levels: 2, 3, 4, 5

Examples (from Grade 2):		0	10	50	100	500	1000
	51 + 53						
	189 + 273						

E3 Estimating Intervals: Subtraction

Abstract: The scale is similar to E2 (except that it involves whole number subtraction) and follows it directly in the test booklets.

Grade Levels: 2, 3, 4

Examples (from Grade 3):		0	10	50	100	500	1000
	93 - 86						
	147 - 99						

E4 Estimating Intervals: Multiplication

Abstract: The scale is similar to E2 and E3 (but is devoted to multiplication with whole numbers for the most part) and follows them in the test booklets.

Grade Levels: 2, 3, 4, 5, 6

Examples (from Grade 4):		0	10	50	100	500	1000
	40 x 10						
	4 x 29						

E5 Estimating Intervals: Division

Abstract: The scale is similar to E2, E3 and E4 (but is devoted to division with whole numbers for the most part) There are only four fixed intervals (0-1, 1-10, 10-20, 20-100) in the response format. It follows E4 in the test booklets.

Grade Level: 5, 6

Examples: 1 ÷ 15 0 1 10 20 100

101 ÷ 9 0 1 10 20 100

E6 Estimating Fractions <, =, >1

Abstract: Given a calculation (+, -, or :) of two numbers (at least one of which is a fraction or mixed number), quickly estimate whether the answer would be less than, equal to or more than 1. Students are encouraged to work quickly and not to compute exact answers before making their choices. A completed sample item is provided.

Grade Level: 6

Examples:

	CHECK ONE		
	Less than 1	Exactly 1	More than 1
$1\frac{5}{8} - \frac{1}{128}$			
$2\frac{1}{2} \div 3$			

Category M: Mental Arithmetic

M1 Whole Number Open Sentences

Abstract: Given an open sentence, where the box may be either on the right or the left of the equal sign, where the numbers are large and easy to work with, and where only one operation is used, put the number in the box which makes the sentence true. By instruction and prompting, students are discouraged from "computing the long way" and are not allowed to do any figuring on paper.

Grade Levels: 2, 3, 4, 5, 6

Examples (from Grade 3) $500 + \square = 800$

$\square - 150 = 50$

$2 \times 200 = \square$

M2 Above and Below Zero

Abstract: Given a starting score (which could be above or below zero), and how much the score went up or down, select the correct final score (multiple choice).

Grade Levels: 2, 3

Examples (from Grade 3)

Score at the start: 3 below zero
Then: Lost 4
Score at the end: 7 below zero 1 below zero 1 above zero 7 above zero

Score at the start: 2 above zero
Then: Lost 4
Score at the end: 6 below zero 2 below zero Zero 2 above zero

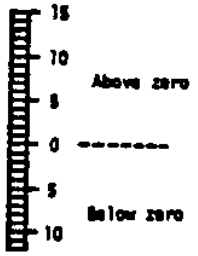
M3 Negative Hits and Misses

Abstract: Given the description of a "game" with two rules (a) each hit means a gain of 5 points and b) each miss means a loss of 1 point) and partial information on the outcome of turns, the student must deduce the missing information. Two sample items are completed collectively.

Grade Levels: 4, 5, 6

Examples:

	Started with a score of	Number of Hits	Number of Misses	Ended with a score of
Pam:	4 above zero		6	3 above zero
John:		2	0	15 below zero



(provided, but not mentioned in instructions)

M4 Fraction Open Sentences

Abstract: Given an open sentence involving at least one fraction, and one of the four arithmetic operations, complete the sentence.

Grade Level: 6

Examples: $\frac{3}{5} \div \square = 1$ $\quad 1 - \square = \frac{3}{4}$

M5 Decimal Open Sentences

Abstract: Given an open sentence involving at least one decimal number and one of the four arithmetic operations, complete the sentence.

Grade Level: 6

Examples: $0.5 + \square = 1$ $\quad 0.75 - \square = 0.5$

Category N: Number Representations

N1 Writing Whole Numbers

Abstract: Part I: The student must write numbers as they are read aloud by the tester.
Part II: Given a number, written in the test booklet, the student must write the number which is 1 (or 10 or 100) more than it. A sample item is worked collectively.

Grade Level: 2

Examples: Part I: Tester says, "Eight hundred twenty" (repeats)

Tester says, "Seven thousand sixty five" (repeats)

Part II: What number is 1 more than 999? _____

What number is 10 more than 495? _____

N2 1, 10, 100 or 1000 More

Abstract: Given two numbers, decide whether the first number is about 1, 10, 100 or 1000 more than the second number. (None is exactly right.) Two sample items are worked collectively.

Grade Level: 3

Examples:

4,265 is about 1 / 10 / 100 / 1000 more than 4,254

1,001 is about 1 / 10 / 100 / 1000 more than 998

N3 Constructing Numbers

Abstract: Given the use of only four digits (2, 5, 7 and 8) and the rule that no digit be used more than once, construct numbers like the smallest (or largest), the second smallest (or largest) or the closest to a given number. The constructed numbers are to be of either 2, 3 or 4 digits and sometimes restricted to a given range of numbers. Collectively, to clarify the rules, two incorrect answers and the correct one are examined for two sample problems.

Grade Level: 4

Examples: What is the second largest four digit number? _____

What is the smallest three digit number between 730 and 850? _____

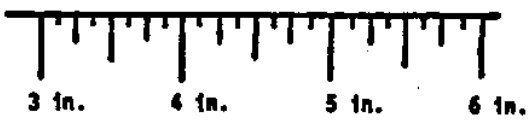
What four digit number between 2,000 and 3,000 is closest to 2,800? _____

N4 Representing Fractions

Abstract: The scale has five short subsections each containing one of two kinds of items: a fraction or mixed number is given in standard form and must be represented in another specific way or else that process is reversed and the response format is multiple choice. Instruction is largely in the form of a written question or command at the beginning of each subsection.

Grade Level: 4

Examples: Put an arrow at $4\frac{1}{4}$ inches.



How much is shaded?

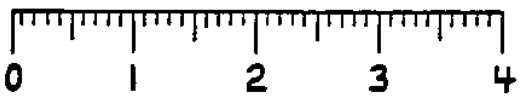
$\frac{1}{2}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{3}{4}$

N5 Representing Fractions and Decimals

Abstract: The scale has five short subsections each containing one of two kinds of items: either a mixed number or decimal is given in standard form and must be represented in another specific way or else that process is reversed and the response format is multiple choice. Instruction is largely in the form of a written question or command at the beginning of each subsection.

Grade Level: 5, 6

Examples: Put an arrow at 1.35 inches.



How much is shaded?

$\frac{1}{3}$ $\frac{1}{2}$ $\frac{2}{3}$ none of these

(A completed sample was given.)

N6 Equivalent Fractions and Decimals

Abstract: Given a fraction (or decimal) determine which members of a set of fractions (or decimals) are equivalent to it. A sample set of four completed items is shown.

Grade Level: 5, 6

Examples: Circle all the fractions that are equal to the one in the box.

$\boxed{\frac{2}{3}}$ $\frac{9}{12}$ $\frac{4}{6}$ $\frac{3}{2}$ $\frac{10}{15}$

Category R: Relationship & Number Patterns

R1 Solving Number Rules

Abstract: Given 3 clues (i.e., pairs of numbers) in a game, determine what the secret method is (i.e., the unique rule relating each of the pairs of numbers) and then use the rule to calculate the missing number from the fourth pair.

Grade Levels: 2, 3, 4, 5, 6

Examples (from Grade 3):

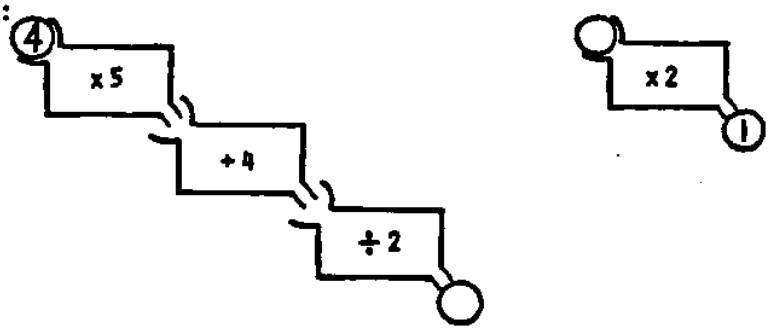
	Maria's Game		Jim's Game	
	Class said:	Maria's answer:	Class said:	Jim's answer:
First clue:	5	10	2	6
Second clue:	7	12	5	9
Third clue:	8	13	10	14
Question:	2	☐	☐	12

R2 Using Number Machines

Abstract: Given labelled "number machines" in sequence and either the initial or the terminating number, determine the other number. There is an introduction showing that "number machines" take in numbers; add, subtract, multiply or divide by a fixed quantity; and give out the resultant number. Then three sample items (each with a "number machine" sequence) are worked collectively.

Grade Levels: 3, 4, 5, 6

Examples (from Grade 4):



R3 Sequences

Abstract: Given an incomplete portion of an additive sequence of numbers, determine the missing number. One sample item is worked collectively.

Grade Level: 2

Examples: 28, 25, ____, 19, 16, 13

1, 1½, 2, ____, 3, 3½, 4

R4 Which Result is Larger

Abstract: Given two quantities (usually similar computation problems using +, -, or x) mark the one which yields the larger result, or mark them both if they are equal. By instruction, format and time limits, students are discouraged from computing exact answers. The correct response should be more easily determined by inspection than by computation. Two sample items are worked collectively.

Grade Levels: 2, 3

Examples (from Grade 2): 585 + 250 ☐ 3 x 31 ☐

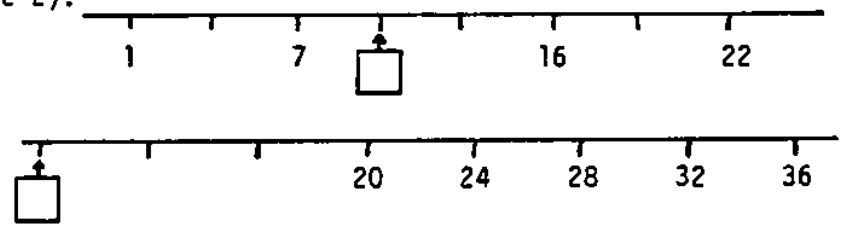
580 + 290 ☐ 31 x 3 ☐

R5 Labelling Number Lines

Abstract: Given partially labelled number lines, with varying increments, determine certain missing numbers. A sample item is worked collectively.

Grade Levels: 2, 3, 4, 5, 6

Examples (from Grade 2):

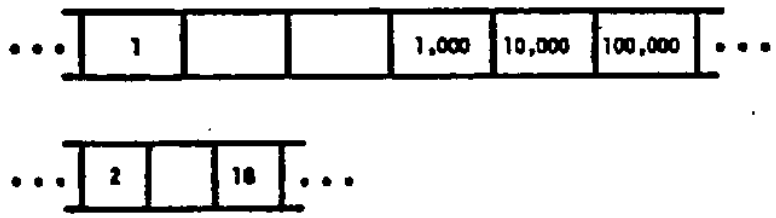


R6 Multiplication Series

Abstract: Given an incomplete portion of a multiplicative series of numbers, determine the constant multiplier involved in order to complete the portion shown. Portions of several series are shown altogether with one, two or three numbers missing from each. A sample series is examined and completed collectively.

Grade Level: 4

Examples:



R7 Which Fraction is Larger

Abstract: Given two non-whole numbers written in fractional form (a proper fraction, an improper fraction or a mixed number), circle the larger one. A completed sample item is shown.

Grade Level: 5, 6

Examples: $\frac{3}{4}$ or $1\frac{1}{4}$

$3\frac{1}{2}$ or $\frac{5}{2}$

R8 Which Decimal is Larger

Abstract: Given two non-whole numbers written in decimal form, circle the larger one. A completed sample item is shown.

Grade Level: 5, 6

Examples: 4.999 or 5.1

1.5 or 0.58

R9 Fractions Between Two Others

Abstract: Given two fractions, write another which is larger than the first and smaller than the second.

Grade Level: 6

Examples: ____ is larger than $\frac{1}{3}$, but smaller than $\frac{1}{8}$

____ is larger than $\frac{1}{4}$, but smaller than $\frac{1}{2}$

R10 Decimals Between Two Others

Abstract: Given two decimal numbers, write another which is larger than the first and smaller than the second.

Examples: ____ is larger than 1.25, but smaller than 2.0

____ is larger than 0.42, but smaller than 0.43

Category U: Elucidation

U1 Number Sentences About 8

Abstract: Students are to produce as many different "sentences about 8" as possible, always in the form "8 = ...". Four correct answers to similar exercises about 9 are examined collectively. (9 = 10 - 1, 9 = 1 + 5 + 3, 9 = 3 x 3, 9 = 18 - 2).

Grade Level: 2

Example: My number sentences about 8.

8 = _____

8 = _____

U2 Producing Many Answers

Abstract: Given several different situations each of which poses a problem for which there are many correct solutions, produce as many of them as possible. For each situation, some potential solutions are accepted or rejected for not following the given rules as inappropriate.

Grade Level: 3, 4, 5, 6

Examples (from Grade 3):

Rules: Take out two balls.
Add the two numbers to get a score.

What are the possible scores? 6, 2, 35

Rules: Write all the two digit numbers you can.
Use only the digits 1, 2, 3.

Give all the numbers that follow the rules. 34, 22

U3 Getting to 12

Abstract: Given a starting point (0), a goal (12) and two rules, invent as many ways of reaching the goal as possible. The rules are that only the numbers 2, 3, 5 & 7 can be used along with addition, subtraction, multiplication or division. Two sample solutions (see below) are worked collectively.

Grade Level: 6

Examples:

Sample 1: 0 + 7 = 7
7 x 2 = 14
14 - 2 = 12

Sample 2: 0 + 5 = 5
5 + 3 = 8
8 + 2 = 4
4 x 3 = 12

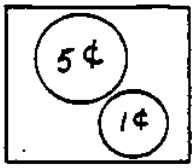
Category W: Word Problems

W1 One Step Word Problems

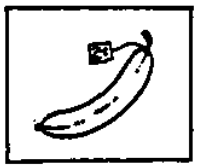
Abstract: Solve word problems in which the story (including the question) is read by the tester while the student looks at a series of cartoons and/or follows the story in the captions beneath the cartoons. Seven items require one-step solutions; two items require two.

Grade Level: 2

Examples:



Jill spent 6¢ to buy some bananas.



Bananas cost 2¢ each.

How many bananas did she buy? _____

Jim found 3 marbles but he lost 4.

And now he has 5 marbles.

How many marbles did he have to begin with? _____

W2 Two Stage Word Problems

Abstract: Solve word problems in which the solutions require two operations. The numbers in the problems are relatively small; the computational and reading requirements are simple.

Grade Levels: 3, 4, 5, 6

Examples (from Grade 4): Pam gets 50¢ each week.
She always spends 30¢ and saves the rest.
How much will she save in 4 weeks? _____

Tom has 3¢ more than Ann.
Tom has 5¢ less than John.
If John has 20¢, how much does Ann have? _____

W3 Miscellaneous Word Problems

Abstract: Solve word problems which are unusual for third graders in one of several ways: requires three-stage solution, requires working backward from a given final state to an unknown initial state, requires more logical analysis than straight computation, involves proportional ratios, involves extraneous data.

Grade Level: 3

Examples: At first, Sally had some marbles.
Then, she lost 3 of them.
Then, she found 2 marbles.
After that, she still had 8 marbles left.
How many did she have at first? _____

Sam has to move 10 boxes.
He can carry 3 boxes each trip.
How many trips will he need to make? _____

W4 Extraneous Information

Abstract: Solve word problems in which extraneous information is given. Once the relevant information is selected, the solutions are simple one-step problems involving small whole numbers.

Grade Level: 4

Examples: A belt costs $4.
A shirt costs $5.
A hat costs $10.
How much more does a hat cost than a belt? _____

Peter has $10.
He needs 4 pounds of candy.
Candy is $2 per pound.
He is buying candy for 6 people.
How much will the candy cost altogether? _____

W5 Fractional Sugar

Abstract: Solve word problems each of which start with cups of sugar. The one-step solutions all require simple computions (+, -, x or -) with fractions or mixed numbers.

Grade Level: 4
Examples: Tina has $4\frac{1}{2}$ cups.
She buys $5\frac{1}{2}$ more cups.
How much sugar will she have then? _____

Kari has $4\frac{1}{2}$ cups.
She gives away half of it.
How many cups of sugar will she have left? _____

W6 Three Stage Word Problems

Abstract: Solve word problems in which the solution requires three operations. The problem is stated in 3 to 5 short sentences and the numbers given in the problems are relatively small.

Grade Level: 5, 6

Examples: Shirts cost $10 each and ties cost $5 each.
Altogether Joe spent $35 for shirts and ties.
He bought 2 shirts.
How many ties did he buy? _____

Bill loads 6 boxes in 2 hours.
John loads 4 boxes in 2 hours.
Together, how many boxes do they load in 6 hours?_____

W7 Decimal Gas

Abstract: Solve word problems each of which start with 6.5 gallons of gas. The one-step solutions all require simple computations (+, -, x, or -) with decimals.

Grade Level: 5

Examples: Peter has 6.5 gallons.
Then he spills 1.2 gallons.
How much gas will he have left? _____

Ron has 6.5 gallons.
Next week he will use ten times this much.
How much gas will he use next week? _____

W8 Novel Word Problems

Abstract: Solve word problems which are novel for sixth graders in one or two of the following ways: involves fractions or decimals, requires more-than-three-stage solution, answer choices are approximate, requires solving for two unknowns, requires the use of data which is common knowledge but not given in the problem. Response format is multiple choice.

Grade Level: 6

Examples:

Ellen saw pepper plants on sale at 3 plants for 40¢.
She bought 12 plants.
She usually bought 3 plants for 50¢.
How much did she save?

20¢ 40¢ 48¢ $1.60 $2.00

George's father gives him 2¢ for every hour he spends in school.
About how much would he have given George for the month of October?

$.50 $1.00 $3.00 $6.00 $10.00

Category A: Algebra

A1 Algebraic Symbols

Abstract: Given the numerical value of a letter (or letters) produce the numerical value of an expression involving that letter (those letters). In written instructions, two sample items are worked out and implied multiplication (e.g. in 3bc or in d^4) is explained. This scale follows A2 in the test booklet.

Grade Level: 6

Examples: If g = 4 and h = 3 then 5gh = ____

If p = 2 then p^5 = ____

A2 Solving Equations

Abstract: Given simple equations in one unknown, solve for the unknown. Three sample items are worked collectively, including one with a parenthesis.

Grade Level: 6

Examples: (7 x h) + 1 = 15, so h = ____

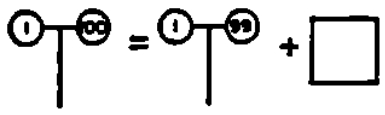
(n + 1) ÷ 3 = 6, so n = ____

A3 Summation Operator

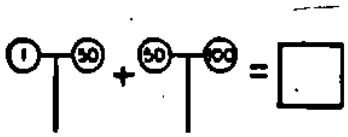
Abstract: Given an open sentence involving one or more summations of consecutive integers, select the answer that completes the sentence. A symbol for such summations ($\overset{\circ}{\top}$) is introduced and explained ($\overset{2}{\top}\overset{6}{} = 2+3+4+5+6$) and two items are worked collectively.

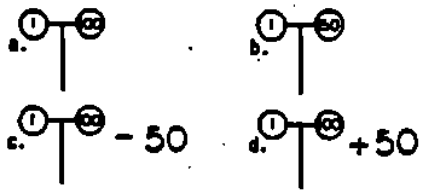
Grade Level: 6

Examples:



a. 1 b. 99 c. 100 d. 199





A4 Transformations

Abstract: Given two different transformations (ㄱ which turns a design clockwise by 90° and ㄷ which reverses the number of symbols at the top and bottom of a design), the scale consists of two different sections: requiring the application of either ㄱ or ㄷ to a design, requiring several applications of ㄱ and/or ㄷ to a design. Several sample items are worked collectively in each section.

Grade Level: 6

Examples: Section I: ㄱ (o x / o x) = ㄷ (x / o o) =

Section II:

Start with		End up with
x x / o	do ㄷ and then ㄱ	
x / o o	do ㄷ twice	

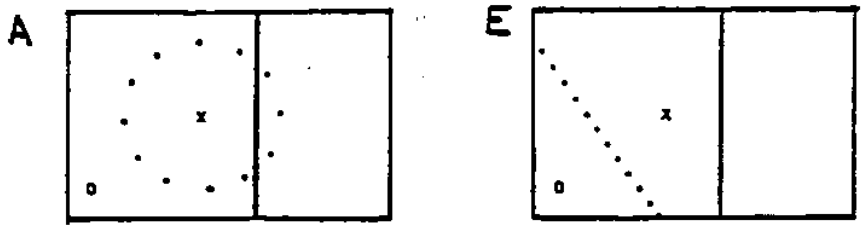
Category G: Geometry

G1 Geometric Loci

Abstract: Determine which picture is described by a given statement, where several pictures are given, each of which has identically placed elements (an 'x,' an 'o' and a line) but a different set of dots, determine which picture a given statement describes. First statement is read by the tester.

Grade Level: 4

Examples:



In which picture are all the dots the same distance from the x? A B C D

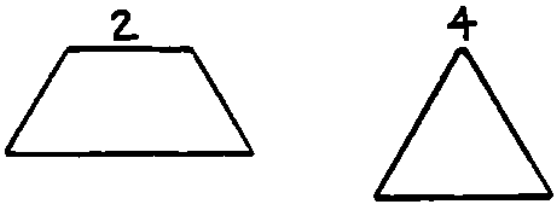
In which picture is each dot just as close to x as to o? A B E F

G2 Geometric Congruencies

Abstract: Given a regular geometric shape dividethe shape into a certain number of congruent parts. The word "congruent"is not used. Three correct and three incorrect solutions to a sample problem are examined collectively.

Grade Level: 5

Examples:



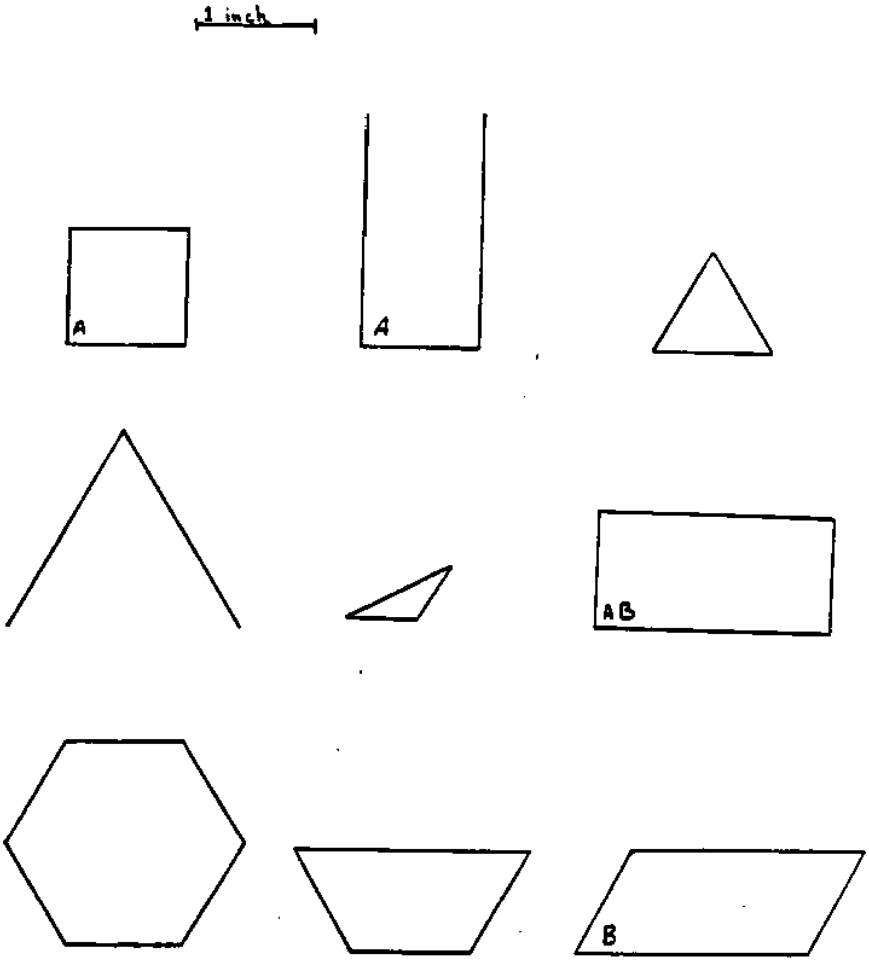
Category G: Geometry

G3 Geometric Categories

Abstract: Given nine different geometric figures, identify a set of 2 to 7 figures that are alike in some way, describe the distinguishing characteristic and label the figures accordingly. Go through this process as many times as possible. Two examples are worked collectively.

Grade Level: 6

Examples:



Sample 1 All the figures with "A" have square angles.

Sample 2 All the figures with "B" have only two sides that are one inch long.

All the figures with "C" _____

All the figures with "D" _____

etc.

Category L: Logic

L1 Logical Identification

Abstract: Given a specific set of individuals, a specific set of characteristics, the fact that each individual has a distinct combination of characteristics, and several facts about some of the characteristics of some of the individuals, identify the characteristics of each individual. A smaller sample problem is worked collectively.

Grade Level: 6

Example :

These are the 4 boys: Bill Tom Ed Pete

These are the 4 leagues: indoor soccer outdoor soccer indoor hockey outdoor hockey

These are the facts: Each boy plays in a different league.
Bill plays indoors
Tom doesn't play hockey
Ed doesn't play outdoors and he doesn't play soccer.

What league does each boy play in? (Circle your answers.)

Bill:	indoor soccer	outdoor soccer	indoor hockey	outdoor hockey
Tom:	indoor soccer	outdoor soccer	indoor hockey	outdoor hockey
Ed:	indoor soccer	outdoor soccer	indoor hockey	outdoor hockey
Pete:	indoor soccer	outdoor soccer	indoor hockey	outdoor hockey

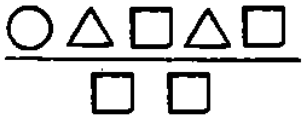
L2 Making Sentences False

Abstract: Given a picture of a set of blocks and a true sentence about them, make the sentence false by changing the blocks. In the first two items, three suggested changes in the blocks are given and the student need only mark which ones would falsify the sentence. In the last three items, the student must write a change in the blocks. An item of the first type is worked collectively.

Grade Level: 6

Examples:

JOE'S BLOCKS



"There are triangles above the line and squares below the line."

a. Take away the triangles.
b. Take away the squares below the line.
c. Add squares above the line.

"Triangles go above the line or circles go below the line."
(You write what Joe could do to make the sentence false.)

Category 0: Organization of Data

01 Graphing Weight

Abstract: Given a graph in which weight (axis labelled at 10 pound increments for each 5 graph units) is plotted against age (axis labelled at 2 year increments for each 2 graph units), determine age per given weights and vice versa. One sample item is worked collectively.

Grade Level: 5

Examples: How much did Bill weigh at 4 1/2 years of age? _____

How old was Bill when he reached 90 pounds? _____

02 Interpolating from a Table

Abstract: Given a table of prices for pipe of 4 different widths and 4 different lengths, interpolate or extrapolate to obtain the price on a pipe of given dimensions: at least one of which is not shown in the table. Two sample items are worked collectively.

Grade Level: 6

Examples:

Cost of Pipe

Width \ Length	100'	300'	600'	1,000'
4"	$50	$150	$300	$500
8"	$70	$210	$420	$700
12"	$90	$270	$540	$900
16"	$110	$330	$660	$1100

HOW MUCH DOES IT COST TO BUY PIPE WHICH IS:

6" x 100' = $_____

20" x 1000' = _____

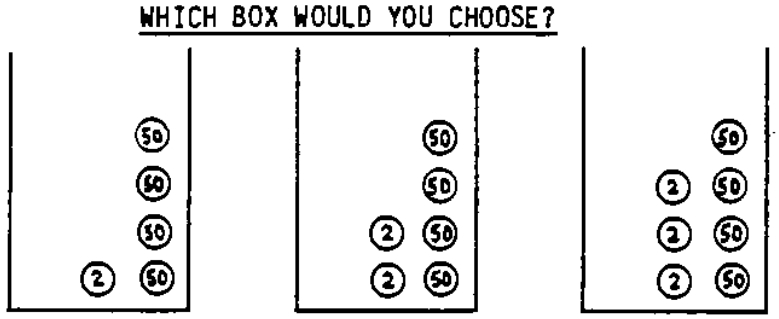
Category P: Probability

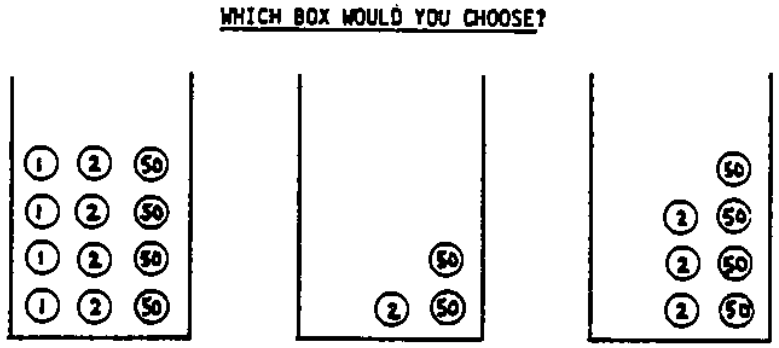
P1 Choosing the Best Box

Abstract: Given three boxes containing different combinations of 1, 2 and 50-cent"balls", determine from which box it would be best tn make a blind draw.

Grade Level: 5, 6

Examples:





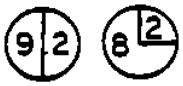
P2 Dependent Outcomes

Abstract: Given two (or three) spinners and an amount (10) to be achieved or exceeded to win, select (from five standard choices) how often a player would win. Collectively it is shown how a player could win or could lose with a specific set of spinners.

Grade Level: 6

two forms, approximately 3.5 minutes.

Examples:



never | less than half the time | half the time | more than half the time | always

never | less than half the time | half the time | more than half the time | always