



# DRILL AND PRACTICE

*at the problem solving level*  
activity pages  
with comments  
for teachers

**CD**math  
**A** ROBERT W. WIRTZ

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### CONTRACT

Curriculum Development Associates, Inc. hereby grants to teachers who have purchased the two-volume set **Drill and Practice at the Problem Solving Level** the right to make duplicate masters of the **Activity Pages** and run copies of them for their own classes.



All of the art work on the Activity Pages in **Drill and Practice at the Problem Solving Level** was done by Frances Thompson with the technique of classical Chinese brush painting (without benefit of a straight edge). This informal style was chosen, in part, so that copies reproduced for use in the classroom will retain some of the informal feeling of teacher-prepared materials.

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To facilitate the use of these activities, two inserts are included at the back of this set of **Activity Pages**. One of them correlates the activities in **Drill and Practice** with the basic arithmetic operations of addition, subtraction, multiplication and division, and indicates the degree of difficulty of each activity. The second insert correlates the **Drill and Practice** activities with the books in the **CDA Math** basic program.



## Introduction

*This resource of varied and flexible activities has been designed for teachers who want to begin taking an alternative approach to the teaching of elementary mathematics. The activities in this book are grounded in the theory of learning and child development described in the companion volume, **Mathematics for Everyone**. It is essential that those involved in the mathematics education of children become familiar with **Mathematics for Everyone** as they investigate this volume of activity pages. For when the rationale and theory of learning that gave rise to these materials are clear, the activities can be used with added insight.*

Teachers who purchase both volumes of the set are granted the right to make masters of the Activity Pages and to run copies of them for use in their classrooms. Ideas for teaching with the Activity Page appear on the back of each page. These brief notes describe the activity, and often pose interesting questions or problems that might be pursued. Sometimes they suggest strategies of instruction. But these notes to the teacher are not meant to be "prescriptive." We encourage teachers and others involved in the mathematics education of children to view the notes as starting points; as activities which can begin to generate a whole new attitude toward mathematics. They are written with a view to inspire teachers and children to go beyond what is in this book, and to ignite a real spirit of inquiry into both mathematics itself and the creative teaching of mathematics.

**Drill and Practice at the Problem Solving Level** is one component in the total CDA mathematics program. With the new edition of this program, Curriculum Development Associates has made a pioneering effort to develop a mathematics curriculum that is responsive to the pluralistic nature of our society. This first effort to build on the almost culture-free nature of mathematics was facilitated by the essentially non-verbal character of the pupil materials in the program.

All language that appears on the pages to be used with children is given in both Spanish and English. These few words introduce a full page of mathematics that everyone can do together regard-

less of language or background. We look forward to the ultimate emergence of a variety of bilingual editions as educators pursue the common goal of transcending the cultural differences among learners.

## Audio Cassette Tape Series

Dr. J. Richard Suchman has prepared a series of taped discussions directly related to the activity pages in this volume. You will find that each line in the table of contents is followed by a number. These notations are the identification numbers of the tapes relating to the activity pages.

Comments on the sections vary in length from about ten minutes to nearly one hour. Each cassette tape describes the use of its corresponding activity pages, suggests possible variations for use of the activities with children, and offers comments about how children might respond to the activities. There is also an introductory tape by Robert Wirtz which runs about 45 minutes.

These tapes are available to those who purchase the two volume set, **Drill and Practice at the Problem Solving Level**. They may be obtained from:

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Curriculum Development Associates, Inc. grants to any public or private school authority which purchases this set of tapes full rights to reproduce them for any or all classroom teachers within its jurisdiction who have purchased or received both volumes of **Drill and Practice at the Problem Solving Level**.

## Organization of the Activity Pages

The activities which make up this volume are grouped into three general categories (see Table of Contents for page numbers).

Part I - Manipulative Activities.

Part II - Representational Activities.

Part III - Activities at the Abstract Level.

These three categories of activities reflect certain stages of development described by the theory of

learning which is explained in more detail in **Mathematics for Everyone**. Important ideas appear and reappear at all three levels in the form of different activities. There is no prescribed way of moving through these activities — no scope and sequence in the usual sense of those words. This volume is a flexible resource, which teachers can use to individualize and personalize the curriculum for each child. There is no expectation that a teacher or child complete **all** the manipulative activities or complete **all** the representational activities before starting some activities at the abstract level. Consequently, the teacher must determine sequences of pages that seem most appropriate for the children involved, and assume the challenge and responsibility for curriculum development in the classroom.

However, there are some principles which should guide the use of these activities. **Our first concern** is that materials be available so children can: (1) meet all the **big ideas** at the manipulative level, (2) move on when they are ready to develop those ideas with the aid of pictures, sketches and diagrams, and (3) eventually explore the same ideas using symbols, at the abstract level.

**Our second concern** is that all children, including beginners and “slow learners,” be given many opportunities and much encouragement to solve problems and make their own investigations.

**Our third concern** is that all children be introduced to activities at gradually increasing levels of complexity and sophistication — always short of that level of frustration that precludes learning.

We are confident that if these principles guide our approach to math education, children will develop a “friendliness with numbers.” We are equally confident that such “friendliness” can guarantee skill development that far surpasses present norms.

There are three different kinds of activity pages in this resource: (1) task cards or pages, (2) report forms, and (3) summary forms. A task card or page specifies a particular activity to be done by the learner. For example, a child may be asked to get three beansticks: a 1-stick, a 3-stick, and a 5-stick, and find which numbers of beans, one through ten, can be picked up using one or more sticks. Provision

is made for keeping a record of which numbers of beans can and cannot be picked up.

A report form differs from a task card in that the **learner** is asked to make choices and then report the outcome of those choices. Children might be asked to make their **own** selections of three beansticks and report which numbers of beans, one through ten, can be picked up using a stick or combination of the sticks they have chosen.

A summary form is used to display the results of a large number of similar investigations. Often, it can lead to further investigations.

Report forms and summary forms are essentially blank. They are designed for learners to keep records of their experiments and investigations. But they also can be used to create new task cards. Teachers can specify particular activities on these forms which might be custom designed to meet the needs of a particular learner or group of learners.

## Investigations and Experiments

The creation and organization of these activities has consistently been guided by our desire to nurture a spirit of investigation and experimentation in young children. For example, the first pages in Part I of this resource book were designed to introduce beginners to the four basic operations (addition, subtraction, multiplication and division) as experiments. These pages were taken from **Individualized Computation, Level A**, a part of the CDA Mathematics Program. Here the assumption is made that when they come to school, children understand the ideas of putting things together, of “take away,” of putting things in piles with the same number in each, and sharing things, or giving the same number to everybody. These are all physical operations involving the manipulations of things. Arithmetic comes on the scene as a way to report about such manipulations; to describe what is done and to record outcomes in a special kind of shorthand.

On each page which introduces one of the four basic operations, space is provided to move counters around and to experiment with different arrangements. Blank record forms enable the experimenters to make their “reports” in the symbols of arithmetic.

Thus the initial use of written numbers and signs is presented to children as a way of describing what they are doing and reporting the results of an experiment as they are looking at it. As soon as they understand each activity, learners can begin making up their own experiments and keeping their own records. They are “on their own,” experimenting timidly or boldly or imaginatively. They are working in their own individual styles.

If they have the ability to count and record, children have a very powerful problem solving tool, and from the beginning can get involved in solving what are real problems to them. For a very young child, the question

$$5 + 8 = ?$$

is a problem. The answer is not in the memory, nor is it obvious to the child. Children must count out five counters, count out eight counters, and count the number of counters all together. They use knowledge that they have to create knowledge that is new to them. Thus, problem solving need not be postponed. It is used in these materials from the outset as a strategy to help all children develop a “friendliness with numbers.”

### Becoming Familiar With This Resource

A glance at the Table of Contents will suggest the novel organization of the activities. Some of the titles reveal the author’s belief that a light touch is consistent with the spirit of good mathematics. Perhaps you will want to note certain items that interest you which you want to investigate further. You should thumb through the pages themselves, noting, among other things, the great variety of format and the almost non-verbal character of the pupil pages. Again, a check mark in the Table of Contents can be a signal to investigate certain areas when time is available.

Stop now and then to try some of the problems yourself. The brief notes on the back of the pages are intended to explain the intent and mechanics of the activity. If the explanations seem inadequate, look at the preceding pages in the same activity grouping where fuller explanations are given. If some of the pages seem interesting and are appropriate for the children with whom you are involved, make a note to try them in the classroom with your

children. Then as time becomes available, you can look more carefully into the material in certain sections of this collection.

Probably at no time will it be very productive to read through this volume page by page. The pupil pages will come to life only as they are brought into the classroom by the teacher to help develop an activity. The responses of the children themselves, as they work with the activities, are indicators of the appropriateness and effectiveness of these materials of instruction.

Exchanges between teachers about their classroom experiences with the activities can also be a productive route to learning about this resource. Teachers will all most probably try different activities, and the sharing of experiences will help everyone gain more familiarity with the various pages, and how they work with children.

### An Open Ended Resource

In order to keep the activity pages always more up to date, each edition of **Drill and Practice** has incorporated significant changes. Activities have been replaced, or the supporting pages redesigned, based on feedback from classroom teachers. New material has been added. Pagination problems, of course, develop. Rather than obsolete the entire numbering system with each revised edition, replacements and additional pages will simply be included within the appropriate sections and be lettered 27A, 27B, etc., where needed.

### A General Approach to New Activities

When teachers approach the teaching of a new activity or a new set of activities from this volume, their plans for instruction often differ markedly. In other words, teachers demonstrate a wide variety of styles as they involve their classes in problem solving situations. We’ve discovered that most teachers try to plan by answering the following questions:

- (1) How does the activity work?
- (2) What is the purpose of the activity?
- (3) Is there a manipulative introduction?
- (4) How shall records be kept?



- (5) When should I introduce a task card, a record form, or a summary page?
- (6) How can I get children working independently?
- (7) How can I extend the activity for those who can go further?

Thus, in the comments on the back of the activity pages, we have tried to help answer these questions.

Let us illustrate this general approach by addressing these seven questions around an activity called "check list arithmetic."

- (1) How does it work?

A series of numbers such as 1, 2, 4, 8, and 16 is presented in a "list." A specific number is given and the problem is to find the combination of numbers from the "list" whose sum is the given number and which uses fewest numbers.

For example:

|    |   |   |   |   |    |
|----|---|---|---|---|----|
| 16 | 8 | 4 | 2 | 1 |    |
| ✓  |   | ✓ | ✓ | ✓ | 23 |

$$16 + 4 + 2 + 1 = 23$$

- (2) What is the purpose of the activity?

One obvious purpose is to provide much practice with addition. The activity also helps develop the basic strategy of "cutting a problem down to size." Because they must use the **fewest** possible numbers in the combination, the learners must figure out the largest number they can use. This process of "cutting the problem down to size" in turn anticipates the division algorithm.

- (3) Is there a manipulative introduction?

There are several manipulative introductions. One could use a set of beansticks with 1, 2, 4 and 8 beans (such as activity page No. 30D). Is there a combination of sticks with exactly 13 beans? If this is too elementary, cartons for a half-pint, a pint, a quart, a half-gallon and a gallon of milk could be used. In terms of glasses of milk, these hold 1, 2, 4, 8 and 16 glasses of milk respectively. Is there a combination of cartons that will provide 23 glasses of milk? Sure — 23 half-pints. But what is the **fewest** combination of cartons? For any number of glasses less than 32

(2 gallons), would the combination of "Fewest cartons" ever require that more than one of each carton be used? (No.)

- (4) How should the records be kept?

There are several possibilities. We can shade in the block under each number we use in the combination, we can use check marks, or we can record in boxes the numbers we used.

|    |   |   |   |   |    |    |   |   |   |   |    |
|----|---|---|---|---|----|----|---|---|---|---|----|
| 16 | 8 | 4 | 2 | 1 |    | 16 | 8 | 4 | 2 | 1 |    |
|    |   |   |   |   | 23 | ✓  |   | ✓ | ✓ | ✓ | 23 |

or

|    |   |   |   |   |    |
|----|---|---|---|---|----|
| 16 | 8 | 4 | 2 | 1 |    |
| 16 |   | 4 | 2 | 1 | 23 |

- (5) When should I introduce a task card, record form, or summary page?

As soon as the children understand the activity, including the notion of "fewest checks," they are ready to solve the problems on a task page. Once they have done problems given them, they can begin to make up their own problems and pursue their own investigations. The record forms can be introduced as soon as children show an interest in doing such independent activities.

The summary pages should be introduced as the need arises in an investigation to keep a more concise record of the results of the experiment, or to organize the results for a new investigation.

- (6) How can I get the children working independently?

In a sense, children are generally working by themselves on this activity from the outset. However, we want to encourage independent investigations on their part as well. When children have completed the first task page, they can begin using the report forms for making up their own examples of a similar kind. This is the beginning of a unique kind of individualization — different children working on the same problem at the same time, each taking it to his or her own level of sophistication.

- (7) How can I extend the activity?

The activity can be extended in many ways. Let's



consider two of them. (1) Children might be asked to find patterns that develop if they use only even numbers or only odd numbers or only multiples of 3. (2) Other sequences of numbers can be used in the list. For example.

|     |    |   |   |
|-----|----|---|---|
| 125 | 25 | 5 | 1 |
|     |    |   |   |

or

|    |   |   |   |
|----|---|---|---|
| 27 | 9 | 3 | 1 |
|    |   |   |   |

Our purpose in illustrating this sequence of questions is not to be prescriptive, but rather to suggest one approach to new activities which some teachers have found helpful. We want to emphasize the importance of an active role for the teacher in planning for new activities. The materials themselves simply assist teachers in implementing their plans for involving children in a meaningful mathematics curriculum.

### Some Notes on the Mechanics of Using the Activity Pages

All the activity pages in this volume are designed in such a way that teachers can reproduce them for use in their classrooms. We have had considerable experience with these processes and thought it might be useful for us to share some "dos and don'ts" on the mechanics of reproduction. We would also like to share our concern about the possible "overuse" of paper at a time when paper is in short supply, and some possible solutions to that potential problem.

### Reproduction and Materials

We have observed that while many systems of reproduction are available, the most widely used copying technique is to make a spirit duplicator master in a heat transfer machine (such as Thermofax). A carrier "sandwich," with one or both sides transparent or translucent, is used. The form to be copied is placed face up in the sandwich. On top of it goes the blank master and carbon. The top of the sandwich is the transparent part of the carrier. This assembly is placed in a heat transfer machine or box. The result is a finished spirit duplicator master ready to produce the required number of copies. The original form is returned to its file.

If additional entries are to be made on the form (as would be the case if you wanted to particularize

a report form), the procedure begins as described above. Entries are then made on the spirit duplicator master before it is run. The master, as it comes from the heat transfer process, is a thin sheet of carbon and a thin piece of master paper. Normally, the carbon is torn off and discarded. Some carbon has been transferred to the "back" of the thin master paper. That is the image that will reproduce and it is backwards. However, it shows through to the "front," and looked at from the front, the image is correct, and can be read.

With the **CARBON STILL ATTACHED**, you can write with pen or pencil on the thin paper master, and carbon (or image material) will be transferred to the back of the thin paper master. Considerable pressure must be used to produce a clear image — and care must be taken not to tear the master. One way to protect the master is to cover it with a thin piece of plastic which can be used over and over again, since the pen or pencil will not leave marks.

Another, and usually a more satisfactory result can be obtained by discarding the thin carbon used in the heat transfer process and replacing it with the carbon that comes as part of the regular unprinted spirit duplicator master. The image material on these sheets transfers better with less pressure. If pencil or pen is used without the protective overlay of plastic, the point should not be sharp because the master paper is thin and soft and can tear easily. One advantage of not using the plastic sheet is that the information you add is legible and can be checked.

In recent years, everyone in schools has felt the impact of the paper shortage. With the rising costs of paper, teachers have begun to devise ways of using these activity pages which do not require continual reproduction of multiple copies. We would like to share some of their suggestions.

Several teachers have begun to provide an acetate pocket (8½" x 11") and a grease pencil to each child. The child inserts a copy of a page into the pocket and writes on the acetate pocket with the grease pencil. When he has finished and checked his work, the acetate pocket can be erased with a paper towel or a rag. The copy in the pocket is unmarked and can be returned to a central classroom file for the next child to use. With this strategy, only a few copies of

each page need to be made, for in most cases, all children in a class will not be using the same activity at the same time. When children make investigations of their own that they want to keep, they can make permanent records of these in their “math notebooks.”

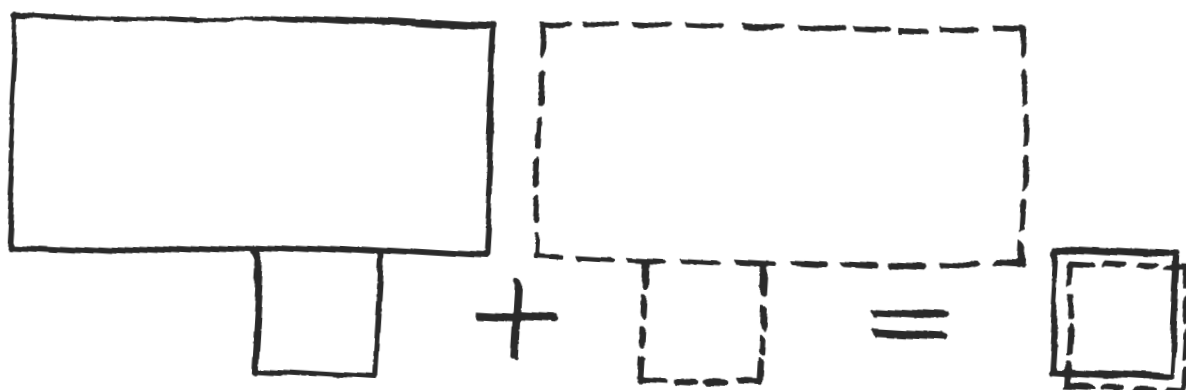
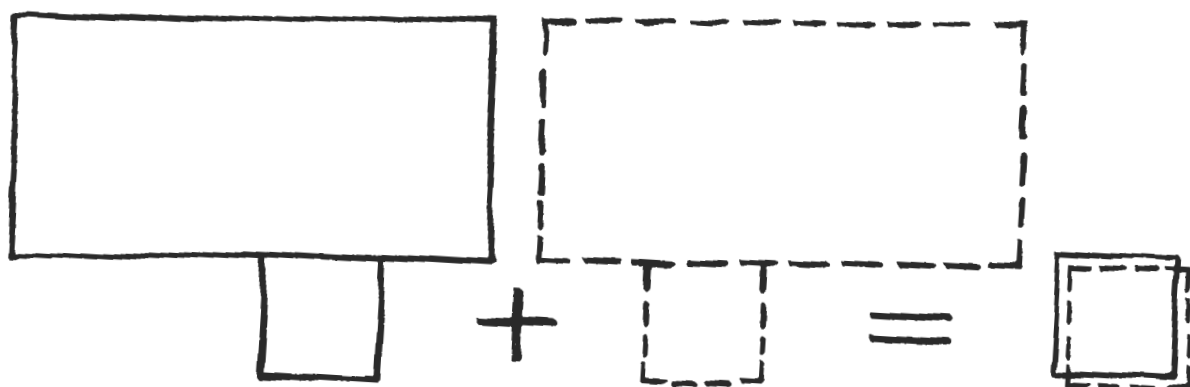
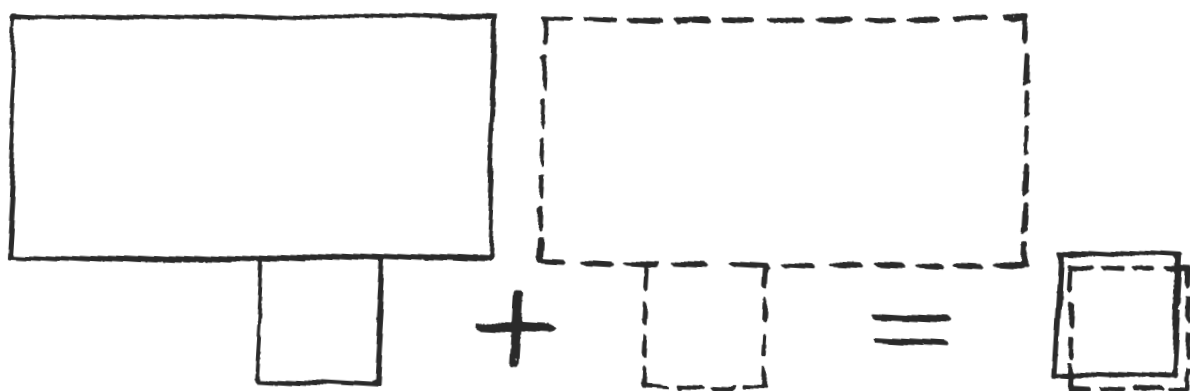
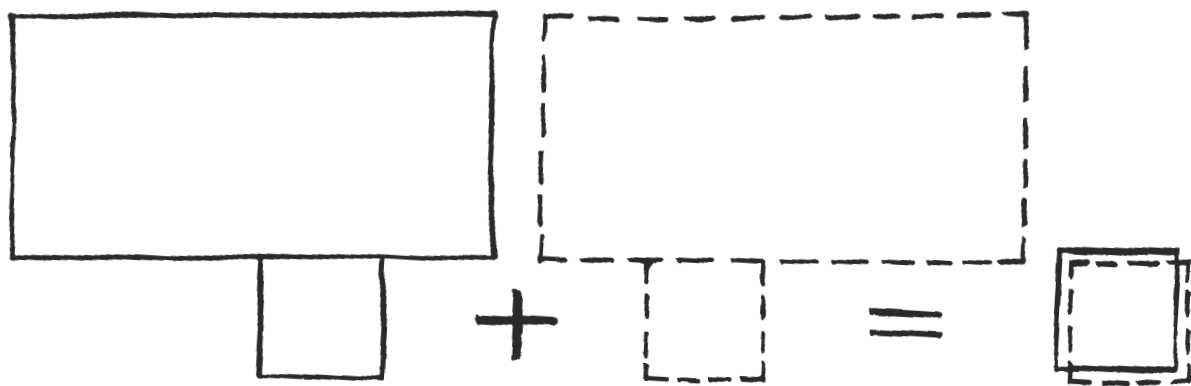
Another possibility which minimizes the use of paper is to laminate copies of the activity pages. Again, children can use a grease pencil and easily remove the markings when they have checked their work and permanently recorded any results that were interesting.

Still another teacher tried making transparency copies of some pages. She clipped a blank piece of paper behind the transparency so the print would show up, and had children work on the page with a grease pencil or flow pen (**not** permanent). Again, they could clean the transparency and return it to the file for the next child.

Many of the pages in this resource can be viewed as **suggestions** for an approach or an activity. In many cases the activity or approach suggested can be done in a format other than using **that** specific page. For example, the activities suggested here for

introducing the four basic operations really suggest an approach which can be implemented in a variety of ways. Some teachers have flat boxes for each child to use for placing counters; some have large pieces of cardboard on which spaces have been drawn for placing counters. Records are kept on regular paper or recycled scrap paper. Many of the activities suggest games which can be made from materials more flexible and permanent than paper. One teacher we know made a game board out of the giant fencing format on page 135 of this book. She used a square of smooth colorful oilcloth and drew the fencing frames on the oilcloth with a permanent marker. Then she cut two squares of clear vinyl plastic (available in many fabric shops) the same size as the oilcloth, and taped one to the oilcloth on four sides with cloth tape. The second sheet of clear plastic was taped on to the other two along just one side, so it could be lifted up. Children could fill in the numbers to be used in the game of the first clear sheet, then flip down the second clear sheet and play the game. Both sheets could be wiped clean after the game.

Teachers have begun to view the paper shortage as a new challenge to create more varied classroom materials. We hope that you will share your own creative ideas with your teacher colleagues and with us at Curriculum Development Associates.





NAME \_\_\_\_\_

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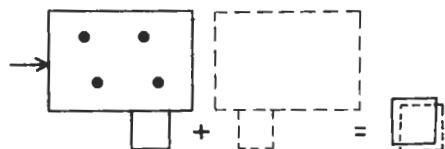
  

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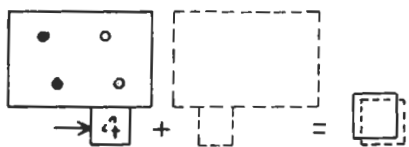
Children need counters.

This activity can be introduced as requests for a series of acts.

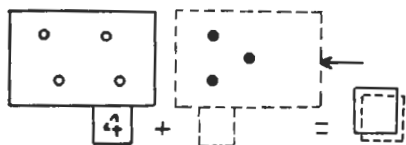
(1) Please put some counters in the large black-line frame.



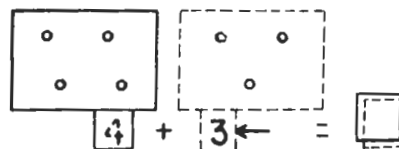
(2) Please write a number in the little square below that frame to show the number of counters in the frame.



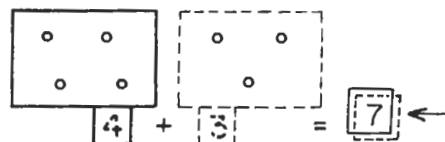
(3) Please put some counters in the other large frame . . . as many as you want.



(4) Please write a number in the little square below that frame showing the number of counters there are in the frame.



(5) Please find the total number of counters you have used in this experiment, and write that number in the double-line square on the right.

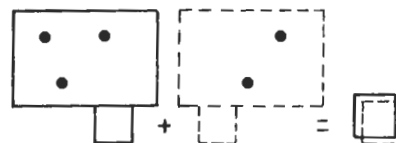


This final report can be read any way the child wants to: "4 and 3 makes 7" or "4 and 3 more is 7," etc.

As each child demonstrates he understands the procedure, he is ready to make up examples on his own—a completely individualized activity from that point on.

While we have considered 5 steps in the activity carried out in a given order, this is not intended as a prescription: rather, the procedure indicated demonstrates that, at no time, is the child confronted with more than one direction at a time and that he is asked to record the results of each separate count he carries out.

In fact, children very soon decide on their own sequence in making up their own examples. Usually they begin by putting counters in both frames.

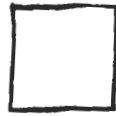
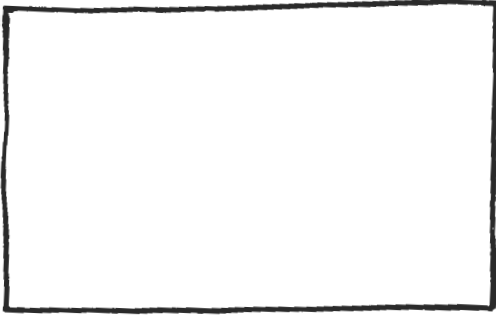


and then fill in the number in each frame—and then find the total and record it.

As long as the reports are reasonable, we are unconcerned about the individual style of reaching conclusions.

We choose to refer to each example the beginner carries out as an "experiment" with things and to the "arithmetic" as his report of his experiment.

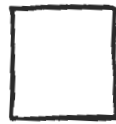
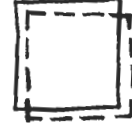
This page can also be particularized to specific experiments the learner is to carry out and report his outcome.



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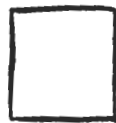
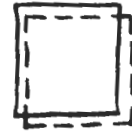
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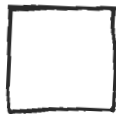
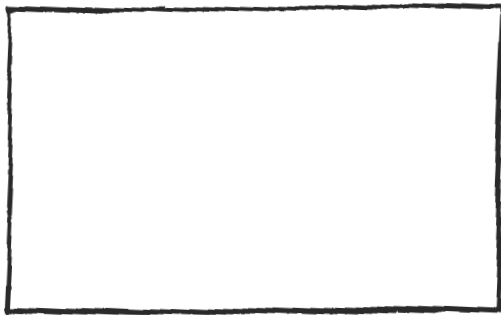
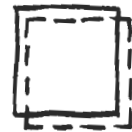
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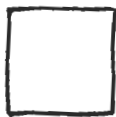
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On the previous page, there was a separate pair of "frames" for each experiment with accompanying blanks for keeping the record.

Experience has shown that for some children the move to using the same pair of frames to carry out more than 1 experiment requires some explanations and examples—some adult intervention. This and the following pages are designed to help minimize the difficulties in making this extension of activity.

This report form calls for 3 and 4 experiments to be carried out and reported with each pair of frames.

The page can be particularized in several ways. For example, any number such as "3" might be added in several of the report boxes.

|                                                 |                                             |
|-------------------------------------------------|---------------------------------------------|
| $\underline{3 + \quad} = \underline{\quad}$     | $\underline{\quad + \quad} = \underline{3}$ |
| $\underline{\quad + 3} = \underline{\quad}$     | $\underline{\quad + \quad} = \underline{3}$ |
| $\underline{\quad + \quad} = \underline{3}$     | $\underline{\quad + \quad} = \underline{3}$ |
| $\underline{\quad + \quad} = \underline{\quad}$ | $\underline{\quad + \quad} = \underline{3}$ |

or

or

$$\begin{array}{r} + 3 = \underline{\quad} \\ + 3 = \underline{\quad} \\ + 3 = \underline{\quad} \end{array}$$

with the rule for each group that all the reports be different in some way—and all true.

The last group of 4 open sentences asks for 4 different ways to arrange 3 counters in the 2 frames:

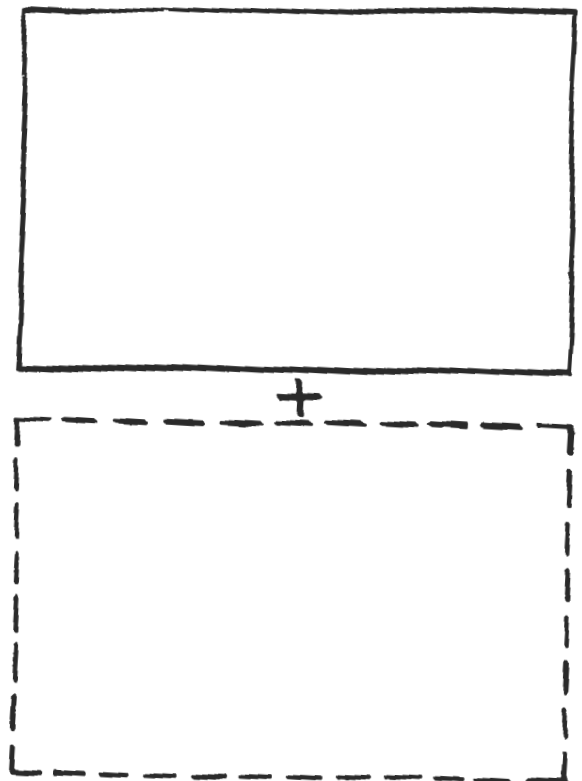
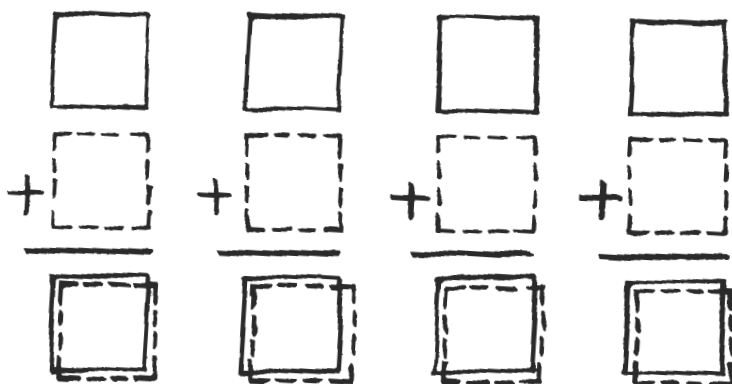
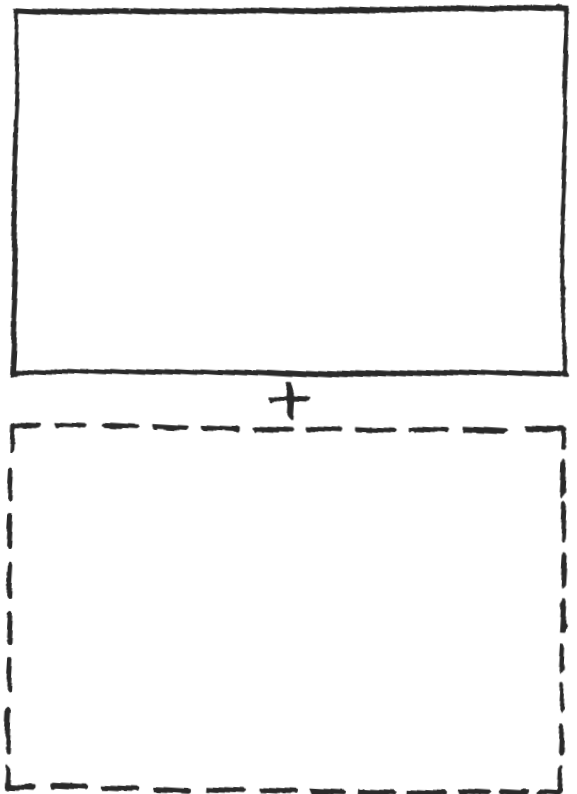
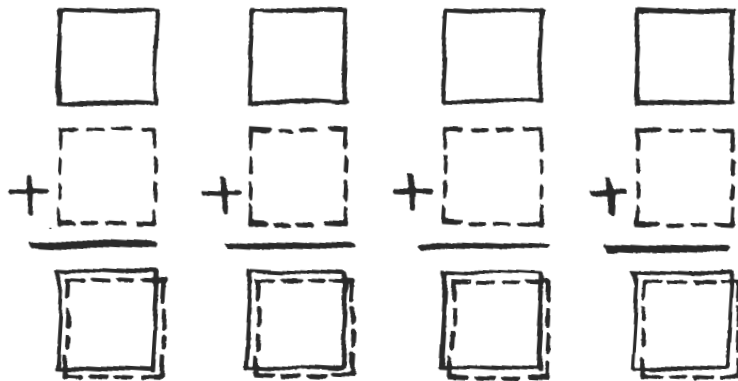
$$\begin{array}{l} 0 + 3 = 3 \\ 1 + 2 = 3 \\ 2 + 1 = 3 \\ 3 + 0 = 3 \end{array}$$

or the page might be more fully particularized.

$$\underline{5 + 2} = \underline{\quad}$$

If the teacher reads such completed reports as "three plus two equals five," but permits children to use any appropriate descriptive language, the children will eventually adopt the teacher's more standard expressions.

Some children may want to skip the counters and write down what they know the record would be. Fine! . . . as long as the records make sense; and, if they don't, try it with counters. "Or, when in doubt, get the counters out."





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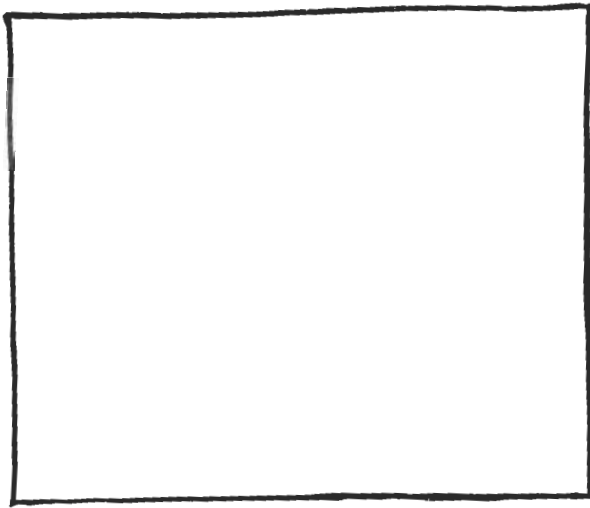
ADDITION

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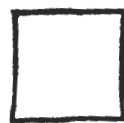
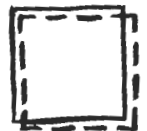
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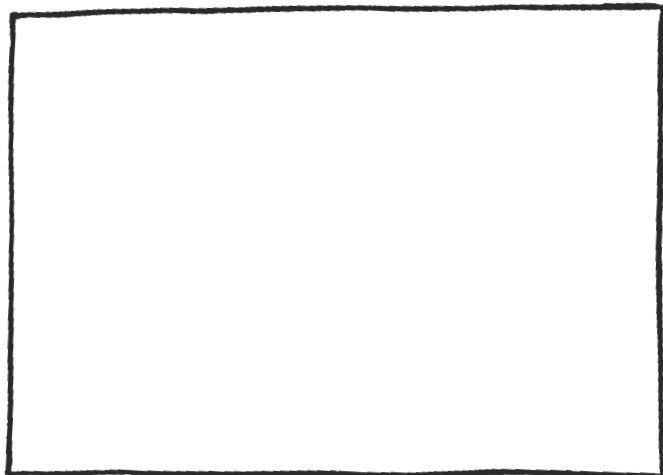
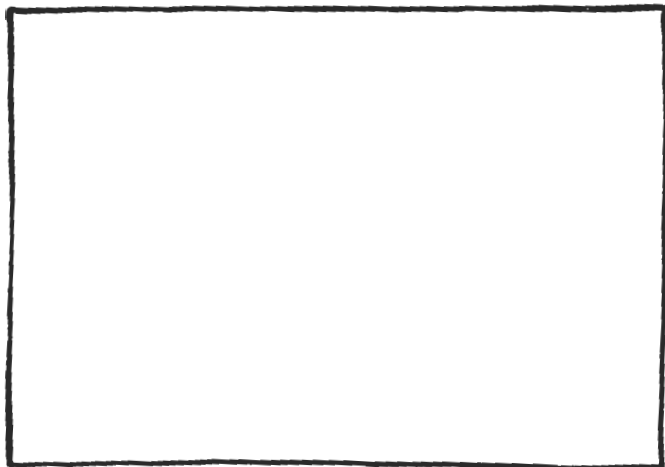
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This Report Form is specially designed using standard notations to facilitate keeping records of individual investigations.

How many different experiments use exactly 5 counters in each? This is equivalent to asking how many different ways can this open sentence be completed as a true statement.

$$\underline{\quad} + \underline{\quad} = \underline{5}$$

(Since the list includes  $0 + 5 = 5$  and  $5 + 0 = 5$ , there are 6 different ways.)

How many different ways to complete

$$\underline{\quad} + \underline{\quad} = \underline{6}$$

(As a true statement, there are 7 ways.)

Can you predict how many different ways there are to complete

$$\underline{\quad} + \underline{\quad} = \underline{7}$$

as a true statement? (There are 8 ways.)

Or, this page can be used to carry out a "chain reaction." Begin with any experiment and record the result.

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The rules for a chain reaction are these:

- (1) You can add one more counter on one side or the other, or
- (2) You can remove one counter from one side or the other, or
- (3) You can switch one counter from either side to the other.
- (4) But you can make only one of these moves at a time.
- (5) Keep records of each change.

A record of such an activity might include this sequence

$$\underline{7} + \underline{7} = \underline{14}$$

$$\underline{6} + \underline{7} = \underline{13}$$

$$\underline{6} + \underline{8} = \underline{14}$$

$$\underline{5} + \underline{9} = \underline{14}$$

$$\underline{6} + \underline{9} = \underline{15}$$

$$\underline{6} + \underline{10} = \underline{16}$$

etc.

Later the rules can be changed requiring 2 counters to be added to one side or the other, removed from one side or the other, or removed from one side to the other. This sequence might occur:

$$\underline{5} + \underline{4} = \underline{9}$$

$$\underline{7} + \underline{4} = \underline{11}$$

$$\underline{7} + \underline{2} = \underline{9}$$

$$\underline{9} + \underline{2} = \underline{11}$$

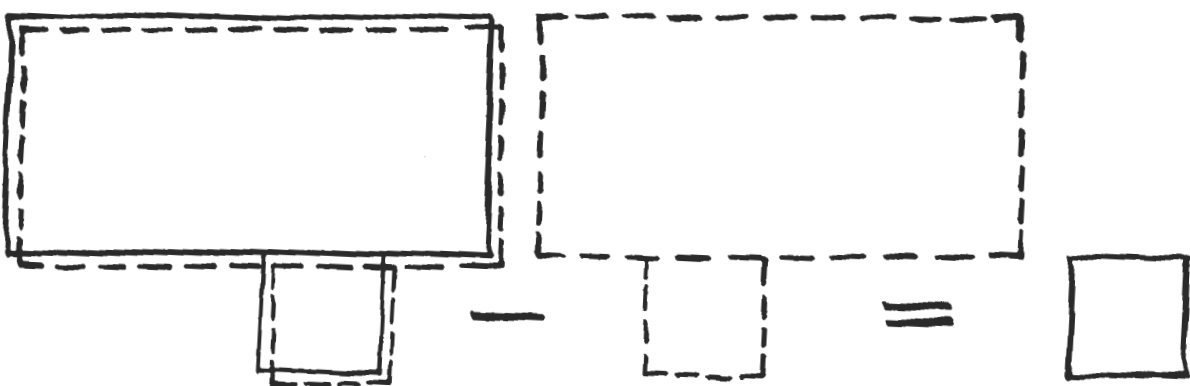
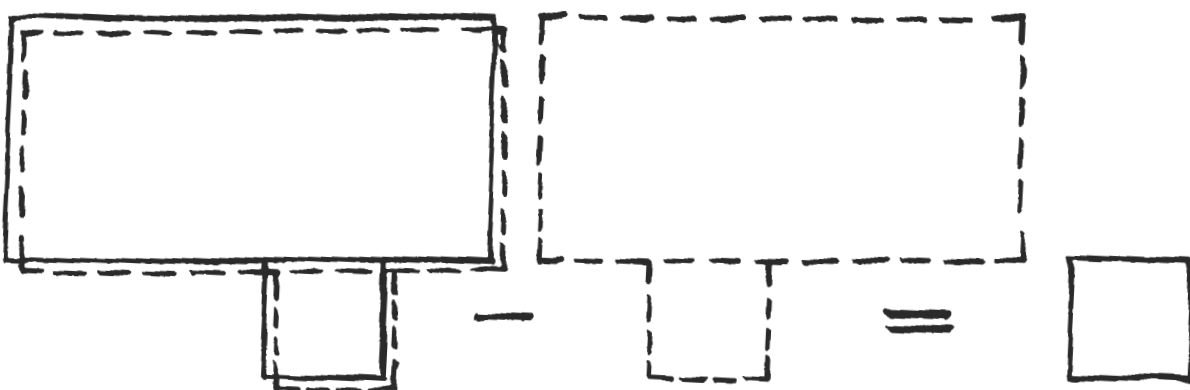
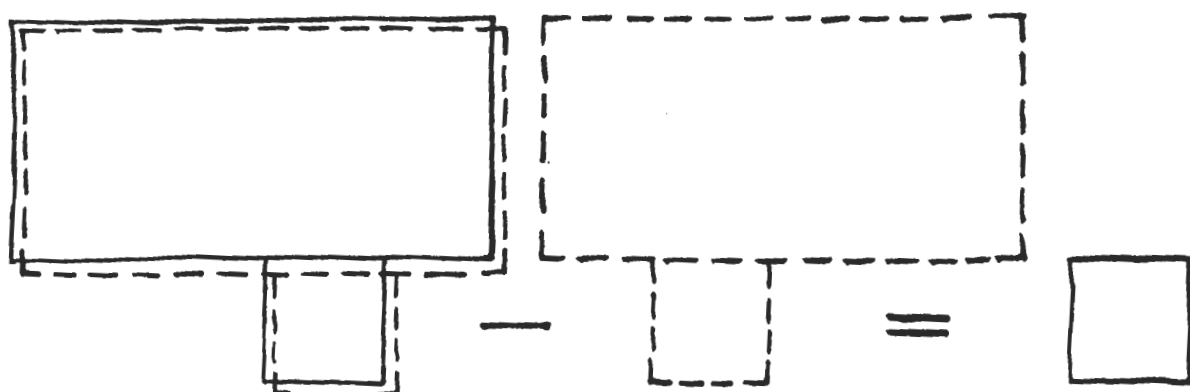
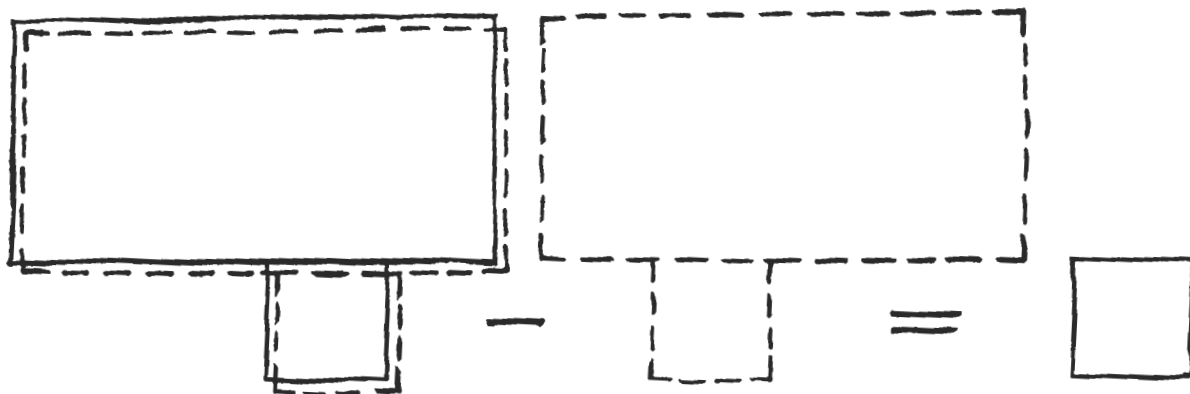
$$\underline{7} + \underline{4} = \underline{11}$$

etc.

Other rules can be made up that lead to chain reactions in which each new result is related to the preceding result.

In all cases, the student is urged to use every short cut he can think of to avoid as much counting as possible.





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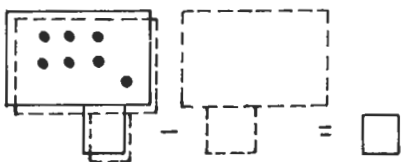
  

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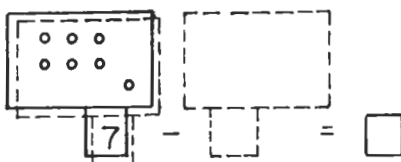
Children need counters.

This activity can be introduced as requests for a series of acts.

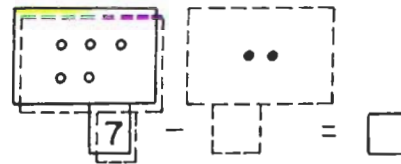
- (1) Please put some counters in the large double-line frame.



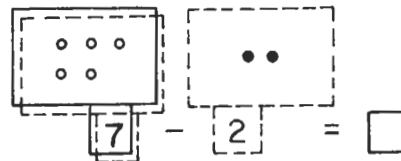
- (2) Please write a number in the little double-line square below to show the numbers of counters in the frame.



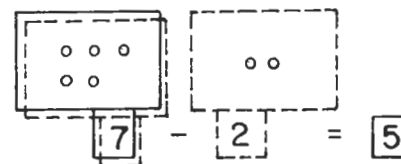
- (3) Please move some counters to the other frame—any number you like—including all of them or none of them.



- (4) Please write a number in the little broken-line square below the large broken-line frame showing the number of counters you moved.



- (5) In the little solid line square on the right, please write a number to show how many counters are left in the double-line frame.



This final report can be read in any way the child wants to describe the experiment such as "I started with 7; moved 2 over and there are 5 left" . . . or "7 take away 2 leaves 5" . . . etc.

We have outlined this activity as a sequence of 5 steps.

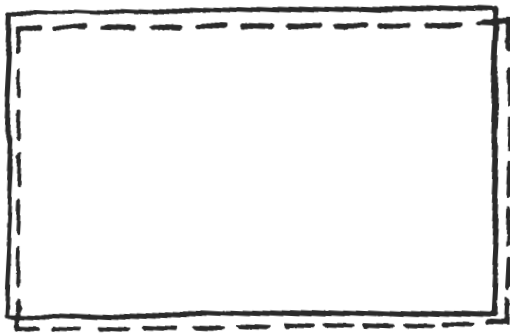
However, as the children begin making up their own experiments, they are free to adopt their own style—as long as the results they report are reasonable.

The children might be asked to suggest a name for these experiments—for this game . . . such as the "move over game," or the "how many left game," or the "take away game"—any name they feel is appropriate.

However, the teacher may want to begin to read

$$7 - 2 = 5$$

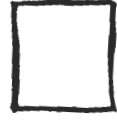
as "seven minus two equals five" but permit the children to use their own, more descriptive language.



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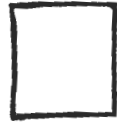
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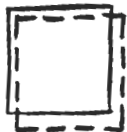
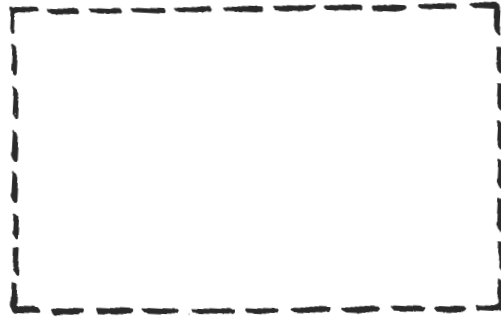
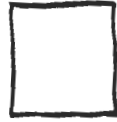
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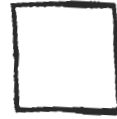
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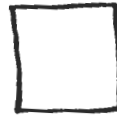
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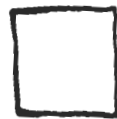
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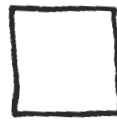
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On this page provision is made for 3 or 4 reports using the same report frames.

The page has several uses in addition to keeping a record of examples the learners make for themselves.

If the reports are partially completed, such as

$$\underline{2 - \quad} = \underline{\quad}$$

$$\underline{2 - \quad} = \underline{\quad}$$

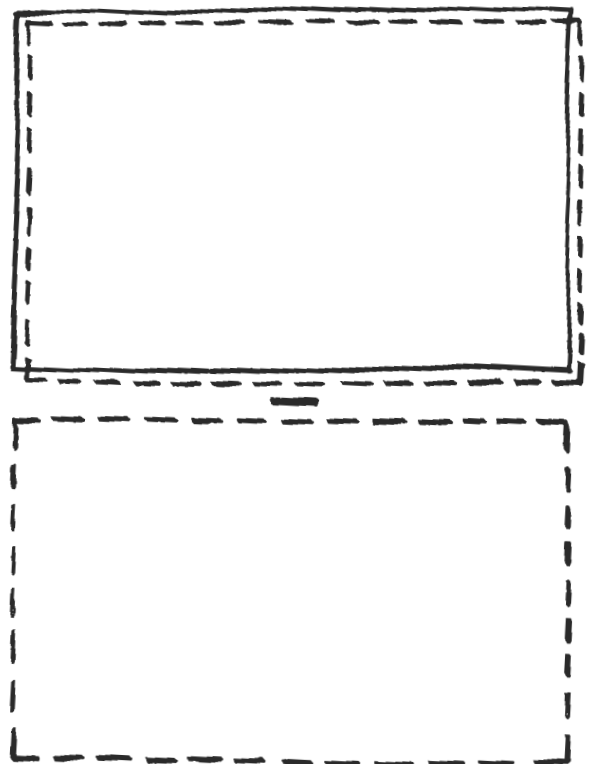
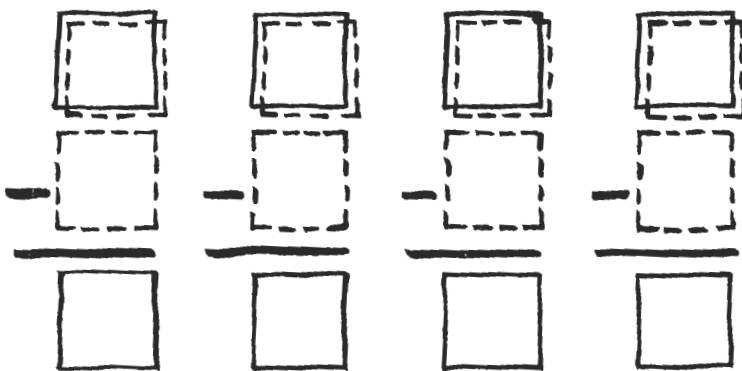
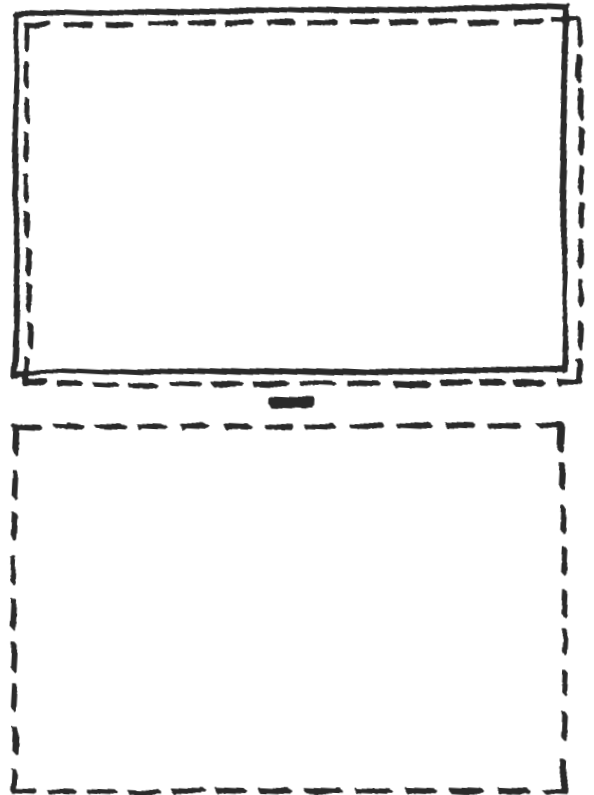
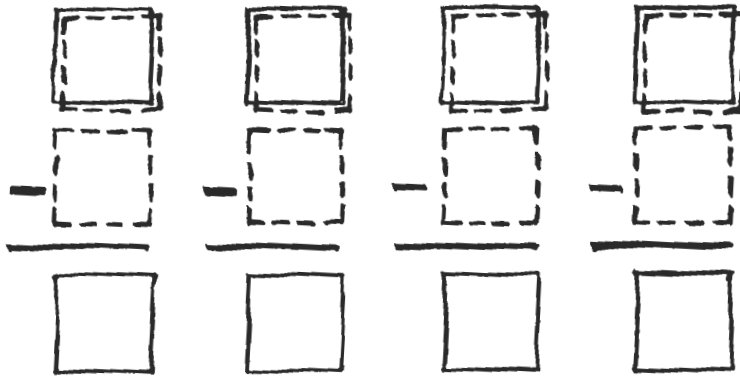
$$\underline{2 - \quad} = \underline{\quad}$$

the question is posed—"Can you make up 3 different experiments of this kind that start with 2 counters in the double-line frame?"

$$\underline{\quad - \quad} = \underline{3}$$

$$\underline{\quad - \quad} = \underline{3}$$

This asks for different experiments in which 3 counters are left in the double line frame.



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SUBTRACTION

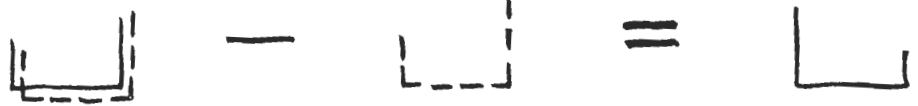
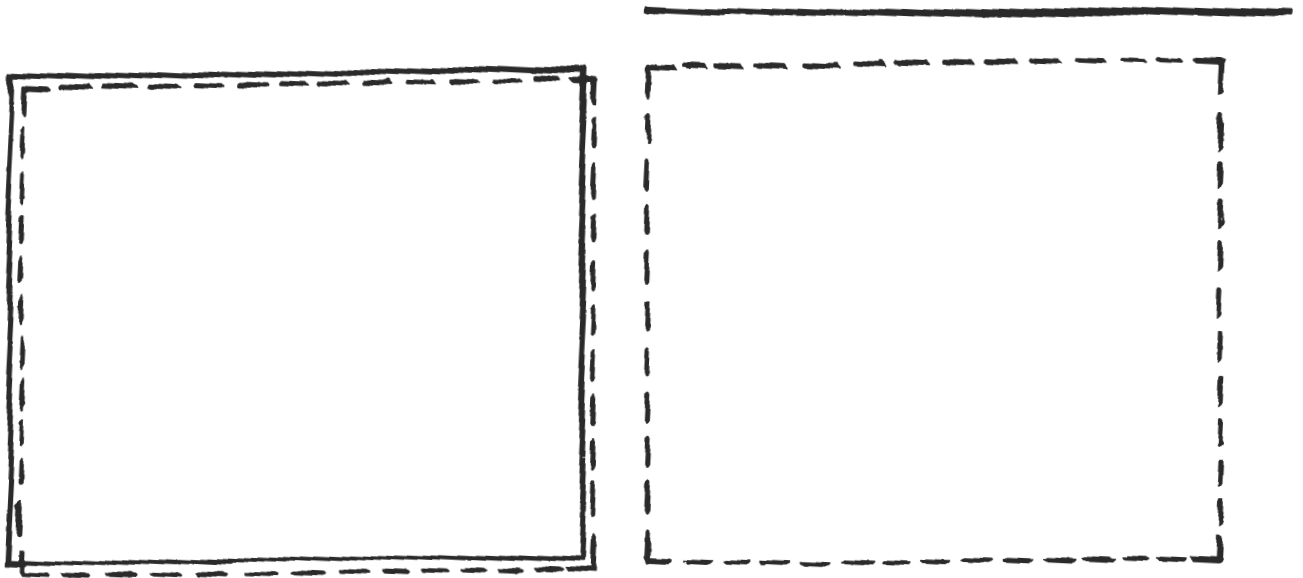
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The format is changed to introduce a change in notation—to the “vertical form.”





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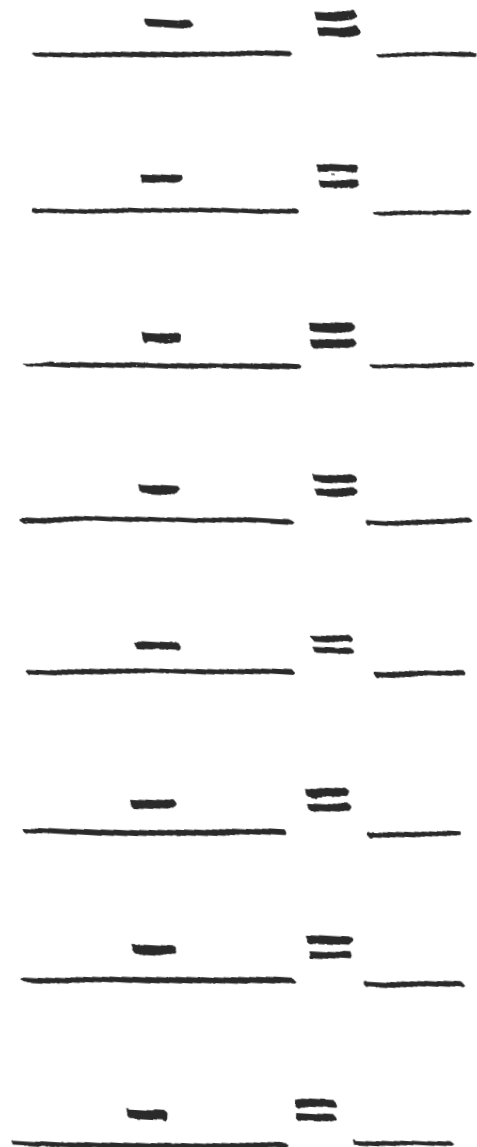
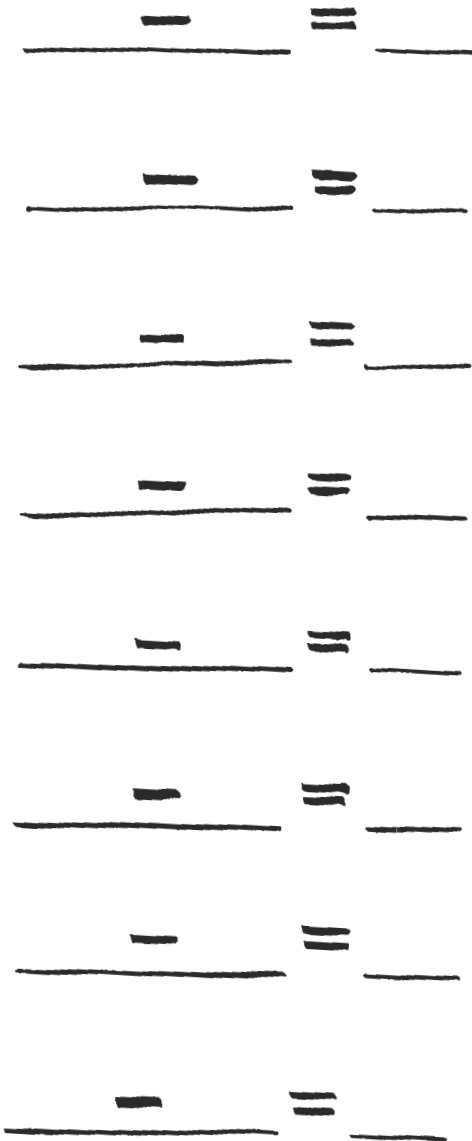
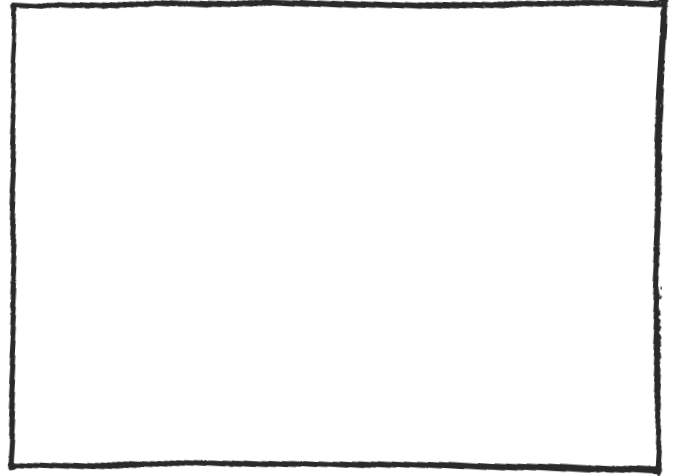
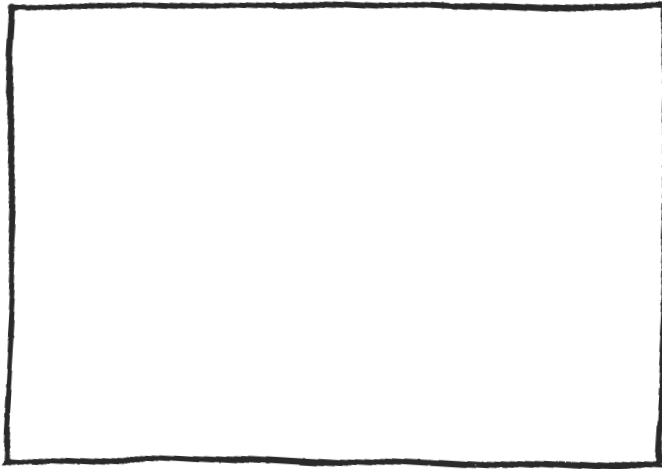
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This page provides for records of 9 experiments using a single pair of frames. And the unique notation that has been employed is phased into the standard notation:

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This Report Form is specially designed to keep records of investigations.

How many different experiments can you carry out in these various forms?

(a)            =   4  

(b)            - 4 =       

(c)     4 -        =       

A little investigation turns up some surprising information.

In form (a), there are an unlimited number of different experiments.

In form (b), the number is also unlimited.

In form (c), there are 5 different records—and no more.

Also consider “chain reactions” with the rules:

(1) Begin with an experiment, record the outcome and leave the counters in the frames.

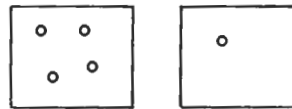
(2) The reaction begins by carrying out one of these instructions:

(a) add a counter to one frame or the other; or

(b) remove a counter from one frame or the other; or

(c) move one counter from one side to the other.

Suppose we start with an experiment that ends with the counters and report as shown:

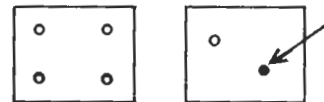


$$\underline{5 - 1 = 4}$$

$$\underline{\quad - \quad = \quad}$$

$$\underline{\quad - \quad = \quad}$$

Suppose we add a counter to the left side. How should we change the report?

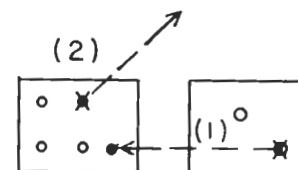


$$\underline{5 - 1 = 4}$$

$$\underline{6 - 2 = 4} \leftarrow$$

$$\underline{\quad - \quad = \quad}$$

Then we made two successive changes: (1) switched one from right to left, and then (2) removed one from the left side;



$$\underline{6 - 2 = 4}$$

(1)  $\underline{6 - 1 = 5}$

(2)  $\underline{5 - 1 = 4}$

This version of “chain reaction” is quite different. Even adults may move quite slowly at first—and counters might help.

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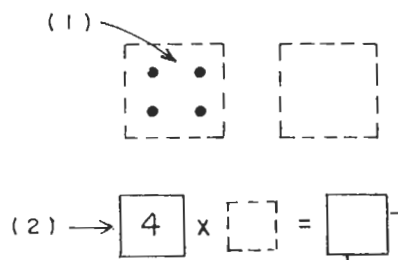
The next series of pages is designed to help introduce the games of multiplication and division to beginners—when they are most willing to experiment with counters.

As soon as a child can count to 12 and write most of the numbers from 0 to 12, he is ready for this game.

Again, we will outline the activity in terms of a series of requests for the child to follow:

(1) Please get some counters and put them in the large frame (in the second example) or in the large frame at the left—if there is more than 1. Choose as many counters as you like. (Suppose he chose 4.)

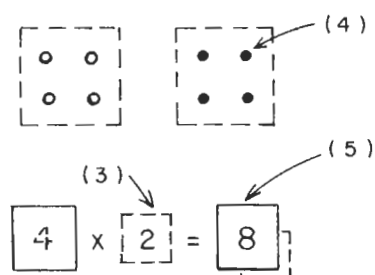
(2) Please write a number in the solid line square showing the number of counters in that frame (4).



(3) Please count the number of frames, and write that number in the smaller broken line square.

(4) Get as many counters as you need so all the frames (both in this case) have this same number of counters as the first—using counters from the general supply.

(5) Please find the total number of counters in the frames, and write a number in the other small square to show the total. The report in this case would be:



This can be read “four **multiplied by** two equals eight”—and that matches the experimenting. We have a definite reason for this that will become apparent when we introduce division—which has to be read “eight **divided by** two equals four” and we want to present multiplication as clearly as possible as “inverse operations”—the one “undoes” the other.

Now if you prefer to describe the activity we described above as “two times four . . .” then change the order of the steps.

(1) Please count frames—and write 2 (in this case) in the first square:

$$\boxed{2} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

(2) Please put some counters in the first frame.

(3) Please write that number in the broken-line box.

(4) Please make all the frames like the first one.

(5) Please count the total and record it, and you can read the expression as “two times four equals eight.”

$$\boxed{2} \times \boxed{4} = \boxed{8}$$

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As the discussion proceeds, we will soon begin to use the former approach only, but please feel perfectly comfortable in adopting the latter sequence and language. Fortunately, the total will always be "12" regardless of how you interpret the statement:

$$\boxed{3} \times \boxed{4} = \boxed{12}$$

In these examples, the "number of boxes" is not indicated. When the child decides how many frames he wants to use, he checks that many or otherwise marks them.

A completed example would look like this:

|               |                 |                   |   |
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| ✓<br>□<br>• • | ✓<br>□<br>• • • | ✓<br>□<br>• • • • | □ |
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$$\boxed{4} \times \boxed{3} = \boxed{12} \text{ (our preference)}$$

or ✓

$$\boxed{3} \times \boxed{4} = \boxed{12} \text{ (optional)}$$

(We have included a small check above the small square with the number indicating the number of boxes—we use the second square; you may use the first square.)

What happens if we decide to put 7 counters in each of the "no boxes"?

$$\boxed{7} \times \overset{\checkmark}{\boxed{0}} = \boxed{\phantom{00}} \text{ or } \overset{\checkmark}{\boxed{0}} \times \boxed{7} = \boxed{\phantom{00}}$$

If no frames are used, then there can be no counters in them—so we would write

$$\boxed{7} \times \overset{\checkmark}{\boxed{0}} = \overset{\checkmark}{\boxed{0}} \text{ or } \overset{\checkmark}{\boxed{0}} \times \boxed{7} = \overset{\checkmark}{\boxed{0}}$$

The total would be the same regardless of how many we chose to put in each frame.

Suppose we chose to put "no counters" in three frames.

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| ✓<br>□ | ✓<br>□ | ✓<br>□ | □ |
|--------|--------|--------|---|

$$\boxed{0} \times \overset{\checkmark}{\boxed{3}} = \overset{\checkmark}{\boxed{0}}$$

or ✓

$$\boxed{3} \times \overset{\checkmark}{\boxed{0}} = \overset{\checkmark}{\boxed{0}}$$

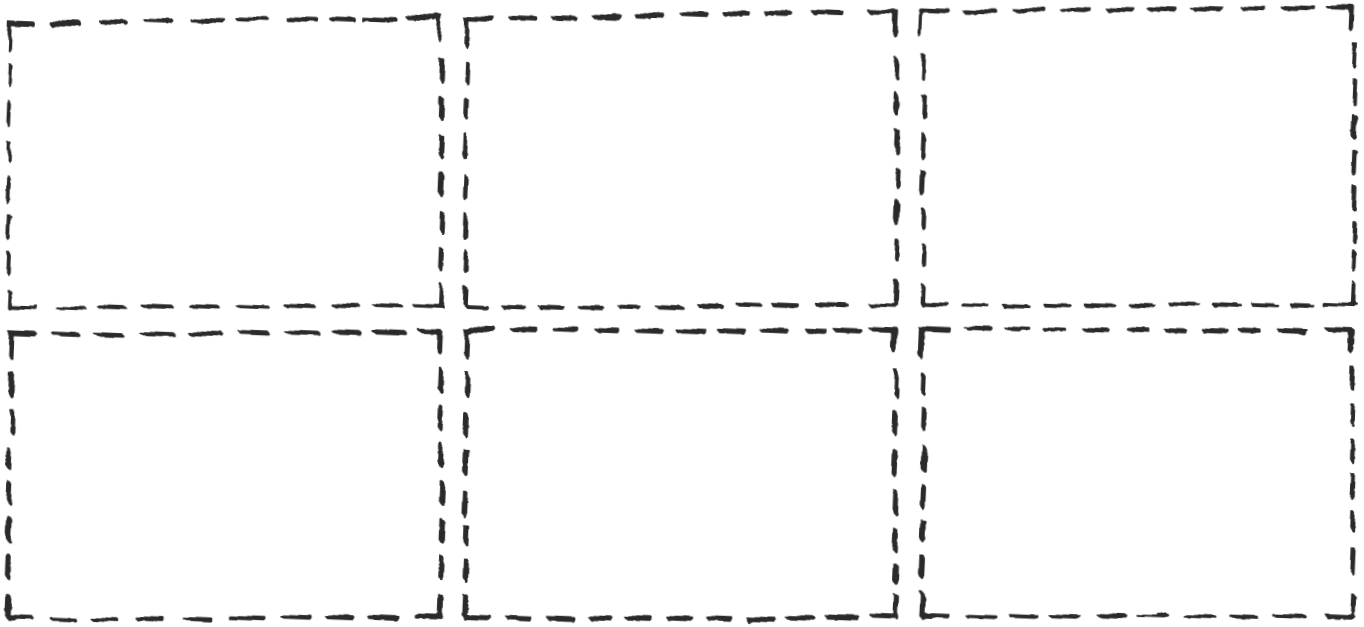
If there are no counters in each of 3 frames, then there is a total of no counters—"0" counters.

No other surprises will arise.

This form might also be used to report the results of all ways exactly 12 counters could be used in experiments (with only 4 frames available).

Or, the report-squares could be particularized to call for specific experiments.

$$\boxed{3} \times \boxed{2} = \boxed{\phantom{00}}$$



$$\square \times \square = \square$$

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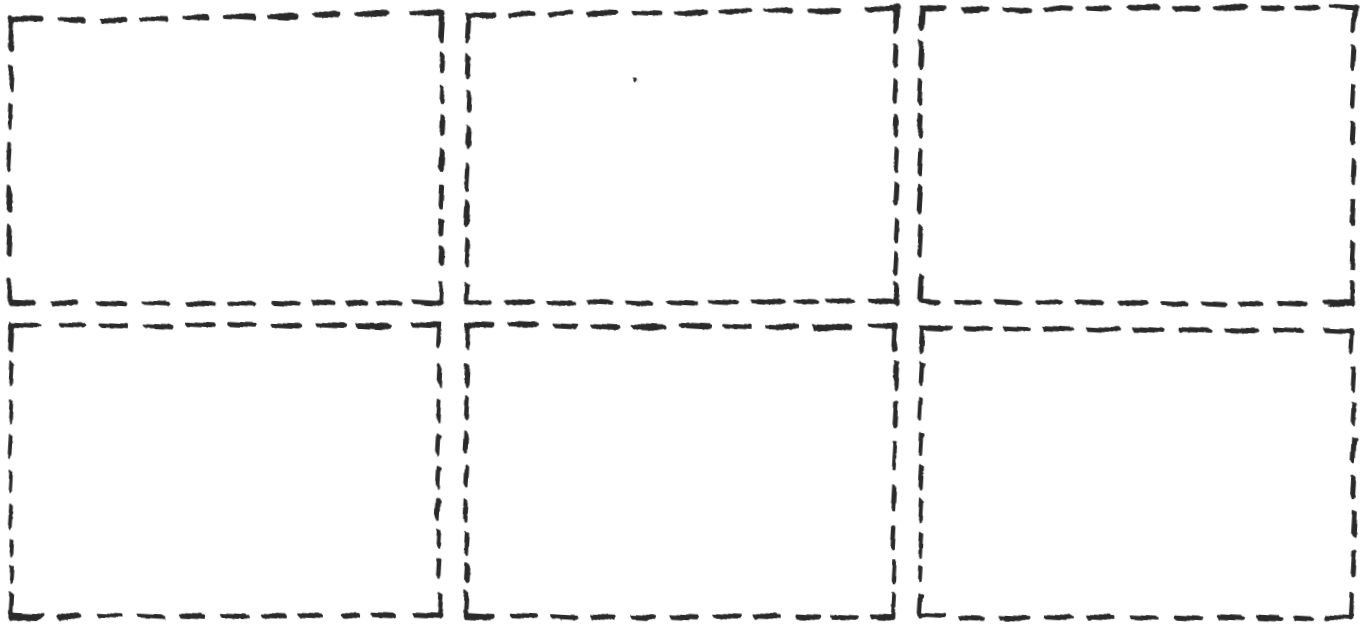
The number of frames provided has increased; and all the experiments are carried out in one group of frames. Further, the unique notation is phased into a standard form—

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

Many children will be finding other reliable ways to find the total number of counters used—the “product.” Some will imagine the experiment and fill out the record knowing what the outcome would be. Others will use tally-marks—often an elaborate system so they can complete many examples in a single group of frames. Still others prefer using scratch paper in many different ways.

All methods that reliably produce sensible statements should be encouraged.

If any child wants to explain his system to others, and discuss it, fine! . . . since we are all involved in one great hunt for reliable shortcuts.



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Six frames again—a standard notation—and room for lots of experimenting.

Some might like to build tables.

or

|                                           |                                           |
|-------------------------------------------|-------------------------------------------|
| $\underline{0 \times 3} = \underline{0}$  | $\underline{3 \times 0} = \underline{0}$  |
| $\underline{1 \times 3} = \underline{3}$  | $\underline{3 \times 1} = \underline{3}$  |
| $\underline{2 \times 3} = \underline{6}$  | $\underline{3 \times 2} = \underline{6}$  |
| $\underline{3 \times 3} = \underline{9}$  | $\underline{3 \times 3} = \underline{9}$  |
| $\underline{4 \times 3} = \underline{12}$ | $\underline{3 \times 4} = \underline{12}$ |

Others can make other investigations.

or

|                                          |                                          |
|------------------------------------------|------------------------------------------|
| $\underline{0 \times 6} = \underline{0}$ | $\underline{0 \times 0} = \underline{0}$ |
| $\underline{1 \times 5} = \underline{5}$ | $\underline{1 \times 1} = \underline{1}$ |
| $\underline{2 \times 4} = \underline{8}$ | $\underline{2 \times 2} = \underline{4}$ |
| $\underline{3 \times 3} = \underline{9}$ | $\underline{3 \times 3} = \underline{9}$ |

$$\underline{4 \times 2} = \underline{8} \qquad \underline{4 \times 4} = \underline{16}$$

$$\underline{5 \times 1} = \underline{5} \qquad \underline{5 \times 5} = \underline{25}$$

$$\underline{6 \times 0} = \underline{0} \qquad \underline{6 \times 6} = \underline{36}$$

Chain reaction can follow from rules such as:

- (1) Start with the report of any experiment.
- (2) Change the experiment in either of these ways:
  - (a) Put one more counter in each frame, or take one out of each frame; or
  - (b) Use one more frame or one less frame.

The reports of a sequence of such a chain reaction would include this:

$$\underline{2 \times 4} = \underline{8}$$

$$\underline{3 \times 4} = \underline{12}$$

$$\underline{3 \times 5} = \underline{15}$$

$$\underline{2 \times 5} = \underline{10}$$

$$\underline{2 \times 6} = \underline{12}$$

Or, the Report Form can be changed to provide specific examples:

$$\underline{2 \times 3} = \underline{\quad}$$

$$\underline{1 \times 4} = \underline{\quad}$$

$$\underline{4 \times 2} = \underline{\quad}$$

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$$\boxed{\phantom{00}} \div \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

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The fourth basic experiment—or “operation”—division.

Now it will become obvious that the second number in the report frame refers to **frames**, not counters, because we have no standard alternative to the formulation “divided by”: thus, no options.

1. Please get some counters.
2. Please write the number in the first (on the left) report-square. If 7 are selected, the report would begin:

$$\boxed{7} \div \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

3. Please find the number of frames shown (later—select a number of frames to use) and write that number in the next report square: (2 in the case of the first example).

$$\boxed{7} \div \boxed{2} = \boxed{\phantom{00}}$$

4. Use as many of the 7 counters as you can, so there are the same number in each of the 2 frames. The result would be:



$$\boxed{7} \div \boxed{2} = \boxed{\phantom{00}}$$

5. Report the outcome showing the largest number of counters you could put in each frame and yet keep the same number in both by writing the number in the third report-square.

$$\boxed{3}$$

6. Finally, show how many of the counters you couldn't use in this way:

$$\boxed{3} \overline{1}$$

... “Three in each frame and one left over.”

From here on there will be no new developments. (Except dividing by “0” . . . which we will consider later.) The question of indicating remainders has been settled. The children are prepared to make up examples of their own.

They might select their own name for this game: such as the “sharing game.” They can talk about it in their own language

At the same time the teacher can find opportunities to read the results of the experiment above: “Seven **divided** by two equals three and one left over”—or “. . . three and a remainder of one.”

(Note that in the third example, there will never be a remainder.)

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$$\boxed{0} \div \overset{\checkmark}{\boxed{3}} = \boxed{0}$$

But suppose a child indicates he wants to distribute 6 counters evenly with "no frames"?

$$\boxed{6} \div \overset{\checkmark}{\boxed{0}} = \boxed{\phantom{0}}$$

Clearly it can't be done!

(Notice we didn't say "you can't divide by zero" . . . simply, you cannot do what you said you had decided to do.) So, there is no way to complete that open sentence as a true statement!

Now it can truly be said, "no more surprises in division."

On this page, the experimenter decides both the total number of counters he wants to use and the number of frames he wants to use.

A completed example using 8 counters and 3 frames might look like this:

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$$\boxed{8} \div \overset{\checkmark}{\boxed{3}} = \boxed{2}^2$$

It's surprising how easy it is to deal with "remainders" when arithmetic is interpreted as a way of keeping a record of what was done and what happened!

Suppose a child decides to use "no" counters and 3 frames. The partial record is:

$$\boxed{0} \div \overset{\checkmark}{\boxed{3}} = \boxed{\phantom{0}}$$

If the counters are distributed equally in the 3 frames, there are none in each frame—and none left over.

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$$\boxed{\phantom{00}} \div \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

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$$\boxed{\phantom{00}} \div \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

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The options increase—6 frames are now provided. More experiments with a single group of frames. And the unique notation we have used phases into a standard notation.

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This report form—with 6 frames available and reports in a “standard form”—is specially designed for investigations.

Because such investigations sometimes create unexpected problems, another twist may be worth looking at. Consider this investigation:

$$4 \div 1 = \underline{\quad}$$

$$3 \div 2 = \underline{\quad}$$

$$2 \div 3 = \underline{\quad} *$$

$$1 \div 4 = \underline{\quad} *$$

The starred questions lead to this problem:

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$$\boxed{2} \div \boxed{3} = \boxed{\quad}$$

There aren't enough counters for even one to a frame. So, a logical way to report the results is:

$$\boxed{2} \div \boxed{3} = \boxed{0} \frac{2}{3}$$

Prepared for this eventuality, no experiment ought to run into any “trouble.”

Which numbers of counters can you start with in this series so there will be “no remainders” or “none left over”?

$$\begin{array}{r} \div 3 = \\ \div 3 = \\ \div 3 = \end{array}$$

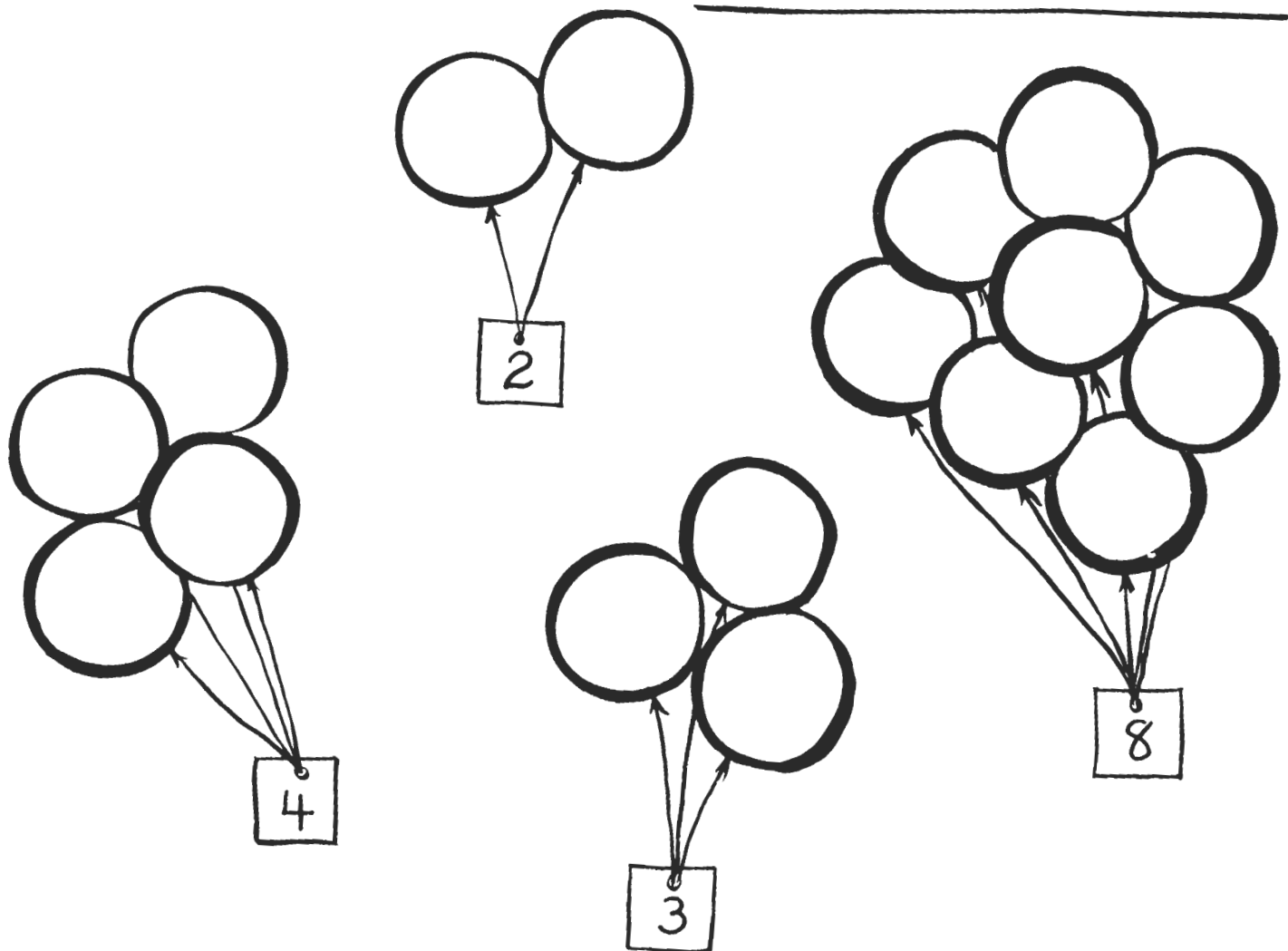
Or, how many ways can you set up experiments so there will be none left over if the reports are in this form:

$$\underline{3 \div \quad} = \underline{\quad}$$

(The answer is: only 2 ways.)

$$\underline{3 \div 3} = \underline{1}$$

$$\underline{3 \div 1} = \underline{3}$$



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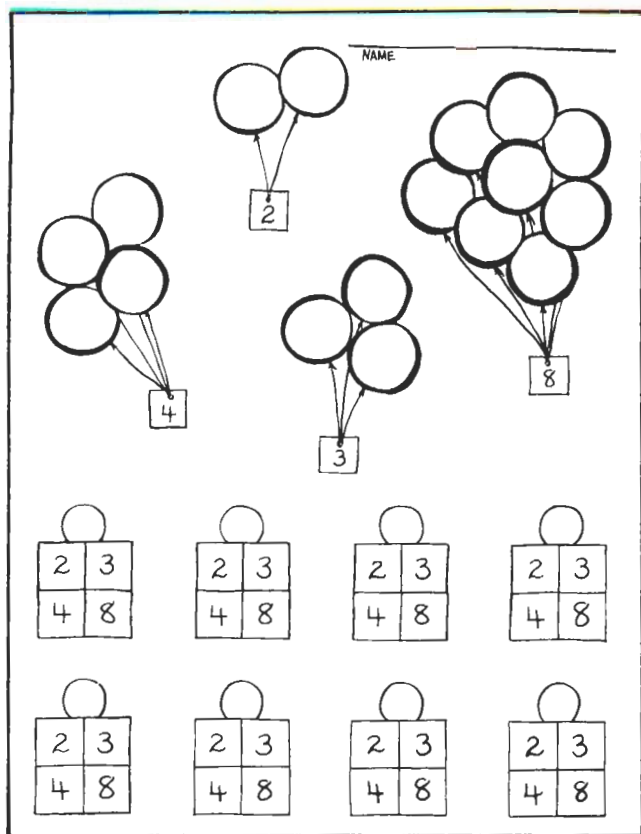
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### Balloons and Bunches (addition)

The learners need counters for this activity. There are two rules:

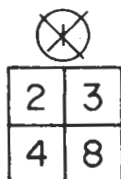
- (1) No more than one counter can be placed in a balloon.
- (2) If you put a counter on one balloon, you must put a counter on each balloon in that bunch.

There are bunches with 2, 3, 4, and 8 balloons. The numbers in the squares refer to the numbers of balloons in the bunches.

The teacher or the learner enters numbers in the circle-spaces in the lower part of the page.

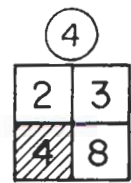
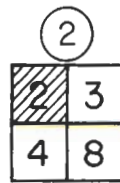
The question is: Following the rules above, can you place each number of beans given in the circles on the array of balloons?

There is no way to place one counter. This result is recorded by crossing out the '1':

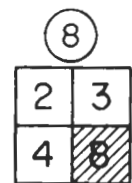
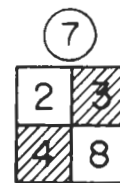
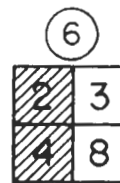


(means: "I can't do it.")

Placing two, three, and four counters is easy. We can record how they were done by coloring in the appropriate blocks:

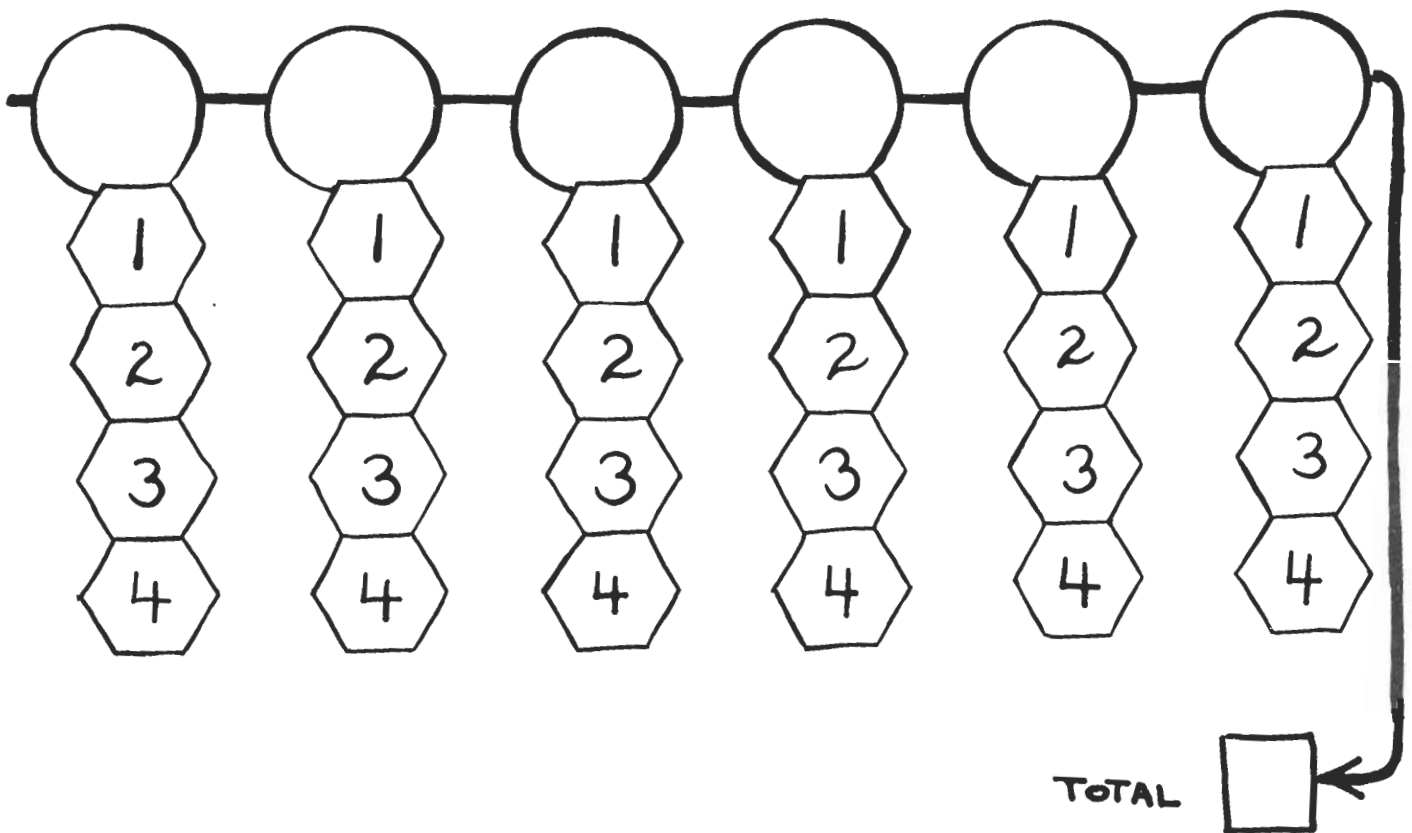
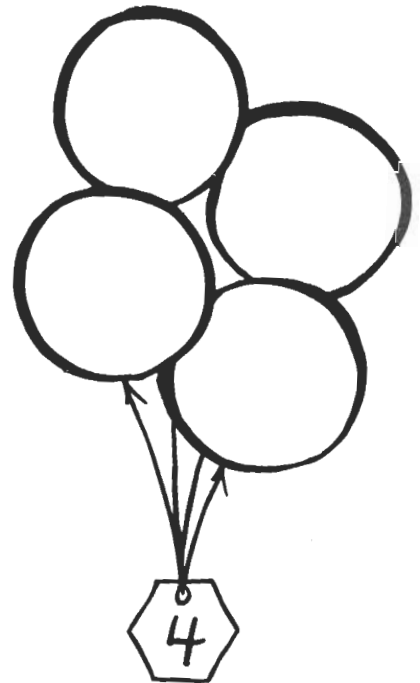
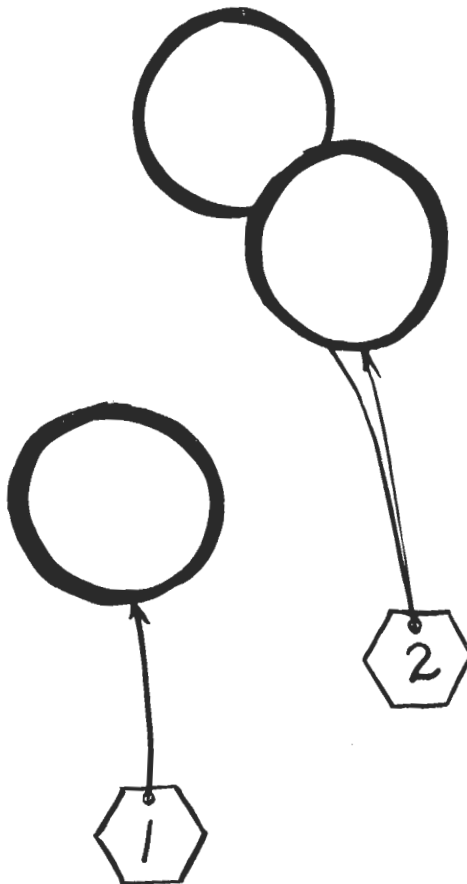
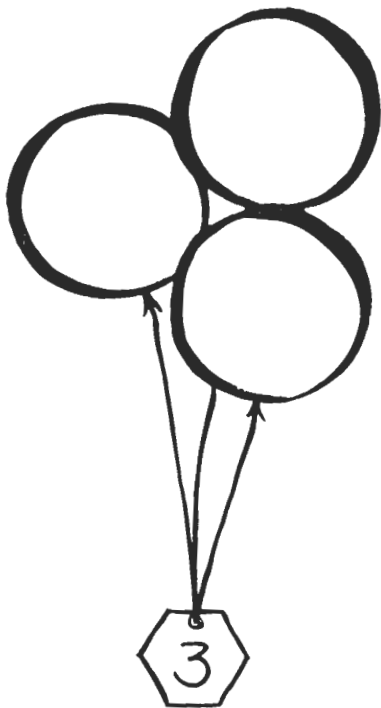


Five counters can be placed by using the bunch of two and the bunch of three, six by using the bunch of two and the bunch of four; etc. These can be recorded:

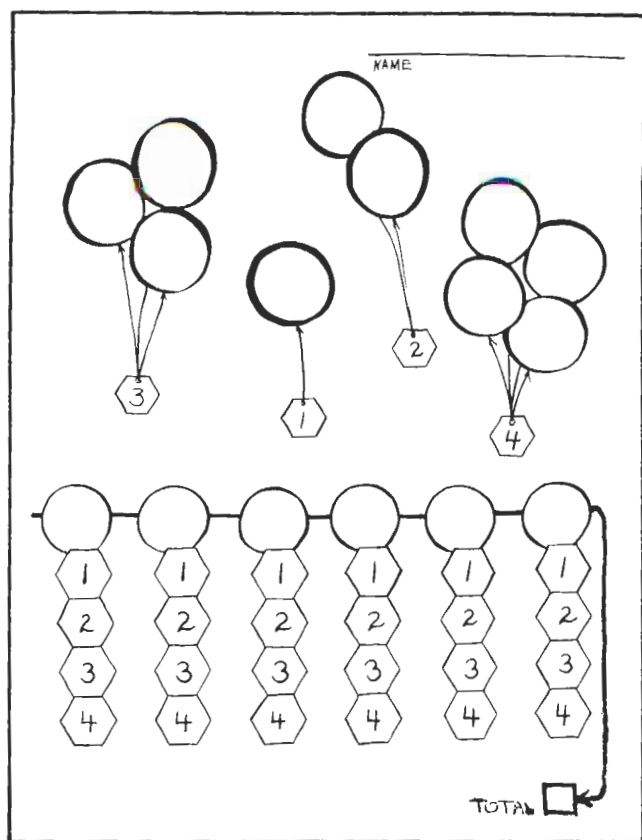


A question to explore:

What numbers, 1 through 17, cannot be done?







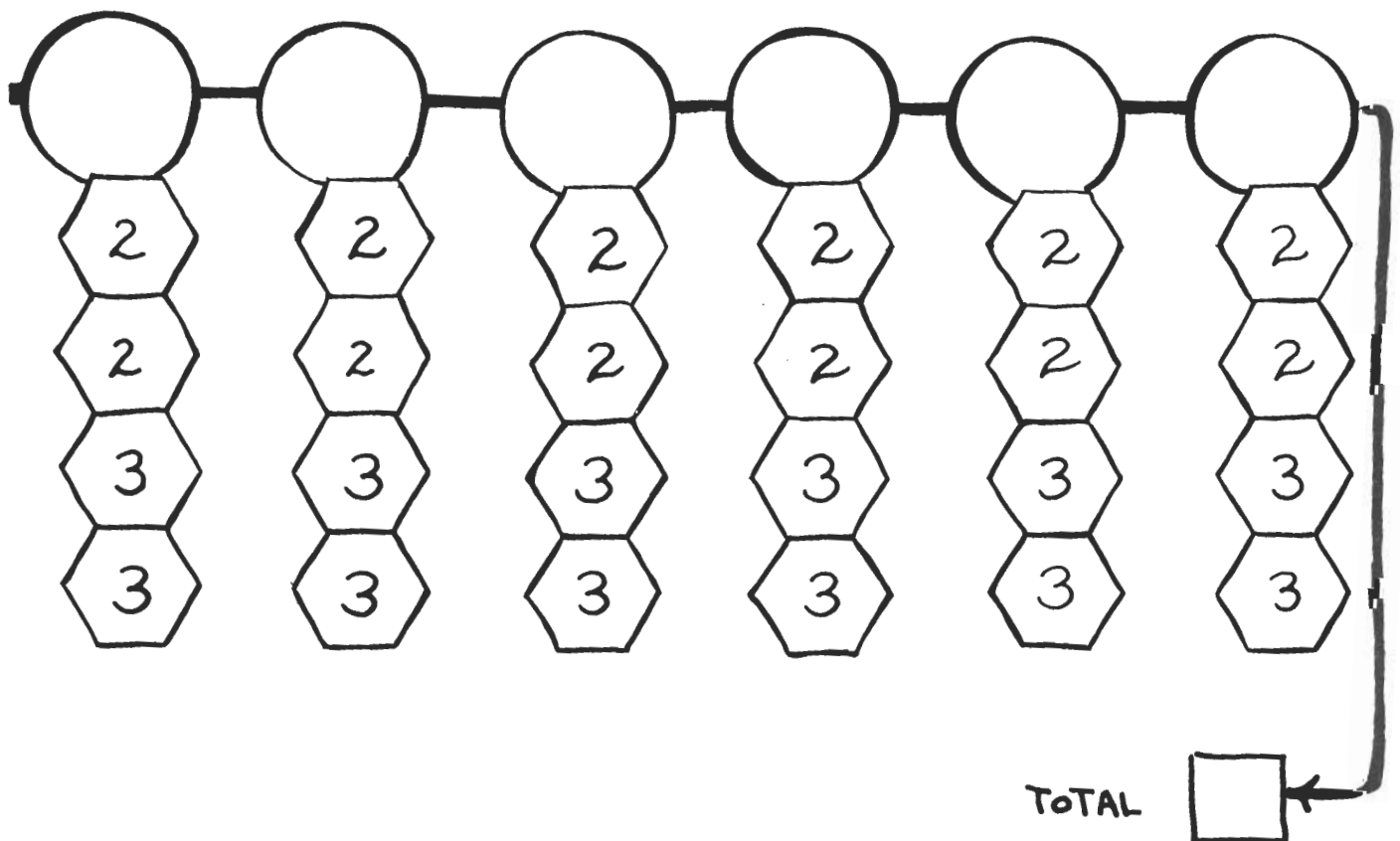
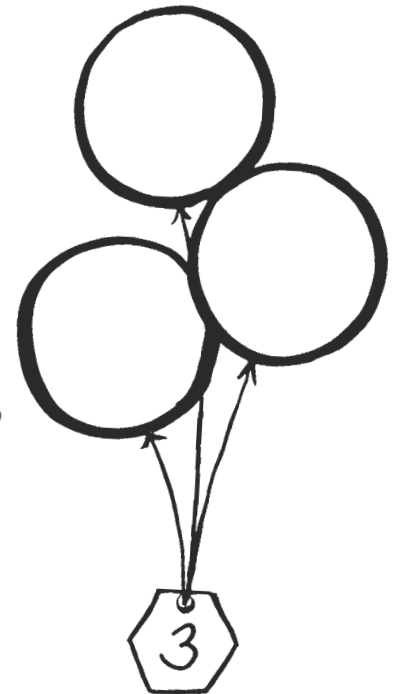
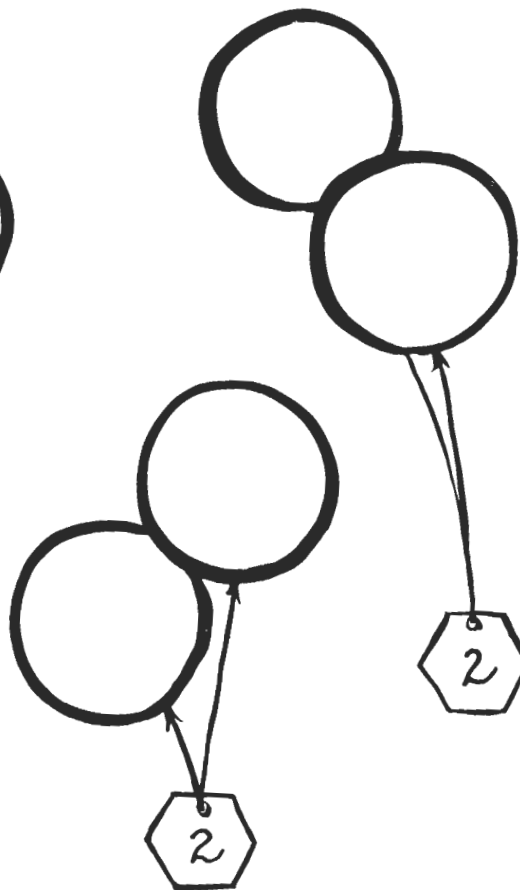
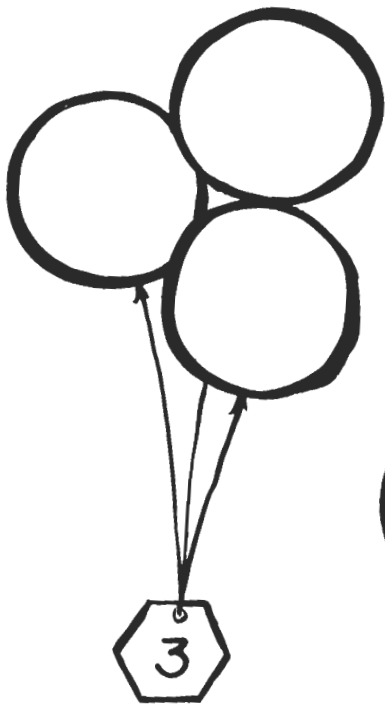
(For a more complete description of the activity, see page 19.)

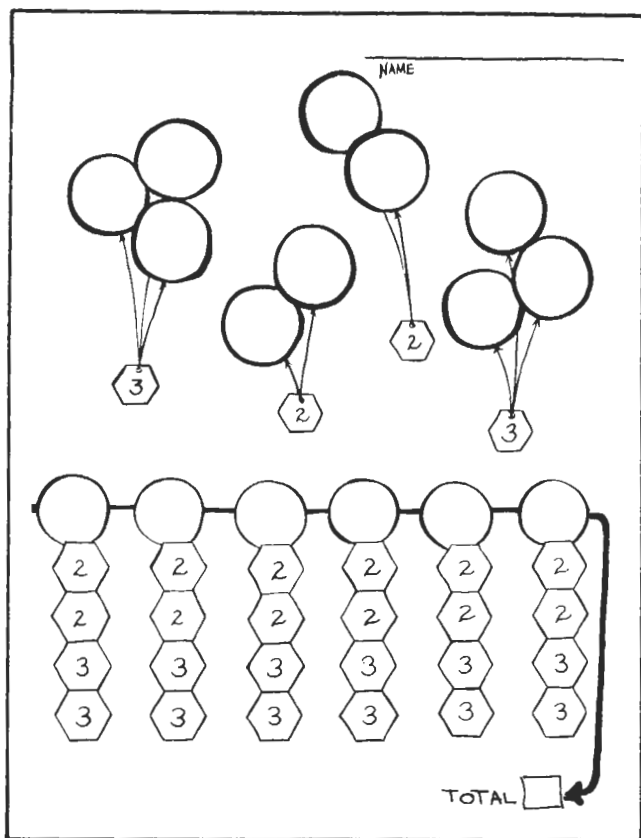
The same rules apply here. Balloons are now in bunches of 1, 2, 3, and 4. Record keeping differs only in that the learner colors the hexagons to indicate use of certain bunches.

Learners or teachers enter numbers in the circle-spaces. Each number that they can "do" gives them that many points toward the total "score." Or, to simplify the arithmetic involved, one point can be given for each number that can be done. The total "score" is entered in the box at the lower right.

A question to explore:

What is the highest possible score you can get?

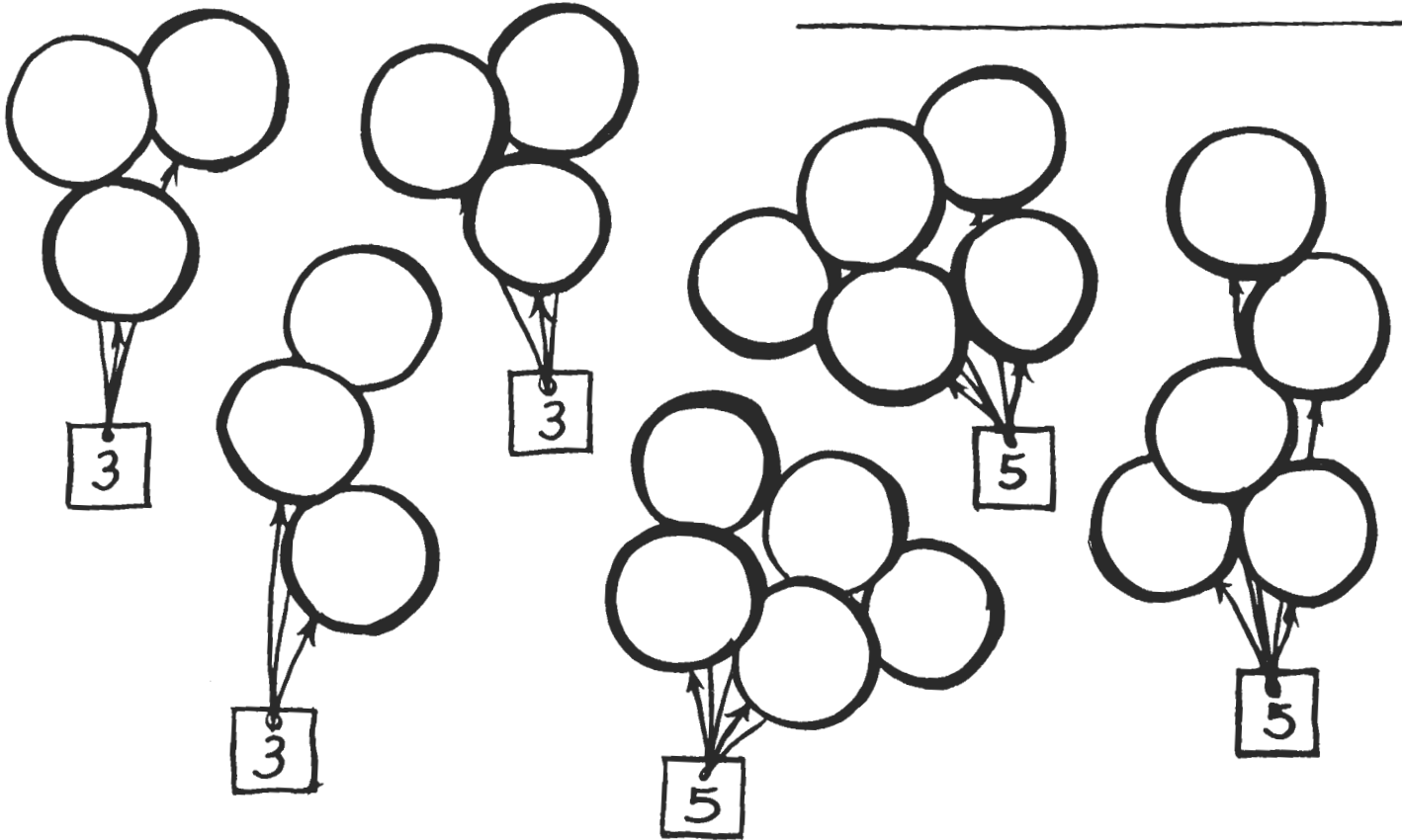




(For a more complete description of this activity, see page 19.)

The rules for placing the counters are the same. Balloons are now in bunches of twos and threes, affording practice with small multiples of those numbers.

A question to explore: What numbers, 1-10, can and cannot be done?



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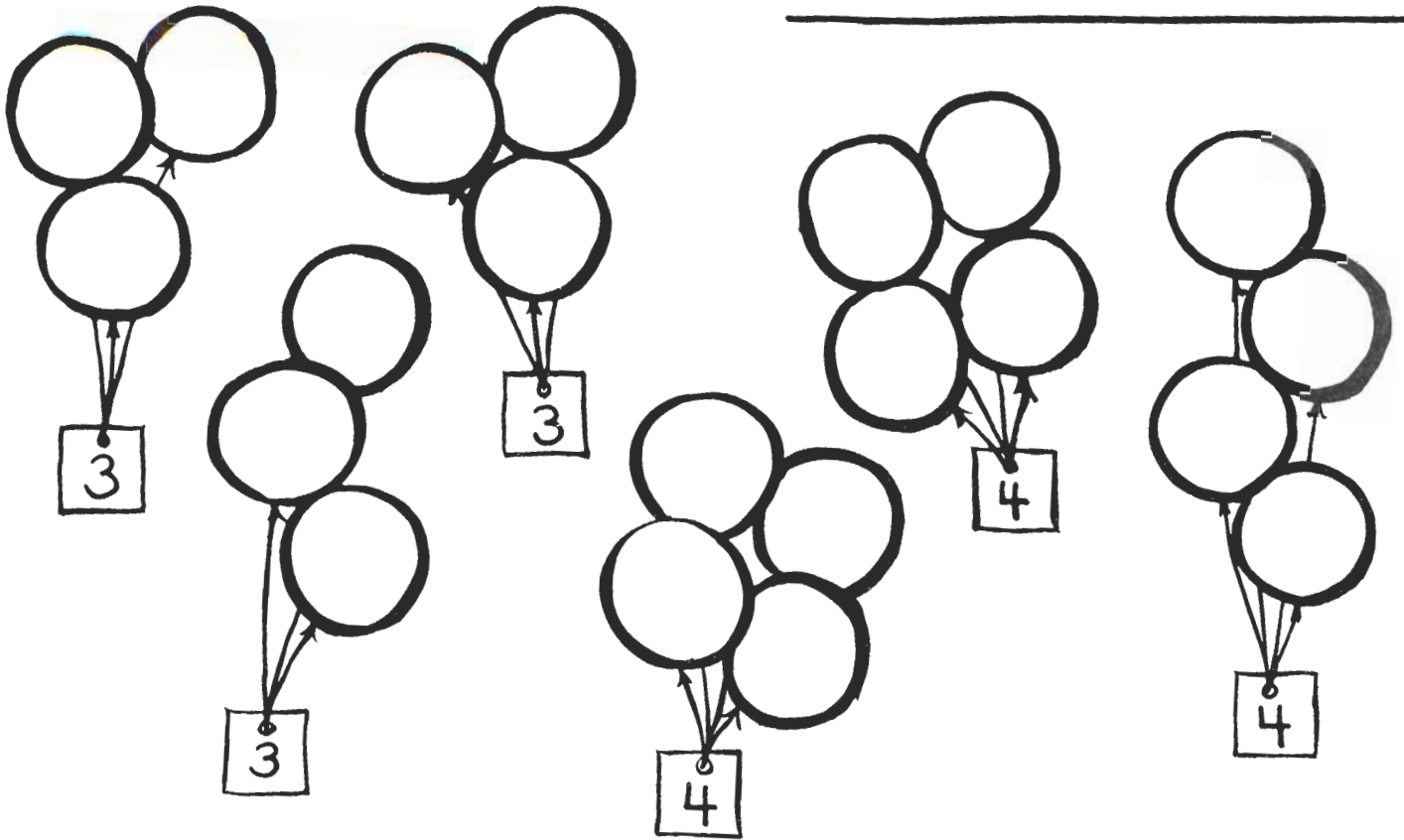
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|   |   |
|---|---|
|   |   |
| 3 | 4 |
| 3 | 4 |
| 3 | 4 |

|   |   |
|---|---|
|   |   |
| 3 | 4 |
| 3 | 4 |
| 3 | 4 |

|   |   |
|---|---|
|   |   |
| 3 | 4 |
| 3 | 4 |
| 3 | 4 |

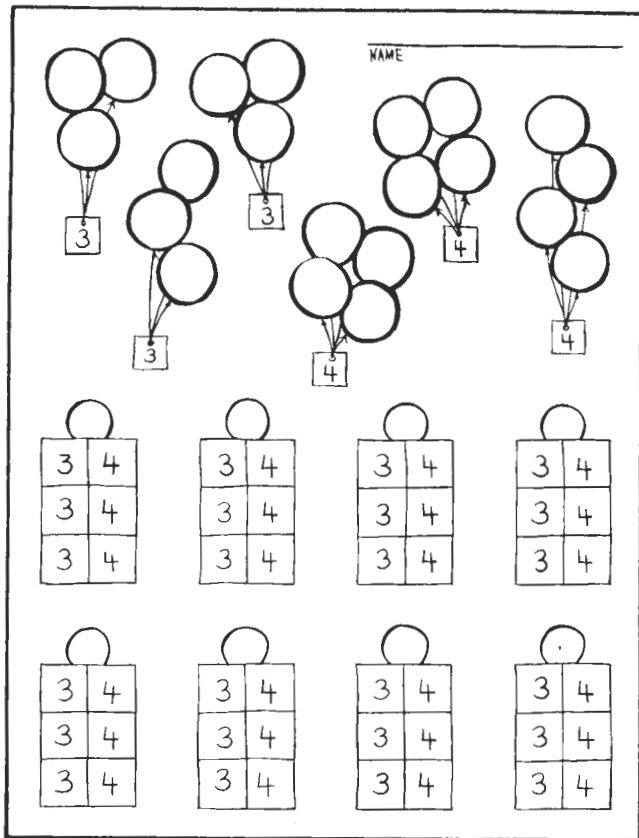
|   |   |
|---|---|
|   |   |
| 3 | 4 |
| 3 | 4 |
| 3 | 4 |

|   |   |
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|   |   |
| 3 | 4 |
| 3 | 4 |
| 3 | 4 |

|   |   |
|---|---|
|   |   |
| 3 | 4 |
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| 3 | 4 |
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| 3 | 4 |

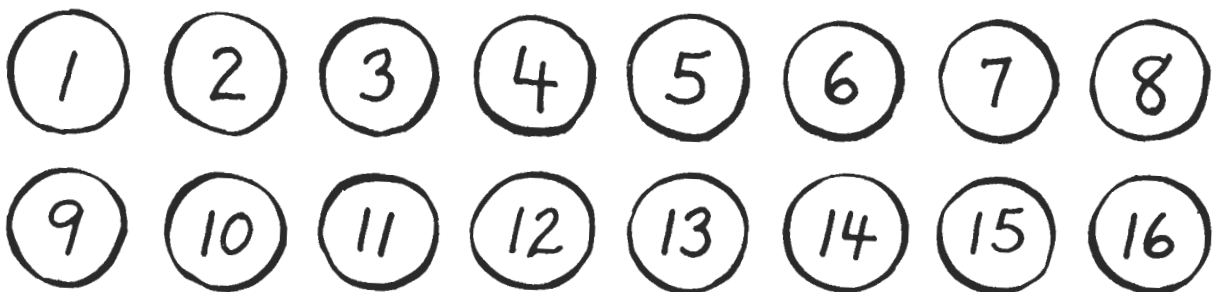
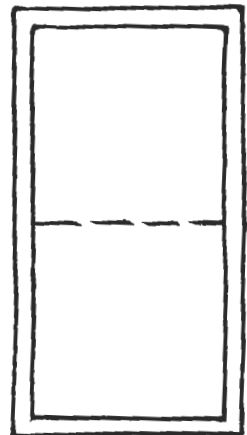
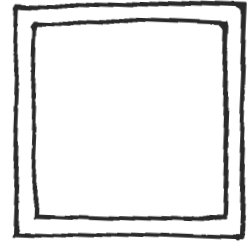
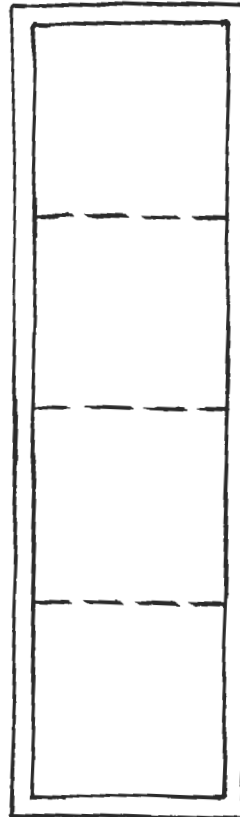
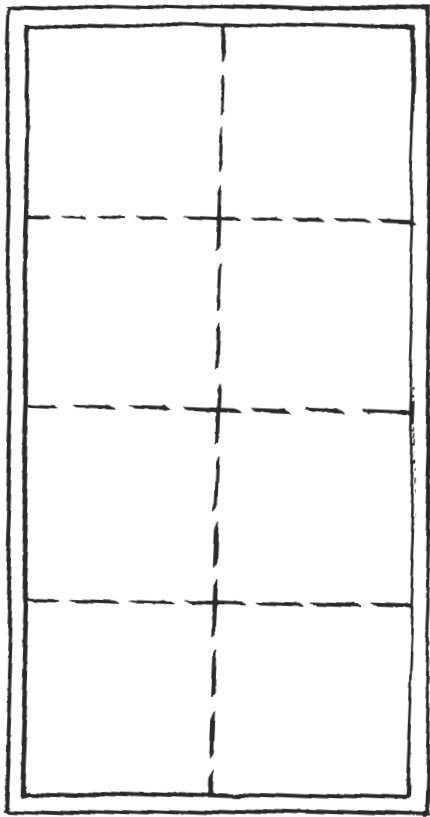
|   |   |
|---|---|
|   |   |
| 3 | 4 |
| 3 | 4 |
| 3 | 4 |



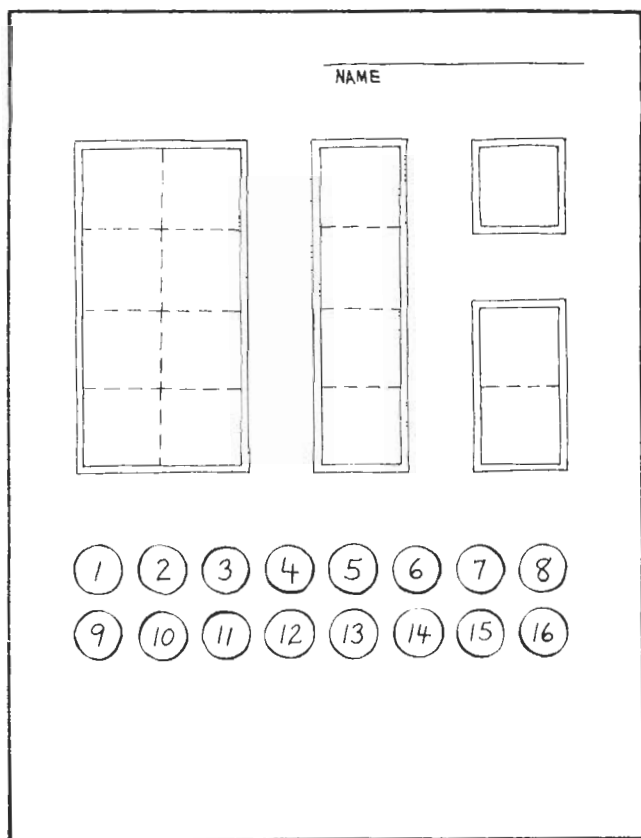
(For a more complete description of this activity, see page 19.)

The same rules apply for placing counters on the balloons. Balloons are now in bunches of threes and fours.

A question: What numbers, 1-21, can and cannot be done?







The name of this activity is "windows and panes." There are 4 "windows"—one with 1 "pane," another with 2 "panes" and others with 4 and 8 panes.

The children will need counters.

The rule is that if you put a counter on one pane in any window, you must put a counter on all panes in that window. Also, you must put only 1 counter on any pane.

The record is kept below:

✓  
① means "I can do it."

⊗ means "I can't do it."

✓  
⑮ } ⑯ squiggle means "There aren't any more panes."

The completed record would show that there is a window or combination of windows for all numbers 1 thru 15—the total number of panes.

A special characteristic of this selection of windows with 1, 2, 4 and 8 panes is that there is a combination (or single window) for each number 1 thru 15—and only 1 combination. If you know the number of counters, then you know which windows are used.

A unique relationship in the series of windows can be demonstrated by having 2 children play a game of "one more." If Sally and Harry are playing and Sally goes first, her task is to put 1 counter on the sketch.

Then Harry puts on one more. He can do it only by moving Sally's counter.

Then Sally puts on a third counter—"one more," the name of the game—on the 1-pane windows.

Harry's job is to put on a fourth counter. He must move all three over to the 4-pane window and add a counter.

Sally adds one on the 1-pane window.

Harry moves that one down and adds one more.

Sally adds one in the 1-pane window.

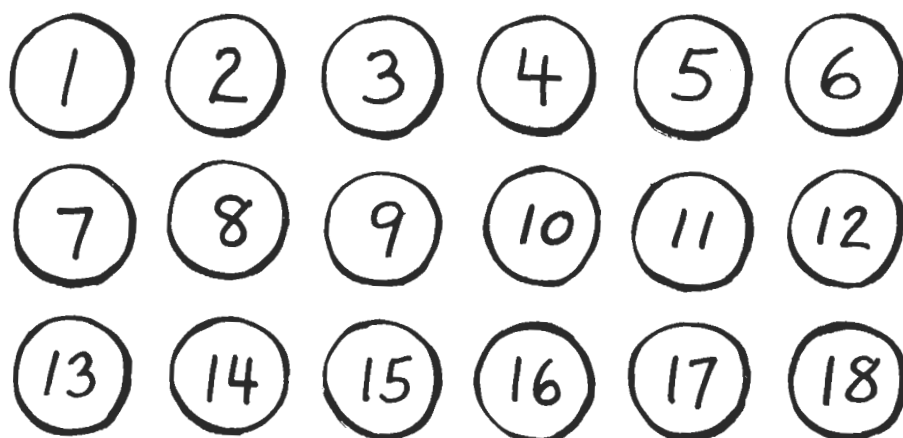
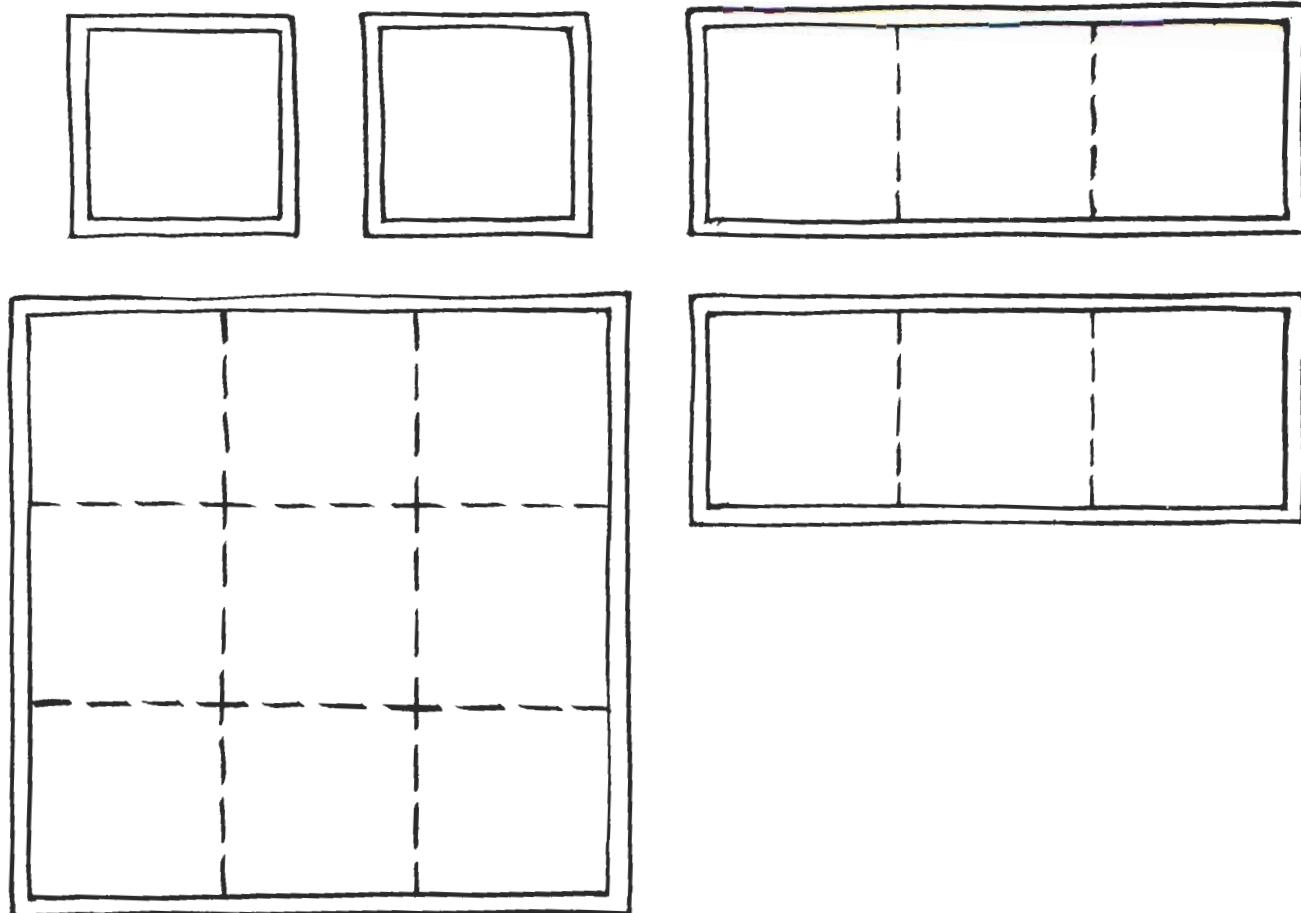
Harry's task: one more.

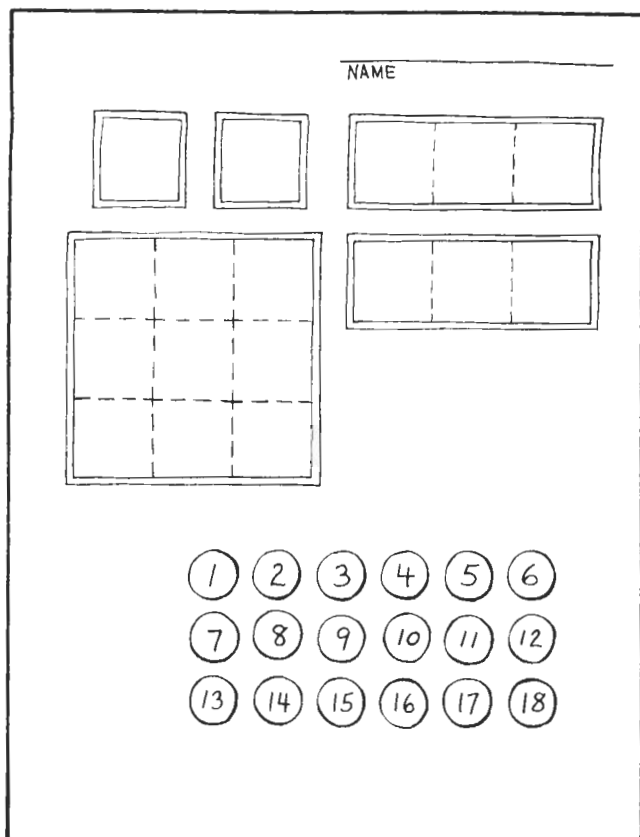
He must move all the counters to the large window and add one more for a total of 8.

About this time Harry has a strong feeling that he is doing all the work while Sally just keeps adding her counter to the 1-pane window.

And everyone involved has a "first encounter" with the "binary system"—after 1, each larger number is twice the next small number:

|   |         |         |         |              |
|---|---------|---------|---------|--------------|
| 1 | 2       | 4       | 8       | 16           |
|   | (2 x 1) | (2 x 2) | (2 x 4) | (2 x 8) etc. |





Al must move them over to the open 3-pane window and add one more to fill it.

Sally and Harry each put a counter on a 1-pane window.

Al—well, Al is doing all of the moving of the counters.

The selection of windows—1, 3, 9—starts with 1 and then each number is 3 times the next smaller number . . . and there are two windows of each kind:

|       |       |       |        |    |
|-------|-------|-------|--------|----|
| 1     | 3     | 9     | 27     | 81 |
| (3x1) | (3x3) | (3x9) | (3x27) |    |

More “windows and panes.”

The children need counters.

Again, the rule is: If you put a counter on a “pane,” you must put a counter on each pane in that “window” . . . and only one counter on any one pane.

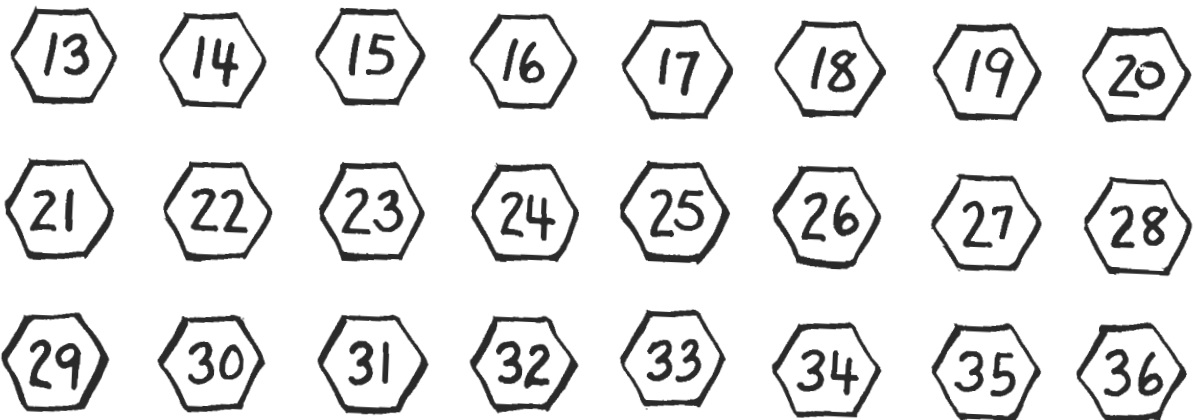
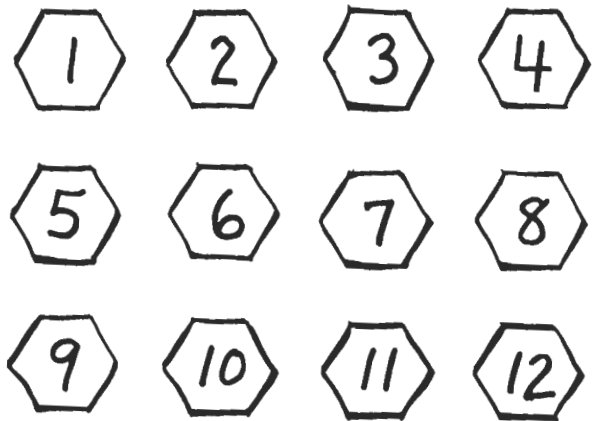
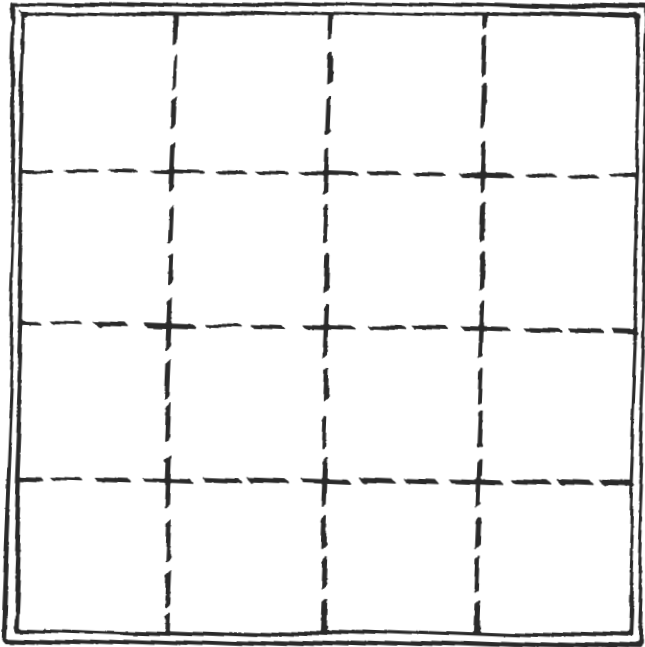
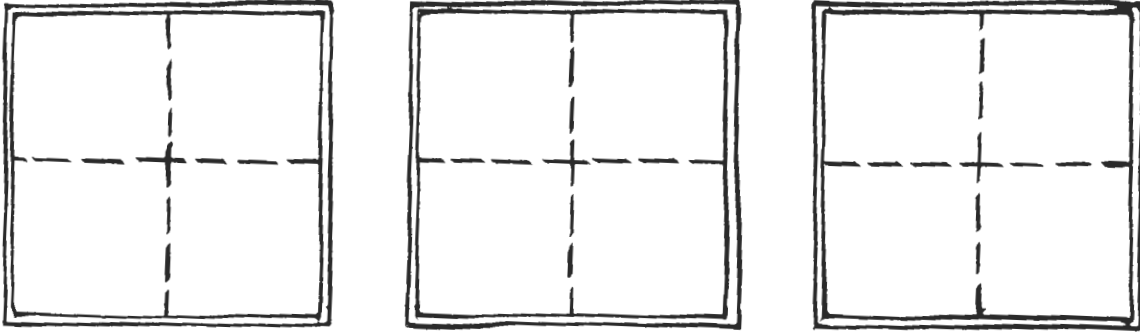
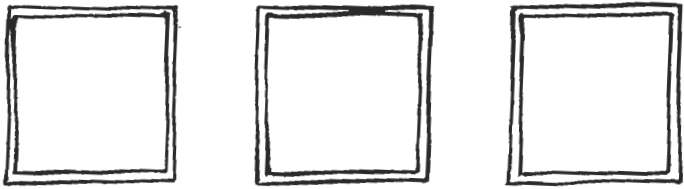
There is a way to put on all counters 1 thru 17. As on the previous page, there is but a single way to put on any given number—except when there is an option to use either one of the 1-pane or 3-pane windows. Counters can be put on one or the other.

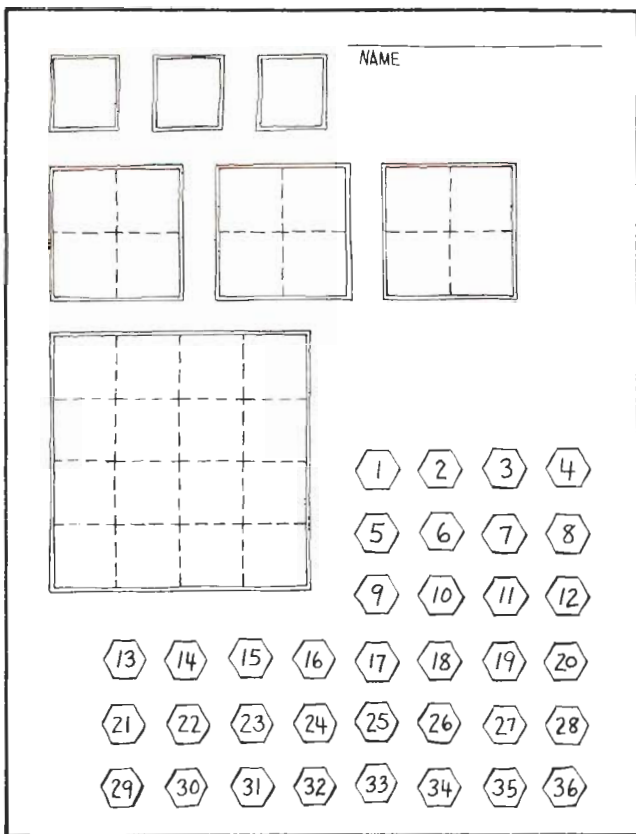
If Harry and Sally were to use this diagram for the game of “one more” described on the previous page, they would find a more equal division of labor.

They might persuade a third player to join the game—Al, perhaps. Sally plays first—on a 1-pane window, Harry plays next—adding one to the 1-pane window.

Al must move those over and add one to fill a 3-pane window.

Sally and Harry each put a counter on a 1-pane window.





Harry, Sally and Al repeat their first plays. Betty must move all three to another 4-pane window and add her "one more."

At the end of the next round, Betty must move all 15 counters to the largest window and add her "one more" . . . and she continues to do all the work of moving counters.

This selection of windows with 1, 4 and 16 panes starts with 1, and each number is 4 times the next window . . . and there are 3 of each kind:

|   |         |         |          |
|---|---------|---------|----------|
| 1 | 4       | 16      | 64       |
|   | (4 x 1) | (4 x 4) | (4 x 16) |
|   |         | etc.    |          |

More "windows and panes."

The children will need counters.

Again the rule is: if you put a counter on a "pane," you must put a counter on each pane in that "window" . . . and only 1 counter on any window.

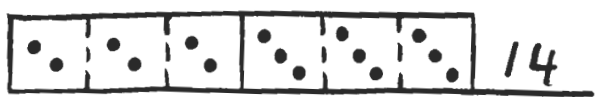
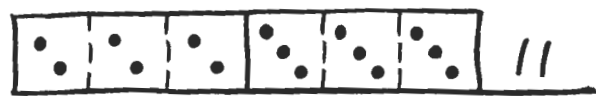
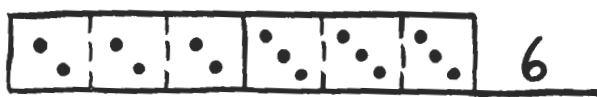
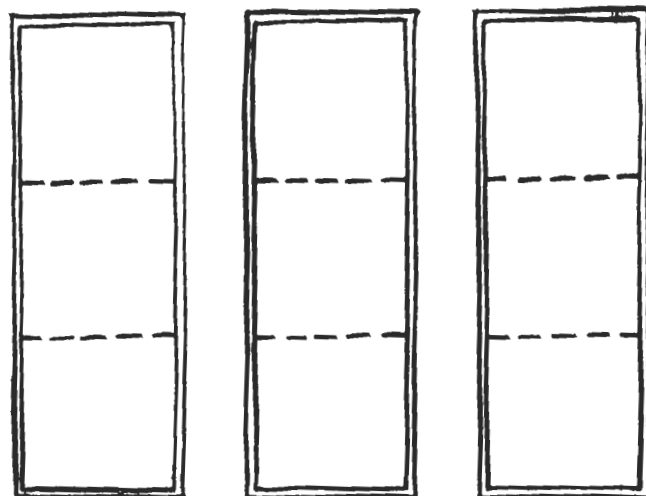
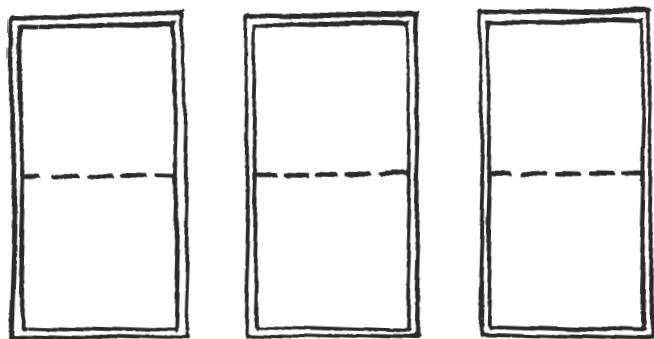
There is a way to put on all numbers of counters 1 thru 31. As on the previous pages, there is but a single way to put on a given number—except when there is an option to use different windows of the same size.

If Harry and Sally and Al were to use this diagram for the game of "one more" described on the previous pages, they would find the labor more equally divided.

They might persuade a fourth person to join the game—Betty, perhaps.

Harry, Sally and Al, playing first, second and third, each would put a counter on a 1-pane window.

Betty must move all three to a 4-pane window and add her "one more."



NAME \_\_\_\_\_

|  |  |  |  |  |  |
|--|--|--|--|--|--|
|  |  |  |  |  |  |
|--|--|--|--|--|--|

|   |    |
|---|----|
| 1 | 10 |
| 2 | 11 |
| 3 | 12 |
| 4 | 13 |
| 5 | 14 |
| 6 | 15 |
| 7 | 16 |
| 8 | 17 |
| 9 |    |

More "windows and panes."

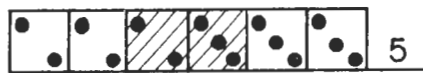
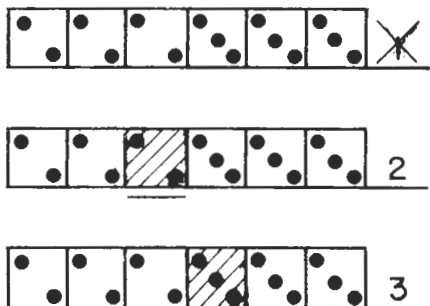
Children need counters.

The rule is: if you put a counter on a "pane," you must put a counter on each "pane" in that "window" ... and only 1 counter on any one window.

Rather than indicate simply those numbers that "can" and "cannot" be done, the combinations for those that can be done are indicated by coloring or shading in a diagram showing three 2's and three 3's.

Some, such as 6, can be done in more than one way—two 3's or three 2's.

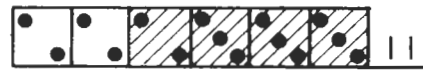
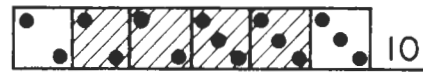
If in the record, the squares nearest the middle are shaded in, a pattern is revealed:



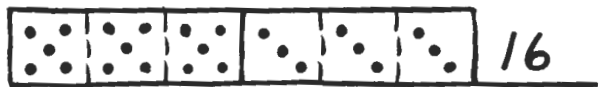
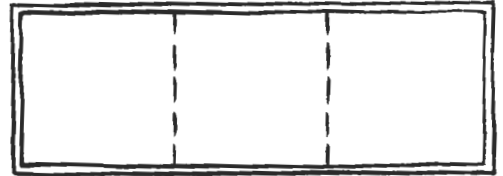
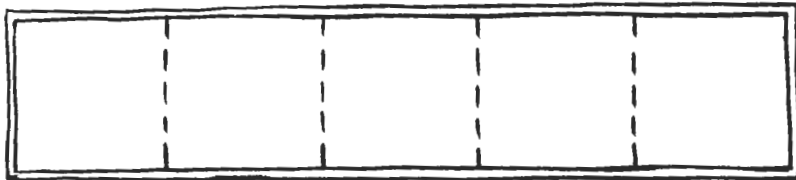
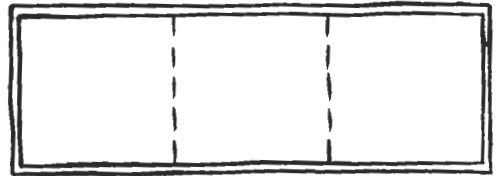
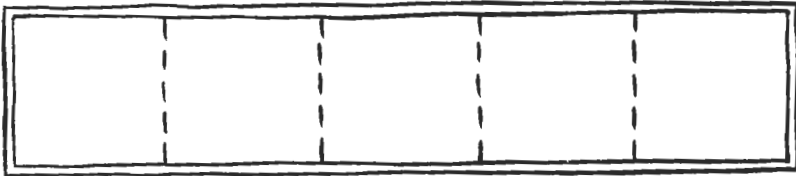
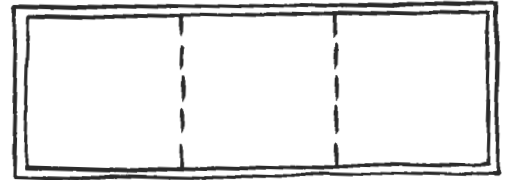
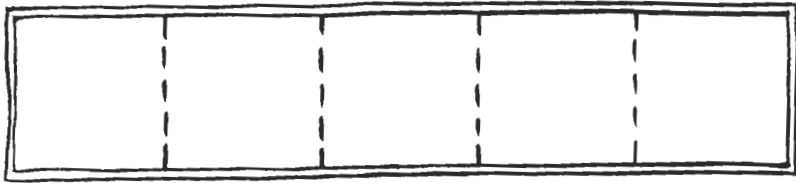
or



or



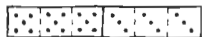
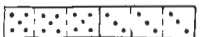
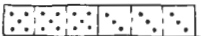
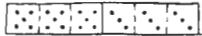
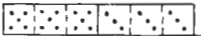
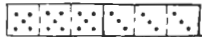
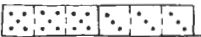
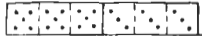
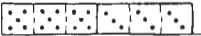
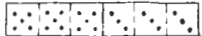
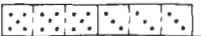
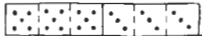
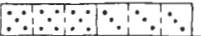
A combination for the "next larger number" can be seen as "trading a 2 for a 3"—shading one less 2 and one more 3.





NAME \_\_\_\_\_

|  |  |
|--|--|
|  |  |
|  |  |
|  |  |

|                                                                                      |                                                                                      |
|--------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
|  5  |  12 |
|  6  |  13 |
|  7  |  14 |
|  8  |  15 |
|  9  |  16 |
|  10 |  17 |
|  11 |  18 |

(See rules and directions on previous pages.)

No nice pattern emerges as in the previous activity except 5 and 6. If one less 5 is shaded in and two more 3's are shaded—the result is “one more.”

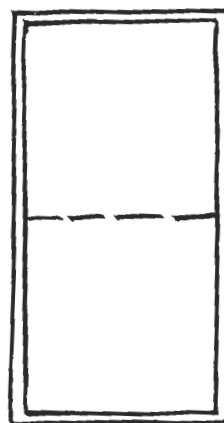
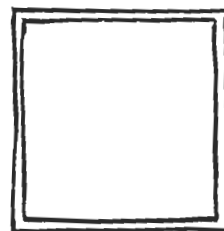
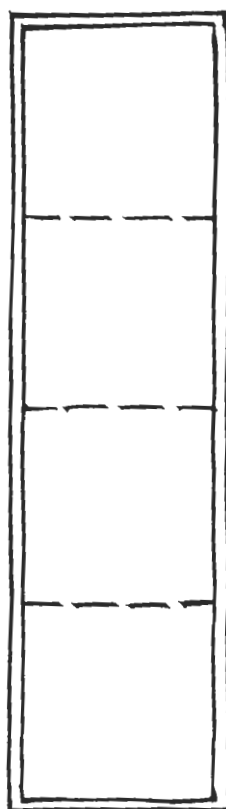
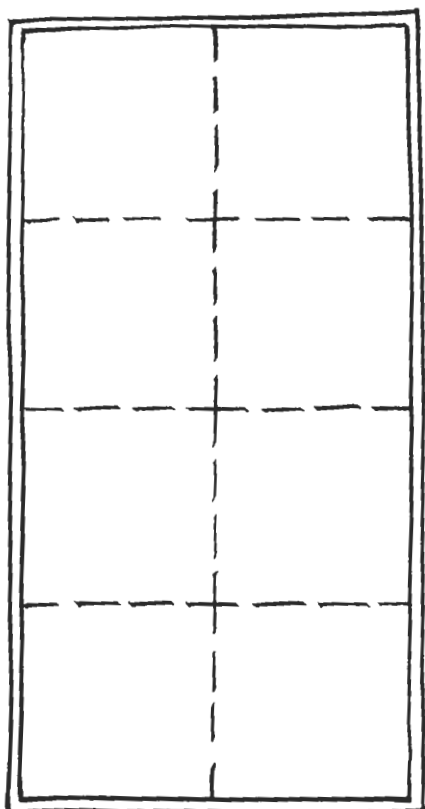


A similar “trade” occurred between 8 and 9



and between





8 4 2 1

8 4 2 1

8 4 2 1

8 4 2 1

8 4 2 1

8 4 2 1

8 4 2 1

8 4 2 1

8 4 2 1

8 4 2 1

8 4 2 1

8 4 2 1

8 4 2 1

8 4 2 1

8 4 2 1

NAME \_\_\_\_\_

Notice the columns of "1's" . . . one looped, one not looped . . . every other "1" is looped.

In the column of "2's," once it is started, two are looped, the next two are not looped, etc.

In the column of "4's," once it is underway, four are looped, and the next four are not.

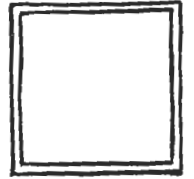
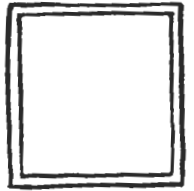
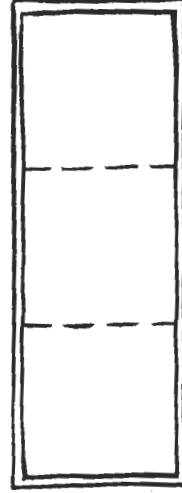
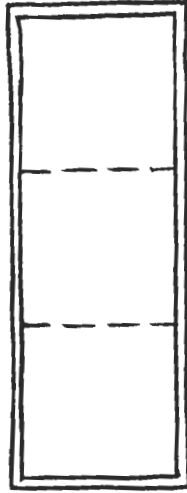
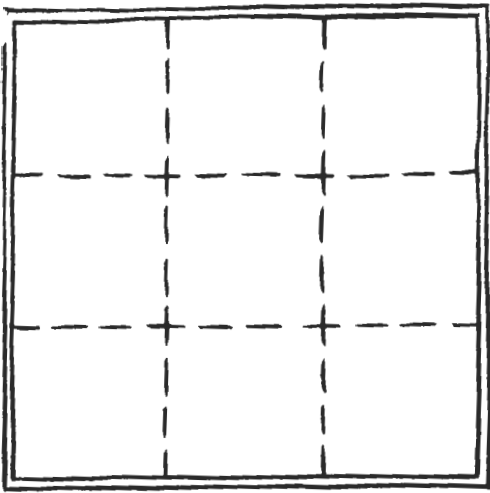
More windows and panes.

Rule: If you place one counter on any "pane," you must place one counter on each pane in that "window" . . . only one counter on each pane.

Records of combinations are indicated by "looping" or otherwise indicating the window's use.

A nice pattern emerges:

|   |   |   |   |   |
|---|---|---|---|---|
| 8 | 4 | 2 | 1 | 0 |
| 8 | 4 | 2 | 1 | 1 |
| 8 | 4 | 2 | 1 | 2 |
| 8 | 4 | 2 | 1 | 3 |
| 8 | 4 | 2 | 1 | 4 |
| 8 | 4 | 2 | 1 | 5 |
| 8 | 4 | 2 | 1 | 6 |
| 8 | 4 | 2 | 1 | 7 |
| 8 | 4 | 2 | 1 | 8 |



9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

NAME \_\_\_\_\_

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

9 3 3 1 1

More "windows and panes."

See previous page for rules.

PLEASE SELECT 3 BEANSTICKS:  
FAVOR DE ESCOGER 3 PALOS CON FRIJOLE:

---



PLEASE SELECT 3 BEANSTICKS:  
FAVOR DE ESCOGER 3 PALOS CON FRIJOLE:



PLEASE SELECT 3 BEANSTICKS: \_\_\_\_\_ NAME \_\_\_\_\_

A \_\_\_\_-STICK, A \_\_\_\_-STICK, AND A \_\_\_\_-STICK.

1 2 3 4 5 6 7 8 9 10

---

PLEASE SELECT 3 BEANSTICKS: \_\_\_\_\_

A \_\_\_\_-STICK, A \_\_\_\_-STICK, AND A \_\_\_\_-STICK.

1 2 3 4 5 6 7 8 9 10

Now, to the page! Ask the children to choose 3 beansticks. See that everyone records the kind of beansticks selected at the top and then proceed to check and cross out the numbers.

For those who are able, have them put two checks on any numbers that can be made in more than one way.

The children may decide to choose 2 or 3 sticks that are the same. Let them explore these possibilities. Some interesting questions:

- (1) Can you pick up 3 sticks to show the sums of 2, 4, 6, 8 and 10?
- (2) Here are the sums possible. 1, 3, 4, 5, 6, 8, 9. What sticks were used?

Opportunity for practicing addition combinations is the thrust of this page. Three beansticks are selected and used singly, in pairs, or all together in an attempt to show the sums from 1 to 10.

In preparation for the page, you might conduct the following activity. From a selection of beansticks showing 1, 2, 3, 4, and 5 beans ask the group to select 3 different ones. Using the sticks chosen, have them try to show you 1 bean, 2 beans, 3 beans, etc. up to the number of beans they have. Suggest a record be used to keep track of the results. On the board write

1 2 3 4 5 6 7 8 9 10

Place a check on top of the numbers they can show and cross out the ones they can't.

#### Example

a 1-stick      a 3-stick      a 5-stick

✓  
1

✓  
2

✓  
3

✓  
4

✓  
5

✓  
6

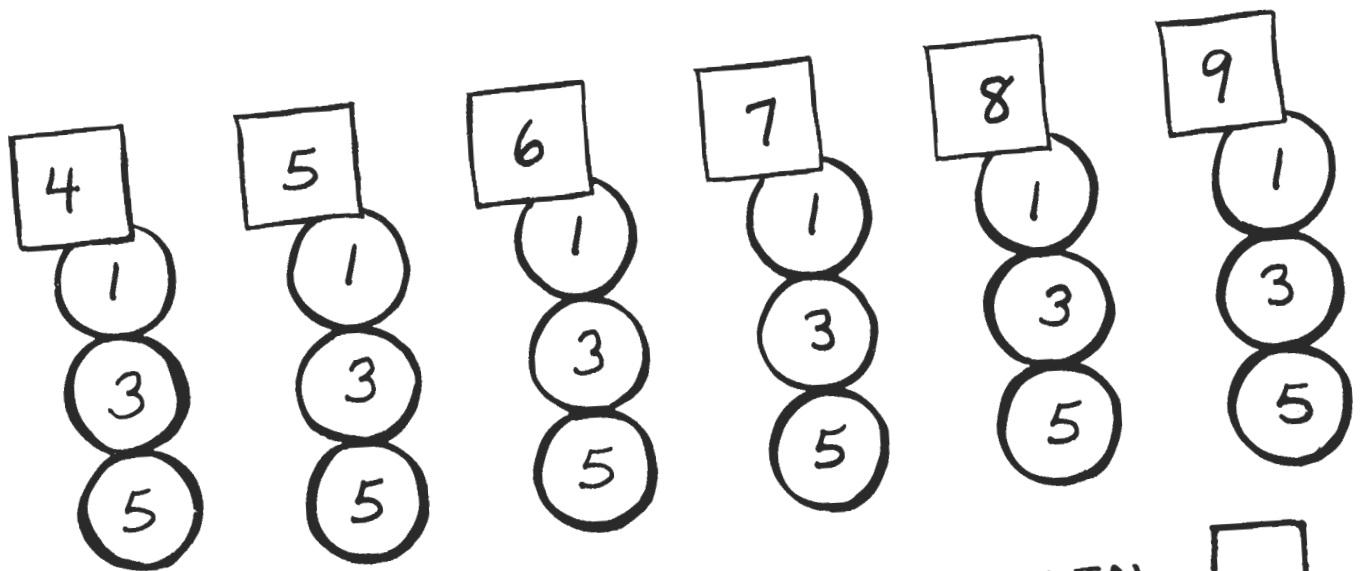
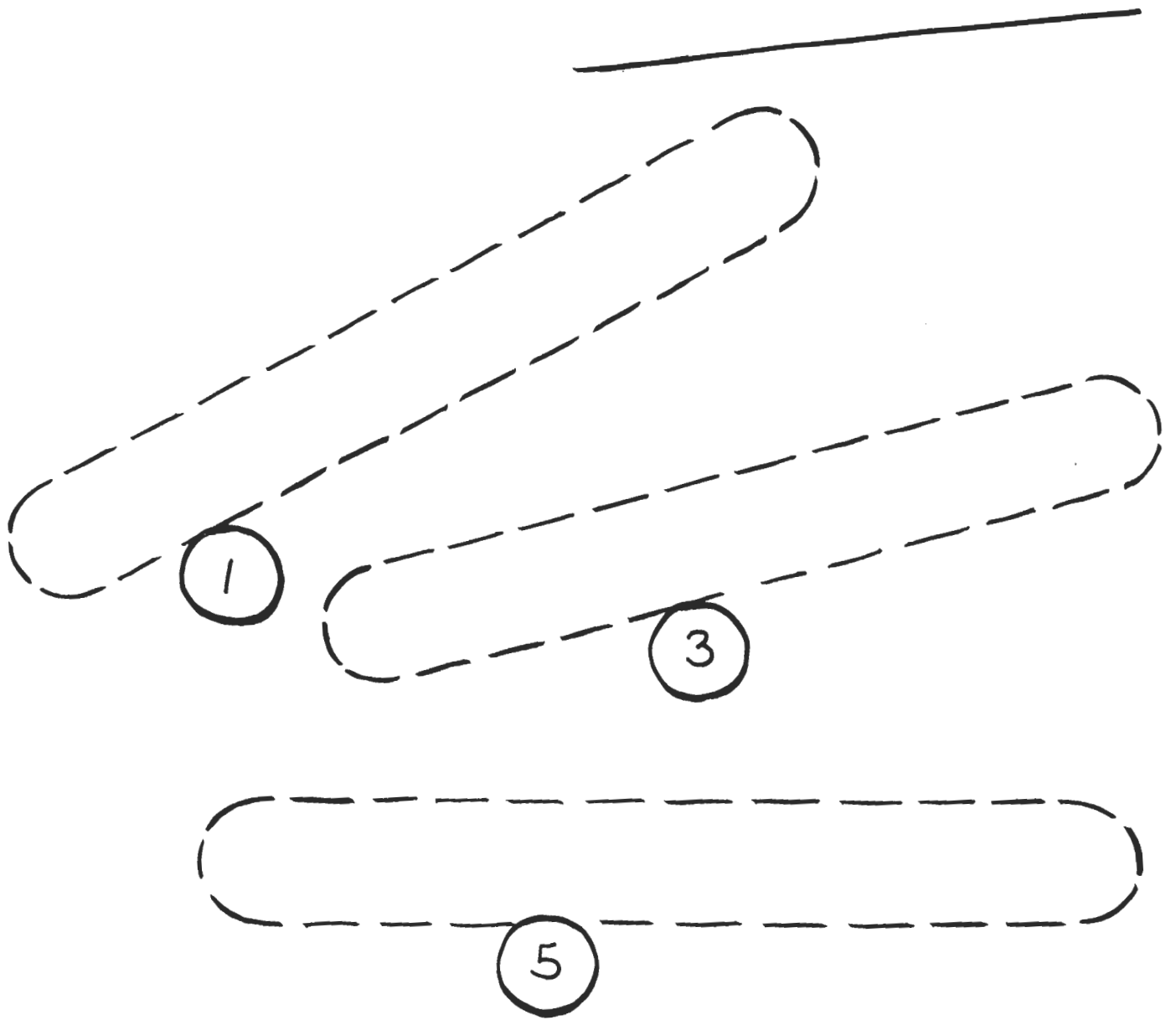
✓  
7

✓  
8

✓  
9

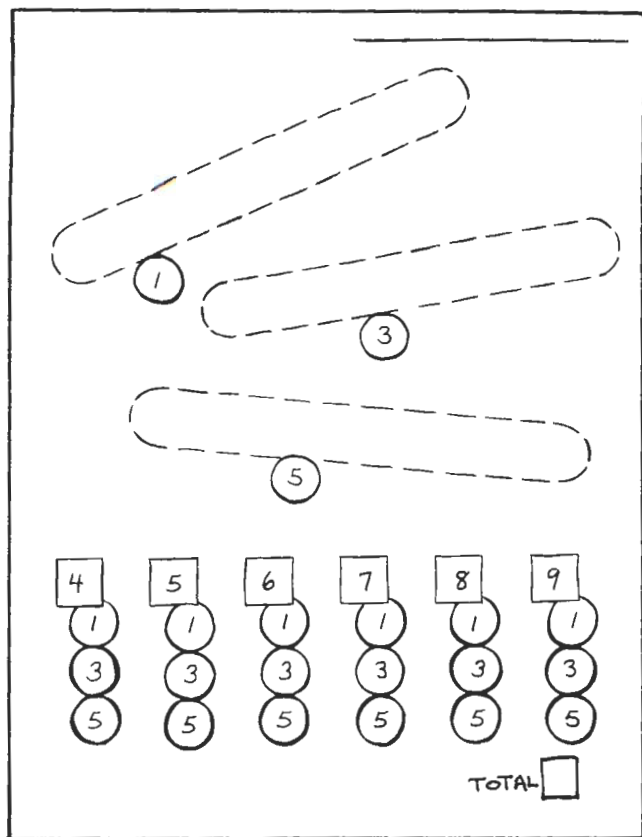
✓  
10

“That’s all the beans there are.” ↑

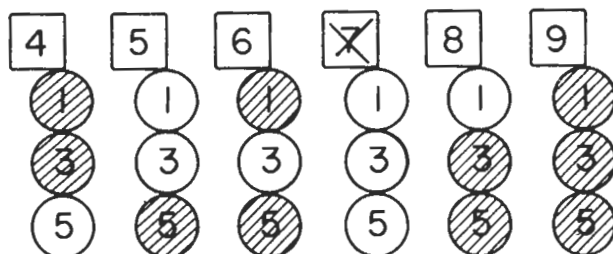


TOTAL





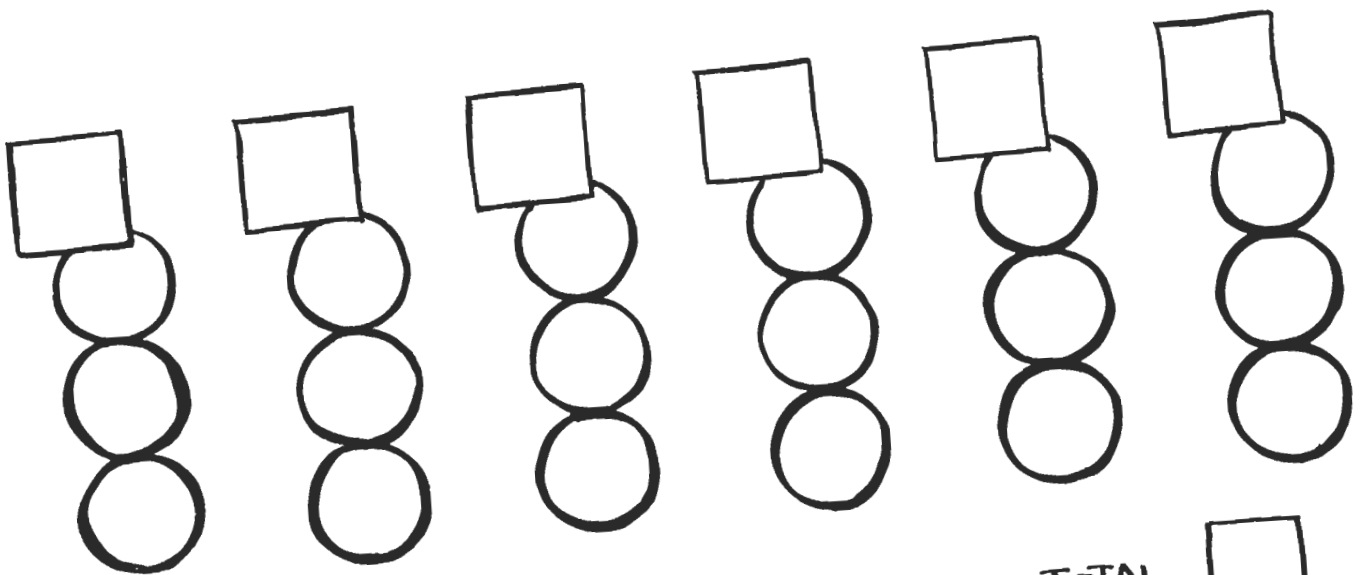
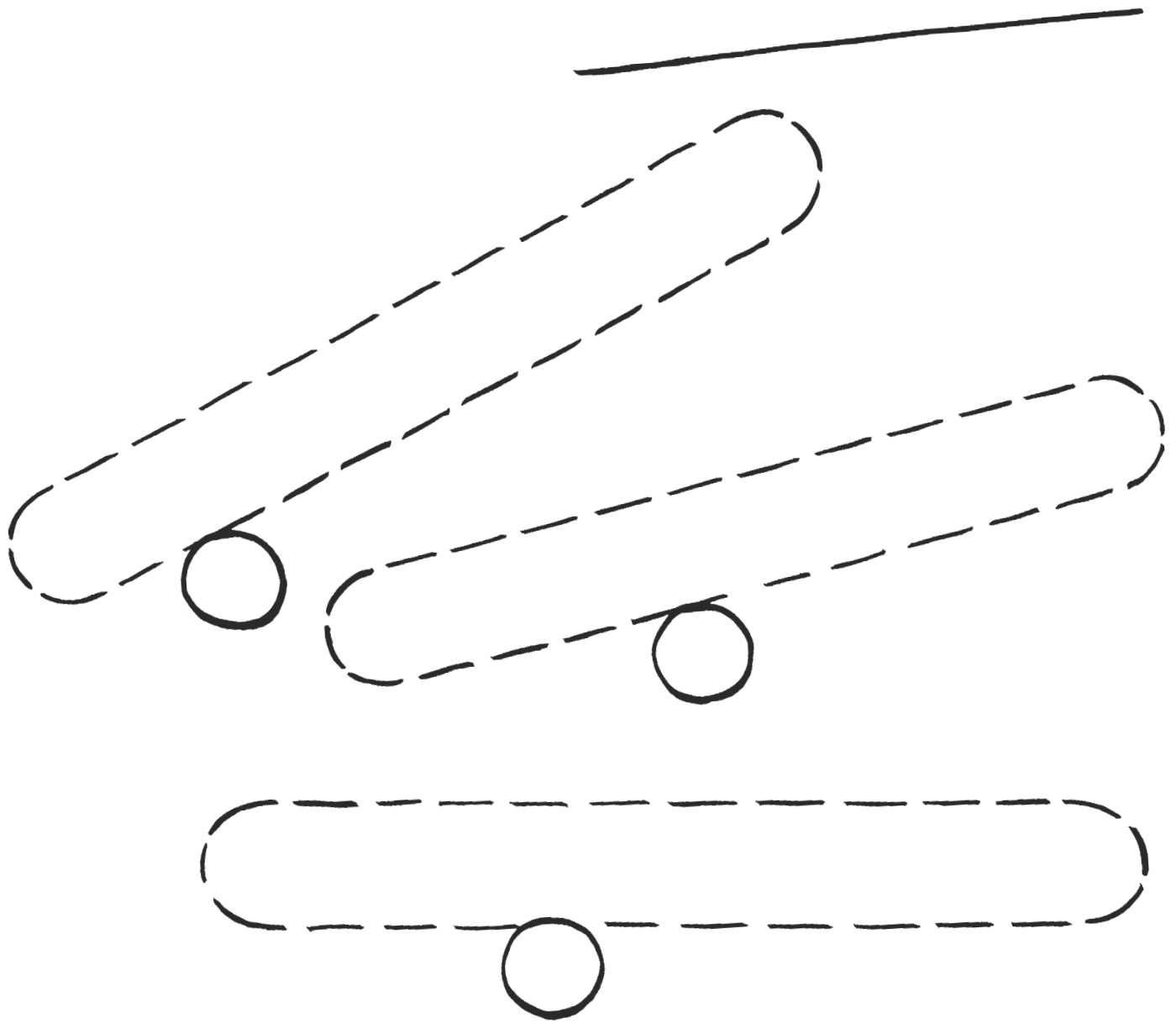
If a number cannot be done, it should be crossed out. Several scoring methods can be used. One point can be given for each success, or each number done can contribute that many points to the score. Other scoring methods also can be used. The completed record for this page would be:



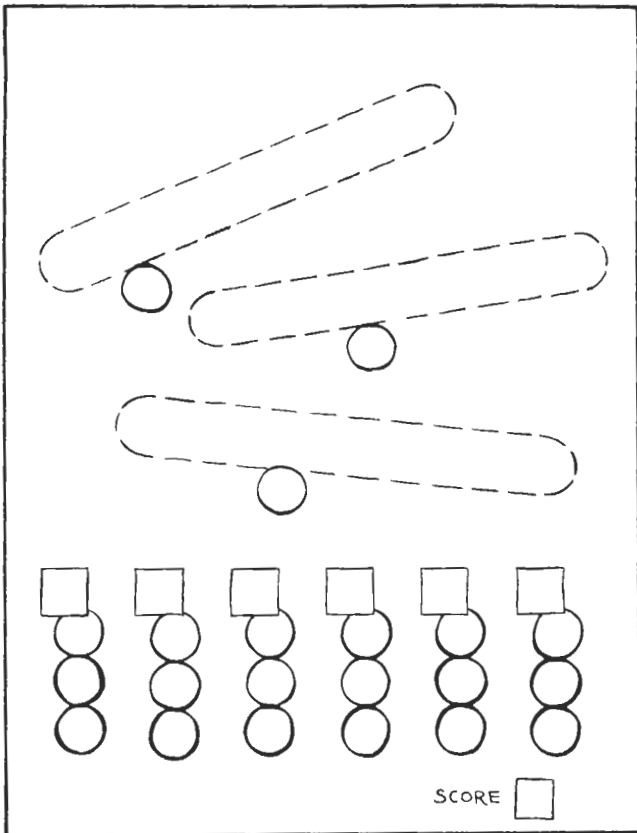
Frames are provided in which the learner can place beansticks. This task page specifies use of a 1-stick, a 3-stick, and a 5-stick. Numbers in the circles refer to numbers of beans on the sticks. Numbers in the squares specify numbers of beans to be picked up. The question is: Can you pick up a stick or a combination of sticks that have on them the number of beans indicated in the square? For example:

4 asks the learner to pick up four beans. This can be done using the 1-stick and the 3-stick, and the record of this result can be made as follows:





TOTAL



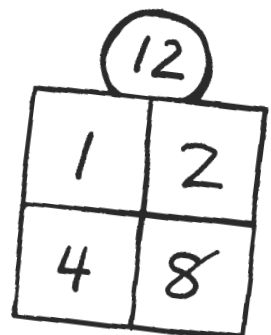
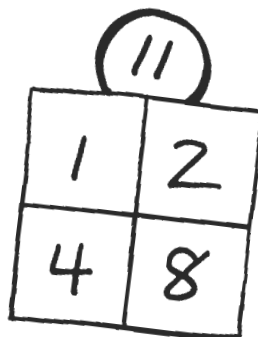
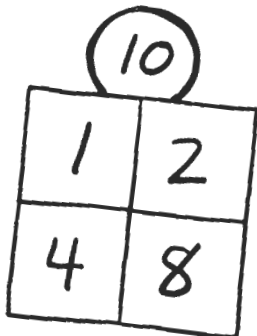
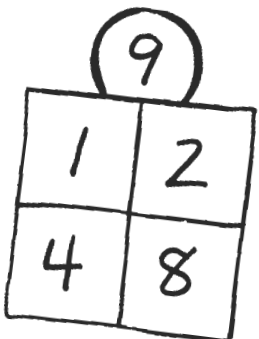
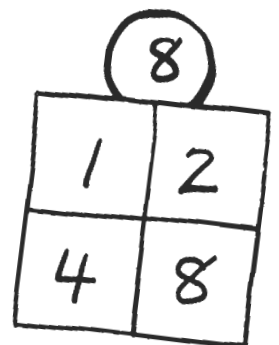
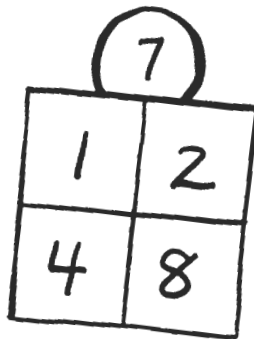
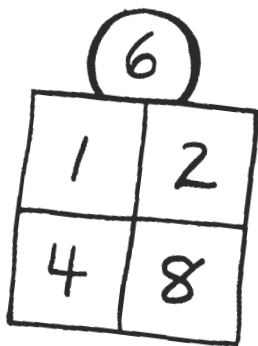
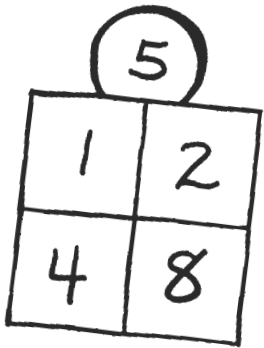
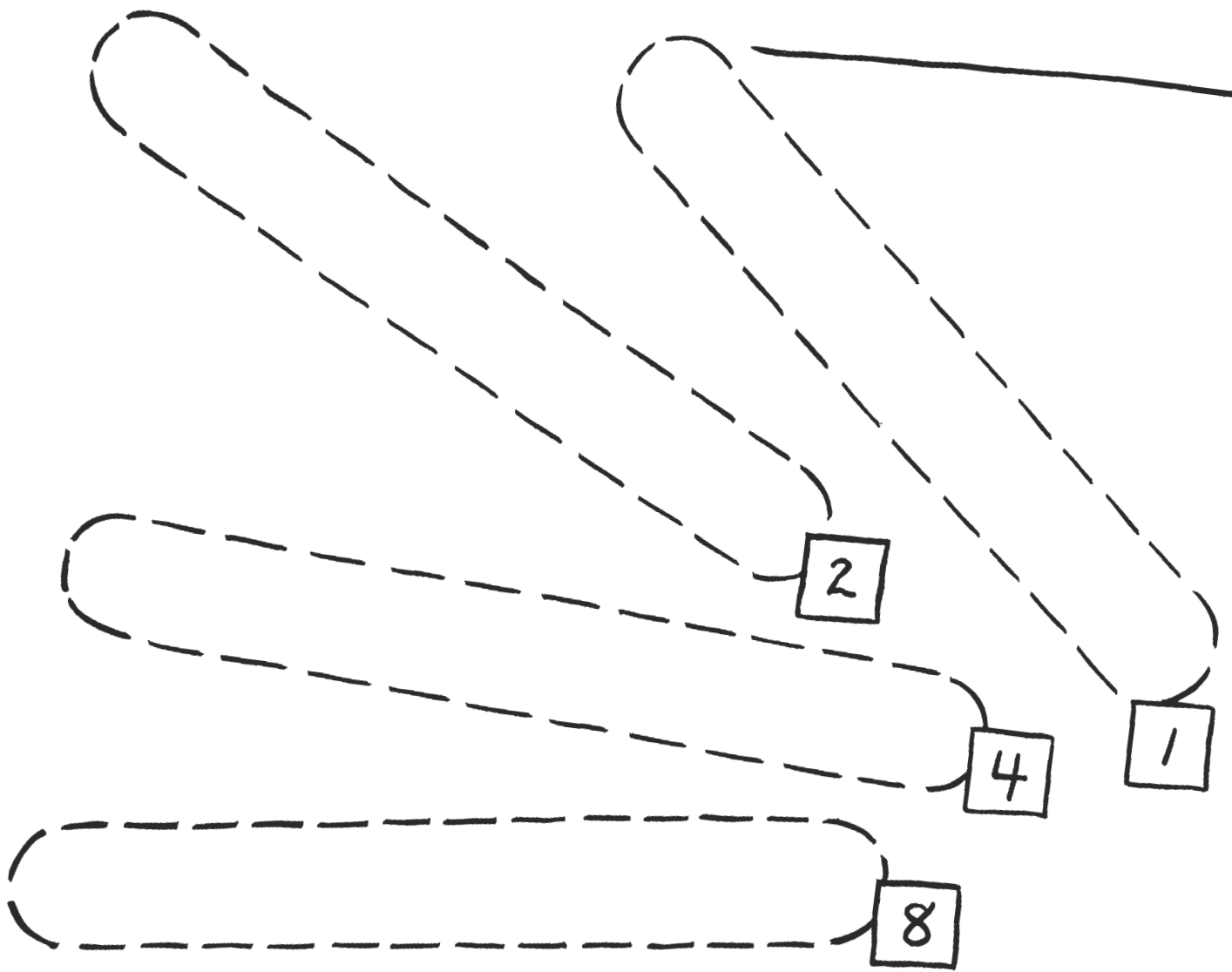
Learners can now choose their own three beansticks. Their choice should be recorded in the circles next to the beanstick frames. Each set of circles in the lower half of the page should be filled in with the three numbers chosen.

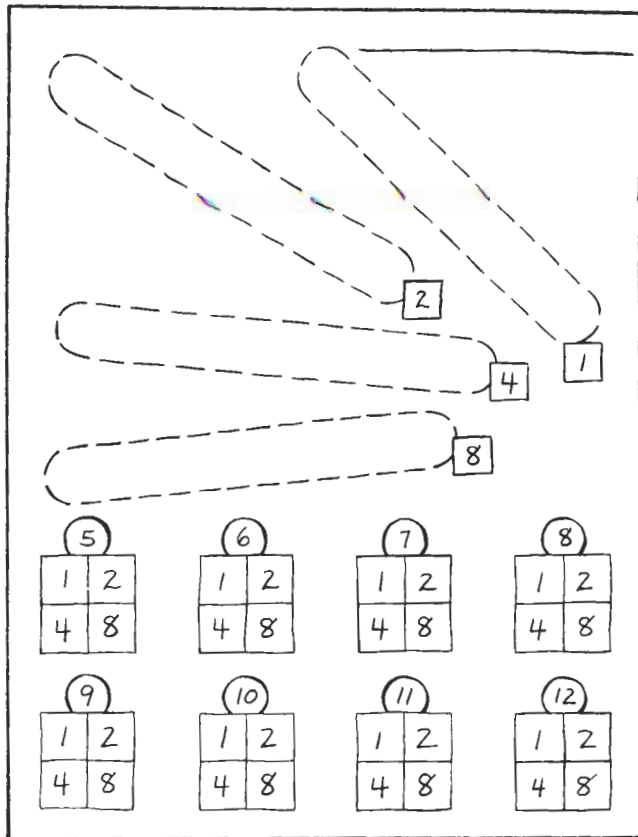
Learners can design their own experiments by choosing numbers to try and writing them in the squares. Each success can count for one point toward the "score," or other methods of scoring can be devised. Numbers that can be made in two ways might be checked, and extra points given toward the score.

Children may want to choose two or three sticks that have the same number of beans. Let them explore the possibilities.

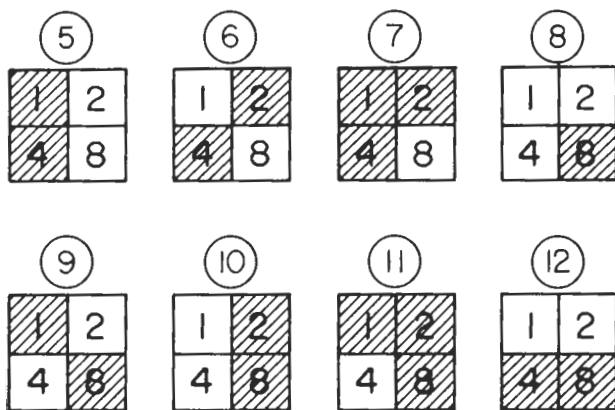
A question to explore:

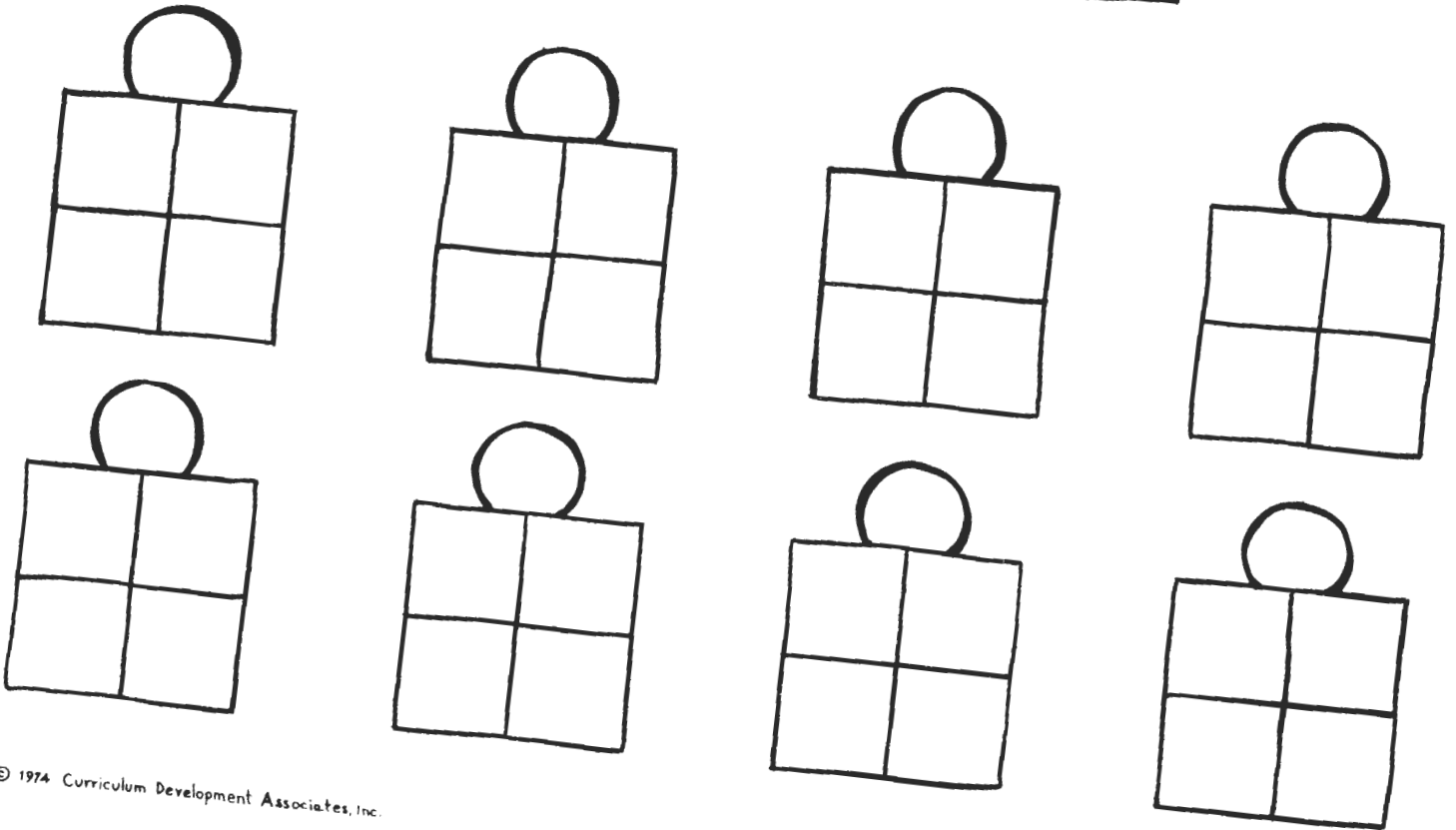
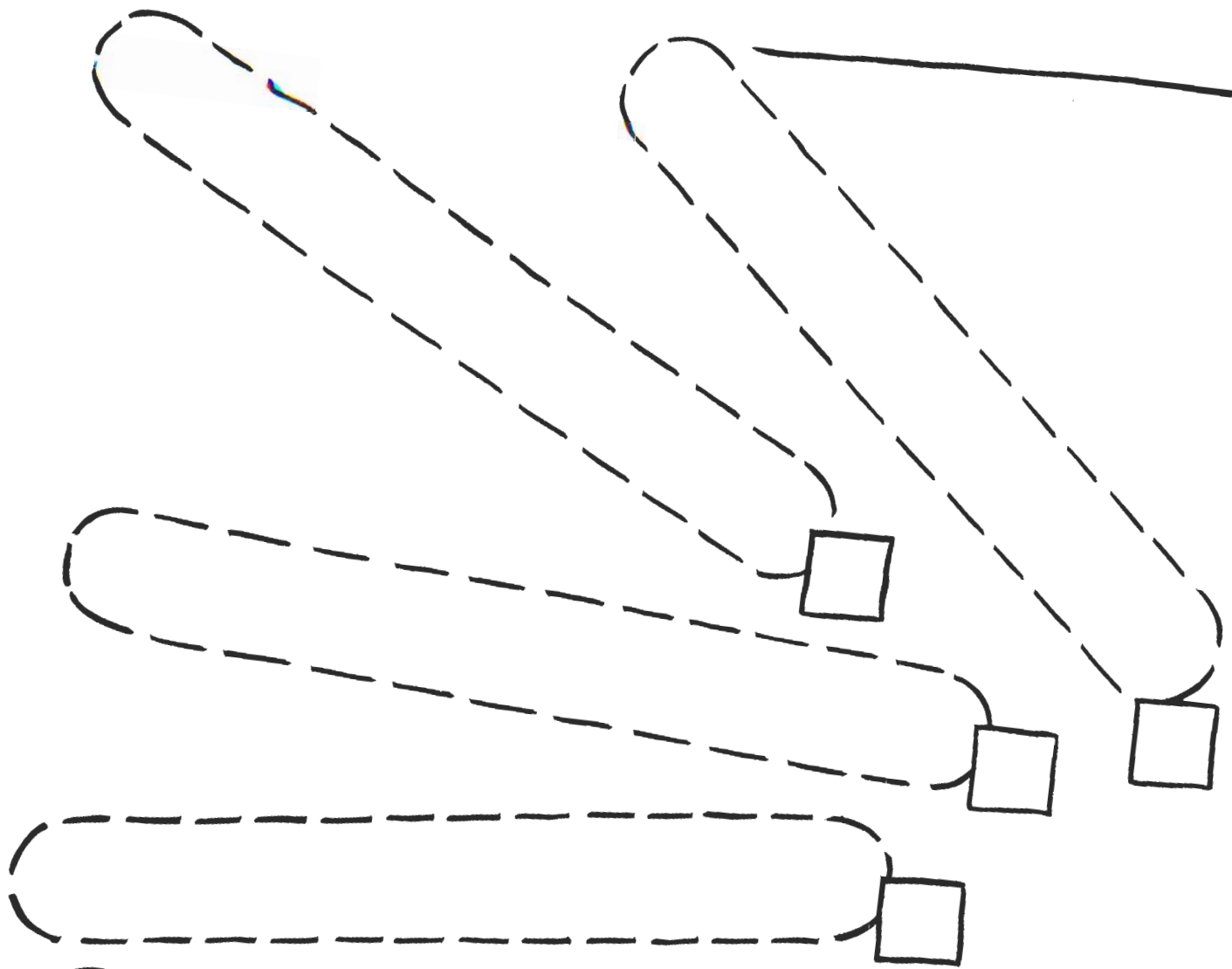
If the only possible sums are 2, 4, 6, 8, and 10, what three sticks were used?

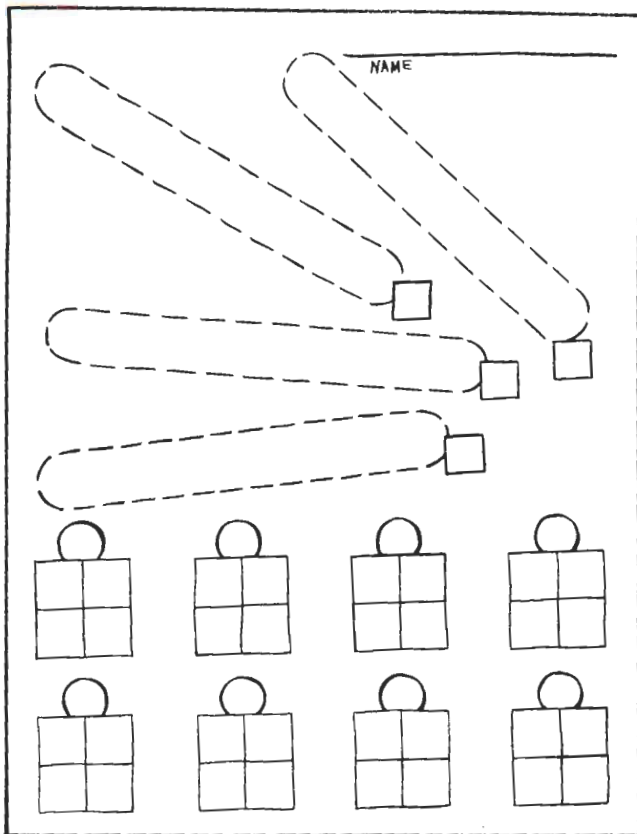




The scope of the investigation is widened to include four beansticks and the possibility of larger sums. Again, the question is: Can you find a stick or a combination of sticks that will have each number of beans specified in the circles? A record of how each is done can be made by coloring in the appropriate boxes. A completed record would be:





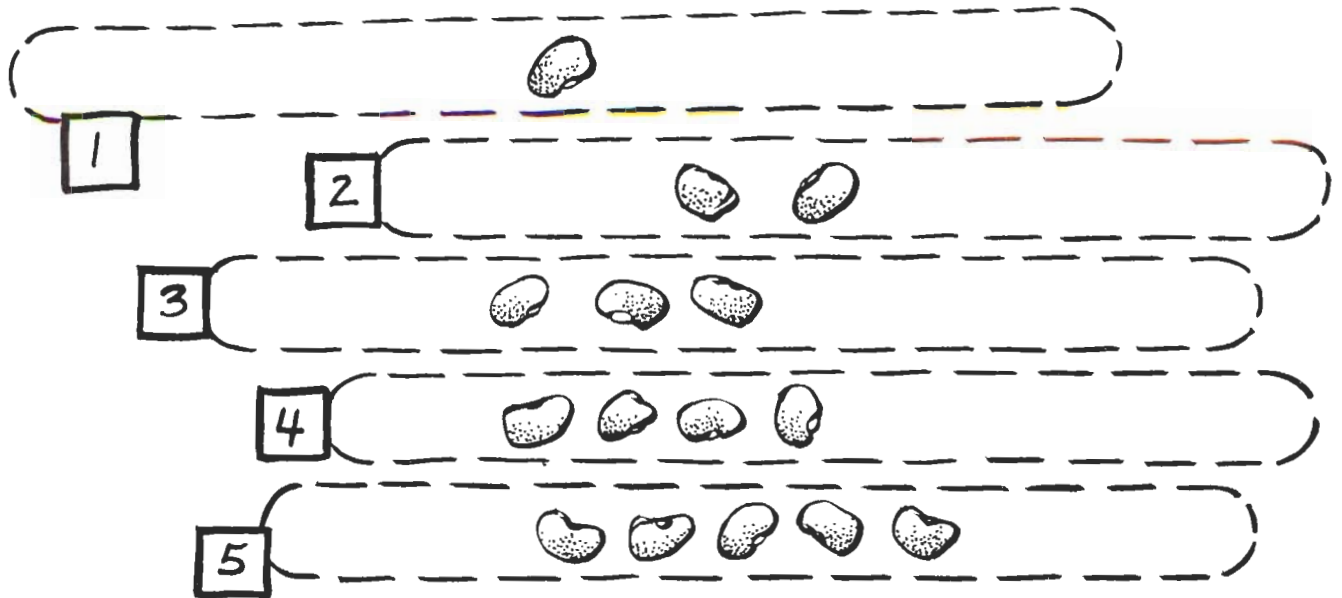


This report form gives the learner the opportunity to make his own investigations using four sticks. He selects four beansticks and records his choices in the squares next to the beanstick frames. These numbers are also entered in the squares in the lower part of the page, as on page 30 C.

Learners can design their own experiments by choosing numbers to try and writing them in the circles. Numbers that can be made in two ways might be checked, or a record of the alternate solutions kept in some way. Children may want to choose two or more sticks that have the same number of beans. Let them explore the possibilities.

A question to explore:

These sums are possible: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. What sticks were used?



| 4 |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |

| 5 |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |

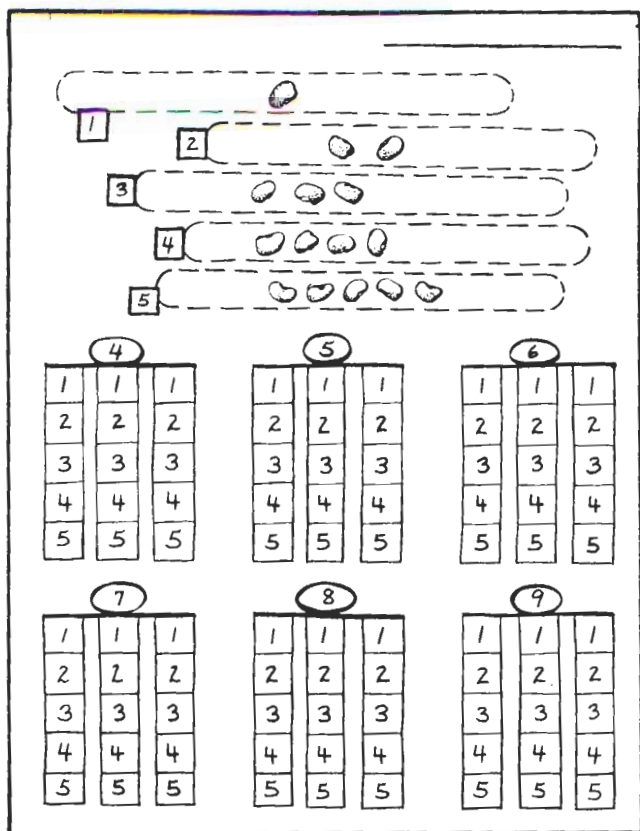
| 6 |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |

| 7 |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |

| 8 |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |

| 9 |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |





|   |   |   |
|---|---|---|
| 6 |   |   |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |

You might suggest that the learners start by trying for one stick, then for combinations of two sticks, then three sticks, then four sticks, etc.

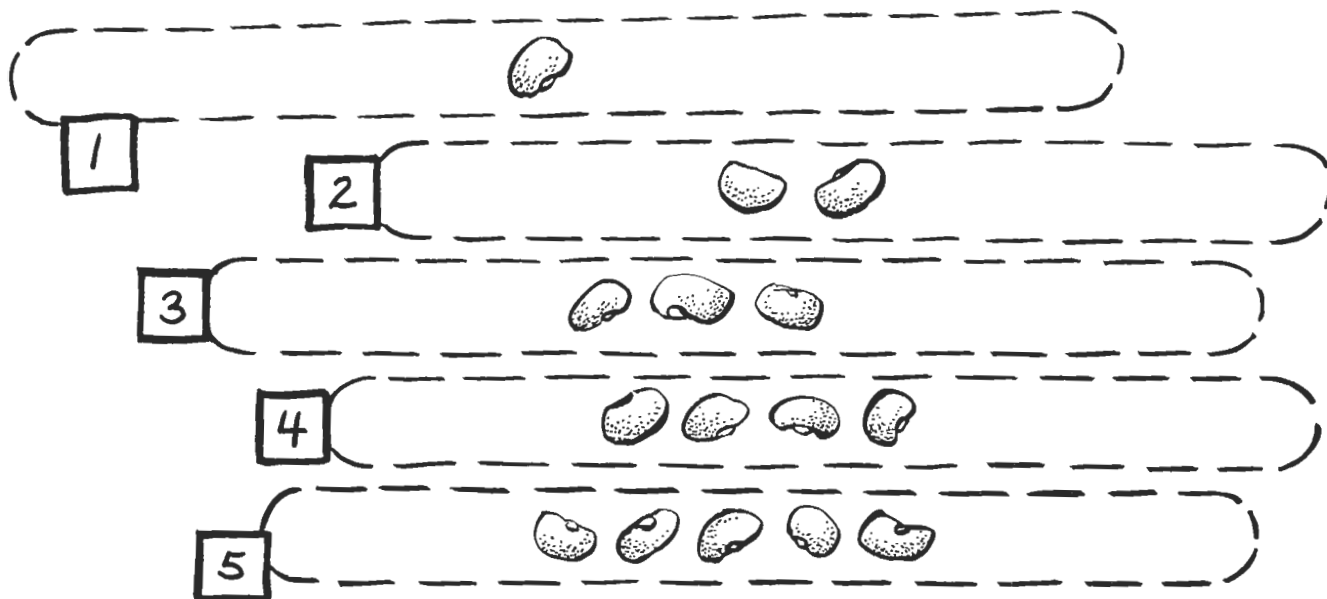
This activity is similar to those on pages 30A - 30C.

Frames are provided for 1-, 2-, 3-, 4-, and 5-sticks. Numbers in the circles are the number of beans to be picked up. There is, however, a shift in the question. The child is now asked: How many **different ways** can you pick up each number of beans?

For example, one way to pick up six beans is to use the 5-stick and the 1-stick. This result can be recorded as follows, by coloring the appropriate boxes.

|   |  |  |
|---|--|--|
| 6 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |

But there are two other ways to pick up 6 beans. We could use the 3-, 2-, and 1-sticks, or the 4-stick and the 2-stick. We can record these results in the other two columns.



| 10 |   |   |
|----|---|---|
| 1  | 1 | 1 |
| 2  | 2 | 2 |
| 3  | 3 | 3 |
| 4  | 4 | 4 |
| 5  | 5 | 5 |

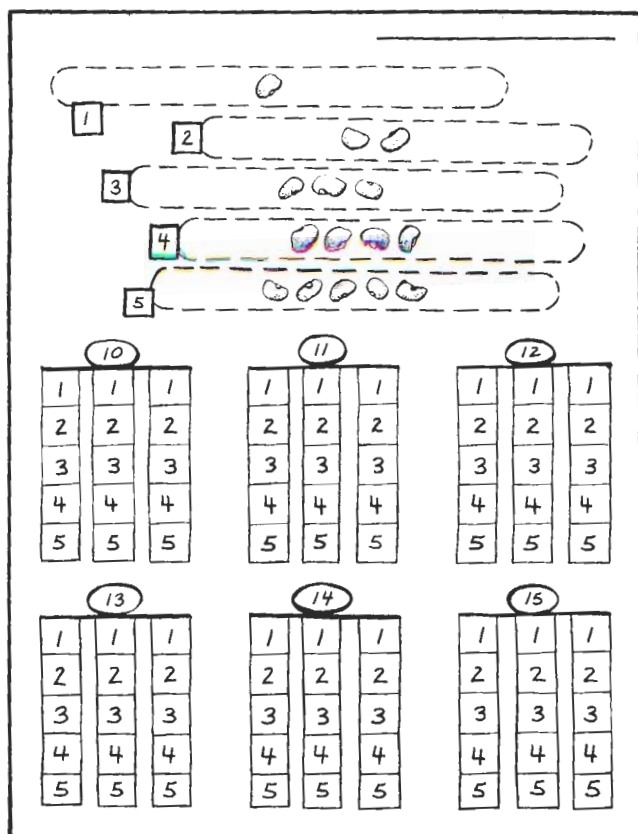
| 11 |   |   |
|----|---|---|
| 1  | 1 | 1 |
| 2  | 2 | 2 |
| 3  | 3 | 3 |
| 4  | 4 | 4 |
| 5  | 5 | 5 |

| 12 |   |   |
|----|---|---|
| 1  | 1 | 1 |
| 2  | 2 | 2 |
| 3  | 3 | 3 |
| 4  | 4 | 4 |
| 5  | 5 | 5 |

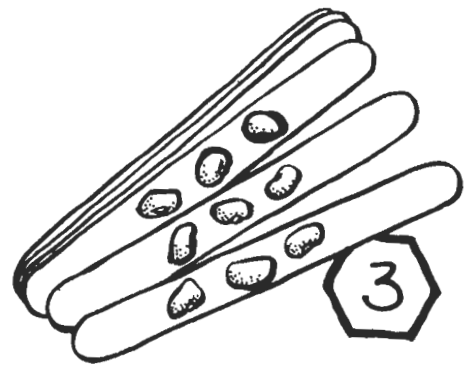
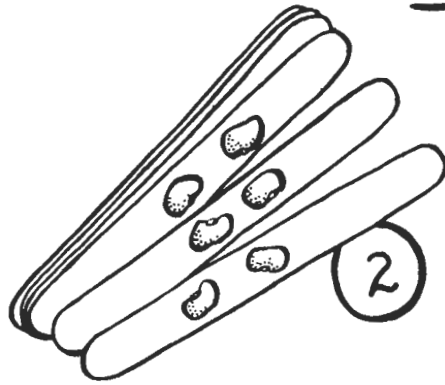
| 13 |   |   |
|----|---|---|
| 1  | 1 | 1 |
| 2  | 2 | 2 |
| 3  | 3 | 3 |
| 4  | 4 | 4 |
| 5  | 5 | 5 |

| 14 |   |   |
|----|---|---|
| 1  | 1 | 1 |
| 2  | 2 | 2 |
| 3  | 3 | 3 |
| 4  | 4 | 4 |
| 5  | 5 | 5 |

| 15 |   |   |
|----|---|---|
| 1  | 1 | 1 |
| 2  | 2 | 2 |
| 3  | 3 | 3 |
| 4  | 4 | 4 |
| 5  | 5 | 5 |



This is an extension of the activity on the previous page. Learners must now try to pick up 10 through 15 beans.



$$\begin{array}{r} \textcircled{2} \quad \text{hexagon } 3 \\ \times \underline{\quad} \quad \times \underline{\quad} \\ \hline \quad + \quad = \end{array}$$

$$\begin{array}{r} \textcircled{2} \quad \text{hexagon } 3 \\ \times \underline{\quad} \quad \times \underline{\quad} \\ \hline \quad + \quad = \end{array}$$

$$\begin{array}{r} \textcircled{2} \quad \text{hexagon } 3 \\ \times \underline{\quad} \quad \times \underline{\quad} \\ \hline \quad + \quad = \end{array}$$

$$\begin{array}{r} \textcircled{2} \quad \text{hexagon } 3 \\ \times \underline{\quad} \quad \times \underline{\quad} \\ \hline \quad + \quad = \end{array}$$

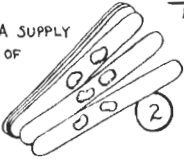
$$\begin{array}{r} \textcircled{2} \quad \text{hexagon } 3 \\ \times \underline{\quad} \quad \times \underline{\quad} \\ \hline \quad + \quad = \end{array}$$

$$\begin{array}{r} \textcircled{2} \quad \text{hexagon } 3 \\ \times \underline{\quad} \quad \times \underline{\quad} \\ \hline \quad + \quad = \end{array}$$

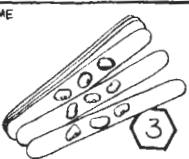
$$\begin{array}{r} \textcircled{2} \quad \text{hexagon } 3 \\ \times \underline{\quad} \quad \times \underline{\quad} \\ \hline \quad + \quad = \end{array}$$

$$\begin{array}{r} \textcircled{2} \quad \text{hexagon } 3 \\ \times \underline{\quad} \quad \times \underline{\quad} \\ \hline \quad + \quad = \end{array}$$

PLEASE HAVE A SUPPLY OF EACH KIND OF BEANSTICK:



NAME \_\_\_\_\_



|                                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                                                                                                    |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">             (2)<br/>x   <br/>——<br/>+   <br/>—— =              </div> <div style="text-align: center;">             (3)<br/>x   <br/>——<br/>+   <br/>—— =              </div> </div> | <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">             (2)<br/>x   <br/>——<br/>+   <br/>—— =              </div> <div style="text-align: center;">             (3)<br/>x   <br/>——<br/>+   <br/>—— =              </div> </div> |
| <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">             (2)<br/>x   <br/>——<br/>+   <br/>—— =              </div> <div style="text-align: center;">             (3)<br/>x   <br/>——<br/>+   <br/>—— =              </div> </div> | <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">             (2)<br/>x   <br/>——<br/>+   <br/>—— =              </div> <div style="text-align: center;">             (3)<br/>x   <br/>——<br/>+   <br/>—— =              </div> </div> |
| <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">             (2)<br/>x   <br/>——<br/>+   <br/>—— =              </div> <div style="text-align: center;">             (3)<br/>x   <br/>——<br/>+   <br/>—— =              </div> </div> | <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">             (2)<br/>x   <br/>——<br/>+   <br/>—— =              </div> <div style="text-align: center;">             (3)<br/>x   <br/>——<br/>+   <br/>—— =              </div> </div> |
| <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">             (2)<br/>x   <br/>——<br/>+   <br/>—— =              </div> <div style="text-align: center;">             (3)<br/>x   <br/>——<br/>+   <br/>—— =              </div> </div> | <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">             (2)<br/>x   <br/>——<br/>+   <br/>—— =              </div> <div style="text-align: center;">             (3)<br/>x   <br/>——<br/>+   <br/>—— =              </div> </div> |

This activity was introduced on the previous page. Here we put together the products of two stick activities and find their sum. The difficulty will be in keeping the record properly. Go over this part very carefully so that all understand it.

Before starting, each child will need 10 or more sticks, 5 or more with 2 beans on them, and 5 or more with 3 beans on them. Use a procedure similar to the following:

- 1) Put out 3 twos. How many beans?
- 2) Put out 4 threes. How many beans?
- 3) How many beans all together?

Do enough so that all understand the process. Then introduce the record.

$$\begin{array}{r}
 \textcircled{2} \quad \textcircled{3} \\
 \times 3 \quad \times 4 \\
 \hline
 6 \quad + \quad 12 = 18
 \end{array}$$

When they understand how to keep the record, they may proceed on their own and complete the page.

## Variations

- 1) Fill in the numbers of sticks.

### Example

$$\begin{array}{r}
 \textcircled{2} \quad \textcircled{3} \\
 \times 5 \quad \times 4 \\
 \hline
 \quad + \quad =
 \end{array}$$

- 2) Fill in the numbers of beans.

### Example

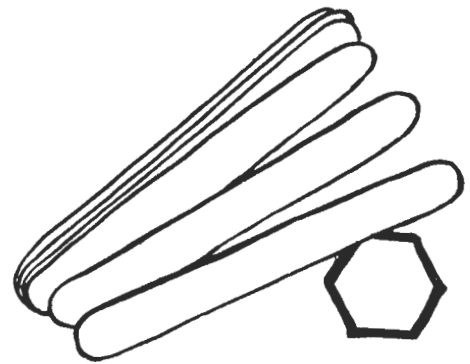
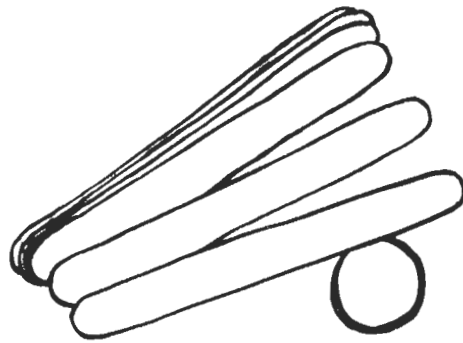
$$\begin{array}{r}
 \textcircled{2} \quad \textcircled{3} \\
 \times \quad \times \\
 \hline
 12 \quad + \quad 9 =
 \end{array}$$

- 3) Fill in the sum only. This introduces possibilities for multiple solutions.

### Example

$$\begin{array}{r}
 \textcircled{2} \quad \textcircled{3} \\
 \times \quad \times \\
 \hline
 \quad + \quad = 17
 \end{array}$$

Note: You can use 1 two and 5 threes or  
4 twos and 3 threes or  
7 twos and 1 three.



$$\begin{array}{cc} \bigcirc & \hexagon \\ \times \underline{\quad} & \times \underline{\quad} \end{array}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\begin{array}{cc} \bigcirc & \hexagon \\ \times \underline{\quad} & \times \underline{\quad} \end{array}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\begin{array}{cc} \bigcirc & \hexagon \\ \times \underline{\quad} & \times \underline{\quad} \end{array}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\begin{array}{cc} \bigcirc & \hexagon \\ \times \underline{\quad} & \times \underline{\quad} \end{array}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\begin{array}{cc} \bigcirc & \hexagon \\ \times \underline{\quad} & \times \underline{\quad} \end{array}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\begin{array}{cc} \bigcirc & \hexagon \\ \times \underline{\quad} & \times \underline{\quad} \end{array}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\begin{array}{cc} \bigcirc & \hexagon \\ \times \underline{\quad} & \times \underline{\quad} \end{array}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\begin{array}{cc} \bigcirc & \hexagon \\ \times \underline{\quad} & \times \underline{\quad} \end{array}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

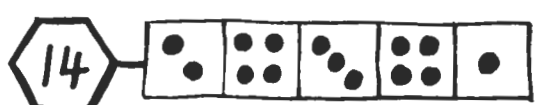
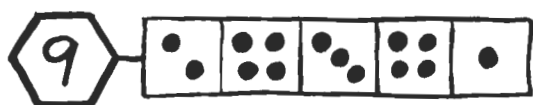
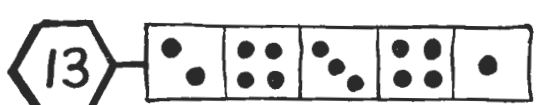
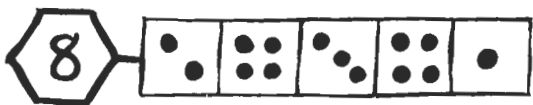
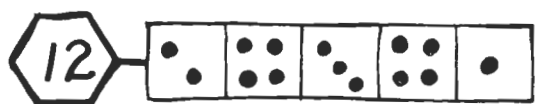
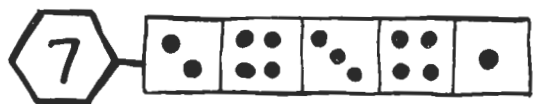
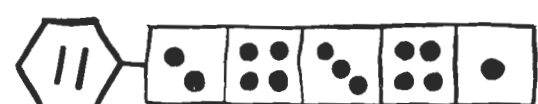
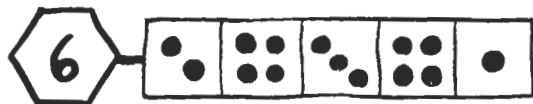
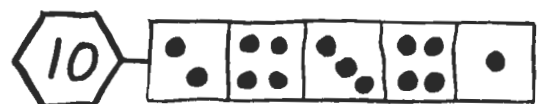
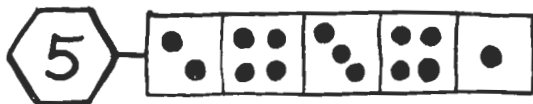
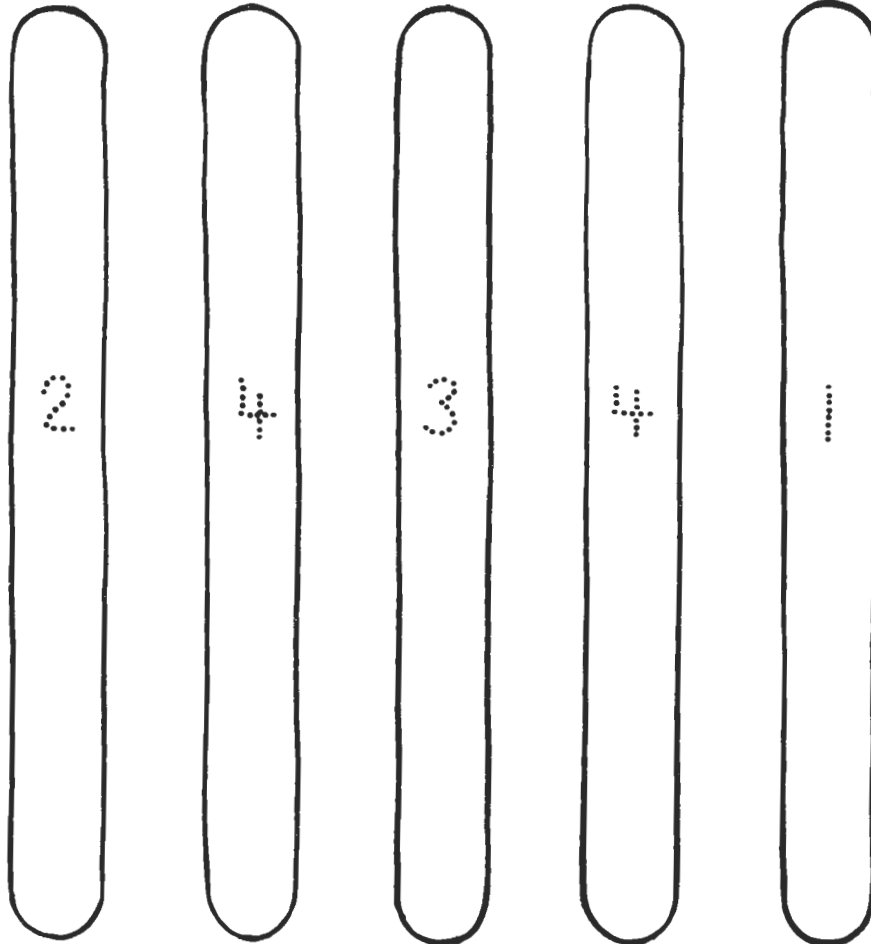
PLEASE HAVE A SUPPLY  
OF EACH KIND OF  
BEANSTICK.

NAME \_\_\_\_\_

|                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                     |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  <br>$\times \underline{\quad}$ $\times \underline{\quad}$<br>$\quad + \quad = \underline{\quad}$ |  <br>$\times \underline{\quad}$ $\times \underline{\quad}$<br>$\quad + \quad = \underline{\quad}$ |
|  <br>$\times \underline{\quad}$ $\times \underline{\quad}$<br>$\quad + \quad = \underline{\quad}$ |  <br>$\times \underline{\quad}$ $\times \underline{\quad}$<br>$\quad + \quad = \underline{\quad}$ |
|  <br>$\times \underline{\quad}$ $\times \underline{\quad}$<br>$\quad + \quad = \underline{\quad}$ |  <br>$\times \underline{\quad}$ $\times \underline{\quad}$<br>$\quad + \quad = \underline{\quad}$ |
|  <br>$\times \underline{\quad}$ $\times \underline{\quad}$<br>$\quad + \quad = \underline{\quad}$ |  <br>$\times \underline{\quad}$ $\times \underline{\quad}$<br>$\quad + \quad = \underline{\quad}$ |

Use this page to individualize with the kinds of activities outlined on previous pages. The added feature is that of not being limited to only twos and threes. A page containing only the number of sticks can be used repeatedly as the kinds of beansticks change.

COMBINATIONS OF NEIGHBORS  
COMBINACIÓN DE VECINOS





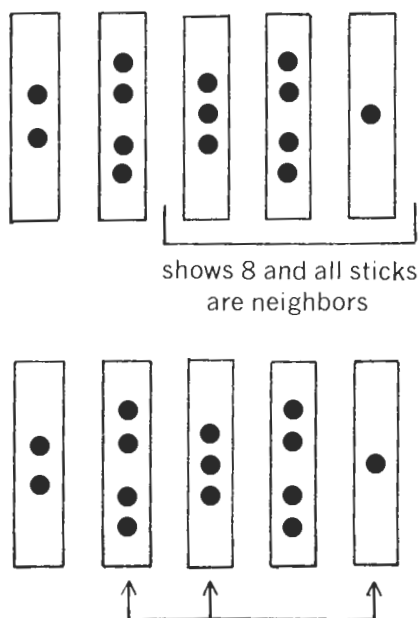
COMBINATIONS OF NEIGHBORS. NAME \_\_\_\_\_

2 4 3 4 1

|   |  |    |  |
|---|--|----|--|
| 5 |  | 10 |  |
| 6 |  | 11 |  |
| 7 |  | 12 |  |
| 8 |  | 13 |  |
| 9 |  | 14 |  |

While this is essentially addition practice, problems develop. Five sticks are placed vertically and then used in an attempt to show different sums. The restriction is that all sticks used to show a sum must be neighbors; that is, they must be in a line next to each other.

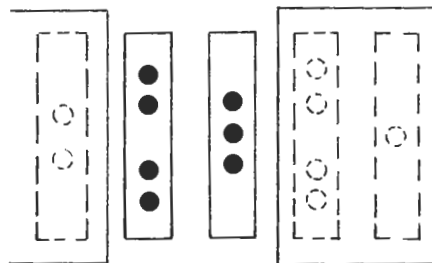
### Example



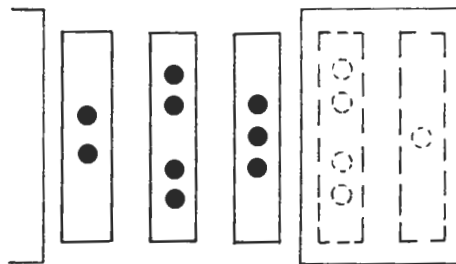
shows 8 but not allowed because sticks not next to each other.

It is very helpful to use two cards or your hands as masks. Sums can be found when they are "trapped" between the cards.

### Example



shows 7

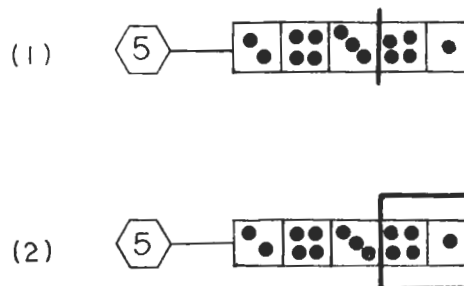


shows 9

When the page is introduced, students should place the appropriate beansticks in the spaces provided. After the rules of trapping neighbors are understood, the record keeping system can be explained.

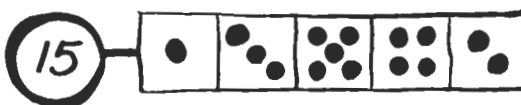
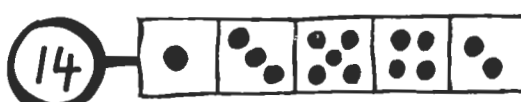
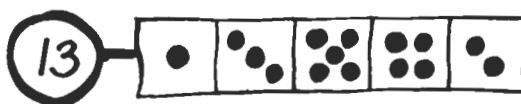
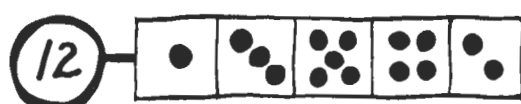
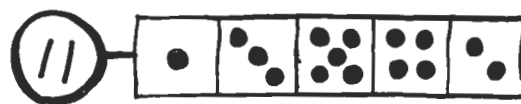
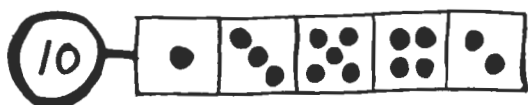
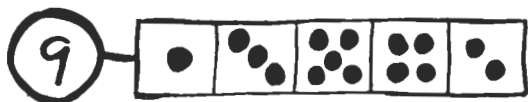
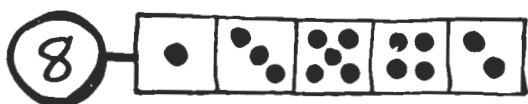
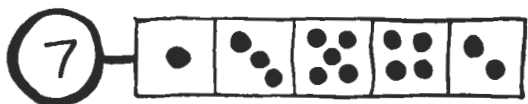
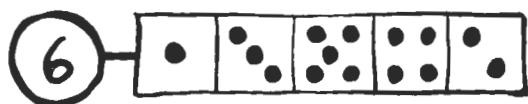
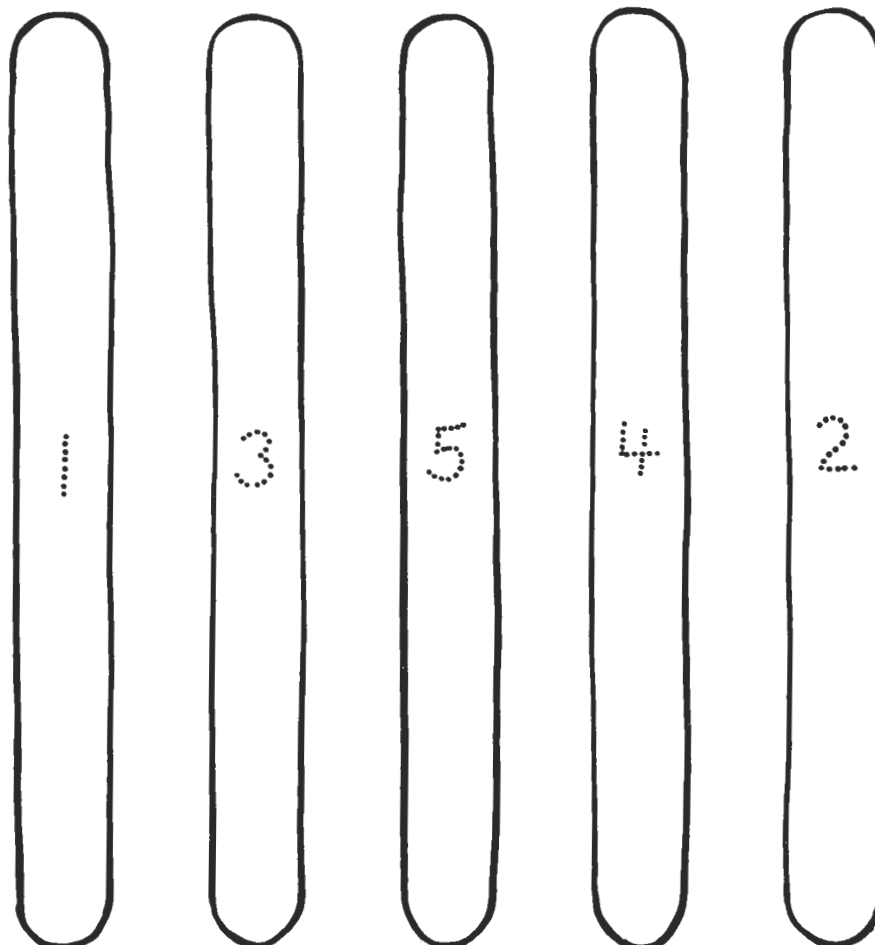
First draw 2 lines to show the "trapped" sum and then "box it in."

### Example



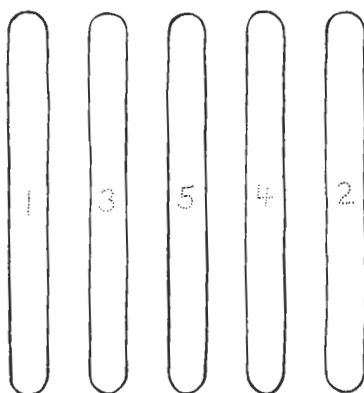
When children are ready let them continue on their own. You may wish to inform them that one sum is impossible (10).

COMBINATIONS OF NEIGHBORS  
COMBINACIÓN DE VECINOS



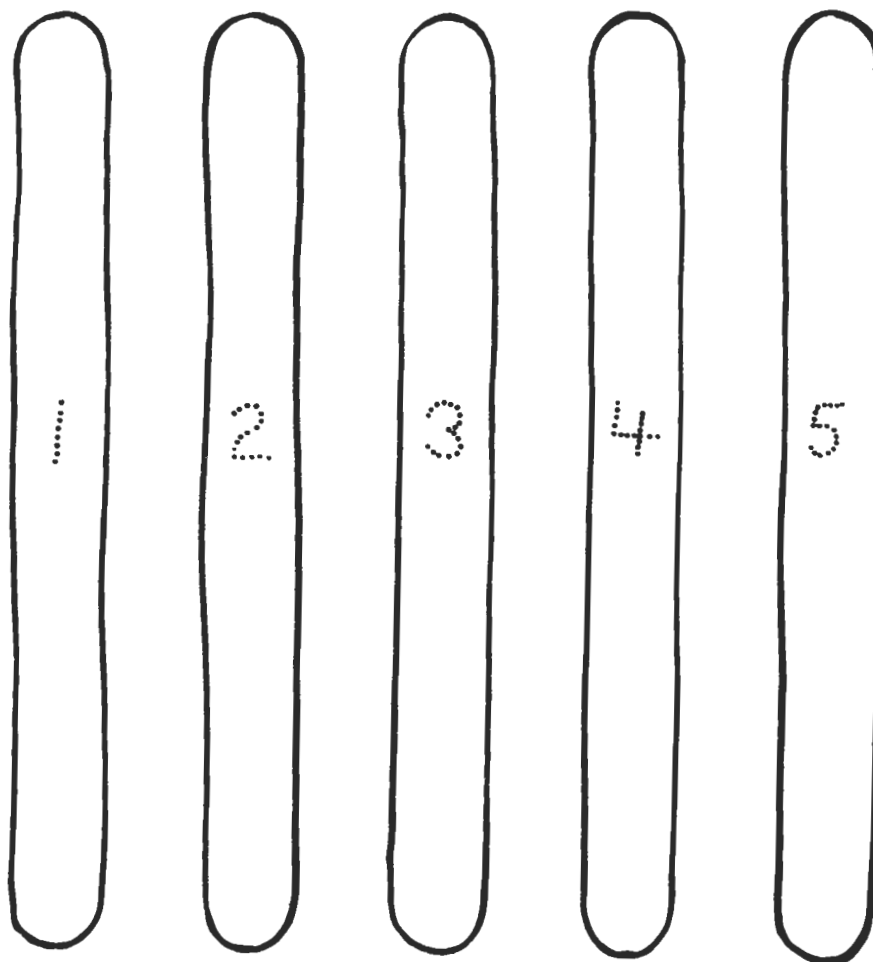
# COMBINATIONS OF NEIGHBORS

NAME \_\_\_\_\_



Use the same strategy as on the previous page. Two sums are impossible (7 and 10).

COMBINATIONS OF NEIGHBORS  
COMBINACIÓN DE VECINOS



6 1 2 3 4 5

7 1 2 3 4 5

8 1 2 3 4 5

9 1 2 3 4 5

10 1 2 3 4 5

11 1 2 3 4 5

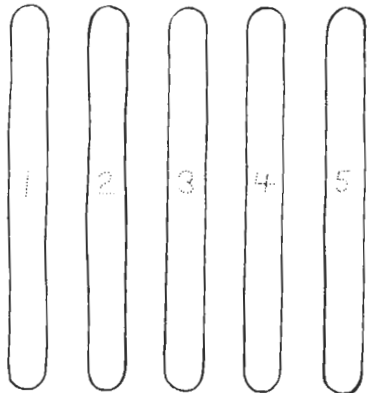
12 1 2 3 4 5

13 1 2 3 4 5

14 1 2 3 4 5

15 1 2 3 4 5

COMBINATIONS OF NEIGHBORS



NAME \_\_\_\_\_

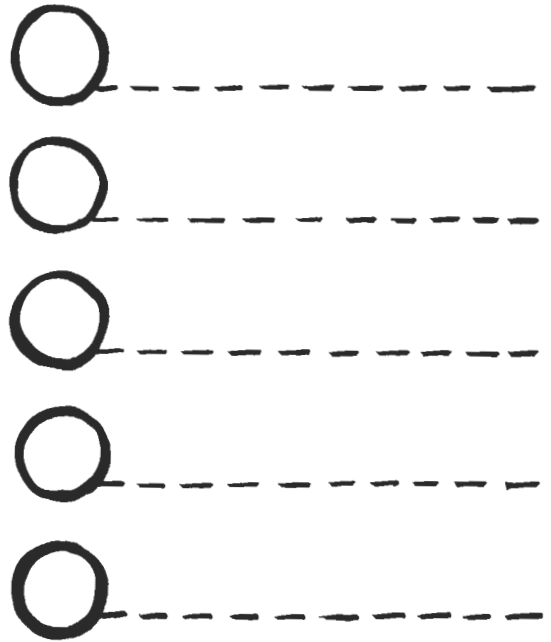
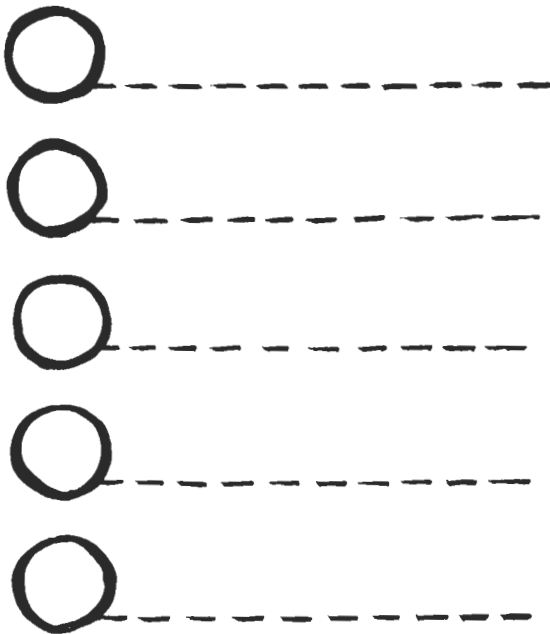
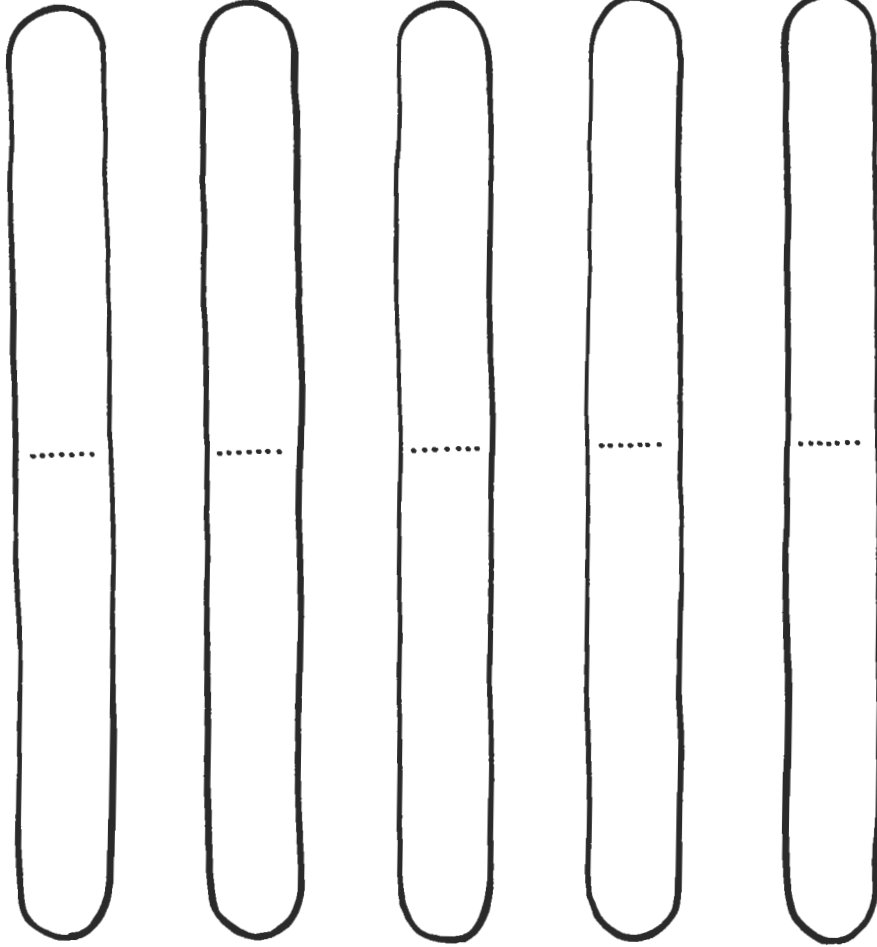
  

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <div style="display: flex; justify-content: space-between;"> <div style="width: 48%;"> <div><div>6</div>1 2 3 4 5</div> <div><div>7</div>1 2 3 4 5</div> <div><div>8</div>1 2 3 4 5</div> <div><div>9</div>1 2 3 4 5</div> <div><div>10</div>1 2 3 4 5</div> </div> <div style="width: 48%;"> <div><div>11</div>1 2 3 4 5</div> <div><div>12</div>1 2 3 4 5</div> <div><div>13</div>1 2 3 4 5</div> <div><div>14</div>1 2 3 4 5</div> <div><div>15</div>1 2 3 4 5</div> </div> </div> |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

The record keeping system changes and numbers are used instead of dots. We still “box in” the neighbors used to show the desired sum. Three sums are impossible here (8, 11, and 13). An interesting investigation is to rearrange the sticks and try to decrease the impossible sums. . . . Do you think the sticks can be rearranged and have all sums possible?

COMBINATIONS OF NEIGHBORS

COMBINACIÓN DE VECINOS



COMBINATIONS OF NEIGHBORS

NAME \_\_\_\_\_

|                       |       |
|-----------------------|-------|
| <input type="radio"/> | _____ |
| <input type="radio"/> | _____ |
| <input type="radio"/> | _____ |
| <input type="radio"/> | _____ |
| <input type="radio"/> | _____ |

|                       |       |
|-----------------------|-------|
| <input type="radio"/> | _____ |
| <input type="radio"/> | _____ |
| <input type="radio"/> | _____ |
| <input type="radio"/> | _____ |
| <input type="radio"/> | _____ |

The activity can be individualized in many ways. Here are a few:

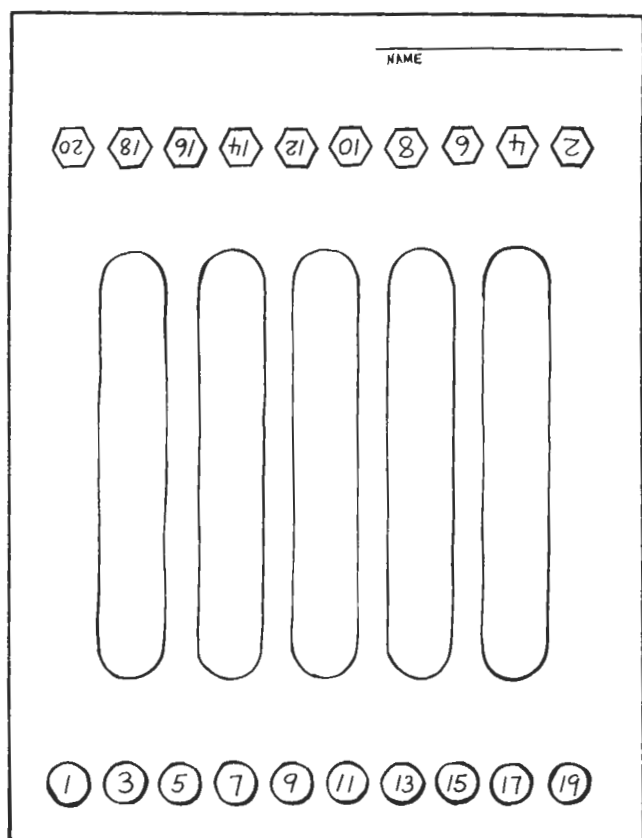
- (1) The teacher can fill in the stick numbers and sums similar to previous pages.
- (2) The teacher fills in the stick numbers only. The children choose the sums.
- (3) The teacher fills in the sums only. The children choose the stick numbers.
- (4) The children choose the stick numbers and the sum. They enjoy doing this and trading with a neighbor for completion.

02 81 91 41 21 01 8 9 4 2

Five vertical oval shapes, each with a rounded top and bottom, arranged in a row. They are intended for writing or drawing.

1 3 5 7 9 11 13 15 17 19





This page is a game board. Two players select 5 beansticks and place them in the channels. One player takes the odd numbers and the other the even numbers. They then take turns trying to "trap" their numbers, starting with 1 and going in order. Score keeping is accomplished by starting each player with 5 counters. Whenever a player can't "do" his number, he gives a counter to the other player. When the possibilities have been exhausted, the players choose new beansticks and the game continues until one player has all the counters or time is up.

After a few games, players will find it more interesting to eliminate using 1 and 2-sticks. They will also see that the kind of beanstick and its placement will affect the game. Have a supply of beansticks and let the players take turns placing the beanstick of their choice in the channel of their choice at the beginning of each round.

Before choosing sticks for the second round, the board is rotated 180° so the player with the "odd" numbers in the previous round now has the "even" numbers.

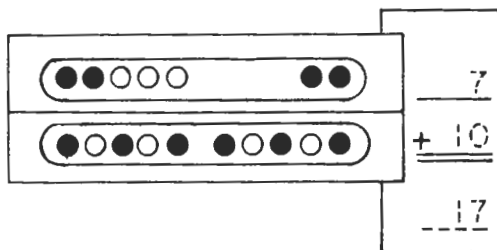


USE 2 BEANSTICKS AT A TIME

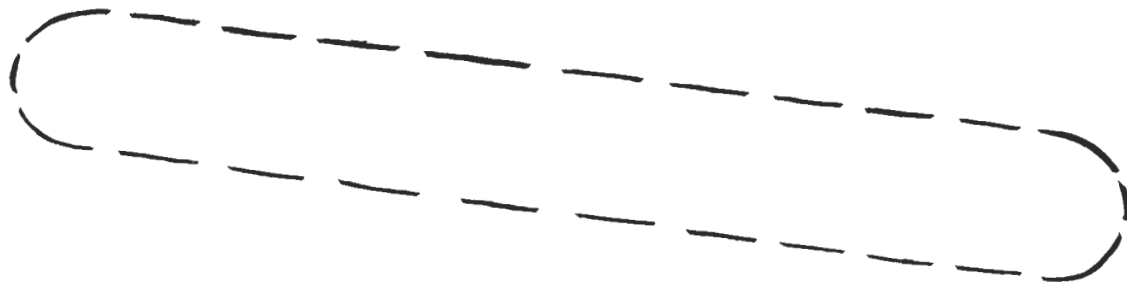
NAME \_\_\_\_\_

Choose two beansticks and place them in the outlines. Count the number of beans on each and record to the right. Count the total number and record in the appropriate place. Encourage the children to count the total by considering the groupings.

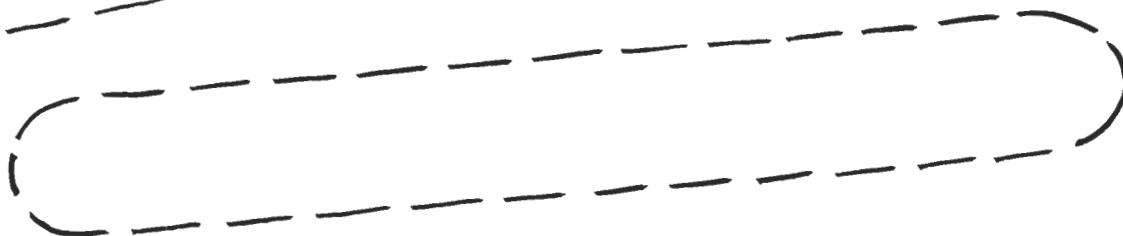
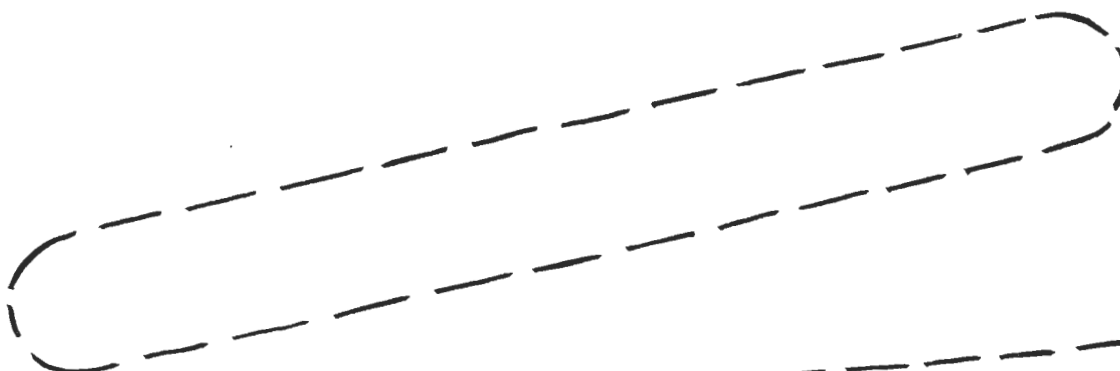
### Example



When the children understand the procedure let them make up their own problems.



|              |     |     |     |     |     |
|--------------|-----|-----|-----|-----|-----|
| TOP          | 2   | 3   | 4   | 5   | 6   |
| BOTTOM       | + 2 | + 3 | + 4 | + 5 | + 6 |
| ALL TOGETHER |     |     |     |     |     |



|              |      |      |      |      |      |
|--------------|------|------|------|------|------|
| ARRIBA       |      |      |      |      |      |
| ABAJO        |      |      |      |      |      |
| TODOS JUNTOS | + 10 | + 10 | + 10 | + 10 | + 10 |

NAME \_\_\_\_\_

|              |     |     |     |     |     |
|--------------|-----|-----|-----|-----|-----|
| TOP          | 2   | 3   | 4   | 5   | 6   |
| BOTTOM       | + 2 | + 3 | + 4 | + 5 | + 6 |
| ALL TOGETHER | 10  | 10  | 10  | 10  | 10  |

|              |    |    |    |    |    |
|--------------|----|----|----|----|----|
| TOP          |    |    |    |    |    |
| BOTTOM       | +  | +  | +  | +  | +  |
| ALL TOGETHER | 10 | 10 | 10 | 10 | 10 |

|     |
|-----|
| 3   |
| + 7 |
| 10  |

and

|     |
|-----|
| 7   |
| + 3 |
| 10  |

can be considered as two different combinations.

A question to explore:

How many combinations of two sticks are there that will have ten beans on them?

There are two different investigations on this page. After making copies of the page, cut them in half.

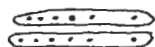
The upper half of the page provides frames for two beansticks. The boxes under the frames tell the learner which two beansticks to place in the frames. The question is: How many beans are there all together on the two sticks? The results are recorded under the double line:

|              |     |
|--------------|-----|
| Top          | 2   |
| Bottom       | + 2 |
| All together | 4   |

|     |
|-----|
| 3   |
| + 3 |
| 6   |

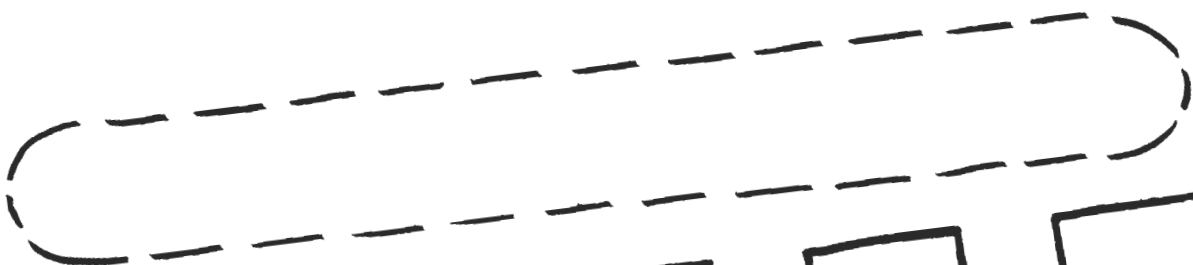
etc.

Encourage children to count the total by considering the groupings on the stick. For example, two 6-sticks might look like this:



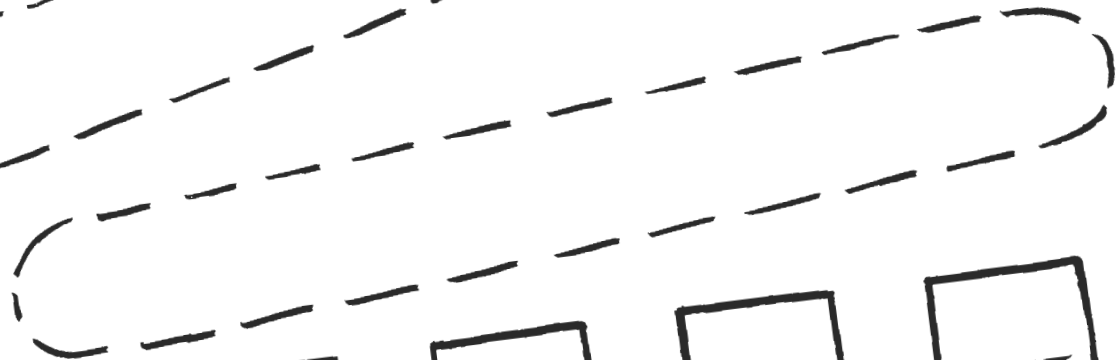
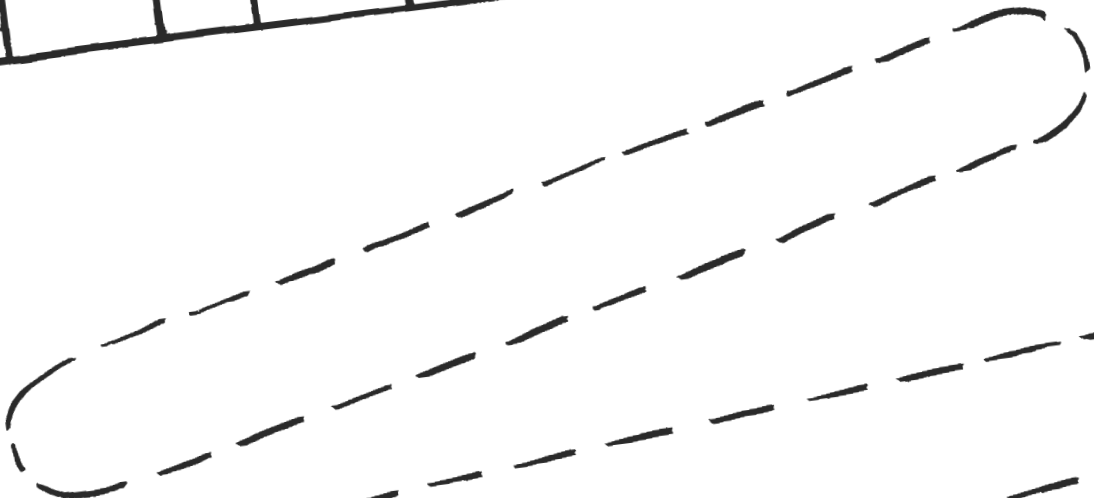
The learner might know that  $5 + 5 = 10$ , and then two more is 12. Try to help them avoid one-by-one counting.

The lower half of the page tells the learner that there are 10 beans all together on the two sticks. How many combinations of two sticks can he find that have a total of 10 beans on them? Since a "top" and a "bottom" stick are specified,



TOP  
BOTTOM  
ALL  
TOGETHER

|                                         |                                         |                                         |                                         |                                         |
|-----------------------------------------|-----------------------------------------|-----------------------------------------|-----------------------------------------|-----------------------------------------|
| <u>        </u><br>+<br><u>        </u> | <u>        </u><br>+<br><u>        </u> | <u>        </u><br>+<br><u>        </u> | <u>        </u><br>+<br><u>        </u> | <u>        </u><br>+<br><u>        </u> |
|-----------------------------------------|-----------------------------------------|-----------------------------------------|-----------------------------------------|-----------------------------------------|



ARRIBA  
ABAJO  
TODOS  
JUNTOS

|                                         |                                         |                                         |                                         |                                         |
|-----------------------------------------|-----------------------------------------|-----------------------------------------|-----------------------------------------|-----------------------------------------|
| <u>        </u><br>+<br><u>        </u> | <u>        </u><br>+<br><u>        </u> | <u>        </u><br>+<br><u>        </u> | <u>        </u><br>+<br><u>        </u> | <u>        </u><br>+<br><u>        </u> |
|-----------------------------------------|-----------------------------------------|-----------------------------------------|-----------------------------------------|-----------------------------------------|

NAME \_\_\_\_\_

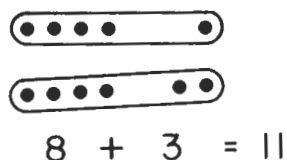
TOP  
 BOTTOM  
 ALL  
 TOGETHER

|  |  |  |  |  |
|--|--|--|--|--|
|  |  |  |  |  |
|--|--|--|--|--|

TOP  
 BOTTOM  
 ALL  
 TOGETHER

|  |  |  |  |  |
|--|--|--|--|--|
|  |  |  |  |  |
|--|--|--|--|--|

This page is a report form for investigations using two beansticks at a time. The learner chooses a beanstick, places it in the top frame, and then records his choice on the dotted line in the record keeping space. He chooses a second beanstick, places it in the bottom frame, and records his choice in the double line of the record keeping spaces. Then he finds the number of beans all together on the two sticks and records that number. Learners should be encouraged to count the total by considering the groupings. For example, if he chooses a 5-stick and a 6-stick, he might consider the two groups of four, and then 3 more.



There are many possibilities for investigations. Learners can:

- (1) Randomly select two beansticks each time, and record the results of the experiments.

- (2) Select two beansticks of the same kind, beginning with 1-sticks, and generate a record of "doubles."
- (3) Select one stick for the top (or bottom), keep it the same throughout the investigation, and use different sticks in the other frame. This leads to a systematic investigation: e.g.,  $5 + 1$ ,  $5 + 2$ ,  $5 + 3$ ,  $5 + 4$ , etc.
- (4) Choose a specific sum and find all possible combinations of sticks for that sum.
- (5) Select a stick for the top (or bottom), choose a sum, and figure out what stick he needs for the second frame. A record keeping device for "I can't find one" or "I need to take beans away" may need to be created for examples like:

|   |
|---|
| 6 |
| + |
| 5 |

# STICKS \_ \_ \_ PALOS

| FRIJOLES<br>BEANS | PALOS<br>STICKS |
|-------------------|-----------------|
|                   | (1)             |
|                   | (2)             |
|                   | (3)             |
|                   | (4)             |
|                   | (5)             |
|                   | (6)             |
|                   | (7)             |
|                   | (8)             |
|                   | (9)             |
|                   | (10)            |

$$\begin{array}{r} \text{---} ( ) \\ \text{---} \\ + \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} ( ) \\ \text{---} \\ + \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} ( ) \\ \text{---} \\ + \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} ( ) \\ \text{---} \\ + \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} ( ) \\ \text{---} \\ + \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} ( ) \\ \text{---} \\ + \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} ( ) \\ \text{---} \\ + \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} ( ) \\ \text{---} \\ + \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$$



USING 3-STICKS ONLY

NAME \_\_\_\_\_

| BEANS | STICKS |
|-------|--------|
|       | (1)    |
|       | (2)    |
|       | (3)    |
|       | (4)    |
|       | (5)    |
|       | (6)    |
|       | (7)    |
|       | (8)    |
|       | (9)    |
|       | (10)   |

— ( )

+ ( )

---

— ( )

— ( )

+ ( )

---

— ( )

— ( )

+ ( )

---

— ( )

— ( )

+ ( )

---

— ( )

— ( )

+ ( )

---

— ( )

— ( )

+ ( )

---

— ( )

— ( )

+ ( )

---

— ( )

— ( )

+ ( )

---

— ( )

Here we introduce the notion of adding sticks. This is accomplished by placing sticks in both boxes, recording the number, and counting the total. The number of beans is recorded and the total number of beans is determined.

Before proceeding, insert a 3 in the blank space at the top of the page (using 3-sticks only), provide a supply of 3-sticks for each child and complete the record at the right. On all pages it is necessary to do this first.

The following sequence may then be used:

- 1) Place 2 sticks in the top box. How many beans? (6)
- 2) Place 1 stick in the bottom box. How many beans? (3)
- 3) How many sticks in both boxes? (3)
- 4) How many beans in both boxes? (9)

Do this a few times, choosing easy examples. The record keeping system may then be introduced in this fashion:

- 1) Place 2 sticks in the top box. On the top line of the first record write 2 in the ( ). (Provide a diagram so the children can follow the recording process.) How many beans? Write 6 before the (2) on the top line.

- 2) Place 3 sticks in the bottom box. On the 2nd line write 3 in the ( ). How many beans? Write 9 before the (2).

- 3) How many sticks altogether? Write 5 on the bottom line in ( ).

- 4) How many beans altogether? Write 15 on the bottom line before the (5). Make sure everyone understands that the number in the ( ) tells the number of sticks. The completed record would look like this:

$$\begin{array}{r}
 \underline{\underline{6}} (2) \\
 + \underline{\underline{9}} (3) \\
 \hline
 15 (5)
 \end{array}$$

It might be wise to do another example (experiment) with the group. Then let them go on independently. It is important to see that each child is keeping the record properly, otherwise it will be difficult to make progress.

Each child makes up his own experiment and records the results.

The page can easily be individualized by writing different numbers in the blank space at the top to indicate a particular kind of beanstick, and doing additional experiments.

# STICKS --- PALOS

+

| FRIJOLES<br>BEANS | PALOS<br>STICKS |
|-------------------|-----------------|
|                   | (1)             |
|                   | (2)             |
|                   | (3)             |
|                   | (4)             |
|                   | (5)             |
|                   | (6)             |
|                   | (7)             |
|                   | (8)             |
|                   | (9)             |
|                   | (10)            |

$$\begin{array}{r} \text{---} (2) \\ + \text{---} (1) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} (5) \\ + \text{---} (2) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} (3) \\ + \text{---} (4) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} (3) \\ + \text{---} (6) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} (7) \\ + \text{---} (1) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} (0) \\ + \text{---} (5) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} (2) \\ + \text{---} (8) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} (6) \\ + \text{---} (4) \\ \hline \text{---} ( ) \end{array}$$

USING — STICKS ONLY

NAME

+

| BEANS | STICKS |
|-------|--------|
|       | (1)    |
|       | (2)    |
|       | (3)    |
|       | (4)    |
|       | (5)    |
|       | (6)    |
|       | (7)    |
|       | (8)    |
|       | (9)    |
|       | (10)   |

|                                                                                            |                                                                                            |                                                                                            |                                                                                            |
|--------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|
| $\begin{array}{r} \text{---} (2) \\ + \text{---} (1) \\ \hline \text{---} ( ) \end{array}$ | $\begin{array}{r} \text{---} (5) \\ + \text{---} (2) \\ \hline \text{---} ( ) \end{array}$ | $\begin{array}{r} \text{---} (3) \\ + \text{---} (4) \\ \hline \text{---} ( ) \end{array}$ | $\begin{array}{r} \text{---} (3) \\ + \text{---} (6) \\ \hline \text{---} ( ) \end{array}$ |
| $\begin{array}{r} \text{---} (7) \\ + \text{---} (1) \\ \hline \text{---} ( ) \end{array}$ | $\begin{array}{r} \text{---} (0) \\ + \text{---} (5) \\ \hline \text{---} ( ) \end{array}$ | $\begin{array}{r} \text{---} (2) \\ + \text{---} (8) \\ \hline \text{---} ( ) \end{array}$ | $\begin{array}{r} \text{---} (6) \\ + \text{---} (4) \\ \hline \text{---} ( ) \end{array}$ |

The experiments are now prescribed but the choice of the kind of stick is still open. The number of sticks to be used in each box are given and the experiment is completed accordingly.

It may be that some children have discovered the connection between the table and the experiment while others are still counting beans and sticks. Please don't be in a hurry to tell everyone the shortcut at the outset. Shortly, however, a question like "Can you do the experiment without counting?" will lead to discussion that may encourage more of the children to look for a faster method.

# STICKS <sup>4</sup> PALOS

+

FRIJOLES PALOS  
BEANS STICKS

|  |      |
|--|------|
|  | (1)  |
|  | (2)  |
|  | (3)  |
|  | (4)  |
|  | (5)  |
|  | (6)  |
|  | (7)  |
|  | (8)  |
|  | (9)  |
|  | (10) |

$$\begin{array}{r} \underline{\underline{8}} ( ) \\ + \underline{\underline{4}} ( ) \\ \hline ( ) \end{array}$$

$$\begin{array}{r} \underline{\underline{16}} ( ) \\ + \underline{\underline{8}} ( ) \\ \hline ( ) \end{array}$$

$$\begin{array}{r} \underline{\underline{20}} ( ) \\ + \underline{\underline{12}} ( ) \\ \hline ( ) \end{array}$$

$$\begin{array}{r} \underline{\underline{24}} ( ) \\ + \underline{\underline{16}} ( ) \\ \hline ( ) \end{array}$$

$$\begin{array}{r} \underline{\underline{20}} ( ) \\ + \underline{\underline{8}} ( ) \\ \hline ( ) \end{array}$$

$$\begin{array}{r} \underline{\underline{32}} ( ) \\ + \underline{\underline{8}} ( ) \\ \hline ( ) \end{array}$$

$$\begin{array}{r} \underline{\underline{12}} ( ) \\ + \underline{\underline{0}} ( ) \\ \hline ( ) \end{array}$$

$$\begin{array}{r} \underline{\underline{20}} ( ) \\ + \underline{\underline{16}} ( ) \\ \hline ( ) \end{array}$$

USING 4 - STICKS ONLY

NAME \_\_\_\_\_



+



BEANS | STICKS

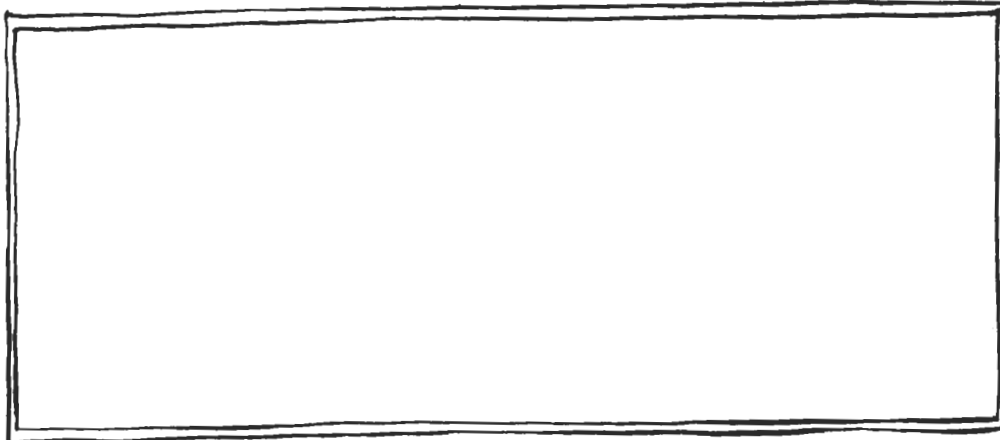
|      |
|------|
| (1)  |
| (2)  |
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| (4)  |
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| (8)  |
| (9)  |
| (10) |

|                |                |                 |                 |
|----------------|----------------|-----------------|-----------------|
| <u>8 ( )</u>   | <u>16 ( )</u>  | <u>20 ( )</u>   | <u>24 ( )</u>   |
| <u>+ 4 ( )</u> | <u>+ 8 ( )</u> | <u>+ 12 ( )</u> | <u>+ 16 ( )</u> |
| <u>    ( )</u> | <u>    ( )</u> | <u>    ( )</u>  | <u>    ( )</u>  |

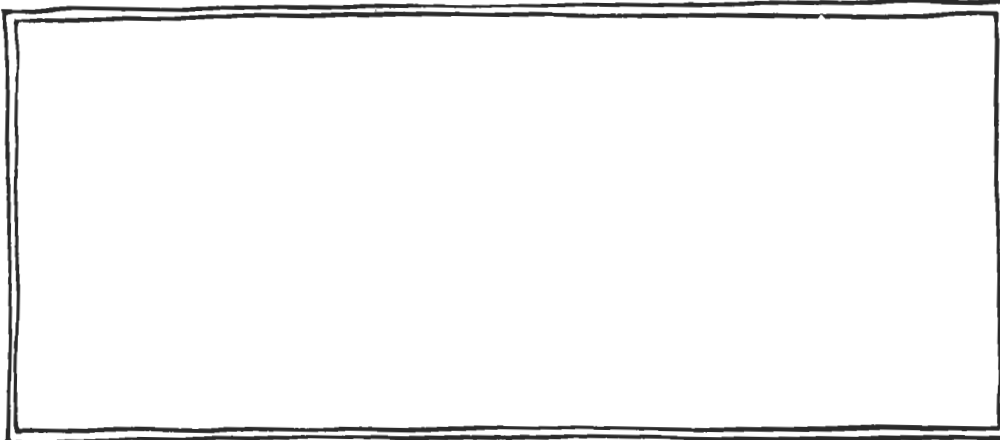
|                |                |                |                 |
|----------------|----------------|----------------|-----------------|
| <u>20 ( )</u>  | <u>32 ( )</u>  | <u>12 ( )</u>  | <u>20 ( )</u>   |
| <u>+ 8 ( )</u> | <u>+ 8 ( )</u> | <u>+ 0 ( )</u> | <u>+ 16 ( )</u> |
| <u>    ( )</u> | <u>    ( )</u> | <u>    ( )</u> | <u>    ( )</u>  |

Here the number of beans is given and the experiment is completed by filling in the missing spaces.

STICKS 5 PALOS



+



| FRIJOLES<br>BEANS | PALOS<br>STICKS |
|-------------------|-----------------|
|                   | (1)             |
|                   | (2)             |
|                   | (3)             |
|                   | (4)             |
|                   | (5)             |
|                   | (6)             |
|                   | (7)             |
|                   | (8)             |
|                   | (9)             |
|                   | (10)            |

$$\begin{array}{r} \text{---} 15 ( ) \\ + \quad (5) \\ \hline ( ) \end{array}$$

$$\begin{array}{r} \text{---} (9) \\ + \quad ( ) \\ \hline (10) \end{array}$$

$$\begin{array}{r} \text{---} 30 ( ) \\ + \quad ( ) \\ \hline 45 ( ) \end{array}$$

$$\begin{array}{r} \text{---} (7) \\ + \quad ( ) \\ \hline 40 ( ) \end{array}$$

$$\begin{array}{r} \text{---} ( ) \\ + \quad (4) \\ \hline (9) \end{array}$$

$$\begin{array}{r} \text{---} ( ) \\ + 35 ( ) \\ \hline 50 ( ) \end{array}$$

$$\begin{array}{r} \text{---} 25 ( ) \\ + \quad ( ) \\ \hline (8) \end{array}$$

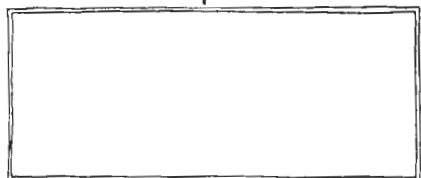
$$\begin{array}{r} \text{---} (8) \\ + \quad ( ) \\ \hline (10) \end{array}$$

USING 5-STICKS ONLY

NAME \_\_\_\_\_



+



| BEANS | STICKS |
|-------|--------|
| _____ | (1)    |
| _____ | (2)    |
| _____ | (3)    |
| _____ | (4)    |
| _____ | (5)    |
| _____ | (6)    |
| _____ | (7)    |
| _____ | (8)    |
| _____ | (9)    |
| _____ | (10)   |

|                                                              |                                                            |                                                                 |                                                              |
|--------------------------------------------------------------|------------------------------------------------------------|-----------------------------------------------------------------|--------------------------------------------------------------|
| $\begin{array}{r} 15 ( ) \\ + (5) \\ \hline ( ) \end{array}$ | $\begin{array}{r} (9) \\ + ( ) \\ \hline (10) \end{array}$ | $\begin{array}{r} 30 ( ) \\ + ( ) \\ \hline 45 ( ) \end{array}$ | $\begin{array}{r} (7) \\ + ( ) \\ \hline 40 ( ) \end{array}$ |
|--------------------------------------------------------------|------------------------------------------------------------|-----------------------------------------------------------------|--------------------------------------------------------------|

|                                                           |                                                                 |                                                              |                                                            |
|-----------------------------------------------------------|-----------------------------------------------------------------|--------------------------------------------------------------|------------------------------------------------------------|
| $\begin{array}{r} ( ) \\ + (4) \\ \hline (9) \end{array}$ | $\begin{array}{r} ( ) \\ + 35 ( ) \\ \hline 50 ( ) \end{array}$ | $\begin{array}{r} 25 ( ) \\ + ( ) \\ \hline (8) \end{array}$ | $\begin{array}{r} (8) \\ + ( ) \\ \hline (10) \end{array}$ |
|-----------------------------------------------------------|-----------------------------------------------------------------|--------------------------------------------------------------|------------------------------------------------------------|

This page gives the child an opportunity to see the many ways experiments can be initiated.

Note that, because of the selection of examples, the sums are all 40 (8), 45 (9), or 50 (10). This suggests an interesting question: "How many ways can you make a given total (such as 50 (10) )?"

STICKS — 3 — PALOS

+

| FRIJOLES<br>BEANS | PALOS<br>STICKS |
|-------------------|-----------------|
| 3                 | (1)             |
| 6                 | (2)             |
|                   | (3)             |
|                   | (4)             |
|                   | (5)             |
|                   | (6)             |
|                   | (7)             |
|                   | (8)             |
|                   | (9)             |
|                   | (10)            |

$$\begin{array}{r} \text{---} 6 \text{ (2)} \\ + 3 \text{ (1)} \\ \hline \text{---} (3) \end{array}$$

$$\begin{array}{r} \text{---} 12 \text{ (4)} \\ + 6 \text{ (2)} \\ \hline \text{---} (6) \end{array}$$

$$\begin{array}{r} \text{---} \text{---} (3) \\ + \text{---} (4) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} 15 \text{ ( )} \\ + 6 \text{ ( )} \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} 15 \text{ ( )} \\ + \text{---} (4) \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} 9 \text{ ( )} \\ + 18 \text{ ( )} \\ \hline \text{---} ( ) \end{array}$$

$$\begin{array}{r} \text{---} \text{---} (8) \\ + \text{---} ( ) \\ \hline \text{---} (10) \end{array}$$

$$\begin{array}{r} \text{---} \text{---} ( ) \\ + \text{---} (7) \\ \hline 30 \text{ ( )} \end{array}$$



USING 3-STICKS ONLY

NAME \_\_\_\_\_

+

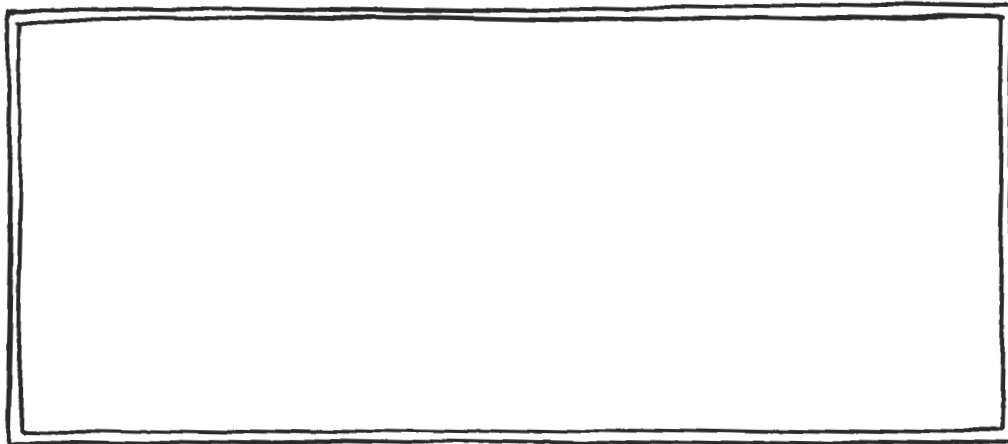
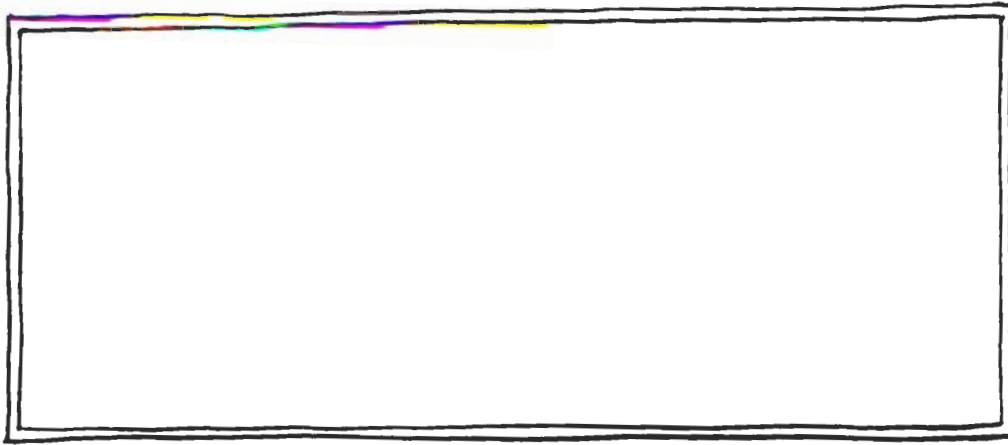
| BEANS | STICKS |
|-------|--------|
| 3     | (1)    |
| 6     | (2)    |
|       | (3)    |
|       | (4)    |
|       | (5)    |
|       | (6)    |
|       | (7)    |
|       | (8)    |
|       | (9)    |
|       | (10)   |

|                 |                 |                 |                 |
|-----------------|-----------------|-----------------|-----------------|
| <u>6</u> (2)    | <u>12</u> (4)   | <u>    </u> (3) | <u>15</u> ( )   |
| <u>+ 3</u> (1)  | <u>+ 6</u> (2)  | <u>+   </u> (4) | <u>+ 6</u> ( )  |
| <u>    </u> (3) | <u>    </u> (6) | <u>    </u> ( ) | <u>    </u> ( ) |

|                 |                 |                  |                 |
|-----------------|-----------------|------------------|-----------------|
| <u>15</u> ( )   | <u>9</u> ( )    | <u>    </u> (8)  | <u>    </u> ( ) |
| <u>+   </u> (4) | <u>+ 18</u> ( ) | <u>+   </u> ( )  | <u>+   </u> (7) |
| <u>    </u> ( ) | <u>    </u> ( ) | <u>    </u> (10) | <u>30</u> ( )   |

This is a continuation of the activity on the previous page.

# STICKS — — PALOS



| FRIJOLES<br>BEANS | PALOS<br>STICKS |
|-------------------|-----------------|
|                   | ( )             |
|                   | ( )             |
|                   | ( )             |
|                   | ( )             |
|                   | ( )             |
|                   | ( )             |
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USING — STICKS      NAME \_\_\_\_\_

|  |  |       |  |        |     |
|--|--|-------|--|--------|-----|
|  |  | BEANS |  | STICKS |     |
|  |  |       |  |        | ( ) |
|  |  |       |  |        | ( ) |
|  |  |       |  |        | ( ) |
|  |  |       |  |        | ( ) |
|  |  |       |  |        | ( ) |
|  |  |       |  |        | ( ) |
|  |  |       |  |        | ( ) |
|  |  |       |  |        | ( ) |
|  |  |       |  |        | ( ) |

|                                                                                            |                                                                                            |                                                                                            |                                                                                            |
|--------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|
| $\begin{array}{r} \text{---} ( ) \\ - \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$ | $\begin{array}{r} \text{---} ( ) \\ - \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$ | $\begin{array}{r} \text{---} ( ) \\ - \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$ | $\begin{array}{r} \text{---} ( ) \\ - \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$ |
| $\begin{array}{r} \text{---} ( ) \\ - \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$ | $\begin{array}{r} \text{---} ( ) \\ - \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$ | $\begin{array}{r} \text{---} ( ) \\ - \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$ | $\begin{array}{r} \text{---} ( ) \\ - \text{---} ( ) \\ \hline \text{---} ( ) \end{array}$ |

Let the children make up problems of this type and work together in solving them. Vary the activity by talking about beans and asking how many sticks they represent. By this time it should not be too difficult to move from beans to sticks or sticks to beans.

The page is introduced following the same format as before, substituting top box and bottom box for left and right hand.

Record keeping is similar to addition. Be certain that the children understand and feel comfortable with the procedures and can work independently.

$$\begin{array}{r} 15 (5) \\ - 9 (3) \\ \hline 6 (2) \end{array}$$

The page may be individualized by filling in numbers at the top or having the children choose their own.

Subtraction of sticks and beans is now introduced. The procedure consists of placing some sticks in the top box, moving some to the bottom box, then looking back at the top box to see what is left.

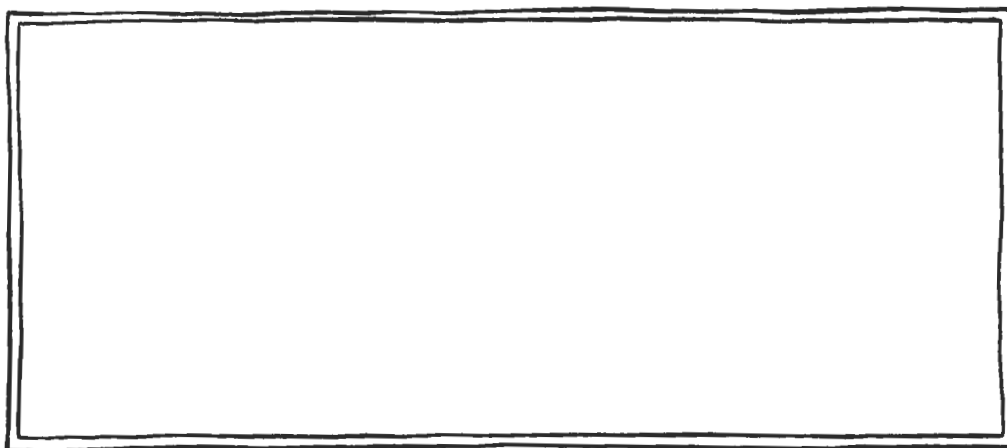
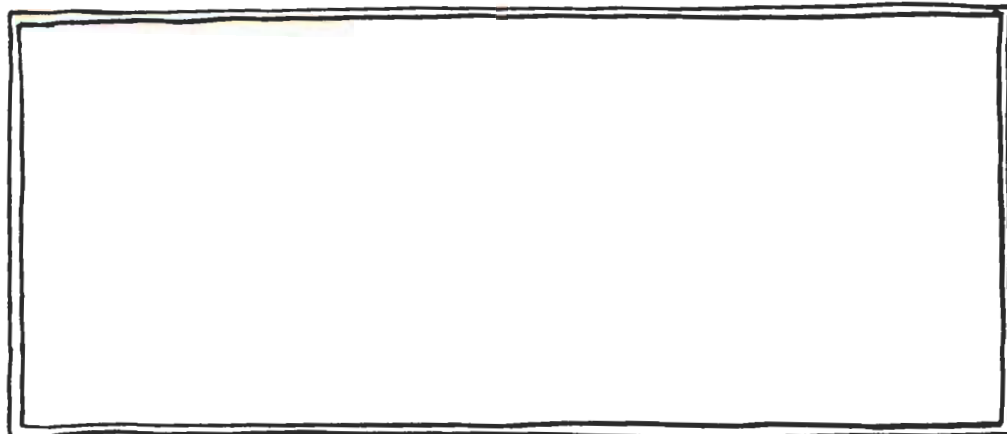
Before proceeding with the experiments have the children manipulate the sticks by introducing this activity using 3-sticks.

First fill out a 3-stick table on the chalkboard and then give instructions such as—

- (1) Pick up 4 sticks in your left hand—How many beans?
- (2) Move 3 sticks to your right hand—How many beans in your right hand?
- (3) How many sticks now in your left hand? How many beans?

The movement here is the same as the activity on the page itself. You may want to have two children working in front of a group, acting out the instructions. One child plays the left hand, the other the right hand. The rest of the group tells what is happening at each stage.

STICKS <sup>4</sup> PALOS



| FRIJOLES<br>BEANS | PALOS<br>STICKS |
|-------------------|-----------------|
|                   | (1)             |
|                   | (2)             |
|                   | (3)             |
|                   | (4)             |
|                   | (5)             |
|                   | (6)             |
|                   | (7)             |
|                   | (8)             |
|                   | (9)             |
|                   | (10)            |

$$\begin{array}{r} \text{---} (5) \\ - 12 ( ) \\ \hline ( ) \end{array}$$

$$\begin{array}{r} 20 ( ) \\ - (2) \\ \hline ( ) \end{array}$$

$$\begin{array}{r} \text{---} (7) \\ - ( ) \\ \hline (4) \end{array}$$

$$\begin{array}{r} 32 ( ) \\ - ( ) \\ \hline (3) \end{array}$$

$$\begin{array}{r} \text{---} ( ) \\ - (3) \\ \hline 24 ( ) \end{array}$$

$$\begin{array}{r} \text{---} ( ) \\ - 8 ( ) \\ \hline (6) \end{array}$$

$$\begin{array}{r} \text{---} ( ) \\ - ( ) \\ \hline 8 ( ) \end{array}$$

$$\begin{array}{r} \text{---} ( ) \\ - ( ) \\ \hline (3) \end{array}$$

USING 4 - STICKS      NAME \_\_\_\_\_

| BEANS | STICKS |
|-------|--------|
| (1)   |        |
| (2)   |        |
| (3)   |        |
| (4)   |        |
| (5)   |        |
| (6)   |        |
| (7)   |        |
| (8)   |        |
| (9)   |        |
| (10)  |        |

|                                                             |                                                              |                                                                |                                                                |
|-------------------------------------------------------------|--------------------------------------------------------------|----------------------------------------------------------------|----------------------------------------------------------------|
| <u>    (5)    </u><br><u>- 12 ( )</u><br><u>    ( )    </u> | <u>  20 ( )  </u><br><u>-   (2)  </u><br><u>    ( )    </u>  | <u>    (7)    </u><br><u>-   ( )    </u><br><u>    (4)    </u> | <u>  32 ( )  </u><br><u>-   ( )    </u><br><u>    (3)    </u>  |
| <u>    ( )    </u><br><u>-   (3)  </u><br><u>  24 ( )  </u> | <u>    ( )    </u><br><u>- 8 ( )  </u><br><u>    (6)    </u> | <u>    ( )    </u><br><u>-   ( )    </u><br><u>    8 ( )  </u> | <u>    ( )    </u><br><u>-   ( )    </u><br><u>    (3)    </u> |

If you find that the children have greater difficulty with one type, use the previous page to work through many examples until they understand how it is done.

A variety of ways to initiate experiments are shown. Work through these yourself, concentrating on the procedure used, and anticipating what problems will be faced by the children.

Before going on to the page, you could do substantially the same experiment (outlined on the previous page) using two children to perform the actions. This gives everyone a chance to see what is going on and can help overcome problems before the page is presented.

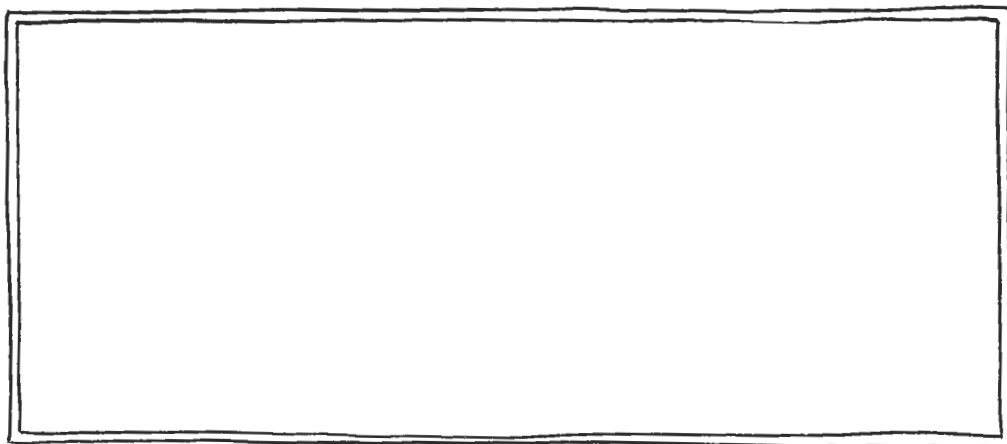
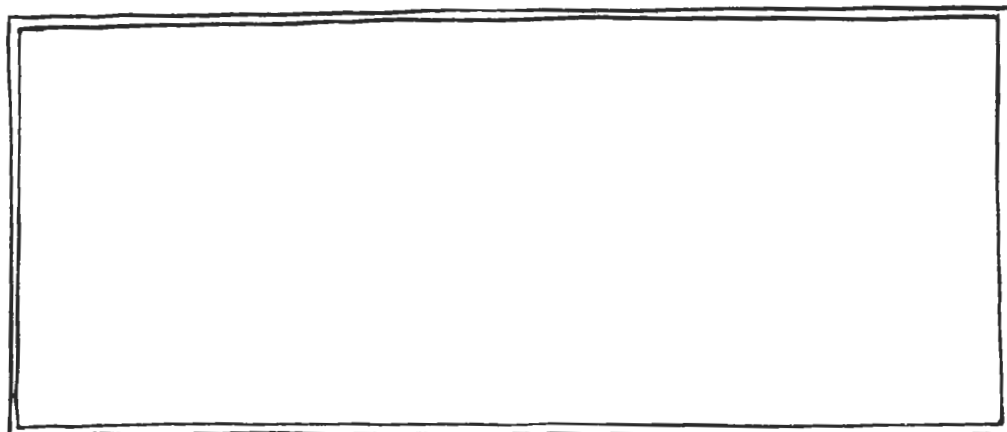
The first two experiments are similar to what was done on the previous page. The question here is, "How many are in the top box after moving some to the bottom box?"

The next two are different in that the question is, "How many were moved to the bottom box?"

The first two in the second row focus on the question, "How many did we start with in the top box before any were moved?"

The last two are more open-ended and we want to know, "How many ways can you carry out the experiment to end up with a given number in the top box?"

STICKS 5 PALOS



|    |      |
|----|------|
| 5  | (1)  |
| 10 | (2)  |
|    | (3)  |
|    | (4)  |
|    | (5)  |
|    | (6)  |
|    | (7)  |
|    | (8)  |
|    | (9)  |
|    | (10) |

$$\begin{array}{r} \underline{\underline{20}} (4) \\ - \underline{\underline{5}} (1) \\ \hline \quad (3) \end{array}$$

$$\begin{array}{r} \underline{\underline{30}} (6) \\ - \underline{\underline{15}} (3) \\ \hline \quad ( ) \end{array}$$

$$\begin{array}{r} \underline{\underline{\quad}} (5) \\ - \underline{\underline{\quad}} (2) \\ \hline \quad ( ) \end{array}$$

$$\begin{array}{r} \underline{\underline{35}} ( ) \\ - \underline{\underline{15}} ( ) \\ \hline \quad ( ) \end{array}$$

$$\begin{array}{r} \underline{\underline{40}} ( ) \\ - \underline{\underline{25}} ( ) \\ \hline \quad ( ) \end{array}$$

$$\begin{array}{r} \underline{\underline{\quad}} (10) \\ - \underline{\underline{\quad}} (7) \\ \hline \quad ( ) \end{array}$$

$$\begin{array}{r} \underline{\underline{40}} ( ) \\ - \underline{\underline{\quad}} (3) \\ \hline \quad ( ) \end{array}$$

$$\begin{array}{r} \underline{\underline{\quad}} ( ) \\ - \underline{\underline{\quad}} (8) \\ \hline \quad (1) \end{array}$$

USING 5-STICKS ONLY

NAME \_\_\_\_\_

|  |  |
|--|--|
|  |  |
|  |  |

|  |  |
|--|--|
|  |  |
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| BEANS | STICKS |
|-------|--------|
| 5     | (1)    |
| 10    | (2)    |
|       | (3)    |
|       | (4)    |
|       | (5)    |
|       | (6)    |
|       | (7)    |
|       | (8)    |
|       | (9)    |
|       | (10)   |

|          |           |           |           |
|----------|-----------|-----------|-----------|
| 20 (4)   | 30 (6)    | ___ (5)   | 35 ( )    |
| - 5 (1)  | - 15 (3)  | - ___ (2) | - 15 ( )  |
| ___ (3)  | ___ ( )   | ___ ( )   | ___ ( )   |
|          |           |           |           |
| 40 ( )   | ___ (10)  | 40 ( )    | ___ ( )   |
| - 25 ( ) | - ___ (7) | - ___ (3) | - ___ (8) |
| ___ ( )  | ___ ( )   | ___ ( )   | ___ (1)   |

This is a continuation of the activity on the previous page.

# STICKS --- PALOS

FRIJOLES  
BEANS

PALOS  
STICKS

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

(9)

(10)

(11)

(12)

(13)

(14)

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USING      STICKS

NAME                     

| BEANS       | STICKS      |             |             |             |
|-------------|-------------|-------------|-------------|-------------|
| (1)         | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |
| (2)         | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |
| (3)         | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |
| (4)         | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |
| (5)         | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |
| (6)         | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |
| (7)         | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |
| (8)         | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |
| (9)         | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |
| (10)        | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |
| (11)        | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |
| (12)        | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |
| (13)        | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |
| (14)        | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |
| <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |
| <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> | <u>    </u> |

This page is used to carry out either addition or subtraction experiments. You can prescribe the kind of sticks to use and provide the numbers for each experiment or leave any part of this to the children. (Simply put a cross through the subtraction sign to make an addition sign.)

Here are some ways to use this page:

(1) Fill in two of the three stick numbers ( ) in each experiment such as:

$$\begin{array}{r}
 \text{---} (7) \\
 - \text{---} (2) \\
 \hline
 \text{---} ( )
 \end{array}
 \quad
 \begin{array}{r}
 \text{---} (5) \\
 - \text{---} ( ) \\
 \hline
 \text{---} (2)
 \end{array}
 \quad
 \begin{array}{r}
 \text{---} ( ) \\
 - \text{---} (3) \\
 \hline
 \text{---} (5)
 \end{array}$$

This provides practice for any multiplication table you wish by merely filling in the number at the top of the page. Use      sticks only.) Make one page for addition and one for subtraction.

$$\begin{array}{r}
 \text{---} (4) \\
 - \text{---} (5) \\
 \hline
 \text{---} ( )
 \end{array}
 \quad
 \begin{array}{r}
 \text{---} (8) \\
 - \text{---} ( ) \\
 \hline
 \text{---} (2)
 \end{array}
 \quad
 \begin{array}{r}
 \text{---} (7) \\
 - \text{---} ( ) \\
 \hline
 \text{---} (5)
 \end{array}$$

(2) Fill in all the bottom lines with the same number (either sticks or beans). The problem is to find as many different addition and subtraction experiments which yield that result.

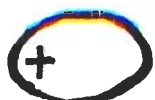
$$\begin{array}{r}
 \text{---} ( ) \\
 - \text{---} ( ) \\
 \hline
 (9)
 \end{array}
 \quad
 \begin{array}{r}
 \text{---} ( ) \\
 - \text{---} ( ) \\
 \hline
 (9)
 \end{array}$$

(3) You may find that a child has difficulty with a particular multiplication combination. On this page have him use that fact in many places. This enables him to see it in a variety of situations and will facilitate its learning. Consider a child who needs more practice on  $4 \times 8 = 32$ :

USING 4 STICKS

$$\begin{array}{r}
 \text{---} (3) \\
 + \text{---} (5) \\
 \hline
 \text{---} ( )
 \end{array}
 \quad
 \begin{array}{r}
 \text{---} (8) \\
 - \text{---} ( ) \\
 \hline
 \text{---} ( )
 \end{array}$$

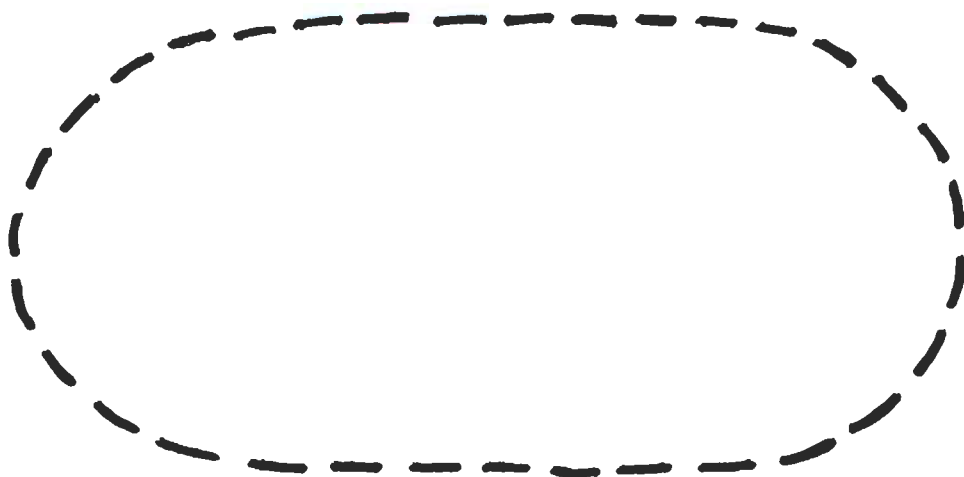
$$\begin{array}{r}
 \text{---} ( ) \\
 - \text{---} (8) \\
 \hline
 \text{---} ( )
 \end{array}
 \quad
 \begin{array}{r}
 \text{---} (9) \\
 - \text{---} (1) \\
 \hline
 \text{---} ( )
 \end{array}$$



PUT  
PON



TAKE  
TOMA



$$4 \text{ (+2) } 6 \text{ (+3) } 9$$

$$10 \text{ (-1) } \text{(-5) } 4$$

$$5 \text{ (-1) } \text{(+3) } 7$$

$$5 \text{ (+3) } \text{(-1) }$$

$$8 \text{ (+2) } \text{(+5) }$$

$$8 \text{ (+5) } \text{(+2) }$$

$$12 \text{ (-4) } \text{(-2) }$$

$$12 \text{ (-2) } \text{(-4) }$$

$$9 \text{ (-4) } \text{(+5) }$$

$$9 \text{ (+5) } \text{(-4) }$$

$$7 \text{ (+3) } \text{(+4) }$$

$$7 \text{ (+4) } \text{(+3) }$$

NAME \_\_\_\_\_

 AND   
 PUT AND TAKE

|                                    |                            |
|------------------------------------|----------------------------|
| $4 \quad (+2) \quad (+3)$<br>START | $10 \quad (-1) \quad (-5)$ |
| $5 \quad (-1) \quad (+3)$          | $5 \quad (+3) \quad (-1)$  |
| $8 \quad (+2) \quad (+5)$          | $8 \quad (+5) \quad (+2)$  |
| $12 \quad (-4) \quad (-2)$         | $12 \quad (-2) \quad (-4)$ |
| $9 \quad (-4) \quad (+5)$          | $9 \quad (+5) \quad (-4)$  |
| $7 \quad (+3) \quad (+4)$          | $7 \quad (+4) \quad (+3)$  |

Beginning with the second line, the examples build toward an idea that the order in which you put or take makes no difference. If children notice this, they should be encouraged to keep testing it as a theory.

Counters are needed for this activity. In each problem a starting number is given. The learner places that many counters in the large dotted shape at the top of the page. Then he follows the "instructions" in the problem. + tells him to **put** a certain number of counters into the space, and - tells him to **take** a certain number out. Spaces are provided for recording results at each step. For example,

$$5 \quad (+3) \quad (-1)$$

tells the learner to start with five counters in the space, put three more in, record the result, take one out, and record that result. Thus the record would be:

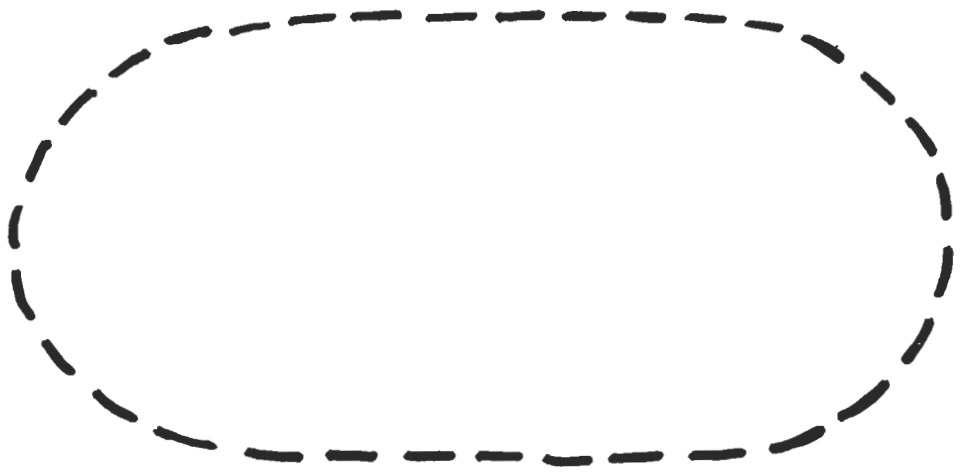
$$5 \quad (+3) \quad 8 \quad (-1) \quad 7$$



PUT  
PON



TAKE  
TOMA



NAME \_\_\_\_\_

 AND   
 PUT AND TAKE

START

(3) "put" and "take" the same number, e.g.,

$$\begin{array}{rcl}
 3 & (+2) & (-2) \\
 \hline
 4 & (+3) & (-3) \\
 \hline
 5 & (+4) & (-4) \\
 \hline
 \end{array}
 \text{ etc.}$$

This report form can be particularized by the teacher or used by the learners for making up their own experiments. There are many possible investigations. Just a few suggestions:

(1) fill in the same "put" and "take" instructions for several examples, and use sequential starting numbers, e.g.,

$$\begin{array}{rcl}
 1 & (+2) & (+3) \\
 \hline
 2 & (+2) & (+3) \\
 \hline
 3 & (+2) & (+3) \\
 \hline
 \end{array}
 \text{ etc.}$$

(2) starting with 1, set up a "put and take" situation to essentially generate a table of triples, e.g.,

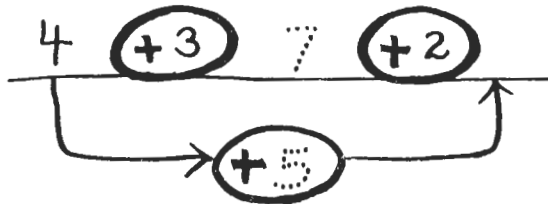
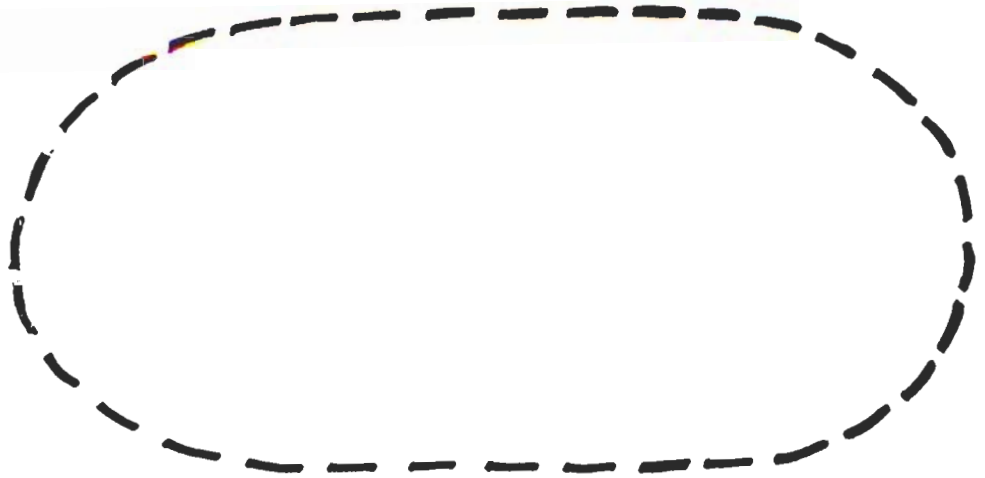
$$\begin{array}{rcl}
 1 & (+1) & (+1) \\
 \hline
 2 & (+2) & (+2) \\
 \hline
 3 & (+3) & (+3) \\
 \hline
 \end{array}
 \text{ etc.}$$



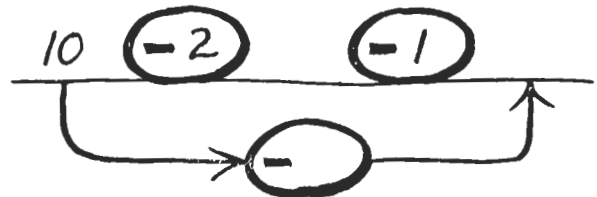
PUT  
PON



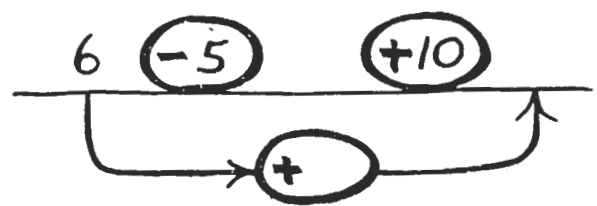
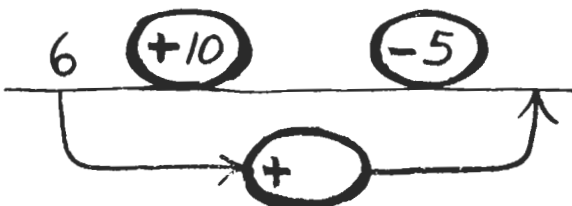
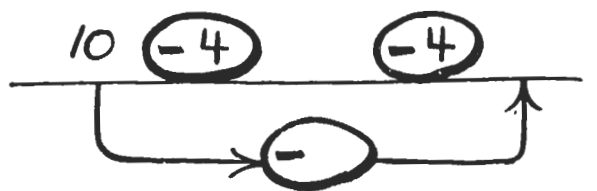
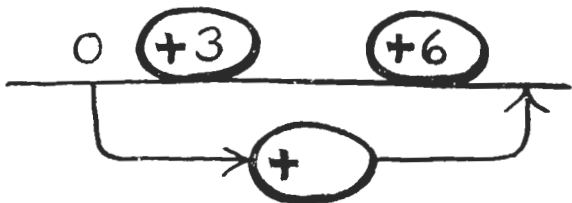
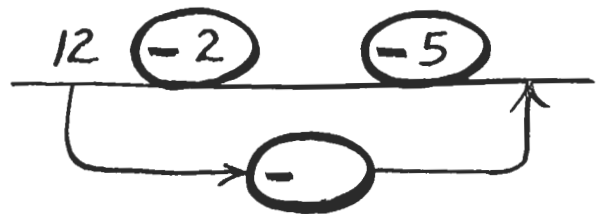
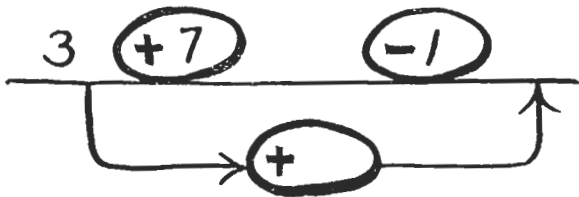
TAKE  
TOMA



SHORTCUT



CAMINO CORTO

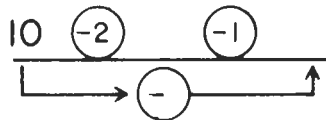


NAME \_\_\_\_\_

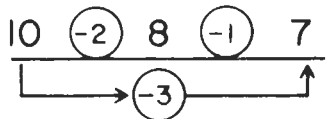
$\oplus$  AND  $\ominus$   
 PUT AND TAKE

|                                         |                                          |
|-----------------------------------------|------------------------------------------|
| $4 \quad \oplus 3 \quad \oplus 2$<br>   | $10 \quad \ominus 2 \quad \ominus 1$<br> |
| $3 \quad \oplus 7 \quad \ominus 1$<br>  | $12 \quad \ominus 2 \quad \ominus 5$<br> |
| $0 \quad \oplus 3 \quad \oplus 6$<br>   | $10 \quad \ominus 4 \quad \ominus 4$<br> |
| $6 \quad \oplus 10 \quad \ominus 5$<br> | $6 \quad \ominus 5 \quad \oplus 10$<br>  |

This activity is similar to that on page 52A. However, on this page, the child is also asked to find **one** “put” or “take” that will have the same effect on the starting number as the two did. Thus



asks the learner to start with 10 counters, take out two of them, record the result (8), take out one and record the result (7). Then he must start with 10 and find **one** “take” that will leave 7 counters. The solution would be:



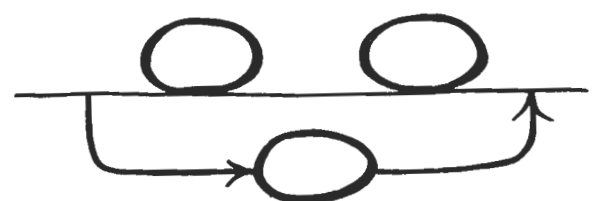
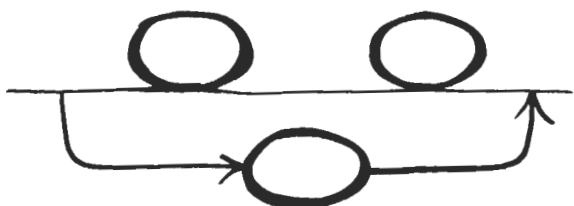
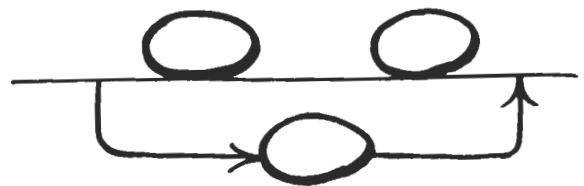
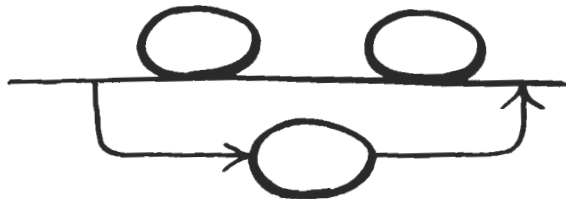
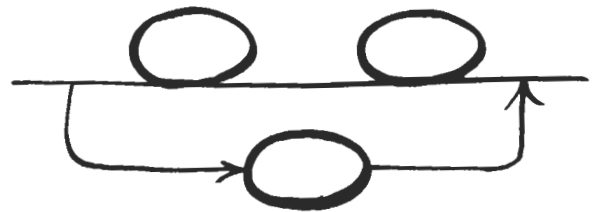
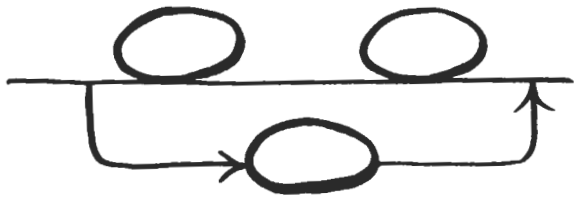
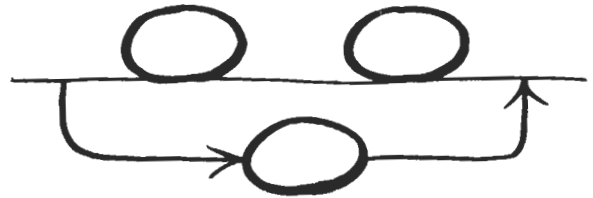
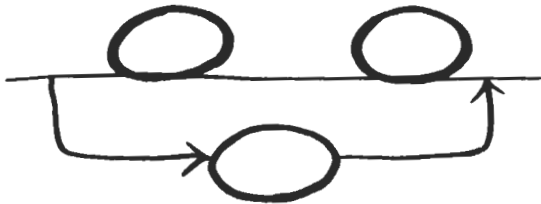
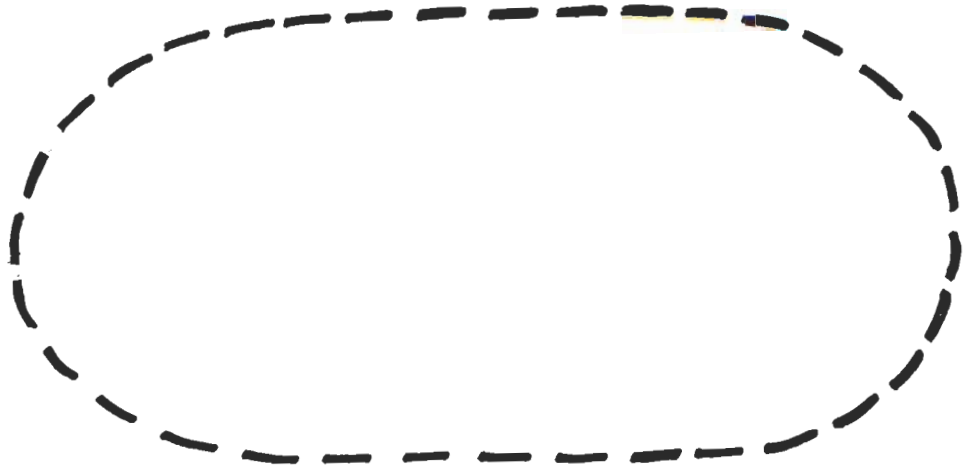
This activity begins building a referent for operations with positive and negative numbers. Taking 2 ( $-2$ ) and then taking 5 ( $-5$ ) is the same as taking 7 ( $-7$ ), ( $-2 - 5 = -7$ ). Taking 5 ( $-5$ ) and then putting 3 ( $+3$ ) is the same as taking 2 ( $-2$ ) ( $-5 + 3 = -2$ ).



PUT  
PON



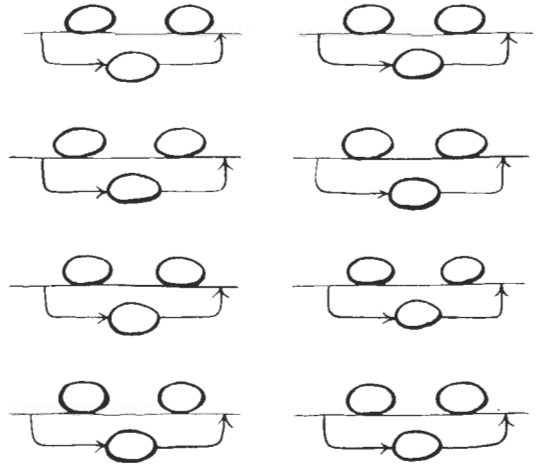
TAKE  
TOMA





NAME \_\_\_\_\_



starting and ending number so that the ending number is larger than the starting number. Then using only "put," how many ways can you get from one number to the other. For example, how many ways are there to get from 4 to 9, using only "put"?

$$4 \quad (+1) \quad 5 \quad (+4) \quad 9$$

and

$$4 \quad (+2) \quad 6 \quad (+3) \quad 9$$

are just two of the possibilities.

Full instructions for this kind of activity are on the previous page.

This report form can be particularized by the teacher or used by the learner for his own experiments. One possible investigation might be to fill in the starting number and ending number and one "put" or "take" in the lower line of each example.

$$3 \quad \bigcirc \quad \bigcirc \quad 7$$

$$\downarrow \quad \rightarrow (+4) \quad \uparrow$$

$$3 \quad \bigcirc \quad \bigcirc \quad 7$$

$$\downarrow \quad \rightarrow (+4) \quad \uparrow$$

$$3 \quad \bigcirc \quad \bigcirc \quad 7$$

$$\downarrow \quad \rightarrow (+4) \quad \uparrow$$

The problem then is: What different "put and take" combinations could have been used?

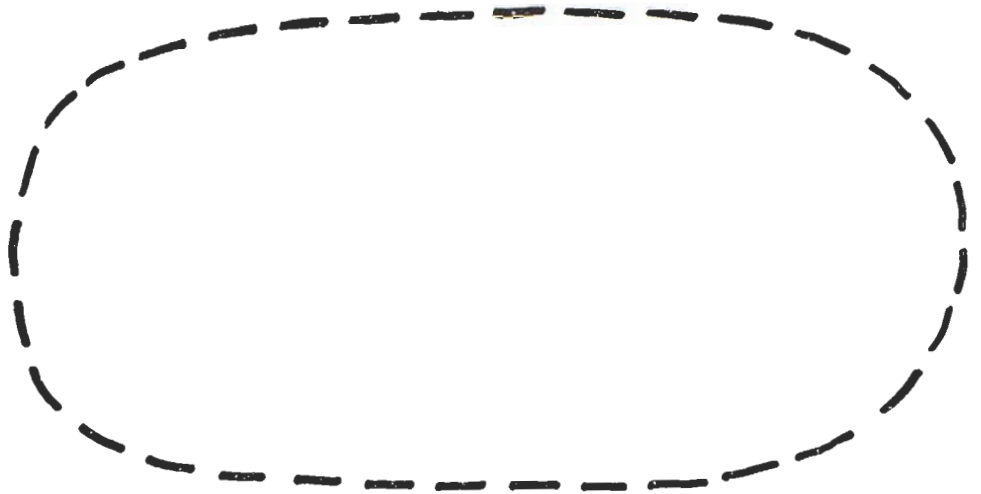
A second investigation might begin by choosing a



PUT  
PON



TAKE  
TOMA



$$4 \quad (+2) (+3) \quad ?$$

$$9 \quad (-2) (-3)$$

$$6 \quad (+3) (-4)$$

$$6 \quad (-4) (+3)$$

$$3 \quad (+5) (+5)$$

$$6 \quad (+3) (-2) \quad (-3) (+2)$$

$$12 \quad (-2) (-3)$$

$$6 \quad (-2) (+3) \quad (+2) (-3)$$

$$7 \quad (+3) (-2)$$

$$8 \quad (+2) (+5) \quad (-2) (-5)$$

$$7 \quad (-2) (+3)$$

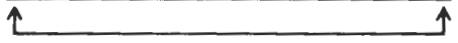
$$1 \quad (+3) (+3) \quad (+3) (-3)$$

 AND   
 PUT AND TAKE

NAME \_\_\_\_\_  


|                             |                                     |
|-----------------------------|-------------------------------------|
| $4 \quad (+2) (+3) \quad ?$ | $9 \quad (-2) (-3)$                 |
| $6 \quad (+3) (-4)$         | $6 \quad (-4) (+3)$                 |
| $3 \quad (+5) (+5)$         | $6 \quad (+3) (-2) \quad (-3) (+2)$ |
| $12 \quad (-2) (-3)$        | $6 \quad (-2) (+3) \quad (+2) (-3)$ |
| $7 \quad (+3) (-2)$         | $8 \quad (+2) (+5) \quad (-2) (-5)$ |
| $7 \quad (-2) (+3)$         | $1 \quad (+3) (+3) \quad (+3) (-3)$ |

This activity is similar to the previous ones, but now asks the learner to do two "put" or "take" operations at a time without separately recording the result of the first one. Several examples on this page may stir curiosity for a further investigation.

- (1)  $4 \quad (+2) (+3) \quad 9$  and  $9 \quad (-2) (-3) \quad 4$
- (2)  $6 \quad (+3) (-4) \quad 5$  and  $6 \quad (-4) (+3) \quad 5$
- (3)  $6 \quad (+3) (-2) \quad 7 \quad (-3) (+2) \quad 6$
- 

Examples (1) and (3) might lead a child to wonder if he can always reverse the starting and ending numbers, change the "puts" to "takes" and vice versa, and end up with a "put and take" statement that works.

Example (2) might lead him to wonder if the order of "put" and "take" makes any difference in the final result.



PUT  
PON



TAKE  
TOMA



This page can be used for independent investigations with "put" and "take" or can be particularized by the teacher or learner. Some suggestions:

- (1) It can incorporate practice in adding or subtracting numbers that may be difficult for a learner. Consider the many possibilities that are available even simply using  $+3$ ,  $-3$ , and  $+2$ ,  $-2$  throughout the page.

$$\begin{array}{rcl}
 3 & (+3) & (-3) \\
 \hline
 3 & (-2) & (+3)
 \end{array}
 \quad
 \begin{array}{rcl}
 2 & (+2) & (-3) \\
 \hline
 2 & (-2) & (+2)
 \end{array}
 \text{ etc.}$$

- (2) Combinations for 10 can be used to give practice with these combinations and to help learners see the pattern produced by adding 10. For example:

$$\begin{array}{rcl}
 2 & (+7) & (+3) & 12 \\
 \hline
 3 & (+9) & (+1) & 13 \\
 \hline
 4 & (+6) & (+4) & 14
 \end{array}$$

- (3) Several pairs of two-step "put and take" examples can be constructed to prompt learners to investigate whether an "in-any-order" rule holds. For example:

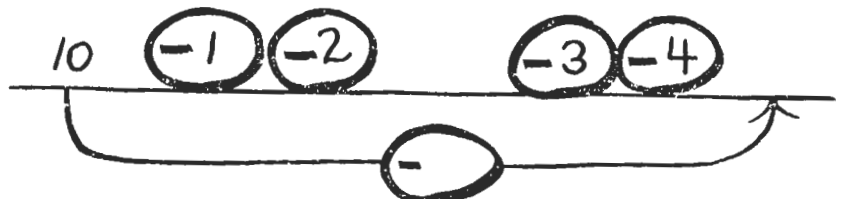
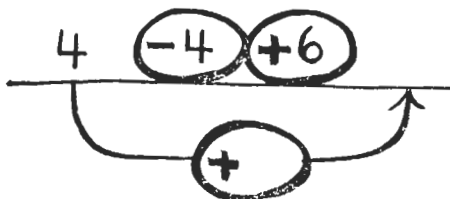
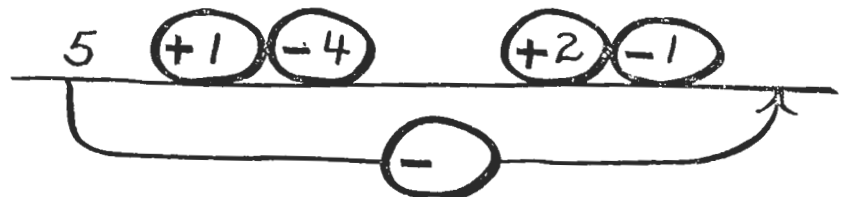
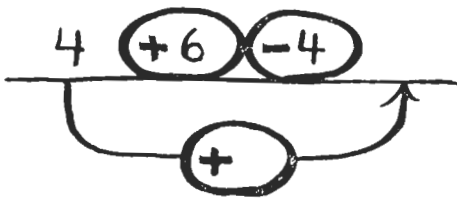
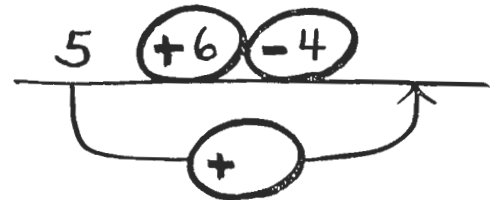
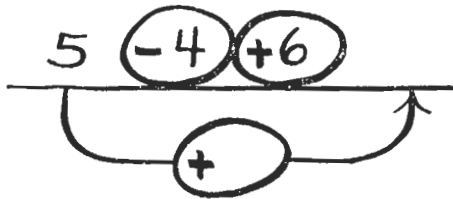
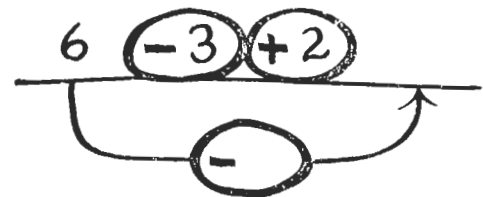
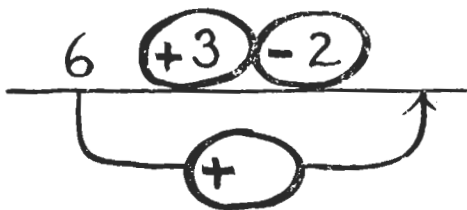
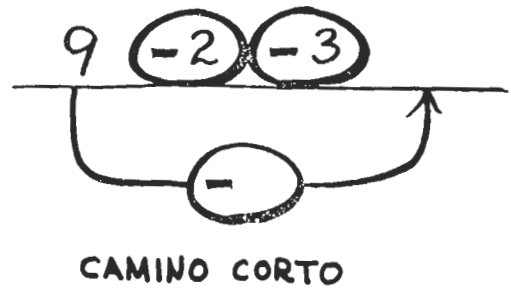
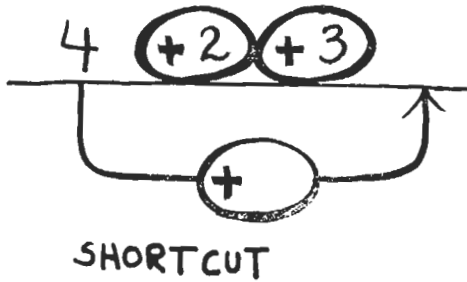
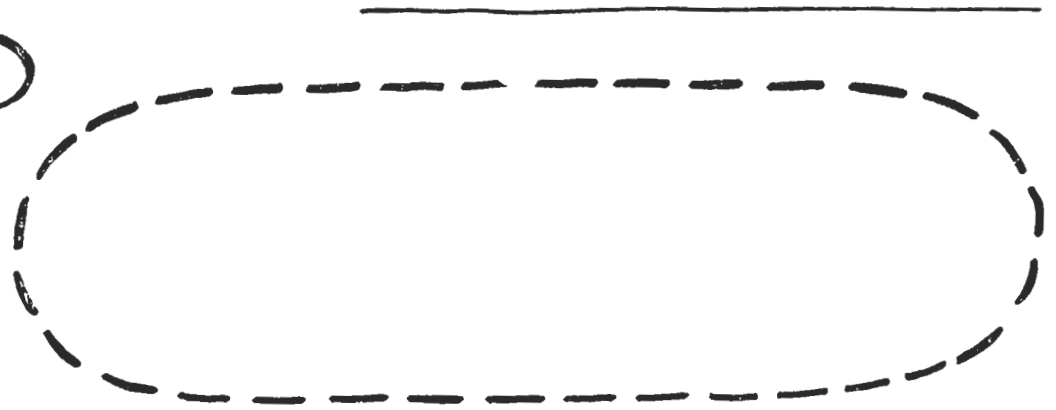
$$\begin{array}{rcl}
 3 & (+2) & (-3) \\
 \hline
 3 & (-3) & (+2)
 \end{array}
 \text{ and }
 \begin{array}{rcl}
 4 & (+1) & (-2) \\
 \hline
 4 & (-2) & (+1)
 \end{array}
 \text{ etc.}$$



PUT  
PON

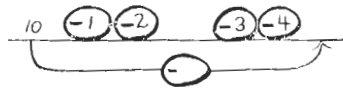
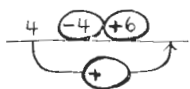
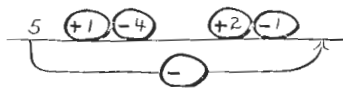
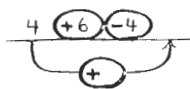
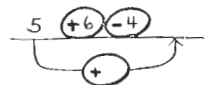
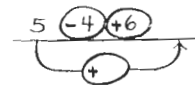
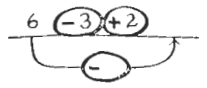
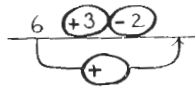
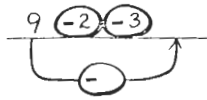
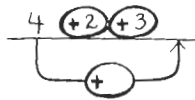


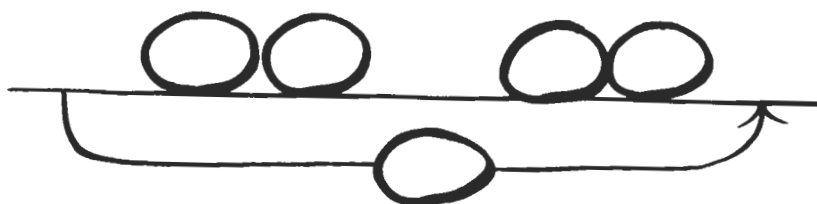
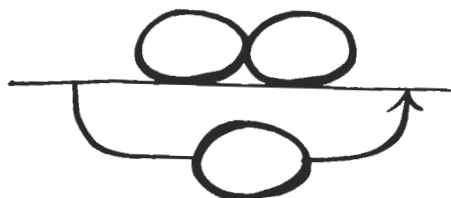
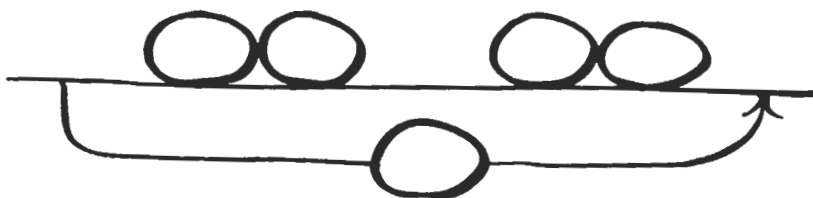
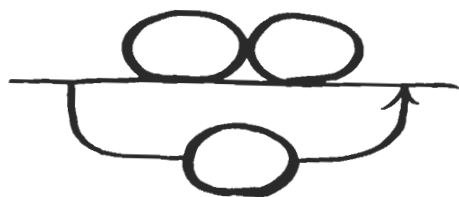
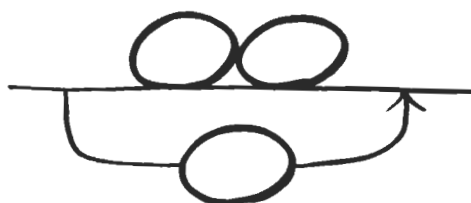
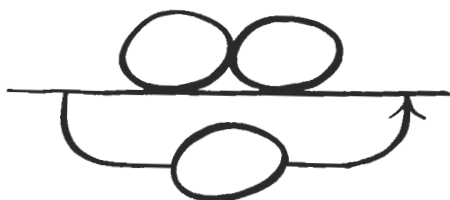
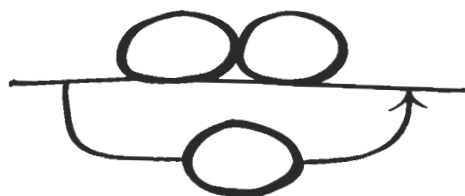
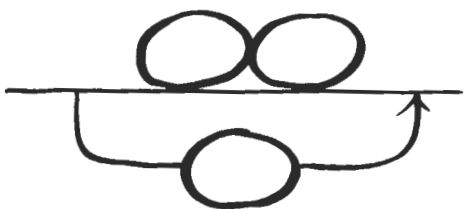
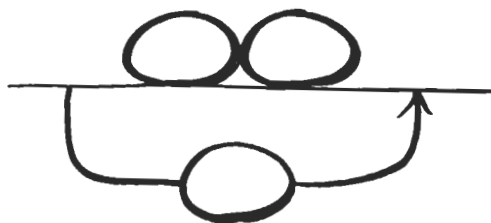
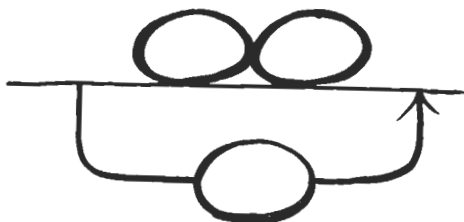
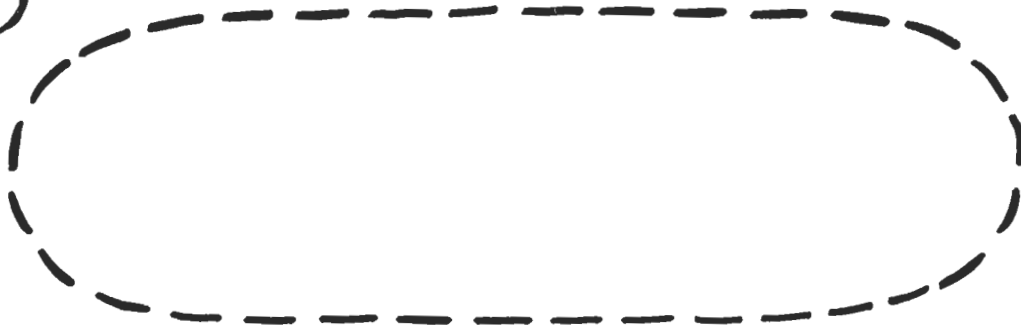
TAKE  
TOMA



$+$  AND  $-$   
PUT AND TAKE

NAME \_\_\_\_\_







+

AND

-

PUT

AND

TAKE

NAME \_\_\_\_\_

|  |  |
|--|--|
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

This form can be used for independent investigations or can be particularized by the teacher. There are again many possible kinds of investigations. A few suggestions:

- (1) Fill in the starting and ending members, and one part (or two parts in the examples with four "puts and takes") of the "put and take" sequence in the top line.

The question is then: Can you figure out what the other "put or take" was?

- (2) Given a starting and an ending number, how many ways can you find to get from one to the other? How many shortcuts can you find?
- (3) Given a starting and an ending number, can you find a two-step and a four-step "put and take" that will work?

+

-

$$\begin{array}{r} 4 \\ +2 \\ +3 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 9 \\ -2 \\ -3 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ +3 \\ -5 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ -8 \\ +5 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ +2 \\ +6 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ -7 \\ -2 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ +2 \\ -3 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ +10 \\ -1 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ +4 \\ +5 \\ \hline \end{array}$$

$$\begin{array}{r} 20 \\ -7 \\ -3 \\ \hline \end{array}$$

$$\begin{array}{r} +5 \\ -3 \\ \hline 10 \end{array}$$

$$\begin{array}{r} -8 \\ +2 \\ \hline 10 \end{array}$$

+

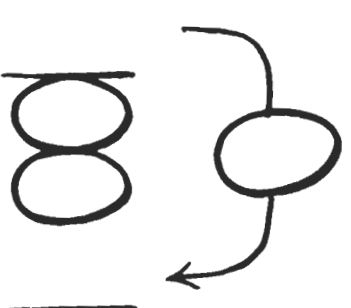
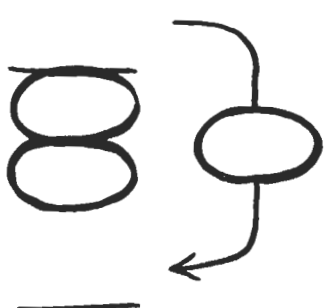
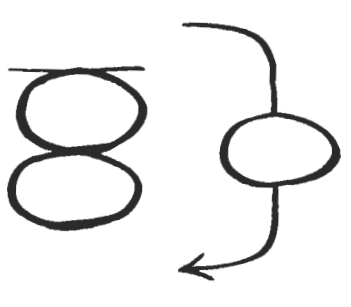
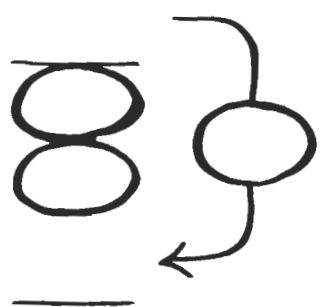
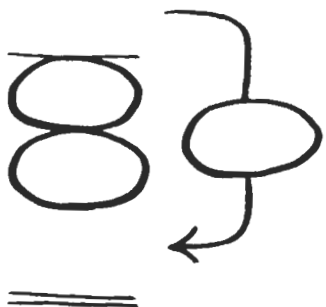
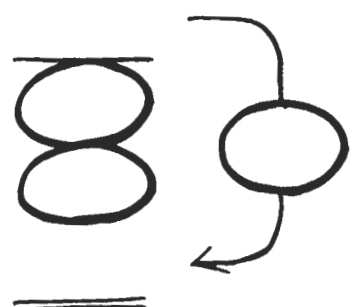
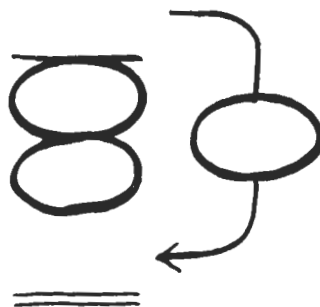
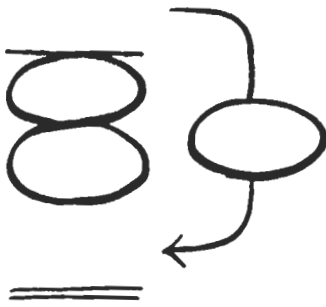
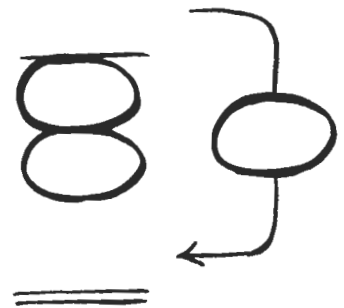
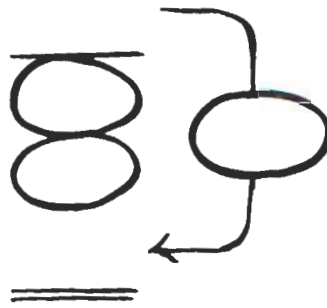
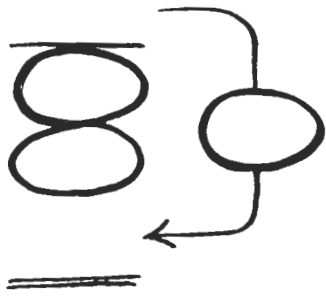
AND

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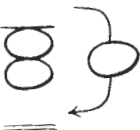
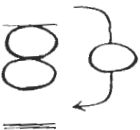
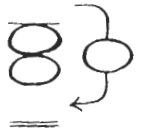
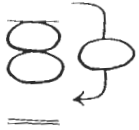
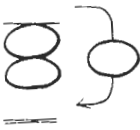
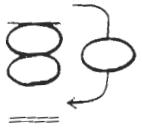
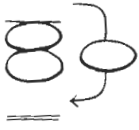
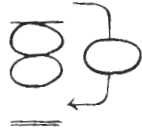
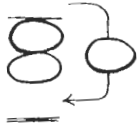
|                                                         |                                                         |                                                         |
|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|
| $\begin{array}{r} 4 \\ +2 \\ +3 \\ \hline \end{array}$  | $\begin{array}{r} 9 \\ -2 \\ -3 \\ \hline \end{array}$  | $\begin{array}{r} 7 \\ +3 \\ -5 \\ \hline \end{array}$  |
| $\begin{array}{r} 10 \\ -8 \\ +5 \\ \hline \end{array}$ | $\begin{array}{r} 7 \\ +2 \\ +6 \\ \hline \end{array}$  | $\begin{array}{r} 17 \\ -7 \\ -2 \\ \hline \end{array}$ |
| $\begin{array}{r} 13 \\ +2 \\ - \\ \hline \end{array}$  | $\begin{array}{r} 7 \\ +10 \\ -1 \\ \hline \end{array}$ | $\begin{array}{r} 6 \\ +4 \\ +5 \\ \hline \end{array}$  |
| $\begin{array}{r} 20 \\ -7 \\ -3 \\ \hline \end{array}$ | $\begin{array}{r} +5 \\ -3 \\ \hline 10 \end{array}$    | $\begin{array}{r} -8 \\ +2 \\ \hline 10 \end{array}$    |

On this page, there is no space for placing counters. However, children who still want or need to use them should have a supply that they can use on their desks. Vertical notation is introduced for the "put" and "take" activity.



O and O

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$\times$

$\div$

$$\begin{array}{r} 4 \\ \times 2 \\ \times 3 \\ \hline 24 \end{array}$$

$\times 6$

$$\begin{array}{r} 3 \\ \times 2 \\ \times 5 \\ \hline \end{array}$$

$\times$

$$\begin{array}{r} 4 \\ \times 3 \\ \times 3 \\ \hline \end{array}$$

$\times$

$$\begin{array}{r} 24 \\ \div 2 \\ \div 3 \\ \hline \end{array}$$

$\div 6$

$$\begin{array}{r} 30 \\ \div 2 \\ \div 5 \\ \hline \end{array}$$

$$\begin{array}{r} 30 \\ \div 5 \\ \div 2 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ \times 6 \\ \div 2 \\ \hline \end{array}$$

$\times 3$

$$\begin{array}{r} 4 \\ \div 2 \\ \times 6 \\ \hline \end{array}$$

$\times$

$$\begin{array}{r} 32 \\ \div 8 \\ \times 2 \\ \hline \end{array}$$

$\div$

$$\begin{array}{r} 20 \\ \div 4 \\ \times 4 \\ \hline \end{array}$$

$\times$

$$\begin{array}{r} 12 \\ \times 2 \\ \div 4 \\ \hline \end{array}$$

$\div$

$$\begin{array}{r} 12 \\ \div 4 \\ \times 2 \\ \hline \end{array}$$

$\div$

 AND 

NAME \_\_\_\_\_

|                                                                       |                                                                    |                                                                    |
|-----------------------------------------------------------------------|--------------------------------------------------------------------|--------------------------------------------------------------------|
| $\begin{array}{r} 4 \\ \times 2 \\ \times 3 \\ \hline 12 \end{array}$ | $\begin{array}{r} 3 \\ \times 2 \\ \times 5 \\ \hline \end{array}$ | $\begin{array}{r} 4 \\ \times 3 \\ \times 3 \\ \hline \end{array}$ |
| $\begin{array}{r} 24 \\ \div 2 \\ \div 3 \\ \hline \end{array}$       | $\begin{array}{r} 30 \\ \div 2 \\ \div 5 \\ \hline \end{array}$    | $\begin{array}{r} 30 \\ \div 5 \\ \div 2 \\ \hline \end{array}$    |
| $\begin{array}{r} 4 \\ \times 6 \\ \div 2 \\ \hline \end{array}$      | $\begin{array}{r} 4 \\ \div 2 \\ \times 6 \\ \hline \end{array}$   | $\begin{array}{r} 32 \\ \div 8 \\ \times 2 \\ \hline \end{array}$  |
| $\begin{array}{r} 20 \\ \div 4 \\ \times 4 \\ \hline \end{array}$     | $\begin{array}{r} 12 \\ \times 2 \\ \div 4 \\ \hline \end{array}$  | $\begin{array}{r} 12 \\ \div 4 \\ \times 2 \\ \hline \end{array}$  |

students may wonder if division always "undoes" multiplication. The blank form on the previous page gives them space to test their thoughts. Other examples lead to consideration of the "in-any-order" rule:

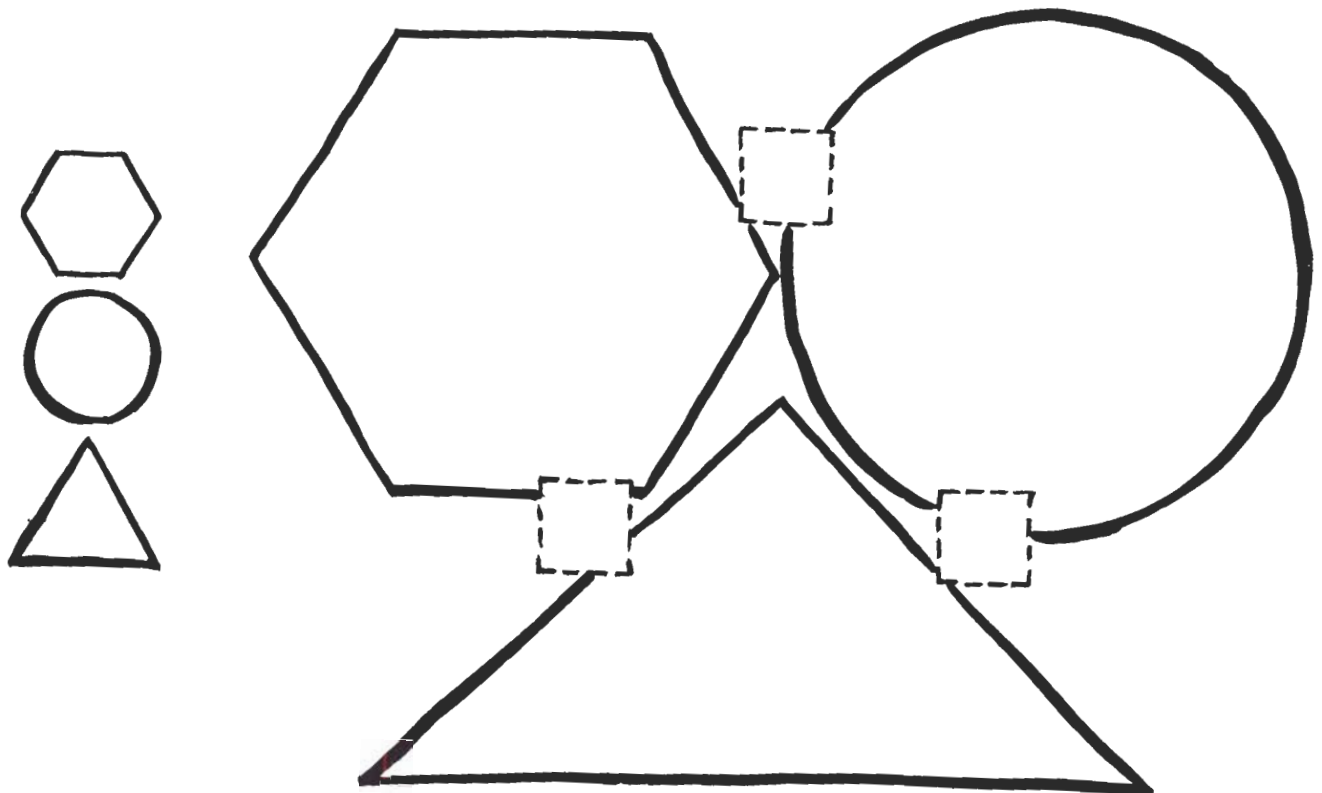
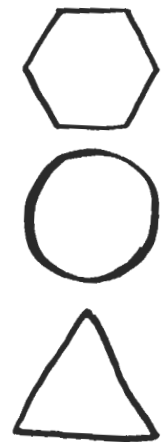
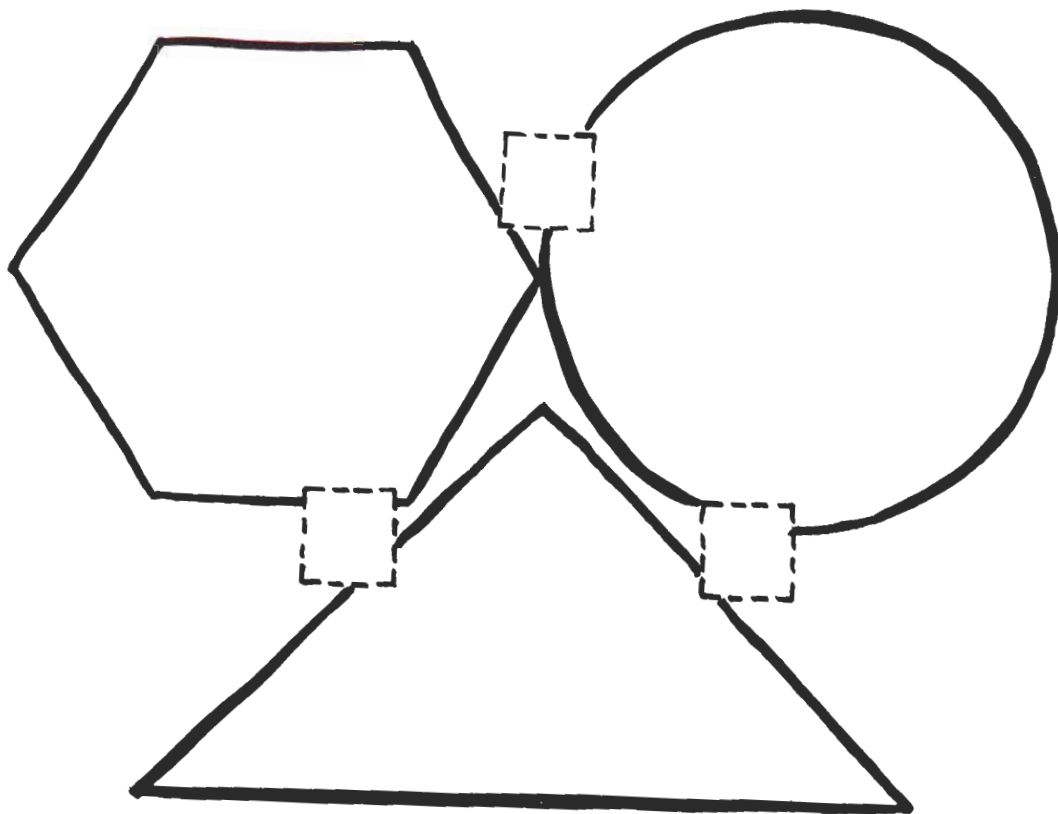
$$\begin{array}{r} 4 \\ \times 6 \\ \div 2 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 4 \\ \div 2 \\ \times 6 \\ \hline 12 \end{array}$$

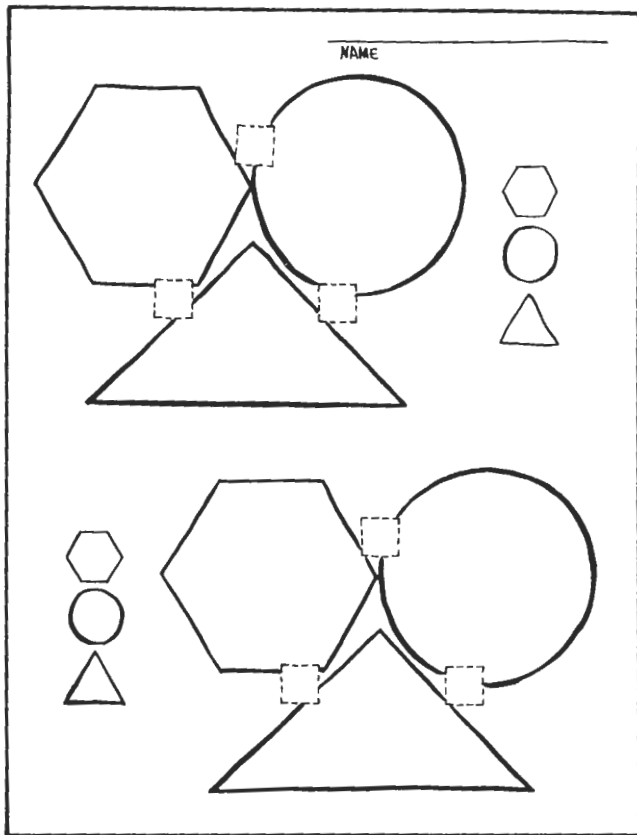
The focus of this page shifts from addition and subtraction to multiplication and division. There are no experiment spaces provided; students work with numbers. The representational counterpart to this activity is found in "Directions to Do Something," pages 77A-77D, and further work at the abstract level can be found in "Chain Reactions," beginning on page 187.

Several of the examples indicate the inverse relationship between multiplication and division. With such problems as

$$\begin{array}{r} 4 \\ \times 2 \\ \times 3 \\ \hline 24 \end{array} \quad \text{and} \quad \begin{array}{r} 24 \\ \div 2 \\ \div 3 \\ \hline 4 \end{array}$$





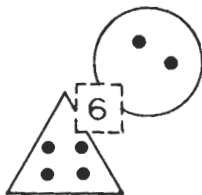


"There are 3 **large** frames—a hexagon, a circle and a triangle on the top half of the page.

"Please put some counters in the large circle . . . and some more in the large triangle. Use any number of counters you like.

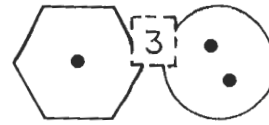
"Now, between the triangle and the circle is a broken line square. In that little square we write the answer to the question—how many are there all together in the circle and the triangle?"

A sketch on the board may be in order:

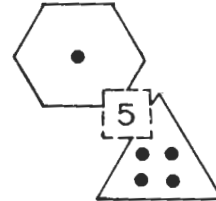


"The frame that's empty is called a hexagon. Please put some counters in it—as many as you please—or leave it empty.

"Now look at the hexagon and the circle. How many counters all together in those two shapes? Write the answer in the little square box between them.



"Next, without moving any counters, look at the hexagon and the triangle. How many together in those two shapes? Show the number in the little square box.



"Now, remove all the counters.

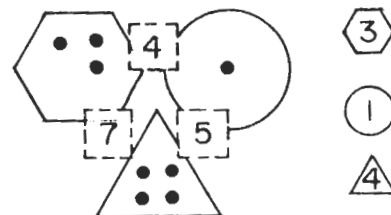
"Let's try another on the lower portion of the page. Use any number of counters you like and fill in all three little squares."

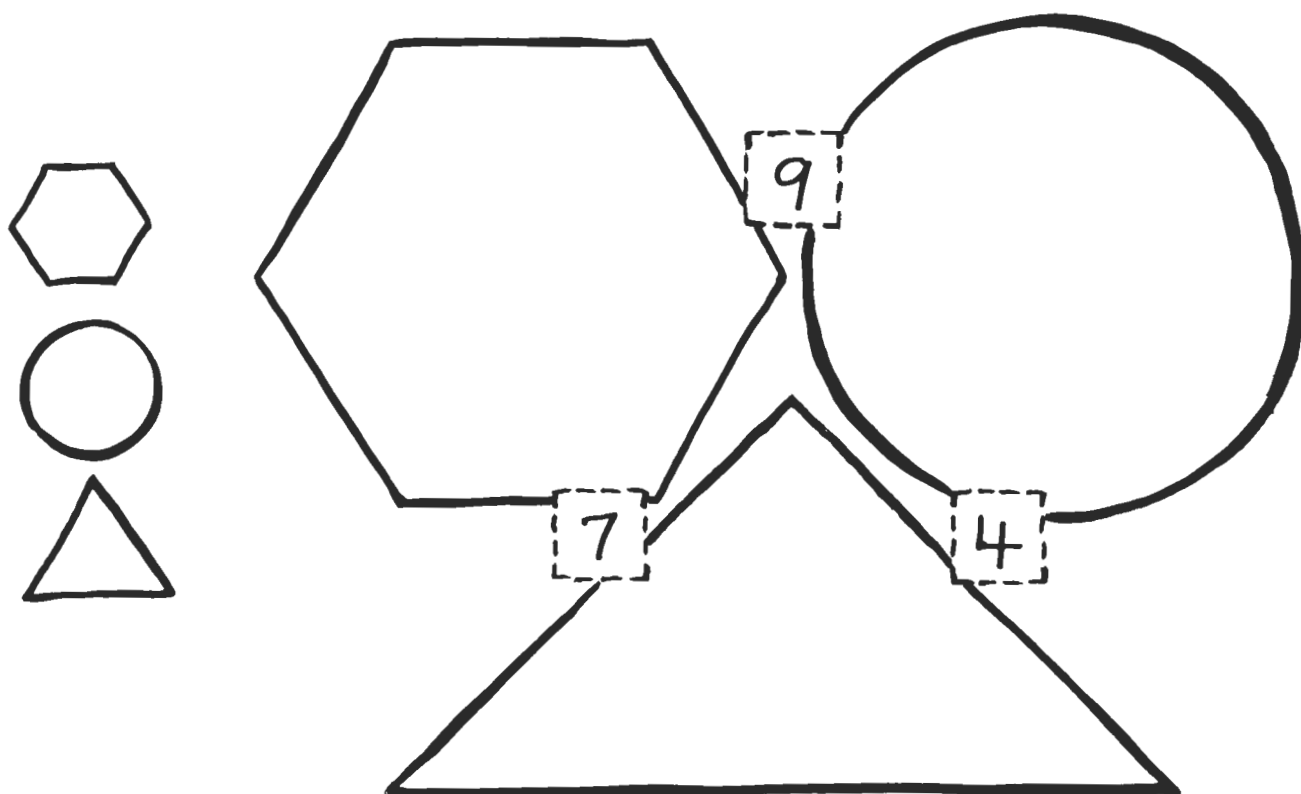
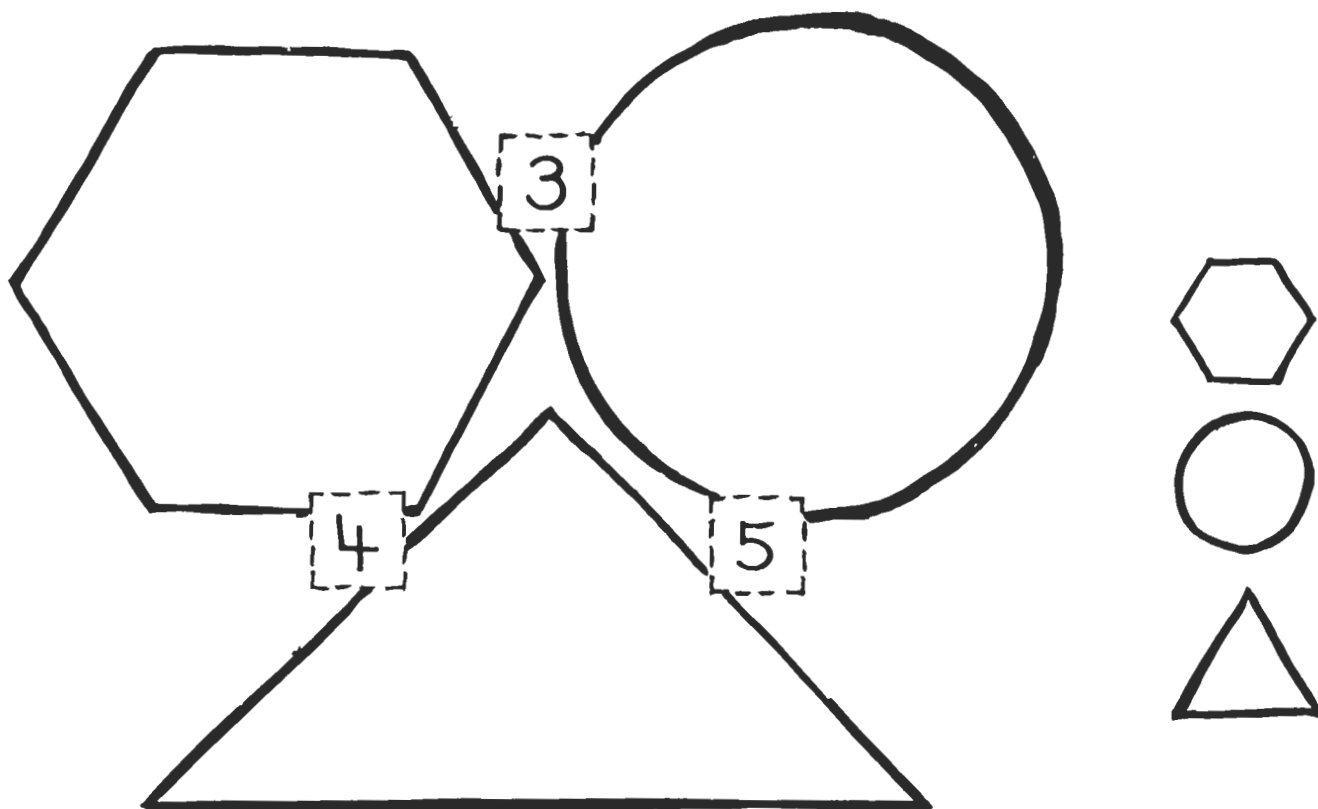
When everyone has finished, ask them to remove all their counters—and be sure their names are on the page.

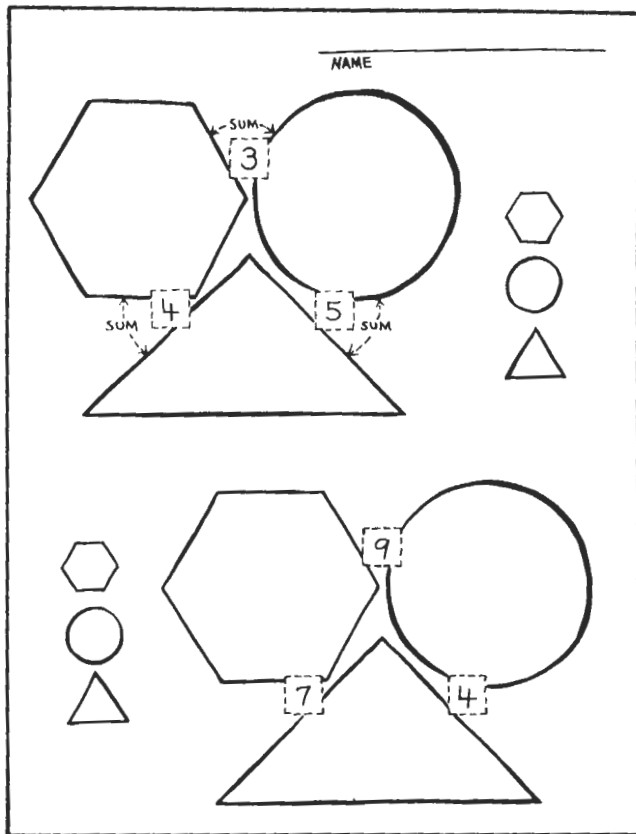
"Please trade your paper with a neighbor. Can you find out how many counters your neighbor used—how many in each large frame? Put counters in the frames—move them around—change them—try again and again until you find a way "that works"—so all the little boxes tell how many counters there are in each pair of frames."

After some work is under way and some solutions have been found, explain the little hexagon, circle and triangle:

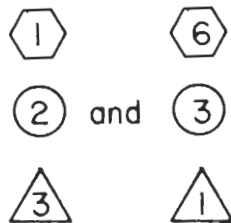
"When you have the right number of counters in a frame, please write that number in the small box at the side that is the same shape."

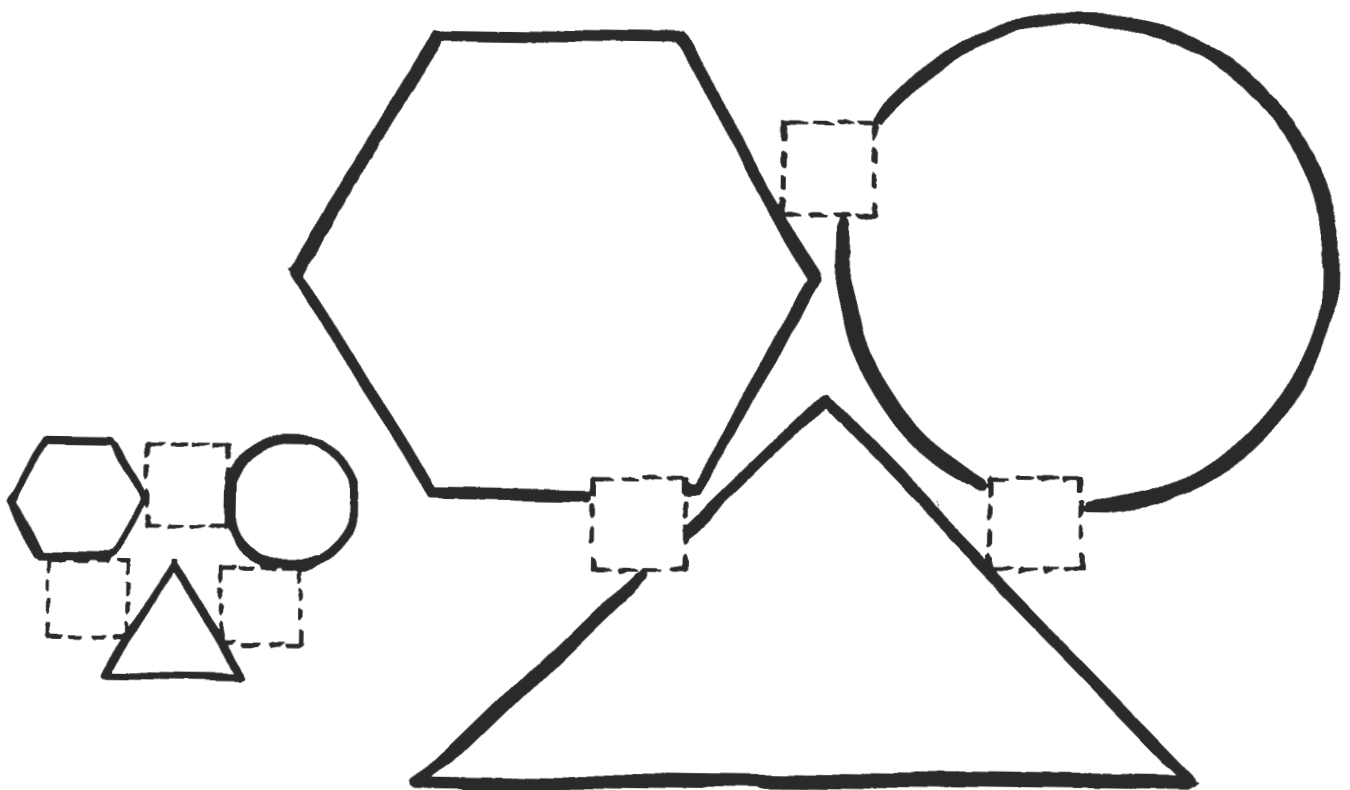
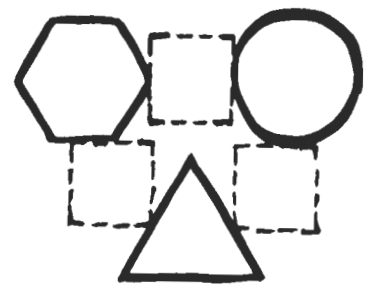
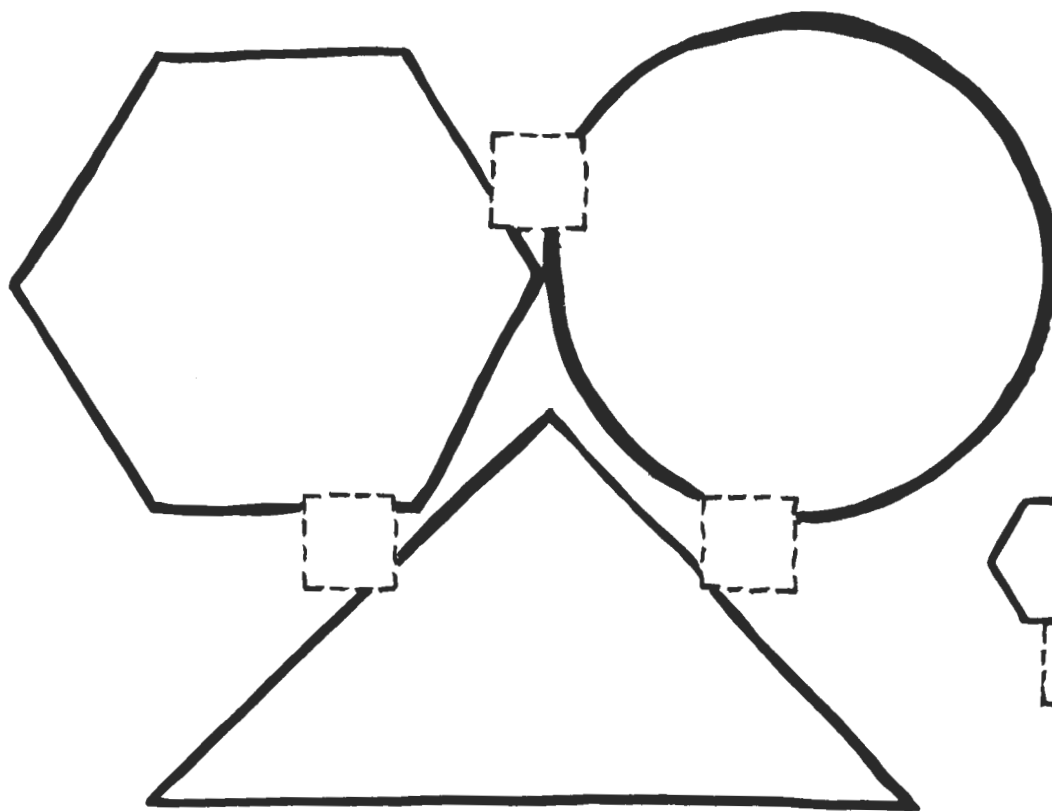


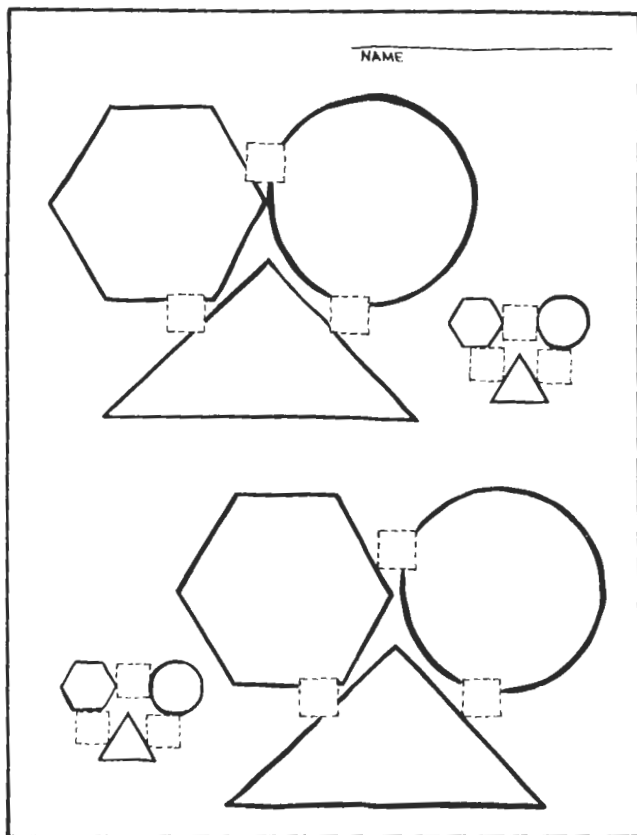




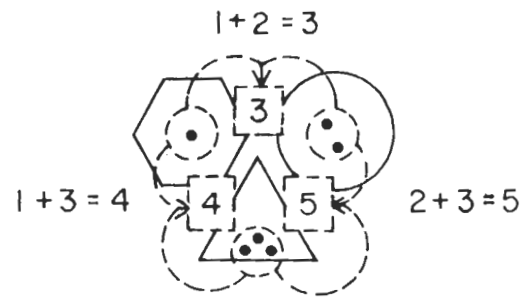
Review the activity on the previous page.  
 Then: "Can you solve these two problems and show  
 the number of counters needed in each frame?"







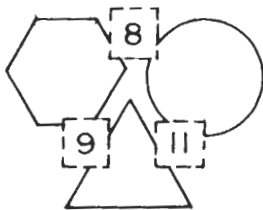
Without finding a solution when only the 3 sums are given, can you predict the number of beans?



In finding the 3 sums, each counter is counted twice so the sum of the sums— $3 + 4 + 5$  or 12—must be twice the number of beans used . . .  $1 + 2 + 3$  or 6, in this case.

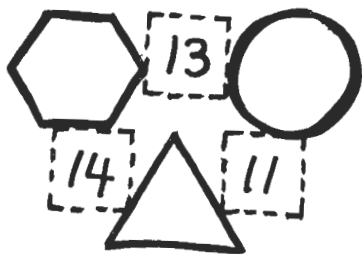
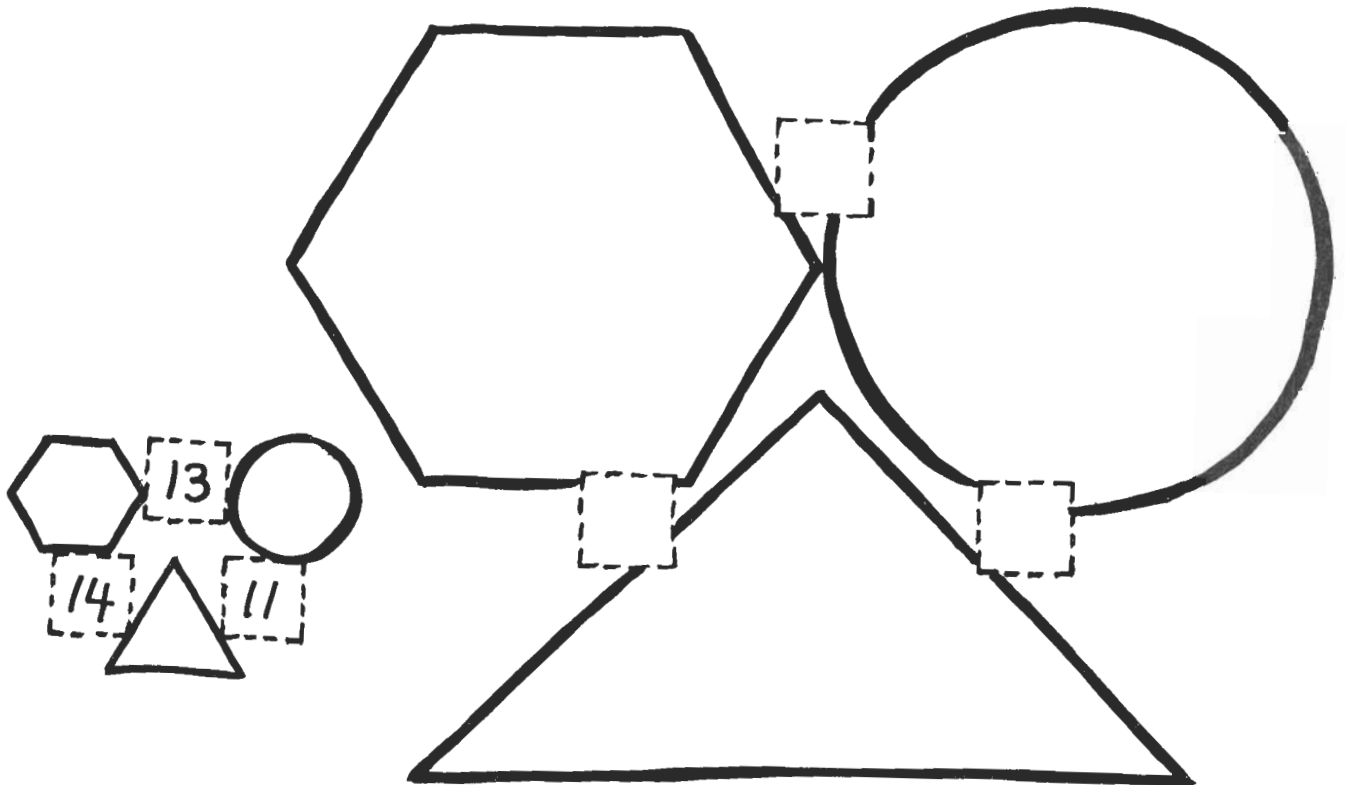
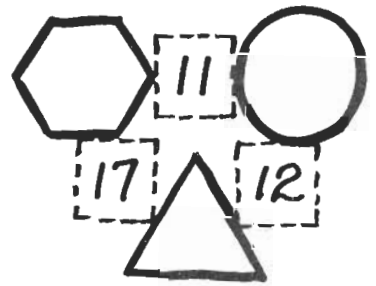
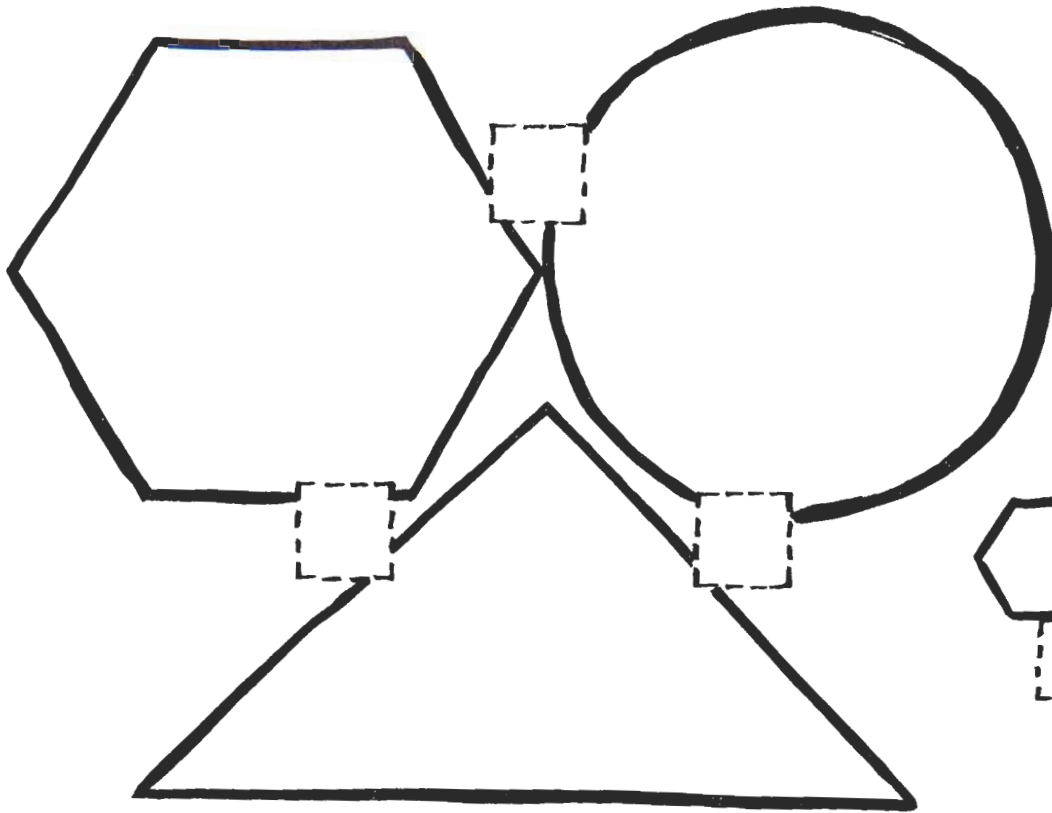
The only change here is in the method of record-keeping. The purpose of this change is evident as you glance at the next two pages . . . a means of providing more problems on the page.

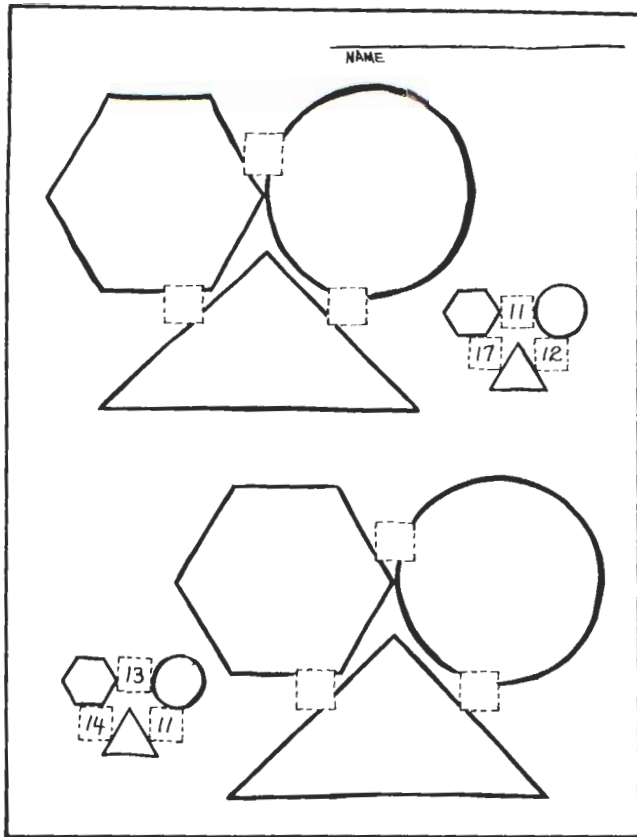
Suggest that the children make up examples, but record the sums only in the full report such as:



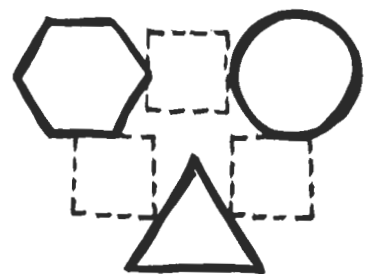
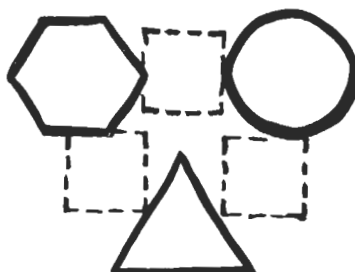
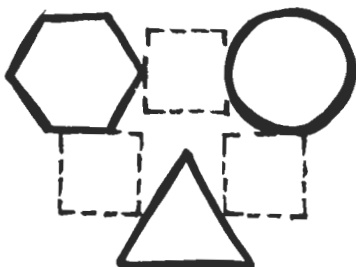
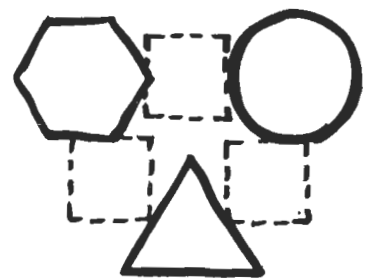
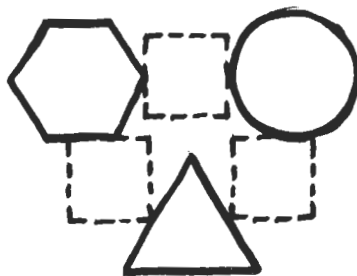
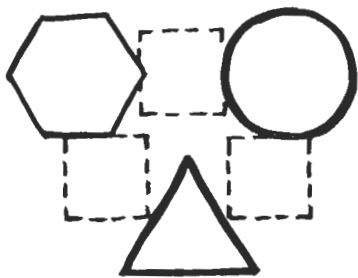
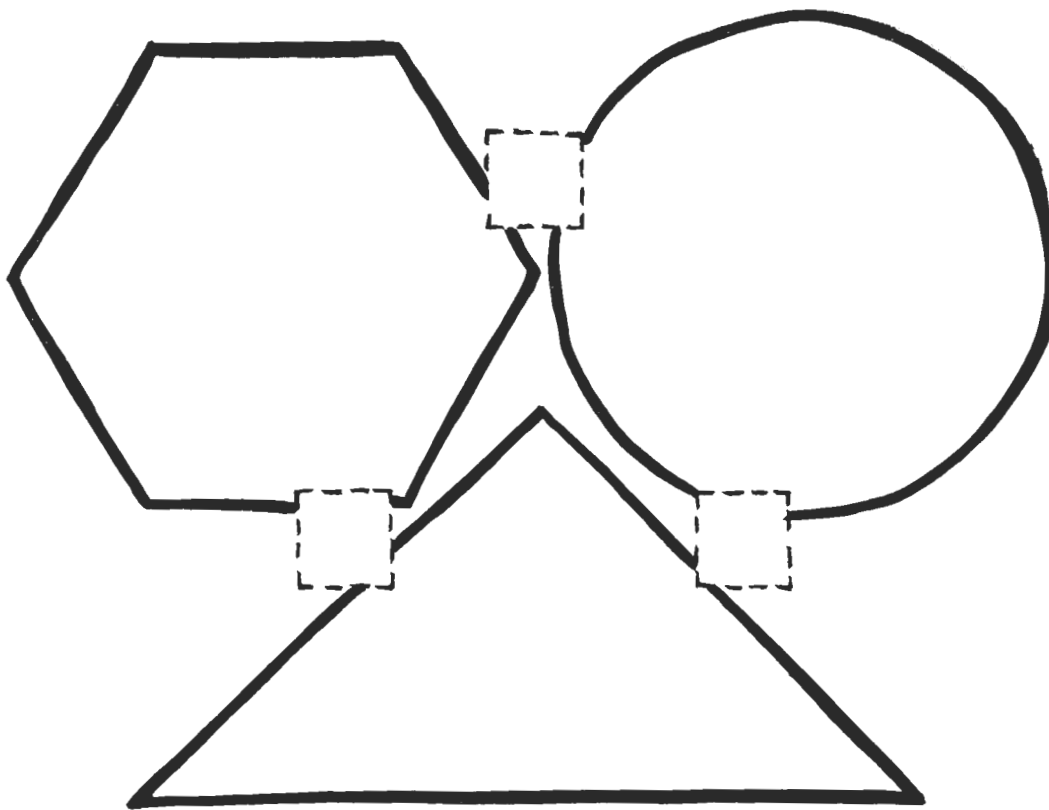
Some might like to see whether they can invent a problem with all sums in the 3 little squares being even numbers. (Yes.) Can they all be odd numbers? (No.) Can there be 2 even and 1 odd number? (No.) Can there be 2 odd and 1 even? (Yes—see example above.)

How can you make up a problem that has 2 sums alike and 1 different? (Answer: two of the frames must be alike.)

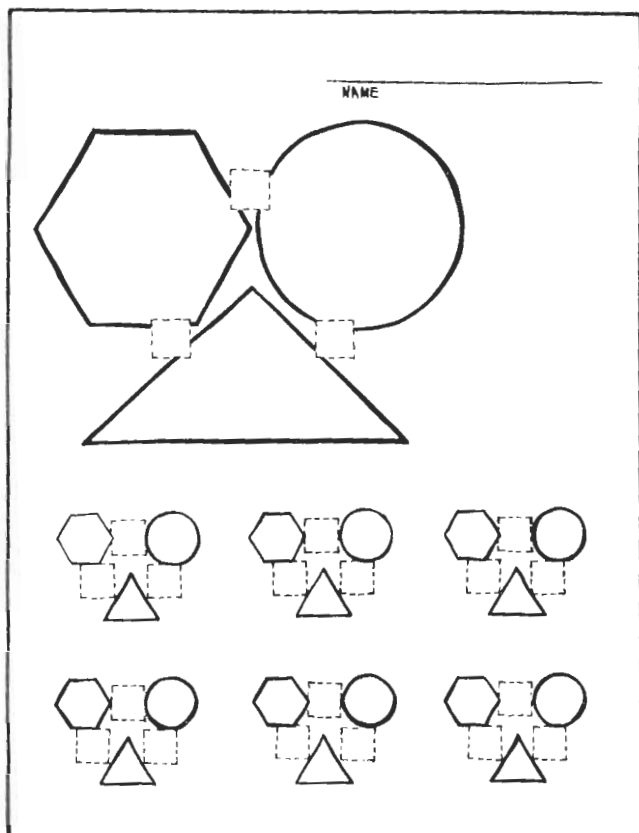




Two additional problems to help prepare the children for the next page.







It now becomes important that the children do not write numbers in the small square between the large frames at the top of the page.

Perhaps, working in pairs, the children can complete 6 examples. Keep records only of the sums of counters in each pair of frames so that the first time through their results are similar to the following page.

Now they are confronted by 6 problems they created. Can they reconstruct the arrangements of counters?

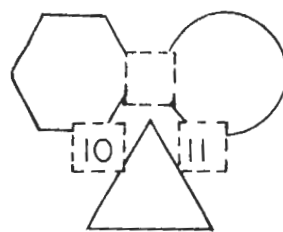
Then, with records only of the sums of counters in each pair of frames, they can exchange papers with others—and solve their problems.

Or, they may want to take their problems home to involve their families.

If some children want to work without counters, then they can use a piece of scratch paper as they try and try again.

Urge the children to find shortcuts to random trial and error.

Suppose you look at 2 sums at a time. What can you conclude from that scant information? For example:

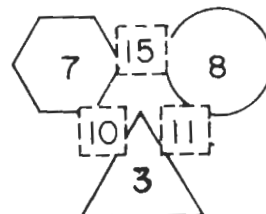


Regardless of how many counters there are in the triangle, what can you say about the number of counters in the hexagon and the circle? Suppose there were no counters in the triangle: then there are 10 in the hexagon and 11 in the circle. If there were 10 in the triangle, what about the hexagon and the circle? . . . 0 and 1. If there were 7 in the triangle, how many in the other two? . . . 3 and 4. Summarizing:

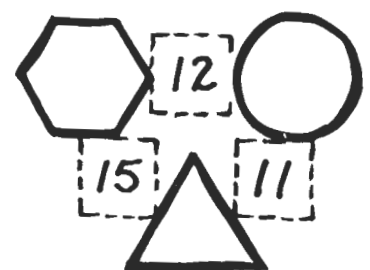
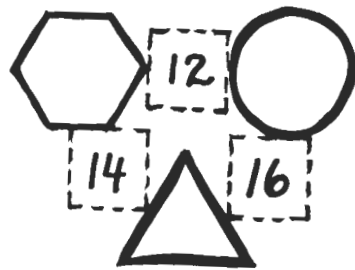
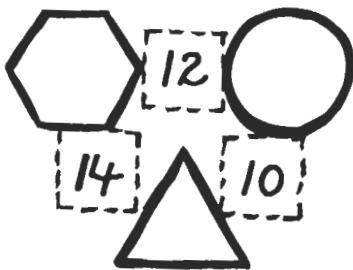
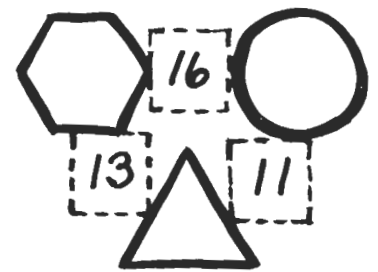
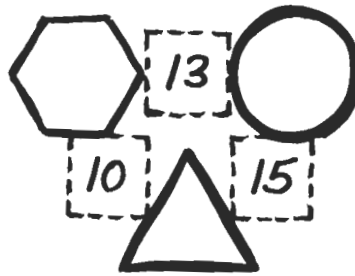
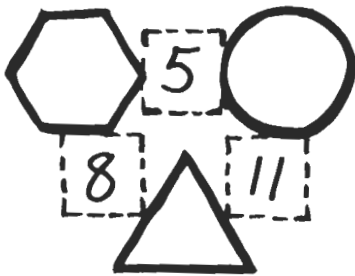
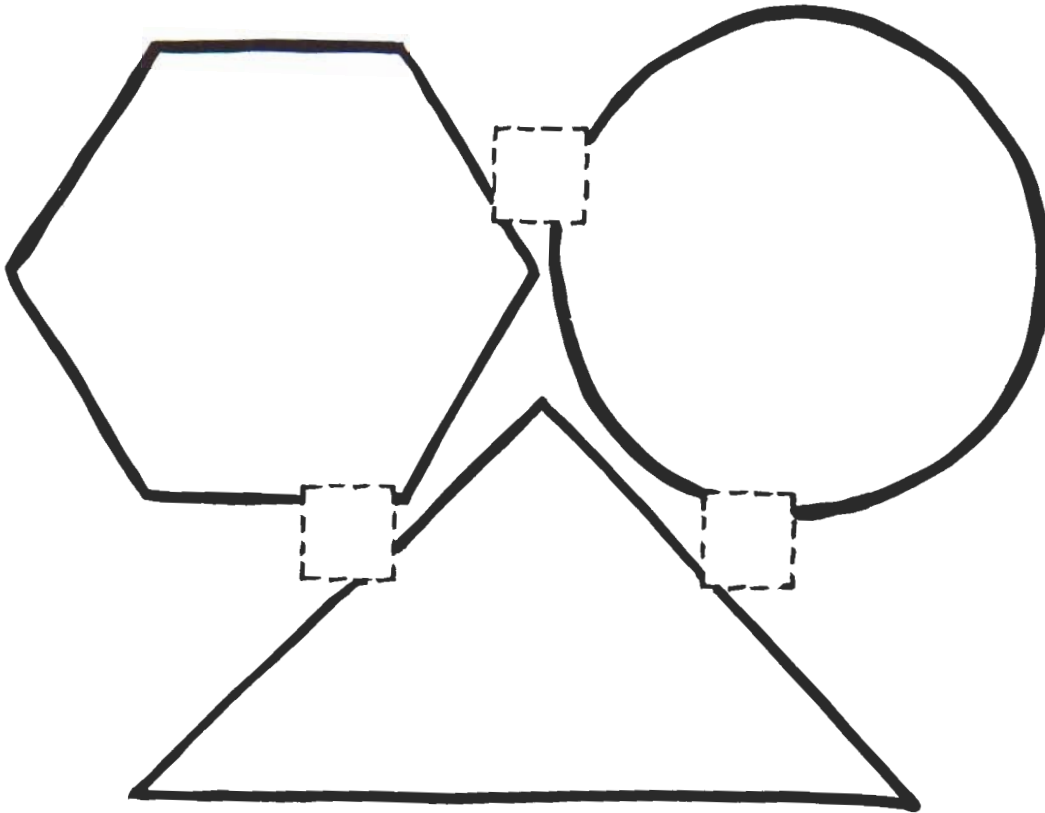
| hexagon |     | circle |
|---------|-----|--------|
| 10      | and | 11     |
| 0       | and | 1      |
| 3       | and | 4      |

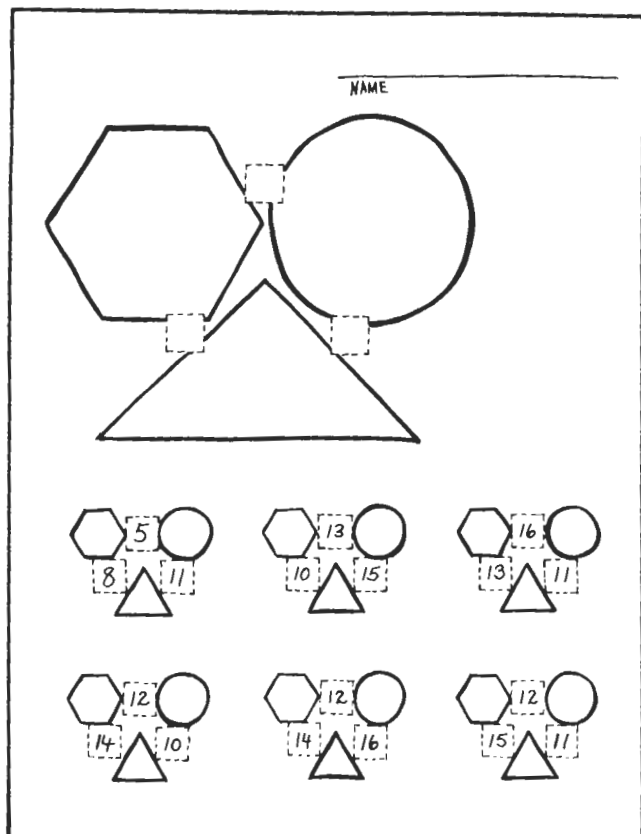
It becomes clear that there will be 1 more in the circle than in the hexagon.

Armed with that information, look at the third sum:



Clearly, there must be 7 in the hexagon and 8 in the circle.





There is now only one large space for doing all the problems on the page. Ask the children not to write anything in the little square boxes between the large frames that hold counters; rather write the number of counters in the appropriate shape in each problem.

Look at the first record and solve by either

(a) trying numbers and adjusting (without counters)  
or

(b) using counters in the larger shapes at the top.

Pupils should choose method with which they feel most comfortable.

Suggest that, after a problem is solved, they might:

- (1) Add sums together in each problem;
- (2) add numbers of counters used in each problem
- (3) and report any conclusions they reach.

---

$$\bigcirc + \text{hexagon} = 3$$

$$\bigcirc + \triangle = 4$$

$$\text{hexagon} + \triangle = 5$$

$$\text{hexagon} + \triangle = 6$$

$$\bigcirc + \text{hexagon} = 9$$

$$\triangle + \bigcirc = 11$$

$$\text{hexagon} + \triangle = 5$$

$$\bigcirc + \triangle = 10$$

$$\bigcirc + \text{hexagon} = 7$$

$$\text{hexagon} + \triangle = 11$$

$$\text{hexagon} + \bigcirc = 9$$

$$\triangle + \bigcirc = 14$$

$$\triangle + \bigcirc = 7$$

$$\text{hexagon} + \triangle = 10$$

$$\bigcirc + \text{hexagon} = 11$$

$$\bigcirc + \text{hexagon} = 8$$

$$\triangle + \text{hexagon} = 12$$

$$\triangle + \bigcirc = 14$$

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$$\bigcirc + \text{hexagon} = 13$$

$$\triangle + \bigcirc = 9$$

$$\text{hexagon} + \triangle = 14$$

$$\triangle + \bigcirc = 8$$

$$\text{hexagon} + \bigcirc = 15$$

$$\text{hexagon} + \triangle = 11$$

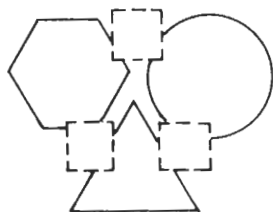
RULES: IN EACH GROUP OF THREE STATEMENTS, IF BOXES ARE ALIKE, NUMBERS WRITTEN IN THEM MUST BE ALIKE... AND ALL STATEMENTS MUST BE TRUE.

NAME \_\_\_\_\_

|                                                                                                       |                                                                                                       |                                                                                                       |
|-------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------|
| $\bigcirc + \hexagon = 3$<br>AND<br>$\bigcirc + \triangle = 4$<br>AND<br>$\hexagon + \triangle = 5$   | $\hexagon + \triangle = 6$<br>AND<br>$\bigcirc + \hexagon = 9$<br>AND<br>$\triangle + \bigcirc = 11$  | $\hexagon + \triangle = 5$<br>AND<br>$\bigcirc + \triangle = 10$<br>AND<br>$\bigcirc + \hexagon = 7$  |
| $\hexagon + \triangle = 11$<br>AND<br>$\hexagon + \bigcirc = 9$<br>AND<br>$\triangle + \bigcirc = 14$ | $\triangle + \bigcirc = 7$<br>AND<br>$\hexagon + \triangle = 10$<br>AND<br>$\bigcirc + \hexagon = 11$ | $\bigcirc + \hexagon = 8$<br>AND<br>$\triangle + \hexagon = 12$<br>AND<br>$\triangle + \bigcirc = 14$ |
| $\triangle + \bigcirc = 10$<br>AND<br>$\triangle + \hexagon = 7$<br>AND<br>$\bigcirc + \hexagon = 13$ | $\bigcirc + \hexagon = 13$<br>AND<br>$\triangle + \bigcirc = 9$<br>AND<br>$\hexagon + \triangle = 14$ | $\triangle + \bigcirc = 8$<br>AND<br>$\hexagon + \bigcirc = 15$<br>AND<br>$\hexagon + \triangle = 11$ |

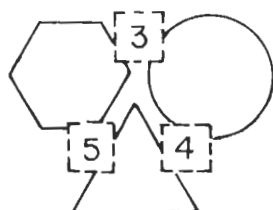
(This extension takes a big jump in complexity. Children who could proceed successfully with the previous work might find this too difficult.)

Most children will need one of the previous pages or a copy of



to facilitate moving counters around to find solutions—while others may not require this assistance.

The first problem on this page could be given in the format of



After finding the solution by moving counters or by trial and error, the solution is:

$$\bigcirc 1 + \hexagon 2 = 3$$

$$\bigcirc 1 + \triangle 3 = 4$$

$$\hexagon 2 + \triangle 3 = 5$$

And that is the only solution.

There is a single solution for every example on this page.

Some children may be able to look at a pair of statements such as

$$\bigcirc + \hexagon = 3$$

$$\bigcirc + \triangle = 4$$

and be able to say that there must be 1 more in the triangle than in the hexagon. Then look at

$$\hexagon + \triangle = 5$$

and argue that there must be 2 in the hexagon and 3 in the triangle.

In the second example, look at

$$\hexagon + \triangle = 6$$

$$\bigcirc + \hexagon = 9$$

Since numbers in the hexagons must be the same, there must be 3 more in the circle than in the triangle. And, since

$$\triangle + \bigcirc = 11$$

there must be 7 in the circle and 4 in the triangle.

$$\bigcirc + \text{hexagon} = 7$$

$$\bigcirc + \triangle = 8$$

$$\text{hexagon} + \triangle = 9$$

$$\text{hexagon} + \bigcirc = 6$$

$$\triangle + \text{hexagon} = 8$$

$$\bigcirc + \triangle = 12$$

$$\triangle + \bigcirc = 8$$

$$\text{hexagon} + \triangle = 11$$

$$\text{hexagon} + \bigcirc = 7$$

$$\triangle + \bigcirc = 10$$

$$\triangle + \text{hexagon} = 15$$

$$\bigcirc + \text{hexagon} = 13$$

$$\triangle + \text{hexagon} = 13$$

$$\bigcirc + \triangle = 8$$

$$\text{hexagon} + \bigcirc = 9$$

$$\triangle + \text{hexagon} = 13$$

$$\triangle + \bigcirc = 17$$

$$\text{hexagon} + \bigcirc = 14$$

$$\text{hexagon} + \bigcirc = 9$$

$$\triangle + \bigcirc = 14$$

$$\triangle + \text{hexagon} = 11$$

$$\text{hexagon} + \bigcirc = 11$$

$$\text{hexagon} + \triangle = 15$$

$$\bigcirc + \triangle = 12$$

$$\text{hexagon} + \triangle = 10$$

$$\bigcirc + \triangle = 16$$

$$\bigcirc + \text{hexagon} = 12$$

NAME \_\_\_\_\_

$$\bigcirc + \text{hexagon} = 7$$

AND

$$\bigcirc + \triangle = 8$$

AND

$$\text{hexagon} + \triangle = 9$$

$$\text{hexagon} + \bigcirc = 6$$

AND

$$\triangle + \text{hexagon} = 8$$

AND

$$\bigcirc + \triangle = 12$$

$$\triangle + \bigcirc = 8$$

AND

$$\text{hexagon} + \triangle = 11$$

AND

$$\text{hexagon} + \bigcirc = 7$$

$$\triangle + \bigcirc = 10$$

AND

$$\triangle + \text{hexagon} = 15$$

AND

$$\bigcirc + \text{hexagon} = 13$$

$$\triangle + \text{hexagon} = 13$$

AND

$$\bigcirc + \triangle = 8$$

AND

$$\text{hexagon} + \bigcirc = 9$$

$$\triangle + \text{hexagon} = 13$$

AND

$$\triangle + \bigcirc = 17$$

AND

$$\text{hexagon} + \bigcirc = 14$$

$$\text{hexagon} + \bigcirc = 9$$

AND

$$\triangle + \bigcirc = 14$$

AND

$$\triangle + \text{hexagon} = 11$$

$$\text{hexagon} + \bigcirc = 11$$

AND

$$\text{hexagon} + \triangle = 15$$

AND

$$\bigcirc + \triangle = 12$$

$$\text{hexagon} + \triangle = 10$$

AND

$$\bigcirc + \triangle = 16$$

AND

$$\bigcirc + \text{hexagon} = 12$$

$$\bigcirc + \text{hexagon} = \underline{\hspace{1cm}}$$

$$\bigcirc + \triangle = \underline{\hspace{1cm}}$$

$$\text{hexagon} + \triangle = \underline{\hspace{1cm}}$$

$$\text{hexagon} + \bigcirc = \underline{\hspace{1cm}}$$

$$\triangle + \text{hexagon} = \underline{\hspace{1cm}}$$

$$\bigcirc + \triangle = \underline{\hspace{1cm}}$$

$$\triangle + \bigcirc = \underline{\hspace{1cm}}$$

$$\text{hexagon} + \triangle = \underline{\hspace{1cm}}$$

$$\text{hexagon} + \bigcirc = \underline{\hspace{1cm}}$$

$$\triangle + \bigcirc = \underline{\hspace{1cm}}$$

$$\triangle + \text{hexagon} = \underline{\hspace{1cm}}$$

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$$\bigcirc + \triangle = \underline{\hspace{1cm}}$$

$$\text{hexagon} + \bigcirc = \underline{\hspace{1cm}}$$

$$\triangle + \text{hexagon} = \underline{\hspace{1cm}}$$

$$\triangle + \bigcirc = \underline{\hspace{1cm}}$$

$$\text{hexagon} + \bigcirc = \underline{\hspace{1cm}}$$

$$\text{hexagon} + \bigcirc = \underline{\hspace{1cm}}$$

$$\triangle + \bigcirc = \underline{\hspace{1cm}}$$

$$\triangle + \text{hexagon} = \underline{\hspace{1cm}}$$

$$\text{hexagon} + \bigcirc = \underline{\hspace{1cm}}$$

$$\text{hexagon} + \triangle = \underline{\hspace{1cm}}$$

$$\bigcirc + \triangle = \underline{\hspace{1cm}}$$

$$\text{hexagon} + \triangle = \underline{\hspace{1cm}}$$

$$\bigcirc + \triangle = \underline{\hspace{1cm}}$$

$$\bigcirc + \text{hexagon} = \underline{\hspace{1cm}}$$

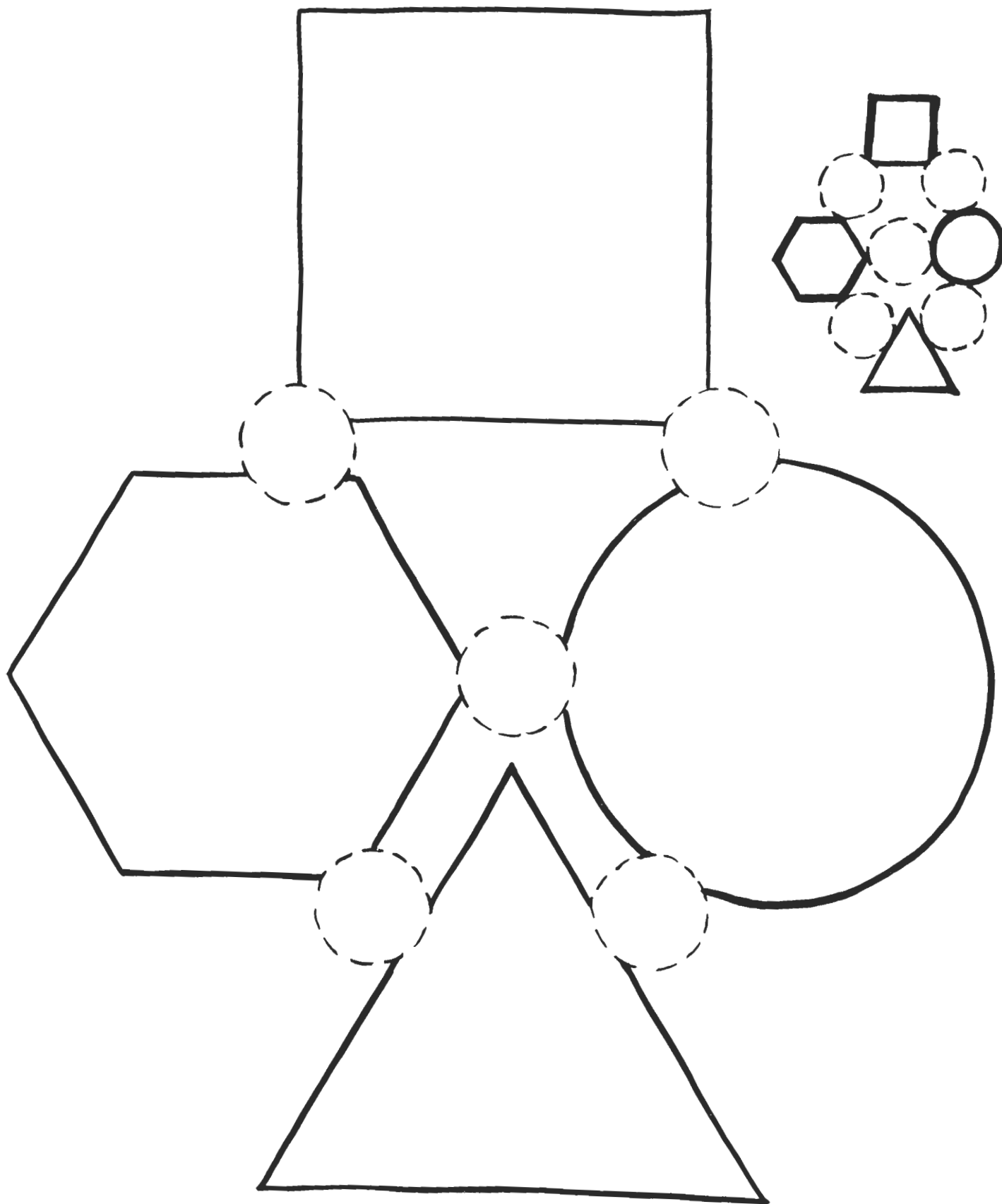


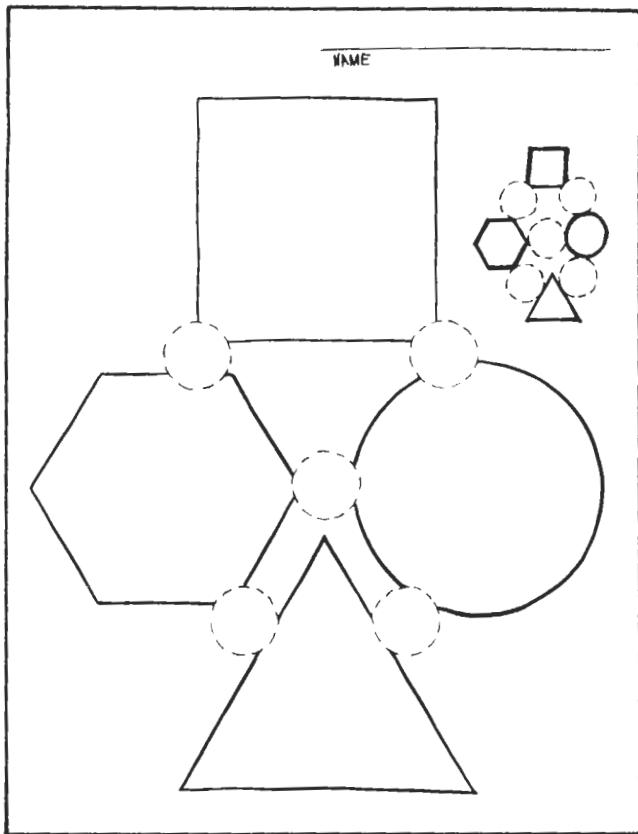
NAME \_\_\_\_\_

|        |        |        |
|--------|--------|--------|
| +  = _ | +  = _ | +  = _ |
| +  = _ | +  = _ | +  = _ |
| +  = _ | +  = _ | +  = _ |
| +  = _ | +  = _ | +  = _ |
| +  = _ | +  = _ | +  = _ |
| +  = _ | +  = _ | +  = _ |
| +  = _ | +  = _ | +  = _ |
| +  = _ | +  = _ | +  = _ |
| +  = _ | +  = _ | +  = _ |

Children can create their own problems—for themselves or for others.

Or, the problems can be specified as they are in the previous two pages.



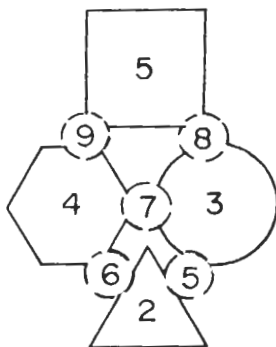


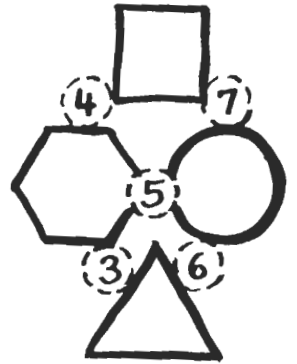
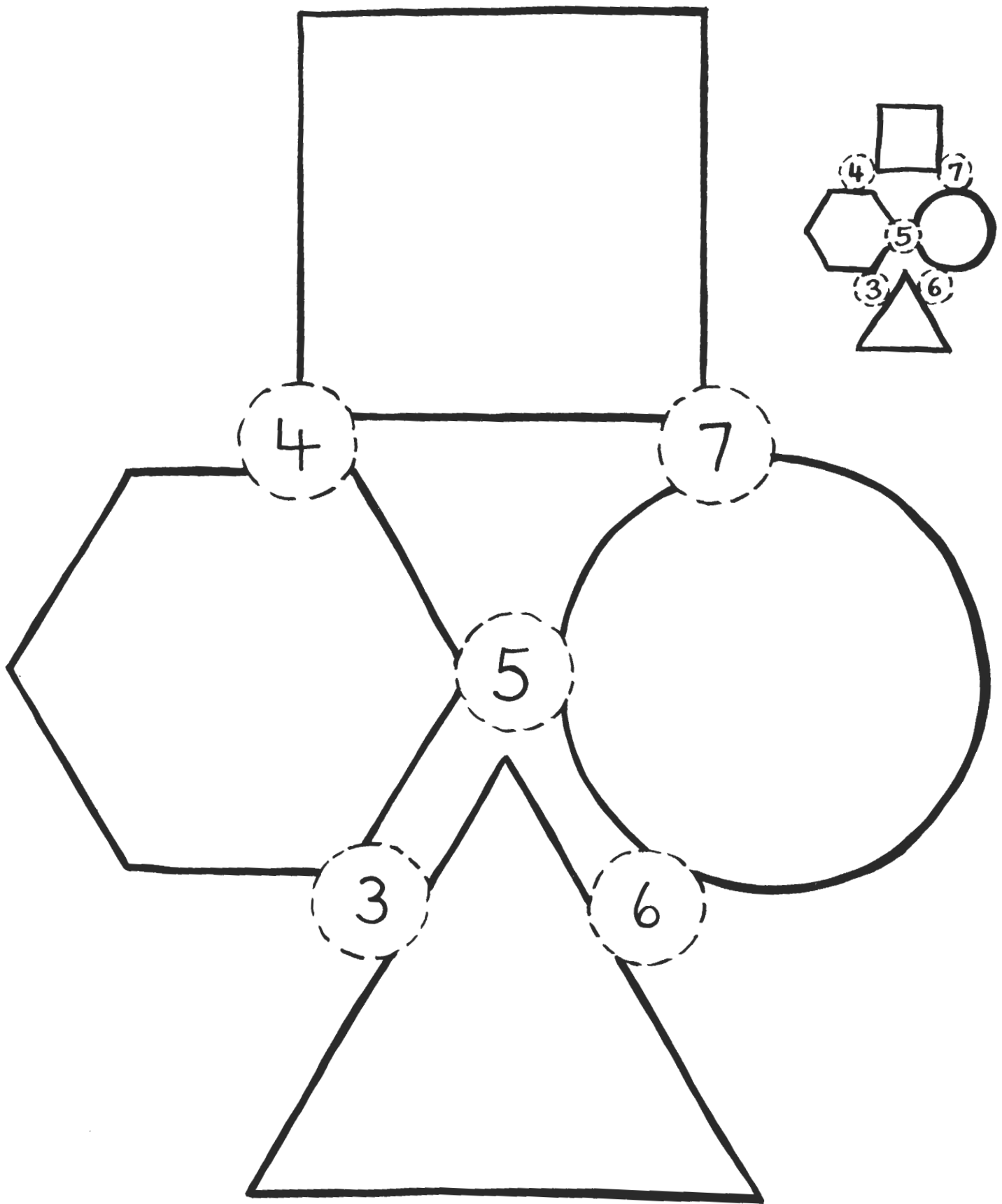
The problem grows up a bit. Now there are 4 frames for counters and 5 sums to write in.

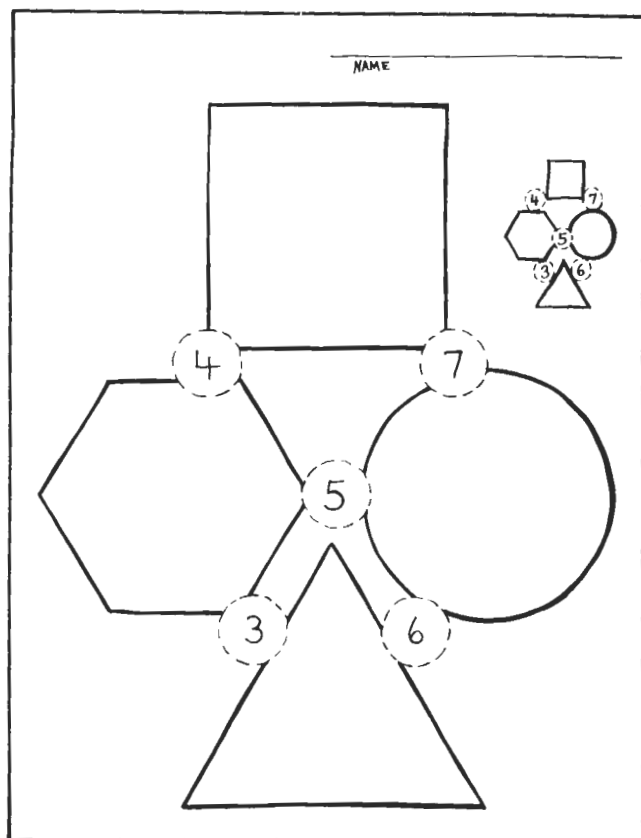
Put counters in the 4 frames. Write in the indicated sums. Remove the counters. Can your neighbor find out how many counters you had in the frame?

A condensed record form is provided.

An example of a completed form is:





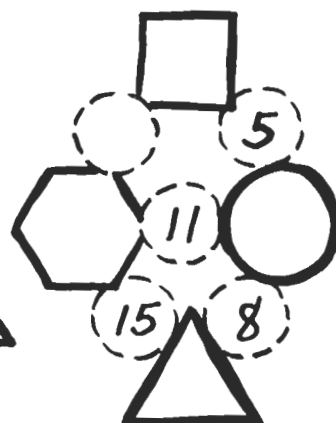
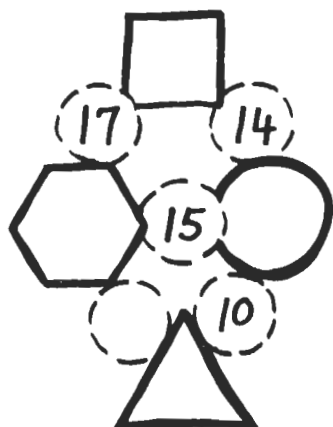
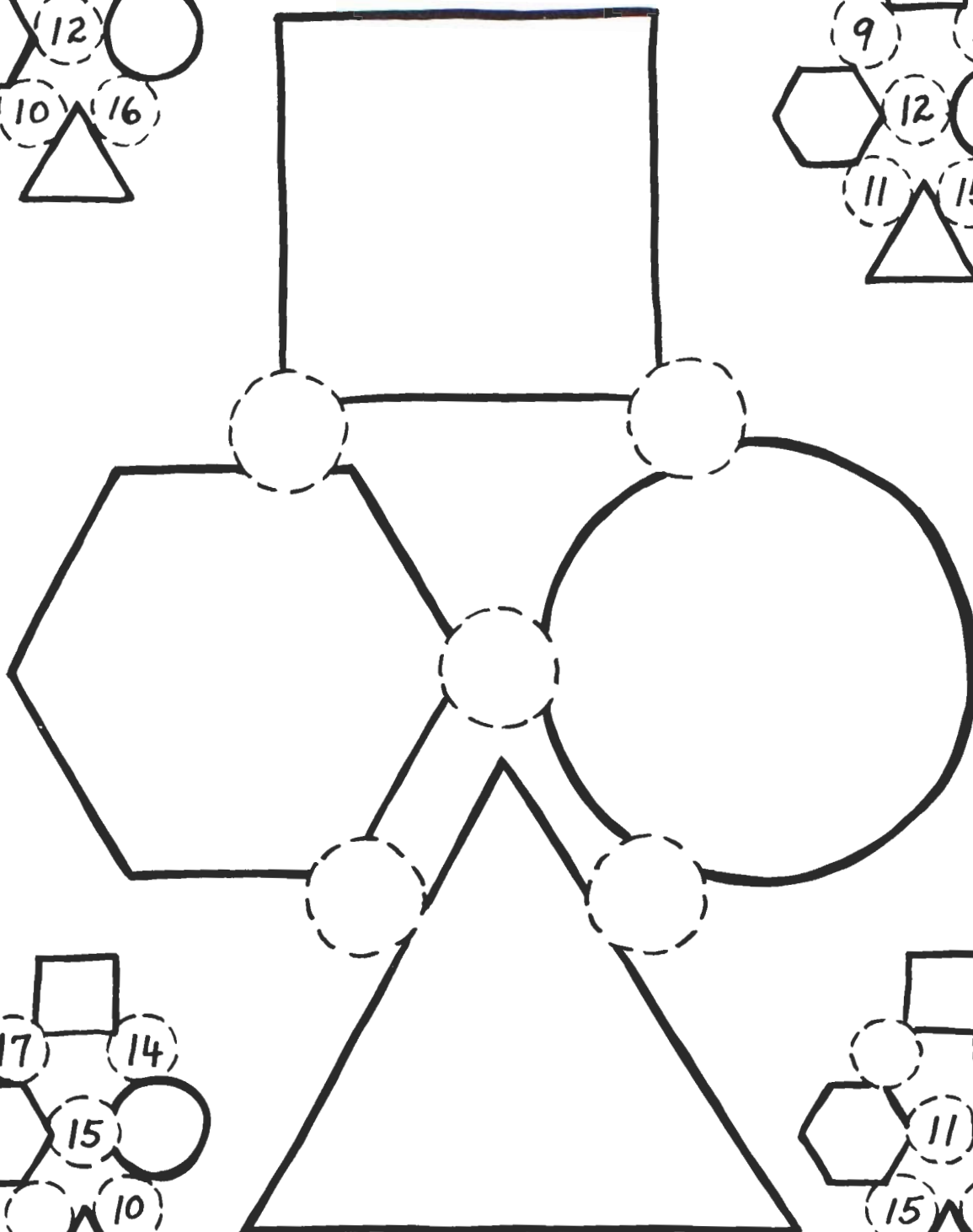
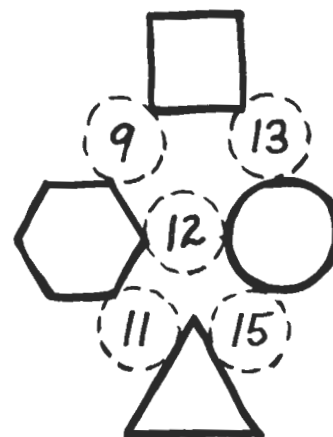
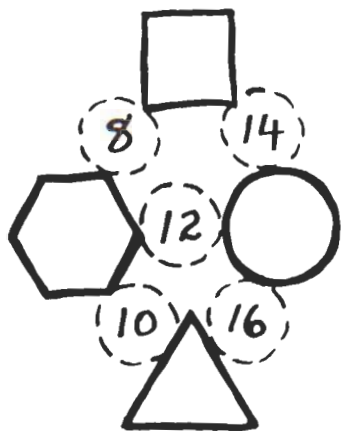


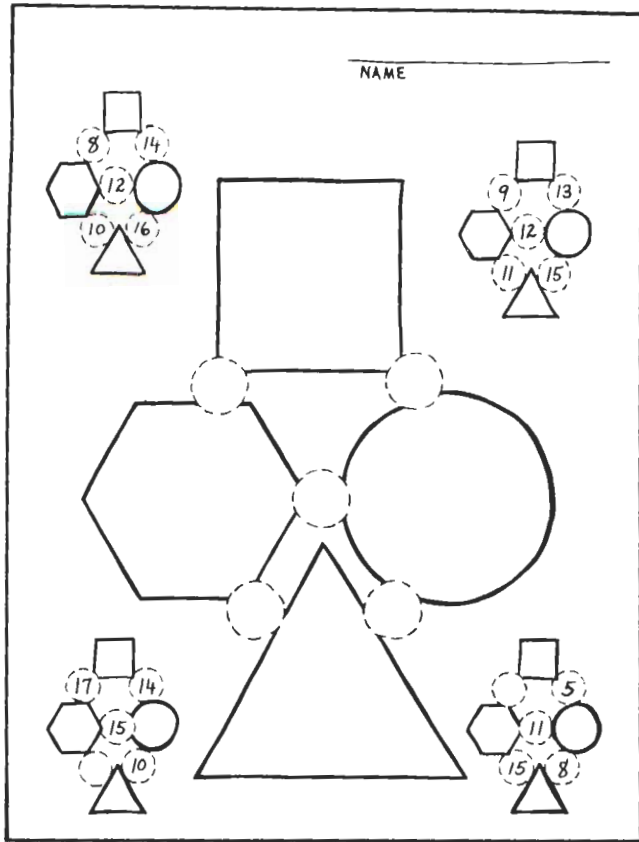
A problem is specified.

The previous form can be similarly particularized to specify the problems.

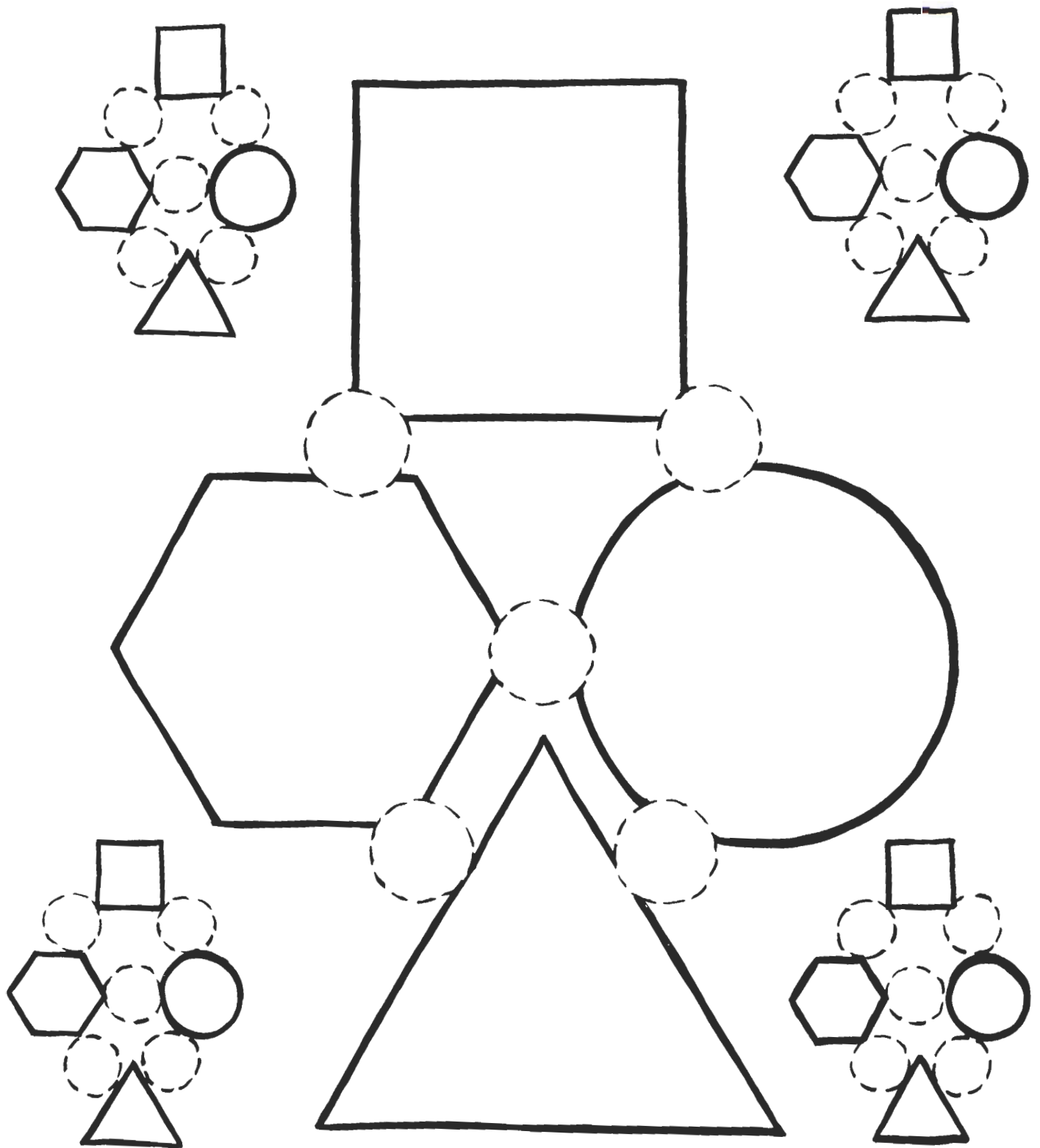
Again, solvers may use counters, scratch paper or arguments to find solutions.

This and the previous page help prepare for the next page—with 4 problems on one page.

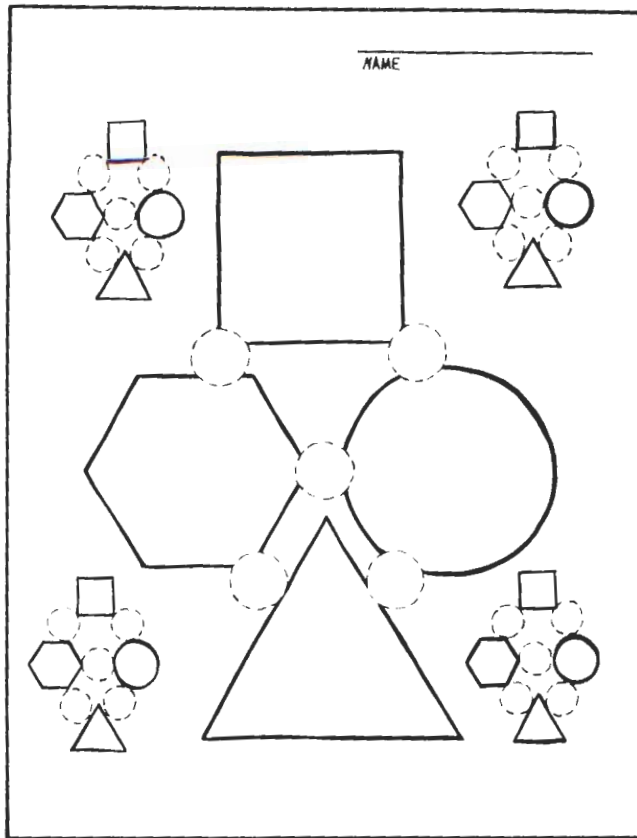




Children should be asked not to write sums in the small broken-line circles connecting pairs of large frames—rather, write them in the report forms.







Children can make up their own examples. If they fill in only the broken-line circles in the report forms, they have created 4 problems for themselves or others—"can you reconstruct the arrangements of counters?"

Question: must all 5 sums be given to determine the arrangement? (No! . . . see last 2 examples on previous page.) If not, which ones can be left out? (Answer: any one except the sums of counters in the hexagon and circle.)

Question: can you leave out 2 of the sums and be sure there is one and only one solution? (No, unless the sums are 0 or 1 in certain patterns.)

$$\text{Hexagon} + \text{Square} = 15$$

$$\text{Hexagon} + \text{Circle} = 11$$

$$\text{Triangle} + \text{Circle} = 13$$

$$\text{Triangle} + \text{Hexagon} = 14$$

$$\text{Hexagon} + \text{Circle} = 12$$

$$\text{Hexagon} + \text{Square} = 9$$

$$\text{Circle} + \text{Square} = 11$$

$$\text{Circle} + \text{Triangle} = 10$$

$$\text{Triangle} + \text{Circle} = 11$$

$$\text{Square} + \text{Circle} = 8$$

$$\text{Hexagon} + \text{Circle} = 7$$

$$\text{Hexagon} + \text{Square} = 5$$

$$\text{Circle} + \text{Square} = 13$$

$$\text{Hexagon} + \text{Square} = 11$$

$$\text{Circle} + \text{Triangle} = 14$$

$$\text{Circle} + \text{Hexagon} = 10$$

$$\text{Circle} + \text{Hexagon} = 7$$

$$\text{Circle} + \text{Square} = 13$$

$$\text{Triangle} + \text{Hexagon} = 9$$

$$\text{Square} + \text{Hexagon} = 12$$

$$\text{Triangle} + \text{Hexagon} = 13$$

$$\text{Circle} + \text{Hexagon} = 17$$

$$\text{Circle} + \text{Square} = 16$$

$$\text{Hexagon} + \text{Square} = 15$$

RULES: IN EACH GROUP OF FOUR STATEMENTS, IF BOXES ARE ALIKE NUMBERS WRITTEN IN THEM MUST BE ALIKE ... AND ALL STATEMENTS MUST BE TRUE.

NAME \_\_\_\_\_

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  +  = 15<br>AND<br> +  = 11<br>AND<br> +  = 13<br>AND<br> +  = 14 |  +  = 12<br>AND<br> +  = 9<br>AND<br> +  = 11<br>AND<br> +  = 10 |  +  = 11<br>AND<br> +  = 8<br>AND<br> +  = 7<br>AND<br> +  = 5    |
|  +  = 13<br>AND<br> +  = 11<br>AND<br> +  = 14<br>AND<br> +  = 10 |  +  = 7<br>AND<br> +  = 13<br>AND<br> +  = 9<br>AND<br> +  = 12  |  +  = 13<br>AND<br> +  = 17<br>AND<br> +  = 16<br>AND<br> +  = 15 |

Again, while the previous page has been used successfully with 6-year olds, this page cannot be used with beginners. However, intermediate grade children have found the previous pages interesting and can handle this activity as well.

Some will need the forms used on previous pages to facilitate moving the counters around. Others will use scratch paper for many "trials." Others will study the statement to see, as in the first example, that

$$\begin{array}{c}
 \text{hexagon} + \text{square} = 15 \\
 \text{and} \\
 \text{hexagon} + \text{circle} = 11
 \end{array}$$

reveals that there are 4 more counters in the square than in the circle, and go on to argue to a solution.

---

$$\text{Hexagon} + \text{Square} = \underline{\hspace{2cm}}$$

$$\text{Hexagon} + \text{Circle} = \underline{\hspace{2cm}}$$

$$\text{Triangle} + \text{Circle} = \underline{\hspace{2cm}}$$

$$\text{Hexagon} + \text{Circle} = \underline{\hspace{2cm}}$$

$$\text{Hexagon} + \text{Square} = \underline{\hspace{2cm}}$$

$$\text{Square} + \text{Circle} = \underline{\hspace{2cm}}$$

$$\text{Triangle} + \text{Circle} = \underline{\hspace{2cm}}$$

$$\text{Circle} + \text{Square} = \underline{\hspace{2cm}}$$

$$\text{Hexagon} + \text{Circle} = \underline{\hspace{2cm}}$$

$$\text{Triangle} + \text{Hexagon} = \underline{\hspace{2cm}}$$

$$\text{Circle} + \text{Triangle} = \underline{\hspace{2cm}}$$

$$\text{Hexagon} + \text{Square} = \underline{\hspace{2cm}}$$

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$$\text{Circle} + \text{Square} = \underline{\hspace{2cm}}$$

$$\text{Circle} + \text{Hexagon} = \underline{\hspace{2cm}}$$

$$\text{Triangle} + \text{Hexagon} = \underline{\hspace{2cm}}$$

$$\text{Hexagon} + \text{Square} = \underline{\hspace{2cm}}$$

$$\text{Circle} + \text{Square} = \underline{\hspace{2cm}}$$

$$\text{Circle} + \text{Hexagon} = \underline{\hspace{2cm}}$$

$$\text{Circle} + \text{Triangle} = \underline{\hspace{2cm}}$$

$$\text{Triangle} + \text{Hexagon} = \underline{\hspace{2cm}}$$

$$\text{Circle} + \text{Square} = \underline{\hspace{2cm}}$$

$$\text{Circle} + \text{Hexagon} = \underline{\hspace{2cm}}$$

$$\text{Square} + \text{Hexagon} = \underline{\hspace{2cm}}$$

$$\text{Hexagon} + \text{Square} = \underline{\hspace{2cm}}$$

NAME \_\_\_\_\_

RULES: IN EACH GROUP OF FOUR STATEMENTS, IF BOXES ARE ALIKE, NUMBERS WRITTEN IN THEM MUST BE ALIKE... AND ALL STATEMENTS MUST BE TRUE.

|                                                                                                                                                                                      |                                                                                                                                                                                      |                                                                                                                                                                                      |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  +  = _____<br>AND |  +  = _____<br>AND |  +  = _____<br>AND |
|  +  = _____<br>AND |  +  = _____<br>AND |  +  = _____<br>AND |
|  +  = _____<br>AND |  +  = _____<br>AND |  +  = _____<br>AND |
|  +  = _____        |  +  = _____        |  +  = _____        |
|  +  = _____        |  +  = _____        |  +  = _____        |
|  +  = _____        |  +  = _____        |  +  = _____        |
|  +  = _____        |  +  = _____        |  +  = _____        |
|  +  = _____        |  +  = _____        |  +  = _____        |

Children can make up their own problems—for themselves, their classmates, or for their families—using numbers with which they are comfortable.

Or, the forms can be particularized by specifying sums—at any level of complexity.

Some children who are not dependent on counters can begin using fractions in this and many previous forms. Consider these three open sentences:

$$\text{hexagon} + \text{square} = 5$$

$$\text{hexagon} + \text{circle} = 6$$

$$\text{square} + \text{circle} = 4$$

From the first 2 sentences you can conclude that there is 1 more in the circle than in the square; and from the third that the sum of the numbers in the square and circle is 4. Only

$$\boxed{1\frac{1}{2}} + \textcircled{2\frac{1}{2}} = 4$$

meets all these conditions.

Problems can be made up by writing fractions in the open sentences making them true statements:

$$\text{hexagon} \left( \frac{1}{2} \right) + \boxed{\frac{3}{4}} = 1\frac{1}{4}$$

$$\text{hexagon} \left( \frac{1}{2} \right) + \text{circle} \left( \frac{1}{4} \right) = \frac{3}{4}$$

$$\text{triangle} \left( \frac{1}{8} \right) + \text{circle} \left( \frac{1}{4} \right) = 1\frac{3}{8}$$

$$\text{triangle} \left( \frac{1}{8} \right) + \text{hexagon} \left( \frac{1}{2} \right) = 1\frac{5}{8}$$

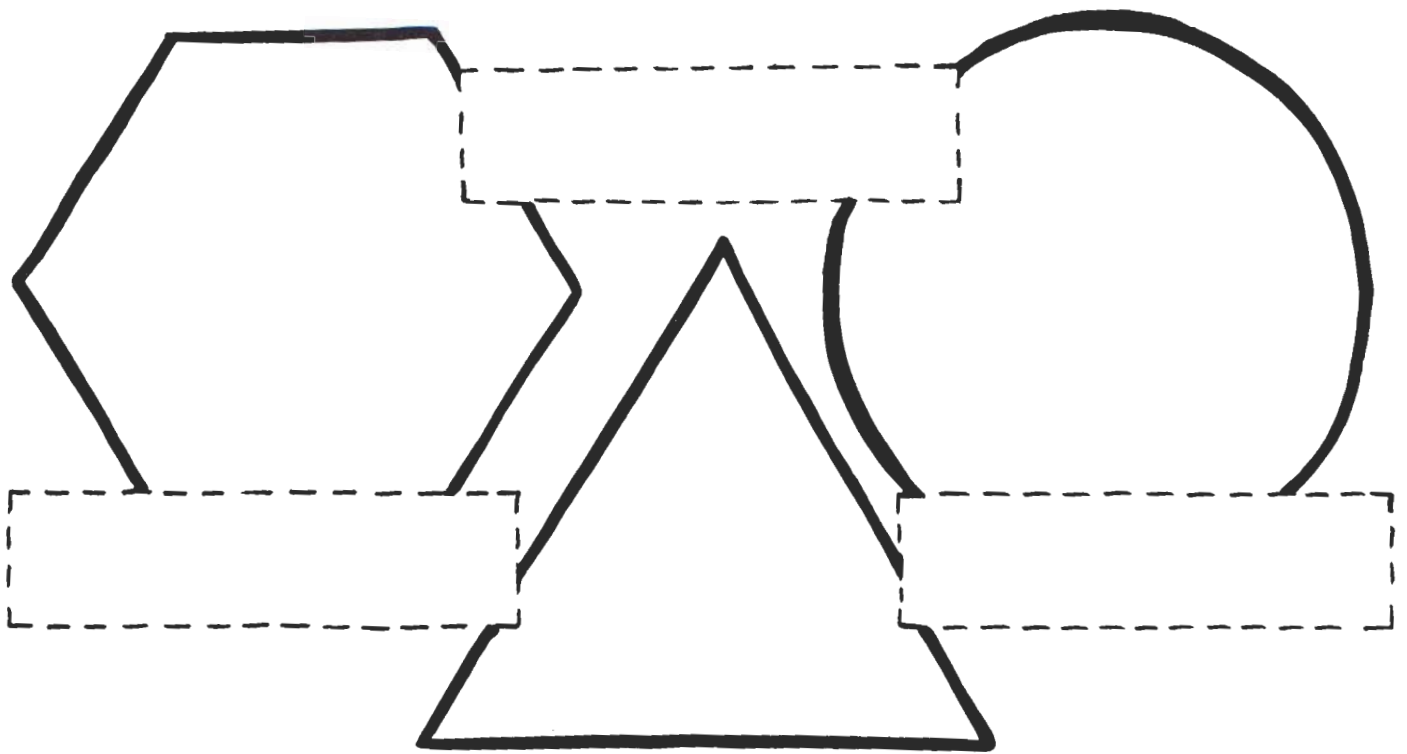
and then omitting the numbers written in the frames:

$$\text{hexagon} + \boxed{\phantom{00}} = 1\frac{1}{4}$$

$$\text{hexagon} + \text{circle} = \frac{3}{4}$$

$$\text{triangle} + \text{circle} = 1\frac{3}{8}$$

$$\text{triangle} + \text{hexagon} = 1\frac{5}{8}$$



$$\bigcirc - \hexagon = 1$$

$$\triangle - \bigcirc = 2$$

$$\hexagon + \triangle = 9$$

$$2 \hexagon - \bigcirc = 1$$

$$\triangle + \hexagon = 7$$

$$\bigcirc - \triangle = 1$$

$$2 \triangle - \hexagon = 0$$

$$\bigcirc - \hexagon = 2$$

$$\triangle + \bigcirc = 8$$

$$2 \bigcirc = \triangle$$

$$\bigcirc + \hexagon = 5$$

$$\triangle - \hexagon = 7$$

NAME \_\_\_\_\_

|                              |                             |
|------------------------------|-----------------------------|
| $\bigcirc - \hexagon = 1$    | $2 \hexagon - \bigcirc = 1$ |
| $\triangle - \bigcirc = 2$   | $\triangle + \hexagon = 7$  |
| $\hexagon + \triangle = 9$   | $\bigcirc - \triangle = 1$  |
| $2 \triangle - \hexagon = 0$ | $2 \bigcirc = \triangle$    |
| $\bigcirc - \hexagon = 2$    | $\bigcirc + \hexagon = 5$   |
| $\triangle + \bigcirc = 8$   | $\triangle - \hexagon = 7$  |

Another jump in complexity . . . further anticipating algebra in a problem solving situation. However, the jump is within the capability of all intermediate grade students.

The problems arise from three statements about arrangements of counters in three frames.

In the first example, the statements can be interpreted as:

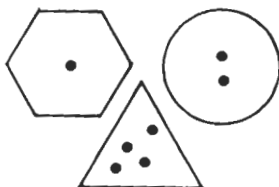
$$\bigcirc - \hexagon = 1$$

There is 1 more counter in the circle than in the hexagon.

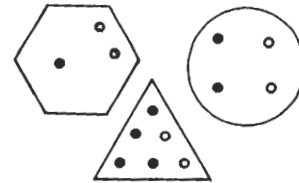
$$\triangle - \bigcirc = 2$$

There are 2 more counters in the triangle than in the circle.

So the triangle has the most, the circle with two less, and the hexagon has one less than the circle. Or, we can say that there are 3 more counters in the triangle than in the hexagon.



This is not the solution, because the third statement says there are a total of 9 in the hexagon and triangle. We show 5; we need 2 more. We can accomplish this without changing the relationship we show above by adding 2 counters to each frame.



Now we can report the result

$$\textcircled{4} - \textcircled{3} = 1$$

$$\textcircled{6} - \textcircled{4} = 2$$

$$\textcircled{3} + \textcircled{6} = 9$$

This is, of course, but one of many ways to tackle the problem.

Trial and error is often productive with a minimum of work—though it can also be most time consuming.

Students may need some explanation of the expression

$$2 \triangle \text{ and } 2 \hexagon \text{ and } 2 \bigcirc$$

This is simply one way of saying "twice the number in the triangle" or hexagon or circle. Other ways of saying the same things are:

$$2 \times \triangle \text{ or } \triangle + \triangle \text{ or } \triangle \times 2$$

In the second example, the final report will be

$$2 \textcircled{3} - \textcircled{5} = 1$$

$$\textcircled{4} + \textcircled{3} = 7$$

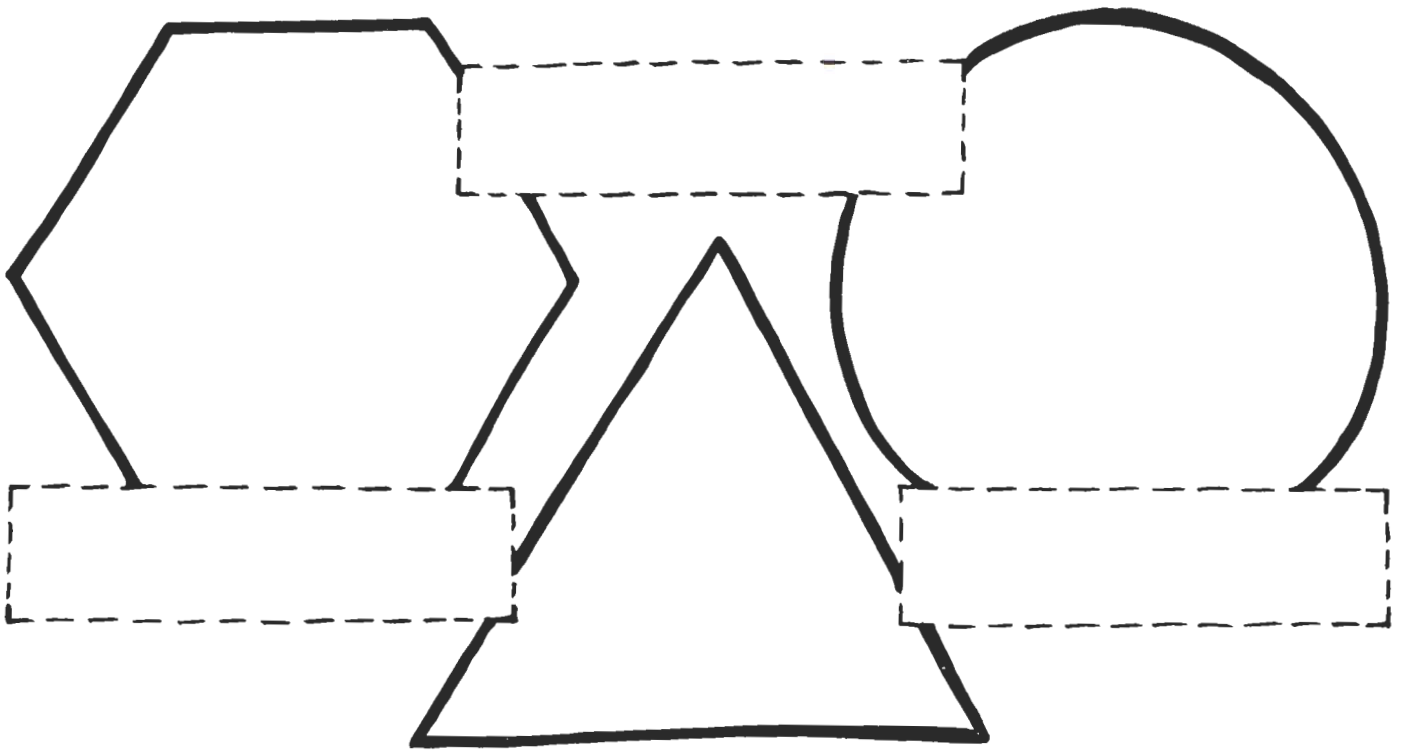
$$\textcircled{5} - \textcircled{4} = 1$$

In the third and fourth examples:

$$\textcircled{2} \quad \textcircled{4}$$

$$\textcircled{6} \text{ and } \textcircled{1}$$

$$\textcircled{4} \quad \textcircled{8}$$



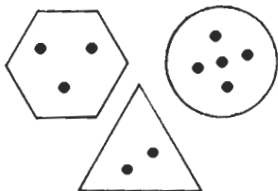
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This activity can be introduced by a general discussion of true statements that can be made about an arrangement of counters in 3 groups. One could be sketched on the board:



What can you say about a relationship between the number of counters? An example is:

$$\text{Hexagon} + \text{Circle} + \text{Triangle} = 10$$

"The number of counters in the boxes all added together would be 10" or, "there are a total of 10 counters in the arrangement."

$$\text{Circle} - \text{Hexagon} = 2$$

"The number in the circle minus the number in the hexagon is 2" . . . or, "There are 2 more in the circle than in the hexagon."

Other examples of statements are:

$$\text{Circle} = \text{Triangle} + \text{Hexagon}$$

$$2 \text{ Circle} = 5 \text{ Triangle}$$

$$2 \text{ Triangle} + 1 = \text{Circle}$$

$$\text{Hexagon} - \text{Triangle} = 1$$

$$3 \text{ Triangle} = 2 \text{ Hexagon}$$

$$\text{Hexagon} + \text{Circle} = 8$$

Next, select three of the statements and write them in on this Report Form as the first problem. For example:

$$2 \text{ Circle} = 5 \text{ Triangle}$$

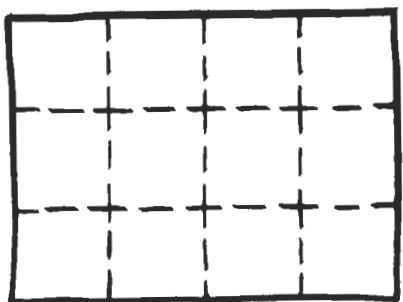
$$\text{Hexagon} - \text{Triangle} = 1$$

$$3 \text{ Triangle} = 2 \text{ Hexagon}$$

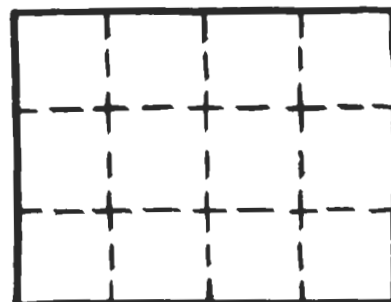
We know, of course, one arrangement that will meet all these conditions—the one we were describing when we made up the statements in the first place. Is there a different arrangement that would meet the same conditions? (Try as you will, there is no such other arrangement.)

After 4 problems are completed and the counters removed from the frame, the students will find it challenging to try to reconstruct the arrangements.

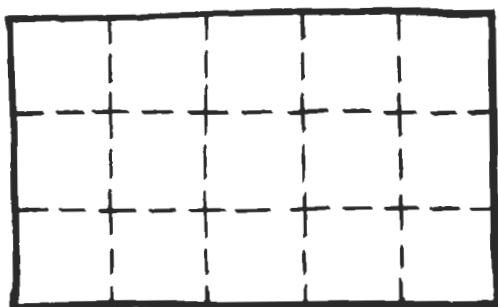
Or, the page may be particularized to present 4 specific problems similar to the previous page—at any desired level of difficulty.



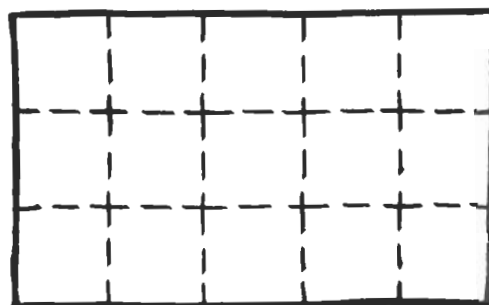
$\boxed{12} (\div 3) \text{ hexagon } (x 2) \text{ triangle}$



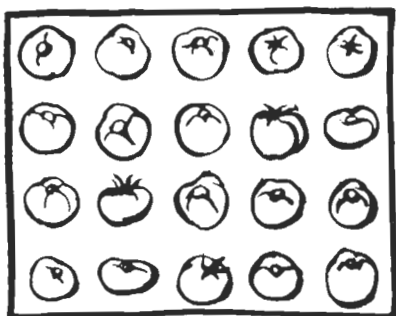
$\boxed{\phantom{00}} (\div 4) \text{ circle } (x 3) \text{ hexagon}$



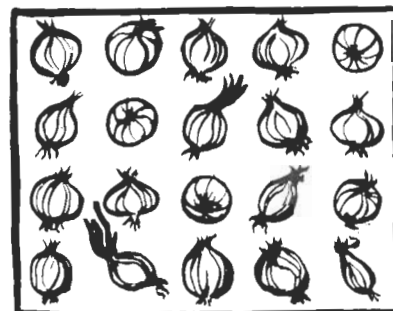
$\boxed{\phantom{00}} (\div 5) \text{ triangle } (x 3) \text{ circle}$



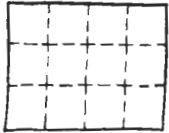
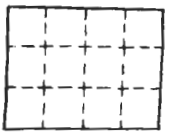
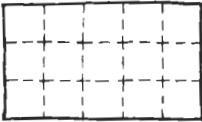
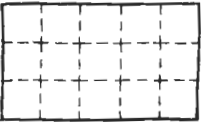
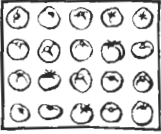
$\boxed{\phantom{00}} (\div 3) \text{ hexagon } (x 1) \text{ triangle}$



$\boxed{\phantom{00}} (\div 4) \text{ circle } (x 2) \text{ hexagon}$



$\boxed{\phantom{00}} (\div 5) \text{ triangle } (x 3) \text{ circle}$

|                                                                                                                                            |                                                                                                                                         |
|--------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <br>$\boxed{12} (\div 3) \text{ hexagon } (x2) \triangle$ | <br>$\square (\div 4) \bigcirc (x3) \text{ hexagon}$   |
| <br>$\square (\div 5) \triangle (x3) \bigcirc$            | <br>$\square (\div 3) \text{ hexagon } (x1) \triangle$ |
| <br>$\square (\div 4) \bigcirc (x2) \text{ hexagon}$      | <br>$\square (\div 5) \triangle (x3) \bigcirc$         |

4—Report the number of blocks shaded in.

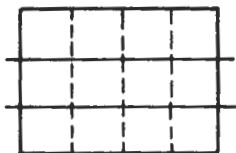
$$\boxed{12} (\div 3) \text{ hexagon } (x2) \triangle 8$$

The first example considers a 3 by 4 arrangement of 12 blocks. The directions:

$$\boxed{12} (\div 3) \text{ hexagon } (x2) \triangle$$

are interpreted to mean:

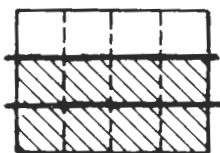
1—Divide the arrangement into 3 equal parts.

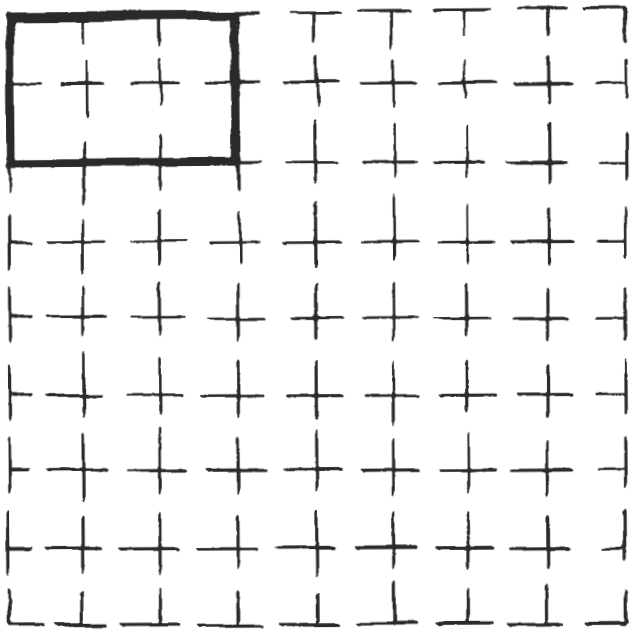


2—Report the number of blocks in each of the 3 parts.

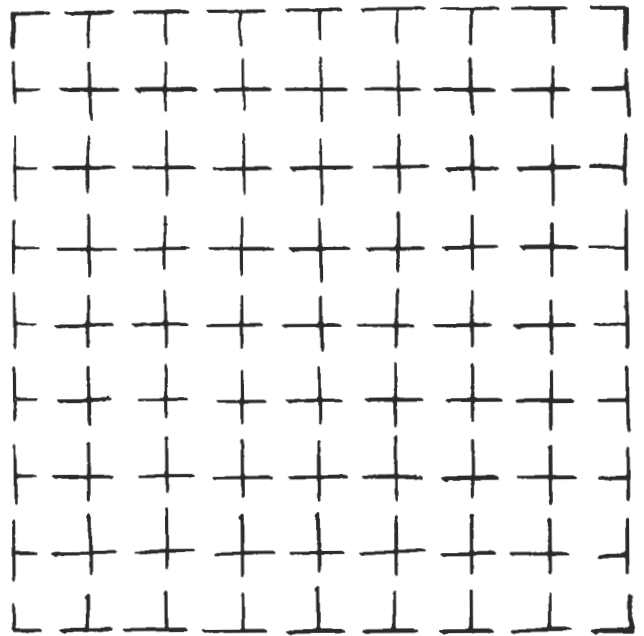
$$\boxed{12} (\div 3) \text{ hexagon}$$

3—Color in or shade two of the parts.

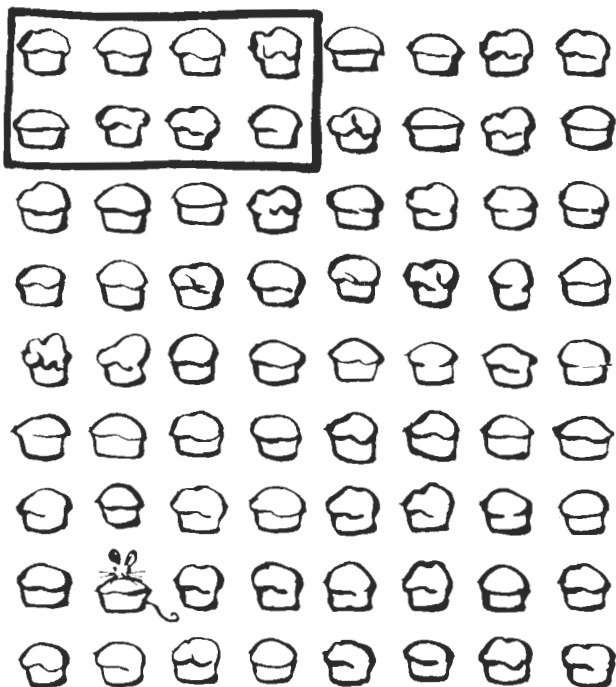




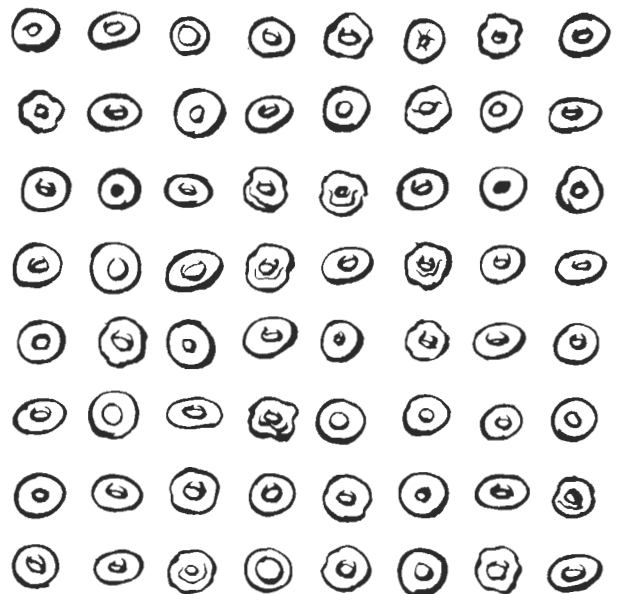
$\square$  (x4)  $\triangle$  ( $\div 3$ )  $\hexagon$



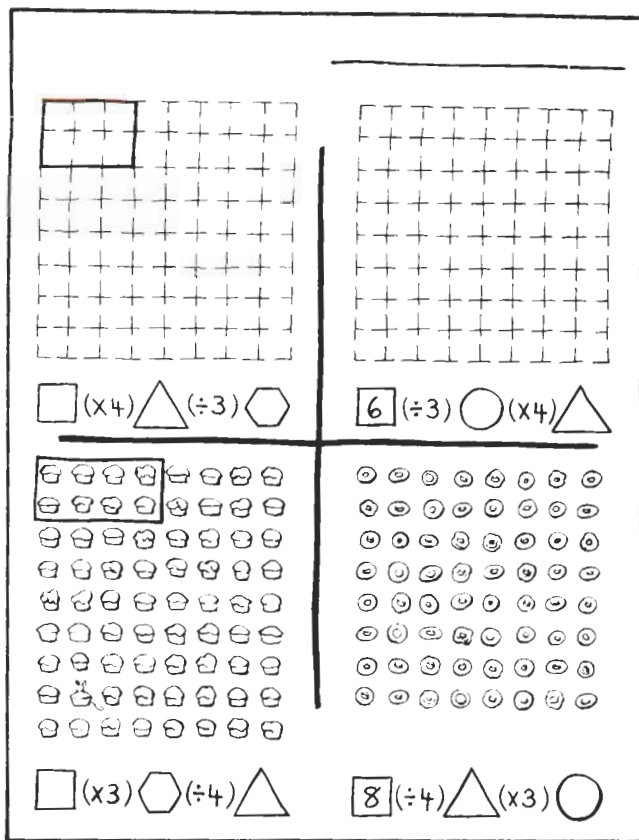
$\boxed{6}$  ( $\div 3$ )  $\bigcirc$  (x4)  $\triangle$



$\square$  (x3)  $\hexagon$  ( $\div 4$ )  $\triangle$



$\boxed{8}$  ( $\div 4$ )  $\triangle$  (x3)  $\bigcirc$



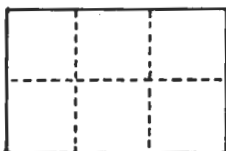
A question is raised in these pairs of examples: does the order of carrying out the multiplication and the division affect the outcome?

$$6(x4)24(\div3)8$$

Reversing the order in the second example (top right) leads to:

$$6(\div3)2(x4)8$$

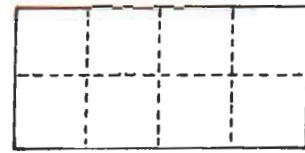
As students work out this example on the front of the page, they may need guidance. The first half of the instruction is clear:

$$6(\div3)2$$


They are asked next to shade in four such parts:

$$6(\div3)2(x4)\triangle$$

This means that they will have to add one more part to their drawing:



Try another example of changing the order:

$$8(x3)24(\div4)6$$

and

$$8(\div4)2(x3)6$$

That is an outcome that is worthy of further examination. If "order" does not affect the result, it seems advisable to carry out the division first: —because it leads to much less work — smaller sketches and smaller numbers.

Also consider the pair of examples on the left:

$$6(x4)24(\div3)8$$

$$8(x3)24(\div4)6$$

To  $(\times 4)$  and  $(\div 3)$  takes 6 to 8.

To  $(\times 3)$  and  $(\div 4)$  takes 8 back to 6.

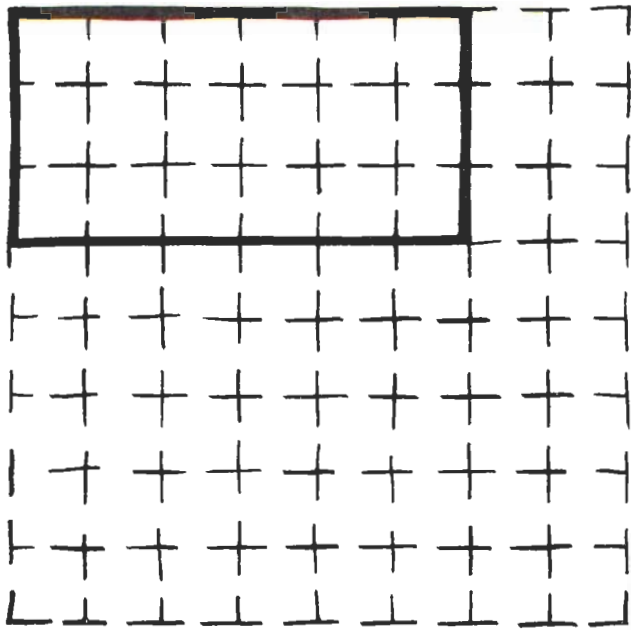
In a sense to  $(\times 3)$  and  $(\div 4)$  "undoes" what  $(\times 4)$  and  $(\div 3)$  has done.

In fact, all four examples are related. We notice that:

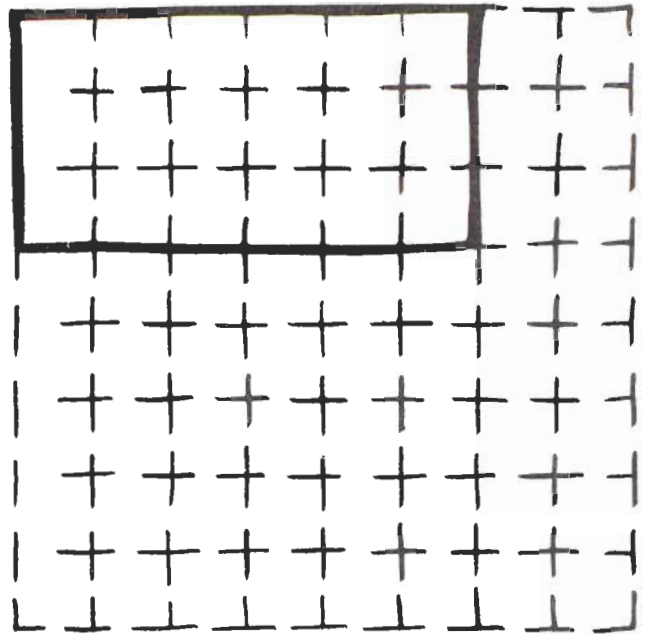
- (1)  $(\times 4)$  and  $(\div 3)$  and  $(\div 3)$  and  $(\times 4)$  produces the same results.
- (2)  $(\div 4)$  and  $(\times 3)$  and  $(\times 3)$  and  $(\div 4)$  produces the same results. Further, either pair in line (2), "undoes" what either pair in line (1) accomplishes.

Will these results hold with other numbers?

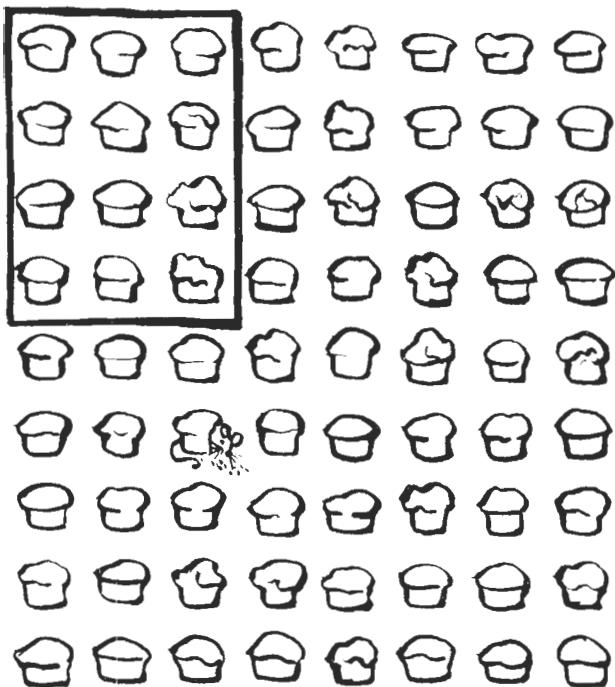
(The following page may help in recording results of such investigations.)



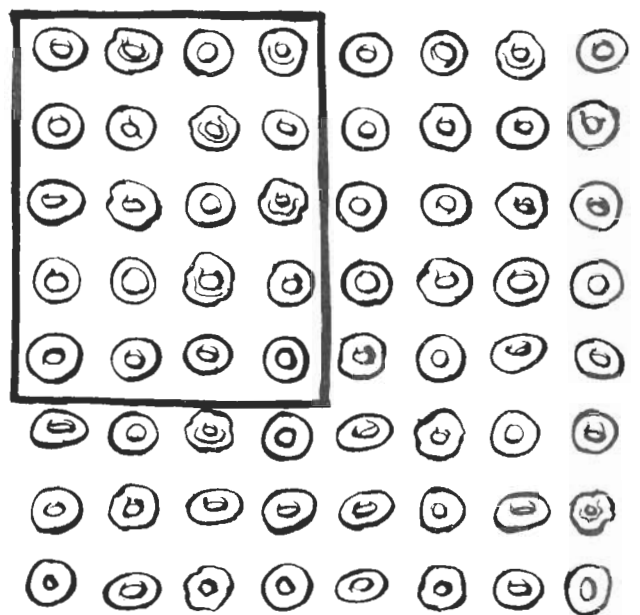
$\square$ 
 $(\times 2)$ 
 $\triangle$ 
 $(\div 3)$ 
 $\hexagon$



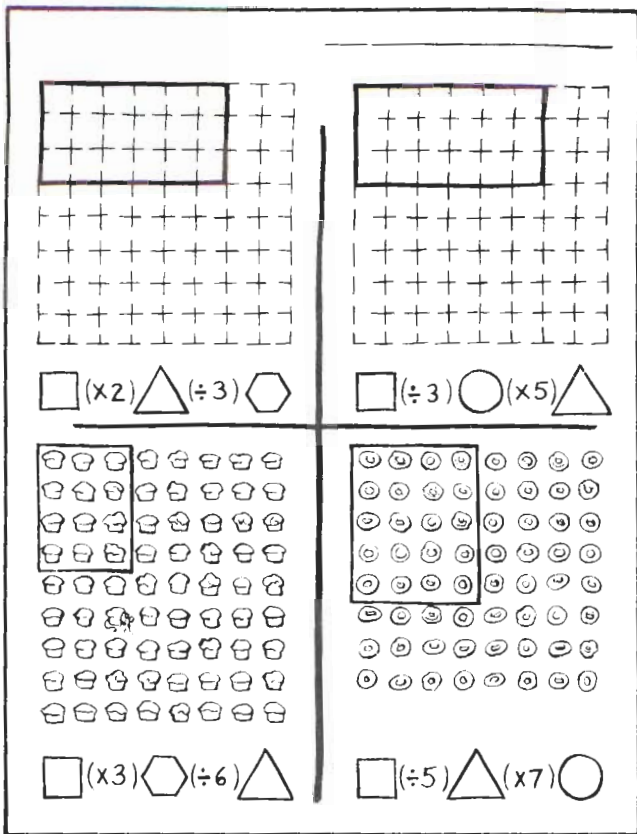
$\square$ 
 $(\div 3)$ 
 $\bigcirc$ 
 $(\times 5)$ 
 $\triangle$



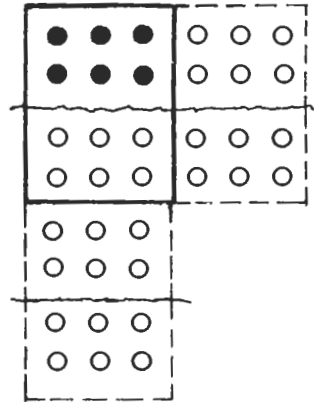
$\square$ 
 $(\times 3)$ 
 $\hexagon$ 
 $(\div 6)$ 
 $\triangle$



$\square$ 
 $(\div 5)$ 
 $\triangle$ 
 $(\times 7)$ 
 $\bigcirc$

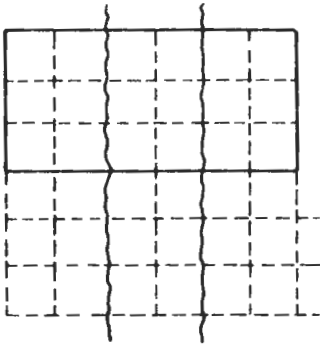


There is no easy way to show the 4 by 3 array "multiplied by 3" in the sketch. One way of carrying out the pair of directions is

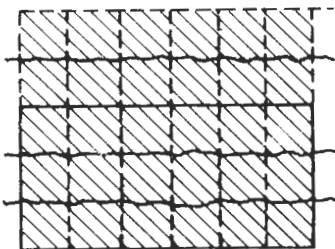


$$\boxed{12} (x3) \hexagon{36} (\div 6) \triangle{6}$$

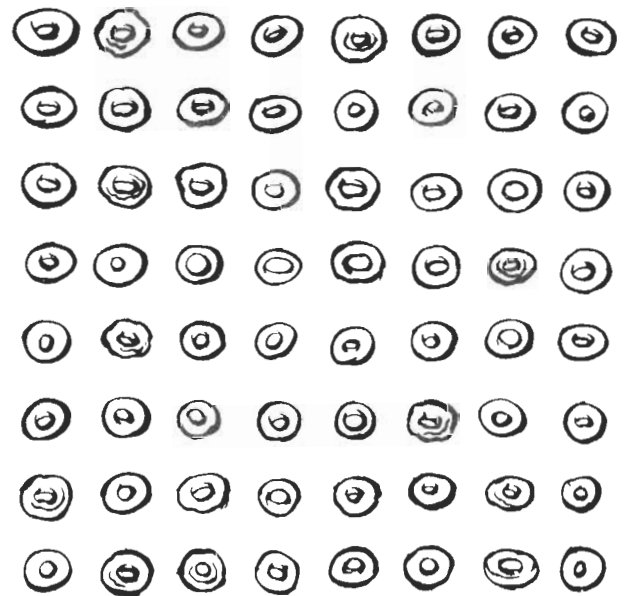
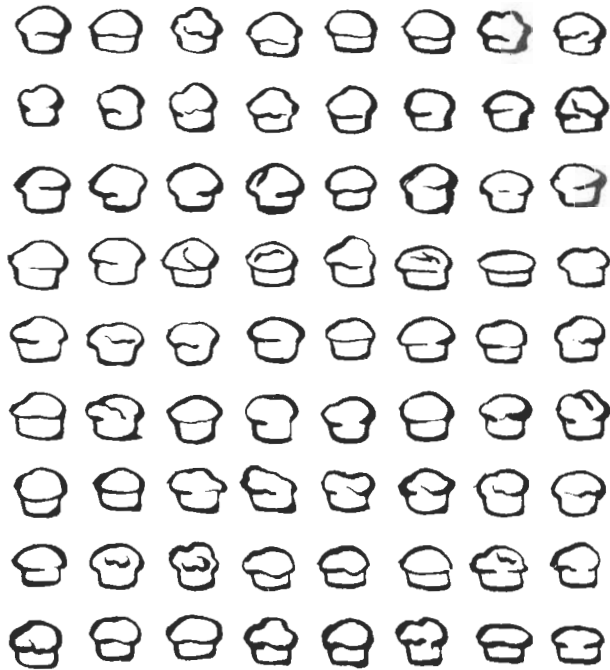
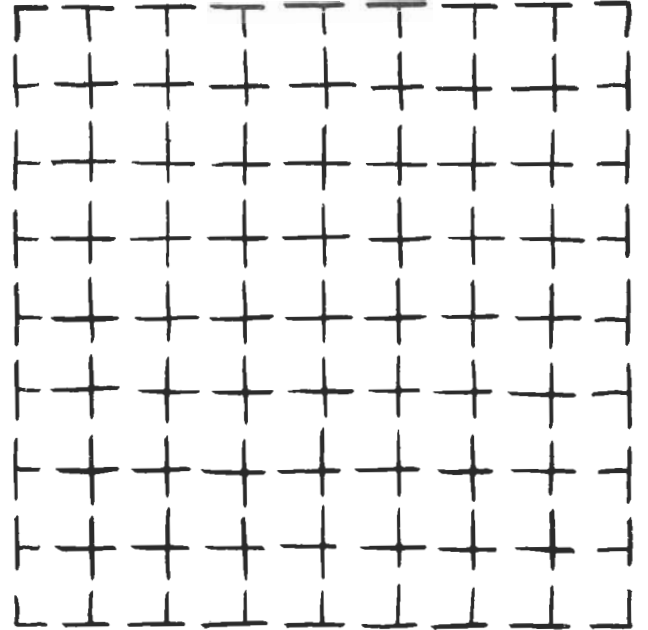
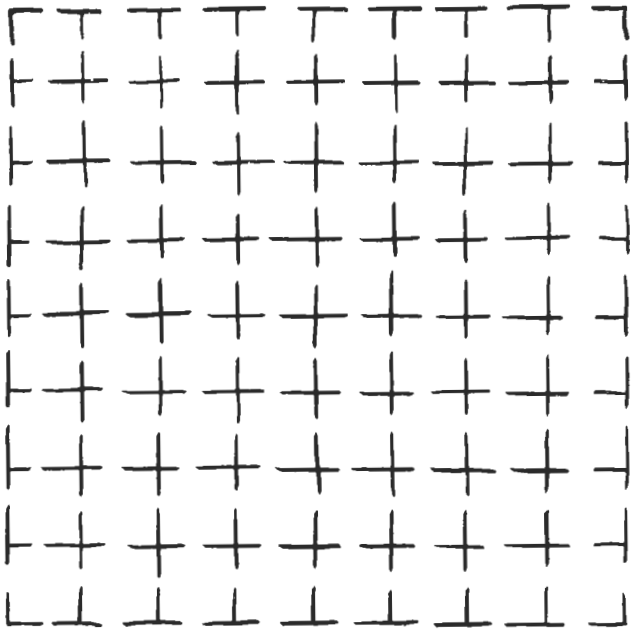
Nothing new arises in the examples at the top of the page



$$\boxed{18} (x2) \triangle{36} (\div 3) \hexagon{12}$$



$$\boxed{18} (\div 3) \triangle{6} (x5) \hexagon{30}$$





NAME \_\_\_\_\_

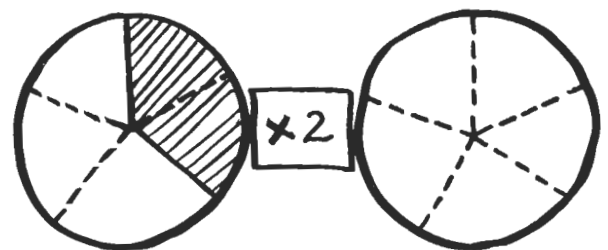
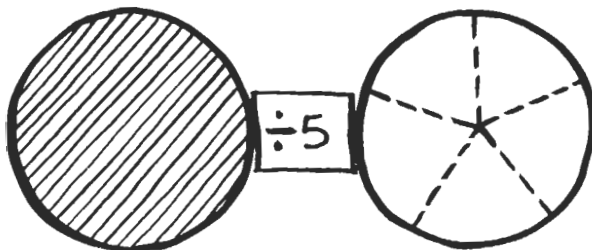
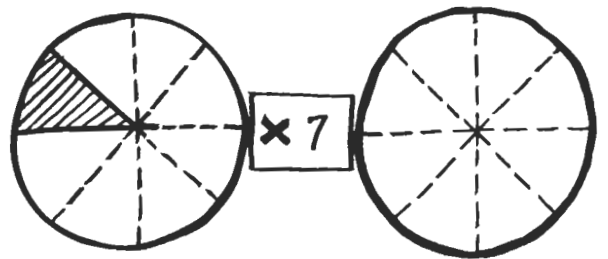
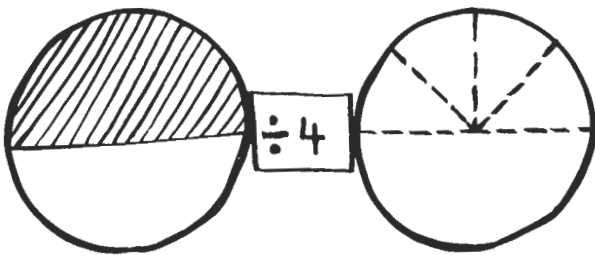
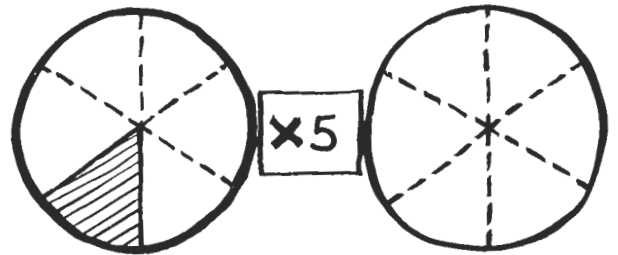
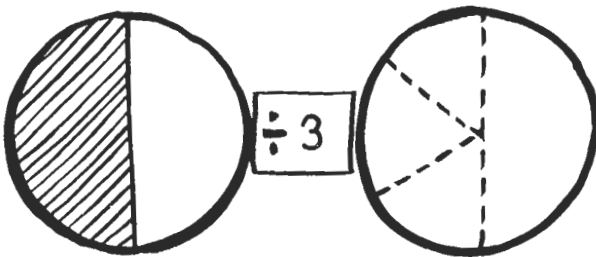
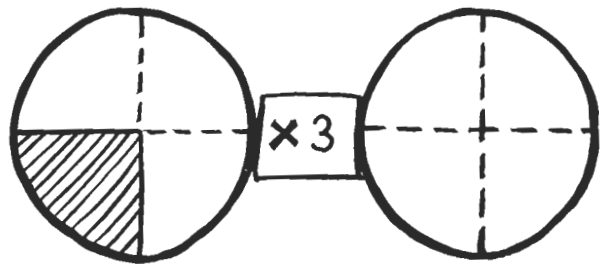
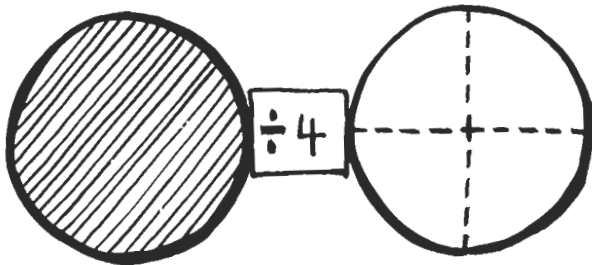
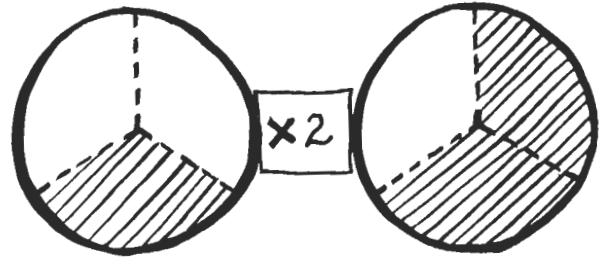
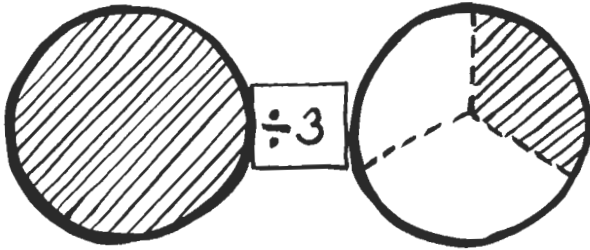
|                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                           |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <div style="border: 1px dashed black; height: 100px; width: 100%; position: relative;"> <!-- Grid of crosses --> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span>□ ( ) △ ( )</span> <span>□ ( ) ○ ( ) △ ( )</span> </div>      | <div style="border: 1px dashed black; height: 100px; width: 100%; position: relative;"> <!-- Grid of crosses --> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span>□ ( ) △ ( )</span> <span>□ ( ) △ ( ) ○ ( )</span> </div>      |
| <div style="border: 1px dashed black; height: 100px; width: 100%; position: relative;"> <!-- Grid of small shapes --> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span>□ ( ) △ ( )</span> <span>□ ( ) △ ( ) ○ ( )</span> </div> | <div style="border: 1px dashed black; height: 100px; width: 100%; position: relative;"> <!-- Grid of small shapes --> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span>□ ( ) △ ( )</span> <span>□ ( ) △ ( ) ○ ( )</span> </div> |

This page enables learners to make up their own examples and carry out investigations. It can also be particularized by the teacher.

$\div 3$

$\times 2$

DIRECTIONS TO "DO SOMETHING"  
INSTRUCCIONES PARA "HACER ALGO"



$\div 3$  AND  $\times 2$  AS

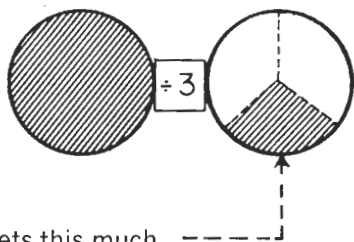
DIRECTIONS TO "DO SOMETHING"

NAME \_\_\_\_\_

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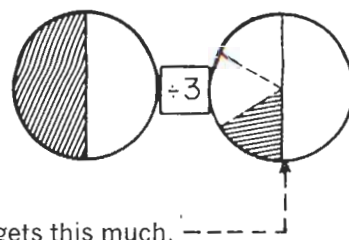
This page begins building toward understanding fractions and multiplication by fractions. Examples in the left hand column ask the learner to divide the shaded area into a certain number of equal pieces and to shade in one of those pieces.

The task can be made clearer for some learners by asking the question as follows: "The shaded area represents a pie (or pizza, or cookie) or some part of a pie. The number in the square tells you how many children are going to share the pie. Please cut up the pie so that each child will get the same amount, and shade one child's portion." Thus, in the first example the solution would be:



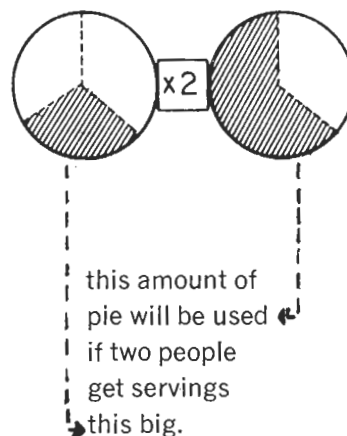
Each child gets this much. -----

The third and fourth examples on the left have a somewhat new twist. There is only half a pie to start with. But the question remains the same. Thus:



Each child gets this much. -----

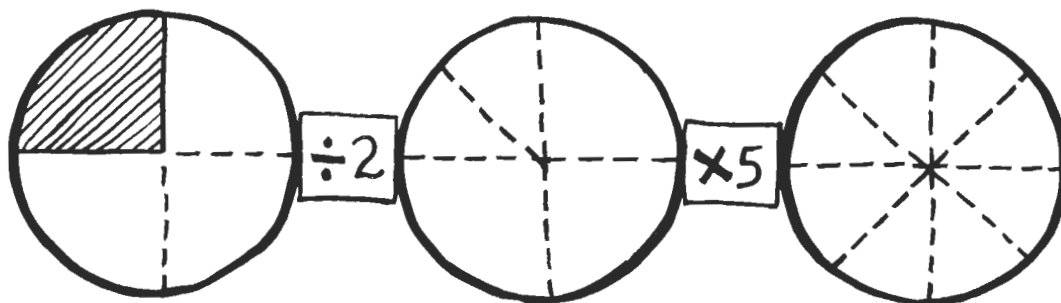
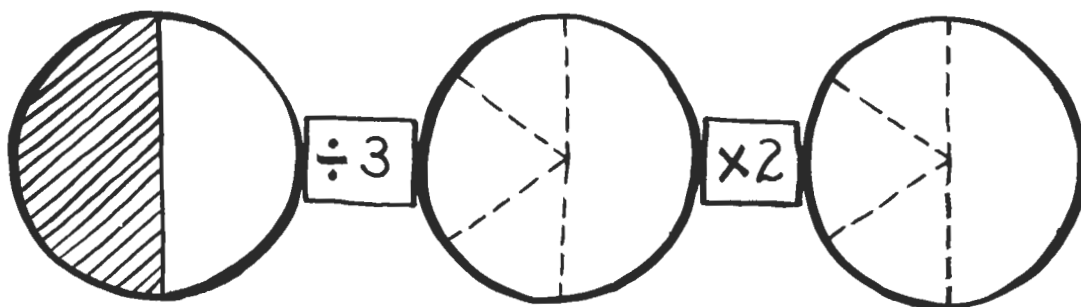
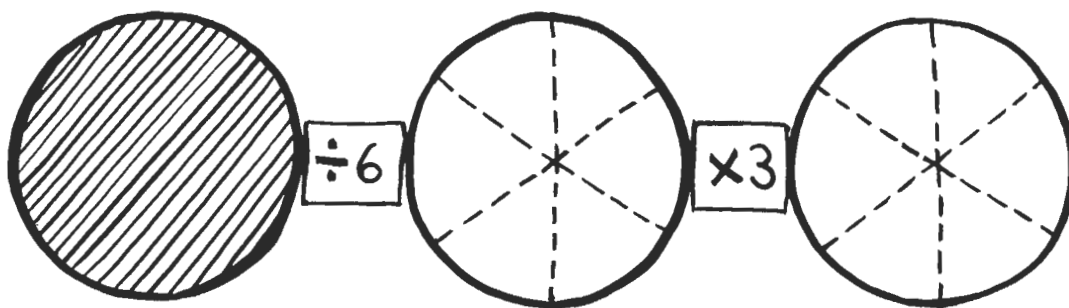
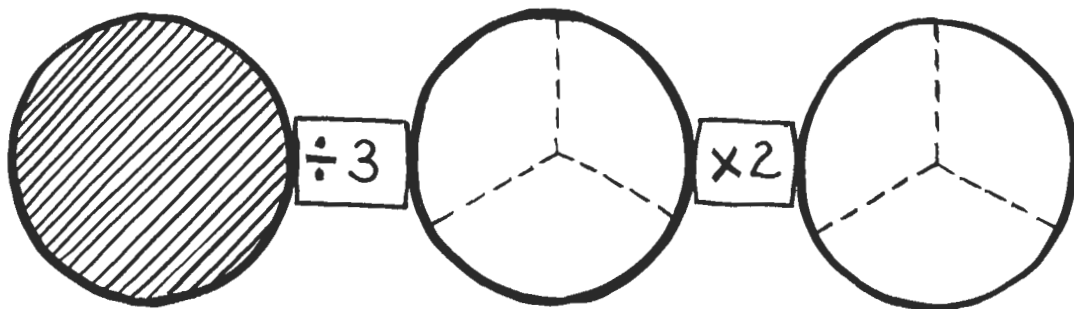
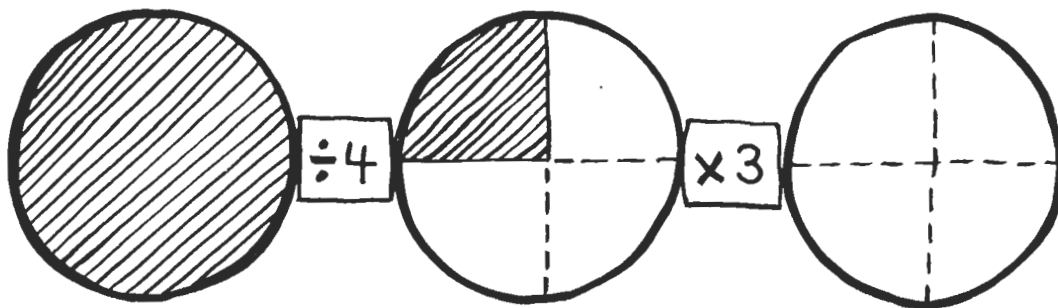
In the right hand column of examples, the circle on the left shows a portion shaded. The number in the box tells the learner how many such portions to shade in the right hand circle. In terms of "pies and people," the question now is: "The shaded part in the left hand circle shows the amount of pie to be given each person. The number in the box tells how many people are to be served. How much of the pie will be used up?" Shade that amount in the circle on the right. Thus:



this amount of  
pie will be used  
if two people  
get servings  
this big.

$\div a$

$\times b$



$\div a$  AND  $\times b$  AS A \_\_\_\_\_ NAME \_\_\_\_\_

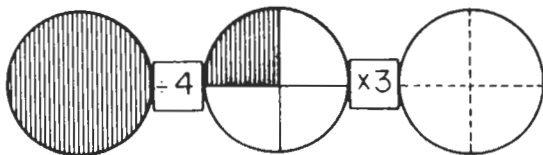
CHAIN REACTION OF "DIRECTIONS TO DO SOMETHING"

The grid contains five chain reaction problems:

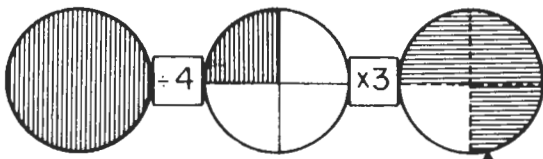
- Problem 1: Circle 1 (shaded 1/4)  $\div 4$  Circle 2 (shaded 1/16)  $\times 3$  Circle 3 (shaded 3/16)
- Problem 2: Circle 1 (shaded 1/2)  $\div 3$  Circle 2 (shaded 1/6)  $\times 2$  Circle 3 (shaded 1/3)
- Problem 3: Circle 1 (shaded 1/2)  $\div 6$  Circle 2 (shaded 1/12)  $\times 3$  Circle 3 (shaded 1/4)
- Problem 4: Circle 1 (shaded 1/2)  $\div 3$  Circle 2 (shaded 1/6)  $\times 2$  Circle 3 (shaded 1/3)
- Problem 5: Circle 1 (shaded 1/4)  $\div 2$  Circle 2 (shaded 1/8)  $\times 5$  Circle 3 (shaded 5/8)

In the last example on this activity page, it turns out that we would need more pie than we started with. In this case, we could add to our story that we have some other pies on hand for when this situation arises.

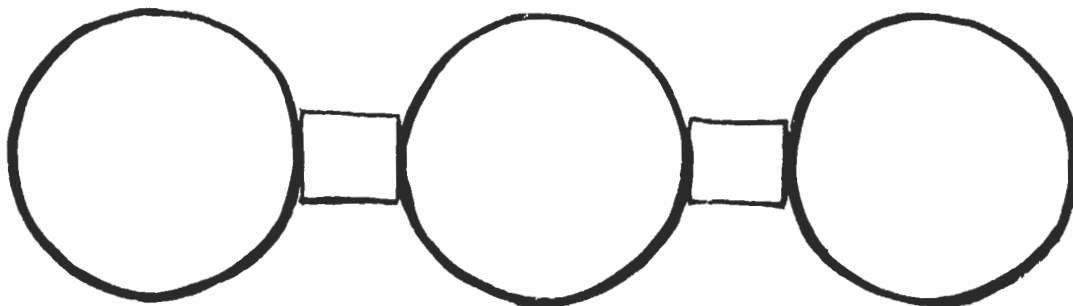
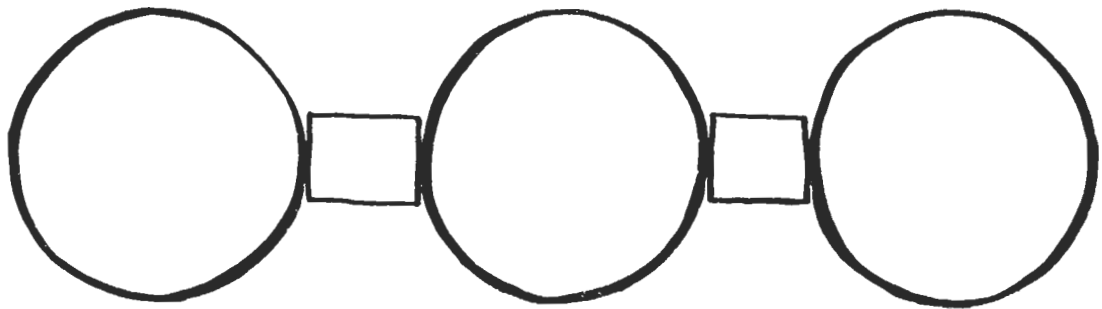
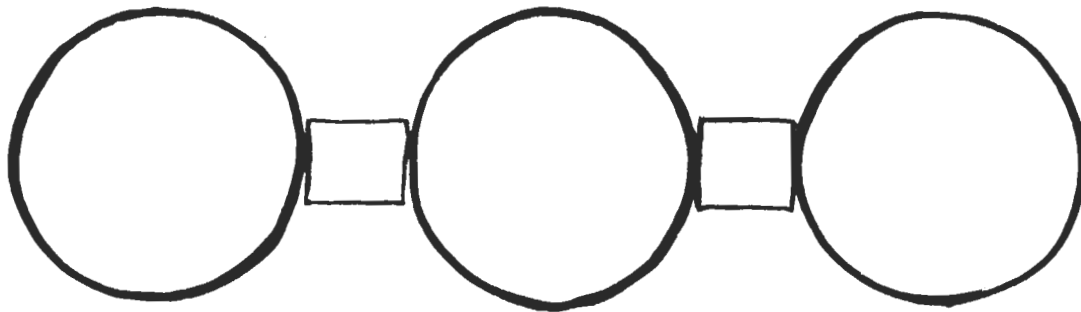
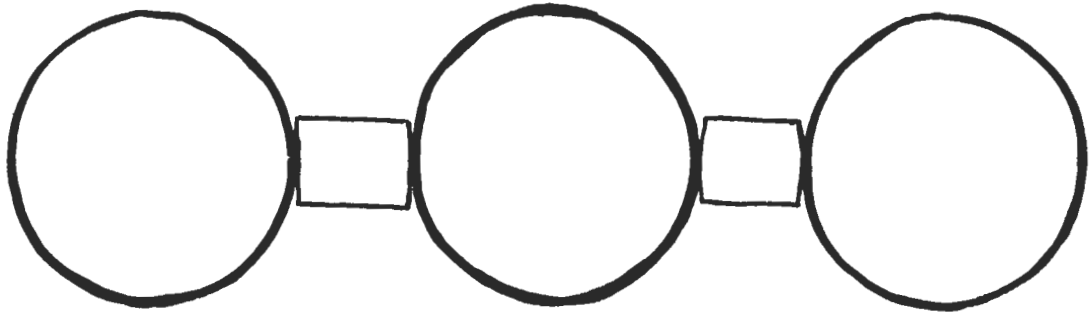
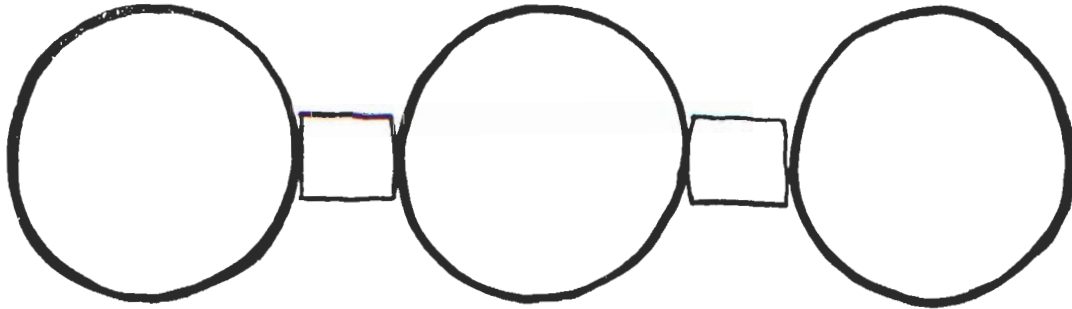
This page compresses the activities on the previous page, so that dividing the circle and "multiplying" a part of it are done all in one "chain reaction." In terms of our "pies and people" scenario, the story might be: "I start with the amount of pie shaded in the first circle. It is to be cut into the number of equal pieces indicated in the box. Please shade in one portion."



"Now, if I want to give that size portion to the number of people in the next box how much pie would I need?"



To serve 3 people I need this much pie. —



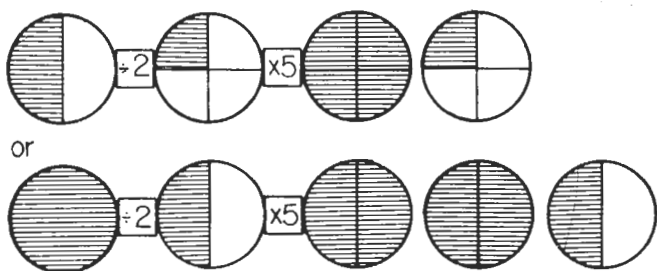


$\div$   $\times$   $\frac{a}{b}$  AND  $\frac{a}{b}$   $\times$   $\frac{b}{a}$  AS A  
 CHAIN REACTION OF "DIRECTIONS TO DO SOMETHING"

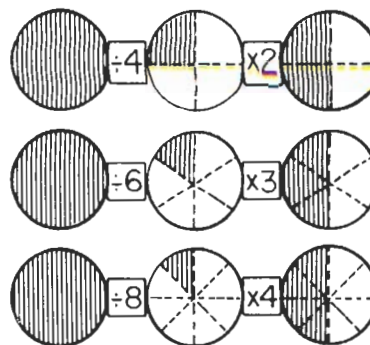
NAME \_\_\_\_\_

This form enables learners to make up their own examples and carry out investigations. It can also be particularized by the teacher.

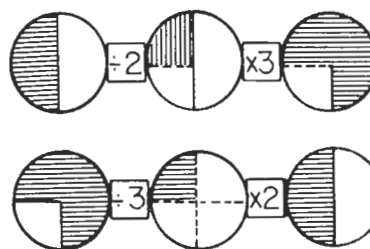
Some new situations might arise in which the learner will have to draw an additional circle or circles to complete the chain reaction. For example:



One possible way to particularize this page might be to structure some examples that build toward the idea that  $1/2 = 2/4 = 3/6$ , etc. For example, sequences like the following could be designed:

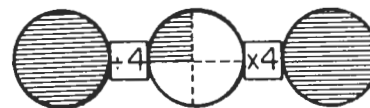


You might also want to structure examples which "undo" each other:



to build some referents for the later introduction of the notion  $a/b \cdot b/a = 1$

Examples that do not change the original portion can be developed:

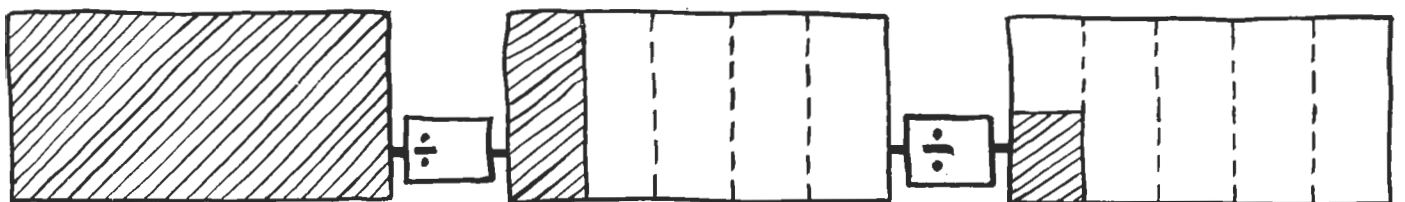
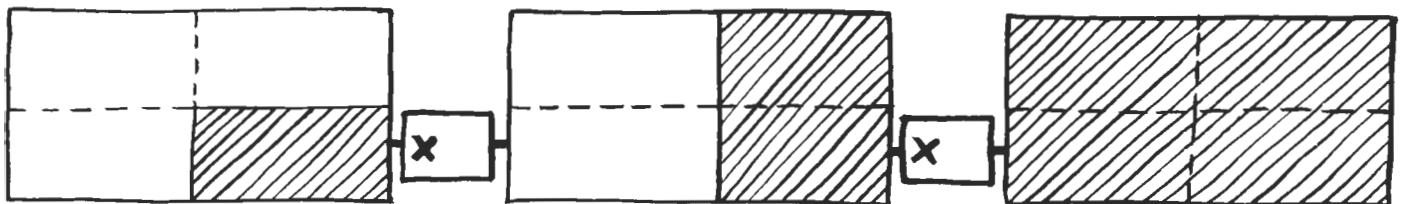
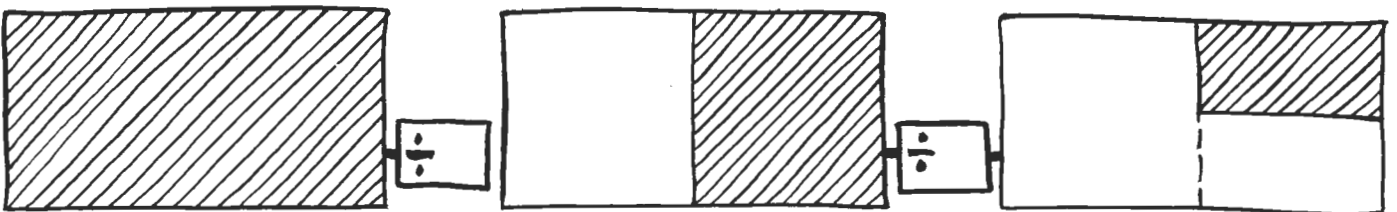
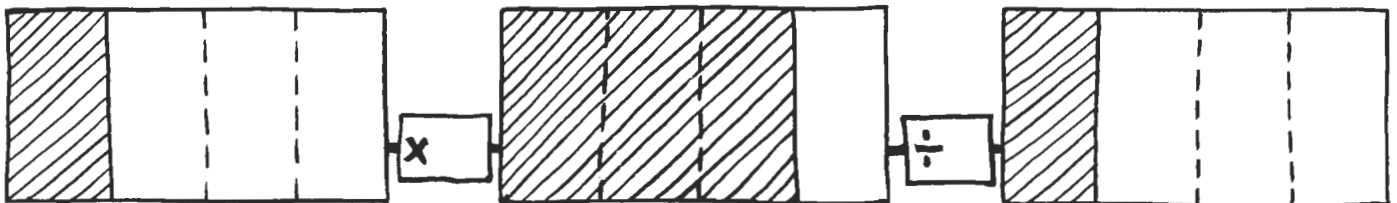
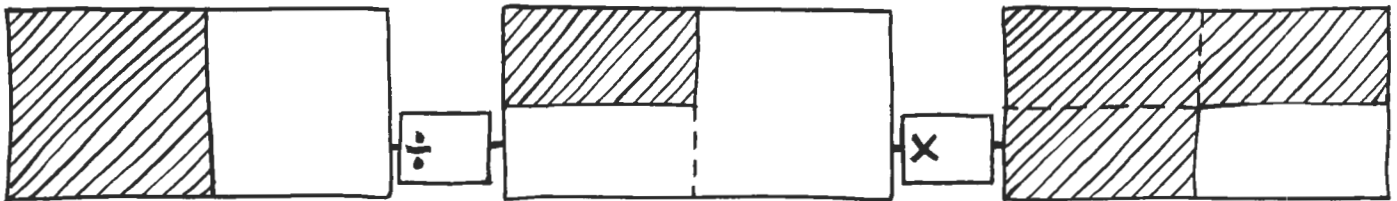
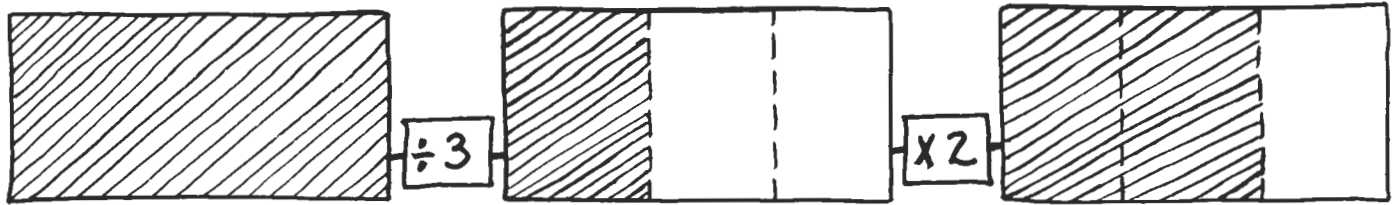


to build toward the later introduction of the idea that  $a/a = 1$ .

Once children see examples of this type, they could investigate to see whether such chain reactions have the same effect when different portions of the circle are used at the start.



"DIRECTIONS TO DO SOMETHING"  
DIRECCIONES PARA "HACER ALGO"





$\div$

c

AND

$\times$

d

AS A

NAME \_\_\_\_\_

CHAIN REACTION OF "DIRECTIONS TO DO SOMETHING"

This activity is similar to those on the previous pages. But now a rectangle is used instead of a circle, and the results of the chain reaction are shown in the diagram. The learner must describe the chain reaction by writing in the appropriate numbers in the small boxes.

$\div c$

$\times d$

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÷ C

AND

x d

AS A

NAME \_\_\_\_\_

CHAIN REACTION OF "DIRECTIONS TO DO SOMETHING"

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This form can be used for a variety of investigations.  
Some possibilities are suggested on Page 78 C.

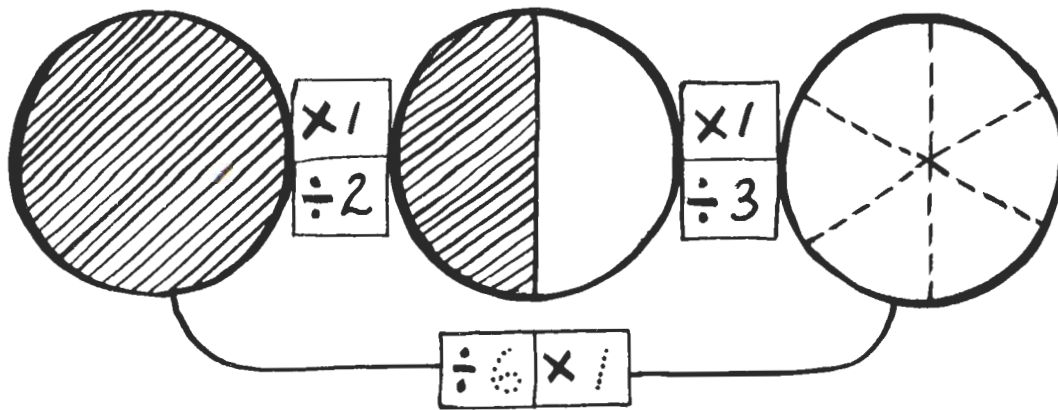
A question to explore:

Start with one half of the rectangle. Can you find  
a chain reaction that will result in doubling it?  
That is, what chain reaction could be written  
to fit:

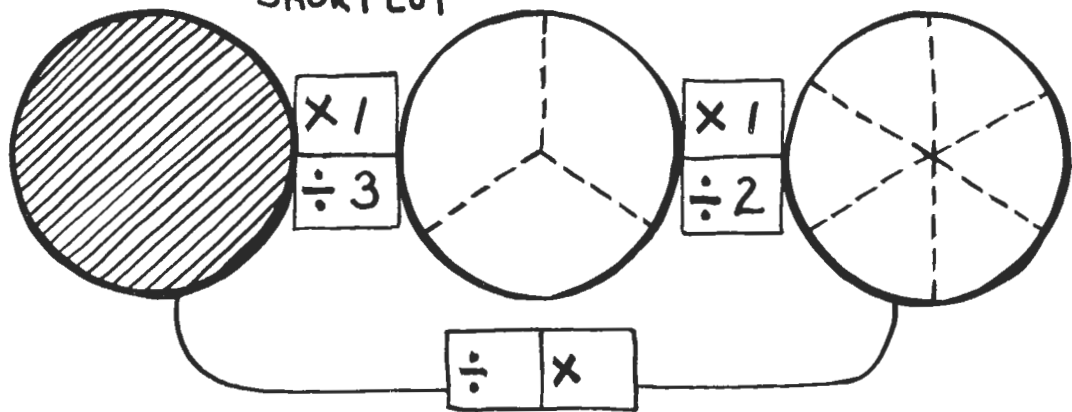


Is there more than one chain reaction that will work?  
Will these chain reactions "double" any area I start  
with?

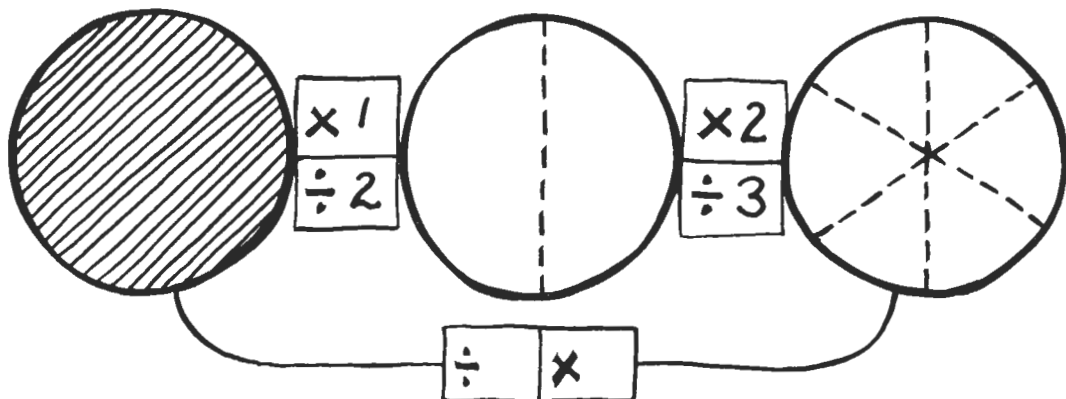
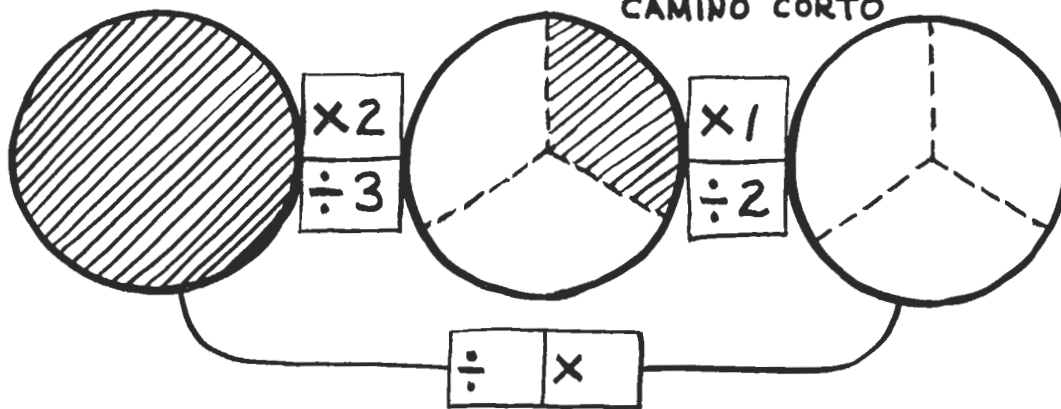
$$\frac{\div a}{\times b}$$

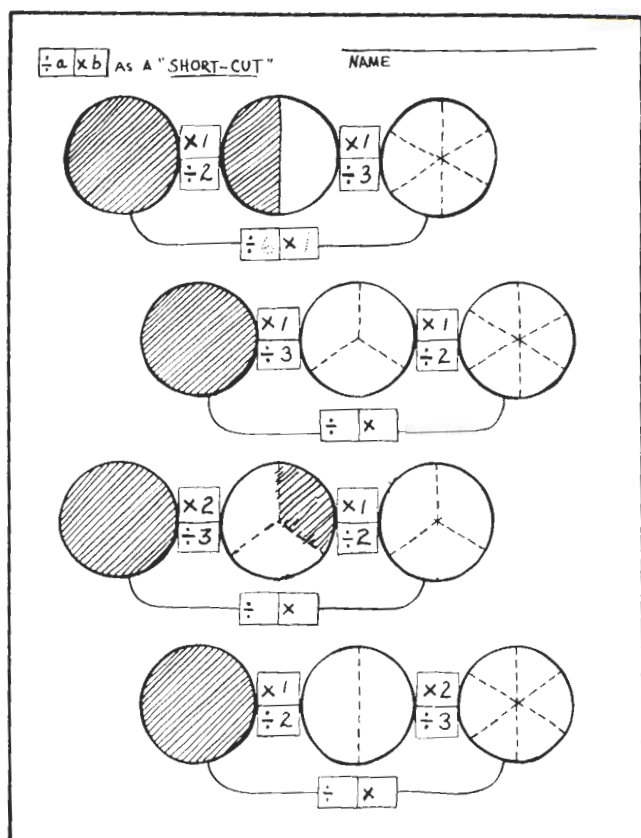


SHORTCUT



CAMINO CORTO





On this page there are two new twists. First, the chain reaction that previously was done in two separate steps must now be done in one step. Secondly, two chain reactions are done in sequence and the learner must find one chain reaction that would produce the same result. Referents are being built for fractions and multiplication by fractions.

If learners need a real situation which can give meaning to the "chain reaction" now expressed by

$$\begin{array}{|c|} \hline \times 1 \\ \hline \div 2 \\ \hline \end{array}$$

it might be the following: "I start with the amount of pie shaded in the first circle. The box tells me

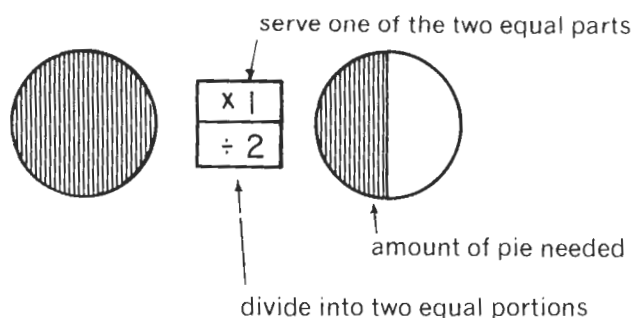
$$\div$$

how many equal parts I must divided the pie into. Each part is one portion of pie. The box tells

$$\times$$

me how many portions I must serve. In the second circle, I should shade in the amount of pie I will need."

Thus:

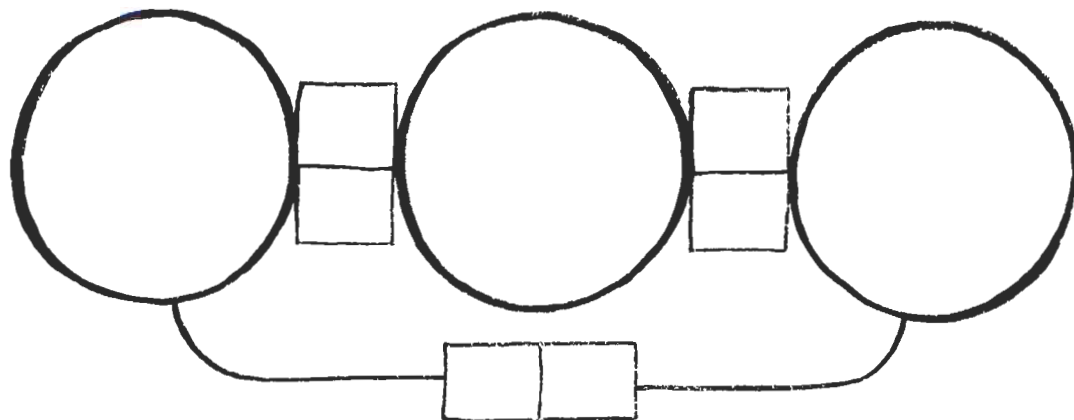


"The second chain reaction can be thought of in the same way, but now I will begin with a different amount of pie — that shown in the second circle."

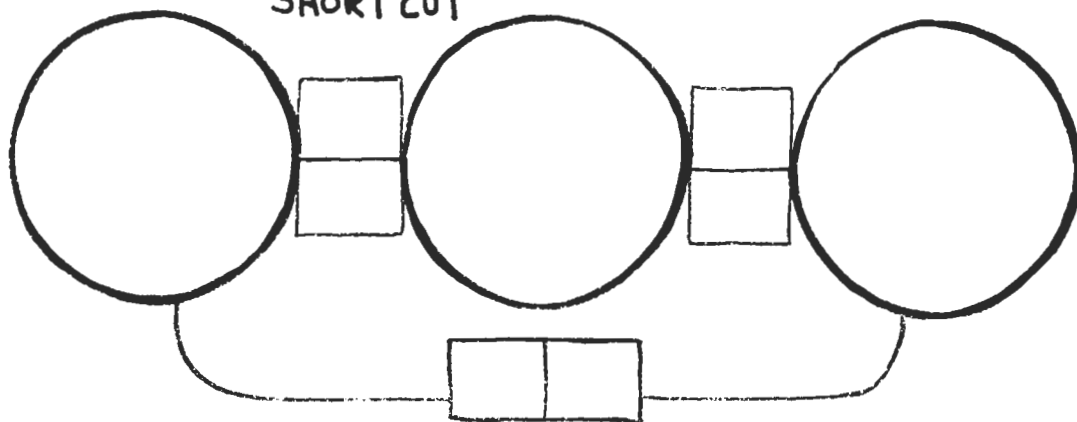
Once the result of the two chain reactions has been found, the problem becomes one of finding **one** chain reaction that would produce the same result.

On this page, the first two examples should be considered together, and the last two together. Each pair suggests an "in-any-order" rule for these chain reactions.

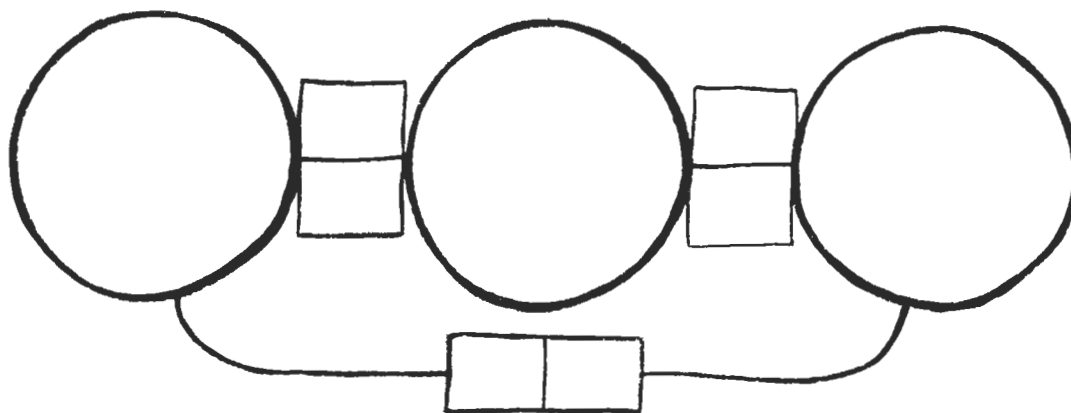
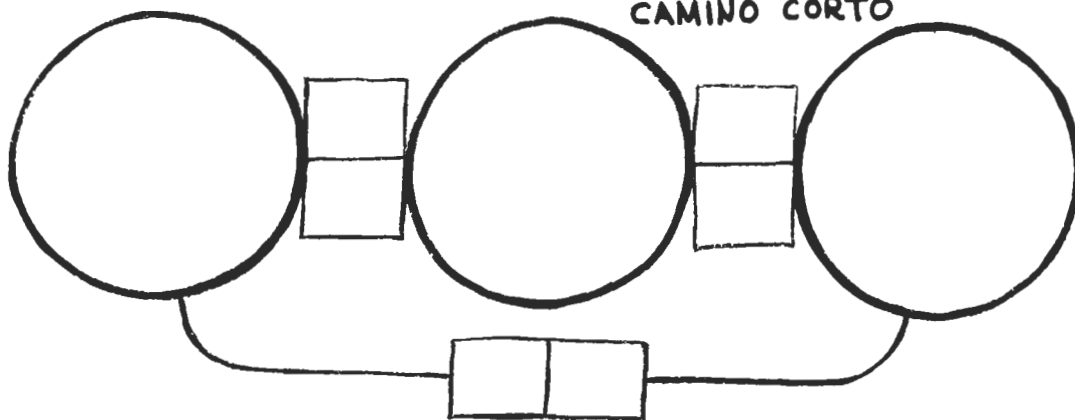
$$\boxed{\div a} \boxed{\times b}$$



SHORTCUT



CAMINO CORTO

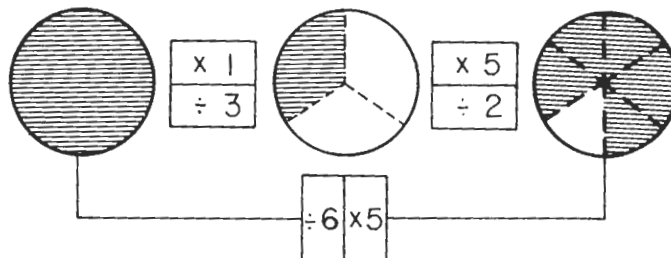


$\div a$   $\times b$

As A "SHORT-CUT"

NAME \_\_\_\_\_

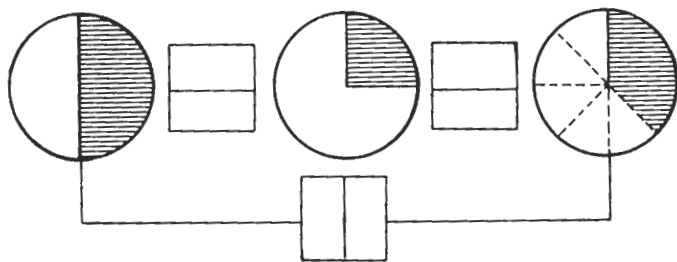
- (2) If the only information given is the shortcut, can you find a "top line" to fit it? For example, if the shortcut were  $\div 6, \times 5$ , one solution would be:



Can you find another?

Learners can make up their own examples using this form, or it can be particularized by the teacher. A few questions to explore:

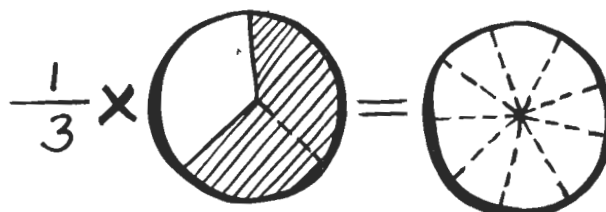
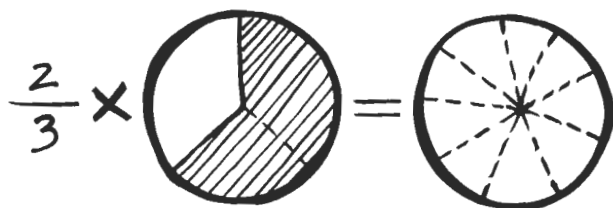
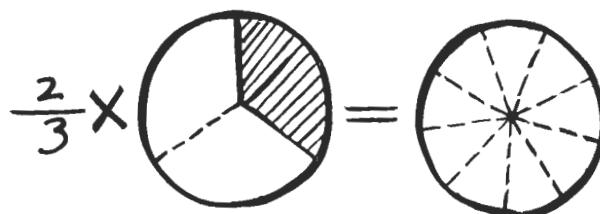
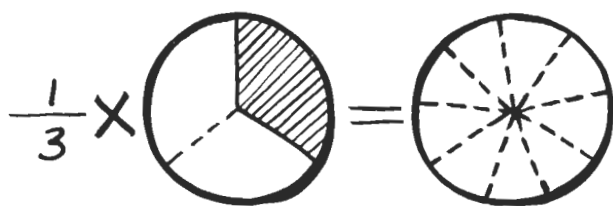
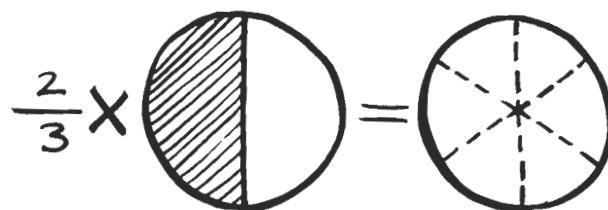
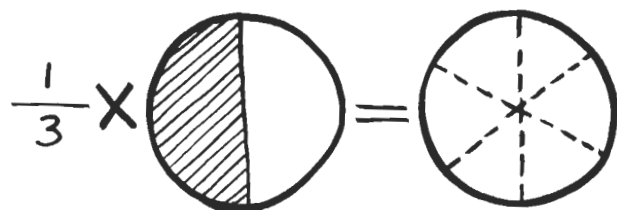
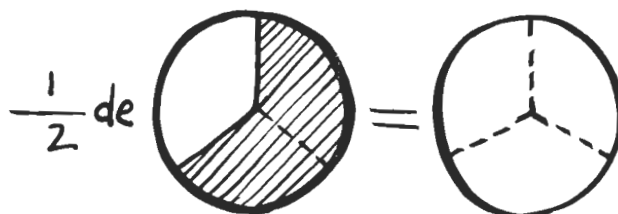
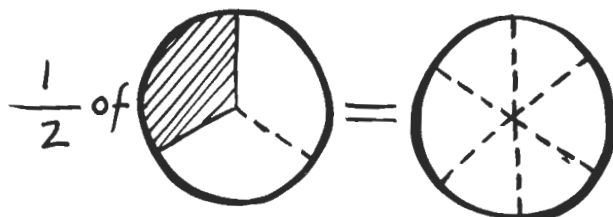
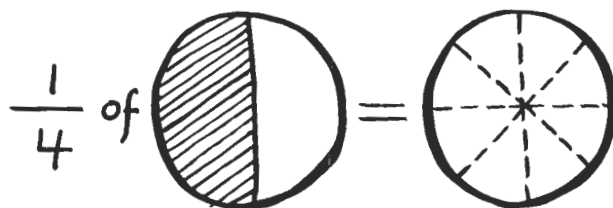
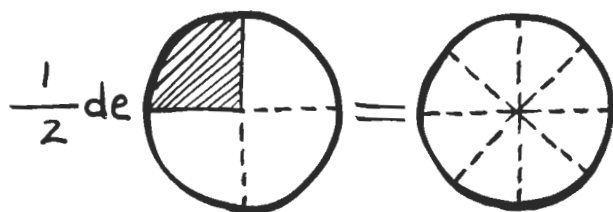
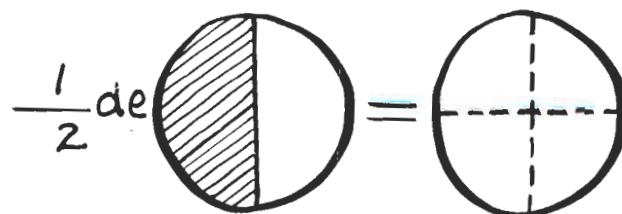
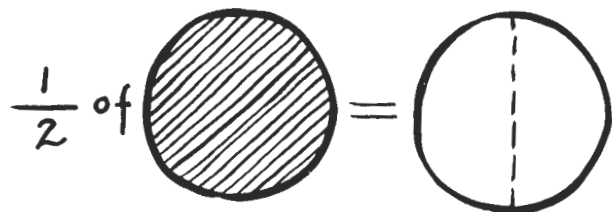
- (1) Shade in parts of the first and third circles. Can you find chain reactions to fit? For example, what chain reactions could lead to this situation?



What shortcuts could be used?

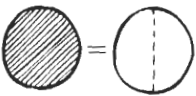
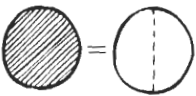
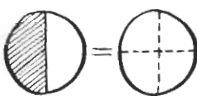
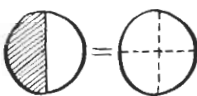
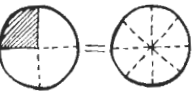
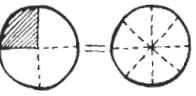
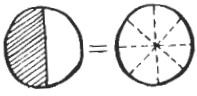
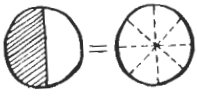
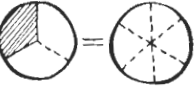
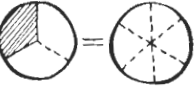
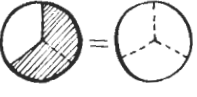
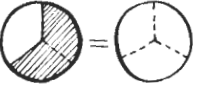
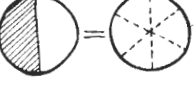
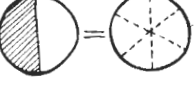
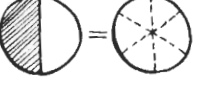
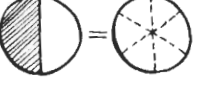
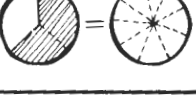
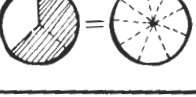
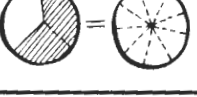
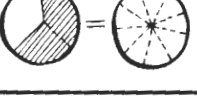
This might be a good opportunity to talk about shortcuts that use the smallest whole numbers, and agree that these shortcuts will be the ones we will record. In the example above,  $\div 2, \times 1$  was one possible shortcut. But we also might have used  $\div 4, \times 2$  or  $\div 8, \times 4$ . But the shortcut  $\div 2, \times 1$  used the smallest whole numbers, so under the new agreement, that would be the shortcut we would record.

$$\frac{1}{2} \text{ of } 14 = 7 \quad \frac{1}{2} \times 14 = 7$$

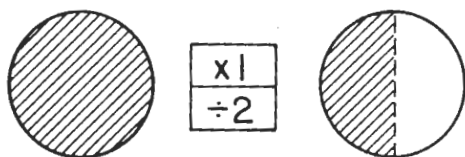




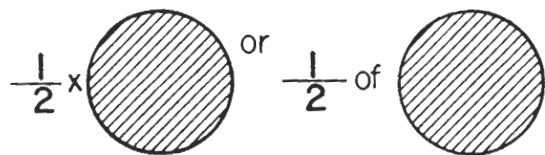
$\frac{1}{2}$  of 14 = 7 AND  $\frac{1}{2} \times 14 = 7$       NAME \_\_\_\_\_

|                                                                                                                                                                                            |                                                                                                                                                                                            |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\frac{1}{2}$ of  =      | $\frac{1}{2}$ of  =      |
| $\frac{1}{2}$ of  =      | $\frac{1}{4}$ of  =      |
| $\frac{1}{2}$ of  =      | $\frac{1}{2}$ of  =      |
| $\frac{1}{3} \times$  =  | $\frac{2}{3} \times$  =  |
| $\frac{1}{3} \times$  =  | $\frac{2}{3} \times$  =  |
| $\frac{2}{3} \times$  =  | $\frac{1}{3} \times$  =  |

The "chain reaction" activities have grown up on this page and learners work with fractions in standard form. Instead of writing



we now write



The meaning is the same, however,

$$\frac{1}{2} \text{ of } 14 = 7 \quad \frac{1}{2} \times 14 = 7$$

$$\text{— de } \bigcirc = \bigcirc$$

$$\text{— of } \bigcirc = \bigcirc$$

$$\text{— de } \bigcirc = \bigcirc$$

$$\text{— } \times \bigcirc = \bigcirc$$

$$\text{— } \times \bigcirc = \bigcirc$$

$$\text{— } \times \bigcirc = \bigcirc$$

---


$$\text{— of } \bigcirc = \bigcirc$$

$$\text{— de } \bigcirc = \bigcirc$$

$$\text{— of } \bigcirc = \bigcirc$$

$$\text{— } \times \bigcirc = \bigcirc$$

$$\text{— } \times \bigcirc = \bigcirc$$

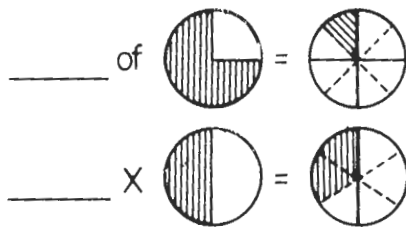
$$\text{— } \times \bigcirc = \bigcirc$$

$\frac{1}{2}$  of 14 = 7 AND  $\frac{1}{2} \times 14 = 7$       NAME \_\_\_\_\_

|                                                                                                                                                                                  |                                                                                                                                                                                  |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| — of  =        | — of  =        |
| — of  =        | — of  =        |
| — of  =        | — of  =        |
| — $\times$  =  | — $\times$  =  |
| — $\times$  =  | — $\times$  =  |
| — $\times$  =  | — $\times$  =  |

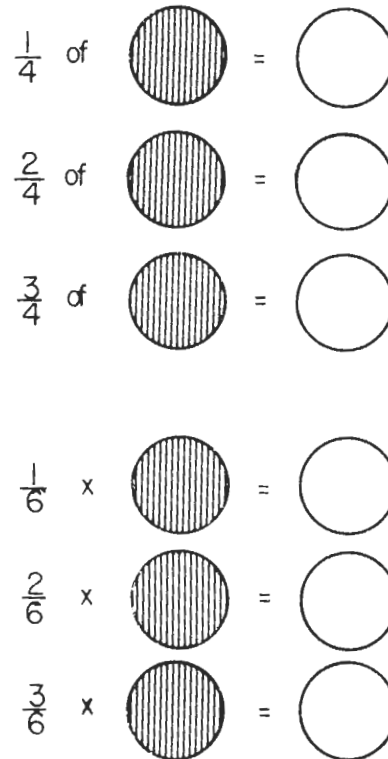
This report form can be used by the learner to make independent investigations or it can be particularized by the teacher.

Problems such as the following might be posed on this page.

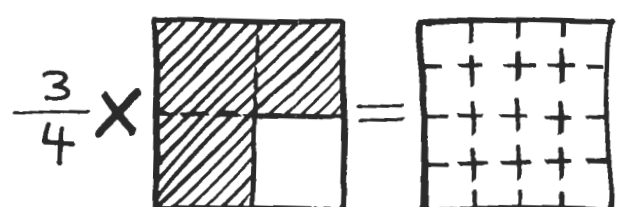
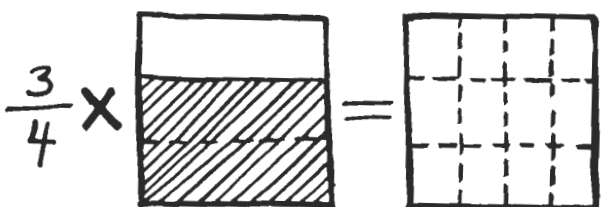
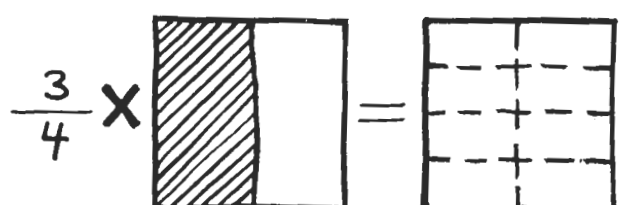
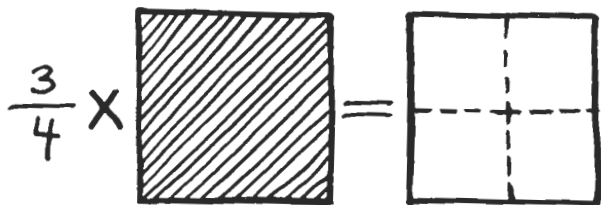
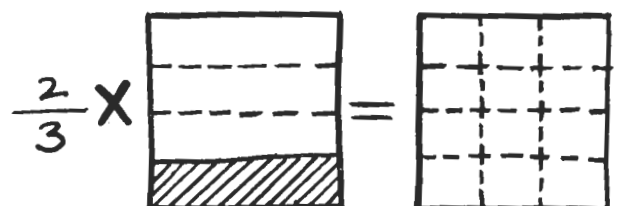
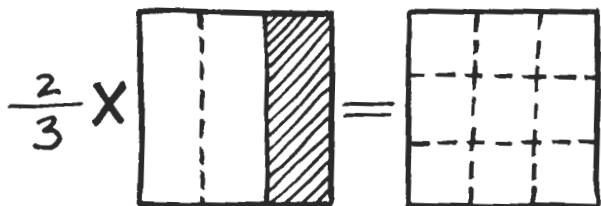
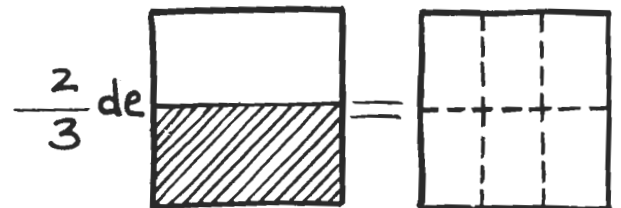
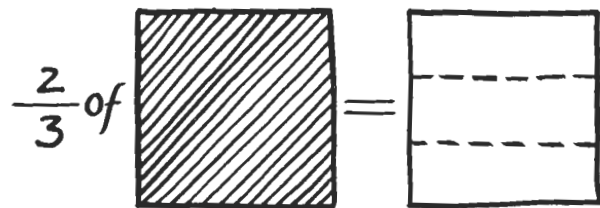
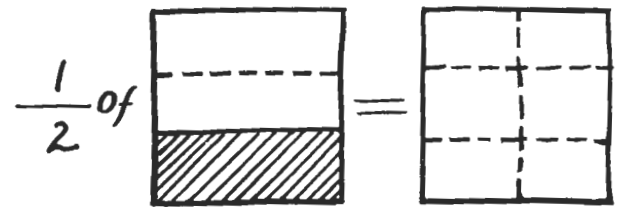
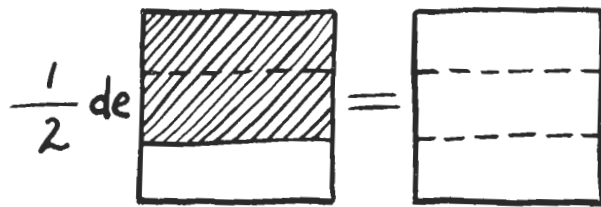
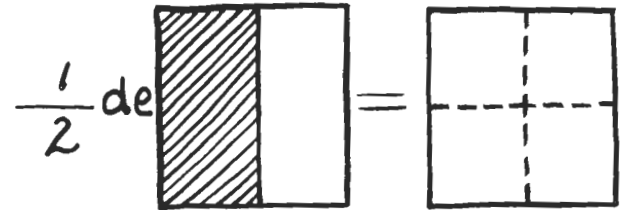
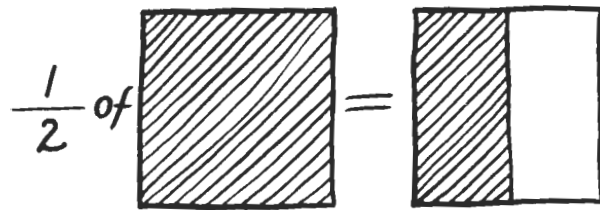


The learner must figure out what fraction can be written on the line to make the statement true.

Sequences of 3 examples might be set up to begin building referents for addition of fractions. For example:

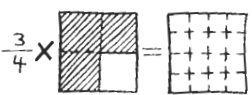
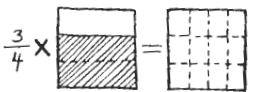
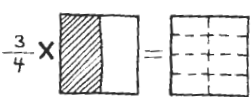
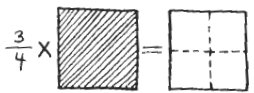
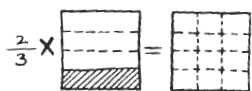
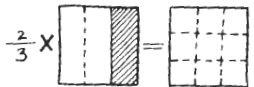
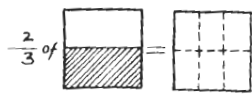
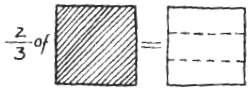
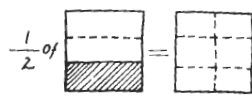
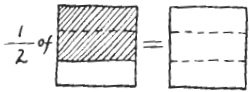
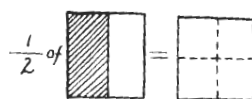
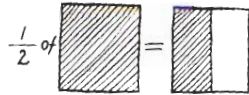


$$\frac{1}{2} \text{ of } 10 = 5 \quad \frac{1}{2} \times 10 = 5$$



$\frac{1}{2}$  of 10 = 5 AND  $\frac{1}{2} \times 10 = 5$

NAME \_\_\_\_\_



This activity is a continuation of previous activities, with the change that a square is now used instead of a circle.

$$\frac{1}{2} \text{ of } 10 = 5 \quad \frac{1}{2} \times 10 = 5$$

$$\text{— de } \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\text{— of } \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\text{— de } \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\text{— x } \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\text{— x } \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\text{— x } \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

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$$\text{— of } \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\text{— de } \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\text{— of } \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\text{— x } \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\text{— x } \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\text{— x } \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\frac{1}{2} \text{ of } 10 = 5 \text{ AND } \frac{1}{2} \times 10 = 5$$

NAME \_\_\_\_\_

$$\text{— of } \square = \square$$

$$\text{— of } \square = \square$$

$$\text{— of } \square = \square$$

$$\text{— of } \square = \square$$

$$\text{— of } \square = \square$$

$$\text{— of } \square = \square$$

$$\text{— } \times \square = \square$$

$$\text{— } \times \square = \square$$

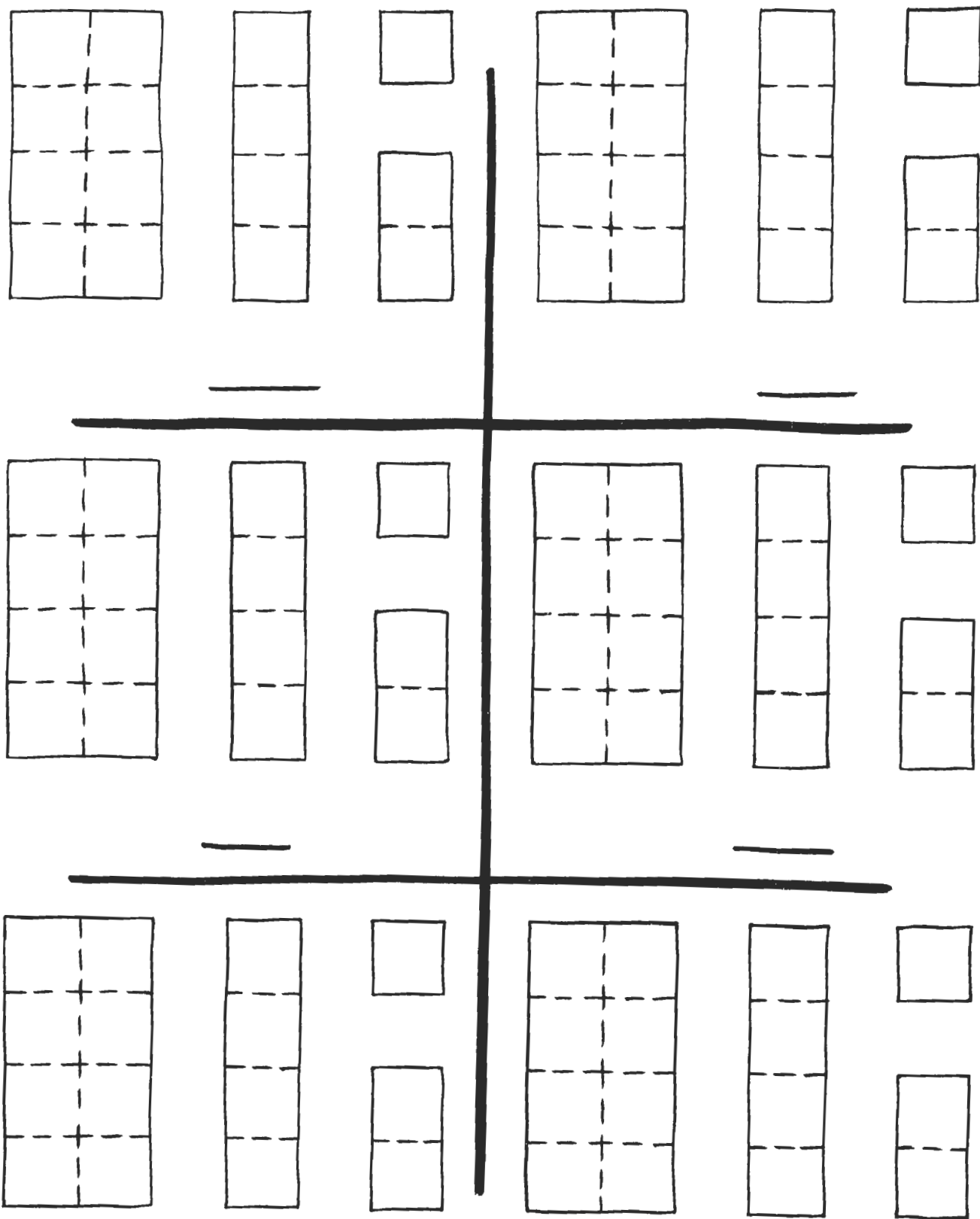
$$\text{— } \times \square = \square$$

$$\text{— } \times \square = \square$$

$$\text{— } \times \square = \square$$

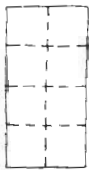
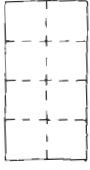
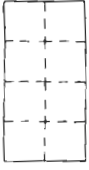
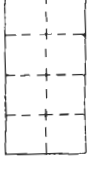
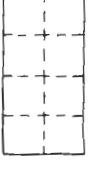
$$\text{— } \times \square = \square$$

Here learners can make up their own examples and do explorations or the teacher can particularize. A few suggestions for activities are given on page 80 D.





NAME \_\_\_\_\_

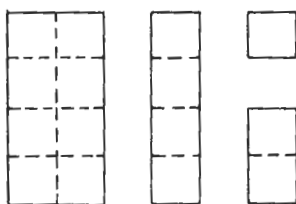
|                                                                                   |                                                                                   |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
| _____ PANES                                                                       | _____ PANES                                                                       |
|  |  |
| _____ PANES                                                                       | _____ PANES                                                                       |
|  |  |
| _____ PANES                                                                       | _____ PANES                                                                       |

This is a "coloring" activity called "windows and panes."

Rules: if you color any pane, you must color all the panes in that window. You may color no windows (or "0" panes), any 1 of the 4 windows, or any combination of windows. There is a provision for a record of the total number of panes.

One way in which this page might be used is to ask children to color in the examples so that each has a different number of panes colored in.

Or, the total numbers can be specified—and combinations found to fit; such as:



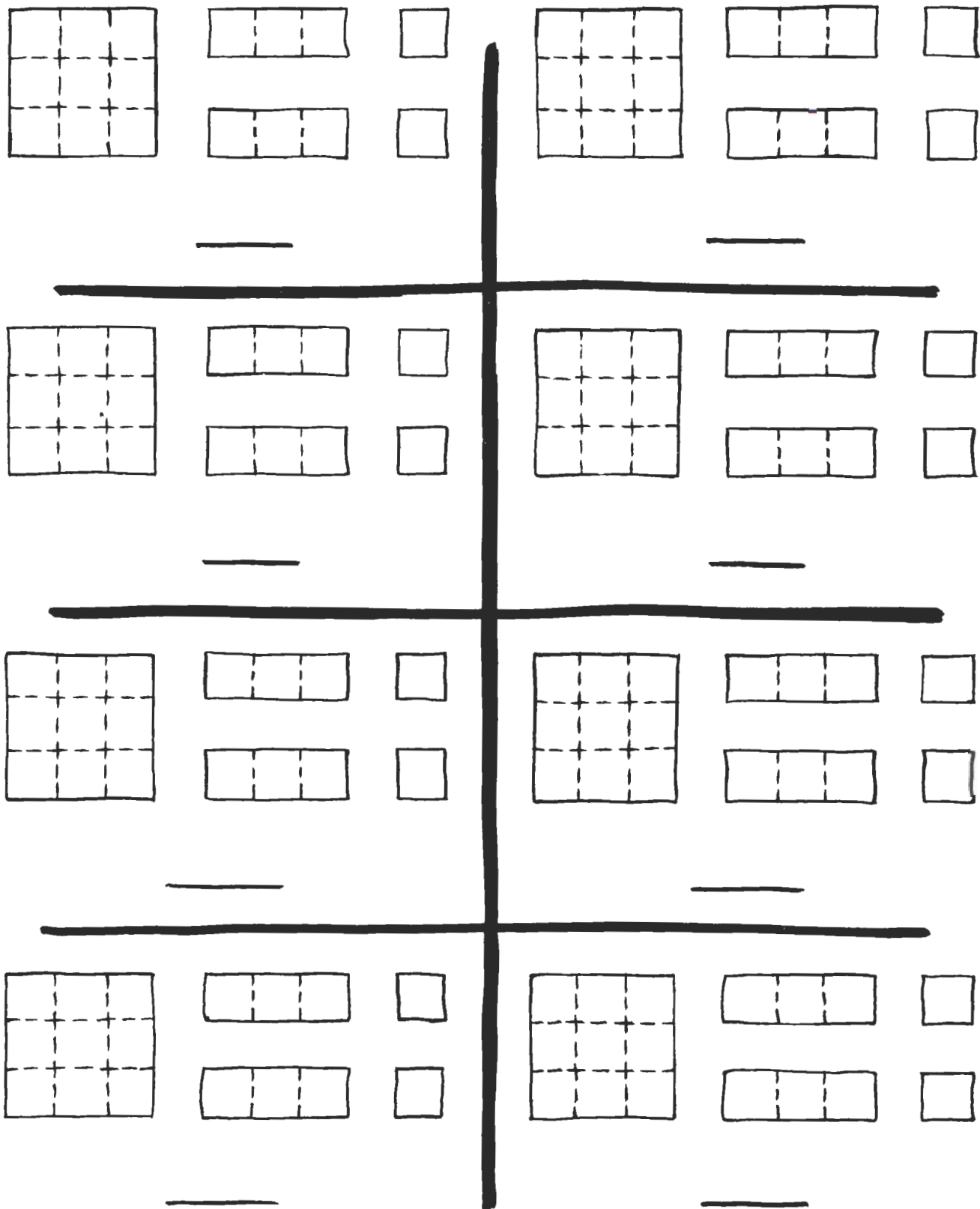
13 panes

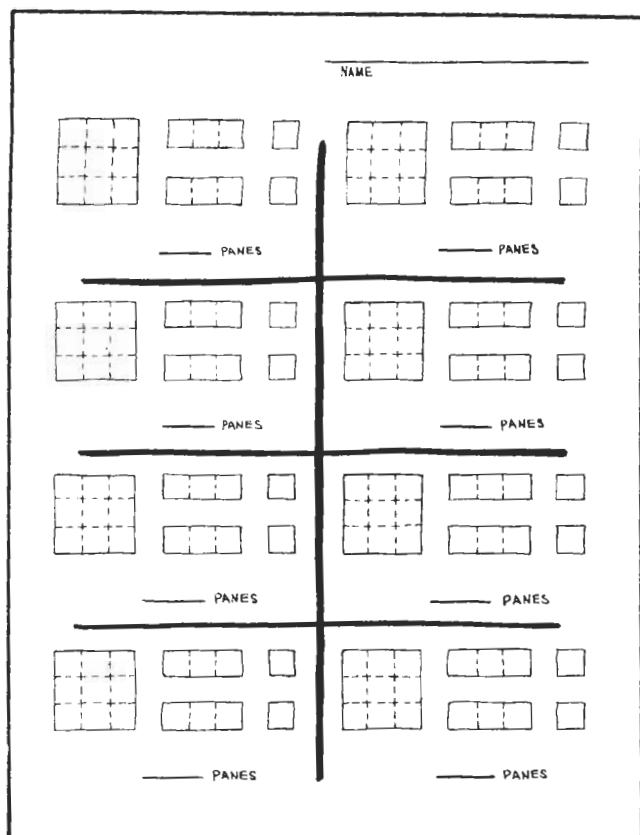
etc.

A more demanding problem is, using 2 sheets (12 examples) make all 12 examples different. (And more demanding yet—all 12 different and each with 4 or more panes.)

The special characteristic of windows with 1, 2, 4 and 8 panes is that there is a combination for each number, 1 thru 15—and only one combination in each case.

Thus a request to find some number that can be colored in two different ways must lead to the conclusion that it can't be done.





and definitions that would consider these as "colored in the same way" would be clumsy and, probably, non-productive. (For example, you might make a rule that if only 1 of any kind is colored, it must be the one on the top.)

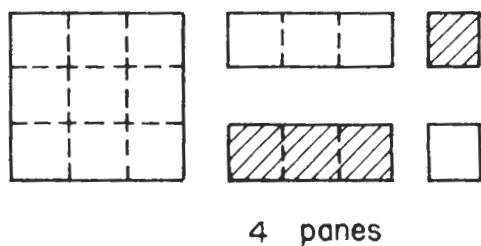
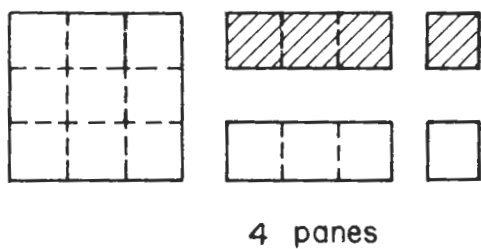
How many different numbers of panes can you color if you are allowed to use only three windows in each example? (You need only one sheet to show 5, 7, 11, 13 and 15. Why are they all odd numbers? Why is "9" missing from the sequence?)

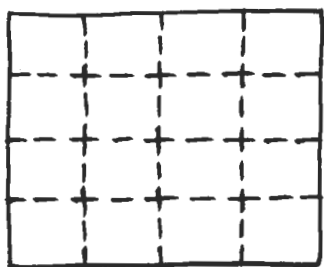
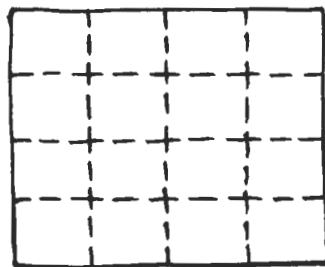
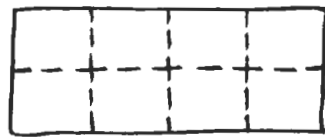
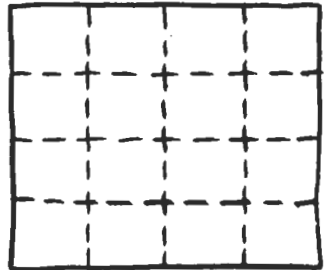
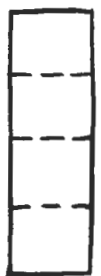
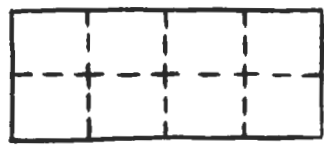
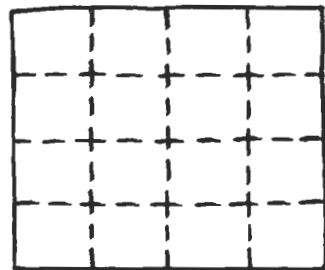
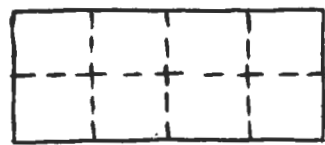
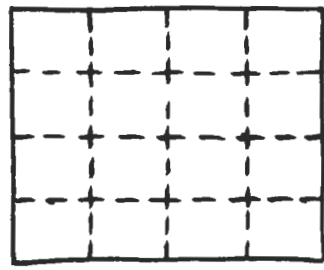
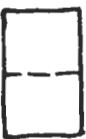
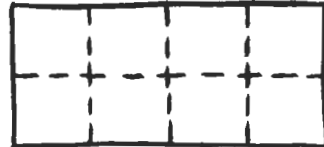
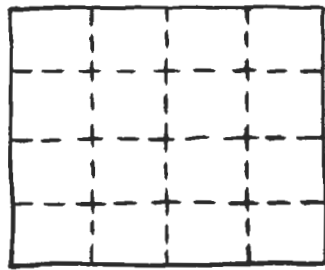
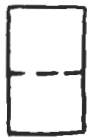
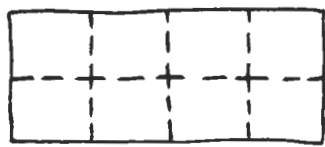
More "windows and panes"—if you color in a pane, you must color in all panes in that window.

"Using 2 sheets (16 examples) can you color all examples so that each has a different total number of panes colored?"

"Yes," is the answer to the question above: there is a way to color in each number of panes 1 thru 17.

To ask whether any number of panes can be colored in 2 different ways may lead to an argument:





NAME \_\_\_\_\_

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
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| <div style="display: flex; justify-content: space-around;"> <div style="border: 1px dashed black; width: 40px; height: 20px; margin-bottom: 5px;"></div> <div style="border: 1px dashed black; width: 15px; height: 20px; margin-bottom: 5px;"></div> <div style="border: 1px dashed black; width: 15px; height: 20px;"></div> </div> <div style="border: 1px dashed black; width: 40px; height: 40px; margin-bottom: 5px;"></div> <div style="border: 1px dashed black; width: 40px; height: 40px;"></div> <div style="text-align: center; margin-top: 5px;">PANES</div> | <div style="display: flex; justify-content: space-around;"> <div style="border: 1px dashed black; width: 40px; height: 20px; margin-bottom: 5px;"></div> <div style="border: 1px dashed black; width: 15px; height: 20px; margin-bottom: 5px;"></div> <div style="border: 1px dashed black; width: 15px; height: 20px;"></div> </div> <div style="border: 1px dashed black; width: 40px; height: 40px; margin-bottom: 5px;"></div> <div style="border: 1px dashed black; width: 40px; height: 40px;"></div> <div style="text-align: center; margin-top: 5px;">PANES</div> |
| <div style="display: flex; justify-content: space-around;"> <div style="border: 1px dashed black; width: 40px; height: 20px; margin-bottom: 5px;"></div> <div style="border: 1px dashed black; width: 15px; height: 20px; margin-bottom: 5px;"></div> <div style="border: 1px dashed black; width: 15px; height: 20px;"></div> </div> <div style="border: 1px dashed black; width: 40px; height: 40px; margin-bottom: 5px;"></div> <div style="border: 1px dashed black; width: 40px; height: 40px;"></div> <div style="text-align: center; margin-top: 5px;">PANES</div> | <div style="display: flex; justify-content: space-around;"> <div style="border: 1px dashed black; width: 40px; height: 20px; margin-bottom: 5px;"></div> <div style="border: 1px dashed black; width: 15px; height: 20px; margin-bottom: 5px;"></div> <div style="border: 1px dashed black; width: 15px; height: 20px;"></div> </div> <div style="border: 1px dashed black; width: 40px; height: 40px; margin-bottom: 5px;"></div> <div style="border: 1px dashed black; width: 40px; height: 40px;"></div> <div style="text-align: center; margin-top: 5px;">PANES</div> |

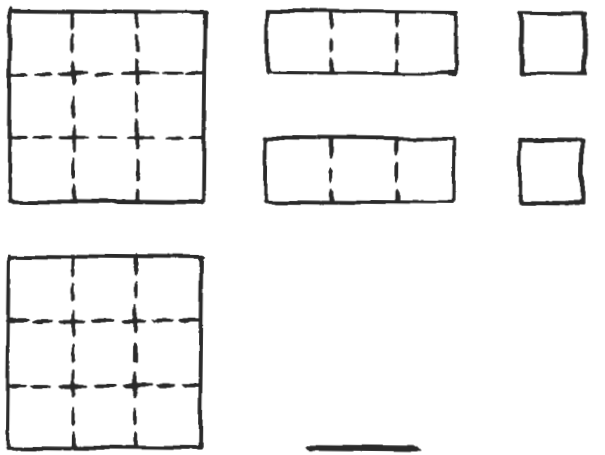
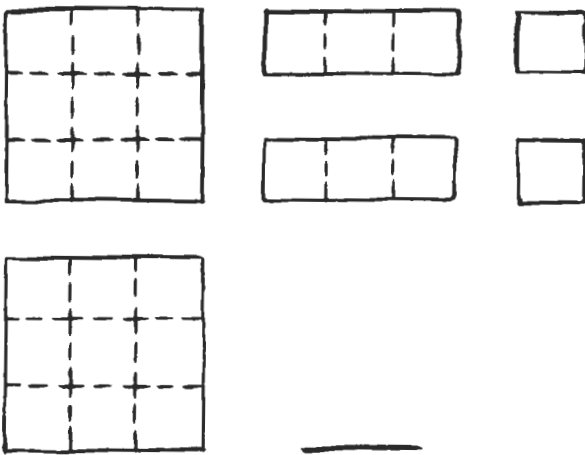
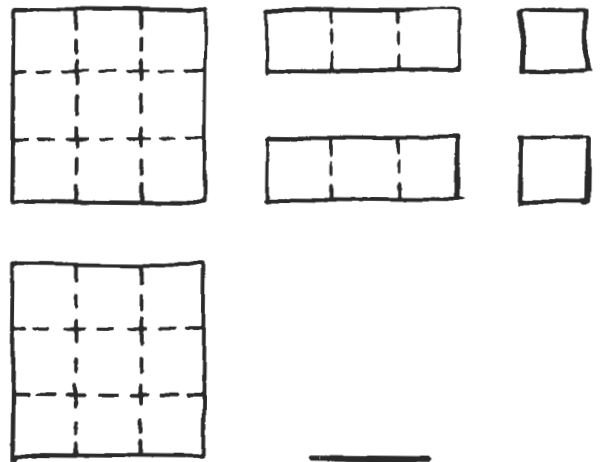
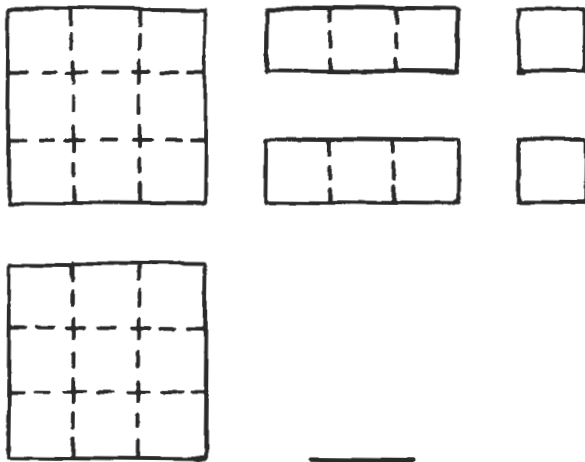
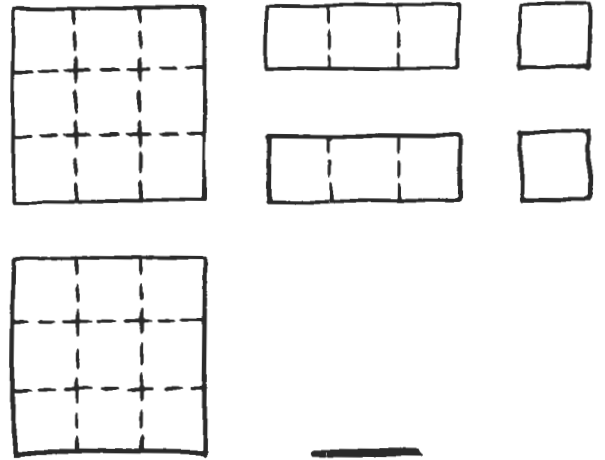
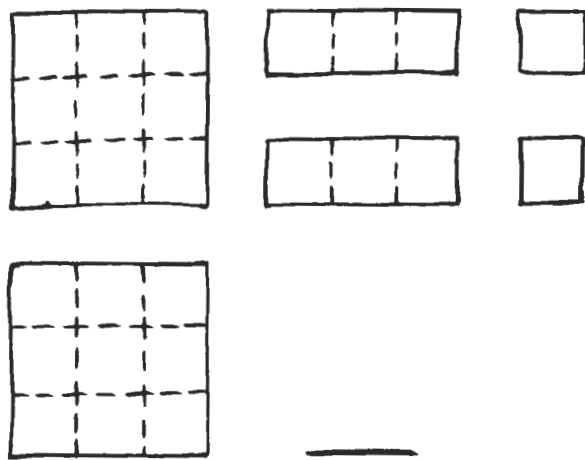
More “windows and panes”—with the rule that, if you color a pane in any window, you must color all panes in that window.

It would require too many pages to find all the different combinations: There is one for each number of panes 0 thru 31.

Is there a combination for all odd numbers larger than 8? . . . which takes 2 pages to display a combination for all such odd numbers.

Is there a combination for each of the numbers 3, 6, 9, 12, 15 and 18? . . . which takes a single sheet.

Is there a combination for each of the numbers 4, 8, 12, 16, 20, 24? . . . which takes a single sheet and reveals a bit of the structure of the sequence 1, 2, 4, 8, 16: all will use a combination of 4, 8 or 16. The smaller windows (1 and 2) will not be used to show a multiple of 4.



NAME \_\_\_\_\_

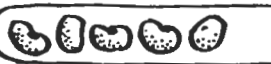
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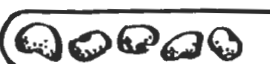
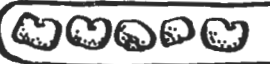
More “windows and panes”—and the rule is that if you color in a pane in any window, you must color all the panes in that window.

There is a way to color in all numbers of panes, 0 thru 26.

“Is there a combination for each of the numbers 4, 8, 12, 16, 20, 24?” Yes.

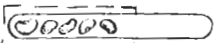
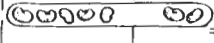
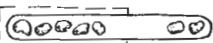
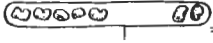
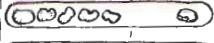
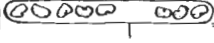
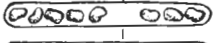
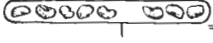
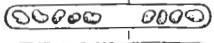
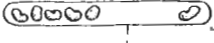
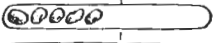
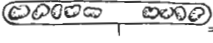
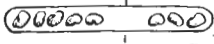
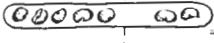
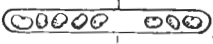
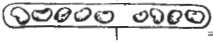
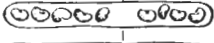
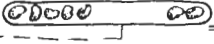
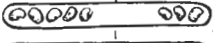
“Is there a combination for each of the numbers 9, 12, 15, 18, 21, 24?” Yes. And, in no example will either of the 1-pane windows be colored: only combinations of 3, 6, and 9 will be used.

|                                                                                     |   |
|-------------------------------------------------------------------------------------|---|
|    | 5 |
|    | 7 |
| TOTAL _____                                                                         |   |
|    |   |
|    |   |
| TOTAL _____                                                                         |   |
|   |   |
|  |   |
| TOTAL _____                                                                         |   |
|  |   |
|  |   |
| TOTAL _____                                                                         |   |
|  |   |
|  |   |
| TOTAL _____                                                                         |   |

|                                                                                      |                                                                                       |
|--------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|
|    |    |
|    |    |
| TOTAL _____                                                                          |                                                                                       |
|    |    |
|    |    |
| TOTAL _____                                                                          |                                                                                       |
|   |                                                                                       |
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| TOTAL _____                                                                          |                                                                                       |
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| TOTAL _____                                                                          |                                                                                       |
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| TOTAL _____                                                                          |                                                                                       |



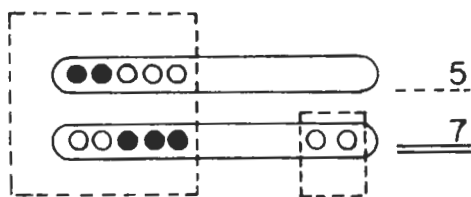
NAME \_\_\_\_\_

|                                                                                                                                                                                           |                                                                                                                                                                                       |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  5<br> 7<br>TOTAL _____ | <br><br>TOTAL _____ |
| <br><br>TOTAL _____     | <br><br>TOTAL _____ |
| <br><br>TOTAL _____     | <br><br>TOTAL _____ |
| <br><br>TOTAL _____     | <br><br>TOTAL _____ |
| <br><br>TOTAL _____     | <br><br>TOTAL _____ |

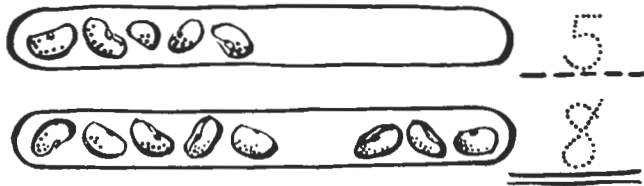
This page provides practice in addition using numbers between 5 and 9. Each stick is shown with 5 beans grouped on the left side and the rest on the right. This is done so a number can be quickly identified as 5 plus some more. The children learn to count on from 5. In a like manner, when the beans on both sticks are to be counted, it should be pointed out that the two 5's can be considered as 10 and then count on from 10 (if necessary).

Proceed by working through the first example.

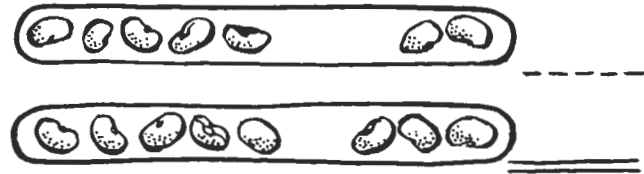
- (1) How many beans on the top stick? (5) Write 5 on the line.
- (2) How many beans on the other stick (5 and 2 more or 7) Write 7 on the line.
- (3) How many beans on both sticks? (5 and 5 are 10 and 2 more is 12) Write 12 under the 7 next to the total.



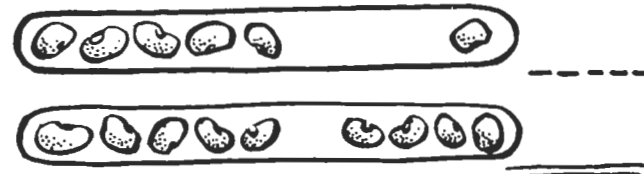
5 and 5 are 10....and 2 more → \_\_\_\_\_



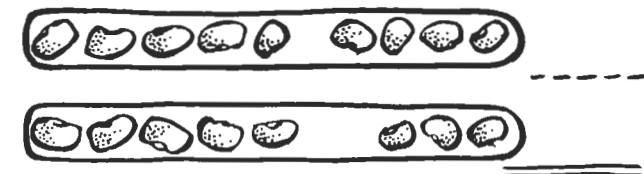
TOTAL \_\_\_\_\_



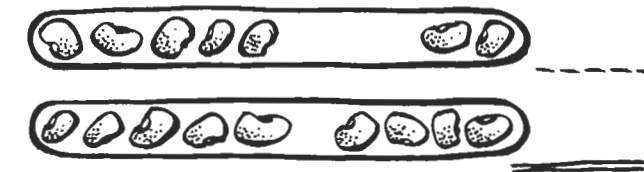
TOTAL \_\_\_\_\_



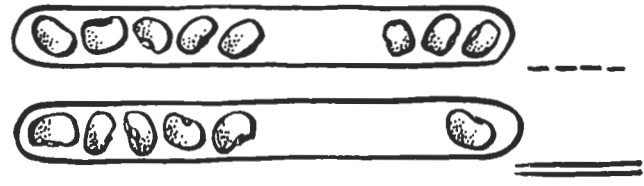
TOTAL \_\_\_\_\_



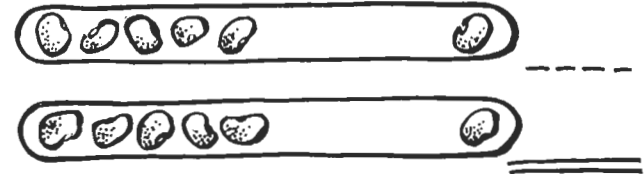
TOTAL \_\_\_\_\_



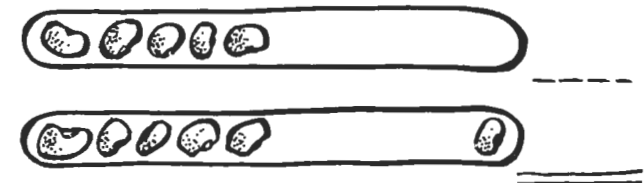
TOTAL \_\_\_\_\_



TOTAL \_\_\_\_\_



TOTAL \_\_\_\_\_



TOTAL \_\_\_\_\_

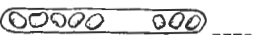
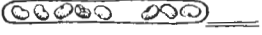
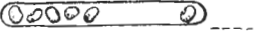
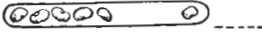
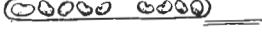
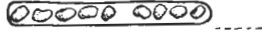
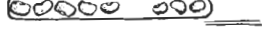
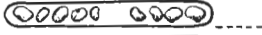


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This page can be particularized using the format on previous pages.

The teacher can draw the beans and have the children complete as previously, or partial records can be provided.

#### Examples

|  |           |
|--|-----------|
|  | _____     |
|  | =====     |
|  | _____     |
|  | =====     |
|  | _____     |
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|  | <u>15</u> |
|  | _____     |
|  | =====     |
|  | <u>16</u> |

|  |           |
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|  | =====     |
|  | <u>15</u> |

Children can make up their own examples and swap with each other. Eventually the children can work with the numbers only, drawing the picture if needed. Or, the page might be used to focus on other ways to avoid one-by-one counting. One strategy would be to work with the doubles whose sums are greater than 10. If a child knows that  $7 + 7 = 14$ , he can use that fact in examples such as:

|  |           |
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|  | =====     |
|  | <u>15</u> |

$14 + 1 = 15$

|  |           |
|--|-----------|
|  | _____     |
|  | =====     |
|  | <u>13</u> |

$14 - 1 = 13$

|  |           |
|--|-----------|
|  | _____     |
|  | =====     |
|  | <u>16</u> |

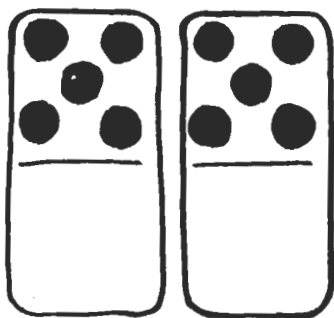
$14 + 2 = 16$

|  |           |
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|  | _____     |
|  | =====     |
|  | <u>14</u> |

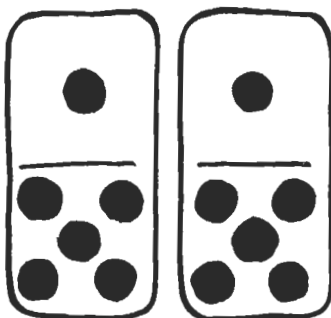
"If I moved one down, there would be two 7's or 14."  
Or, perhaps, the activity might be to alter the sketch to agree with the partial report and complete the report.

|  |           |
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|  | _____     |
|  | =====     |
|  | <u>15</u> |

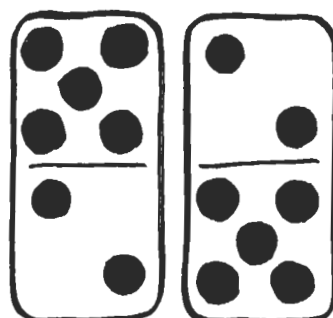
Again, children might use the Report Form for making up their own examples or carrying out investigations.



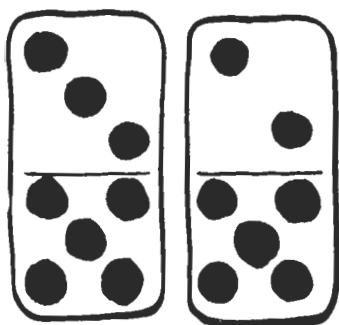
$$5 + 5 = \underline{\hspace{2cm}}$$



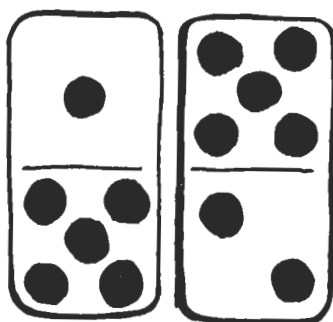
$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$



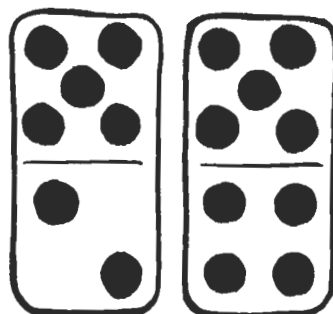
$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$



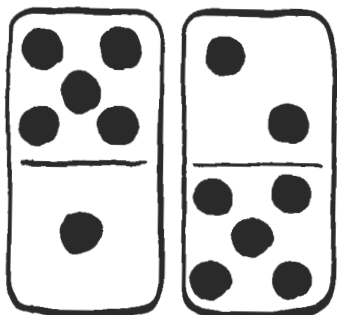
$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$



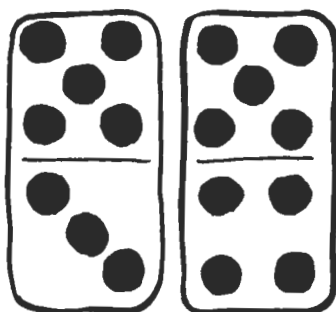
$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$



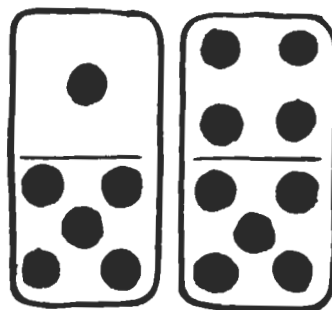
$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$



$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

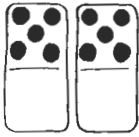
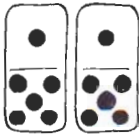
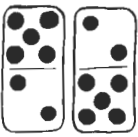
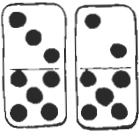
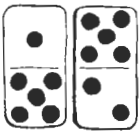
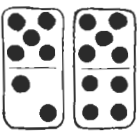
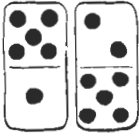
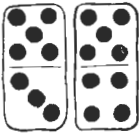
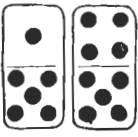


$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

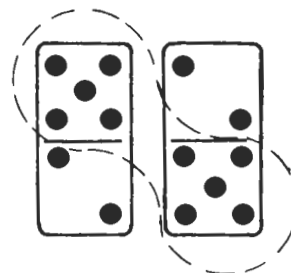


$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

NAME \_\_\_\_\_

|                                                                                                                          |                                                                                                                          |                                                                                                                          |
|--------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| <br>$5 + 5 = \underline{\quad}$         | <br>$\quad + \quad = \underline{\quad}$ | <br>$\quad + \quad = \underline{\quad}$ |
| <br>$\quad + \quad = \underline{\quad}$ | <br>$\quad + \quad = \underline{\quad}$ | <br>$\quad + \quad = \underline{\quad}$ |
| <br>$\quad + \quad = \underline{\quad}$ | <br>$\quad + \quad = \underline{\quad}$ | <br>$\quad + \quad = \underline{\quad}$ |

Consider putting the top row on the chalkboard to complete as a group. Point out the record keeping method. The last example on the top row is the first time that the 5's are not next to each other but the same strategy can be used.



5 and 5 are 10 and 4 more is 14.

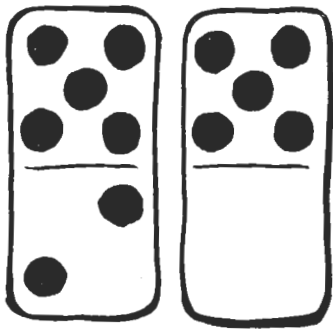
"Domino addition" points out again that all addition facts with both addend 5 or larger can be reduced to a question of  $10 + \text{---} = \text{---}$ .

The student simply records the number of spots on each domino, and the total of spots on the pair of dominoes.

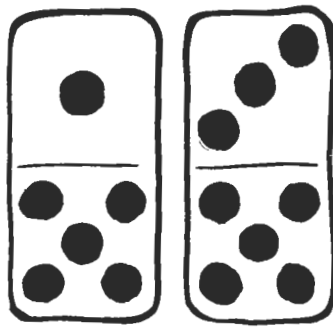
A transparency could be made for the overhead. Then, as a pair of dominoes is pointed out, "How can you tell at a glance how many spots there are?"

In each case there are two familiar 5's: that's 10 . . . and how many more?

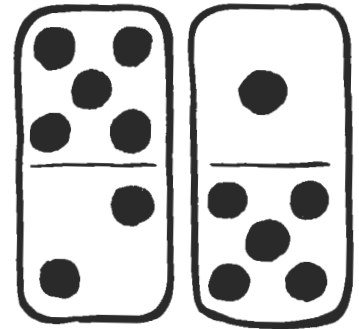
This is a natural "choral response" situation. "When I point to a pair of dominoes, think about how many—but wait for the signal. Then, when I lower my arm, call out the answer." The "thinking time" can be reduced and the whole activity speeded up.



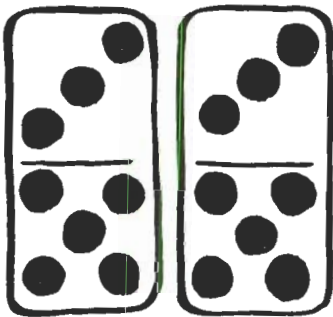
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



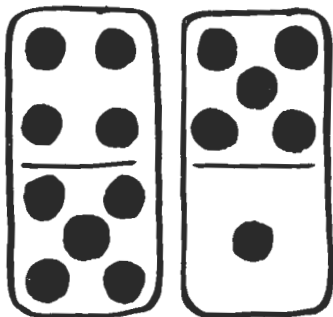
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



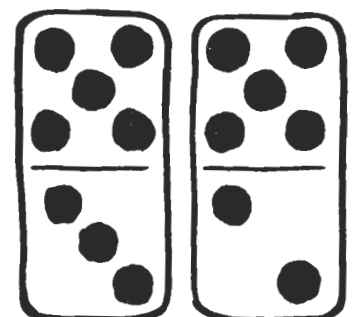
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



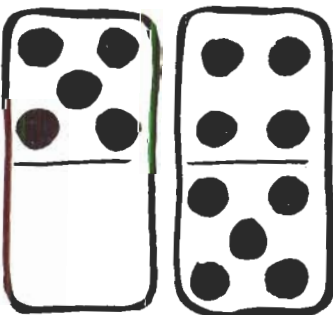
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



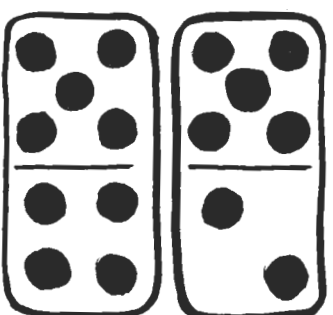
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



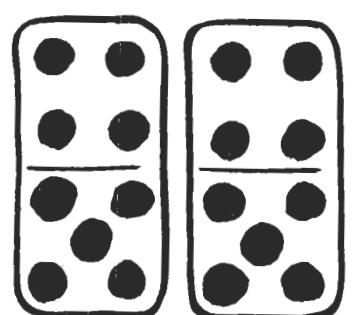
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



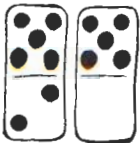
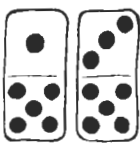
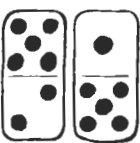
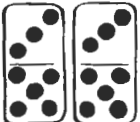
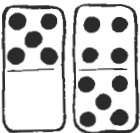
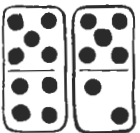
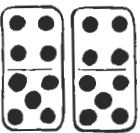
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



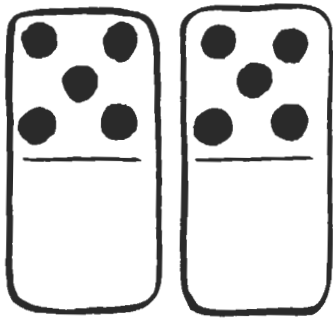
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



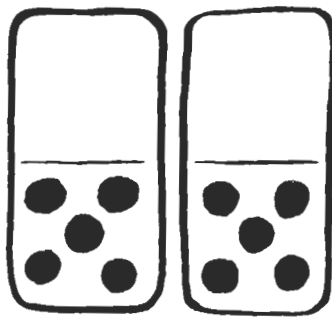
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| <br>$\underline{\quad} + \underline{\quad} = \underline{\quad}$ | <br>$\underline{\quad} + \underline{\quad} = \underline{\quad}$ | <br>$\underline{\quad} + \underline{\quad} = \underline{\quad}$ |
| <br>$\underline{\quad} + \underline{\quad} = \underline{\quad}$ | <br>$\underline{\quad} + \underline{\quad} = \underline{\quad}$ | <br>$\underline{\quad} + \underline{\quad} = \underline{\quad}$ |
| <br>$\underline{\quad} + \underline{\quad} = \underline{\quad}$ | <br>$\underline{\quad} + \underline{\quad} = \underline{\quad}$ | <br>$\underline{\quad} + \underline{\quad} = \underline{\quad}$ |

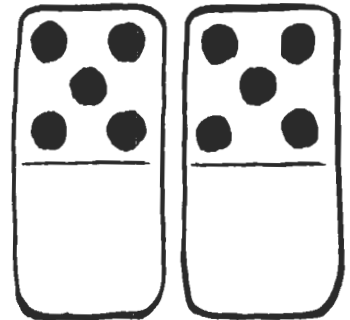
This is an extension of the previous page with more of the larger sums.



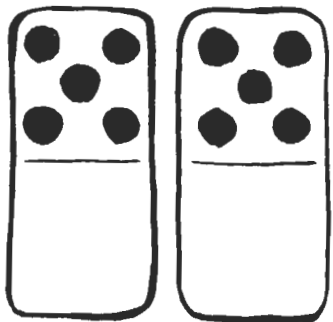
$$6 + 7 = \underline{\quad}$$



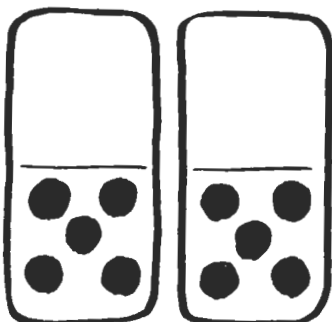
$$9 + 5 = \underline{\quad}$$



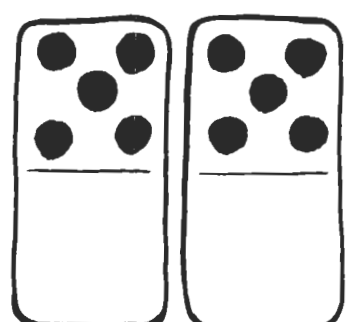
$$8 + 8 = \underline{\quad}$$



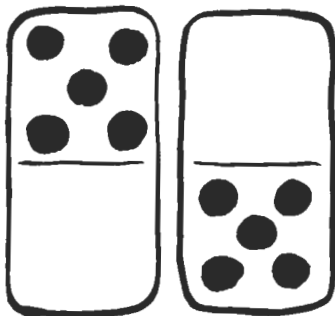
$$7 + 7 = \underline{\quad}$$



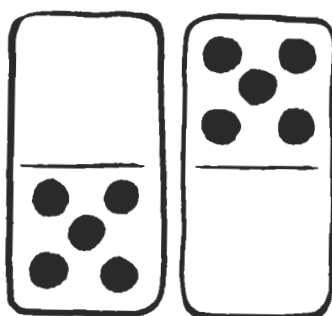
$$9 + 9 = \underline{\quad}$$



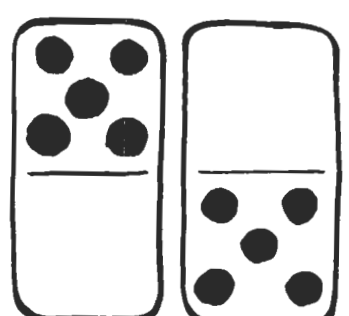
$$7 + 8 = \underline{\quad}$$



$$9 + 8 = \underline{\quad}$$

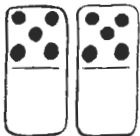
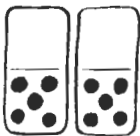
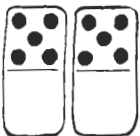
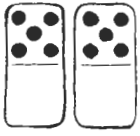
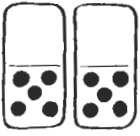
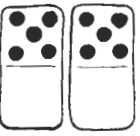
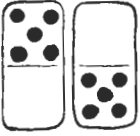
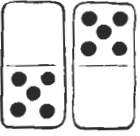
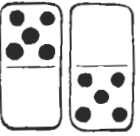


$$9 + 6 = \underline{\quad}$$



$$7 + 9 = \underline{\quad}$$

NAME \_\_\_\_\_

|                                                                                   |                                                                                   |                                                                                   |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |  |
| $6 + 7 = \underline{\quad}$                                                       | $9 + 5 = \underline{\quad}$                                                       | $8 + 8 = \underline{\quad}$                                                       |
|  |  |  |
| $7 + 7 = \underline{\quad}$                                                       | $9 + 9 = \underline{\quad}$                                                       | $7 + 8 = \underline{\quad}$                                                       |
|  |  |  |
| $9 + 8 = \underline{\quad}$                                                       | $9 + 6 = \underline{\quad}$                                                       | $7 + 9 = \underline{\quad}$                                                       |

Each child then draws one spot in the blank space on the domino. Ask similar questions about the second domino and draw in the necessary spots. Count the total and record in the proper place. Do others if necessary and then let children complete the page on their own.

On this page a new dimension is added. We are told how many spots are supposed to be on each domino and are asked to draw additional spots where needed. Each domino already has 5 spots and the focus is directly on the idea of adding on from 5.

Have the children look at the first domino and pose the following questions:

(1) How many dots on the domino? (5)

(2) How many dots are supposed to be on the domino? (6)

(If necessary, direct them to look at the number under the domino.)

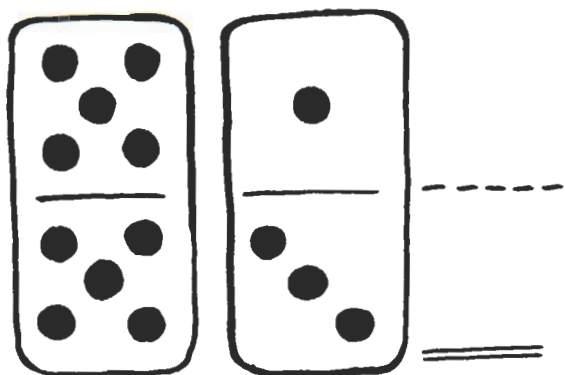
(3) How many more spots are needed? (1)

or

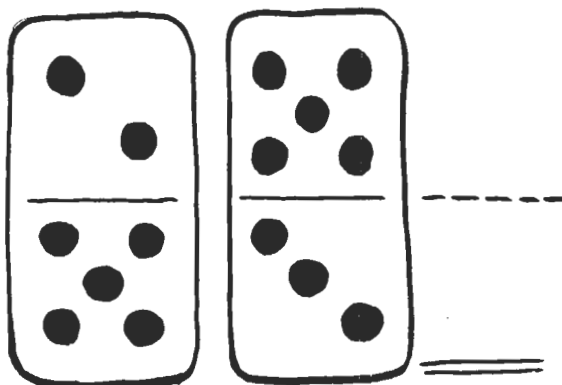
How many spots need to be added on to 5 to show 6? (1)

or

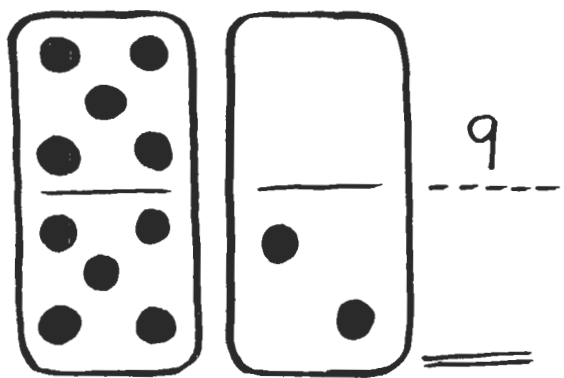
Five and how many make 6? (1)



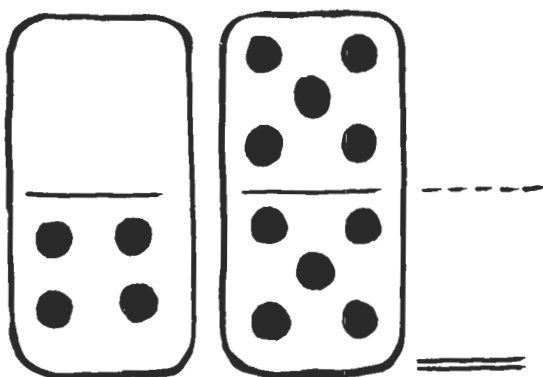
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



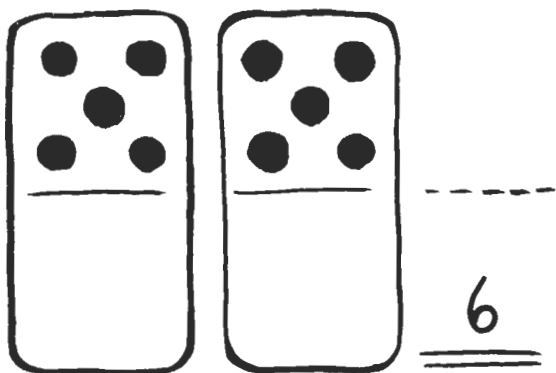
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



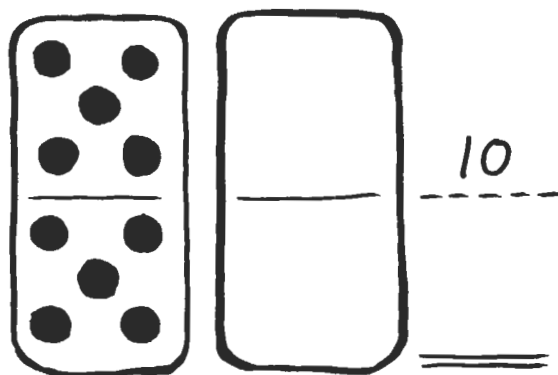
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



$$\underline{8} + \underline{\quad} = \underline{\quad}$$



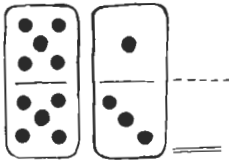
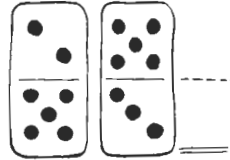
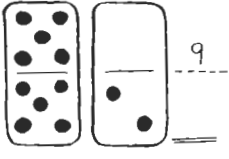
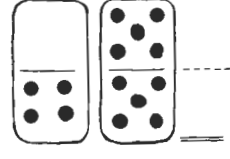
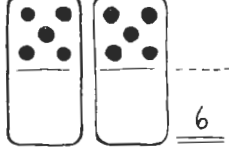
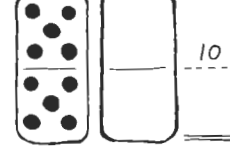
$$\underline{8} + \underline{\quad} = \underline{\quad}$$



$$\underline{\quad} + \underline{\quad} = \underline{19}$$

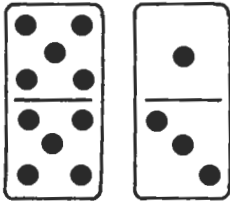
PLEASE ADD BOTH WAYS  
AND FILL IN MISSING SPOTS.

NAME \_\_\_\_\_

|                                                                                                                                                  |                                                                                                                                                  |
|--------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <br>$\underline{\quad} + \underline{\quad} = \underline{\quad}$ | <br>$\underline{\quad} + \underline{\quad} = \underline{\quad}$ |
| <br>$\underline{\quad} + \underline{\quad} = \underline{\quad}$ | <br>$8 + \underline{\quad} = \underline{\quad}$                 |
| <br>$8 + \underline{\quad} = \underline{\quad}$                 | <br>$\underline{\quad} + \underline{\quad} = 19$                |

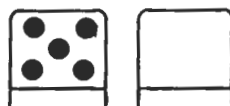
The page will help clear up questions that would otherwise come up on the pages that follow.

The activity is broadened to "adding both ways." The spots on domino-halves are added horizontally and vertically.

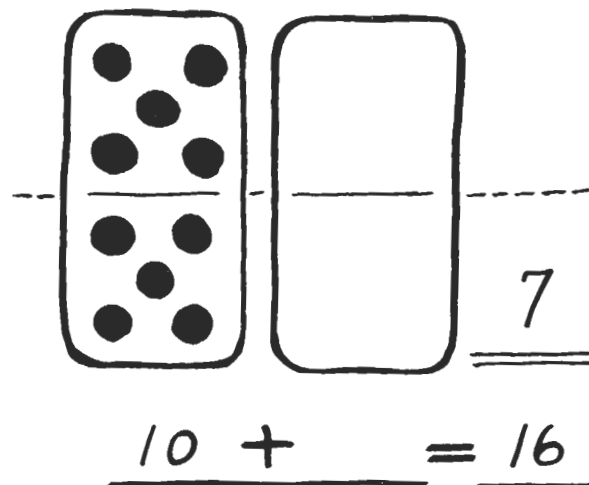
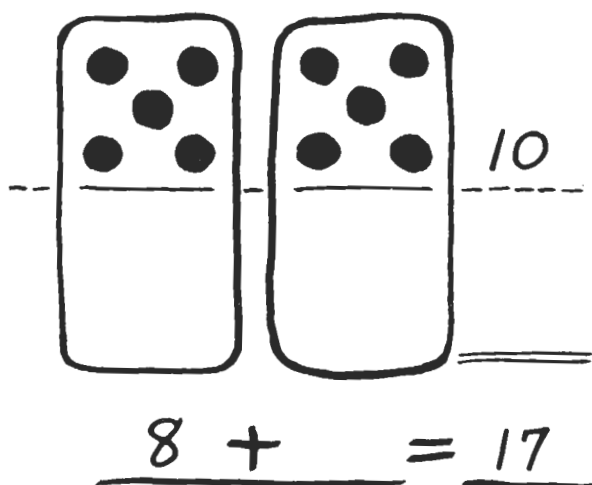
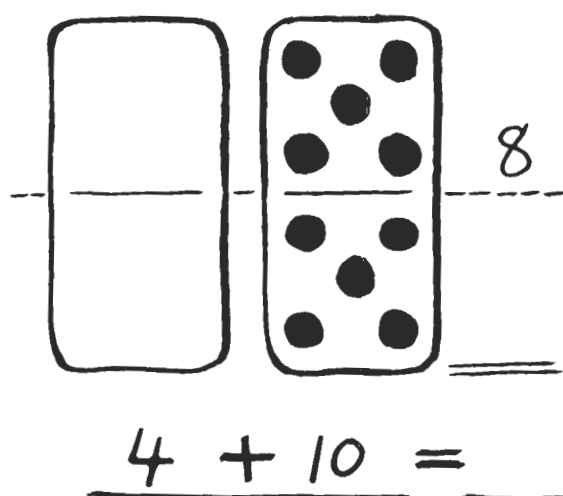
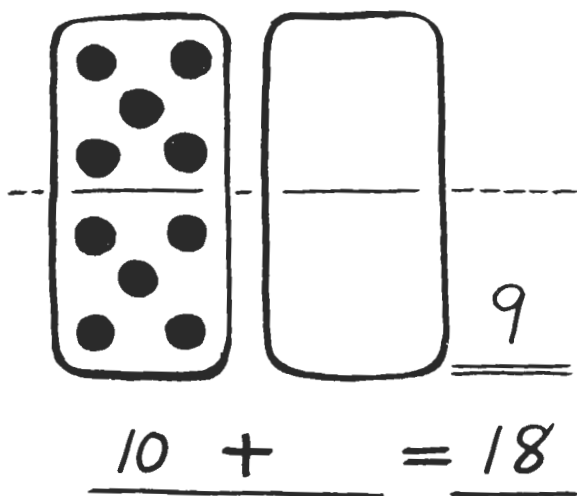
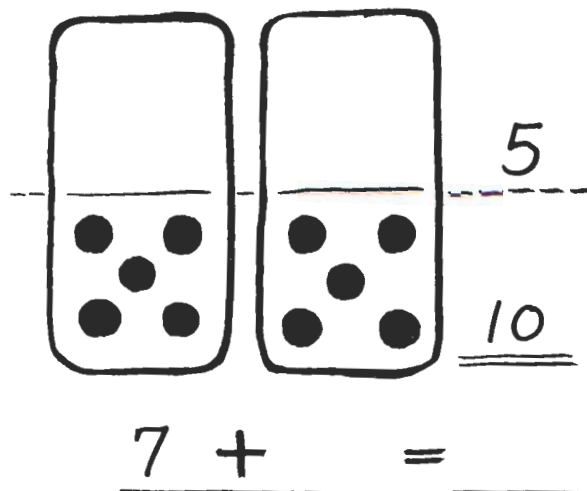
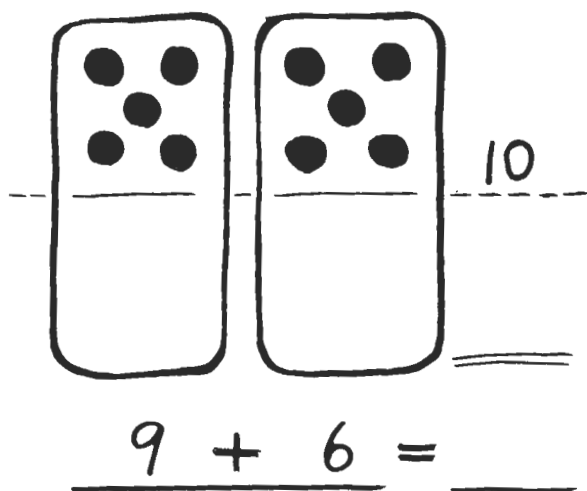
|                                                                                                      |                                                             |
|------------------------------------------------------------------------------------------------------|-------------------------------------------------------------|
| <br>$10 + 4 = 14$ | $\underline{\quad} + \underline{\quad} = \underline{\quad}$ |
|------------------------------------------------------------------------------------------------------|-------------------------------------------------------------|

Since a new twist is added in each row of examples, consider having volunteers complete the first example in each row or similar examples.

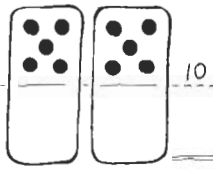
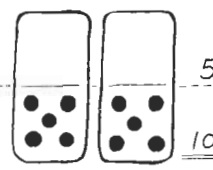
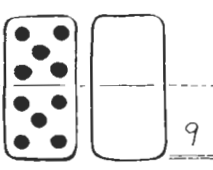
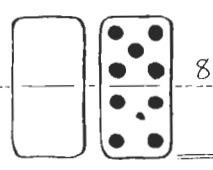
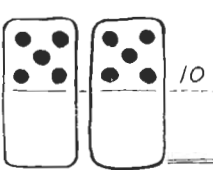
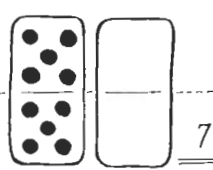
In the second row, spots are missing from the domino-halves. How many are missing? Enough so the two top halves together have 9 spots.

|                                                                                                                                                    |                                                             |
|----------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------|
| <br>$\underline{\quad} + \underline{\quad} = \underline{\quad}$ | $\underline{\quad} + \underline{\quad} = \underline{\quad}$ |
|----------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------|

In the third row, spots are missing in two domino-halves. We are told that there are a total of 6 spots, but that isn't enough. However, we are told there are 8 spots on the left hand domino—but there are only 5 shown. How many more so there will be a total of 8?



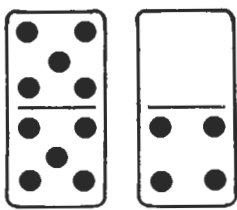
NAME \_\_\_\_\_

|                                                                                                                                |                                                                                                                                  |
|--------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------|
| <br>$9 + 6 = \underline{\quad}$               | <br>$7 + \underline{\quad} = \underline{\quad}$ |
| <br>$10 + \underline{\quad} = \underline{18}$ | <br>$4 + 10 = \underline{\quad}$                |
| <br>$8 + \underline{\quad} = \underline{17}$  | <br>$10 + \underline{\quad} = \underline{16}$   |

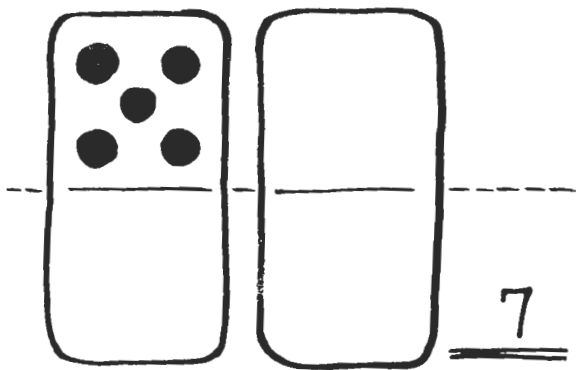
More “domino addition” with different parts of the record missing.

In examples where the total number is given there may at first seem to be insufficient information: and there is unless one uses the given total to help.

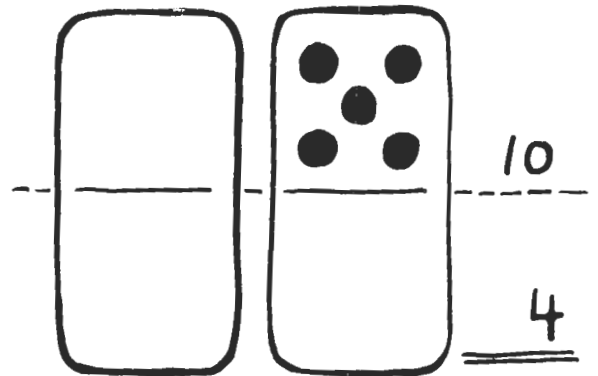
The first example in the middle row reaches this problem:

|                                                                                     |                                           |
|-------------------------------------------------------------------------------------|-------------------------------------------|
|  | $10 + \underline{\quad} = \underline{18}$ |
|-------------------------------------------------------------------------------------|-------------------------------------------|

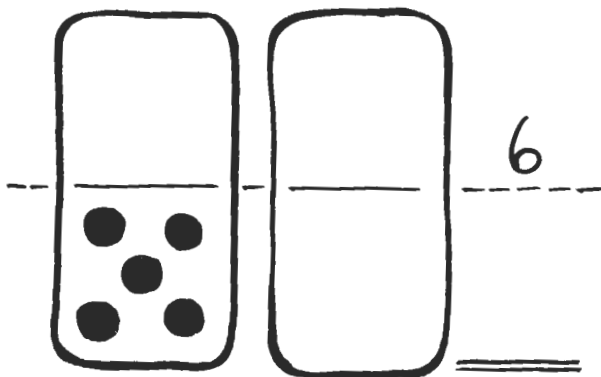
But there are only 14 spots—and we need 18. Then 4 must be drawn in the blank. Or, if one notices that  $10 + 8 = 18$ , then the need for 4 spots in the blank becomes obvious. Or, if one notices that  $9 + 9 = 18$ , the same need becomes obvious.



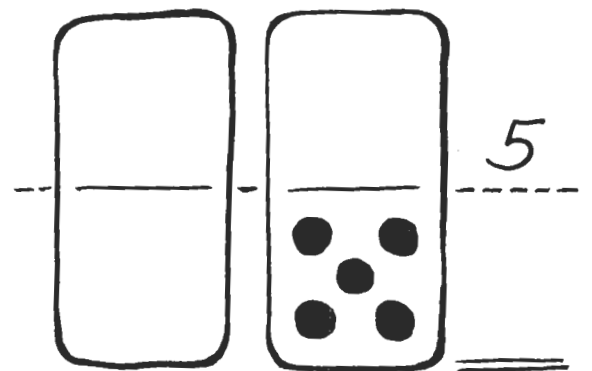
$$9 + 5 = \underline{\quad}$$



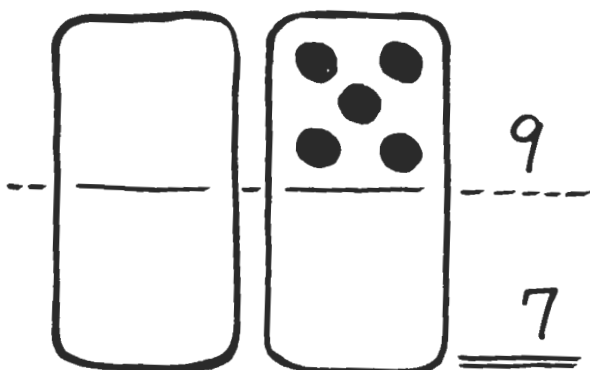
$$6 + \underline{\quad} = \underline{\quad}$$



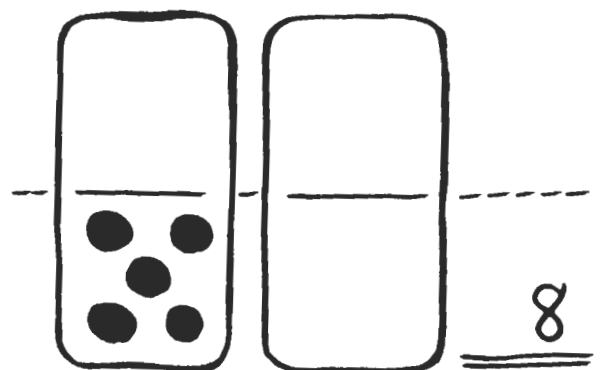
$$8 + \underline{\quad} = \underline{13}$$



$$3 + \underline{\quad} = \underline{12}$$



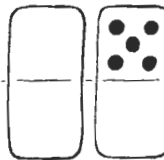
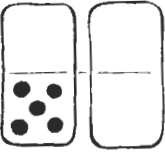
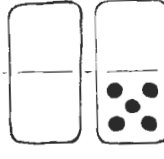
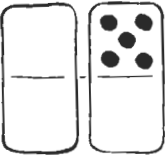
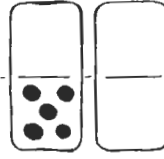
$$8 + \underline{\quad} = \underline{\quad}$$



$$9 + \underline{\quad} = \underline{15}$$



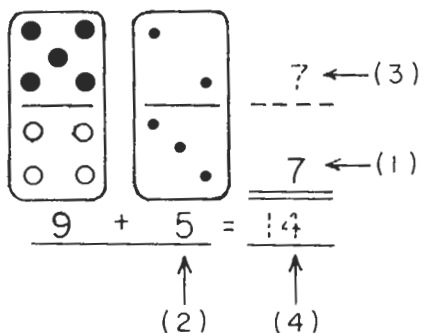
NAME \_\_\_\_\_

|                                                                                                                                  |                                                                                                                                  |
|----------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------|
| <br>$9 + 5 = \underline{\quad}$                 | <br>$6 + \underline{\quad} = \underline{\quad}$ |
| <br>$8 + \underline{\quad} = \underline{13}$    | <br>$3 + \underline{\quad} = \underline{12}$    |
| <br>$8 + \underline{\quad} = \underline{\quad}$ | <br>$9 + \underline{\quad} = \underline{15}$    |

Notice that in each example there are 5 spots in one of the domino halves—the other halves are blank. And spots added must be drawn in the halves that are blank in the beginning.

Each of these is a “problem.” Where do you begin? . . . Answer: in general, add spots when there is no question about how many are needed.

The only starting point in the first example is noticing that there have to be 9 spots on a domino that already has 5 spots. Four spots must be added to the blank half of that domino. The result helps move ahead.



$\underline{9} + \underline{5} = \underline{14}$   
 (2)      (3)

Now the pieces needed for the full solution fall in place.

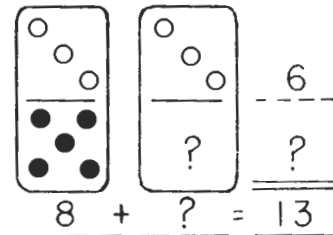
(1) There must be 3 spots in the bottom half of the domino on the right,

(2) and 2 spots in the top half.

(3) There are a total of 7 in the top halves, and

(4) a total of 14 on both dominoes.

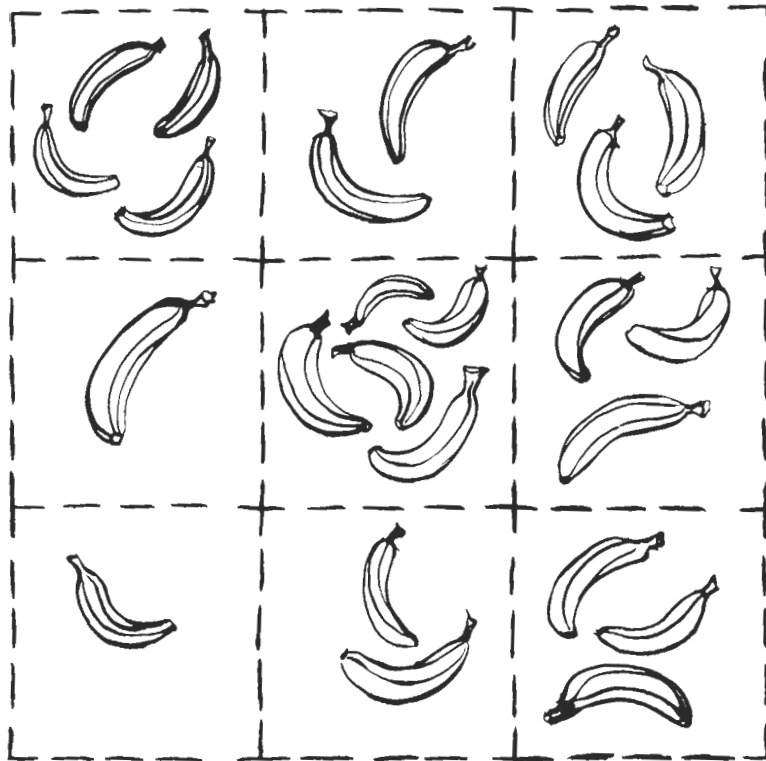
The examples in which the total is given are perhaps more difficult than the others. This situation develops in the first example in the middle



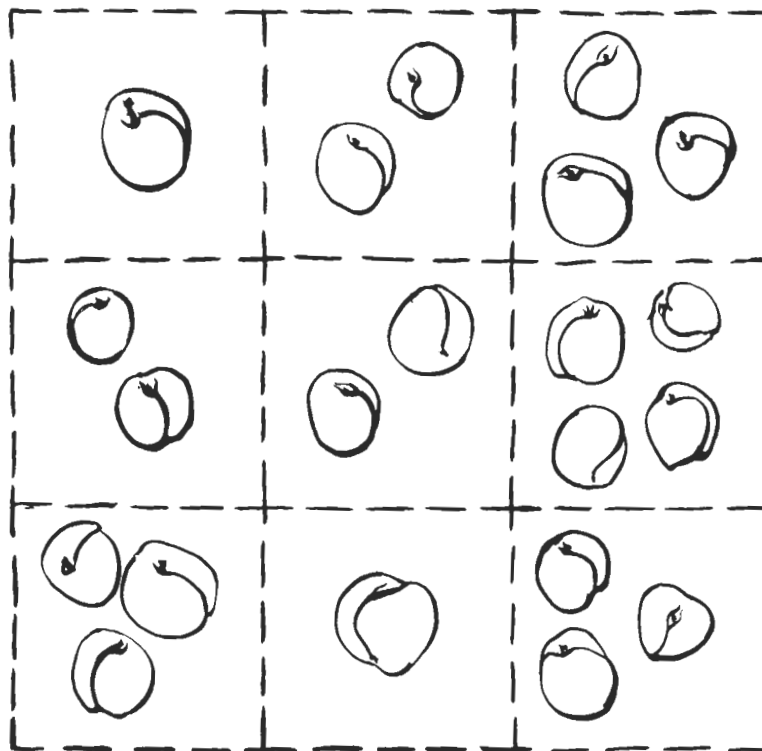
$\underline{8} + \underline{\quad} = \underline{13}$

One way out is to count the spots already drawn in the 3 halves—a total of 11. Then there must be 2 spots in the blank.

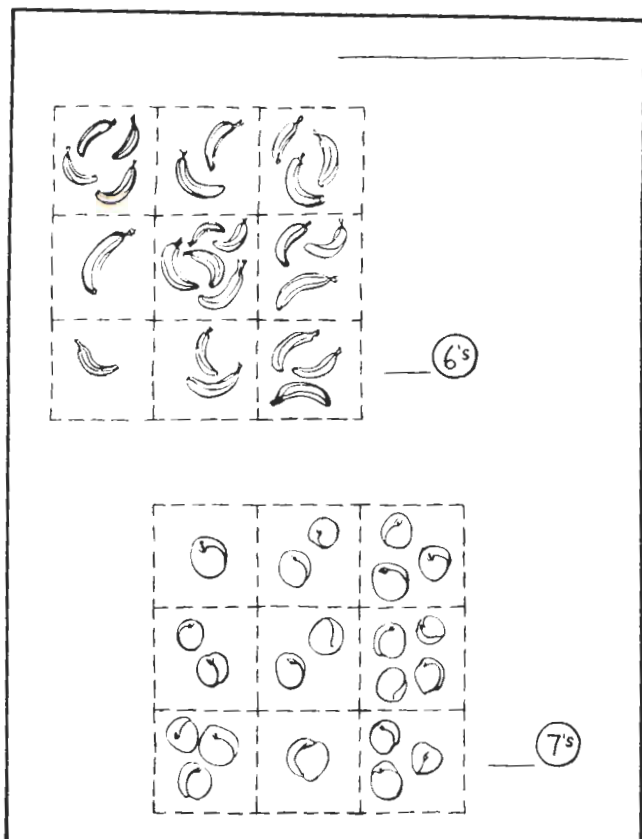
Another approach is to complete either of the open sentences  $8 + \underline{\quad} = 13$  or  $6 + \underline{\quad} = 13$ . Either piece of missing information would lead to the solution.



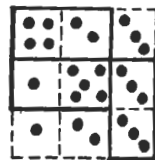
— (6's)



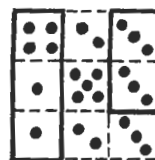
— (7's)



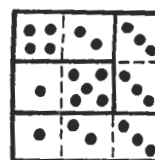
Each example is planned so all shapes can be fenced in—and that is our goal:



3 (6's)



2 (6's)

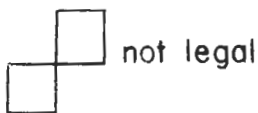


4 (6's)

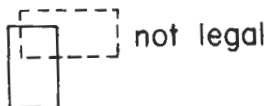
(There is only 1 way to fence in 4 6's in this example.) If necessary, do the bottom square following the same procedure. Those who are ready should work it on their own. Discuss the possible solutions. The children should know that fences need not be rectangular.

We call this activity "Fence Arithmetic." In each case a sum is chosen. Then a "fence" is drawn around that number of shapes.

Rule: Adjacent boxes must touch side by side.

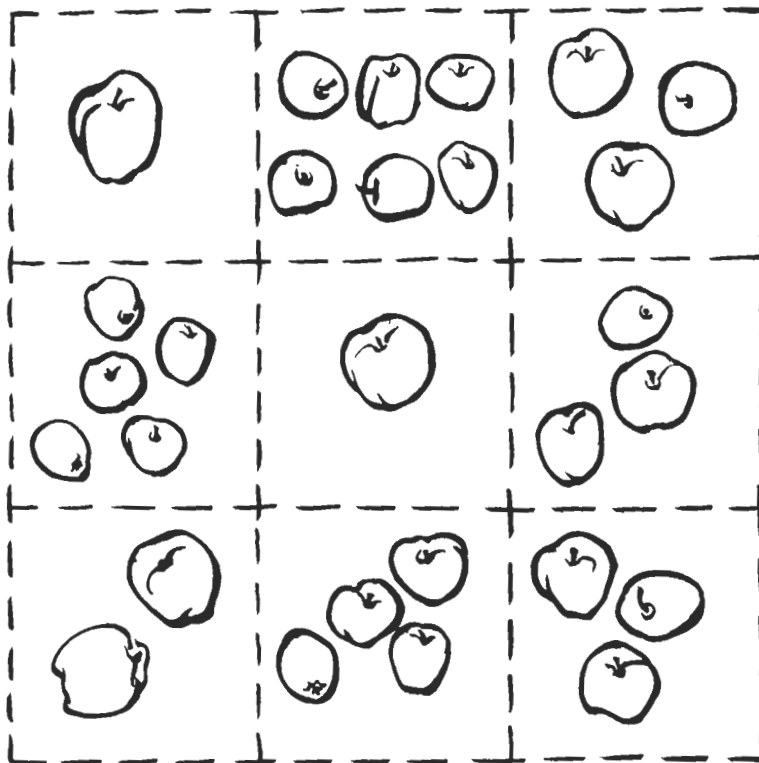


Rule: one fence cannot cross another.

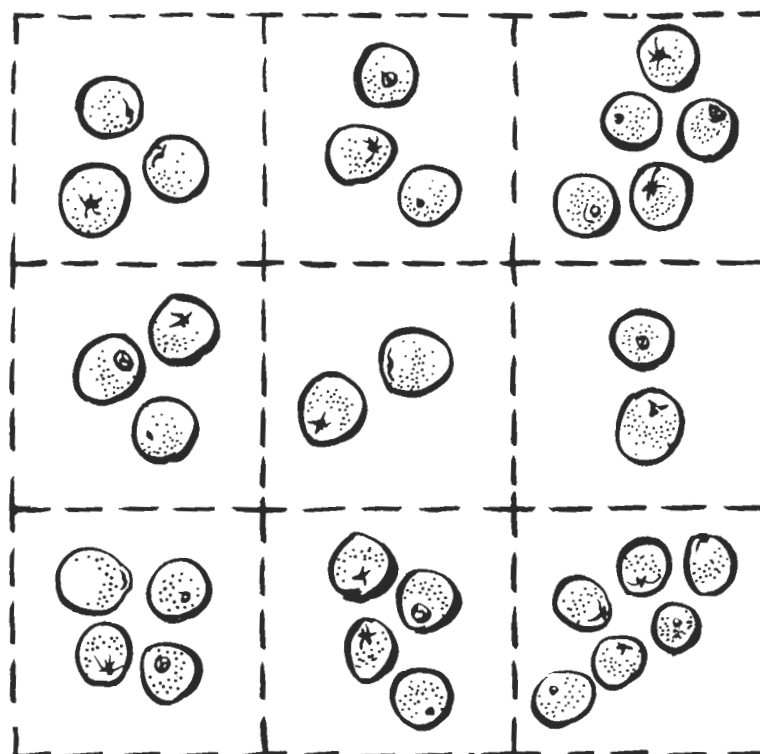


You may want to use the overhead to project the top square on the chalkboard or draw the square directly on the board. You could start with a question similar to this, "Can someone come to the board and draw a fence around some boxes to show 6 bananas?"

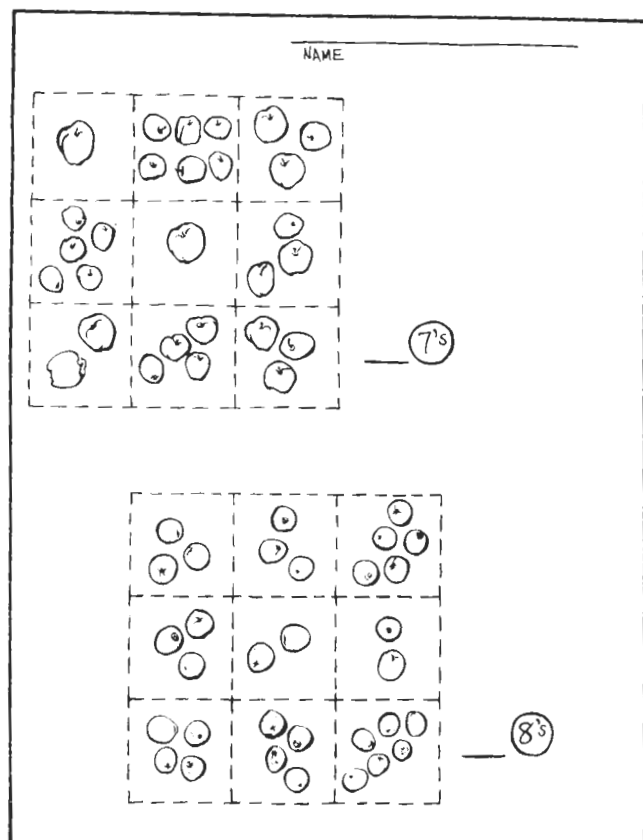
There are many possible correct "fences" and any one of them will do. After the first fence is drawn, check to see that it is correct and then have the children draw more fences until no more are possible. Fill in the record with the number of fences made.



— (7's)



— (8's)



Let the children work on their own with your help as needed. You might wish to tell them that

$$\underline{4} \text{ } \textcircled{7's} \text{ and } \underline{4} \text{ } \textcircled{8's}$$

are possible or you can wait for this to come out in a discussion.

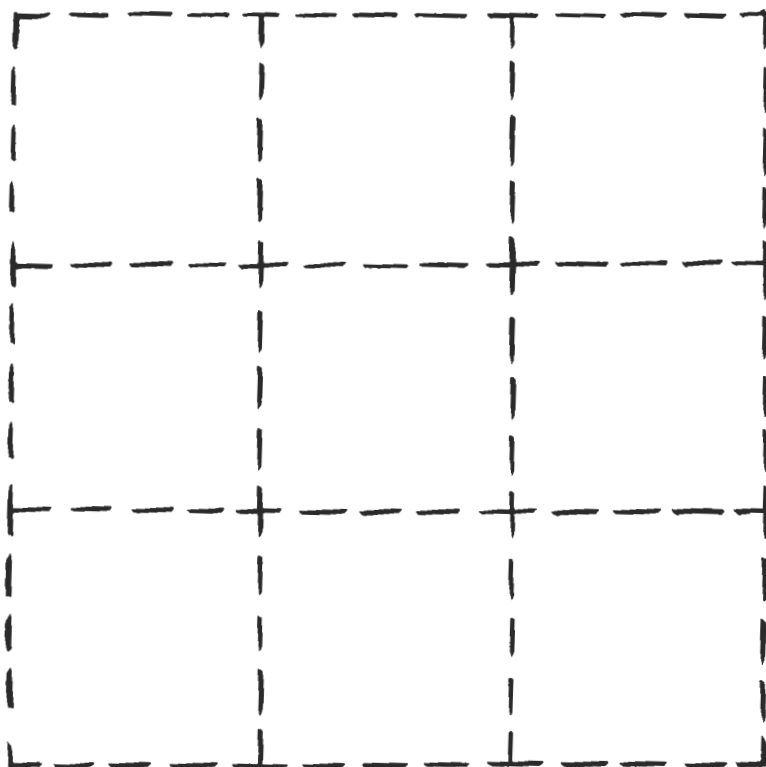
|   |   |   |
|---|---|---|
| 1 | 6 | 3 |
| 5 | 1 | 3 |
| 2 | 4 | 3 |

$\underline{4} \text{ } \textcircled{7's}$

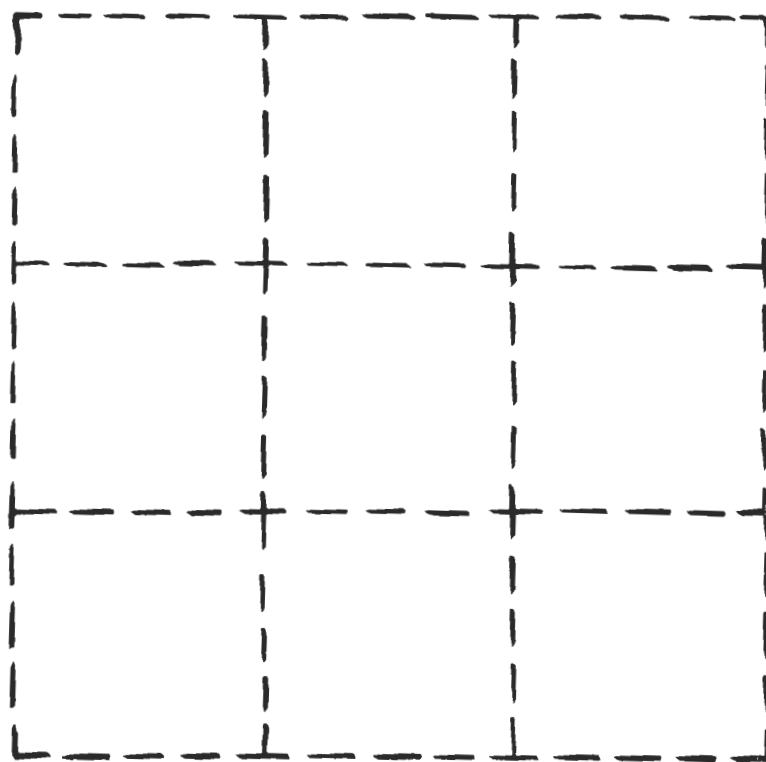
|   |   |   |
|---|---|---|
| 3 | 3 | 5 |
| 3 | 2 | 2 |
| 4 | 4 | 6 |

$\underline{4} \text{ } \textcircled{8's}$

These are the only ways to fence in all 7's and all 8's.



— (s)



— (s)

NAME \_\_\_\_\_

|  |  |  |
|--|--|--|
|  |  |  |
|  |  |  |
|  |  |  |

s

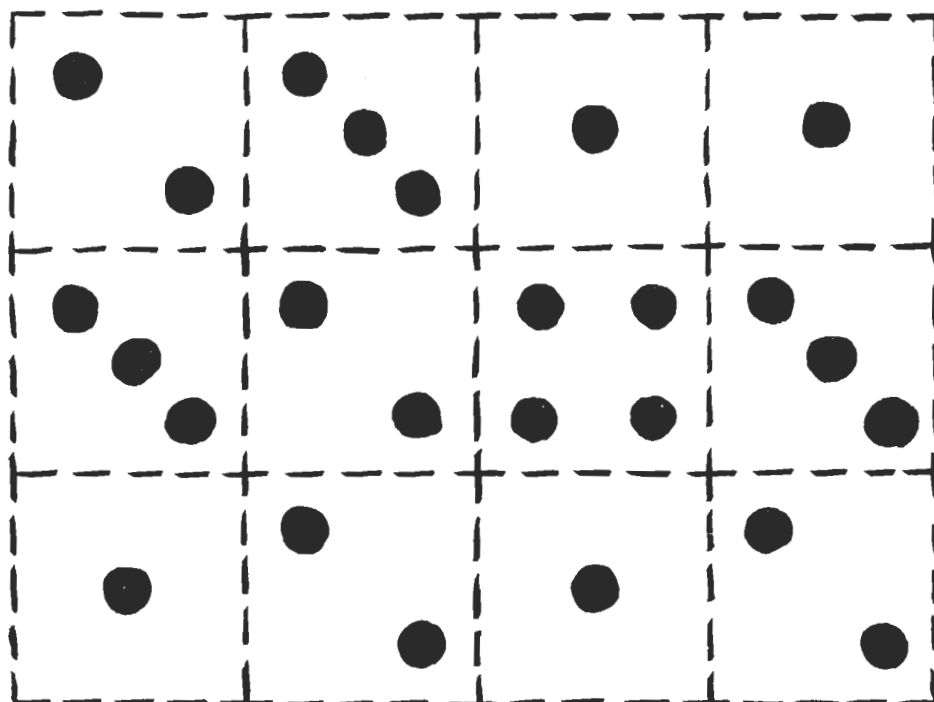
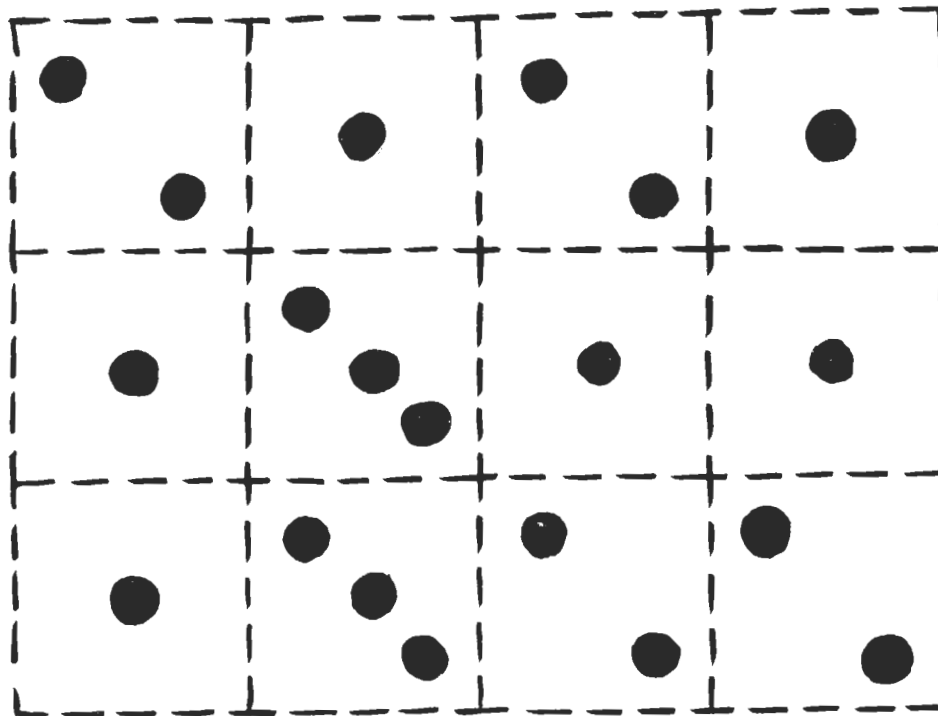
|  |  |  |
|--|--|--|
|  |  |  |
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s

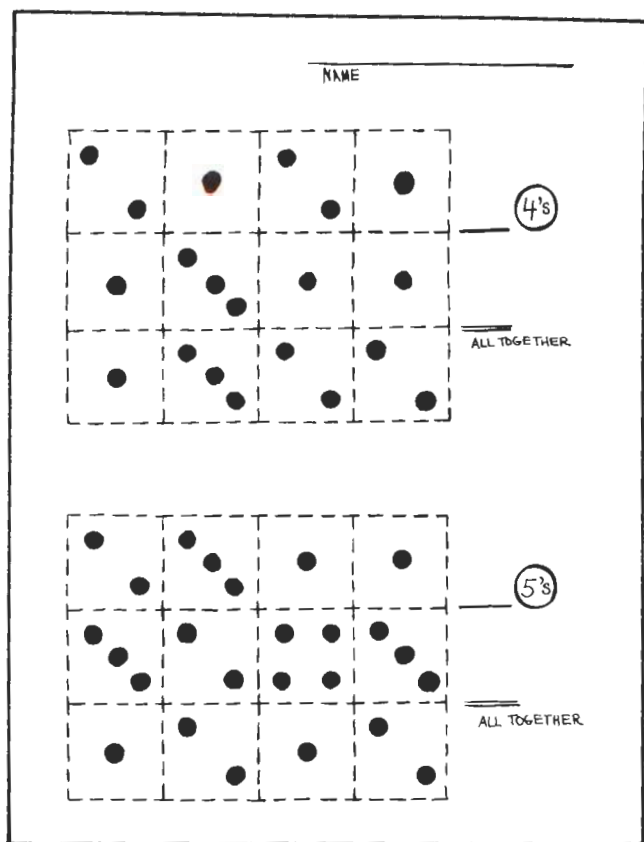
This page is used to individualize the activities introduced so far.

- (1) Draw some shapes in each box and enter the sums in the
- (2) Draw some shapes in each box and let the child choose his own sums.
- (3) Enter the sums and let the child draw his own shapes in each box.
- (4) Let the child draw his own shapes in each box and also choose his own sums.

Up to now, the greatest number of fences has always used all the boxes in the square. It may not be possible now to use all the squares in each case. If some children are upset by this, assure them that it is all right not to use all the boxes.



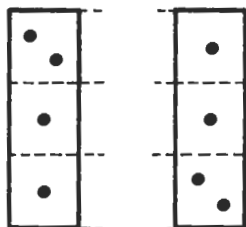




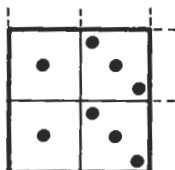
More "Fence Arithmetic."

The first example is not as easy as it would appear at first—"fencing in" all dots in groups of 4 each.

There are many false starts possible. Either side column is attractive.



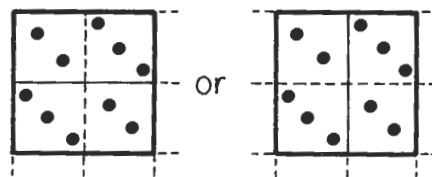
Here is another attractive situation:



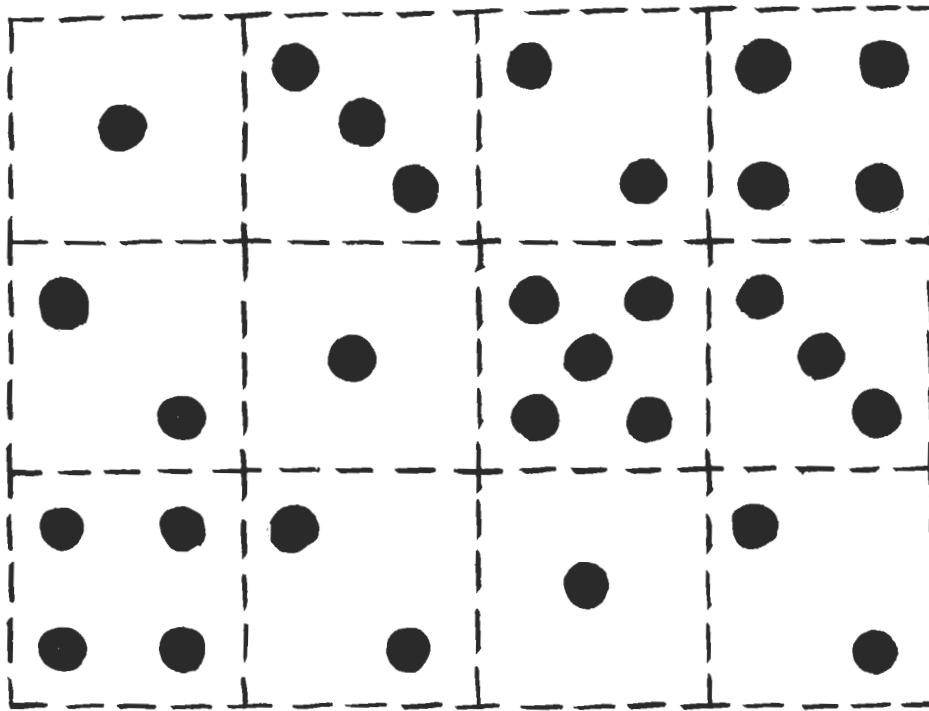
But all of these will leave some of the dots not "fenced in."

Perhaps, because of these difficulties, the first example might well be done on the chalkboard or on a piece of scratch paper—where errors can be more easily changed.

In the second example there are still opportunities for false starts:

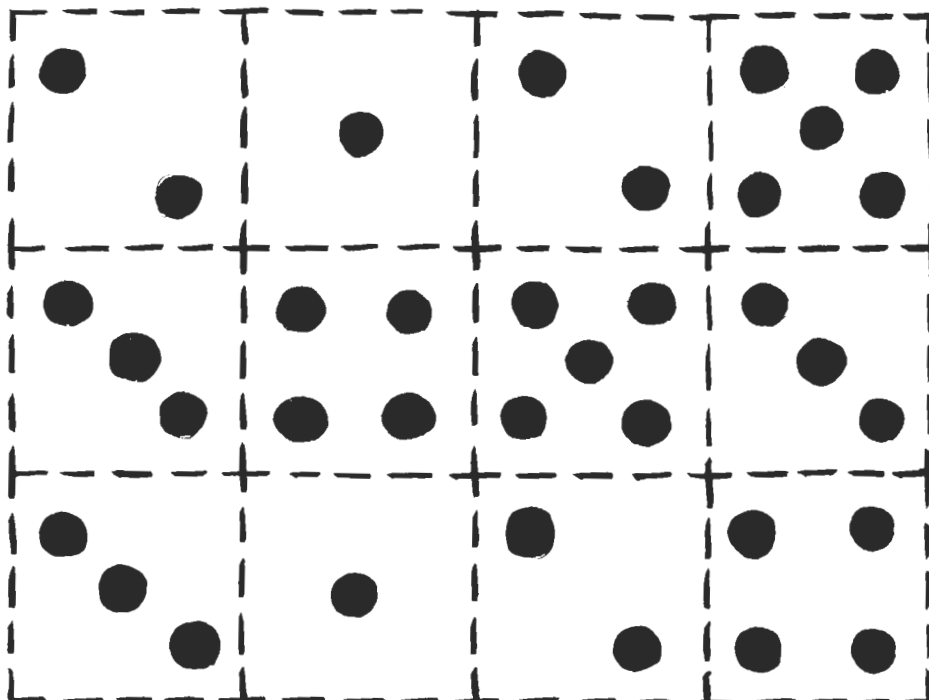


will lead to trouble.



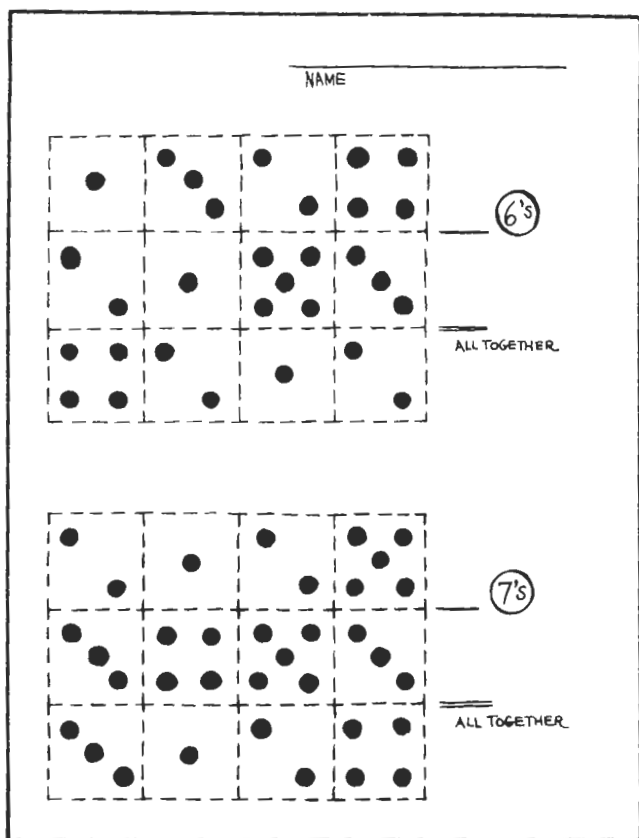
6's

TOTAL



7's

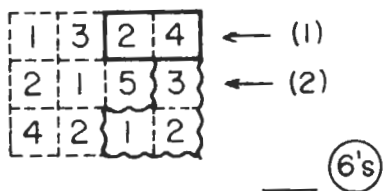
TOTAL



Similarly each of the problems on this and the previous page admit to but a single solution.

Can either of these "fence" problems or those on the previous page be solved in more than 1 way?

Consider the first one on this page—using numbers in place of dots.

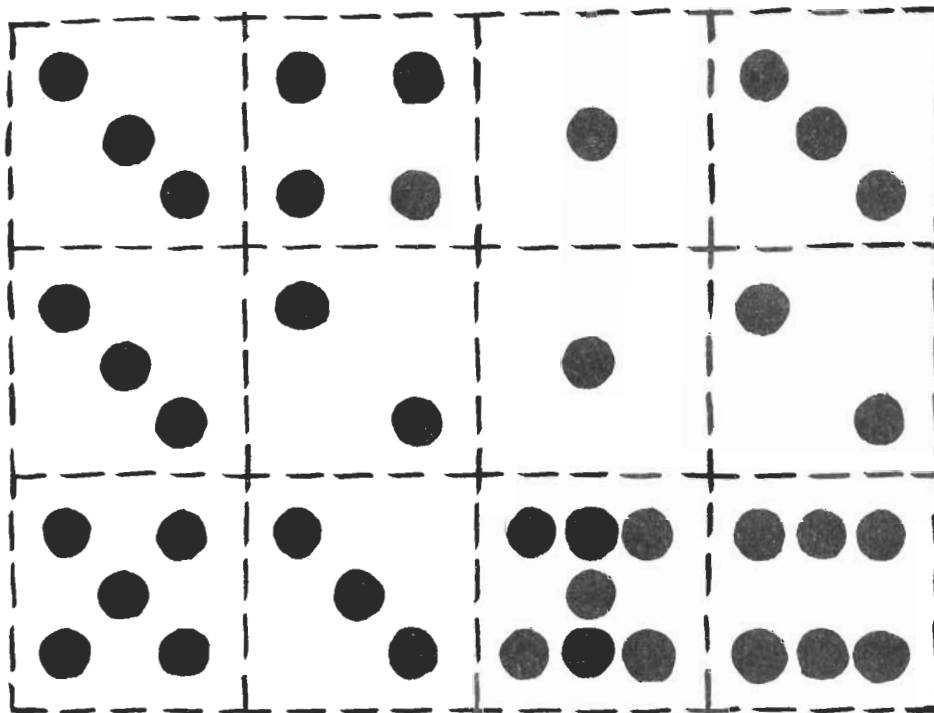


Consider "5": there are two ways to enclose it with "1" in a fence to make 6.

(1) Consider the "4" in the upper right corner: There is only one way to enclose it with 2 more to make 6.

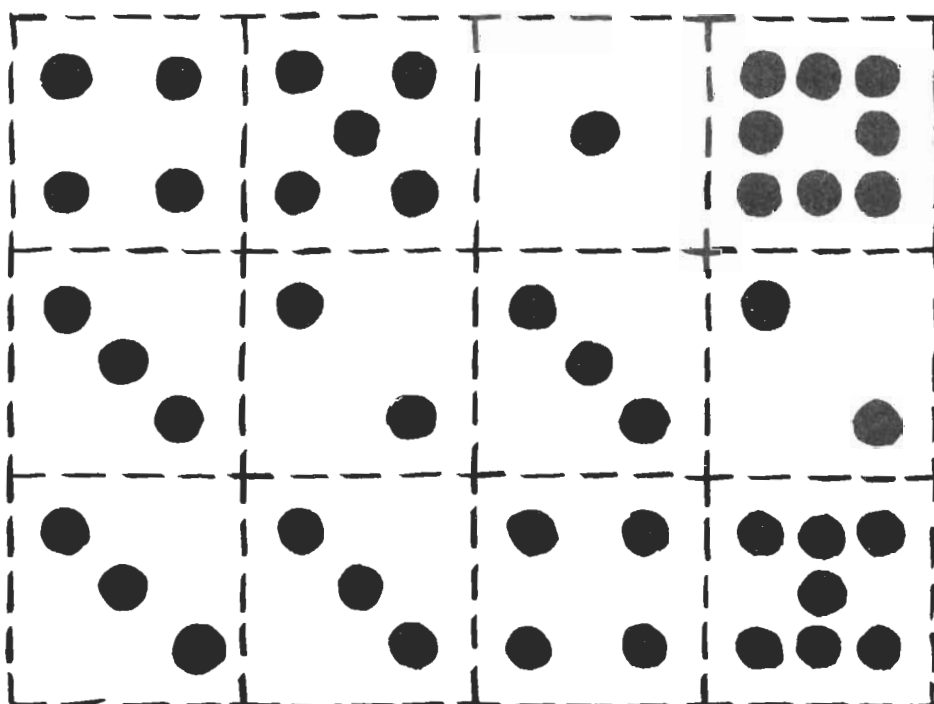
(2) Next consider the "3" in the middle row and right hand column. It can be enclosed in a fence with 3 more in only one way—with the "2" and "1".

The rest fall into place, and it is clear there is no other arrangement of fences that can include all in groups of 6.



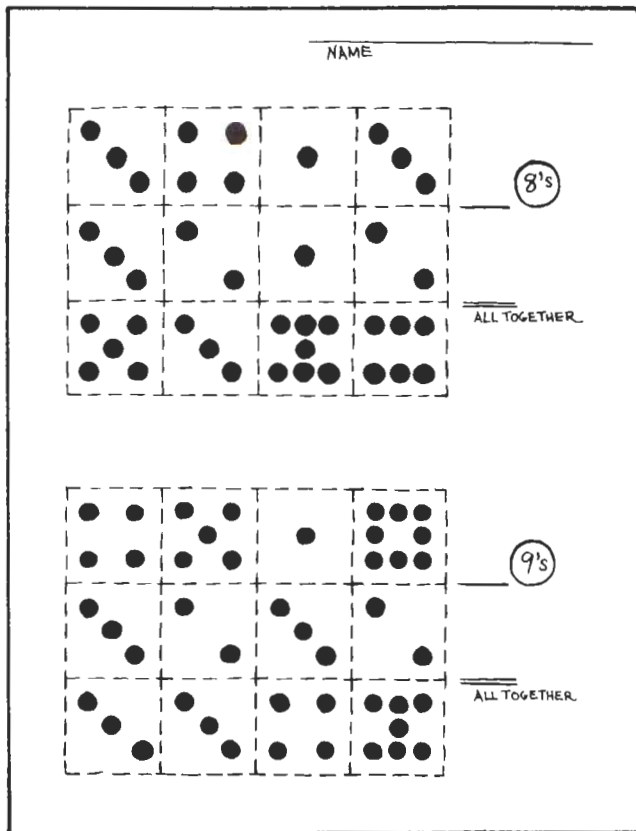
8's

TOTAL

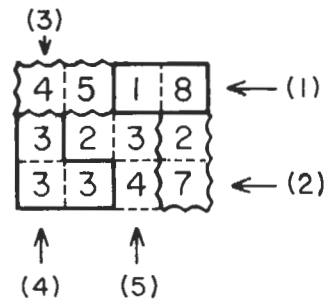


9's

TOTAL



The second example also has but a single solution.

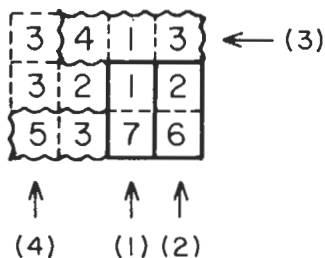


Were "5" selected as a starting point, there would be 2 ways to trap 4 more.

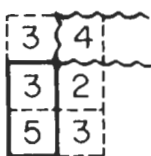


But, considering "8" and "7" first produces no options and solves the "5" quandary.

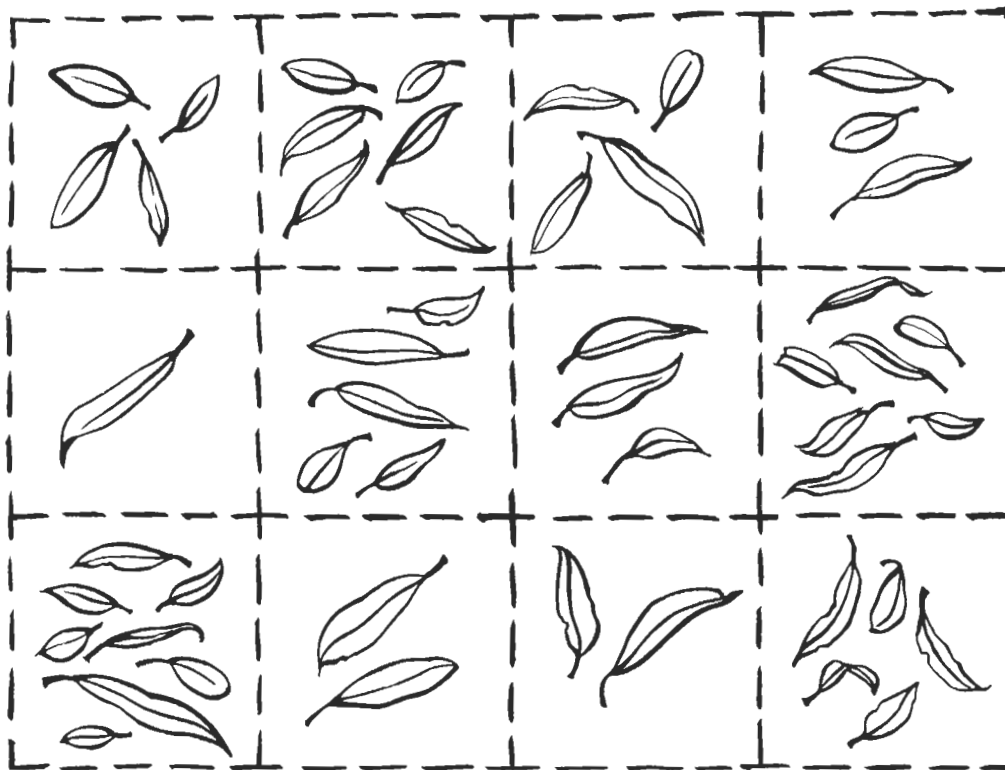
Consider a strategy in each example on this page to find out whether there is more than 1 way.



- (1) There is but a single "1" that can be fenced with "7."
- (2) There is but a single "2" that can be fenced with "6."
- (3) The "3" and "1" require "4" in the top row.
- (4) If the "5" were fenced with the "3" above it, then there would be a trap.

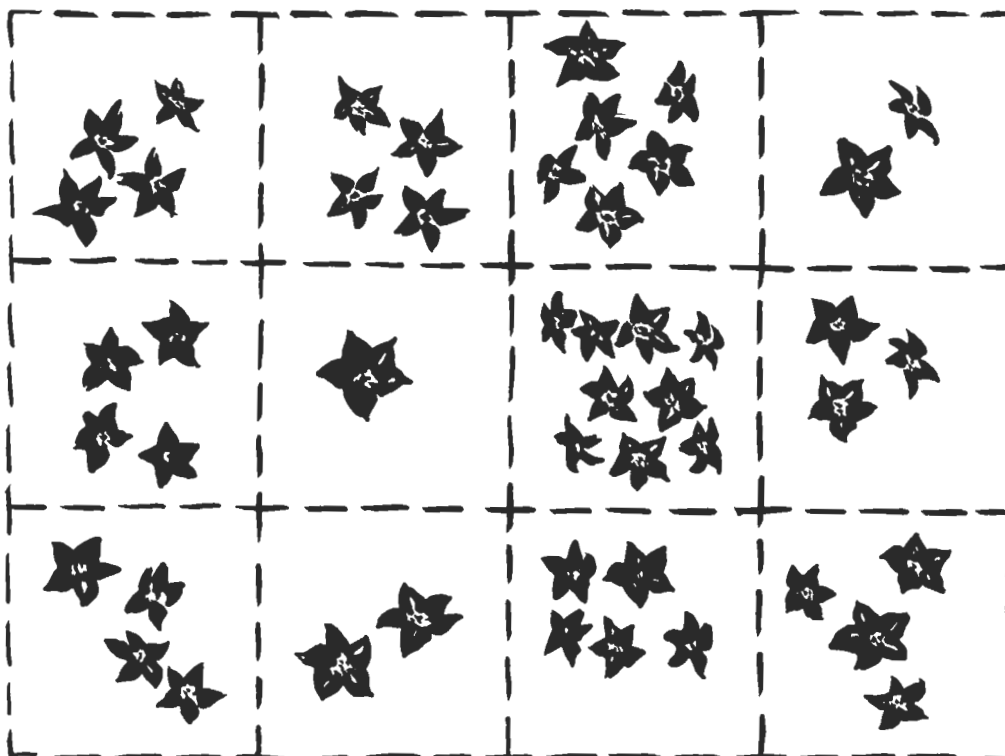


There are no alternatives—a single solution.



10's

TOTAL



12's

TOTAL







NAME \_\_\_\_\_

|  |  |  |  |
|--|--|--|--|
|  |  |  |  |
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( 's

ALL TOGETHER

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|  |  |  |  |
|  |  |  |  |

( 's

ALL TOGETHER

Now, the master can be particularized:

|   |   |   |   |
|---|---|---|---|
| 2 | 3 | 1 | 6 |
| 5 | 2 | 8 | 3 |
| 4 | 1 | 2 | 3 |

and

|   |   |   |   |
|---|---|---|---|
| 1 | 2 | 2 | 6 |
| 9 | 1 | 3 | 4 |
| 7 | 2 | 7 | 6 |

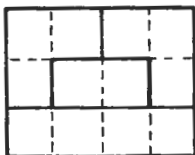
and you have constructed two interesting problems.

The solver must "practice" or "use" at least 9 combinations for 10. And he just may "use" many more than that to find solutions.

Now, if the question arises as to whether either has more than 1 solution, the **use** multiplies.

There are many uses for this page.

- (1) it can be used as scratch paper in working out problems on preceding pages.
- (2) it can be "particularized" by the teacher to make up more examples for combinations that need more drill and practice. An easy way to do this is to use a piece of scratch paper and draw the fences first.

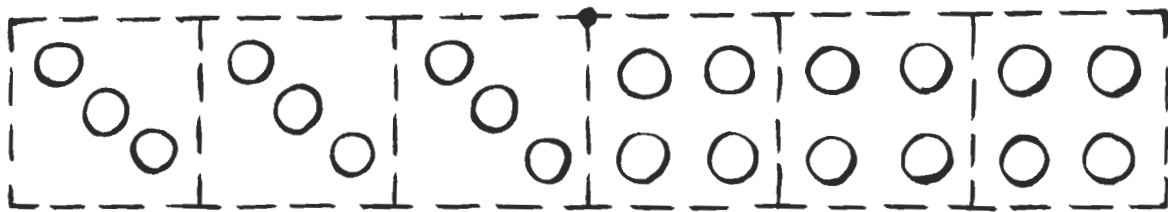


Inside each fence, jot down any combination for a selected number—such as 10. The result might be:

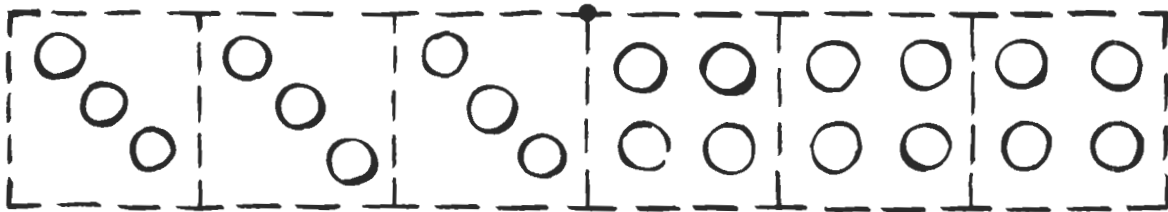
|   |   |   |   |
|---|---|---|---|
| 2 | 3 | 1 | 6 |
| 5 | 2 | 8 | 3 |
| 4 | 1 | 2 | 3 |

and

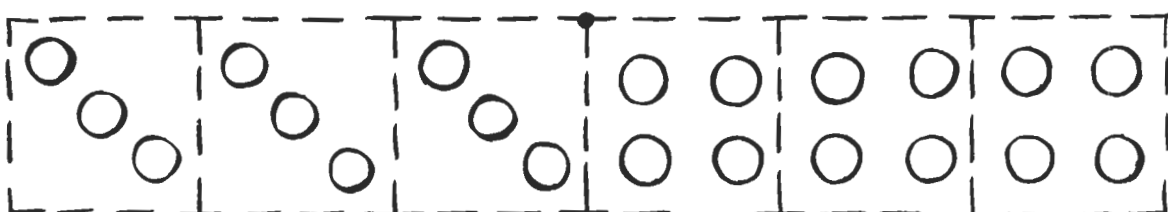
|   |   |   |   |
|---|---|---|---|
| 1 | 2 | 2 | 6 |
| 9 | 1 | 3 | 4 |
| 7 | 2 | 7 | 6 |



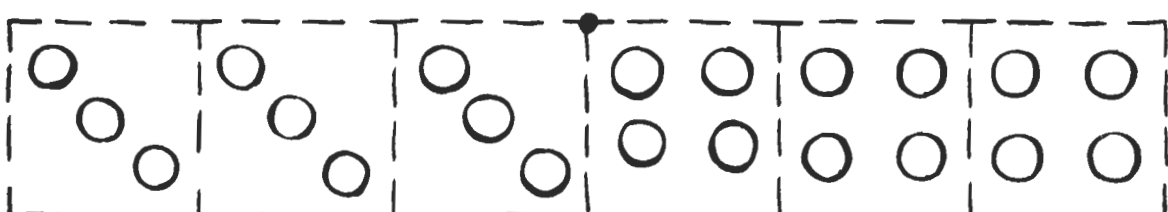
7



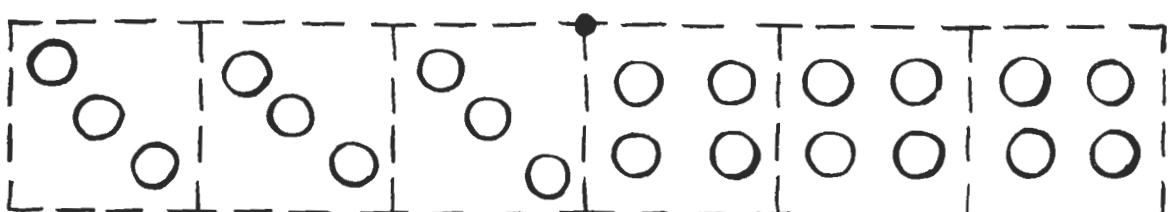
8



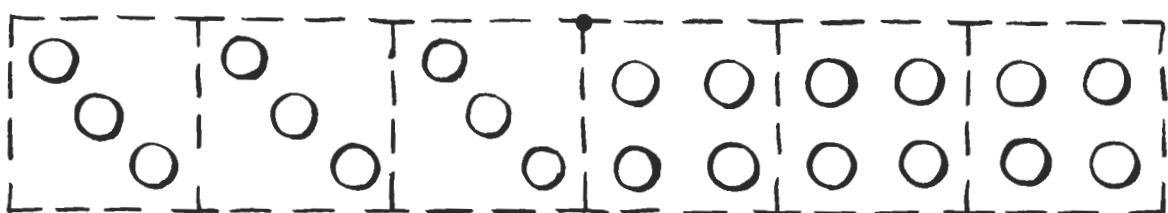
9



10



11



12

PLEASE START FENCE AT THE DOT (•). NAME \_\_\_\_\_

START

|  |           |
|--|-----------|
|  | <u>7</u>  |
|  | <u>8</u>  |
|  | <u>9</u>  |
|  | <u>10</u> |
|  | <u>11</u> |
|  | <u>12</u> |

To talk about this with the children, the following language might be appropriate.

**Show 8**—Move one trap two boxes to the right.

Keep the other trap at start.

8 dots are now trapped. Draw in the fence.

**Show 10**—Move one trap two boxes to the left. Move the other one box to the right.

10 dots are now fenced.

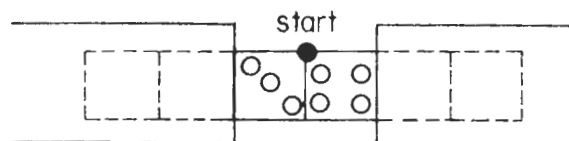
Draw in the fence.

Children have called this “trap arithmetic.”

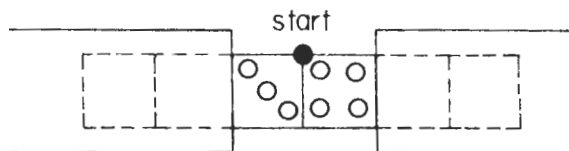
### Procedure

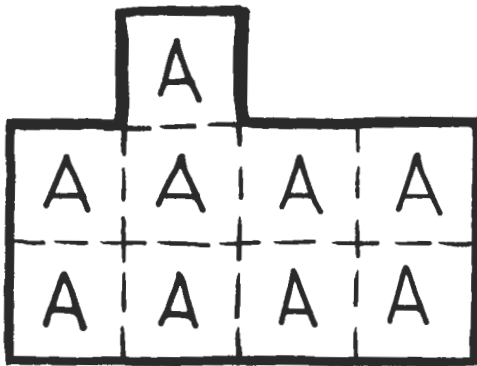
Each child should have two pieces of tagboard or other stiff paper about 3 inches long and 1 inch wide. These will be used as “traps.” Look at the first problem—how can you use your tagboard pieces to trap 7 dots? (Edges of traps must lie along vertical lines.) Notice the dot marked “start.” Both traps start there and then move left or right until the correct number of dots is “trapped” or until it is determined that it is impossible.

Solution:



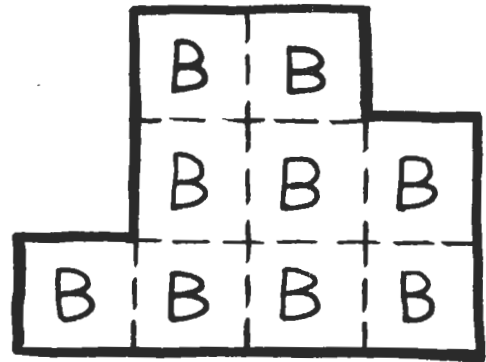
After solving with the tagboard traps, use the starting point and with a pencil draw a fence around the dots that were trapped.





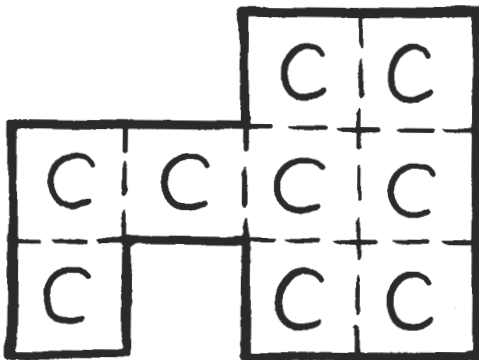
1 2 3 4 5 6  
7 8 9 10 11 12

==



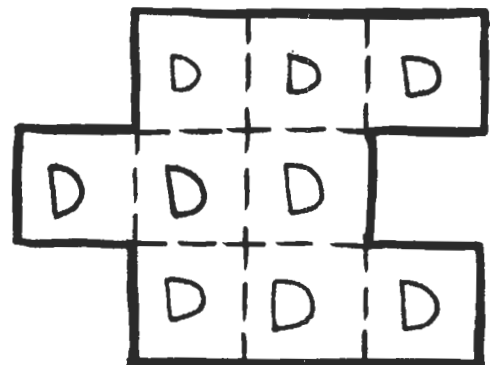
1 2 3 4 5 6  
7 8 9 10 11 12

==



1 2 3 4 5 6  
7 8 9 10 11 12

==



1 2 3 4 5 6  
7 8 9 10 11 12

==

NAME \_\_\_\_\_

1 2 3 4 5 6  
7 8 9 10 11 12

1 2 3 4 5 6  
7 8 9 10 11 12

---

1 2 3 4 5 6  
7 8 9 10 11 12

1 2 3 4 5 6  
7 8 9 10 11 12

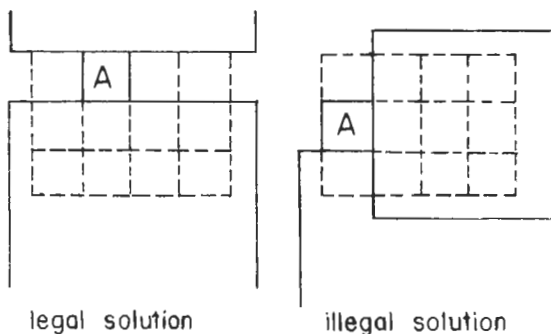
This is another trapping activity with a two dimensional aspect to it. There are more choices in using the masks. The masks may both be vertical as in the previous activity or both horizontal.

You might well use an overhead projector or chalkboard drawing to introduce this page. The placing of masks can be tricky.

Children will need their tagboard masks.

Ask "Can you find a way to show 1 A by trapping it in?" (Yes, by masks whose parallel edges are horizontal.)

There is one solution that is legal.



The solution must always be found between the masks. This may need to be emphasized as it comes up in other instances. Since 1 A can be trapped, circle the 1 in the record beneath the sketch.

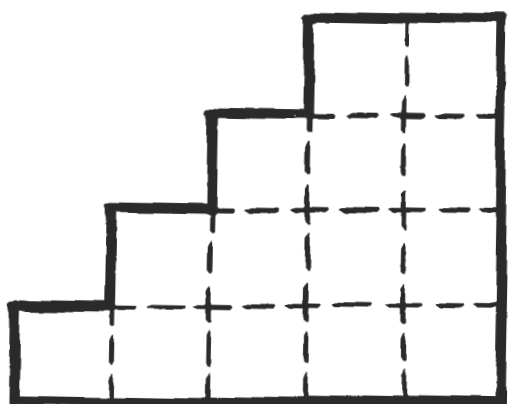
Continue trying to trap 2 A's, 3 A's, 4 A's, etc. Circle the ones that are possible and cross out the ones that cannot be done. Since there are only 9 A's in the problem, a line can be placed after the nine to show that there is no need to go on.

Here is the A record completed:

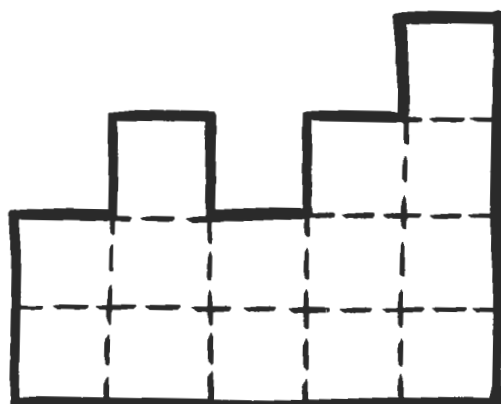
(1) (2) (3) (4) (5) ~~6~~  
 (7) (8) (9) || 10 11 12  
 13 etc. 8

The 8 on the double line shows that we were able to get 8 out of the possible 9.

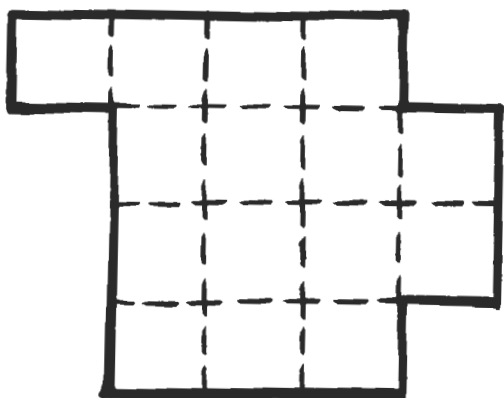
Proceed in the same manner with the B's and C's.



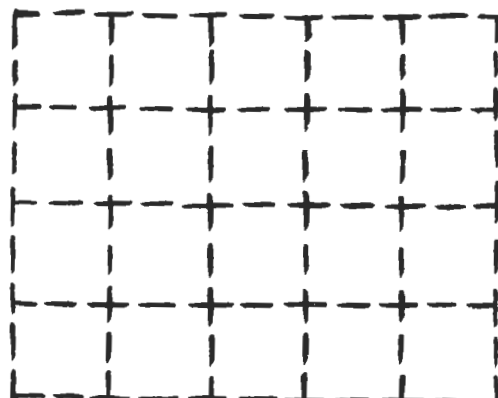
1 2 3 4 5 6  
7 8 9 10 11 12  
13 14 15 16 17 18  
19 20     



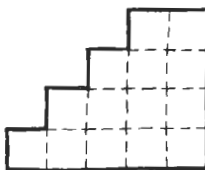
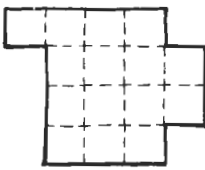
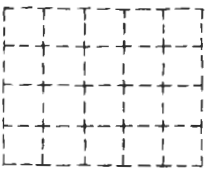
1 2 3 4 5 6  
7 8 9 10 11 12  
13 14 15 16 17 18  
19 20     



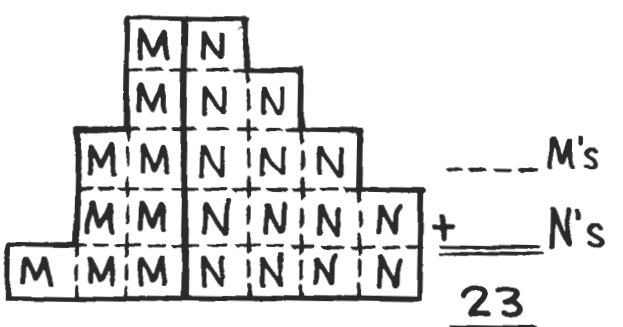
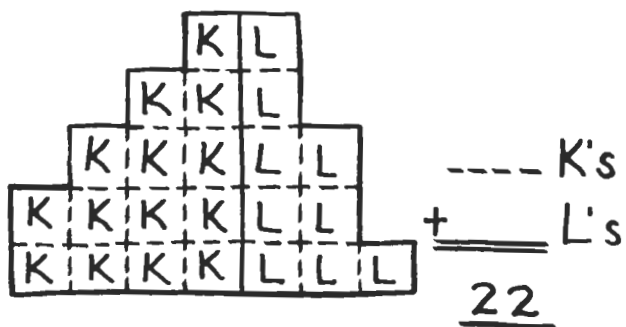
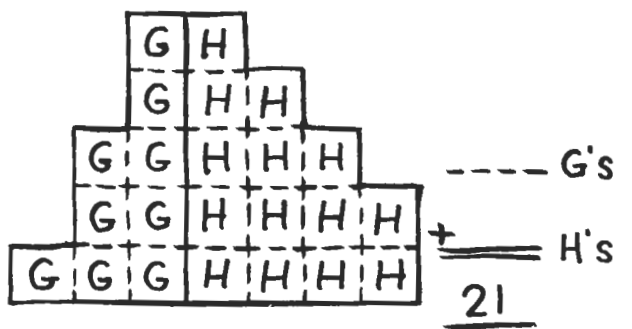
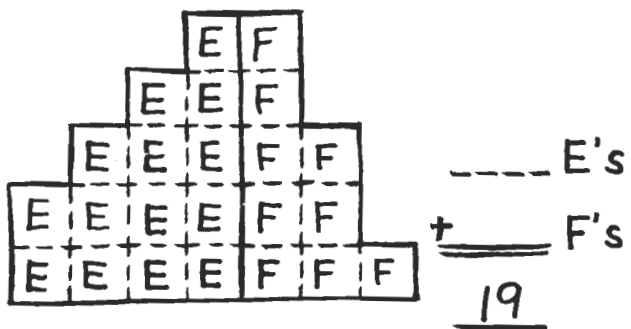
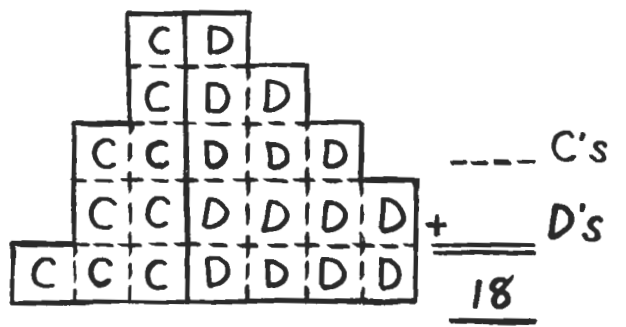
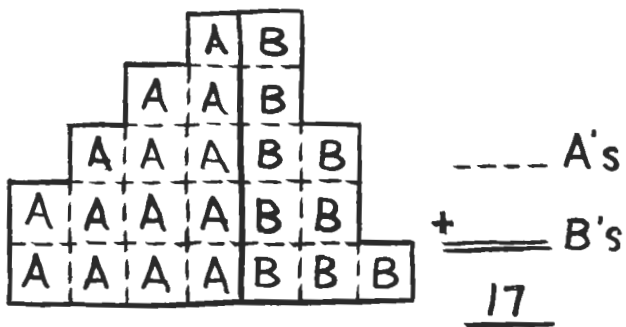
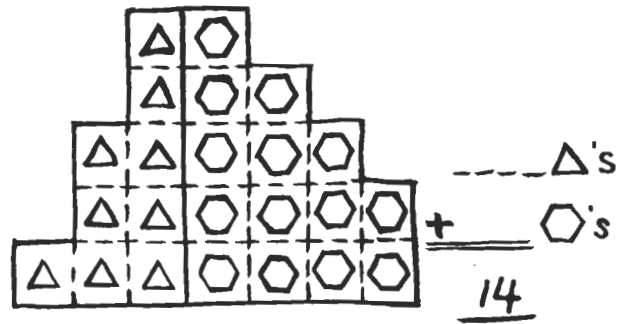
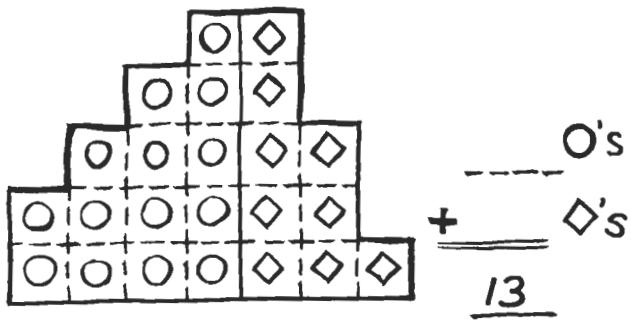
1 2 3 4 5 6  
7 8 9 10 11 12  
13 14 15 16 17 18  
19 20     



1 2 3 4 5 6  
7 8 9 10 11 12  
13 14 15 16 17 18  
19 20

| NAME _____                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                                                    |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  <div style="margin-top: 10px;"> <div style="display: flex; justify-content: space-between; width: 100%;"> <span>1 2 3 4 5 6</span> <span>7 8 9 10 11 12</span> <span>13 14 15 16 17 18</span> <span>19 20</span> </div> </div> |  <div style="margin-top: 10px;"> <div style="display: flex; justify-content: space-between; width: 100%;"> <span>1 2 3 4 5 6</span> <span>7 8 9 10 11 12</span> <span>13 14 15 16 17 18</span> <span>19 20</span> </div> </div>                                                                                   |
|  <div style="margin-top: 10px;"> <div style="display: flex; justify-content: space-between; width: 100%;"> <span>1 2 3 4 5 6</span> <span>7 8 9 10 11 12</span> <span>13 14 15 16 17 18</span> <span>19 20</span> </div> </div> | <p style="text-align: center; font-size: small;">PLEASE DRAW YOUR OWN DIAGRAM</p>  <div style="margin-top: 10px;"> <div style="display: flex; justify-content: space-between; width: 100%;"> <span>1 2 3 4 5 6</span> <span>7 8 9 10 11 12</span> <span>13 14 15 16 17 18</span> <span>19 20</span> </div> </div> |

Only the boxes enclosed within the thick lines can be trapped. In the last problem, children can make up their own diagram.





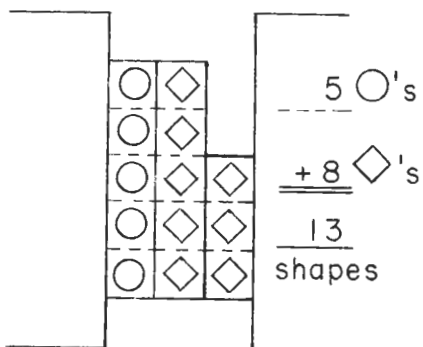
NAME \_\_\_\_\_

|                                                                                                                               |                                                                                                                                 |
|-------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} \text{--- } \bigcirc\text{'s} \\ + \text{--- } \bigcirc\text{'s} \\ \hline 13 \\ \text{SHAPES} \end{array}$ | $\begin{array}{r} \text{--- } \triangle\text{'s} \\ + \text{--- } \triangle\text{'s} \\ \hline 14 \\ \text{SHAPES} \end{array}$ |
| $\begin{array}{r} \text{--- } \text{A's} \\ + \text{--- } \text{B's} \\ \hline 17 \\ \text{LETTERS} \end{array}$              | $\begin{array}{r} \text{--- } \text{C's} \\ + \text{--- } \text{D's} \\ \hline 18 \\ \text{LETTERS} \end{array}$                |
| $\begin{array}{r} \text{--- } \text{E's} \\ + \text{--- } \text{F's} \\ \hline 19 \\ \text{LETTERS} \end{array}$              | $\begin{array}{r} \text{--- } \text{G's} \\ + \text{--- } \text{H's} \\ \hline 21 \\ \text{LETTERS} \end{array}$                |
| $\begin{array}{r} \text{--- } \text{K's} \\ + \text{--- } \text{L's} \\ \hline 22 \\ \text{LETTERS} \end{array}$              | $\begin{array}{r} \text{--- } \text{M's} \\ + \text{--- } \text{N's} \\ \hline 23 \\ \text{LETTERS} \end{array}$                |

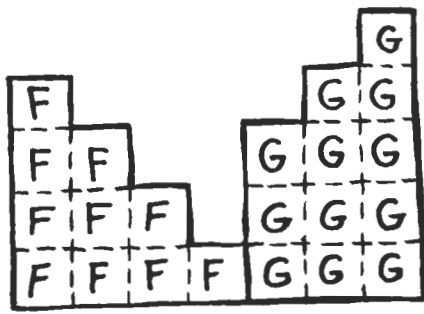
There are 23 boxes in each diagram. Can all numbers of boxes 1 thru 23 be trapped? No: there is no way to trap 6 boxes. Are there others? (Yes: 11 and 16.)

More "trap arithmetic" with a pair of parallel masks. The rule here is that the edges of the masks must always be vertical.

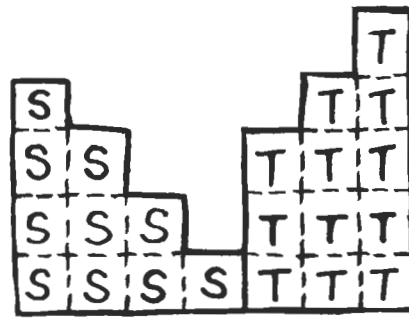
In each of the 8 examples, a certain number of shapes or letters to be trapped is given. After that number is "trapped," a record is kept showing the number of each shape or letter in the trap. The first example calls for 13 shapes; its only solution and the report would be:



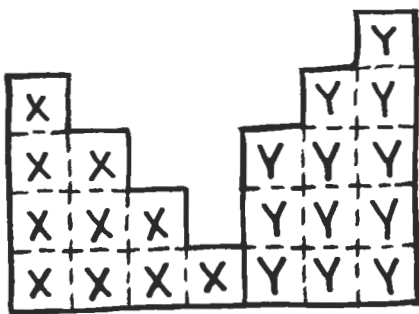
In the second example, 14 shapes can be trapped in two different ways. Can any other number of shapes or letters given be "trapped" in at least one way?



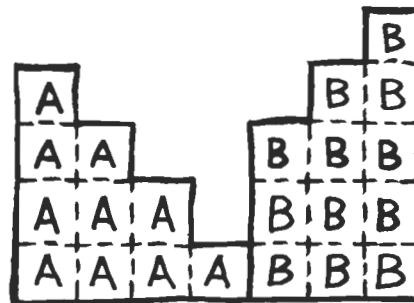
$$\begin{array}{r} \text{---} F's \\ + \text{---} G's \\ \hline 10 \end{array}$$



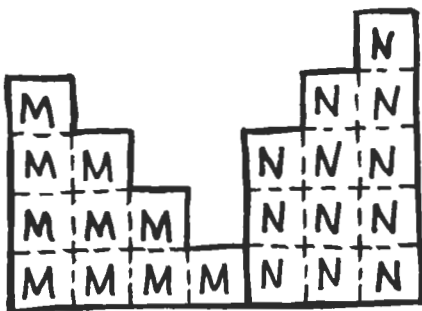
$$\begin{array}{r} \text{---} S's \\ + \text{---} T's \\ \hline 15 \end{array}$$



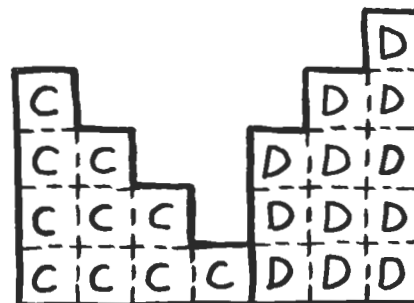
$$\begin{array}{r} \text{---} X's \\ + \text{---} Y's \\ \hline 12 \end{array}$$



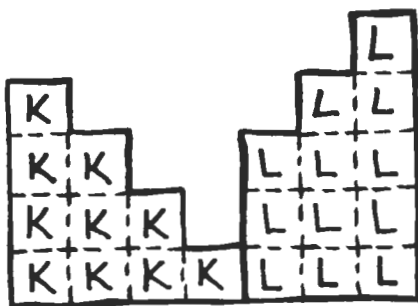
$$\begin{array}{r} \text{---} A's \\ + \text{---} B's \\ \hline 16 \end{array}$$



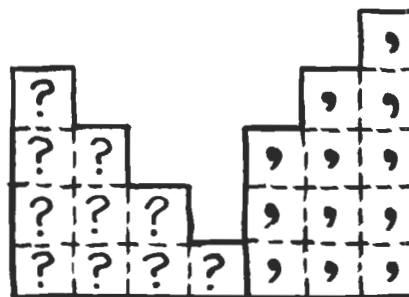
$$\begin{array}{r} \text{---} M's \\ + \text{---} N's \\ \hline 13 \end{array}$$



$$\begin{array}{r} \text{---} C's \\ + \text{---} D's \\ \hline 17 \end{array}$$

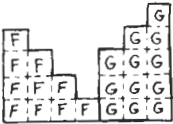
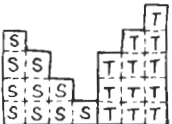
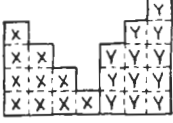
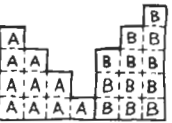
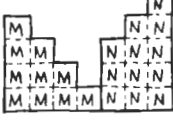
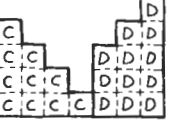
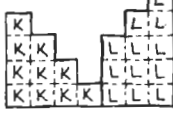
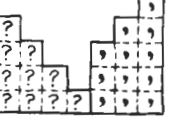


$$\begin{array}{r} \text{---} K's \\ + \text{---} L's \\ \hline 14 \end{array}$$



$$\begin{array}{r} \text{---} ?'s \\ + \text{---} , 's \\ \hline 18 \end{array}$$

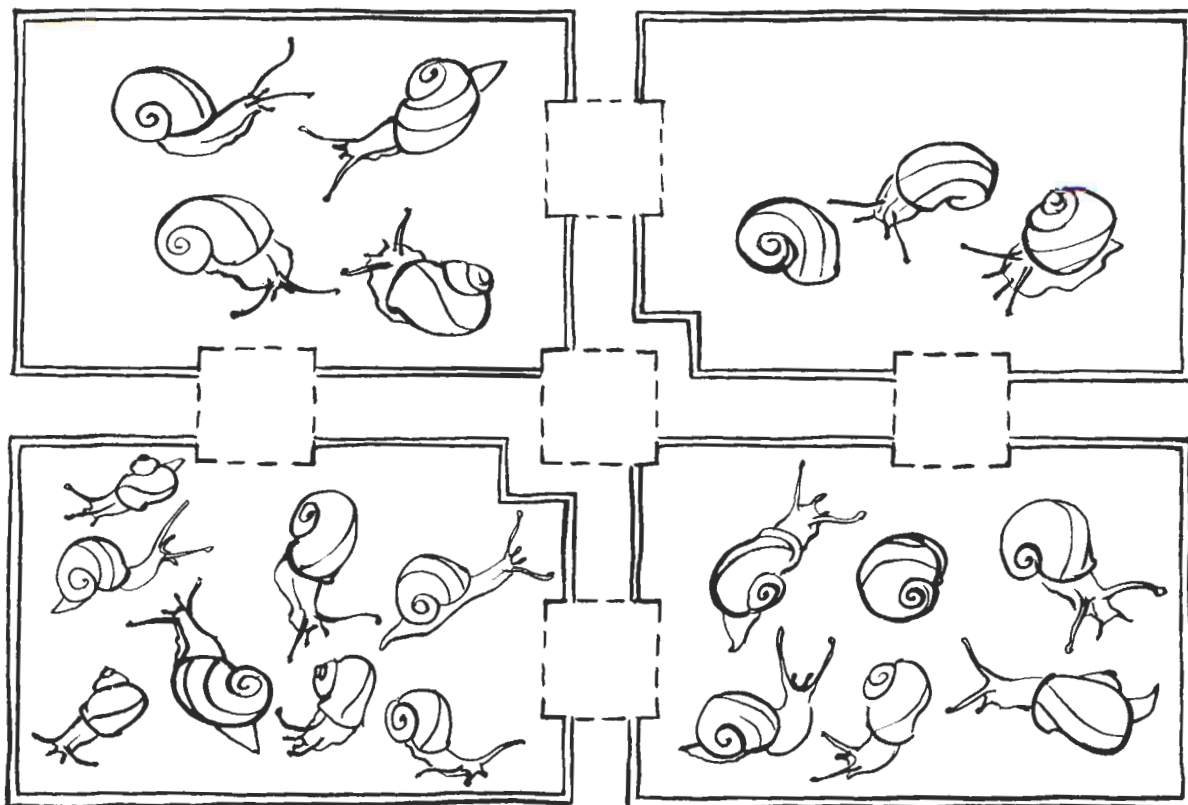
NAME \_\_\_\_\_

|                                                                                                                                                                                    |                                                                                                                                                                                    |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  $\begin{array}{r} \text{--- F's} \\ + \text{--- G's} \\ \hline 10 \\ \text{LETTERS} \end{array}$ |  $\begin{array}{r} \text{--- S's} \\ + \text{--- T's} \\ \hline 15 \\ \text{LETTERS} \end{array}$ |
|  $\begin{array}{r} \text{--- X's} \\ + \text{--- Y's} \\ \hline 12 \\ \text{LETTERS} \end{array}$ |  $\begin{array}{r} \text{--- A's} \\ + \text{--- B's} \\ \hline 16 \\ \text{LETTERS} \end{array}$ |
|  $\begin{array}{r} \text{--- M's} \\ + \text{--- N's} \\ \hline 13 \\ \text{LETTERS} \end{array}$ |  $\begin{array}{r} \text{--- C's} \\ + \text{--- D's} \\ \hline 17 \\ \text{LETTERS} \end{array}$ |
|  $\begin{array}{r} \text{--- K's} \\ + \text{--- L's} \\ \hline 14 \\ \text{LETTERS} \end{array}$ |  $\begin{array}{r} \text{--- ?'s} \\ + \text{--- ?'s} \\ \hline 18 \\ \text{MARKS} \end{array}$   |

More "trap arithmetic" with a pair of parallel masks with vertical edges. Record keeping is the same as the previous page.

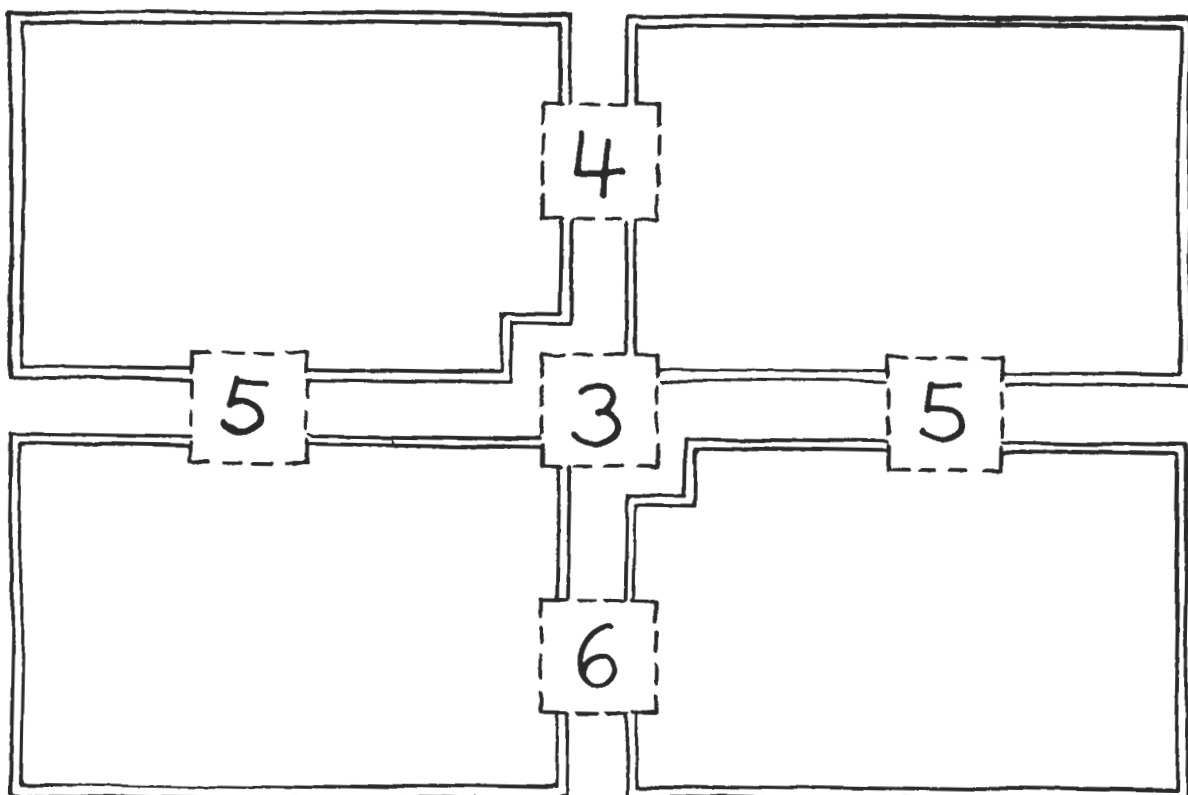
Some of the 8 problems have no solutions. Which ones are they? (14 and 16 cannot be trapped.)

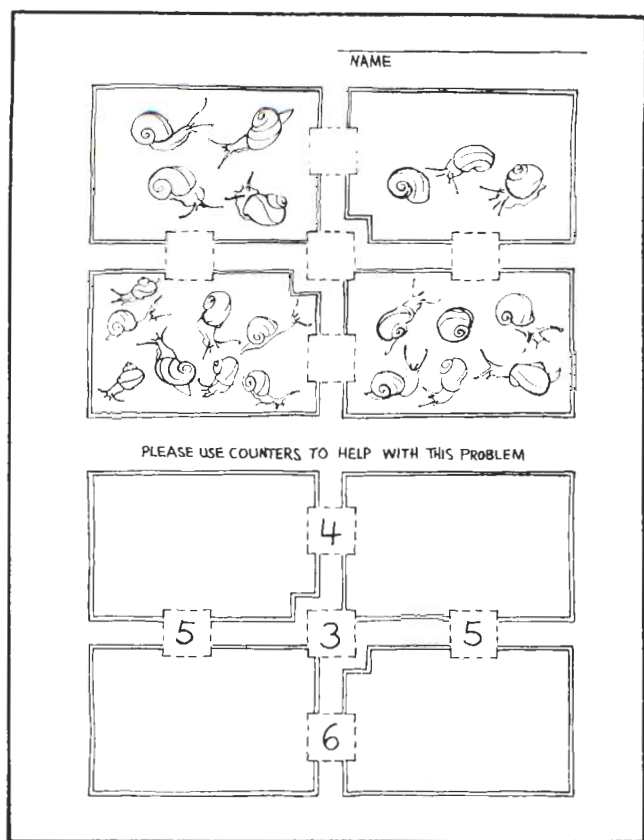
Are there any number of boxes 1 thru 22 that cannot be trapped? (Yes: 11, 19, 20 and 21.)



PLEASE USE COUNTERS.

FAVOR DE USAR OBJETOS CONTABLES.

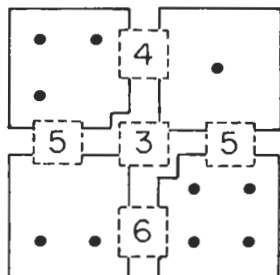


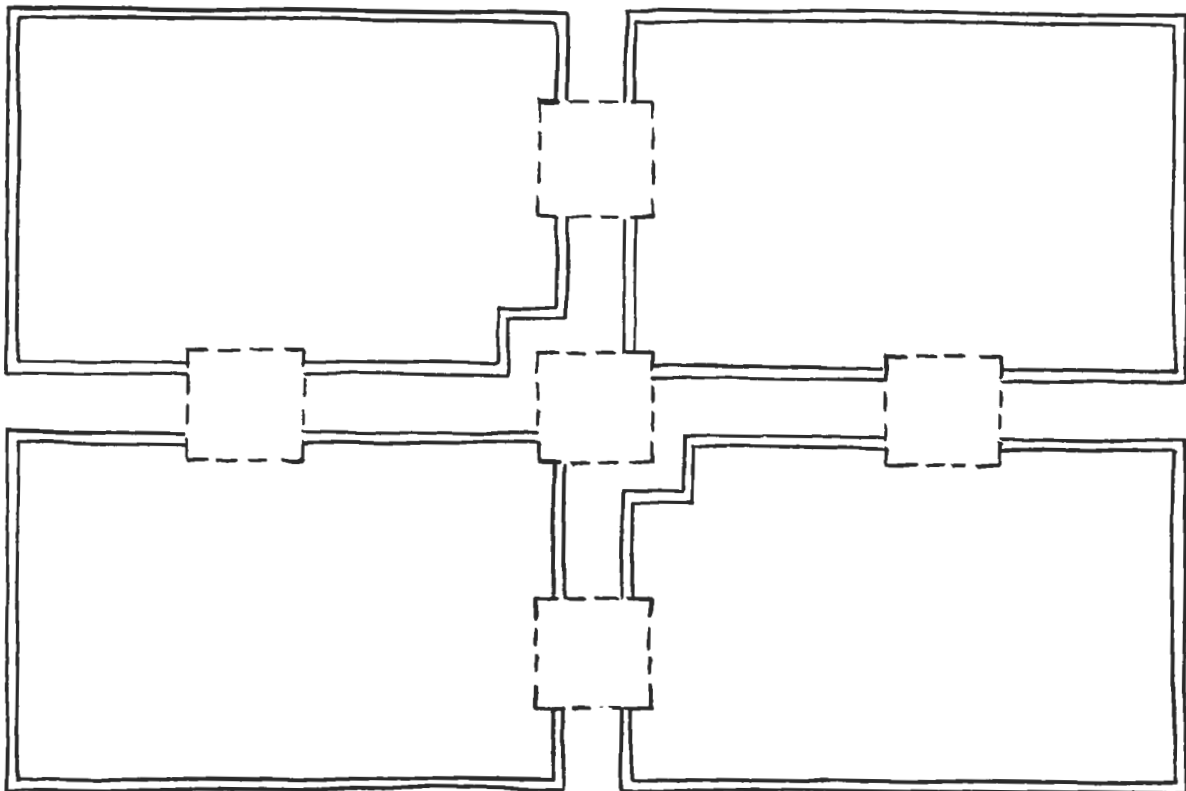
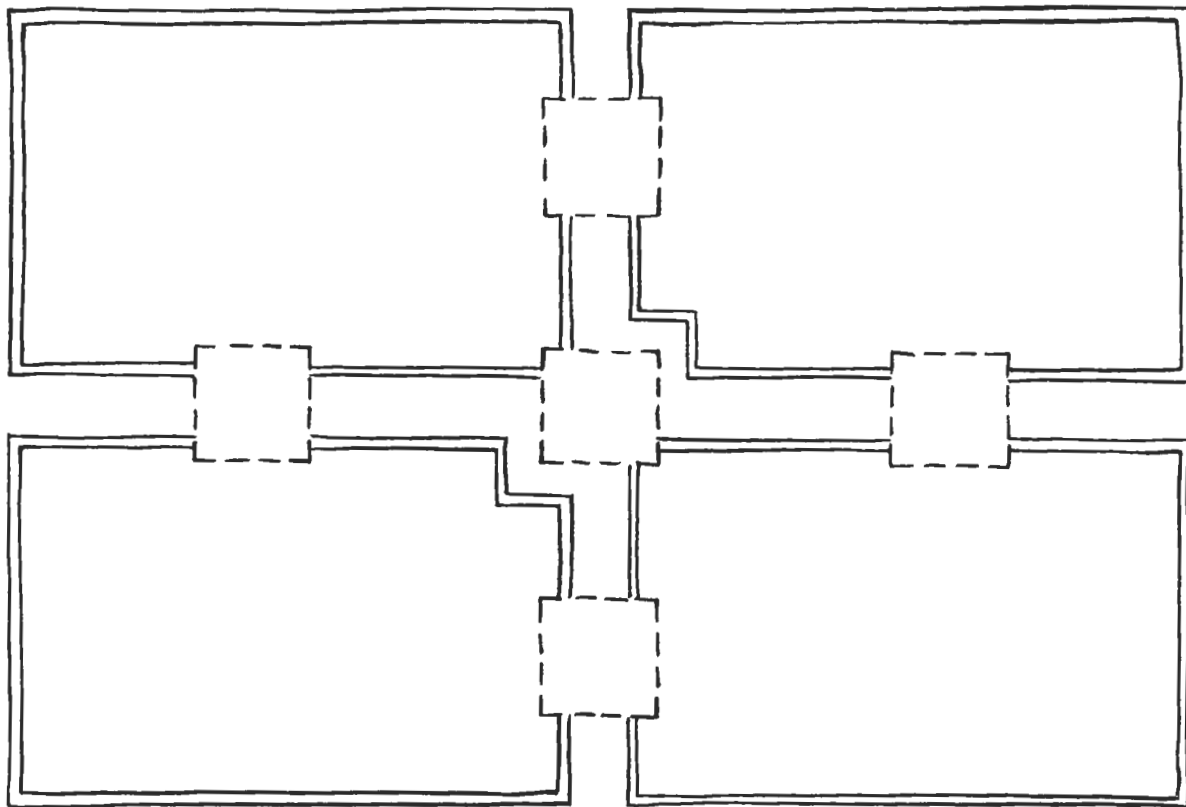


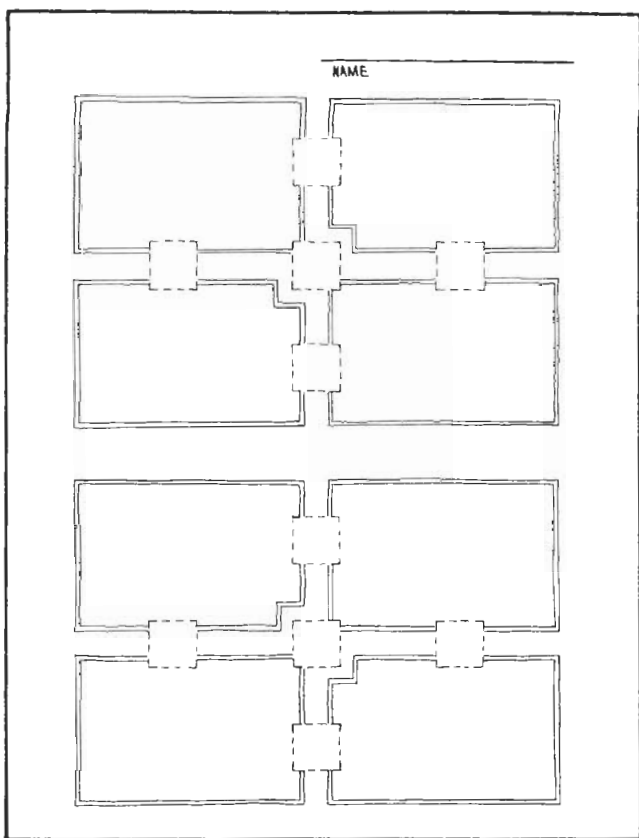
In each broken line square, write the number of "insects" there are in the two frames that are "connected."

This is not a problem. Rather it introduces the second activity on the page—which is a problem!

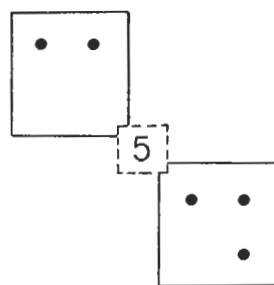
Because most solutions are preceded by much trial and error, students should be encouraged to use counters, trying to find a combination that fits all of the numbers given. There is only one solution.







Finally, two frames are connected by the center small square.



Next, repeat the activity on the bottom of the page with as many in each as the student wants.

**Then remove all counters.**

Now, there's a real problem! Can you put the counters back the way they were?

Partners can exchange papers: Can they solve each other's problems?

Perhaps some children will want to use big numbers and challenge others to solve their problems.

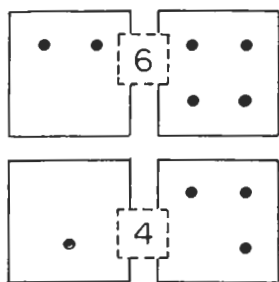
Or, the page can be particularized as a pair of problems for everybody. Here is one:

This activity calls for the use of counters.

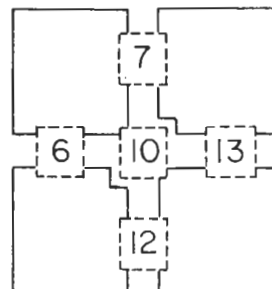
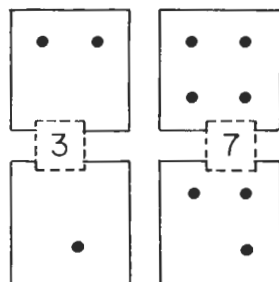
"Please put some counters in each of the 4 large frames in the top half of the page."

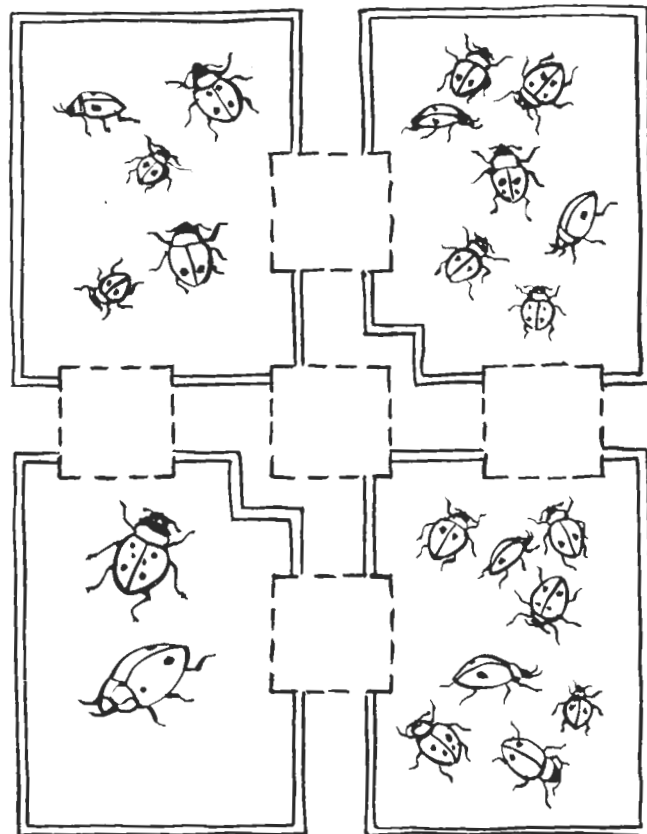
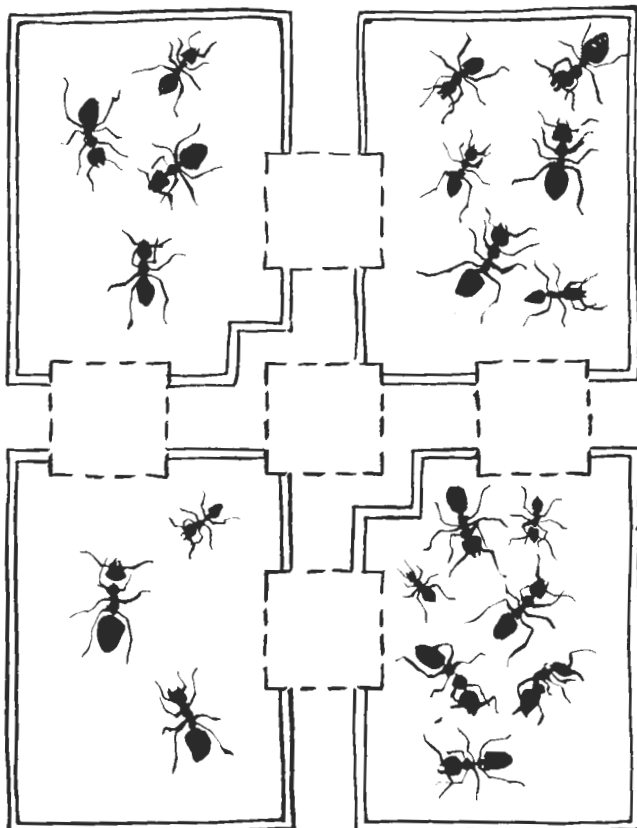
"Pairs of these frames are connected by a small broken-line box. In this box, please write the sum of the numbers of counters in those two frames."

An illustration on the board or overhead might help.



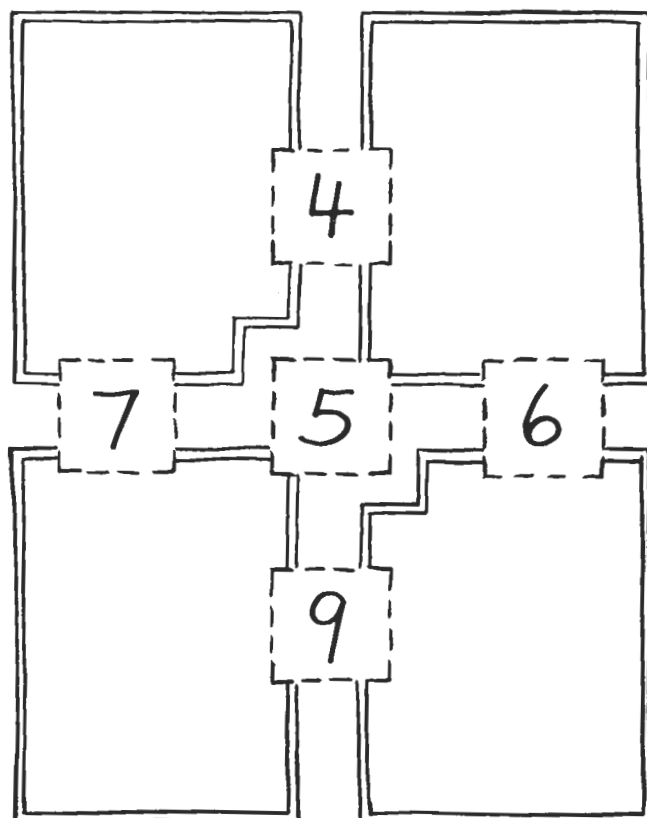
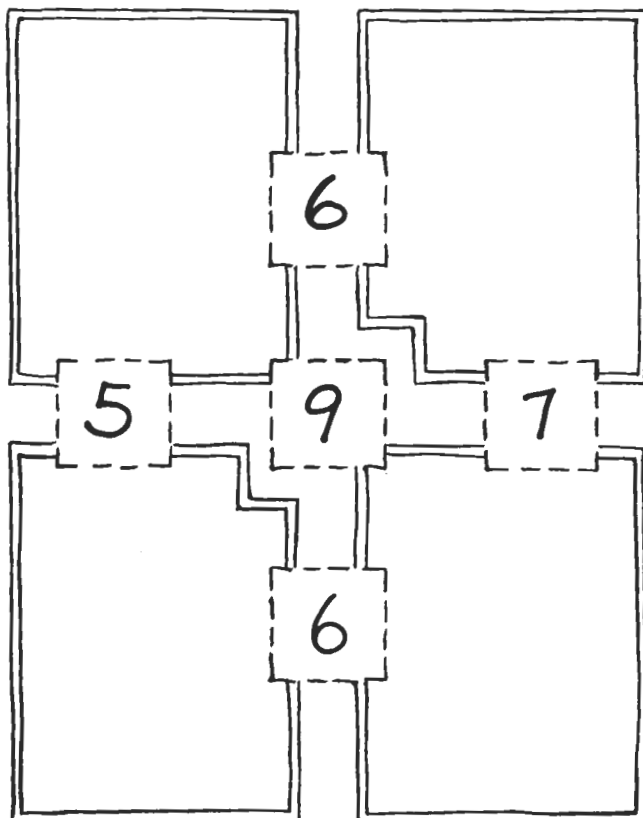
Pairs are also connected in columns.





PLEASE USE COUNTERS.

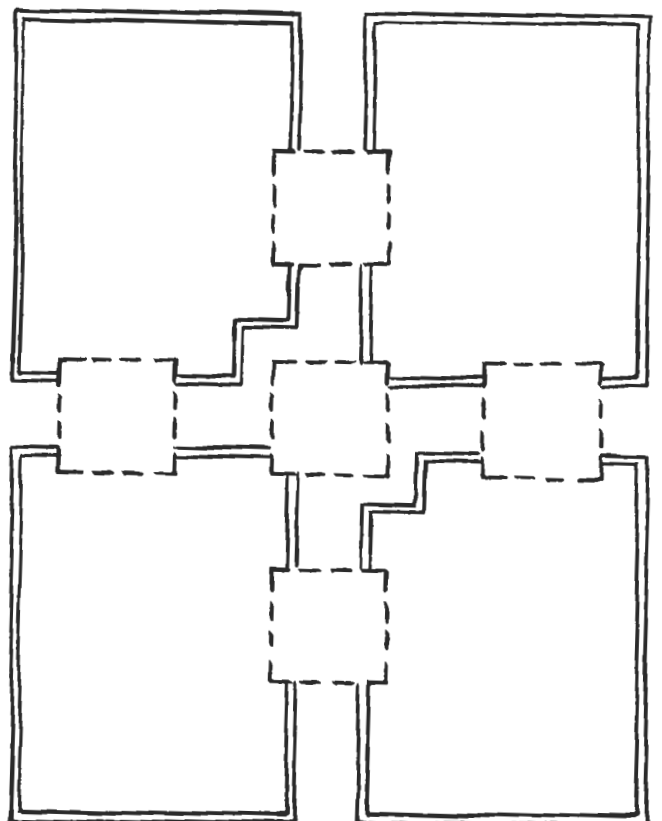
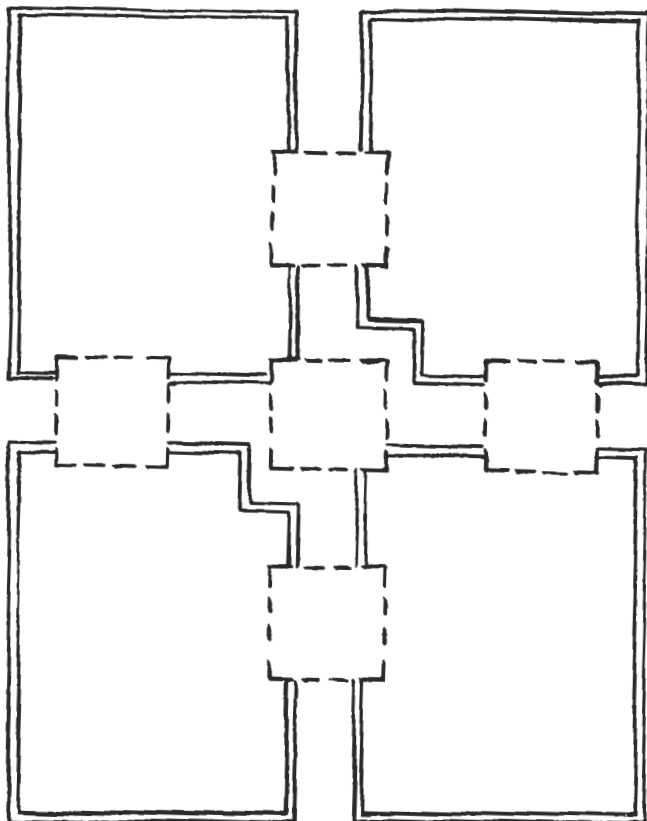
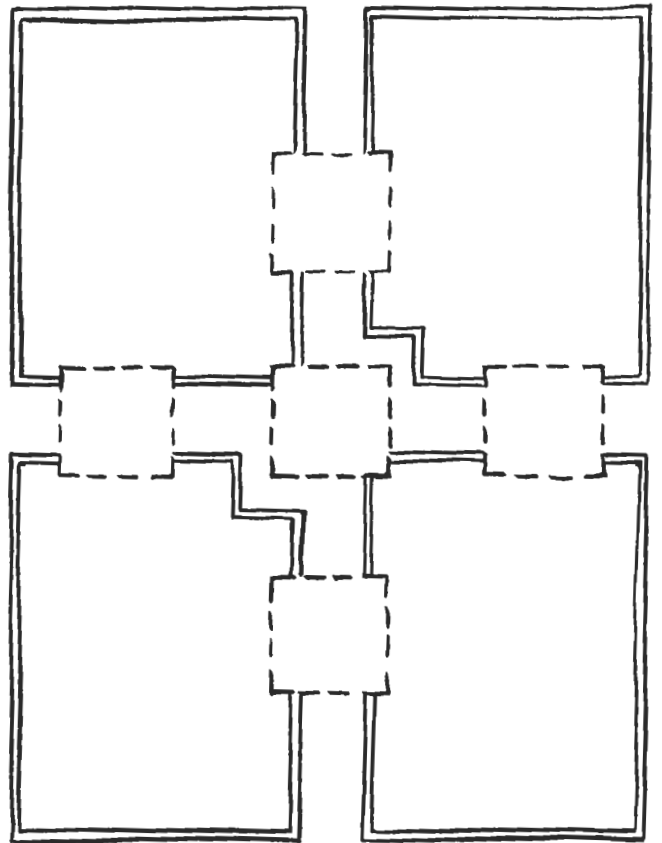
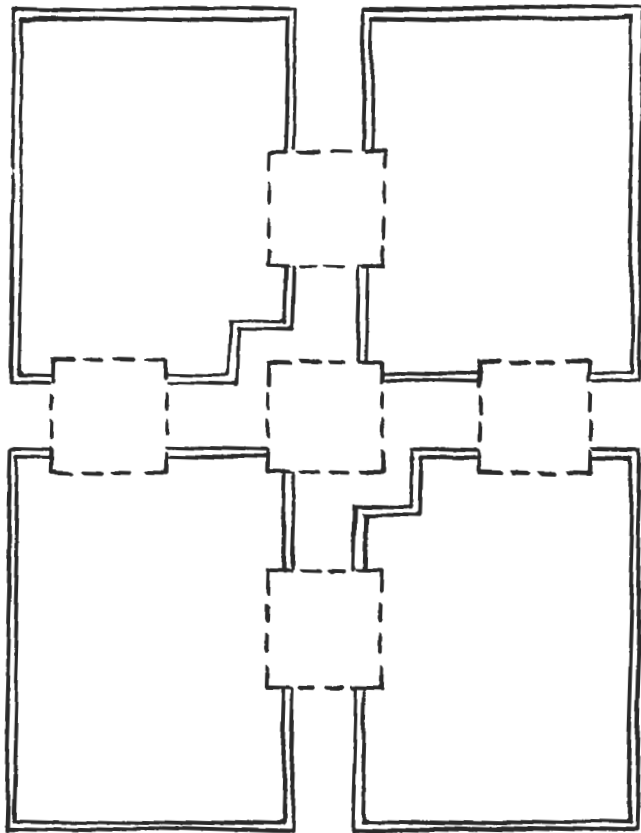
FAVOR DE USAR OBJETOS CONTABLES.



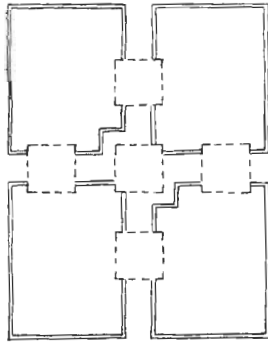
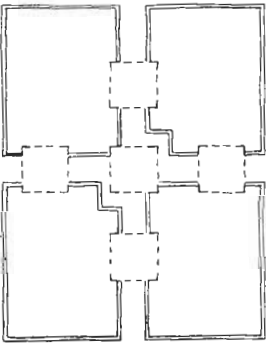
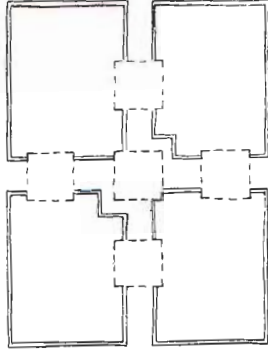
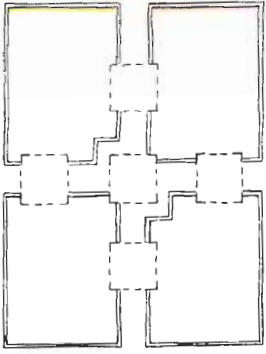


PLEASE USE COUNTERS TO HELP WITH THESE PROBLEMS

Because of the "trial and error" usually required in problems on the bottom half of the page, children should be encouraged to use counters.



NAME \_\_\_\_\_



PALOS A 10  
10 STICKS

FRIJOLE SUELTOS  
LOOSE BEANS

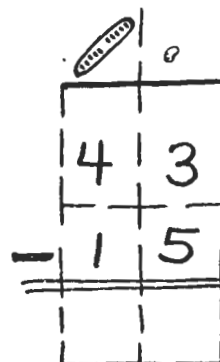
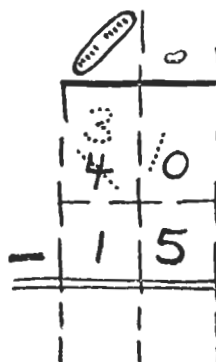
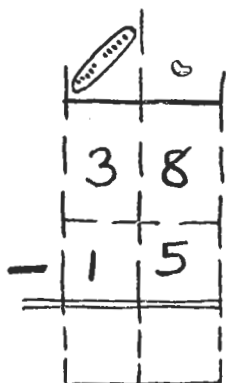
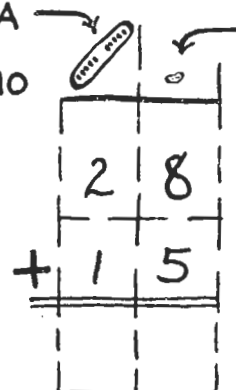
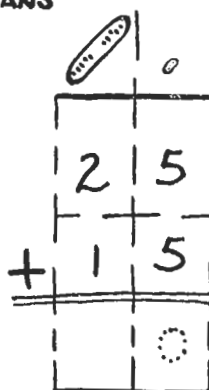
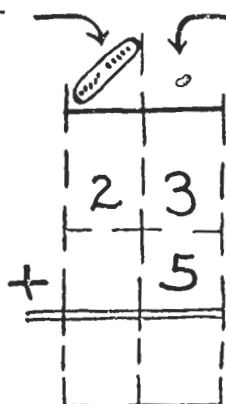
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10-STICK  
COLUMN

LOOSE BEANS  
COLUMN

COLUMNA  
PARA  
PALOS A 10

COLUMNA  
PARA  
FRIJOLE SUELTOS



PLEASE USE 10-STICKS AND LOOSE BEANS  
MAKING UP YOUR OWN EXAMPLES

| 10 STICKS | LOOSE BEANS |
|-----------|-------------|
|           |             |

10-STICK COLUMN      LOOSE BEANS COLUMN

|      |   |
|------|---|
| 2    | 3 |
| + 5  |   |
| 3    | 8 |
| - 15 |   |
| 2    | 3 |

10-STICK COLUMN      LOOSE BEANS COLUMN

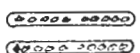
|      |   |
|------|---|
| 2    | 5 |
| + 15 |   |
| 3    | 0 |
| - 15 |   |
| 2    | 5 |

10-STICK COLUMN      LOOSE BEANS COLUMN

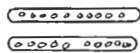
|      |   |
|------|---|
| 2    | 8 |
| + 15 |   |
| 3    | 3 |
| - 15 |   |
| 2    | 8 |

This activity helps children see how the algorithms for “carrying” and “borrowing” (or “regrouping”) emerge as a record of experiments.

Assume that we have passed a rule that we cannot use more than nine loose beans. The activity might proceed as follows: “Please get me 23 beans, and put them in the top space.”

| 10-sticks                                                                           | loose beans                                                                         |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
|                                                                                     |                                                                                     |

“Now get me five more beans and put them in the lower space.”

| 10-sticks                                                                           | loose beans                                                                         |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |
|                                                                                     |                                                                                     |

“How many beans are there all together?” There are 8 loose beans, which we record in the “loose beans” column, and two 10-sticks, which we record in the 10-stick column as follows:

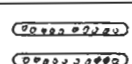
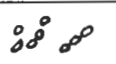
|                 |                   |
|-----------------|-------------------|
|                 |                   |
| 10-stick column | loose bean column |
| 2               | 3                 |
|                 | 5                 |
| 2               | 8                 |

There is a total of 28 beans.

Now, consider an example like

|                 |                   |
|-----------------|-------------------|
|                 |                   |
| 10-stick column | loose bean column |
| 2               | 8                 |
| 1               | 5                 |
|                 |                   |

“Please get me 28 beans. Put them in the top space.”

| 10-sticks                                                                           | loose beans                                                                          |
|-------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
|  |  |
|                                                                                     |                                                                                      |

“Now get me 15 beans, and put them in the bottom space.”

| 10-sticks                                                                            | loose beans                                                                           |
|--------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|
|  |  |
|                                                                                      |                                                                                       |

“How many are there all together?” There are 13 loose beans. But we said no more than nine loose beans could be used, so we can trade — ten loose beans for one stick, and record the trade by putting a small “1” in the 10-stick column.

one more 10-stick from the trade

|                 |                   |
|-----------------|-------------------|
|                 |                   |
| 10-stick column | loose bean column |
| 2               | 8                 |
| 1               | 5                 |
|                 |                   |

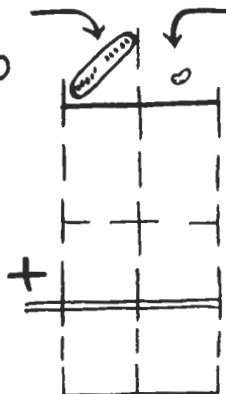
(continued on page 110B)

PALOS A 10  
10 STICKS

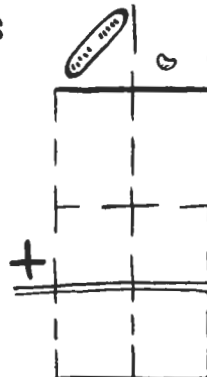
FRIJOLES SUELTOS  
LOOSE BEANS

|  |  |
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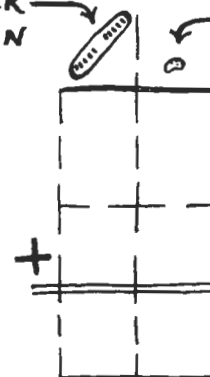
COLUMNA  
PARA  
PALOS A 10



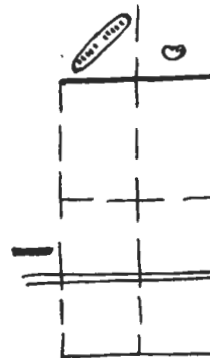
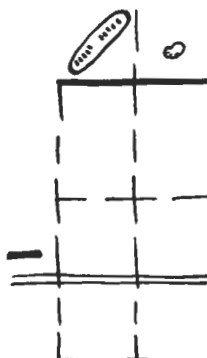
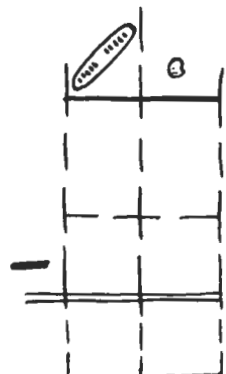
COLUMNA  
PARA  
FRIJOLES  
SUELTOS

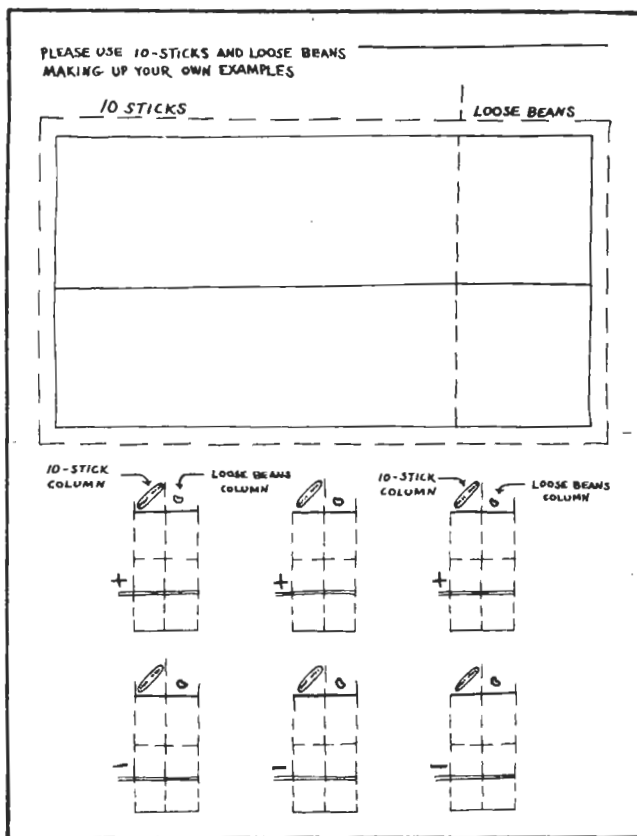


10-STICK  
COLUMN



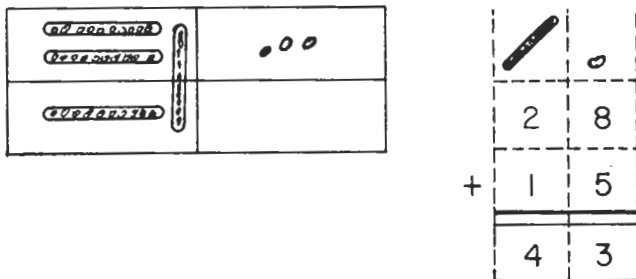
LOOSE BEANS  
COLUMN





(continued from Page 110A)

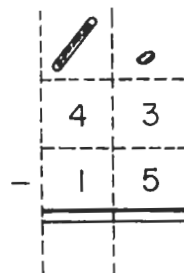
Now the array is:



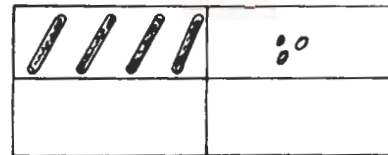
There are 3 loose beans and four 10-sticks, and this result can be recorded

It is wise to establish the rule for addition that we "clean up the little ones first" and do any necessary trading before writing the total.

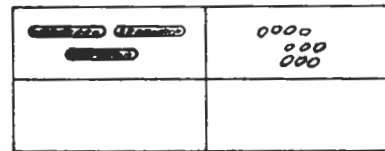
Consider a subtraction problem:



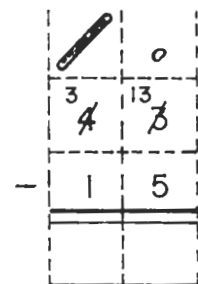
"Please get 43 beans and put them in the upper space."



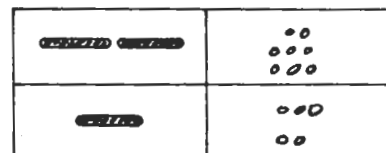
"Now we move 15 of them into the lower space." There is no way to move 15 without breaking a stick, so we must make a trade — one 10-stick for 10 loose beans:



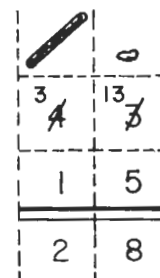
We keep a record of the trade:



Now it is easy to move 15 beans:

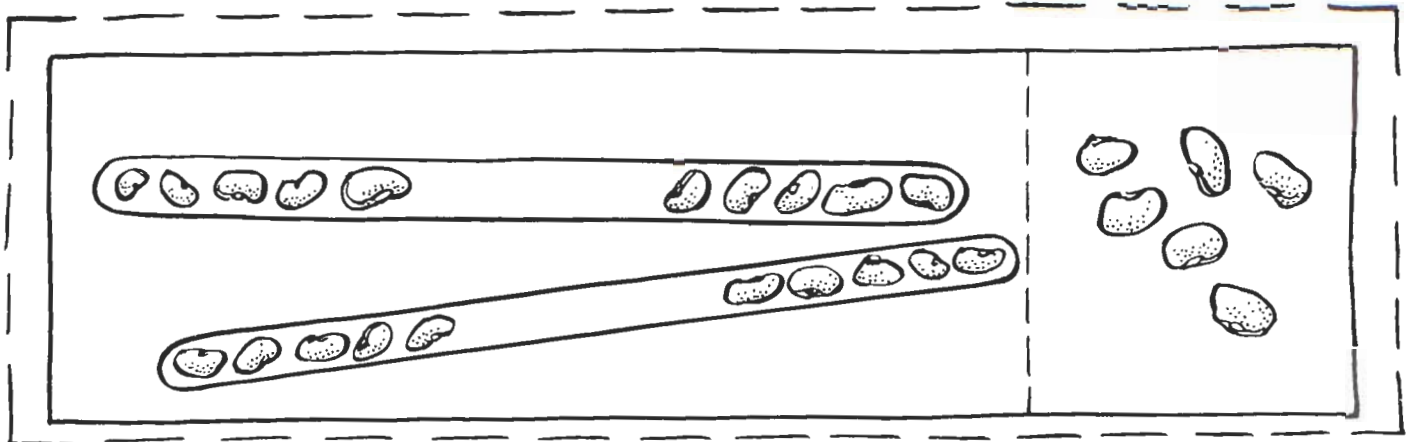


"How many are left in the top space?" There are 8 loose beans and two 10-sticks. This result can be recorded:



A total of 28 beans remain in the upper space.

Talk about tens and ones is abstract talk. Talk about 10-sticks and loose beans is talk about things. Children can see what is happening, and the algorithms develop naturally as a way of keeping a record of what is happening. We prefer to use a word like "re-packaging" instead of "borrowing", "carrying", or "regrouping", for it gives a more graphic sense of what is happening.

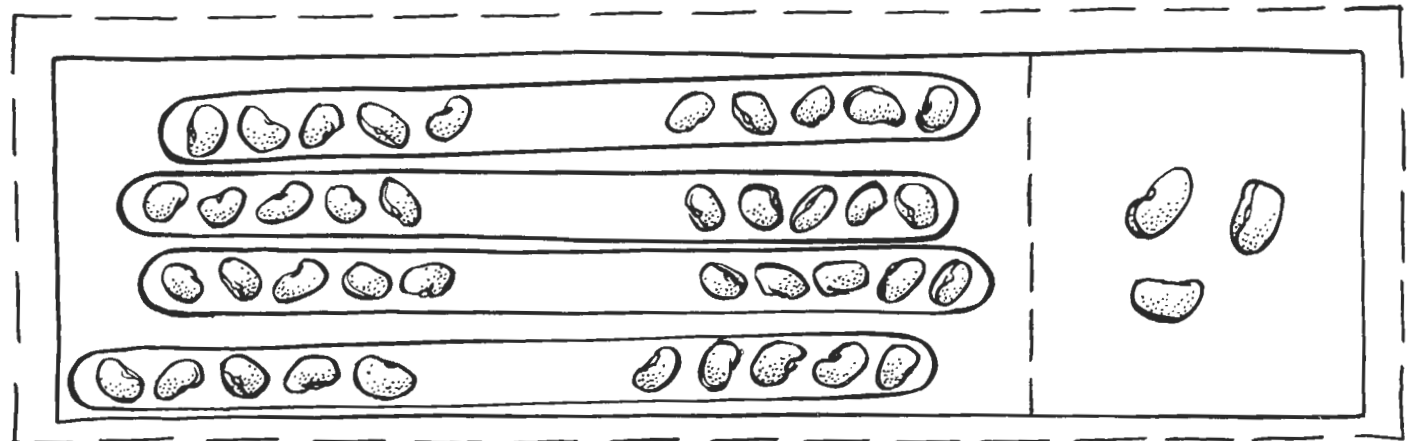


|       |   |   |
|-------|---|---|
| 100   |   |   |
| 2     | 6 |   |
| +     | 1 | 8 |
| ----- |   |   |
|       |   | 4 |

|       |   |   |
|-------|---|---|
| 100   |   |   |
| 2     | 6 |   |
| -     | 1 | 8 |
| ----- |   |   |
|       |   |   |

|       |   |   |
|-------|---|---|
| 100   |   |   |
| 2     | 6 |   |
| +     | 2 | 4 |
| ----- |   |   |
|       |   |   |

|       |   |   |
|-------|---|---|
| 100   |   |   |
| 2     | 6 |   |
| -     | 1 | 4 |
| ----- |   |   |
|       |   |   |



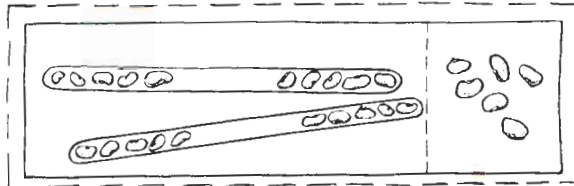
|       |   |   |
|-------|---|---|
| 100   |   |   |
| 4     | 3 |   |
| +     | 1 | 5 |
| ----- |   |   |
|       |   |   |

|       |   |   |
|-------|---|---|
| 100   |   |   |
| 4     | 3 |   |
| -     | 2 | 6 |
| ----- |   |   |
|       |   |   |

|       |   |   |
|-------|---|---|
| 100   |   |   |
| 4     | 3 |   |
| +     | 3 | 7 |
| ----- |   |   |
|       |   |   |

|       |   |   |
|-------|---|---|
| 100   |   |   |
| 4     | 3 |   |
| -     | 1 | 9 |
| ----- |   |   |
|       |   |   |



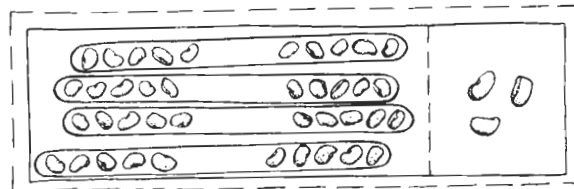


$$\begin{array}{r} 26 \\ + 18 \\ \hline \end{array}$$

$$\begin{array}{r} 26 \\ - 18 \\ \hline \end{array}$$

$$\begin{array}{r} 26 \\ + 24 \\ \hline \end{array}$$

$$\begin{array}{r} 26 \\ - 14 \\ \hline \end{array}$$



$$\begin{array}{r} 43 \\ + 15 \\ \hline \end{array}$$

$$\begin{array}{r} 43 \\ - 26 \\ \hline \end{array}$$

$$\begin{array}{r} 43 \\ + 37 \\ \hline \end{array}$$

$$\begin{array}{r} 43 \\ - 19 \\ \hline \end{array}$$

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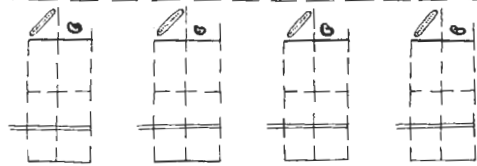
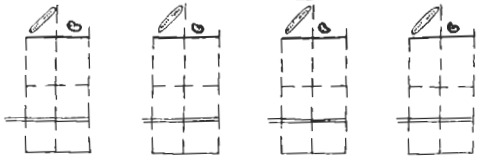
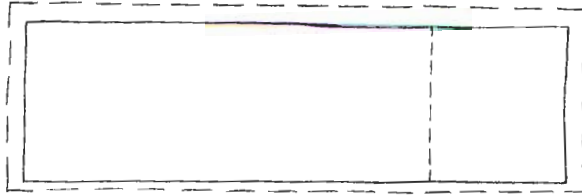
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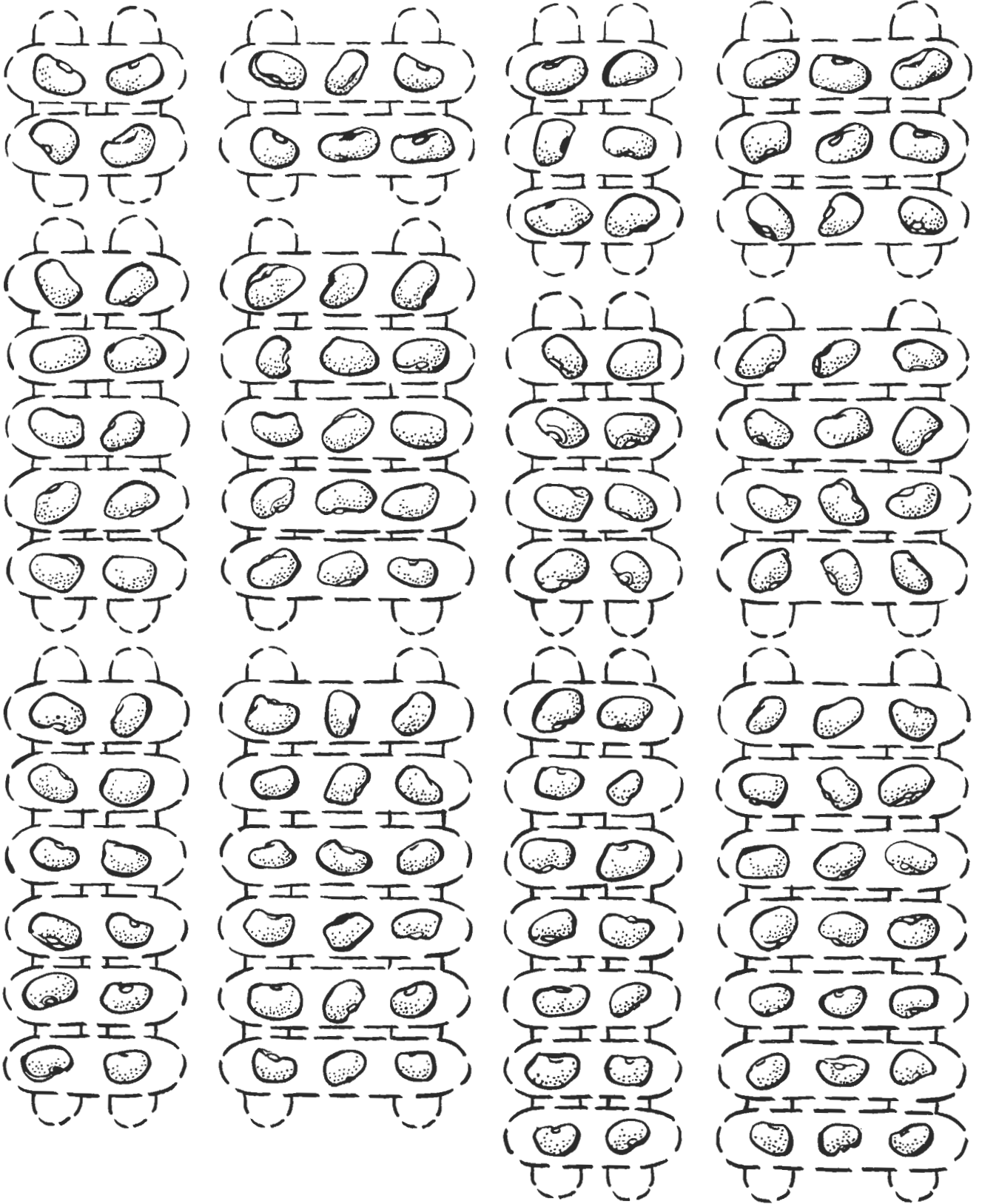
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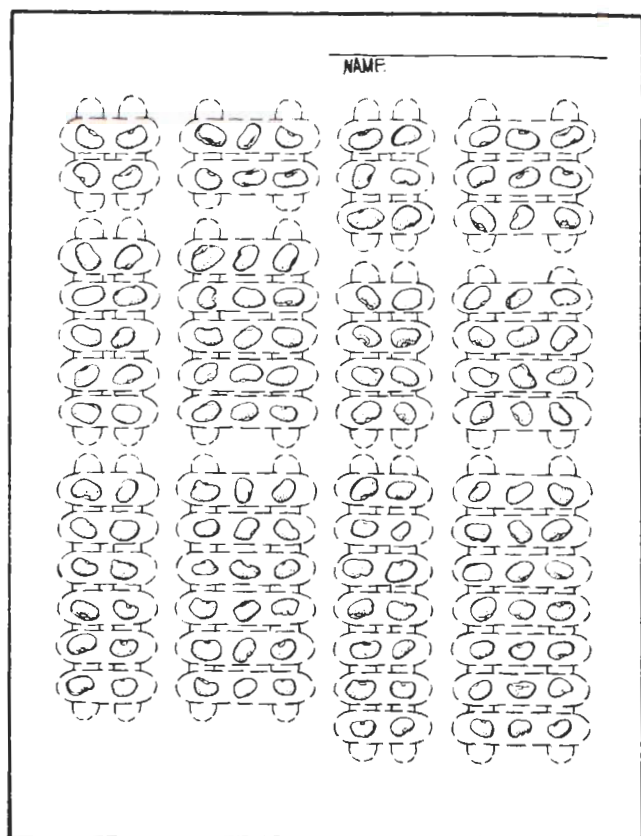
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10-STICKS AND LOOSE BEANS







These diagrams are meant to represent "beanstick rafts."

We can talk about "rafts" in terms of the number of logs and the number of beans on each log. The largest sketch shows a raft of 7 logs with 3 beans on each. We will refer to this as a "7 by 3 raft" or a "3 by 7 raft." It has a total of 21 beans. It is a "21 bean raft."

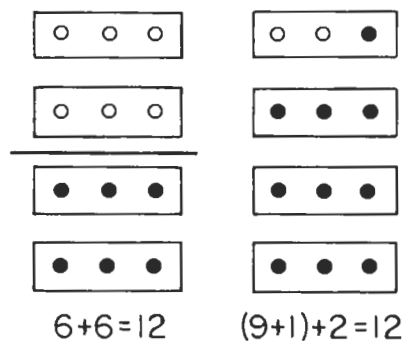
How many beans on each raft? The answer, in some instances, can be determined by a quick glance. This is certainly true in the case of the 2 by 2 raft. Is it true of the 2 by 3 rafts? Yes, but only because one knows that  $3 + 3 = 6$  or  $2 \times 3 = 6$ . Likewise, the 3 by 3 raft with its domino pattern is surely a 9-bean raft.

As the raft becomes larger, the number of beans becomes less obvious. If a student has committed the multiplication fact to memory, he need only know the number of beans on each log and the number of logs—but he should not be making rafts (unless he wants to join in).

As soon as a raft has too many beans to take in at a glance, we want to use different colored beans and/or spacing of beans and logs to provide short cuts to the answer.

A 3 by 4 raft can be built using these devices in several ways.

If one knows the 3 by 3 domino pattern has 9 beans and that  $9 + 1 = 10$  and that  $10 + 2 = 12$ , or if he knows that two 3's are 6 and  $6 + 6 = 12$ , then one of the following plans would help him know there are 12 beans on a 3 by 4 raft.



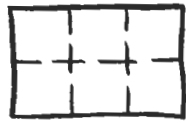
Crayons can be used to show the different colors of beans on the diagrams. A heavy line can indicate uses of spacings to make groups.

This page can be used to introduce the idea of planning rafts so they will reduce the problem of knowing how many there are on a raft—for those who have not yet committed all the facts to memory.

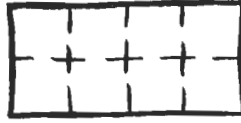
The pages that follow can be used to plan rafts of all sizes with 2 thru 9 beans on each log and 2 thru 9 logs—to represent all the basic multiplication facts with factors 2 thru 9.



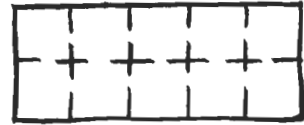
$$\underline{2 \times 2} = \underline{\quad}$$



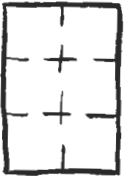
$$\underline{2 \times 3} = \underline{\quad}$$



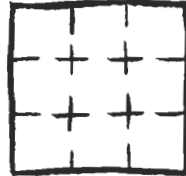
$$\underline{2 \times 4} = \underline{\quad}$$



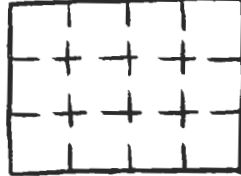
$$\underline{2 \times 5} = \underline{\quad}$$



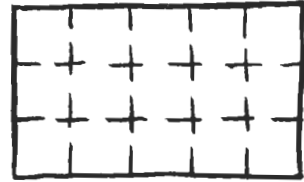
$$\underline{3 \times 2} = \underline{\quad}$$



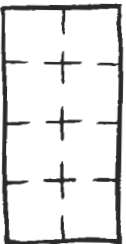
$$\underline{3 \times 3} = \underline{\quad}$$



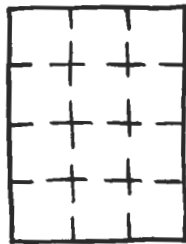
$$\underline{3 \times 4} = \underline{\quad}$$



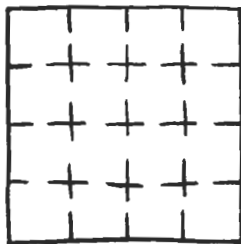
$$\underline{3 \times 5} = \underline{\quad}$$



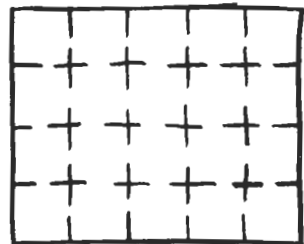
$$\underline{4 \times 2} = \underline{\quad}$$



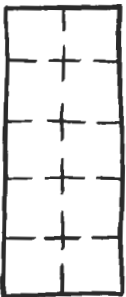
$$\underline{4 \times 3} = \underline{\quad}$$



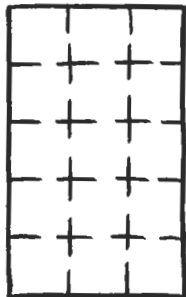
$$\underline{4 \times 4} = \underline{\quad}$$



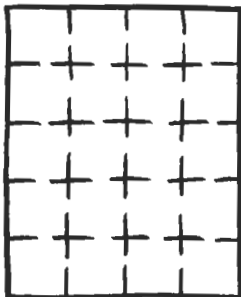
$$\underline{4 \times 5} = \underline{\quad}$$



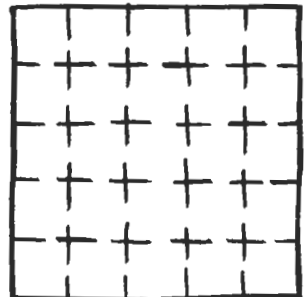
$$\underline{5 \times 2} = \underline{\quad}$$



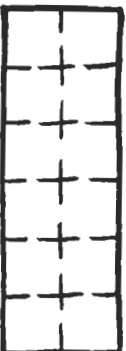
$$\underline{5 \times 3} = \underline{\quad}$$



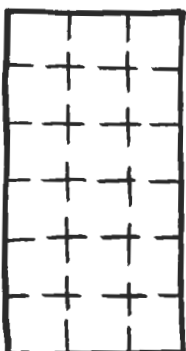
$$\underline{5 \times 4} = \underline{\quad}$$



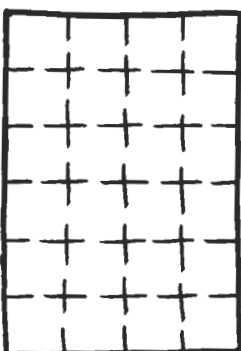
$$\underline{5 \times 5} = \underline{\quad}$$



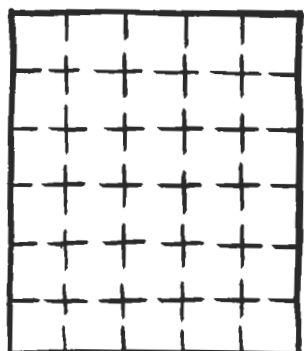
$$\underline{6 \times 2} = \underline{\quad}$$



$$\underline{6 \times 3} = \underline{\quad}$$

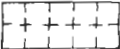
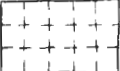
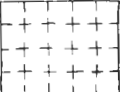
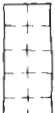
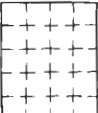
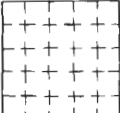
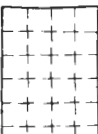
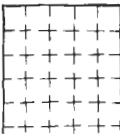


$$\underline{6 \times 4} = \underline{\quad}$$



$$\underline{6 \times 5} = \underline{\quad}$$

NAME \_\_\_\_\_

|                                                                                                           |                                                                                                           |                                                                                                           |                                                                                                           |
|-----------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| <br>$2 \times 2 =$ _____ | <br>$2 \times 3 =$ _____ | <br>$2 \times 4 =$ _____ | <br>$2 \times 5 =$ _____ |
| <br>$3 \times 2 =$ _____ | <br>$3 \times 3 =$ _____ | <br>$3 \times 4 =$ _____ | <br>$3 \times 5 =$ _____ |
| <br>$4 \times 2 =$ _____ | <br>$4 \times 3 =$ _____ | <br>$4 \times 4 =$ _____ | <br>$4 \times 5 =$ _____ |
| <br>$5 \times 2 =$ _____ | <br>$5 \times 3 =$ _____ | <br>$5 \times 4 =$ _____ | <br>$5 \times 5 =$ _____ |
| <br>$6 \times 2 =$ _____ | <br>$6 \times 3 =$ _____ | <br>$6 \times 4 =$ _____ | <br>$6 \times 5 =$ _____ |

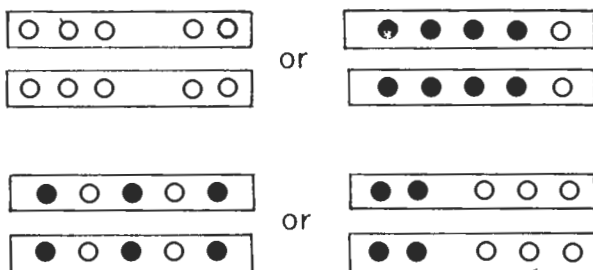
One purpose of this page is to help in planning "beanstick rafts."

A "raft" is defined by the rule that each log has the same number of beans on it.

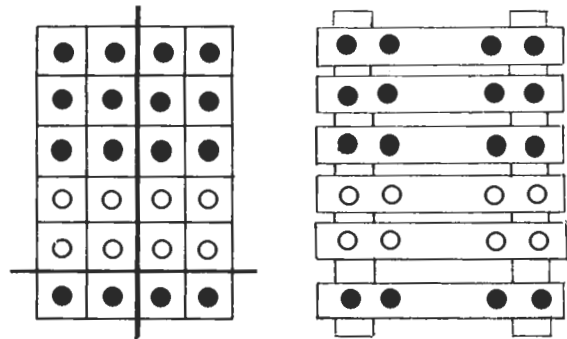
A further requirement of rafts is that it be planned to encourage a minimum of one-by-one counting. This, of course, refers to rafts with more than 3 or 4 beans on a log and with more than 3 or 4 logs. (See previous page.)

Tactics used in planning may utilize spacing or grouping of beans on each log, spacing the logs, different colors of beans or any combination.

A 2 x 5 raft might show a variety of these devices:



Here are plans for a 4 by 6 raft—and the raft it suggests:

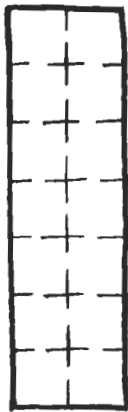


We might talk about the plan as thinking about 24 as  $10 + 10 + 4$ . Further that 10 can be seen as  $6 + 4$  or as  $5 + 5$ .

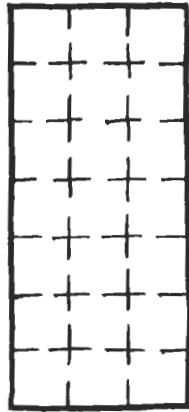
A reasonable goal of a group of children is to make all rafts with 9 or less logs and 9 or less beans on the log. And to qualify as a "raft," there must be at least 2 logs and at least 2 beans on each log.

That would be 64 rafts. If the group plans for each member to contribute toward this goal, the burden on any one member is modest.

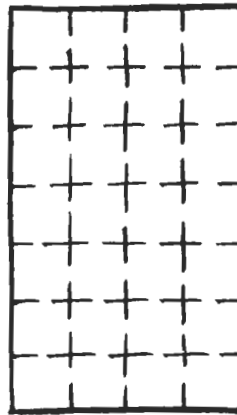
As the project is underway, a most valuable aspect of it is wide open discussion of various tactics for displaying each collection of beans in a way that helps most to reduce requirements for one-by-one counting.



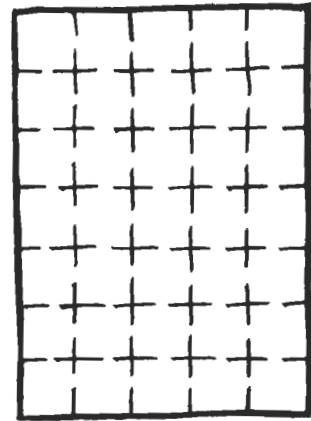
$$7 \times 2 = \underline{\quad}$$



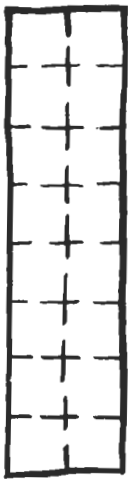
$$7 \times 3 = \underline{\quad}$$



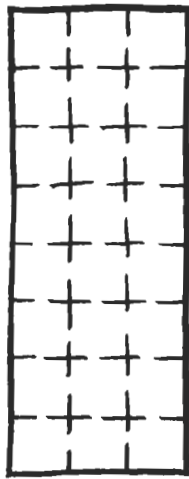
$$7 \times 4 = \underline{\quad}$$



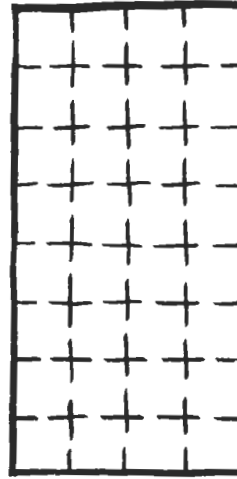
$$7 \times 5 = \underline{\quad}$$



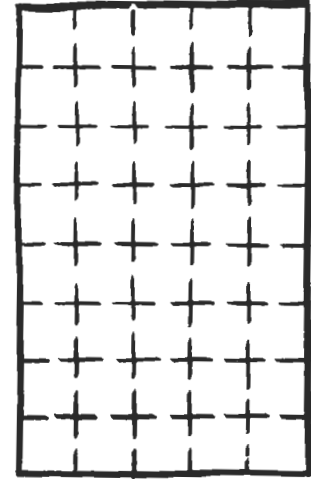
$$8 \times 2 = \underline{\quad}$$



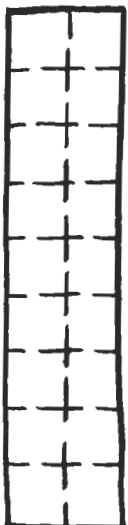
$$8 \times 3 = \underline{\quad}$$



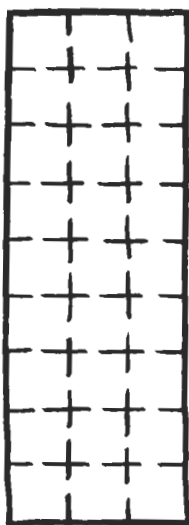
$$8 \times 4 = \underline{\quad}$$



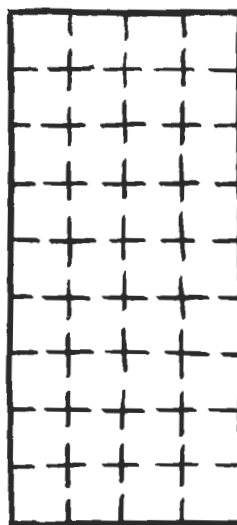
$$8 \times 5 = \underline{\quad}$$



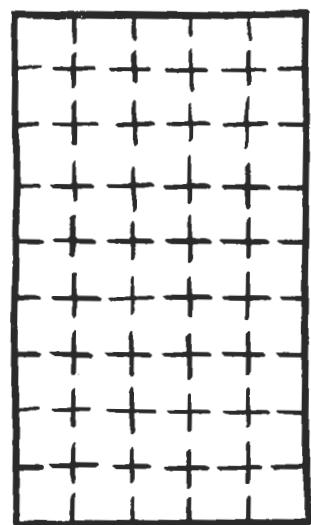
$$9 \times 2 = \underline{\quad}$$



$$9 \times 3 = \underline{\quad}$$



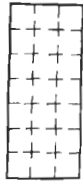
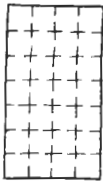
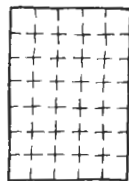
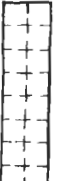
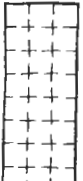
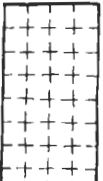
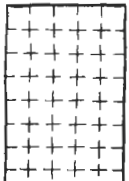
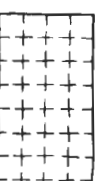
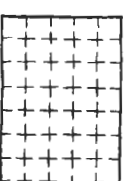
$$9 \times 4 = \underline{\quad}$$

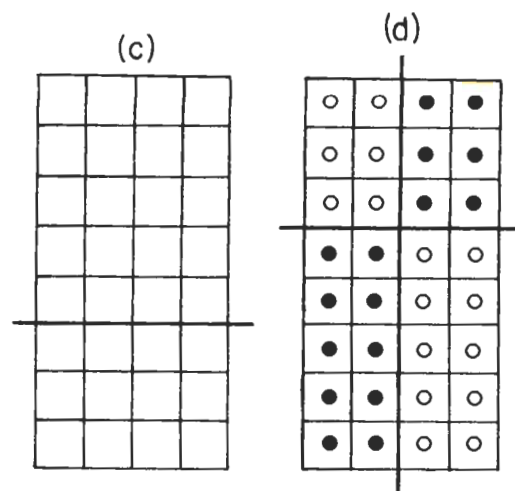


$$9 \times 5 = \underline{\quad}$$



NAME \_\_\_\_\_

|                                                                                   |                                                                                   |                                                                                   |                                                                                   |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |  |  |
| $7 \times 2 = \underline{\quad}$                                                  | $7 \times 3 = \underline{\quad}$                                                  | $7 \times 4 = \underline{\quad}$                                                  | $7 \times 5 = \underline{\quad}$                                                  |
|  |  |  |  |
| $8 \times 2 = \underline{\quad}$                                                  | $8 \times 3 = \underline{\quad}$                                                  | $8 \times 4 = \underline{\quad}$                                                  | $8 \times 5 = \underline{\quad}$                                                  |
|  |  |  |  |
| $9 \times 2 = \underline{\quad}$                                                  | $9 \times 3 = \underline{\quad}$                                                  | $9 \times 4 = \underline{\quad}$                                                  | $9 \times 5 = \underline{\quad}$                                                  |



Plans (a) and (b) seem to think about 32 as  $10 + 10 + 12 \dots$  in different arrangements.

Plan (c) relies on knowing the  $4 \times 5 = 20$  and  $3 \times 4 = 12$  and  $20 + 12 = 32$ .

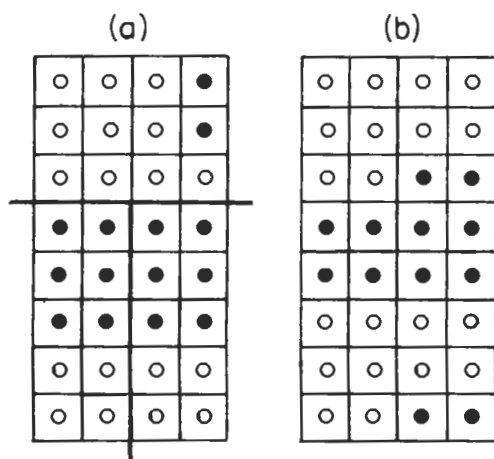
Plan (d) assumes less information—showing the solution is two 10's and two 6's leading to  $10 + 10 = 20$ ,  $6 + 6 = 12$ , and  $20 + 12 = 32$ .

As the rafts become larger, the need for special arrangements becomes more necessary and the opportunities grow.

Differences of opinion arise. Arguments get heated. The search for "a better" means of display is more intense.

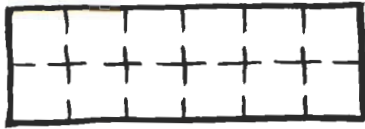
And all of the differences and disputes are focused on representations of the "facts" children need to commit to memory: "drill and practice" at the problem solving level!

Consider  $8 \times 4 = 32$

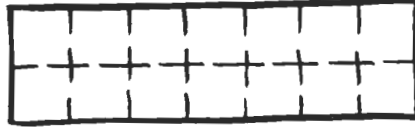


Which is more useful?

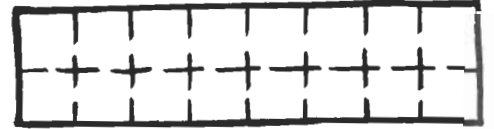
The discussion and debate can only help fix the fact that  $4 \times 8 = 32$  and  $8 \times 4 = 32$  more firmly in the memory.



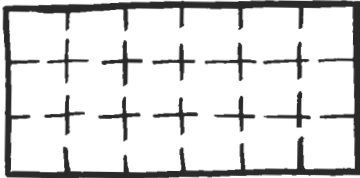
$$2 \times 6 = \underline{\quad}$$



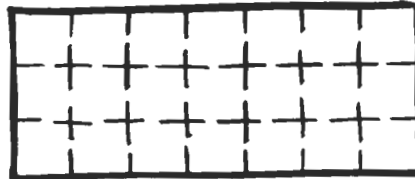
$$2 \times 7 = \underline{\quad}$$



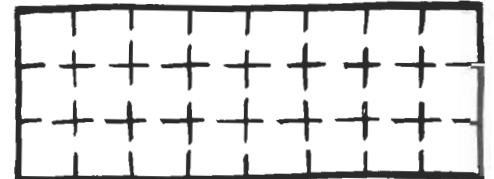
$$2 \times 8 = \underline{\quad}$$



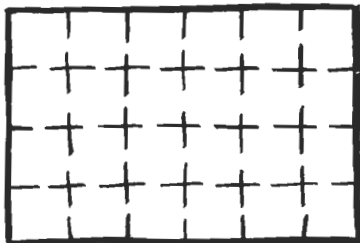
$$3 \times 6 = \underline{\quad}$$



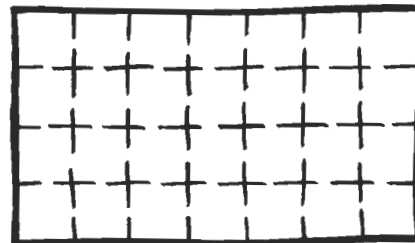
$$3 \times 7 = \underline{\quad}$$



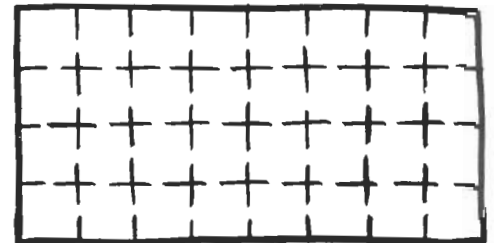
$$3 \times 8 = \underline{\quad}$$



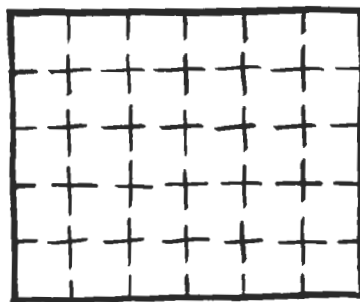
$$4 \times 6 = \underline{\quad}$$



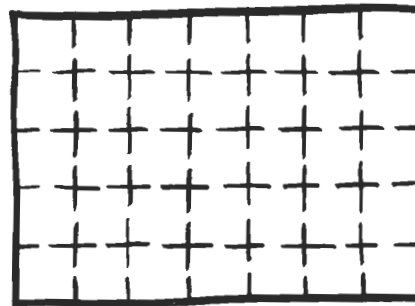
$$4 \times 7 = \underline{\quad}$$



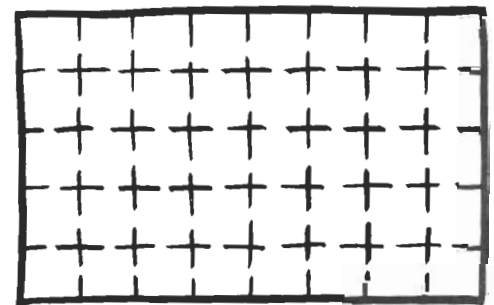
$$4 \times 8 = \underline{\quad}$$



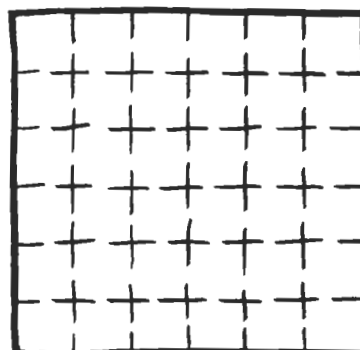
$$5 \times 6 = \underline{\quad}$$



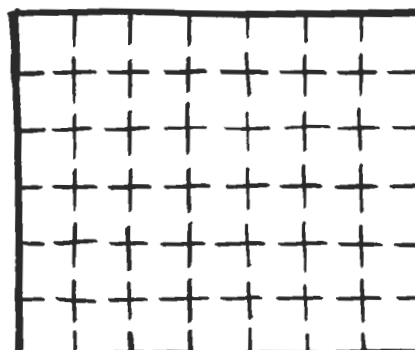
$$5 \times 7 = \underline{\quad}$$



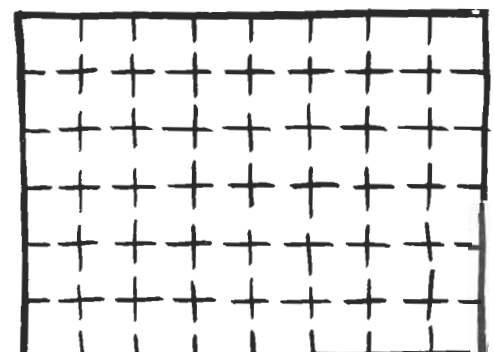
$$5 \times 8 = \underline{\quad}$$



$$6 \times 6 = \underline{\quad}$$

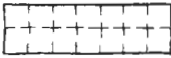
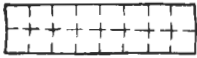
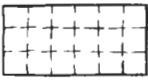
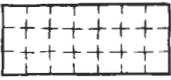
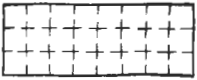
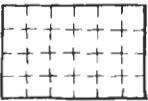
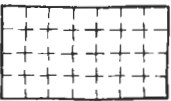
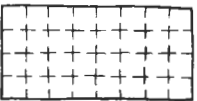
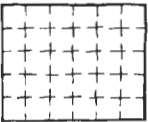
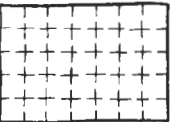
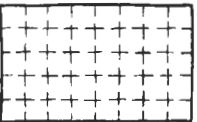
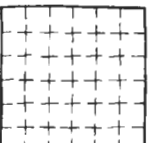
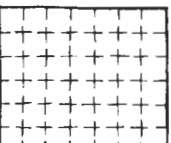
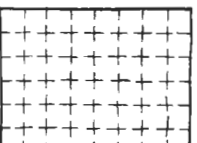


$$6 \times 7 = \underline{\quad}$$

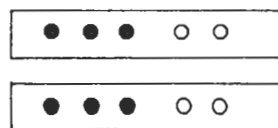


$$6 \times 8 = \underline{\quad}$$

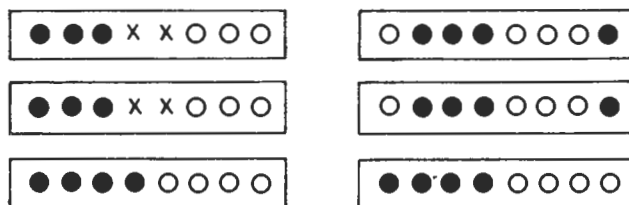
NAME \_\_\_\_\_

|                                                                                                                       |                                                                                                                       |                                                                                                                       |
|-----------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------|
| <br>$2 \times 6 = \underline{\quad}$ | <br>$2 \times 7 = \underline{\quad}$ | <br>$2 \times 8 = \underline{\quad}$ |
| <br>$3 \times 6 = \underline{\quad}$ | <br>$3 \times 7 = \underline{\quad}$ | <br>$3 \times 8 = \underline{\quad}$ |
| <br>$4 \times 6 = \underline{\quad}$ | <br>$4 \times 7 = \underline{\quad}$ | <br>$4 \times 8 = \underline{\quad}$ |
| <br>$5 \times 6 = \underline{\quad}$ | <br>$5 \times 7 = \underline{\quad}$ | <br>$5 \times 8 = \underline{\quad}$ |
| <br>$6 \times 6 = \underline{\quad}$ | <br>$6 \times 7 = \underline{\quad}$ | <br>$6 \times 8 = \underline{\quad}$ |

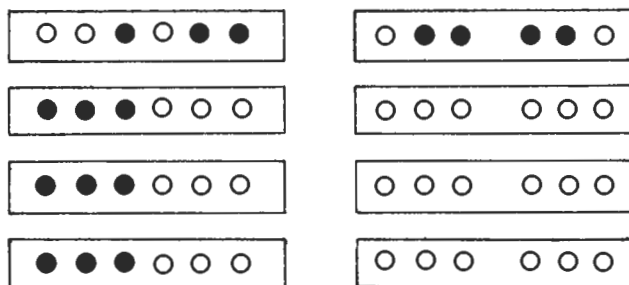
Or, a plan can include the idea of  $4 + 6 = 10$  as well as  $2 \times 5 = 10$ .



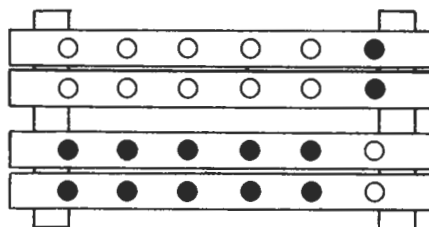
Another "shape" for 10 might be called the 9 + 1 arrangement. Here is an example in a 3 x 8 raft:



Similarly in a 4 x 6 raft:



Using a 2 x 5 scheme, a 4 by 6 raft might look like this:



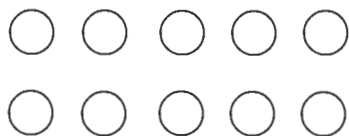
All planning and discussions of planning help build toward automatic recall of the basic multiplication and division facts represented.

Children usually depend on two basic schemes for planning beanstick rafts:

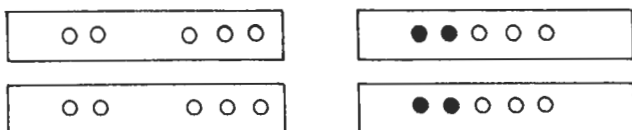
- (1) point out as many 10's as possible and
- (2) break up arrangements into easily recognizable parts that can be added.

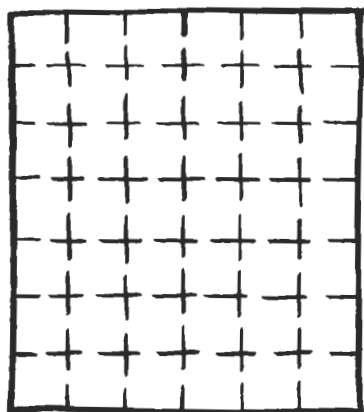
What arrangements of 10 are easily recognizable? . . . and a healthy discussion follows.

Of course, a 2 x 5 array is helpful:

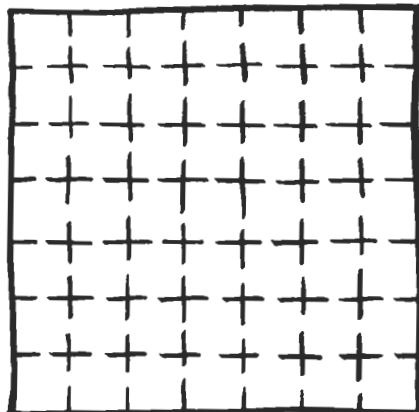


But since 5 is a bit many to see at a glance, variations in spacing or color can represent 5 as the sum of 3 and 2—both of which can be taken in at a glance.

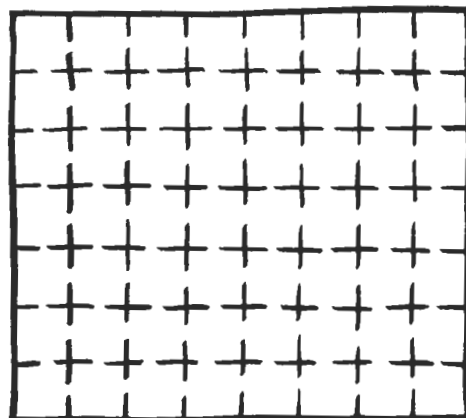




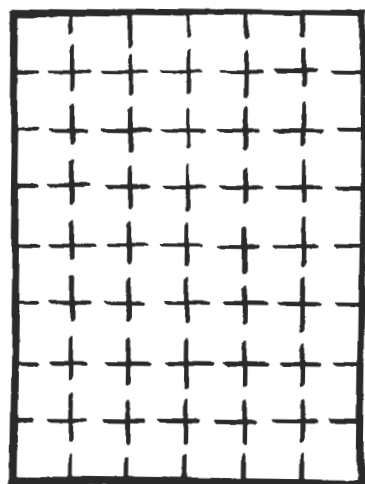
$$7 \times 6 = \underline{\quad}$$



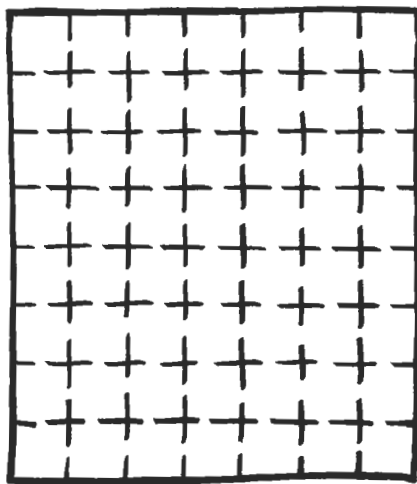
$$7 \times 7 = \underline{\quad}$$



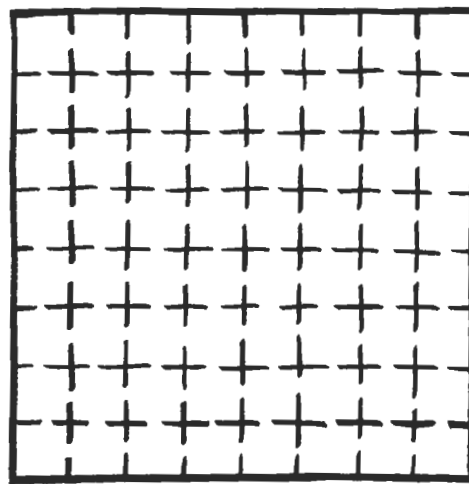
$$7 \times 8 = \underline{\quad}$$



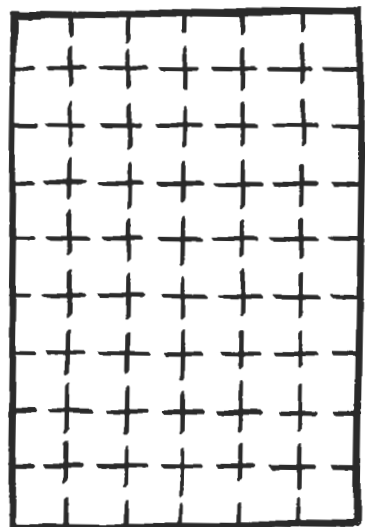
$$8 \times 6 = \underline{\quad}$$



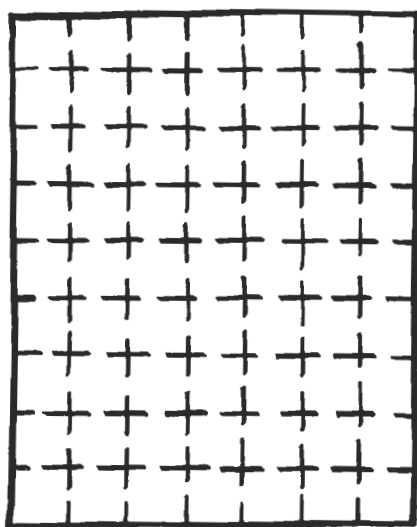
$$8 \times 7 = \underline{\quad}$$



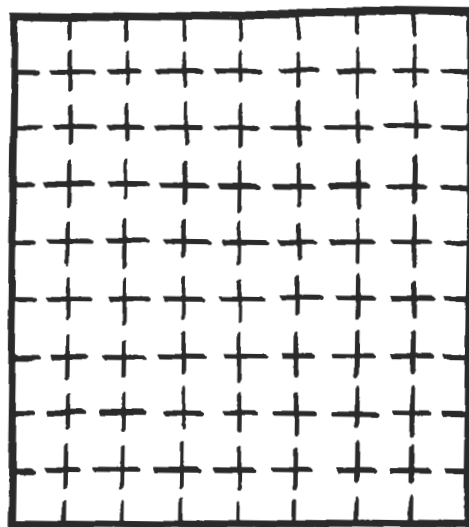
$$8 \times 8 = \underline{\quad}$$



$$9 \times 6 = \underline{\quad}$$

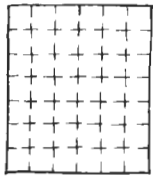
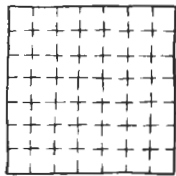
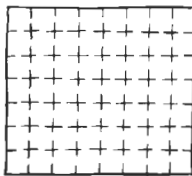
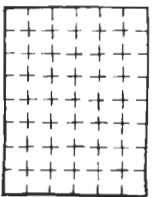
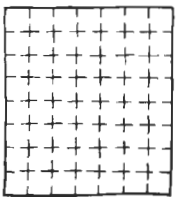
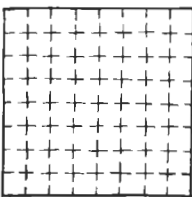
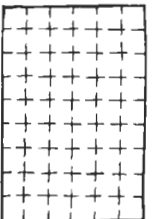
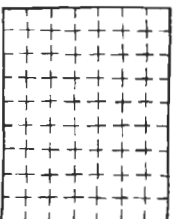
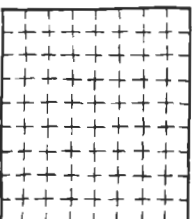


$$9 \times 7 = \underline{\quad}$$

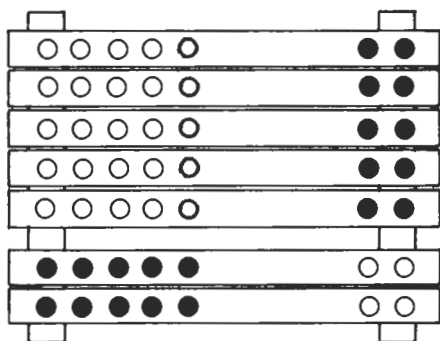


$$9 \times 8 = \underline{\quad}$$

NAME \_\_\_\_\_

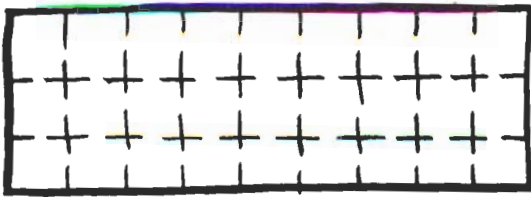
|                                                                                                                       |                                                                                                                       |                                                                                                                       |
|-----------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------|
| <br>$7 \times 6 = \underline{\quad}$ | <br>$7 \times 7 = \underline{\quad}$ | <br>$7 \times 8 = \underline{\quad}$ |
| <br>$8 \times 6 = \underline{\quad}$ | <br>$8 \times 7 = \underline{\quad}$ | <br>$8 \times 8 = \underline{\quad}$ |
| <br>$9 \times 6 = \underline{\quad}$ | <br>$9 \times 7 = \underline{\quad}$ | <br>$9 \times 8 = \underline{\quad}$ |

As the arrays become larger, plans may become a bit more complex. Here is a plan for a 7 by 7 raft:

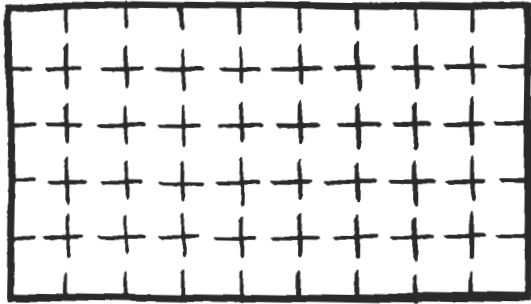


This, of course, depends on knowing that five 5's are 25, that two 5's are 10, and that two 2's are 4. Also, a bit of "mental addition" is called for:  $25 + 10 = 35$ ,  $35 + 10 = 45$  and  $45 + 4 = 49$ .

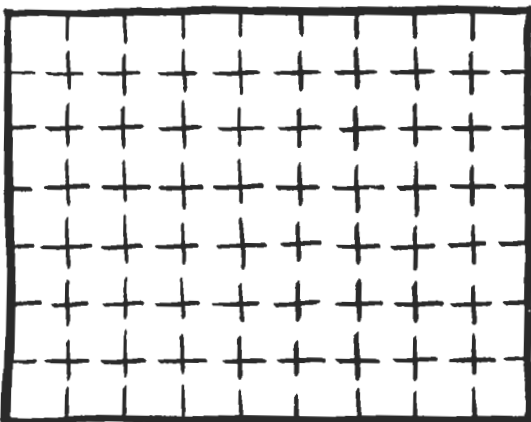
In addition to "using" (and thereby "practicing") the basic facts, the learner is also using a strategy that will pay off handsomely later—breaking up problems into manageable parts and then reassembling those parts.



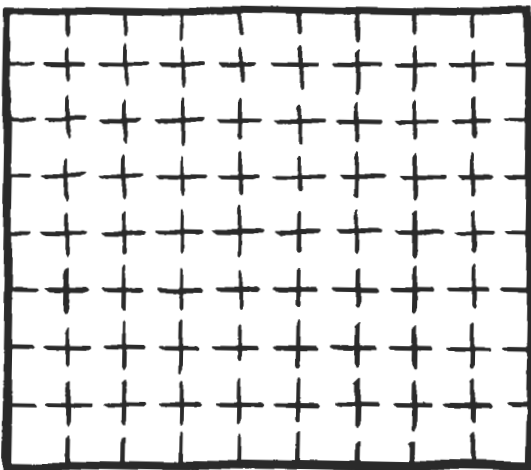
$$3 \times 9 = \underline{\quad}$$



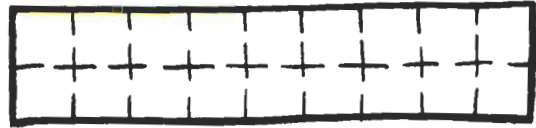
$$5 \times 9 = \underline{\quad}$$



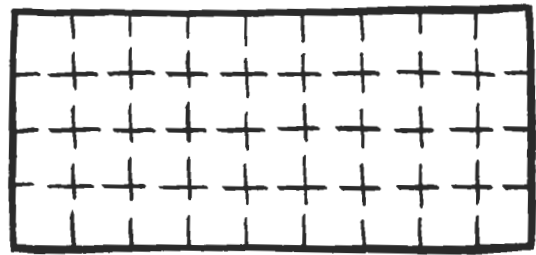
$$7 \times 9 = \underline{\quad}$$



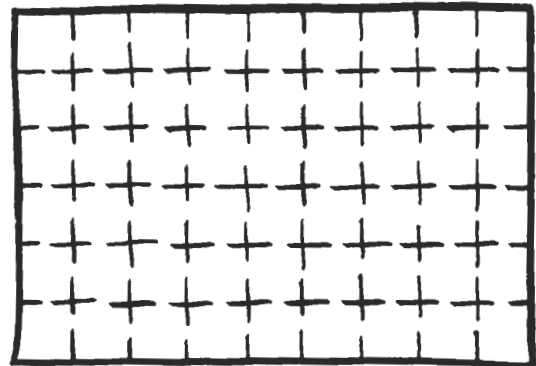
$$8 \times 9 = \underline{\quad}$$



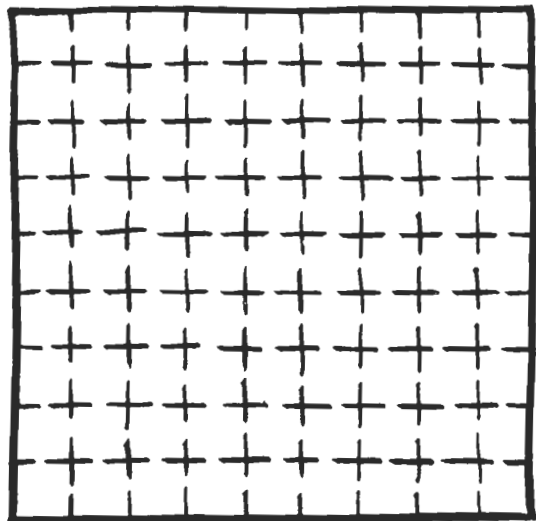
$$2 \times 9 = \underline{\quad}$$



$$4 \times 9 = \underline{\quad}$$

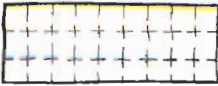
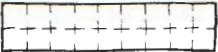
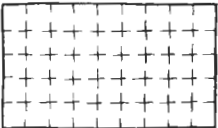
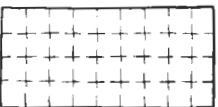
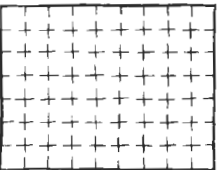
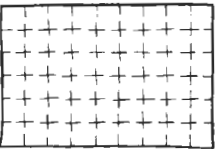
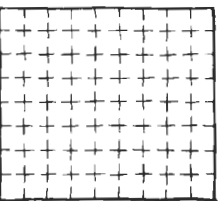
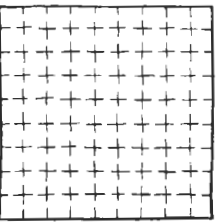


$$6 \times 9 = \underline{\quad}$$



$$9 \times 9 = \underline{\quad}$$

NAME \_\_\_\_\_

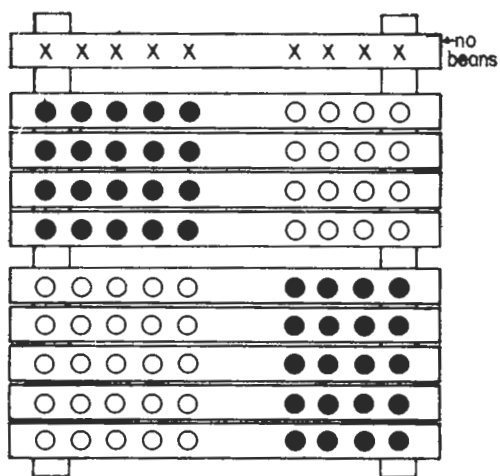
|                                                                                                                       |                                                                                                                       |
|-----------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------|
| <br>$3 \times 9 = \underline{\quad}$ | <br>$2 \times 9 = \underline{\quad}$ |
| <br>$5 \times 9 = \underline{\quad}$ | <br>$4 \times 9 = \underline{\quad}$ |
| <br>$7 \times 9 = \underline{\quad}$ | <br>$6 \times 9 = \underline{\quad}$ |
| <br>$8 \times 9 = \underline{\quad}$ | <br>$9 \times 9 = \underline{\quad}$ |

Or, disregarding the blank stick, there are four parts—5 by 5, 4 by 5, 4 by 5, and 4 by 4. Thus by adding:

$$\begin{array}{r} 25 \\ 20 \\ 20 \\ + 16 \\ \hline 81 \end{array}$$

This page helps plan rafts that have 9 beans to the log.

Here is a plan that uses old ideas and introduces a new one:



There are 2 groups of 5 logs—10 logs in all. But one stick has no beans—just marks. If there were beans on that stick it would be a 9 by 10 array—with 90 beans. Actually, there are 90 — 9 beans or 81.

# FENCING CERCANDO

10

(7's)       

  x          x  

└─┘      └─┘

10

(3's)       

  x          x  

└─┘      └─┘

10

(5's)       

  x          x  

└─┘      └─┘

10

(4's)       

  x          x  

└─┘      └─┘



FENCING NAME \_\_\_\_\_

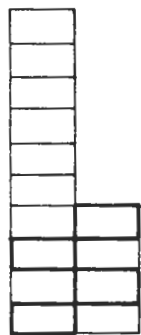
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|--|--|
|  |  |
|  |  |

The next eight pages present fencing activities. Sketches of blocks piled 10-high are used. We can agree on two rules for fencing:

- (1) A fence can only be drawn around blocks that share a common side (no diagonal fencing). For example, when fencing groups of four blocks:



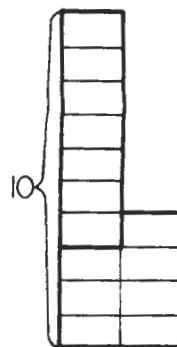
This



Not this

- (2) Once a block is fenced in, it cannot be fenced again.

The number in the circle tells the learner how many blocks are to be fenced. Space is provided for recording the results. In the first example, groups of seven blocks are to be fenced. This can be done as follows:



Two groups of seven were fenced. Fourteen blocks were fenced all together. This can be recorded in the recording device provided:

this many groups 2 (7's) 14 this many all together

In the other record-keeping forms the child expresses the result in alternate ways. First, he writes a multiplication fact which he "sees" in the result, and a division fact which he "sees" in the result. For the first example, a child might see:

$$\begin{array}{r} 7 \\ \times 2 \\ \hline 14 \end{array} \quad \text{and} \quad \begin{array}{r} 2 \\ 7 \overline{) 14} \\ \hline \end{array}$$

Once he knows these, he knows two other related facts which he should also write:

$$\begin{array}{r} 2 \\ \times 7 \\ \hline 14 \end{array} \quad \text{and} \quad \begin{array}{r} 7 \\ 2 \overline{) 14} \\ \hline \end{array}$$

Another child might "see" the result as:

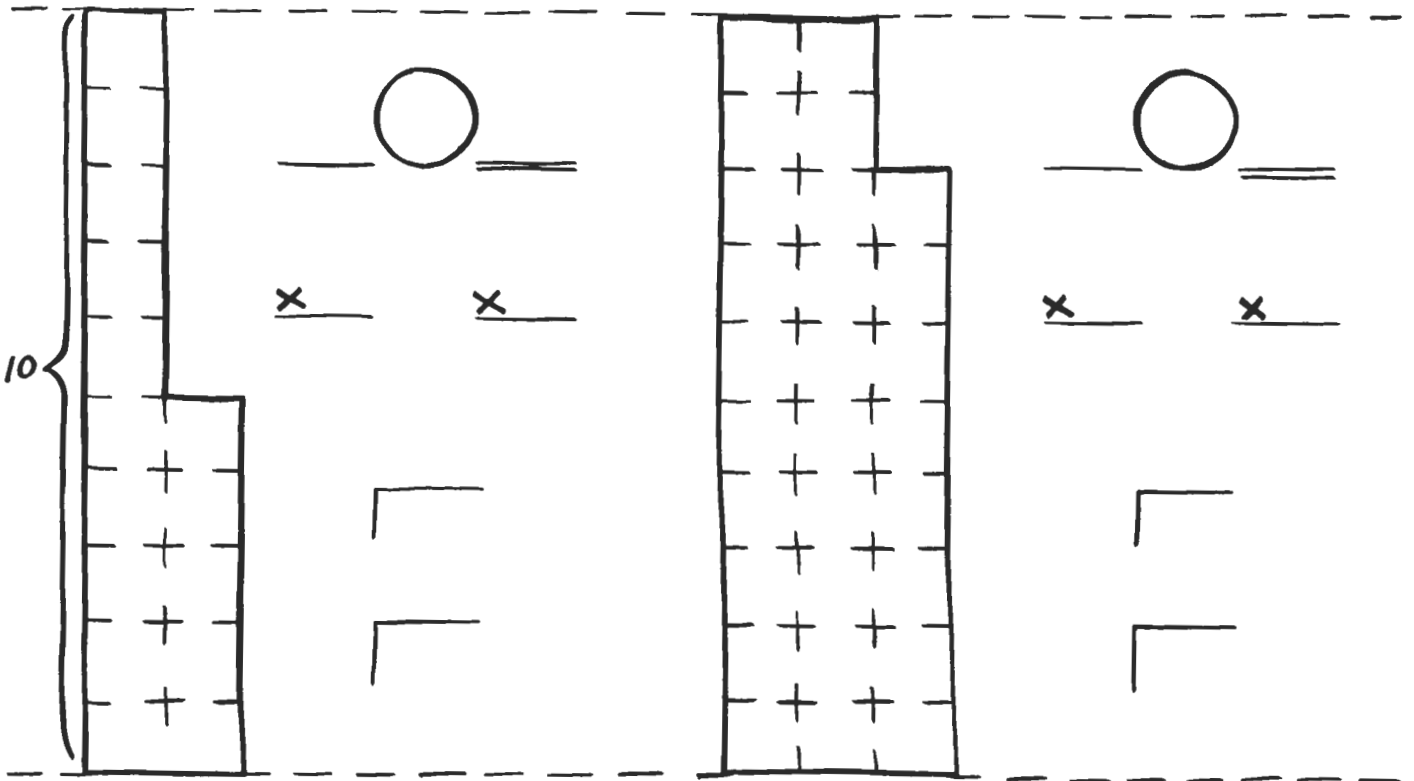
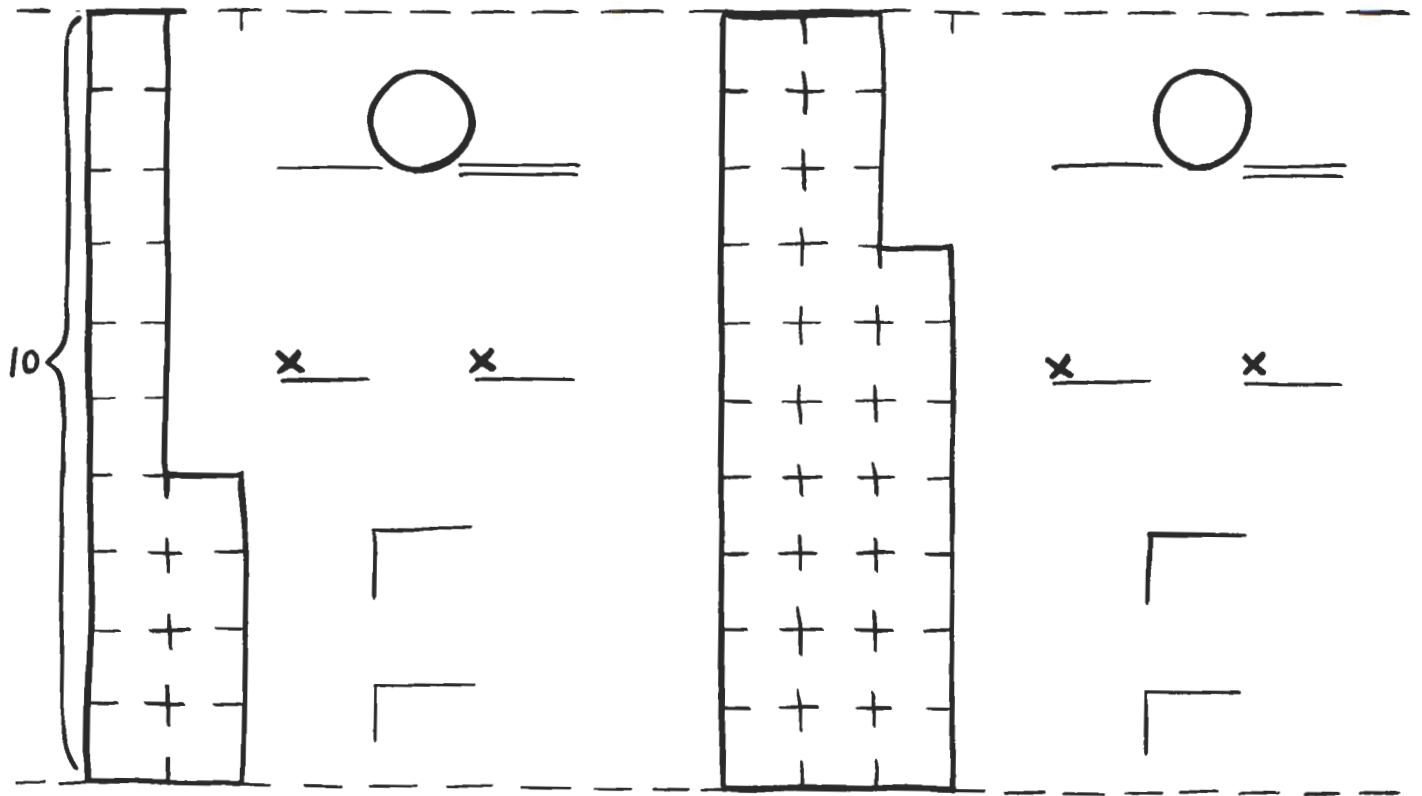
$$\begin{array}{r} 2 \\ \times 7 \\ \hline 14 \end{array} \quad \text{and} \quad \begin{array}{r} 7 \\ 2 \overline{) 14} \\ \hline \end{array}$$

and then write the related facts:

$$\begin{array}{r} 7 \\ \times 2 \\ \hline 14 \end{array} \quad \text{and} \quad \begin{array}{r} 2 \\ 7 \overline{) 14} \\ \hline \end{array}$$

In either case, he ends up with the entire "fact team."

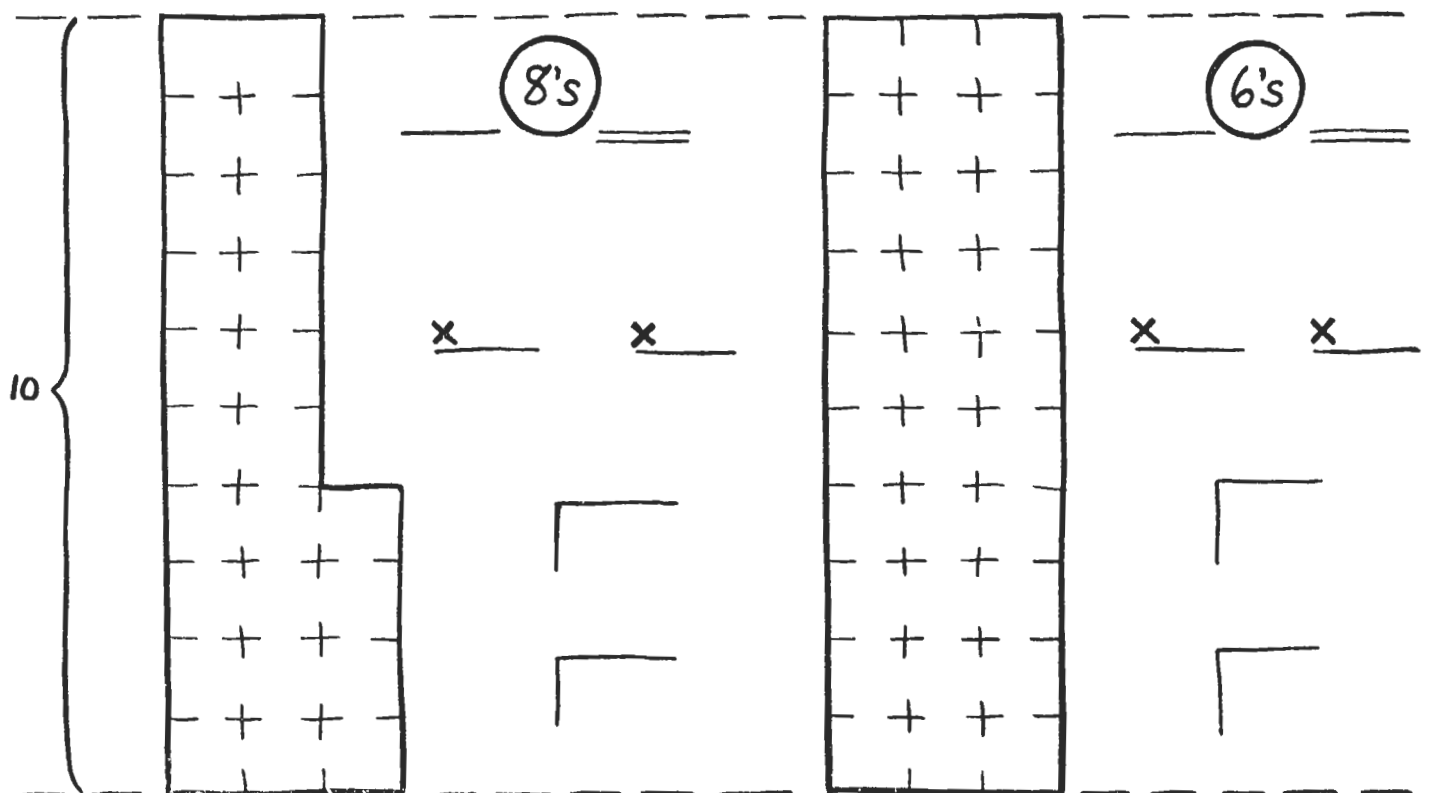
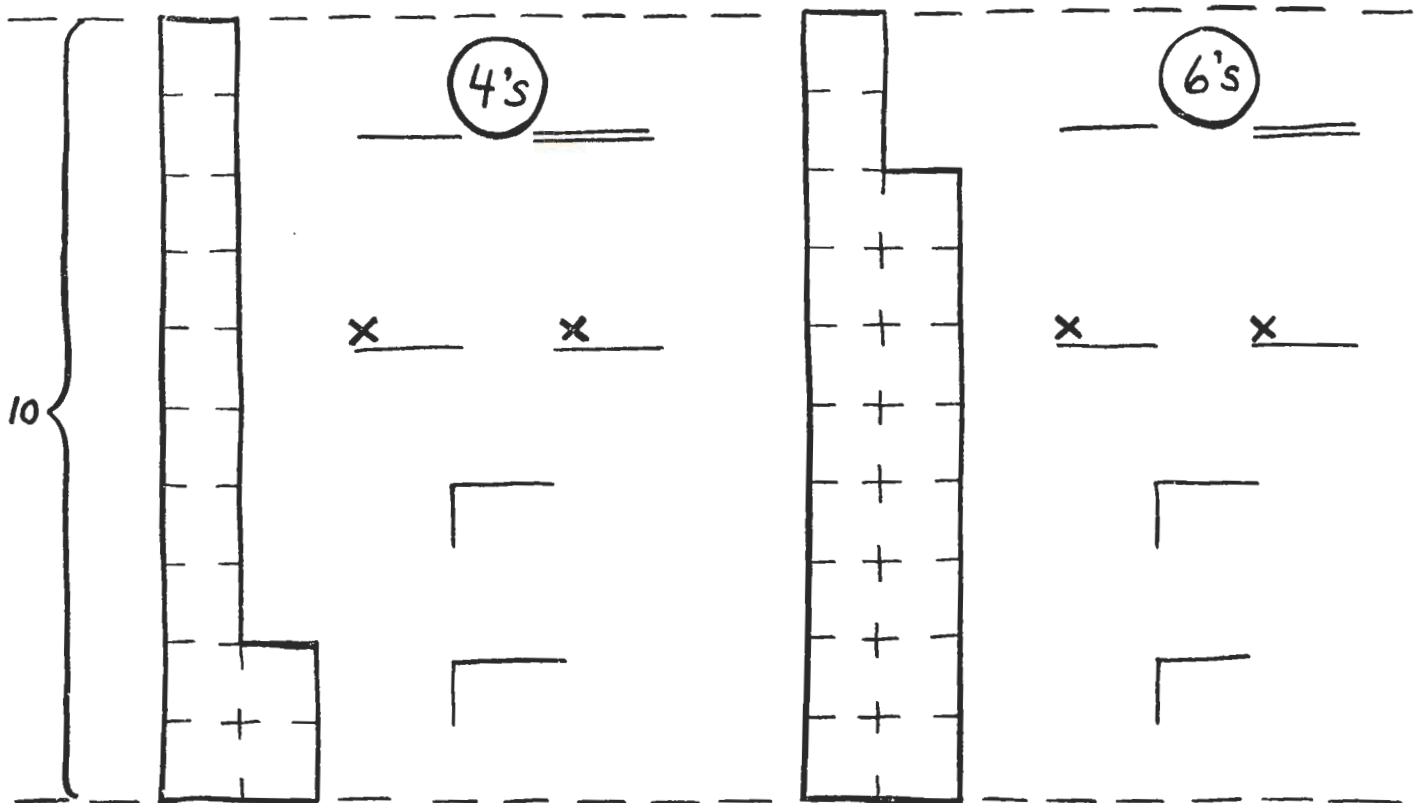
# FENCING CERCANDO



| FENCING |  | NAME |  |
|---------|--|------|--|
|         |  |      |  |
|         |  |      |  |

Learners can make up their own examples on this page, or teachers can particularize it. When experimenting on their own, learners might make up examples which do not work out evenly. In such cases, they will have to devise a notation for recording the results — including the remainder. However, you might want to simply make it a constraint of the problem that the fencing **must** come out exactly. Problems do involve constraints — and this is one place where you have the option to tighten those constraints.

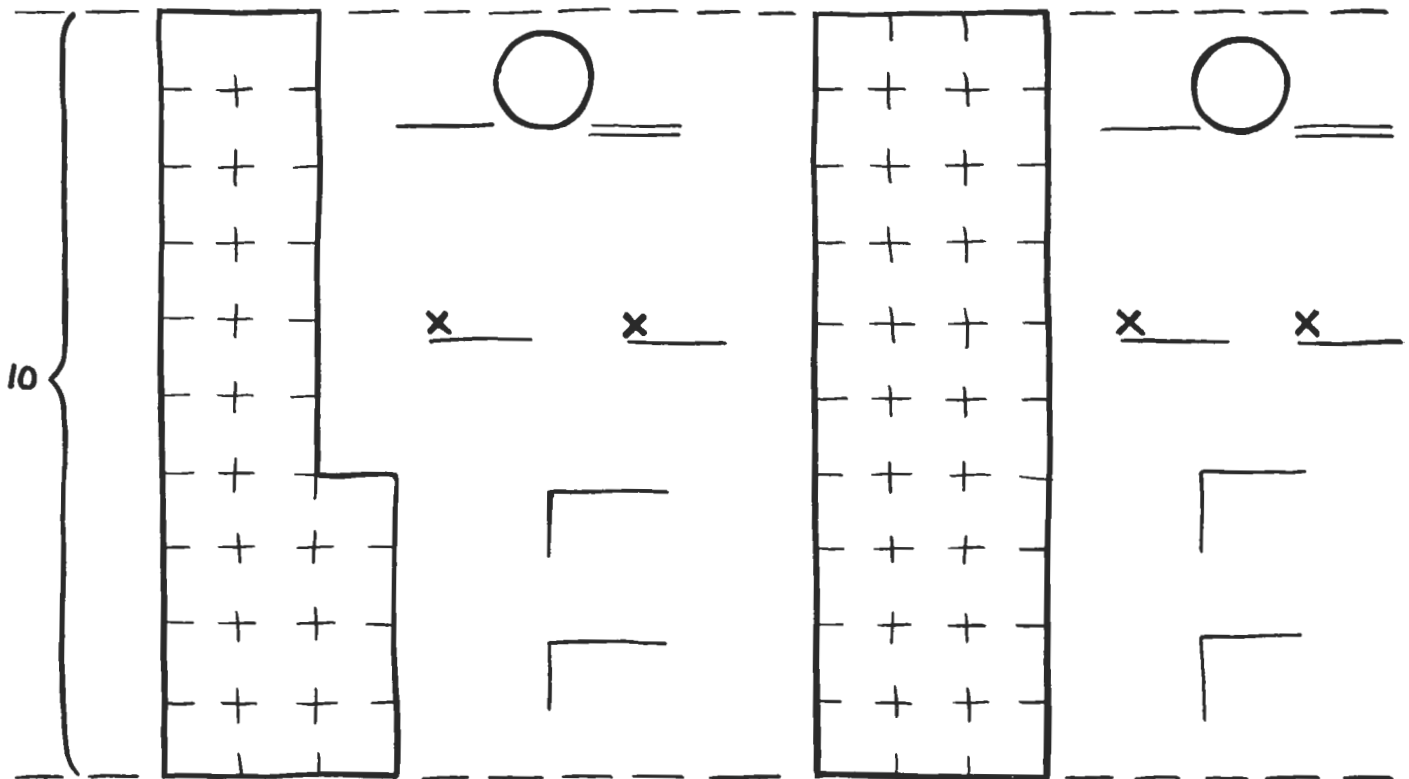
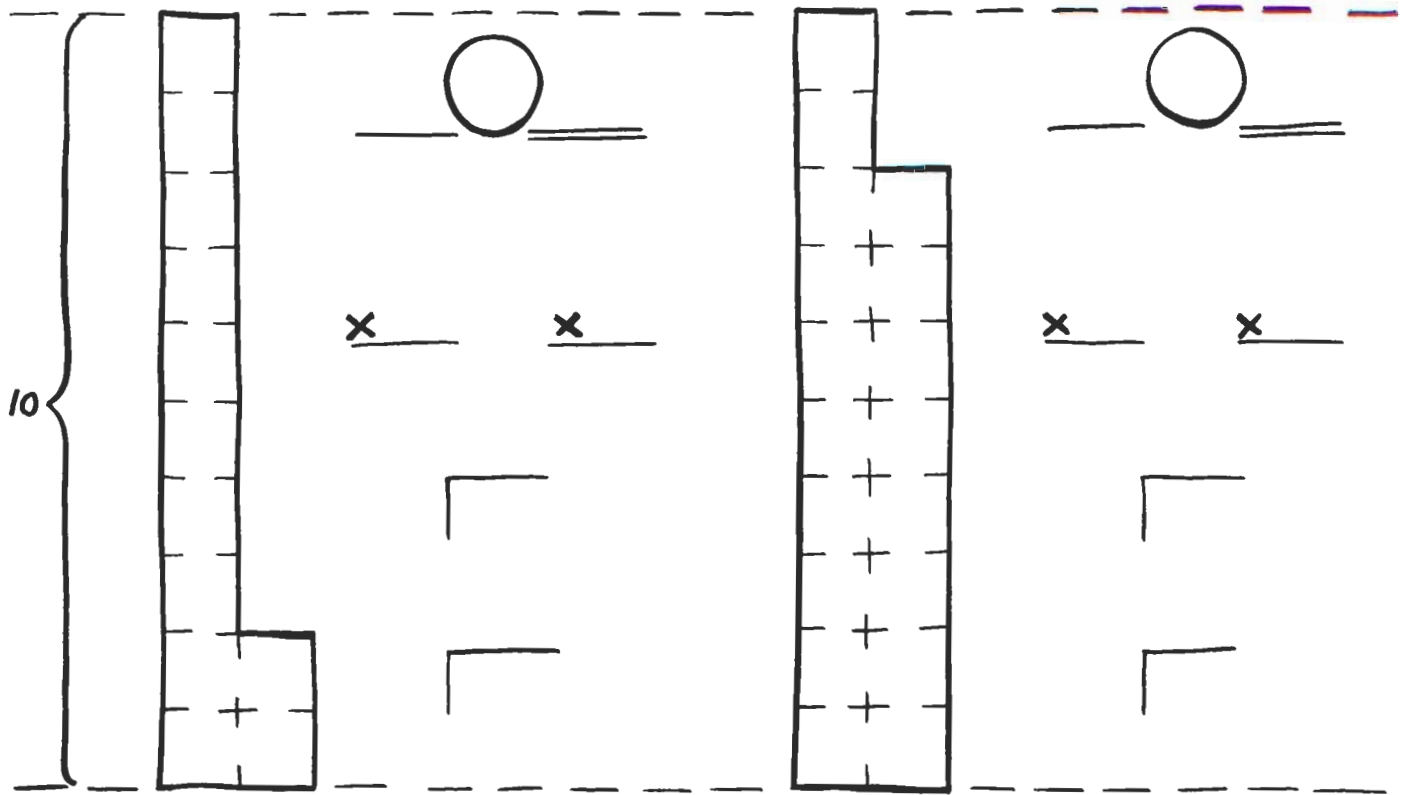
# FENCING CERCANDO

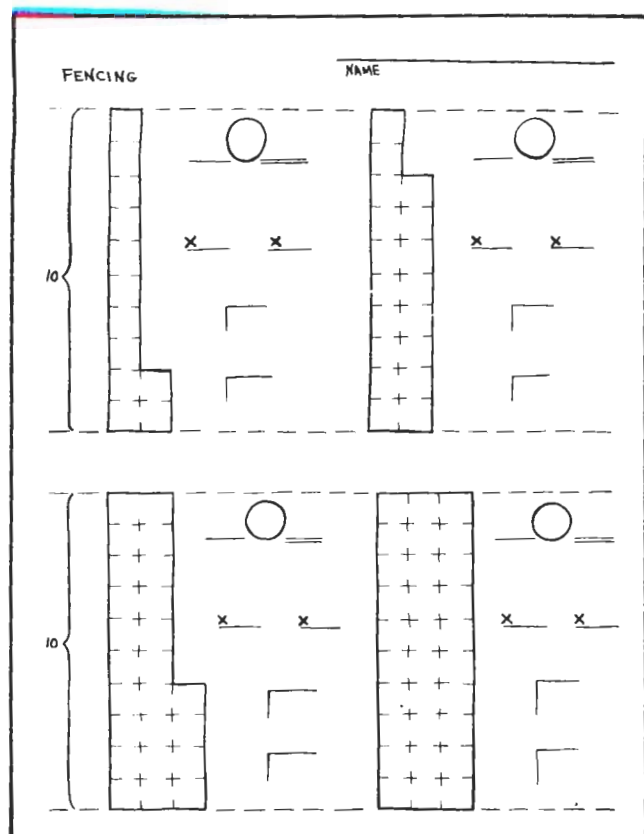


| FENCING                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | NAME _____                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
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| <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <div style="border-left: 1px solid black; height: 100px; position: relative;"> <div style="position: absolute; left: -20px; top: 0; bottom: 0; border-left: 1px solid black; border-right: 1px solid black; width: 10px;"></div> <div style="position: absolute; left: 0; top: 0; bottom: 0; width: 100%; height: 100%;"></div> </div> </div> <div style="text-align: center;"> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; line-height: 30px; margin: 0 auto;">4's</div> <div style="border-bottom: 1px solid black; width: 100px; margin: 5px auto;"></div> <div style="display: flex; justify-content: space-around; width: 100px;"> <div style="text-align: center;">x</div> <div style="text-align: center;">x</div> </div> <div style="margin-top: 20px;"> <div style="border-bottom: 1px solid black; width: 40px; margin: 0 auto;"></div> <div style="border-bottom: 1px solid black; width: 40px; margin: 0 auto;"></div> </div> </div> </div> | <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <div style="border-left: 1px solid black; height: 100px; position: relative;"> <div style="position: absolute; left: -20px; top: 0; bottom: 0; border-left: 1px solid black; border-right: 1px solid black; width: 10px;"></div> <div style="position: absolute; left: 0; top: 0; bottom: 0; width: 100%; height: 100%;"></div> </div> </div> <div style="text-align: center;"> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; line-height: 30px; margin: 0 auto;">6's</div> <div style="border-bottom: 1px solid black; width: 100px; margin: 5px auto;"></div> <div style="display: flex; justify-content: space-around; width: 100px;"> <div style="text-align: center;">x</div> <div style="text-align: center;">x</div> </div> <div style="margin-top: 20px;"> <div style="border-bottom: 1px solid black; width: 40px; margin: 0 auto;"></div> <div style="border-bottom: 1px solid black; width: 40px; margin: 0 auto;"></div> </div> </div> </div> |
| <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <div style="border-left: 1px solid black; height: 100px; position: relative;"> <div style="position: absolute; left: -20px; top: 0; bottom: 0; border-left: 1px solid black; border-right: 1px solid black; width: 10px;"></div> <div style="position: absolute; left: 0; top: 0; bottom: 0; width: 100%; height: 100%;"></div> </div> </div> <div style="text-align: center;"> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; line-height: 30px; margin: 0 auto;">8's</div> <div style="border-bottom: 1px solid black; width: 100px; margin: 5px auto;"></div> <div style="display: flex; justify-content: space-around; width: 100px;"> <div style="text-align: center;">x</div> <div style="text-align: center;">x</div> </div> <div style="margin-top: 20px;"> <div style="border-bottom: 1px solid black; width: 40px; margin: 0 auto;"></div> <div style="border-bottom: 1px solid black; width: 40px; margin: 0 auto;"></div> </div> </div> </div> | <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <div style="border-left: 1px solid black; height: 100px; position: relative;"> <div style="position: absolute; left: -20px; top: 0; bottom: 0; border-left: 1px solid black; border-right: 1px solid black; width: 10px;"></div> <div style="position: absolute; left: 0; top: 0; bottom: 0; width: 100%; height: 100%;"></div> </div> </div> <div style="text-align: center;"> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; line-height: 30px; margin: 0 auto;">6's</div> <div style="border-bottom: 1px solid black; width: 100px; margin: 5px auto;"></div> <div style="display: flex; justify-content: space-around; width: 100px;"> <div style="text-align: center;">x</div> <div style="text-align: center;">x</div> </div> <div style="margin-top: 20px;"> <div style="border-bottom: 1px solid black; width: 40px; margin: 0 auto;"></div> <div style="border-bottom: 1px solid black; width: 40px; margin: 0 auto;"></div> </div> </div> </div> |

This is a continuation of the activity on the previous pages. Different piles of blocks and different numbers of blocks to fence are used.

# FENCING CERCANDO





This is a continuation of the activity on the previous pages. Different stacks of blocks are used. You might want to particularize the form by drawing in the fences and then having the learners complete the entire report.

A question to explore:

How many **different** reports can be done for each example on this page?

For example, a stock of 24 blocks could lead to the reports:

$$\begin{array}{r} \underline{8} \quad \textcircled{3's} \quad \underline{\underline{24}} \\ \underline{12} \quad \textcircled{2's} \quad \underline{\underline{24}} \\ \underline{6} \quad \textcircled{4's} \quad \underline{\underline{24}} \end{array}$$

Are there others? How many are there?

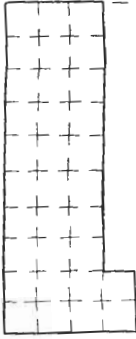
$\underline{8} \textcircled{3's} \underline{\underline{24}}$  and  $\underline{3} \textcircled{8's} \underline{\underline{24}}$  are not considered different reports because they lead to the same four facts.)

# FENCING CERCANDO

|  |                                                         |  |                                                         |
|--|---------------------------------------------------------|--|---------------------------------------------------------|
|  | <p>(4's)</p> <p>—</p> <p>x      x</p> <p>—</p> <p>—</p> |  | <p>(8's)</p> <p>—</p> <p>x      x</p> <p>—</p> <p>—</p> |
|--|---------------------------------------------------------|--|---------------------------------------------------------|

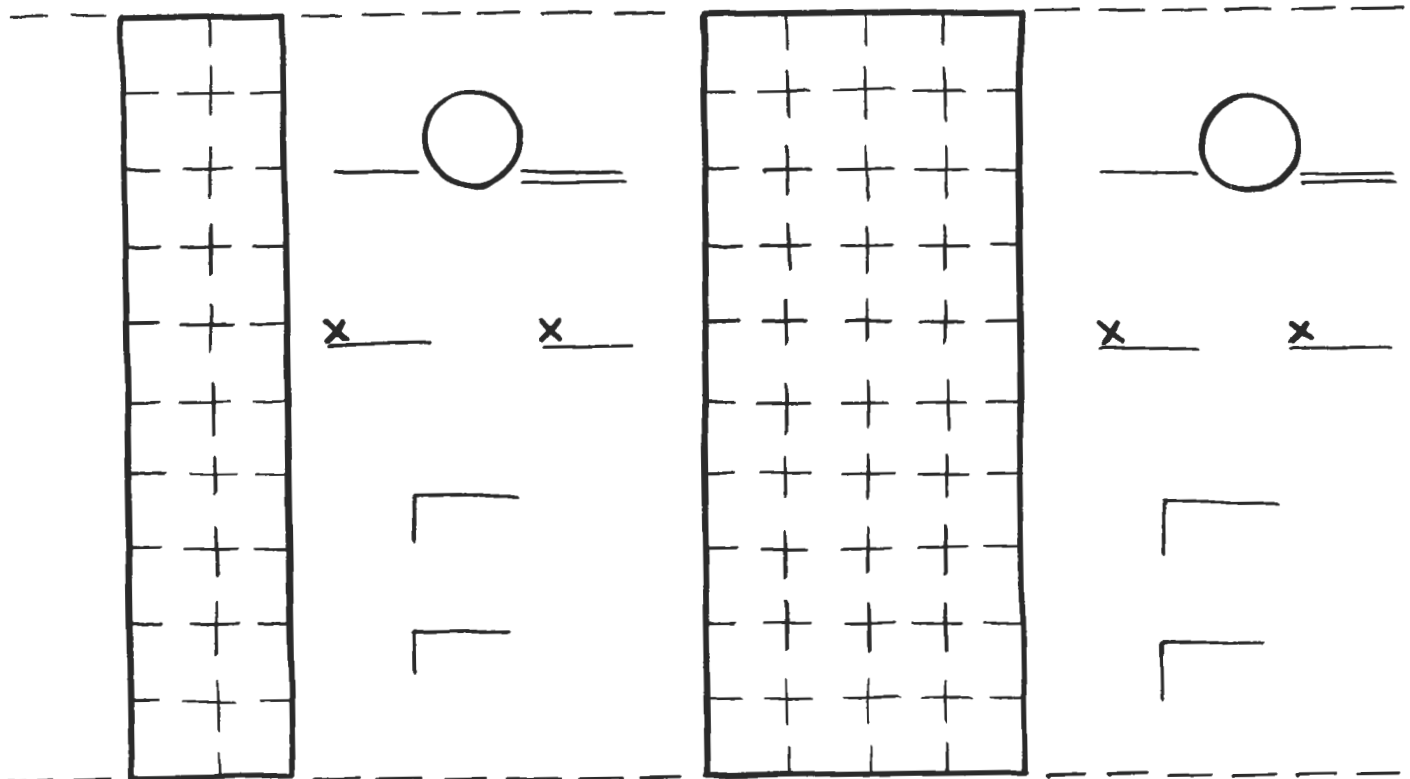
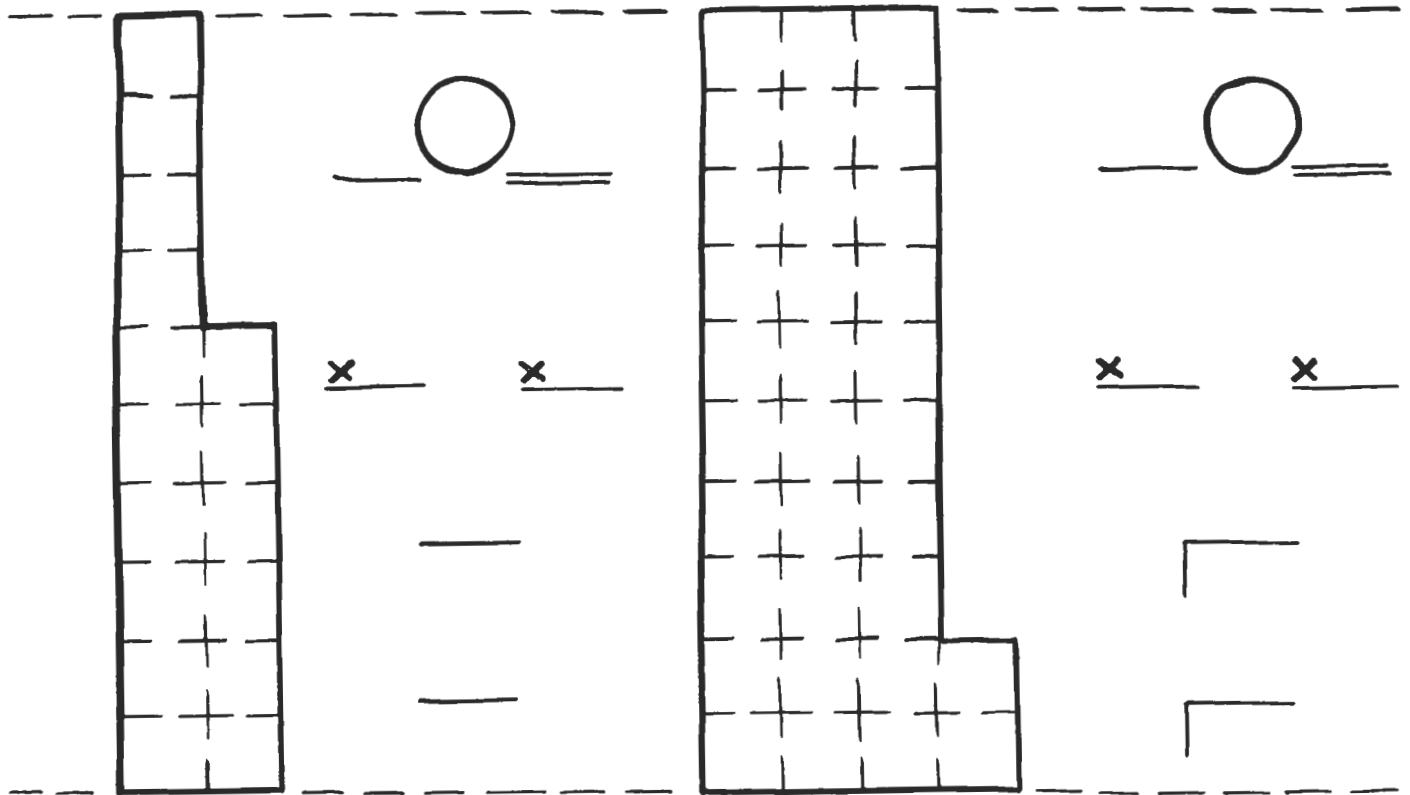
|  |                                                         |  |                                                         |
|--|---------------------------------------------------------|--|---------------------------------------------------------|
|  | <p>(5's)</p> <p>—</p> <p>x      x</p> <p>—</p> <p>—</p> |  | <p>(8's)</p> <p>—</p> <p>x      x</p> <p>—</p> <p>—</p> |
|--|---------------------------------------------------------|--|---------------------------------------------------------|

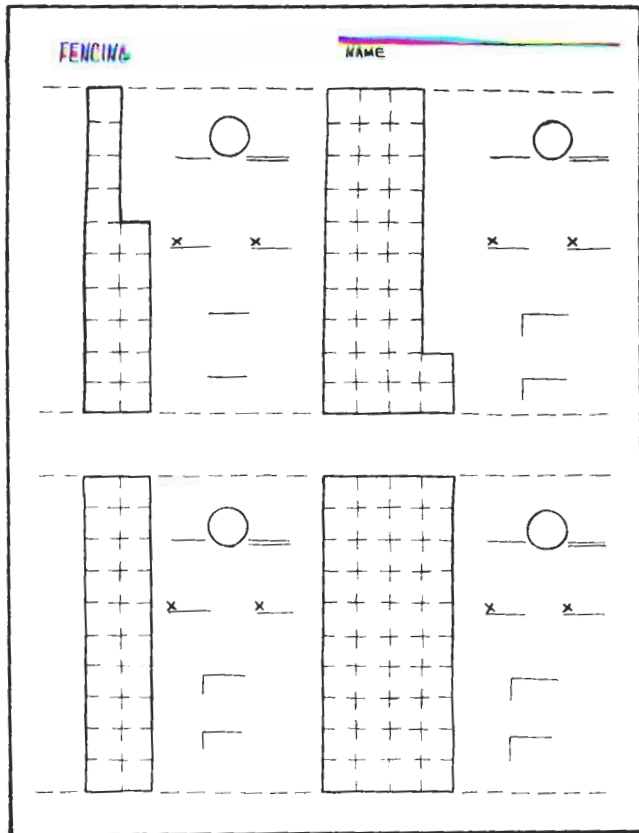


| FENCING                                                                           |                                                | NAME                                                                              |                                                |
|-----------------------------------------------------------------------------------|------------------------------------------------|-----------------------------------------------------------------------------------|------------------------------------------------|
|  | <p>(4's)</p> <p>x      x</p> <p>—</p> <p>—</p> |  | <p>(8's)</p> <p>x      x</p> <p>—</p> <p>—</p> |
|  | <p>(5's)</p> <p>x      x</p> <p>—</p> <p>—</p> |  | <p>(8's)</p> <p>x      x</p> <p>—</p> <p>—</p> |

This is a continuation of the activities on the previous pages.

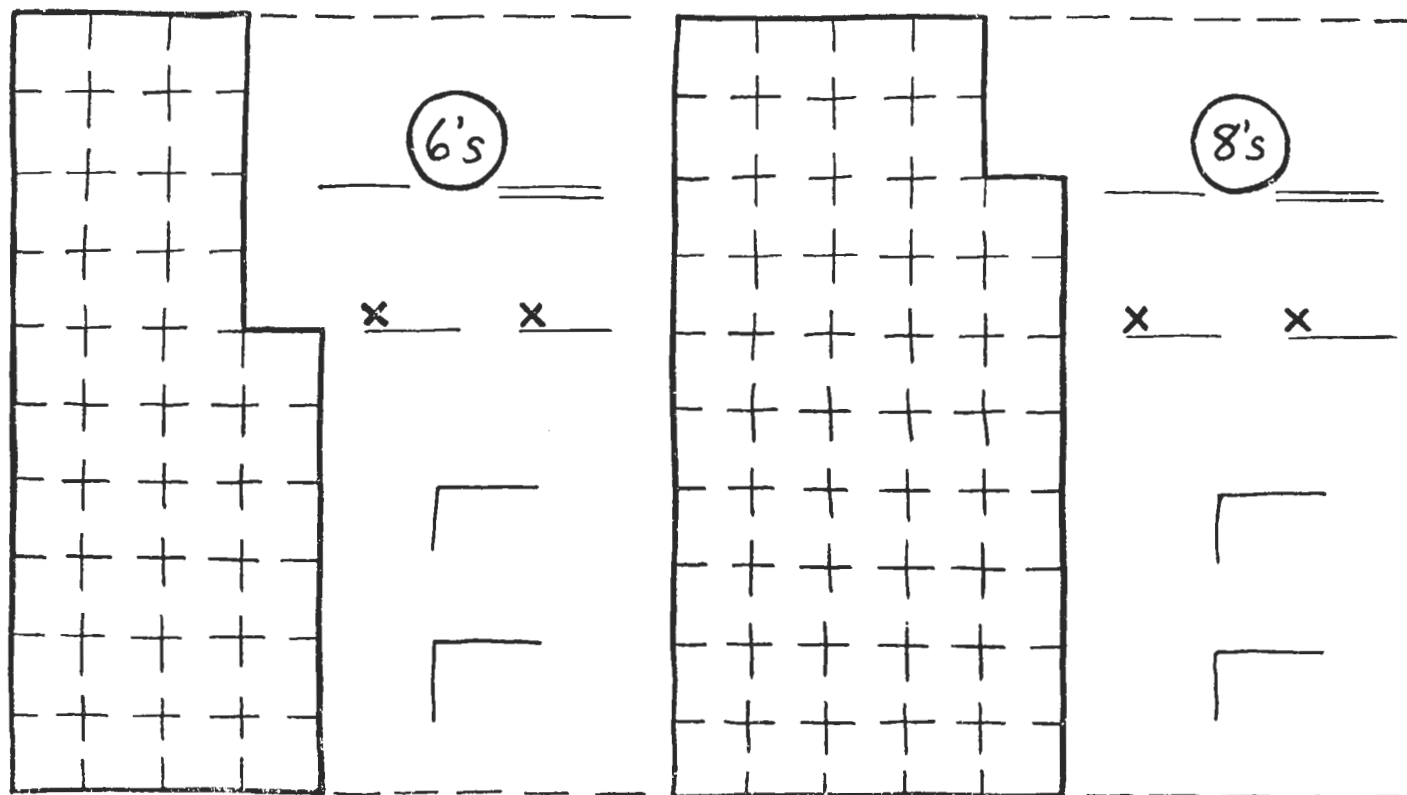
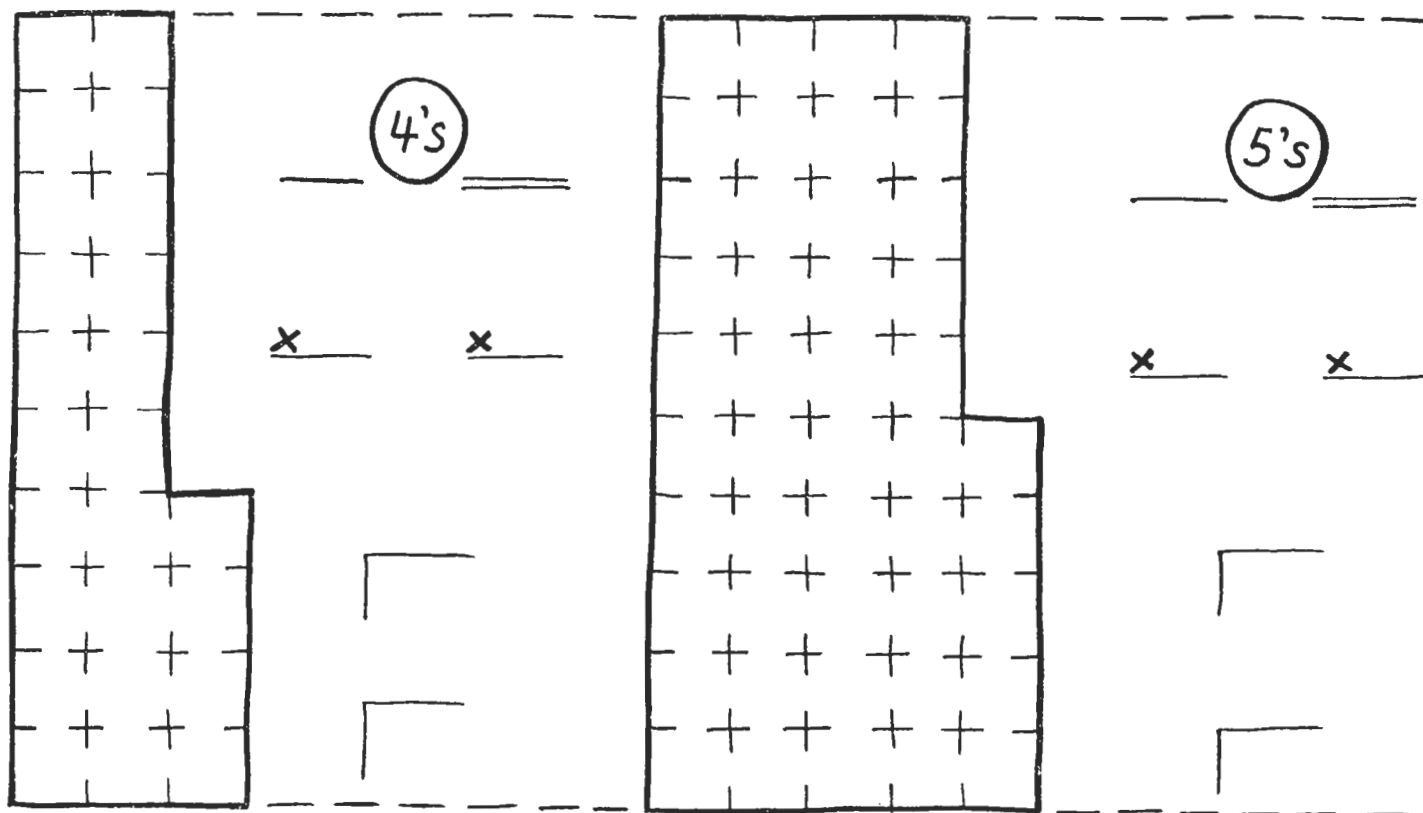
# FENCING CERCANDO

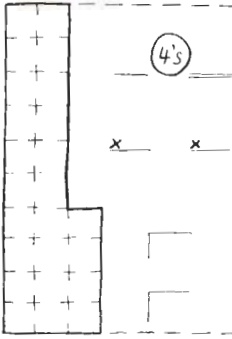
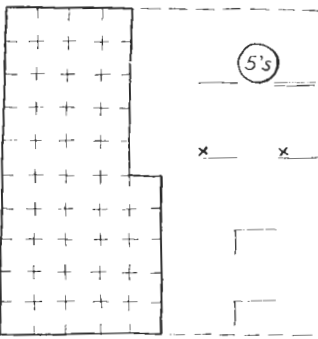
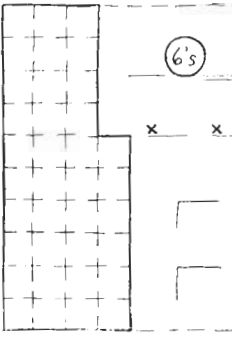
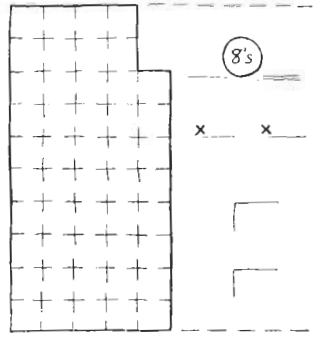




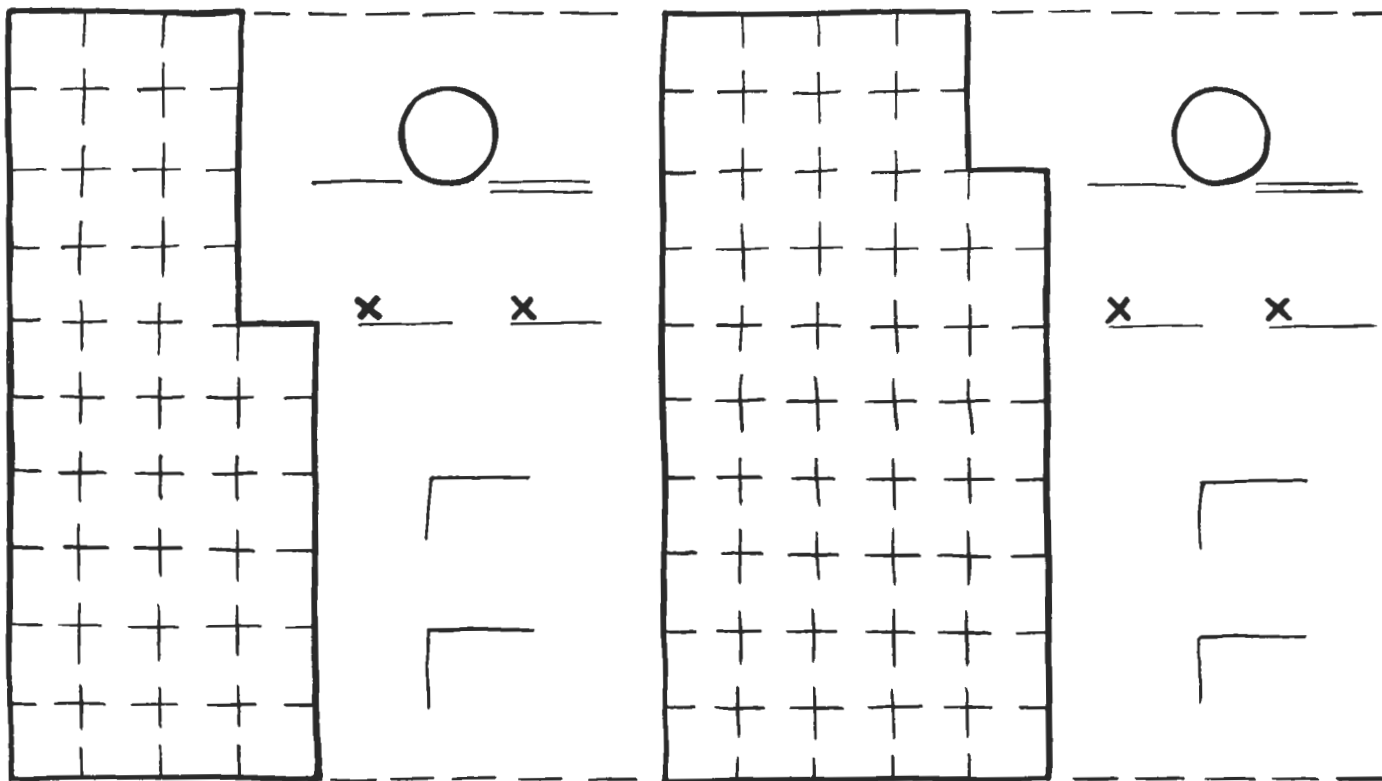
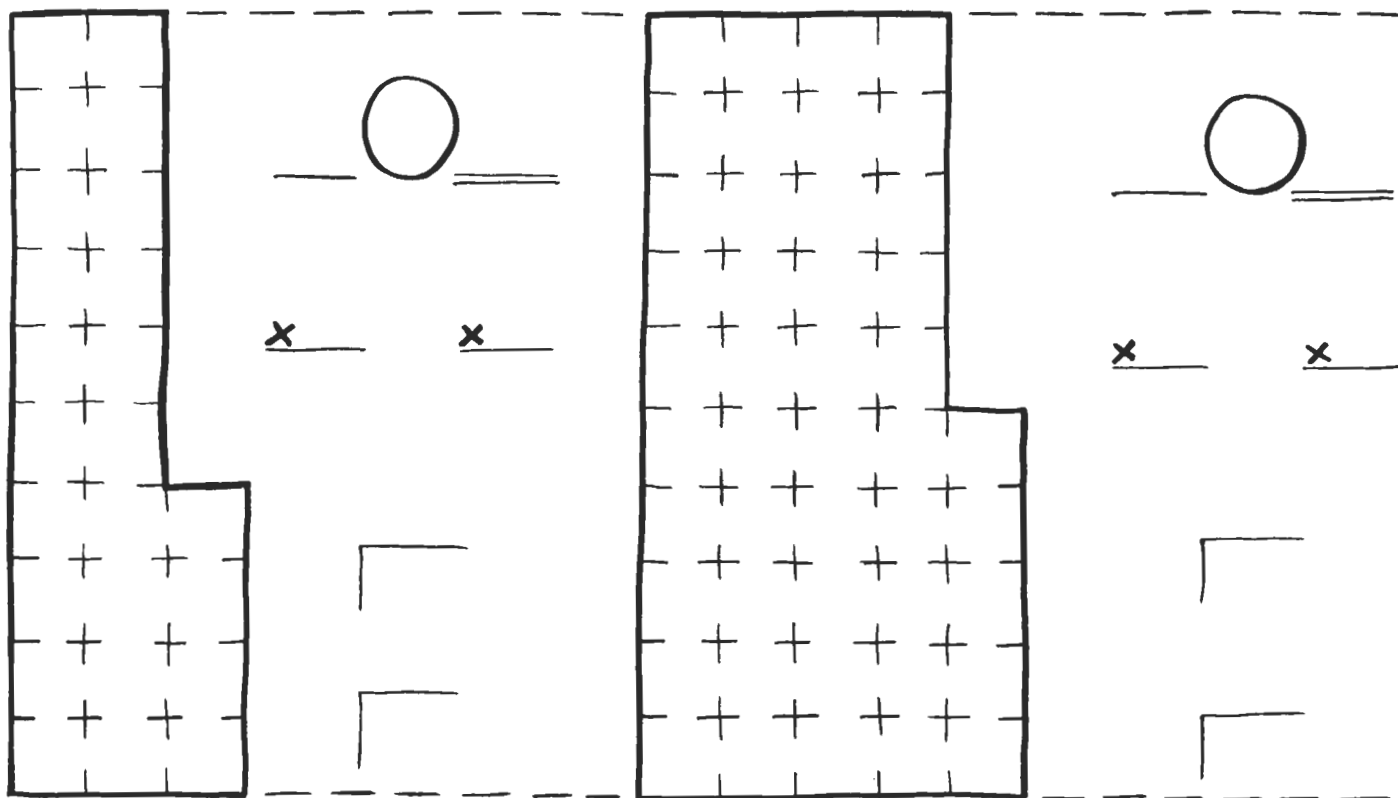
Again, a continuation of previous activities. Larger arrays of blocks are used. See previous pages for suggestions for use.

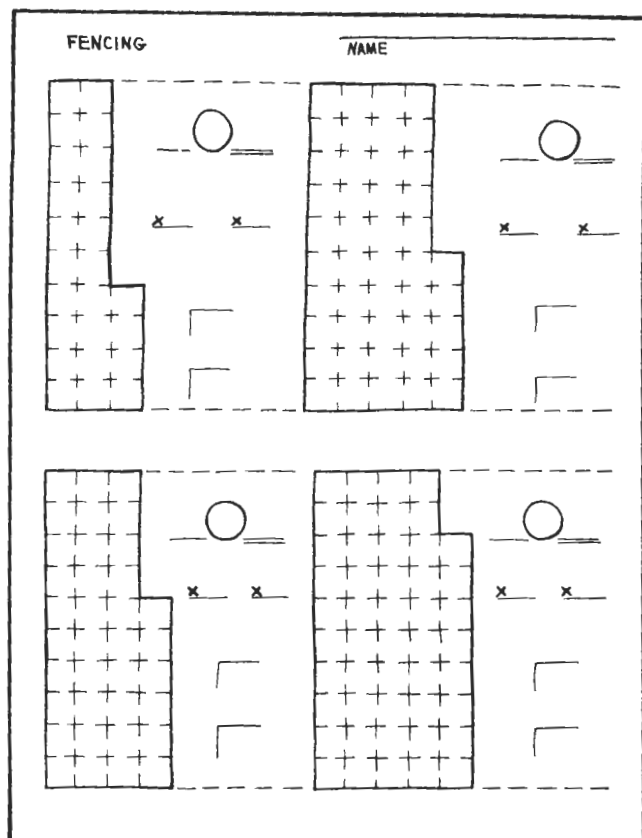
# FENCING CERCANDO



| FENCING                                                                           | NAME _____                                                                        |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
|  |  |

This is a continuation of previous activities using larger arrays of blocks.





This is a continuation of previous activities. Again, learners might try to find how many different ways each array can be fenced. For example, 36 blocks has these possibilities:

|           |        |           |
|-----------|--------|-----------|
| <u>1</u>  | (36's) | <u>36</u> |
| <u>6</u>  | (6's)  | <u>36</u> |
| <u>12</u> | (3's)  | <u>36</u> |
| <u>9</u>  | (4's)  | <u>36</u> |
| <u>18</u> | (2's)  | <u>36</u> |

¿QUÉ PUEDES VER?  
WHAT CAN YOU SEE?

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| $\begin{array}{r} 13 \\ \times 2 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 3 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 4 \\ \hline \end{array}$ |
|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|

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|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|
| $\begin{array}{r} 15 \\ \times 2 \\ \hline \end{array}$ | $\begin{array}{r} 15 \\ \times 3 \\ \hline \end{array}$ | $\begin{array}{r} 15 \\ \times 4 \\ \hline \end{array}$ |
|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|

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$$\begin{array}{r} 23 \\ \times 2 \\ \hline \end{array}$$

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$$\begin{array}{r} 23 \\ \times 3 \\ \hline \end{array}$$

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$$\begin{array}{r} 23 \\ \times 4 \\ \hline \end{array}$$



WHAT CAN YOU SEE ?

NAME \_\_\_\_\_

13

x2

13

x3

13

x4

15

x2

15

x3

15

x4

23

x2

23

x3

23

x4

### What Can You See? (multiplication)

On this task page, learners have a chance to visualize large multiplications. In the diagrams, a  can be thought of as either a Dienes block rod or a 10-stick, and a  stands for a small cube or a bean. In any case  represents 10 units and  represents one unit.

The learner can look at the arrays and count the total beans (or cubes) by making use of the groupings.

For the first example, 
$$\begin{array}{r} 13 \\ \times 2 \\ \hline \end{array}$$

two rows are shown with 13 in each row.

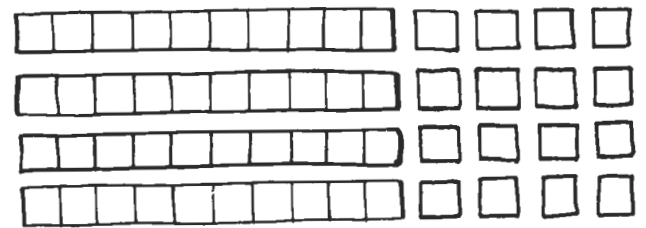
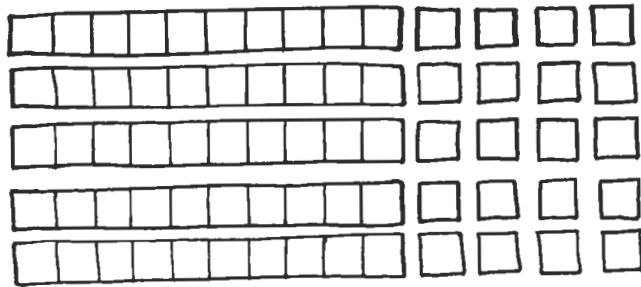
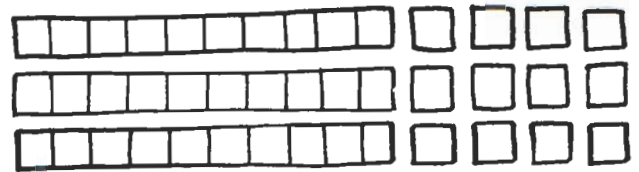
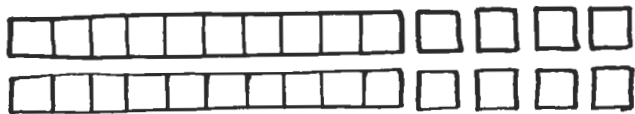
There are 2 rods, or 20 cubes, and six loose cubes for a total of 26 cubes. Thus, the result can be recorded:

$$\begin{array}{r} 13 \\ \times 2 \\ \hline 26 \end{array}$$

Sketches are provided for each example. Children should be encouraged to count loose blocks by considering the groupings rather than by counting by ones. If children have difficulty, beansticks and loose beans or rods and cubes should be brought out.

WHAT CAN YOU SEE?

¿QUÉ PUEDES VER?



$$\begin{array}{r} 14 \\ \times 2 \\ \hline \end{array}$$

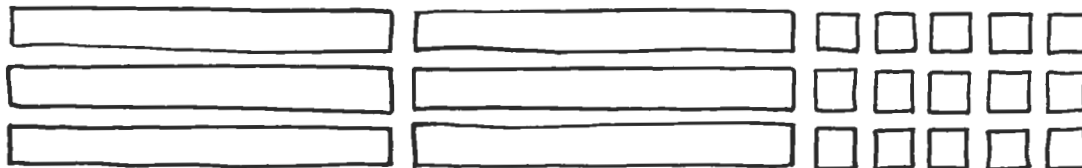
$$\begin{array}{r} 14 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 14 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 14 \\ \times 4 \\ \hline \end{array}$$



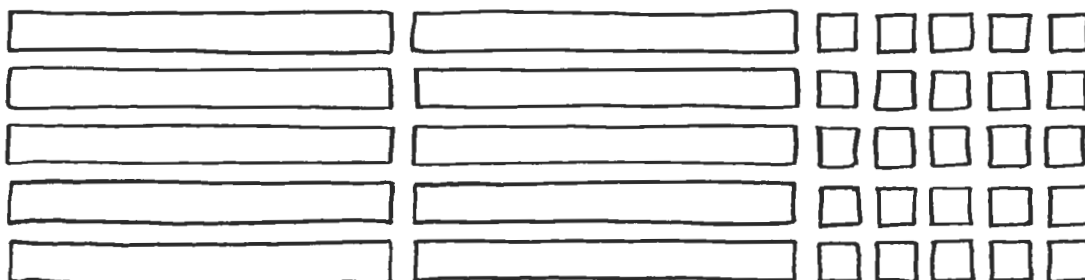
$$\begin{array}{r} 25 \\ \times 2 \\ \hline \end{array}$$



$$\begin{array}{r} 25 \\ \times 3 \\ \hline \end{array}$$



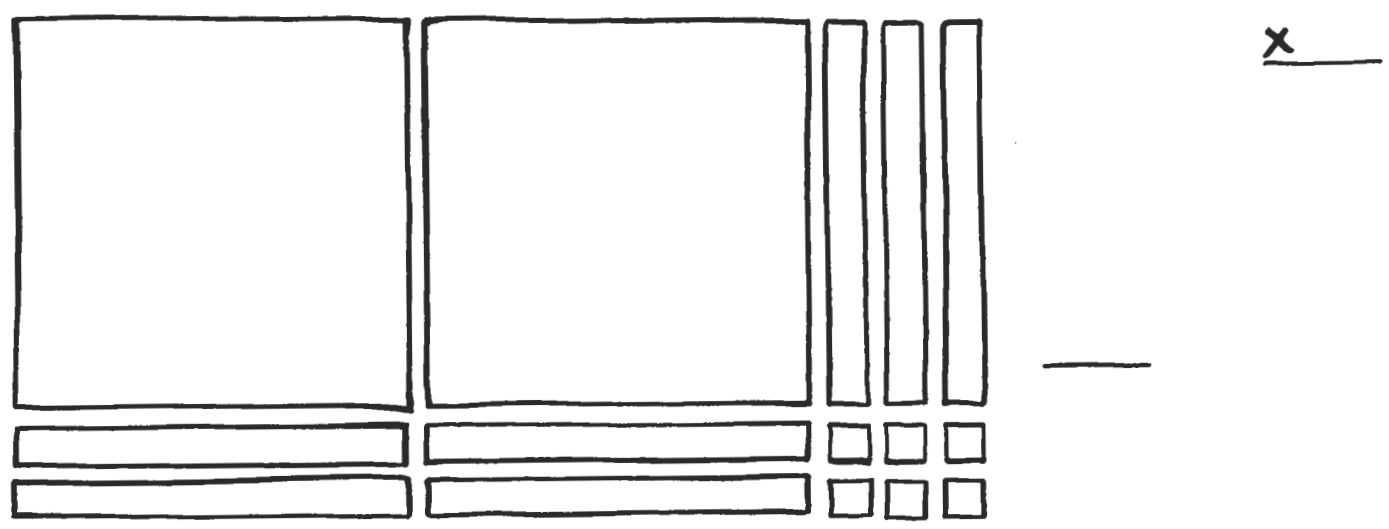
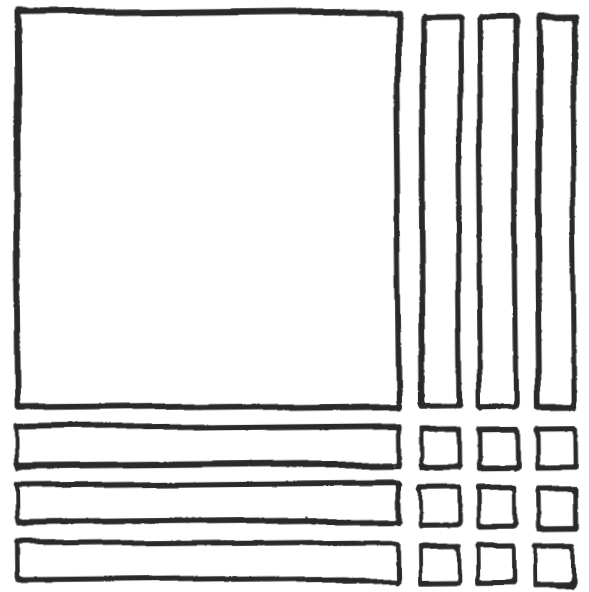
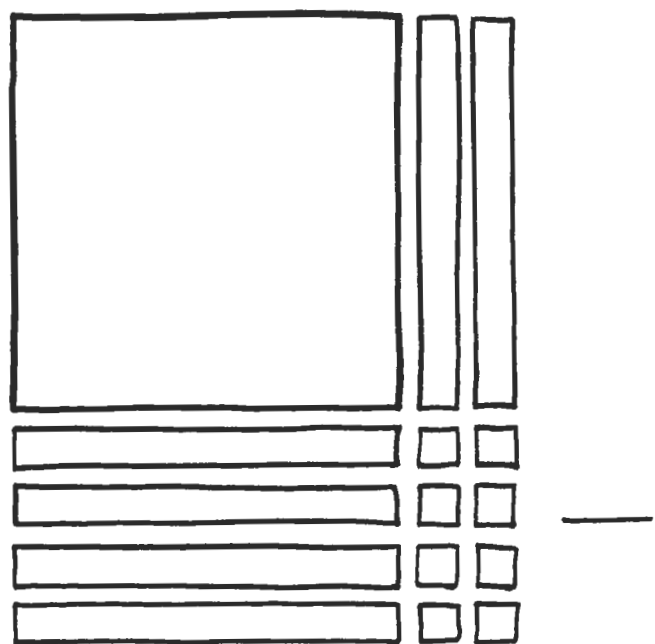
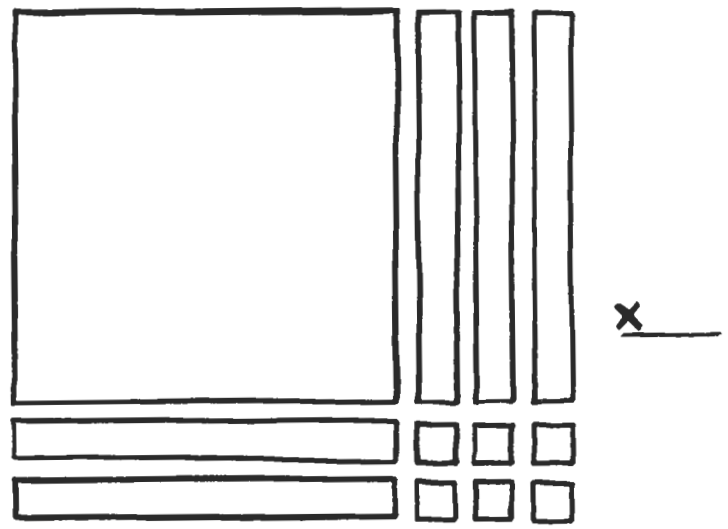
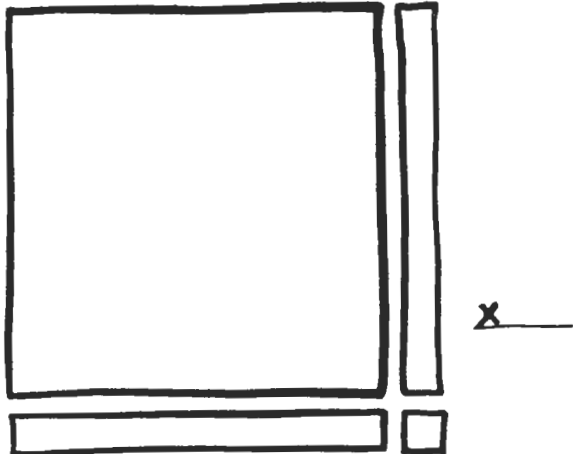
$$\begin{array}{r} 25 \\ \times 4 \\ \hline \end{array}$$

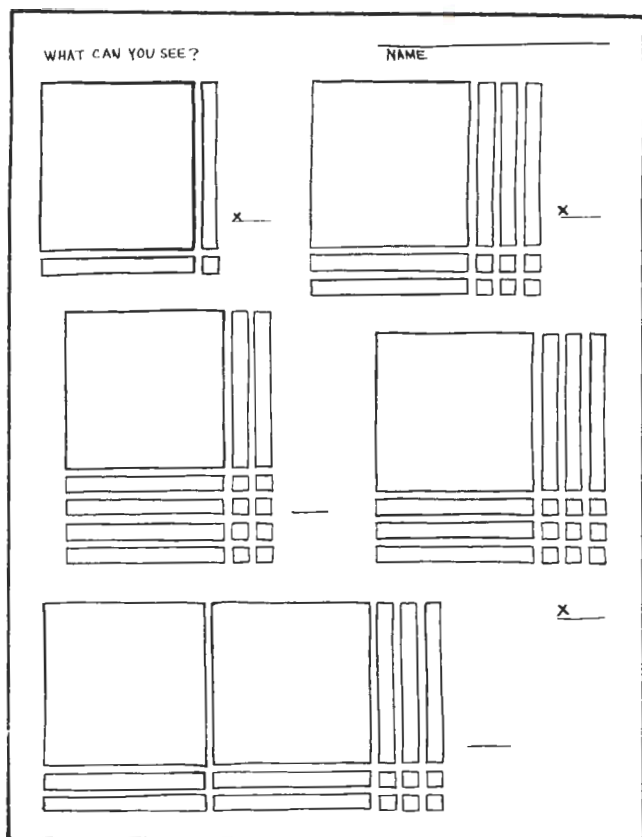


$$\begin{array}{r} 25 \\ \times 5 \\ \hline \end{array}$$

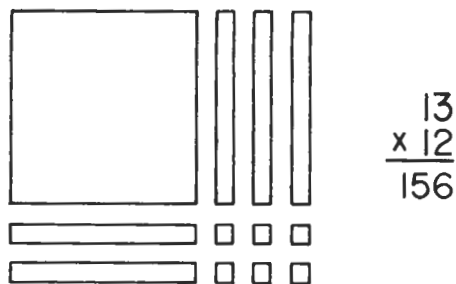


¿QUÉ PUEDES VER?  
WHAT CAN YOU SEE?





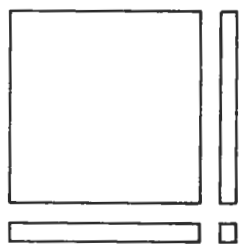
The second diagram shows an array of 12 rows with 13 in each row. There is a total of  $100 + 50 + 6$ , or 156 units. Thus,



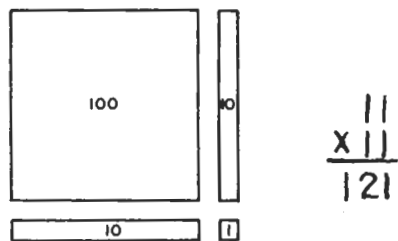
In these examples, we are simply concerned with making and counting arrays. No push toward the use of the algorithm and partial products is made. Children should be encouraged to count the units in the array by considering the rafts, then the sticks, and then the loose beans.

### What Can You See? (multiplication)

On this page,  is introduced. It can be thought of as a Dienes 'flat' (100 little cubes or 10 rods) or a "hundred raft" (ten 10-sticks glued together). The learner must now look at the diagram and record the multiplication it illustrates. For example, the first diagram,



shows an array of eleven rows with eleven units in each row. There is a total of 100 (the raft) + 20 (two 10-sticks) + 1 (loose block), or 121 units all together. Thus, we can record:



WHAT CAN YOU SEE?  
¿QUÉ PUEDES VER?

$$\begin{array}{c} \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 13 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{c} \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 15 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 13 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 16 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 13 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 15 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 15 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 17 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 24 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 25 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 24 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 27 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 24 \\ \times 5 \\ \hline \end{array}$$

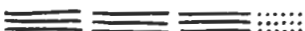
$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 27 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 36 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \dots \begin{array}{r} 48 \\ \times 4 \\ \hline \end{array}$$

WHAT CAN YOU SEE?

NAME \_\_\_\_\_

|                                                                                                         |                                                                                                         |
|---------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------|
|  $\frac{13}{\times 2}$ |  $\frac{15}{\times 2}$ |
|  $\frac{13}{\times 3}$ |  $\frac{16}{\times 3}$ |
|  $\frac{13}{\times 4}$ |  $\frac{15}{\times 4}$ |
|  $\frac{15}{\times 5}$ |  $\frac{17}{\times 5}$ |
|  $\frac{24}{\times 2}$ |  $\frac{25}{\times 2}$ |
|  $\frac{24}{\times 3}$ |  $\frac{27}{\times 3}$ |
|  $\frac{24}{\times 5}$ |  $\frac{27}{\times 5}$ |
|  $\frac{36}{\times 3}$ |                                                                                                         |
|  $\frac{48}{\times 4}$ |                                                                                                         |

### What Can You See? (multiplication)

This activity is similar to the activity on page 121B. The diagrams have become more of a shorthand notation in which  stands for a rod or a 10-stick, and  stands for a small cube or a loose bean.

Each multiplication is sketched as a number of rows of beans with the same number in each row. The learner counts the total number of beans (or cubes) in the array. He should be encouraged to use shortcuts and count loose beans by multiples rather than by ones.

# ¿QUÉ PUEDES VER? WHAT CAN YOU SEE?

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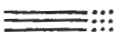
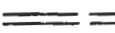
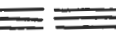
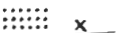
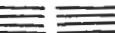
===== : : : : : X

===== : : : : : X

===== : : : : : X



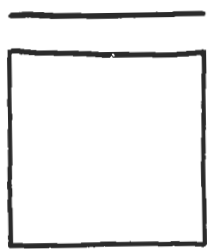
WHAT CAN YOU SEE?
NAME

|                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                       |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  x                                                                                                                                                                                                                                                       |  x                                                                                   |
|  x                                                                                                                                                                                                                                                       |  x                                                                                   |
|  x                                                                                                                                                                                                                                                       |  x                                                                                   |
|  x                                                                                                                                                                                                                                                       |  x                                                                                   |
|   x                                                                                                                                                                     |   x |
|   x                                                                                                                                                                     |   x |
|   x                                                                                                                                                                     |   x |
|    x                                                                                   |                                                                                                                                                                       |
|     x |                                                                                                                                                                       |

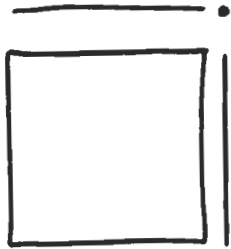
### What Can You See? (multiplication)

This is a continuation of the activity on the previous page. Note that children can check their work by comparing this page with page 122A.

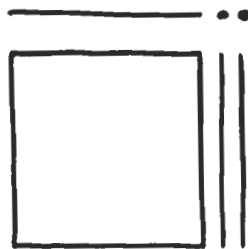
WHAT CAN YOU SEE?  
¿QUÉ PUEDES VER?



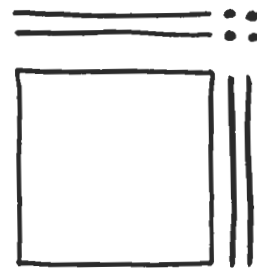
$$\begin{array}{r} 10 \\ \times 11 \\ \hline \end{array}$$



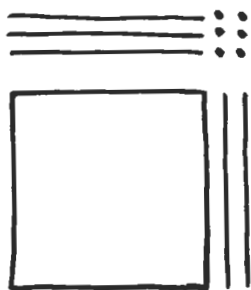
$$\begin{array}{r} 11 \\ \times 11 \\ \hline \end{array}$$



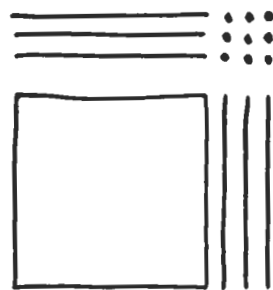
$$\begin{array}{r} 12 \\ \times 11 \\ \hline \end{array}$$



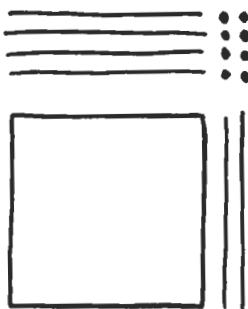
$$\begin{array}{r} 12 \\ \times 12 \\ \hline \end{array}$$



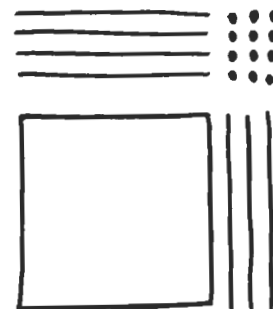
$$\begin{array}{r} 12 \\ \times 13 \\ \hline \end{array}$$



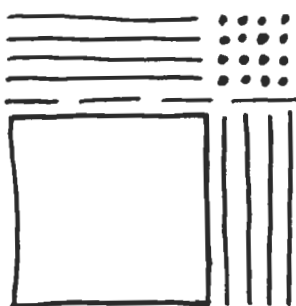
$$\begin{array}{r} 13 \\ \times 13 \\ \hline \end{array}$$



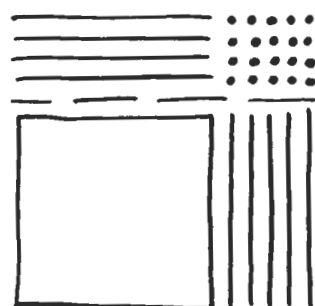
$$\begin{array}{r} 12 \\ \times 14 \\ \hline \end{array}$$



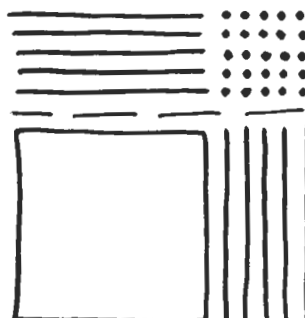
$$\begin{array}{r} 13 \\ \times 14 \\ \hline \end{array}$$



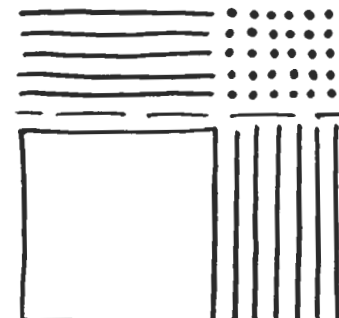
$$\begin{array}{r} 14 \\ \times 14 \\ \hline \end{array}$$



$$\begin{array}{r} 15 \\ \times 14 \\ \hline \end{array}$$



$$\begin{array}{r} 15 \\ \times 15 \\ \hline \end{array}$$



$$\begin{array}{r} 16 \\ \times 15 \\ \hline \end{array}$$

56 ABOVE  
BELOW  
TOTAL

-----  
=====

-----  
=====

----- ARRIBA  
----- ABAJO

WHAT CAN YOU SEE? NAME \_\_\_\_\_

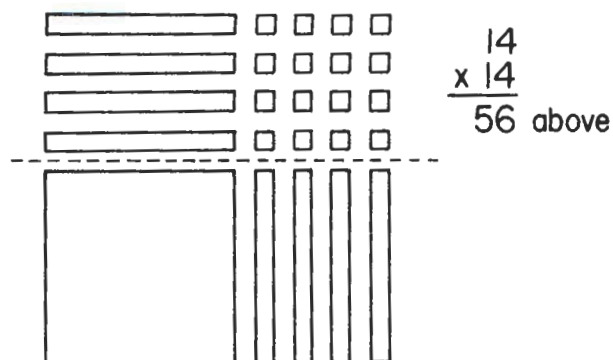
|                                                                                                                                               |                                                                                                                                               |                                                                                                                                               |                                                                                                                                               |
|-----------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| <br>$\begin{array}{r} 10 \\ \times 11 \\ \hline \end{array}$ | <br>$\begin{array}{r} 11 \\ \times 11 \\ \hline \end{array}$ | <br>$\begin{array}{r} 12 \\ \times 11 \\ \hline \end{array}$ | <br>$\begin{array}{r} 12 \\ \times 12 \\ \hline \end{array}$ |
| <br>$\begin{array}{r} 12 \\ \times 13 \\ \hline \end{array}$ | <br>$\begin{array}{r} 13 \\ \times 13 \\ \hline \end{array}$ | <br>$\begin{array}{r} 12 \\ \times 14 \\ \hline \end{array}$ | <br>$\begin{array}{r} 13 \\ \times 14 \\ \hline \end{array}$ |
| <br>$\begin{array}{r} 14 \\ \times 14 \\ \hline \end{array}$ | <br>$\begin{array}{r} 15 \\ \times 14 \\ \hline \end{array}$ | <br>$\begin{array}{r} 15 \\ \times 15 \\ \hline \end{array}$ | <br>$\begin{array}{r} 16 \\ \times 15 \\ \hline \end{array}$ |

$\begin{array}{r} 56 \text{ ABOVE} \\ \hline \text{BELOW} \\ \hline \text{TOTAL} \end{array}$

This activity introduces a  as a sketch of a Dienes flat (100 cubes) or a hundred raft (ten 10-sticks glued together). As on previous pages, each multiplication is sketched as a number of rows with the same number of beans (blocks) in each row. The learner counts the total number of beans (cubes) in the array. Again, he should be encouraged to count by multiples rather than by ones when counting loose beans.

The last row of examples here begins to build toward the use of partial products and the multiplication algorithm. However, children should have many experiences with sketches of this type before the algorithm is formally introduced. (Some children may even discover it.) In these examples the sketches are divided into two parts by a dotted line. The learner should consider the sketch in these two parts. The number of beans (blocks) **above** the line should be counted and recorded **above** the dotted line under

the example. For example:



The number **below** the line should be counted and recorded **below** the dotted line. Then the total number of units in the sketch can be found by adding these two numbers.

$$\begin{array}{r} 14 \\ \times 14 \\ \hline 56 \text{ above} \\ 140 \text{ below} \\ \hline 196 \text{ Total} \end{array}$$

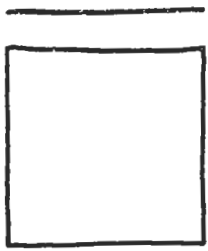
Later on, when you feel it appropriate to press on toward the algorithm, you might have children develop a table of multiples to use in conjunction with the diagram (see page 209). For example, the table:

|            |            |            |            |            |            |            |            |            |    |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|----|
| 13         | 13         | 13         | 13         | 13         | 13         | 13         | 13         | 13         | 13 |
| <u>x1</u>  | <u>x2</u>  | <u>x3</u>  | <u>x4</u>  | <u>x5</u>  | <u>x6</u>  | <u>x7</u>  | <u>x8</u>  | <u>x9</u>  |    |
| 13         | 26         | 29         | 52         | 65         | 78         | 91         | 104        | 117        |    |
| 13         | 13         | 13         | 13         | 13         | 13         | 13         | 13         | 13         | 13 |
| <u>x10</u> | <u>x20</u> | <u>x30</u> | <u>x40</u> | <u>x50</u> | <u>x60</u> | <u>x70</u> | <u>x80</u> | <u>x90</u> |    |
| 130        | 260        | 390        | 520        | 650        | 780        | 910        | 1040       | 1170       |    |

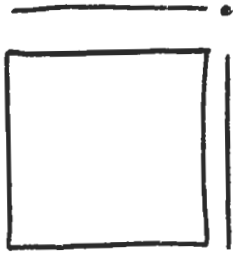
would be useful when considering examples like:

$$\begin{array}{r} 13 \\ \times 14 \\ \hline 52 \text{ above} \\ 130 \text{ below} \\ \hline 182 \text{ Total} \end{array}$$

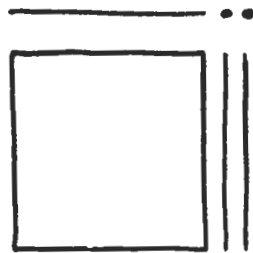
¿QUÉ PUEDES VER?  
WHAT CAN YOU SEE?



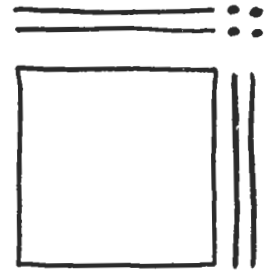
x



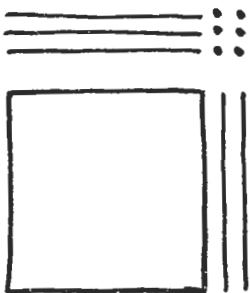
x



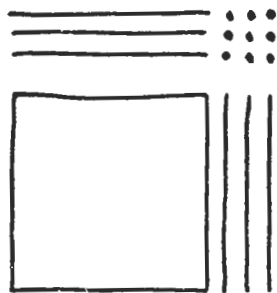
x



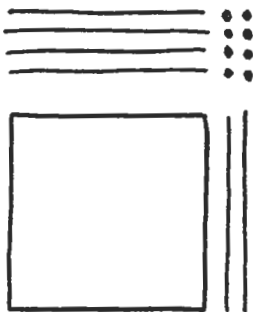
x



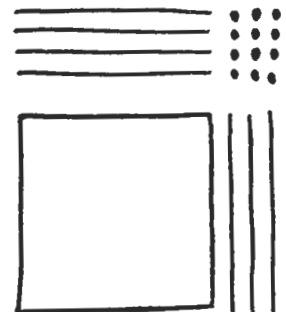
x



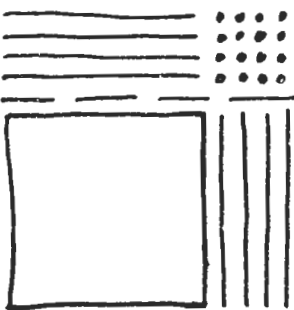
x



x

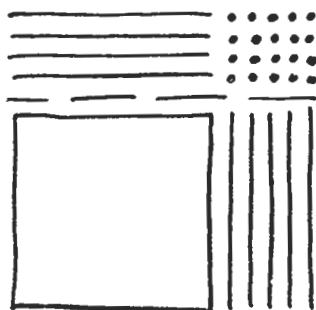


x



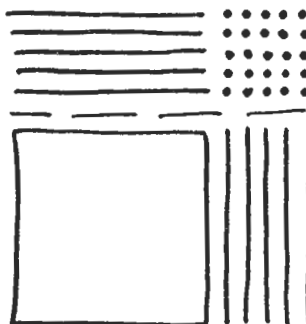
14  
x 14

56 ABOVE  
BELOW  
TOTAL



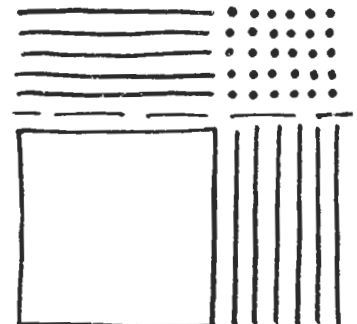
x

-----  
=====



x

-----  
=====



x

-----  
=====

ARRIBA  
ABAJO

WHAT CAN YOU SEE? NAME \_\_\_\_\_

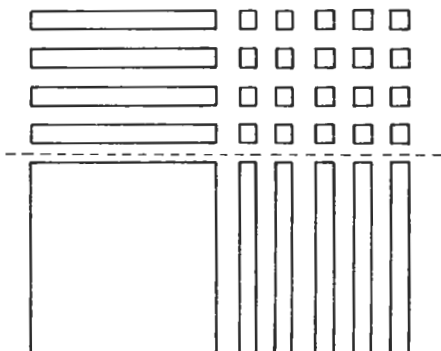
|                                                                                   |                                                                                   |                                                                                   |                                                                                   |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |  |  |
| x _____                                                                           | x _____                                                                           | x _____                                                                           | x _____                                                                           |
|  |  |  |  |
| x _____                                                                           | x _____                                                                           | x _____                                                                           | x _____                                                                           |
|  |  |  |  |
|  | x _____                                                                           | x _____                                                                           | x _____                                                                           |
| $\begin{array}{r} 14 \\ \times 14 \\ \hline \end{array}$                          |                                                                                   |                                                                                   |                                                                                   |
| Above: _____<br>Below: _____<br>Total: _____                                      |                                                                                   |                                                                                   |                                                                                   |

Above the dotted line, there are 60 beans. Below the line are 150 beans. Thus, the learner would record:

$$\begin{array}{r} 15 \\ \times 14 \\ \hline 60 \text{ above} \\ 150 \text{ below} \\ \hline 210 \text{ Total} \end{array}$$

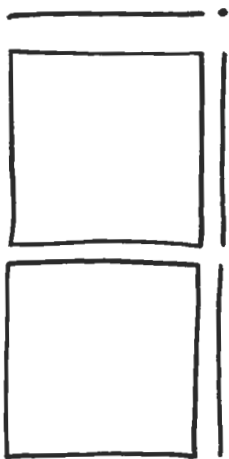
Note that children can check their own work by comparing this page to page 126A.

Here, the learner must look at each sketch and figure out which multiplication it illustrates. Once again, examples are given in the last row which begin to build toward the use of partial products and the multiplication algorithm. Once the learner figures out what multiplication is indicated by the array, he can count the number of beans above the dotted line and below the dotted line and then add these numbers to get the total. For example:

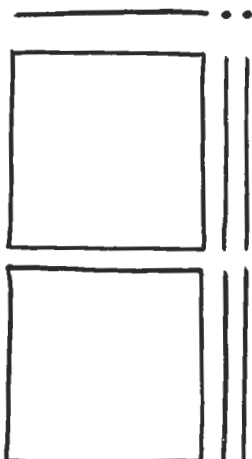


shows 14 rows with 15 beans (blocks) in each row. Thus it represents the multiplication:

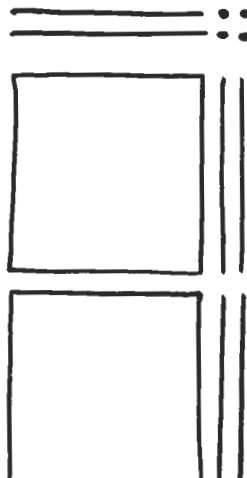
$$\begin{array}{r} 15 \\ \times 14 \end{array}$$



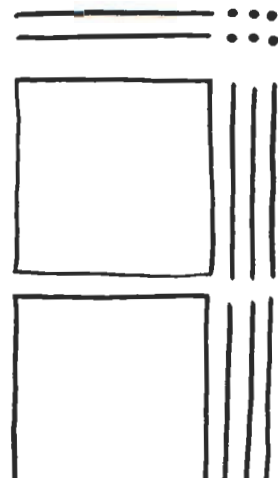
$$\begin{array}{r} 11 \\ \times 21 \\ \hline \end{array}$$



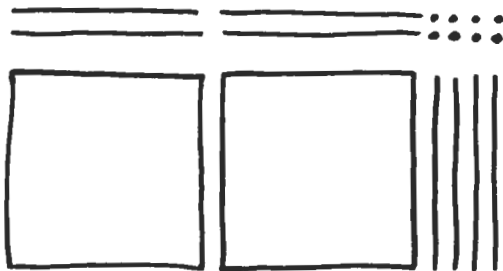
$$\begin{array}{r} 12 \\ \times 21 \\ \hline \end{array}$$



$$\begin{array}{r} 12 \\ \times 22 \\ \hline \end{array}$$



$$\begin{array}{r} 13 \\ \times 22 \\ \hline \end{array}$$



$$\begin{array}{r} 24 \\ \times 12 \\ \hline \end{array}$$

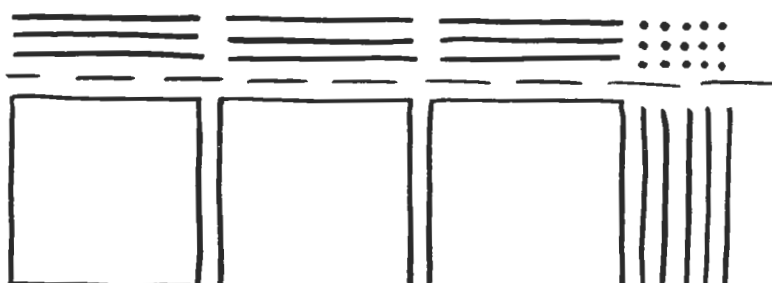


$$\begin{array}{r} 33 \\ \times 12 \\ \hline \end{array}$$



$$\begin{array}{r} 24 \\ \times 13 \\ \hline \end{array}$$

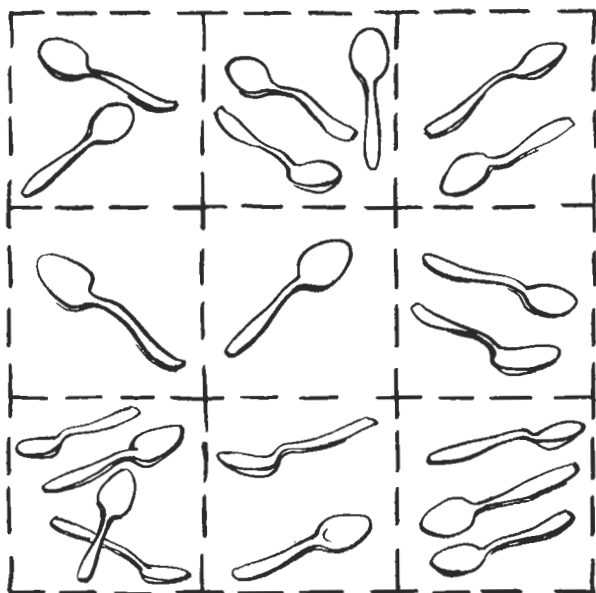
----- ABOVE  
----- BELOW  
===== TOTAL



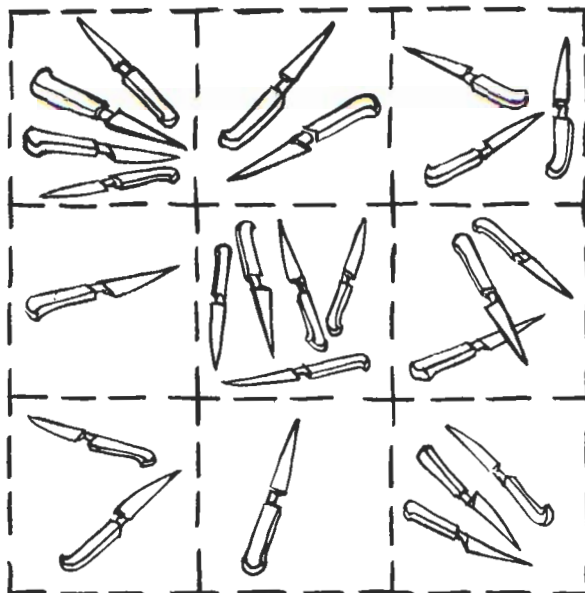
$$\begin{array}{r} 35 \\ \times 13 \\ \hline \end{array}$$

----- ARRIBA  
----- ABAJO  
===== TOTAL





— (5's)



— (6's)

|   |   |   |
|---|---|---|
| 2 | 3 | 2 |
| 3 | 1 | 4 |
| 2 | 2 | 1 |

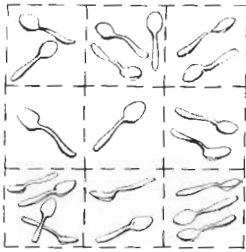
— (5's)

|   |   |   |
|---|---|---|
| 1 | 2 | 4 |
| 3 | 2 | 2 |
| 2 | 1 | 1 |

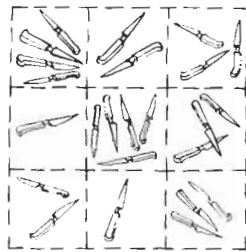
— (6's)



NAME \_\_\_\_\_



— (5's)



— (6's)

|   |   |   |
|---|---|---|
| 2 | 3 | 2 |
| 3 | 1 | 4 |
| 2 | 2 | 1 |

— (5's)

|   |   |   |
|---|---|---|
| 1 | 2 | 4 |
| 3 | 2 | 2 |
| 2 | 1 | 1 |

— (6's)

For other fencing activities, see pages 94-103.

On this page we make the transition from pictures to numerals. The children should have little difficulty making the transition. Tell them to do the bottom squares the same way they do the top squares.

Remember, try to get the greatest number of fences.

---

|   |   |   |
|---|---|---|
| 1 | 3 | 4 |
| 6 | 5 | 2 |
| 1 | 2 | 4 |

— (7's)

|   |   |   |
|---|---|---|
| 4 | 6 | 2 |
| 1 | 1 | 5 |
| 3 | 7 | 3 |

— (8's)

|   |   |   |
|---|---|---|
| 1 | 6 | 2 |
| 5 | 2 | 3 |
| 2 | 3 | 3 |

— (9's)

|   |   |   |
|---|---|---|
| 8 | 2 | 7 |
| 4 | 3 | 3 |
| 6 | 2 | 5 |

— (10's)

NAME \_\_\_\_\_

|   |   |   |
|---|---|---|
| 1 | 3 | 4 |
| 6 | 5 | 2 |
| 1 | 2 | 4 |

— (7's)

|   |   |   |
|---|---|---|
| 4 | 6 | 2 |
| 1 | 1 | 5 |
| 3 | 7 | 3 |

— (8's)

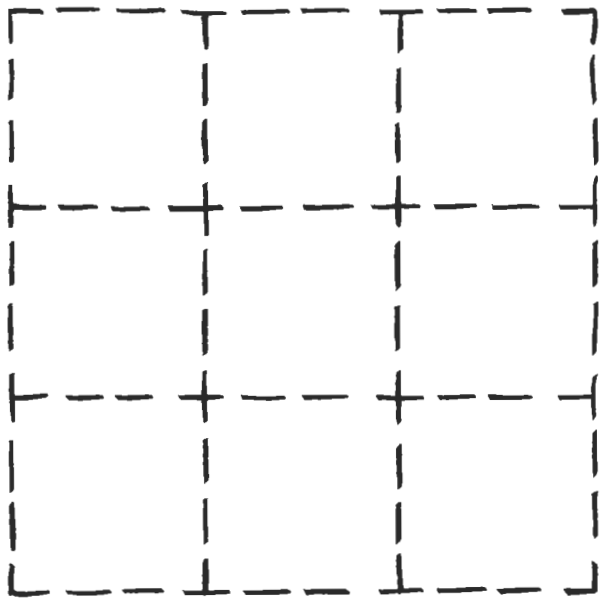
|   |   |   |
|---|---|---|
| 1 | 6 | 2 |
| 5 | 2 | 3 |
| 2 | 3 | 3 |

— (9's)

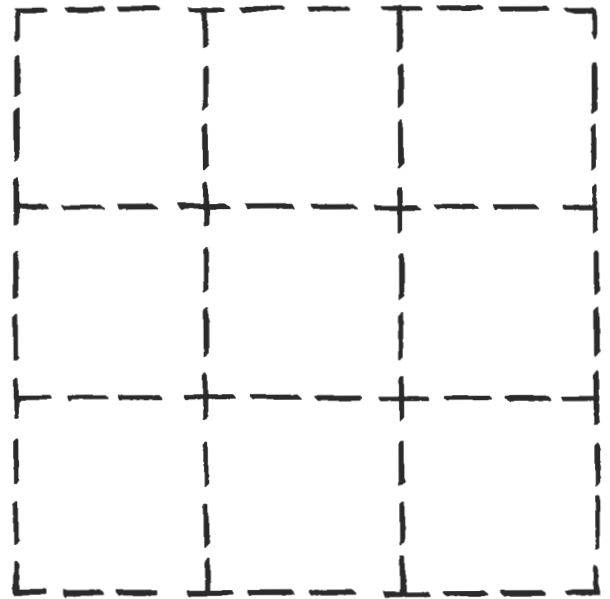
|   |   |   |
|---|---|---|
| 8 | 2 | 7 |
| 4 | 3 | 3 |
| 6 | 2 | 5 |

— (10's)

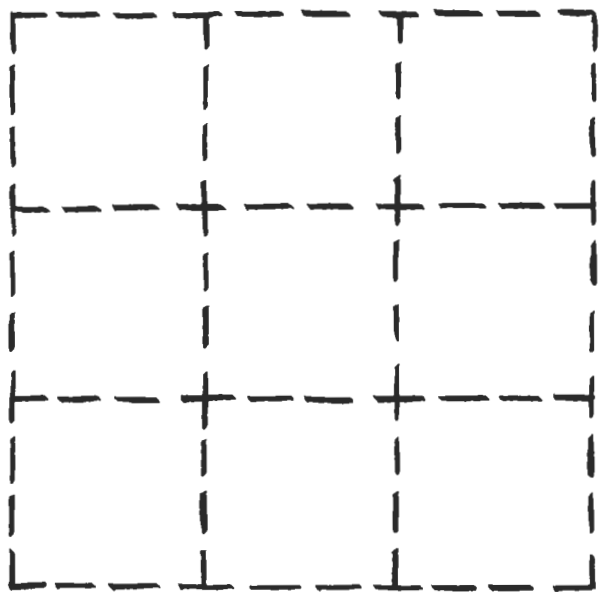
The numerals in the boxes and the sums get larger.



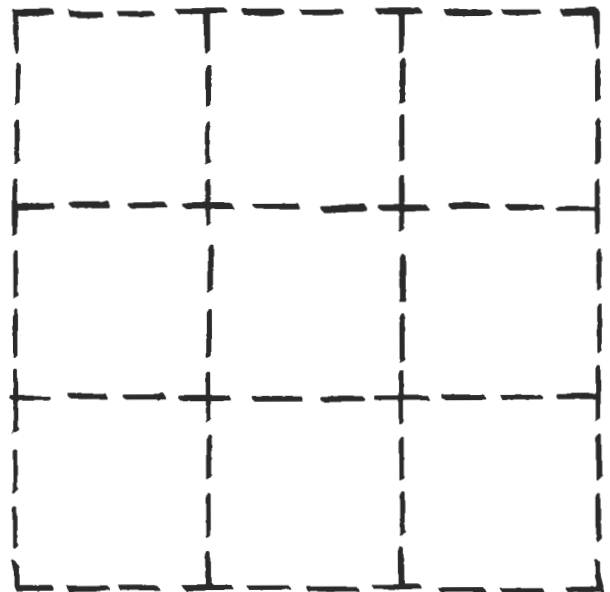
— (s)



— (s)

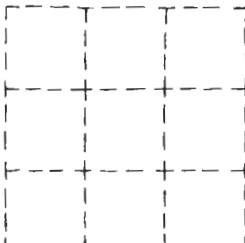


— (s)

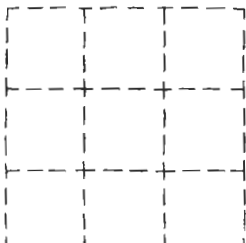


— (s)

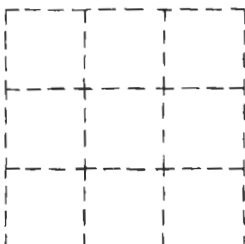
NAME \_\_\_\_\_



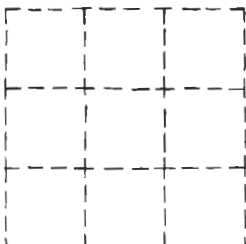
— (s)



— (s)



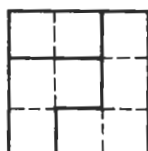
— (s)



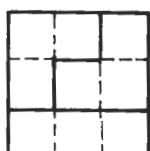
— (s)

And you can be sure there is at least 1 way to get all the numbers fenced in—though there may be other ways than the originating fences.

To make up examples of "fence arithmetic" of this kind, start with a 3 by 3 arrangement with fences drawn.



or



After choosing the sum desired, such as 9, put combinations of 9 in each fence such as:

|   |   |   |
|---|---|---|
| 6 | 3 | 3 |
| 2 | 4 | 3 |
| 3 | 1 | 2 |

or

|   |   |   |
|---|---|---|
| 3 | 3 | 2 |
| 3 | 4 | 3 |
| 6 | 1 | 2 |

Then write in the numbers without the fences:

|   |   |   |
|---|---|---|
| 6 | 3 | 3 |
| 2 | 4 | 3 |
| 3 | 1 | 2 |

or

|   |   |   |
|---|---|---|
| 3 | 3 | 2 |
| 3 | 4 | 3 |
| 6 | 1 | 2 |

|   |   |   |
|---|---|---|
| 1 | 2 | 4 |
| 5 | 3 | 3 |
| 3 | 2 | 1 |

— (6's)

|   |   |   |
|---|---|---|
| 1 | 4 | 2 |
| 2 | 2 | 3 |
| 1 | 2 | 4 |

— (7's)

|   |   |   |
|---|---|---|
| 4 | 1 | 6 |
| 3 | 2 | 2 |
| 1 | 2 | 3 |

— (8's)

|   |   |   |
|---|---|---|
| 4 | 3 | 1 |
| 3 | 2 | 5 |
| 2 | 3 | 4 |

— (9's)

|   |   |   |
|---|---|---|
| 1 | 4 | 6 |
| 2 | 4 | 5 |
| 3 | 3 | 2 |

— (10's)

|   |   |   |
|---|---|---|
| 2 | 3 | 1 |
| 1 | 4 | 2 |
| 1 | 1 | 6 |

— (7's)

|   |   |   |
|---|---|---|
| 2 | 6 | 1 |
| 3 | 1 | 7 |
| 4 | 5 | 3 |

— (8's)

|   |   |   |
|---|---|---|
| 5 | 2 | 7 |
| 4 | 3 | 3 |
| 1 | 8 | 3 |

— (9's)

|   |   |   |
|---|---|---|
| 4 | 7 | 3 |
| 3 | 5 | 1 |
| 2 | 1 | 4 |

— (10's)

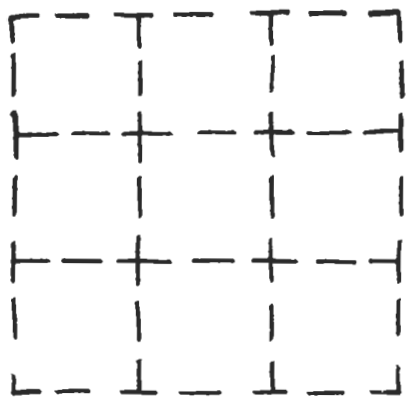
NAME \_\_\_\_\_

|                         |                         |                         |
|-------------------------|-------------------------|-------------------------|
| 1 2 4<br>5 3 3<br>3 2 1 | 1 4 2<br>2 2 3<br>1 2 4 | 4 1 6<br>3 2 2<br>1 2 3 |
| — (6's)                 | — (7's)                 | — (8's)                 |
| 4 3 1<br>3 2 5<br>2 3 4 | 1 4 6<br>2 4 5<br>3 3 2 | 2 3 1<br>1 4 2<br>1 1 6 |
| — (9's)                 | — (10's)                | — (7's)                 |
| 2 6 1<br>3 1 7<br>4 5 3 | 5 2 7<br>4 3 3<br>1 8 3 | 4 7 3<br>3 5 1<br>2 1 4 |
| — (8's)                 | — (9's)                 | — (10's)                |

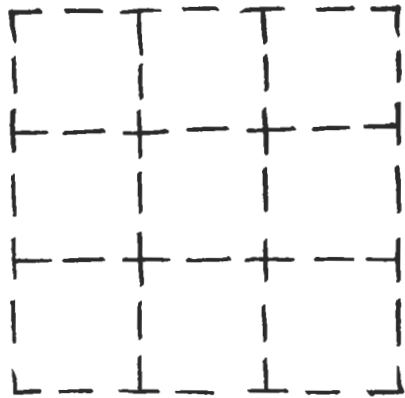
After finding one way to fence all numbers in all examples, consider providing another copy of this or the following page and the question:

"In how many examples are there two different ways to fence all numbers in?" (There is only 1—fencing the (9's) in the second row.)

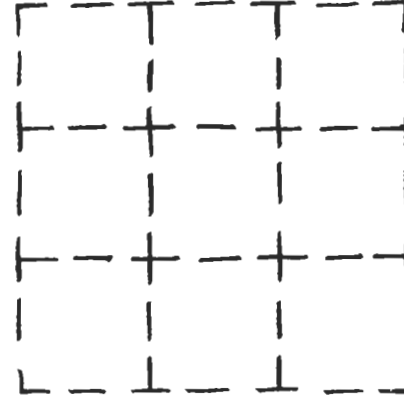
"Can you make up examples so that all numbers can be fenced in more than one way?"



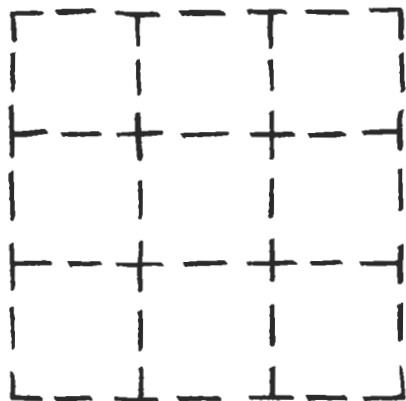
— (s)



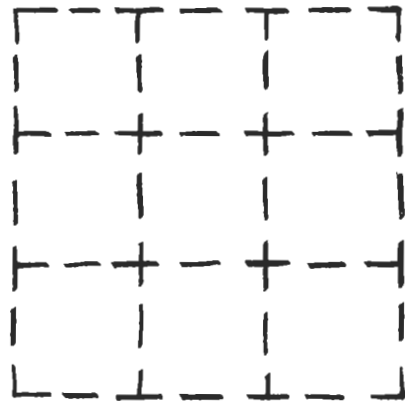
— (s)



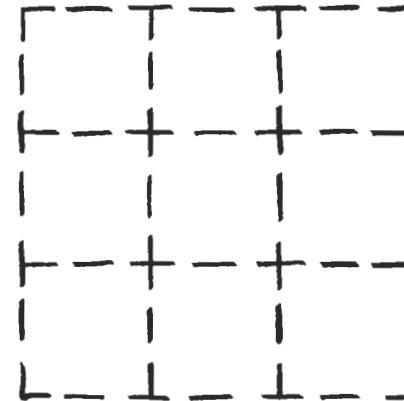
— (s)



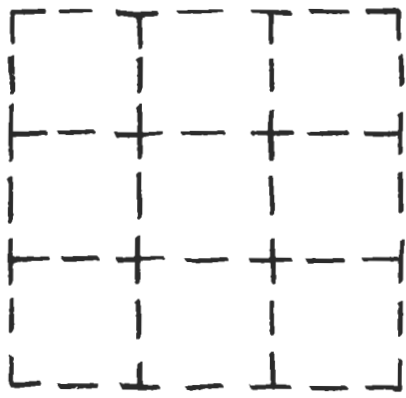
— (s)



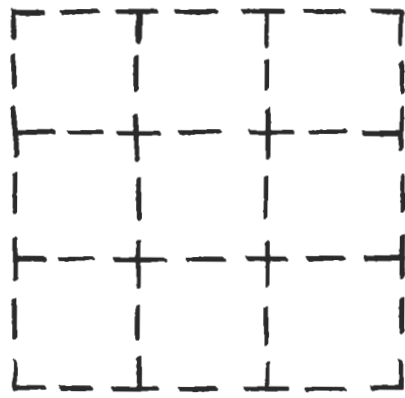
— (s)



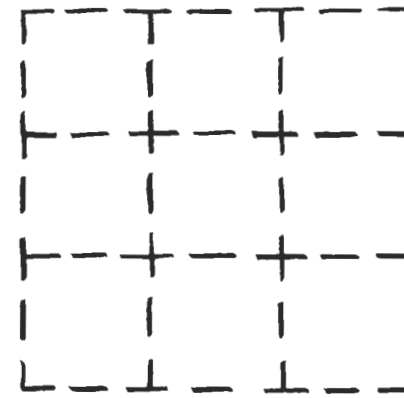
— (s)



— (s)



— (s)



— (s)



NAME \_\_\_\_\_

|                                                                                                                   |                                                                                                                   |                                                                                                                   |
|-------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|
| <div style="position: absolute; top: 10px; left: 10px; width: 80%; height: 80%; border: 1px dashed black;"></div> | <div style="position: absolute; top: 10px; left: 10px; width: 80%; height: 80%; border: 1px dashed black;"></div> | <div style="position: absolute; top: 10px; left: 10px; width: 80%; height: 80%; border: 1px dashed black;"></div> |
| — ( ) 's                                                                                                          | — ( ) 's                                                                                                          | — ( ) 's                                                                                                          |
| <div style="position: absolute; top: 10px; left: 10px; width: 80%; height: 80%; border: 1px dashed black;"></div> | <div style="position: absolute; top: 10px; left: 10px; width: 80%; height: 80%; border: 1px dashed black;"></div> | <div style="position: absolute; top: 10px; left: 10px; width: 80%; height: 80%; border: 1px dashed black;"></div> |
| — ( ) 's                                                                                                          | — ( ) 's                                                                                                          | — ( ) 's                                                                                                          |
| <div style="position: absolute; top: 10px; left: 10px; width: 80%; height: 80%; border: 1px dashed black;"></div> | <div style="position: absolute; top: 10px; left: 10px; width: 80%; height: 80%; border: 1px dashed black;"></div> | <div style="position: absolute; top: 10px; left: 10px; width: 80%; height: 80%; border: 1px dashed black;"></div> |
| — ( ) 's                                                                                                          | — ( ) 's                                                                                                          | — ( ) 's                                                                                                          |

If two numbers are omitted and the final record completed, the problem grows.

|   |   |   |
|---|---|---|
|   | 6 | 1 |
| 3 | 1 | 7 |
| 4 |   | 3 |

4 — (8's)

The sum of the given number is 25—so the missing pair must have a sum of 7: so the pair is far from determined yet.

The problem can be extended. If you find a pair that leads to fencing in 4 (8's), could some other pair of numbers be used?

A variation of the "fencing problem" leaves some of the numbers out.

|   |   |   |
|---|---|---|
| 2 | 3 | 1 |
| 1 | 4 | 2 |
| 1 | 1 |   |

or

|   |   |   |
|---|---|---|
| 4 |   | 3 |
| 3 | 5 | 1 |
| 2 | 1 | 4 |

— (7's)

— (10's)

These are harder than they look at first glance—if all numbers are to be fenced. Try them.

They are 2 examples from the previous page with the largest number missing. Other difficult examples can be made in a similar fashion.

Of course, there is a simple way to find these numbers. If all are to be fenced in, then the sum of all numbers must be a multiple of the sum that is specified—7's and 10's in the example above.

The total of numbers given in the first example is 15—or 6 less than the next largest multiple of 7.

The total of numbers given in the second example is 23—or 7 less than the next largest multiple of 10.

|   |   |   |   |
|---|---|---|---|
| 2 | 1 | 3 | 5 |
| 5 | 1 | 5 | 7 |
| 6 | 3 | 2 | 1 |
| 2 | 5 | 4 | 4 |

— (8's)

|   |   |   |   |
|---|---|---|---|
| 5 | 4 | 2 | 3 |
| 3 | 2 | 6 | 4 |
| 6 | 1 | 7 | 6 |
| 4 | 5 | 2 | 3 |

— (9's)

|   |   |   |   |
|---|---|---|---|
| 1 | 9 | 3 | 3 |
| 6 | 2 | 8 | 4 |
| 4 | 5 | 2 | 3 |
| 5 | 5 | 3 | 7 |

— (10's)

|   |   |   |   |
|---|---|---|---|
| 4 | 4 | 7 | 2 |
| 4 | 3 | 3 | 9 |
| 3 | 3 | 2 | 3 |
| 5 | 3 | 3 | 8 |

— (11's)

|   |   |   |   |
|---|---|---|---|
| 4 | 8 | 5 | 5 |
| 7 | 9 | 3 | 2 |
| 2 | 5 | 6 | 8 |
| 3 | 7 | 6 | 4 |

— (12's)

|   |   |   |   |
|---|---|---|---|
| 3 | 3 | 4 | 6 |
| 3 | 8 | 5 | 3 |
| 1 | 1 | 2 | 7 |
| 3 | 5 | 3 | 6 |

— (9's)

|   |   |   |   |
|---|---|---|---|
| 6 | 4 | 2 | 8 |
| 4 | 5 | 3 | 1 |
| 5 | 1 | 2 | 9 |
| 7 | 3 | 4 | 6 |

— (10's)

|   |   |   |   |
|---|---|---|---|
| 2 | 5 | 6 | 8 |
| 9 | 6 | 2 | 3 |
| 3 | 3 | 4 | 7 |
| 4 | 4 | 9 | 2 |

— (11's)

|   |   |   |   |
|---|---|---|---|
| 4 | 8 | 3 | 9 |
| 6 | 6 | 3 | 5 |
| 4 | 5 | 4 | 7 |
| 4 | 4 | 8 | 4 |

— (12's)

NAME \_\_\_\_\_

|          |   |   |   |          |   |   |   |          |   |   |   |
|----------|---|---|---|----------|---|---|---|----------|---|---|---|
| 2        | 1 | 3 | 5 | 5        | 4 | 2 | 3 | 1        | 9 | 3 | 3 |
| 5        | 1 | 5 | 7 | 3        | 2 | 6 | 4 | 6        | 2 | 8 | 4 |
| 6        | 3 | 2 | 1 | 6        | 1 | 7 | 6 | 4        | 5 | 2 | 3 |
| 2        | 5 | 4 | 4 | 4        | 5 | 2 | 3 | 5        | 5 | 3 | 7 |
| — (8's)  |   |   |   | — (9's)  |   |   |   | — (10's) |   |   |   |
|          |   |   |   |          |   |   |   |          |   |   |   |
| 4        | 4 | 7 | 2 | 4        | 8 | 5 | 5 | 3        | 3 | 4 | 6 |
| 4        | 3 | 3 | 9 | 7        | 9 | 3 | 2 | 3        | 8 | 5 | 3 |
| 3        | 3 | 2 | 3 | 2        | 5 | 6 | 8 | 1        | 1 | 2 | 7 |
| 5        | 3 | 3 | 8 | 3        | 7 | 6 | 4 | 3        | 5 | 3 | 6 |
| — (11's) |   |   |   | — (12's) |   |   |   | — (9's)  |   |   |   |
|          |   |   |   |          |   |   |   |          |   |   |   |
| 6        | 4 | 2 | 8 | 2        | 5 | 6 | 8 | 4        | 8 | 3 | 9 |
| 4        | 5 | 3 | 1 | 9        | 6 | 2 | 3 | 6        | 6 | 3 | 5 |
| 5        | 1 | 2 | 9 | 3        | 3 | 4 | 7 | 4        | 5 | 4 | 7 |
| 7        | 3 | 4 | 6 | 4        | 4 | 9 | 2 | 4        | 4 | 8 | 4 |
| — (10's) |   |   |   | — (11's) |   |   |   | — (12's) |   |   |   |

(1) There is only a single 1 neighboring 7 to make 8.

(2) After that "5 needs 3"

(3) and "4 needs 4."

Then everything falls into place: "6 needs 2" and "5 needs 3" and the two 5, 2 and 1 combinations are left.

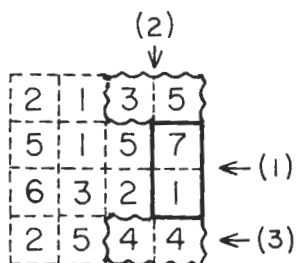
These are not "secrets" we hope children don't uncover. Rather, finding such useful strategies is the essence of mathematics. And group discussion of these systems should be encouraged as wise use of "instructional time."

Not only do the numbers in the boxes and the sums get larger, but the 9 box square grows to a 16 box square. As a child works through the square, many decisions are made and there will often be a need to go back and change some fences. To avoid the need for erasing constantly a "scribble sheet" is provided as the next page.

However, there is a useful strategy someone may discover. It might be explained this way:

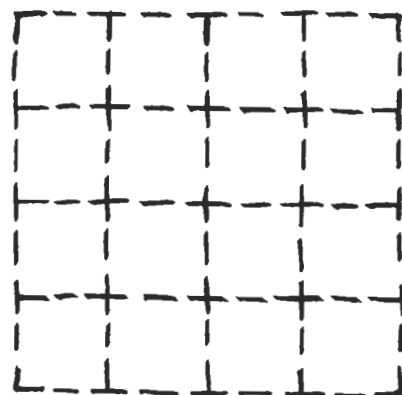
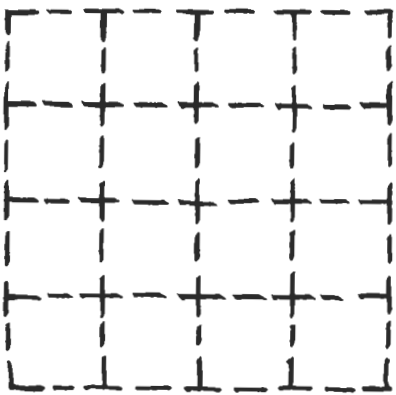
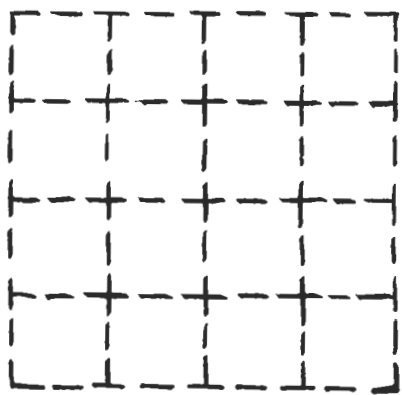
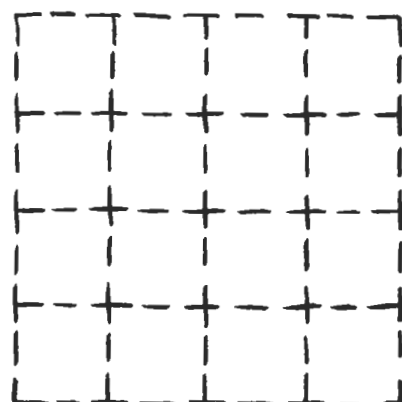
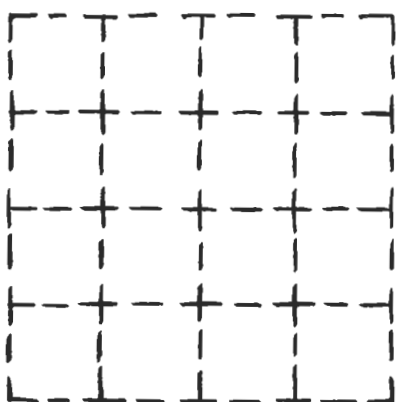
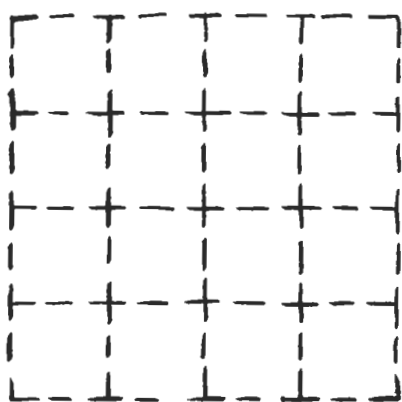
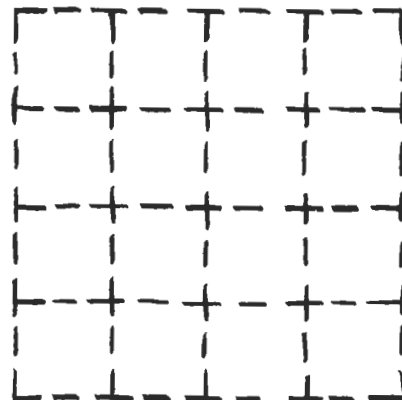
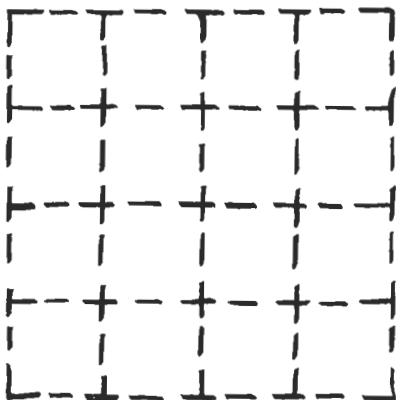
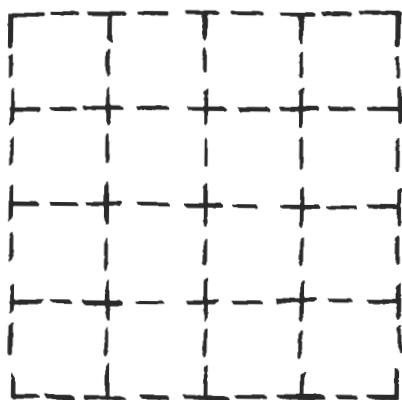
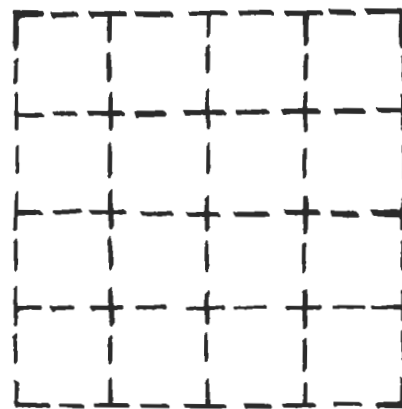
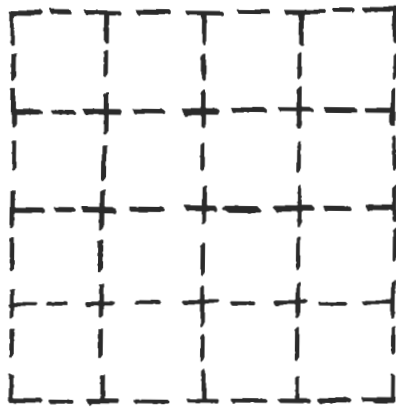
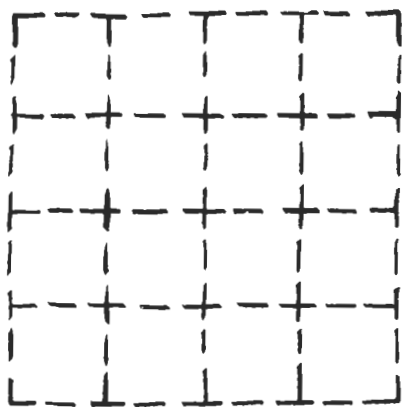
First look for the larger numbers. If there is only one way to fence them with a neighbor, that's a good beginning.

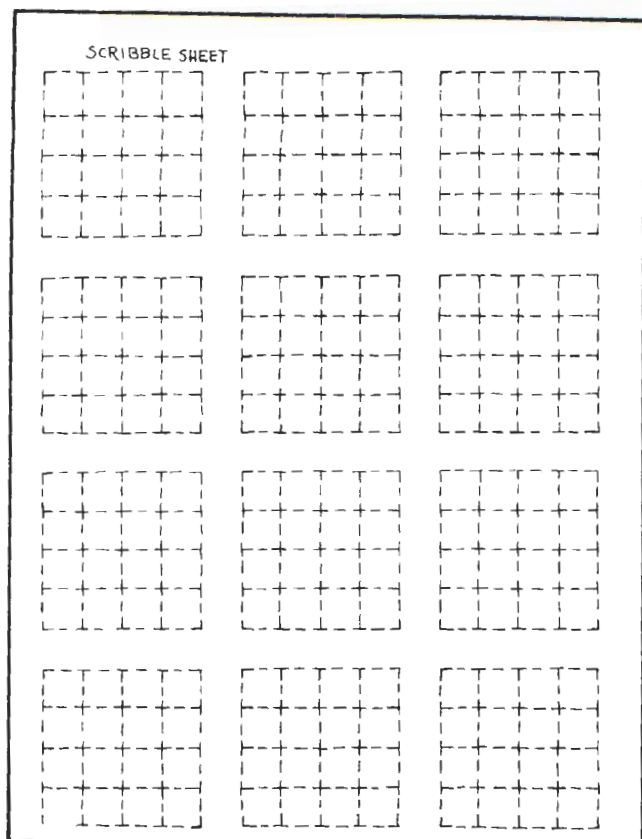
In the first example:



PAPEL PARA PRACTICAR

SCRIBBLE SHEET





This scribble sheet can be used with the previous page and with those that follow.

Its purpose is to give the child a place to experiment before making the final decision as to where the fences should be. Its use will become more evident as the sums get larger. This sheet may be introduced in conjunction with the previous page or when you see erasures beginning to appear.

|   |   |   |   |
|---|---|---|---|
| 2 | 7 | 6 | 4 |
| 1 | 1 | 3 | 3 |
| 3 | 2 | 5 | 7 |
| 6 | 2 | 6 | 2 |

— (10's)

|   |   |   |   |
|---|---|---|---|
| 2 | 6 | 3 | 2 |
| 3 | 8 | 1 | 3 |
| 5 | 2 | 2 | 4 |
| 3 | 2 | 7 | 2 |

— (11's)

|   |   |   |   |
|---|---|---|---|
| 3 | 4 | 4 | 4 |
| 3 | 3 | 4 | 6 |
| 5 | 3 | 2 | 5 |
| 2 | 5 | 3 | 4 |

— (12's)

|   |   |   |   |
|---|---|---|---|
| 7 | 3 | 3 | 6 |
| 3 | 3 | 2 | 4 |
| 2 | 8 | 9 | 2 |
| 3 | 3 | 3 | 4 |

— (13's)

|   |   |   |   |
|---|---|---|---|
| 9 | 3 | 4 | 6 |
| 2 | 2 | 5 | 4 |
| 3 | 5 | 2 | 6 |
| 7 | 4 | 6 | 2 |

— (14's)

|   |   |   |   |
|---|---|---|---|
| 7 | 2 | 3 | 9 |
| 3 | 5 | 5 | 1 |
| 5 | 5 | 5 | 5 |
| 8 | 2 | 4 | 6 |

— (15's)

|   |   |   |   |
|---|---|---|---|
| 2 | 6 | 6 | 7 |
| 3 | 5 | 4 | 3 |
| 6 | 6 | 6 | 6 |
| 2 | 8 | 1 | 9 |

— (16's)

|   |   |    |   |
|---|---|----|---|
| 8 | 3 | 4  | 9 |
| 3 | 3 | 4  | 4 |
| 7 | 3 | 10 | 1 |
| 4 | 7 | 8  | 8 |

— (17's)

|   |   |   |    |
|---|---|---|----|
| 7 | 3 | 5 | 13 |
| 8 | 9 | 6 | 8  |
| 5 | 9 | 4 | 3  |
| 4 | 9 | 9 | 6  |

— (18's)

NAME \_\_\_\_\_

|          |   |   |   |          |   |    |   |          |   |   |    |
|----------|---|---|---|----------|---|----|---|----------|---|---|----|
| 2        | 7 | 6 | 4 | 2        | 6 | 3  | 2 | 3        | 4 | 4 | 4  |
| 1        | 1 | 3 | 3 | 3        | 8 | 1  | 3 | 3        | 3 | 4 | 6  |
| 3        | 2 | 5 | 7 | 5        | 2 | 2  | 4 | 5        | 3 | 2 | 5  |
| 6        | 2 | 6 | 2 | 3        | 2 | 7  | 2 | 2        | 5 | 3 | 4  |
| — (10's) |   |   |   | — (11's) |   |    |   | — (12's) |   |   |    |
|          |   |   |   |          |   |    |   |          |   |   |    |
| 7        | 3 | 3 | 6 | 9        | 3 | 4  | 6 | 7        | 2 | 3 | 9  |
| 3        | 3 | 2 | 4 | 2        | 2 | 5  | 4 | 3        | 5 | 5 | 1  |
| 2        | 8 | 9 | 2 | 3        | 5 | 2  | 6 | 5        | 5 | 5 | 5  |
| 3        | 3 | 3 | 4 | 7        | 4 | 6  | 2 | 8        | 2 | 4 | 6  |
| — (13's) |   |   |   | — (14's) |   |    |   | — (15's) |   |   |    |
|          |   |   |   |          |   |    |   |          |   |   |    |
| 2        | 6 | 6 | 7 | 8        | 3 | 4  | 9 | 7        | 3 | 5 | 13 |
| 3        | 5 | 4 | 3 | 3        | 3 | 4  | 4 | 8        | 9 | 6 | 8  |
| 6        | 6 | 6 | 6 | 7        | 3 | 10 | 1 | 5        | 9 | 4 | 3  |
| 2        | 8 | 1 | 9 | 4        | 7 | 8  | 8 | 4        | 9 | 9 | 6  |
| — (16's) |   |   |   | — (17's) |   |    |   | — (18's) |   |   |    |

An extra challenge this time: there is one problem that can't be done — center problem in bottom row.

|   |   |   |   |
|---|---|---|---|
| 2 | 2 | 1 | 3 |
| 3 | 4 | 1 | 4 |
| 5 | 5 | 4 | 6 |
| 2 | 8 | 3 | 7 |

— (10's)

|   |   |   |   |
|---|---|---|---|
| 4 | 7 | 3 | 6 |
| 9 | 2 | 8 | 5 |
| 2 | 7 | 5 | 6 |
| 9 | 4 | 8 | 3 |

— (11's)

|   |   |   |   |
|---|---|---|---|
| 5 | 3 | 2 | 1 |
| 2 | 4 | 2 | 6 |
| 3 | 3 | 5 | 1 |
| 3 | 3 | 1 | 4 |

— (12's)

|   |   |    |   |
|---|---|----|---|
| 3 | 5 | 6  | 7 |
| 5 | 5 | 8  | 4 |
| 2 | 7 | 6  | 9 |
| 8 | 3 | 10 | 3 |

— (13's)

|   |   |   |   |
|---|---|---|---|
| 3 | 4 | 1 | 9 |
| 7 | 7 | 2 | 5 |
| 5 | 9 | 4 | 6 |
| 3 | 5 | 6 | 8 |

— (14's)

|   |   |   |   |
|---|---|---|---|
| 2 | 3 | 4 | 5 |
| 7 | 8 | 4 | 1 |
| 6 | 6 | 5 | 4 |
| 2 | 7 | 8 | 3 |

— (15's)

|   |   |   |    |
|---|---|---|----|
| 8 | 5 | 5 | 9  |
| 3 | 5 | 7 | 2  |
| 3 | 4 | 2 | 1  |
| 7 | 6 | 3 | 10 |

— (16's)

|   |   |    |    |
|---|---|----|----|
| 5 | 6 | 11 | 8  |
| 6 | 7 | 6  | 9  |
| 3 | 7 | 5  | 12 |
| 9 | 8 | 10 | 7  |

— (17's)

|    |   |   |    |
|----|---|---|----|
| 4  | 5 | 7 | 9  |
| 3  | 5 | 6 | 9  |
| 6  | 6 | 6 | 13 |
| 10 | 8 | 6 | 5  |

— (18's)



NAME \_\_\_\_\_

|   |   |   |   |
|---|---|---|---|
| 2 | 2 | 1 | 3 |
| 3 | 4 | 1 | 4 |
| 5 | 5 | 4 | 6 |
| 2 | 8 | 3 | 7 |

— (10's)

|   |   |   |   |
|---|---|---|---|
| 4 | 7 | 3 | 6 |
| 9 | 2 | 8 | 5 |
| 2 | 7 | 5 | 6 |
| 9 | 4 | 8 | 3 |

— (11's)

|   |   |   |   |
|---|---|---|---|
| 5 | 3 | 2 | 1 |
| 2 | 4 | 2 | 6 |
| 3 | 3 | 5 | 1 |
| 3 | 3 | 1 | 4 |

— (12's)

|   |   |    |   |
|---|---|----|---|
| 3 | 5 | 6  | 7 |
| 5 | 5 | 8  | 4 |
| 2 | 7 | 6  | 9 |
| 8 | 3 | 10 | 3 |

— (13's)

|   |   |   |   |
|---|---|---|---|
| 3 | 4 | 1 | 9 |
| 7 | 7 | 2 | 5 |
| 5 | 9 | 4 | 6 |
| 3 | 5 | 6 | 8 |

— (14's)

|   |   |   |   |
|---|---|---|---|
| 2 | 3 | 4 | 5 |
| 7 | 8 | 4 | 1 |
| 6 | 6 | 5 | 4 |
| 2 | 7 | 8 | 3 |

— (15's)

|   |   |   |    |
|---|---|---|----|
| 8 | 5 | 5 | 9  |
| 3 | 5 | 7 | 2  |
| 3 | 4 | 2 | 1  |
| 7 | 6 | 3 | 10 |

— (16's)

|   |   |    |    |
|---|---|----|----|
| 5 | 6 | 11 | 8  |
| 6 | 7 | 6  | 9  |
| 3 | 7 | 5  | 12 |
| 9 | 8 | 10 | 7  |

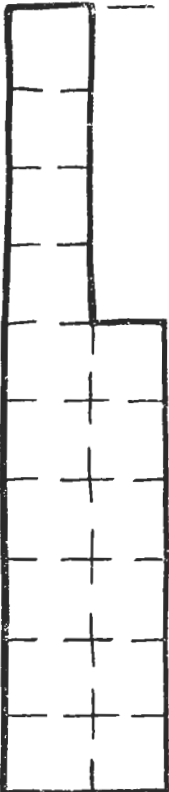
— (17's)

|    |   |   |    |
|----|---|---|----|
| 4  | 5 | 7 | 9  |
| 3  | 5 | 6 | 9  |
| 6  | 6 | 6 | 13 |
| 10 | 8 | 6 | 5  |

— (18's)

More fun with fence arithmetic. Discussions regarding strategies would be beneficial. How can you tell when the greatest number of fences are made?

# FENCING CERCANDO

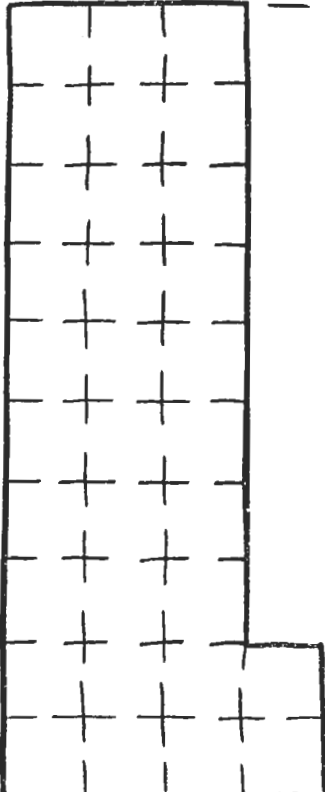


4's

x      x

—

—

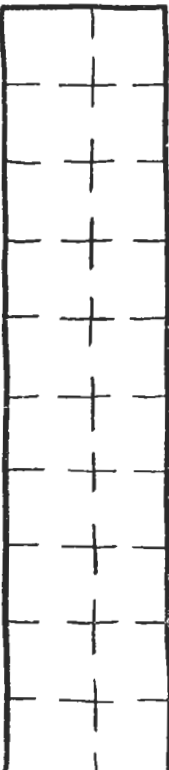


8's

x      x

└

└

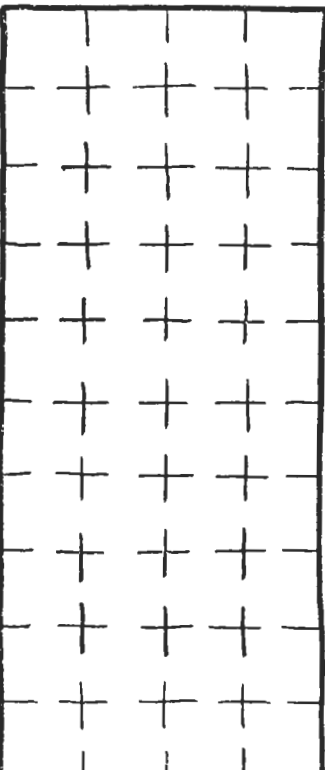


5's

x      x

└

└

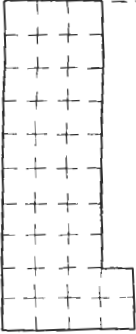
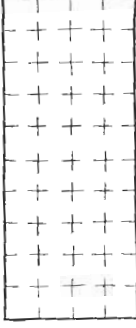


8's

x      x

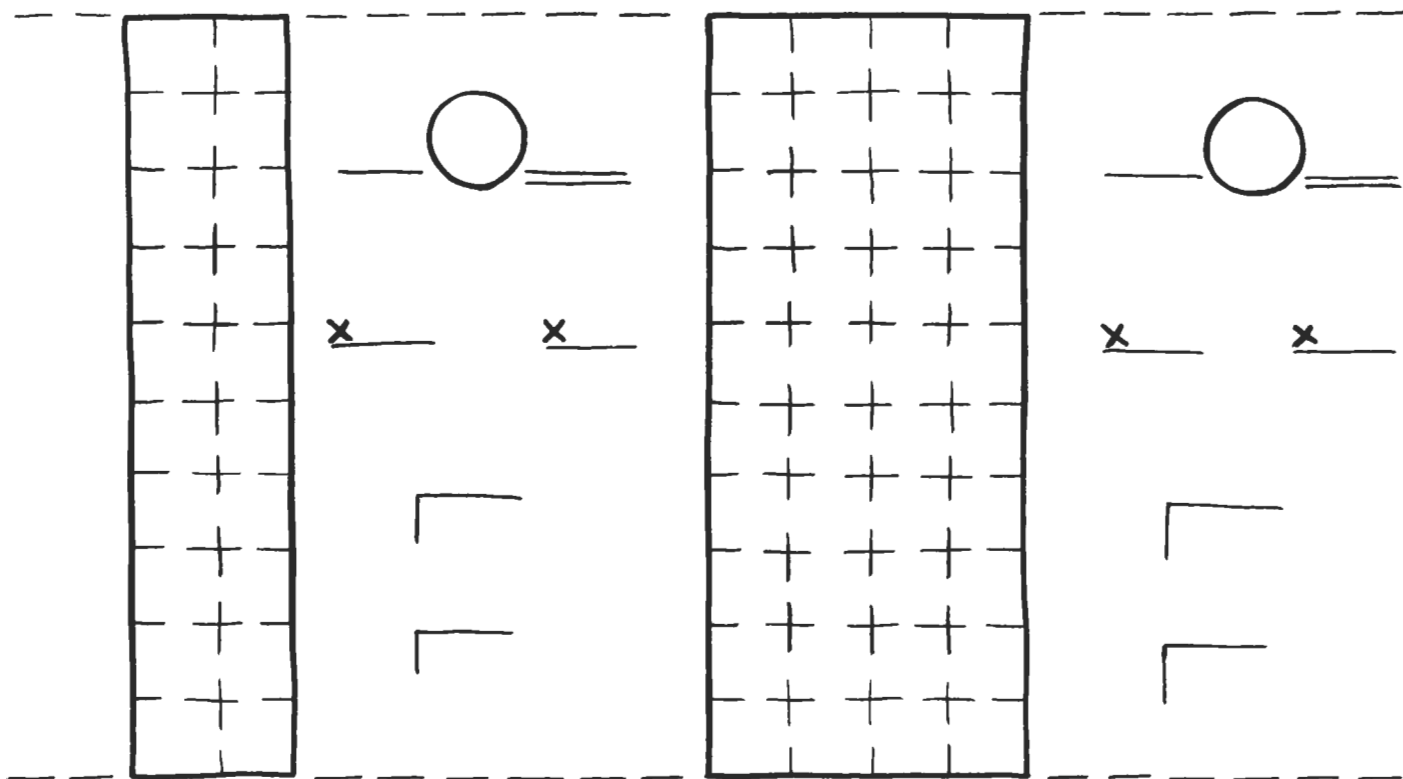
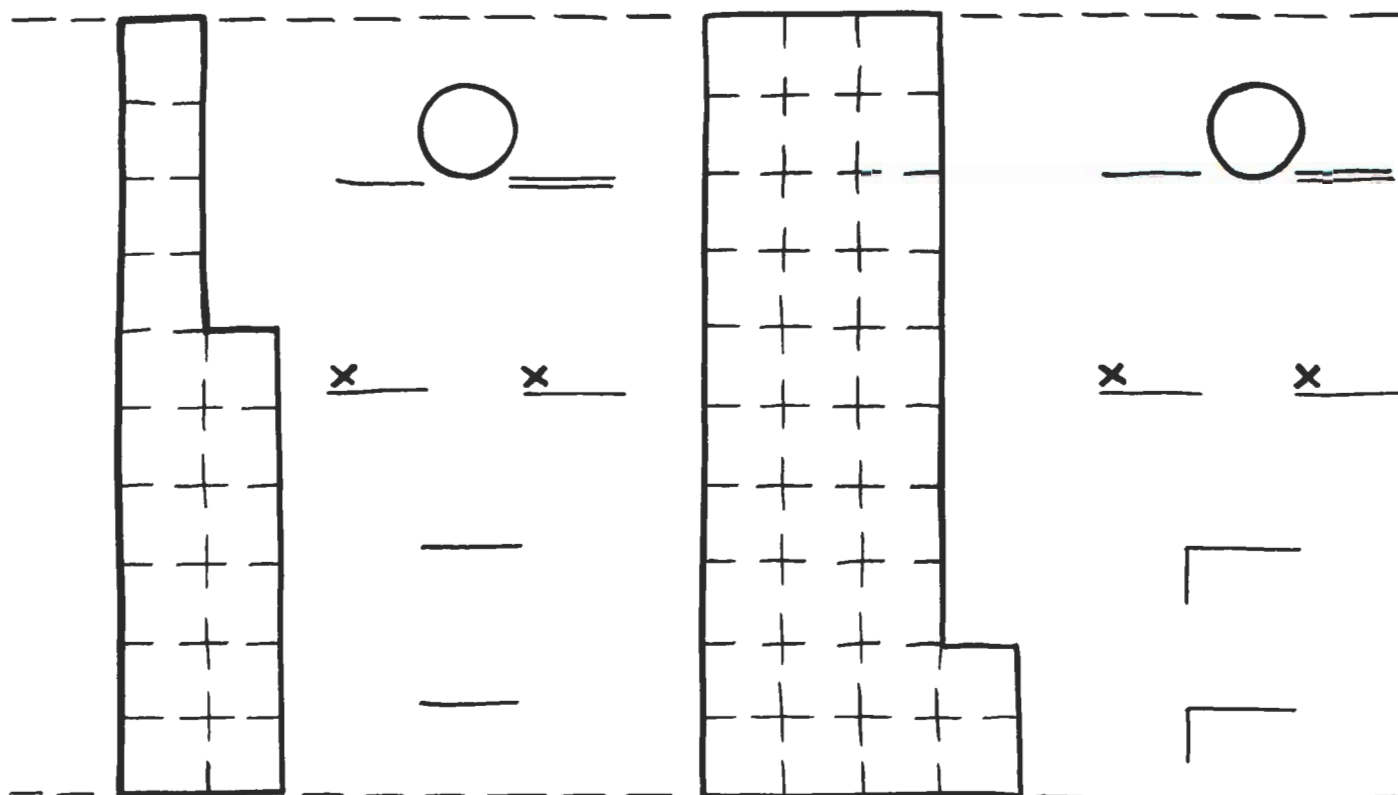
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└

| FENCING                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                   | NAME                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|-----------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  | <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; line-height: 30px; margin: 0 auto;">4's</div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span>x</span> <span>x</span> </div> <div style="margin-top: 10px;"> <div style="border-bottom: 1px solid black; width: 40px; margin-bottom: 5px;"></div> <div style="border-bottom: 1px solid black; width: 40px;"></div> </div> |  | <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; line-height: 30px; margin: 0 auto;">8's</div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span>x</span> <span>x</span> </div> <div style="margin-top: 10px;"> <div style="border-bottom: 1px solid black; width: 40px; margin-bottom: 5px;"></div> <div style="border-bottom: 1px solid black; width: 40px;"></div> </div> |
|  | <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; line-height: 30px; margin: 0 auto;">5's</div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span>x</span> <span>x</span> </div> <div style="margin-top: 10px;"> <div style="border-bottom: 1px solid black; width: 40px; margin-bottom: 5px;"></div> <div style="border-bottom: 1px solid black; width: 40px;"></div> </div> |  | <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; line-height: 30px; margin: 0 auto;">8's</div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span>x</span> <span>x</span> </div> <div style="margin-top: 10px;"> <div style="border-bottom: 1px solid black; width: 40px; margin-bottom: 5px;"></div> <div style="border-bottom: 1px solid black; width: 40px;"></div> </div> |

This is a continuation of the activities on the previous pages.

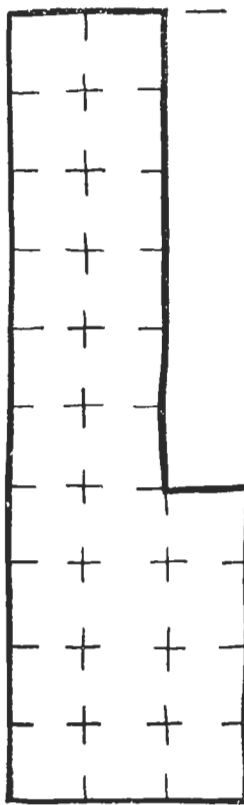
# FENCING CERCANDO



| FENCING |  | NAME _____ |  |
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Again, a continuation of previous activities. Larger arrays of blocks are used. See previous pages for suggestions for use.

# FENCING CERCANDO

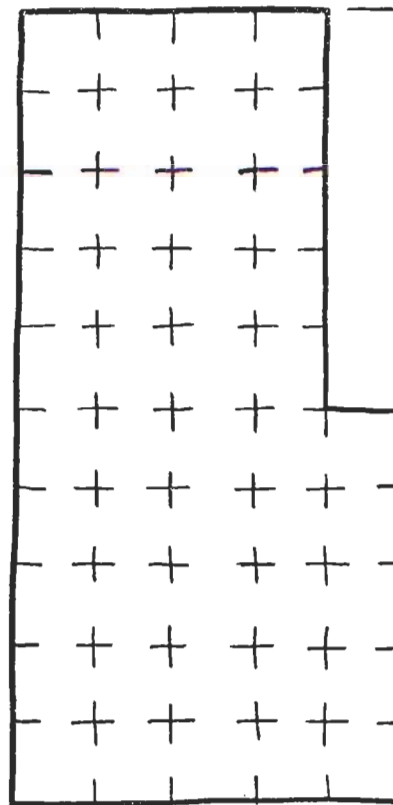


4's

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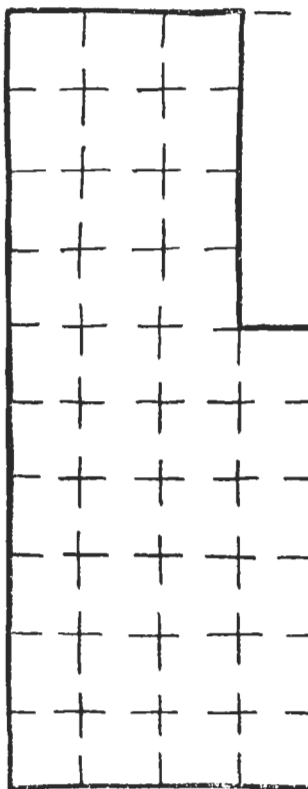


5's

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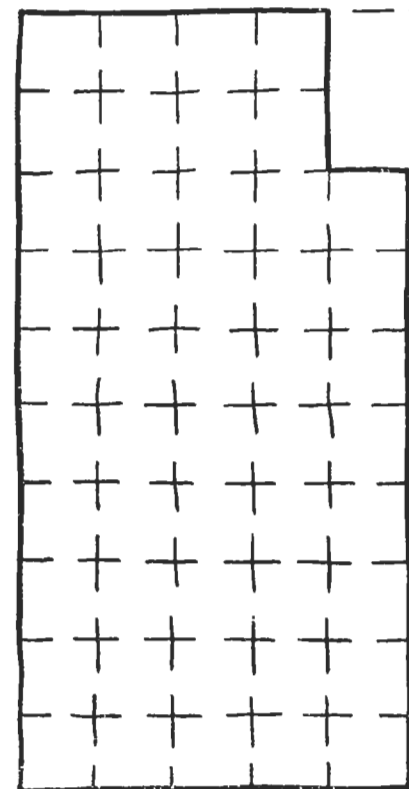


6's

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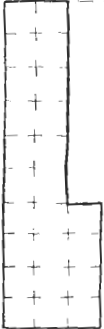
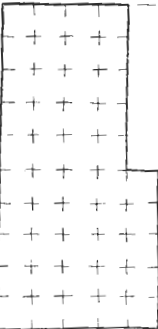
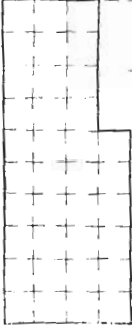
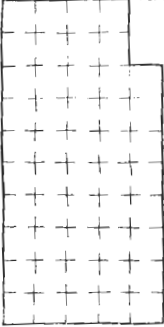


8's

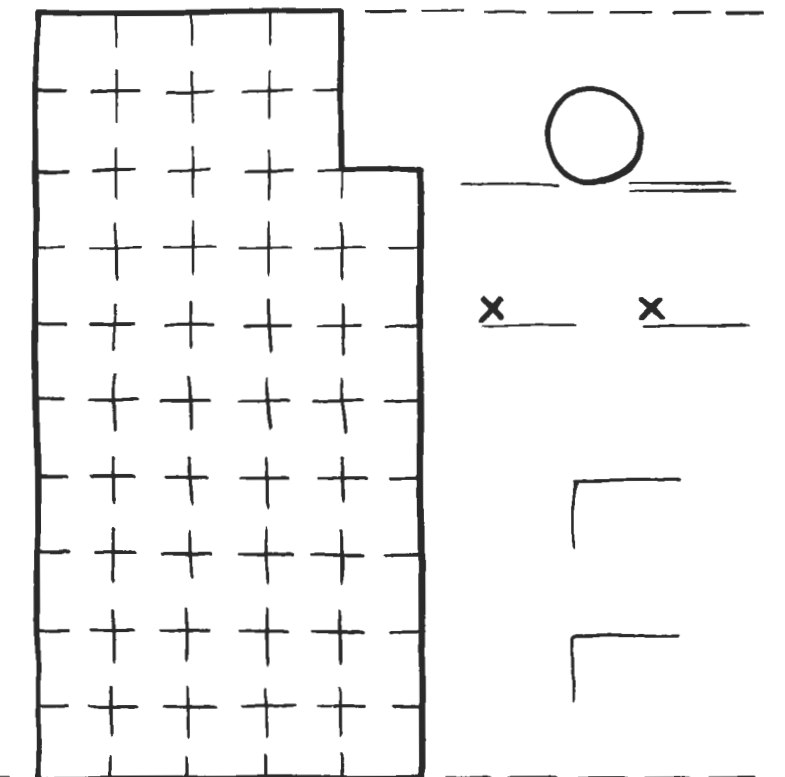
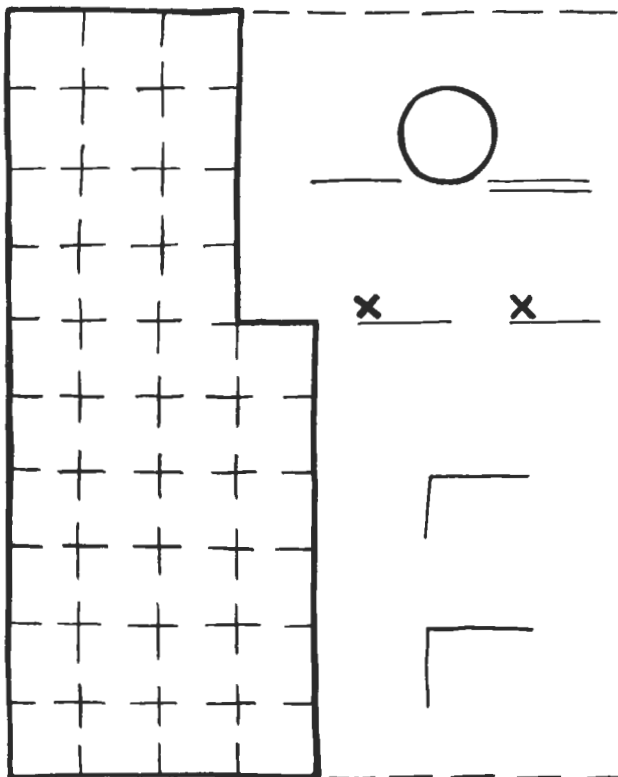
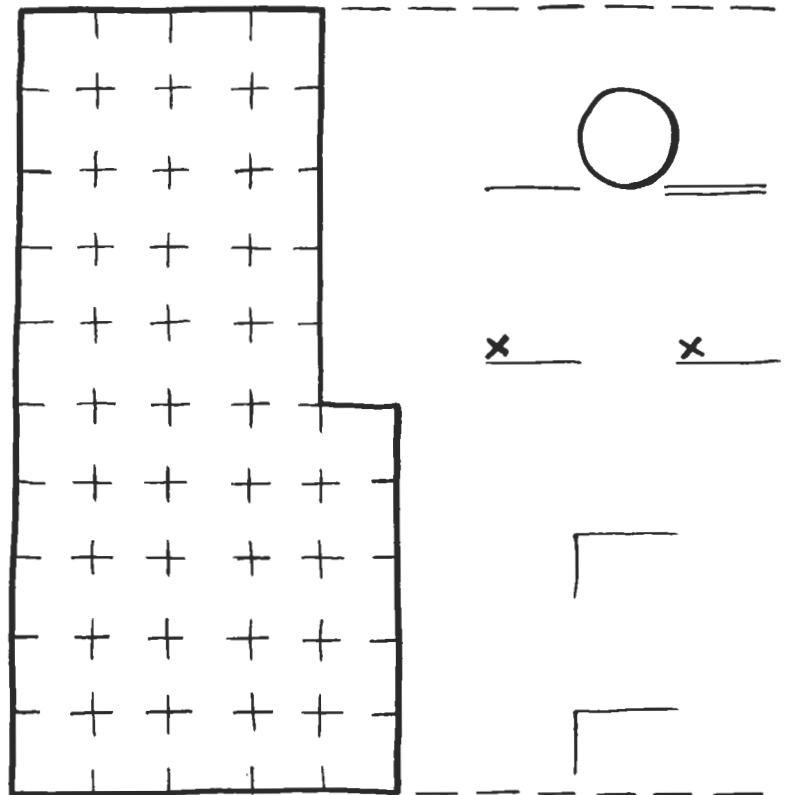
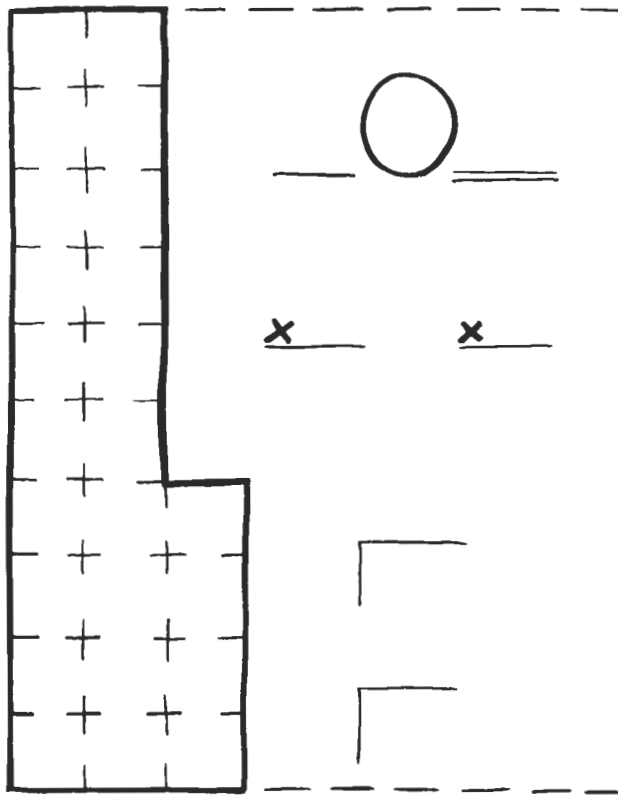
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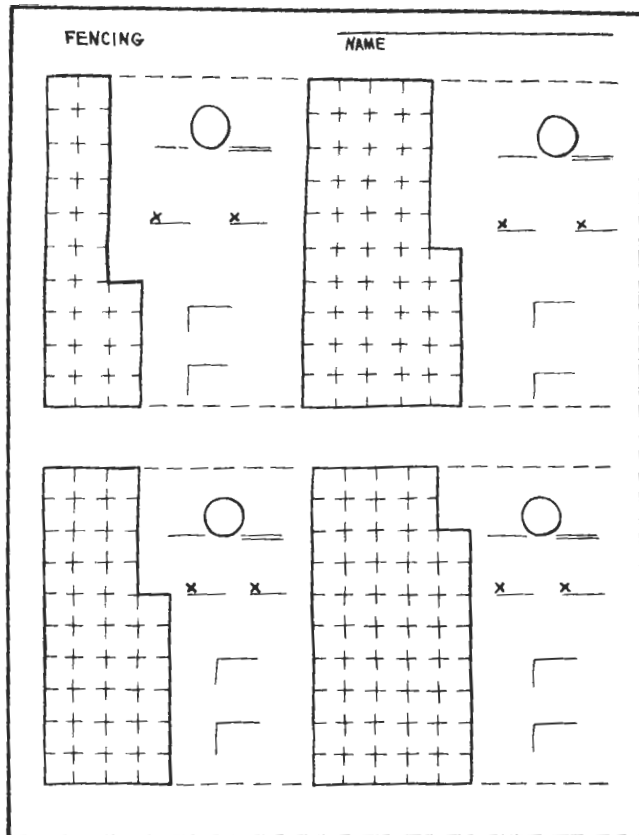
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| FENCING                                                                           |                                             | NAME _____                                                                        |                                             |
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|  | <p>(4's)</p> <p>x   x</p> <p>┌</p> <p>└</p> |  | <p>(5's)</p> <p>x   x</p> <p>┌</p> <p>└</p> |
|  | <p>(6's)</p> <p>x   x</p> <p>┌</p> <p>└</p> |  | <p>(8's)</p> <p>x   x</p> <p>┌</p> <p>└</p> |

This is a continuation of previous activities using larger arrays of blocks.







This is a continuation of previous activities. Again, learners might try to find how many different ways each array can be fenced. For example, 36 blocks has these possibilities:

|           |        |           |
|-----------|--------|-----------|
| <u>1</u>  | (36's) | <u>36</u> |
| <u>6</u>  | (6's)  | <u>36</u> |
| <u>12</u> | (3's)  | <u>36</u> |
| <u>9</u>  | (4's)  | <u>36</u> |
| <u>18</u> | (2's)  | <u>36</u> |

¿QUÉ PUEDES VER?  
WHAT CAN YOU SEE?

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$$\begin{array}{r} 13 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ \times 4 \\ \hline \end{array}$$

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$$\begin{array}{r} 15 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} 15 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 15 \\ \times 4 \\ \hline \end{array}$$

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$$\begin{array}{r} 23 \\ \times 2 \\ \hline \end{array}$$

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$$\begin{array}{r} 23 \\ \times 3 \\ \hline \end{array}$$

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$$\begin{array}{r} 23 \\ \times 4 \\ \hline \end{array}$$

| WHAT CAN YOU SEE ?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | NAME _____                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
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| <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> </div> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> </div>                                                                                                                                                                                                                                                                                                                                            | <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> </div> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> </div>                                                                                                                                                                                                                                                                                                                         |
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| <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> </div> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> </div> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> </div> | <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> </div> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> </div> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> </div> |
| <div style="display: flex; justify-content: space-around; align-items: flex-end;"> <div style="text-align: center;"> <math>\begin{array}{r} 13 \\ \times 2 \\ \hline \end{array}</math> </div> <div style="text-align: center;"> <math>\begin{array}{r} 13 \\ \times 3 \\ \hline \end{array}</math> </div> <div style="text-align: center;"> <math>\begin{array}{r} 13 \\ \times 4 \\ \hline \end{array}</math> </div> </div>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | <div style="display: flex; justify-content: space-around; align-items: flex-end;"> <div style="text-align: center;"> <math>\begin{array}{r} 15 \\ \times 2 \\ \hline \end{array}</math> </div> <div style="text-align: center;"> <math>\begin{array}{r} 15 \\ \times 3 \\ \hline \end{array}</math> </div> <div style="text-align: center;"> <math>\begin{array}{r} 15 \\ \times 4 \\ \hline \end{array}</math> </div> </div>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |

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|  |  |  |  |  | $\begin{array}{r} 23 \\ \times 4 \\ \hline \end{array}$ |

### What Can You See? (multiplication)

On this task page, learners have a chance to visualize large multiplications. In the diagrams, a  can be thought of as either a Dienes block rod or a 10-stick, and a  stands for a small cube or a bean. In any case  represents 10 units and  represents one unit.

The learner can look at the arrays and count the total beans (or cubes) by making use of the groupings.

For the first example,  $\begin{array}{r} 13 \\ \times 2 \\ \hline \end{array}$

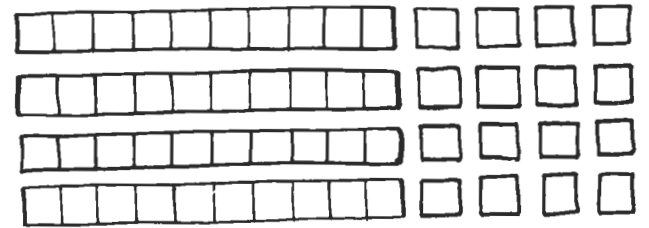
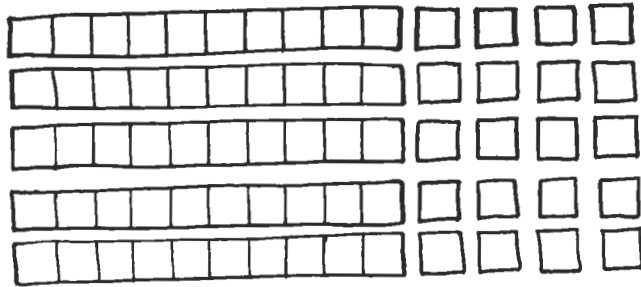
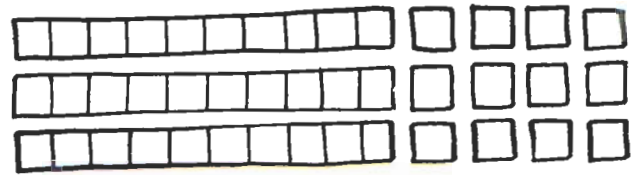
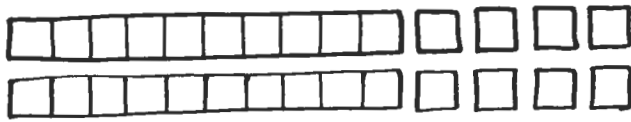
two rows are shown with 13 in each row.

There are 2 rods, or 20 cubes, and six loose cubes for a total of 26 cubes. Thus, the result can be recorded:

$$\begin{array}{r} 13 \\ \times 2 \\ \hline 26 \end{array}$$

Sketches are provided for each example. Children should be encouraged to count loose blocks by considering the groupings rather than by counting by ones. If children have difficulty, beansticks and loose beans or rods and cubes should be brought out.

WHAT CAN YOU SEE?  
¿QUÉ PUEDES VER?



$$\begin{array}{r} 14 \\ \times 2 \\ \hline \end{array}$$

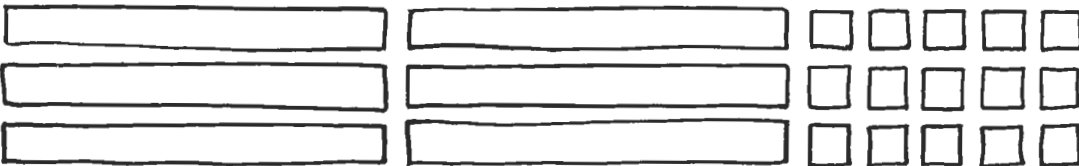
$$\begin{array}{r} 14 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 14 \\ \times 3 \\ \hline \end{array}$$

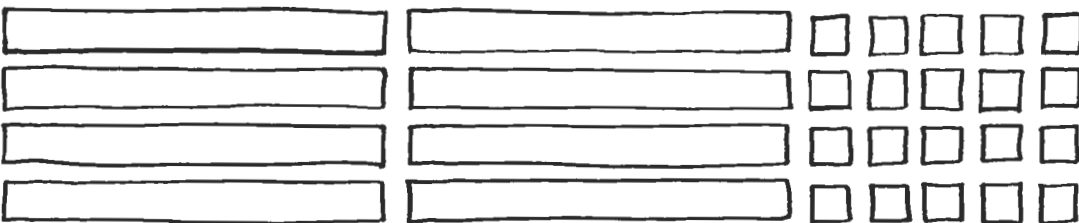
$$\begin{array}{r} 14 \\ \times 4 \\ \hline \end{array}$$



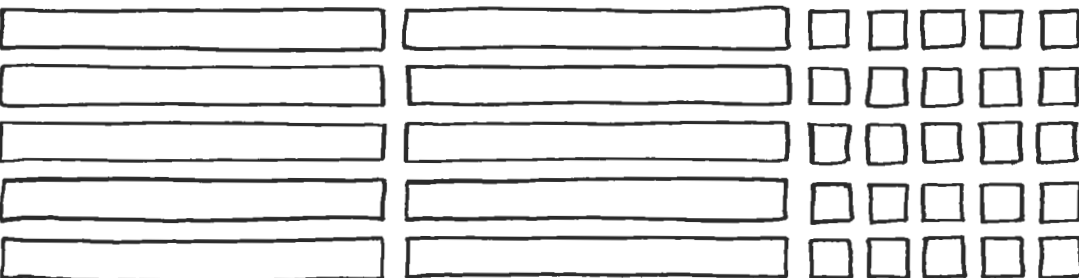
$$\begin{array}{r} 25 \\ \times 2 \\ \hline \end{array}$$



$$\begin{array}{r} 25 \\ \times 3 \\ \hline \end{array}$$

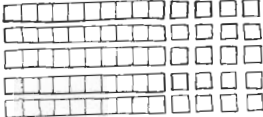
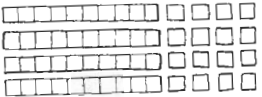


$$\begin{array}{r} 25 \\ \times 4 \\ \hline \end{array}$$

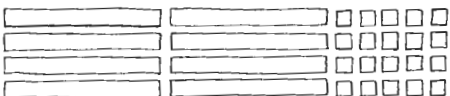


$$\begin{array}{r} 25 \\ \times 5 \\ \hline \end{array}$$

WHAT CAN YOU SEE? NAME \_\_\_\_\_

|                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                           |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <br><br>$\begin{array}{r} 14 \\ \times 2 \\ \hline \end{array}$ $\begin{array}{r} 14 \\ \times 5 \\ \hline \end{array}$ | <br><br>$\begin{array}{r} 14 \\ \times 3 \\ \hline \end{array}$ $\begin{array}{r} 14 \\ \times 4 \\ \hline \end{array}$ |
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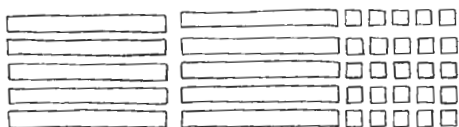
  

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| <br><br><br> | $\begin{array}{r} 25 \\ \times 2 \\ \hline \end{array}$ $\begin{array}{r} 25 \\ \times 3 \\ \hline \end{array}$ $\begin{array}{r} 25 \\ \times 4 \\ \hline \end{array}$ $\begin{array}{r} 25 \\ \times 5 \\ \hline \end{array}$ |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

### What Can You See? (multiplication)

This is a continuation of the activity on the previous page.

Here the learner must look at each sketch and figure out which multiplication it illustrates. For example, in the sketch



five rows are shown with 25 in each row. Thus, the multiplication being illustrated is

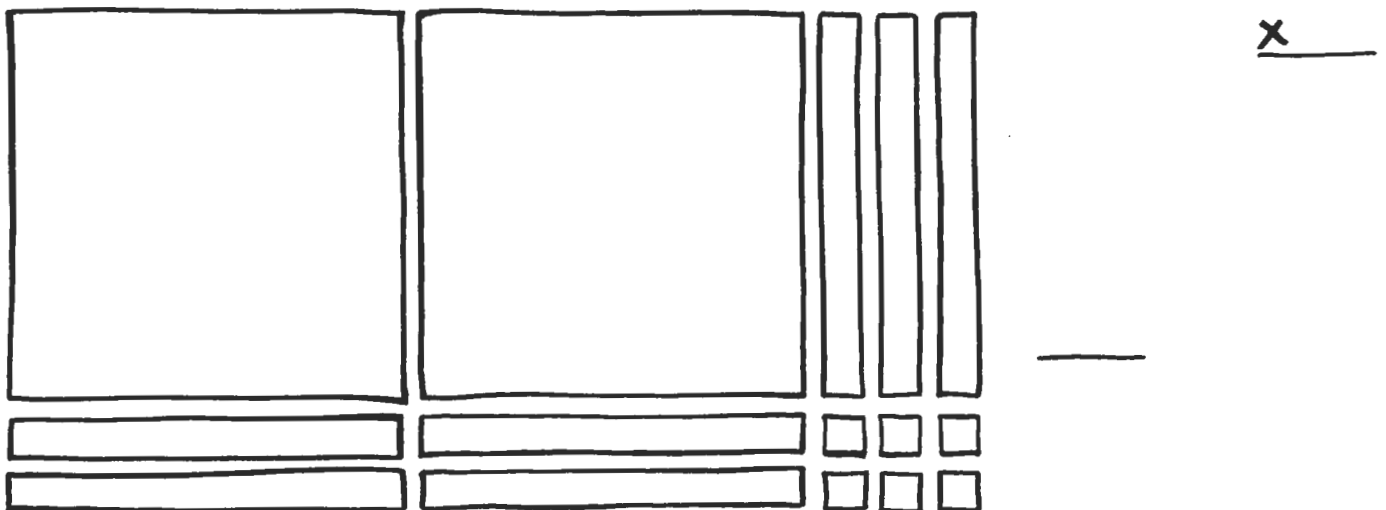
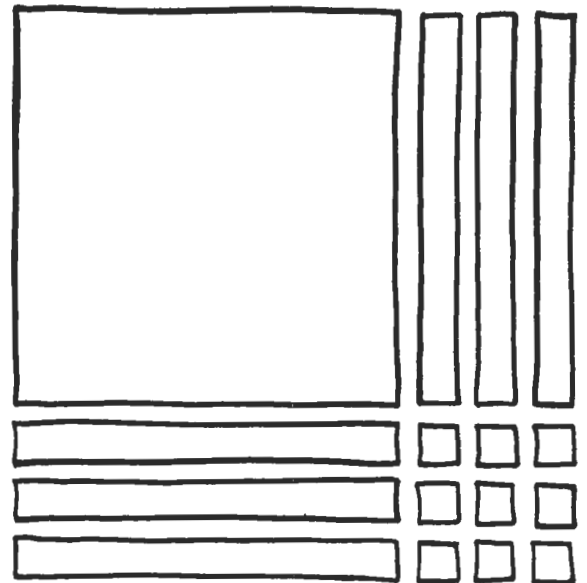
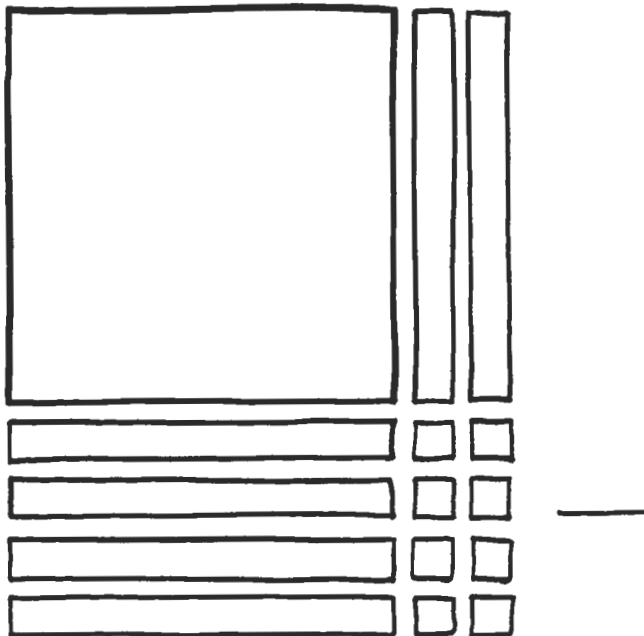
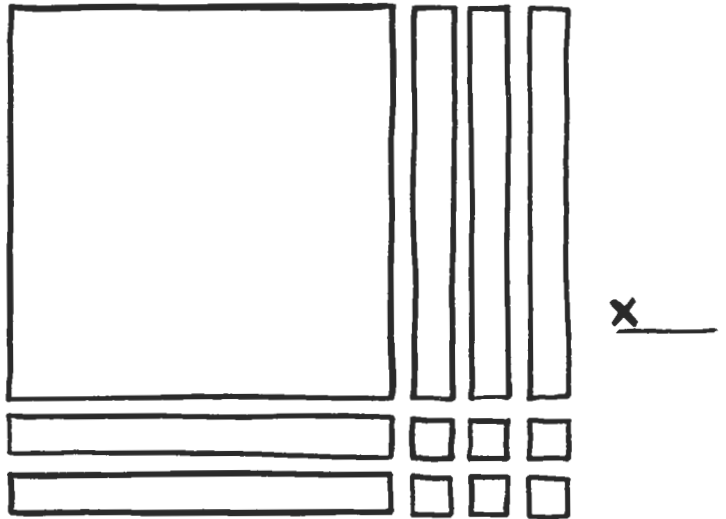
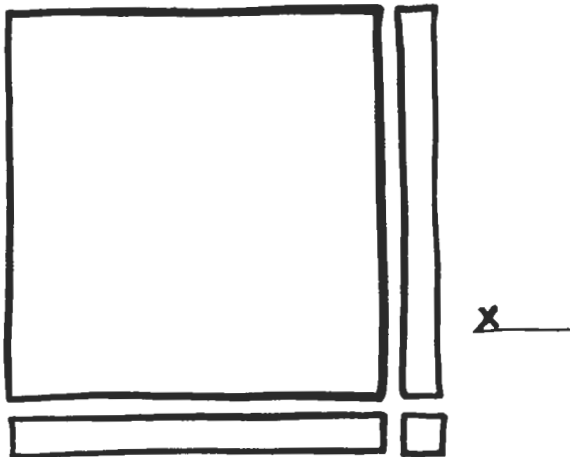
$$\begin{array}{r} 25 \\ \times 5 \\ \hline \end{array}$$

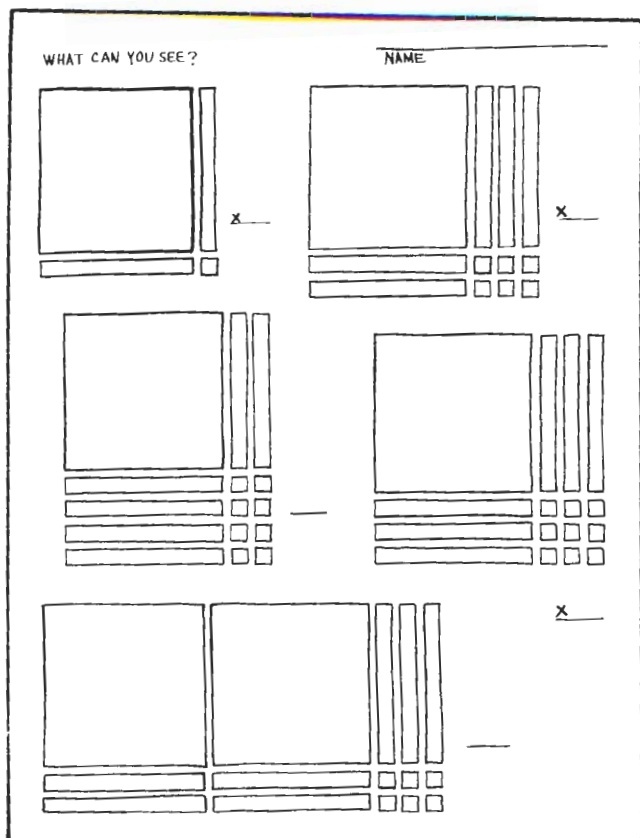
There are a total of 125 beans (or blocks) in the array, so we complete the record

$$\begin{array}{r} 25 \\ \times 5 \\ \hline 125 \end{array}$$

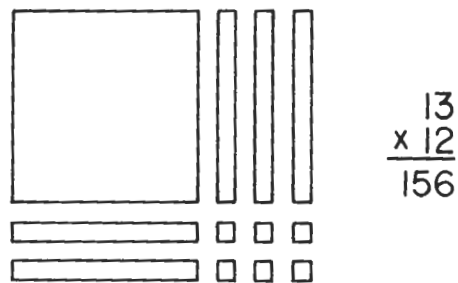
Children should be encouraged to use shortcuts to count loose beans. For example, after determining there are five loose beans in each column, a child should be encouraged to count the beans by fives, rather than by ones.

¿QUÉ PUEDES VER?  
WHAT CAN YOU SEE?





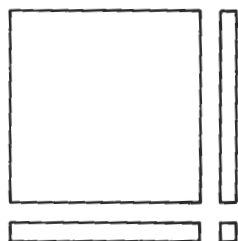
The second diagram shows an array of 12 rows with 13 in each row. There is a total of  $100 + 50 + 6$ , or 156 units. Thus,



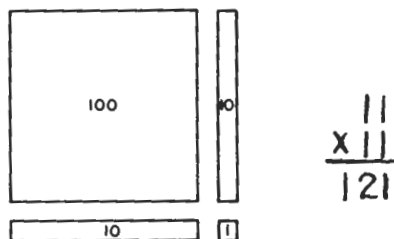
In these examples, we are simply concerned with making and counting arrays. No push toward the use of the algorithm and partial products is made. Children should be encouraged to count the units in the array by considering the rafts, then the sticks, and then the loose beans.

### What Can You See? (multiplication)

On this page, is introduced. It can be thought of as a Dienes 'flat' (100 little cubes or 10 rods) or a "hundred raft" (ten 10-sticks glued together). The learner must now look at the diagram and record the multiplication it illustrates. For example, the first diagram,



shows an array of eleven rows with eleven units in each row. There is a total of 100 (the raft) + 20 (two 10-sticks) + 1 (loose block), or 121 units all together. Thus, we can record:



WHAT CAN YOU SEE?  
¿QUÉ PUEDES VER?

$$\begin{array}{c} \text{=====} \text{:::} \\ \underline{\times 2} \end{array} \quad \begin{array}{c} 13 \\ \times 2 \end{array}$$

---


$$\begin{array}{c} \text{=====} \text{::::} \\ \underline{\times 2} \end{array} \quad \begin{array}{c} 15 \\ \times 2 \end{array}$$

$$\begin{array}{c} \text{=====} \text{:::} \\ \underline{\times 3} \end{array} \quad \begin{array}{c} 13 \\ \times 3 \end{array}$$

$$\begin{array}{c} \text{=====} \text{::::} \\ \underline{\times 3} \end{array} \quad \begin{array}{c} 16 \\ \times 3 \end{array}$$

$$\begin{array}{c} \text{=====} \text{:::} \\ \underline{\times 4} \end{array} \quad \begin{array}{c} 13 \\ \times 4 \end{array}$$

$$\begin{array}{c} \text{=====} \text{:::} \\ \underline{\times 4} \end{array} \quad \begin{array}{c} 15 \\ \times 4 \end{array}$$

$$\begin{array}{c} \text{=====} \text{:::} \\ \underline{\times 5} \end{array} \quad \begin{array}{c} 15 \\ \times 5 \end{array}$$

$$\begin{array}{c} \text{=====} \text{:::} \\ \underline{\times 5} \end{array} \quad \begin{array}{c} 17 \\ \times 5 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{::::} \\ \underline{\times 2} \end{array} \quad \begin{array}{c} 24 \\ \times 2 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{::::} \\ \underline{\times 2} \end{array} \quad \begin{array}{c} 25 \\ \times 2 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{:::} \\ \underline{\times 3} \end{array} \quad \begin{array}{c} 24 \\ \times 3 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{:::} \\ \underline{\times 3} \end{array} \quad \begin{array}{c} 27 \\ \times 3 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{:::} \\ \underline{\times 5} \end{array} \quad \begin{array}{c} 24 \\ \times 5 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{:::} \\ \underline{\times 5} \end{array} \quad \begin{array}{c} 27 \\ \times 5 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{=====} \text{:::} \\ \underline{\times 3} \end{array} \quad \begin{array}{c} 36 \\ \times 3 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{=====} \text{=====} \text{:::} \\ \underline{\times 4} \end{array} \quad \begin{array}{c} 48 \\ \times 4 \end{array}$$



WHAT CAN YOU SEE?



$$\begin{array}{r} 13 \\ \times 2 \\ \hline \end{array}$$



$$\begin{array}{r} 15 \\ \times 2 \\ \hline \end{array}$$



$$\begin{array}{r} 13 \\ \times 3 \\ \hline \end{array}$$



$$\begin{array}{r} 16 \\ \times 3 \\ \hline \end{array}$$



$$\begin{array}{r} 13 \\ \times 4 \\ \hline \end{array}$$



$$\begin{array}{r} 15 \\ \times 4 \\ \hline \end{array}$$



$$\begin{array}{r} 15 \\ \times 5 \\ \hline \end{array}$$



$$\begin{array}{r} 17 \\ \times 5 \\ \hline \end{array}$$



$$\begin{array}{r} 24 \\ \times 2 \\ \hline \end{array}$$



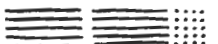
$$\begin{array}{r} 25 \\ \times 2 \\ \hline \end{array}$$



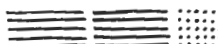
$$\begin{array}{r} 24 \\ \times 3 \\ \hline \end{array}$$



$$\begin{array}{r} 27 \\ \times 3 \\ \hline \end{array}$$



$$\begin{array}{r} 24 \\ \times 5 \\ \hline \end{array}$$



$$\begin{array}{r} 27 \\ \times 5 \\ \hline \end{array}$$



$$\begin{array}{r} 36 \\ \times 3 \\ \hline \end{array}$$



$$\begin{array}{r} 48 \\ \times 4 \\ \hline \end{array}$$

NAME \_\_\_\_\_

### What Can You See? (multiplication)

This activity is similar to the activity on page 121B. The diagrams have become more of a shorthand notation in which  stands for a rod or a 10-stick, and  stands for a small cube or a loose bean.

Each multiplication is sketched as a number of rows of beans with the same number in each row. The learner counts the total number of beans (or cubes) in the array. He should be encouraged to use shortcuts and count loose beans by multiples rather than by ones.

¿QUÉ PUEDES VER?  
WHAT CAN YOU SEE?

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

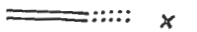
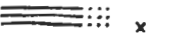
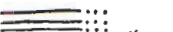
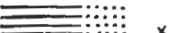
\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

WHAT CAN YOU SEE?

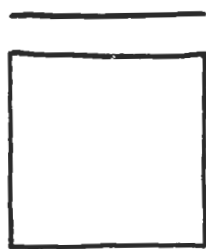
NAME \_\_\_\_\_

|                                                                                   |                                                                                   |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |
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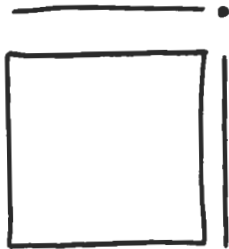
### What Can You See? (multiplication)

This is a continuation of the activity on the previous page. Note that children can check their work by comparing this page with page 122A.

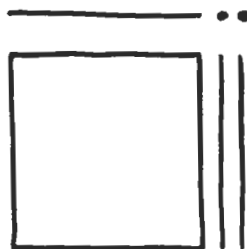
WHAT CAN YOU SEE?  
¿QUÉ PUEDES VER?



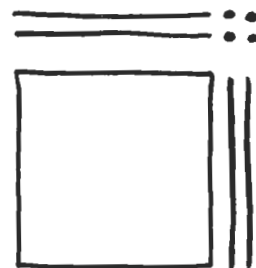
$$\begin{array}{r} 10 \\ \times 11 \\ \hline \end{array}$$



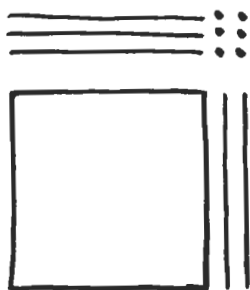
$$\begin{array}{r} 11 \\ \times 11 \\ \hline \end{array}$$



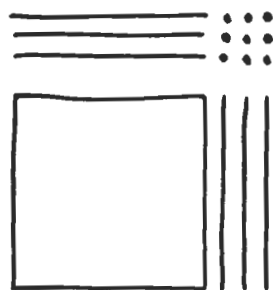
$$\begin{array}{r} 12 \\ \times 11 \\ \hline \end{array}$$



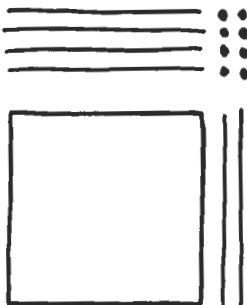
$$\begin{array}{r} 12 \\ \times 12 \\ \hline \end{array}$$



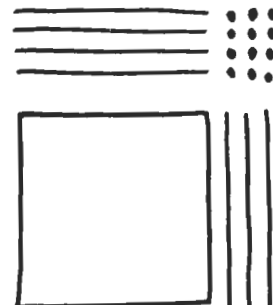
$$\begin{array}{r} 12 \\ \times 13 \\ \hline \end{array}$$



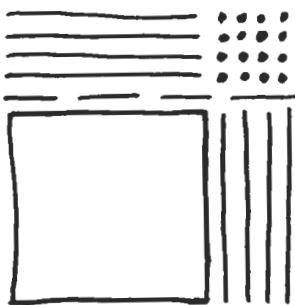
$$\begin{array}{r} 13 \\ \times 13 \\ \hline \end{array}$$



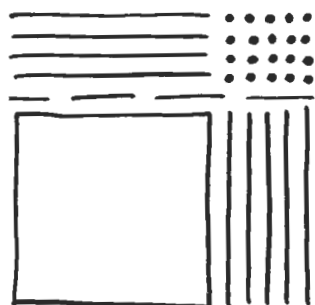
$$\begin{array}{r} 12 \\ \times 14 \\ \hline \end{array}$$



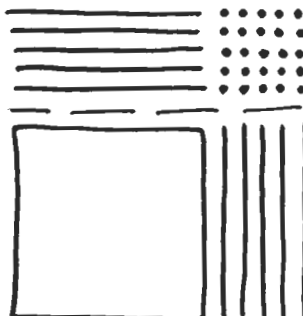
$$\begin{array}{r} 13 \\ \times 14 \\ \hline \end{array}$$



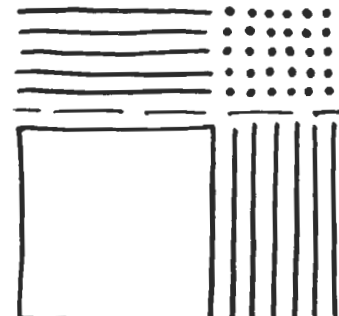
$$\begin{array}{r} 14 \\ \times 14 \\ \hline \end{array}$$



$$\begin{array}{r} 15 \\ \times 14 \\ \hline \end{array}$$

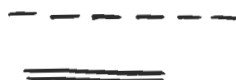


$$\begin{array}{r} 15 \\ \times 15 \\ \hline \end{array}$$



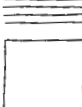
$$\begin{array}{r} 16 \\ \times 15 \\ \hline \end{array}$$

  
 ABOVE  
 BELOW  
 TOTAL



-----  
 ARRIBA  
 ABAJO

WHAT CAN YOU SEE? NAME \_\_\_\_\_

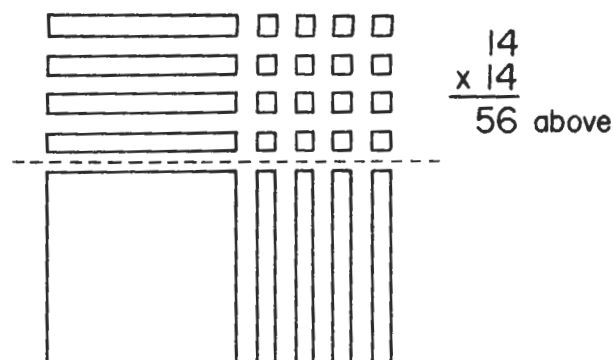
|                                                                                                                                              |                                                                                                                                              |                                                                                                                                              |                                                                                                                                              |
|----------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| <br>$\begin{array}{r} 10 \\ \times 1 \\ \hline \end{array}$ | <br>$\begin{array}{r} 11 \\ \times 1 \\ \hline \end{array}$ | <br>$\begin{array}{r} 12 \\ \times 1 \\ \hline \end{array}$ | <br>$\begin{array}{r} 12 \\ \times 2 \\ \hline \end{array}$ |
| <br>$\begin{array}{r} 12 \\ \times 3 \\ \hline \end{array}$ | <br>$\begin{array}{r} 13 \\ \times 3 \\ \hline \end{array}$ | <br>$\begin{array}{r} 12 \\ \times 4 \\ \hline \end{array}$ | <br>$\begin{array}{r} 13 \\ \times 4 \\ \hline \end{array}$ |
| <br>$\begin{array}{r} 14 \\ \times 4 \\ \hline \end{array}$ | <br>$\begin{array}{r} 15 \\ \times 4 \\ \hline \end{array}$ | <br>$\begin{array}{r} 15 \\ \times 5 \\ \hline \end{array}$ | <br>$\begin{array}{r} 16 \\ \times 5 \\ \hline \end{array}$ |

ABOVE  
 BELOW  
 TOTAL

This activity introduces a  as a sketch of a Dienes flat (100 cubes) or a hundred raft (ten 10-sticks glued together). As on previous pages, each multiplication is sketched as a number of rows with the same number of beans (blocks) in each row. The learner counts the total number of beans (cubes) in the array. Again, he should be encouraged to count by multiples rather than by ones when counting loose beans.

The last row of examples here begins to build toward the use of partial products and the multiplication algorithm. However, children should have many experiences with sketches of this type before the algorithm is formally introduced. (Some children may even discover it.) In these examples the sketches are divided into two parts by a dotted line. The learner should consider the sketch in these two parts. The number of beans (blocks) **above** the line should be counted and recorded **above** the dotted line under

the example. For example:



The number **below** the line should be counted and recorded **below** the dotted line. Then the total number of units in the sketch can be found by adding these two numbers.

$$\begin{array}{r}
 14 \\
 \times 14 \\
 \hline
 56 \text{ above} \\
 140 \text{ below} \\
 \hline
 196 \text{ Total}
 \end{array}$$

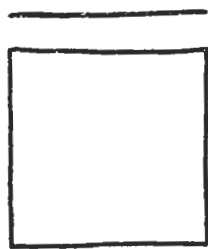
Later on, when you feel it appropriate to press on toward the algorithm, you might have children develop a table of multiples to use in conjunction with the diagram (see page 209). For example, the table:

|             |             |             |             |             |             |             |             |             |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 13          | 13          | 13          | 13          | 13          | 13          | 13          | 13          | 13          |
| $\times 1$  | $\times 2$  | $\times 3$  | $\times 4$  | $\times 5$  | $\times 6$  | $\times 7$  | $\times 8$  | $\times 9$  |
| 13          | 26          | 39          | 52          | 65          | 78          | 91          | 104         | 117         |
| 13          | 13          | 13          | 13          | 13          | 13          | 13          | 13          | 13          |
| $\times 10$ | $\times 20$ | $\times 30$ | $\times 40$ | $\times 50$ | $\times 60$ | $\times 70$ | $\times 80$ | $\times 90$ |
| 130         | 260         | 390         | 520         | 650         | 780         | 910         | 1040        | 1170        |

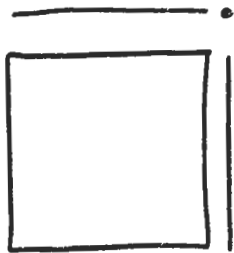
would be useful when considering examples like:

$$\begin{array}{r}
 13 \\
 \times 14 \\
 \hline
 52 \text{ above} \\
 130 \text{ below} \\
 \hline
 182 \text{ Total}
 \end{array}$$

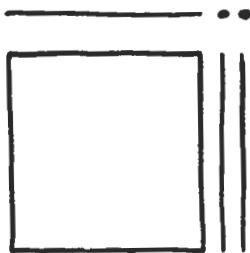
¿QUÉ PUEDES VER?  
WHAT CAN YOU SEE?



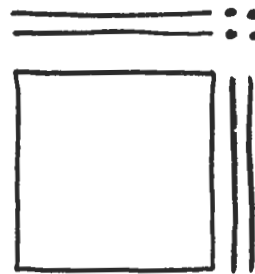
x



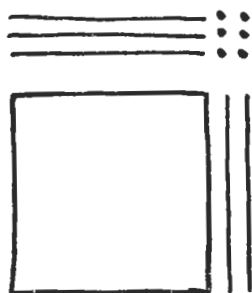
x



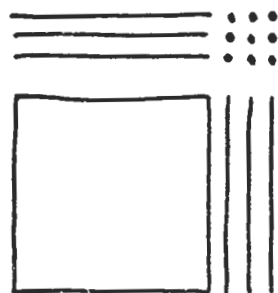
x



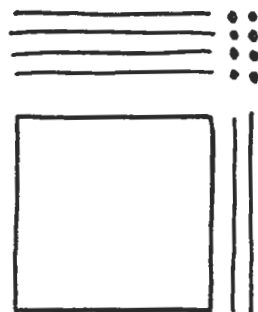
x



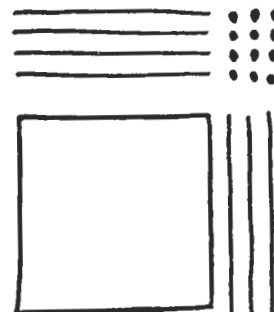
x



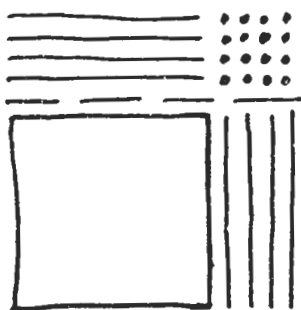
x



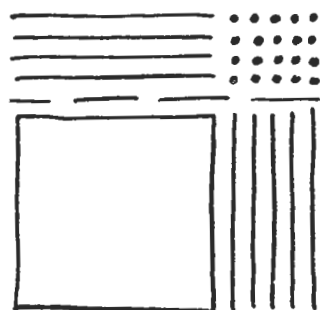
x



x

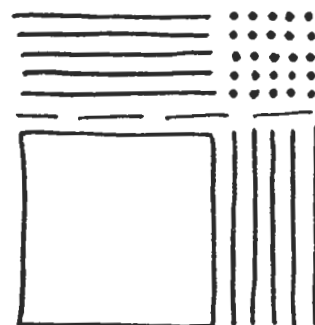


14  
x 14  
56 ABOVE  
BELOW  
TOTAL



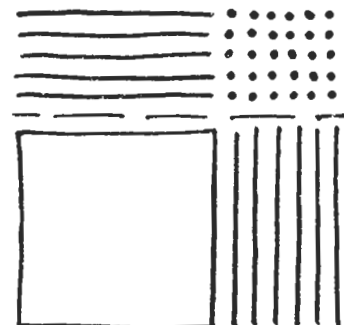
x

-----  
=====



x

-----  
=====

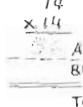


x

-----  
=====

ARRIBA  
ABAJO

WHAT CAN YOU SEE? NAME \_\_\_\_\_

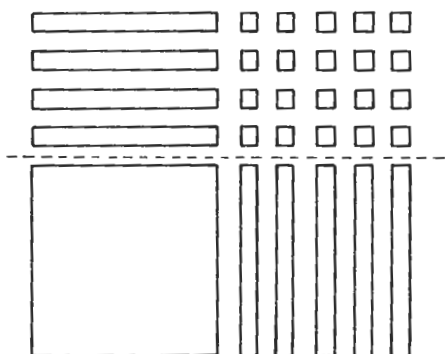
|                                                                                   |                                                                                   |                                                                                   |                                                                                   |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |  |  |
| x _____                                                                           | x _____                                                                           | x _____                                                                           | x _____                                                                           |
|  |  |  |  |
| x _____                                                                           | x _____                                                                           | x _____                                                                           | x _____                                                                           |
|  |  |  |  |
|  | x _____                                                                           | x _____                                                                           | x _____                                                                           |
| 14<br>x 14                                                                        |                                                                                   |                                                                                   |                                                                                   |
| ABOVE                                                                             |                                                                                   |                                                                                   |                                                                                   |
| BELOW                                                                             |                                                                                   |                                                                                   |                                                                                   |
| TOTAL                                                                             |                                                                                   |                                                                                   |                                                                                   |

Above the dotted line, there are 60 beans. Below the line are 150 beans. Thus, the learner would record:

$$\begin{array}{r}
 15 \\
 \times 14 \\
 \hline
 60 \text{ above} \\
 150 \text{ below} \\
 \hline
 210 \text{ Total}
 \end{array}$$

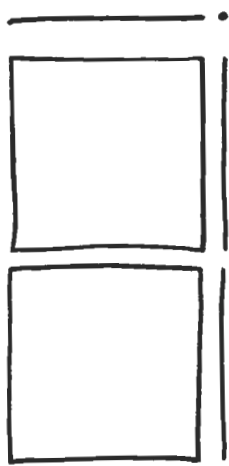
Note that children can check their own work by comparing this page to page 126A.

Here, the learner must look at each sketch and figure out which multiplication it illustrates. Once again, examples are given in the last row which begin to build toward the use of partial products and the multiplication algorithm. Once the learner figures out what multiplication is indicated by the array, he can count the number of beans above the dotted line and below the dotted line and then add these numbers to get the total. For example:

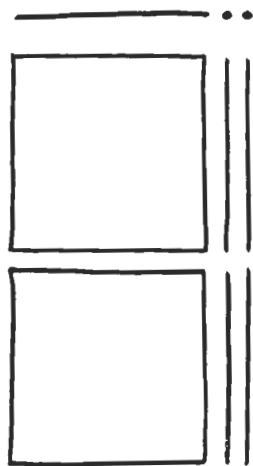


shows 14 rows with 15 beans (blocks) in each row. Thus it represents the multiplication:

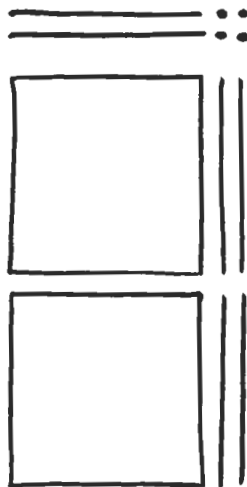
$$\begin{array}{r}
 15 \\
 \times 14 \\
 \hline
 \end{array}$$



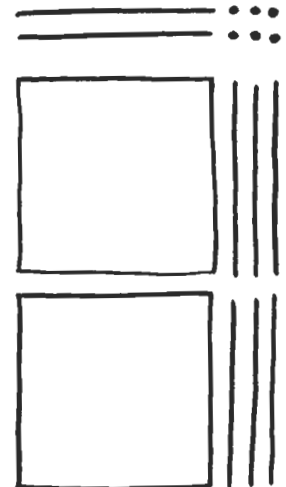
$$\begin{array}{r} 11 \\ \times 21 \\ \hline \end{array}$$



$$\begin{array}{r} 12 \\ \times 21 \\ \hline \end{array}$$



$$\begin{array}{r} 12 \\ \times 22 \\ \hline \end{array}$$



$$\begin{array}{r} 13 \\ \times 22 \\ \hline \end{array}$$



$$\begin{array}{r} 24 \\ \times 12 \\ \hline \end{array}$$

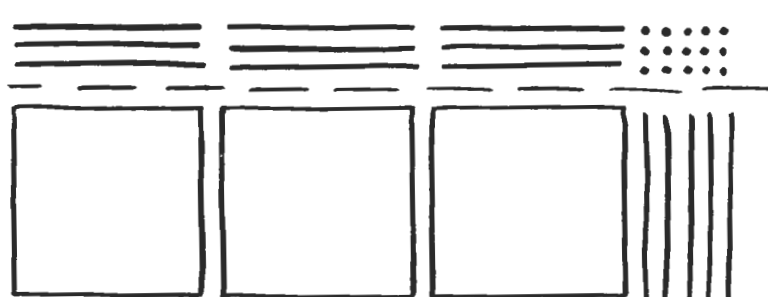


$$\begin{array}{r} 33 \\ \times 12 \\ \hline \end{array}$$



$$\begin{array}{r} 24 \\ \times 13 \\ \hline \end{array}$$

----- ABOVE  
----- BELOW  
===== TOTAL



$$\begin{array}{r} 35 \\ \times 13 \\ \hline \end{array}$$

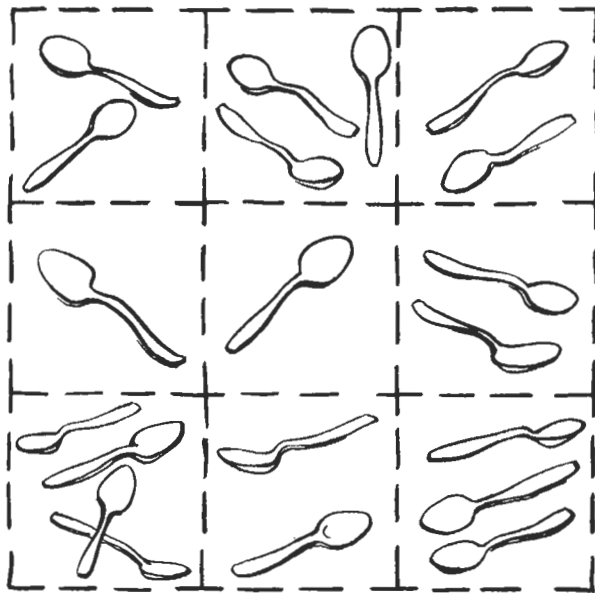
----- ARRIBA  
----- ABAJO  
===== TOTAL



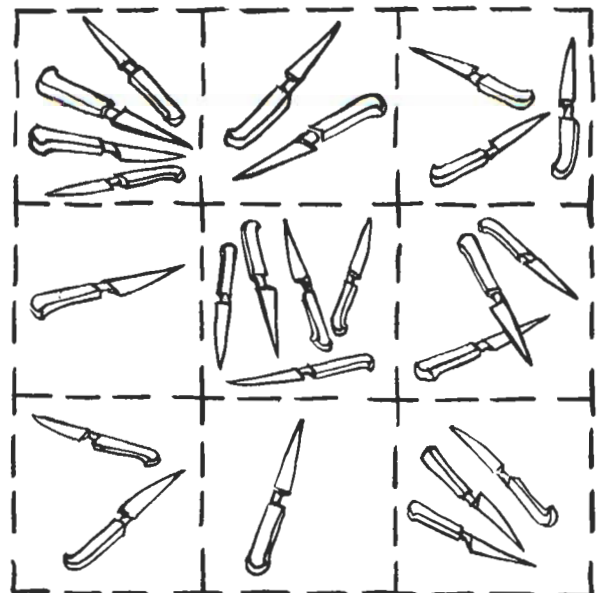
WHAT CAN YOU SEE? NAME \_\_\_\_\_

|                                                                                                                                               |                                                                                                                                               |                                                                                                                                               |                                                                                                                                               |
|-----------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| <br>$\begin{array}{r} 11 \\ \times 21 \\ \hline \end{array}$ | <br>$\begin{array}{r} 12 \\ \times 21 \\ \hline \end{array}$ | <br>$\begin{array}{r} 12 \\ \times 22 \\ \hline \end{array}$ | <br>$\begin{array}{r} 13 \\ \times 22 \\ \hline \end{array}$ |
| <br>$\begin{array}{r} 24 \\ \times 12 \\ \hline \end{array}$ | <br>$\begin{array}{r} 33 \\ \times 12 \\ \hline \end{array}$ |                                                                                                                                               |                                                                                                                                               |
| <br>$\begin{array}{r} 24 \\ \times 13 \\ \hline \end{array}$ | <br>$\begin{array}{r} 35 \\ \times 13 \\ \hline \end{array}$ |                                                                                                                                               |                                                                                                                                               |
| <p>ABOVE</p> <p>BELOW</p> <p>TOTAL</p>                                                                                                        | <p>ABOVE</p> <p>BELOW</p> <p>TOTAL</p>                                                                                                        |                                                                                                                                               |                                                                                                                                               |

This activity is a continuation of the activities on the previous pages. The multiplications use larger numbers.



— (5's)



— (6's)

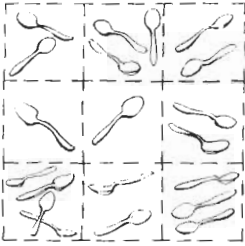
|   |   |   |
|---|---|---|
| 2 | 3 | 2 |
| 3 | 1 | 4 |
| 2 | 2 | 1 |

— (5's)

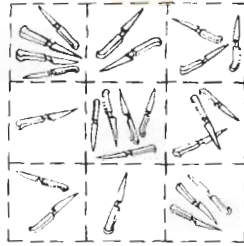
|   |   |   |
|---|---|---|
| 1 | 2 | 4 |
| 3 | 2 | 2 |
| 2 | 1 | 1 |

— (6's)

NAME \_\_\_\_\_



— (5's)



— (6's)

|   |   |   |
|---|---|---|
| 2 | 3 | 2 |
| 3 | 1 | 4 |
| 2 | 2 | 1 |

— (5's)

|   |   |   |
|---|---|---|
| 1 | 2 | 4 |
| 3 | 2 | 2 |
| 2 | 1 | 1 |

— (6's)

For other fencing activities, see pages 94-103.

On this page we make the transition from pictures to numerals. The children should have little difficulty making the transition. Tell them to do the bottom squares the same way they do the top squares.

Remember, try to get the greatest number of fences.

|   |   |   |
|---|---|---|
| 1 | 3 | 4 |
| 6 | 5 | 2 |
| 1 | 2 | 4 |

— (7's)

|   |   |   |
|---|---|---|
| 4 | 6 | 2 |
| 1 | 1 | 5 |
| 3 | 7 | 3 |

— (8's)

|   |   |   |
|---|---|---|
| 1 | 6 | 2 |
| 5 | 2 | 3 |
| 2 | 3 | 3 |

— (9's)

|   |   |   |
|---|---|---|
| 8 | 2 | 7 |
| 4 | 3 | 3 |
| 6 | 2 | 5 |

— (10's)

NAME \_\_\_\_\_

|   |   |   |
|---|---|---|
| 1 | 3 | 4 |
| 6 | 5 | 2 |
| 1 | 2 | 4 |

— (7's)

|   |   |   |
|---|---|---|
| 4 | 6 | 2 |
| 1 | 1 | 5 |
| 3 | 7 | 3 |

— (8's)

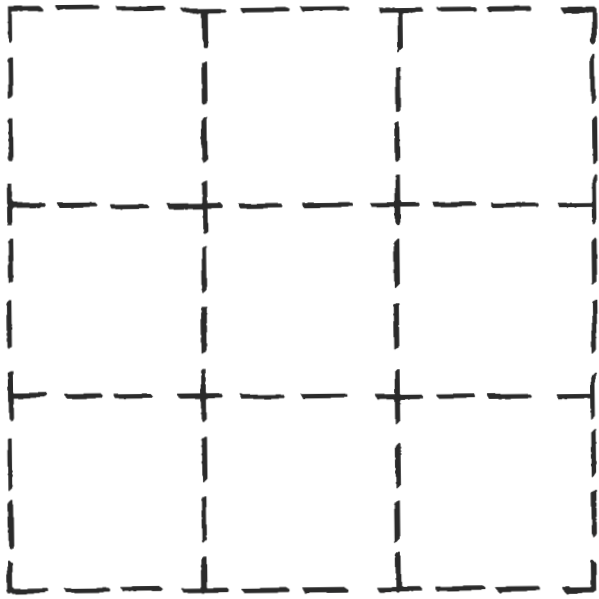
|   |   |   |
|---|---|---|
| 1 | 6 | 2 |
| 5 | 2 | 3 |
| 2 | 3 | 3 |

— (9's)

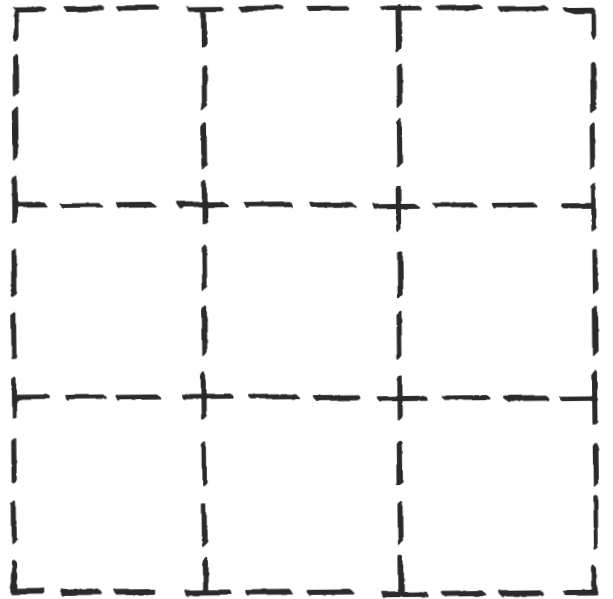
|   |   |   |
|---|---|---|
| 8 | 2 | 7 |
| 4 | 3 | 3 |
| 6 | 2 | 5 |

— (10's)

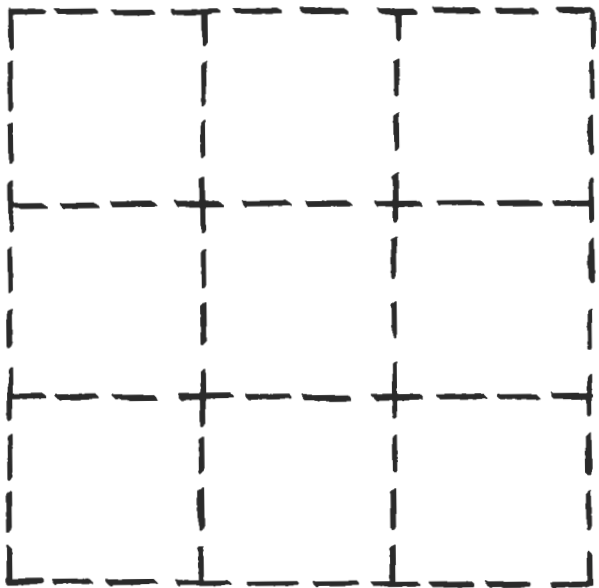
The numerals in the boxes and the sums get larger.



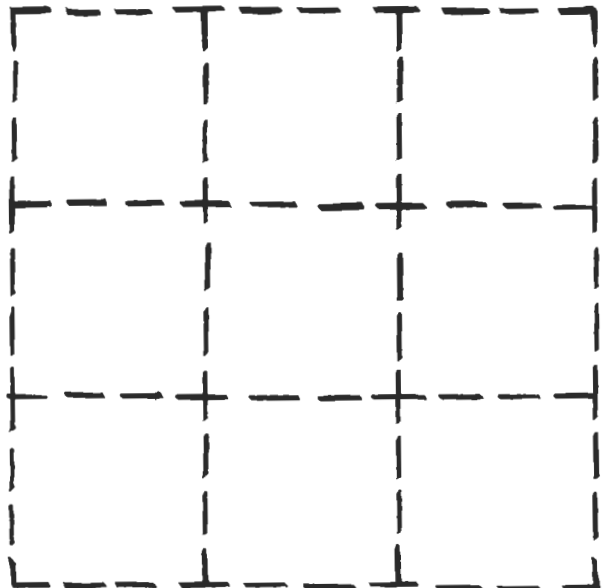
— (s)



— (s)

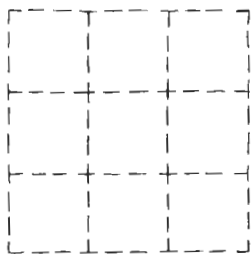
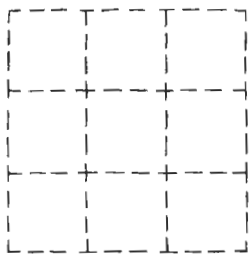
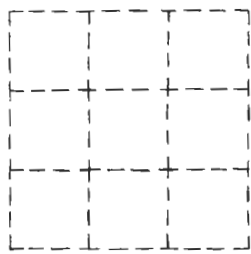
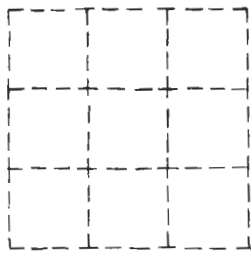


— (s)



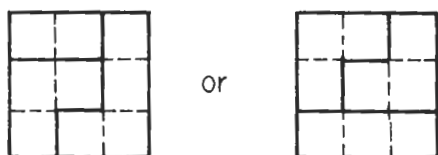
— (s)

NAME \_\_\_\_\_

|                                                                                                |                                                                                                |
|------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------|
| <br>_____ (s) | <br>_____ (s) |
| <br>_____ (s) | <br>_____ (s) |

And you can be sure there is at least 1 way to get all the numbers fenced in—though there may be other ways than the originating fences.

To make up examples of "fence arithmetic" of this kind, start with a 3 by 3 arrangement with fences drawn.



After choosing the sum desired, such as 9, put combinations of 9 in each fence such as:

|   |   |   |
|---|---|---|
| 6 | 3 | 3 |
| 2 | 4 | 3 |
| 3 | 1 | 2 |

or

|   |   |   |
|---|---|---|
| 3 | 3 | 2 |
| 3 | 4 | 3 |
| 6 | 1 | 2 |

Then write in the numbers without the fences:

|   |   |   |
|---|---|---|
| 6 | 3 | 3 |
| 2 | 4 | 3 |
| 3 | 1 | 2 |

or

|   |   |   |
|---|---|---|
| 3 | 3 | 2 |
| 3 | 4 | 3 |
| 6 | 1 | 2 |

|   |   |   |
|---|---|---|
| 1 | 2 | 4 |
| 5 | 3 | 3 |
| 3 | 2 | 1 |

\_\_\_ (6's)

|   |   |   |
|---|---|---|
| 1 | 4 | 2 |
| 2 | 2 | 3 |
| 1 | 2 | 4 |

\_\_\_ (7's)

|   |   |   |
|---|---|---|
| 4 | 1 | 6 |
| 3 | 2 | 2 |
| 1 | 2 | 3 |

\_\_\_ (8's)

|   |   |   |
|---|---|---|
| 4 | 3 | 1 |
| 3 | 2 | 5 |
| 2 | 3 | 4 |

\_\_\_ (9's)

|   |   |   |
|---|---|---|
| 1 | 4 | 6 |
| 2 | 4 | 5 |
| 3 | 3 | 2 |

\_\_\_ (10's)

|   |   |   |
|---|---|---|
| 2 | 3 | 1 |
| 1 | 4 | 2 |
| 1 | 1 | 6 |

\_\_\_ (7's)

|   |   |   |
|---|---|---|
| 2 | 6 | 1 |
| 3 | 1 | 7 |
| 4 | 5 | 3 |

\_\_\_ (8's)

|   |   |   |
|---|---|---|
| 5 | 2 | 7 |
| 4 | 3 | 3 |
| 1 | 8 | 3 |

\_\_\_ (9's)

|   |   |   |
|---|---|---|
| 4 | 7 | 3 |
| 3 | 5 | 1 |
| 2 | 1 | 4 |

\_\_\_ (10's)



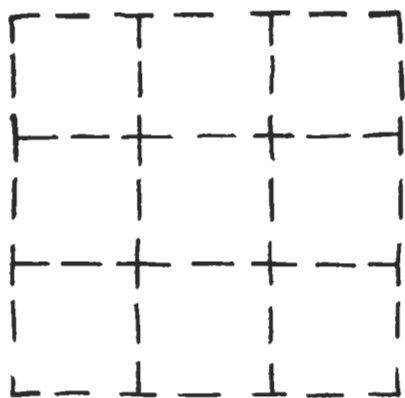
NAME \_\_\_\_\_

|         |          |          |
|---------|----------|----------|
| 1 2 4   | 1 4 2    | 4 1 6    |
| 5 3 3   | 2 2 3    | 3 2 2    |
| 3 2 1   | 1 2 4    | 1 2 3    |
| — (6's) | — (7's)  | — (8's)  |
| 4 3 1   | 1 4 6    | 2 3 1    |
| 3 2 5   | 2 4 5    | 1 4 2    |
| 2 3 4   | 3 3 2    | 1 1 6    |
| — (9's) | — (10's) | — (7's)  |
| 2 6 1   | 5 2 7    | 4 7 3    |
| 3 1 7   | 4 3 3    | 3 5 1    |
| 4 5 3   | 1 8 3    | 2 1 4    |
| — (8's) | — (9's)  | — (10's) |

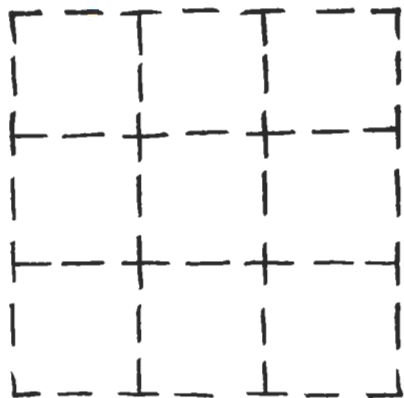
After finding one way to fence all numbers in all examples, consider providing another copy of this or the following page and the question:

“In how many examples are there two different ways to fence all numbers in?” (There is only 1—fencing the (9's) in the second row.)

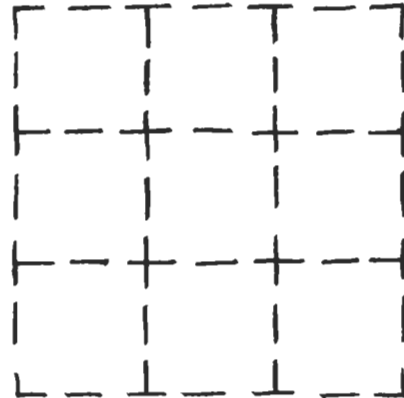
“Can you make up examples so that all numbers can be fenced in more than one way?”



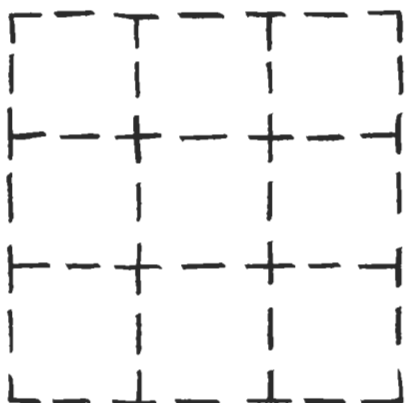
— (s)



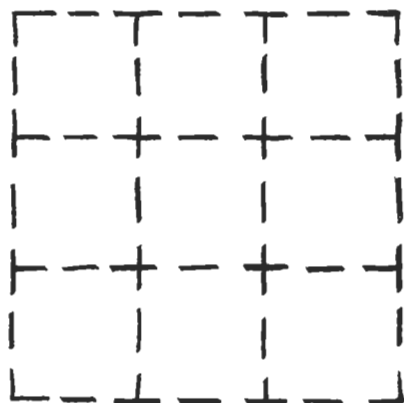
— (s)



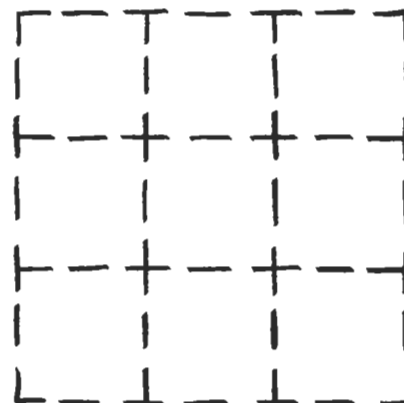
— (s)



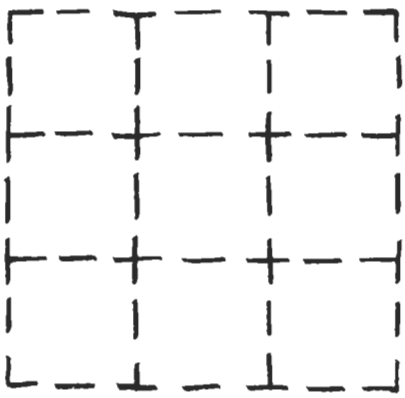
— (s)



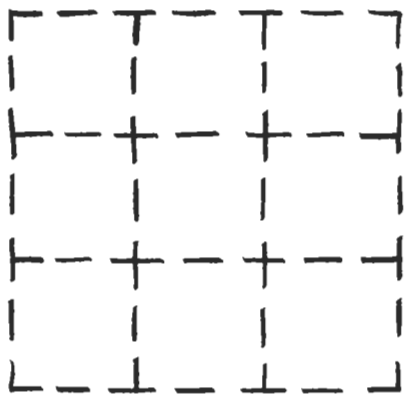
— (s)



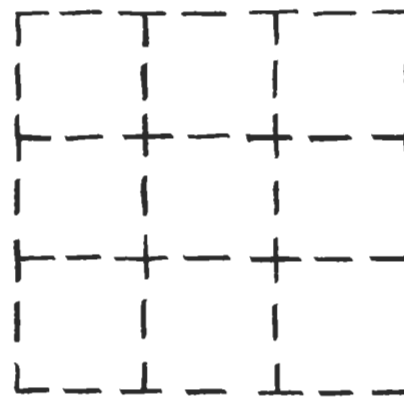
— (s)



— (s)



— (s)



— (s)

NAME \_\_\_\_\_

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;"> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center;"> <span style="font-size: 1.2em;">—</span> <span style="font-size: 1.2em;">'s</span> </div> </div> | <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;"> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center;"> <span style="font-size: 1.2em;">—</span> <span style="font-size: 1.2em;">'s</span> </div> </div> | <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;"> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center;"> <span style="font-size: 1.2em;">—</span> <span style="font-size: 1.2em;">'s</span> </div> </div> |
| <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;"> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center;"> <span style="font-size: 1.2em;">—</span> <span style="font-size: 1.2em;">'s</span> </div> </div> | <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;"> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center;"> <span style="font-size: 1.2em;">—</span> <span style="font-size: 1.2em;">'s</span> </div> </div> | <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;"> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center;"> <span style="font-size: 1.2em;">—</span> <span style="font-size: 1.2em;">'s</span> </div> </div> |
| <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;"> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center;"> <span style="font-size: 1.2em;">—</span> <span style="font-size: 1.2em;">'s</span> </div> </div> | <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;"> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center;"> <span style="font-size: 1.2em;">—</span> <span style="font-size: 1.2em;">'s</span> </div> </div> | <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;"> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center;"> <span style="font-size: 1.2em;">—</span> <span style="font-size: 1.2em;">'s</span> </div> </div> |

If two numbers are omitted and the final record completed, the problem grows.

|   |   |   |
|---|---|---|
|   | 6 | 1 |
| 3 | 1 | 7 |
| 4 |   | 3 |

4 8's

The sum of the given numbers is 25—so the missing pair must have a sum of 7: so the pair is far from determined yet.

The problem can be extended. If you find a pair that leads to fencing in 4 (8's), could some other pair of numbers be used?

A variation of the "fencing problem" leaves some of the numbers out.

|   |   |   |
|---|---|---|
| 2 | 3 | 1 |
| 1 | 4 | 2 |
| 1 | 1 |   |

or

|   |   |   |
|---|---|---|
| 4 |   | 3 |
| 3 | 5 | 1 |
| 2 | 1 | 4 |

— 7's

— 10's

These are harder than they look at first glance—if all numbers are to be fenced. Try them.

They are 2 examples from the previous page with the largest number missing. Other difficult examples can be made in a similar fashion.

Of course, there is a simple way to find these numbers. If all are to be fenced in, then the sum of all numbers must be a multiple of the sum that is specified—7's and 10's in the example above.

The total of numbers given in the first example is 15—or 6 less than the next largest multiple of 7.

The total of numbers given in the second example is 23—or 7 less than the next largest multiple of 10.

|   |   |   |   |
|---|---|---|---|
| 2 | 1 | 3 | 5 |
| 5 | 1 | 5 | 7 |
| 6 | 3 | 2 | 1 |
| 2 | 5 | 4 | 4 |

— (8's)

|   |   |   |   |
|---|---|---|---|
| 5 | 4 | 2 | 3 |
| 3 | 2 | 6 | 4 |
| 6 | 1 | 7 | 6 |
| 4 | 5 | 2 | 3 |

— (9's)

|   |   |   |   |
|---|---|---|---|
| 1 | 9 | 3 | 3 |
| 6 | 2 | 8 | 4 |
| 4 | 5 | 2 | 3 |
| 5 | 5 | 3 | 7 |

— (10's)

|   |   |   |   |
|---|---|---|---|
| 4 | 4 | 7 | 2 |
| 4 | 3 | 3 | 9 |
| 3 | 3 | 2 | 3 |
| 5 | 3 | 3 | 8 |

— (11's)

|   |   |   |   |
|---|---|---|---|
| 4 | 8 | 5 | 5 |
| 7 | 9 | 3 | 2 |
| 2 | 5 | 6 | 8 |
| 3 | 7 | 6 | 4 |

— (12's)

|   |   |   |   |
|---|---|---|---|
| 3 | 3 | 4 | 6 |
| 3 | 8 | 5 | 3 |
| 1 | 1 | 2 | 7 |
| 3 | 5 | 3 | 6 |

— (9's)

|   |   |   |   |
|---|---|---|---|
| 6 | 4 | 2 | 8 |
| 4 | 5 | 3 | 1 |
| 5 | 1 | 2 | 9 |
| 7 | 3 | 4 | 6 |

— (10's)

|   |   |   |   |
|---|---|---|---|
| 2 | 5 | 6 | 8 |
| 9 | 6 | 2 | 3 |
| 3 | 3 | 4 | 7 |
| 4 | 4 | 9 | 2 |

— (11's)

|   |   |   |   |
|---|---|---|---|
| 4 | 8 | 3 | 9 |
| 6 | 6 | 3 | 5 |
| 4 | 5 | 4 | 7 |
| 4 | 4 | 8 | 4 |

— (12's)

NAME \_\_\_\_\_

|          |   |   |   |          |   |   |   |          |   |   |   |
|----------|---|---|---|----------|---|---|---|----------|---|---|---|
| 2        | 1 | 3 | 5 | 5        | 4 | 2 | 3 | 1        | 9 | 3 | 3 |
| 5        | 1 | 5 | 7 | 3        | 2 | 6 | 4 | 6        | 2 | 8 | 4 |
| 6        | 3 | 2 | 1 | 6        | 1 | 7 | 6 | 4        | 5 | 2 | 3 |
| 2        | 5 | 4 | 4 | 4        | 5 | 2 | 3 | 5        | 5 | 3 | 7 |
| — (8's)  |   |   |   | — (9's)  |   |   |   | — (10's) |   |   |   |
| 4        | 4 | 7 | 2 | 4        | 8 | 5 | 5 | 3        | 3 | 4 | 6 |
| 4        | 3 | 3 | 9 | 7        | 9 | 3 | 2 | 3        | 8 | 5 | 3 |
| 3        | 3 | 2 | 3 | 2        | 5 | 6 | 8 | 1        | 1 | 2 | 7 |
| 5        | 3 | 3 | 8 | 3        | 7 | 6 | 4 | 3        | 5 | 3 | 6 |
| — (11's) |   |   |   | — (12's) |   |   |   | — (9's)  |   |   |   |
| 6        | 4 | 2 | 8 | 2        | 5 | 6 | 8 | 4        | 8 | 3 | 9 |
| 4        | 5 | 3 | 1 | 9        | 6 | 2 | 3 | 6        | 6 | 3 | 5 |
| 5        | 1 | 2 | 9 | 3        | 3 | 4 | 7 | 4        | 5 | 4 | 7 |
| 7        | 3 | 4 | 6 | 4        | 4 | 9 | 2 | 4        | 4 | 8 | 4 |
| — (10's) |   |   |   | — (11's) |   |   |   | — (12's) |   |   |   |

(1) There is only a single 1 neighboring 7 to make 8.

(2) After that "5 needs 3"

(3) and "4 needs 4."

Then everything falls into place: "6 needs 2" and "5 needs 3" and the two 5, 2 and 1 combinations are left.

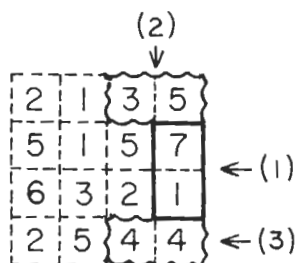
These are not "secrets" we hope children don't uncover. Rather, finding such useful strategies is the essence of mathematics. And group discussion of these systems should be encouraged as wise use of "instructional time."

Not only do the numbers in the boxes and the sums get larger, but the 9 box square grows to a 16 box square. As a child works through the square, many decisions are made and there will often be a need to go back and change some fences. To avoid the need for erasing constantly a "scribble sheet" is provided as the next page.

However, there is a useful strategy someone may discover. It might be explained this way:

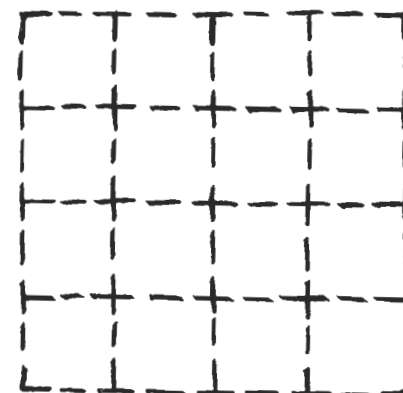
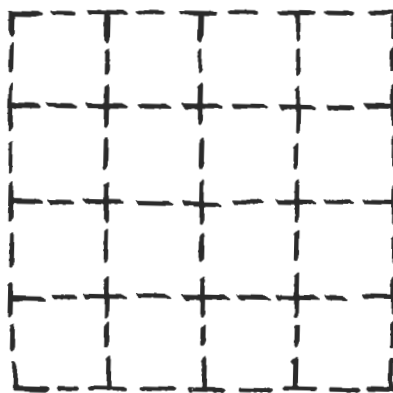
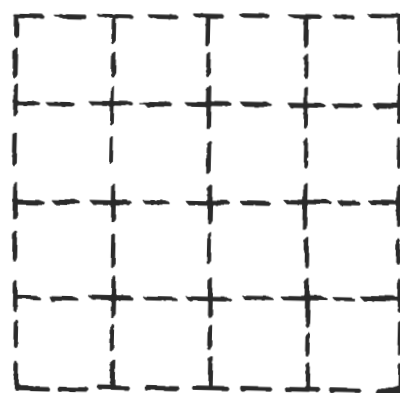
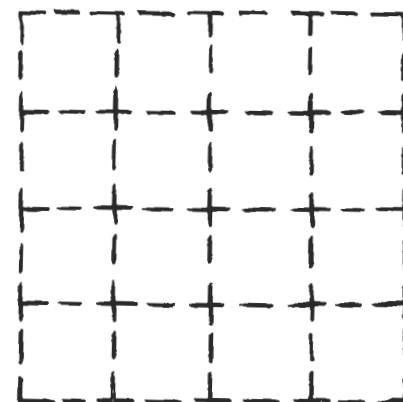
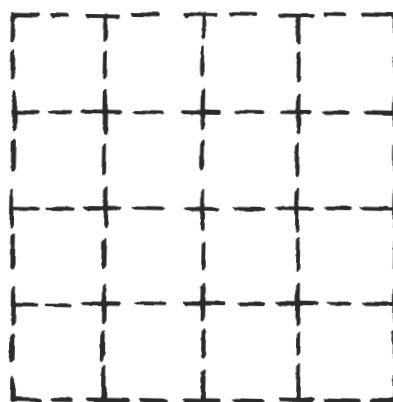
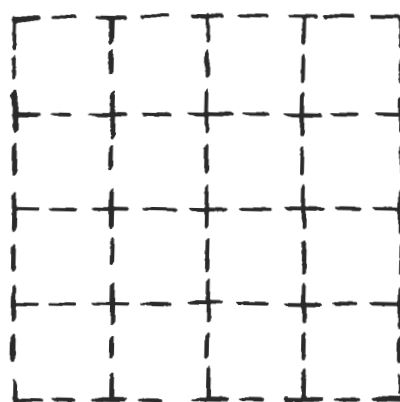
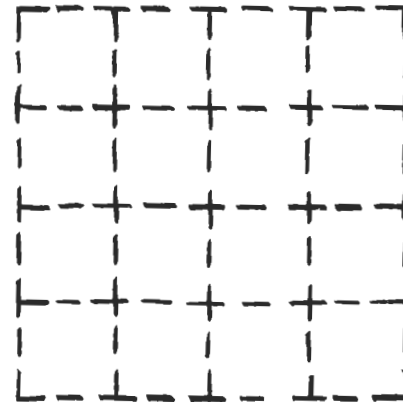
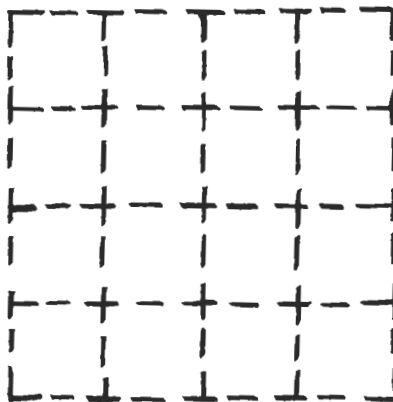
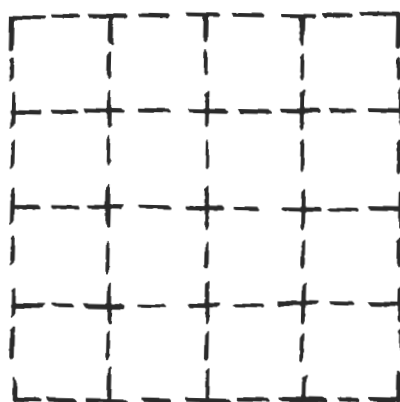
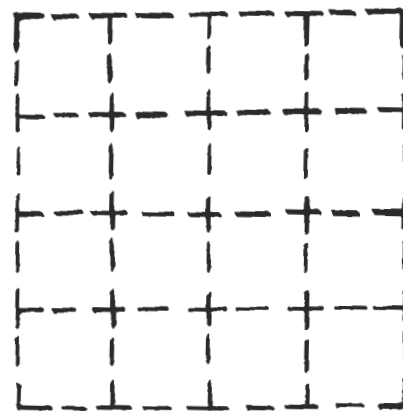
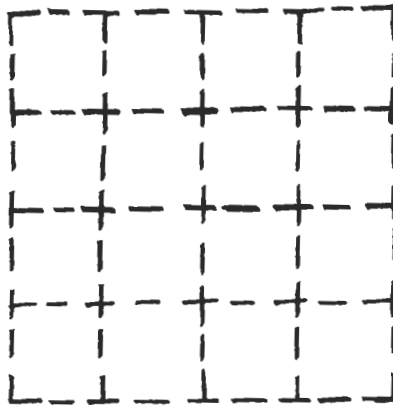
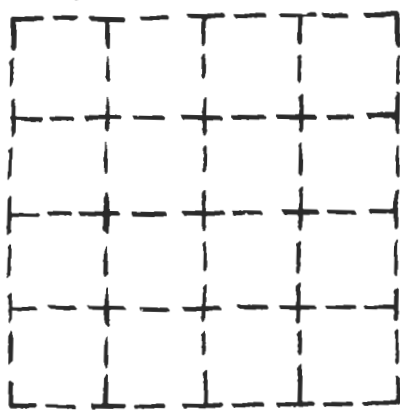
First look for the larger numbers. If there is only one way to fence them with a neighbor, that's a good beginning.

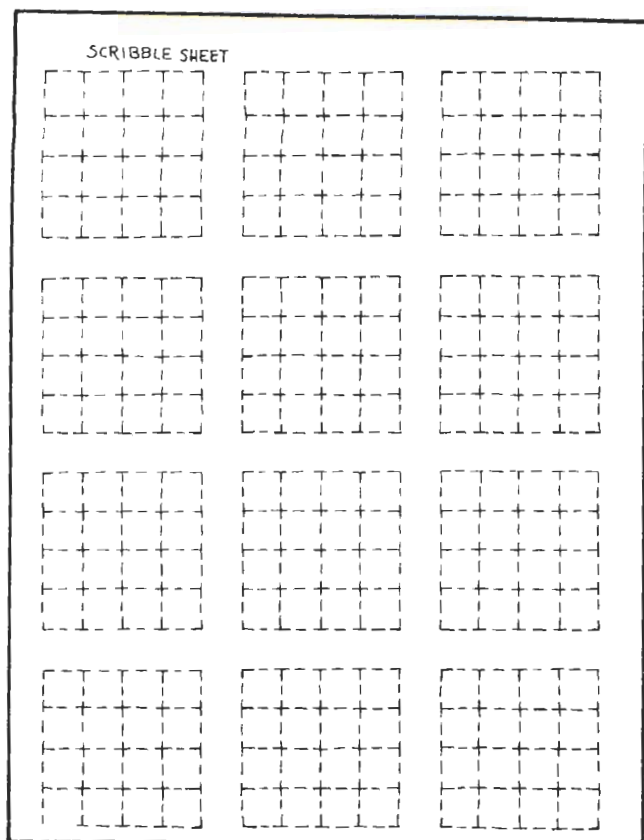
In the first example:



PAPEL PARA PRACTICAR

SCRIBBLE SHEET





This scribble sheet can be used with the previous page and with those that follow.

Its purpose is to give the child a place to experiment before making the final decision as to where the fences should be. Its use will become more evident as the sums get larger. This sheet may be introduced in conjunction with the previous page or when you see erasures beginning to appear.

|   |   |   |   |
|---|---|---|---|
| 2 | 7 | 6 | 4 |
| 1 | 1 | 3 | 3 |
| 3 | 2 | 5 | 7 |
| 6 | 2 | 6 | 2 |

— (10's)

|   |   |   |   |
|---|---|---|---|
| 2 | 6 | 3 | 2 |
| 3 | 8 | 1 | 3 |
| 5 | 2 | 2 | 4 |
| 3 | 2 | 7 | 2 |

— (11's)

|   |   |   |   |
|---|---|---|---|
| 3 | 4 | 4 | 4 |
| 3 | 3 | 4 | 6 |
| 5 | 3 | 2 | 5 |
| 2 | 5 | 3 | 4 |

— (12's)

|   |   |   |   |
|---|---|---|---|
| 7 | 3 | 3 | 6 |
| 3 | 3 | 2 | 4 |
| 2 | 8 | 9 | 2 |
| 3 | 3 | 3 | 4 |

— (13's)

|   |   |   |   |
|---|---|---|---|
| 9 | 3 | 4 | 6 |
| 2 | 2 | 5 | 4 |
| 3 | 5 | 2 | 6 |
| 7 | 4 | 6 | 2 |

— (14's)

|   |   |   |   |
|---|---|---|---|
| 7 | 2 | 3 | 9 |
| 3 | 5 | 5 | 1 |
| 5 | 5 | 5 | 5 |
| 8 | 2 | 4 | 6 |

— (15's)

|   |   |   |   |
|---|---|---|---|
| 2 | 6 | 6 | 7 |
| 3 | 5 | 4 | 3 |
| 6 | 6 | 6 | 6 |
| 2 | 8 | 1 | 9 |

— (16's)

|   |   |    |   |
|---|---|----|---|
| 8 | 3 | 4  | 9 |
| 3 | 3 | 4  | 4 |
| 7 | 3 | 10 | 1 |
| 4 | 7 | 8  | 8 |

— (17's)

|   |   |   |    |
|---|---|---|----|
| 7 | 3 | 5 | 13 |
| 8 | 9 | 6 | 8  |
| 5 | 9 | 4 | 3  |
| 4 | 9 | 9 | 6  |

— (18's)



NAME \_\_\_\_\_

|                                                                                                                                                                                                                                                                                                     |   |    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|----|----|---|---|---|---|---|---|---|---|---|---|---|---|---|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|---|---|---|---|---|---|---|---|----|---|---|---|---|---|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|---|----|---|---|---|---|---|---|---|---|---|---|---|---|
| <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>2</td><td>7</td><td>6</td><td>4</td></tr> <tr><td>1</td><td>1</td><td>3</td><td>3</td></tr> <tr><td>3</td><td>2</td><td>5</td><td>7</td></tr> <tr><td>6</td><td>2</td><td>6</td><td>2</td></tr> </table> <p>— (10's)</p> | 2 | 7  | 6  | 4 | 1 | 1 | 3 | 3 | 3 | 2 | 5 | 7 | 6 | 2 | 6 | 2 | <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>2</td><td>6</td><td>3</td><td>2</td></tr> <tr><td>3</td><td>8</td><td>1</td><td>3</td></tr> <tr><td>5</td><td>2</td><td>2</td><td>4</td></tr> <tr><td>3</td><td>2</td><td>7</td><td>2</td></tr> </table> <p>— (11's)</p>  | 2 | 6 | 3 | 2 | 3 | 8 | 1 | 3 | 5 | 2 | 2  | 4 | 3 | 2 | 7 | 2 | <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>3</td><td>4</td><td>4</td><td>4</td></tr> <tr><td>3</td><td>3</td><td>4</td><td>6</td></tr> <tr><td>5</td><td>3</td><td>2</td><td>5</td></tr> <tr><td>2</td><td>5</td><td>3</td><td>4</td></tr> </table> <p>— (12's)</p>  | 3 | 4 | 4 | 4  | 3 | 3 | 4 | 6 | 5 | 3 | 2 | 5 | 2 | 5 | 3 | 4 |
| 2                                                                                                                                                                                                                                                                                                   | 7 | 6  | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                                                                                                                                                                                                                                                                                                   | 1 | 3  | 3  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                                   | 2 | 5  | 7  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 6                                                                                                                                                                                                                                                                                                   | 2 | 6  | 2  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 2                                                                                                                                                                                                                                                                                                   | 6 | 3  | 2  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                                   | 8 | 1  | 3  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 5                                                                                                                                                                                                                                                                                                   | 2 | 2  | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                                   | 2 | 7  | 2  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                                   | 4 | 4  | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                                   | 3 | 4  | 6  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 5                                                                                                                                                                                                                                                                                                   | 3 | 2  | 5  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 2                                                                                                                                                                                                                                                                                                   | 5 | 3  | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>7</td><td>3</td><td>3</td><td>6</td></tr> <tr><td>3</td><td>3</td><td>2</td><td>4</td></tr> <tr><td>2</td><td>8</td><td>9</td><td>2</td></tr> <tr><td>3</td><td>3</td><td>3</td><td>4</td></tr> </table> <p>— (13's)</p> | 7 | 3  | 3  | 6 | 3 | 3 | 2 | 4 | 2 | 8 | 9 | 2 | 3 | 3 | 3 | 4 | <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>9</td><td>3</td><td>4</td><td>6</td></tr> <tr><td>2</td><td>2</td><td>5</td><td>4</td></tr> <tr><td>3</td><td>5</td><td>2</td><td>6</td></tr> <tr><td>7</td><td>4</td><td>6</td><td>2</td></tr> </table> <p>— (14's)</p>  | 9 | 3 | 4 | 6 | 2 | 2 | 5 | 4 | 3 | 5 | 2  | 6 | 7 | 4 | 6 | 2 | <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>7</td><td>2</td><td>3</td><td>9</td></tr> <tr><td>3</td><td>5</td><td>5</td><td>1</td></tr> <tr><td>5</td><td>5</td><td>5</td><td>5</td></tr> <tr><td>8</td><td>2</td><td>4</td><td>6</td></tr> </table> <p>— (15's)</p>  | 7 | 2 | 3 | 9  | 3 | 5 | 5 | 1 | 5 | 5 | 5 | 5 | 8 | 2 | 4 | 6 |
| 7                                                                                                                                                                                                                                                                                                   | 3 | 3  | 6  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                                   | 3 | 2  | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 2                                                                                                                                                                                                                                                                                                   | 8 | 9  | 2  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                                   | 3 | 3  | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 9                                                                                                                                                                                                                                                                                                   | 3 | 4  | 6  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 2                                                                                                                                                                                                                                                                                                   | 2 | 5  | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                                   | 5 | 2  | 6  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 7                                                                                                                                                                                                                                                                                                   | 4 | 6  | 2  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 7                                                                                                                                                                                                                                                                                                   | 2 | 3  | 9  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                                   | 5 | 5  | 1  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 5                                                                                                                                                                                                                                                                                                   | 5 | 5  | 5  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 8                                                                                                                                                                                                                                                                                                   | 2 | 4  | 6  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
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| 2                                                                                                                                                                                                                                                                                                   | 6 | 6  | 7  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                                   | 5 | 4  | 3  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 6                                                                                                                                                                                                                                                                                                   | 6 | 6  | 6  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 2                                                                                                                                                                                                                                                                                                   | 8 | 1  | 9  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 8                                                                                                                                                                                                                                                                                                   | 3 | 4  | 9  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                                   | 3 | 4  | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 7                                                                                                                                                                                                                                                                                                   | 3 | 10 | 1  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 4                                                                                                                                                                                                                                                                                                   | 7 | 8  | 8  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 7                                                                                                                                                                                                                                                                                                   | 3 | 5  | 13 |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 8                                                                                                                                                                                                                                                                                                   | 9 | 6  | 8  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 5                                                                                                                                                                                                                                                                                                   | 9 | 4  | 3  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 4                                                                                                                                                                                                                                                                                                   | 9 | 9  | 6  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |

An extra challenge this time: there is one problem that can't be done — center problem in bottom row.

|   |   |   |   |
|---|---|---|---|
| 2 | 2 | 1 | 3 |
| 3 | 4 | 1 | 4 |
| 5 | 5 | 4 | 6 |
| 2 | 8 | 3 | 7 |

— (10's)

|   |   |   |   |
|---|---|---|---|
| 4 | 7 | 3 | 6 |
| 9 | 2 | 8 | 5 |
| 2 | 7 | 5 | 6 |
| 9 | 4 | 8 | 3 |

— (11's)

|   |   |   |   |
|---|---|---|---|
| 5 | 3 | 2 | 1 |
| 2 | 4 | 2 | 6 |
| 3 | 3 | 5 | 1 |
| 3 | 3 | 1 | 4 |

— (12's)

|   |   |    |   |
|---|---|----|---|
| 3 | 5 | 6  | 7 |
| 5 | 5 | 8  | 4 |
| 2 | 7 | 6  | 9 |
| 8 | 3 | 10 | 3 |

— (13's)

|   |   |   |   |
|---|---|---|---|
| 3 | 4 | 1 | 9 |
| 7 | 7 | 2 | 5 |
| 5 | 9 | 4 | 6 |
| 3 | 5 | 6 | 8 |

— (14's)

|   |   |   |   |
|---|---|---|---|
| 2 | 3 | 4 | 5 |
| 7 | 8 | 4 | 1 |
| 6 | 6 | 5 | 4 |
| 2 | 7 | 8 | 3 |

— (15's)

|   |   |   |    |
|---|---|---|----|
| 8 | 5 | 5 | 9  |
| 3 | 5 | 7 | 2  |
| 3 | 4 | 2 | 1  |
| 7 | 6 | 3 | 10 |

— (16's)

|   |   |    |    |
|---|---|----|----|
| 5 | 6 | 11 | 8  |
| 6 | 7 | 6  | 9  |
| 3 | 7 | 5  | 12 |
| 9 | 8 | 10 | 7  |

— (17's)

|    |   |   |    |
|----|---|---|----|
| 4  | 5 | 7 | 9  |
| 3  | 5 | 6 | 9  |
| 6  | 6 | 6 | 13 |
| 10 | 8 | 6 | 5  |

— (18's)

NAME \_\_\_\_\_

|          |   |    |    |          |   |    |    |          |   |   |    |
|----------|---|----|----|----------|---|----|----|----------|---|---|----|
| 2        | 2 | 1  | 3  | 4        | 7 | 3  | 6  | 5        | 3 | 2 | 1  |
| 3        | 4 | 1  | 4  | 9        | 2 | 8  | 5  | 2        | 4 | 2 | 6  |
| 5        | 5 | 4  | 6  | 2        | 7 | 5  | 6  | 3        | 3 | 5 | 1  |
| 2        | 8 | 3  | 7  | 9        | 4 | 8  | 3  | 3        | 3 | 1 | 4  |
| — (10's) |   |    |    | — (11's) |   |    |    | — (12's) |   |   |    |
|          |   |    |    |          |   |    |    |          |   |   |    |
| 3        | 5 | 6  | 7  | 3        | 4 | 1  | 9  | 2        | 3 | 4 | 5  |
| 5        | 5 | 8  | 4  | 7        | 7 | 2  | 5  | 7        | 8 | 4 | 1  |
| 2        | 7 | 6  | 9  | 5        | 9 | 4  | 6  | 6        | 6 | 5 | 4  |
| 8        | 3 | 10 | 3  | 3        | 5 | 6  | 8  | 2        | 7 | 8 | 3  |
| — (13's) |   |    |    | — (14's) |   |    |    | — (15's) |   |   |    |
|          |   |    |    |          |   |    |    |          |   |   |    |
| 8        | 5 | 5  | 9  | 5        | 6 | 11 | 8  | 4        | 5 | 7 | 9  |
| 3        | 5 | 7  | 2  | 6        | 7 | 6  | 9  | 3        | 5 | 6 | 9  |
| 3        | 4 | 2  | 1  | 3        | 7 | 5  | 12 | 6        | 6 | 6 | 13 |
| 7        | 6 | 3  | 10 | 9        | 8 | 10 | 7  | 10       | 8 | 6 | 5  |
| — (16's) |   |    |    | — (17's) |   |    |    | — (18's) |   |   |    |

More fun with fence arithmetic. Discussions regarding strategies would be beneficial. How can you tell when the greatest number of fences are made?

|   |    |    |   |
|---|----|----|---|
| 7 | 10 | 8  | 8 |
| 6 | 7  | 7  | 8 |
| 9 | 9  | 10 | 7 |
| 6 | 6  | 6  | 6 |

— (24's)

|   |   |   |    |
|---|---|---|----|
| 5 | 6 | 6 | 12 |
| 7 | 4 | 4 | 4  |
| 7 | 4 | 4 | 7  |
| 5 | 4 | 9 | 8  |

— (24's)

|    |   |    |    |
|----|---|----|----|
| 3  | 8 | 10 | 6  |
| 12 | 9 | 7  | 14 |
| 8  | 7 | 3  | 3  |
| 9  | 5 | 7  | 9  |

— (24's)

|    |   |    |    |
|----|---|----|----|
| 6  | 6 | 5  | 13 |
| 4  | 9 | 10 | 7  |
| 15 | 8 | 7  | 7  |
| 5  | 5 | 7  | 11 |

— (25's)

|    |   |    |    |
|----|---|----|----|
| 10 | 2 | 3  | 5  |
| 10 | 5 | 5  | 7  |
| 3  | 5 | 12 | 10 |
| 5  | 5 | 6  | 7  |

— (25's)

|   |   |   |   |
|---|---|---|---|
| 3 | 9 | 4 | 8 |
| 7 | 9 | 8 | 8 |
| 7 | 7 | 6 | 1 |
| 4 | 6 | 6 | 7 |

— (25's)

|    |    |    |    |
|----|----|----|----|
| 10 | 8  | 15 | 15 |
| 15 | 12 | 11 | 23 |
| 8  | 9  | 10 | 7  |
| 7  | 10 | 10 | 10 |

— (30's)

|    |    |    |    |
|----|----|----|----|
| 5  | 10 | 15 | 4  |
| 5  | 20 | 6  | 20 |
| 5  | 12 | 8  | 7  |
| 13 | 5  | 6  | 9  |

— (30's)

|    |    |    |    |
|----|----|----|----|
| 10 | 13 | 22 | 8  |
| 7  | 14 | 8  | 15 |
| 26 | 16 | 4  | 7  |
| 4  | 6  | 12 | 8  |

— (30's)

NAME \_\_\_\_\_

|   |    |    |   |
|---|----|----|---|
| 7 | 10 | 8  | 8 |
| 6 | 7  | 7  | 8 |
| 9 | 9  | 10 | 7 |
| 6 | 6  | 6  | 6 |

— (24's)

|   |   |   |    |
|---|---|---|----|
| 5 | 6 | 6 | 12 |
| 7 | 4 | 4 | 4  |
| 7 | 4 | 4 | 7  |
| 5 | 4 | 9 | 8  |

— (24's)

|    |   |    |    |
|----|---|----|----|
| 3  | 8 | 10 | 6  |
| 12 | 9 | 7  | 14 |
| 8  | 7 | 3  | 3  |
| 9  | 5 | 7  | 9  |

— (24's)

|    |   |    |    |
|----|---|----|----|
| 6  | 6 | 5  | 13 |
| 4  | 9 | 10 | 7  |
| 15 | 8 | 7  | 7  |
| 5  | 5 | 7  | 11 |

— (25's)

|    |   |    |    |
|----|---|----|----|
| 10 | 2 | 3  | 5  |
| 10 | 5 | 5  | 7  |
| 3  | 5 | 12 | 10 |
| 5  | 5 | 6  | 7  |

— (25's)

|   |   |   |   |
|---|---|---|---|
| 3 | 9 | 4 | 8 |
| 7 | 9 | 8 | 8 |
| 7 | 7 | 6 | 1 |
| 4 | 6 | 6 | 7 |

— (25's)

|    |    |    |    |
|----|----|----|----|
| 10 | 8  | 15 | 15 |
| 15 | 12 | 11 | 23 |
| 8  | 9  | 10 | 7  |
| 7  | 10 | 10 | 10 |

— (30's)

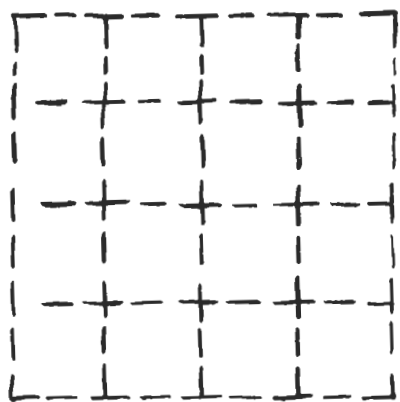
|    |    |    |    |
|----|----|----|----|
| 5  | 10 | 15 | 4  |
| 5  | 20 | 6  | 20 |
| 5  | 12 | 8  | 7  |
| 13 | 5  | 6  | 9  |

— (30's)

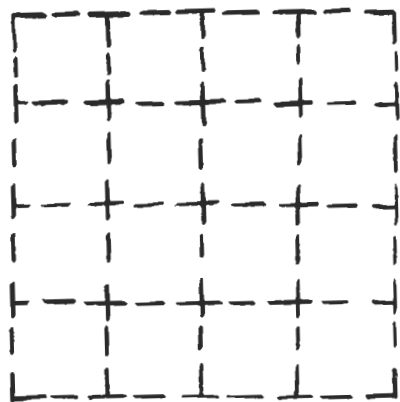
|    |    |    |    |
|----|----|----|----|
| 10 | 13 | 22 | 8  |
| 7  | 14 | 8  | 15 |
| 26 | 16 | 4  | 7  |
| 4  | 6  | 12 | 8  |

— (30's)

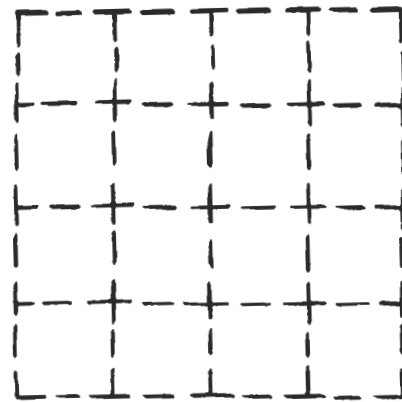
Sums here are considerably larger. Get out the scribble sheet and go to work.



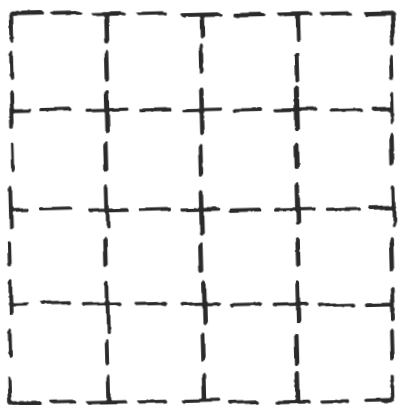
— (s)



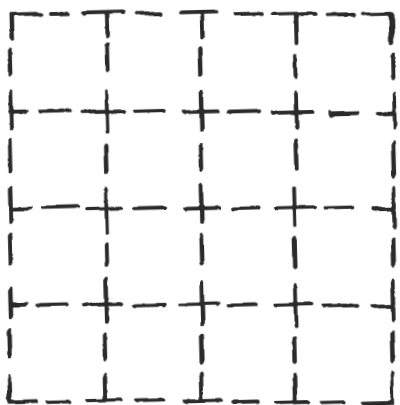
— (s)



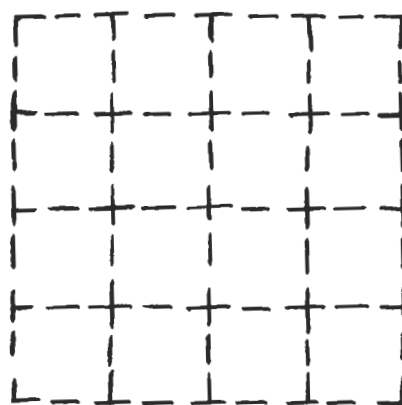
— (s)



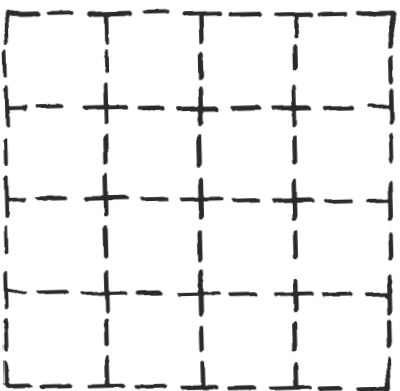
— (s)



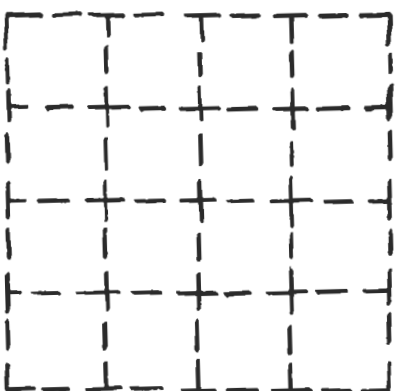
— (s)



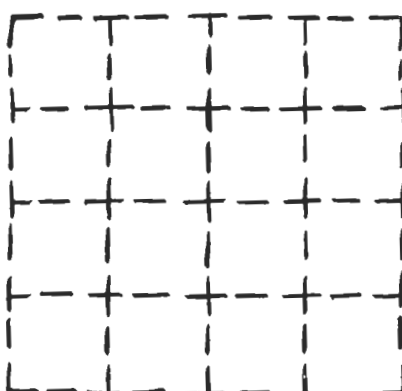
— (s)



— (s)

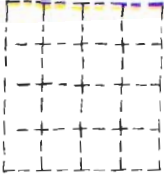
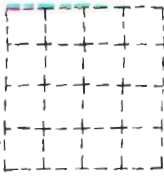
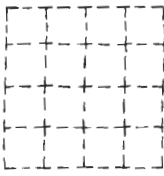
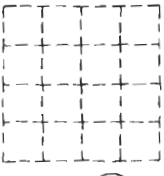
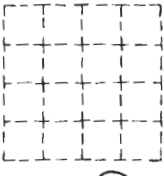
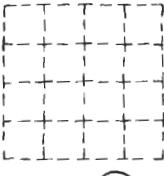
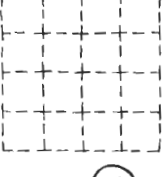
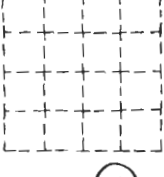
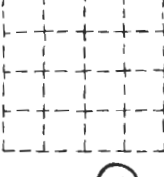


— (s)



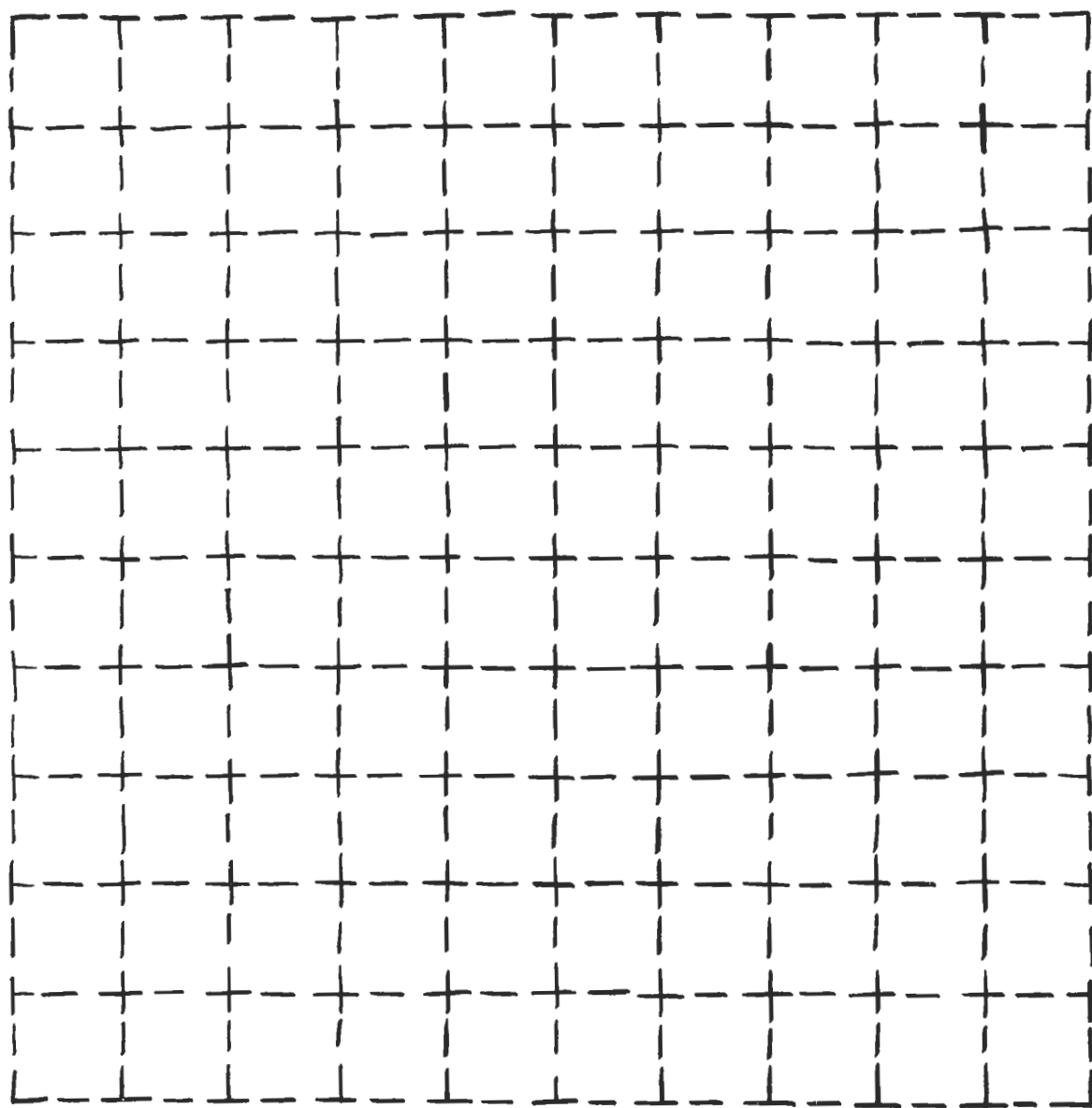
— (s)

NAME \_\_\_\_\_

|                                                                                            |                                                                                            |                                                                                            |
|--------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|
| <br>— (s) | <br>— (s) | <br>— (s) |
| <br>— (s) | <br>— (s) | <br>— (s) |
| <br>— (s) | <br>— (s) | <br>— (s) |

Uses for this page have been considered earlier with a 3 by 3 Report Form.

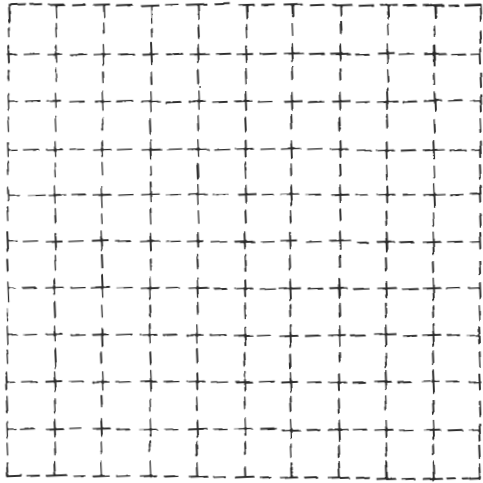
\_\_\_\_\_



— (s)



NAME \_\_\_\_\_



— (5)

A giant "fence arithmetic" problem for the brave and persevering types.

---

|   |   |
|---|---|
| 5 | 8 |
| 4 | 8 |

|   |   |
|---|---|
| 6 | 9 |
| 3 | 5 |

|    |   |
|----|---|
| 10 | 7 |
| 5  | 2 |

|   |   |
|---|---|
| 9 | 9 |
| 4 | 1 |

|   |   |
|---|---|
| 6 | 4 |
| 7 | 4 |

|   |    |
|---|----|
| 1 | 2  |
| 3 | 10 |

|   |   |   |
|---|---|---|
| 6 |   | 2 |
| 5 | 7 |   |

|    |   |
|----|---|
| 10 | 9 |
|    | 3 |
| 2  |   |

|   |   |   |
|---|---|---|
| 8 | 4 |   |
| 9 |   | 2 |

|   |   |   |
|---|---|---|
|   | 6 |   |
| 4 | 1 |   |
|   |   | 3 |

|   |   |   |
|---|---|---|
|   | 4 | 6 |
| 8 |   | 7 |

|  |   |   |
|--|---|---|
|  | 7 | 0 |
|  | 3 |   |
|  |   | 2 |

INTRODUCING "DIFFERENCING" NAME \_\_\_\_\_

|   |   |  |
|---|---|--|
| 5 | 8 |  |
| 4 | 8 |  |
|   |   |  |

|   |   |  |
|---|---|--|
| 6 | 9 |  |
| 3 | 5 |  |
|   |   |  |

|    |   |  |
|----|---|--|
| 10 | 7 |  |
| 5  | 2 |  |
|    |   |  |

|   |   |  |
|---|---|--|
| 9 | 9 |  |
| 4 | 1 |  |
|   |   |  |

|   |   |  |
|---|---|--|
| 6 | 4 |  |
| 7 | 4 |  |
|   |   |  |

|   |    |  |
|---|----|--|
| 1 | 2  |  |
| 3 | 10 |  |
|   |    |  |

|   |   |   |
|---|---|---|
| 6 |   | 2 |
| 5 | 7 |   |
|   |   |   |

|    |   |  |
|----|---|--|
| 10 | 9 |  |
|    | 3 |  |
| 2  |   |  |

|   |   |   |
|---|---|---|
| 8 | 4 |   |
| 9 |   | 2 |
|   |   |   |

|   |   |   |
|---|---|---|
|   | 6 |   |
| 4 | 1 |   |
|   |   | 3 |

|   |   |   |
|---|---|---|
|   | 4 | 6 |
| 8 |   | 7 |
|   |   |   |

|  |   |   |
|--|---|---|
|  | 7 | 0 |
|  | 3 |   |
|  |   | 2 |

We call this the "differencing" game to distinguish it from "subtraction." In this format

|   |  |  |
|---|--|--|
| 5 |  |  |
| 4 |  |  |
|   |  |  |

|   |  |  |
|---|--|--|
| 6 |  |  |
| 7 |  |  |
|   |  |  |

|   |   |   |
|---|---|---|
| 5 | 8 | 3 |
|   |   |   |
|   |   |   |

|   |   |   |
|---|---|---|
|   |   |   |
| 9 | 9 | 0 |
|   |   |   |

We mean to ask: what is the difference between 5 and 4, 6 and 7, 5 and 8, 9 and 9? In "subtraction" the larger number is written first (from left to right and from top to bottom—if our investigation is limited to whole numbers). In "differencing," the order is immaterial.

There is a rather surprising outcome in each example on this page:

|   |   |   |
|---|---|---|
| 5 | 8 | 3 |
| 4 | 8 | 4 |
| 1 | 0 |   |

The difference between the differences is the same—1 in this example.

|   |    |   |
|---|----|---|
| 1 | 2  | 1 |
| 3 | 10 | 7 |
| 2 | 8  | 6 |

The difference between each pair of differences is 6.

When all four initiating numbers are not given, problems arise:

|   |   |   |
|---|---|---|
| 6 |   | 2 |
| 5 | 7 |   |
|   |   |   |

Without looking further, the number in the upper right hand box could be either 4 or 8.

However, if we fill in all the differences we can, the choice becomes obvious.

|   |   |   |
|---|---|---|
| 6 |   | 2 |
| 5 | 7 | 2 |
|   |   | 0 |

As part of the top row the missing number can be 4 or 8; as part of the middle column, the missing number can be 6 or 8 . . . so the missing number must be 8.

The hardest problem is the last one. From the information given, the following entries (1), (2), and (3) are obvious.

|       |   |   |         |
|-------|---|---|---------|
| (1) → | ? | 7 | 0       |
| (4) → |   | 3 | 2 ← (3) |
| (5) → |   | 4 | 2       |

↑  
(2)

Entry 4 must be 1 or 5: if it is 5, then entry (5) would be 2; if it is 1, then entry (5) would be 6 . . . and the difference in the bottom row between 2 and 4 is 2 (a solution); but the difference between 6 and 4 is also 2 (another solution).

All examples on this and the following pages "work"—they "difference" all the way into the bottom right corner. If anyone wonders whether any 4 numbers in the four initiating boxes will "work," he ought to be encouraged to find out for himself. (He can use form 138 as a scratch pad.)

|   |   |   |
|---|---|---|
| 3 | 7 | 4 |
| 2 | 4 | 2 |
| 1 | 3 | 2 |

|   |   |  |
|---|---|--|
| 1 | 6 |  |
| 8 | 9 |  |
|   |   |  |

|    |   |  |
|----|---|--|
| 12 | 7 |  |
| 5  | 3 |  |
|    |   |  |

|   |    |  |
|---|----|--|
| 5 | 15 |  |
| 3 | 6  |  |
|   |    |  |

|    |   |  |
|----|---|--|
| 6  | 4 |  |
| 13 | 9 |  |
|    |   |  |

|    |    |  |
|----|----|--|
| 3  | 6  |  |
| 10 | 14 |  |
|    |    |  |

|    |   |  |
|----|---|--|
| 12 | 6 |  |
|    | 3 |  |
| 8  |   |  |

|    |    |   |
|----|----|---|
| 4  |    | 1 |
| 10 | 14 |   |
|    |    |   |

|   |   |   |
|---|---|---|
| 9 |   | 2 |
|   | 8 |   |
| 5 | 3 |   |

|   |   |   |
|---|---|---|
| 7 |   | 5 |
| 1 | 9 |   |
|   | 3 |   |

|   |   |   |
|---|---|---|
|   | 8 | 5 |
| 6 |   |   |
| 7 | 3 |   |

|   |    |   |
|---|----|---|
| 5 | 10 |   |
| 2 |    |   |
|   |    | 0 |

ANOTHER "DIFFERENCING" NAME \_\_\_\_\_

|   |   |   |
|---|---|---|
| 3 | 7 | 4 |
| 2 | 4 | 2 |
| 1 | 3 | 2 |

|   |   |  |
|---|---|--|
| 1 | 6 |  |
| 8 | 9 |  |
|   |   |  |

|    |   |  |
|----|---|--|
| 12 | 7 |  |
| 5  | 3 |  |
|    |   |  |

|   |    |  |
|---|----|--|
| 5 | 15 |  |
| 3 | 6  |  |
|   |    |  |

|    |   |  |
|----|---|--|
| 6  | 4 |  |
| 13 | 9 |  |
|    |   |  |

|    |    |  |
|----|----|--|
| 3  | 6  |  |
| 10 | 14 |  |
|    |    |  |

|    |   |  |
|----|---|--|
| 12 | 6 |  |
|    | 3 |  |
| 8  |   |  |

|    |    |   |
|----|----|---|
| 4  |    | 1 |
| 10 | 14 |   |
|    |    |   |

|   |   |   |
|---|---|---|
| 9 |   | 2 |
|   | 8 |   |
| 5 | 3 |   |

|   |   |   |
|---|---|---|
| 7 |   | 5 |
| 1 | 9 |   |
|   | 3 |   |

|   |   |   |
|---|---|---|
|   | 8 | 5 |
| 6 |   |   |
| 7 | 3 |   |

|   |    |   |
|---|----|---|
| 5 | 10 |   |
| 2 |    |   |
|   |    | 0 |

Nothing new arises on this page, but the numbers have grown in size and the last six examples require some trial and error . . . so they "difference" all the way into the corner.

We would hope that several children are wondering whether any numbers in the four initiating boxes will "work." They will find that such is not the case. Here are two counter examples.

|   |   |   |
|---|---|---|
| 3 | 4 | 1 |
| 6 | 5 | 1 |
| 3 | 1 | ? |

|   |   |   |
|---|---|---|
| 7 | 5 | 2 |
| 3 | 8 | 5 |
| 4 | 3 | ? |

So, the problem is to find a way to create your own examples that "do work." This and the previous page have given 24 different solutions. What is similar about them all?

Children will come up with a great variety of guesses . . . all odd numbers, all even numbers, numbers in sequence such as 2, 3, 4 and 5 or 2, 4, 6 and 8, etc.

The answer is always the same: make up some examples and test the theory.

Is there anything interesting about the examples that "don't work"? Here are two provocative examples:

|   |   |   |
|---|---|---|
| 2 | 5 | 3 |
| 9 | 6 | 3 |
| 7 | 1 | ? |

|   |   |   |
|---|---|---|
| 3 | 6 | 3 |
| 5 | 2 | 3 |
| 2 | 4 | ? |

Neither works: but it is noteworthy that  $3 + 3 = 6$ ,  $2 + 4 = 6$  and  $7 - 1 = 6$ .

If that outcome suggests any ideas, try them out using the next page.

Suppose we pick out 4 numbers and try different arrangements in the 4 initiating boxes. Any 4 will do, but let's consider using 1, 2, 3 and 4, separating those that "do work" and "don't work" in the sense of differencing all the way into the corner.

| do work |   |   |
|---------|---|---|
| 1       | 2 | 1 |
| 3       | 4 | 1 |
| 2       | 2 | 0 |

| don't work |   |   |
|------------|---|---|
| 1          | 2 | 1 |
| 4          | 3 | 1 |
| 3          | 1 |   |

|   |   |   |
|---|---|---|
| 3 | 1 | 2 |
| 4 | 2 | 2 |
| 1 | 1 | 0 |

|   |   |   |
|---|---|---|
| 3 | 2 | 1 |
| 4 | 1 | 3 |
| 1 | 1 |   |

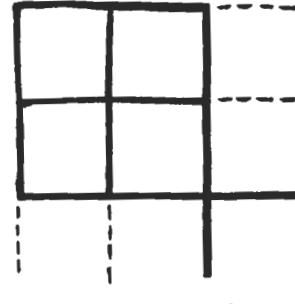
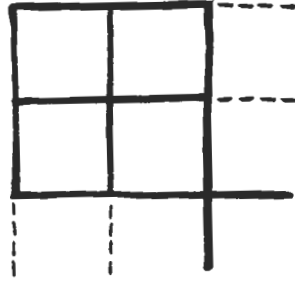
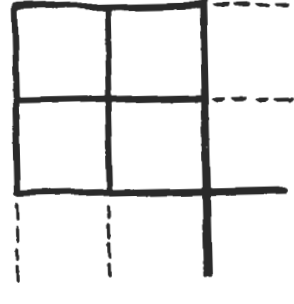
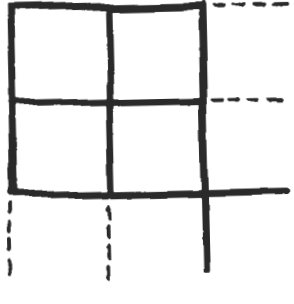
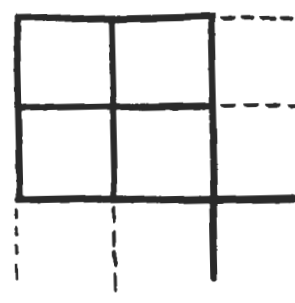
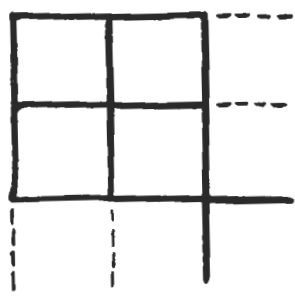
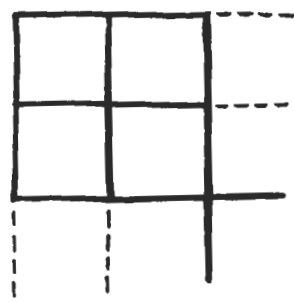
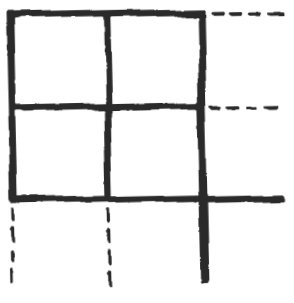
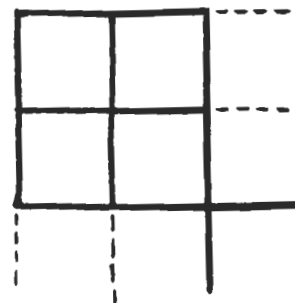
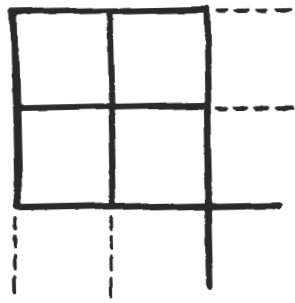
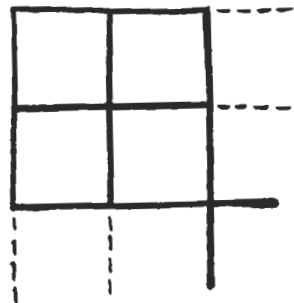
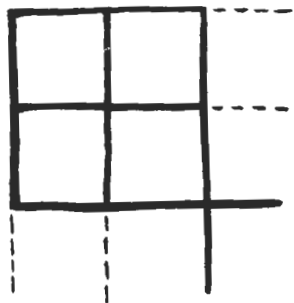
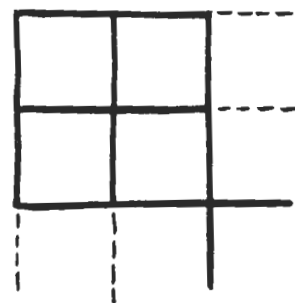
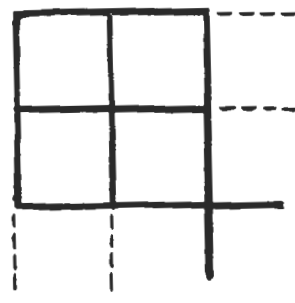
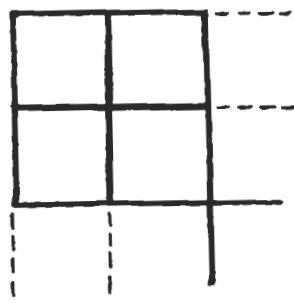
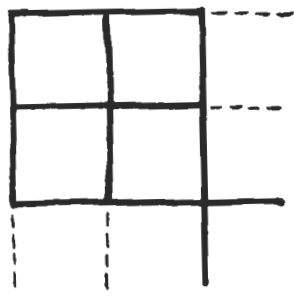
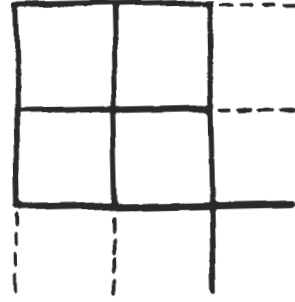
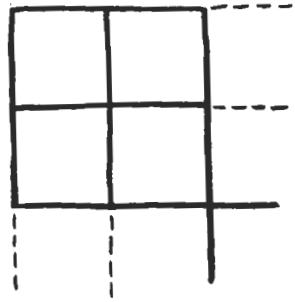
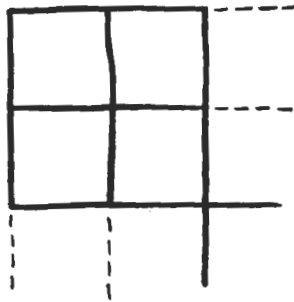
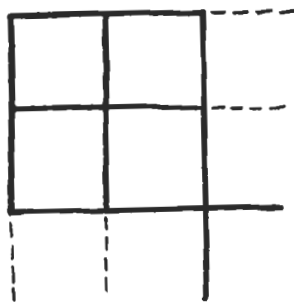
|   |   |   |
|---|---|---|
| 3 | 4 | 1 |
| 1 | 2 | 1 |
| 2 | 2 | 0 |

|   |   |   |
|---|---|---|
| 2 | 3 | 1 |
| 4 | 1 | 3 |
| 2 | 2 |   |

|   |   |   |
|---|---|---|
| 4 | 3 | 1 |
| 2 | 1 | 1 |
| 2 | 2 | 0 |

|   |   |   |
|---|---|---|
| 4 | 1 | 3 |
| 2 | 3 | 1 |
| 2 | 2 |   |

The next page is designed to facilitate such investigations.

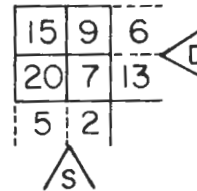


NAME \_\_\_\_\_

|  |  |  |  |
|--|--|--|--|
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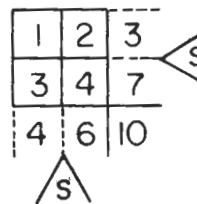
Will your theories hold when larger numbers are used? . . . when the numbers are not in sequence? After much experimenting most children will become convinced that the answer is "Yes" to all those questions.

A handy notation to indicate the operations used with "D" standing for "Difference" and "S" for "Sum" is:



This anticipates the next page.

Suppose we change the game to an "adding game": under what conditions can we "add all the way to the corner"? Consider this example:

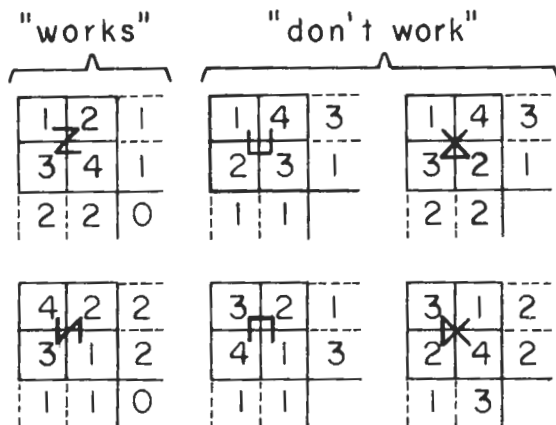


Can you find other examples in which "adding" can be used at each move toward the bottom right corner? Children will have no trouble finding several examples each. Perhaps each child can contribute an example for the chalk-board.

Finally—"Let's erase the board and fill it again with examples that **don't work**."

If anyone comes up with such an example, he will find an error in his additions someplace along the way.

Suppose we draw a line from the smallest to the next largest, etc., in three pairs of examples:

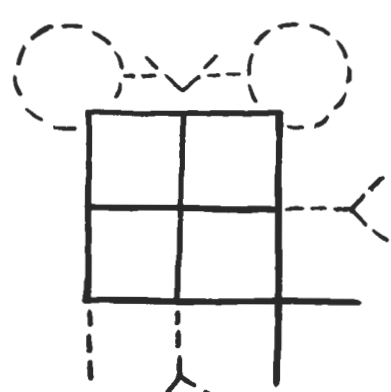
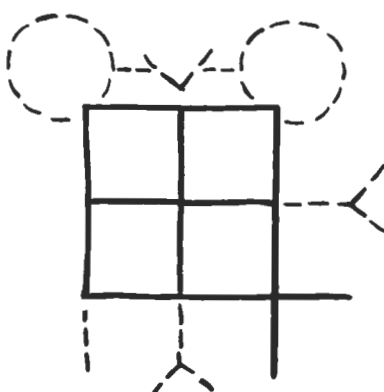
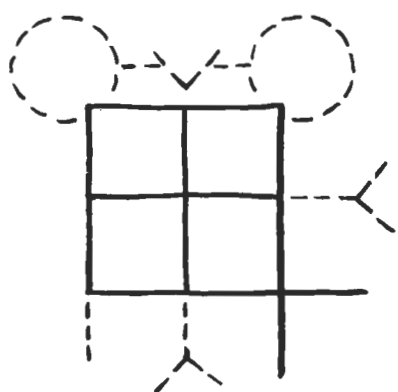
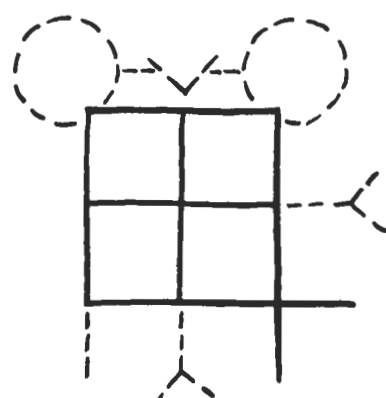
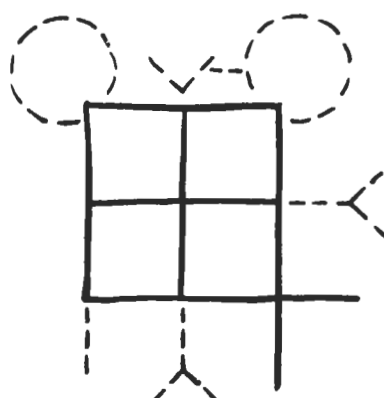
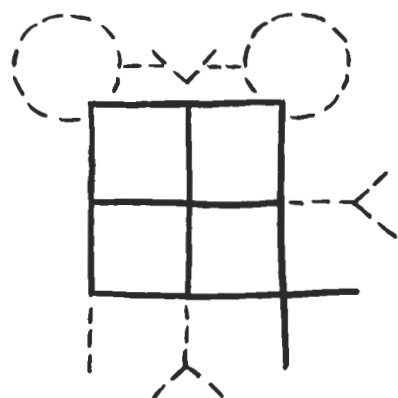
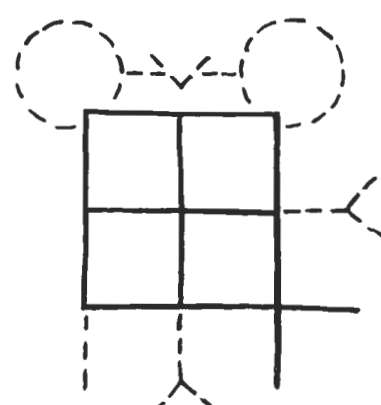
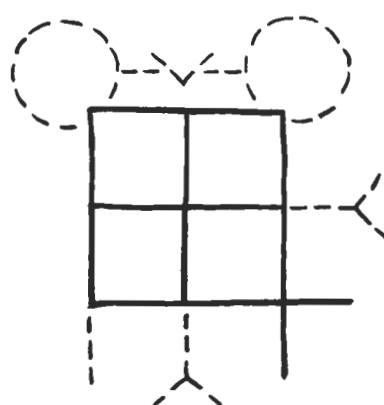
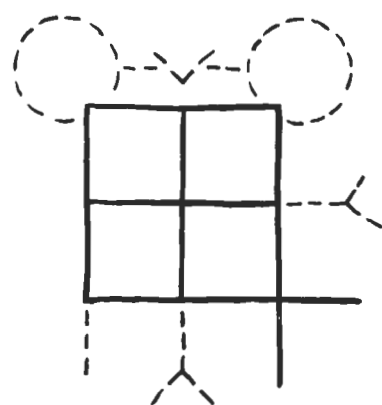
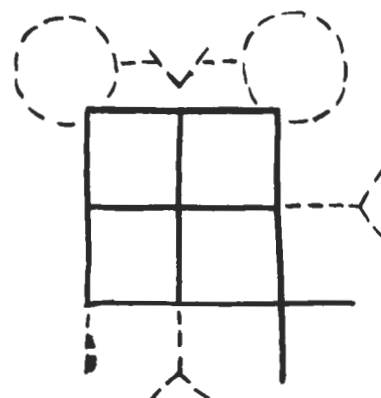
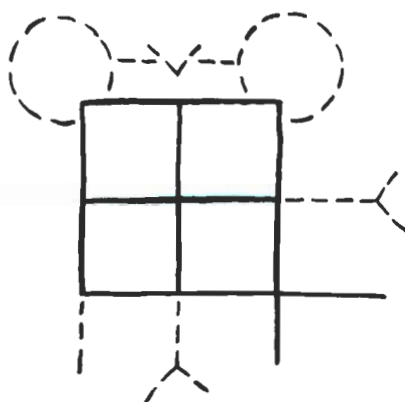
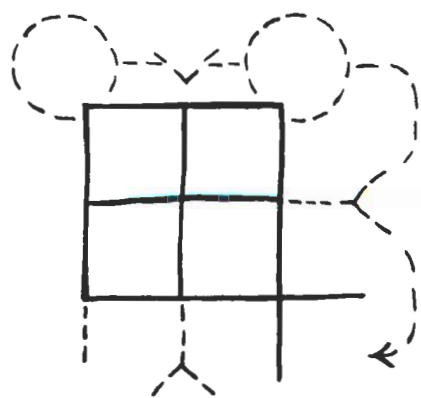


We might refer to these as the "Z," "U" and "X" configurations. Are there any others that are not simply rotations of these three?

Will the "Z" configuration "always work"?

Will the "U" configuration always require one addition and one "difference" to get into the corner?

Will the "X" configuration always require two additions to get into the corner?

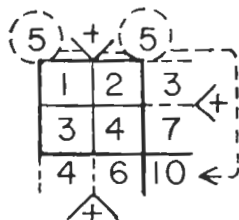




NAME \_\_\_\_\_

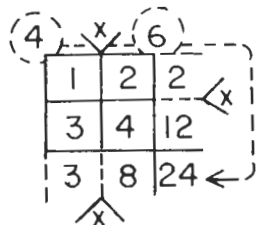
This form is designed to extend the investigation.

We now can consider the diagonals. Here is a completed example—"adding all the way."

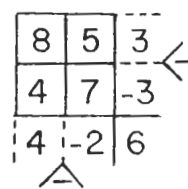
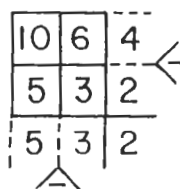


The children who first began considering the diagonals called the circles "ears" ... and they have been called ears from that time on.

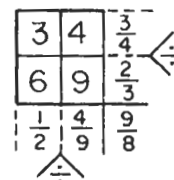
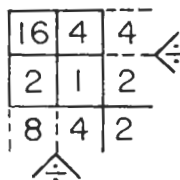
Suppose we change the game to "multiplying all the way" and use the same example:



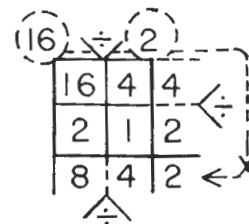
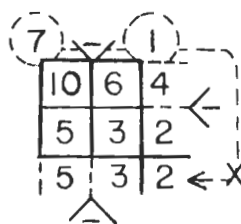
Subtraction presents some problems unless positive and negative numbers are available.



And division presents the same problems sometimes unless fractions are available.



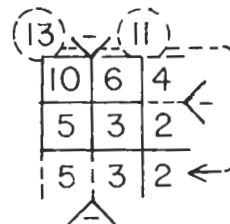
Further troubles arise in subtraction and division when the "ears" are considered:



But

$$7 - 1 \neq 2 \text{ and } 16 \div 2 \neq 2$$

A fourth grade child suggested a "way out" in subtraction: Just let me **add** the diagonals.



"Will it always work," the teacher asked.

"Why don't you try some and find out," was the inventor's challenge.

A fifth grader, when he heard this story, closed his eyes and made the following conjecture:

"Addition and subtraction are related in the same way multiplication and division are related. I'll bet we'll run into trouble with the 'ears' in division; and to get out of trouble, we can go back to multiplication on the diagonals!"

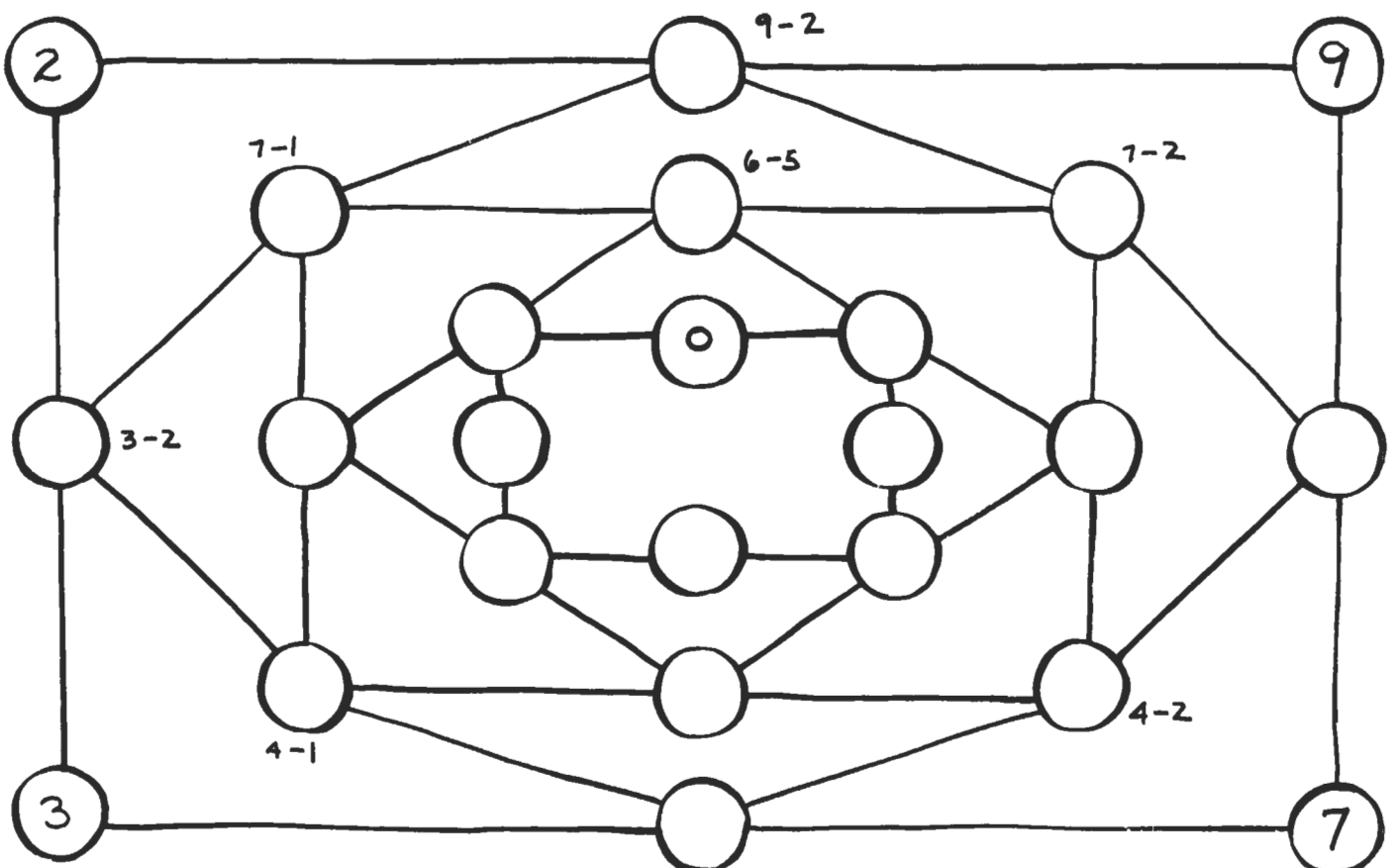
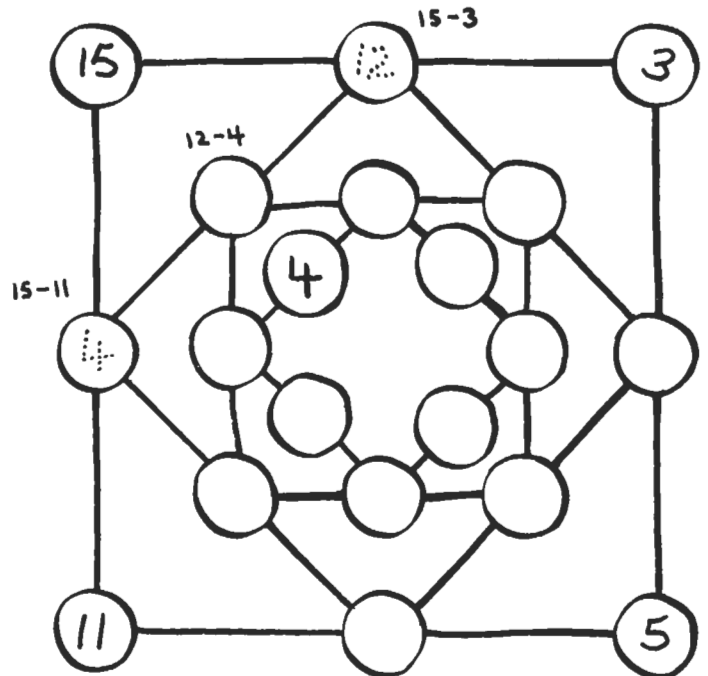
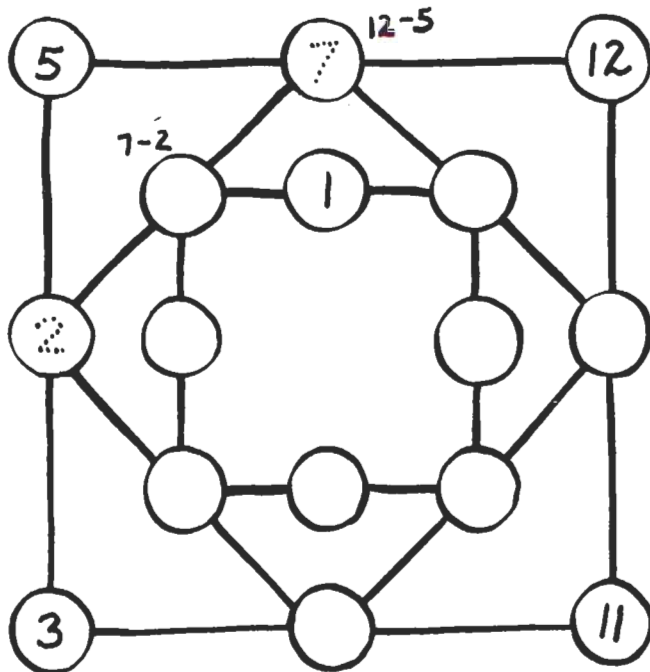
What an insight into the "structure of arithmetic"!

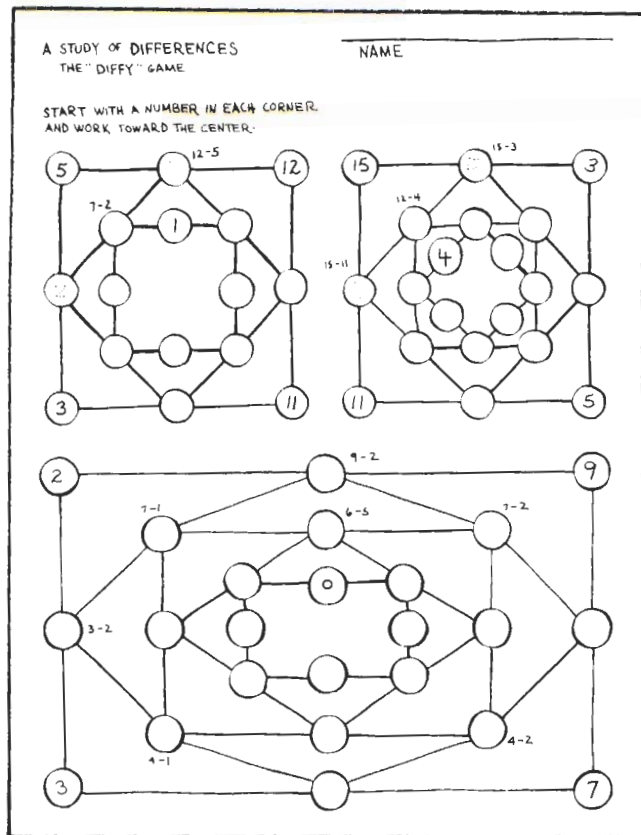
# "DIFFY" GAMES

## JUEGOS DE "RESTAS"

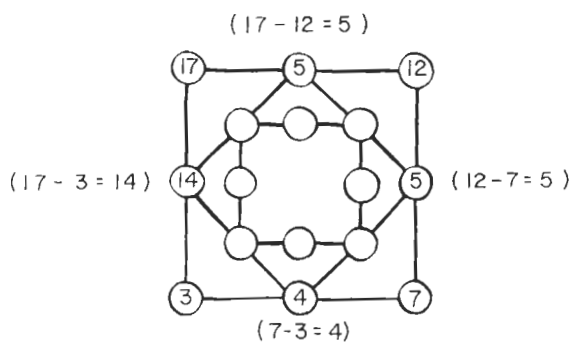
START WITH A NUMBER IN EACH CORNER  
AND WORK TOWARD THE CENTER.

COMIENZA CON UN NÚMERO EN CADA  
ESQUINA RESUELVE HACIA EL CENTRO.

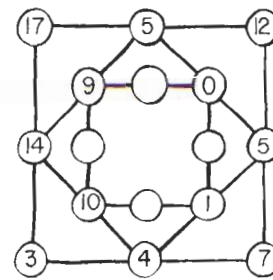




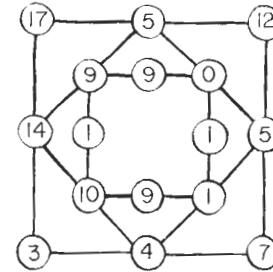
This page introduces the "Diffy" game. To play the game, one starts by writing a number in each of the four circles in the corners of the array. Two corner numbers are then considered at a time, and their difference is recorded in the circle between them. For example, with starting numbers of 17, 12, 7, and 3, we would have



The process of "differencing" continues. Now the previous "differences" are considered in pairs. The difference of each pair is again recorded in the circle between them. Continuing with our example above,



The "differencing" is done once again, and the final result is:



After working a few of these problems, some questions might arise. These should be investigated by individuals or groups of children, and records of results should be kept.

Questions to explore:

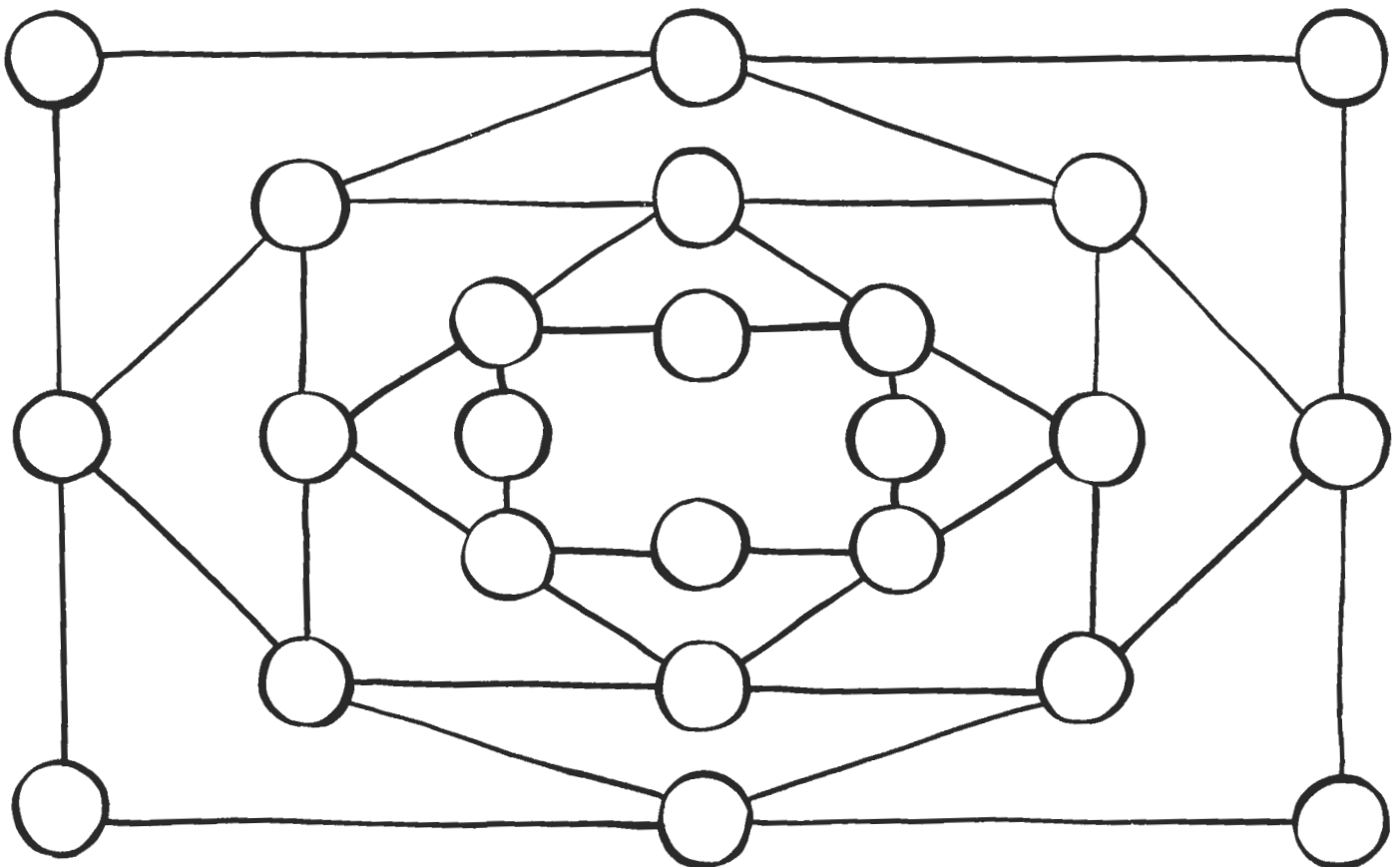
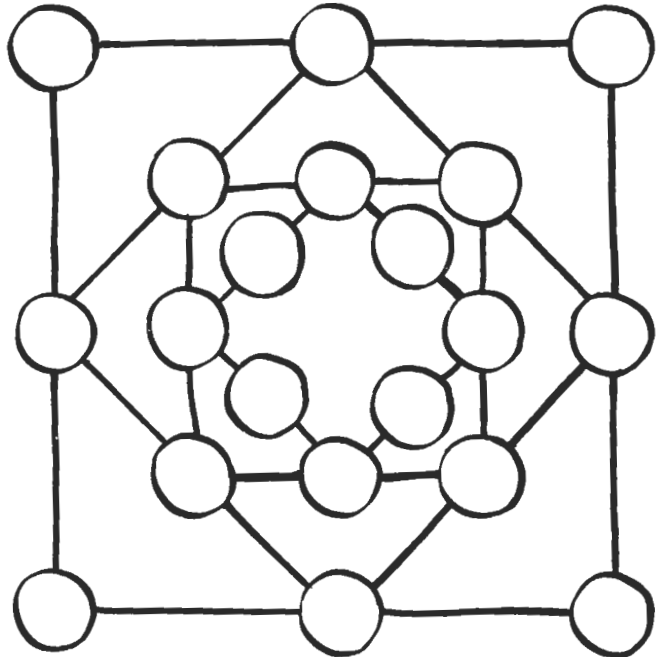
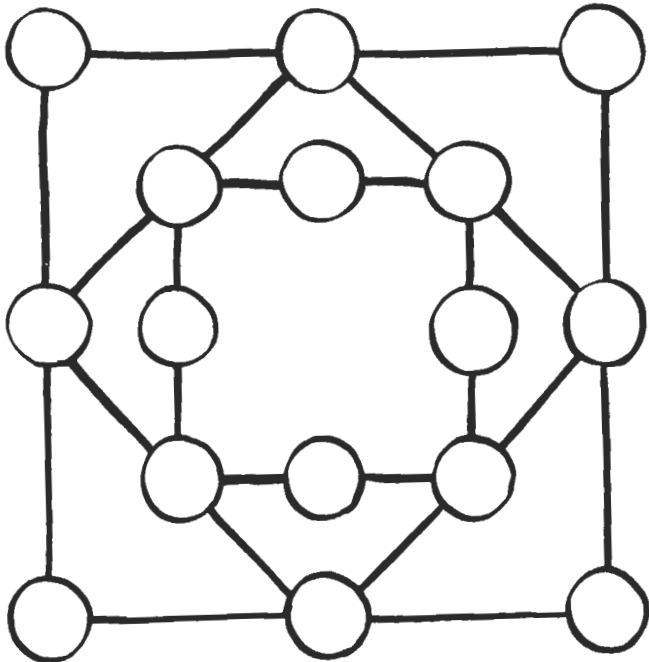
- (1) If you kept "differencing" long enough, would all arrays eventually end up with four zeroes? Can anybody find one that doesn't?
- (2) Is there any way to predict how many "differencings" must be done before the four "differences" are all zeroes?

# "DIFFY" GAMES

## JUEGOS DE "RESTAS"

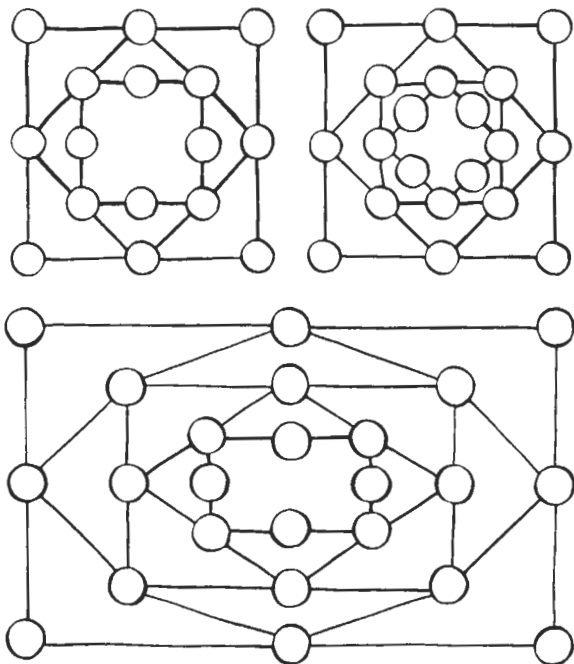
START WITH A NUMBER IN EACH CORNER  
AND WORK TOWARD THE CENTER.

COMIENZA CON UN NÚMERO EN CADA  
ESQUINA RESUELVE HACIA EL CENTRO.

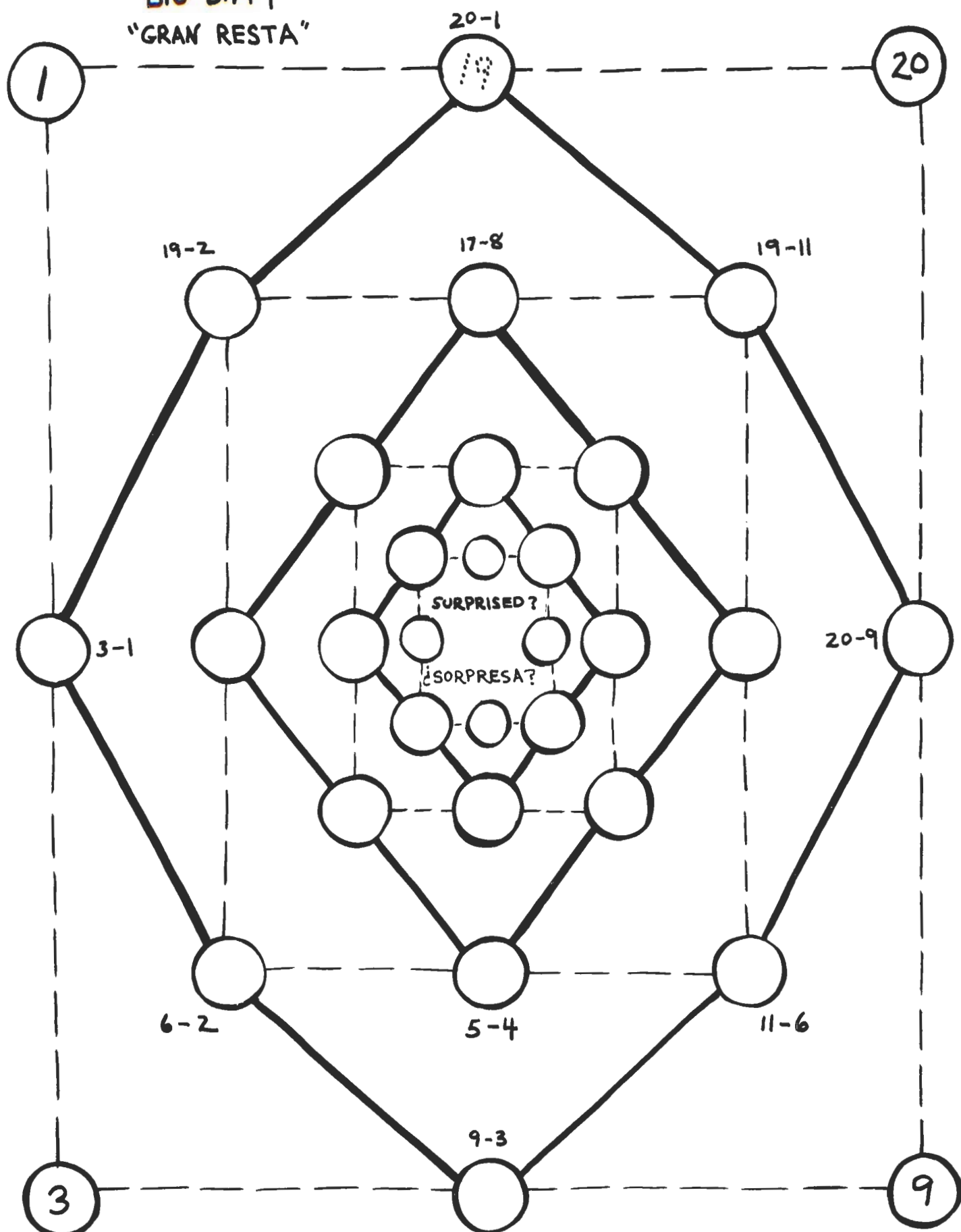


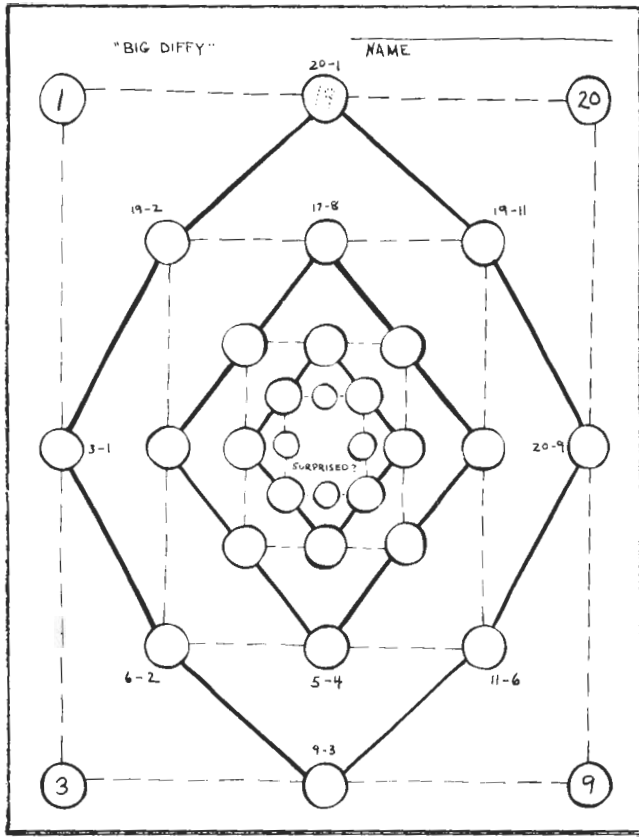
A STUDY OF DIFFERENCES  
THE "DIFFY" GAME

START WITH A NUMBER IN EACH CORNER.  
AND WORK TOWARD THE CENTER.

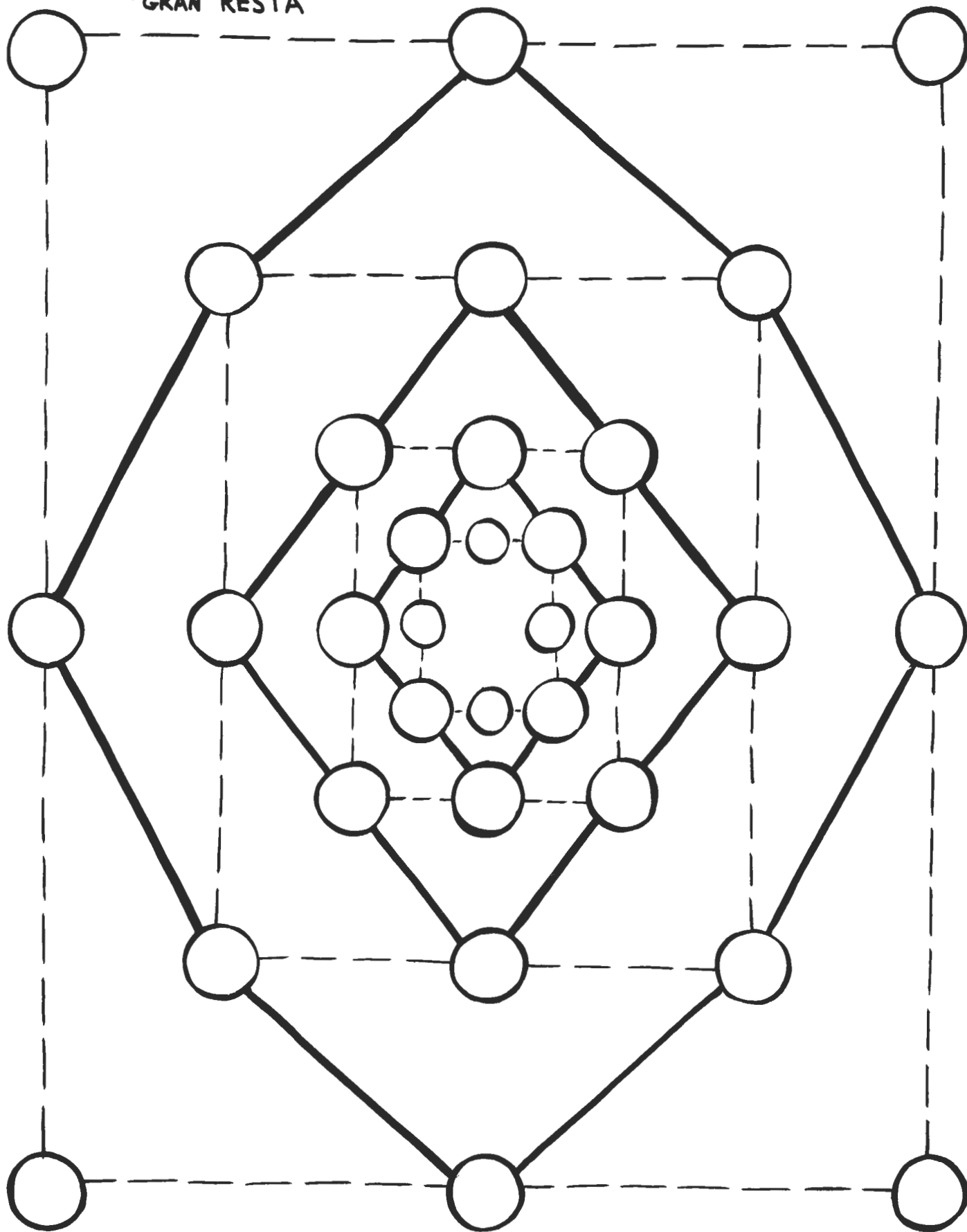


"BIG DIFFY"  
"GRAN RESTA"

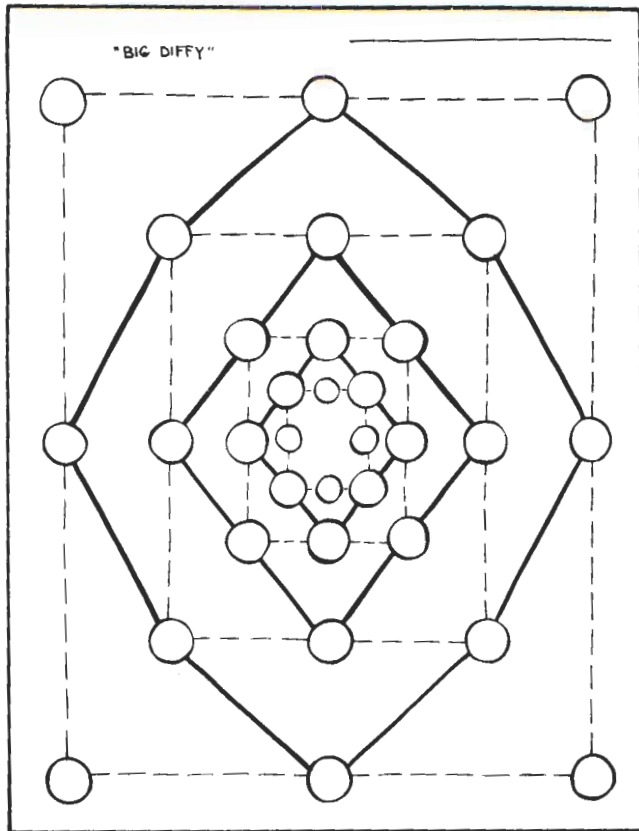




"BIG DIFFY"  
"GRAN RESTA"



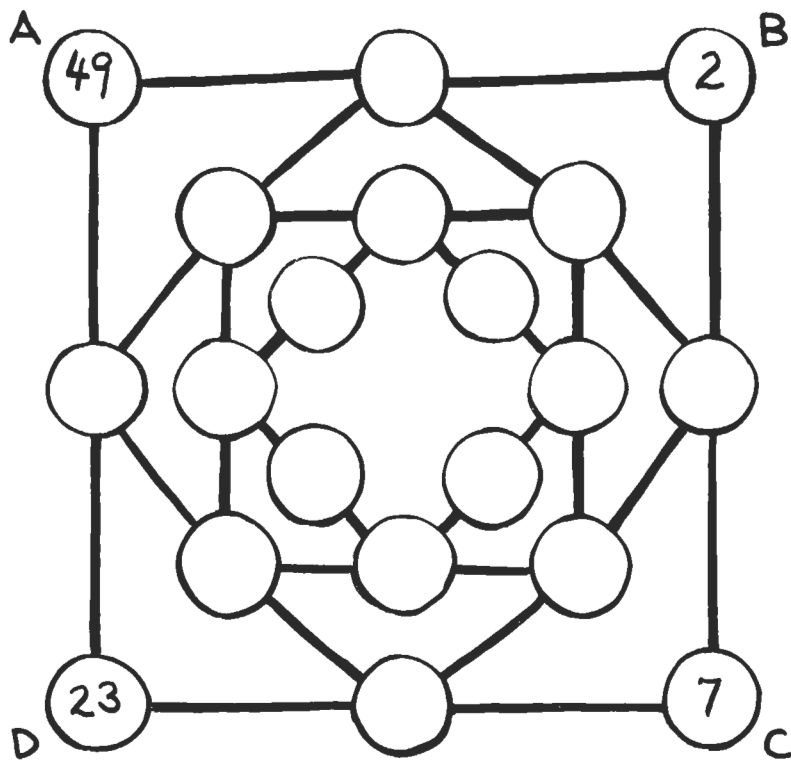




This page presents a format for a "differencing" array like those on the previous pages. The problem: Can you find four starting numbers which will difference all the way into the center array so that (1) there are four zeroes in the center array, **and** (2) the center set of differences is the **only** set with four zeros?

Obviously, if we started with a 7 in each corner, we would have differences of zero right away, and these could be repeated all the way into the center. But then the center four differences would not be the **only** set with four zeroes.

How many solutions to "Big Diffy" can you find?



|     | A  | B | C | D  | A  |
|-----|----|---|---|----|----|
|     | 49 | 2 | 7 | 23 | 49 |
|     | 47 | 5 |   |    | 47 |
| (2) | 42 |   |   |    | 42 |
| (3) |    |   |   |    |    |
| (4) |    |   |   |    |    |
| (5) |    |   |   |    |    |
| (6) |    |   |   |    |    |
| (7) |    |   |   |    |    |

|     | A | B | C | D | A |
|-----|---|---|---|---|---|
|     |   |   |   |   |   |
| (1) |   |   |   |   |   |
| (2) |   |   |   |   |   |
| (3) |   |   |   |   |   |
| (4) |   |   |   |   |   |
| (5) |   |   |   |   |   |
| (6) |   |   |   |   |   |

|     | A | B | C | D | A |
|-----|---|---|---|---|---|
|     |   |   |   |   |   |
| (1) |   |   |   |   |   |
| (2) |   |   |   |   |   |
| (3) |   |   |   |   |   |
| (4) |   |   |   |   |   |
| (5) |   |   |   |   |   |
| (6) |   |   |   |   |   |

|     | A | B | C | D | A |
|-----|---|---|---|---|---|
|     |   |   |   |   |   |
| (1) |   |   |   |   |   |
| (2) |   |   |   |   |   |
| (3) |   |   |   |   |   |
| (4) |   |   |   |   |   |
| (5) |   |   |   |   |   |
| (6) |   |   |   |   |   |

A SPACE-SAVING VARIATION FOR KEEPING SCORE.

NAME \_\_\_\_\_

A B C D A

49 2 7 23 49

1<sup>ST</sup> DIFFERENCE

(2) (3) (4) (5) (6) (7)

PICK ANY 4 NUMBERS AS STARTERS

A B C D A

(1) (2) (3) (4) (5) (6)

A B C D A

(1) (2) (3) (4) (5) (6)

A B C D A

(1) (2) (3) (4) (5) (6)

Question: How many different solutions can you find for the array on this page so that after six "differencings" all differences are zero?

This page provides a new format for keeping track of the differencing game. Again, choose any four numbers to start. Write them in the blocks labelled A, B, C, and D respectively. Then in the fifth block, repeat the number in the first block labelled A. By doing this, we can conveniently record the four differences. Suppose the starting numbers were 1, 2, 3, and 4. We would write:

| A | B | C | D | A |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 1 |
| 1 | 1 | 1 | 3 |   |

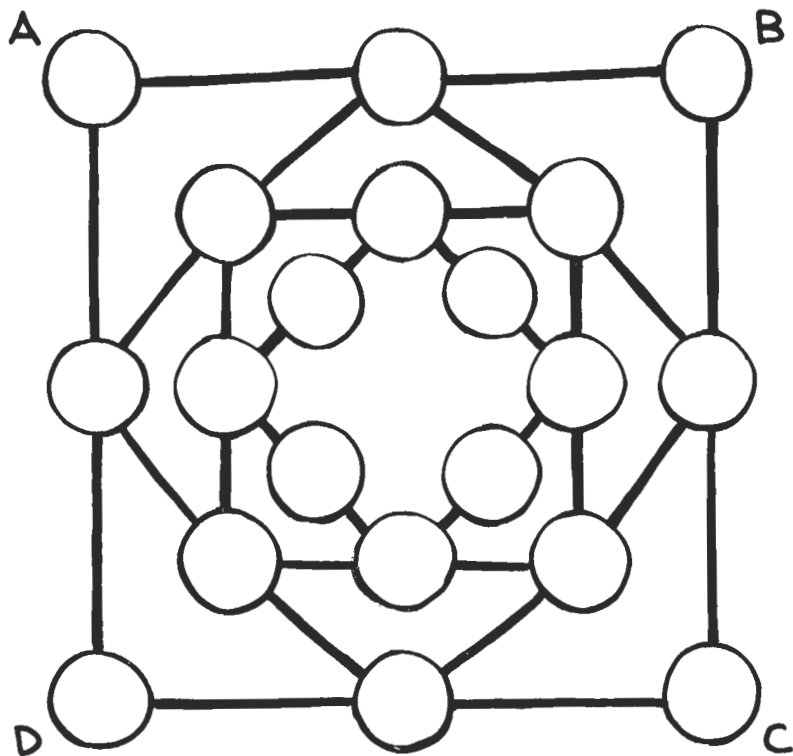
difference of A,B      difference of B,C      difference of C,D      difference of D,A

In the last block in the second row, we repeat the first number in the row, so we can easily find the next four differences:

| A | B | C | D | A |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 1 |
| 1 | 1 | 1 | 3 | 1 |
| 0 | 0 | 2 | 2 |   |

repeat of first number in this row

This procedure is repeated until all differences are zero.



|     | A | B | C | D | A |
|-----|---|---|---|---|---|
|     |   |   |   |   |   |
|     |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |

|     | A | B | C | D | A |
|-----|---|---|---|---|---|
|     |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |

|     | A | B | C | D | A |
|-----|---|---|---|---|---|
|     |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |

|     | A | B | C | D | A |
|-----|---|---|---|---|---|
|     |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |
| ( ) |   |   |   |   |   |



**RULE: THE SUM OF THE 2 OR MORE  
NUMBERS LOOPED IS THE NUMBER IN THE BOX.**

**REGLA: LA SUMA DE 2 O MÁS NÚMEROS  
ES EL NÚMERO QUE ESTÁ EN LA CAJA.**

**CAN ALL THE SUMS BE DIFFERENT?**

**¿PUEDEN SER DIFERENTES TODAS  
LAS SUMAS?**

|                                                                                          |                                                                                          |                                                                                          |                                                                                          |                                                                                          |                                                                                          |
|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| 4                                                                                        | 4                                                                                        | 4                                                                                        | 4                                                                                        | 4                                                                                        | 4                                                                                        |
| 2                                                                                        | 2                                                                                        | 2                                                                                        | 2                                                                                        | 2                                                                                        | 2                                                                                        |
| 1                                                                                        | 1                                                                                        | 1                                                                                        | 1                                                                                        | 1                                                                                        | 1                                                                                        |
| 3                                                                                        | 3                                                                                        | 3                                                                                        | 3                                                                                        | 3                                                                                        | 3                                                                                        |
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|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| 2                                                                                        | 2                                                                                        | 2                                                                                        | 2                                                                                        | 2                                                                                        | 2                                                                                        |
| 5                                                                                        | 5                                                                                        | 5                                                                                        | 5                                                                                        | 5                                                                                        | 5                                                                                        |
| 3                                                                                        | 3                                                                                        | 3                                                                                        | 3                                                                                        | 3                                                                                        | 3                                                                                        |
| 4                                                                                        | 4                                                                                        | 4                                                                                        | 4                                                                                        | 4                                                                                        | 4                                                                                        |
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|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| 3                                                                                        | 3                                                                                        | 3                                                                                        | 3                                                                                        | 3                                                                                        | 3                                                                                        |
| 6                                                                                        | 6                                                                                        | 6                                                                                        | 6                                                                                        | 6                                                                                        | 6                                                                                        |
| 4                                                                                        | 4                                                                                        | 4                                                                                        | 4                                                                                        | 4                                                                                        | 4                                                                                        |
| 5                                                                                        | 5                                                                                        | 5                                                                                        | 5                                                                                        | 5                                                                                        | 5                                                                                        |
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RULE: THE SUM OF THE 2 OR MORE NUMBERS LOOPED IS THE NUMBER IN THE BOX.

NAME \_\_\_\_\_

COULD YOU COMPLETE THE PAGE SO ALL NUMBERS IN THE BOXES ARE DIFFERENT?

|                      |                      |                      |                      |                      |                      |
|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| 4<br>2<br>1<br>3     | 4<br>2<br>1<br>3     | 4<br>2<br>1<br>3     | 4<br>2<br>1<br>3     | 4<br>2<br>1<br>3     | 4<br>2<br>1<br>3     |
| <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> |
| 2<br>5<br>3<br>4     | 2<br>5<br>3<br>4     | 2<br>5<br>3<br>4     | 2<br>5<br>3<br>4     | 2<br>5<br>3<br>4     | 2<br>5<br>3<br>4     |
| <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> |
| 3<br>6<br>4<br>5     | 3<br>6<br>4<br>5     | 3<br>6<br>4<br>5     | 3<br>6<br>4<br>5     | 3<br>6<br>4<br>5     | 3<br>6<br>4<br>5     |
| <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> |

- (4) Can all sums between the smallest and the largest be made?
- (5) How many different loop problems can be made using 4 numbers?

The objective here is to provide drill with numbers less than 10. Numbers are looped and their sum is placed in the box. The activity may be introduced in a variety of ways. Each child takes 4 beansticks containing 1, 2, 3, and 4 beans respectively. He picks up two or more beansticks and tells the sum. Instead of beansticks, cards with the numbers written on them can be used. In either case the teacher introduces the list of numbers and shows how to loop them as the children show their sticks or cards.

For those who are ready, start by showing the list from the top row on this page and asking someone to come up and loop two or more numbers. Record the sum and then have each child complete the top line on his paper looping different numbers each time. Review results emphasizing that more than 2 numbers may be looped.

Looking at any line of examples, here are some questions for discussion:

- (1) What is the smallest sum possible?
- (2) What is the largest sum possible?
- (3) Can any sums be made in more than one way?

RULE: THE SUM OF THE 2 OR MORE  
NUMBERS LOOPED IS THE NUMBER IN THE BOX.

REGLA: LA SUMA DE 2 O MÁS NÚMEROS  
ES EL NÚMERO QUE ESTÁ EN LA CAJA.

FAVOR DE HACER DIFERENTES TODOS  
LOS RESULTADOS.

PLEASE MAKE ALL RESULTS DIFFERENT.

|                                                                                          |                                                                                          |                                                                                          |                                                                                          |                                                                                          |                                                                                          |
|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| 3                                                                                        | 3                                                                                        | 3                                                                                        | 3                                                                                        | 3                                                                                        | 3                                                                                        |
| 4                                                                                        | 4                                                                                        | 4                                                                                        | 4                                                                                        | 4                                                                                        | 4                                                                                        |
| 7                                                                                        | 7                                                                                        | 7                                                                                        | 7                                                                                        | 7                                                                                        | 7                                                                                        |
| 8                                                                                        | 8                                                                                        | 8                                                                                        | 8                                                                                        | 8                                                                                        | 8                                                                                        |
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|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| 3                                                                                        | 3                                                                                        | 3                                                                                        | 3                                                                                        | 3                                                                                        | 3                                                                                        |
| 5                                                                                        | 5                                                                                        | 5                                                                                        | 5                                                                                        | 5                                                                                        | 5                                                                                        |
| 8                                                                                        | 8                                                                                        | 8                                                                                        | 8                                                                                        | 8                                                                                        | 8                                                                                        |
| 9                                                                                        | 9                                                                                        | 9                                                                                        | 9                                                                                        | 9                                                                                        | 9                                                                                        |
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|                                                                                          |                                                                                          |                                                                                          |                                                                                          |                                                                                          |                                                                                          |
|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| 4                                                                                        | 4                                                                                        | 4                                                                                        | 4                                                                                        | 4                                                                                        | 4                                                                                        |
| 5                                                                                        | 5                                                                                        | 5                                                                                        | 5                                                                                        | 5                                                                                        | 5                                                                                        |
| 6                                                                                        | 6                                                                                        | 6                                                                                        | 6                                                                                        | 6                                                                                        | 6                                                                                        |
| 7                                                                                        | 7                                                                                        | 7                                                                                        | 7                                                                                        | 7                                                                                        | 7                                                                                        |
| 9                                                                                        | 9                                                                                        | 9                                                                                        | 9                                                                                        | 9                                                                                        | 9                                                                                        |
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RULE: THE SUM OF THE 2 OR MORE NUMBERS LOOPED IS THE NUMBER IN THE BOX.

NAME \_\_\_\_\_

COULD YOU COMPLETE THE PAGE SO ALL NUMBERS IN THE BOXES ARE DIFFERENT?

|                      |                      |                      |                      |                      |                      |
|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| 3                    | 3                    | 3                    | 3                    | 3                    | 3                    |
| 4                    | 4                    | 4                    | 4                    | 4                    | 4                    |
| 7                    | 7                    | 7                    | 7                    | 7                    | 7                    |
| 8                    | 8                    | 8                    | 8                    | 8                    | 8                    |
| <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> |
|                      |                      |                      |                      |                      |                      |
| 3                    | 3                    | 3                    | 3                    | 3                    | 3                    |
| 5                    | 5                    | 5                    | 5                    | 5                    | 5                    |
| 8                    | 8                    | 8                    | 8                    | 8                    | 8                    |
| 9                    | 9                    | 9                    | 9                    | 9                    | 9                    |
| <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> |
|                      |                      |                      |                      |                      |                      |
| 4                    | 4                    | 4                    | 4                    | 4                    | 4                    |
| 5                    | 5                    | 5                    | 5                    | 5                    | 5                    |
| 6                    | 6                    | 6                    | 6                    | 6                    | 6                    |
| 7                    | 7                    | 7                    | 7                    | 7                    | 7                    |
| 9                    | 9                    | 9                    | 9                    | 9                    | 9                    |
| <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> | <input type="text"/> |

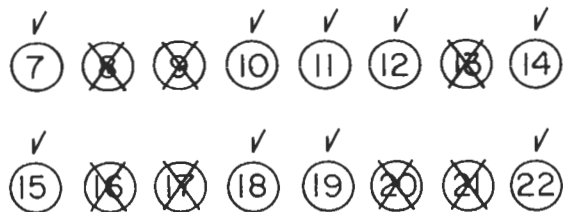
The last line on the page contains 5 numbers and there are 26 different ways to loop 2 or more numbers. You may want to offer more practice with this combination. You might make up your own page containing only 4, 5, 6, 7, and 9. Challenge the group to find all 26 possibilities.

Of course, you or your students can construct other problems of this type and investigate the sums.

Some groups may find it interesting to keep a list of possible sums.

### Example

$$\begin{array}{r}
 3 \\
 4 \\
 7 \\
 8 \\
 \hline
 \square
 \end{array}$$



Check the possible sums—cross out the impossible sums.

If desired, place 2 checks by any sum that can be made in more than one way. In the above example, 11 and 15 would get two checks. (7 can be looped in 2 different ways—but not when there is a requirement to “loop” 2 or more.)

FAVOR DE HACER DIFERENTES  
TODOS LOS RESULTADOS.

PLEASE MAKE ALL RESULTS DIFFERENT.

|          |          |          |          |          |          |          |
|----------|----------|----------|----------|----------|----------|----------|
| 2        | 2        | 2        | 2        | 2        | 2        | 2        |
| 3        | 3        | 3        | 3        | 3        | 3        | 3        |
| 4        | 4        | 4        | 4        | 4        | 4        | 4        |
| 5        | 5        | 5        | 5        | 5        | 5        | 5        |
| 6        | 6        | 6        | 6        | 6        | 6        | 6        |
| <u>7</u> | <u>7</u> | <u>8</u> | <u>8</u> | <u>9</u> | <u>9</u> | <u>9</u> |

|           |           |           |           |           |           |           |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 2         | 2         | 2         | 2         | 2         | 2         | 2         |
| 3         | 3         | 3         | 3         | 3         | 3         | 3         |
| 4         | 4         | 4         | 4         | 4         | 4         | 4         |
| 5         | 5         | 5         | 5         | 5         | 5         | 5         |
| 6         | 6         | 6         | 6         | 6         | 6         | 6         |
| <u>10</u> | <u>10</u> | <u>11</u> | <u>11</u> | <u>11</u> | <u>12</u> | <u>12</u> |

|           |           |           |           |           |           |           |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 2         | 2         | 2         | 2         | 2         | 2         | 2         |
| 3         | 3         | 3         | 3         | 3         | 3         | 3         |
| 4         | 4         | 4         | 4         | 4         | 4         | 4         |
| 5         | 5         | 5         | 5         | 5         | 5         | 5         |
| 6         | 6         | 6         | 6         | 6         | 6         | 6         |
| <u>13</u> | <u>13</u> | <u>14</u> | <u>14</u> | <u>15</u> | <u>15</u> | <u>16</u> |

RULE: THE SUM OF THE 2 OR MORE NUMBERS LOOPED IS THE NUMBER IN THE BOX.

NAME \_\_\_\_\_

WHEN SUMS ARE REPEATED, PLEASE FIND DIFFERENT COMBINATIONS.

|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 7 | 7 | 8 | 8 | 9 | 9 | 9 |

|    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|
| 2  | 2  | 2  | 2  | 2  | 2  | 2  |
| 3  | 3  | 3  | 3  | 3  | 3  | 3  |
| 4  | 4  | 4  | 4  | 4  | 4  | 4  |
| 5  | 5  | 5  | 5  | 5  | 5  | 5  |
| 6  | 6  | 6  | 6  | 6  | 6  | 6  |
| 10 | 10 | 11 | 11 | 11 | 12 | 12 |

|    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|
| 2  | 2  | 2  | 2  | 2  | 2  | 2  |
| 3  | 3  | 3  | 3  | 3  | 3  | 3  |
| 4  | 4  | 4  | 4  | 4  | 4  | 4  |
| 5  | 5  | 5  | 5  | 5  | 5  | 5  |
| 6  | 6  | 6  | 6  | 6  | 6  | 6  |
| 13 | 13 | 14 | 14 | 15 | 15 | 16 |

The format changes and now two or more numbers are looped to find a sum already given. Note the rule: if a sum is repeated, loop different numbers to make that sum.

All examples on this page use the same numbers. You might ask the children to find the possible sums that are not given. There are 5 of them. (5, 6, 17, 18, and 20—there is no combination for 19).

To expand the activity, prepare a list of 5 numbers. Ask the children to find all the ways to loop 2 or more numbers and report all the sums. Using the results of their research, you might make a page similar to this one and distribute it to the rest of the class for completion.

$$\underline{4 + 2 + 1 = \underline{\quad}}$$

$$\underline{4 - 2 + 1 = \underline{\quad}}$$

$$\underline{4 - 2 - 1 = \underline{\quad}}$$

$$\underline{4 + 2 - 1 = \underline{\quad}}$$

$$\underline{7 + 4 - 1 = \underline{\quad}}$$

$$\underline{7 + 4 \quad 1 = \underline{12}}$$

$$\underline{7 \quad 4 - 1 = \underline{2}}$$

$$\underline{7 - 4 \quad 1 = \underline{4}}$$

$$\underline{3 \quad 2 \quad 1 = \underline{6}}$$

$$\underline{3 \quad 2 \quad 1 = \underline{4}}$$

$$\underline{3 \quad 2 \quad 1 = \underline{2}}$$

$$\underline{3 \quad 2 \quad 1 = \underline{0}}$$

$$\underline{6 - 3 - 2 = \underline{\quad}}$$

$$\underline{6 + 3 \quad 2 = \underline{11}}$$

$$\underline{6 + 3 \quad 2 = \underline{7}}$$

$$\underline{6 \quad 3 + 2 = \underline{5}}$$

$$\underline{5 - 3 + 2 = \underline{\quad}}$$

$$\underline{5 + 3 \quad 2 = \underline{6}}$$

$$\underline{5 - 3 \quad 2 = \underline{0}}$$

$$\underline{5 \quad 3 \quad 2 = \underline{10}}$$

$$\underline{6 \quad 4 \quad 1 = \underline{11}}$$

$$\underline{6 \quad 4 \quad 1 = \underline{9}}$$

$$\underline{6 \quad 4 \quad 1 = \underline{3}}$$

$$\underline{6 \quad 4 \quad 1 = \underline{1}}$$

| NAME _____                      |                                 |
|---------------------------------|---------------------------------|
| $4 + 2 + 1 = \underline{\quad}$ | $6 - 3 - 2 = \underline{\quad}$ |
| $4 - 2 + 1 = \underline{\quad}$ | $6 + 3 - 2 = \underline{11}$    |
| $4 - 2 - 1 = \underline{\quad}$ | $6 + 3 - 2 = \underline{7}$     |
| $4 + 2 - 1 = \underline{\quad}$ | $6 - 3 + 2 = \underline{5}$     |
| $7 + 4 - 1 = \underline{\quad}$ | $5 - 3 + 2 = \underline{\quad}$ |
| $7 + 4 - 1 = \underline{12}$    | $5 + 3 - 2 = \underline{6}$     |
| $7 - 4 - 1 = \underline{2}$     | $5 - 3 - 2 = \underline{0}$     |
| $7 - 4 - 1 = \underline{4}$     | $5 - 3 + 2 = \underline{10}$    |
| $3 - 2 - 1 = \underline{6}$     | $6 - 4 - 1 = \underline{11}$    |
| $3 - 2 - 1 = \underline{4}$     | $6 - 4 - 1 = \underline{9}$     |
| $3 - 2 - 1 = \underline{2}$     | $6 - 4 - 1 = \underline{3}$     |
| $3 - 2 - 1 = \underline{0}$     | $6 - 4 - 1 = \underline{1}$     |

We suggest agreement on a rule that applies to this particular activity: rather than introducing "parentheses," we prefer the rule that operations are to be carried out from left to right, one at a time.

For example: we interpret

$$4 - 2 - 1 = \underline{\quad}$$

as a way of saying "start with 4; subtract 2; and then subtract 1 from the result"

$$4 - 2 = 2 \text{ and } 2 - 1 = 1$$

Similarly

$$4 - 2 + 1 = \underline{\quad}$$

$$\text{because } 4 - 2 = 2 \text{ and } 2 + 1 = 3$$

Later on the danger of ambiguity is avoided by using parentheses:

$$(4 - 2) - 1 = \underline{1}$$

and

$$(4 + 2) - 1 = \underline{5}$$

If the "left to right" rule disturbs anyone, then he can in all cases leave no doubt he's enclosing the first part of each statement in parentheses:

$$(3 + 2) + 1 = \underline{6}$$

$$(3 + 2) - 1 = \underline{4}$$

$$(3 - 2) + 1 = \underline{2}$$

$$(3 - 2) - 1 = \underline{0}$$

A standard agreement in algebra is that when all operational signs are the same, work from left to right if there are no parentheses. Another is that, when addition and subtraction are included, then carry out the addition first.

With these rules, then parentheses are required in only the third of the four examples above:

$$(3 - 2) + 1 = \underline{2}$$

We hope our temporary "left to right rule" is sufficient at this time.

---

$$7 + 4 - 3 = \underline{\quad}$$

$$7 \quad 4 \quad 3 = 14$$

$$7 \quad 4 \quad 3 = 6$$

$$7 \quad 4 \quad 3 = 0$$

$$9 \quad 5 \quad 3 = \underline{\quad}$$

$$9 \quad 5 \quad 3 = \underline{\quad}$$

$$9 \quad 5 \quad 3 = \underline{\quad}$$

$$9 \quad 5 \quad 3 = \underline{\quad}$$

$$11 \quad 7 \quad 3 = \underline{\quad}$$

$$11 \quad 7 \quad 3 = \underline{\quad}$$

$$11 \quad 7 \quad 3 = \underline{\quad}$$

$$11 \quad 7 \quad 3 = \underline{\quad}$$

---

$$8 \quad 5 \quad 2 = \underline{\quad}$$

$$8 \quad 5 \quad 2 = \underline{\quad}$$

$$8 \quad 5 \quad 2 = \underline{\quad}$$

$$8 \quad 5 \quad 2 = \underline{\quad}$$

$$10 \quad 6 \quad 4 = \underline{\quad}$$

$$10 \quad 6 \quad 4 = \underline{\quad}$$

$$10 \quad 6 \quad 4 = \underline{\quad}$$

$$10 \quad 6 \quad 4 = \underline{\quad}$$

$$12 \quad 5 \quad 2 = \underline{\quad}$$

$$12 \quad 5 \quad 2 = \underline{\quad}$$

$$12 \quad 5 \quad 2 = \underline{\quad}$$

$$12 \quad 5 \quad 2 = \underline{\quad}$$

NAME \_\_\_\_\_

IN EACH GROUP OF FOUR SENTENCES,  
MAKE THEM ALL DIFFERENT, IF POSSIBLE.

|                                  |                                  |
|----------------------------------|----------------------------------|
| $7 + 4 - 3 = \underline{\quad}$  | $8 \ 5 \ 2 = \underline{\quad}$  |
| $7 \ 4 \ 3 = 14$                 | $8 \ 5 \ 2 = \underline{\quad}$  |
| $7 \ 4 \ 3 = 6$                  | $8 \ 5 \ 2 = \underline{\quad}$  |
| $7 \ 4 \ 3 = 0$                  | $8 \ 5 \ 2 = \underline{\quad}$  |
| $9 \ 5 \ 3 = \underline{\quad}$  | $10 \ 6 \ 4 = \underline{\quad}$ |
| $9 \ 5 \ 3 = \underline{\quad}$  | $10 \ 6 \ 4 = \underline{\quad}$ |
| $9 \ 5 \ 3 = \underline{\quad}$  | $10 \ 6 \ 4 = \underline{\quad}$ |
| $9 \ 5 \ 3 = \underline{\quad}$  | $10 \ 6 \ 4 = \underline{\quad}$ |
| $11 \ 7 \ 3 = \underline{\quad}$ | $12 \ 5 \ 2 = \underline{\quad}$ |
| $11 \ 7 \ 3 = \underline{\quad}$ | $12 \ 5 \ 2 = \underline{\quad}$ |
| $11 \ 7 \ 3 = \underline{\quad}$ | $12 \ 5 \ 2 = \underline{\quad}$ |
| $11 \ 7 \ 3 = \underline{\quad}$ | $12 \ 5 \ 2 = \underline{\quad}$ |

They are not to be discouraged from this shortcut: it is an important pattern to discover. Further, they must still compute the final result in each case.

Again the rule for "order" is—work step by step from left to right.

Thus

$$\underline{7 \ 4 \ 3} = \underline{6}$$

can be completed as a true statement in this way:

$$\underline{7 - 4 + 3} = \underline{6}$$

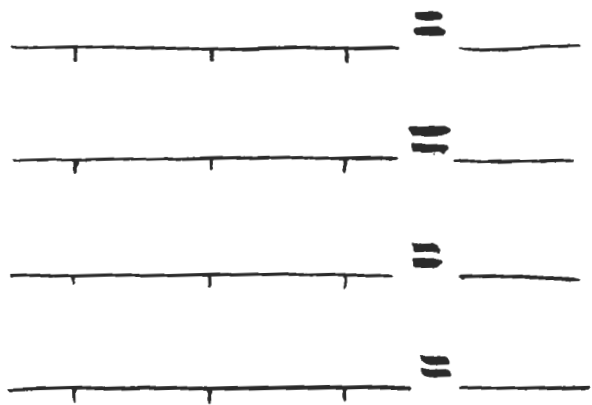
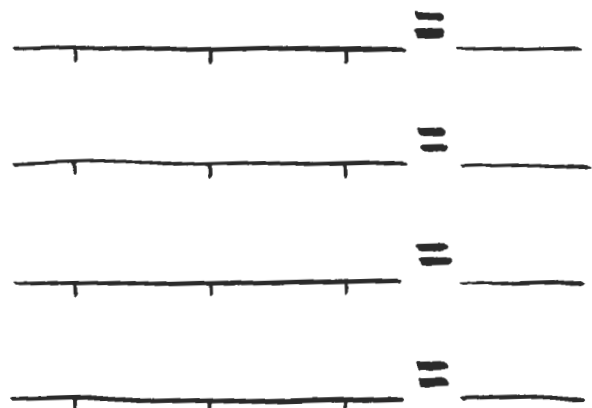
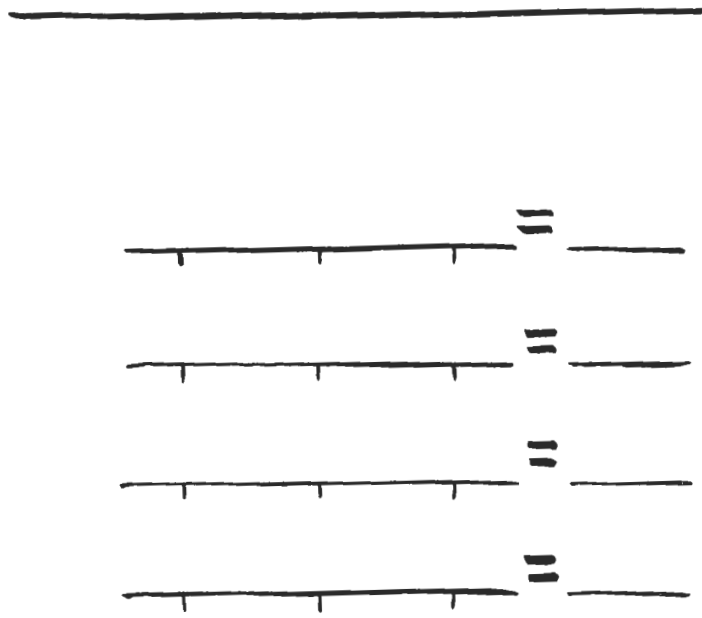
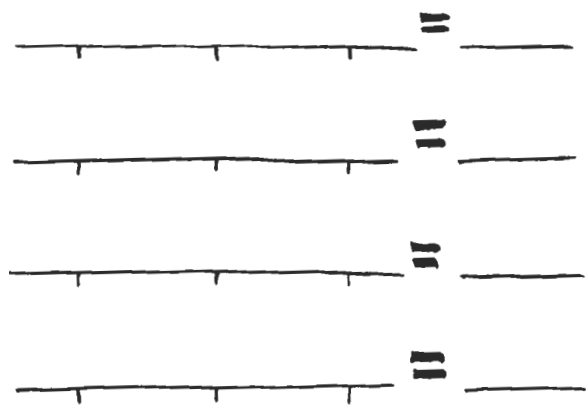
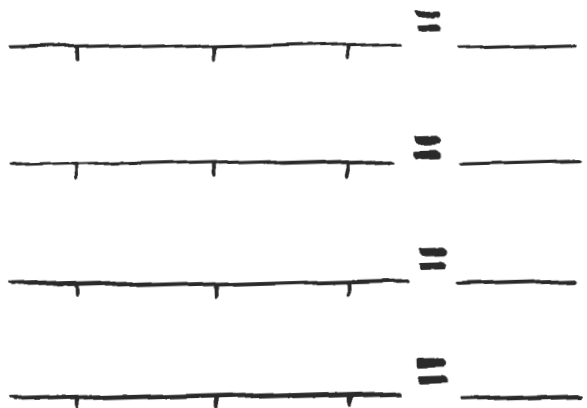
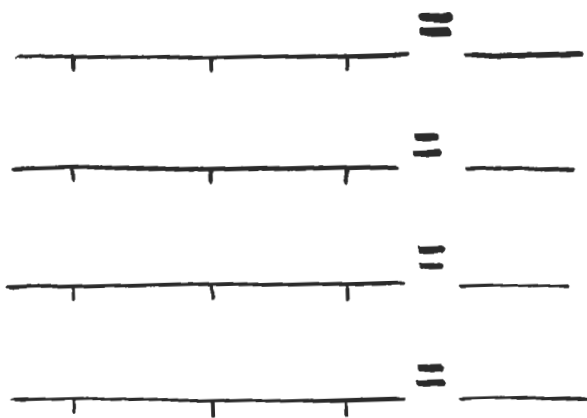
Some children might notice that when the result is not given, they can use a simple pattern to produce 4 different statements by using the 4 combinations possible with 2 different operations:

$$\square + \triangle + \bigcirc = \underline{\quad}$$

$$\square + \triangle - \bigcirc = \underline{\quad}$$

$$\square - \triangle + \bigcirc = \underline{\quad}$$

$$\square - \triangle - \bigcirc = \underline{\quad}$$







$$\underline{8 + 4 + 2 + 1 = \underline{\quad}}$$

$$\underline{8 + 4 - 2 + 1 = \underline{\quad}}$$

$$\underline{8 + 4 + 2 \quad 1 = \underline{13}}$$

$$\underline{8 - 4 - 2 - 1 = \underline{\quad}}$$

$$\underline{8 \quad 4 \quad 2 \quad 1 = \underline{9}}$$

$$\underline{8 \quad 4 \quad 2 \quad 1 = \underline{7}}$$

$$\underline{8 \quad 4 \quad 2 \quad 1 = \underline{5}}$$

$$\underline{8 \quad 4 \quad 2 \quad 1 = \underline{3}}$$

$$\underline{10 + 4 + 3 + 2 = \underline{\quad}}$$

$$\underline{10 + 4 + 3 - 2 = \underline{\quad}}$$

$$\underline{10 + 4 - 3 \quad 2 = \underline{13}}$$

$$\underline{10 \quad 4 \quad 3 \quad 2 = \underline{\quad}}$$

$$\underline{10 \quad 4 - 3 - 2 = \underline{9}}$$

$$\underline{10 \quad 4 \quad 3 \quad 2 = \underline{7}}$$

$$\underline{10 \quad 4 \quad 3 \quad 2 = \underline{5}}$$

$$\underline{10 \quad 4 \quad 3 \quad 2 = \underline{1}}$$

$$\underline{12 \quad 6 \quad 3 \quad 1 = \underline{22}}$$

$$\underline{12 \quad 6 \quad 3 \quad 1 = \underline{20}}$$

$$\underline{12 \quad 6 \quad 3 \quad 1 = \underline{16}}$$

$$\underline{12 \quad 6 \quad 3 \quad 1 = \underline{14}}$$

$$\underline{12 \quad 6 \quad 3 \quad 1 = \underline{10}}$$

$$\underline{12 \quad 6 \quad 3 \quad 1 = \underline{8}}$$

$$\underline{12 \quad 6 \quad 3 \quad 1 = \underline{4}}$$

$$\underline{12 \quad 6 \quad 3 \quad 1 = \underline{2}}$$

FAVOR DE HACER DIFERENTES TODOS  
LOS RESULTADOS.

PLEASE MAKE ALL RESULTS DIFFERENT

$$\underline{9 + 7 + 5 + 3 = \underline{24}}$$

$$\underline{9 \quad 7 \quad 5 \quad 3 = \underline{18}}$$

$$\underline{9 \quad 7 \quad 5 \quad 3 = \underline{14}}$$

$$\underline{9 \quad 7 \quad 5 \quad 3 = \underline{10}}$$

$$\underline{9 \quad 7 \quad 5 \quad 3 = \underline{8}}$$

$$\underline{7 \quad 5 \quad 9 \quad 3 = \underline{6}}$$

$$\underline{9 \quad 7 \quad 5 \quad 3 = \underline{4}}$$

$$\underline{7 \quad 5 \quad 9 \quad 3 = \underline{0}}$$

$$\underline{12 \quad 5 \quad 4 \quad 2 = \underline{\quad}}$$

$$\underline{12 \quad 5 \quad 4 \quad 2 = \underline{\quad}}$$

$$\underline{12 \quad 5 \quad 4 \quad 2 = \underline{\quad}}$$

$$\underline{12 \quad 5 \quad 4 \quad 2 = \underline{\quad}}$$

$$\underline{12 \quad 5 \quad 4 \quad 2 = \underline{\quad}}$$

$$\underline{12 \quad 5 \quad 4 \quad 2 = \underline{\quad}}$$

$$\underline{12 \quad 5 \quad 4 \quad 2 = \underline{\quad}}$$

$$\underline{12 \quad 5 \quad 4 \quad 2 = \underline{\quad}}$$

NAME \_\_\_\_\_

|                                                                                     |                                                                                          |
|-------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| $8 + 4 + 2 + 1 =$<br>$8 + 4 - 2 + 1 =$<br>$8 + 4 + 2 - 1 = 13$<br>$8 - 4 - 2 - 1 =$ | $8 \ 4 \ 2 \ 1 = 9$<br>$8 \ 4 \ 2 \ 1 = 7$<br>$8 \ 4 \ 2 \ 1 = 5$<br>$8 \ 4 \ 2 \ 1 = 3$ |
|-------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|

---

|                                                                                                                                                                                         |                                                                                                                                                                                                   |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $10 + 4 + 3 + 2 =$<br>$10 + 4 + 3 - 2 =$<br>$10 + 4 - 3 - 2 = 13$<br>$10 \ 4 \ 3 \ 2 =$<br>$10 \ 4 - 3 - 2 = 9$<br>$10 \ 4 \ 3 \ 2 = 7$<br>$10 \ 4 \ 3 \ 2 = 5$<br>$10 \ 4 \ 3 \ 2 = 1$ | $12 \ 6 \ 3 \ 1 = 22$<br>$12 \ 6 \ 3 \ 1 = 20$<br>$12 \ 6 \ 3 \ 1 = 16$<br>$12 \ 6 \ 3 \ 1 = 14$<br>$12 \ 6 \ 3 \ 1 = 10$<br>$12 \ 6 \ 3 \ 1 = 8$<br>$12 \ 6 \ 3 \ 1 = 4$<br>$12 \ 6 \ 3 \ 1 = 2$ |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

PLEASE MAKE ALL RESULTS DIFFERENT →

|                                                                                                                                                                                          |                                                                                                                                                                              |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $9 + 7 + 5 + 3 = 24$<br>$9 \ 7 \ 5 \ 3 = 18$<br>$9 \ 7 \ 5 \ 3 = 14$<br>$9 \ 7 \ 5 \ 3 = 10$<br>$9 \ 7 \ 5 \ 3 = 8$<br>$7 \ 5 \ 9 \ 3 = 6$<br>$9 \ 7 \ 5 \ 3 = 4$<br>$7 \ 5 \ 9 \ 3 = 0$ | $12 \ 5 \ 4 \ 2 =$<br>$12 \ 5 \ 4 \ 2 =$<br>$12 \ 5 \ 4 \ 2 =$<br>$12 \ 5 \ 4 \ 2 =$<br>$12 \ 5 \ 4 \ 2 =$<br>$12 \ 5 \ 4 \ 2 =$<br>$12 \ 5 \ 4 \ 2 =$<br>$12 \ 5 \ 4 \ 2 =$ |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

... an extension of an important pattern mentioned earlier. However, they must still carry out the indicated computations.

In each set of 8 examples on this page results are all odd or they are all even. Why?

Also, in each set of 8 examples, all results are different. Why? Could a set of 4 numbers be selected that would not lead to 8 different results?

Notice what happens in the following cases using 12, 6, 3, and 3:

$$\begin{aligned}
 12 + 6 - 3 - 3 &= 12 \\
 12 - 6 + 3 + 3 &= 12 \\
 12 + 6 - 3 + 3 &= 18 \\
 12 + 6 + 3 - 3 &= 18 \\
 12 - 6 - 3 + 3 &= 6 \\
 12 - 6 + 3 - 3 &= 6
 \end{aligned}$$

However, they will be different if (1) no number is repeated and (2) if none of the numbers after the first is the sum of the other two.

(Note: in the example depending on 9, 7, 5, and 3, the order of these numbers is changed to avoid the necessity for negative numbers.)

We hope our general "left to right" rule for order of operations is sufficient. Otherwise, to avoid ambiguity we would need double parentheses in such cases as:

$$[(8 - 4) - 2] + 1 = 3$$

or

$$(8 - 4) - (2 - 1) = 3$$

Again, when asked to "make all results different" some may see a method for producing all combinations of different operational signs taken 3 at a time:

|                                                                                                        |                                                                                                        |
|--------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|
| $\begin{array}{ccc} + & + & + \\ \hline + & + & - \\ \hline + & - & + \\ \hline - & + & + \end{array}$ | $\begin{array}{ccc} - & - & - \\ \hline - & - & + \\ \hline - & + & - \\ \hline + & - & - \end{array}$ |
|--------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|

FAVOR DE HACER DIFERENTES TODOS  
LOS RESULTADOS.

PLEASE MAKE ALL RESULTS DIFFERENT.

$$11 - 6 + 3 + 2 = \underline{\quad}$$

$$11 + 6 + 3 - 2 = 18$$

$$11 - 6 - 3 - 2 = 0$$

$$11 - 6 - 3 - 2 = \underline{\quad}$$

$$11 - 6 - 3 - 2 = \underline{\quad}$$

$$11 - 6 - 3 - 2 = \underline{\quad}$$

$$11 - 6 - 3 - 2 = \underline{\quad}$$

$$11 - 6 - 3 - 2 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

IN EACH GROUP OF 8 SENTENCES,  
PLEASE MAKE THEM ALL DIFFERENT.

NAME \_\_\_\_\_

$$11 - 6 + 3 + 2 = \underline{\quad}$$

$$11 + 6 + 3 - 2 = 18$$

$$11 - 6 - 3 - 2 = 0$$

$$11 - 6 - 3 - 2 = \underline{\quad}$$

$$11 - 6 - 3 - 2 = \underline{\quad}$$

$$11 - 6 - 3 - 2 = \underline{\quad}$$

$$11 - 6 - 3 - 2 = \underline{\quad}$$

$$11 - 6 - 3 - 2 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$12 - 7 - 4 - 1 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$13 - 6 - 4 - 2 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$14 - 8 - 5 - 1 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

$$15 - 6 - 5 - 3 = \underline{\quad}$$

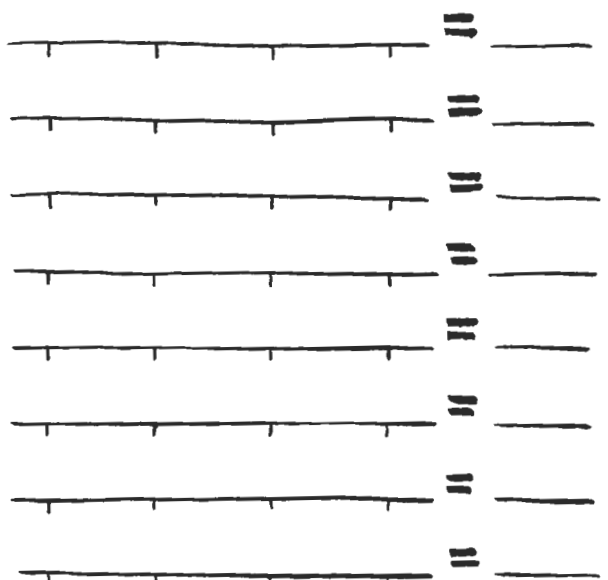
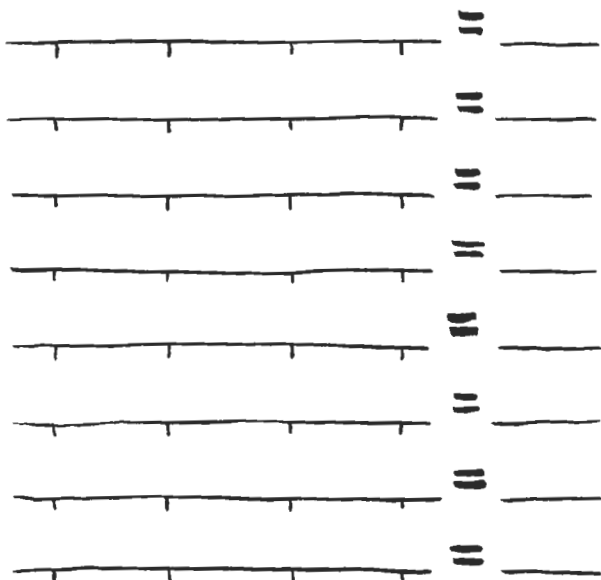
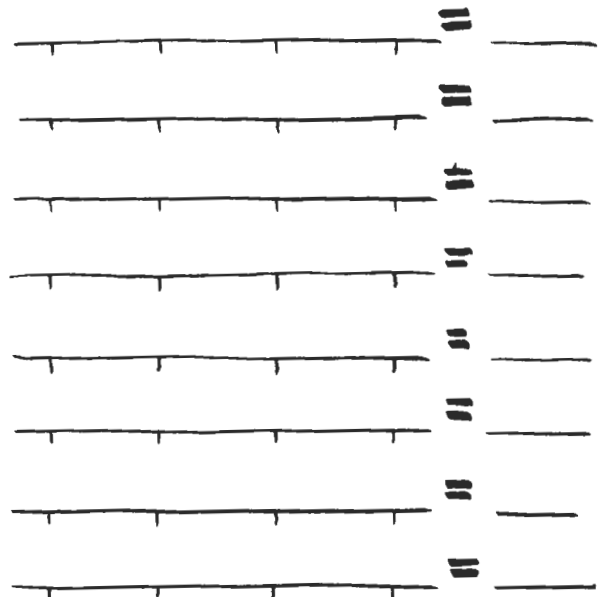
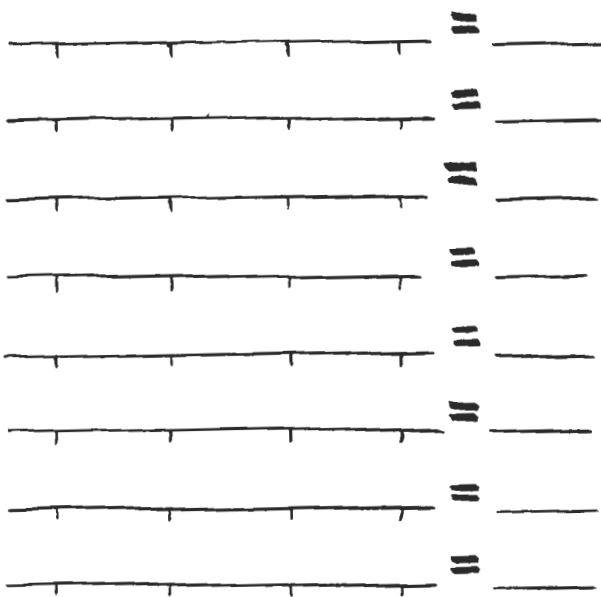
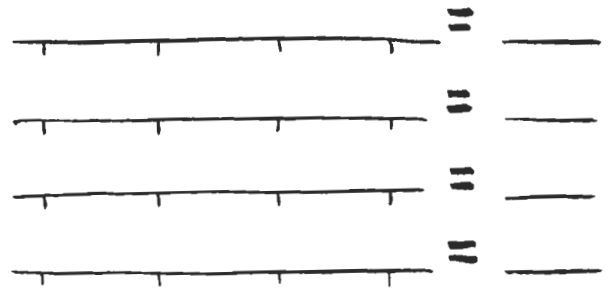
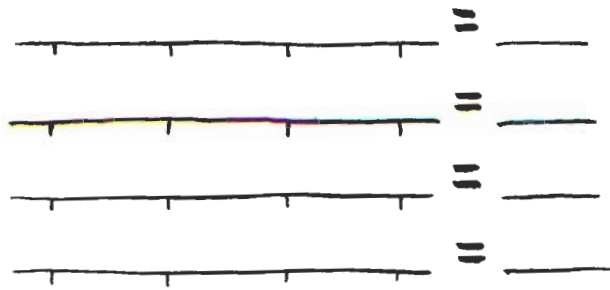
Again the "left to right" rule for order.

In the third example is the first instance in this activity in which all results are not different.

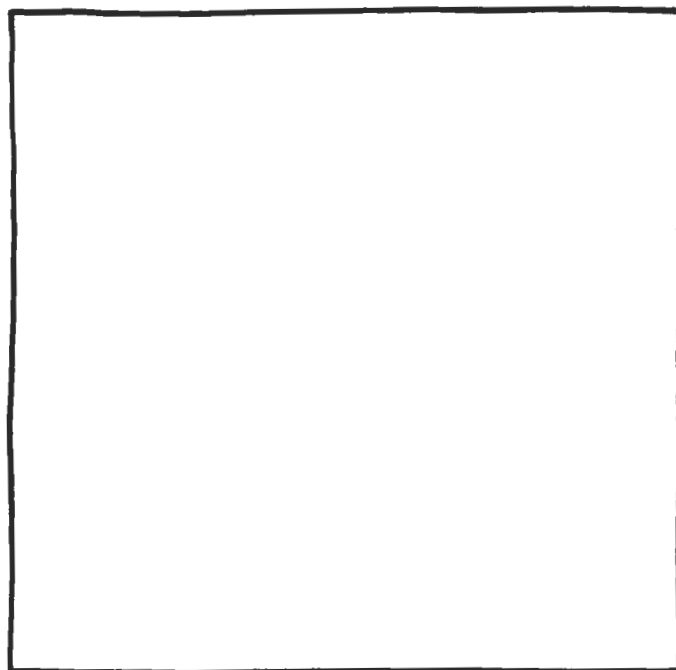
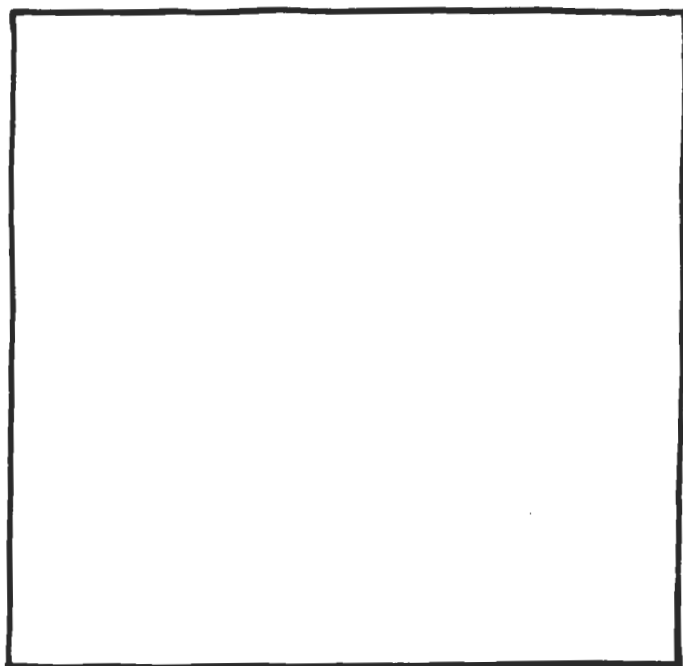
$$13 + 6 - 4 - 2 = 13$$

$$13 - 6 + 4 + 2 = 13$$

In the 1st, 2nd, and 4th examples, "0" appears as a result. Why? Why not in the 3rd and 5th examples?







|   |  |  |
|---|--|--|
| 3 |  |  |
| 4 |  |  |

11

|   |  |  |
|---|--|--|
| 3 |  |  |
| 4 |  |  |

12

|   |  |  |
|---|--|--|
| 3 |  |  |
| 4 |  |  |

7

|   |  |  |
|---|--|--|
| 3 |  |  |
| 4 |  |  |

8

|   |  |  |
|---|--|--|
| 3 |  |  |
| 4 |  |  |

9

|   |  |  |
|---|--|--|
| 3 |  |  |
| 4 |  |  |

10

|   |  |  |
|---|--|--|
| 3 |  |  |
| 4 |  |  |

13

|   |  |  |
|---|--|--|
| 3 |  |  |
| 4 |  |  |

17

|   |  |  |
|---|--|--|
| 3 |  |  |
| 4 |  |  |

14

|   |  |  |
|---|--|--|
| 3 |  |  |
| 4 |  |  |

18

|   |  |  |
|---|--|--|
| 3 |  |  |
| 4 |  |  |

21

|   |  |  |
|---|--|--|
| 3 |  |  |
| 4 |  |  |

6



FEWEST STICKS

NAME \_\_\_\_\_

THIS SIDE FOR 3-STICKS

THIS SIDE FOR 4-STICKS

|                                                                        |                                                                        |                                                                        |                                                                        |
|------------------------------------------------------------------------|------------------------------------------------------------------------|------------------------------------------------------------------------|------------------------------------------------------------------------|
| $\begin{array}{r l} 3 & \\ \hline 4 & \\ \hline \hline 11 \end{array}$ | $\begin{array}{r l} 3 & \\ \hline 4 & \\ \hline \hline 12 \end{array}$ | $\begin{array}{r l} 3 & \\ \hline 4 & \\ \hline \hline 1 \end{array}$  | $\begin{array}{r l} 3 & \\ \hline 4 & \\ \hline \hline 8 \end{array}$  |
| $\begin{array}{r l} 3 & \\ \hline 4 & \\ \hline \hline 9 \end{array}$  | $\begin{array}{r l} 3 & \\ \hline 4 & \\ \hline \hline 10 \end{array}$ | $\begin{array}{r l} 3 & \\ \hline 4 & \\ \hline \hline 13 \end{array}$ | $\begin{array}{r l} 3 & \\ \hline 4 & \\ \hline \hline 17 \end{array}$ |
| $\begin{array}{r l} 3 & \\ \hline 4 & \\ \hline \hline 14 \end{array}$ | $\begin{array}{r l} 3 & \\ \hline 4 & \\ \hline \hline 18 \end{array}$ | $\begin{array}{r l} 3 & \\ \hline 4 & \\ \hline \hline 21 \end{array}$ | $\begin{array}{r l} 3 & \\ \hline 4 & \\ \hline \hline 6 \end{array}$  |

In the next example there are 2 possible solutions.

$$\begin{array}{r|l} 3 & 4 \\ \hline 4 & 0 \\ \hline \hline 12 \end{array} \quad \text{and} \quad \begin{array}{r|l} 3 & 0 \\ \hline 4 & 3 \\ \hline \hline 12 \end{array}$$

The second is more appropriate because it uses the "fewest sticks."

Each child or group needs a supply of at least four 4-sticks and three 3-sticks for the examples on this page. There are separate areas for the 3-sticks and 4-sticks used in an example.

The first problem is given in this form:

$$\begin{array}{r|l} \text{sticks} & \text{beans} \\ \hline 3 & \\ \hline 4 & \\ \hline \hline 11 \end{array}$$

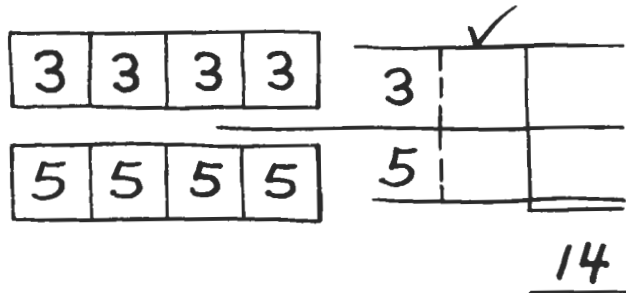
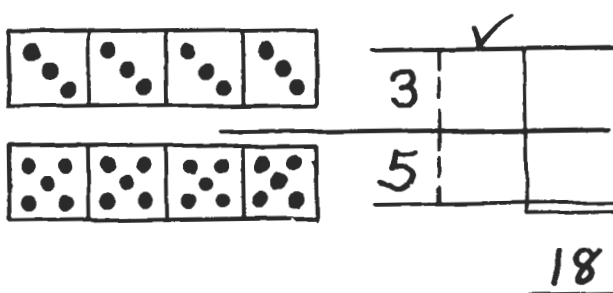
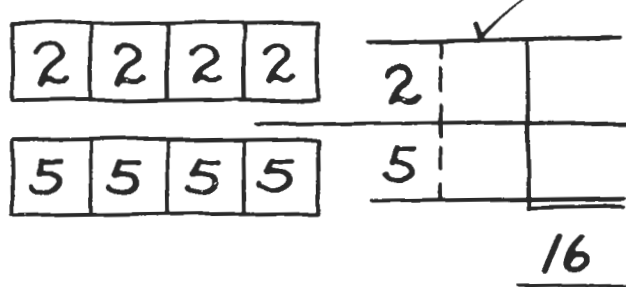
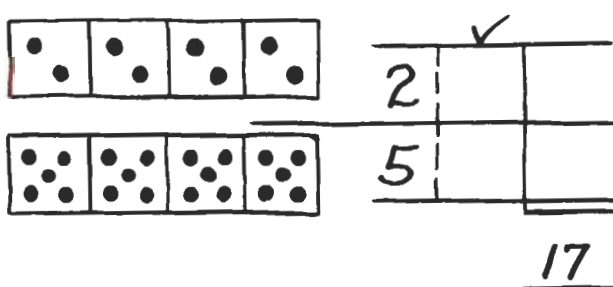
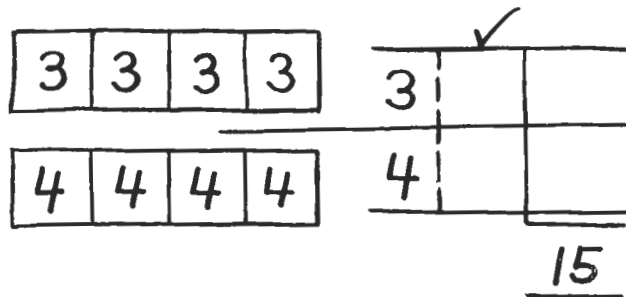
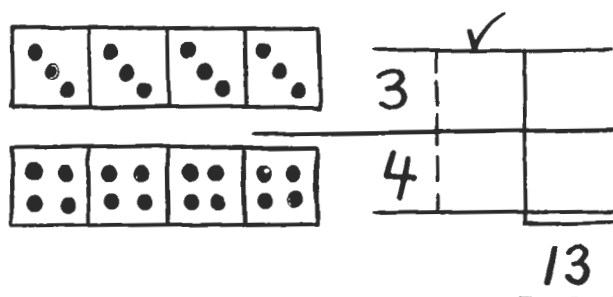
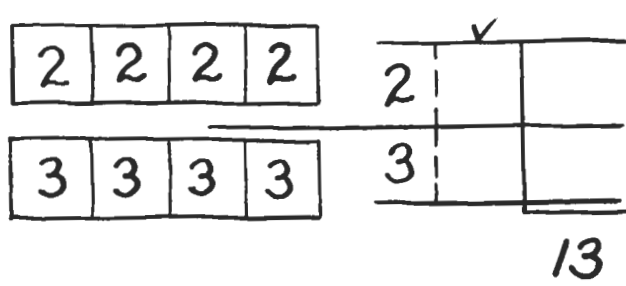
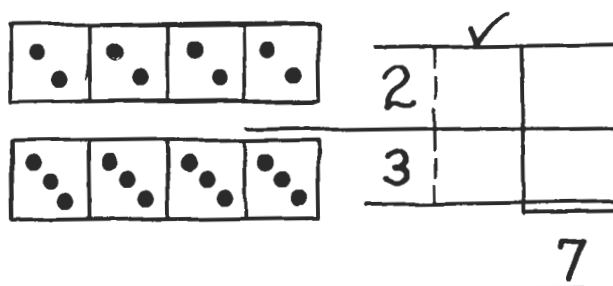
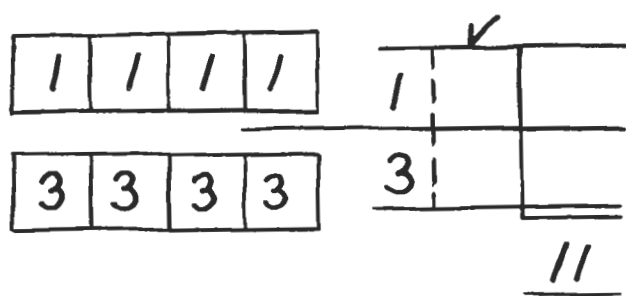
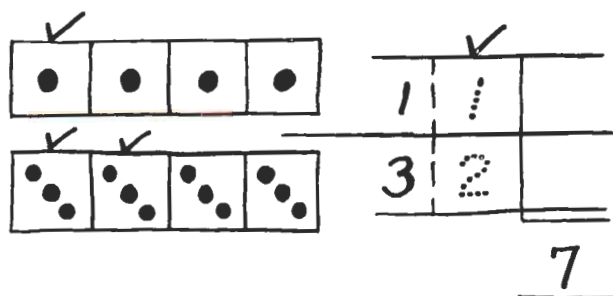
—please find a combination of 3-sticks and 4-sticks with a total of exactly 11 beans. The note at the top indicates that the most appropriate solution uses the fewest sticks.

In this case the answer is 1 3-stick with a total of 3 beans and 2 4-sticks with a total of 8 beans; and  $3 + 8 = 11$ . That solution is indicated in this way:

$$\begin{array}{r|l} \text{sticks} & \text{beans} \\ \hline 3 & 1 \\ \hline 4 & 2 \\ \hline \hline 11 \end{array}$$

# "FEWEST CHECK-MARKS"

"MENOS MARCAS"



**FEWEST CHECK-MARKS** NAME \_\_\_\_\_

|                                                                                                                                                                                            |                                                                                                                                                                                            |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                                                                                                                                                                            |                                                                                                                                                                                            |
| <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;">1</div> <div style="border: 1px solid black; padding: 2px;">2</div> </div> | <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;">3</div> <div style="border: 1px solid black; padding: 2px;">4</div> </div> |
| 7                                                                                                                                                                                          | 11                                                                                                                                                                                         |
| <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;">2</div> <div style="border: 1px solid black; padding: 2px;">3</div> </div> | <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;">2</div> <div style="border: 1px solid black; padding: 2px;">3</div> </div> |
| 13                                                                                                                                                                                         | 13                                                                                                                                                                                         |
| <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;">3</div> <div style="border: 1px solid black; padding: 2px;">4</div> </div> | <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;">3</div> <div style="border: 1px solid black; padding: 2px;">4</div> </div> |
| 13                                                                                                                                                                                         | 15                                                                                                                                                                                         |
| <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;">2</div> <div style="border: 1px solid black; padding: 2px;">5</div> </div> | <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;">2</div> <div style="border: 1px solid black; padding: 2px;">5</div> </div> |
| 17                                                                                                                                                                                         | 16                                                                                                                                                                                         |
| <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;">3</div> <div style="border: 1px solid black; padding: 2px;">5</div> </div> | <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;">3</div> <div style="border: 1px solid black; padding: 2px;">5</div> </div> |
| 18                                                                                                                                                                                         | 14                                                                                                                                                                                         |

Notice that there are many different things for the student to do. They include:

- (1) Seeing what combination of numbers is to be used.
- (2) Seeing what sum is asked for.
- (3) Determining how to make the number using the fewest checkmarks.
- (4) Recording the number of checkmarks used for each one.
- (5) Recording the total number for each one.
- (6) Adding the numbers to see if they make the sum asked for.

To facilitate learning all these steps we'll start by using beansticks, or squares of paper with either dots or numerals on them. Each child should have four each of the numbers 1, 2, 3, 4 and 5.

#### 4 each of any set of the above

The activity is described as if beansticks were used, but the same format is used with squares containing dots or numerals.

Ask everyone to show 7 beans using only 1-sticks and 3-sticks.

Discuss the results, asking how many sticks were used in each case and which way used the fewest sticks.

Try some others. Discuss the number of sticks used and focus on which used the least sticks.

a) Show 8—Using 1-sticks and 3-sticks.

b) Show 8—Using 2-sticks and 3-sticks.

c) Show 13—Using 3-sticks and 4-sticks.

Make up some more of your own and let the students make up some of their own.

Start to keep a record of results on the board in the same form used on the page. As they tell you the results, you record until they see how to do it. Do at the chalkboard so the whole group can observe.

#### Example

Show 16 using 2-sticks and 5-sticks

| sticks |   | totals |
|--------|---|--------|
| 2      | 3 | 6      |
| 5      | 2 | 10     |
|        |   | 16     |

When you see they are ready to record let them come to the board and fill in the spaces. Remember, always use the least number of sticks.

After this they ought to be ready to go ahead on their own.

Watch for these two examples:

|    |  |  |    |  |  |
|----|--|--|----|--|--|
| 2  |  |  | 3  |  |  |
| 5  |  |  | 4  |  |  |
| 17 |  |  | 15 |  |  |

There are two solutions. We want the one with the fewest check marks.

"FEWEST CHECK-MARKS"

\* MENOS MARCAS \*

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

11

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

13

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

14

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

16

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

17

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

19

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

22

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

23

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

26

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

28

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

29

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

31

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

32

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

34

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

37

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

38

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

41

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

43

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

44

|   |  |  |
|---|--|--|
| 3 |  |  |
| 5 |  |  |

46

"FEWEST CHECK - MARKS" NAME \_\_\_\_\_

|                  |                  |                  |                  |
|------------------|------------------|------------------|------------------|
| 3  <br>5  <br>11 | 3  <br>5  <br>13 | 3  <br>5  <br>14 | 3  <br>5  <br>16 |
| 3  <br>5  <br>17 | 3  <br>5  <br>19 | 3  <br>5  <br>22 | 3  <br>5  <br>23 |
| 3  <br>5  <br>26 | 3  <br>5  <br>28 | 3  <br>5  <br>29 | 3  <br>5  <br>31 |
| 3  <br>5  <br>32 | 3  <br>5  <br>34 | 3  <br>5  <br>37 | 3  <br>5  <br>38 |
| 3  <br>5  <br>41 | 3  <br>5  <br>43 | 3  <br>5  <br>44 | 3  <br>5  <br>46 |

Practice in using combinations of 3 and 5 is provided. Some children may still need to use the squares of paper with dots or numerals on them. Encourage them to give up this aid as soon as possible. It is time to consider the strategy one uses to solve these problems. As you work through, think about how you are doing it. Discuss with the children to see if they are approaching the task in a systematic way. Get them to describe their method to you and to each other.

- 1) When using 3's and 5's, we never use more than four 3's. Why?
- 2) When using any two numbers what is the greatest amount of the smaller ones we will use? Try 4 and 5. 3 and 7.
- 3) What happens when you use two even numbers?
- 4) When using 3's and 5's are there any sums which cannot be made? (In addition to 1, 2 and 4 only 7 is impossible.)

"FEWEST CHECK-MARKS"

"MENOS MARCAS"

|   |   |    |
|---|---|----|
| 2 | 8 | 16 |
| 9 | 1 | 9  |
|   |   | 25 |

|   |  |    |
|---|--|----|
| 3 |  |    |
| 8 |  |    |
|   |  | 25 |

|   |  |    |
|---|--|----|
| 4 |  |    |
| 7 |  |    |
|   |  | 25 |

|   |  |    |
|---|--|----|
| 5 |  |    |
| 6 |  |    |
|   |  | 25 |

|   |  |    |
|---|--|----|
| 2 |  |    |
| 9 |  |    |
|   |  | 26 |

|   |  |    |
|---|--|----|
| 3 |  |    |
| 8 |  |    |
|   |  | 26 |

|   |  |    |
|---|--|----|
| 4 |  |    |
| 7 |  |    |
|   |  | 26 |

|   |  |    |
|---|--|----|
| 5 |  |    |
| 6 |  |    |
|   |  | 26 |

|   |  |    |
|---|--|----|
| 2 |  |    |
| 9 |  |    |
|   |  | 27 |

|   |  |    |
|---|--|----|
| 3 |  |    |
| 8 |  |    |
|   |  | 27 |

|   |  |    |
|---|--|----|
| 4 |  |    |
| 7 |  |    |
|   |  | 27 |

|   |  |    |
|---|--|----|
| 5 |  |    |
| 6 |  |    |
|   |  | 27 |

|   |  |    |
|---|--|----|
| 2 |  |    |
| 9 |  |    |
|   |  | 28 |

|   |  |    |
|---|--|----|
| 3 |  |    |
| 8 |  |    |
|   |  | 28 |

|   |  |    |
|---|--|----|
| 4 |  |    |
| 7 |  |    |
|   |  | 28 |

|   |  |    |
|---|--|----|
| 5 |  |    |
| 6 |  |    |
|   |  | 28 |

|   |  |    |
|---|--|----|
| 2 |  |    |
| 9 |  |    |
|   |  | 29 |

|   |  |    |
|---|--|----|
| 3 |  |    |
| 8 |  |    |
|   |  | 29 |

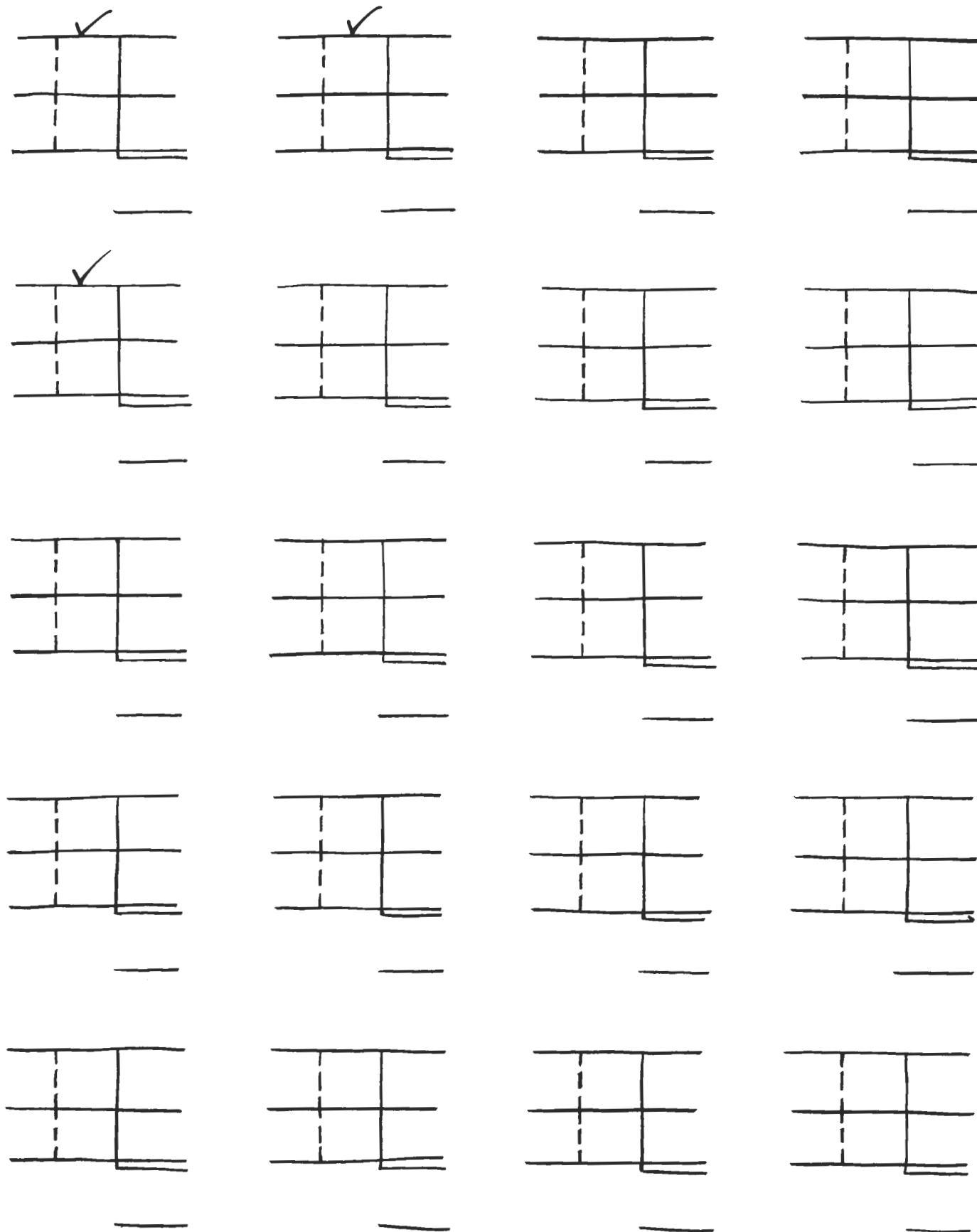
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|---|--|----|
| 4 |  |    |
| 7 |  |    |
|   |  | 29 |

|   |  |    |
|---|--|----|
| 5 |  |    |
| 6 |  |    |
|   |  | 29 |

| "FEWEST CHECK - MARKS"                                                                                                 |                                                                                                                   | NAME _____                                                                                                        |                                                                                                                   |
|------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|
| <div> <div>2</div> <div>8</div> <div>16</div> </div> <div> <div>9</div> <div>7</div> <div>9</div> </div> <div>25</div> | <div> <div>3</div> <div></div> <div></div> </div> <div> <div>8</div> <div></div> <div></div> </div> <div>25</div> | <div> <div>4</div> <div></div> <div></div> </div> <div> <div>7</div> <div></div> <div></div> </div> <div>25</div> | <div> <div>5</div> <div></div> <div></div> </div> <div> <div>6</div> <div></div> <div></div> </div> <div>25</div> |
| <div> <div>2</div> <div></div> <div></div> </div> <div> <div>9</div> <div></div> <div></div> </div> <div>26</div>      | <div> <div>3</div> <div></div> <div></div> </div> <div> <div>8</div> <div></div> <div></div> </div> <div>26</div> | <div> <div>4</div> <div></div> <div></div> </div> <div> <div>7</div> <div></div> <div></div> </div> <div>26</div> | <div> <div>5</div> <div></div> <div></div> </div> <div> <div>6</div> <div></div> <div></div> </div> <div>26</div> |
| <div> <div>2</div> <div></div> <div></div> </div> <div> <div>9</div> <div></div> <div></div> </div> <div>27</div>      | <div> <div>3</div> <div></div> <div></div> </div> <div> <div>8</div> <div></div> <div></div> </div> <div>27</div> | <div> <div>4</div> <div></div> <div></div> </div> <div> <div>7</div> <div></div> <div></div> </div> <div>27</div> | <div> <div>5</div> <div></div> <div></div> </div> <div> <div>6</div> <div></div> <div></div> </div> <div>27</div> |
| <div> <div>2</div> <div></div> <div></div> </div> <div> <div>9</div> <div></div> <div></div> </div> <div>28</div>      | <div> <div>3</div> <div></div> <div></div> </div> <div> <div>8</div> <div></div> <div></div> </div> <div>28</div> | <div> <div>4</div> <div></div> <div></div> </div> <div> <div>7</div> <div></div> <div></div> </div> <div>28</div> | <div> <div>5</div> <div></div> <div></div> </div> <div> <div>6</div> <div></div> <div></div> </div> <div>28</div> |
| <div> <div>2</div> <div></div> <div></div> </div> <div> <div>9</div> <div></div> <div></div> </div> <div>29</div>      | <div> <div>3</div> <div></div> <div></div> </div> <div> <div>8</div> <div></div> <div></div> </div> <div>29</div> | <div> <div>4</div> <div></div> <div></div> </div> <div> <div>7</div> <div></div> <div></div> </div> <div>29</div> | <div> <div>5</div> <div></div> <div></div> </div> <div> <div>6</div> <div></div> <div></div> </div> <div>29</div> |

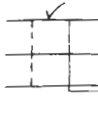
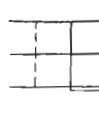
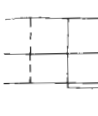
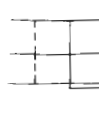
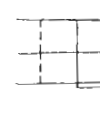
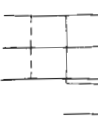
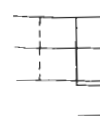
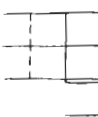
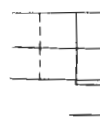
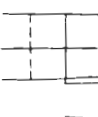
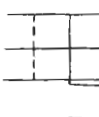
Four different combinations of numbers are used. On each line the same sum is given. Note once again that there are strategies to use. "Get as many of the larger numbers as possible and make up the difference with the smaller numbers" . . . etc.

The following Report Forms can be used to provide more problems of this type. Choose the same combinations as shown here and write different sums or pick different combinations with the same or different sums.





"FEWEST CHECK-MARKS" NAME \_\_\_\_\_

|                                                                                   |                                                                                   |                                                                                   |                                                                                   |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

- 3) Write in the check marks and the sum. Children try to determine what pair of numbers were used and if there was more than one pair possible.

Example:

|    |
|----|
| 2  |
| 3  |
| 23 |

Again there is more than one solution.

We have suggested activities for this page on previous pages. Here are some others.

- 1) Choose a pair of numbers and a sum. Use the same pair and increase the sum by one. Continue until you find a sum that cannot be done or you get tired of looking for one.

Example: Choose 3 and 8 and the sum of 20. This is possible so try a sum of 21 and keep going until a sum cannot be found . . . or you become convinced all larger sums will be possible.

- 2) Write in the sums only. Have the children choose the pair of numbers and complete the problem.

Example:

|    |
|----|
|    |
|    |
| 17 |

There is more than one correct solution. Write the same sum two or three times and ask for different solutions.

# SMALLEST NUMBERS NÚMEROS MÁS PEQUEÑOS

$$\begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline 3 & 2 & 6 \\ \hline 9 & 1 & 9 \\ \hline \end{array}$$

15

$$\begin{array}{|c|c|c|} \hline 1 & & \\ \hline 3 & & \\ \hline 9 & & \\ \hline \end{array}$$

23

$$\begin{array}{|c|c|c|} \hline 1 & & \\ \hline 3 & & \\ \hline 9 & & \\ \hline \end{array}$$

26

$$\begin{array}{|c|c|c|} \hline 1 & & \\ \hline 3 & & \\ \hline 9 & & \\ \hline \end{array}$$

31

$$\begin{array}{|c|c|c|} \hline 1 & & \\ \hline 4 & & \\ \hline 16 & & \\ \hline \end{array}$$

15

$$\begin{array}{|c|c|c|} \hline 1 & & \\ \hline 4 & & \\ \hline 16 & & \\ \hline \end{array}$$

23

$$\begin{array}{|c|c|c|} \hline 1 & & \\ \hline 4 & & \\ \hline 16 & & \\ \hline \end{array}$$

26

$$\begin{array}{|c|c|c|} \hline 1 & & \\ \hline 4 & & \\ \hline 16 & & \\ \hline \end{array}$$

31

$$\begin{array}{|c|c|c|} \hline 1 & & \\ \hline 5 & & \\ \hline 8 & & \\ \hline \end{array}$$

15

$$\begin{array}{|c|c|c|} \hline 1 & & \\ \hline 5 & & \\ \hline 8 & & \\ \hline \end{array}$$

23

$$\begin{array}{|c|c|c|} \hline 1 & & \\ \hline 5 & & \\ \hline 8 & & \\ \hline \end{array}$$

26

$$\begin{array}{|c|c|c|} \hline 1 & & \\ \hline 5 & & \\ \hline 8 & & \\ \hline \end{array}$$

31

$$\begin{array}{|c|c|c|} \hline 2 & & \\ \hline 6 & & \\ \hline 7 & & \\ \hline \end{array}$$

15

$$\begin{array}{|c|c|c|} \hline 2 & & \\ \hline 6 & & \\ \hline 7 & & \\ \hline \end{array}$$

23

$$\begin{array}{|c|c|c|} \hline 2 & & \\ \hline 6 & & \\ \hline 7 & & \\ \hline \end{array}$$

26

$$\begin{array}{|c|c|c|} \hline 2 & & \\ \hline 6 & & \\ \hline 7 & & \\ \hline \end{array}$$

31

NAME \_\_\_\_\_

FEWEST CHECK MARKS"  
OR "SMALLEST NUMBERS"

|   |   |    |    |   |   |    |    |   |   |    |    |   |   |    |    |
|---|---|----|----|---|---|----|----|---|---|----|----|---|---|----|----|
| 1 | 3 | 9  | 15 | 1 | 3 | 9  | 23 | 1 | 3 | 9  | 26 | 1 | 3 | 9  | 31 |
| 1 | 4 | 16 | 15 | 1 | 4 | 16 | 23 | 1 | 4 | 16 | 26 | 1 | 4 | 16 | 31 |
| 1 | 5 | 8  | 15 | 1 | 5 | 8  | 23 | 1 | 5 | 8  | 26 | 1 | 5 | 8  | 31 |
| 2 | 6 | 7  | 15 | 2 | 6 | 7  | 23 | 2 | 6 | 7  | 26 | 2 | 6 | 7  | 31 |

The activity is now enlarged to a combination of three numbers. Rules are the same and the strategy is similar. The two top rows have a definite pattern to them.

|         |   |  |  |
|---------|---|--|--|
| (3 x 1) | 1 |  |  |
| (3 x 3) | 3 |  |  |
|         | 9 |  |  |

|         |    |  |  |
|---------|----|--|--|
| (4 x 1) | 1  |  |  |
| (4 x 4) | 4  |  |  |
|         | 16 |  |  |

The bottom two rows do not have the pattern described above.

The following page may be used to provide more practice. Consider such examples as:

|         |    |  |  |
|---------|----|--|--|
| (5 x 1) | 1  |  |  |
| (5 x 5) | 5  |  |  |
|         | 25 |  |  |

|         |    |  |  |
|---------|----|--|--|
| (6 x 1) | 1  |  |  |
| (6 x 6) | 6  |  |  |
|         | 36 |  |  |

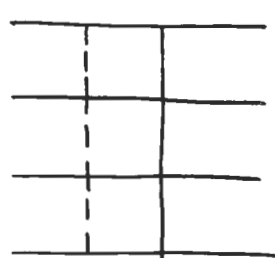
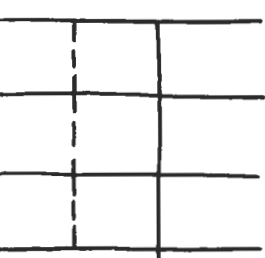
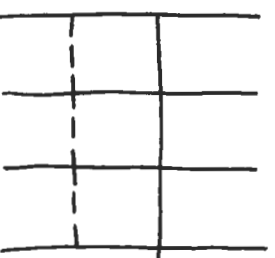
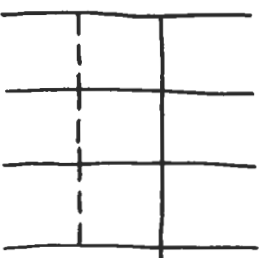
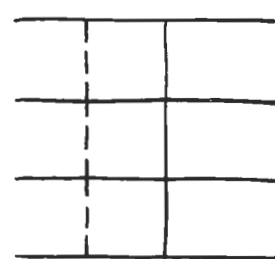
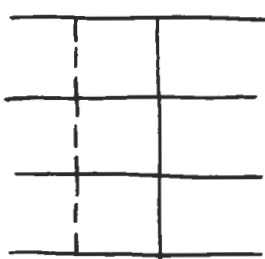
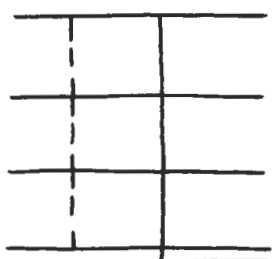
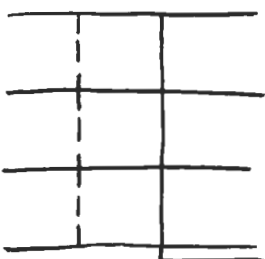
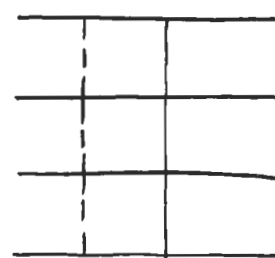
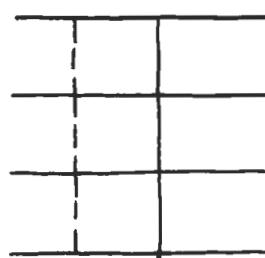
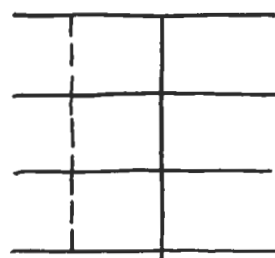
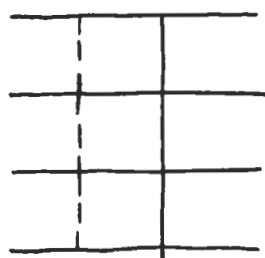
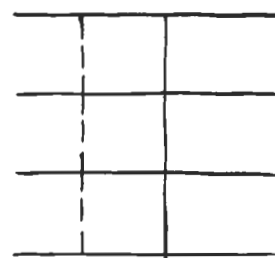
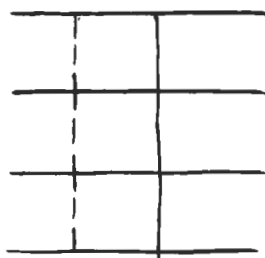
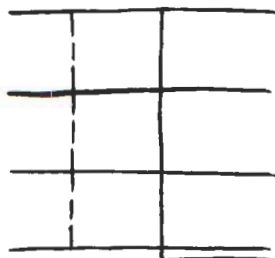
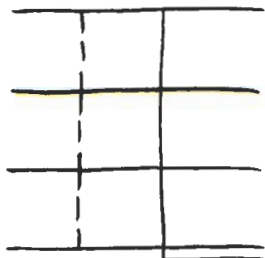
|   |  |  |
|---|--|--|
| 1 |  |  |
| 4 |  |  |
| 9 |  |  |

|   |  |  |
|---|--|--|
| 2 |  |  |
| 5 |  |  |
| 7 |  |  |

What happens if you use three even numbers?

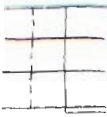
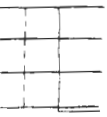
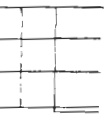
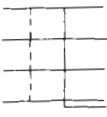
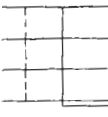
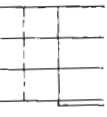
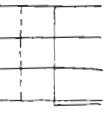
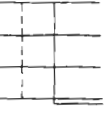
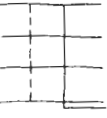
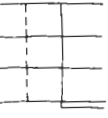
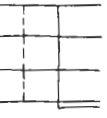
# SMALLEST NUMBERS

## NÚMEROS MAS PEQUEÑOS



'FEWEST CHECK MARKS'  
OR  
'SMALLEST NUMBER.'

NAME \_\_\_\_\_

|                                                                                   |                                                                                   |                                                                                   |                                                                                   |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |  |  |
| _____                                                                             | _____                                                                             | _____                                                                             | _____                                                                             |
|  |  |  |  |
| _____                                                                             | _____                                                                             | _____                                                                             | _____                                                                             |
|  |  |  |  |
| _____                                                                             | _____                                                                             | _____                                                                             | _____                                                                             |
|  |  |  |  |
| _____                                                                             | _____                                                                             | _____                                                                             | _____                                                                             |

Use as indicated on previous page. Here is another idea. Given a combination of three numbers, are there any impossible sums?

Example:

|   |  |  |
|---|--|--|
|   |  |  |
| 1 |  |  |
| 4 |  |  |
| 7 |  |  |
|   |  |  |

\_\_\_\_\_

Start at 0 and continue until you find impossible sums, determine all sums are possible, or get tired. Notice that if 1 is one of the three numbers, all sums are possible.

Try:

|   |  |  |
|---|--|--|
|   |  |  |
| 2 |  |  |
| 4 |  |  |
| 7 |  |  |
|   |  |  |

\_\_\_\_\_

Can you find any impossible sums?

Let the children pick any three numbers and do the same activity.

# "CHECK-LIST" ADDITION SUMAS DE LA LISTA.

|    |   |   |   |   |   |
|----|---|---|---|---|---|
| 16 | 8 | 4 | 2 | 1 |   |
|    |   |   |   |   | 3 |

|   |   |   |    |
|---|---|---|----|
| 9 | 3 | 1 |    |
|   |   |   | 10 |

|    |   |   |    |
|----|---|---|----|
| 25 | 5 | 1 |    |
|    |   |   | 12 |

|    |   |   |   |   |   |
|----|---|---|---|---|---|
| 16 | 8 | 4 | 2 | 1 |   |
|    |   |   |   |   | 6 |

|   |   |   |    |
|---|---|---|----|
| 9 | 3 | 1 |    |
|   |   |   | 12 |

|    |   |   |    |
|----|---|---|----|
| 25 | 5 | 1 |    |
|    |   |   | 16 |

|    |   |   |   |   |   |
|----|---|---|---|---|---|
| 16 | 8 | 4 | 2 | 1 |   |
|    |   |   |   |   | 9 |

|   |   |   |    |
|---|---|---|----|
| 9 | 3 | 1 |    |
|   |   |   | 14 |

|    |   |   |    |
|----|---|---|----|
| 25 | 5 | 1 |    |
|    |   |   | 20 |

|    |   |   |   |   |    |
|----|---|---|---|---|----|
| 16 | 8 | 4 | 2 | 1 |    |
|    |   |   |   |   | 12 |

|   |   |   |    |
|---|---|---|----|
| 9 | 3 | 1 |    |
|   |   |   | 16 |

|    |   |   |    |
|----|---|---|----|
| 25 | 5 | 1 |    |
|    |   |   | 24 |

|    |   |   |   |   |    |
|----|---|---|---|---|----|
| 16 | 8 | 4 | 2 | 1 |    |
|    |   |   |   |   | 15 |

|   |   |   |    |
|---|---|---|----|
| 9 | 3 | 1 |    |
|   |   |   | 18 |

|    |   |   |    |
|----|---|---|----|
| 25 | 5 | 1 |    |
|    |   |   | 28 |

|    |   |   |   |   |    |
|----|---|---|---|---|----|
| 16 | 8 | 4 | 2 | 1 |    |
|    |   |   |   |   | 18 |

|   |   |   |    |
|---|---|---|----|
| 9 | 3 | 1 |    |
|   |   |   | 20 |

|    |   |   |    |
|----|---|---|----|
| 25 | 5 | 1 |    |
|    |   |   | 32 |

|    |   |   |   |   |    |
|----|---|---|---|---|----|
| 16 | 8 | 4 | 2 | 1 |    |
|    |   |   |   |   | 21 |

|   |   |   |    |
|---|---|---|----|
| 9 | 3 | 1 |    |
|   |   |   | 22 |

|    |   |   |    |
|----|---|---|----|
| 25 | 5 | 1 |    |
|    |   |   | 36 |

|    |   |   |   |   |    |
|----|---|---|---|---|----|
| 16 | 8 | 4 | 2 | 1 |    |
|    |   |   |   |   | 24 |

|   |   |   |    |
|---|---|---|----|
| 9 | 3 | 1 |    |
|   |   |   | 24 |

|    |   |   |    |
|----|---|---|----|
| 25 | 5 | 1 |    |
|    |   |   | 40 |

|    |   |   |   |   |    |
|----|---|---|---|---|----|
| 16 | 8 | 4 | 2 | 1 |    |
|    |   |   |   |   | 27 |

|   |   |   |    |
|---|---|---|----|
| 9 | 3 | 1 |    |
|   |   |   | 26 |

|    |   |   |    |
|----|---|---|----|
| 25 | 5 | 1 |    |
|    |   |   | 44 |

**"CHECK-LIST" ADDITION**  
PLEASE USE THE FEWEST CHECKMARKS  
TO INDICATE NUMBERS WHOSE SUM IS GIVEN ON THE RIGHT.

NAME \_\_\_\_\_

|                            |                   |                    |
|----------------------------|-------------------|--------------------|
| 16 8 4 2 1<br>          3  | 9 3 1<br>      10 | 25 5 1<br>      12 |
| 16 8 4 2 1<br>          6  | 9 3 1<br>      12 | 25 5 1<br>      16 |
| 16 8 4 2 1<br>          9  | 9 3 1<br>      14 | 25 5 1<br>      20 |
| 16 8 4 2 1<br>          12 | 9 3 1<br>      16 | 25 5 1<br>      24 |
| 16 8 4 2 1<br>          15 | 9 3 1<br>      18 | 25 5 1<br>      28 |
| 16 8 4 2 1<br>          18 | 9 3 1<br>      20 | 25 5 1<br>      32 |
| 16 8 4 2 1<br>          21 | 9 3 1<br>      22 | 25 5 1<br>      36 |
| 16 8 4 2 1<br>          24 | 9 3 1<br>      24 | 25 5 1<br>      40 |
| 16 8 4 2 1<br>          27 | 9 3 1<br>      26 | 25 5 1<br>      44 |

Use checks in the boxes below this list of numbers to indicate numbers whose sum is the number on the right hand side.

|   |   |   |    |
|---|---|---|----|
| 9 | 3 | 1 |    |
|   |   | 2 | 14 |

The numbers used in the lists have been chosen by following a specific pattern.

|    |    |    |    |   |
|----|----|----|----|---|
| 8  | 4  | 2  | 1  |   |
| x2 | x2 | x2 | x2 |   |
| 16 | 8  | 4  | 2  | 1 |
|    |    |    |    |   |

|    |    |   |  |
|----|----|---|--|
| 3  | 1  |   |  |
| x3 | x3 |   |  |
| 9  | 3  | 1 |  |
|    |    |   |  |

|    |    |   |  |
|----|----|---|--|
| 5  | 1  |   |  |
| x5 | x5 |   |  |
| 25 | 5  | 1 |  |
|    |    |   |  |

All combinations begin with a 1 on the right side. The second number is then used as a multiplier for succeeding numbers as shown above.

A few examples of completed records:

|    |   |   |   |   |    |
|----|---|---|---|---|----|
| 16 | 8 | 4 | 2 | 1 |    |
| ✓  |   | ✓ | ✓ | ✓ | 23 |

|   |   |   |  |    |
|---|---|---|--|----|
| 9 | 3 | 1 |  |    |
| ✓ | ✓ | ✓ |  | 23 |

|    |   |   |   |    |
|----|---|---|---|----|
| 25 | 5 | 1 |   |    |
|    | ✓ | ✓ | ✓ | 23 |

Each example has but a single solution if the rule of the "fewest checks" is followed. Without that rule, there would be many solutions.

Soon, the children will prefer "numbers" to "checks."

|    |   |   |   |   |    |
|----|---|---|---|---|----|
| 16 | 8 | 4 | 2 | 1 |    |
| 1  | 0 | 1 | 1 | 1 | 23 |

|   |   |   |  |    |
|---|---|---|--|----|
| 9 | 3 | 1 |  |    |
| 2 | 1 | 2 |  | 23 |

|    |   |   |  |    |
|----|---|---|--|----|
| 25 | 5 | 1 |  |    |
|    | 4 | 3 |  | 23 |

# "CHECK-LIST" ADDITION

## SUMAS DE LA LISTA.

|    |   |   |   |   |
|----|---|---|---|---|
| 16 | 8 | 4 | 2 | 1 |
|    |   |   |   |   |

|   |   |   |
|---|---|---|
| 9 | 3 | 1 |
|   |   |   |

|    |   |   |
|----|---|---|
| 25 | 5 | 1 |
|    |   |   |

|    |   |   |   |   |
|----|---|---|---|---|
| 16 | 8 | 4 | 2 | 1 |
|    |   |   |   |   |

|   |   |   |
|---|---|---|
| 9 | 3 | 1 |
|   |   |   |

|    |   |   |
|----|---|---|
| 25 | 5 | 1 |
|    |   |   |

|    |   |   |   |   |
|----|---|---|---|---|
| 16 | 8 | 4 | 2 | 1 |
|    |   |   |   |   |

|   |   |   |
|---|---|---|
| 9 | 3 | 1 |
|   |   |   |

|    |   |   |
|----|---|---|
| 25 | 5 | 1 |
|    |   |   |

|    |   |   |   |   |
|----|---|---|---|---|
| 16 | 8 | 4 | 2 | 1 |
|    |   |   |   |   |

|   |   |   |
|---|---|---|
| 9 | 3 | 1 |
|   |   |   |

|    |   |   |
|----|---|---|
| 25 | 5 | 1 |
|    |   |   |

|    |   |   |   |   |
|----|---|---|---|---|
| 16 | 8 | 4 | 2 | 1 |
|    |   |   |   |   |

|   |   |   |
|---|---|---|
| 9 | 3 | 1 |
|   |   |   |

|    |   |   |
|----|---|---|
| 25 | 5 | 1 |
|    |   |   |

|    |   |   |   |   |
|----|---|---|---|---|
| 16 | 8 | 4 | 2 | 1 |
|    |   |   |   |   |

|   |   |   |
|---|---|---|
| 9 | 3 | 1 |
|   |   |   |

|    |   |   |
|----|---|---|
| 25 | 5 | 1 |
|    |   |   |

|    |   |   |   |   |
|----|---|---|---|---|
| 16 | 8 | 4 | 2 | 1 |
|    |   |   |   |   |

|   |   |   |
|---|---|---|
| 9 | 3 | 1 |
|   |   |   |

|    |   |   |
|----|---|---|
| 25 | 5 | 1 |
|    |   |   |

|    |   |   |   |   |
|----|---|---|---|---|
| 16 | 8 | 4 | 2 | 1 |
|    |   |   |   |   |

|   |   |   |
|---|---|---|
| 9 | 3 | 1 |
|   |   |   |

|    |   |   |
|----|---|---|
| 25 | 5 | 1 |
|    |   |   |

|    |   |   |   |   |
|----|---|---|---|---|
| 16 | 8 | 4 | 2 | 1 |
|    |   |   |   |   |

|   |   |   |
|---|---|---|
| 9 | 3 | 1 |
|   |   |   |

|    |   |   |
|----|---|---|
| 25 | 5 | 1 |
|    |   |   |



**"CHECK-LIST" ADDITION**  
PLEASE USE THE FEWEST CHECKMARKS  
TO INDICATE NUMBERS WHOSE SUM IS GIVEN ON THE RIGHT.

NAME \_\_\_\_\_

|            |       |        |
|------------|-------|--------|
| 16 8 4 2 1 | 9 3 1 | 25 5 1 |
| 16 8 4 2 1 | 9 3 1 | 25 5 1 |
| 16 8 4 2 1 | 9 3 1 | 25 5 1 |
| 16 8 4 2 1 | 9 3 1 | 25 5 1 |
| 16 8 4 2 1 | 9 3 1 | 25 5 1 |
| 16 8 4 2 1 | 9 3 1 | 25 5 1 |
| 16 8 4 2 1 | 9 3 1 | 25 5 1 |
| 16 8 4 2 1 | 9 3 1 | 25 5 1 |

This page continues the activity introduced on the previous page. Some ideas for use:

- 1) Provide sums—children fill in the record.
- 2) Children choose sums and then fill in the record.
- 3) Specify all sums odd or all sums even.
- 4) Specify sums between 30 and 40 only, or any span.

Questions to raise are:

- 1) Looking at each record can you determine the maximum number of check marks for each column of examples?
- 2) In the first column of examples is there more than 1 way to indicate sums of any number less than 32, using "fewest check marks"? (No)
- 3) Same question in the middle column for any number less than 27? (No)
- 4) Same question in the third column for any number less than 125? (No)

"CHECK-LIST" ADDITION  
SUMAS DE LA LISTA.

|    |   |   |    |
|----|---|---|----|
| 16 | 4 | 1 |    |
|    |   |   | 23 |

|    |   |   |    |
|----|---|---|----|
| 36 | 6 | 1 |    |
|    |   |   | 23 |

|    |   |   |    |
|----|---|---|----|
| 49 | 7 | 1 |    |
|    |   |   | 23 |

|    |   |   |    |
|----|---|---|----|
| 16 | 4 | 1 |    |
|    |   |   | 29 |

|    |   |   |    |
|----|---|---|----|
| 36 | 6 | 1 |    |
|    |   |   | 29 |

|    |   |   |    |
|----|---|---|----|
| 49 | 7 | 1 |    |
|    |   |   | 29 |

|    |   |   |    |
|----|---|---|----|
| 16 | 4 | 1 |    |
|    |   |   | 31 |

|    |   |   |    |
|----|---|---|----|
| 36 | 6 | 1 |    |
|    |   |   | 31 |

|    |   |   |    |
|----|---|---|----|
| 49 | 7 | 1 |    |
|    |   |   | 31 |

|    |   |   |    |
|----|---|---|----|
| 16 | 4 | 1 |    |
|    |   |   | 37 |

|    |   |   |    |
|----|---|---|----|
| 36 | 6 | 1 |    |
|    |   |   | 37 |

|    |   |   |    |
|----|---|---|----|
| 49 | 7 | 1 |    |
|    |   |   | 37 |

|    |   |   |    |
|----|---|---|----|
| 16 | 4 | 1 |    |
|    |   |   | 41 |

|    |   |   |    |
|----|---|---|----|
| 36 | 6 | 1 |    |
|    |   |   | 41 |

|    |   |   |    |
|----|---|---|----|
| 49 | 7 | 1 |    |
|    |   |   | 41 |

|    |   |   |    |
|----|---|---|----|
| 16 | 4 | 1 |    |
|    |   |   | 43 |

|    |   |   |    |
|----|---|---|----|
| 36 | 6 | 1 |    |
|    |   |   | 43 |

|    |   |   |    |
|----|---|---|----|
| 49 | 7 | 1 |    |
|    |   |   | 43 |

|    |   |   |    |
|----|---|---|----|
| 16 | 4 | 1 |    |
|    |   |   | 47 |

|    |   |   |    |
|----|---|---|----|
| 36 | 6 | 1 |    |
|    |   |   | 47 |

|    |   |   |    |
|----|---|---|----|
| 49 | 7 | 1 |    |
|    |   |   | 47 |

|    |   |   |    |
|----|---|---|----|
| 16 | 4 | 1 |    |
|    |   |   | 53 |

|    |   |   |    |
|----|---|---|----|
| 36 | 6 | 1 |    |
|    |   |   | 53 |

|    |   |   |    |
|----|---|---|----|
| 49 | 7 | 1 |    |
|    |   |   | 53 |

|    |   |   |    |
|----|---|---|----|
| 16 | 4 | 1 |    |
|    |   |   | 59 |

|    |   |   |    |
|----|---|---|----|
| 36 | 6 | 1 |    |
|    |   |   | 59 |

|    |   |   |    |
|----|---|---|----|
| 49 | 7 | 1 |    |
|    |   |   | 59 |

**"CHECK-LIST" ADDITION** NAME \_\_\_\_\_

PLEASE USE THE FEWEST CHECKMARKS  
TO INDICATE NUMBERS WHOSE SUM IS GIVEN ON THE RIGHT.

|              |              |              |
|--------------|--------------|--------------|
| 16 4 1<br>23 | 36 6 1<br>23 | 49 7 1<br>23 |
| 16 4 1<br>29 | 36 6 1<br>29 | 49 7 1<br>29 |
| 16 4 1<br>31 | 36 6 1<br>31 | 49 7 1<br>31 |
| 16 4 1<br>37 | 36 6 1<br>37 | 49 7 1<br>37 |
| 16 4 1<br>41 | 36 6 1<br>41 | 49 7 1<br>41 |
| 16 4 1<br>43 | 36 6 1<br>43 | 49 7 1<br>43 |
| 16 4 1<br>47 | 36 6 1<br>47 | 49 7 1<br>47 |
| 16 4 1<br>53 | 36 6 1<br>53 | 49 7 1<br>53 |
| 16 4 1<br>59 | 36 6 1<br>59 | 49 7 1<br>59 |

|    |    |    |    |
|----|----|----|----|
| 16 | 4  | 1  |    |
| ✓✓ | ✓✓ | ✓✓ | 59 |

 or
 

|    |   |   |    |
|----|---|---|----|
| 16 | 4 | 1 |    |
| 3  | 2 | 3 | 59 |

A continuation of the activities begun on the previous page with different combinations.

|                |                |                |
|----------------|----------------|----------------|
| 4 1<br>x 4 x 4 | 6 1<br>x 6 x 6 | 7 1<br>x 7 x 7 |
| 16 4 1         | 36 6 1         | 49 7 1         |

Ask the children how to get the number in each case if we added another column. ( $4 \times 16 = 64$ ;  $6 \times 36 = 216$ ;  $7 \times 49 = 343$ )

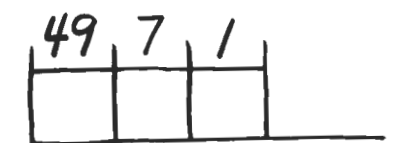
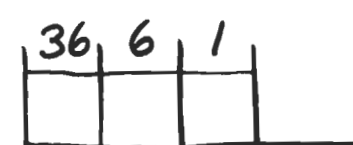
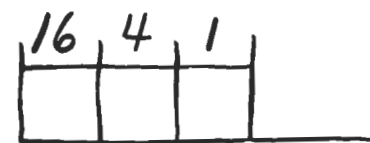
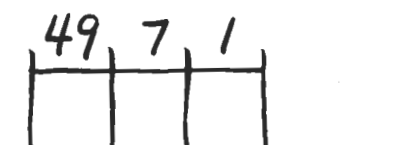
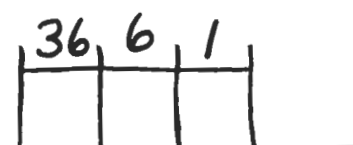
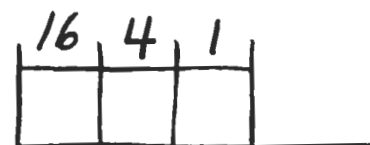
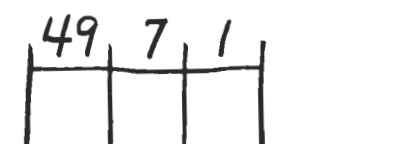
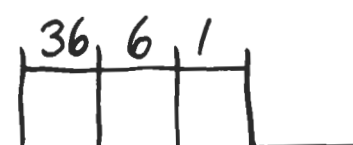
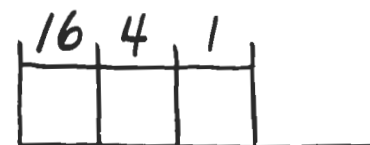
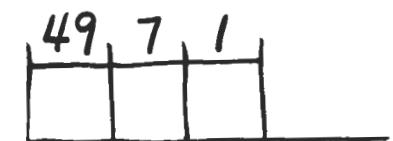
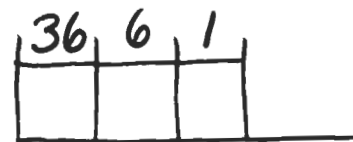
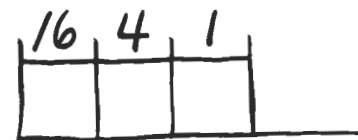
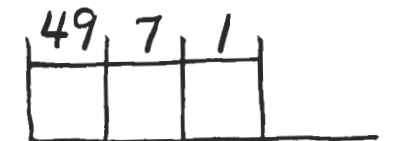
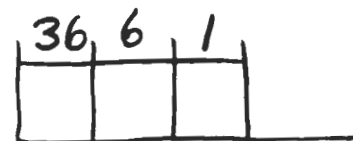
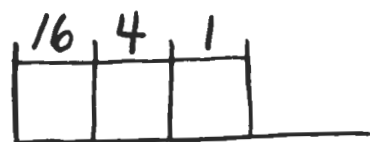
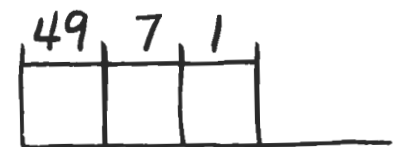
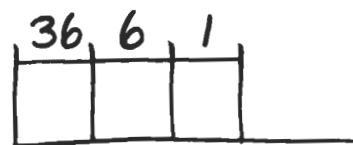
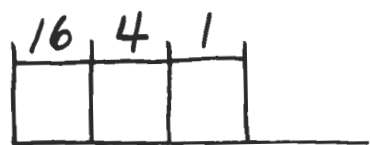
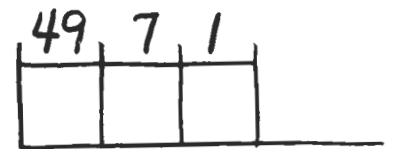
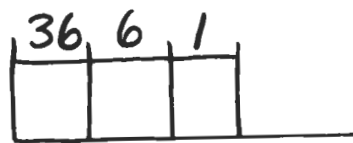
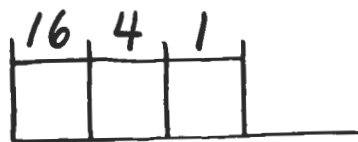
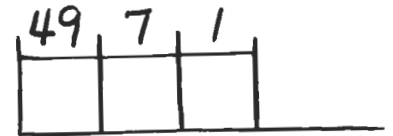
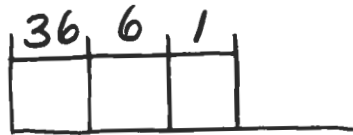
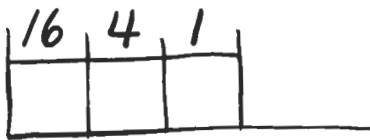
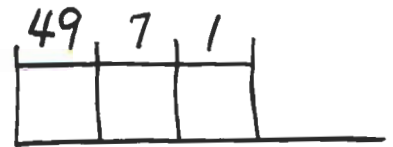
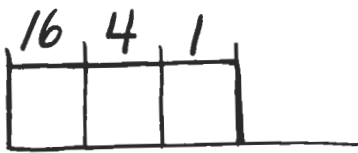
In the example such as

|    |   |   |    |
|----|---|---|----|
| 16 | 4 | 1 |    |
|    |   |   | 59 |

how does one begin? By working from left to right. How many 16's can I use without going beyond 59? Three 16's are 48 and four 16's are 64: so, the record is begun.

|     |   |   |     |
|-----|---|---|-----|
| 16  | 4 | 1 | 59  |
| ✓✓✓ |   |   | -48 |
|     |   |   | 11  |

And the problem has been cut down in size: how to indicate 11 more? . . . two 4's and three 1's.



"CHECK-LIST" ADDITION NAME \_\_\_\_\_

|        |        |        |
|--------|--------|--------|
| 16 4 1 | 36 6 1 | 49 7 1 |
| 16 4 1 | 36 6 1 | 49 7 1 |
| 16 4 1 | 36 6 1 | 49 7 1 |
| 16 4 1 | 36 6 1 | 49 7 1 |
| 16 4 1 | 36 6 1 | 49 7 1 |
| 16 4 1 | 36 6 1 | 49 7 1 |
| 16 4 1 | 36 6 1 | 49 7 1 |
| 16 4 1 | 36 6 1 | 49 7 1 |

Each different selection leads to different patterns.

Use this page to individualize with problems similar to the previous page.

Numbers should replace checks in boxes.

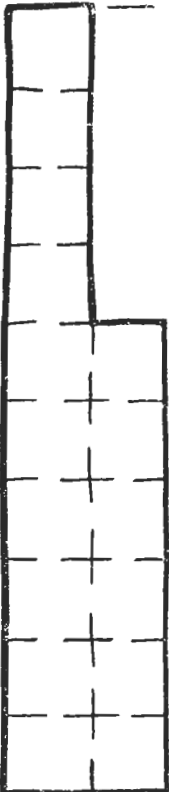
Different members of the group might pick a number from 2 to 10 and specify all sums by writing a multiplication table of that number times all numbers 1 thru 27 (there are 27 examples on the page).

A choice of 8 leads to these three columns of numbers

| 16 4 1 | 36 6 1   | 49 7 1    |
|--------|----------|-----------|
| 2 0 8  | 2 1 2 80 | 3 0 5 152 |
| 16     | 88       | 160       |
| 24     | 96       | 168       |
| 32     | 104      | 176       |
| 40     | 112      | 184       |
| 48     | 120      | 192       |
| 56     | 128      | 200       |
| 64     | 136      | 208       |
| 72     | 144      | 216       |

In each final report, there would be patterns to talk about. If 8 were chosen (as above) the first column would have a very simple pattern; the next column a more complex pattern; and the third column would have almost no obvious patterns . . . but patterns none the less.

# FENCING CERCANDO

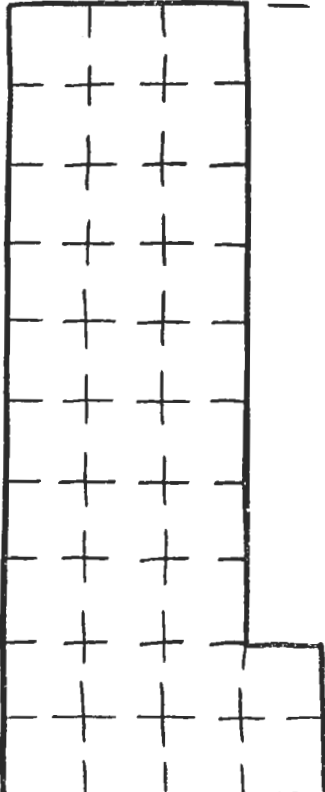


4's

x      x

—

—

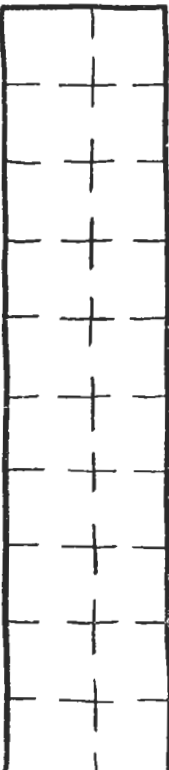


8's

x      x

└

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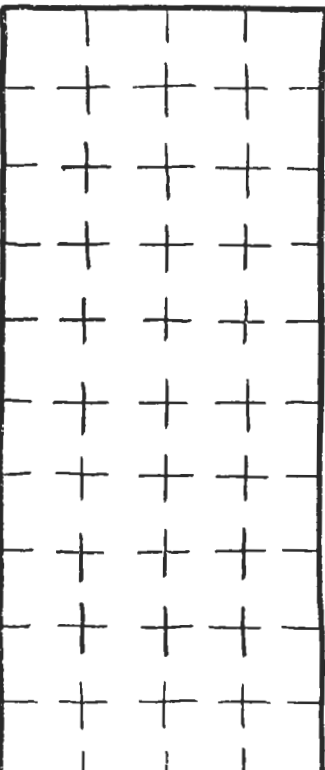


5's

x      x

└

└

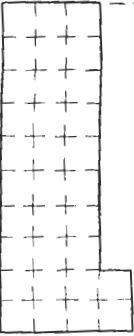
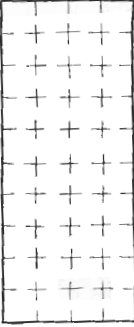


8's

x      x

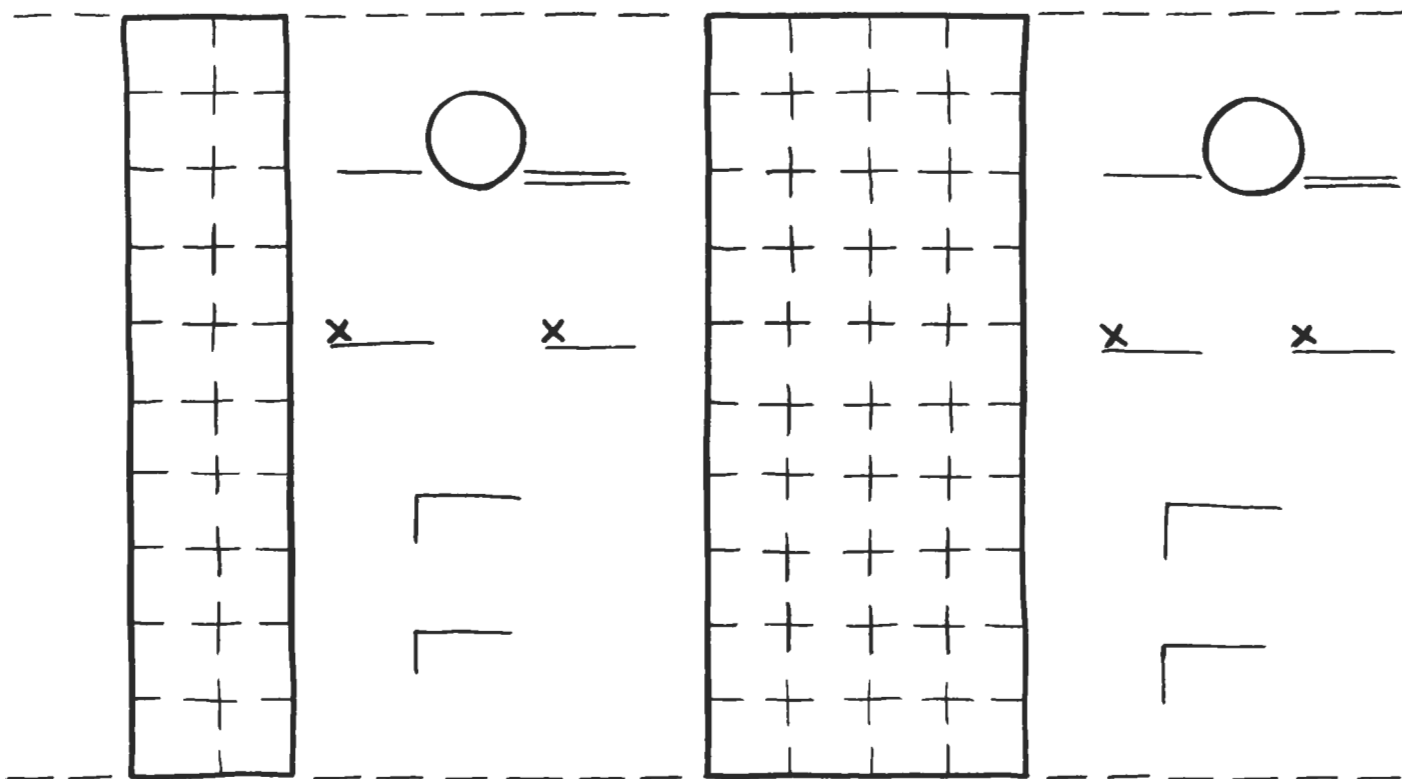
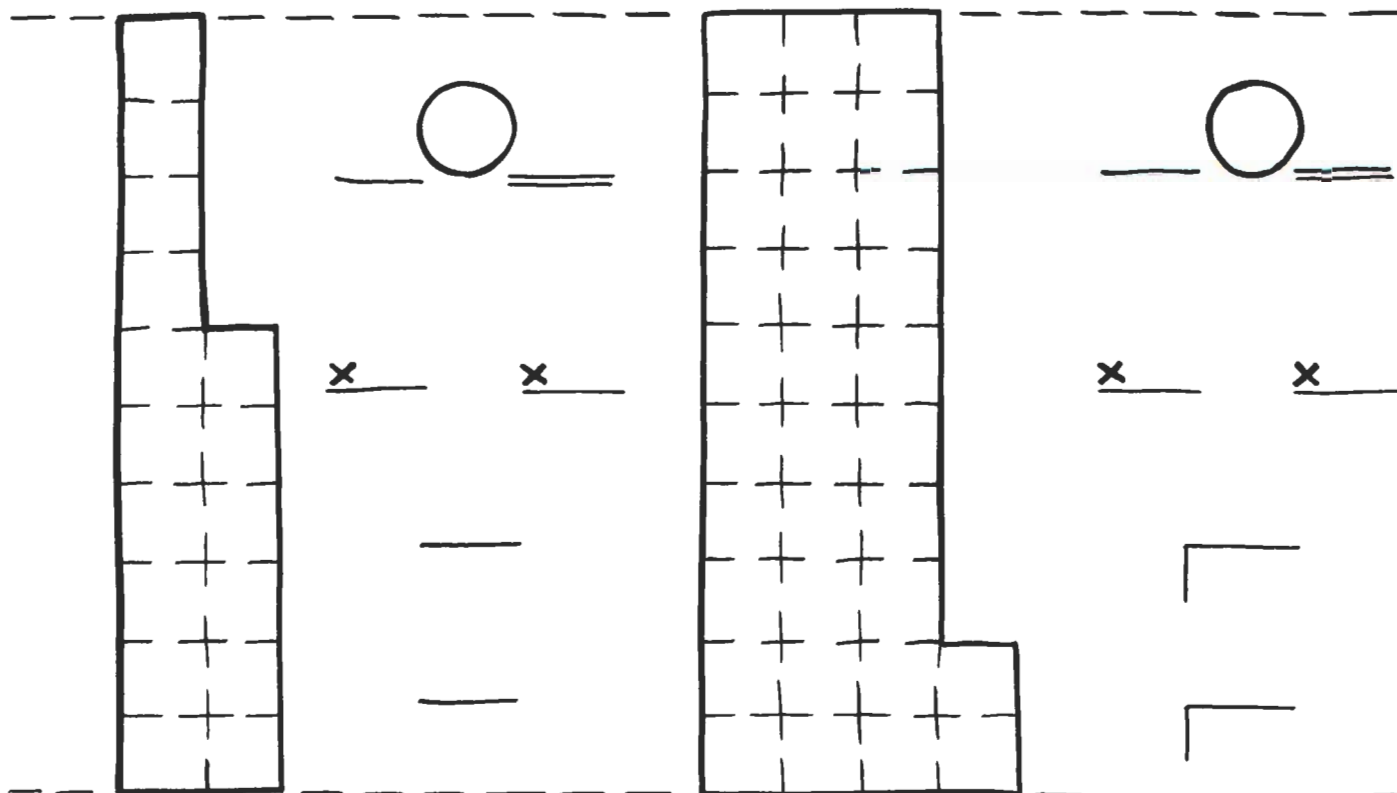
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| FENCING                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                   | NAME _____                                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|-----------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  | <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; line-height: 30px; margin: 0 auto;">4's</div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span>x</span> <span>x</span> </div> <div style="margin-top: 10px;"> <div style="border-bottom: 1px solid black; width: 40px; margin-bottom: 5px;"></div> <div style="border-bottom: 1px solid black; width: 40px;"></div> </div> |  | <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; line-height: 30px; margin: 0 auto;">8's</div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span>x</span> <span>x</span> </div> <div style="margin-top: 10px;"> <div style="border-bottom: 1px solid black; width: 40px; margin-bottom: 5px;"></div> <div style="border-bottom: 1px solid black; width: 40px;"></div> </div> |
|  | <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; line-height: 30px; margin: 0 auto;">5's</div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span>x</span> <span>x</span> </div> <div style="margin-top: 10px;"> <div style="border-bottom: 1px solid black; width: 40px; margin-bottom: 5px;"></div> <div style="border-bottom: 1px solid black; width: 40px;"></div> </div> |  | <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; line-height: 30px; margin: 0 auto;">8's</div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span>x</span> <span>x</span> </div> <div style="margin-top: 10px;"> <div style="border-bottom: 1px solid black; width: 40px; margin-bottom: 5px;"></div> <div style="border-bottom: 1px solid black; width: 40px;"></div> </div> |

This is a continuation of the activities on the previous pages.

# FENCING CERCANDO

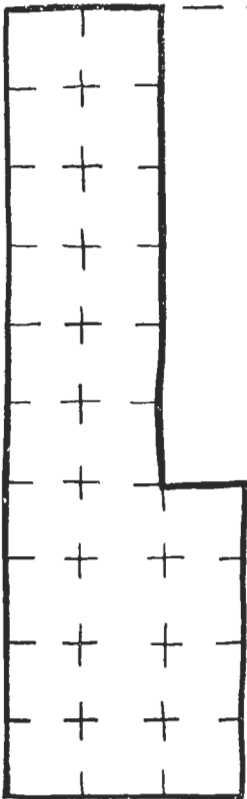




| FENCING |  | NAME _____ |  |
|---------|--|------------|--|
|         |  |            |  |
|         |  |            |  |

Again, a continuation of previous activities. Larger arrays of blocks are used. See previous pages for suggestions for use.

# FENCING CERCANDO

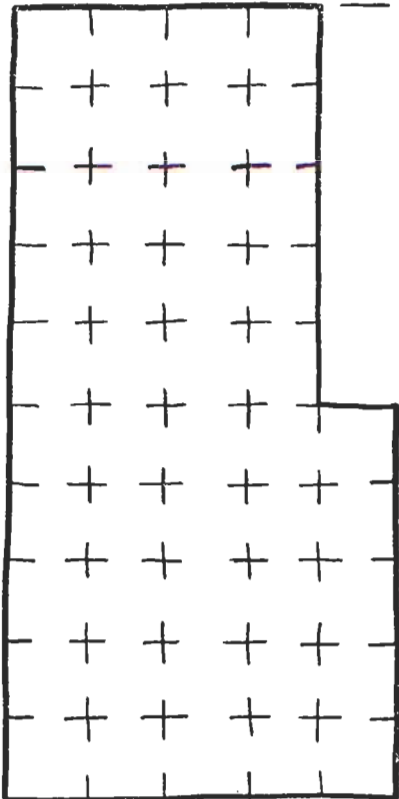


4's

x      x

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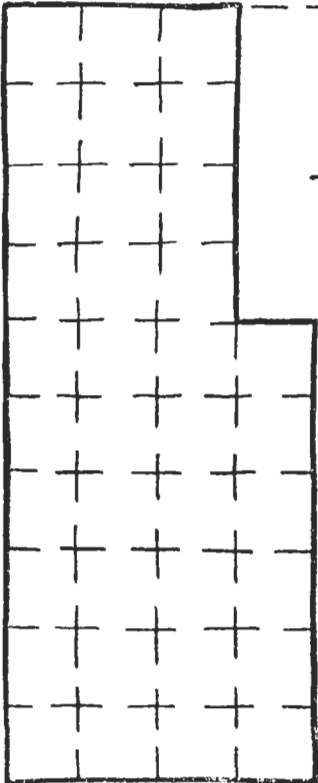


5's

x      x

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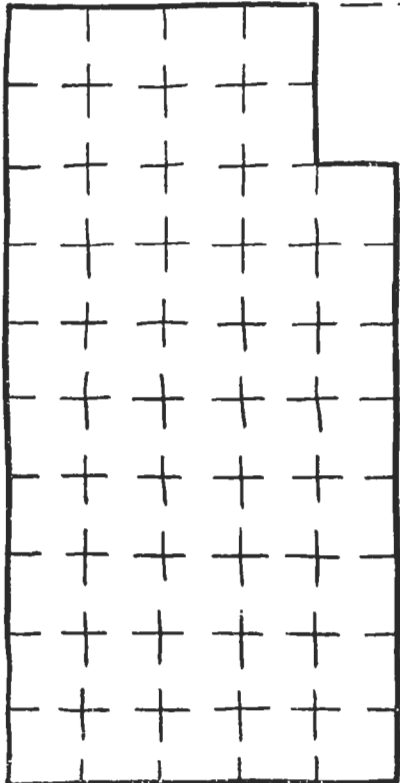


6's

x      x

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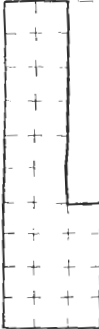
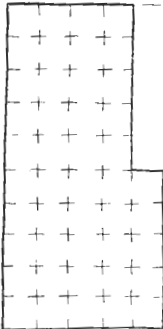
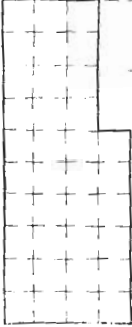
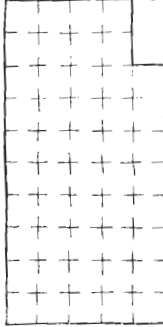


8's

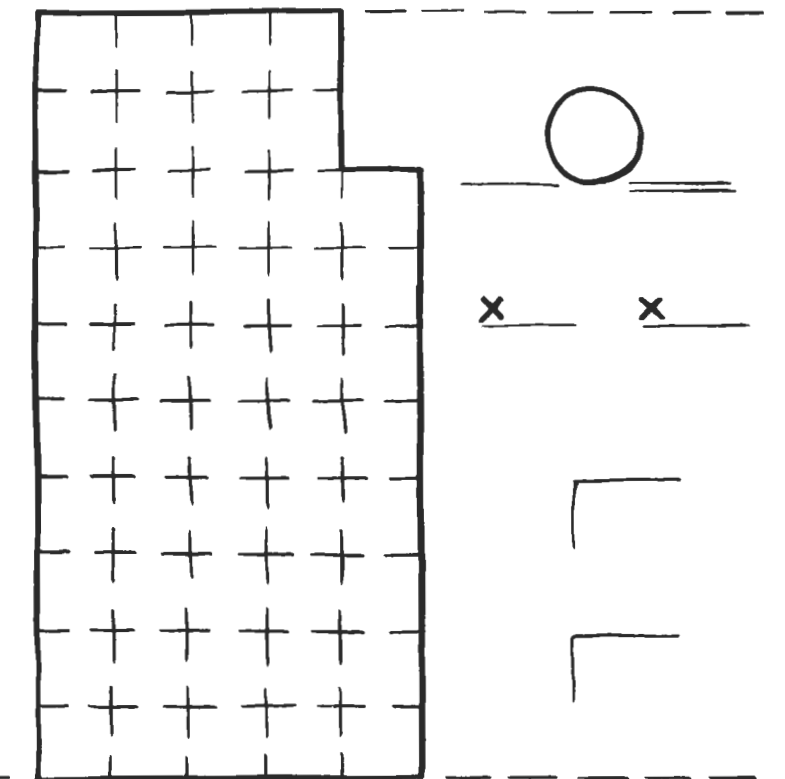
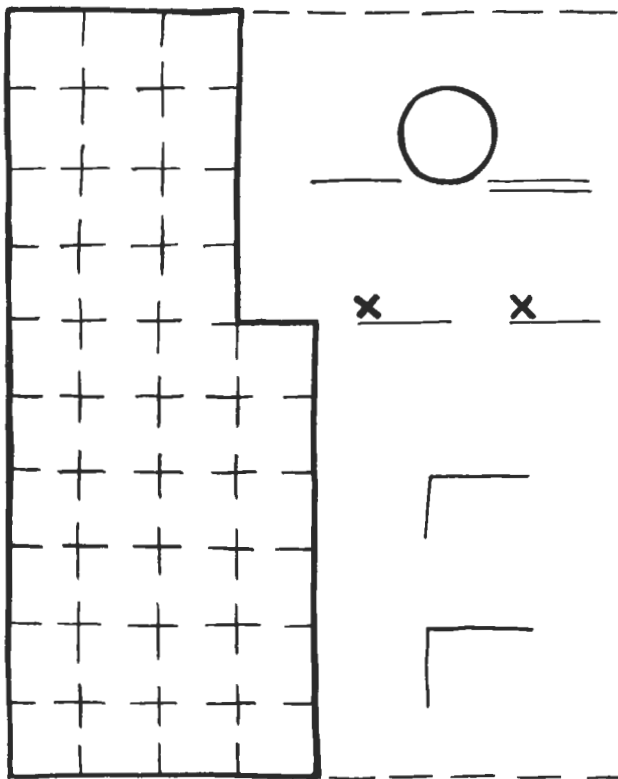
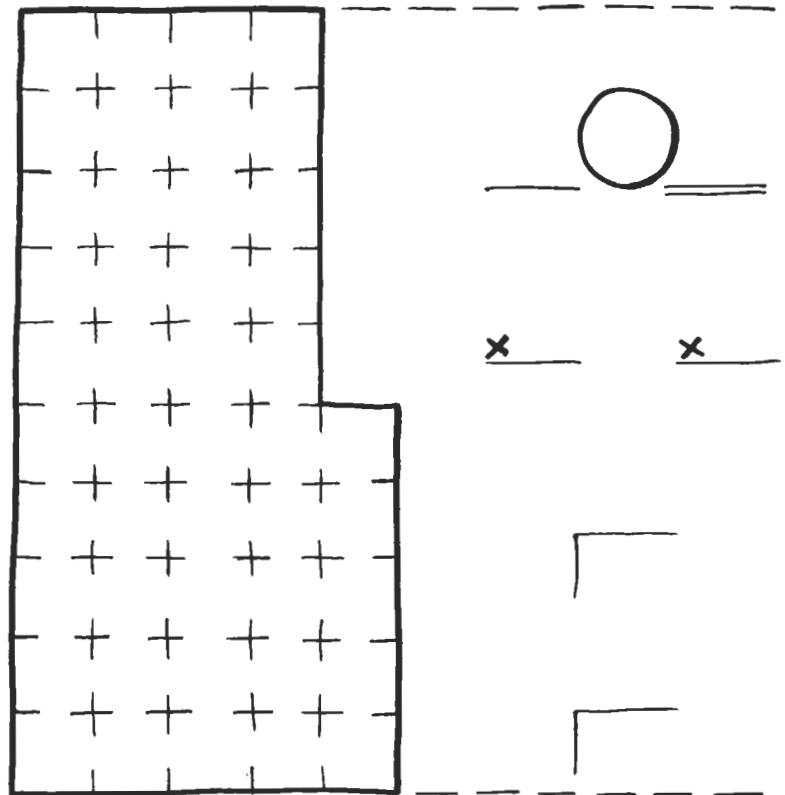
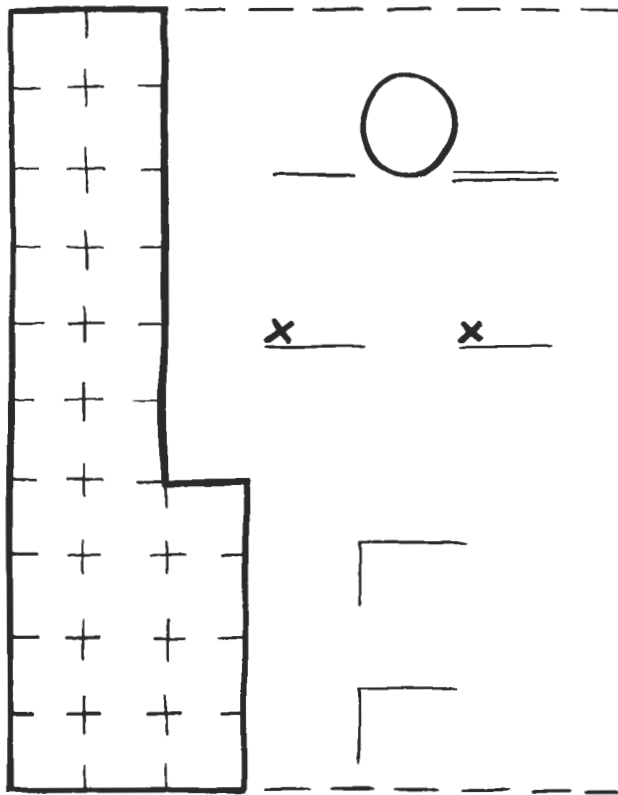
x      x

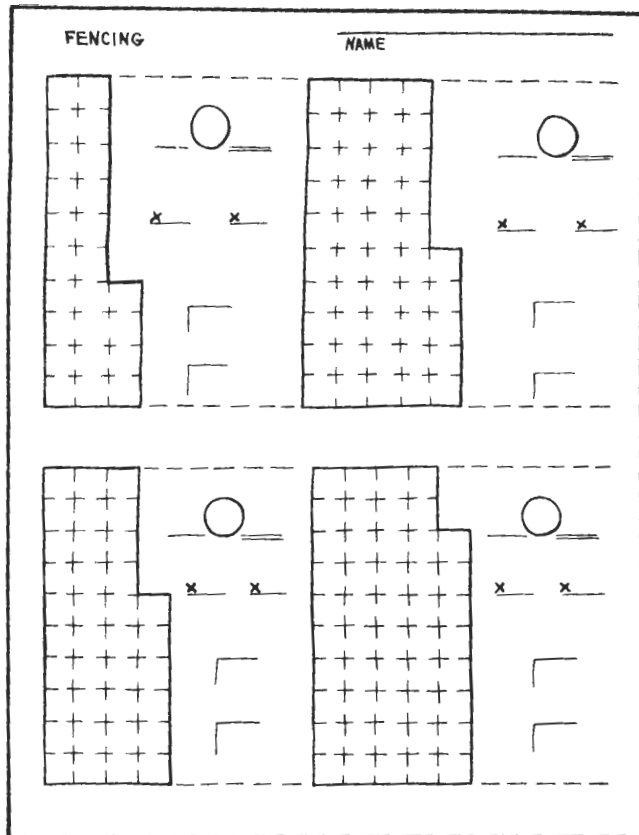
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| FENCING                                                                           |                                                | NAME _____                                                                        |                                                |
|-----------------------------------------------------------------------------------|------------------------------------------------|-----------------------------------------------------------------------------------|------------------------------------------------|
|  | <p>(4's)</p> <p>x      x</p> <p>┌</p> <p>└</p> |  | <p>(5's)</p> <p>x      x</p> <p>┌</p> <p>└</p> |
|  | <p>(6's)</p> <p>x      x</p> <p>┌</p> <p>└</p> |  | <p>(8's)</p> <p>x      x</p> <p>┌</p> <p>└</p> |

This is a continuation of previous activities using larger arrays of blocks.





This is a continuation of previous activities. Again, learners might try to find how many different ways each array can be fenced. For example, 36 blocks has these possibilities:

|           |        |           |
|-----------|--------|-----------|
| <u>1</u>  | (36's) | <u>36</u> |
| <u>6</u>  | (6's)  | <u>36</u> |
| <u>12</u> | (3's)  | <u>36</u> |
| <u>9</u>  | (4's)  | <u>36</u> |
| <u>18</u> | (2's)  | <u>36</u> |

¿QUÉ PUEDES VER?  
WHAT CAN YOU SEE?

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$$\begin{array}{r} 13 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ \times 4 \\ \hline \end{array}$$

|  |  |  |  |  |
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$$\begin{array}{r} 15 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} 15 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 15 \\ \times 4 \\ \hline \end{array}$$

|  |  |  |  |
|--|--|--|--|
|  |  |  |  |
|  |  |  |  |

$$\begin{array}{r} 23 \\ \times 2 \\ \hline \end{array}$$

|  |  |  |  |
|--|--|--|--|
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

$$\begin{array}{r} 23 \\ \times 3 \\ \hline \end{array}$$

|  |  |  |  |
|--|--|--|--|
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

$$\begin{array}{r} 23 \\ \times 4 \\ \hline \end{array}$$

| WHAT CAN YOU SEE ?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | NAME _____                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> </div> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> </div> | <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> </div> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> </div> |
| <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> </div> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> </div> | <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> </div> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> </div> |
| <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> </div> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> </div> | <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> </div> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; width: 100px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-right: 5px;"></div> </div> |
| <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"><math>\begin{array}{r} 13 \\ \times 2 \\ \hline \end{array}</math></div> <div style="text-align: center;"><math>\begin{array}{r} 13 \\ \times 3 \\ \hline \end{array}</math></div> <div style="text-align: center;"><math>\begin{array}{r} 13 \\ \times 4 \\ \hline \end{array}</math></div> </div>                                                                                                                                                                                                                                                                    | <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"><math>\begin{array}{r} 15 \\ \times 2 \\ \hline \end{array}</math></div> <div style="text-align: center;"><math>\begin{array}{r} 15 \\ \times 3 \\ \hline \end{array}</math></div> <div style="text-align: center;"><math>\begin{array}{r} 15 \\ \times 4 \\ \hline \end{array}</math></div> </div>                                                                                                                                                                                                                                                                                                                                                                                                                                                          |

|  |  |  |  |  |                                                         |
|--|--|--|--|--|---------------------------------------------------------|
|  |  |  |  |  |                                                         |
|  |  |  |  |  | $\begin{array}{r} 23 \\ \times 2 \\ \hline \end{array}$ |
|  |  |  |  |  | $\begin{array}{r} 23 \\ \times 3 \\ \hline \end{array}$ |
|  |  |  |  |  | $\begin{array}{r} 23 \\ \times 4 \\ \hline \end{array}$ |

### What Can You See? (multiplication)

On this task page, learners have a chance to visualize large multiplications. In the diagrams, a  can be thought of as either a Dienes block rod or a 10-stick, and a  stands for a small cube or a bean. In any case  represents 10 units and  represents one unit.

The learner can look at the arrays and count the total beans (or cubes) by making use of the groupings.

For the first example,  $\begin{array}{r} 13 \\ \times 2 \\ \hline \end{array}$

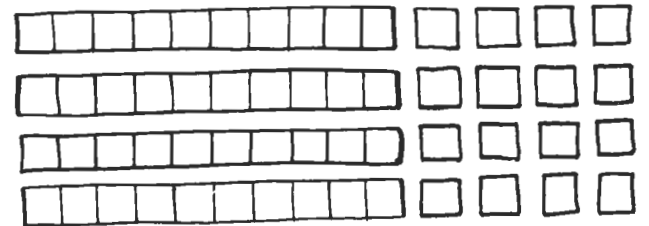
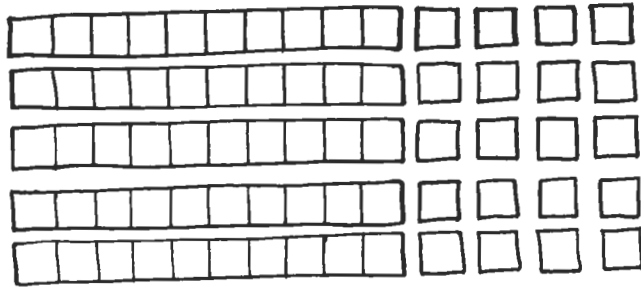
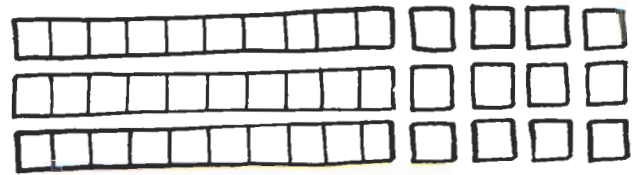
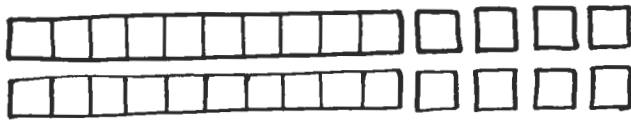
two rows are shown with 13 in each row.

There are 2 rods, or 20 cubes, and six loose cubes for a total of 26 cubes. Thus, the result can be re-

corded:  $\begin{array}{r} 13 \\ \times 2 \\ \hline 26 \end{array}$

Sketches are provided for each example. Children should be encouraged to count loose blocks by considering the groupings rather than by counting by ones. If children have difficulty, beansticks and loose beans or rods and cubes should be brought out.

WHAT CAN YOU SEE?  
¿QUÉ PUEDES VER?



$$\begin{array}{r} 14 \\ \times 2 \\ \hline \end{array}$$

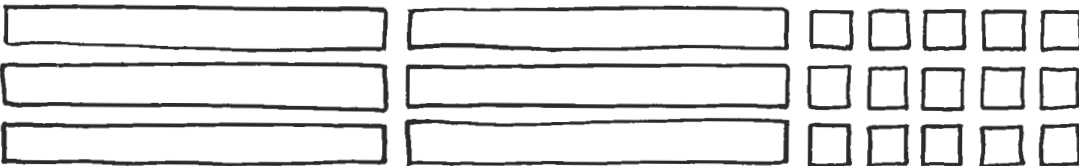
$$\begin{array}{r} 14 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 14 \\ \times 3 \\ \hline \end{array}$$

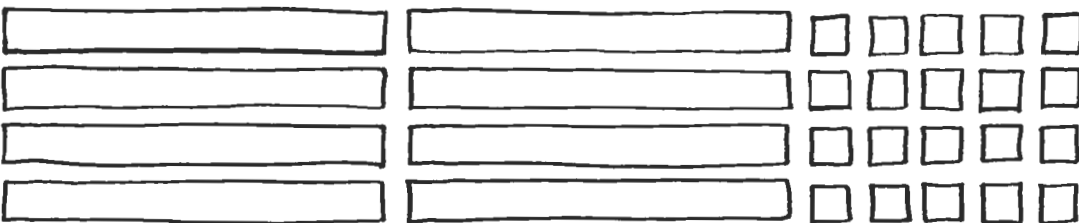
$$\begin{array}{r} 14 \\ \times 4 \\ \hline \end{array}$$



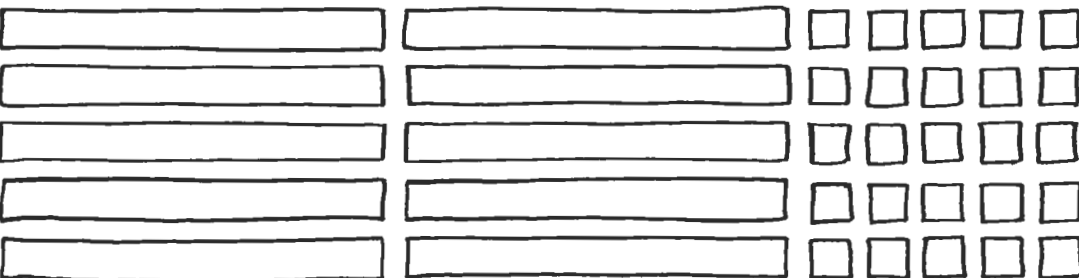
$$\begin{array}{r} 25 \\ \times 2 \\ \hline \end{array}$$



$$\begin{array}{r} 25 \\ \times 3 \\ \hline \end{array}$$



$$\begin{array}{r} 25 \\ \times 4 \\ \hline \end{array}$$

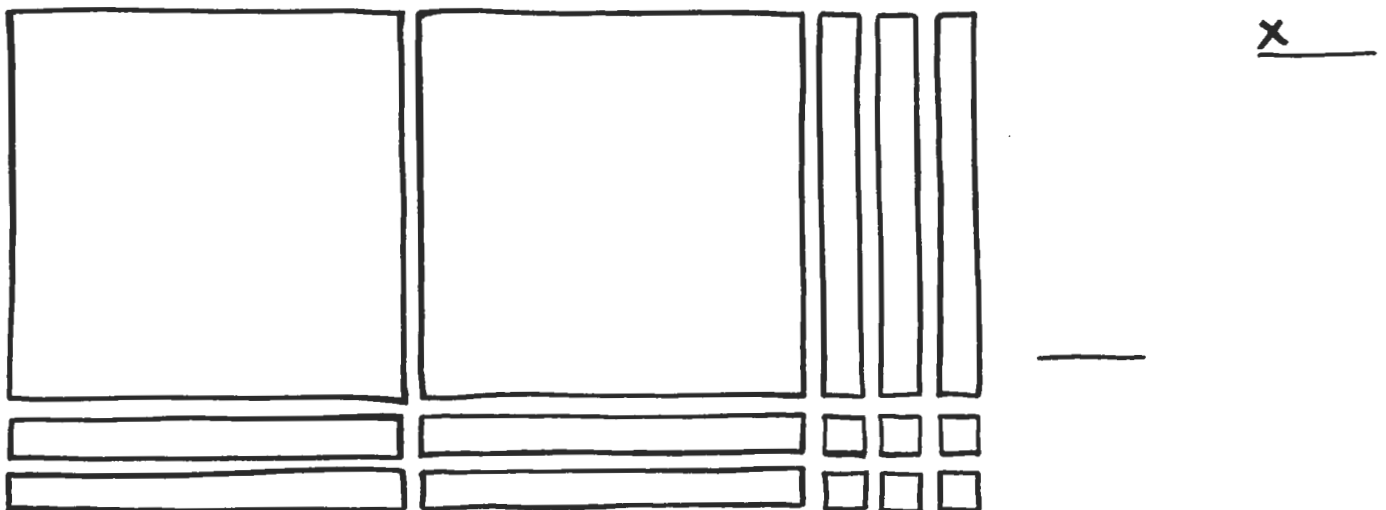
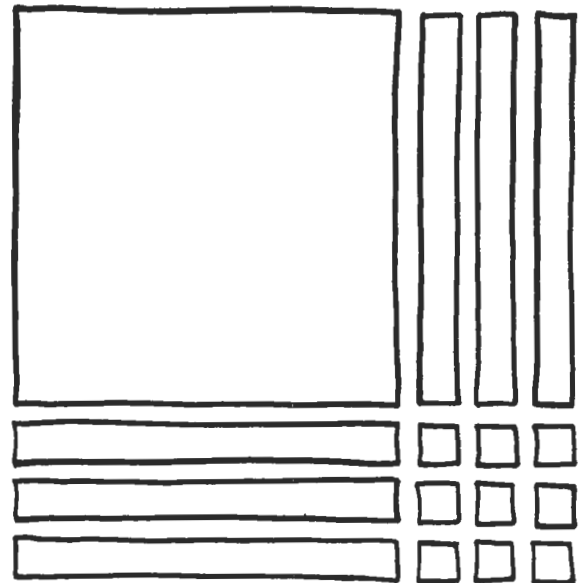
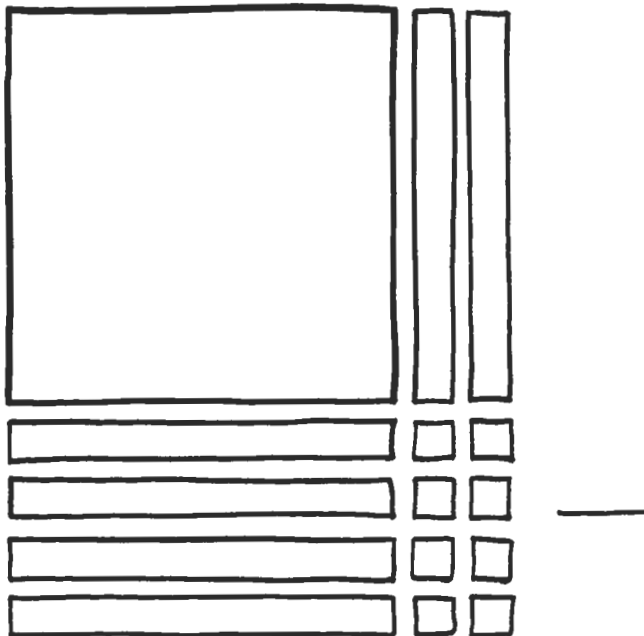
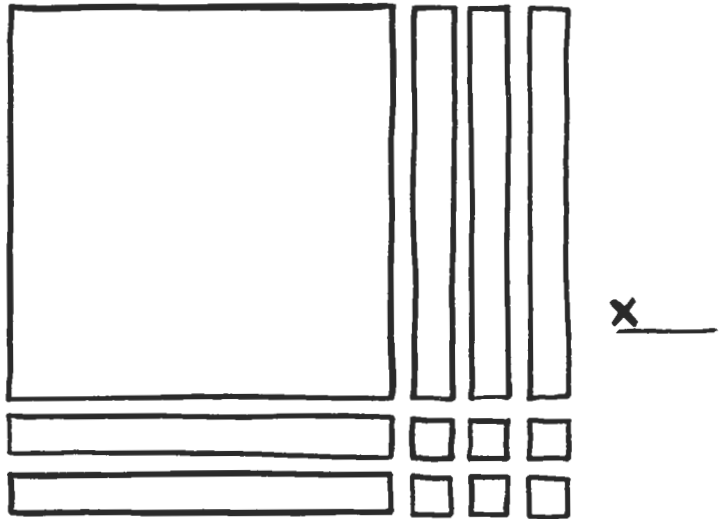
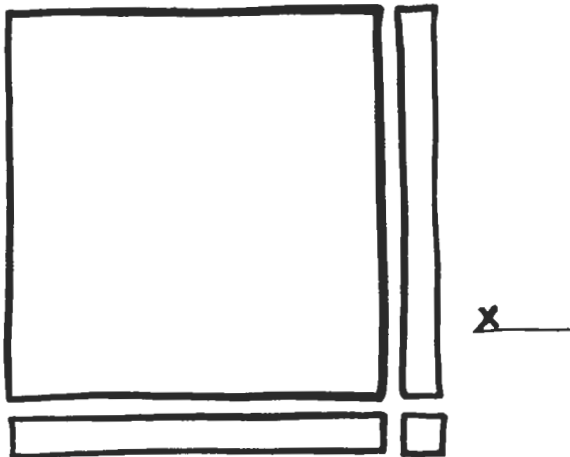


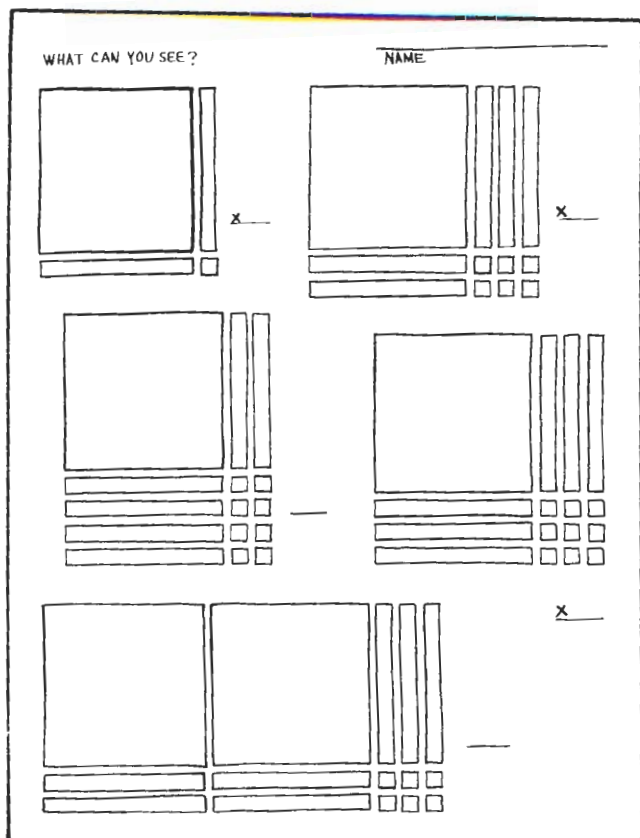
$$\begin{array}{r} 25 \\ \times 5 \\ \hline \end{array}$$





¿QUÉ PUEDES VER?  
WHAT CAN YOU SEE?





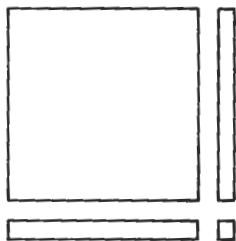
The second diagram shows an array of 12 rows with 13 in each row. There is a total of  $100 + 50 + 6$ , or 156 units. Thus,

$$\begin{array}{r} 13 \\ \times 12 \\ \hline 156 \end{array}$$

In these examples, we are simply concerned with making and counting arrays. No push toward the use of the algorithm and partial products is made. Children should be encouraged to count the units in the array by considering the rafts, then the sticks, and then the loose beans.

### What Can You See? (multiplication)

On this page,  is introduced. It can be thought of as a Dienes 'flat' (100 little cubes or 10 rods) or a "hundred raft" (ten 10-sticks glued together). The learner must now look at the diagram and record the multiplication it illustrates. For example, the first diagram,



shows an array of eleven rows with eleven units in each row. There is a total of 100 (the raft) + 20 (two 10-sticks) + 1 (loose block), or 121 units all together. Thus, we can record:

$$\begin{array}{r} 11 \\ \times 11 \\ \hline 121 \end{array}$$

WHAT CAN YOU SEE?  
¿QUÉ PUEDES VER?

$$\begin{array}{c} \text{=====} \text{:::} \\ \underline{\times 2} \end{array} \quad \begin{array}{c} 13 \\ \times 2 \end{array}$$

---


$$\begin{array}{c} \text{=====} \text{::::} \\ \underline{\times 2} \end{array} \quad \begin{array}{c} 15 \\ \times 2 \end{array}$$

$$\begin{array}{c} \text{=====} \text{:::} \\ \underline{\times 3} \end{array} \quad \begin{array}{c} 13 \\ \times 3 \end{array}$$

$$\begin{array}{c} \text{=====} \text{::::} \\ \underline{\times 3} \end{array} \quad \begin{array}{c} 16 \\ \times 3 \end{array}$$

$$\begin{array}{c} \text{=====} \text{:::} \\ \underline{\times 4} \end{array} \quad \begin{array}{c} 13 \\ \times 4 \end{array}$$

$$\begin{array}{c} \text{=====} \text{:::} \\ \underline{\times 4} \end{array} \quad \begin{array}{c} 15 \\ \times 4 \end{array}$$

$$\begin{array}{c} \text{=====} \text{:::} \\ \underline{\times 5} \end{array} \quad \begin{array}{c} 15 \\ \times 5 \end{array}$$

$$\begin{array}{c} \text{=====} \text{:::} \\ \underline{\times 5} \end{array} \quad \begin{array}{c} 17 \\ \times 5 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{::::} \\ \underline{\times 2} \end{array} \quad \begin{array}{c} 24 \\ \times 2 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{::::} \\ \underline{\times 2} \end{array} \quad \begin{array}{c} 25 \\ \times 2 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{:::} \\ \underline{\times 3} \end{array} \quad \begin{array}{c} 24 \\ \times 3 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{:::} \\ \underline{\times 3} \end{array} \quad \begin{array}{c} 27 \\ \times 3 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{:::} \\ \underline{\times 5} \end{array} \quad \begin{array}{c} 24 \\ \times 5 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{:::} \\ \underline{\times 5} \end{array} \quad \begin{array}{c} 27 \\ \times 5 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{=====} \text{:::} \\ \underline{\times 3} \end{array} \quad \begin{array}{c} 36 \\ \times 3 \end{array}$$

$$\begin{array}{c} \text{=====} \text{=====} \text{=====} \text{=====} \text{:::} \\ \underline{\times 4} \end{array} \quad \begin{array}{c} 48 \\ \times 4 \end{array}$$

WHAT CAN YOU SEE?



$$\begin{array}{r} 13 \\ \times 2 \\ \hline \end{array}$$



$$\begin{array}{r} 15 \\ \times 2 \\ \hline \end{array}$$



$$\begin{array}{r} 13 \\ \times 3 \\ \hline \end{array}$$



$$\begin{array}{r} 16 \\ \times 3 \\ \hline \end{array}$$



$$\begin{array}{r} 13 \\ \times 4 \\ \hline \end{array}$$



$$\begin{array}{r} 15 \\ \times 4 \\ \hline \end{array}$$



$$\begin{array}{r} 15 \\ \times 5 \\ \hline \end{array}$$



$$\begin{array}{r} 17 \\ \times 5 \\ \hline \end{array}$$



$$\begin{array}{r} 24 \\ \times 2 \\ \hline \end{array}$$



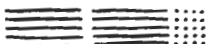
$$\begin{array}{r} 25 \\ \times 2 \\ \hline \end{array}$$



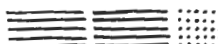
$$\begin{array}{r} 24 \\ \times 3 \\ \hline \end{array}$$



$$\begin{array}{r} 27 \\ \times 3 \\ \hline \end{array}$$



$$\begin{array}{r} 24 \\ \times 5 \\ \hline \end{array}$$



$$\begin{array}{r} 27 \\ \times 5 \\ \hline \end{array}$$



$$\begin{array}{r} 36 \\ \times 3 \\ \hline \end{array}$$



$$\begin{array}{r} 48 \\ \times 4 \\ \hline \end{array}$$

NAME \_\_\_\_\_

### What Can You See? (multiplication)

This activity is similar to the activity on page 121B. The diagrams have become more of a shorthand notation in which  stands for a rod or a 10-stick, and  stands for a small cube or a loose bean.

Each multiplication is sketched as a number of rows of beans with the same number in each row. The learner counts the total number of beans (or cubes) in the array. He should be encouraged to use shortcuts and count loose beans by multiples rather than by ones.

¿QUÉ PUEDES VER?  
WHAT CAN YOU SEE?

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

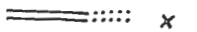
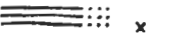
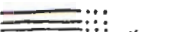
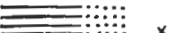
\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

\_\_\_\_\_ x \_\_\_\_\_

WHAT CAN YOU SEE?

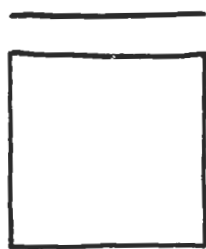
NAME \_\_\_\_\_

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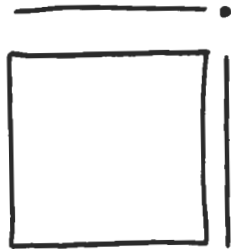
### What Can You See? (multiplication)

This is a continuation of the activity on the previous page. Note that children can check their work by comparing this page with page 122A.

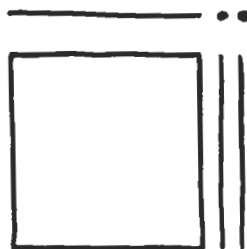
WHAT CAN YOU SEE?  
¿QUÉ PUEDES VER?



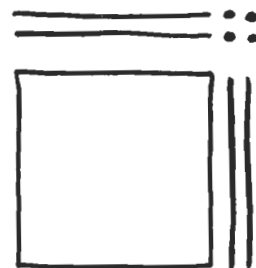
$$\begin{array}{r} 10 \\ \times 11 \\ \hline \end{array}$$



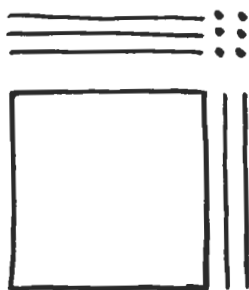
$$\begin{array}{r} 11 \\ \times 11 \\ \hline \end{array}$$



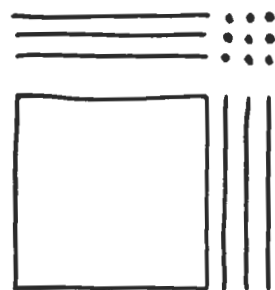
$$\begin{array}{r} 12 \\ \times 11 \\ \hline \end{array}$$



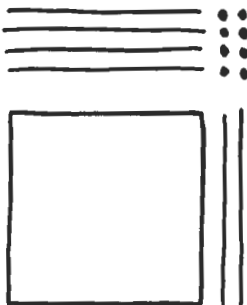
$$\begin{array}{r} 12 \\ \times 12 \\ \hline \end{array}$$



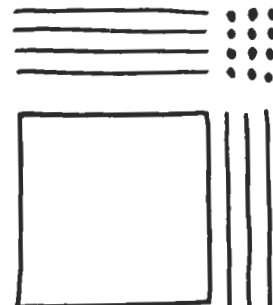
$$\begin{array}{r} 12 \\ \times 13 \\ \hline \end{array}$$



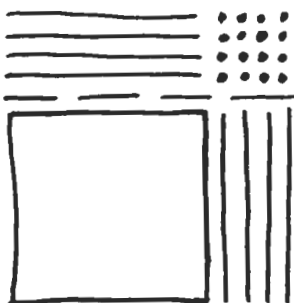
$$\begin{array}{r} 13 \\ \times 13 \\ \hline \end{array}$$



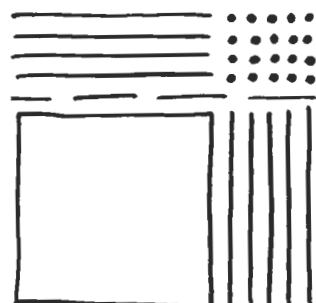
$$\begin{array}{r} 12 \\ \times 14 \\ \hline \end{array}$$



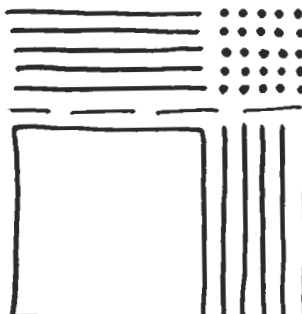
$$\begin{array}{r} 13 \\ \times 14 \\ \hline \end{array}$$



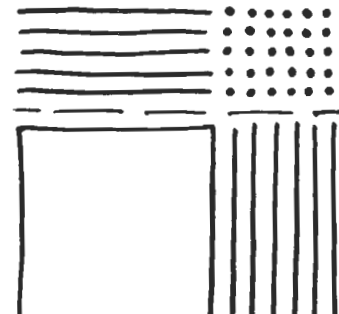
$$\begin{array}{r} 14 \\ \times 14 \\ \hline \end{array}$$



$$\begin{array}{r} 15 \\ \times 14 \\ \hline \end{array}$$

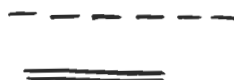


$$\begin{array}{r} 15 \\ \times 15 \\ \hline \end{array}$$



$$\begin{array}{r} 16 \\ \times 15 \\ \hline \end{array}$$

 ABOVE  
 BELOW  
 TOTAL



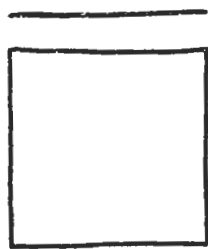
ARriba  
 Abajo



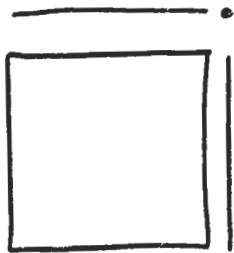
WHAT CAN YOU SEE? NAME \_\_\_\_\_

|                                                                                                                                              |                                                                                                                                              |                                                                                                                                              |                                                                                                                                              |
|----------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| <br>$\begin{array}{r} 10 \\ \times 1 \\ \hline \end{array}$ | <br>$\begin{array}{r} 11 \\ \times 1 \\ \hline \end{array}$ | <br>$\begin{array}{r} 12 \\ \times 1 \\ \hline \end{array}$ | <br>$\begin{array}{r} 12 \\ \times 2 \\ \hline \end{array}$ |
| <br>$\begin{array}{r} 12 \\ \times 3 \\ \hline \end{array}$ | <br>$\begin{array}{r} 13 \\ \times 3 \\ \hline \end{array}$ | <br>$\begin{array}{r} 12 \\ \times 4 \\ \hline \end{array}$ | <br>$\begin{array}{r} 13 \\ \times 4 \\ \hline \end{array}$ |
| <br>$\begin{array}{r} 14 \\ \times 4 \\ \hline \end{array}$ | <br>$\begin{array}{r} 15 \\ \times 4 \\ \hline \end{array}$ | <br>$\begin{array}{r} 15 \\ \times 5 \\ \hline \end{array}$ | <br>$\begin{array}{r} 16 \\ \times 5 \\ \hline \end{array}$ |

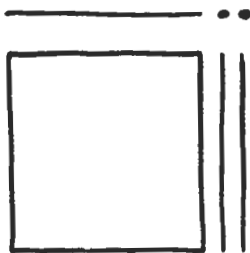
¿QUÉ PUEDES VER?  
WHAT CAN YOU SEE?



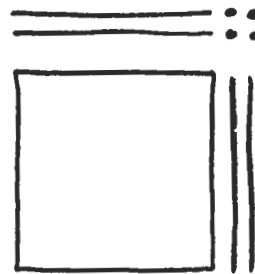
x



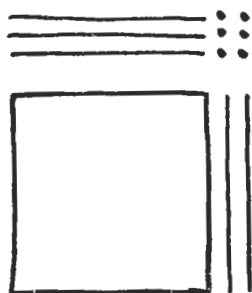
x



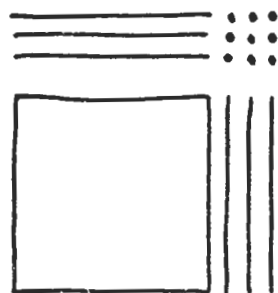
x



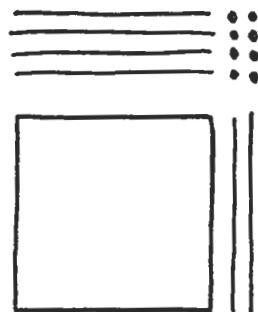
x



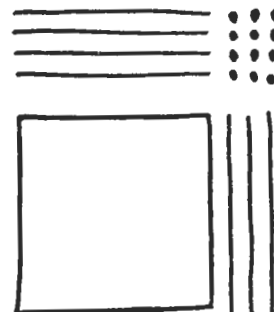
x



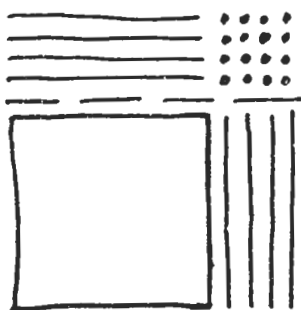
x



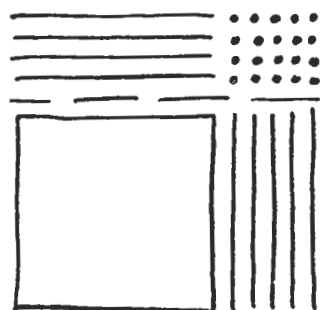
x



x

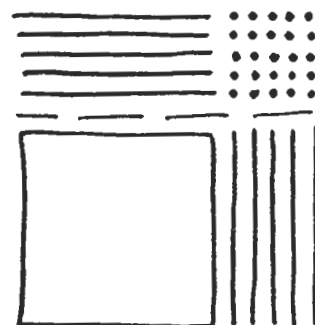


14  
x 14  
56 ABOVE  
BELOW  
TOTAL



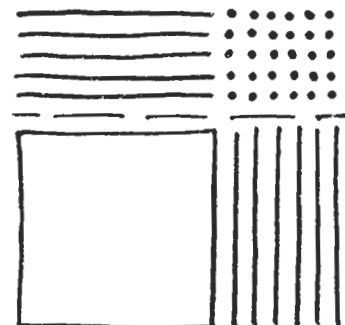
x

-----  
=====



x

-----  
=====

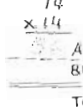


x

-----  
=====

ARRIBA  
ABAJO

WHAT CAN YOU SEE? NAME \_\_\_\_\_

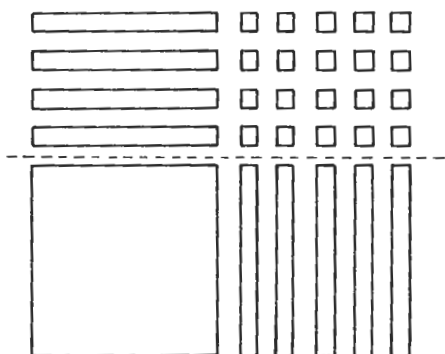
|                                                                                   |                                                                                   |                                                                                   |                                                                                   |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |  |  |
| x _____                                                                           | x _____                                                                           | x _____                                                                           | x _____                                                                           |
|  |  |  |  |
| x _____                                                                           | x _____                                                                           | x _____                                                                           | x _____                                                                           |
|  |  |  |  |
|  | x _____                                                                           | x _____                                                                           | x _____                                                                           |
| 14<br>x 14                                                                        |                                                                                   |                                                                                   |                                                                                   |
| ABOVE                                                                             |                                                                                   |                                                                                   |                                                                                   |
| BELOW                                                                             |                                                                                   |                                                                                   |                                                                                   |
| TOTAL                                                                             |                                                                                   |                                                                                   |                                                                                   |

Above the dotted line, there are 60 beans. Below the line are 150 beans. Thus, the learner would record:

$$\begin{array}{r}
 15 \\
 \times 14 \\
 \hline
 60 \text{ above} \\
 150 \text{ below} \\
 \hline
 210 \text{ Total}
 \end{array}$$

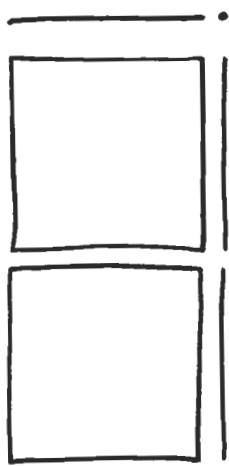
Note that children can check their own work by comparing this page to page 126A.

Here, the learner must look at each sketch and figure out which multiplication it illustrates. Once again, examples are given in the last row which begin to build toward the use of partial products and the multiplication algorithm. Once the learner figures out what multiplication is indicated by the array, he can count the number of beans above the dotted line and below the dotted line and then add these numbers to get the total. For example:

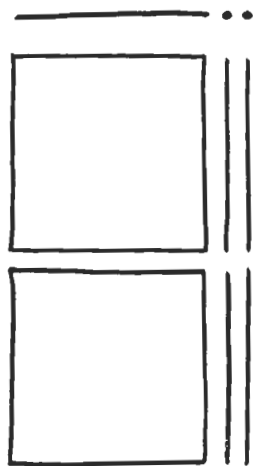


shows 14 rows with 15 beans (blocks) in each row. Thus it represents the multiplication:

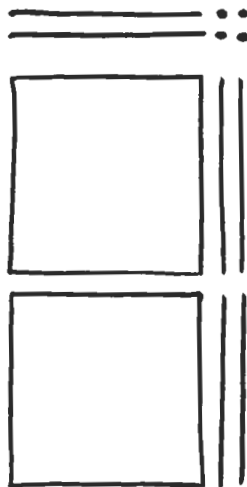
$$\begin{array}{r}
 15 \\
 \times 14 \\
 \hline
 \end{array}$$



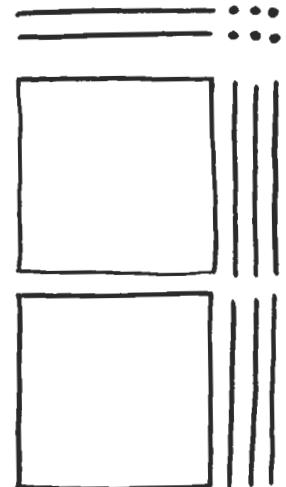
$$\begin{array}{r} 11 \\ \times 21 \\ \hline \end{array}$$



$$\begin{array}{r} 12 \\ \times 21 \\ \hline \end{array}$$



$$\begin{array}{r} 12 \\ \times 22 \\ \hline \end{array}$$



$$\begin{array}{r} 13 \\ \times 22 \\ \hline \end{array}$$



$$\begin{array}{r} 24 \\ \times 12 \\ \hline \end{array}$$

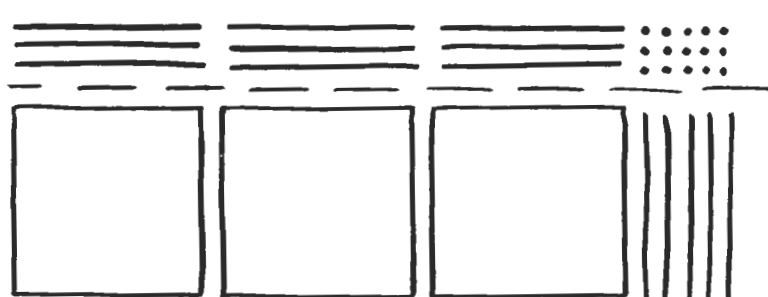


$$\begin{array}{r} 33 \\ \times 12 \\ \hline \end{array}$$



$$\begin{array}{r} 24 \\ \times 13 \\ \hline \end{array}$$

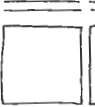
----- ABOVE  
----- BELOW  
===== TOTAL



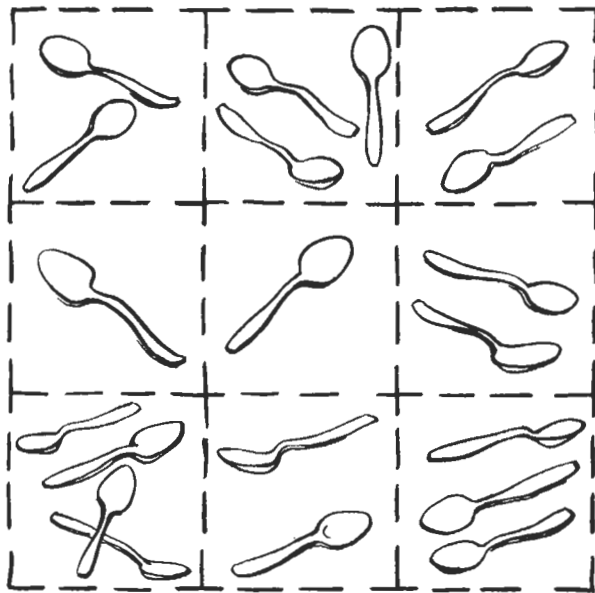
$$\begin{array}{r} 35 \\ \times 13 \\ \hline \end{array}$$

----- ARRIBA  
----- ABAJO  
===== TOTAL

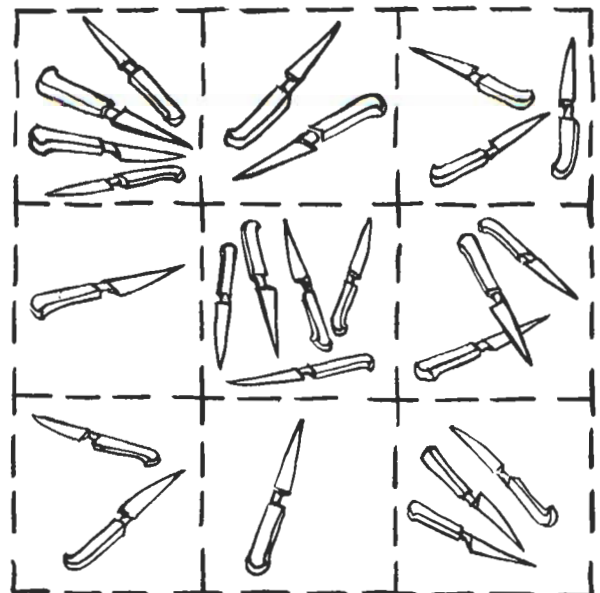
WHAT CAN YOU SEE? NAME \_\_\_\_\_

|                                                                                                                                               |                                                                                                                                               |                                                                                                                                               |                                                                                                                                               |
|-----------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| <br>$\begin{array}{r} 11 \\ \times 21 \\ \hline \end{array}$ | <br>$\begin{array}{r} 12 \\ \times 21 \\ \hline \end{array}$ | <br>$\begin{array}{r} 12 \\ \times 22 \\ \hline \end{array}$ | <br>$\begin{array}{r} 13 \\ \times 22 \\ \hline \end{array}$ |
| <br>$\begin{array}{r} 24 \\ \times 12 \\ \hline \end{array}$ | <br>$\begin{array}{r} 33 \\ \times 12 \\ \hline \end{array}$ |                                                                                                                                               |                                                                                                                                               |
| <br>$\begin{array}{r} 24 \\ \times 13 \\ \hline \end{array}$ | <br>$\begin{array}{r} 35 \\ \times 13 \\ \hline \end{array}$ |                                                                                                                                               |                                                                                                                                               |
| <p>ABOVE</p> <p>BELOW</p> <p>TOTAL</p>                                                                                                        | <p>ABOVE</p> <p>BELOW</p> <p>TOTAL</p>                                                                                                        |                                                                                                                                               |                                                                                                                                               |

This activity is a continuation of the activities on the previous pages. The multiplications use larger numbers.



— (5's)



— (6's)

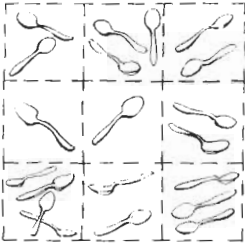
|   |   |   |
|---|---|---|
| 2 | 3 | 2 |
| 3 | 1 | 4 |
| 2 | 2 | 1 |

— (5's)

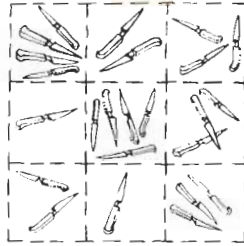
|   |   |   |
|---|---|---|
| 1 | 2 | 4 |
| 3 | 2 | 2 |
| 2 | 1 | 1 |

— (6's)

NAME \_\_\_\_\_



— (5's)



— (6's)

|   |   |   |
|---|---|---|
| 2 | 3 | 2 |
| 3 | 1 | 4 |
| 2 | 2 | 1 |

— (5's)

|   |   |   |
|---|---|---|
| 1 | 2 | 4 |
| 3 | 2 | 2 |
| 2 | 1 | 1 |

— (6's)

For other fencing activities, see pages 94-103.

On this page we make the transition from pictures to numerals. The children should have little difficulty making the transition. Tell them to do the bottom squares the same way they do the top squares.

Remember, try to get the greatest number of fences.

|   |   |   |
|---|---|---|
| 1 | 3 | 4 |
| 6 | 5 | 2 |
| 1 | 2 | 4 |

— (7's)

|   |   |   |
|---|---|---|
| 4 | 6 | 2 |
| 1 | 1 | 5 |
| 3 | 7 | 3 |

— (8's)

|   |   |   |
|---|---|---|
| 1 | 6 | 2 |
| 5 | 2 | 3 |
| 2 | 3 | 3 |

— (9's)

|   |   |   |
|---|---|---|
| 8 | 2 | 7 |
| 4 | 3 | 3 |
| 6 | 2 | 5 |

— (10's)



NAME \_\_\_\_\_

|   |   |   |
|---|---|---|
| 1 | 3 | 4 |
| 6 | 5 | 2 |
| 1 | 2 | 4 |

— (7's)

|   |   |   |
|---|---|---|
| 4 | 6 | 2 |
| 1 | 1 | 5 |
| 3 | 7 | 3 |

— (8's)

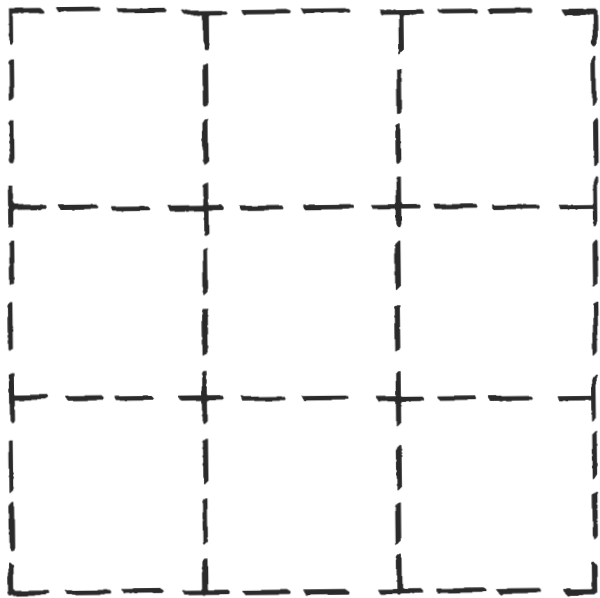
|   |   |   |
|---|---|---|
| 1 | 6 | 2 |
| 5 | 2 | 3 |
| 2 | 3 | 3 |

— (9's)

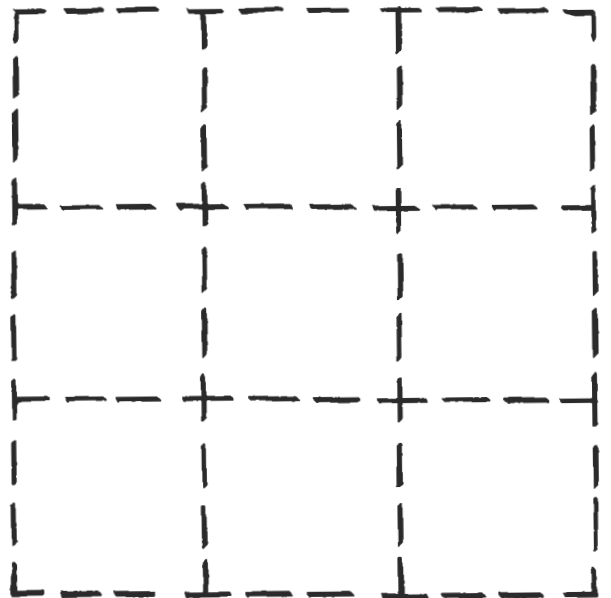
|   |   |   |
|---|---|---|
| 8 | 2 | 7 |
| 4 | 3 | 3 |
| 6 | 2 | 5 |

— (10's)

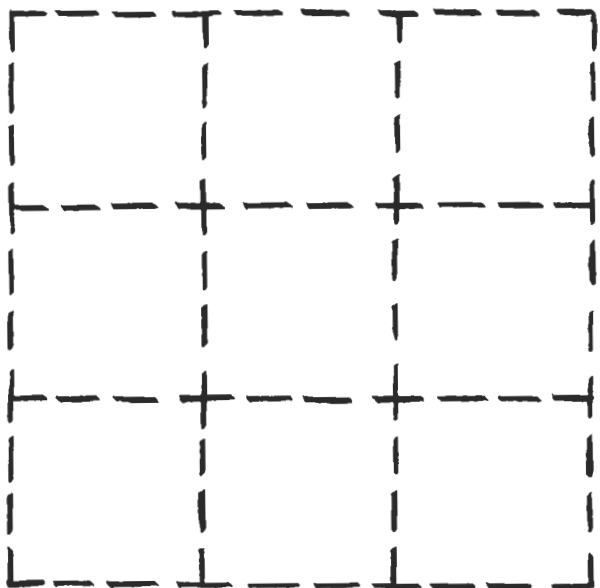
The numerals in the boxes and the sums get larger.



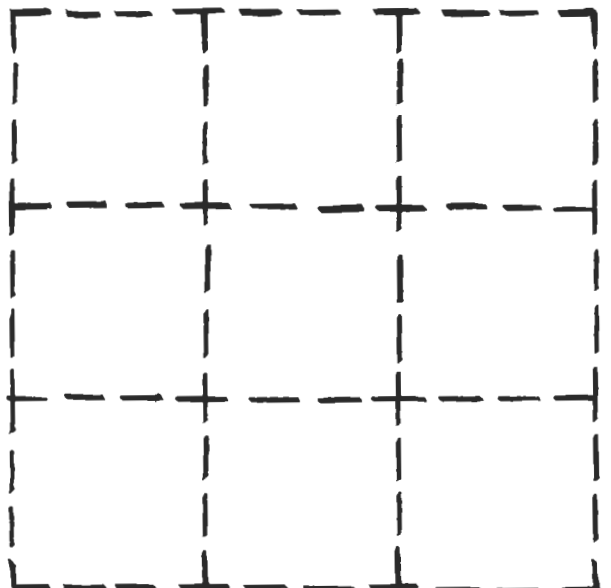
— (s)



— (s)

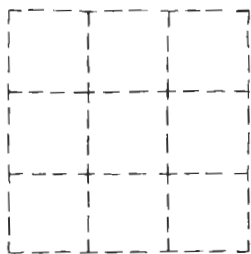
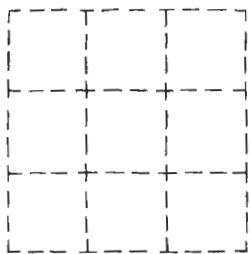
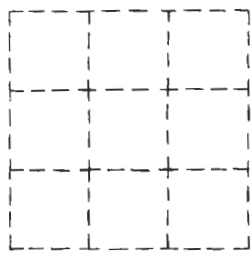
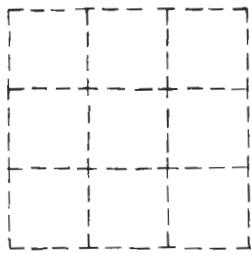


— (s)



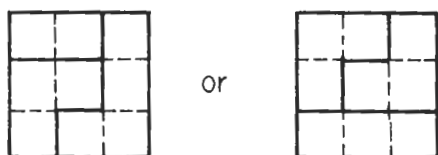
— (s)

NAME \_\_\_\_\_

|                                                                                                |                                                                                                |
|------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------|
| <br>_____ (s) | <br>_____ (s) |
| <br>_____ (s) | <br>_____ (s) |

And you can be sure there is at least 1 way to get all the numbers fenced in—though there may be other ways than the originating fences.

To make up examples of "fence arithmetic" of this kind, start with a 3 by 3 arrangement with fences drawn.



After choosing the sum desired, such as 9, put combinations of 9 in each fence such as:

|   |   |   |
|---|---|---|
| 6 | 3 | 3 |
| 2 | 4 | 3 |
| 3 | 1 | 2 |

or

|   |   |   |
|---|---|---|
| 3 | 3 | 2 |
| 3 | 4 | 3 |
| 6 | 1 | 2 |

Then write in the numbers without the fences:

|   |   |   |
|---|---|---|
| 6 | 3 | 3 |
| 2 | 4 | 3 |
| 3 | 1 | 2 |

or

|   |   |   |
|---|---|---|
| 3 | 3 | 2 |
| 3 | 4 | 3 |
| 6 | 1 | 2 |

|   |   |   |
|---|---|---|
| 1 | 2 | 4 |
| 5 | 3 | 3 |
| 3 | 2 | 1 |

\_\_\_ (6's)

|   |   |   |
|---|---|---|
| 1 | 4 | 2 |
| 2 | 2 | 3 |
| 1 | 2 | 4 |

\_\_\_ (7's)

|   |   |   |
|---|---|---|
| 4 | 1 | 6 |
| 3 | 2 | 2 |
| 1 | 2 | 3 |

\_\_\_ (8's)

|   |   |   |
|---|---|---|
| 4 | 3 | 1 |
| 3 | 2 | 5 |
| 2 | 3 | 4 |

\_\_\_ (9's)

|   |   |   |
|---|---|---|
| 1 | 4 | 6 |
| 2 | 4 | 5 |
| 3 | 3 | 2 |

\_\_\_ (10's)

|   |   |   |
|---|---|---|
| 2 | 3 | 1 |
| 1 | 4 | 2 |
| 1 | 1 | 6 |

\_\_\_ (7's)

|   |   |   |
|---|---|---|
| 2 | 6 | 1 |
| 3 | 1 | 7 |
| 4 | 5 | 3 |

\_\_\_ (8's)

|   |   |   |
|---|---|---|
| 5 | 2 | 7 |
| 4 | 3 | 3 |
| 1 | 8 | 3 |

\_\_\_ (9's)

|   |   |   |
|---|---|---|
| 4 | 7 | 3 |
| 3 | 5 | 1 |
| 2 | 1 | 4 |

\_\_\_ (10's)

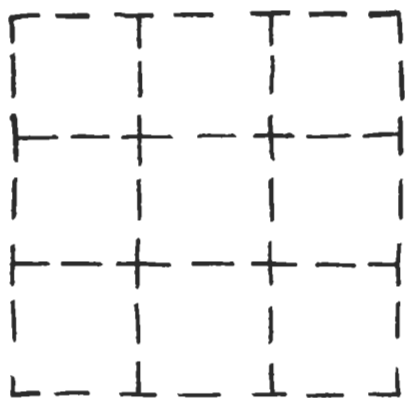
NAME \_\_\_\_\_

|         |          |          |
|---------|----------|----------|
| 1 2 4   | 1 4 2    | 4 1 6    |
| 5 3 3   | 2 2 3    | 3 2 2    |
| 3 2 1   | 1 2 4    | 1 2 3    |
| — (6's) | — (7's)  | — (8's)  |
| 4 3 1   | 1 4 6    | 2 3 1    |
| 3 2 5   | 2 4 5    | 1 4 2    |
| 2 3 4   | 3 3 2    | 1 1 6    |
| — (9's) | — (10's) | — (7's)  |
| 2 6 1   | 5 2 7    | 4 7 3    |
| 3 1 7   | 4 3 3    | 3 5 1    |
| 4 5 3   | 1 8 3    | 2 1 4    |
| — (8's) | — (9's)  | — (10's) |

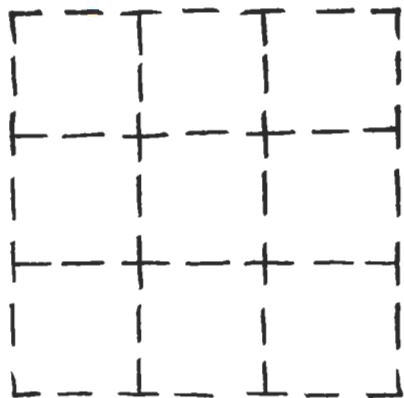
After finding one way to fence all numbers in all examples, consider providing another copy of this or the following page and the question:

“In how many examples are there two different ways to fence all numbers in?” (There is only 1—fencing the (9's) in the second row.)

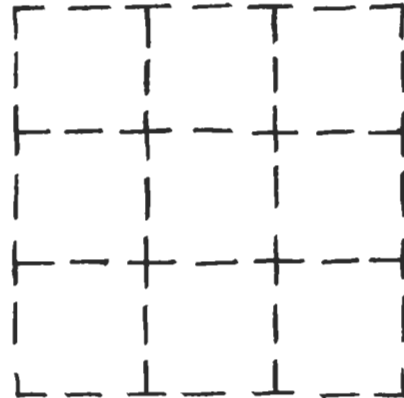
“Can you make up examples so that all numbers can be fenced in more than one way?”



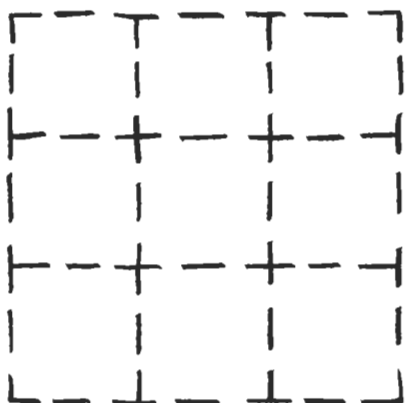
— (s)



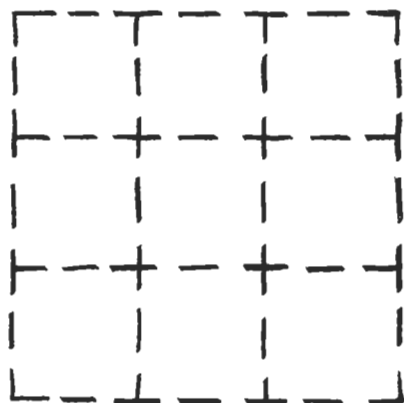
— (s)



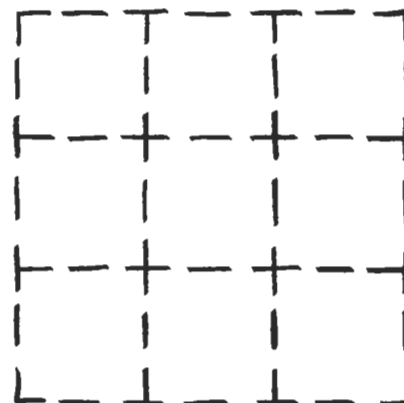
— (s)



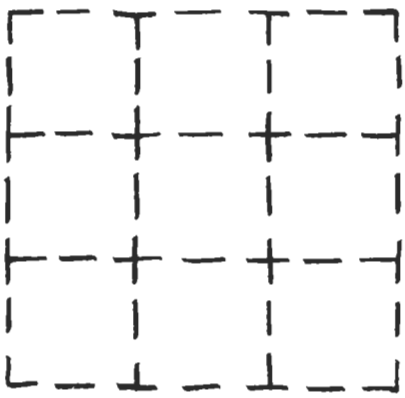
— (s)



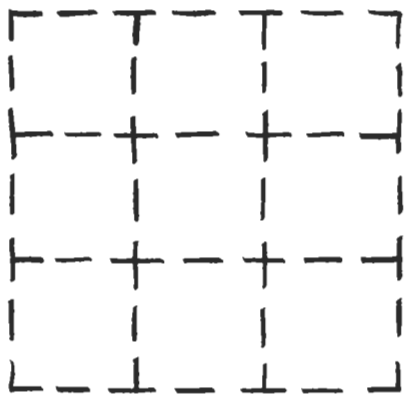
— (s)



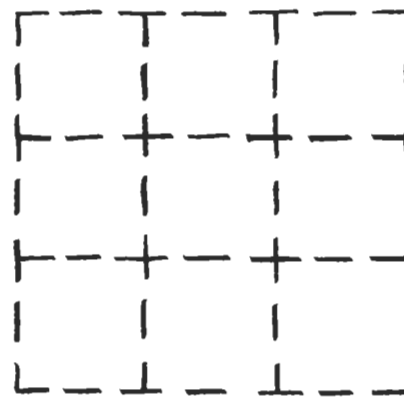
— (s)



— (s)



— (s)



— (s)

NAME \_\_\_\_\_

|                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                      |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;">       _____ (s)     </div> | <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;">       _____ (s)     </div> | <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;">       _____ (s)     </div> |
| <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;">       _____ (s)     </div> | <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;">       _____ (s)     </div> | <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;">       _____ (s)     </div> |
| <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;">       _____ (s)     </div> | <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;">       _____ (s)     </div> | <div style="border: 1px dashed black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; border: 1px dashed black;"></div> </div> <div style="text-align: center; margin-top: 10px;">       _____ (s)     </div> |

If two numbers are omitted and the final record completed, the problem grows.

|   |   |   |
|---|---|---|
|   | 6 | 1 |
| 3 | 1 | 7 |
| 4 |   | 3 |

4 \_\_\_\_\_ (8's)

The sum of the given numbers is 25—so the missing pair must have a sum of 7: so the pair is far from determined yet.

The problem can be extended. If you find a pair that leads to fencing in 4 (8's), could some other pair of numbers be used?

A variation of the "fencing problem" leaves some of the numbers out.

|   |   |   |
|---|---|---|
| 2 | 3 | 1 |
| 1 | 4 | 2 |
| 1 | 1 |   |

or

|   |   |   |
|---|---|---|
| 4 |   | 3 |
| 3 | 5 | 1 |
| 2 | 1 | 4 |

\_\_\_\_\_ (7's)

\_\_\_\_\_ (10's)

These are harder than they look at first glance—if all numbers are to be fenced. Try them.

They are 2 examples from the previous page with the largest number missing. Other difficult examples can be made in a similar fashion.

Of course, there is a simple way to find these numbers. If all are to be fenced in, then the sum of all numbers must be a multiple of the sum that is specified—7's and 10's in the example above.

The total of numbers given in the first example is 15—or 6 less than the next largest multiple of 7.

The total of numbers given in the second example is 23—or 7 less than the next largest multiple of 10.

|   |   |   |   |
|---|---|---|---|
| 2 | 1 | 3 | 5 |
| 5 | 1 | 5 | 7 |
| 6 | 3 | 2 | 1 |
| 2 | 5 | 4 | 4 |

— (8's)

|   |   |   |   |
|---|---|---|---|
| 5 | 4 | 2 | 3 |
| 3 | 2 | 6 | 4 |
| 6 | 1 | 7 | 6 |
| 4 | 5 | 2 | 3 |

— (9's)

|   |   |   |   |
|---|---|---|---|
| 1 | 9 | 3 | 3 |
| 6 | 2 | 8 | 4 |
| 4 | 5 | 2 | 3 |
| 5 | 5 | 3 | 7 |

— (10's)

|   |   |   |   |
|---|---|---|---|
| 4 | 4 | 7 | 2 |
| 4 | 3 | 3 | 9 |
| 3 | 3 | 2 | 3 |
| 5 | 3 | 3 | 8 |

— (11's)

|   |   |   |   |
|---|---|---|---|
| 4 | 8 | 5 | 5 |
| 7 | 9 | 3 | 2 |
| 2 | 5 | 6 | 8 |
| 3 | 7 | 6 | 4 |

— (12's)

|   |   |   |   |
|---|---|---|---|
| 3 | 3 | 4 | 6 |
| 3 | 8 | 5 | 3 |
| 1 | 1 | 2 | 7 |
| 3 | 5 | 3 | 6 |

— (9's)

|   |   |   |   |
|---|---|---|---|
| 6 | 4 | 2 | 8 |
| 4 | 5 | 3 | 1 |
| 5 | 1 | 2 | 9 |
| 7 | 3 | 4 | 6 |

— (10's)

|   |   |   |   |
|---|---|---|---|
| 2 | 5 | 6 | 8 |
| 9 | 6 | 2 | 3 |
| 3 | 3 | 4 | 7 |
| 4 | 4 | 9 | 2 |

— (11's)

|   |   |   |   |
|---|---|---|---|
| 4 | 8 | 3 | 9 |
| 6 | 6 | 3 | 5 |
| 4 | 5 | 4 | 7 |
| 4 | 4 | 8 | 4 |

— (12's)



NAME \_\_\_\_\_

|          |   |   |   |          |   |   |   |          |   |   |   |
|----------|---|---|---|----------|---|---|---|----------|---|---|---|
| 2        | 1 | 3 | 5 | 5        | 4 | 2 | 3 | 1        | 9 | 3 | 3 |
| 5        | 1 | 5 | 7 | 3        | 2 | 6 | 4 | 6        | 2 | 8 | 4 |
| 6        | 3 | 2 | 1 | 6        | 1 | 7 | 6 | 4        | 5 | 2 | 3 |
| 2        | 5 | 4 | 4 | 4        | 5 | 2 | 3 | 5        | 5 | 3 | 7 |
| — (8's)  |   |   |   | — (9's)  |   |   |   | — (10's) |   |   |   |
| 4        | 4 | 7 | 2 | 4        | 8 | 5 | 5 | 3        | 3 | 4 | 6 |
| 4        | 3 | 3 | 9 | 7        | 9 | 3 | 2 | 3        | 8 | 5 | 3 |
| 3        | 3 | 2 | 3 | 2        | 5 | 6 | 8 | 1        | 1 | 2 | 7 |
| 5        | 3 | 3 | 8 | 3        | 7 | 6 | 4 | 3        | 5 | 3 | 6 |
| — (11's) |   |   |   | — (12's) |   |   |   | — (9's)  |   |   |   |
| 6        | 4 | 2 | 8 | 2        | 5 | 6 | 8 | 4        | 8 | 3 | 9 |
| 4        | 5 | 3 | 1 | 9        | 6 | 2 | 3 | 6        | 6 | 3 | 5 |
| 5        | 1 | 2 | 9 | 3        | 3 | 4 | 7 | 4        | 5 | 4 | 7 |
| 7        | 3 | 4 | 6 | 4        | 4 | 9 | 2 | 4        | 4 | 8 | 4 |
| — (10's) |   |   |   | — (11's) |   |   |   | — (12's) |   |   |   |

- (1) There is only a single 1 neighboring 7 to make 8.
- (2) After that "5 needs 3"
- (3) and "4 needs 4."

Then everything falls into place: "6 needs 2" and "5 needs 3" and the two 5, 2 and 1 combinations are left.

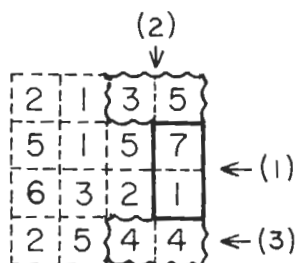
These are not "secrets" we hope children don't uncover. Rather, finding such useful strategies is the essence of mathematics. And group discussion of these systems should be encouraged as wise use of "instructional time."

Not only do the numbers in the boxes and the sums get larger, but the 9 box square grows to a 16 box square. As a child works through the square, many decisions are made and there will often be a need to go back and change some fences. To avoid the need for erasing constantly a "scribble sheet" is provided as the next page.

However, there is a useful strategy someone may discover. It might be explained this way:

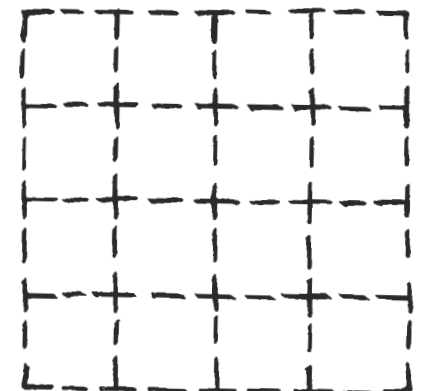
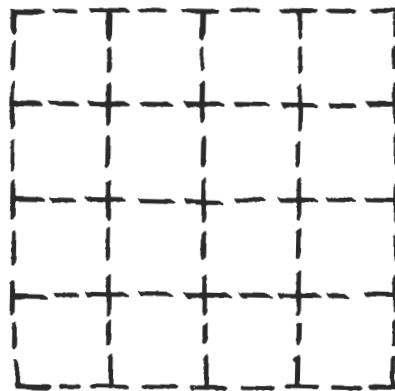
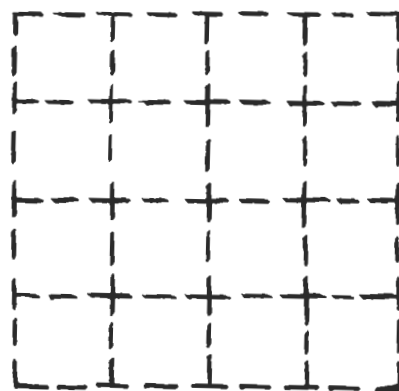
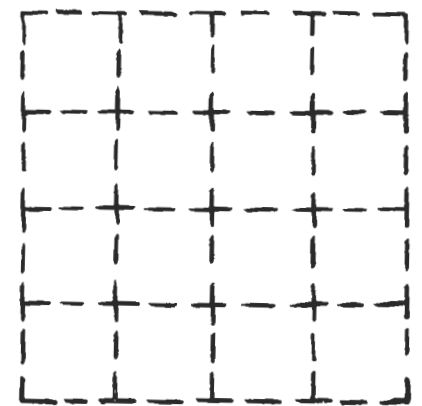
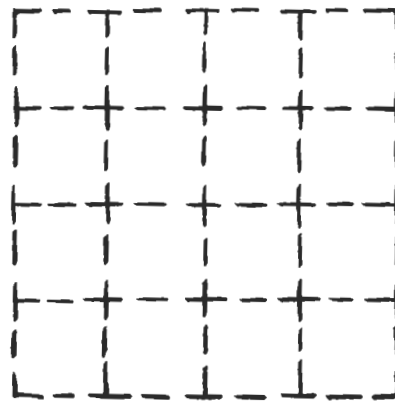
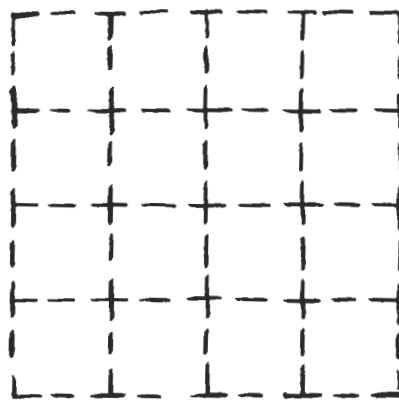
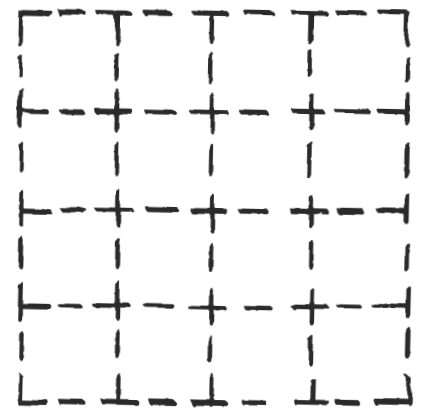
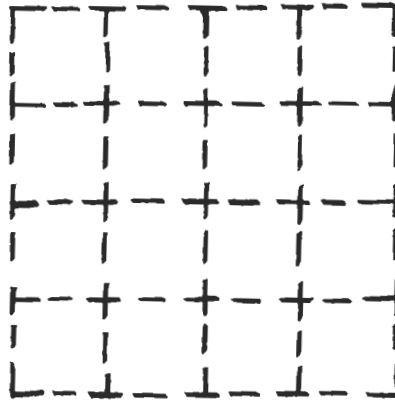
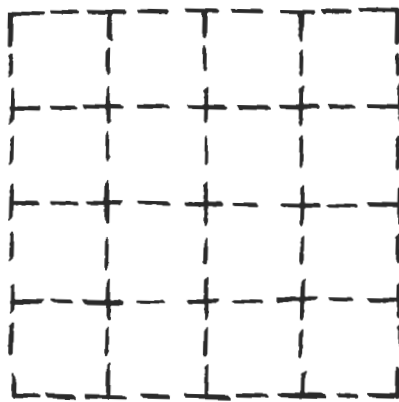
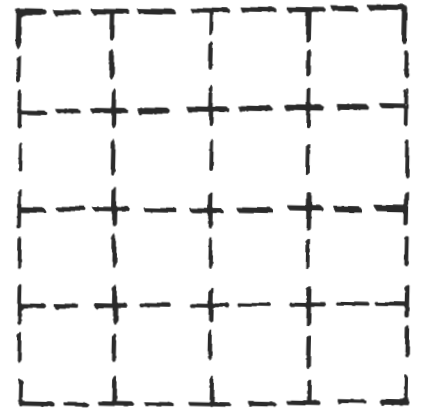
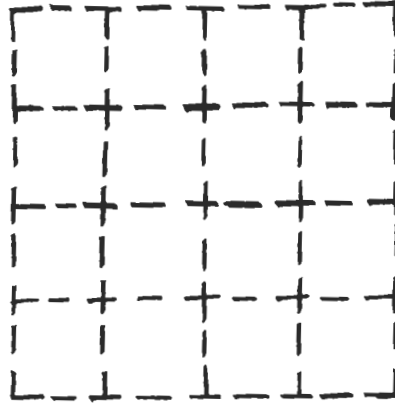
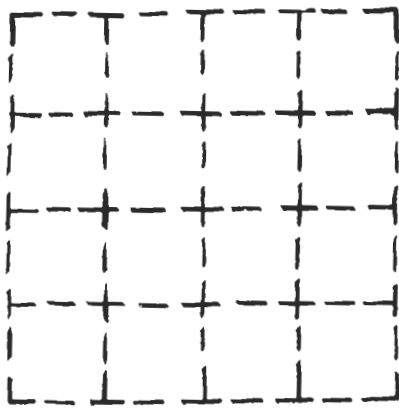
First look for the larger numbers. If there is only one way to fence them with a neighbor, that's a good beginning.

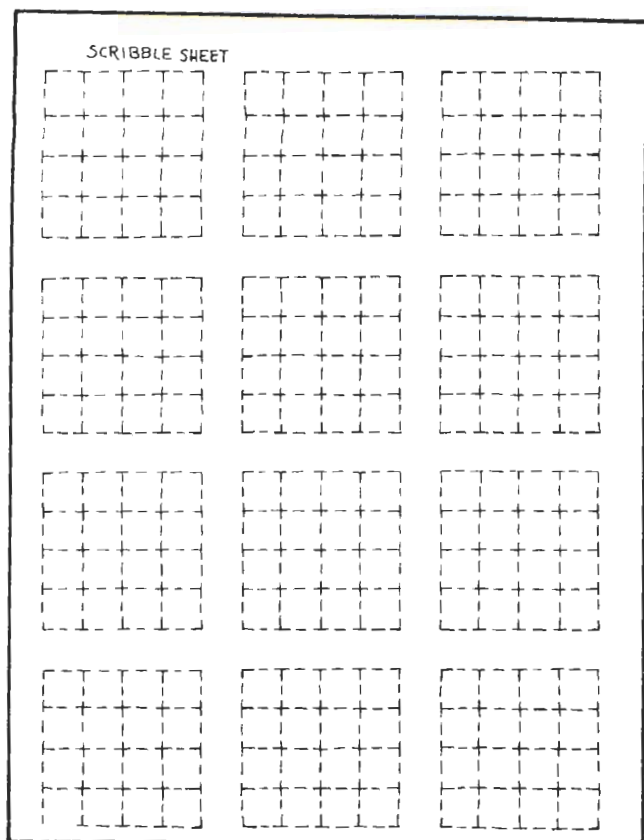
In the first example:



PAPEL PARA PRACTICAR

SCRIBBLE SHEET





This scribble sheet can be used with the previous page and with those that follow.

Its purpose is to give the child a place to experiment before making the final decision as to where the fences should be. Its use will become more evident as the sums get larger. This sheet may be introduced in conjunction with the previous page or when you see erasures beginning to appear.

|   |   |   |   |
|---|---|---|---|
| 2 | 7 | 6 | 4 |
| 1 | 1 | 3 | 3 |
| 3 | 2 | 5 | 7 |
| 6 | 2 | 6 | 2 |

— (10's)

|   |   |   |   |
|---|---|---|---|
| 2 | 6 | 3 | 2 |
| 3 | 8 | 1 | 3 |
| 5 | 2 | 2 | 4 |
| 3 | 2 | 7 | 2 |

— (11's)

|   |   |   |   |
|---|---|---|---|
| 3 | 4 | 4 | 4 |
| 3 | 3 | 4 | 6 |
| 5 | 3 | 2 | 5 |
| 2 | 5 | 3 | 4 |

— (12's)

|   |   |   |   |
|---|---|---|---|
| 7 | 3 | 3 | 6 |
| 3 | 3 | 2 | 4 |
| 2 | 8 | 9 | 2 |
| 3 | 3 | 3 | 4 |

— (13's)

|   |   |   |   |
|---|---|---|---|
| 9 | 3 | 4 | 6 |
| 2 | 2 | 5 | 4 |
| 3 | 5 | 2 | 6 |
| 7 | 4 | 6 | 2 |

— (14's)

|   |   |   |   |
|---|---|---|---|
| 7 | 2 | 3 | 9 |
| 3 | 5 | 5 | 1 |
| 5 | 5 | 5 | 5 |
| 8 | 2 | 4 | 6 |

— (15's)

|   |   |   |   |
|---|---|---|---|
| 2 | 6 | 6 | 7 |
| 3 | 5 | 4 | 3 |
| 6 | 6 | 6 | 6 |
| 2 | 8 | 1 | 9 |

— (16's)

|   |   |    |   |
|---|---|----|---|
| 8 | 3 | 4  | 9 |
| 3 | 3 | 4  | 4 |
| 7 | 3 | 10 | 1 |
| 4 | 7 | 8  | 8 |

— (17's)

|   |   |   |    |
|---|---|---|----|
| 7 | 3 | 5 | 13 |
| 8 | 9 | 6 | 8  |
| 5 | 9 | 4 | 3  |
| 4 | 9 | 9 | 6  |

— (18's)

NAME \_\_\_\_\_

|                                                                                                                                                                                                                                                                                     |          |          |    |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------|----------|----|---|---|---|---|---|---|---|---|---|---|---|---|---|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|---|---|---|---|---|---|---|---|----|---|---|---|---|---|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|---|----|---|---|---|---|---|---|---|---|---|---|---|---|
| <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>2</td><td>7</td><td>6</td><td>4</td></tr> <tr><td>1</td><td>1</td><td>3</td><td>3</td></tr> <tr><td>3</td><td>2</td><td>5</td><td>7</td></tr> <tr><td>6</td><td>2</td><td>6</td><td>2</td></tr> </table> | 2        | 7        | 6  | 4 | 1 | 1 | 3 | 3 | 3 | 2 | 5 | 7 | 6 | 2 | 6 | 2 | <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>2</td><td>6</td><td>3</td><td>2</td></tr> <tr><td>3</td><td>8</td><td>1</td><td>3</td></tr> <tr><td>5</td><td>2</td><td>2</td><td>4</td></tr> <tr><td>3</td><td>2</td><td>7</td><td>2</td></tr> </table>  | 2 | 6 | 3 | 2 | 3 | 8 | 1 | 3 | 5 | 2 | 2  | 4 | 3 | 2 | 7 | 2 | <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>3</td><td>4</td><td>4</td><td>4</td></tr> <tr><td>3</td><td>3</td><td>4</td><td>6</td></tr> <tr><td>5</td><td>3</td><td>2</td><td>5</td></tr> <tr><td>2</td><td>5</td><td>3</td><td>4</td></tr> </table>  | 3 | 4 | 4 | 4  | 3 | 3 | 4 | 6 | 5 | 3 | 2 | 5 | 2 | 5 | 3 | 4 |
| 2                                                                                                                                                                                                                                                                                   | 7        | 6        | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                                                                                                                                                                                                                                                                                   | 1        | 3        | 3  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                   | 2        | 5        | 7  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 6                                                                                                                                                                                                                                                                                   | 2        | 6        | 2  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 2                                                                                                                                                                                                                                                                                   | 6        | 3        | 2  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                   | 8        | 1        | 3  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 5                                                                                                                                                                                                                                                                                   | 2        | 2        | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                   | 2        | 7        | 2  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                   | 4        | 4        | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                   | 3        | 4        | 6  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 5                                                                                                                                                                                                                                                                                   | 3        | 2        | 5  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 2                                                                                                                                                                                                                                                                                   | 5        | 3        | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| — (10's)                                                                                                                                                                                                                                                                            | — (11's) | — (12's) |    |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>7</td><td>3</td><td>3</td><td>6</td></tr> <tr><td>3</td><td>3</td><td>2</td><td>4</td></tr> <tr><td>2</td><td>8</td><td>9</td><td>2</td></tr> <tr><td>3</td><td>3</td><td>3</td><td>4</td></tr> </table> | 7        | 3        | 3  | 6 | 3 | 3 | 2 | 4 | 2 | 8 | 9 | 2 | 3 | 3 | 3 | 4 | <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>9</td><td>3</td><td>4</td><td>6</td></tr> <tr><td>2</td><td>2</td><td>5</td><td>4</td></tr> <tr><td>3</td><td>5</td><td>2</td><td>6</td></tr> <tr><td>7</td><td>4</td><td>6</td><td>2</td></tr> </table>  | 9 | 3 | 4 | 6 | 2 | 2 | 5 | 4 | 3 | 5 | 2  | 6 | 7 | 4 | 6 | 2 | <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>7</td><td>2</td><td>3</td><td>9</td></tr> <tr><td>3</td><td>5</td><td>5</td><td>1</td></tr> <tr><td>5</td><td>5</td><td>5</td><td>5</td></tr> <tr><td>8</td><td>2</td><td>4</td><td>6</td></tr> </table>  | 7 | 2 | 3 | 9  | 3 | 5 | 5 | 1 | 5 | 5 | 5 | 5 | 8 | 2 | 4 | 6 |
| 7                                                                                                                                                                                                                                                                                   | 3        | 3        | 6  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                   | 3        | 2        | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 2                                                                                                                                                                                                                                                                                   | 8        | 9        | 2  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                   | 3        | 3        | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 9                                                                                                                                                                                                                                                                                   | 3        | 4        | 6  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 2                                                                                                                                                                                                                                                                                   | 2        | 5        | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                   | 5        | 2        | 6  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 7                                                                                                                                                                                                                                                                                   | 4        | 6        | 2  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 7                                                                                                                                                                                                                                                                                   | 2        | 3        | 9  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                   | 5        | 5        | 1  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 5                                                                                                                                                                                                                                                                                   | 5        | 5        | 5  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 8                                                                                                                                                                                                                                                                                   | 2        | 4        | 6  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| — (13's)                                                                                                                                                                                                                                                                            | — (14's) | — (15's) |    |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>2</td><td>6</td><td>6</td><td>7</td></tr> <tr><td>3</td><td>5</td><td>4</td><td>3</td></tr> <tr><td>6</td><td>6</td><td>6</td><td>6</td></tr> <tr><td>2</td><td>8</td><td>1</td><td>9</td></tr> </table> | 2        | 6        | 6  | 7 | 3 | 5 | 4 | 3 | 6 | 6 | 6 | 6 | 2 | 8 | 1 | 9 | <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>8</td><td>3</td><td>4</td><td>9</td></tr> <tr><td>3</td><td>3</td><td>4</td><td>4</td></tr> <tr><td>7</td><td>3</td><td>10</td><td>1</td></tr> <tr><td>4</td><td>7</td><td>8</td><td>8</td></tr> </table> | 8 | 3 | 4 | 9 | 3 | 3 | 4 | 4 | 7 | 3 | 10 | 1 | 4 | 7 | 8 | 8 | <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>7</td><td>3</td><td>5</td><td>13</td></tr> <tr><td>8</td><td>9</td><td>6</td><td>8</td></tr> <tr><td>5</td><td>9</td><td>4</td><td>3</td></tr> <tr><td>4</td><td>9</td><td>9</td><td>6</td></tr> </table> | 7 | 3 | 5 | 13 | 8 | 9 | 6 | 8 | 5 | 9 | 4 | 3 | 4 | 9 | 9 | 6 |
| 2                                                                                                                                                                                                                                                                                   | 6        | 6        | 7  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                   | 5        | 4        | 3  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 6                                                                                                                                                                                                                                                                                   | 6        | 6        | 6  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 2                                                                                                                                                                                                                                                                                   | 8        | 1        | 9  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 8                                                                                                                                                                                                                                                                                   | 3        | 4        | 9  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 3                                                                                                                                                                                                                                                                                   | 3        | 4        | 4  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 7                                                                                                                                                                                                                                                                                   | 3        | 10       | 1  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 4                                                                                                                                                                                                                                                                                   | 7        | 8        | 8  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 7                                                                                                                                                                                                                                                                                   | 3        | 5        | 13 |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 8                                                                                                                                                                                                                                                                                   | 9        | 6        | 8  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 5                                                                                                                                                                                                                                                                                   | 9        | 4        | 3  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| 4                                                                                                                                                                                                                                                                                   | 9        | 9        | 6  |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |
| — (16's)                                                                                                                                                                                                                                                                            | — (17's) | — (18's) |    |   |   |   |   |   |   |   |   |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |                                                                                                                                                                                                                                                                                      |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |

An extra challenge this time: there is one problem that can't be done — center problem in bottom row.

|   |   |   |   |
|---|---|---|---|
| 2 | 2 | 1 | 3 |
| 3 | 4 | 1 | 4 |
| 5 | 5 | 4 | 6 |
| 2 | 8 | 3 | 7 |

— (10's)

|   |   |   |   |
|---|---|---|---|
| 4 | 7 | 3 | 6 |
| 9 | 2 | 8 | 5 |
| 2 | 7 | 5 | 6 |
| 9 | 4 | 8 | 3 |

— (11's)

|   |   |   |   |
|---|---|---|---|
| 5 | 3 | 2 | 1 |
| 2 | 4 | 2 | 6 |
| 3 | 3 | 5 | 1 |
| 3 | 3 | 1 | 4 |

— (12's)

|   |   |    |   |
|---|---|----|---|
| 3 | 5 | 6  | 7 |
| 5 | 5 | 8  | 4 |
| 2 | 7 | 6  | 9 |
| 8 | 3 | 10 | 3 |

— (13's)

|   |   |   |   |
|---|---|---|---|
| 3 | 4 | 1 | 9 |
| 7 | 7 | 2 | 5 |
| 5 | 9 | 4 | 6 |
| 3 | 5 | 6 | 8 |

— (14's)

|   |   |   |   |
|---|---|---|---|
| 2 | 3 | 4 | 5 |
| 7 | 8 | 4 | 1 |
| 6 | 6 | 5 | 4 |
| 2 | 7 | 8 | 3 |

— (15's)

|   |   |   |    |
|---|---|---|----|
| 8 | 5 | 5 | 9  |
| 3 | 5 | 7 | 2  |
| 3 | 4 | 2 | 1  |
| 7 | 6 | 3 | 10 |

— (16's)

|   |   |    |    |
|---|---|----|----|
| 5 | 6 | 11 | 8  |
| 6 | 7 | 6  | 9  |
| 3 | 7 | 5  | 12 |
| 9 | 8 | 10 | 7  |

— (17's)

|    |   |   |    |
|----|---|---|----|
| 4  | 5 | 7 | 9  |
| 3  | 5 | 6 | 9  |
| 6  | 6 | 6 | 13 |
| 10 | 8 | 6 | 5  |

— (18's)

NAME \_\_\_\_\_

|          |   |    |    |          |   |    |    |          |   |   |    |
|----------|---|----|----|----------|---|----|----|----------|---|---|----|
| 2        | 2 | 1  | 3  | 4        | 7 | 3  | 6  | 5        | 3 | 2 | 1  |
| 3        | 4 | 1  | 4  | 9        | 2 | 8  | 5  | 2        | 4 | 2 | 6  |
| 5        | 5 | 4  | 6  | 2        | 7 | 5  | 6  | 3        | 3 | 5 | 1  |
| 2        | 8 | 3  | 7  | 9        | 4 | 8  | 3  | 3        | 3 | 1 | 4  |
| — (10's) |   |    |    | — (11's) |   |    |    | — (12's) |   |   |    |
|          |   |    |    |          |   |    |    |          |   |   |    |
| 3        | 5 | 6  | 7  | 3        | 4 | 1  | 9  | 2        | 3 | 4 | 5  |
| 5        | 5 | 8  | 4  | 7        | 7 | 2  | 5  | 7        | 8 | 4 | 1  |
| 2        | 7 | 6  | 9  | 5        | 9 | 4  | 6  | 6        | 6 | 5 | 4  |
| 8        | 3 | 10 | 3  | 3        | 5 | 6  | 8  | 2        | 7 | 8 | 3  |
| — (13's) |   |    |    | — (14's) |   |    |    | — (15's) |   |   |    |
|          |   |    |    |          |   |    |    |          |   |   |    |
| 8        | 5 | 5  | 9  | 5        | 6 | 11 | 8  | 4        | 5 | 7 | 9  |
| 3        | 5 | 7  | 2  | 6        | 7 | 6  | 9  | 3        | 5 | 6 | 9  |
| 3        | 4 | 2  | 1  | 3        | 7 | 5  | 12 | 6        | 6 | 6 | 13 |
| 7        | 6 | 3  | 10 | 9        | 8 | 10 | 7  | 10       | 8 | 6 | 5  |
| — (16's) |   |    |    | — (17's) |   |    |    | — (18's) |   |   |    |

More fun with fence arithmetic. Discussions regarding strategies would be beneficial. How can you tell when the greatest number of fences are made?

|    |   |   |
|----|---|---|
| 64 | 8 | 1 |
|    |   |   |

16

|   |   |   |
|---|---|---|
| 8 | 9 | 1 |
|   |   |   |

 16

|     |    |   |
|-----|----|---|
| 100 | 10 | 1 |
|     |    |   |

16

|    |   |   |
|----|---|---|
| 64 | 8 | 1 |
|    |   |   |

 25

|    |   |   |
|----|---|---|
| 81 | 9 | 1 |
|    |   |   |

25

| 100 | 10 | 1 |
|-----|----|---|
|     |    |   |

25

|    |   |   |
|----|---|---|
| 64 | 8 | 1 |
|    |   |   |

36

|    |   |   |
|----|---|---|
| 81 | 9 | 1 |
|    |   |   |

 36

|     |    |   |
|-----|----|---|
| 100 | 10 | 1 |
|     |    |   |

36

|    |   |   |
|----|---|---|
| 64 | 8 | 1 |
|    |   |   |

49

|   |   |   |
|---|---|---|
| 8 | 9 | 1 |
|   |   |   |

 49

| 100 | 10 | 1 |
|-----|----|---|
|     |    |   |

49

|    |   |   |
|----|---|---|
| 64 | 8 | 1 |
|    |   |   |

64

|    |   |   |
|----|---|---|
| 81 | 9 | 1 |
|    |   |   |

64

| 100 | 10 | 1 |
|-----|----|---|
|     |    |   |

64

|    |   |   |    |
|----|---|---|----|
| 64 | 8 | 1 |    |
|    |   |   | 81 |

|   |   |   |
|---|---|---|
| 8 | 9 | 1 |
|   |   |   |

81

|     |    |   |
|-----|----|---|
| 100 | 10 | 1 |
|     |    |   |

81

|    |   |   |
|----|---|---|
| 64 | 8 | 1 |
|    |   |   |

100

|    |   |   |
|----|---|---|
| 81 | 9 | 1 |
|    |   |   |

 100

|     |    |   |     |
|-----|----|---|-----|
| 100 | 10 | 1 |     |
|     |    |   | 100 |

|    |   |   |
|----|---|---|
| 64 | 8 | 1 |
|    |   |   |

121

|   |   |   |   |
|---|---|---|---|
| 8 | 1 | 9 | 1 |
|   |   |   |   |

 121

| 100 | 10 | 1 |
|-----|----|---|
|     |    |   |

121

|    |   |   |
|----|---|---|
| 64 | 8 | 1 |
|    |   |   |

144

|    |   |   |
|----|---|---|
| 81 | 9 | 1 |
|    |   |   |

 144

|     |    |   |
|-----|----|---|
| 100 | 10 | 1 |
|     |    |   |

144



"CHECK-LIST" ADDITION

NAME \_\_\_\_\_

|                     |                     |                       |
|---------------------|---------------------|-----------------------|
| 64 8 1<br>      16  | 81 9 1<br>      16  | 100 10 1<br>      16  |
| 64 8 1<br>      25  | 81 9 1<br>      25  | 100 10 1<br>      25  |
| 64 8 1<br>      36  | 81 9 1<br>      36  | 100 10 1<br>      36  |
| 64 8 1<br>      49  | 81 9 1<br>      49  | 100 10 1<br>      49  |
| 64 8 1<br>      64  | 81 9 1<br>      64  | 100 10 1<br>      64  |
| 64 8 1<br>      81  | 81 9 1<br>      81  | 100 10 1<br>      81  |
| 64 8 1<br>      100 | 81 9 1<br>      100 | 100 10 1<br>      100 |
| 64 8 1<br>      121 | 81 9 1<br>      121 | 100 10 1<br>      121 |
| 64 8 1<br>      144 | 81 9 1<br>      144 | 100 10 1<br>      144 |

An extension of the same ideas presented on previous pages. Records are established in the same way. Checks have been replaced by numbers.

Try to have children tell you the next number in each record if you made another column.

Some children will be startled by the results of using the 1, 10, 100 record. Since it comes up unexpectedly it may give further insight into our numeration system.

The strategy that has developed is worthy of discussion—"cutting the problem down to size"—because it is precisely this strategy we use in the standard "division algorithms."

One way to focus on this method is to ask children to complete the page the first time through by writing in only the first number in each solution.

|                       |                       |                         |
|-----------------------|-----------------------|-------------------------|
| 64 8 1<br>0       16  | 81 9 1<br>0       16  | 100 10 1<br>0       16  |
| 64 8 1<br>1       100 | 81 9 1<br>1       100 | 100 10 1<br>1       100 |

|                     |                     |                       |
|---------------------|---------------------|-----------------------|
| 64 8 1<br>      144 | 81 9 1<br>      144 | 100 10 1<br>      144 |
|---------------------|---------------------|-----------------------|

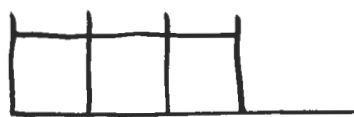
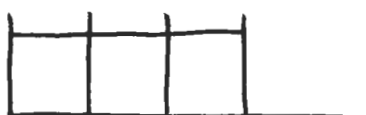
What is the next step? . . . find how many are left to be accounted for. In examples that do not have "0" in the lefthand box, this number can be found and noted in each example. Here is one way:

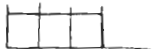
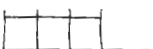
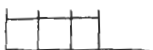
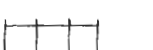
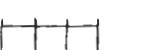
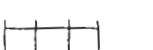
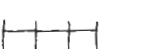
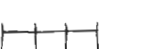
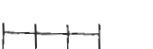
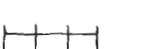
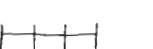
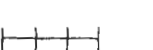
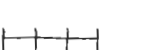
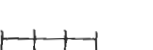
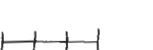
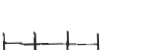
$$\begin{array}{r}
 64 \ 8 \ 1 \\
 | \quad | \quad | \quad 100 \\
 -64 \\
 \hline
 36
 \end{array}$$

After this is completed for each example, write the appropriate number in the center box of all the examples.

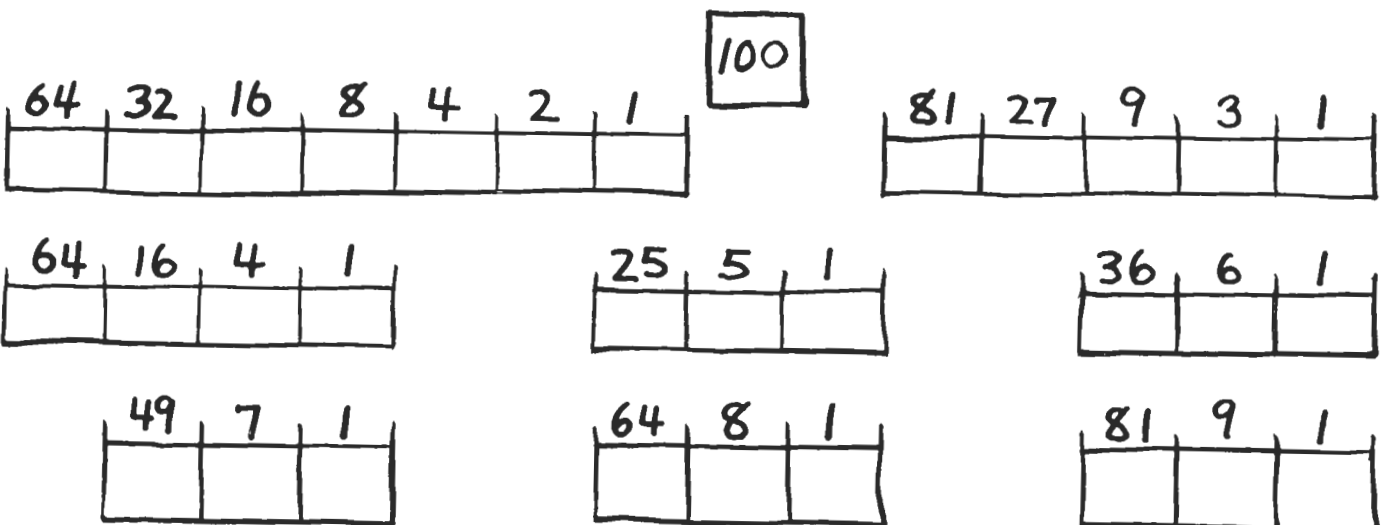
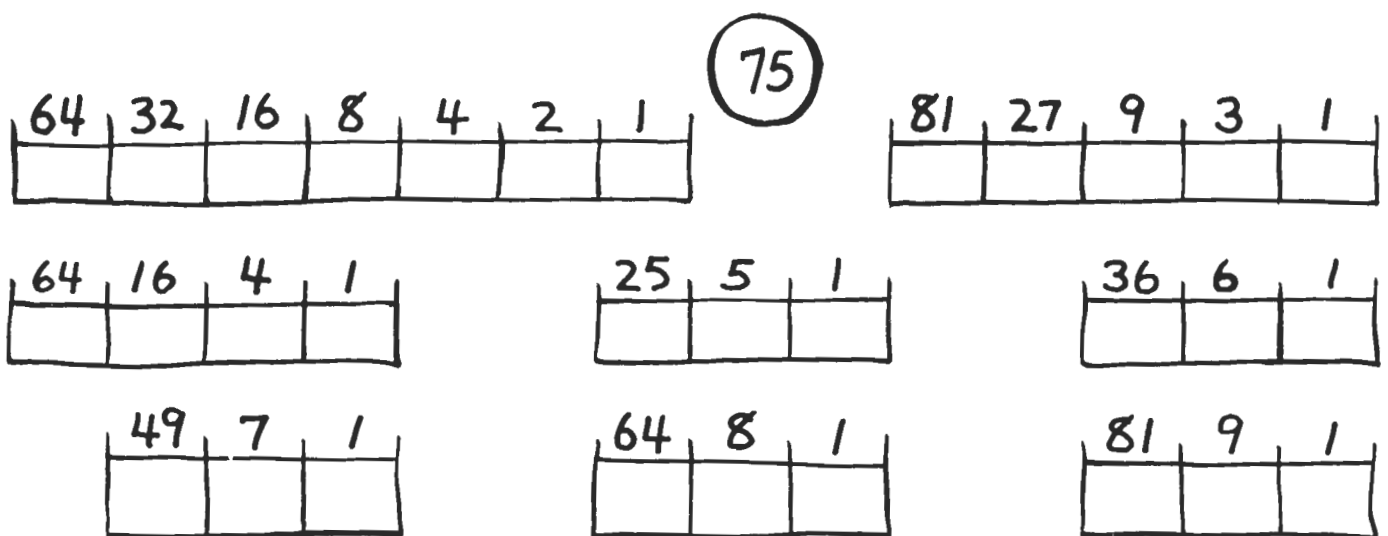
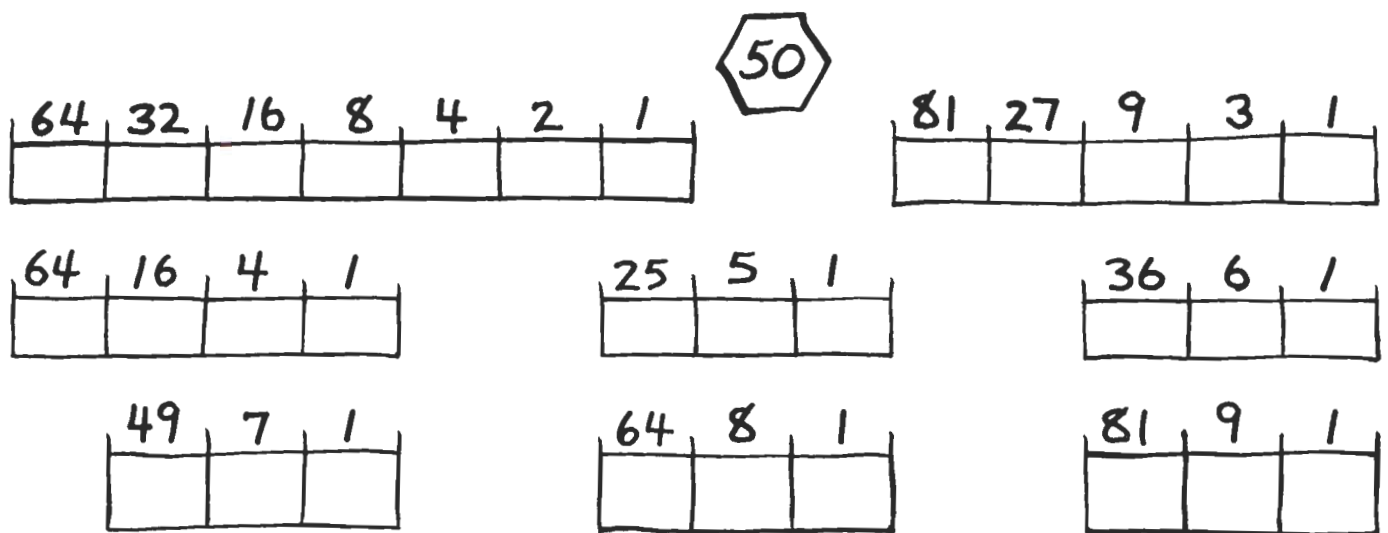
Finally, go through the examples completing the solution.

\_\_\_\_\_



|                                                                                   |                                                                                   |                                                                                   |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

This page can be used for an extension of the activity on the previous page.



**"CHECK-LIST" ADDITION** NAME \_\_\_\_\_

PLEASE SHOW EACH NUMBER IN EIGHT DIFFERENT WAYS.

50

|                  |             |        |
|------------------|-------------|--------|
| 64 32 16 8 4 2 1 | 81 27 9 3 1 |        |
| 64 16 4 1        | 25 5 1      | 36 6 1 |
| 49 7 1           | 64 8 1      | 81 9 1 |

---

75

|                  |             |        |
|------------------|-------------|--------|
| 64 32 16 8 4 2 1 | 81 27 9 3 1 |        |
| 64 16 4 1        | 25 5 1      | 36 6 1 |
| 49 7 1           | 64 8 1      | 81 9 1 |

---

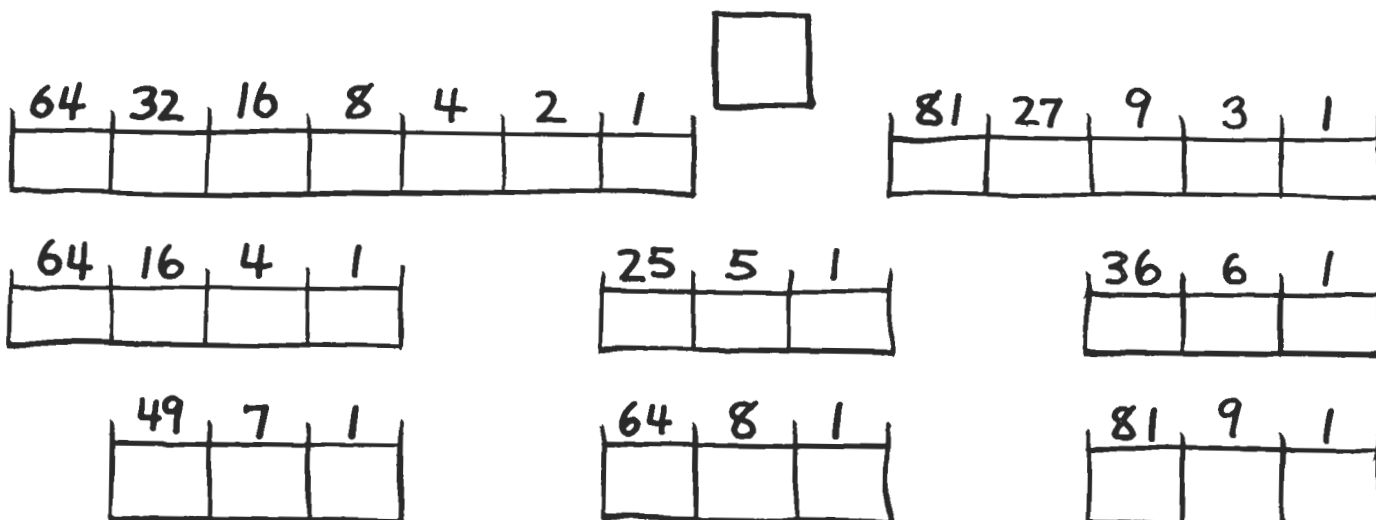
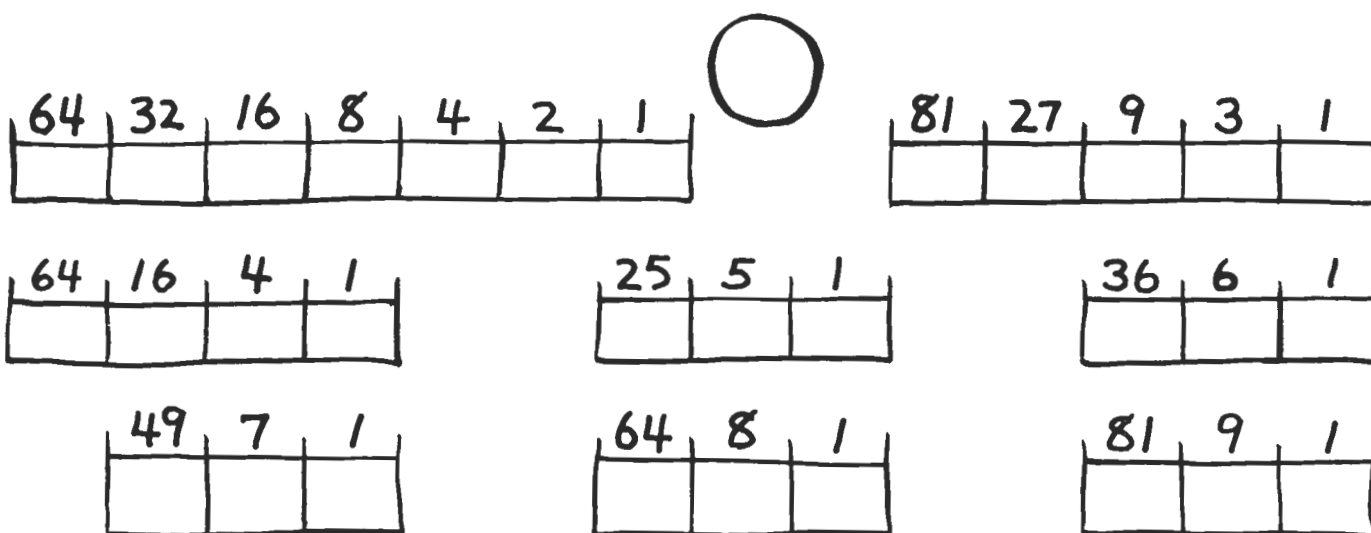
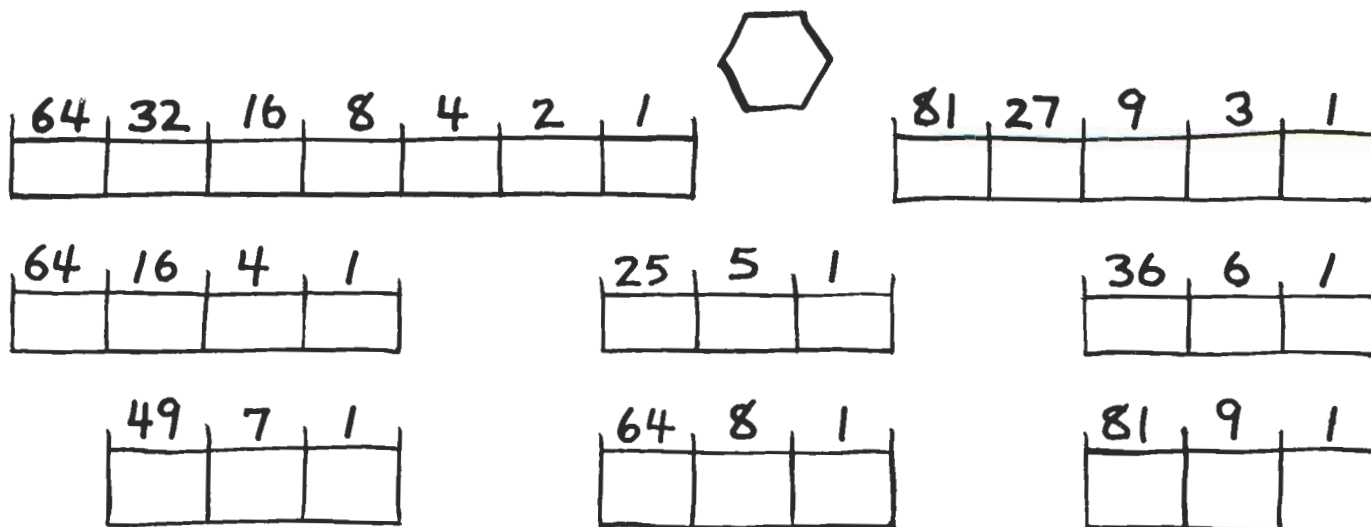
100

|                  |             |        |
|------------------|-------------|--------|
| 64 32 16 8 4 2 1 | 81 27 9 3 1 |        |
| 64 16 4 1        | 25 5 1      | 36 6 1 |
| 49 7 1           | 64 8 1      | 81 9 1 |

This page provides cumulative practice from the previous pages.

Again, the strategy of cutting the problem down in size can be emphasized by filling in all first numbers in each example. The second time through, find out the numbers left to be accounted for, and fill in the next box; then the third box in each example, etc., until the page is completed.

Children are making continual use of all the multiplication tables—but also anticipating the “long division algorithm.”



**"CHECK-LIST" ADDITION**  
PLEASE SHOW EACH NUMBER IN EIGHT DIFFERENT WAYS.

NAME \_\_\_\_\_







|    |    |    |   |   |   |   |  |    |    |   |   |   |  |
|----|----|----|---|---|---|---|--|----|----|---|---|---|--|
| 64 | 32 | 16 | 8 | 4 | 2 | 1 |  | 81 | 27 | 9 | 3 | 1 |  |
| 64 | 16 | 4  | 1 |   |   |   |  | 25 | 5  | 1 |   |   |  |
| 49 | 7  | 1  |   |   |   |   |  | 64 | 8  | 1 |   |   |  |
|    |    |    |   |   |   |   |  |    |    |   |   |   |  |

|    |    |    |   |   |   |   |  |    |    |   |   |   |  |
|----|----|----|---|---|---|---|--|----|----|---|---|---|--|
| 64 | 32 | 16 | 8 | 4 | 2 | 1 |  | 81 | 27 | 9 | 3 | 1 |  |
| 64 | 16 | 4  | 1 |   |   |   |  | 25 | 5  | 1 |   |   |  |
| 49 | 7  | 1  |   |   |   |   |  | 64 | 8  | 1 |   |   |  |
|    |    |    |   |   |   |   |  |    |    |   |   |   |  |

|    |    |    |   |   |   |   |  |    |    |   |   |   |  |
|----|----|----|---|---|---|---|--|----|----|---|---|---|--|
| 64 | 32 | 16 | 8 | 4 | 2 | 1 |  | 81 | 27 | 9 | 3 | 1 |  |
| 64 | 16 | 4  | 1 |   |   |   |  | 25 | 5  | 1 |   |   |  |
| 49 | 7  | 1  |   |   |   |   |  | 64 | 8  | 1 |   |   |  |
|    |    |    |   |   |   |   |  |    |    |   |   |   |  |

Checklist:

|    |    |   |   |     |
|----|----|---|---|-----|
| 64 | 16 | 4 | 1 |     |
| 1  | 3  | 2 | 3 | 123 |

Never more than 3 in any box (common multiplier: 4).

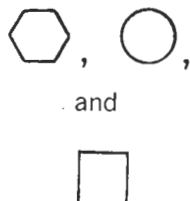
Checklist:

|    |    |   |   |     |
|----|----|---|---|-----|
| 25 | 25 | 5 | 1 |     |
|    | 4  | 4 | 4 | 124 |

Never more than 4 in any box (common multiplier: 5).

In the last example the pattern would be broken in finding a sum for 125. Before breaking the pattern, we would extend the list to 1, 5, 25 and 125.

Insert numbers in the



and have children complete, or let them pick their own numbers.

If numbers selected are all less than 125, a certain pattern can be noticed: the largest number of check marks indicated in any box will always be 1 less than the common multiplier used to generate the numbers in the "check-lists".

Check-list:

|    |    |    |   |   |   |   |     |
|----|----|----|---|---|---|---|-----|
| 64 | 32 | 16 | 8 | 4 | 2 | 1 |     |
| 1  | 1  | 1  | 1 | 0 | 1 | 1 | 123 |

Never more than 1 in any box (common multiplier: 2).

Checklist:

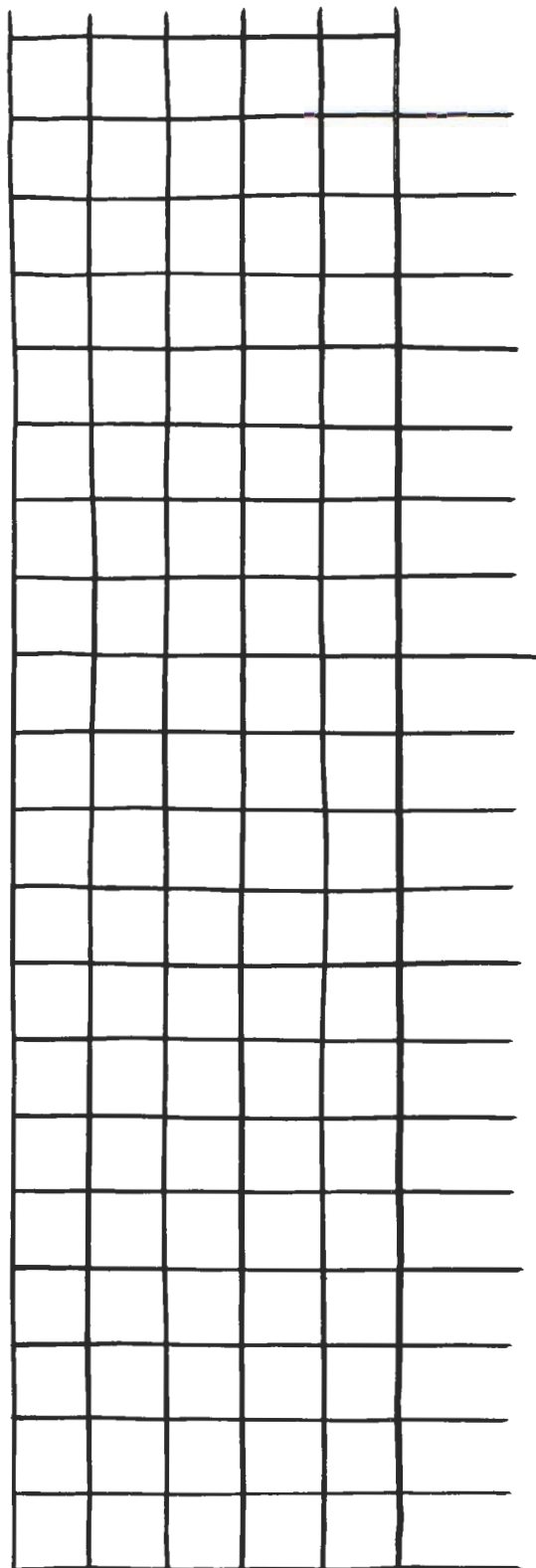
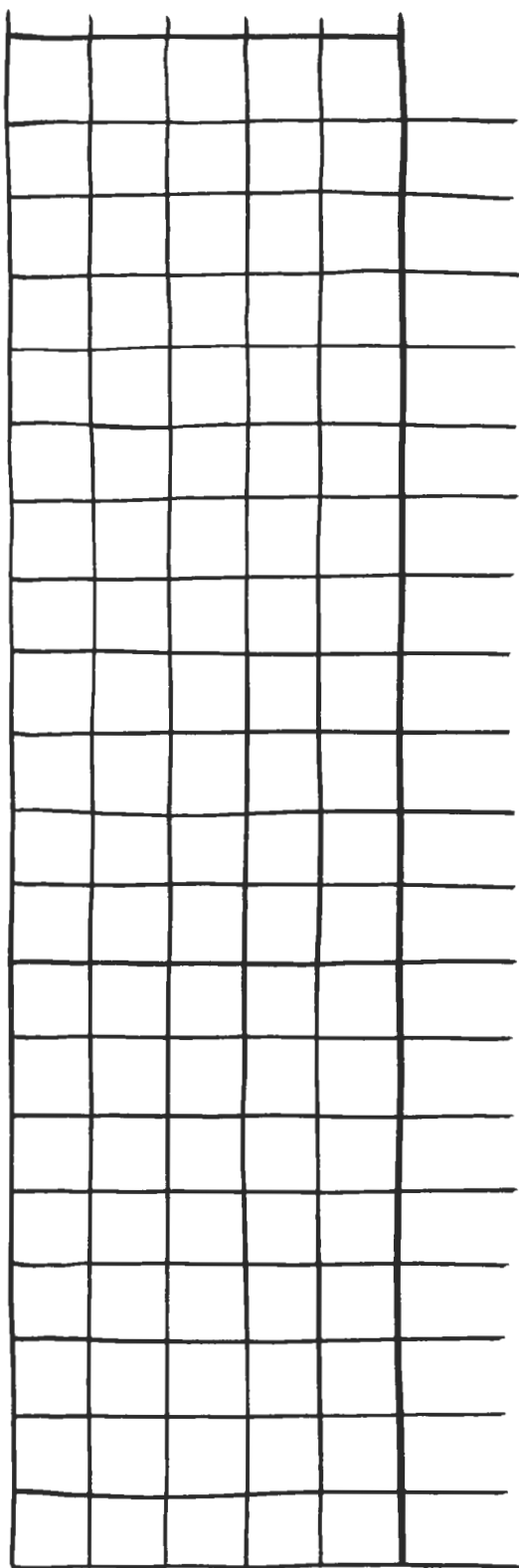
|    |    |   |   |   |  |     |
|----|----|---|---|---|--|-----|
| 81 | 27 | 9 | 3 | 1 |  |     |
| 1  | 1  | 1 | 2 | 0 |  | 123 |

Never more than 2 in any box (common multiplier: 3).











**RULES: USE ALL DIGITS GIVEN ONCE AND ONLY ONCE IN EACH SUM; AND NO NUMBERS LARGER THAN 100.**

**REGLAS: USA TODAS LAS CIFRAS DADAS UNA VEZ Y SÓLO UNA VEZ EN CADA SUMA; Y NO USES NÚMEROS MAYORES QUE 100.**



$$0 + 1 + 2 = 3$$

$$12 + 0 = 12$$

$$=$$

$$=$$

$$=$$

$$=$$

$$=$$

$$=$$

**PLEASE LIST NUMBERS FROM SMALLEST TO LARGEST.**

**FAVOR DE DAR LAS SUMAS, DE LA MENOR A LA MAYOR.**

3 12 \_\_\_\_\_

**FAVOR DE ANOTAR LAS DIFERENCIAS:**  
**PLEASE NOTE DIFFERENCES:**



$$=$$

$$=$$

$$=$$

$$=$$

$$=$$

$$=$$

$$=$$

$$=$$

**SUMAS:**

**SUMS:** \_\_\_\_\_

**DIFERENCIAS:**

**DIFFERENCES:** \_\_\_\_\_

**SHIFTING DIGITS** NAME \_\_\_\_\_

RULES: USE ALL DIGITS GIVEN ONCE AND ONLY ONCE IN EACH SUM; AND NO SUMS LARGER THAN 100.

**0 1 2**

$0 + 1 + 2 = 3$  \_\_\_\_\_ = \_\_\_\_\_

$12 + 0 = 12$  \_\_\_\_\_ = \_\_\_\_\_

\_\_\_\_\_ = \_\_\_\_\_

\_\_\_\_\_ = \_\_\_\_\_

PLEASE LIST SUMS FROM SMALLEST TO LARGEST:

3 12 \_\_\_\_\_

PLEASE NOTE DIFFERENCES: 9 \_\_\_\_\_

---

**1 2 3**

\_\_\_\_\_ = \_\_\_\_\_

\_\_\_\_\_ = \_\_\_\_\_

\_\_\_\_\_ = \_\_\_\_\_

\_\_\_\_\_ = \_\_\_\_\_

SUMS: \_\_\_\_\_

DIFFERENCES: \_\_\_\_\_

Sums listed in order:  $\rightarrow$  6 15 24 33

Difference between neighbors:  $\rightarrow$  9 9 9

That is a surprising outcome. In both investigations there was a common difference of 9.

Why does this happen?

Will it happen regardless of the digits selected?

Some may want to investigate these questions on their own. Others may want to try the example on the next page.

Suppose we are limited to the digits **0, 1, and 2**. The directions are to use these digits once and only once in each sum (less than 100). Here are some possibilities:

$$\begin{array}{ll} 0 + 1 + 2 = 3 & 10 + 2 = 12 \\ 12 + 0 = 12 & 0 + 21 = 21 \\ 20 + 1 = 21 & \end{array}$$

Different sums listed in order are:  $\rightarrow$  3 12 21

Difference between neighbors:  $\rightarrow$  9 9

$$12 - 3 = 9 \quad \text{and} \quad 21 - 12 = 9$$

Repeating the activity limited to digits **1, 2, and 3**, we find:

$$\begin{array}{ll} 1 + 2 + 3 = 6 & 23 + 1 = 24 \\ 12 + 3 = 15 & 13 + 2 = 15 \\ 21 + 3 = 24 & 32 + 1 = 33 \\ 31 + 2 = 33 & \end{array}$$

There are four different sums.

0 1 2 4

\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_

\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_

SUMAS:

SUMS:

DIFERENCIAS:

DIFFERENCES:

0 2 3 5 6

\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_

\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_  
\_\_\_\_ = \_\_\_\_

SUMAS:

SUMS:

DIFERENCIAS:

DIFFERENCES:

**MORE SHIFTING DIGITS** NAME \_\_\_\_\_

0 1 2 4

\_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_\_

\_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_\_

\_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_\_

\_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_\_

SUMS: \_\_\_\_\_

DIFFERENCES: \_\_\_\_\_

---

0 2 3 5 6

\_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_\_

\_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_\_

\_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_\_

\_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_\_

\_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_\_

SUMS: \_\_\_\_\_

DIFFERENCES: \_\_\_\_\_

Continuing the activity on the previous pages, we consider using the digits **0, 1, 2, 4**. Sums can be produced under the rules such as:

$$0 + 1 + 2 + 4 = 7$$

$$10 + 2 + 4 = 16$$

$$40 + 1 + 2 = 43$$

$$24 + 10 = 34$$

$$24 + 0 + 1 = 25$$

$$42 + 10 = 52$$

$$14 + 20 = 34$$

$$12 + 0 + 4 = 16$$

$$40 + 21 = 61$$

$$20 + 1 + 4 = 25$$

etc.

sums: 7 16 25 34 43 52 61

difference: 9 9 9 9 9 9

Using digits **0, 2, 3, 5 and 6** once and only once, the two smallest sums to be made are:

$$0 + 2 + 3 + 5 + 6 = 16$$

$$20 + 3 + 5 + 6 = 34$$

The difference is not the expected **9**: rather, the difference is 18.

A complete list of sums and differences is:

16 34 43 61 70 79

18 9 18 9 9

Will the difference always be 9 or 18 regardless of the digits chosen?

Before trying larger series of digits, it might be more helpful to see what happens with only 3 digits, one of them being 0.

0, 1 and 2

$$0 + 1 + 2 = 3$$

$$10 + 2 = 12$$

$$20 + 1 = 21$$

3 12 21

9 9

0, 1 and 3

$$0 + 1 + 3 = 4$$

$$0 + 13 = 13$$

$$0 + 31 = 31$$

4 13 31

9 18

0, 1 and 4

$$0 + 1 + 4 = 5$$

$$10 + 4 = 14$$

$$40 + 1 = 41$$

5 14 41

9 27

0, 3 and 6

$$0 + 3 + 6 = 9$$

$$30 + 6 = 36$$

$$60 + 3 = 63$$

9 36 63

27 27

DIGITS USED  
CIFRAS USADAS

|       |   |       |   |       |
|-------|---|-------|---|-------|
| _____ | " | _____ | " | _____ |
| _____ | " | _____ | " | _____ |
| _____ | " | _____ | " | _____ |
| _____ | " | _____ | " | _____ |
| _____ | " | _____ | " | _____ |
| _____ | " | _____ | " | _____ |
| _____ | " | _____ | " | _____ |

SUMAS:

SUMS:

DIFERENCIAS:

DIFFERENCES:

DIGITS USED  
CIFRAS USADAS

|       |   |       |   |       |
|-------|---|-------|---|-------|
| _____ | " | _____ | " | _____ |
| _____ | " | _____ | " | _____ |
| _____ | " | _____ | " | _____ |
| _____ | " | _____ | " | _____ |
| _____ | " | _____ | " | _____ |
| _____ | " | _____ | " | _____ |
| _____ | " | _____ | " | _____ |



**MORE SHIFTING DIGITS**      NAME \_\_\_\_\_

DIGITS USED \_\_\_\_\_

|               |               |
|---------------|---------------|
| _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ |

SUMS: \_\_\_\_\_

DIFFERENCES: \_\_\_\_\_

DIGITS USED \_\_\_\_\_

|               |               |
|---------------|---------------|
| _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ |

\_\_\_\_\_

\_\_\_\_\_

This page can be used as a report form for many different investigations.

Can you predict what will happen if we select all even numbers less than 10 as digits?

**0, 2, 4, 6, 8**

$$\begin{aligned}
 0 + 2 + 4 + 6 + 8 &= 20 \\
 20 + 4 + 6 + 8 &= 38 \\
 48 + 2 + 6 + 0 &= 56 \\
 46 + 20 + 8 &= 74 \\
 80 + 2 + 4 + 6 &= 92
 \end{aligned}$$

$$\begin{array}{r}
 20 \quad 38 \quad 56 \quad 74 \quad 92 \\
 \underline{18} \quad \underline{18} \quad \underline{18} \quad \underline{18}
 \end{array}$$

Can you predict what will happen if we select all odd numbers less than 10 as digits?

**1, 3, 5, 7, 9**

$$\begin{aligned}
 1 + 3 + 5 + 7 + 9 &= 25 \\
 13 + 5 + 7 + 9 &= 34 \\
 31 + 5 + 7 + 9 &= 52 \\
 57 + 1 + 3 + 9 &= 70 \\
 75 + 1 + 3 + 9 &= 88
 \end{aligned}$$

$$\begin{array}{r}
 25 \quad 34 \quad 52 \quad 70 \quad 88 \\
 \underline{9} \quad \underline{18} \quad \underline{18} \quad \underline{18}
 \end{array}$$

An illuminating side trip is to consider the differences as a digit is moved from the units column to the tens column.

$$\begin{array}{r}
 10 \\
 -1 \\
 \hline
 9
 \end{array}
 \quad
 \begin{array}{r}
 20 \\
 -2 \\
 \hline
 18
 \end{array}
 \quad
 \begin{array}{r}
 30 \\
 -3 \\
 \hline
 27
 \end{array}
 \quad
 \begin{array}{r}
 70 \\
 -7 \\
 \hline
 63
 \end{array}
 \quad
 \begin{array}{r}
 90 \\
 -9 \\
 \hline
 81
 \end{array}$$

And the difference between "reversals"

$$\begin{array}{r}
 21 \\
 -12 \\
 \hline
 9
 \end{array}
 \quad
 \begin{array}{r}
 54 \\
 -45 \\
 \hline
 9
 \end{array}
 \quad
 \begin{array}{r}
 64 \\
 -46 \\
 \hline
 18
 \end{array}
 \quad
 \begin{array}{r}
 52 \\
 -25 \\
 \hline
 27
 \end{array}
 \quad
 \begin{array}{r}
 81 \\
 -18 \\
 \hline
 63
 \end{array}$$

A very interesting problem is this: using all 10 digits—0 thru 9—once and only once, can you fashion a sum of exactly 100?

For example, here is a near miss:

$$10 + 24 + 35 + 6 + 7 + 8 + 9 = 99$$

Here are two wider misses:

$$59 + 0 + 1 + 2 + 3 + 4 + 6 + 7 + 8 = 90$$

and

$$10 + 23 + 45 + 6 + 7 + 8 + 9 = 108$$

Do these "misses help"?

(They ought to: their differences are the familiar "9" or "multiple of 9." When a digit is shifted from the units to the tens column, the effect is adding a "multiple of 9." Because the sum 99 can be made by shifting digits, how can the sum be increased by only 1—from 99 to 100? No way!)

# OPERACIONES REVUELTAS SCRAMBLED SENTENCES

|   |   |   |    |
|---|---|---|----|
| 6 | 9 | 6 | 15 |
| 5 | 5 | 2 | 2  |
| 1 | 3 | 4 | 8  |
| 4 | 2 | 2 | 3  |

$$6 + 5 + 5 - 9 = 7$$

$$15 - 6 - 2 = 7$$

$$1 + 2 + 4 = 7$$

$$\underline{\hspace{2cm}} = 7$$

$$\underline{\hspace{2cm}} = 7$$

|   |    |    |   |
|---|----|----|---|
| 5 | 2  | 7  | 2 |
| 2 | 3  | 13 | 4 |
| 7 | 4  | 1  | 9 |
| 4 | 16 | 3  | 4 |

$$\underline{\hspace{2cm}} = 8$$

$$\underline{\hspace{2cm}} = 8$$

$$\underline{\hspace{2cm}} = 8$$

$$\underline{\hspace{2cm}} = 8$$

$$\underline{\hspace{2cm}} = 8$$

|   |   |    |    |
|---|---|----|----|
| 7 | 3 | 10 | 5  |
| 5 | 7 | 9  | 1  |
| 1 | 7 | 8  | 7  |
| 2 | 3 | 2  | 15 |

$$\underline{\hspace{2cm}} = 6$$

$$\underline{\hspace{2cm}} = 6$$

$$\underline{\hspace{2cm}} = 6$$

$$\underline{\hspace{2cm}} = 6$$

$$\underline{\hspace{2cm}} = 6$$

|    |   |   |   |
|----|---|---|---|
| 2  | 3 | 7 | 4 |
| 10 | 4 | 7 | 2 |
| 4  | 8 | 6 | 1 |
| 7  | 6 | 6 | 2 |

$$\underline{\hspace{2cm}} = 5$$

$$\underline{\hspace{2cm}} = 5$$

$$\underline{\hspace{2cm}} = 5$$

$$\underline{\hspace{2cm}} = 5$$

$$\underline{\hspace{2cm}} = 5$$

|    |   |   |   |
|----|---|---|---|
| 9  | 6 | 7 | 3 |
| 5  | 2 | 3 | 5 |
| 10 | 8 | 6 | 7 |
| 3  | 7 | 6 | 9 |

$$\underline{\hspace{2cm}} = 4$$

$$\underline{\hspace{2cm}} = 4$$

$$\underline{\hspace{2cm}} = 4$$

$$\underline{\hspace{2cm}} = 4$$

$$\underline{\hspace{2cm}} = 4$$

|    |   |   |    |
|----|---|---|----|
| 2  | 5 | 6 | 5  |
| 10 | 9 | 4 | 6  |
| 6  | 7 | 5 | 13 |
| 5  | 4 | 9 | 7  |

$$\underline{\hspace{2cm}} = 3$$

$$\underline{\hspace{2cm}} = 3$$

$$\underline{\hspace{2cm}} = 3$$

$$\underline{\hspace{2cm}} = 3$$

$$\underline{\hspace{2cm}} = 3$$

SCRAMBLED SENTENCES NAME \_\_\_\_\_  
(FENCED-IN NUMBERS WITH "+" AND/OR "-" = "S")

|   |   |   |    |
|---|---|---|----|
| 6 | 9 | 6 | 15 |
| 5 | 5 | 2 | 2  |
| 1 | 3 | 4 | 8  |
| 4 | 2 | 2 | 3  |

$6+5+5-9=7$   
 $15-6-2=7$   
 $1+2+4=7$   
 \_\_\_\_\_ = 7  
 \_\_\_\_\_ = 7

|   |    |    |   |
|---|----|----|---|
| 5 | 2  | 7  | 2 |
| 2 | 3  | 13 | 4 |
| 7 | 4  | 1  | 9 |
| 4 | 16 | 3  | 4 |

\_\_\_\_\_ = 8  
 \_\_\_\_\_ = 8  
 \_\_\_\_\_ = 8  
 \_\_\_\_\_ = 8  
 \_\_\_\_\_ = 8

|   |   |    |    |
|---|---|----|----|
| 7 | 3 | 10 | 5  |
| 5 | 7 | 9  | 1  |
| 1 | 7 | 8  | 7  |
| 2 | 3 | 2  | 15 |

\_\_\_\_\_ = 6  
 \_\_\_\_\_ = 6  
 \_\_\_\_\_ = 6  
 \_\_\_\_\_ = 6  
 \_\_\_\_\_ = 6

|    |   |   |   |
|----|---|---|---|
| 2  | 3 | 7 | 4 |
| 10 | 4 | 7 | 2 |
| 4  | 8 | 6 | 1 |
| 7  | 6 | 6 | 2 |

\_\_\_\_\_ = 5  
 \_\_\_\_\_ = 5  
 \_\_\_\_\_ = 5  
 \_\_\_\_\_ = 5  
 \_\_\_\_\_ = 5

|    |   |   |   |
|----|---|---|---|
| 9  | 6 | 7 | 3 |
| 5  | 2 | 3 | 5 |
| 10 | 8 | 6 | 7 |
| 3  | 7 | 6 | 9 |

\_\_\_\_\_ = 4  
 \_\_\_\_\_ = 4  
 \_\_\_\_\_ = 4  
 \_\_\_\_\_ = 4  
 \_\_\_\_\_ = 4

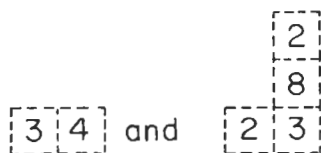
|    |   |   |    |
|----|---|---|----|
| 2  | 5 | 6 | 5  |
| 10 | 9 | 4 | 6  |
| 6  | 7 | 5 | 13 |
| 5  | 4 | 9 | 7  |

\_\_\_\_\_ = 3  
 \_\_\_\_\_ = 3  
 \_\_\_\_\_ = 3  
 \_\_\_\_\_ = 3  
 \_\_\_\_\_ = 3

In the mathematical sentences, we suggest an agreement to avoid necessity of parentheses: the temporary rule that operations are to be carried out from "left to right."

"Scrambled Sentences" is a recreational form of "Fence Arithmetic."

When the "fences" are drawn in, then the problem is to arrange the numbers so fenced in separated by "+" and/or "-" signs to indicate a given result. For example, in the first problem answers are provided for numbers inside 3 of the fences. The situations not completed are:



each set of numbers must be arranged to indicate 7. The first is obvious:

$$\underline{3+4=7}$$

After a little trial and error, the other problem can also be solved

$$\underline{8+3-2-2=7}$$

Or, an alert solver might argue that  $8+3+2+2=15$  or 8 too much. That can be removed by changing 4 (2 and 2) from "+" to "-." And he has the answer. Also, he can see that there is no other answer—except for changes in order.

The second example quickly yields both to trial and error and to the argument in the preceding paragraph. That argument is, for example:

$$\underline{13+9+4=26}$$

That is 18 too much—18 more than 8. To reduce by 18, half that much, or 9, must be changed from "+" to "-." So

$$\underline{13-9+4=8}$$

In the rest of the examples, only some of the fences are shown. Now trial and error is the order of the day and scratch paper almost a necessity. Fencings that lead to solutions are:

|   |   |    |    |
|---|---|----|----|
| 7 | 3 | 10 | 5  |
| 5 | 7 | 9  | 1  |
| 1 | 7 | 8  | 7  |
| 2 | 3 | 2  | 15 |

$$\underline{7+7-5-3=6}$$

$$\underline{1+2+3=6}$$

$$\underline{10+5+1=6}$$

etc.

|    |   |   |   |
|----|---|---|---|
| 2  | 3 | 7 | 4 |
| 10 | 4 | 7 | 2 |
| 4  | 8 | 6 | 1 |
| 7  | 6 | 6 | 2 |

$$\underline{10-2-3=5}$$

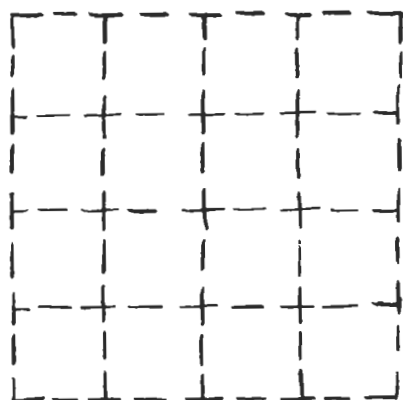
$$\underline{7-4+2=5}$$

$$\underline{6-2+1=5}$$

etc.

# SCRAMBLED SENTENCES

## OPERACIONES REVUELTAS



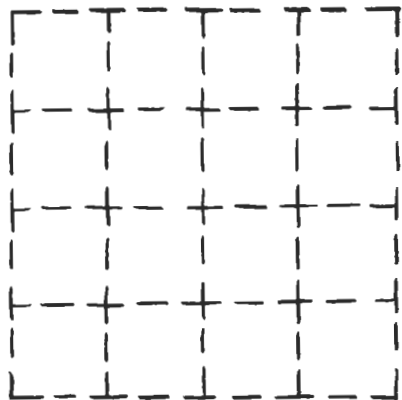
\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "



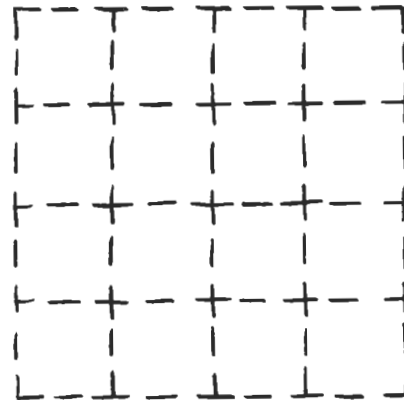
\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "



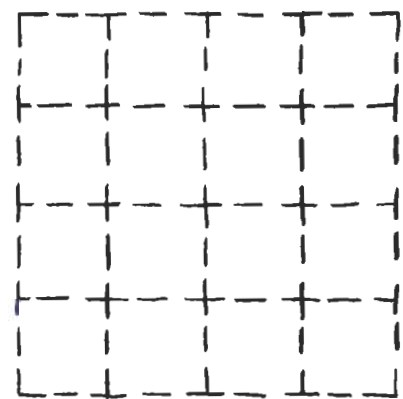
\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "



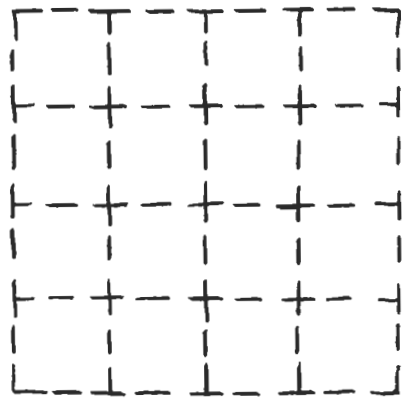
\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "



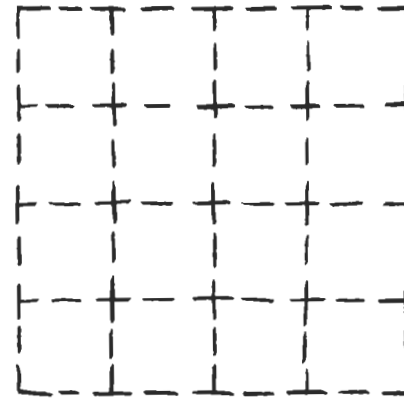
\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "



\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "

\_\_\_\_\_ "

SCRAMBLED SENTENCES      NAME \_\_\_\_\_

|               |               |               |
|---------------|---------------|---------------|
|               |               |               |
| _____ = _____ | _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ | _____ = _____ |

|               |               |               |
|---------------|---------------|---------------|
|               |               |               |
| _____ = _____ | _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ | _____ = _____ |
| _____ = _____ | _____ = _____ | _____ = _____ |

Children or teachers can “make up” these puzzlers with a very simple procedure.

Step. 1. Draw in the fences

|   |   |   |   |
|---|---|---|---|
| E |   |   | A |
|   |   | B |   |
|   | D |   |   |
|   |   |   | C |

Step 2. Select any number for the common result  
... 9 for example.

Step 3. Make up a sentence to fit each of the fenced areas using as many numbers as the fenced area indicates such as:

- A:  $13 - 4 = 9$       (2 numbers fenced in)  
 B:  $15 - 8 + 2 = 9$       (3 numbers fenced in)  
 C:  $2 + 3 + 4 = 9$       (3 numbers fenced in)  
 D:  $14 - 6 + 2 - 1 = 9$  (4 numbers fenced in)  
 E:  $1 + 2 + 3 + 3 = 9$  (4 numbers fenced in)

Step 4. Write the numbers inside the fences:

|   |    |    |    |
|---|----|----|----|
| 3 | 2  | 3  | 4  |
| 1 | 2  | 15 | 13 |
| 1 | 14 | 8  | 3  |
| 6 | 2  | 4  | 2  |

Step 5. Remove the fences and show the number of expression equal to the selected number—9 in this case.

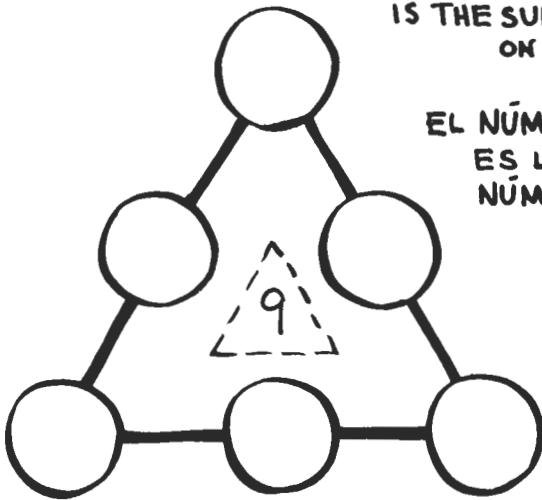
|   |    |    |    |
|---|----|----|----|
| 3 | 2  | 3  | 4  |
| 1 | 2  | 15 | 13 |
| 1 | 14 | 8  | 3  |
| 6 | 2  | 4  | 2  |

\_\_\_\_\_ = 9  
 \_\_\_\_\_ = 9  
 \_\_\_\_\_ = 9  
 \_\_\_\_\_ = 9  
 \_\_\_\_\_ = 9

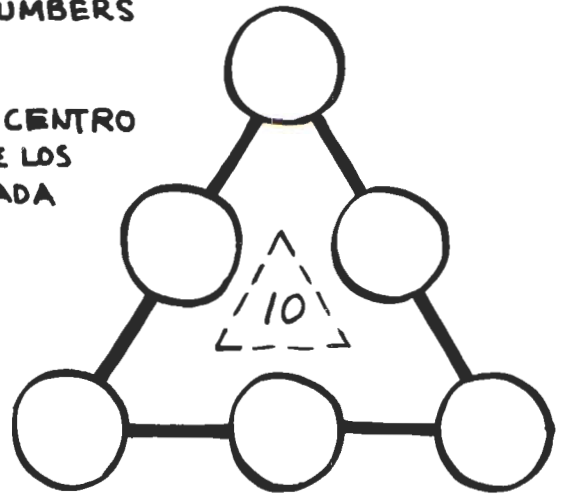
and that's a problem! (The complexity can be scaled down by showing all or some of the fences.)

(Note: It is very often the case that several solutions can be found—different fencings and different statements. However, the above recipe assures the problem maker that there is at least 1 solution.)

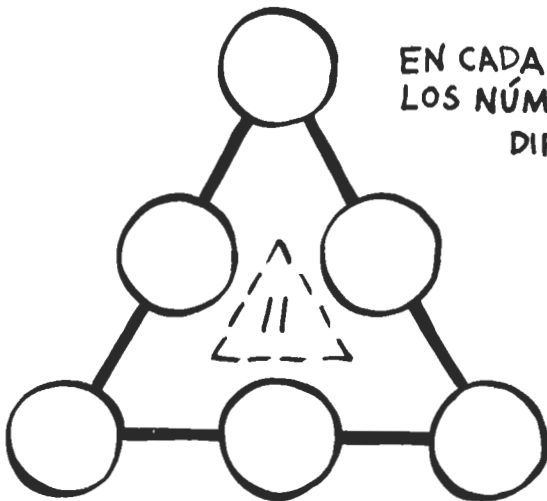
THE NUMBER IN THE CENTER  
IS THE SUM OF THE NUMBERS  
ON EACH SIDE.



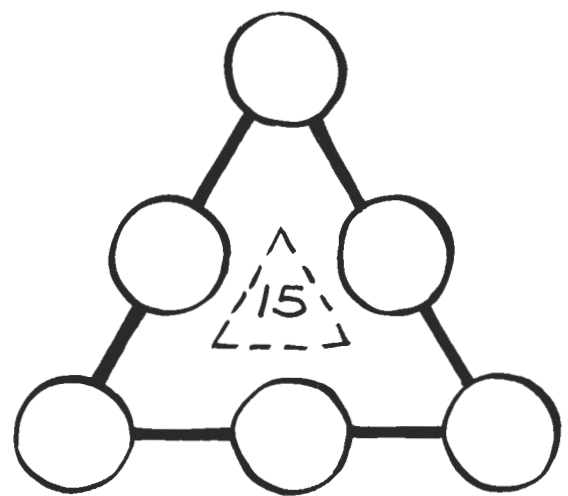
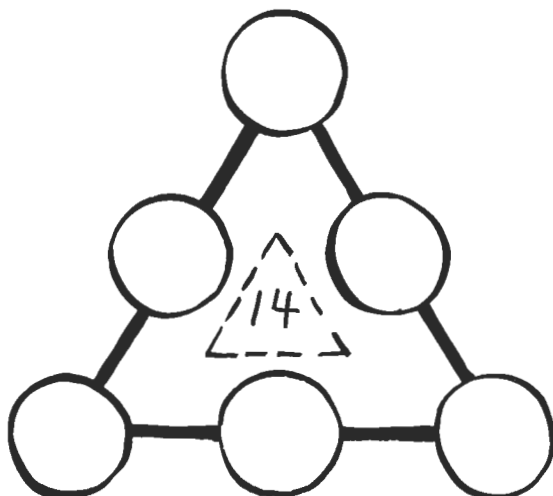
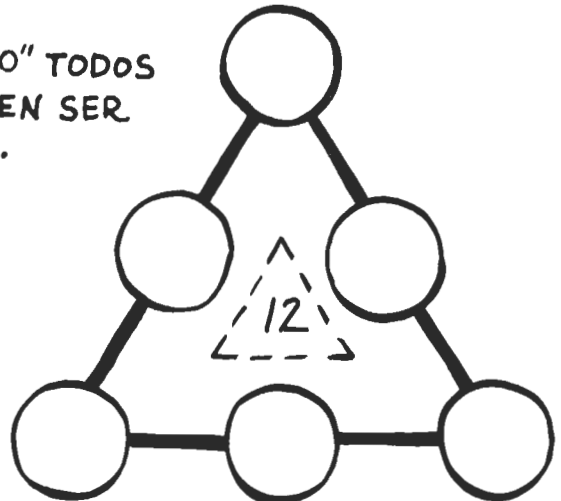
EL NÚMERO EN EL CENTRO  
ES LA SUMA DE LOS  
NÚMEROS EN CADA  
LADO.



IN EACH "TRIANGLE" ALL NUMBERS  
SHOULD BE DIFFERENT.



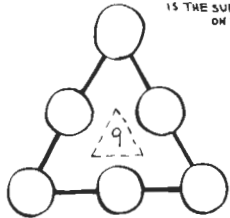
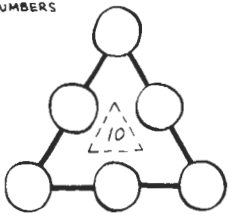
EN CADA "TRIÁNGULO" TODOS  
LOS NÚMEROS DEBEN SER  
DIFERENTES.



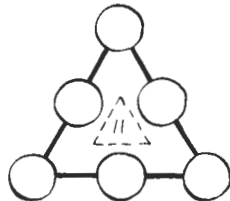
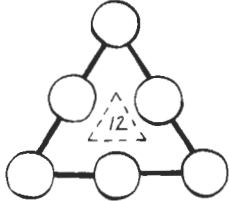
NUMBER TRIANGLES

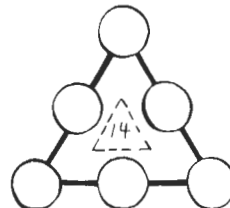
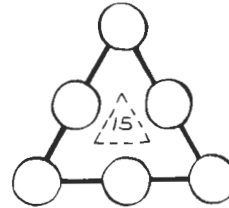
NAME \_\_\_\_\_

THE NUMBER IN THE CENTER  
IS THE SUM OF THE NUMBERS  
ON EACH SIDE.

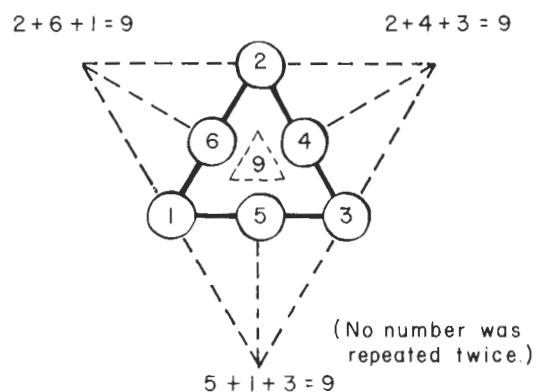



IN EACH "TRIANGLE" ALL NUMBERS  
SHOULD BE DIFFERENT.

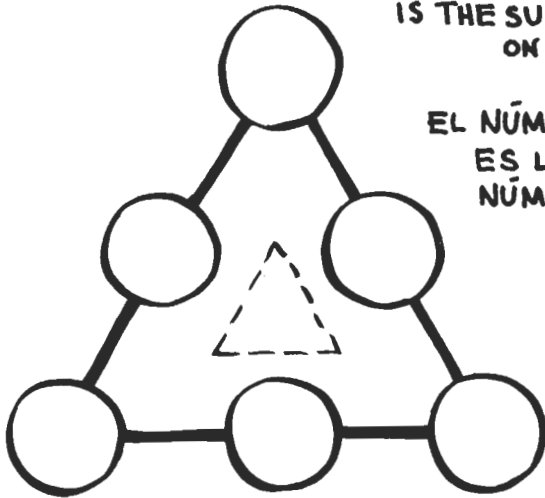
On this task page the learner must enter a number in each circle on the triangle. No one number may be repeated within the same triangle. For example, the following is a possible solution to the first problem:



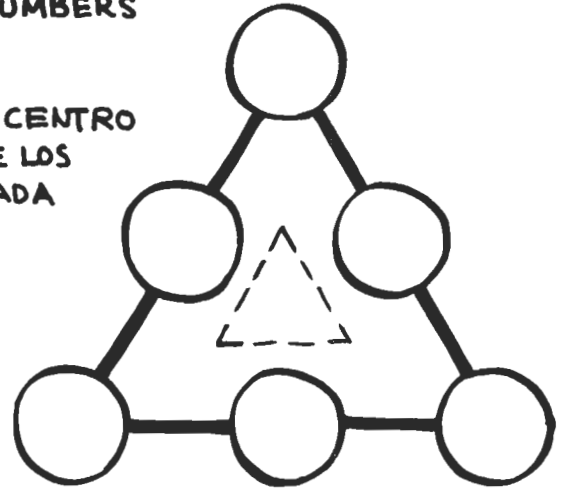
A question to explore:

Can you find a method for making up number triangles?

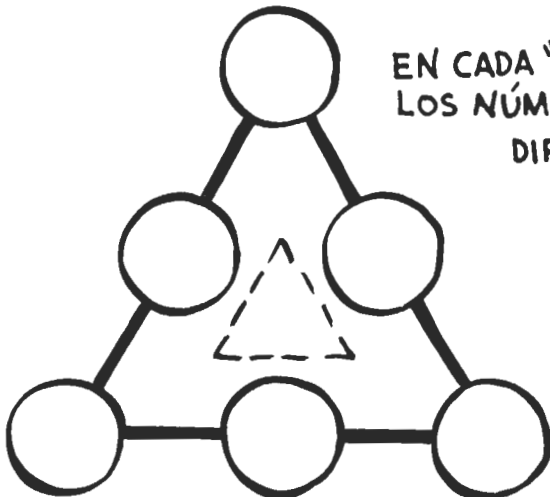
THE NUMBER IN THE CENTER  
IS THE SUM OF THE NUMBERS  
ON EACH SIDE.



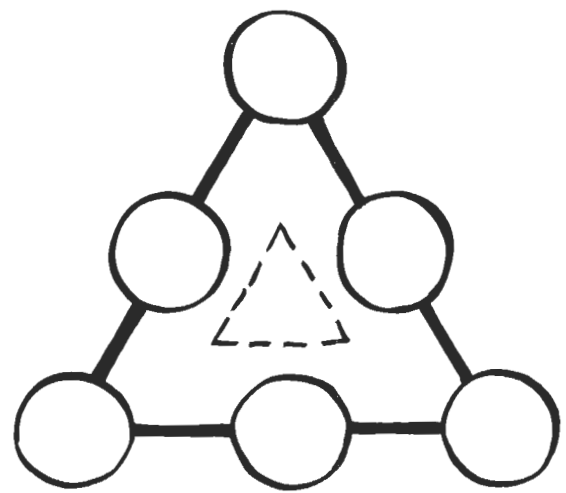
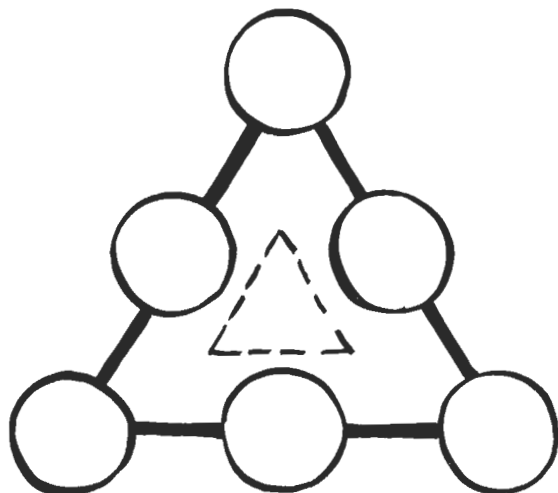
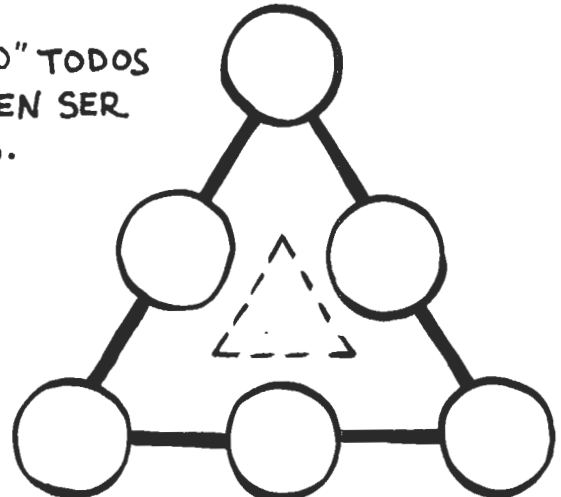
EL NÚMERO EN EL CENTRO  
ES LA SUMA DE LOS  
NÚMEROS EN CADA  
LADO.



IN EACH "TRIANGLE" ALL NUMBERS  
SHOULD BE DIFFERENT.



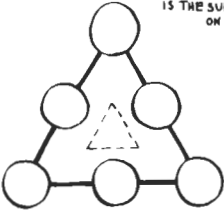
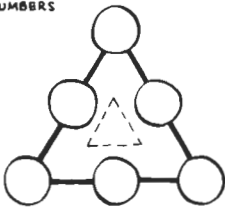
EN CADA "TRIÁNGULO" TODOS  
LOS NÚMEROS DEBEN SER  
DIFERENTES.



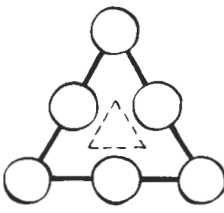
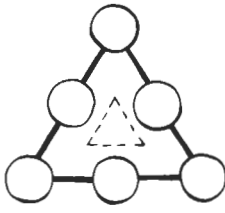
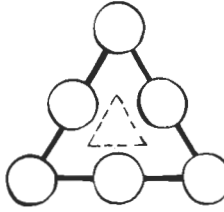
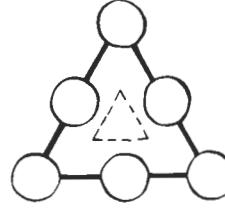


**NUMBER TRIANGLES** NAME \_\_\_\_\_

THE NUMBER IN THE CENTER  
IS THE SUM OF THE NUMBERS  
ON EACH SIDE.

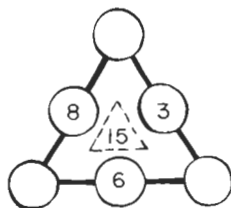



IN EACH "TRIANGLE" ALL NUMBERS  
SHOULD BE DIFFERENT.

This is a continuation of the activities on the previous page. Here, learners can make up their own examples.

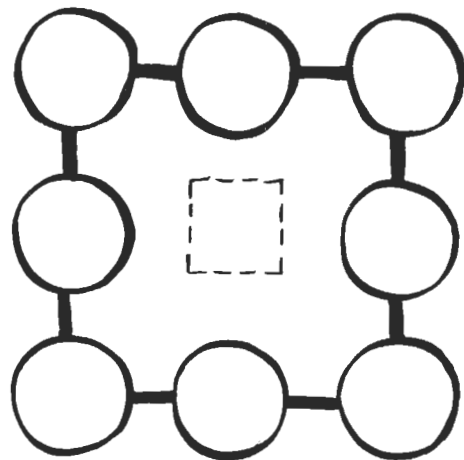
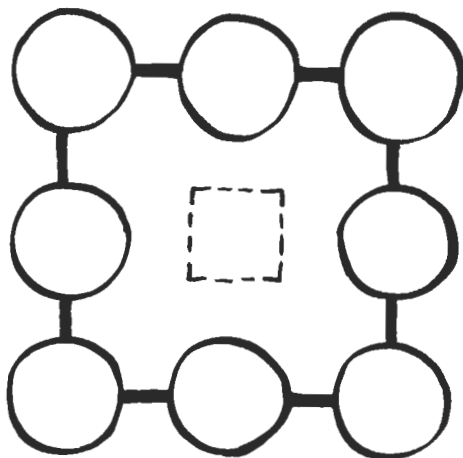
Problems that are somewhat less complex can be created by providing different bits of information. For example, given



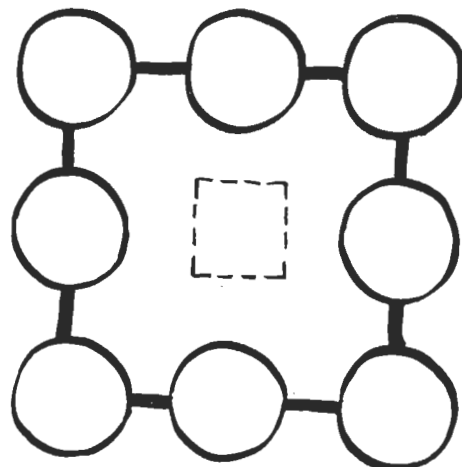
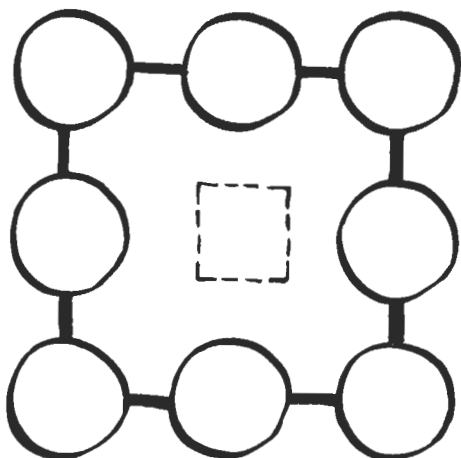
can you complete the triangle?

THE NUMBER IN THE CENTER IS THE SUM  
OF THE NUMBERS ON EACH SIDE.

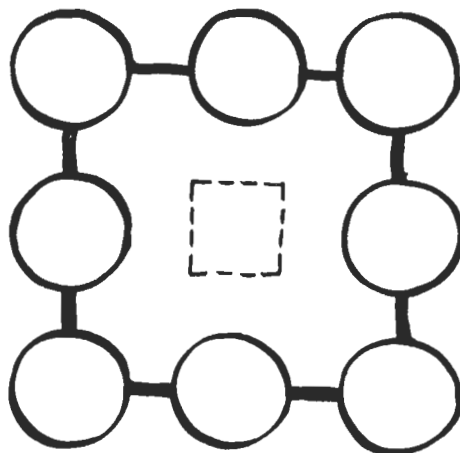
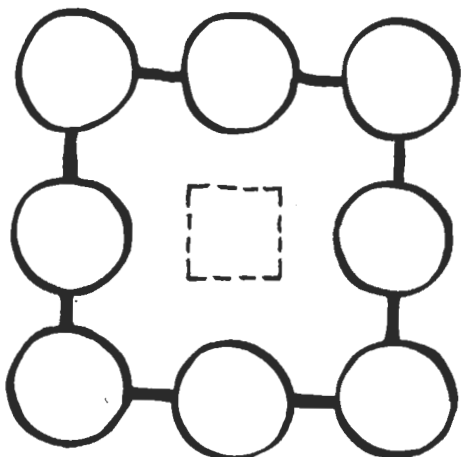
EL NÚMERO EN EL CENTRO ES LA  
SUMA DE LOS NÚMEROS EN CADA LADO.



IN EACH "SQUARE" ALL NUMBERS  
SHOULD BE DIFFERENT.



EN CADA "RECTÁNGULO" TODOS LOS  
NÚMEROS DEBEN SER DIFERENTES.

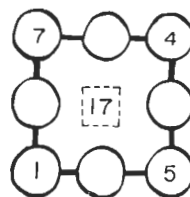


**NUMBER SQUARES**  
THE NUMBER IN THE CENTER IS THE SUM OF THE NUMBERS ON EACH SIDE.

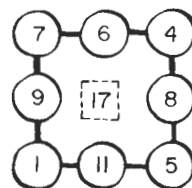
NAME \_\_\_\_\_

IN EACH "SQUARE" ALL NUMBERS SHOULD BE DIFFERENT.

sum in the center. For example:



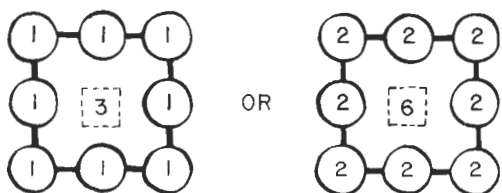
Then, simply find the missing numbers to make each side add up to 17:



Does this theory always work? Can you find a case when it does not? Can you find another method for making these number squares?

On this page, the object is to find numbers to write in the circles so that the numbers are all different and the sum of the three numbers on each side is the same.

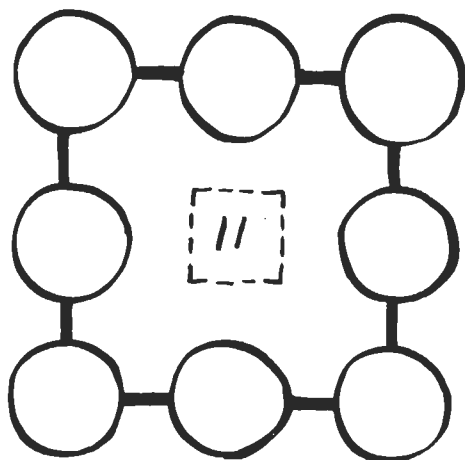
We require that all numbers be different, because if they could be the same, simple solutions abound:



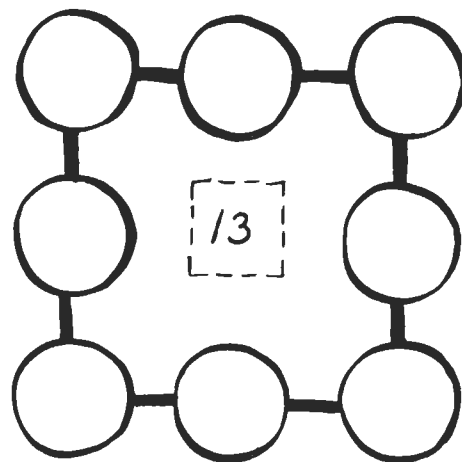
Many theories for ways of creating these number squares will probably be suggested. Each must be tried — and tried — and tried. If examples are found which don't fit the theory, it will have to be revised.

After much experimenting, one learner suggested this theory: Choose any four numbers for the corner circles, making sure that (1) no two of them have the same sum as any other two, and (2) no two of the corner numbers add up to another corner number. Now, add up the four corner numbers, and enter that

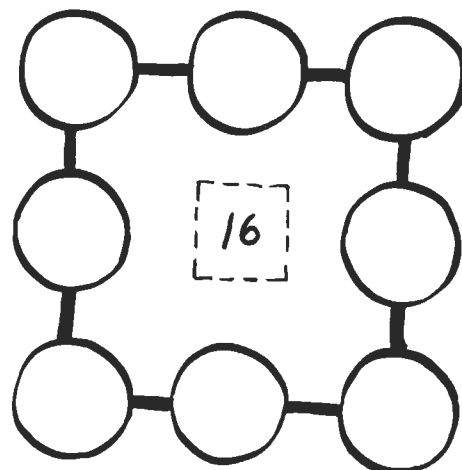
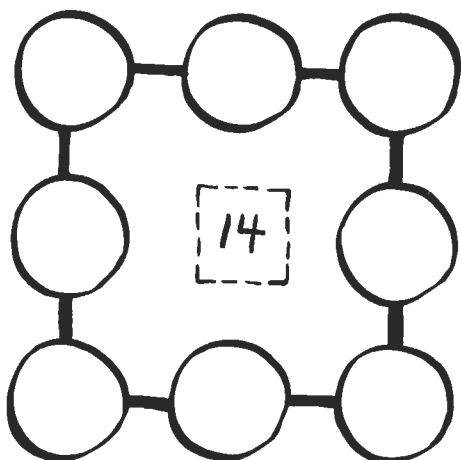
THE NUMBER IN THE CENTER IS THE SUM  
OF THE NUMBERS ON EACH SIDE.



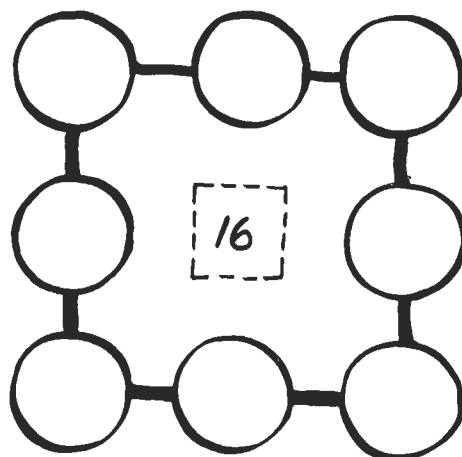
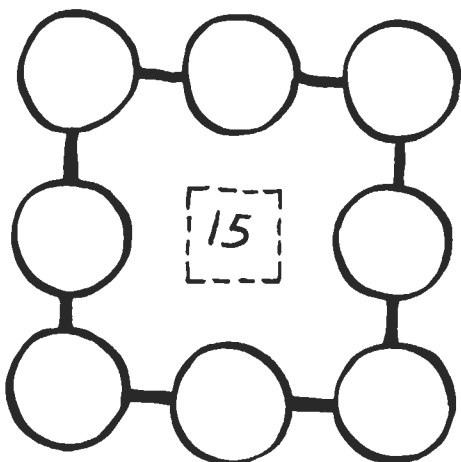
EL NÚMERO EN EL CENTRO ES LA  
SUMA DE LOS NÚMEROS EN CADA LADO.



IN EACH "SQUARE" ALL NUMBERS  
SHOULD BE DIFFERENT.



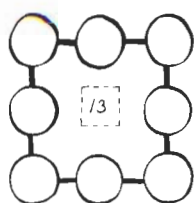
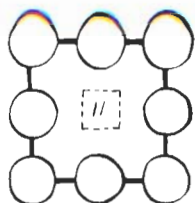
EN CADA "RECTÁNGULO" TODOS LOS  
NÚMEROS DEBEN SER DIFERENTES.



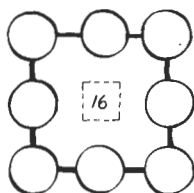
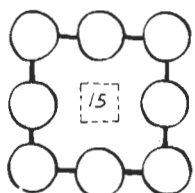
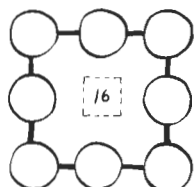
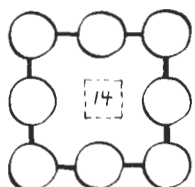
# NUMBER SQUARES

THE NUMBER IN THE CENTER IS THE SUM OF THE NUMBERS ON EACH SIDE.

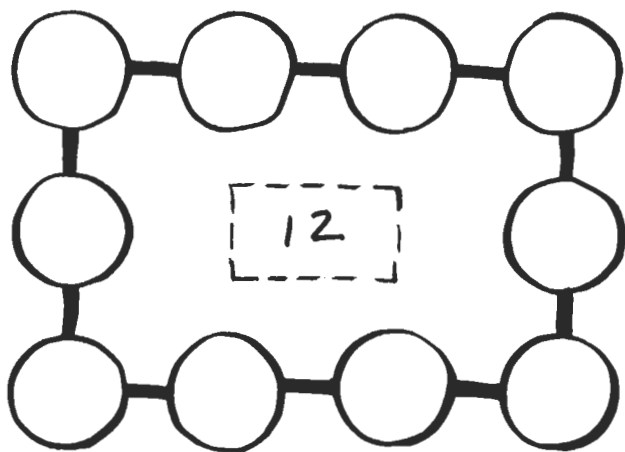
NAME \_\_\_\_\_



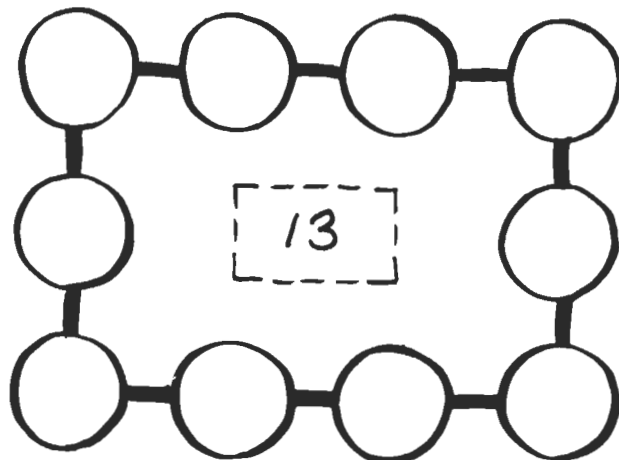
IN EACH SQUARE ALL NUMBERS SHOULD BE DIFFERENT.



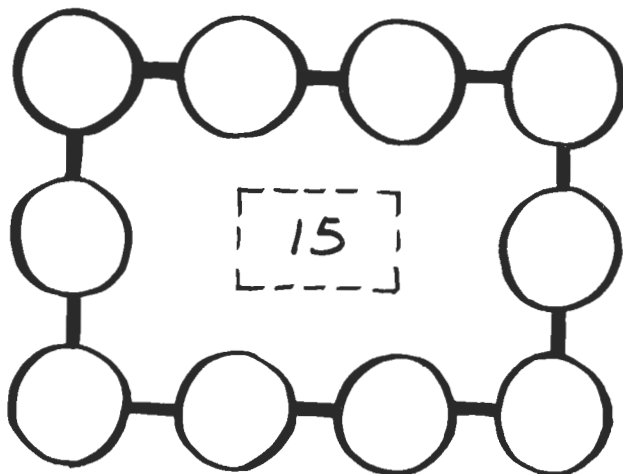
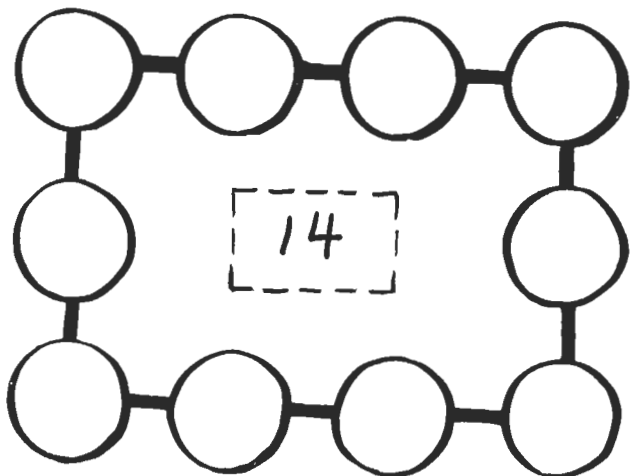
THE NUMBER IN THE CENTER IS THE SUM  
OF THE NUMBERS ON EACH SIDE.



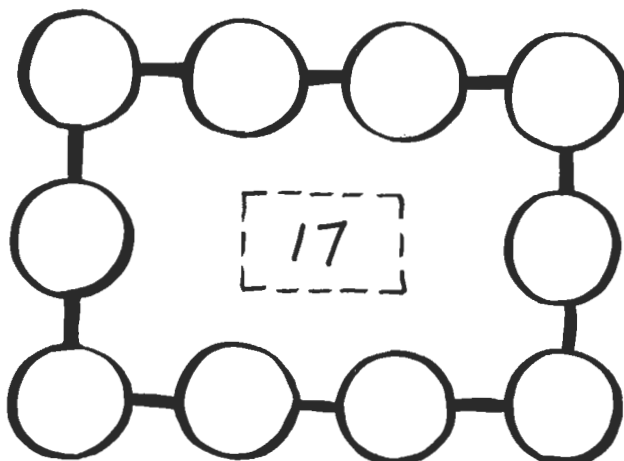
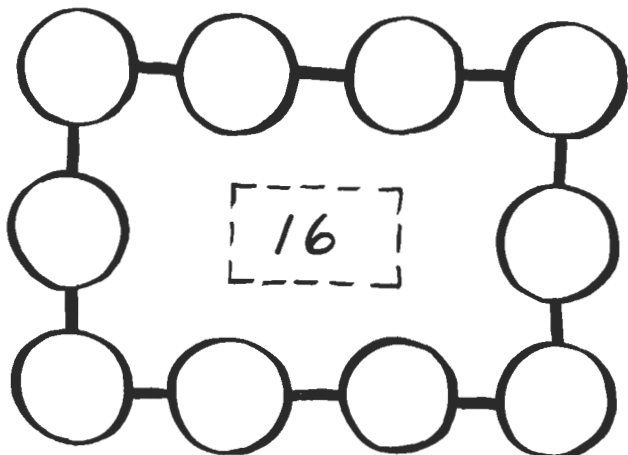
EL NÚMERO EN EL CENTRO ES LA  
SUMA DE LOS NÚMEROS EN CADA LADO.



IN EACH "RECTANGLE" ALL NUMBERS  
SHOULD BE DIFFERENT.



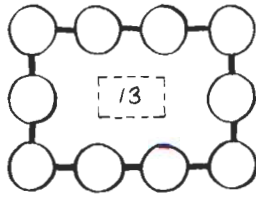
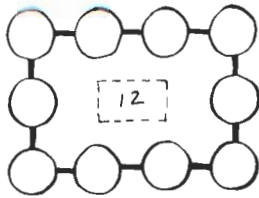
EN CADA "RECTÁNGULO" TODOS LOS  
NÚMEROS DEBEN SER DIFERENTES.



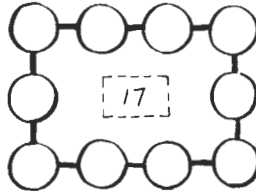
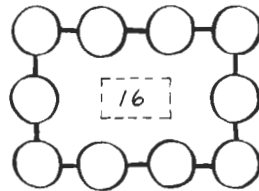
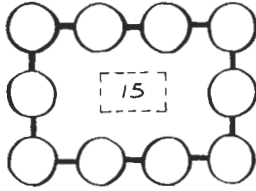
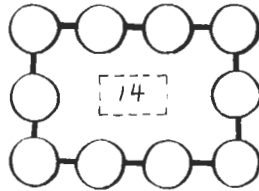
# NUMBER RECTANGLES

THE NUMBER IN THE CENTER IS THE SUM OF THE NUMBERS ON EACH SIDE.

NAME \_\_\_\_\_

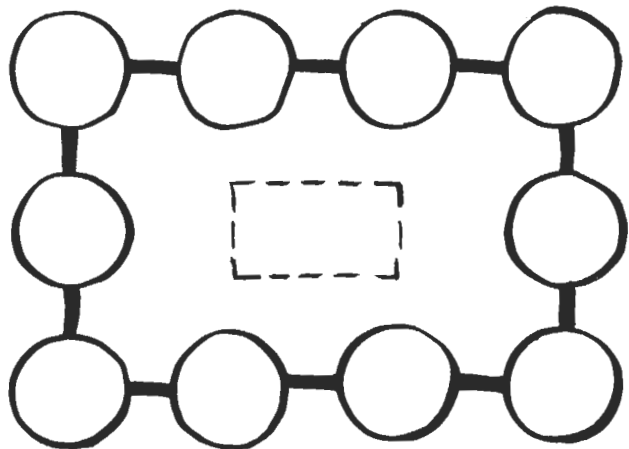


IN EACH "RECTANGLE" ALL NUMBERS SHOULD BE DIFFERENT.

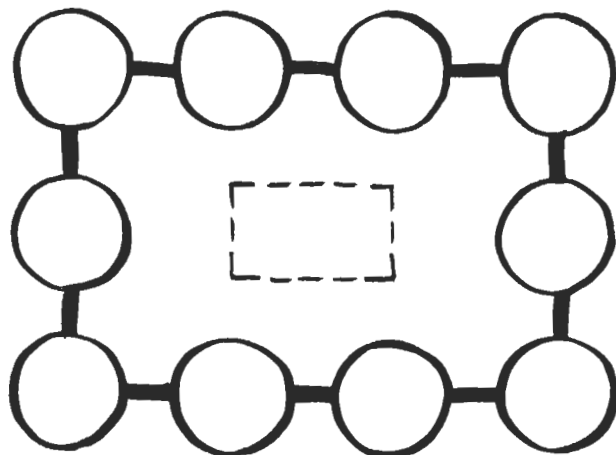


"Number rectangles" are introduced here. Again, the problem is to find different numbers to enter in the circles such that the sum of the numbers on each side of the rectangle is the number in the center.

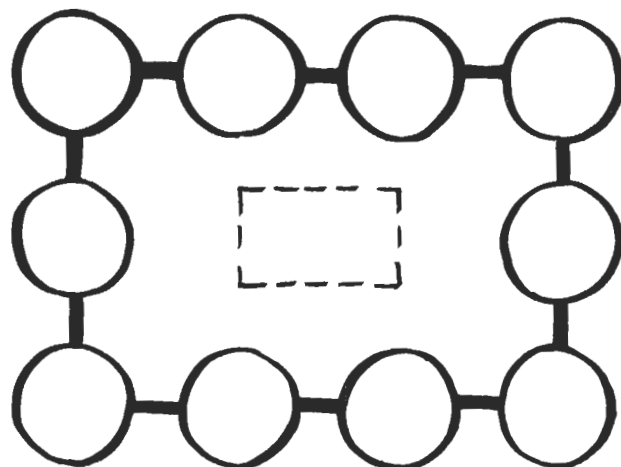
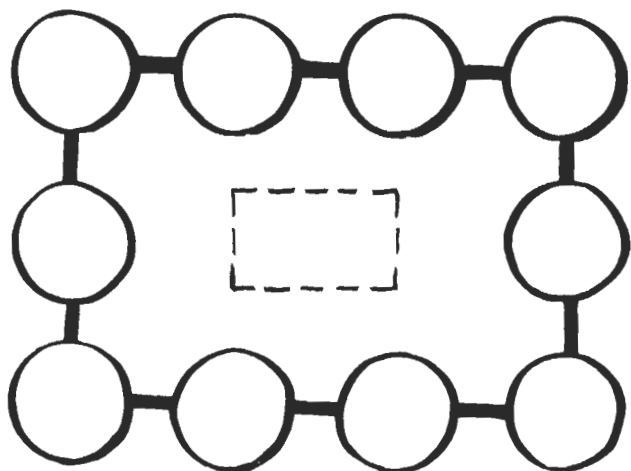
THE NUMBER IN THE CENTER IS THE SUM  
OF THE NUMBERS ON EACH SIDE.



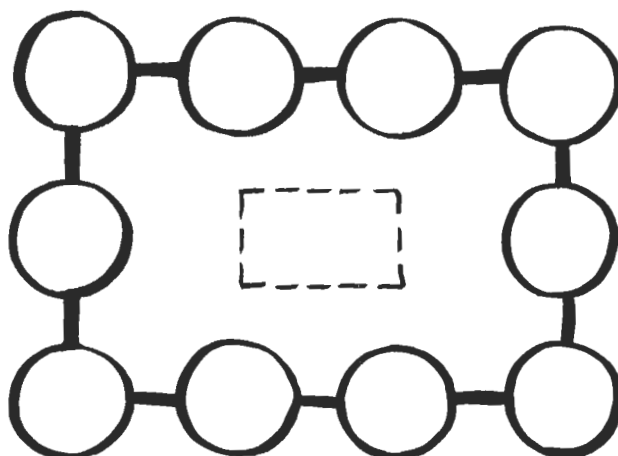
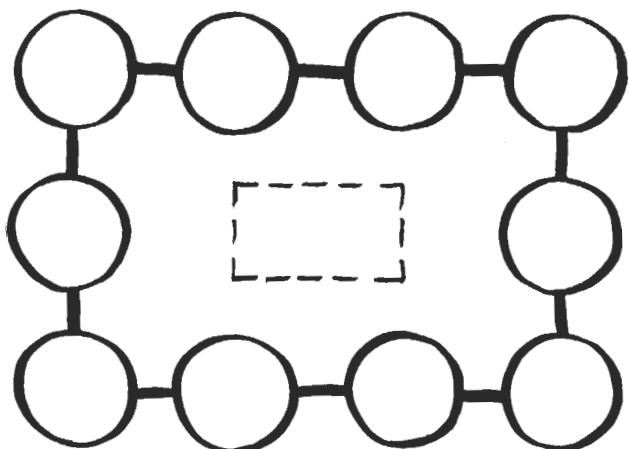
EL NÚMERO EN EL CENTRO ES LA  
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IN EACH "RECTANGLE" ALL NUMBERS  
SHOULD BE DIFFERENT.



EN CADA "RECTÁNGULO" TODOS LOS  
NÚMEROS DEBEN SER DIFERENTES.

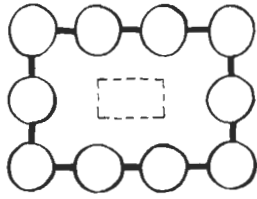
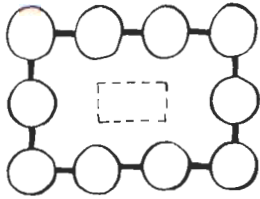




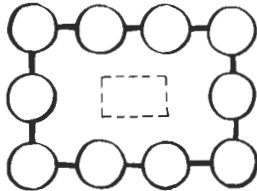
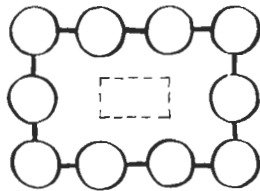
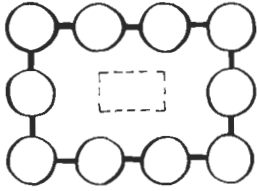
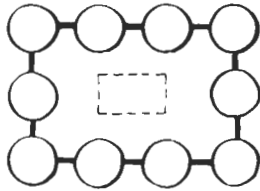
# NUMBER RECTANGLES

THE NUMBER IN THE CENTER IS THE SUM  
OF THE NUMBERS ON EACH SIDE.

NAME \_\_\_\_\_



IN EACH "RECTANGLE" ALL NUMBERS  
SHOULD BE DIFFERENT.



**RULES: SHOW A COMBINATION OF COINS OR STAMPS FOR AMOUNTS SHOWN ..... WITH EXACTLY THE NUMBER OF COINS OR STAMPS INDICATED.**

| 50¢ | 25¢ | 10¢ | 5¢ | 1¢ | NUMBER OF COINS |
|-----|-----|-----|----|----|-----------------|
|     |     |     | 4  | 3  | 23¢ (7)         |
|     |     |     |    |    | 23¢ (10)        |
|     |     |     |    |    | 23¢ (15)        |

**REGLAS: INDICA UNA COMBINACIÓN DE MONEDAS O ESTAMPILLAS PARA LAS CANTIDADES MOSTRADAS.... CON EL NÚMERO EXACTO DE MONEDAS O ESTAMPILLAS INDICADOS.**

| 81¢ | 27¢ | 9¢ | 3¢ | 1¢ | NUMBER OF STAMPS |
|-----|-----|----|----|----|------------------|
|     |     |    |    |    | 17¢ (5)          |
|     |     |    |    |    | 26¢ (6)          |
|     |     |    |    |    | 40¢ (4)          |

| 50¢ | 25¢ | 10¢ | 5¢ | 1¢ | NÚMERO DE MONEDAS |
|-----|-----|-----|----|----|-------------------|
|     |     |     |    |    | 15¢ (6)           |
|     |     |     |    |    | 30¢ (6)           |
|     |     |     |    |    | 92¢ (6)           |

| 81¢ | 27¢ | 9¢ | 3¢ | 1¢ | NÚMERO DE ESTAMPILLAS |
|-----|-----|----|----|----|-----------------------|
|     |     |    |    |    | 80¢ (8)               |
|     |     |    |    |    | 72¢ (8)               |
|     |     |    |    |    | \$1.00 (8)            |

| 50¢ | 25¢ | 10¢ | 5¢ | 1¢ |             |
|-----|-----|-----|----|----|-------------|
|     |     |     |    |    | \$1.25 (5)  |
|     |     |     |    |    | \$1.25 (8)  |
|     |     |     |    |    | \$1.25 (19) |

| 81¢ | 27¢ | 9¢ | 3¢ | 1¢ |             |
|-----|-----|----|----|----|-------------|
|     |     |    |    |    | \$1.00 (4)  |
|     |     |    |    |    | \$1.00 (8)  |
|     |     |    |    |    | \$1.00 (12) |

| 50¢ | 25¢ | 10¢ | 5¢ | 1¢ |             |
|-----|-----|-----|----|----|-------------|
|     |     |     |    |    | 55¢ (10)    |
|     |     |     |    |    | \$1.03 (10) |
|     |     |     |    |    | 90¢ (10)    |

| 81¢ | 27¢ | 9¢ | 3¢ | 1¢ |          |
|-----|-----|----|----|----|----------|
|     |     |    |    |    | 97¢ (11) |
|     |     |    |    |    | 96¢ (11) |
|     |     |    |    |    | 95¢ (11) |

RULES: SHOW A COMBINATION OF COINS OR STAMPS FOR THE AMOUNTS SHOWN... WITH EXACTLY THE NUMBER OF COINS OR STAMPS INDICATED.

The figure shows two place value charts. The first chart is for coins, with columns labeled 50¢, 25¢, 10¢, 5¢, and 1¢. It has three rows. The second chart is for stamps, with columns labeled \$1, 50¢, 25¢, 10¢, and 5¢. It also has three rows.

|     |     |     |    |    |         |  |  |
|-----|-----|-----|----|----|---------|--|--|
| 50¢ | 25¢ | 10¢ | 5¢ | 1¢ |         |  |  |
|     |     |     |    |    | 15¢ (6) |  |  |
|     |     |     |    |    | 30¢ (6) |  |  |
|     |     |     |    |    | 92¢ (6) |  |  |

|     |     |     |    |    |
|-----|-----|-----|----|----|
| 50¢ | 25¢ | 10¢ | 5¢ | 1¢ |
|     |     |     |    |    |
|     |     |     |    |    |
|     |     |     |    |    |

\$1.25 (5)

\$1.25 (8)

\$1.25 (19)

|     |     |     |    |    |
|-----|-----|-----|----|----|
| 50¢ | 25¢ | 10¢ | 5¢ | 1¢ |
|     |     |     |    |    |
|     |     |     |    |    |
|     |     |     |    |    |

\$1.00 (4)

\$1.00 (8)

\$1.00 (12)

|     |     |     |    |    |             |
|-----|-----|-----|----|----|-------------|
| 50¢ | 25¢ | 10¢ | 5¢ | 1¢ |             |
|     |     |     |    |    | 55¢ (10)    |
|     |     |     |    |    | \$1.03 (10) |
|     |     |     |    |    | 90¢ (10)    |

|     |    |    |    |    |          |
|-----|----|----|----|----|----------|
| 10¢ | 2¢ | 1¢ | 1¢ | 1¢ |          |
|     |    |    |    |    | 97¢ (11) |
|     |    |    |    |    | 96¢ (11) |
|     |    |    |    |    | 95¢ (11) |

On this task page the learner must consider ways to make certain amounts of money using only half-dollars, quarters, dimes, nickels, and pennies. He must also consider ways to get a certain amount of postage using stamps in denominations of 1¢, 3¢, 9¢, 27¢, 81¢. The number of coins or stamps to be used in each example is indicated in parentheses.

For example:

|     |     |    |    |    |         |
|-----|-----|----|----|----|---------|
| 81¢ | 27¢ | 9¢ | 3¢ | 1¢ |         |
|     |     |    |    |    | 17¢ (5) |

asks the learner how to get 17¢ worth of postage using just 5 stamps. This can be done with one 9¢ stamp, two 3¢ stamps, and two 1¢ stamps. This result can be recorded:

|     |     |    |    |    |
|-----|-----|----|----|----|
| 81¢ | 27¢ | 9¢ | 3¢ | 1¢ |
|     |     | 1  | 2  | 2  |

17¢ (5)

Some problems ask the learner to find several different ways to make the same amount. Others ask the learner to find combinations that use the same **number** of coins or stamps to get different amounts.

IN EACH EXAMPLE THE SAME NUMBER  
MUST BE WRITTEN IN SHAPES THAT  
ARE ALIKE.

EN CADA EJEMPLO HAY QUE ESCRIBIR  
EL MISMO NÚMERO EN LAS FORMAS QUE  
SEAN IDÉNTICAS.

$$\frac{1}{2} \text{ of } (8) = (4)$$

$$\otimes \frac{1}{2} \text{ of } (4) = (2)$$


---


$$\text{— of } (8) = (2)$$

$$\frac{1}{2} \text{ of } (6) = ( )$$

$$\otimes \frac{1}{3} \text{ of } (3) = ( )$$


---


$$\text{— of } (6) = ( )$$

$$\frac{1}{3} \text{ of } (9) = ( )$$

$$\otimes \frac{1}{3} \text{ of } (3) = ( )$$


---


$$\text{— of } (9) = ( )$$

$$\frac{1}{4} \times (12) = ( )$$

$$\otimes \frac{1}{3} \times ( ) = ( )$$


---


$$\text{—} \times (12) = ( )$$

$$\frac{1}{2} \times (6) = ( )$$

$$\otimes \frac{2}{3} \times ( ) = ( )$$


---


$$\text{—} \times (6) = ( )$$

$$\frac{1}{3} \times (9) = ( )$$

$$\otimes \frac{2}{3} \times ( ) = ( )$$


---


$$\text{—} \times (9) = ( )$$

$$\frac{2}{5} \times (10) = ( )$$

$$\otimes \frac{1}{2} \times ( ) = ( )$$


---


$$\text{—} \times (10) = ( )$$

$$\frac{2}{3} \times (6) = ( )$$

$$\otimes \frac{1}{2} \times ( ) = ( )$$


---


$$\text{—} \times (6) = ( )$$

$$\frac{2}{3} \times (9) = ( )$$

$$\otimes \frac{2}{3} \times ( ) = ( )$$


---


$$\text{—} \times (9) = ( )$$

$$\frac{1}{2} \times (10) = ( )$$

$$\otimes \frac{2}{5} \times ( ) = ( )$$


---


$$\text{—} \times (10) = ( )$$

$$\frac{1}{2} \times (12) = ( )$$

$$\otimes \frac{2}{3} \times ( ) = ( )$$


---


$$\text{—} \times (12) = ( )$$

$$\frac{2}{3} \times (12) = ( )$$

$$\otimes \frac{1}{2} \times ( ) = ( )$$


---


$$\text{—} \times (12) = ( )$$

IN EACH GROUP OF 3 STATEMENTS,  
THE SAME NUMBER MUST BE WRITTEN  
IN SHAPES THAT ARE ALIKE.

NAME \_\_\_\_\_

|                                                |                                                |                                                |
|------------------------------------------------|------------------------------------------------|------------------------------------------------|
| $\frac{1}{2}$ of $\textcircled{8} = \text{⬡}$  | $\frac{1}{2}$ of $\text{⬡} = \text{⬡}$         | $\frac{1}{3}$ of $\text{⬡} = \text{⬡}$         |
| $\otimes \frac{1}{2}$ of $\text{⬡} = \text{⬡}$ | $\otimes \frac{1}{3}$ of $\text{⬡} = \text{⬡}$ | $\otimes \frac{1}{3}$ of $\text{⬡} = \text{⬡}$ |
| $\text{— of } \text{⬡} = \text{⬡}$             | $\text{— of } \text{⬡} = \text{⬡}$             | $\text{— of } \text{⬡} = \text{⬡}$             |

|                                                  |                                                  |                                                  |
|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|
| $\frac{1}{4} \times \text{⬡} = \text{⬡}$         | $\frac{1}{2} \times \text{⬡} = \text{⬡}$         | $\frac{1}{3} \times \text{⬡} = \text{⬡}$         |
| $\otimes \frac{1}{3} \times \text{⬡} = \text{⬡}$ | $\otimes \frac{2}{3} \times \text{⬡} = \text{⬡}$ | $\otimes \frac{2}{3} \times \text{⬡} = \text{⬡}$ |
| $\text{—} \times \text{⬡} = \text{⬡}$            | $\text{—} \times \text{⬡} = \text{⬡}$            | $\text{—} \times \text{⬡} = \text{⬡}$            |

|                                                  |                                                  |                                                  |
|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|
| $\frac{2}{5} \times \text{⬡} = \text{⬡}$         | $\frac{2}{3} \times \text{⬡} = \text{⬡}$         | $\frac{2}{3} \times \text{⬡} = \text{⬡}$         |
| $\otimes \frac{1}{2} \times \text{⬡} = \text{⬡}$ | $\otimes \frac{1}{2} \times \text{⬡} = \text{⬡}$ | $\otimes \frac{2}{3} \times \text{⬡} = \text{⬡}$ |
| $\text{—} \times \text{⬡} = \text{⬡}$            | $\text{—} \times \text{⬡} = \text{⬡}$            | $\text{—} \times \text{⬡} = \text{⬡}$            |

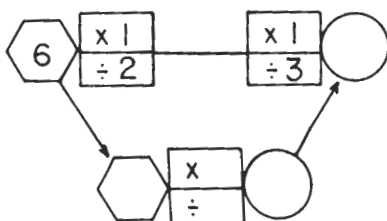
|                                                  |                                                  |                                                  |
|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|
| $\frac{1}{2} \times \text{⬡} = \text{⬡}$         | $\frac{1}{2} \times \text{⬡} = \text{⬡}$         | $\frac{2}{3} \times \text{⬡} = \text{⬡}$         |
| $\otimes \frac{2}{5} \times \text{⬡} = \text{⬡}$ | $\otimes \frac{2}{3} \times \text{⬡} = \text{⬡}$ | $\otimes \frac{1}{2} \times \text{⬡} = \text{⬡}$ |
| $\text{—} \times \text{⬡} = \text{⬡}$            | $\text{—} \times \text{⬡} = \text{⬡}$            | $\text{—} \times \text{⬡} = \text{⬡}$            |

Here we introduce the rule that within each group of three statements, the same number must be written in shapes that are alike.

On this page we are essentially using a more conventional notation for the problem encountered previously in these pages of finding a shortcut for two chain reactions (see pages 197-200). That is to say that

$$\begin{array}{l} \frac{1}{2} \text{ of } \text{⬡} = \text{⬡} \\ \frac{1}{3} \text{ of } \text{⬡} = \text{⬡} \\ \hline \text{⬡} = \text{⬡} \end{array}$$

is the **same** problem as:



The new notation and organization of the problem might be difficult for some children; it does require some ability to reason logically. If frustration occurs for these children, the page may be best put off until another time.

A reasonable approach to this type of problem would be to do each line in succession. Once the first line is done, we know one of the numbers to fill in the second line. When the second line is done, we know a number to put in the third line. The third line will always present the problem of "finding a shortcut."

Of course, these examples are building referents for the introduction of multiplication by fractions. Children may already have noticed a relationship between the two fractions in the first two lines and that in the third. If they have a "guess" about the relationship, let them test it out. A likely guess is that "you multiply the top numbers and the bottom numbers." This theory will always work until you require that the shortcut use the smallest possible whole numbers. Then a revision is necessary—and we have begun to build some referents for the later introduction of "reducing fractions." Learners can use the next page to test and revise their theories.

EN CADA EJEMPLO HAY QUE ESCRIBIR  
EL MISMO NÚMERO EN LAS FORMAS QUE  
SEAN IDÉNTICAS.

IN EACH EXAMPLE THE SAME NUMBER  
MUST BE WRITTEN IN SHAPES THAT  
ARE ALIKE.

$$\begin{array}{l} \_ \text{ of } \bigcirc = \text{hexagon} \\ \otimes \_ \text{ of } \text{hexagon} = \square \\ \hline \_ \text{ of } \bigcirc = \square \end{array}$$

$$\begin{array}{l} \_ \text{ of } \text{hexagon} = \square \\ \otimes \_ \text{ of } \square = \bigcirc \\ \hline \_ \text{ of } \text{hexagon} = \bigcirc \end{array}$$

$$\begin{array}{l} \_ \text{ of } \square = \bigcirc \\ \otimes \_ \text{ of } \bigcirc = \text{hexagon} \\ \hline \_ \text{ of } \square = \text{hexagon} \end{array}$$

$$\begin{array}{l} \_ \times \square = \bigcirc \\ \otimes \_ \times \bigcirc = \text{hexagon} \\ \hline \_ \times \square = \text{hexagon} \end{array}$$

$$\begin{array}{l} \_ \times \bigcirc = \text{hexagon} \\ \otimes \_ \times \text{hexagon} = \square \\ \hline \_ \times \bigcirc = \square \end{array}$$

$$\begin{array}{l} \_ \times \text{hexagon} = \square \\ \otimes \_ \times \square = \bigcirc \\ \hline \_ \times \text{hexagon} = \bigcirc \end{array}$$

$$\begin{array}{l} \_ \times \text{hexagon} = \square \\ \otimes \_ \times \square = \bigcirc \\ \hline \_ \times \text{hexagon} = \bigcirc \end{array}$$

$$\begin{array}{l} \_ \times \square = \bigcirc \\ \otimes \_ \times \bigcirc = \text{hexagon} \\ \hline \_ \times \square = \text{hexagon} \end{array}$$

$$\begin{array}{l} \_ \times \bigcirc = \text{hexagon} \\ \otimes \_ \times \text{hexagon} = \square \\ \hline \_ \times \bigcirc = \square \end{array}$$

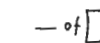
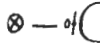
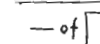
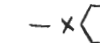
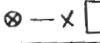
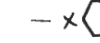
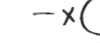
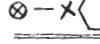
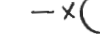
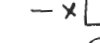
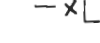
$$\begin{array}{l} \_ \times \bigcirc = \text{hexagon} \\ \otimes \_ \times \text{hexagon} = \square \\ \hline \_ \times \bigcirc = \square \end{array}$$

$$\begin{array}{l} \_ \times \text{hexagon} = \square \\ \otimes \_ \times \square = \bigcirc \\ \hline \_ \times \text{hexagon} = \bigcirc \end{array}$$

$$\begin{array}{l} \_ \times \square = \bigcirc \\ \otimes \_ \times \bigcirc = \text{hexagon} \\ \hline \_ \times \square = \text{hexagon} \end{array}$$

IN EACH GROUP OF 3 STATEMENTS,  
THE SAME NUMBER MUST BE WRITTEN  
IN SHAPES THAT ARE ALIKE.

NAME \_\_\_\_\_

|                                                                                                                                                                                      |                                                                                                                                                                                      |                                                                                                                                                                                      |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| — of  =            | — of  =            | — of  =            |
| $\otimes$ — of  =  | $\otimes$ — of  =  | $\otimes$ — of  =  |
| — of  =            | — of  =            | — of  =            |
| — x  =             | — x  =             | — x  =             |
| $\otimes$ — x  =   | $\otimes$ — x  =   | $\otimes$ — x  =   |
| — x  =             | — x  =             | — x  =             |
| — x  =             | — x  =             | — x  =             |
| $\otimes$ — x  =   | $\otimes$ — x  =   | $\otimes$ — x  =   |
| — x  =             | — x  =             | — x  =             |
| — x  =             | — x  =             | — x  =             |
| $\otimes$ — x  =   | $\otimes$ — x  =   | $\otimes$ — x  =   |
| — x  =             | — x  =             | — x  =             |

Will the fraction  $\frac{1}{5}$  still be used as the shortcut  
if he starts with:

$$\begin{array}{l} \frac{2}{5} \times \text{hexagon}(20) = \square \\ \frac{1}{2} \times \square = \text{circle} \\ \hline \text{—} \times \text{hexagon} = \text{circle} \end{array}$$

Will it work if he starts with 30 in the hexagon?

This form can be particularized or used for explorations by the learner. Some variations might be:

(1) Make up problems of this type:

$$\begin{array}{l} \text{— of hexagon}(4) = \square \\ \text{— of } \square = \text{circle}(1) \\ \hline \text{— of hexagon}(4) = \text{circle}(1) \end{array}$$

(2) The learner might want to test some of the "shortcuts" from the previous page. For example, on that page, he found:

$$\begin{array}{l} \frac{2}{5} \times \text{hexagon}(10) = \text{square}(4) \\ \frac{1}{2} \times \text{square}(4) = \text{circle}(2) \\ \hline \frac{1}{5} \times \text{hexagon}(10) = \text{circle}(2) \end{array}$$



$$\frac{1}{2} \text{ of } (8) = 4$$

$$\oplus \frac{1}{4} \text{ of } (8) = 2$$


---


$$\frac{1}{4} \text{ of } (8) = 2$$

$$\frac{1}{6} \text{ of } (18) = 3$$

$$\oplus \frac{1}{3} \text{ of } (18) = 6$$


---


$$\frac{1}{3} \text{ of } (18) = 6$$

$$\frac{1}{3} \text{ of } [9] = 3$$

$$\oplus \frac{1}{3} \text{ of } [9] = 3$$


---


$$\frac{1}{3} \text{ of } [9] = 3$$

$$\frac{1}{5} \times [15] = 3$$

$$\oplus \frac{2}{5} \times [15] = 6$$


---


$$\frac{3}{5} \times [15] = 9$$

$$\frac{1}{2} \times (6) = 3$$

$$\oplus \frac{1}{3} \times (6) = 2$$


---


$$\frac{5}{6} \times (6) = 5$$

$$\frac{1}{2} \times (30) = 15$$

$$\oplus \frac{1}{3} \times (30) = 10$$


---


$$\frac{5}{6} \times (30) = 25$$

$$\frac{3}{4} \times (16) = 12$$

$$\ominus \frac{1}{2} \times (16) = 8$$


---


$$\frac{5}{4} \times (16) = 20$$

$$\frac{3}{4} \times [100] = 75$$

$$\ominus \frac{1}{2} \times [100] = 50$$


---


$$\frac{5}{4} \times [100] = 125$$

$$\frac{3}{4} \times (36) = 27$$

$$\ominus \frac{1}{2} \times (36) = 18$$


---


$$\frac{5}{4} \times (36) = 45$$

$$\frac{2}{5} \times (10) = 4$$

$$\ominus \frac{1}{10} \times (10) = 1$$


---


$$\frac{1}{5} \times (10) = 2$$

$$\frac{2}{5} \times (50) = 20$$

$$\ominus \frac{1}{10} \times (50) = 5$$


---


$$\frac{1}{5} \times (50) = 10$$

$$\frac{2}{5} \times [100] = 40$$

$$\ominus \frac{1}{10} \times [100] = 10$$


---


$$\frac{1}{5} \times [100] = 20$$



NAME \_\_\_\_\_

$$\frac{1}{2} \text{ of } (8) = \underline{4} \quad \frac{1}{6} \text{ of } (18) = \underline{\quad} \quad \frac{1}{3} \text{ of } (9) = \underline{\quad}$$

$$+ \frac{1}{4} \text{ of } (8) = \underline{2} \quad + \frac{1}{3} \text{ of } (18) = \underline{\quad} \quad + \frac{1}{3} \text{ of } (9) = \underline{\quad}$$

$$\text{— of } (8) = \underline{6} \quad \text{— of } (18) = \underline{7} \quad \text{— of } (9) = \underline{\quad}$$

$$\frac{1}{5} \times (15) = \underline{\quad} \quad \frac{1}{2} \times (6) = \underline{\quad} \quad \frac{1}{2} \times (30) = \underline{\quad}$$

$$+ \frac{2}{5} \times (15) = \underline{\quad} \quad + \frac{1}{3} \times (6) = \underline{\quad} \quad + \frac{1}{3} \times (30) = \underline{\quad}$$

$$\text{—} \times (15) = \underline{\quad} \quad \text{—} \times (6) = \underline{\quad} \quad \text{—} \times (30) = \underline{\quad}$$

$$\frac{3}{4} \times (16) = \underline{\quad} \quad \frac{3}{4} \times (100) = \underline{\quad} \quad \frac{3}{4} \times (36) = \underline{\quad}$$

$$\ominus \frac{1}{2} \times (16) = \underline{\quad} \quad \ominus \frac{1}{2} \times (100) = \underline{\quad} \quad \ominus \frac{1}{2} \times (36) = \underline{\quad}$$

$$\text{—} \times (16) = \underline{\quad} \quad \text{—} \times (100) = \underline{\quad} \quad \text{—} \times (36) = \underline{\quad}$$

$$\frac{2}{5} \times (10) = \underline{\quad} \quad \frac{2}{5} \times (50) = \underline{\quad} \quad \frac{2}{5} \times (100) = \underline{\quad}$$

$$\ominus \frac{1}{10} \times (10) = \underline{\quad} \quad \ominus \frac{1}{10} \times (50) = \underline{\quad} \quad \ominus \frac{1}{10} \times (100) = \underline{\quad}$$

$$\text{—} \times (10) = \underline{\quad} \quad \text{—} \times (50) = \underline{\quad} \quad \text{—} \times (100) = \underline{\quad}$$

On this page we make the rule that in each group of three statements, the number on the right of “=” in the third line is the sum or difference (indicated by “+” or “—”) of the two numbers directly above it.

For example in:

$$\begin{array}{r} \frac{1}{6} \text{ of } (18) = \underline{3} \\ + \frac{1}{3} \text{ of } (18) = \underline{6} \\ \hline \text{— of } (18) = \underline{9} \end{array}$$

9 is the sum of 3 and 6.

The problem is once again to “find a shortcut for getting 18 to 9,” or more conventionally, to “find that fraction which when multiplied by 18 will produce 9.”

Children should be encouraged to test any guess they have about relationships among the three fractions in each group. The next page provides a report form for doing these investigations.

$$\begin{array}{l} \text{— of } \bigcirc = \text{—} \\ \bigcirc \text{— of } \bigcirc = \text{—} \\ \hline \text{— of } \bigcirc = \text{—} \end{array}$$

$$\begin{array}{l} \text{— of } \heptagon = \text{—} \\ \bigcirc \text{— of } \heptagon = \text{—} \\ \hline \text{— of } \heptagon = \text{—} \end{array}$$

$$\begin{array}{l} \text{— of } \square = \text{—} \\ \bigcirc \text{— of } \square = \text{—} \\ \hline \text{— of } \square = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \square = \text{—} \\ \bigcirc \text{—} \times \square = \text{—} \\ \hline \text{—} \times \square = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \bigcirc = \text{—} \\ \bigcirc \text{—} \times \bigcirc = \text{—} \\ \hline \text{—} \times \bigcirc = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \heptagon = \text{—} \\ \bigcirc \text{—} \times \heptagon = \text{—} \\ \hline \text{—} \times \heptagon = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \heptagon = \text{—} \\ \bigcirc \text{—} \times \heptagon = \text{—} \\ \hline \text{—} \times \heptagon = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \square = \text{—} \\ \bigcirc \text{—} \times \square = \text{—} \\ \hline \text{—} \times \square = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \bigcirc = \text{—} \\ \bigcirc \text{—} \times \bigcirc = \text{—} \\ \hline \text{—} \times \bigcirc = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \bigcirc = \text{—} \\ \bigcirc \text{—} \times \bigcirc = \text{—} \\ \hline \text{—} \times \bigcirc = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \heptagon = \text{—} \\ \bigcirc \text{—} \times \heptagon = \text{—} \\ \hline \text{—} \times \heptagon = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \square = \text{—} \\ \bigcirc \text{—} \times \square = \text{—} \\ \hline \text{—} \times \square = \text{—} \end{array}$$

NAME \_\_\_\_\_
$$\begin{array}{l} \frac{1}{3} \times \boxed{12} = \underline{\hspace{2cm}} \\ \frac{2}{6} \times \boxed{12} = \underline{\hspace{2cm}} \\ + \frac{1}{2} \times \boxed{12} = \underline{\hspace{2cm}} \\ \hline \quad \times \boxed{12} = \underline{\hspace{2cm}} \end{array} \quad \text{and} \quad \begin{array}{l} \frac{3}{6} \times \boxed{12} = \underline{\hspace{2cm}} \\ \frac{1}{4} \times \boxed{8} = \underline{\hspace{2cm}} \\ + \frac{2}{4} \times \boxed{8} = \underline{\hspace{2cm}} \\ \hline \quad \times \boxed{8} = \underline{\hspace{2cm}} \end{array}$$

with the constraint that you must find the fraction for the last line that uses the smallest whole numbers. Learners can also use this page to make up their own examples, and pursue their own investigations.

HUEVOS  
EGGS  
DOCENAS  
DOZENS

|                |               |               |               |   |               |   |   |   |    |    |    |                |
|----------------|---------------|---------------|---------------|---|---------------|---|---|---|----|----|----|----------------|
| 1              | 2             | 3             | 4             | 5 | 6             | 7 | 8 | 9 | 10 | 11 | 12 | 13             |
| $\frac{1}{12}$ | $\frac{1}{6}$ | $\frac{1}{4}$ | $\frac{1}{3}$ |   | $\frac{1}{2}$ |   |   |   |    |    |    | $\frac{1}{12}$ |

$$\begin{array}{r} 3 + 3 = 6 \\ \hline \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \end{array}$$

$$\begin{array}{r} 4 + \quad = \quad \\ \hline \frac{1}{3} + \frac{1}{3} = \end{array}$$

$$\begin{array}{r} \quad + \quad = \quad \\ \hline \frac{1}{4} + \quad = \end{array}$$

$$\begin{array}{r} 5 + 2 = \quad \\ \hline \quad + \quad = \end{array}$$

$$\begin{array}{r} \quad + 1 = \quad \\ \hline \frac{2}{3} + \quad = \end{array}$$

$$\begin{array}{r} 4 + \quad = \quad \\ \hline \quad + \frac{1}{6} = \end{array}$$

$$\begin{array}{r} 6 - 3 = \quad \\ \hline \frac{1}{2} - \frac{1}{4} = \end{array}$$

$$\begin{array}{r} \quad - \quad = \quad \\ \hline \frac{3}{4} - \frac{1}{4} = \end{array}$$

$$\begin{array}{r} 8 - \quad = \quad \\ \hline \quad - \frac{1}{3} = \end{array}$$

$$\begin{array}{r} \quad - \quad = \quad \\ \hline \frac{7}{12} - \frac{1}{12} = \end{array}$$

$$\begin{array}{r} 10 - \quad = \quad \\ \hline \quad - \frac{1}{4} = \end{array}$$

$$\begin{array}{r} 12 - \quad = \quad \\ \hline 1 - \frac{1}{12} = \end{array}$$

$$\frac{1}{2} + \frac{1}{4} = \quad$$

$$\frac{5}{12} + \frac{1}{12} = \quad$$

$$\frac{1}{4} + \frac{3}{4} = \quad$$

$$1 - \frac{1}{2} = \quad$$

$$\frac{2}{3} - \frac{1}{3} = \quad$$

$$\frac{5}{6} - \frac{1}{2} = \quad$$

| EGGS   | 1              | 2             | 3             | 4             | 5 | 6             | 7 | 8 | 9 | 10 | 11 | 12 | 13             |
|--------|----------------|---------------|---------------|---------------|---|---------------|---|---|---|----|----|----|----------------|
| DOZENS | $\frac{1}{12}$ | $\frac{2}{6}$ | $\frac{1}{4}$ | $\frac{1}{3}$ |   | $\frac{1}{2}$ |   |   |   |    |    |    | $\frac{1}{12}$ |

|                                 |                                 |                                 |
|---------------------------------|---------------------------------|---------------------------------|
| $\frac{3}{4} + \frac{3}{4} =$   | $\frac{4}{3} + \frac{1}{3} =$   | $\frac{1}{4} + \frac{1}{4} =$   |
| $\frac{5}{2} + \frac{2}{2} =$   | $\frac{2}{3} + \frac{1}{3} =$   | $\frac{4}{6} + \frac{1}{6} =$   |
| $\frac{6}{2} - \frac{3}{4} =$   | $\frac{3}{4} - \frac{1}{4} =$   | $\frac{8}{3} - \frac{1}{3} =$   |
| $\frac{7}{12} - \frac{1}{12} =$ | $\frac{10}{4} - \frac{1}{4} =$  | $\frac{12}{1} - \frac{1}{12} =$ |
| $\frac{1}{2} + \frac{1}{4} =$   | $\frac{5}{12} + \frac{1}{12} =$ | $\frac{1}{4} + \frac{3}{4} =$   |
| $1 - \frac{1}{2} =$             | $\frac{2}{3} - \frac{1}{3} =$   | $\frac{5}{6} - \frac{1}{2} =$   |

This page begins a sequence of activities which might be termed a "double-talk approach to fractions".

Using some real referent within his experience, the learner creates a table involving both fractions and whole numbers to describe the whole and its parts.

HUEVOS  
EGGS  
DOCENAS  
DOZENS

|                |                |    |                 |    |    |    |    |    |    |    |                |    |
|----------------|----------------|----|-----------------|----|----|----|----|----|----|----|----------------|----|
| 14             | 15             | 16 | 17              | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25             | 26 |
| $1\frac{1}{2}$ | $1\frac{1}{4}$ |    | $1\frac{5}{12}$ |    |    |    |    |    |    | 2  | $2\frac{1}{2}$ |    |

$$\begin{array}{|c|} \hline 4 \\ \hline \frac{1}{3} \\ \hline \end{array} \times 2 = \begin{array}{|c|} \hline 8 \\ \hline \frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 9 \\ \hline \frac{3}{4} \\ \hline \end{array} \times 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \frac{5}{12} \\ \hline \end{array} \times 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \frac{1}{6} \\ \hline \end{array} \times 3 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \frac{1}{3} \\ \hline \end{array} \times 3 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \frac{1}{3} \\ \hline \end{array} \times 4 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 6 \\ \hline \frac{1}{2} \\ \hline \end{array} \div 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \frac{2}{3} \\ \hline \end{array} \div 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \frac{5}{6} \\ \hline \end{array} \div 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline 1\frac{1}{2} \\ \hline \end{array} \div 3 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 8 \\ \hline \\ \hline \end{array} \div 4 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 24 \\ \hline \\ \hline \end{array} \div 3 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \frac{1}{4} \\ \hline \end{array} \times 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \frac{2}{3} \\ \hline \end{array} \times 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \frac{1}{2} \\ \hline \end{array} \times 3 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 1\frac{1}{3} \\ \hline \end{array} \div 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \frac{1}{2} \\ \hline \end{array} \div 3 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 1 \\ \hline \end{array} \div 4 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

|        |               |               |               |               |    |    |    |    |    |    |    |                |    |
|--------|---------------|---------------|---------------|---------------|----|----|----|----|----|----|----|----------------|----|
| EGGS   | 14            | 15            | 16            | 17            | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25             | 26 |
| DOZENS | $\frac{1}{6}$ | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{1}{2}$ |    |    |    |    |    |    | 2  | $2\frac{1}{2}$ |    |

|                                      |                                       |                                         |
|--------------------------------------|---------------------------------------|-----------------------------------------|
| $\frac{4}{1} \times 2 = \frac{8}{1}$ | $\frac{9}{3} \times 2 = \frac{18}{3}$ | $\frac{5}{12} \times 2 = \frac{10}{12}$ |
| $\frac{1}{3} \times 2 = \frac{2}{3}$ | $\frac{3}{4} \times 2 = \frac{6}{4}$  | $\frac{5}{12} \times 2 = \frac{10}{12}$ |
| $\frac{1}{6} \times 3 = \frac{3}{6}$ | $\frac{1}{3} \times 3 = \frac{3}{3}$  | $\frac{1}{3} \times 4 = \frac{4}{3}$    |
| $\frac{10}{2} \div 2 = \frac{5}{1}$  | $\frac{2}{3} \div 2 = \frac{1}{3}$    | $\frac{5}{6} \div 2 = \frac{5}{12}$     |
| $\frac{1}{2} \div 3 = \frac{1}{6}$   | $\frac{8}{4} \div 4 = \frac{2}{1}$    | $\frac{24}{3} \div 3 = \frac{8}{1}$     |
| $\frac{1}{4} \times 2 = \frac{2}{4}$ | $\frac{2}{3} \times 2 = \frac{4}{3}$  | $\frac{1}{2} \times 3 = \frac{3}{2}$    |
| $\frac{1}{3} \div 2 = \frac{1}{6}$   | $\frac{1}{2} \div 3 = \frac{1}{6}$    | $1 \div 4 = \frac{1}{4}$                |

This page is a continuation of the previous page. "Double talk" multiplication and division occur here. These examples are done in the same way. For example, in

$$\frac{5}{1} \times 2 = \frac{10}{1}$$

5 eggs is 5/12 of a dozen.  $5 \times 2 = 10$ , and 10 eggs is 5/6 dozen. Thus:

$$\frac{5}{12} \times 2 = \frac{10}{12}$$

$$\frac{5}{12} \times 2 = \frac{5}{6}$$

Consider the division example

$$\frac{15}{1} \div 3 = \frac{5}{1}$$

$$\frac{5}{4} \div 3 = \frac{5}{12}$$

15 eggs is 5/4 of a dozen.  $15 \div 3 = 5$ , and 5 eggs is 5/12 of a dozen. Thus,

$$\frac{15}{4} \div 3 = \frac{5}{12}$$

$$\frac{5}{4} \div 3 = \frac{5}{12}$$



|  |   |  |   |  |
|--|---|--|---|--|
|  | + |  | = |  |
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|  |   |  |   |  |
|--|---|--|---|--|
|  | + |  | = |  |
|  | + |  | = |  |

$$\begin{array}{r} \square - \square = \square \\ \square - \square = \square \end{array}$$

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} - \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$



NAME \_\_\_\_\_

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|

|                               |                               |                               |
|-------------------------------|-------------------------------|-------------------------------|
| $\square + \square = \square$ | $\square + \square = \square$ | $\square + \square = \square$ |
|-------------------------------|-------------------------------|-------------------------------|

|                               |                               |                               |
|-------------------------------|-------------------------------|-------------------------------|
| $\square + \square = \square$ | $\square + \square = \square$ | $\square + \square = \square$ |
|-------------------------------|-------------------------------|-------------------------------|

|                               |                               |                               |
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| $\square - \square = \square$ | $\square - \square = \square$ | $\square - \square = \square$ |
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|                               |                               |                               |
|-------------------------------|-------------------------------|-------------------------------|
| $\square - \square = \square$ | $\square - \square = \square$ | $\square - \square = \square$ |
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|                                  |                                  |                                  |
|----------------------------------|----------------------------------|----------------------------------|
| $\square \div \square = \square$ | $\square \div \square = \square$ | $\square \div \square = \square$ |
|----------------------------------|----------------------------------|----------------------------------|

|                                  |                                  |                                  |
|----------------------------------|----------------------------------|----------------------------------|
| $\square \div \square = \square$ | $\square \div \square = \square$ | $\square \div \square = \square$ |
|----------------------------------|----------------------------------|----------------------------------|

This page continues the type of activities on the previous pages. Learners can make up their own tables about whatever they want that can be expressed in "double talk". Some possibilities could be:

- (1) Pies cut with a 6-piece cutter. The table could be in terms of pieces and fractional parts of the pie.
- (2) Candy (or butter, or nuts) measured in ounces. The table could be in terms of ounces and fractional parts of a pound.
- (3) Time measured in hours and days. The table could be in terms of hours and fractional parts parts of a day.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bd \\ ce+dg & cf+dh \end{bmatrix}$$
$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \begin{array}{c} \times \\ \times \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$
$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \times \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$
$$\begin{array}{r} \boxed{\phantom{00}} \div \phantom{00} = \boxed{\phantom{00}} \\ \hline \boxed{\phantom{00}} \div \phantom{00} = \boxed{\phantom{00}} \end{array}$$

$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \div \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$$

$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \div \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$$

$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \div \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$$

$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \div \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$$

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|  | ÷ |  | = |  |

$$\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$$
$$\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$$
$$\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\boxed{\phantom{00}} \div \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\boxed{\phantom{00}} \div \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\frac{\quad}{\quad} \div \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

\_\_\_\_\_

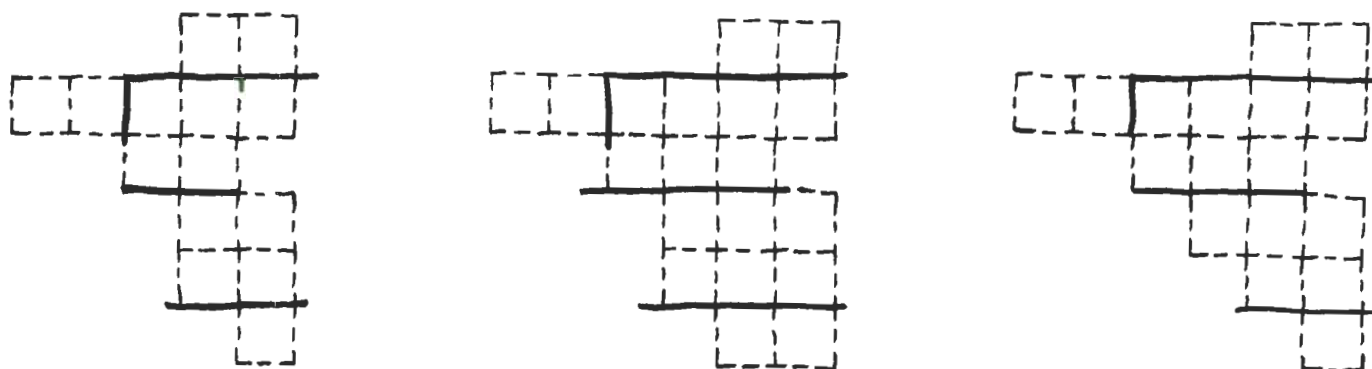
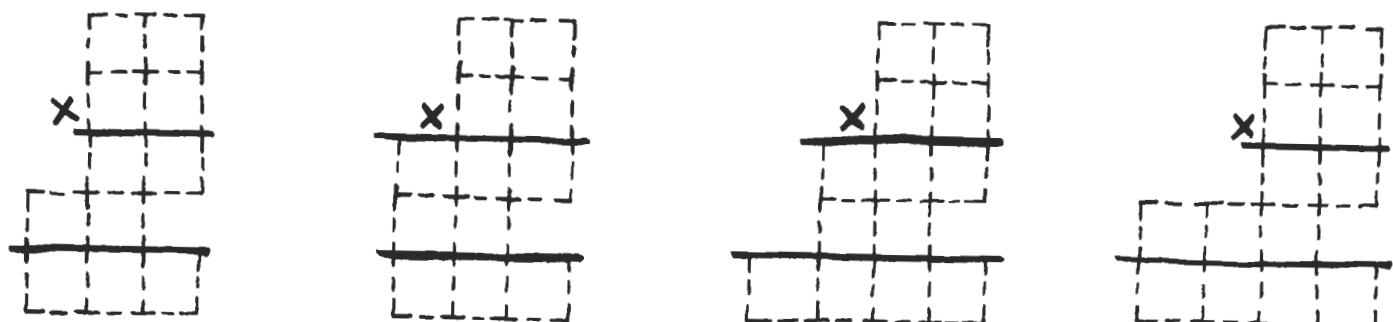
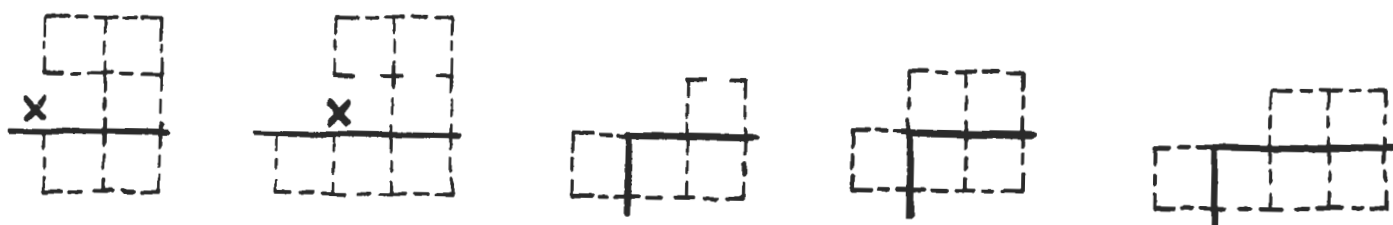
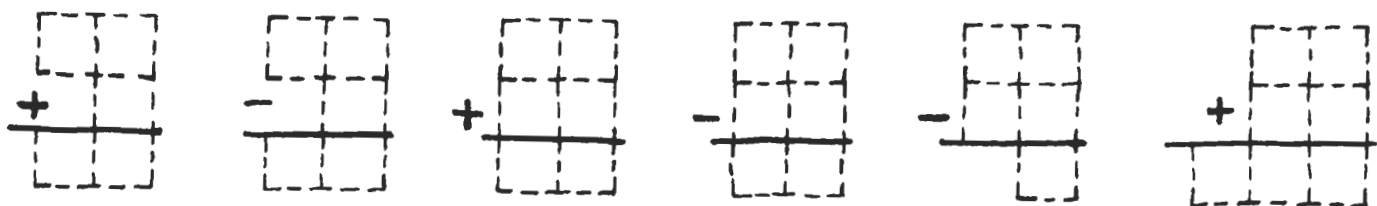
[illegible]
$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \begin{array}{c} \times \\ \times \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$
$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \times \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$
$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \begin{array}{l} \times \\ \times \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$



$$\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$
$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \begin{array}{c} + \\ - \end{array} \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$
$$\begin{array}{|c|} \hline \square \div \square = \square \\ \hline \square \div \square = \square \\ \hline \end{array}$$
$$\boxed{\phantom{00}} \div \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

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Row 1:  $\begin{array}{|c|c|} \hline \text{+} & \\ \hline \hline \end{array} + \begin{array}{|c|c|} \hline \text{+} & \\ \hline \hline \end{array} = \begin{array}{|c|c|} \hline \text{+} & \\ \hline \hline \end{array}$

Row 2:  $\begin{array}{|c|c|} \hline \text{x} & \\ \hline \hline \end{array} \times \begin{array}{|c|c|} \hline \text{x} & \\ \hline \hline \end{array} = \begin{array}{|c|c|} \hline \text{x} & \\ \hline \hline \end{array}$

Row 3:  $\begin{array}{|c|c|c|} \hline \text{x} & & \\ \hline \hline \hline \end{array} \times \begin{array}{|c|c|c|} \hline \text{x} & & \\ \hline \hline \hline \end{array} = \begin{array}{|c|c|c|} \hline \text{x} & & \\ \hline \hline \hline \end{array}$

Row 4:  $\begin{array}{|c|c|c|c|} \hline \text{x} & & & \\ \hline \hline \hline \hline \end{array} \times \begin{array}{|c|c|c|c|} \hline \text{x} & & & \\ \hline \hline \hline \hline \end{array} = \begin{array}{|c|c|c|c|} \hline \text{x} & & & \\ \hline \hline \hline \hline \end{array}$

On this page, learners make up examples using addition, multiplication, subtraction, and division to fit the forms given. Success on this page requires a good understanding of the algorithms.

Children will probably need some scrap "scribble sheets" to try out their examples before entering them in the forms.

Of course, the form can also be particularized in many ways as well, giving different bits of information as is done on the next page.

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$$\begin{array}{|c|c|} \hline 3 & \\ \hline + & \\ \hline 4 & \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & \\ \hline - & \\ \hline 5 & \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 8 & 9 \\ \hline + & \\ \hline & \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 2 & 0 \\ \hline - & \\ \hline & \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 3 & 8 \\ \hline - & \\ \hline & \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & \\ \hline + & 4 \\ \hline & \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline & \\ \hline \times & \\ \hline 3 & 9 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline & & \\ \hline \times & & \\ \hline 1 & 2 & 5 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline & \\ \hline 7 & 9 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 1 & 3 \\ \hline & 5 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|} \hline & & & \\ \hline 9 & 1 & 1 & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline & & \\ \hline \times & & \\ \hline & & \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline & & \\ \hline \times & & \\ \hline 1 & 0 & 0 \\ \hline 4 & 0 & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline & & \\ \hline \times & & \\ \hline 1 & 3 & 2 \\ \hline 9 & 9 & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|} \hline & & & \\ \hline \times & & & \\ \hline & 9 & 4 & \\ \hline 1 & 5 & 0 & 4 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline & & \\ \hline 7 & 5 & \\ \hline 4 & 5 & \\ \hline 5 & & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|} \hline 1 & 6 & & \\ \hline 1 & 1 & 2 & \\ \hline 1 & 2 & & \\ \hline 1 & 1 & & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|} \hline & 5 & & \\ \hline 3 & & & \\ \hline 1 & 0 & 0 & \\ \hline & & & 8 \\ \hline \end{array}$$

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|                                                                                                     |                                                                                                                   |                                                                                                                |                                                                                                        |                                                                                     |                                                                                     |
|-----------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| $\begin{array}{ c c } \hline 3 & \\ \hline + & \\ \hline 4 & \\ \hline \end{array}$                 | $\begin{array}{ c c } \hline 6 & \\ \hline - & \\ \hline 5 & \\ \hline \end{array}$                               | $\begin{array}{ c c } \hline 8 & 9 \\ \hline + & \\ \hline & \\ \hline \end{array}$                            | $\begin{array}{ c c } \hline 2 & 0 \\ \hline - & \\ \hline & \\ \hline \end{array}$                    | $\begin{array}{ c c } \hline 3 & 8 \\ \hline - & \\ \hline & \\ \hline \end{array}$ | $\begin{array}{ c c } \hline 5 & \\ \hline + & 4 \\ \hline & \\ \hline \end{array}$ |
| $\begin{array}{ c c } \hline & \\ \hline \times & \\ \hline 3 & 9 \\ \hline \end{array}$            | $\begin{array}{ c c } \hline & \\ \hline \times & \\ \hline 1 & 2 & 5 \\ \hline \end{array}$                      | $\begin{array}{ c c } \hline 7 & \\ \hline \times & 9 \\ \hline & \\ \hline \end{array}$                       | $\begin{array}{ c c } \hline 1 & 3 \\ \hline \times & 5 \\ \hline & \\ \hline \end{array}$             | $\begin{array}{ c c } \hline & \\ \hline 9 & 1 & 1 \\ \hline \end{array}$           |                                                                                     |
| $\begin{array}{ c c } \hline & \\ \hline \times & \\ \hline 1 & 2 & 1 \\ \hline \end{array}$        | $\begin{array}{ c c } \hline & \\ \hline \times & \\ \hline 1 & 0 & 0 \\ \hline 4 & 0 & \\ \hline \end{array}$    | $\begin{array}{ c c } \hline & \\ \hline \times & \\ \hline 1 & 3 & 2 \\ \hline 9 & 9 & \\ \hline \end{array}$ | $\begin{array}{ c c } \hline & \\ \hline \times & 9 & 4 \\ \hline 1 & 5 & 0 & 8 \\ \hline \end{array}$ |                                                                                     |                                                                                     |
| $\begin{array}{ c c } \hline & \\ \hline 7 & 5 \\ \hline 4 & 5 \\ \hline 5 & \\ \hline \end{array}$ | $\begin{array}{ c c } \hline 1 & 6 \\ \hline 1 & 1 & 2 \\ \hline 1 & 2 & \\ \hline 1 & 1 & \\ \hline \end{array}$ | $\begin{array}{ c c } \hline 3 & \\ \hline 1 & 0 & 0 \\ \hline 8 & \\ \hline \end{array}$                      |                                                                                                        |                                                                                     |                                                                                     |

On this page, the constraints of the problem are a little tighter. Here the learner must complete the addition, subtraction, multiplication, or division problem to fit the form and the bits of information given. Consider the problem:

$$\begin{array}{|c|c|} \hline 5 & \\ \hline + & 4 \\ \hline & \\ \hline \end{array}$$

One clue is that the answer must have three digits. Since each addend has only two digits, we need to complete the example so that when we add in the tens column we will have a two-digit number. Since we have  $5 + 4$ , or 9 there already, we will have to have "carried" one from the units column. Thus we must use numbers in the units column which add to ten or more. One possibility would be:

$$\begin{array}{|c|c|} \hline 5 & 7 \\ \hline + & 4 & 4 \\ \hline 1 & 0 & 1 \\ \hline \end{array}$$

A good question for each example might be: "Can you find a different way to do it?" Some can be done in more than one way; some cannot.

Consider:

$$\begin{array}{|c|c|} \hline 9 & 1 & 1 \\ \hline \end{array}$$

We know that  $9 \overline{)11}^{1r2}$ . Thus the first digit in the quotient has to be 1 and the next number to be divided by 9 must be a two digit number that begins with 2. What two-digit multiples of 9 begin with 2? 27 is the only one, and  $9 \overline{)27}^3$ . Thus the completed example:

$$\begin{array}{|c|c|} \hline 1 & 3 \\ \hline 9 & 1 & 1 & 7 \\ \hline \end{array}$$

Some examples get more difficult:

$$\begin{array}{|c|c|} \hline & \\ \hline \times & \\ \hline & 9 & 4 \\ \hline 1 & 5 & 0 & 4 \\ \hline \end{array}$$

The clues are skimpy. Before we start guessing, we should probably fill in all the information we can derive from what is given. Because we know the first partial product and the total, we can find the second partial product:

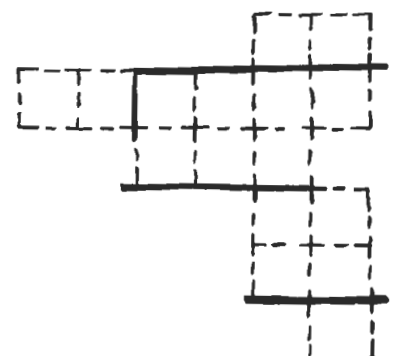
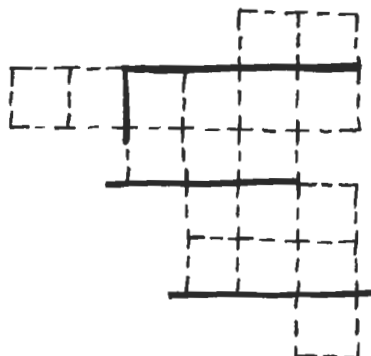
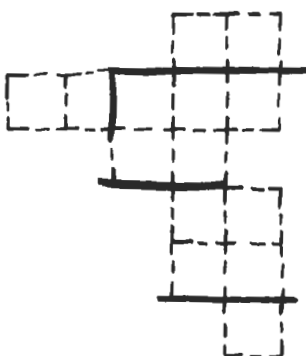
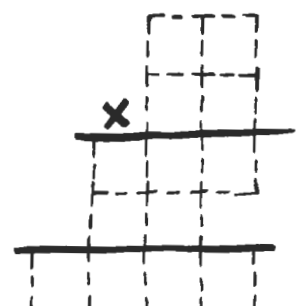
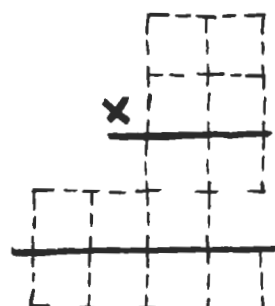
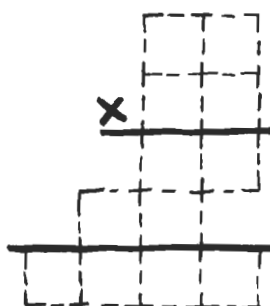
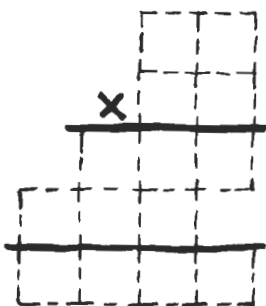
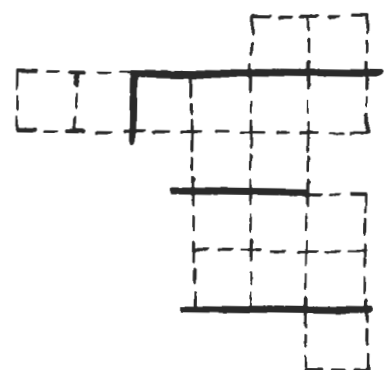
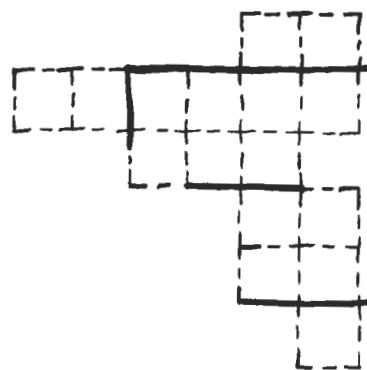
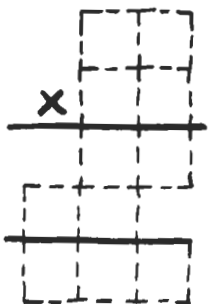
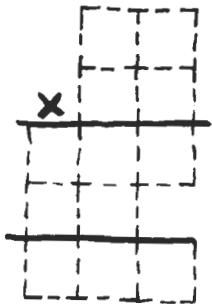
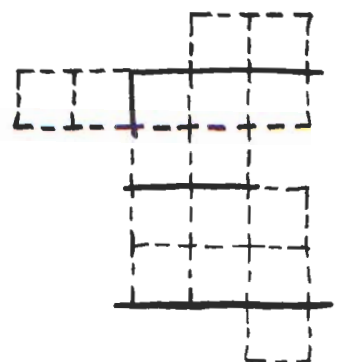
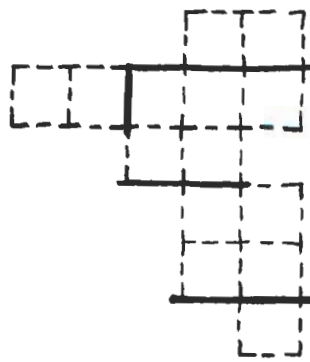
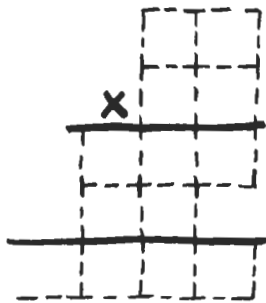
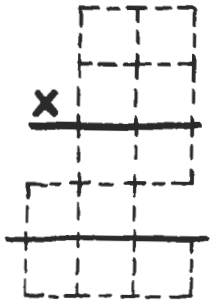
$$\begin{array}{|c|c|} \hline & \\ \hline \times & \\ \hline & 9 & 4 \\ \hline 1 & 4 & 1 \\ \hline 1 & 5 & 0 & 4 \\ \hline \end{array}$$

Now the question becomes: What two digit number can be multiplied by a number to get 94 and by another number to get 141? (Essentially, what numbers are common factors of 94 and 141?) Some trial and error will get the final result:

$$\begin{array}{|c|c|} \hline 4 & 7 \\ \hline \times & 3 & 2 \\ \hline & 9 & 4 \\ \hline 1 & 4 & 1 \\ \hline 1 & 5 & 0 & 4 \\ \hline \end{array}$$

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We could take away the "1" in the top number:

$$\begin{array}{r} \phantom{1}9 \\ \times 13 \\ \hline 4 \phantom{0} \end{array}$$

Still the problem solver could solve it — and only one solution is possible.

This is a continuation of the activities on the previous two pages. Different forms for the examples are given:

A question to explore:

Make up a problem to fit a form. What are the **fewest** clues you could give and still lead the problem solver to exactly your problem?

For example, if we make up the problem:

$$\begin{array}{r} 17 \\ \times 29 \\ \hline 153 \\ 34 \phantom{0} \\ \hline 493 \end{array}$$

we could erase all but the following:

$$\begin{array}{r} \phantom{1}9 \\ \times \phantom{1}3 \\ \hline 4 \phantom{0} \end{array}$$

The problem solver would be able to figure only one solution — the problem we originally stated.

The question is — can we take away any of those "clues" and **still** have the situation where the problem solver could only find **one** solution?

# "FROM THE LISTS" "DE LAS LISTA"

A

|                                                        |                                                        |                                                        |                                                        |                                                        |                                                        |                                                        |                                                        |                                                        |                                                        |
|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|
| $\begin{array}{r} 0 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 1 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 2 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 3 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 4 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 9 \\ \times 7 \\ \hline \end{array}$ |
|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|

B

|                                                        |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|
| $\begin{array}{r} 0 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 10 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 20 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 30 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 40 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 50 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 60 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 70 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 80 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 90 \\ \times 7 \\ \hline \end{array}$ |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|

| A                                                 | B                                                 | A+B                                                     |
|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 25 \\ \times 7 \\ \hline \end{array}$ |

|                                                   |                                                   |                                                         |
|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 18 \\ \times 7 \\ \hline \end{array}$ |
|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------------|

|                                                        |                                                         |                                                   |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------|
| $\begin{array}{r} 3 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 70 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------|

|                                                      |                                                       |                                                   |
|------------------------------------------------------|-------------------------------------------------------|---------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline 42 \end{array}$ | $\begin{array}{r} \times 7 \\ \hline 630 \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ |
|------------------------------------------------------|-------------------------------------------------------|---------------------------------------------------|

|                                                   |                                                   |                                                       |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline 224 \end{array}$ |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|

|                                                        |                                                         |                                                   |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------|
| $\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 40 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------|

|                                                   |                                                   |                                                         |
|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 59 \\ \times 7 \\ \hline \end{array}$ |
|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------------|

|                                                   |                                                   |                                                         |
|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 86 \\ \times 7 \\ \hline \end{array}$ |
|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------------|

|                                                   |                                                   |                                                       |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline 588 \end{array}$ |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|

|                                                        |                                                         |                                                   |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------|
| $\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 40 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------|

|                                                   |                                                   |                                                       |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline 273 \end{array}$ |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|

|                                                   |                                                   |                                                       |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline 497 \end{array}$ |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|

The obvious purpose behind this activity is to focus on the "multiplication and division algorithms"—not on "multiplication facts." Even the slowest computer can move ahead quickly and without errors—provided he can use the tables and carry out rather elementary additions (and subtractions).

"DE LAS LISTA" "FROM THE LISTS"

|          |          |          |          |          |          |          |          |          |          |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0        | 1        | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        |
| <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> |

|          |          |          |          |          |          |          |          |          |          |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0        | 10       | 20       | 30       | 40       | 50       | 60       | 70       | 80       | 90       |
| <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> |

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

"FROM THE LISTS" NAME \_\_\_\_\_

|   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| x | x | x | x | x | x | x | x | x | x |

|   |    |    |    |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|----|----|
| 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| x | x  | x  | x  | x  | x  | x  | x  | x  | x  |

|   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|
| x | x | x | x | x | x | x | x | x |
| x | x | x | x | x | x | x | x | x |
| x | x | x | x | x | x | x | x | x |
| x | x | x | x | x | x | x | x | x |

This can be particularized by using any common multiplier to complete the table, and indicating specific examples. Children can also make up their own examples with the simple rule that all must be different.

Or, the page can be used to explore this idea: suppose the two items were "subtracted" and the "difference" noted in the third space? For example:

We will assume the "x 7" tables from the previous page:

| A          | B          | B-A        |
|------------|------------|------------|
| 2          | 50         | 48         |
| $\times 7$ | $\times 7$ | $\times 7$ |
| 14         | 350        | 336        |

or

| A          | B          | B-A        |
|------------|------------|------------|
| 1          | 40         | 39         |
| $\times 7$ | $\times 7$ | $\times 7$ |
| 7          | 280        | 273        |

Or, perhaps the children would rather reverse the "A" and "B":

| B          | A          | B-A        |
|------------|------------|------------|
| 70         | 2          | 68         |
| $\times 7$ | $\times 7$ | $\times 7$ |
| 490        | 14         | 476        |

Could you make up examples if the tables were turned under?

Could you make up examples if the top table (A) were turned under or not filled in?

Suppose we only complete the entries in the chart for 5 thru 9 and 50 thru 90: could you still find the answer to any example (with one 2-digit factor and the "common" multiplier in the table?) Or, can you remember the smaller or easier entries?

Some might enjoy participating in a hunt for examples in which all of the digits in the third space are different and can be arranged in a consecutive sequence. Here is an example:

|            |            |            |
|------------|------------|------------|
| 2          | 50         | 52         |
| $\times 7$ | $\times 7$ | $\times 7$ |
| 14         | 350        | 364        |

2, 3, 4, 5, 6, 7

|   |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |
|---|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|
| A | $\begin{array}{r} 13 \\ \times 1 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 2 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 3 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 4 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 5 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 6 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 8 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 9 \\ \hline \end{array}$ |
|---|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|

|   |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |
|---|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|
| B | $\begin{array}{r} 13 \\ \times 10 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 20 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 30 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 40 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 50 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 60 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 70 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 80 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 90 \\ \hline \end{array}$ |
|---|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|

|                                                                                                                     |                                                                                |                                                                                |                                                                                |
|---------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------|--------------------------------------------------------------------------------|--------------------------------------------------------------------------------|
| $\begin{array}{r} 13 \\ \times 42 \\ \hline \end{array}$<br>-----<br>+ A<br>-----<br>+ B<br>-----<br>+ A+B<br>----- | $\begin{array}{r} 13 \\ \times 67 \\ \hline \end{array}$<br>-----<br><br>----- | $\begin{array}{r} 13 \\ \times 58 \\ \hline \end{array}$<br>-----<br><br>----- | $\begin{array}{r} 13 \\ \times 16 \\ \hline \end{array}$<br>-----<br><br>----- |
| $\begin{array}{r} 13 \\ \times 95 \\ \hline \end{array}$<br>-----<br><br>-----                                      | $\begin{array}{r} 13 \\ \times 29 \\ \hline \end{array}$<br>-----<br><br>----- | $\begin{array}{r} 13 \\ \times 83 \\ \hline \end{array}$<br>-----<br><br>----- | $\begin{array}{r} 13 \\ \times 78 \\ \hline \end{array}$<br>-----<br><br>----- |
| $\begin{array}{r} 13 \\ \times 84 \\ \hline \end{array}$<br>-----<br><br>-----                                      | $\begin{array}{r} 13 \\ \times 38 \\ \hline \end{array}$<br>-----<br><br>----- | $\begin{array}{r} 13 \\ \times 97 \\ \hline \end{array}$<br>-----<br><br>----- | $\begin{array}{r} 13 \\ \times 79 \\ \hline \end{array}$<br>-----<br><br>----- |

|   |  | NAME _____                                               |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |  |
|---|--|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|--|
| A |  | $\begin{array}{r} 13 \\ \times 1 \\ \hline \end{array}$  | $\begin{array}{r} 13 \\ \times 2 \\ \hline \end{array}$  | $\begin{array}{r} 13 \\ \times 3 \\ \hline \end{array}$  | $\begin{array}{r} 13 \\ \times 4 \\ \hline \end{array}$  | $\begin{array}{r} 13 \\ \times 5 \\ \hline \end{array}$  | $\begin{array}{r} 13 \\ \times 6 \\ \hline \end{array}$  | $\begin{array}{r} 13 \\ \times 7 \\ \hline \end{array}$  | $\begin{array}{r} 13 \\ \times 8 \\ \hline \end{array}$  | $\begin{array}{r} 13 \\ \times 9 \\ \hline \end{array}$  |  |
|   |  | $\begin{array}{r} 13 \\ \times 10 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 20 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 30 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 40 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 50 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 60 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 70 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 80 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 90 \\ \hline \end{array}$ |  |
| B |  | $\begin{array}{r} 13 \\ \times 10 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 20 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 30 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 40 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 50 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 60 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 70 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 80 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 90 \\ \hline \end{array}$ |  |
|   |  | $\begin{array}{r} 13 \\ \times 10 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 20 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 30 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 40 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 50 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 60 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 70 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 80 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 90 \\ \hline \end{array}$ |  |

|                                                                                                |                                                                                     |                                                                                     |                                                                                     |
|------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| $\begin{array}{r} 13 \\ \times 42 \\ \hline \end{array}$<br>----- +A<br>----- +B<br>----- +A+B | $\begin{array}{r} 13 \\ \times 67 \\ \hline \end{array}$<br>-----<br>-----<br>----- | $\begin{array}{r} 13 \\ \times 58 \\ \hline \end{array}$<br>-----<br>-----<br>----- | $\begin{array}{r} 13 \\ \times 16 \\ \hline \end{array}$<br>-----<br>-----<br>----- |
| $\begin{array}{r} 13 \\ \times 95 \\ \hline \end{array}$<br>-----<br>-----<br>-----            | $\begin{array}{r} 13 \\ \times 29 \\ \hline \end{array}$<br>-----<br>-----<br>----- | $\begin{array}{r} 13 \\ \times 83 \\ \hline \end{array}$<br>-----<br>-----<br>----- | $\begin{array}{r} 13 \\ \times 78 \\ \hline \end{array}$<br>-----<br>-----<br>----- |
| $\begin{array}{r} 13 \\ \times 84 \\ \hline \end{array}$<br>-----<br>-----<br>-----            | $\begin{array}{r} 13 \\ \times 38 \\ \hline \end{array}$<br>-----<br>-----<br>----- | $\begin{array}{r} 13 \\ \times 97 \\ \hline \end{array}$<br>-----<br>-----<br>----- | $\begin{array}{r} 13 \\ \times 79 \\ \hline \end{array}$<br>-----<br>-----<br>----- |

"From the list" grows up. The "common factor" now appears in the top of each entry box.

Now the advantages of knowing several ways to get each entry offers opportunities for short cuts and for checking.

Since the "addition" aspect of the activity has become more complex, a new format is introduced.

$$\begin{array}{r}
 13 \\
 \times 42 \\
 \hline
 26 \leftarrow A \\
 520 \leftarrow B \\
 \hline
 546 \leftarrow A + B
 \end{array}$$

Some students may want to demonstrate their ability to work with only list B or with neither list. They can fold under as much of the page as they like or leave the lists blank.

Others may have the same urge, but feel more comfortable with the list available for checking.

Others have a very unsure grasp on the basic "multiplication skills" involved and would make a discouraging number of errors. By using the lists, most

errors are eliminated, and they will increase their familiarity with facts they have not yet committed to memory.

In this way, all members of the group can focus on the mechanics of the algorithm without being unduly distracted or burdened . . . and each at his own level of development.

Students ought to save this page so that after they have completed the next page, they can compare the two pages.

AB

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NAME \_\_\_\_\_

|   |                        |                        |                        |                        |                        |                        |                        |                        |                        |
|---|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| A | $\frac{13}{\times 1}$  | $\frac{13}{\times 2}$  | $\frac{13}{\times 3}$  | $\frac{13}{\times 4}$  | $\frac{13}{\times 5}$  | $\frac{13}{\times 6}$  | $\frac{13}{\times 7}$  | $\frac{13}{\times 8}$  | $\frac{13}{\times 9}$  |
| B | $\frac{13}{\times 10}$ | $\frac{13}{\times 20}$ | $\frac{13}{\times 30}$ | $\frac{13}{\times 40}$ | $\frac{13}{\times 50}$ | $\frac{13}{\times 60}$ | $\frac{13}{\times 70}$ | $\frac{13}{\times 80}$ | $\frac{13}{\times 90}$ |

|                                                                                                                                         |                                                                                                   |                                                                                                    |                                                                                                    |
|-----------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad +A \\ \quad \quad \quad +B \\ \hline 546 \quad +A+B \end{array}$ | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 871 \end{array}$ | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 754 \end{array}$  | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 208 \end{array}$  |
| $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 1235 \end{array}$                                      | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 377 \end{array}$ | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 1079 \end{array}$ | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 1014 \end{array}$ |
| $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 1092 \end{array}$                                      | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 494 \end{array}$ | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 1261 \end{array}$ | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 1027 \end{array}$ |

A comparison with the previous page shows that all examples from that page are repeated here, but in their equivalent "division" mode.

The problem is to find two entries, one from each list whose sum is indicated already.

$$\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \leftarrow A \\ \hline \quad \quad \quad \leftarrow B \\ \hline 546 \quad \leftarrow A + B \end{array}$$

A little study of this problem leads to the strategy of beginning by finding the largest number in list B that is not larger than the final sum. (This is often referred to as "finding the trial divisor.")

In this case, the number from list B is 520:

$$\begin{array}{r} 13 \\ \times 40 \\ \hline \quad \quad \quad \leftarrow A \\ \hline 520 \quad \leftarrow B \\ \hline 546 \quad \leftarrow A + B \end{array}$$

That entry is  $13 \times 40$  (or  $40 \times 13$ ), so "4" is written in the "tens place."

How many more to be accounted for? . . . The difference between 546 and 520—or 26. That number occurs in list A— $13 \times 2 = 26$  (or  $2 \times 13 = 26$ ), so the example is completed.

$$\begin{array}{r} 13 \\ \times 42 \\ \hline 26 \quad \leftarrow A \\ \hline 520 \quad \leftarrow B \\ \hline 546 \quad \leftarrow A + B \end{array}$$

Compare this procedure with the standard division algorithm:

$$\begin{array}{r} 4 \\ 13 \overline{) 546} \\ \underline{520} \\ 26 \end{array} \rightarrow \begin{array}{r} 42 \\ 13 \overline{) 546} \\ \underline{520} \\ 26 \\ \underline{26} \end{array}$$

The steps and reasons for each are precisely those called for in the activities on this page.

If students compare this page with the previous page they will see that the only difference, example for example, is in the information that was given . . . and that is the difference between multiplication and division.

Multiplication: Given two factors, find the product.

Division: Given the product and one factor, find the other factor.

AB

$\leftarrow A$   
 $\leftarrow B$   
 $\leftarrow A+B$

| NAME _____ |                        |                        |                        |                        |                        |                        |                        |                        |                        |
|------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| A          | $\overline{\times 1}$  | $\overline{\times 2}$  | $\overline{\times 3}$  | $\overline{\times 4}$  | $\overline{\times 5}$  | $\overline{\times 6}$  | $\overline{\times 7}$  | $\overline{\times 8}$  | $\overline{\times 9}$  |
| B          | $\overline{\times 10}$ | $\overline{\times 20}$ | $\overline{\times 30}$ | $\overline{\times 40}$ | $\overline{\times 50}$ | $\overline{\times 60}$ | $\overline{\times 70}$ | $\overline{\times 80}$ | $\overline{\times 90}$ |

|                                                                                                                    |                                                                                        |                                                                                        |                                                                                        |
|--------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|
| $\begin{array}{r} \overline{\times 42} \\ \hline \text{---} + A \\ \text{---} + B \\ \text{---} + A+B \end{array}$ | $\begin{array}{r} \overline{\times 67} \\ \hline \text{---} \\ \text{---} \end{array}$ | $\begin{array}{r} \overline{\times 51} \\ \hline \text{---} \\ \text{---} \end{array}$ | $\begin{array}{r} \overline{\times 16} \\ \hline \text{---} \\ \text{---} \end{array}$ |
| $\begin{array}{r} \overline{\times 95} \\ \hline \text{---} \\ \text{---} \end{array}$                             | $\begin{array}{r} \overline{\times 89} \\ \hline \text{---} \\ \text{---} \end{array}$ | $\begin{array}{r} \overline{\times 83} \\ \hline \text{---} \\ \text{---} \end{array}$ | $\begin{array}{r} \overline{\times 79} \\ \hline \text{---} \\ \text{---} \end{array}$ |
| $\begin{array}{r} \overline{\times 84} \\ \hline \text{---} \\ \text{---} \end{array}$                             | $\begin{array}{r} \overline{\times 38} \\ \hline \text{---} \\ \text{---} \end{array}$ | $\begin{array}{r} \overline{\times 97} \\ \hline \text{---} \\ \text{---} \end{array}$ | $\begin{array}{r} \overline{\times 78} \\ \hline \text{---} \\ \text{---} \end{array}$ |

The second table (B) is so easily made up by referring to the first table (A), it can become an easy bit of arithmetic. Multiplying by 10 is accomplished by including another "0" on the right side of the other factor. This has the effect on the number of "moving it over 1 place":  $1700 \times 10 = 17000$ .

Any 2-digit or 3-digit number can be selected as the missing factor (multiplicand).

$$\begin{array}{r}
 425 \\
 \times 42 \\
 \hline
 850 \quad \leftarrow \text{from table A} \\
 17000 \quad \leftarrow \text{from table B} \\
 \hline
 17850
 \end{array}$$

At some point, the question of using a single table can be explored. What problems arise?

In the example above, such "moving over" can be shown by omitting the "0" in the units place of all entries in table B. The example given above would then appear this way:

$$\begin{array}{r}
 425 \\
 \times 42 \\
 \hline
 850 \quad \leftarrow A \\
 1700 \quad \leftarrow B \\
 \hline
 17850
 \end{array}$$

and this is the standard form.

|   |            |            |            |            |            |            |            |            |            |
|---|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| A | <u>x 1</u> | <u>x 2</u> | <u>x 3</u> | <u>x 4</u> | <u>x 5</u> | <u>x 6</u> | <u>x 7</u> | <u>x 8</u> | <u>x 9</u> |
|---|------------|------------|------------|------------|------------|------------|------------|------------|------------|

|   |             |             |             |             |             |             |             |             |             |
|---|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| B | <u>x 10</u> | <u>x 20</u> | <u>x 30</u> | <u>x 40</u> | <u>x 50</u> | <u>x 60</u> | <u>x 70</u> | <u>x 80</u> | <u>x 90</u> |
|---|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|

|                                                           |                                    |                                    |                                    |
|-----------------------------------------------------------|------------------------------------|------------------------------------|------------------------------------|
| <u>x</u><br>----- ← A<br><u>      </u> ← B<br>----- ← A+B | <u>x</u><br>-----<br><u>      </u> | <u>x</u><br>-----<br><u>      </u> | <u>x</u><br>-----<br><u>      </u> |
| <u>x</u><br>-----<br><u>      </u>                        | <u>x</u><br>-----<br><u>      </u> | <u>x</u><br>-----<br><u>      </u> | <u>x</u><br>-----<br><u>      </u> |
| <u>x</u><br>-----<br><u>      </u>                        | <u>x</u><br>-----<br><u>      </u> | <u>x</u><br>-----<br><u>      </u> | <u>x</u><br>-----<br><u>      </u> |



|   |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |
|---|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|
| A | $\begin{array}{r} 27 \\ \times 1 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 2 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 3 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 4 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 5 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 6 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 8 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 9 \\ \hline \end{array}$ |
|---|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|

|   |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |
|---|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|
| B | $\begin{array}{r} 27 \\ \times 10 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 20 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 30 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 40 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 50 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 60 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 70 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 80 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 90 \\ \hline \end{array}$ |
|---|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|

$$\begin{array}{r} 27 \overline{) 1917} \\ B \rightarrow \end{array}$$

$$\begin{array}{r} 27 \overline{) 2214} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 459} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 1566} \\ \hline \end{array}$$

$$\begin{array}{r} A \rightarrow \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 1701} \\ \hline \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 756} \\ \hline \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 2538} \\ \hline \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 1053} \\ \hline \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 2565} \\ \hline \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 1539} \\ \hline \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 2322} \\ \hline \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 1323} \\ \hline \text{---} \\ \hline \end{array}$$

|   |           | NAME _____ |           |           |           |           |           |           |           |  |  |
|---|-----------|------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|--|--|
| A | 27<br>x1  | 27<br>x2   | 27<br>x3  | 27<br>x4  | 27<br>x5  | 27<br>x6  | 27<br>x7  | 27<br>x8  | 27<br>x9  |  |  |
|   |           |            |           |           |           |           |           |           |           |  |  |
| B | 27<br>x10 | 27<br>x20  | 27<br>x30 | 27<br>x40 | 27<br>x50 | 27<br>x60 | 27<br>x70 | 27<br>x80 | 27<br>x90 |  |  |
|   |           |            |           |           |           |           |           |           |           |  |  |

|                                                                                                           |                                                                                                           |                                                                                                           |                                                                                                           |
|-----------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 27 \overline{) 1917} \\ \underline{B \rightarrow} \\ \text{A} \rightarrow \end{array}$  | $\begin{array}{r} 27 \overline{) 2214} \\ \underline{\phantom{B \rightarrow}} \\ \phantom{A} \end{array}$ | $\begin{array}{r} 27 \overline{) 459} \\ \underline{\phantom{B \rightarrow}} \\ \phantom{A} \end{array}$  | $\begin{array}{r} 27 \overline{) 1566} \\ \underline{\phantom{B \rightarrow}} \\ \phantom{A} \end{array}$ |
| $\begin{array}{r} 27 \overline{) 1701} \\ \underline{\phantom{B \rightarrow}} \\ \phantom{A} \end{array}$ | $\begin{array}{r} 27 \overline{) 756} \\ \underline{\phantom{B \rightarrow}} \\ \phantom{A} \end{array}$  | $\begin{array}{r} 27 \overline{) 2538} \\ \underline{\phantom{B \rightarrow}} \\ \phantom{A} \end{array}$ | $\begin{array}{r} 27 \overline{) 1053} \\ \underline{\phantom{B \rightarrow}} \\ \phantom{A} \end{array}$ |
| $\begin{array}{r} 27 \overline{) 2565} \\ \underline{\phantom{B \rightarrow}} \\ \phantom{A} \end{array}$ | $\begin{array}{r} 27 \overline{) 1539} \\ \underline{\phantom{B \rightarrow}} \\ \phantom{A} \end{array}$ | $\begin{array}{r} 27 \overline{) 2322} \\ \underline{\phantom{B \rightarrow}} \\ \phantom{A} \end{array}$ | $\begin{array}{r} 27 \overline{) 1323} \\ \underline{\phantom{B \rightarrow}} \\ \phantom{A} \end{array}$ |

The actual operations are the same as previous pages, but the format is changed—using the standard long-division notation.

As before, the first task is to complete the tables at the top of the page.

There are 4 entries that require a minimum of effort:

$$\begin{array}{r} 27 \\ \times 1 \\ \hline 27 \end{array} \quad \begin{array}{r} 27 \\ \times 2 \\ \hline 54 \end{array} \quad \begin{array}{r} 27 \\ \times 4 \\ \hline 108 \end{array} \quad \begin{array}{r} 27 \\ \times 8 \\ \hline 216 \end{array}$$

This is a familiar “doubling” series.

The first 2 entries can be “added,” that entry doubled—and the two new entries added:

$$(27 + 54) \rightarrow \begin{array}{r} 27 \\ \times 3 \\ \hline 81 \end{array} \quad \begin{array}{r} 27 \\ \times 6 \\ \hline 162 \end{array} \quad \begin{array}{r} 27 \\ \times 9 \\ \hline 243 \end{array}$$

Leaving only  $27 \times 5$  and  $27 \times 7$ . The first one can be found by adding 27 to  $27 \times 4$ :  $108 + 27 = 135$ . The second is,  $27 \times 6 + 27$ ;  $162 + 27 = 189$ .

The second table can be completed as fast as the products can be written after glances at the first table.

And, of course, there are many alternatives to the shortcuts we took above.

The first problem asks: how many 27's in 1917? The second table quickly reveals a good approximation: there are seventy 27's in 1917 and a little more. We can write down that partial answer and find out how many more of those 1917 we must still account for

$$\begin{array}{r} \text{list B} \rightarrow \begin{array}{r} 27 \overline{) 1917} \\ \underline{1890} \\ 27 \\ 27 \end{array} \quad \text{or} \quad \begin{array}{r} \begin{array}{c} 71 \\ 70 \end{array} \overline{) 1917} \\ \underline{1890} \\ 27 \\ 27 \end{array} \end{array}$$

and complete the example as shown with broken-line numbers. The first method is standard; the second may help as a brief transitional method of keeping track.

Notice that by the use of tables the children have worked out in advance, the need for the rather painful and inefficient “scaffold” or “ladder” algorism is avoided . . . to the great relief of teachers and parents, as well as children.

Because all necessary multiplications are carried out in advance, this activity focuses in an uncluttered way on the algorism itself: the possibility for careless or computational error is minimized. The children have a far greater opportunity to understand the process—the “way it works” and “why it works.”

|   |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |
|---|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|
| A | $\begin{array}{r} 85 \\ \times 1 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 2 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 3 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 4 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 5 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 6 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 8 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 9 \\ \hline \end{array}$ |
|   |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |

|   |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |
|---|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|
| B | $\begin{array}{r} 85 \\ \times 10 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 20 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 30 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 40 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 50 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 60 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 70 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 80 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 90 \\ \hline \end{array}$ |
|   |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |

|                        |                                                                               |                        |                                                                               |
|------------------------|-------------------------------------------------------------------------------|------------------------|-------------------------------------------------------------------------------|
| $85 \overline{) 1955}$ | $85 \overline{) \quad \quad \quad} \begin{array}{r} 74 \\ \hline \end{array}$ | $85 \overline{) 3060}$ | $85 \overline{) \quad \quad \quad} \begin{array}{r} 85 \\ \hline \end{array}$ |
| B →                    |                                                                               |                        |                                                                               |
|                        |                                                                               |                        |                                                                               |
| A →                    |                                                                               |                        |                                                                               |
|                        |                                                                               |                        |                                                                               |

|                        |                                                                               |                        |                                                                               |
|------------------------|-------------------------------------------------------------------------------|------------------------|-------------------------------------------------------------------------------|
| $85 \overline{) 4080}$ | $85 \overline{) \quad \quad \quad} \begin{array}{r} 96 \\ \hline \end{array}$ | $85 \overline{) 4420}$ | $85 \overline{) \quad \quad \quad} \begin{array}{r} 79 \\ \hline \end{array}$ |
|                        |                                                                               |                        |                                                                               |
|                        |                                                                               |                        |                                                                               |
|                        |                                                                               |                        |                                                                               |
|                        |                                                                               |                        |                                                                               |

|                        |                                                                               |                        |                                                                               |
|------------------------|-------------------------------------------------------------------------------|------------------------|-------------------------------------------------------------------------------|
| $85 \overline{) 7310}$ | $85 \overline{) \quad \quad \quad} \begin{array}{r} 78 \\ \hline \end{array}$ | $85 \overline{) 8245}$ | $85 \overline{) \quad \quad \quad} \begin{array}{r} 64 \\ \hline \end{array}$ |
|                        |                                                                               |                        |                                                                               |
|                        |                                                                               |                        |                                                                               |
|                        |                                                                               |                        |                                                                               |
|                        |                                                                               |                        |                                                                               |



|   |                                                | NAME _____                                     |                                                |                                                |                                                |                                                |                                                |                                                |                                                |
|---|------------------------------------------------|------------------------------------------------|------------------------------------------------|------------------------------------------------|------------------------------------------------|------------------------------------------------|------------------------------------------------|------------------------------------------------|------------------------------------------------|
| A | $\begin{array}{r} 85 \\ \times 1 \end{array}$  | $\begin{array}{r} 85 \\ \times 2 \end{array}$  | $\begin{array}{r} 85 \\ \times 3 \end{array}$  | $\begin{array}{r} 85 \\ \times 4 \end{array}$  | $\begin{array}{r} 85 \\ \times 5 \end{array}$  | $\begin{array}{r} 85 \\ \times 6 \end{array}$  | $\begin{array}{r} 85 \\ \times 7 \end{array}$  | $\begin{array}{r} 85 \\ \times 8 \end{array}$  | $\begin{array}{r} 85 \\ \times 9 \end{array}$  |
|   |                                                |                                                |                                                |                                                |                                                |                                                |                                                |                                                |                                                |
| B | $\begin{array}{r} 85 \\ \times 10 \end{array}$ | $\begin{array}{r} 85 \\ \times 20 \end{array}$ | $\begin{array}{r} 85 \\ \times 30 \end{array}$ | $\begin{array}{r} 85 \\ \times 40 \end{array}$ | $\begin{array}{r} 85 \\ \times 50 \end{array}$ | $\begin{array}{r} 85 \\ \times 60 \end{array}$ | $\begin{array}{r} 85 \\ \times 70 \end{array}$ | $\begin{array}{r} 85 \\ \times 80 \end{array}$ | $\begin{array}{r} 85 \\ \times 90 \end{array}$ |
|   |                                                |                                                |                                                |                                                |                                                |                                                |                                                |                                                |                                                |

|                                                                       |                                                                        |                                                    |                                                                        |
|-----------------------------------------------------------------------|------------------------------------------------------------------------|----------------------------------------------------|------------------------------------------------------------------------|
| $\begin{array}{r} 85 \overline{)1955} \\ B+ \\ \hline A+ \end{array}$ | $\begin{array}{r} 74 \\ 85 \overline{) \quad \quad \quad} \end{array}$ | $\begin{array}{r} 85 \overline{)3060} \end{array}$ | $\begin{array}{r} 85 \overline{) \quad \quad \quad} \end{array}$       |
| $\begin{array}{r} 85 \overline{)4080} \\ \hline \hline \end{array}$   | $\begin{array}{r} 96 \\ 85 \overline{) \quad \quad \quad} \end{array}$ | $\begin{array}{r} 85 \overline{)4420} \end{array}$ | $\begin{array}{r} 79 \\ 85 \overline{) \quad \quad \quad} \end{array}$ |
| $\begin{array}{r} 85 \overline{)7310} \\ \hline \hline \end{array}$   | $\begin{array}{r} 78 \\ 85 \overline{) \quad \quad \quad} \end{array}$ | $\begin{array}{r} 85 \overline{)8245} \end{array}$ | $\begin{array}{r} 64 \\ 85 \overline{) \quad \quad \quad} \end{array}$ |

Two columns of the examples use the usual long-division format to ask "division" questions. The other columns use the same form but ask "multiplication" questions. Using the same forms and the same tables can you answer both kinds of questions?

$$85 \overline{)1955} \quad \text{and} \quad 85 \overline{) \quad \quad \quad}^{74}$$

If you put 1955 books on 85 shelves with the same number of books on each shelf, how many would there be on each shelf?

If you had 85 shelves and 74 books on each shelf, how many books would you have?

How can you use the same tables (multiples of 85) to answer these two different questions?

The flow of these two related algorithms and their close relationships are usually lost in a heavy sea of difficult and incidental computations.

If a child has difficulties with the question of

$$\begin{array}{r} 85 \\ \times 4 \end{array} \quad \text{and} \quad \begin{array}{r} 85 \\ \times 70 \end{array}$$

how could he understand the standard algorithm for finding the answer or product:

$$\begin{array}{r} 85 \\ \times 74 \\ \hline \end{array}$$

If a child has difficulty with the questions of

$$\begin{array}{r} 85 \\ \times 8 \end{array} \quad \text{and} \quad \begin{array}{r} 85 \\ \times 40 \end{array}$$

how could he understand the "why" and "how" of the standard algorithm for finding the answer or quotient.

$$85 \overline{)4080}$$

which when completed in standard form is

$$\begin{array}{r} 48 \\ 85 \overline{)4080} \\ \underline{340} \phantom{00} \\ 680 \\ \underline{680} \\ 0 \end{array}$$

If the focus is on the algorithm, then remove every distraction and all complexity to let the central problem be the central task.

If our goal is friendliness with numbers, we will remove every possible threat that would distract any child from the particular we are looking at. When he has created necessary multiplication tables in advance, then, and only then for many children, does a division example become a reasonable problem.

A

|           |           |           |           |           |           |           |           |           |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| <u>x1</u> | <u>x2</u> | <u>x3</u> | <u>x4</u> | <u>x5</u> | <u>x6</u> | <u>x7</u> | <u>x8</u> | <u>x9</u> |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|

B

|            |            |            |            |            |            |            |            |            |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| <u>x10</u> | <u>x20</u> | <u>x30</u> | <u>x40</u> | <u>x50</u> | <u>x60</u> | <u>x70</u> | <u>x80</u> | <u>x90</u> |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|

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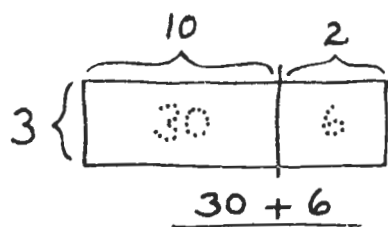
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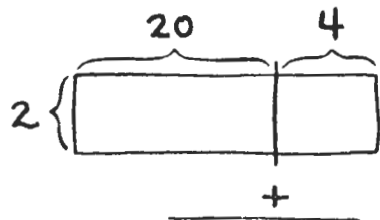
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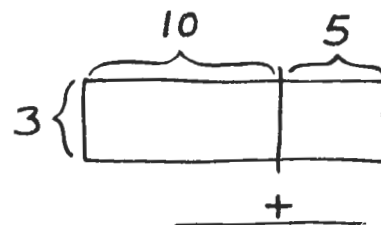


$$\begin{array}{r} 12 \\ \times 3 \\ \hline \end{array}$$

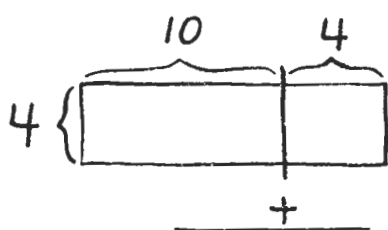
$$\begin{array}{r} 12 \\ 3 \overline{) 36} \end{array}$$



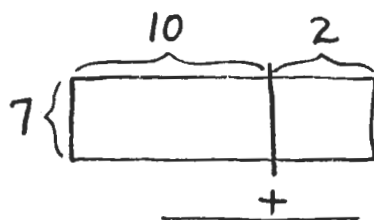
$$\begin{array}{r} \times \\ \hline \end{array}$$



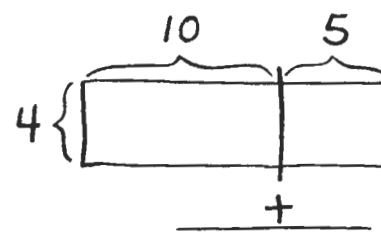
$$\begin{array}{r} \times \\ \hline \end{array}$$



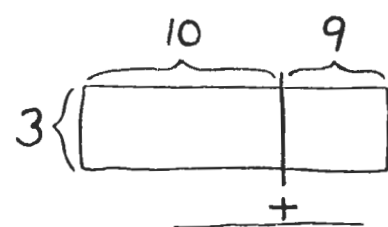
$$\begin{array}{r} \times \\ \hline \end{array}$$



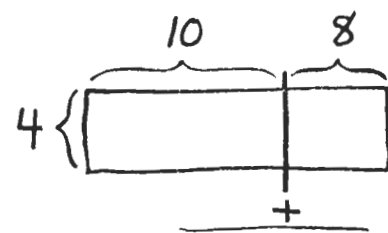
$$\begin{array}{r} \times \\ \hline \end{array}$$



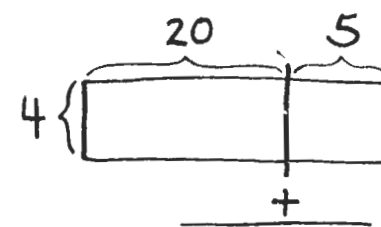
$$\begin{array}{r} \times \\ \hline \end{array}$$



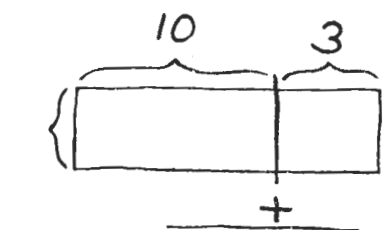
$$\begin{array}{r} \times \\ \hline \end{array}$$



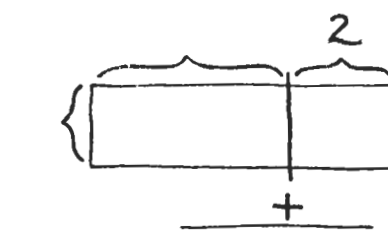
$$\begin{array}{r} \times \\ \hline \end{array}$$



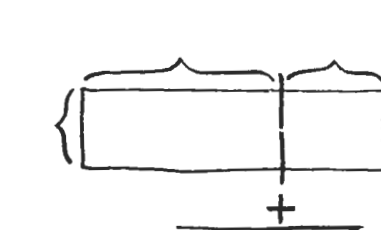
$$\begin{array}{r} \times \\ \hline \end{array}$$



$$\begin{array}{r} 13 \\ \times 8 \\ \hline \end{array}$$



$$\begin{array}{r} 12 \\ \times 9 \\ \hline \end{array}$$



$$\begin{array}{r} 18 \\ \times 6 \\ \hline \end{array}$$

NAME \_\_\_\_\_

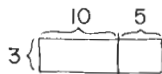
|                                                                                                                                                                                      |                                                                                                                                                |                                                                                                                                              |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 3 \left\{ \begin{array}{ c c } \hline 10 & 2 \\ \hline \end{array} \right. \\ \hline 30 + 6 \\ \hline \times \frac{12}{3} \\ \hline 3 \overline{) 36} \end{array}$ | $\begin{array}{r} 2 \left\{ \begin{array}{ c c } \hline 20 & 4 \\ \hline \end{array} \right. \\ \hline + \\ \hline \times \end{array}$         | $\begin{array}{r} 3 \left\{ \begin{array}{ c c } \hline 10 & 5 \\ \hline \end{array} \right. \\ \hline + \\ \hline \times \end{array}$       |
| $\begin{array}{r} 4 \left\{ \begin{array}{ c c } \hline 10 & 4 \\ \hline \end{array} \right. \\ \hline + \\ \hline \times \end{array}$                                               | $\begin{array}{r} 7 \left\{ \begin{array}{ c c } \hline 10 & 2 \\ \hline \end{array} \right. \\ \hline + \\ \hline \times \end{array}$         | $\begin{array}{r} 4 \left\{ \begin{array}{ c c } \hline 10 & 5 \\ \hline \end{array} \right. \\ \hline + \\ \hline \times \end{array}$       |
| $\begin{array}{r} 3 \left\{ \begin{array}{ c c } \hline 10 & 9 \\ \hline \end{array} \right. \\ \hline + \\ \hline \times \end{array}$                                               | $\begin{array}{r} 4 \left\{ \begin{array}{ c c } \hline 10 & 8 \\ \hline \end{array} \right. \\ \hline + \\ \hline \times \end{array}$         | $\begin{array}{r} 4 \left\{ \begin{array}{ c c } \hline 20 & 5 \\ \hline \end{array} \right. \\ \hline + \\ \hline \times \end{array}$       |
| $\begin{array}{r} \left\{ \begin{array}{ c c } \hline 10 & 3 \\ \hline \end{array} \right. \\ \hline + \\ \hline \times \frac{13}{8} \end{array}$                                    | $\begin{array}{r} \left\{ \begin{array}{ c c } \hline & 2 \\ \hline \end{array} \right. \\ \hline + \\ \hline \times \frac{12}{9} \end{array}$ | $\begin{array}{r} \left\{ \begin{array}{ c c } \hline & \\ \hline \end{array} \right. \\ \hline + \\ \hline \times \frac{18}{6} \end{array}$ |

This page helps the learner build a useful technique for cutting some multiplication examples "down to size".

We have already established that:

$$\begin{array}{r} 15 \\ \times 3 \\ \hline \end{array}$$

in terms of real things means three rows with 15 counters in each row. As we look at such an array, we notice that there is a convenient way to break it up to count the total number of counters. We can see three rows of 10 and then 3 rows of 5, as in this diagram.



It is easy to count each group.  $3 \times 10$  is 30 and  $3 \times 5$  is 15, so there are  $30 + 15$ , or 45 counters in the array.

Essentially we have said that

$$3 \times 15 = 3(10 + 5) = 3 \times 10 + 3 \times 5 = 30 + 15 = 45$$

In formal terms, this is the "distributive principle of multiplication over addition", and it is one of the least obvious of the basic principles of arithmetic. We prefer to let children do many examples of the type on this page and have lots of experiences where they build some real referents for this principle before they ever hear the words "distributive principle". In fact at this level, if the need arises to give a name to this new "rule", children ought to think of a name that makes sense to them.

This principle is quite useful. Essentially it tells us: If we don't know the product of 15 and 3, we can "regroup" 15 in any way we want, multiply each new group by 3, and finally add the two results to find the product of 15 and 3. Because multiplication by 10 and multiples of 10 are usually easy, it is convenient to "regroup" the number into 10 or a multiple of 10 and some other number. (e.g.,  $18 = 10 + 8$ ,  $26 = 20 + 6$ ,  $32 = 30 + 2$ .)

Thus, to multiply

$$\begin{array}{r} 24 \\ \times 3 \\ \hline \end{array}$$

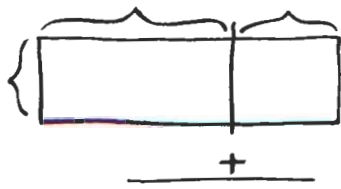
it is convenient to think:  $3 \times 20 = 60$ ,  $3 \times 4 = 12$ ,  $60 + 12 = 72$

so 
$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \end{array}$$

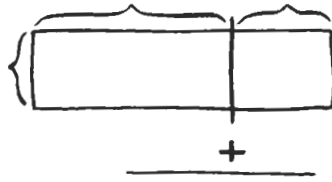
On this page there are diagrams and record-keeping devices. The learner records the number of counters in each group shown in the array and then writes the multiplication and division he sees illustrated by the array. The first example indicates how the record-keeping devices are used.

In the last row, there is a new twist to the examples. The multiplication fact is given and some bit of information about the array. The learner must figure the missing information.

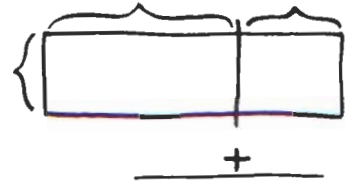
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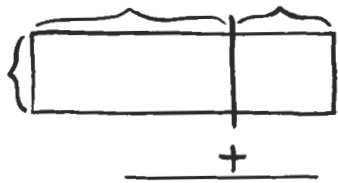
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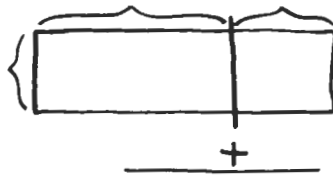
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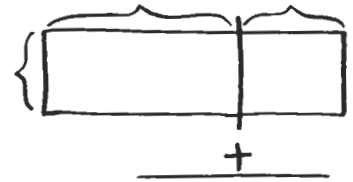
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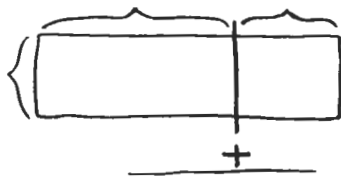
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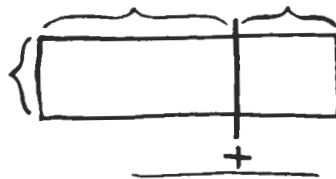
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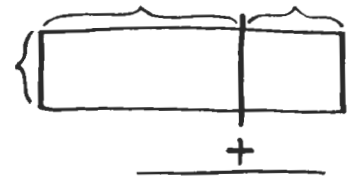
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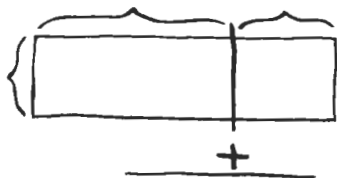
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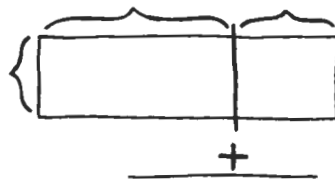
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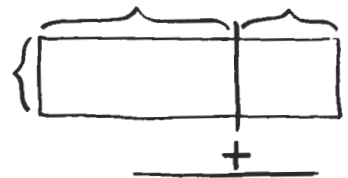
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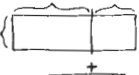
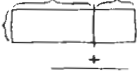
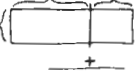
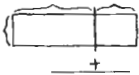
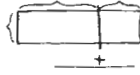
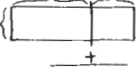
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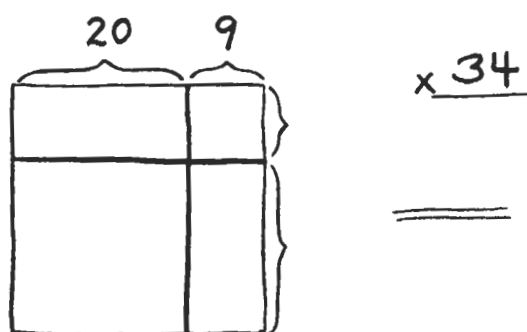
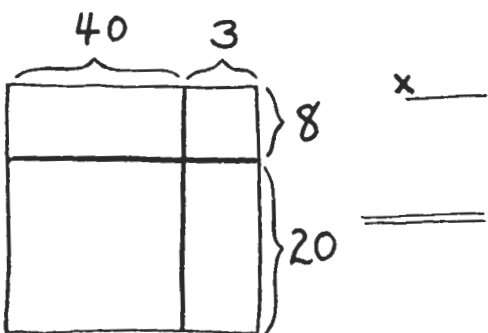
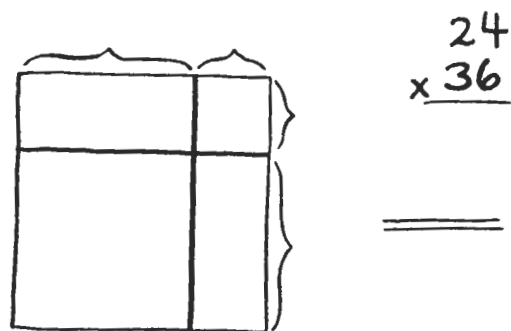
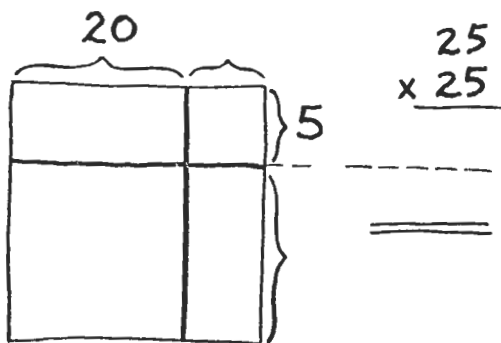
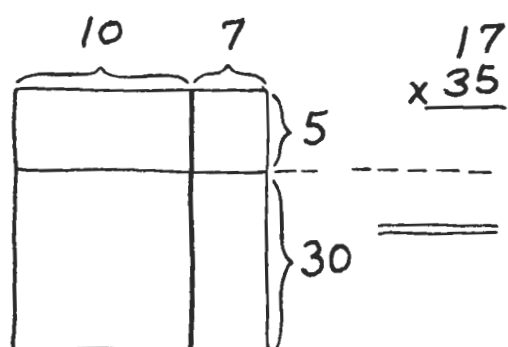
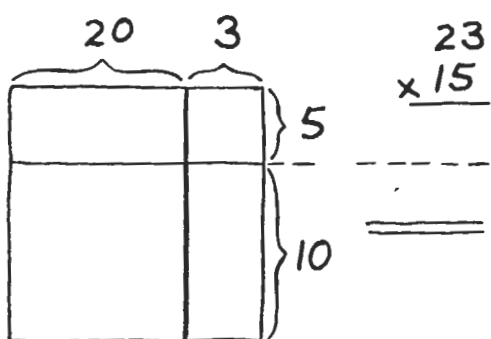
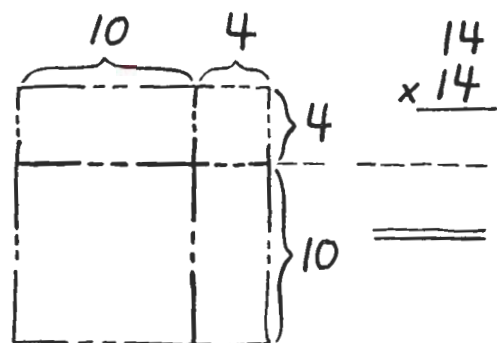
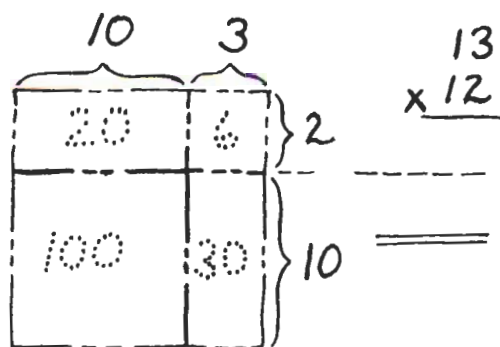
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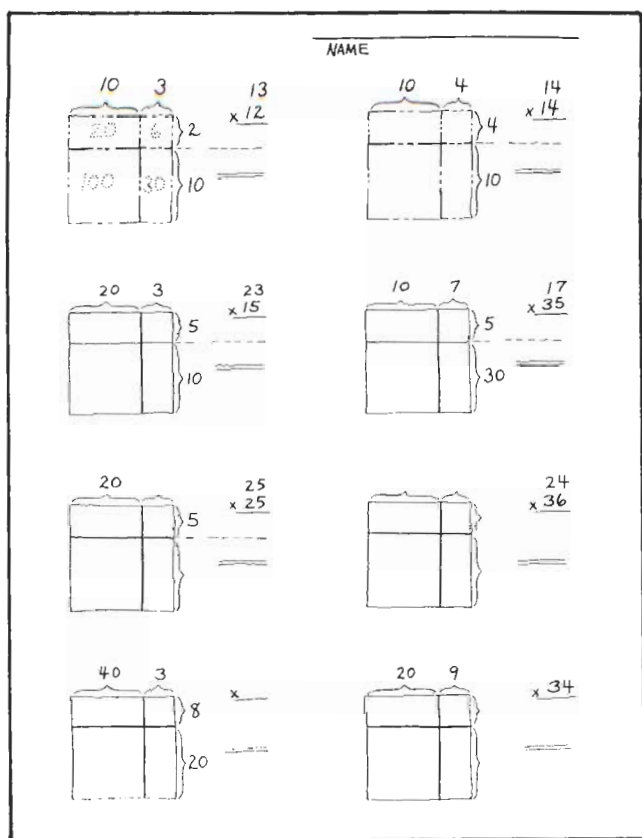
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| NAME _____                                                                        |                                                                                   |                                                                                   |
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| X _____                                                                           | X _____                                                                           | X _____                                                                           |
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| X _____                                                                           | X _____                                                                           | X _____                                                                           |
|  |  |  |
| X _____                                                                           | X _____                                                                           | X _____                                                                           |
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| X _____                                                                           | X _____                                                                           | X _____                                                                           |

This page is a continuation of activities on the previous page. Learners can make up their own examples and problems. It could also be particularized by the teacher.







On this page we extend the activities of the previous pages and use rough sketches to help the learner understand the multiplication algorithm more fully. We make use of our discovery that in multiplication a number can be "regrouped" in any way that makes the multiplication easier. Here, we essentially lead the learner to do that twice. For example, if we don't know how much  $13 \times 12$  is, we can think of the problem as  $13 \times (10 + 2)$ , or  $(13 \times 10) + (13 \times 2)$ .  $13 \times 10$  is easy. If we don't know  $13 \times 2$ , we can apply the new rule again.  $13 \times 2 = (10 + 3) \times 2$ , or  $(10 \times 2) + (3 \times 2)$ .

In the diagram, we have illustrated an array of counters in 12 rows with 13 in each row. The top part of the array illustrates  $13 \times 2$ . If we consider only that part of the array, we have an example like one on the previous page. We have considered  $13 \times 2$  as  $(10 + 3) \times 2$ , or  $(10 \times 2) + (3 \times 2)$ . There are 26 counters in this "part" of the diagram. This is recorded as a "partial" product on the dotted line:

$$\begin{array}{r} 13 \\ \times 12 \\ \hline 26 \end{array}$$

The lower part of the array illustrates  $13 \times 10$ . In this part, we have another example like those on the previous page.  $13 \times 10$  is shown as  $(10 + 3) \times 10$ , or  $(10 \times 10) + (10 \times 3)$ . The number of counters in this part of the diagram is 130, and is recorded on the double line.

$$\begin{array}{r} 13 \\ \times 12 \\ \hline 26 \\ \hline 130 \end{array}$$

We add the two "partial" products to get 156, our final result.

$$\begin{array}{r} 13 \\ \times 12 \\ \hline 26 \\ \hline 130 \\ \hline 156 \end{array}$$

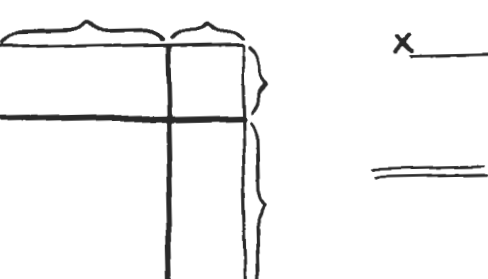
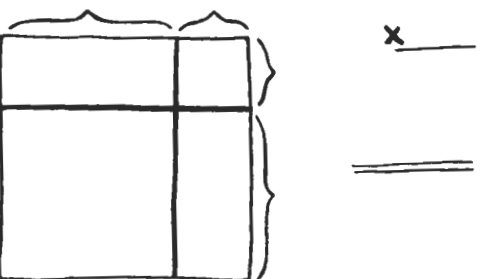
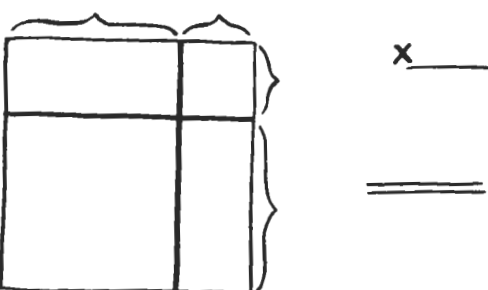
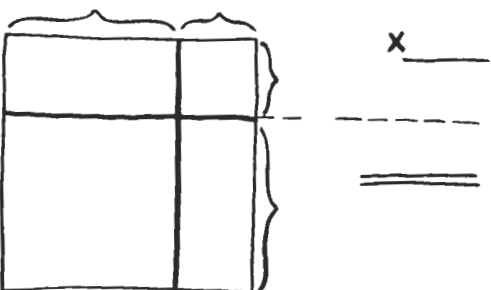
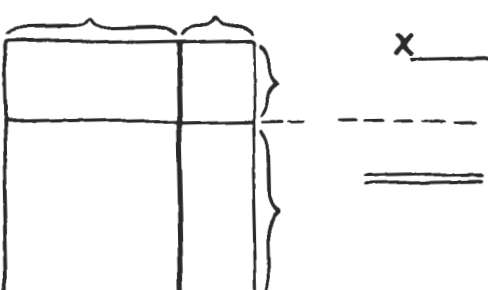
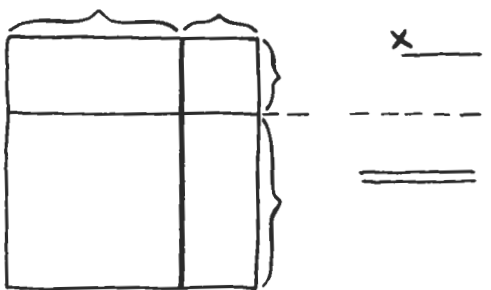
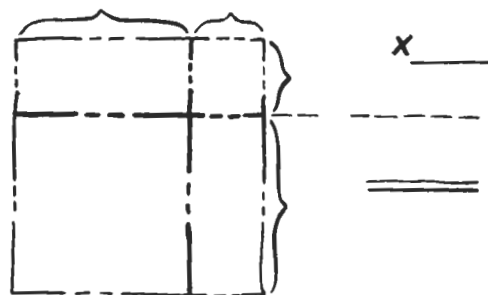
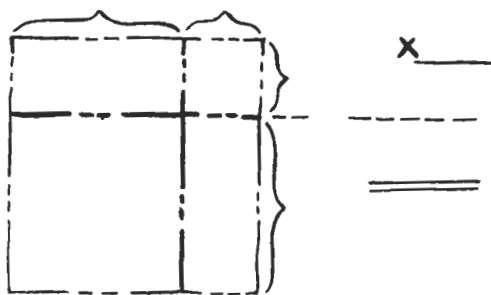
Children do not need such a logical explanation at this point. It is sufficient if they understand that, for example

$$\begin{array}{r} 13 \\ \times 12 \\ \hline \end{array}$$

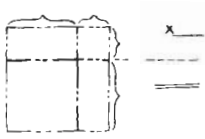
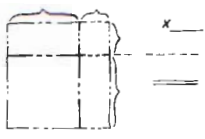
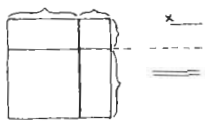
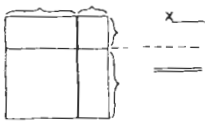
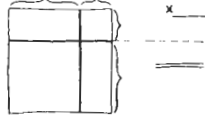
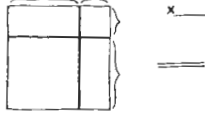
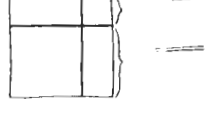
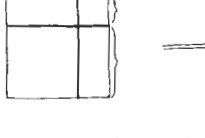
is the same thing as  $13 \times (2 + 10)$ , or  $(13 \times 2) + (13 \times 10)$ . This can best be explained using the diagram. Once they understand this, the partial products become simply the record of the result of counting the counters in each part.

After many experiences with examples of this type, children may begin to suggest that they could take a "shortcut" and not write a "0" at the end of the second partial product. But at that point they will understand where it came from and why such a shortcut can be used. Many children simply learn the rule that "you move over a place when you write the second partial product" and never have any notion that the number they write there is a "shorthand" for really writing that number times 10.

Some examples on this page require that the learner label the diagram, and some ask that he figure out what multiplication is represented by the diagram. It should be pointed out that each diagram should be considered separately. In some, a certain amount of space might represent 40 units, and in another, the same space might represent 20 units. The point is that we are not trying to make scale drawings. We want to make quick rough sketches to help with an understanding of the multiplication algorithm.



NAME \_\_\_\_\_

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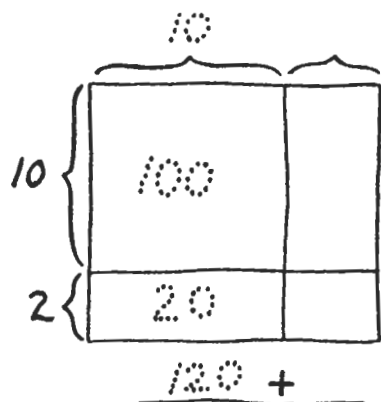
Learners can make up their own experiments on this page or teachers can particularize it.

You might want to try experimenting with providing different bits of information. For example:

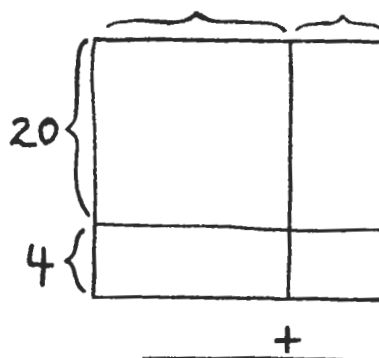
- (1) What multiplication would lead to the following situation?

|    |    |
|----|----|
| 80 | 12 |
|    | 30 |

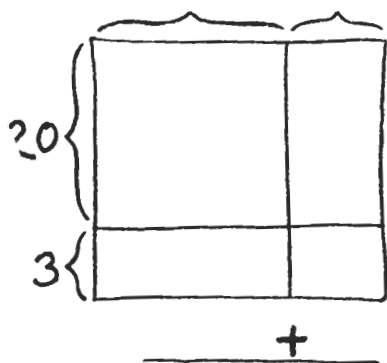
$\times$  \_\_\_\_\_  
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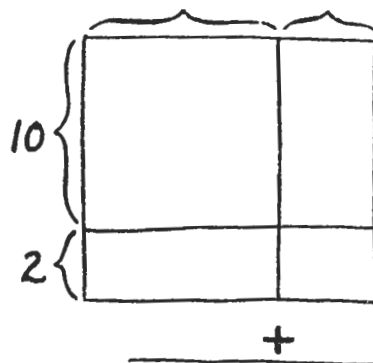
$$\begin{array}{r} 12 \overline{) 156} \\ \underline{120} \\ 36 \end{array}$$



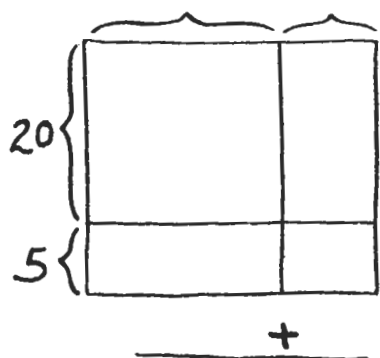
$$24 \overline{) 384}$$



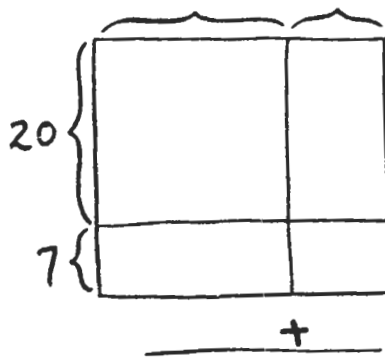
$$23 \overline{) 598}$$



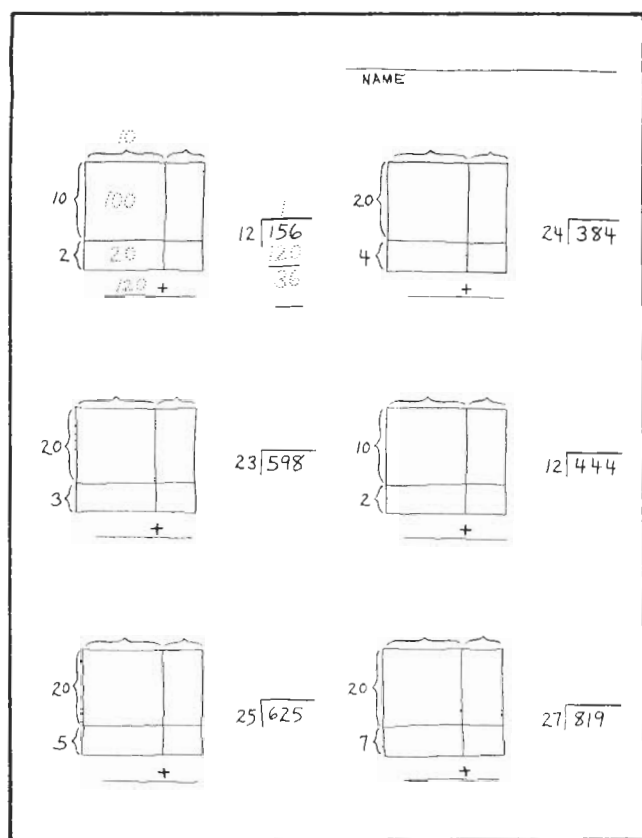
$$12 \overline{) 444}$$



$$25 \overline{) 625}$$



$$27 \overline{) 837}$$

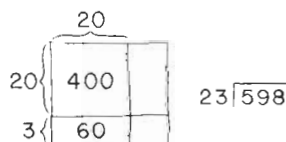


The activities on this page help the learner focus on the long division algorithm through the use of sketches. The essential question for division becomes "Given an array of a certain number of counters arranged in a certain number of rows, how many counters are there in each row?"

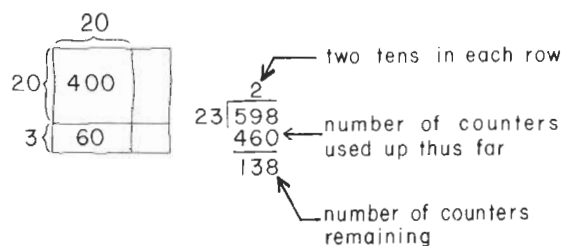
Consider the example in which 598 counters are arranged in 23 rows.

First we should decide how many tens (10-sticks or rods) could be put in each row.

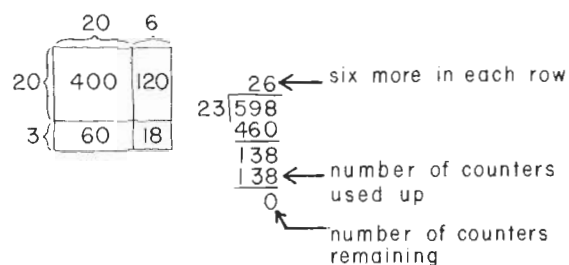
We could certainly put one ten in each row to start. But that only uses up 230 of the counters. If we put another ten in each row, we use 460 of the counters.



Putting another ten in each row would require another 230 counters, or 690 all together. We only have 598 counters, so we have to settle for two tens in each row. That means we have 138 counters still to arrange. We can record this partial result:



Now we must decide how many of the remaining counters go in each row. Five in each row would use up 115 . . . but six in each row would use up 138 . . . exactly the number we have. Thus:



Once again, we must recognize a different strategy used in division. In addition, subtraction and multiplication, the first step could be described as "taking care of the units first". Here are examples of such first steps:

$$\begin{array}{r} 1 \\ 14 \\ + 28 \\ \hline 2 \end{array} \quad \begin{array}{r} 2 \\ 34 \\ - 19 \\ \hline 5 \end{array} \quad \begin{array}{r} 2 \\ 18 \\ \times 3 \\ \hline 4 \end{array}$$

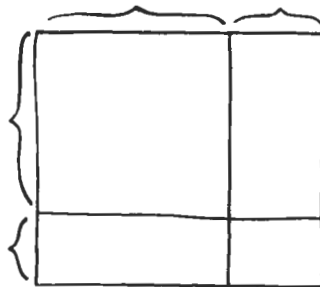
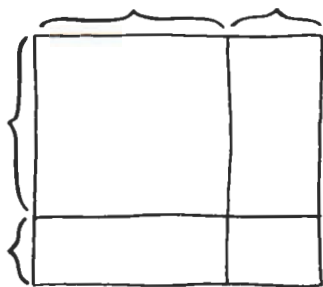
Compare this with the first step of our division strategy:

$$23 \overline{) 598} \quad 2$$

In addition, subtraction, and multiplication, we worry about loose beans first and make a trade so we can report the number of loose beans that will appear in our final report. In division, we worry first about how many sticks there will be in each row — and then, distribute the rest as loose beans.

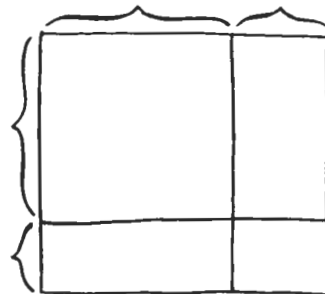
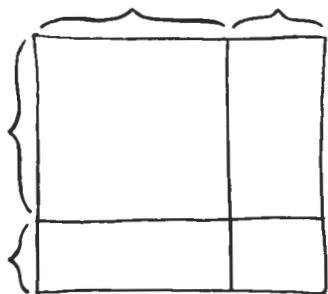
When such a fundamental shift in strategy occurs, we should afford time to discuss this shift with children.

This strategy of cutting a problem down to size is a most powerful mathematical technique. A tactic most engineers rely on in difficult problems is trial and error. They guess, and record results. Their next guess is influenced by the outcome of the first guess. Soon they begin trapping their elusive answer between "too much" and "too little".



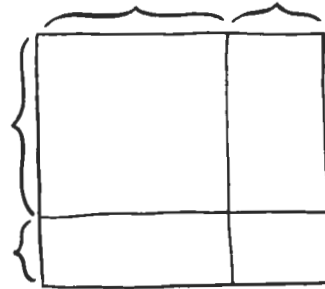
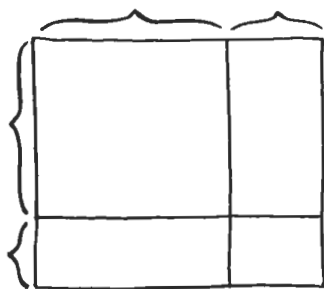
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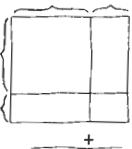
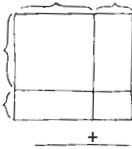
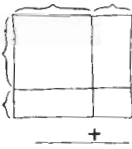
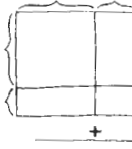
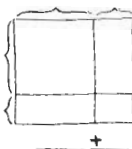
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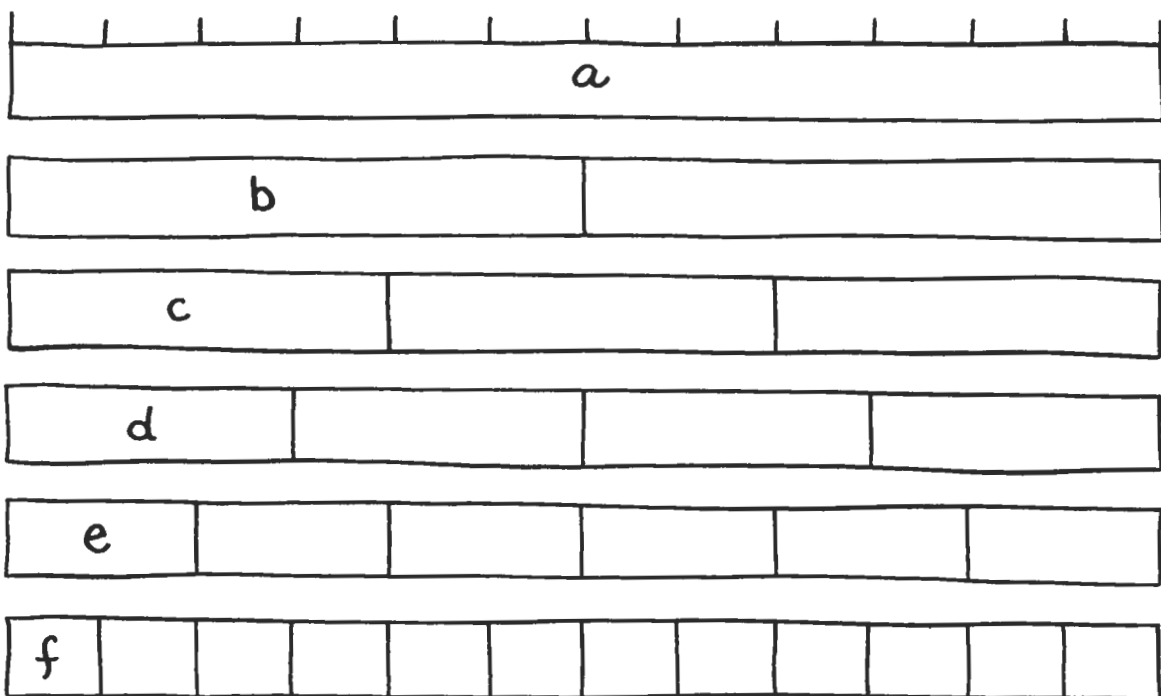
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+

NAME \_\_\_\_\_

|                                                                                   |                                                                                   |                                                                                   |                                                                                   |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

This is a general form for the type of activity on the previous page.



$$\frac{b + b}{+} = \underline{\quad}$$

$$\frac{2 \times b}{\times} = \underline{\quad}$$

$$\frac{a - b}{-} = \underline{\quad}$$

$$\frac{a \div 2}{\div} = \underline{\quad}$$

$$\frac{e + f}{+} = \underline{\quad}$$

$$\frac{b + e}{+} = \underline{\quad}$$

$$\frac{b \div 3}{\div} = \underline{\quad}$$

$$\frac{\frac{1}{2} \times c}{\times} = \underline{\quad}$$

$$\frac{2c \div 4}{\div} = \underline{\quad}$$

$$\frac{b + c}{+} = \underline{\quad}$$

$$\frac{f + 5f}{+} = \underline{\quad}$$

$$\frac{b + 2d}{+} = \underline{\quad}$$

$$\frac{a \div c}{\div} = \underline{\quad}$$

$$\frac{b \div d}{\div} = \underline{\quad}$$

$$\frac{d \div b}{\div} = \underline{\quad}$$



WHAT CAN YOU SEE? NAME \_\_\_\_\_

|                                                    |                                                                  |                                                     |
|----------------------------------------------------|------------------------------------------------------------------|-----------------------------------------------------|
| $\frac{b+b}{+} = \underline{\hspace{2cm}}$         | $\frac{2 \times b}{\times} = \underline{\hspace{2cm}}$           | $\frac{a-b}{-} = \underline{\hspace{2cm}}$          |
| $\frac{a \div 2}{\div} = \underline{\hspace{2cm}}$ | $\frac{e+f}{+} = \underline{\hspace{2cm}}$                       | $\frac{b+e}{+} = \underline{\hspace{2cm}}$          |
| $\frac{b \div 3}{\div} = \underline{\hspace{2cm}}$ | $\frac{\frac{1}{2} \times c}{\times} = \underline{\hspace{2cm}}$ | $\frac{2c \div 4}{\div} = \underline{\hspace{2cm}}$ |
| $\frac{b+c}{+} = \underline{\hspace{2cm}}$         | $\frac{f+5f}{+} = \underline{\hspace{2cm}}$                      | $\frac{b+2d}{+} = \underline{\hspace{2cm}}$         |
| $\frac{a \div c}{\div} = \underline{\hspace{2cm}}$ | $\frac{b \div d}{\div} = \underline{\hspace{2cm}}$               | $\frac{d \div b}{\div} = \underline{\hspace{2cm}}$  |

This page can be used in many different ways. We suggest one, but are certain you will invent many more.

- (1) Learners can consider the bar labelled "a" as a unit, and then figure out what fractional part of "a" is represented by the "b" bar, the "c" bar, etc. This could be determined by measuring or by comparing. One easy way to compare sizes is to simply count the number of bars of each size that it takes to make up the length of the "a" bar. There are two "b" bars in the length of one "a" bar, so "b" can be considered  $1/2$ . Likewise, "c" is  $1/3$ , "d" is  $1/4$ , "e" is  $1/6$ , and "f" is  $1/12$ .

Then we can begin a "double talk" approach to operations. In each problem, we can work with letters above the lines, and numbers below the lines. To solve

$$\begin{array}{r} e + f = \\ \hline + \quad = \end{array}$$

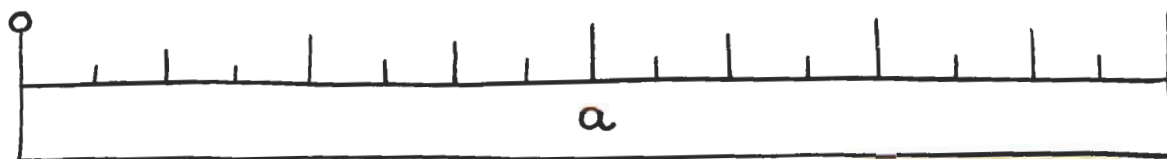
we can do several things. First, we could simply use our previous investigation and write fractional equivalents below the line.

$$\begin{array}{r} e + f = \\ \hline 1/6 + 1/12 = \end{array}$$

Then we could add  $1/6 + 1/12$  and get  $3/12$ , or  $1/4$ . Or, if we don't know how much  $1/6 + 1/12$  is, we could use another strategy. We might use a card, or folded piece of paper, and hold it so that an edge is directly on the vertical line at the end of the first "e" bar (so the first "e" bar is to the left of the card's edge). We could look down to the "f" row, and see that the edge of the card is at the end of two "f" blocks. To those two "f" blocks, which are equivalent to one "e" block I want to add one more "f", so I could move the card to the right one block. I would then show three "f" blocks, which I know is  $3/12$ . If we want to push toward fractions in lowest terms, we might have the rule that the final answer must be in terms of the largest blocks which have an end coinciding with the edge of the card. In this case, "d" does the trick. One "d" is the same length as 3 "f", and one "d" is  $1/4$ . Thus, I can record

$$\begin{array}{r} e + f = \quad d \\ \hline 1/6 + 1/12 = \quad 1/4 \end{array}$$

Other examples are done in a similar way.



$$\frac{c + c}{+} = \underline{\quad}$$

$$\frac{a \div 8}{\div} = \underline{\quad}$$

$$\frac{2d + c}{+} = \underline{\quad}$$

$$\frac{c - e}{-} = \underline{\quad}$$

$$\frac{b \div 8}{\div} = \underline{\quad}$$

$$\frac{a - 3c}{-} = \underline{\quad}$$

$$\frac{2c - 3e}{-} = \underline{\quad}$$

$$\frac{3d + 3d}{+} = \underline{\quad}$$

$$\frac{7e - 3e}{-} = \underline{\quad}$$

$$\frac{c + d}{+} = \underline{\quad}$$

$$\frac{d + 3e}{+} = \underline{\quad}$$

$$\frac{a - 5d}{-} = \underline{\quad}$$

$$\frac{a \div 4}{\div} = \underline{\quad}$$

$$\frac{3d \div 2}{\div} = \underline{\quad}$$

$$\frac{b \div 2e}{\div} = \underline{\quad}$$

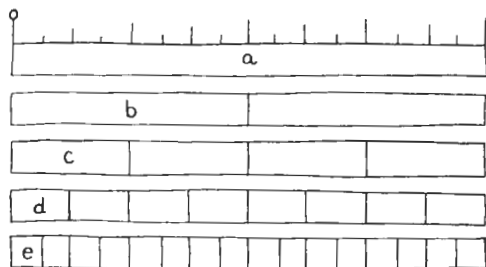
$$\frac{a \div c}{\div} = \underline{\quad}$$

$$\frac{3d \div 3e}{\div} = \underline{\quad}$$

$$\frac{5d \div c}{\div} = \underline{\quad}$$

WHAT CAN YOU SEE?

NAME \_\_\_\_\_



$$\frac{c+c}{+} = \underline{\hspace{2cm}} \quad \frac{a \div 8}{\div} = \underline{\hspace{2cm}} \quad \frac{2d+c}{+} = \underline{\hspace{2cm}}$$

$$\frac{c-e}{-} = \underline{\hspace{2cm}} \quad \frac{b \div 8}{\div} = \underline{\hspace{2cm}} \quad \frac{a-3c}{-} = \underline{\hspace{2cm}}$$

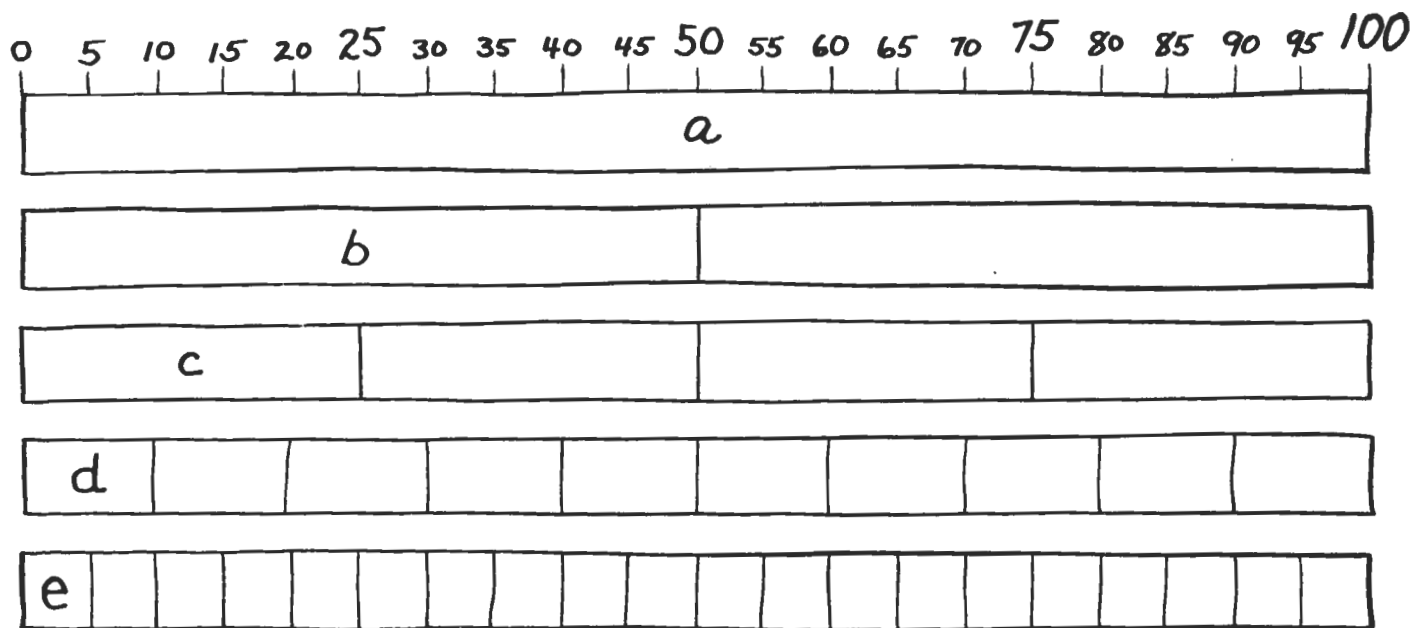
$$\frac{2c-3e}{-} = \underline{\hspace{2cm}} \quad \frac{3d+3d}{+} = \underline{\hspace{2cm}} \quad \frac{7e-3e}{-} = \underline{\hspace{2cm}}$$

$$\frac{c+d}{+} = \underline{\hspace{2cm}} \quad \frac{d+3e}{+} = \underline{\hspace{2cm}} \quad \frac{a-5d}{-} = \underline{\hspace{2cm}}$$

$$\frac{a \div 4}{\div} = \underline{\hspace{2cm}} \quad \frac{3d \div 2}{\div} = \underline{\hspace{2cm}} \quad \frac{b \div 2e}{\div} = \underline{\hspace{2cm}}$$

$$\frac{a \div c}{\div} = \underline{\hspace{2cm}} \quad \frac{3d \div 3e}{\div} = \underline{\hspace{2cm}} \quad \frac{5d \div c}{\div} = \underline{\hspace{2cm}}$$

This is a continuation of the activity on the previous page. Bars of different lengths are used.



$$\frac{a - c}{-} = \underline{\hspace{2cm}}$$

$$\frac{b + c}{+} = \underline{\hspace{2cm}}$$

$$\frac{3 \times c}{\times} = \underline{\hspace{2cm}}$$

$$\frac{b - 4e}{-} = \underline{\hspace{2cm}}$$

$$\frac{5 \times d}{\times} = \underline{\hspace{2cm}}$$

$$\frac{3d + 4e}{+} = \underline{\hspace{2cm}}$$

$$\frac{2c - 3e}{-} = \underline{\hspace{2cm}}$$

$$\frac{3c - b}{-} = \underline{\hspace{2cm}}$$

$$\frac{b + 2c}{+} = \underline{\hspace{2cm}}$$

$$\frac{a - 11e}{-} = \underline{\hspace{2cm}}$$

$$\frac{b - 2d}{-} = \underline{\hspace{2cm}}$$

$$\frac{c - e}{-} = \underline{\hspace{2cm}}$$

$$\frac{c \div e}{\div} = \underline{\hspace{2cm}}$$

$$\frac{b \div c}{\div} = \underline{\hspace{2cm}}$$

$$\frac{b \div d}{\div} = \underline{\hspace{2cm}}$$

$$\frac{c \div d}{\div} = \underline{\hspace{2cm}}$$

$$\frac{5d \div c}{\div} = \underline{\hspace{2cm}}$$

$$\frac{3c - 3e}{-} = \underline{\hspace{2cm}}$$

WHAT CAN YOU SEE? NAME \_\_\_\_\_

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100

a \_\_\_\_\_

b \_\_\_\_\_

c \_\_\_\_\_

d \_\_\_\_\_

e \_\_\_\_\_

$\frac{a-c}{-} =$  \_\_\_\_\_  $\frac{b+c}{+} =$  \_\_\_\_\_  $\frac{3 \times c}{\times} =$  \_\_\_\_\_

$\frac{b-4e}{-} =$  \_\_\_\_\_  $\frac{5 \times d}{\times} =$  \_\_\_\_\_  $\frac{3d+4e}{+} =$  \_\_\_\_\_

$\frac{2c-3e}{-} =$  \_\_\_\_\_  $\frac{3c-b}{-} =$  \_\_\_\_\_  $\frac{b+2c}{+} =$  \_\_\_\_\_

$\frac{a-11e}{-} =$  \_\_\_\_\_  $\frac{b-2d}{-} =$  \_\_\_\_\_  $\frac{c-e}{-} =$  \_\_\_\_\_

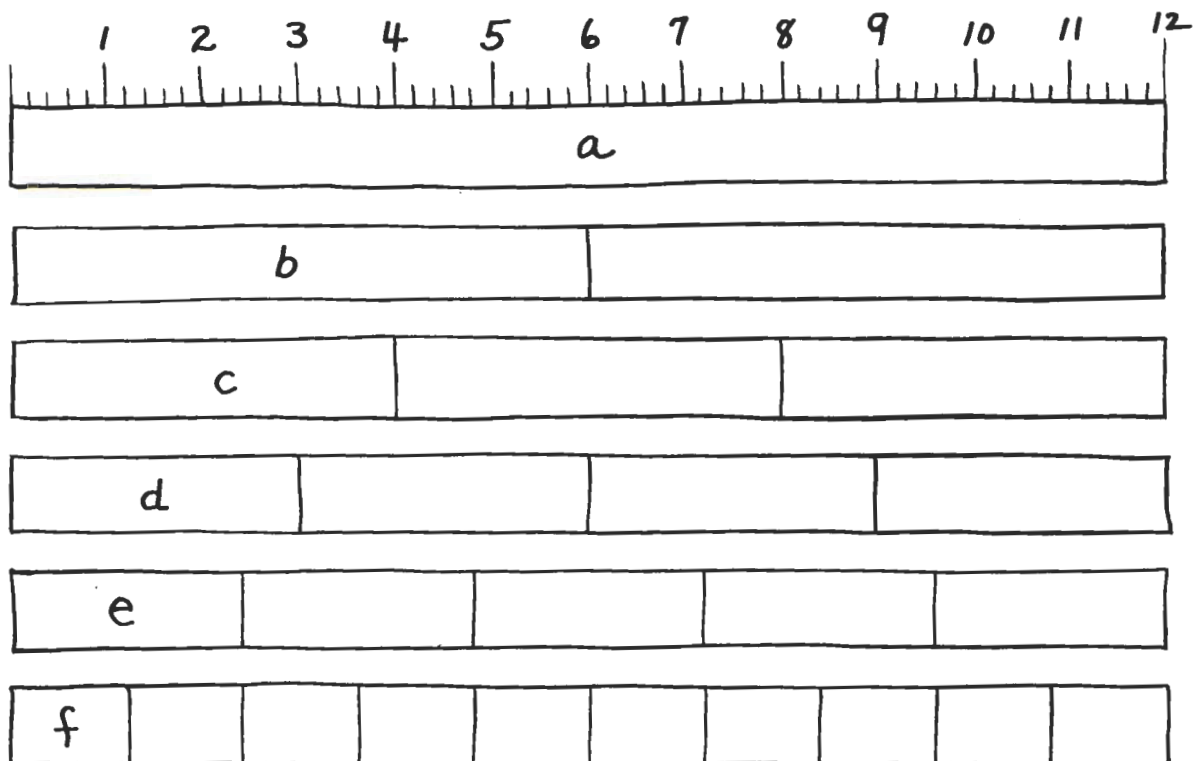
$\frac{c \div e}{\div} =$  \_\_\_\_\_  $\frac{b \div c}{\div} =$  \_\_\_\_\_  $\frac{b \div d}{\div} =$  \_\_\_\_\_

$\frac{c \div d}{\div} =$  \_\_\_\_\_  $\frac{5d \div c}{\div} =$  \_\_\_\_\_  $\frac{3c-3e}{-} =$  \_\_\_\_\_

This is a continuation of the activity on the previous pages. Bars of different lengths are used.

On this page, a scale is used at the top of bar "a", and you might want learners to "measure" the bars by using the card again and recording where the edge of the card cut the scale. (e.g.  $c = 25$ ,  $e = 5$ , etc.) The exercises could be done with whole numbers as well as with fractions.

By giving children exposure to a variety of systems of measurement and giving them opportunities to combine and compare lengths we are trying to build a "sense of measurement" which will facilitate the introduction of the metric or any other system of measurement.



$$\frac{2e + f}{+} = \underline{\quad}$$

$$\frac{b + d}{+} = \underline{\quad}$$

$$\frac{7f - e}{-} = \underline{\quad}$$

$$\frac{3e \div 2}{\div} = \underline{\quad}$$

$$\frac{a \div 3}{\div} = \underline{\quad}$$

$$\frac{2d - f}{-} = \underline{\quad}$$

$$\frac{5f + d}{+} = \underline{\quad}$$

$$\frac{a \div c}{\div} = \underline{\quad}$$

$$\frac{10f - c}{-} = \underline{\quad}$$

$$\frac{5f - d}{-} = \underline{\quad}$$

$$\frac{3f + e}{+} = \underline{\quad}$$

$$\frac{2d + 2e}{+} = \underline{\quad}$$

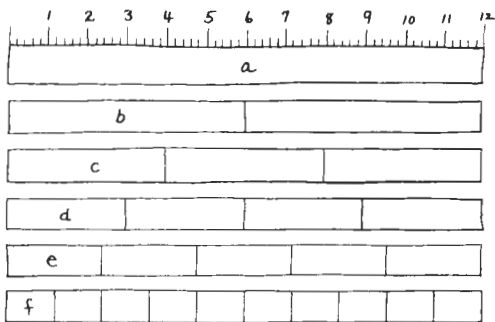
$$\frac{f \div e}{\div} = \underline{\quad}$$

$$\frac{3e \div 3f}{\div} = \underline{\quad}$$

$$\frac{a \div 2e}{\div} = \underline{\quad}$$

WHAT CAN YOU SEE?

NAME \_\_\_\_\_



$$\frac{2e + f}{+} = \quad \frac{b + d}{+} = \quad \frac{7f - e}{-} =$$

$$\frac{3e \div 2}{\div} = \quad \frac{a \div 3}{\div} = \quad \frac{2d - f}{-} =$$

$$\frac{3f + d}{+} = \quad \frac{a \div c}{\div} = \quad \frac{10f - c}{-} =$$

$$\frac{5f - d}{-} = \quad \frac{3f + e}{+} = \quad \frac{2d + 2e}{+} =$$

$$\frac{f \div e}{\div} = \quad \frac{3e \div 3f}{\div} = \quad \frac{a \div 2e}{\div} =$$

This activity is a continuation of those on the previous pages. A different measurement scale is given, and bars of different lengths are used.

IN EACH EXAMPLE THE SAME NUMBER  
MUST BE WRITTEN IN SHAPES THAT  
ARE ALIKE.

EN CADA EJEMPLO HAY QUE ESCRIBIR  
EL MISMO NÚMERO EN LAS FORMAS QUE  
SEAN IDÉNTICAS.

$$\frac{1}{2} \text{ of } (8) = (4)$$

$$\otimes \frac{1}{2} \text{ of } (4) = (2)$$


---


$$\text{— of } (8) = (2)$$

$$\frac{1}{2} \text{ of } (6) = ( )$$

$$\otimes \frac{1}{3} \text{ of } (3) = ( )$$


---


$$\text{— of } (6) = ( )$$

$$\frac{1}{3} \text{ of } (9) = ( )$$

$$\otimes \frac{1}{3} \text{ of } (3) = ( )$$


---


$$\text{— of } (9) = ( )$$

$$\frac{1}{4} \times (12) = ( )$$

$$\otimes \frac{1}{3} \times ( ) = ( )$$


---


$$\text{—} \times (12) = ( )$$

$$\frac{1}{2} \times (6) = ( )$$

$$\otimes \frac{2}{3} \times ( ) = ( )$$


---


$$\text{—} \times (6) = ( )$$

$$\frac{1}{3} \times (9) = ( )$$

$$\otimes \frac{2}{3} \times ( ) = ( )$$


---


$$\text{—} \times (9) = ( )$$

$$\frac{2}{5} \times (10) = ( )$$

$$\otimes \frac{1}{2} \times ( ) = ( )$$


---


$$\text{—} \times (10) = ( )$$

$$\frac{2}{3} \times (6) = ( )$$

$$\otimes \frac{1}{2} \times ( ) = ( )$$


---


$$\text{—} \times (6) = ( )$$

$$\frac{2}{3} \times (9) = ( )$$

$$\otimes \frac{2}{3} \times ( ) = ( )$$


---


$$\text{—} \times (9) = ( )$$

$$\frac{1}{2} \times (10) = ( )$$

$$\otimes \frac{2}{5} \times ( ) = ( )$$


---


$$\text{—} \times (10) = ( )$$

$$\frac{1}{2} \times (12) = ( )$$

$$\otimes \frac{2}{3} \times ( ) = ( )$$


---


$$\text{—} \times (12) = ( )$$

$$\frac{2}{3} \times (12) = ( )$$

$$\otimes \frac{1}{2} \times ( ) = ( )$$


---


$$\text{—} \times (12) = ( )$$



IN EACH GROUP OF 3 STATEMENTS,  
THE SAME NUMBER MUST BE WRITTEN  
IN SHAPES THAT ARE ALIKE.

NAME \_\_\_\_\_

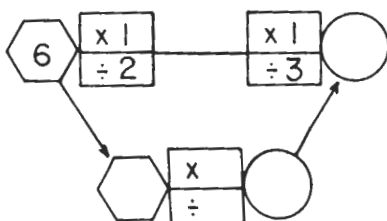
|                                                  |                                                  |                                                  |
|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|
| $\frac{1}{2}$ of $\textcircled{8} = \text{⬡}$    | $\frac{1}{2}$ of $\text{⬡} = \text{⬡}$           | $\frac{1}{3}$ of $\text{⬡} = \text{⬡}$           |
| $\otimes \frac{1}{2}$ of $\text{⬡} = \text{⬡}$   | $\otimes \frac{1}{3}$ of $\text{⬡} = \text{⬡}$   | $\otimes \frac{1}{3}$ of $\text{⬡} = \text{⬡}$   |
| $\text{— of } \text{⬡} = \text{⬡}$               | $\text{— of } \text{⬡} = \text{⬡}$               | $\text{— of } \text{⬡} = \text{⬡}$               |
| $\frac{1}{4} \times \text{⬡} = \text{⬡}$         | $\frac{1}{2} \times \text{⬡} = \text{⬡}$         | $\frac{1}{3} \times \text{⬡} = \text{⬡}$         |
| $\otimes \frac{1}{3} \times \text{⬡} = \text{⬡}$ | $\otimes \frac{2}{3} \times \text{⬡} = \text{⬡}$ | $\otimes \frac{2}{3} \times \text{⬡} = \text{⬡}$ |
| $\text{—} \times \text{⬡} = \text{⬡}$            | $\text{—} \times \text{⬡} = \text{⬡}$            | $\text{—} \times \text{⬡} = \text{⬡}$            |
| $\frac{2}{5} \times \text{⬡} = \text{⬡}$         | $\frac{2}{3} \times \text{⬡} = \text{⬡}$         | $\frac{2}{3} \times \text{⬡} = \text{⬡}$         |
| $\otimes \frac{1}{2} \times \text{⬡} = \text{⬡}$ | $\otimes \frac{1}{2} \times \text{⬡} = \text{⬡}$ | $\otimes \frac{2}{3} \times \text{⬡} = \text{⬡}$ |
| $\text{—} \times \text{⬡} = \text{⬡}$            | $\text{—} \times \text{⬡} = \text{⬡}$            | $\text{—} \times \text{⬡} = \text{⬡}$            |
| $\frac{1}{2} \times \text{⬡} = \text{⬡}$         | $\frac{1}{2} \times \text{⬡} = \text{⬡}$         | $\frac{2}{3} \times \text{⬡} = \text{⬡}$         |
| $\otimes \frac{2}{5} \times \text{⬡} = \text{⬡}$ | $\otimes \frac{2}{3} \times \text{⬡} = \text{⬡}$ | $\otimes \frac{1}{2} \times \text{⬡} = \text{⬡}$ |
| $\text{—} \times \text{⬡} = \text{⬡}$            | $\text{—} \times \text{⬡} = \text{⬡}$            | $\text{—} \times \text{⬡} = \text{⬡}$            |

Here we introduce the rule that within each group of three statements, the same number must be written in shapes that are alike.

On this page we are essentially using a more conventional notation for the problem encountered previously in these pages of finding a shortcut for two chain reactions (see pages 197-200). That is to say that

$$\begin{array}{rcl} \frac{1}{2} \text{ of } \text{⬡} & = & \text{⬡} \\ \frac{1}{3} \text{ of } \text{⬡} & = & \text{⬡} \\ \hline \text{⬡} & = & \text{⬡} \end{array}$$

is the **same** problem as:



The new notation and organization of the problem might be difficult for some children; it does require some ability to reason logically. If frustration occurs for these children, the page may be best put off until another time.

A reasonable approach to this type of problem would be to do each line in succession. Once the first line is done, we know one of the numbers to fill in the second line. When the second line is done, we know a number to put in the third line. The third line will always present the problem of "finding a shortcut."

Of course, these examples are building referents for the introduction of multiplication by fractions. Children may already have noticed a relationship between the two fractions in the first two lines and that in the third. If they have a "guess" about the relationship, let them test it out. A likely guess is that "you multiply the top numbers and the bottom numbers." This theory will always work until you require that the shortcut use the smallest possible whole numbers. Then a revision is necessary—and we have begun to build some referents for the later introduction of "reducing fractions." Learners can use the next page to test and revise their theories.

EN CADA EJEMPLO HAY QUE ESCRIBIR  
EL MISMO NÚMERO EN LAS FORMAS QUE  
SEAN IDÉNTICAS.

IN EACH EXAMPLE THE SAME NUMBER  
MUST BE WRITTEN IN SHAPES THAT  
ARE ALIKE.

$$\begin{array}{l} \_ \text{ of } \bigcirc = \text{hexagon} \\ \otimes \_ \text{ of } \text{hexagon} = \square \\ \hline \_ \text{ of } \bigcirc = \square \end{array}$$

$$\begin{array}{l} \_ \text{ of } \text{hexagon} = \square \\ \otimes \_ \text{ of } \square = \bigcirc \\ \hline \_ \text{ of } \text{hexagon} = \bigcirc \end{array}$$

$$\begin{array}{l} \_ \text{ of } \square = \bigcirc \\ \otimes \_ \text{ of } \bigcirc = \text{hexagon} \\ \hline \_ \text{ of } \square = \text{hexagon} \end{array}$$

$$\begin{array}{l} \_ \times \square = \bigcirc \\ \otimes \_ \times \bigcirc = \text{hexagon} \\ \hline \_ \times \square = \text{hexagon} \end{array}$$

$$\begin{array}{l} \_ \times \bigcirc = \text{hexagon} \\ \otimes \_ \times \text{hexagon} = \square \\ \hline \_ \times \bigcirc = \square \end{array}$$

$$\begin{array}{l} \_ \times \text{hexagon} = \square \\ \otimes \_ \times \square = \bigcirc \\ \hline \_ \times \text{hexagon} = \bigcirc \end{array}$$

$$\begin{array}{l} \_ \times \text{hexagon} = \square \\ \otimes \_ \times \square = \bigcirc \\ \hline \_ \times \text{hexagon} = \bigcirc \end{array}$$

$$\begin{array}{l} \_ \times \square = \bigcirc \\ \otimes \_ \times \bigcirc = \text{hexagon} \\ \hline \_ \times \square = \text{hexagon} \end{array}$$

$$\begin{array}{l} \_ \times \bigcirc = \text{hexagon} \\ \otimes \_ \times \text{hexagon} = \square \\ \hline \_ \times \bigcirc = \square \end{array}$$

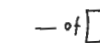
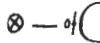
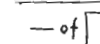
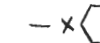
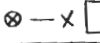
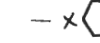
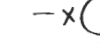
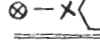
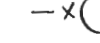
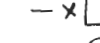
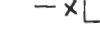
$$\begin{array}{l} \_ \times \bigcirc = \text{hexagon} \\ \otimes \_ \times \text{hexagon} = \square \\ \hline \_ \times \bigcirc = \square \end{array}$$

$$\begin{array}{l} \_ \times \text{hexagon} = \square \\ \otimes \_ \times \square = \bigcirc \\ \hline \_ \times \text{hexagon} = \bigcirc \end{array}$$

$$\begin{array}{l} \_ \times \square = \bigcirc \\ \otimes \_ \times \bigcirc = \text{hexagon} \\ \hline \_ \times \square = \text{hexagon} \end{array}$$

IN EACH GROUP OF 3 STATEMENTS,  
THE SAME NUMBER MUST BE WRITTEN  
IN SHAPES THAT ARE ALIKE.

NAME \_\_\_\_\_

|                                                                                                                                                                                      |                                                                                                                                                                                      |                                                                                                                                                                                      |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| — of  =            | — of  =            | — of  =            |
| $\otimes$ — of  =  | $\otimes$ — of  =  | $\otimes$ — of  =  |
| — of  =            | — of  =            | — of  =            |
| — x  =             | — x  =             | — x  =             |
| $\otimes$ — x  =   | $\otimes$ — x  =   | $\otimes$ — x  =   |
| — x  =             | — x  =             | — x  =             |
| — x  =             | — x  =             | — x  =             |
| $\otimes$ — x  =   | $\otimes$ — x  =   | $\otimes$ — x  =   |
| — x  =             | — x  =             | — x  =             |
| — x  =             | — x  =             | — x  =             |
| $\otimes$ — x  =   | $\otimes$ — x  =   | $\otimes$ — x  =   |
| — x  =             | — x  =             | — x  =             |

Will the fraction  $\frac{1}{5}$  still be used as the shortcut  
if he starts with:

$$\begin{array}{l} \frac{2}{5} \times \text{hexagon}(20) = \text{square} \\ \frac{1}{2} \times \text{square} = \text{circle} \\ \hline \text{—} \times \text{hexagon} = \text{circle} \end{array}$$

Will it work if he starts with 30 in the hexagon?

This form can be particularized or used for explorations by the learner. Some variations might be:

(1) Make up problems of this type:

$$\begin{array}{l} \text{— of hexagon}(4) = \text{square} \\ \text{— of square} = \text{circle}(1) \\ \hline \text{— of hexagon}(4) = \text{circle}(1) \end{array}$$

(2) The learner might want to test some of the "shortcuts" from the previous page. For example, on that page, he found:

$$\begin{array}{l} \frac{2}{5} \times \text{hexagon}(10) = \text{square}(4) \\ \frac{1}{2} \times \text{square}(4) = \text{circle}(2) \\ \hline \frac{1}{5} \times \text{hexagon}(10) = \text{circle}(2) \end{array}$$

$$\frac{1}{2} \text{ of } (8) = \underline{4}$$

$$\oplus \frac{1}{4} \text{ of } (8) = \underline{2}$$


---


$$\underline{\quad} \text{ of } (8) = \underline{6}$$

$$\frac{1}{6} \text{ of } \langle 18 \rangle = \underline{\quad}$$

$$\oplus \frac{1}{3} \text{ of } \langle 18 \rangle = \underline{\quad}$$


---


$$\underline{\quad} \text{ of } \langle 18 \rangle = \underline{9}$$

$$\frac{1}{3} \text{ of } [9] = \underline{\quad}$$

$$\oplus \frac{1}{3} \text{ of } [9] = \underline{\quad}$$


---


$$\underline{\quad} \text{ of } [9] = \underline{\quad}$$

$$\frac{1}{5} \times [15] = \underline{\quad}$$

$$\oplus \frac{2}{5} \times [15] = \underline{\quad}$$


---


$$\underline{\quad} \times [15] = \underline{\quad}$$

$$\frac{1}{2} \times (6) = \underline{\quad}$$

$$\oplus \frac{1}{3} \times (6) = \underline{\quad}$$


---


$$\underline{\quad} \times (6) = \underline{\quad}$$

$$\frac{1}{2} \times \langle 30 \rangle = \underline{\quad}$$

$$\oplus \frac{1}{3} \times \langle 30 \rangle = \underline{\quad}$$


---


$$\underline{\quad} \times \langle 30 \rangle = \underline{\quad}$$

$$\frac{3}{4} \times \langle 16 \rangle = \underline{\quad}$$

$$\ominus \frac{1}{2} \times \langle 16 \rangle = \underline{\quad}$$


---


$$\underline{\quad} \times \langle 16 \rangle = \underline{4}$$

$$\frac{3}{4} \times [100] = \underline{\quad}$$

$$\ominus \frac{1}{2} \times [100] = \underline{\quad}$$


---


$$\underline{\quad} \times [100] = \underline{\quad}$$

$$\frac{3}{4} \times (36) = \underline{\quad}$$

$$\ominus \frac{1}{2} \times (36) = \underline{\quad}$$


---


$$\underline{\quad} \times (36) = \underline{\quad}$$

$$\frac{2}{5} \times (10) = \underline{\quad}$$

$$\ominus \frac{1}{10} \times (10) = \underline{\quad}$$


---


$$\underline{\quad} \times (10) = \underline{\quad}$$

$$\frac{2}{5} \times \langle 50 \rangle = \underline{\quad}$$

$$\ominus \frac{1}{10} \times \langle 50 \rangle = \underline{\quad}$$


---


$$\underline{\quad} \times \langle 50 \rangle = \underline{\quad}$$

$$\frac{2}{5} \times [100] = \underline{\quad}$$

$$\ominus \frac{1}{10} \times [100] = \underline{\quad}$$


---


$$\underline{\quad} \times [100] = \underline{\quad}$$

NAME \_\_\_\_\_

$$\frac{1}{2} \text{ of } (8) = \underline{4} \quad \frac{1}{6} \text{ of } (18) = \underline{\quad} \quad \frac{1}{3} \text{ of } (9) = \underline{\quad}$$

$$+ \frac{1}{4} \text{ of } (8) = \underline{2} \quad + \frac{1}{3} \text{ of } (18) = \underline{\quad} \quad + \frac{1}{3} \text{ of } (9) = \underline{\quad}$$

$$\underline{\quad} \text{ of } (8) = \underline{6} \quad \underline{\quad} \text{ of } (18) = \underline{9} \quad \underline{\quad} \text{ of } (9) = \underline{\quad}$$

$$\frac{1}{5} \times (15) = \underline{\quad} \quad \frac{1}{2} \times (6) = \underline{\quad} \quad \frac{1}{2} \times (30) = \underline{\quad}$$

$$+ \frac{2}{5} \times (15) = \underline{\quad} \quad + \frac{1}{3} \times (6) = \underline{\quad} \quad + \frac{1}{3} \times (30) = \underline{\quad}$$

$$\underline{\quad} \times (15) = \underline{\quad} \quad \underline{\quad} \times (6) = \underline{\quad} \quad \underline{\quad} \times (30) = \underline{\quad}$$

$$\frac{3}{4} \times (16) = \underline{\quad} \quad \frac{3}{4} \times (100) = \underline{\quad} \quad \frac{3}{4} \times (36) = \underline{\quad}$$

$$\ominus \frac{1}{2} \times (16) = \underline{\quad} \quad \ominus \frac{1}{2} \times (100) = \underline{\quad} \quad \ominus \frac{1}{2} \times (36) = \underline{\quad}$$

$$\underline{\quad} \times (16) = \underline{\quad} \quad \underline{\quad} \times (100) = \underline{\quad} \quad \underline{\quad} \times (36) = \underline{\quad}$$

$$\frac{2}{5} \times (10) = \underline{\quad} \quad \frac{2}{5} \times (50) = \underline{\quad} \quad \frac{2}{5} \times (100) = \underline{\quad}$$

$$\ominus \frac{1}{10} \times (10) = \underline{\quad} \quad \ominus \frac{1}{10} \times (50) = \underline{\quad} \quad \ominus \frac{1}{10} \times (100) = \underline{\quad}$$

$$\underline{\quad} \times (10) = \underline{\quad} \quad \underline{\quad} \times (50) = \underline{\quad} \quad \underline{\quad} \times (100) = \underline{\quad}$$

On this page we make the rule that in each group of three statements, the number on the right of “=” in the third line is the sum or difference (indicated by “+” or “-”) of the two numbers directly above it.

For example in:

$$\begin{array}{r} \frac{1}{6} \text{ of } (18) = 3 \\ + \frac{1}{3} \text{ of } (18) = 6 \\ \hline \underline{\quad} \text{ of } (18) = 9 \end{array}$$

9 is the sum of 3 and 6.

The problem is once again to “find a shortcut for getting 18 to 9,” or more conventionally, to “find that fraction which when multiplied by 18 will produce 9.”

Children should be encouraged to test any guess they have about relationships among the three fractions in each group. The next page provides a report form for doing these investigations.

$$\begin{array}{l} \text{— of } \bigcirc = \text{—} \\ \bigcirc \text{— of } \bigcirc = \text{—} \\ \hline \text{— of } \bigcirc = \text{—} \end{array}$$

$$\begin{array}{l} \text{— of } \heptagon = \text{—} \\ \bigcirc \text{— of } \heptagon = \text{—} \\ \hline \text{— of } \heptagon = \text{—} \end{array}$$

$$\begin{array}{l} \text{— of } \square = \text{—} \\ \bigcirc \text{— of } \square = \text{—} \\ \hline \text{— of } \square = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \square = \text{—} \\ \bigcirc \text{—} \times \square = \text{—} \\ \hline \text{—} \times \square = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \bigcirc = \text{—} \\ \bigcirc \text{—} \times \bigcirc = \text{—} \\ \hline \text{—} \times \bigcirc = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \heptagon = \text{—} \\ \bigcirc \text{—} \times \heptagon = \text{—} \\ \hline \text{—} \times \heptagon = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \heptagon = \text{—} \\ \bigcirc \text{—} \times \heptagon = \text{—} \\ \hline \text{—} \times \heptagon = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \square = \text{—} \\ \bigcirc \text{—} \times \square = \text{—} \\ \hline \text{—} \times \square = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \bigcirc = \text{—} \\ \bigcirc \text{—} \times \bigcirc = \text{—} \\ \hline \text{—} \times \bigcirc = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \bigcirc = \text{—} \\ \bigcirc \text{—} \times \bigcirc = \text{—} \\ \hline \text{—} \times \bigcirc = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \heptagon = \text{—} \\ \bigcirc \text{—} \times \heptagon = \text{—} \\ \hline \text{—} \times \heptagon = \text{—} \end{array}$$

$$\begin{array}{l} \text{—} \times \square = \text{—} \\ \bigcirc \text{—} \times \square = \text{—} \\ \hline \text{—} \times \square = \text{—} \end{array}$$

You might want to use this form to create some sequences of examples that build some referents from using common denominators in addition and subtraction of fractions. Several pairs of problems might be designed such as

$$\begin{array}{rcl} \frac{1}{4} \times 8 & = & \underline{\hspace{1cm}} \\ + \frac{1}{2} \times 8 & = & \underline{\hspace{1cm}} \\ \hline \text{---} \times 8 & = & \underline{\hspace{1cm}} \end{array} \qquad \text{and} \qquad \begin{array}{rcl} \frac{1}{4} \times 8 & = & \underline{\hspace{1cm}} \\ + \frac{2}{4} \times 8 & = & \underline{\hspace{1cm}} \\ \hline \text{---} \times 8 & = & \underline{\hspace{1cm}} \end{array}$$

with the constraint that you must find the fraction for the last line that uses the smallest whole numbers. Learners can also use this page to make up their own examples, and pursue their own investigations.

HUEVOS  
EGGS  
DOCENAS  
DOZENS

|                |               |               |               |   |               |   |   |   |    |    |    |                |
|----------------|---------------|---------------|---------------|---|---------------|---|---|---|----|----|----|----------------|
| 1              | 2             | 3             | 4             | 5 | 6             | 7 | 8 | 9 | 10 | 11 | 12 | 13             |
| $\frac{1}{12}$ | $\frac{1}{6}$ | $\frac{1}{4}$ | $\frac{1}{3}$ |   | $\frac{1}{2}$ |   |   |   |    |    |    | $\frac{1}{12}$ |

$$\begin{array}{r} 3 + 3 = 6 \\ \hline \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \end{array}$$

$$\begin{array}{r} 4 + \quad = \quad \\ \hline \frac{1}{3} + \frac{1}{3} = \end{array}$$

$$\begin{array}{r} \quad + \quad = \quad \\ \hline \frac{1}{4} + \quad = \end{array}$$

$$\begin{array}{r} 5 + 2 = \quad \\ \hline \quad + \quad = \end{array}$$

$$\begin{array}{r} \quad + 1 = \quad \\ \hline \frac{2}{3} + \quad = \end{array}$$

$$\begin{array}{r} 4 + \quad = \quad \\ \hline \quad + \frac{1}{6} = \end{array}$$

$$\begin{array}{r} 6 - 3 = \quad \\ \hline \frac{1}{2} - \frac{1}{4} = \end{array}$$

$$\begin{array}{r} \quad - \quad = \quad \\ \hline \frac{3}{4} - \frac{1}{4} = \end{array}$$

$$\begin{array}{r} 8 - \quad = \quad \\ \hline \quad - \frac{1}{3} = \end{array}$$

$$\begin{array}{r} \quad - \quad = \quad \\ \hline \frac{7}{12} - \frac{1}{12} = \end{array}$$

$$\begin{array}{r} 10 - \quad = \quad \\ \hline \quad - \frac{1}{4} = \end{array}$$

$$\begin{array}{r} 12 - \quad = \quad \\ \hline 1 - \frac{1}{12} = \end{array}$$

$$\frac{1}{2} + \frac{1}{4} = \quad$$

$$\frac{5}{12} + \frac{1}{12} = \quad$$

$$\frac{1}{4} + \frac{3}{4} = \quad$$

$$1 - \frac{1}{2} = \quad$$

$$\frac{2}{3} - \frac{1}{3} = \quad$$

$$\frac{5}{6} - \frac{1}{2} = \quad$$



| EGGS   | 1              | 2             | 3             | 4             | 5 | 6             | 7 | 8 | 9 | 10 | 11 | 12 | 13             |
|--------|----------------|---------------|---------------|---------------|---|---------------|---|---|---|----|----|----|----------------|
| DOZENS | $\frac{1}{12}$ | $\frac{2}{6}$ | $\frac{1}{4}$ | $\frac{1}{3}$ |   | $\frac{1}{2}$ |   |   |   |    |    |    | $\frac{1}{12}$ |

|                                              |                                              |                                               |
|----------------------------------------------|----------------------------------------------|-----------------------------------------------|
| $\frac{3}{4} + \frac{3}{4} = \frac{6}{4}$    | $\frac{4}{3} + \frac{1}{3} = \frac{5}{3}$    | $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$     |
| $\frac{5}{2} + \frac{2}{2} = \frac{7}{2}$    | $\frac{2}{3} + \frac{1}{3} = \frac{3}{3}$    | $\frac{4}{6} + \frac{1}{6} = \frac{5}{6}$     |
| $\frac{6}{2} - \frac{3}{2} = \frac{3}{2}$    | $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$    | $\frac{8}{3} - \frac{1}{3} = \frac{7}{3}$     |
| $\frac{7}{12} - \frac{1}{12} = \frac{6}{12}$ | $\frac{10}{4} - \frac{1}{4} = \frac{9}{4}$   | $\frac{12}{1} - \frac{1}{12} = \frac{11}{12}$ |
| $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$    | $\frac{5}{12} + \frac{1}{12} = \frac{6}{12}$ | $\frac{1}{4} + \frac{3}{4} = \frac{4}{4}$     |
| $1 - \frac{1}{2} = \frac{1}{2}$              | $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$    | $\frac{5}{6} - \frac{1}{2} = \frac{1}{6}$     |

This page begins a sequence of activities which might be termed a "double-talk approach to fractions".

Using some real referent within his experience, the learner creates a table involving both fractions and whole numbers to describe the whole and its parts.

HUEVOS  
EGGS  
DOCENAS  
DOZENS

|                |                |    |                 |    |    |    |    |    |    |    |                |    |
|----------------|----------------|----|-----------------|----|----|----|----|----|----|----|----------------|----|
| 14             | 15             | 16 | 17              | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25             | 26 |
| $1\frac{1}{2}$ | $1\frac{1}{4}$ |    | $1\frac{5}{12}$ |    |    |    |    |    |    | 2  | $2\frac{1}{2}$ |    |

$$\begin{array}{|c|} \hline 4 \\ \hline \frac{1}{3} \\ \hline \end{array} \times 2 = \begin{array}{|c|} \hline 8 \\ \hline \frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 9 \\ \hline \frac{3}{4} \\ \hline \end{array} \times 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \frac{5}{12} \\ \hline \end{array} \times 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \frac{1}{6} \\ \hline \end{array} \times 3 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \frac{1}{3} \\ \hline \end{array} \times 3 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \frac{1}{3} \\ \hline \end{array} \times 4 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 6 \\ \hline \frac{1}{2} \\ \hline \end{array} \div 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \frac{2}{3} \\ \hline \end{array} \div 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \frac{5}{6} \\ \hline \end{array} \div 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline 1\frac{1}{2} \\ \hline \end{array} \div 3 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 8 \\ \hline \\ \hline \end{array} \div 4 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 24 \\ \hline \\ \hline \end{array} \div 3 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \frac{1}{4} \\ \hline \end{array} \times 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \frac{2}{3} \\ \hline \end{array} \times 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \frac{1}{2} \\ \hline \end{array} \times 3 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 1\frac{1}{3} \\ \hline \end{array} \div 2 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \frac{1}{2} \\ \hline \end{array} \div 3 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 1 \\ \hline \end{array} \div 4 = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

|        |               |               |    |               |    |    |    |    |    |    |    |                |    |
|--------|---------------|---------------|----|---------------|----|----|----|----|----|----|----|----------------|----|
| EGGS   | 14            | 15            | 16 | 17            | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25             | 26 |
| DOZENS | $\frac{1}{6}$ | $\frac{1}{4}$ |    | $\frac{1}{2}$ |    |    |    |    |    |    | 2  | $1\frac{1}{2}$ |    |

|                                      |                                       |                                         |
|--------------------------------------|---------------------------------------|-----------------------------------------|
| $\frac{4}{1} \times 2 = \frac{8}{1}$ | $\frac{9}{3} \times 2 = \frac{18}{3}$ | $\frac{5}{12} \times 2 = \frac{10}{12}$ |
| $\frac{1}{3} \times 2 = \frac{2}{3}$ | $\frac{3}{4} \times 2 = \frac{6}{4}$  | $\frac{5}{12} \times 2 = \frac{10}{12}$ |
| $\frac{1}{6} \times 3 = \frac{3}{6}$ | $\frac{1}{3} \times 3 = \frac{3}{3}$  | $\frac{1}{3} \times 4 = \frac{4}{3}$    |
| $\frac{10}{2} \div 2 = \frac{5}{1}$  | $\frac{2}{3} \div 2 = \frac{1}{3}$    | $\frac{5}{6} \div 2 = \frac{5}{12}$     |
| $\frac{1}{2} \div 3 = \frac{1}{6}$   | $\frac{8}{4} \div 4 = \frac{2}{1}$    | $\frac{24}{3} \div 3 = \frac{8}{1}$     |
| $\frac{1}{4} \times 2 = \frac{2}{4}$ | $\frac{2}{3} \times 2 = \frac{4}{3}$  | $\frac{1}{2} \times 3 = \frac{3}{2}$    |
| $\frac{1}{3} \div 2 = \frac{1}{6}$   | $\frac{1}{2} \div 3 = \frac{1}{6}$    | $1 \div 4 = \frac{1}{4}$                |

This page is a continuation of the previous page. "Double talk" multiplication and division occur here. These examples are done in the same way. For example, in

$$\frac{5}{12} \times 2 = \frac{10}{12}$$

5 eggs is 5/12 of a dozen.  $5 \times 2 = 10$ , and 10 eggs is 10/12 of a dozen. Thus:

$$\frac{5}{12} \times 2 = \frac{10}{12}$$

Consider the division example

$$\frac{15}{4} \div 3 = \frac{5}{4}$$

15 eggs is 15/4 of a dozen.  $15 \div 3 = 5$ , and 5 eggs is 5/4 of a dozen. Thus,

$$\frac{15}{4} \div 3 = \frac{5}{4}$$



|  |   |  |   |  |
|--|---|--|---|--|
|  | + |  | = |  |
|  | + |  | = |  |

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} + \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} - \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} - \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$

NAME \_\_\_\_\_

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|

|                               |                               |                               |
|-------------------------------|-------------------------------|-------------------------------|
| $\square + \square = \square$ | $\square + \square = \square$ | $\square + \square = \square$ |
|-------------------------------|-------------------------------|-------------------------------|

|                               |                               |                               |
|-------------------------------|-------------------------------|-------------------------------|
| $\square + \square = \square$ | $\square + \square = \square$ | $\square + \square = \square$ |
|-------------------------------|-------------------------------|-------------------------------|

|                               |                               |                               |
|-------------------------------|-------------------------------|-------------------------------|
| $\square - \square = \square$ | $\square - \square = \square$ | $\square - \square = \square$ |
|-------------------------------|-------------------------------|-------------------------------|

|                               |                               |                               |
|-------------------------------|-------------------------------|-------------------------------|
| $\square - \square = \square$ | $\square - \square = \square$ | $\square - \square = \square$ |
|-------------------------------|-------------------------------|-------------------------------|

|                                  |                                  |                                  |
|----------------------------------|----------------------------------|----------------------------------|
| $\square \div \square = \square$ | $\square \div \square = \square$ | $\square \div \square = \square$ |
|----------------------------------|----------------------------------|----------------------------------|

|                                  |                                  |                                  |
|----------------------------------|----------------------------------|----------------------------------|
| $\square \div \square = \square$ | $\square \div \square = \square$ | $\square \div \square = \square$ |
|----------------------------------|----------------------------------|----------------------------------|

This page continues the type of activities on the previous pages. Learners can make up their own tables about whatever they want that can be expressed in "double talk". Some possibilities could be:

- (1) Pies cut with a 6-piece cutter. The table could be in terms of pieces and fractional parts of the pie.
- (2) Candy (or butter, or nuts) measured in ounces. The table could be in terms of ounces and fractional parts of a pound.
- (3) Time measured in hours and days. The table could be in terms of hours and fractional parts parts of a day.

Diagram illustrating the multiplication of two 2x2 matrices. The first matrix has a solid top row and a dashed bottom row. The second matrix has a solid top row and a dashed bottom row. The result is a 2x2 matrix with a solid top row and a dashed bottom row.

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \begin{array}{c} \times \\ \times \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$
$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \times = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$
$$\begin{array}{r} \boxed{\phantom{00}} \div \phantom{00} = \boxed{\phantom{00}} \\ \hline \boxed{\phantom{00}} \div \phantom{00} = \boxed{\phantom{00}} \end{array}$$

$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \div \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$$

$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \div \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$$

$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \div \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$$

$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \div \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$$

|  |        |     |  |
|--|--------|-----|--|
|  | $\div$ | $=$ |  |
|  | $\div$ | $=$ |  |

$$\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$$
$$\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$$
$$\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\boxed{\phantom{00}} \div \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

$$\boxed{\quad} \div \boxed{\quad} = \boxed{\quad}$$

$$\frac{\quad}{\quad} \div \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

\_\_\_\_\_

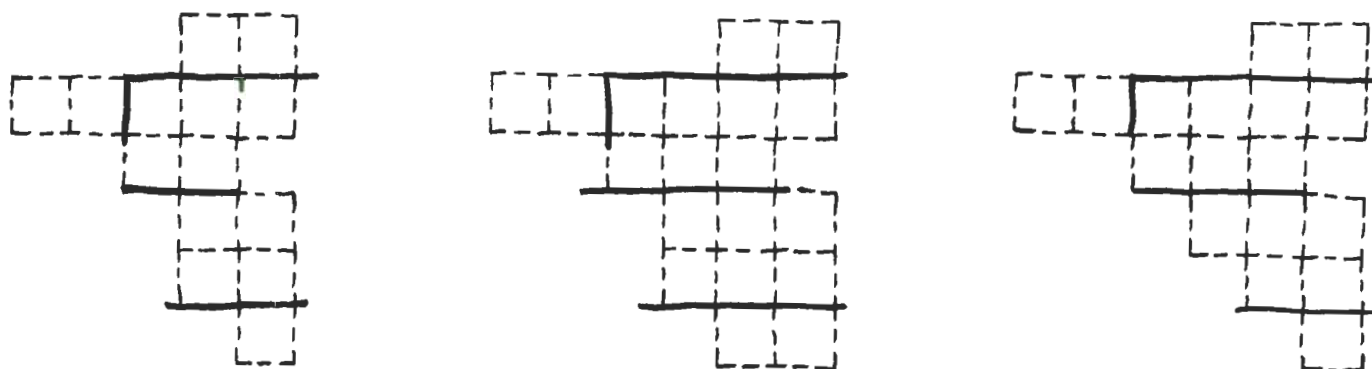
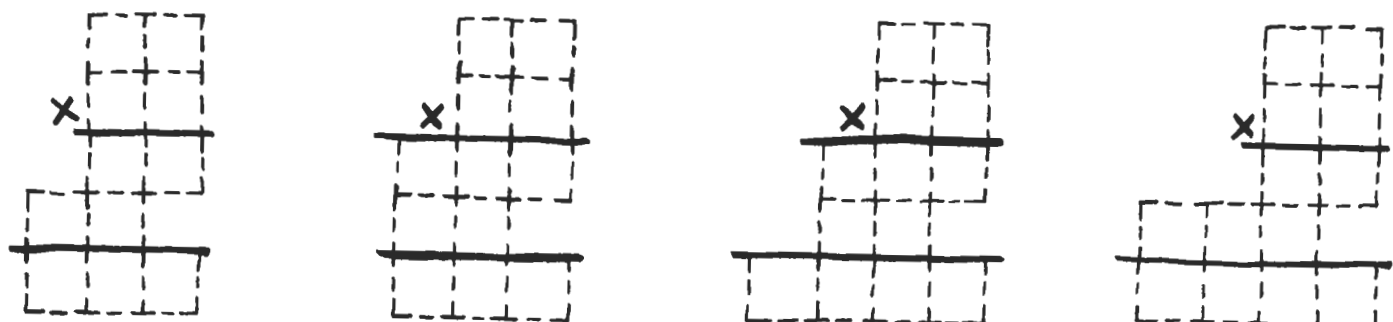
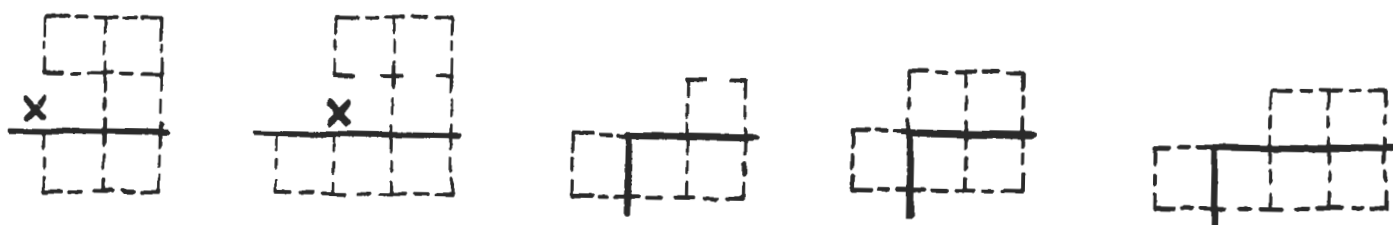
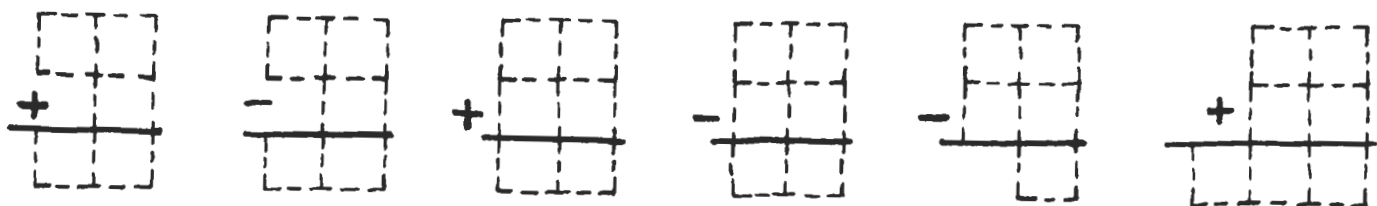
$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \begin{array}{c} \times \\ \times \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$
$$\begin{array}{|c|} \hline \square \times \\ \hline \square \times \\ \hline \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$
$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \begin{array}{l} \times \\ \\ \times \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$



$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \begin{array}{c} + \\ - \end{array} \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$
$$\begin{array}{|c|} \hline \square \div \square = \square \\ \hline \square \div \square = \square \\ \hline \end{array}$$
$$\boxed{\phantom{00}} \div \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

PLEASE COMPLETE THE EXAMPLES  
TO FIT THE FORMS.

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MODO QUE CONCUERDEN.





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Row 1:  $\begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} + \begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} = \begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array}$   $\begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} - \begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} = \begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array}$

Row 2:  $\begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} \times \begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} = \begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array}$   $\begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} \div \begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} = \begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array}$

Row 3:  $\begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \hline \end{array} \times \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \hline \end{array} = \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \hline \end{array}$   $\begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \hline \end{array} \div \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \hline \end{array} = \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \hline \end{array}$

Row 4:  $\begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \hline \end{array} \times \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \hline \end{array} = \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \hline \end{array}$   $\begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \hline \end{array} \div \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \hline \end{array} = \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \hline \end{array}$

On this page, learners make up examples using addition, multiplication, subtraction, and division to fit the forms given. Success on this page requires a good understanding of the algorithms.

Children will probably need some scrap "scribble sheets" to try out their examples before entering them in the forms.

Of course, the form can also be particularized in many ways as well, giving different bits of information as is done on the next page.

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$$\begin{array}{|c|c|} \hline 3 & \\ \hline + & \\ \hline 4 & \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & \\ \hline - & \\ \hline 5 & \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 8 & 9 \\ \hline + & \\ \hline & \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 2 & 0 \\ \hline - & \\ \hline & \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 3 & 8 \\ \hline - & \\ \hline & \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & \\ \hline + & 4 \\ \hline & \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline & \\ \hline \times & \\ \hline 3 & 9 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline & & \\ \hline \times & & \\ \hline 1 & 2 & 5 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline & \\ \hline 7 & 9 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 1 & 3 \\ \hline & 5 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|} \hline & & & \\ \hline 9 & 1 & 1 & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline & & \\ \hline \times & & \\ \hline & & \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline & & \\ \hline \times & & \\ \hline 1 & 0 & 0 \\ \hline 4 & 0 & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline & & \\ \hline \times & & \\ \hline 1 & 3 & 2 \\ \hline 9 & 9 & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|} \hline & & & \\ \hline \times & & & \\ \hline & 9 & 4 & \\ \hline 1 & 5 & 0 & 4 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline & & \\ \hline 7 & 5 & \\ \hline 4 & 5 & \\ \hline 5 & & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|} \hline 1 & 6 & & \\ \hline 1 & 1 & 2 & \\ \hline 1 & 2 & & \\ \hline 1 & 1 & & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|} \hline & 5 & & \\ \hline 3 & & & \\ \hline 1 & 0 & 0 & \\ \hline & & & 8 \\ \hline \end{array}$$

PLEASE COMPLETE THE EXAMPLES TO FIT THE FORMS. NAME \_\_\_\_\_

|                                                                                                     |                                                                                                                   |                                                                                                                |                                                                                                        |                                                                                     |                                                                                     |
|-----------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| $\begin{array}{ c c } \hline 3 & \\ \hline + & \\ \hline 4 & \\ \hline \end{array}$                 | $\begin{array}{ c c } \hline 6 & \\ \hline - & \\ \hline 5 & \\ \hline \end{array}$                               | $\begin{array}{ c c } \hline 8 & 9 \\ \hline + & \\ \hline & \\ \hline \end{array}$                            | $\begin{array}{ c c } \hline 2 & 0 \\ \hline - & \\ \hline & \\ \hline \end{array}$                    | $\begin{array}{ c c } \hline 3 & 8 \\ \hline - & \\ \hline & \\ \hline \end{array}$ | $\begin{array}{ c c } \hline 5 & \\ \hline + & 4 \\ \hline & \\ \hline \end{array}$ |
| $\begin{array}{ c c } \hline & \\ \hline \times & \\ \hline 3 & 9 \\ \hline \end{array}$            | $\begin{array}{ c c } \hline & \\ \hline \times & \\ \hline 1 & 2 & 5 \\ \hline \end{array}$                      | $\begin{array}{ c c } \hline 7 & \\ \hline \times & 9 \\ \hline & \\ \hline \end{array}$                       | $\begin{array}{ c c } \hline 1 & 3 \\ \hline \times & 5 \\ \hline & \\ \hline \end{array}$             | $\begin{array}{ c c } \hline & \\ \hline 9 & 1 & 1 \\ \hline \end{array}$           |                                                                                     |
| $\begin{array}{ c c } \hline & \\ \hline \times & \\ \hline 1 & 2 & 1 \\ \hline \end{array}$        | $\begin{array}{ c c } \hline & \\ \hline \times & \\ \hline 1 & 0 & 0 \\ \hline 4 & 0 & \\ \hline \end{array}$    | $\begin{array}{ c c } \hline & \\ \hline \times & \\ \hline 1 & 3 & 2 \\ \hline 9 & 9 & \\ \hline \end{array}$ | $\begin{array}{ c c } \hline & \\ \hline \times & 9 & 4 \\ \hline 1 & 5 & 0 & 8 \\ \hline \end{array}$ |                                                                                     |                                                                                     |
| $\begin{array}{ c c } \hline & \\ \hline 7 & 5 \\ \hline 4 & 5 \\ \hline 5 & \\ \hline \end{array}$ | $\begin{array}{ c c } \hline 1 & 6 \\ \hline 1 & 1 & 2 \\ \hline 1 & 2 & \\ \hline 1 & 1 & \\ \hline \end{array}$ | $\begin{array}{ c c } \hline 3 & \\ \hline 1 & 0 & 0 \\ \hline 8 & \\ \hline \end{array}$                      |                                                                                                        |                                                                                     |                                                                                     |

On this page, the constraints of the problem are a little tighter. Here the learner must complete the addition, subtraction, multiplication, or division problem to fit the form and the bits of information given. Consider the problem:

$$\begin{array}{|c|c|} \hline 5 & \\ \hline + & 4 \\ \hline & \\ \hline \end{array}$$

One clue is that the answer must have three digits. Since each addend has only two digits, we need to complete the example so that when we add in the tens column we will have a two-digit number. Since we have  $5 + 4$ , or 9 there already, we will have to have "carried" one from the units column. Thus we must use numbers in the units column which add to ten or more. One possibility would be:

$$\begin{array}{|c|c|} \hline 5 & 7 \\ \hline + & 4 & 4 \\ \hline 1 & 0 & 1 \\ \hline \end{array}$$

A good question for each example might be: "Can you find a different way to do it?" Some can be done in more than one way; some cannot.

Consider:

$$\begin{array}{|c|c|} \hline 9 & 1 & 1 \\ \hline \end{array}$$

We know that  $9 \overline{)11}^{1r2}$ . Thus the first digit in the quotient has to be 1 and the next number to be divided by 9 must be a two digit number that begins with 2. What two-digit multiples of 9 begin with 2? 27 is the only one, and  $9 \overline{)27}^3$ . Thus the completed example:

$$\begin{array}{|c|c|} \hline 1 & 3 \\ \hline 9 & 1 & 1 & 7 \\ \hline \end{array}$$

Some examples get more difficult:

$$\begin{array}{|c|c|} \hline & \\ \hline \times & \\ \hline & 9 & 4 \\ \hline 1 & 5 & 0 & 4 \\ \hline \end{array}$$

The clues are skimpy. Before we start guessing, we should probably fill in all the information we can derive from what is given. Because we know the first partial product and the total, we can find the second partial product:

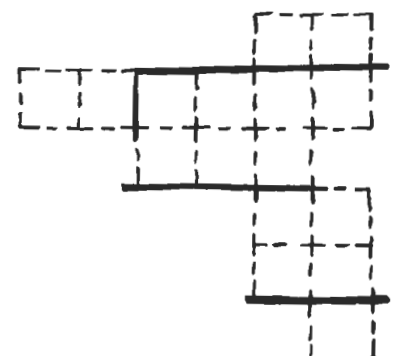
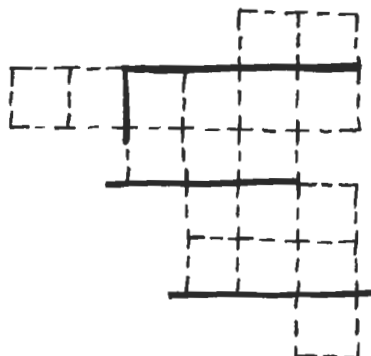
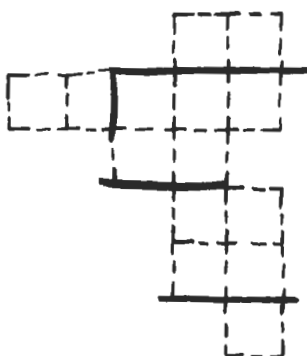
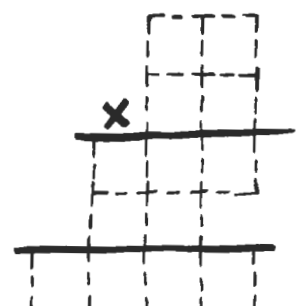
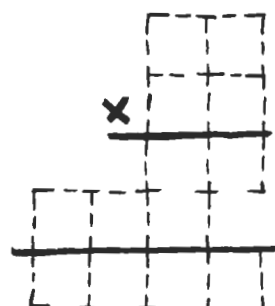
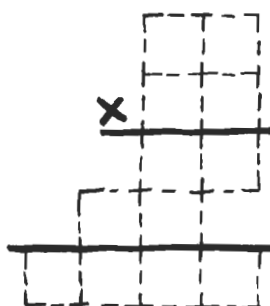
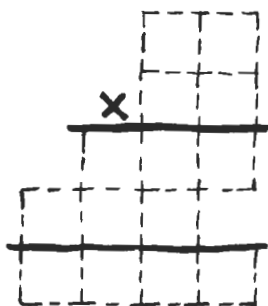
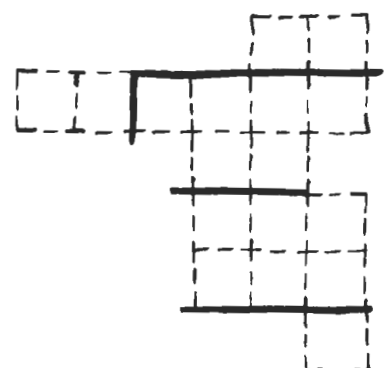
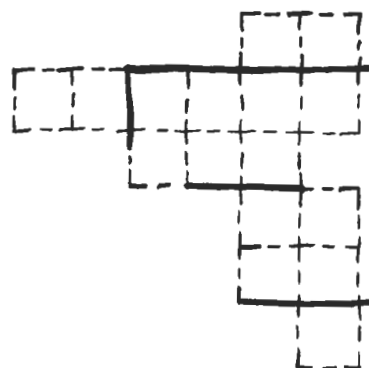
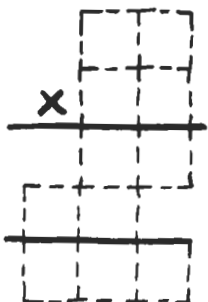
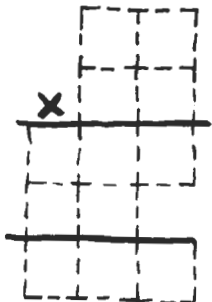
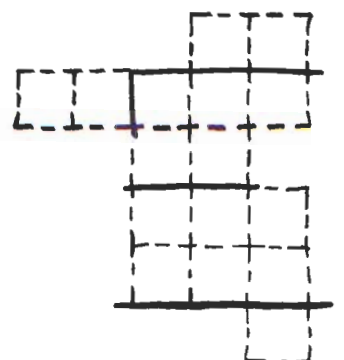
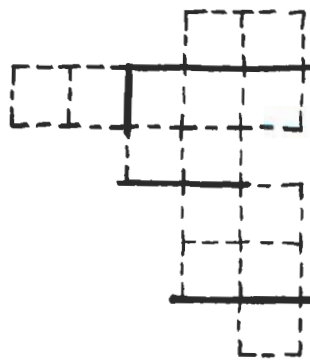
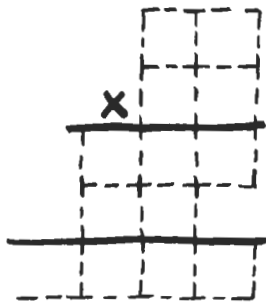
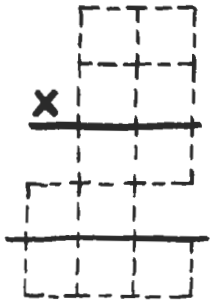
$$\begin{array}{|c|c|} \hline & \\ \hline \times & \\ \hline & 9 & 4 \\ \hline 1 & 4 & 1 \\ \hline 1 & 5 & 0 & 4 \\ \hline \end{array}$$

Now the question becomes: What two digit number can be multiplied by a number to get 94 and by another number to get 141? (Essentially, what numbers are common factors of 94 and 141?) Some trial and error will get the final result:

$$\begin{array}{|c|c|} \hline 4 & 7 \\ \hline \times & 3 & 2 \\ \hline & 9 & 4 \\ \hline 1 & 4 & 1 \\ \hline 1 & 5 & 0 & 4 \\ \hline \end{array}$$

PLEASE FIND EXAMPLES TO FIT  
THE FORMS

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DE MODO QUE CONCUERDEN



PLEASE FIND EXAMPLES  
TO FIT THE FORMS.

NAME \_\_\_\_\_

We could take away the "1" in the top number:

$$\begin{array}{r} \text{x} \quad 9 \\ 1 \quad 3 \\ \hline 4 \end{array}$$

Still the problem solver could solve it — and only one solution is possible.

This is a continuation of the activities on the previous two pages. Different forms for the examples are given:

A question to explore:

Make up a problem to fit a form. What are the **fewest** clues you could give and still lead the problem solver to exactly your problem?

For example, if we make up the problem:

$$\begin{array}{r} 1 \quad 7 \\ \text{x} \quad 2 \quad 9 \\ 1 \quad 5 \quad 3 \\ 3 \quad 4 \\ \hline 4 \quad 9 \quad 3 \end{array}$$

we could erase all but the following:

$$\begin{array}{r} \text{x} \quad 9 \\ 1 \quad 3 \\ \hline 4 \end{array}$$

The problem solver would be able to figure only one solution — the problem we originally stated.

The question is — can we take away any of those "clues" and **still** have the situation where the problem solver could only find **one** solution?

# "FROM THE LISTS" "DE LAS LISTA"

A

|                                                        |                                                        |                                                        |                                                        |                                                        |                                                        |                                                        |                                                        |                                                        |                                                        |
|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|
| $\begin{array}{r} 0 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 1 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 2 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 3 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 4 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 9 \\ \times 7 \\ \hline \end{array}$ |
|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|

B

|                                                        |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|
| $\begin{array}{r} 0 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 10 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 20 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 30 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 40 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 50 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 60 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 70 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 80 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 90 \\ \times 7 \\ \hline \end{array}$ |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|

| A                                                 | B                                                 | A+B                                                     |
|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 25 \\ \times 7 \\ \hline \end{array}$ |

|                                                   |                                                   |                                                         |
|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 18 \\ \times 7 \\ \hline \end{array}$ |
|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------------|

|                                                        |                                                         |                                                   |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------|
| $\begin{array}{r} 3 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 70 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------|

|                                                      |                                                       |                                                   |
|------------------------------------------------------|-------------------------------------------------------|---------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline 42 \end{array}$ | $\begin{array}{r} \times 7 \\ \hline 630 \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ |
|------------------------------------------------------|-------------------------------------------------------|---------------------------------------------------|

|                                                   |                                                   |                                                       |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline 224 \end{array}$ |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|

|                                                        |                                                         |                                                   |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------|
| $\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 40 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------|

|                                                   |                                                   |                                                         |
|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 59 \\ \times 7 \\ \hline \end{array}$ |
|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------------|

|                                                   |                                                   |                                                         |
|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 86 \\ \times 7 \\ \hline \end{array}$ |
|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------------|

|                                                   |                                                   |                                                       |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline 588 \end{array}$ |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|

|                                                        |                                                         |                                                   |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------|
| $\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 40 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ |
|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------|

|                                                   |                                                   |                                                       |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline 273 \end{array}$ |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|

|                                                   |                                                   |                                                       |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline 497 \end{array}$ |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------------|

"FROM THE LISTS" NAME \_\_\_\_\_

|   |                                                        |                                                        |                                                        |                                                        |                                                        |                                                        |                                                        |                                                        |                                                        |                                                        |
|---|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|
| A | 0                                                      | 1                                                      | 2                                                      | 3                                                      | 4                                                      | 5                                                      | 6                                                      | 7                                                      | 8                                                      | 9                                                      |
|   | $\begin{array}{r} 0 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 1 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 2 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 3 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 4 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 9 \\ \times 7 \\ \hline \end{array}$ |

|   |                                                        |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |
|---|--------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|
| B | 0                                                      | 10                                                      | 20                                                      | 30                                                      | 40                                                      | 50                                                      | 60                                                      | 70                                                      | 80                                                      | 90                                                      |
|   | $\begin{array}{r} 0 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 10 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 20 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 30 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 40 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 50 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 60 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 70 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 80 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 90 \\ \times 7 \\ \hline \end{array}$ |

|                                                   |                                                   |                                                          |
|---------------------------------------------------|---------------------------------------------------|----------------------------------------------------------|
| A                                                 | B                                                 | A+B                                                      |
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 25 \\ \times 7 \\ \hline \end{array}$  |
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 18 \\ \times 7 \\ \hline \end{array}$  |
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 3 \\ \times 7 \\ \hline \end{array}$   |
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 70 \\ \times 7 \\ \hline \end{array}$  |
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 18 \\ \times 7 \\ \hline \end{array}$  |
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$   |
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 40 \\ \times 7 \\ \hline \end{array}$  |
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 224 \\ \times 7 \\ \hline \end{array}$ |
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 59 \\ \times 7 \\ \hline \end{array}$  |
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 86 \\ \times 7 \\ \hline \end{array}$  |
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 588 \\ \times 7 \\ \hline \end{array}$ |
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$   |
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 40 \\ \times 7 \\ \hline \end{array}$  |
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 273 \\ \times 7 \\ \hline \end{array}$ |
| $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 497 \\ \times 7 \\ \hline \end{array}$ |

Completing the tables at the top of the page can be much more than routine activity.

After the first 3 entries—which are routine—we can talk about different ways to get the other entries.

|                                                          |                                                          |                                                           |                                                           |                                                        |                                                        |
|----------------------------------------------------------|----------------------------------------------------------|-----------------------------------------------------------|-----------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|
| 0                                                        | 1                                                        | 2                                                         | 3                                                         | 4                                                      | 5                                                      |
| $\begin{array}{r} 0 \\ \times 7 \\ \hline 0 \end{array}$ | $\begin{array}{r} 1 \\ \times 7 \\ \hline 7 \end{array}$ | $\begin{array}{r} 2 \\ \times 7 \\ \hline 14 \end{array}$ | $\begin{array}{r} 3 \\ \times 7 \\ \hline 21 \end{array}$ | $\begin{array}{r} 4 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$ |

If someone forgot that  $3 \times 7$  (or  $7 \times 3$ ) = 21, how could he find the right number? He could "count on from 14" or "add the 2 previous entries—7 and 14."

What about  $5 \times 7$  (or  $7 \times 5$ )? " $28 + 7 = 35$ " or "two 7's are 14 and 3 7's are 21, and  $14 + 21 = 35$ " or "five is half of 10; ten 7's is 70; and half of 70 is 35."

What about  $6 \times 7$  (or  $7 \times 6$ )? " $35 + 7 = 42$ " or "6 is twice 3; three 7's are 21, and twice 21 is 42" or "four 7's are 28 and two 7's are 14; and  $28 + 14 = 42$ " or "I remember that  $7 \times 7 = 49$ , and  $49 - 7 = 42$ , so six 7's must be 42," etc.

We are not asking "what is the best way"—rather "mention some different ways." The more talk there is about each item, the more apt the participants will be to "remember" next time. Further, such discussions emphasize the great variety of ways there usually are to solve problems in arithmetic. This search for "different ways" often becomes more spirited—a healthy development.

The second row in the table can be approached in the same way, but very soon every one would rather rattle off the answers by looking at the corresponding entry in the top row—"210, 280, 350, 420, etc." as fast as they can be written down.

Using the table may be considered as "close to cheating" by some . . . and they are free to disregard the table. However, using work you have already done is a legitimate and most useful habit developed by too few learners.

|                                                           |                                                             |                                                             |
|-----------------------------------------------------------|-------------------------------------------------------------|-------------------------------------------------------------|
| A                                                         | B                                                           | A+B                                                         |
| $\begin{array}{r} 5 \\ \times 7 \\ \hline 35 \end{array}$ | $\begin{array}{r} 20 \\ \times 7 \\ \hline 140 \end{array}$ | $\begin{array}{r} 25 \\ \times 7 \\ \hline 175 \end{array}$ |

|                                                           |                                                             |                                                             |
|-----------------------------------------------------------|-------------------------------------------------------------|-------------------------------------------------------------|
| 6                                                         | 90                                                          | 96                                                          |
| $\begin{array}{r} 6 \\ \times 7 \\ \hline 42 \end{array}$ | $\begin{array}{r} 90 \\ \times 7 \\ \hline 630 \end{array}$ | $\begin{array}{r} 96 \\ \times 7 \\ \hline 672 \end{array}$ |

The obvious purpose behind this activity is to focus on the "multiplication and division algorithms"—not on "multiplication facts." Even the slowest computer can move ahead quickly and without errors—provided he can use the tables and carry out rather elementary additions (and subtractions).

"DE LAS LISTA" "FROM THE LISTS"

|          |          |          |          |          |          |          |          |          |          |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0        | 1        | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        |
| <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> |

|          |          |          |          |          |          |          |          |          |          |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0        | 10       | 20       | 30       | 40       | 50       | 60       | 70       | 80       | 90       |
| <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> | <u>x</u> |

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|

|          |          |          |
|----------|----------|----------|
| <u>x</u> | <u>x</u> | <u>x</u> |
|----------|----------|----------|



"FROM THE LISTS" NAME \_\_\_\_\_

|   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| x | x | x | x | x | x | x | x | x | x |

|   |    |    |    |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|----|----|
| 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| x | x  | x  | x  | x  | x  | x  | x  | x  | x  |

|   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|
| x | x | x | x | x | x | x | x | x |
| x | x | x | x | x | x | x | x | x |
| x | x | x | x | x | x | x | x | x |
| x | x | x | x | x | x | x | x | x |

This can be particularized by using any common multiplier to complete the table, and indicating specific examples. Children can also make up their own examples with the simple rule that all must be different.

Or, the page can be used to explore this idea: suppose the two items were "subtracted" and the "difference" noted in the third space? For example:

We will assume the "x 7" tables from the previous page:

| A          | B          | B-A        |
|------------|------------|------------|
| 2          | 50         | 48         |
| $\times 7$ | $\times 7$ | $\times 7$ |
| 14         | 350        | 336        |

or

| A          | B          | B-A        |
|------------|------------|------------|
| 1          | 40         | 39         |
| $\times 7$ | $\times 7$ | $\times 7$ |
| 7          | 280        | 273        |

Or, perhaps the children would rather reverse the "A" and "B":

| B          | A          | B-A        |
|------------|------------|------------|
| 70         | 2          | 68         |
| $\times 7$ | $\times 7$ | $\times 7$ |
| 490        | 14         | 476        |

Could you make up examples if the tables were turned under?

Could you make up examples if the top table (A) were turned under or not filled in?

Suppose we only complete the entries in the chart for 5 thru 9 and 50 thru 90: could you still find the answer to any example (with one 2-digit factor and the "common" multiplier in the table?) Or, can you remember the smaller or easier entries?

Some might enjoy participating in a hunt for examples in which all of the digits in the third space are different and can be arranged in a consecutive sequence. Here is an example:

|            |            |            |
|------------|------------|------------|
| 2          | 50         | 52         |
| $\times 7$ | $\times 7$ | $\times 7$ |
| 14         | 350        | 364        |

2, 3, 4, 5, 6, 7

|   |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |
|---|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|
| A | $\begin{array}{r} 13 \\ \times 1 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 2 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 3 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 4 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 5 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 6 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 8 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 9 \\ \hline \end{array}$ |
|---|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|

|   |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |
|---|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|
| B | $\begin{array}{r} 13 \\ \times 10 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 20 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 30 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 40 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 50 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 60 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 70 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 80 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 90 \\ \hline \end{array}$ |
|---|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|

|                                                                                                                     |                                                                                |                                                                                |                                                                                |
|---------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------|--------------------------------------------------------------------------------|--------------------------------------------------------------------------------|
| $\begin{array}{r} 13 \\ \times 42 \\ \hline \end{array}$<br>-----<br>+ A<br>-----<br>+ B<br>-----<br>+ A+B<br>----- | $\begin{array}{r} 13 \\ \times 67 \\ \hline \end{array}$<br>-----<br><br>----- | $\begin{array}{r} 13 \\ \times 58 \\ \hline \end{array}$<br>-----<br><br>----- | $\begin{array}{r} 13 \\ \times 16 \\ \hline \end{array}$<br>-----<br><br>----- |
| $\begin{array}{r} 13 \\ \times 95 \\ \hline \end{array}$<br>-----<br><br>-----                                      | $\begin{array}{r} 13 \\ \times 29 \\ \hline \end{array}$<br>-----<br><br>----- | $\begin{array}{r} 13 \\ \times 83 \\ \hline \end{array}$<br>-----<br><br>----- | $\begin{array}{r} 13 \\ \times 78 \\ \hline \end{array}$<br>-----<br><br>----- |
| $\begin{array}{r} 13 \\ \times 84 \\ \hline \end{array}$<br>-----<br><br>-----                                      | $\begin{array}{r} 13 \\ \times 38 \\ \hline \end{array}$<br>-----<br><br>----- | $\begin{array}{r} 13 \\ \times 97 \\ \hline \end{array}$<br>-----<br><br>----- | $\begin{array}{r} 13 \\ \times 79 \\ \hline \end{array}$<br>-----<br><br>----- |

|   |                                                                                                                    | NAME _____                                                                                  |                                                                                             |                                                                                             |                                                          |                                                          |                                                          |                                                          |                                                          |  |  |
|---|--------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|--|--|
| A | $\begin{array}{r} 13 \\ \times 1 \\ \hline \end{array}$                                                            | $\begin{array}{r} 13 \\ \times 2 \\ \hline \end{array}$                                     | $\begin{array}{r} 13 \\ \times 3 \\ \hline \end{array}$                                     | $\begin{array}{r} 13 \\ \times 4 \\ \hline \end{array}$                                     | $\begin{array}{r} 13 \\ \times 5 \\ \hline \end{array}$  | $\begin{array}{r} 13 \\ \times 6 \\ \hline \end{array}$  | $\begin{array}{r} 13 \\ \times 7 \\ \hline \end{array}$  | $\begin{array}{r} 13 \\ \times 8 \\ \hline \end{array}$  | $\begin{array}{r} 13 \\ \times 9 \\ \hline \end{array}$  |  |  |
|   | $\begin{array}{r} 13 \\ \times 10 \\ \hline \end{array}$                                                           | $\begin{array}{r} 13 \\ \times 20 \\ \hline \end{array}$                                    | $\begin{array}{r} 13 \\ \times 30 \\ \hline \end{array}$                                    | $\begin{array}{r} 13 \\ \times 40 \\ \hline \end{array}$                                    | $\begin{array}{r} 13 \\ \times 50 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 60 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 70 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 80 \\ \hline \end{array}$ | $\begin{array}{r} 13 \\ \times 90 \\ \hline \end{array}$ |  |  |
|   |                                                                                                                    |                                                                                             |                                                                                             |                                                                                             |                                                          |                                                          |                                                          |                                                          |                                                          |  |  |
|   | $\begin{array}{r} 13 \\ \times 42 \\ \hline \end{array}$                                                           | $\begin{array}{r} 13 \\ \times 67 \\ \hline \end{array}$                                    | $\begin{array}{r} 13 \\ \times 58 \\ \hline \end{array}$                                    | $\begin{array}{r} 13 \\ \times 16 \\ \hline \end{array}$                                    |                                                          |                                                          |                                                          |                                                          |                                                          |  |  |
|   | $\begin{array}{r} \text{-----} \\ + A \\ \text{-----} \\ + B \\ \text{-----} \\ + A+B \\ \text{-----} \end{array}$ | $\begin{array}{r} \text{-----} \\ \text{-----} \\ \text{-----} \\ \text{-----} \end{array}$ | $\begin{array}{r} \text{-----} \\ \text{-----} \\ \text{-----} \\ \text{-----} \end{array}$ | $\begin{array}{r} \text{-----} \\ \text{-----} \\ \text{-----} \\ \text{-----} \end{array}$ |                                                          |                                                          |                                                          |                                                          |                                                          |  |  |
|   | $\begin{array}{r} 13 \\ \times 95 \\ \hline \end{array}$                                                           | $\begin{array}{r} 13 \\ \times 29 \\ \hline \end{array}$                                    | $\begin{array}{r} 13 \\ \times 83 \\ \hline \end{array}$                                    | $\begin{array}{r} 13 \\ \times 78 \\ \hline \end{array}$                                    |                                                          |                                                          |                                                          |                                                          |                                                          |  |  |
|   | $\begin{array}{r} \text{-----} \\ \text{-----} \\ \text{-----} \end{array}$                                        | $\begin{array}{r} \text{-----} \\ \text{-----} \\ \text{-----} \end{array}$                 | $\begin{array}{r} \text{-----} \\ \text{-----} \\ \text{-----} \end{array}$                 | $\begin{array}{r} \text{-----} \\ \text{-----} \\ \text{-----} \end{array}$                 |                                                          |                                                          |                                                          |                                                          |                                                          |  |  |
|   | $\begin{array}{r} 13 \\ \times 84 \\ \hline \end{array}$                                                           | $\begin{array}{r} 13 \\ \times 38 \\ \hline \end{array}$                                    | $\begin{array}{r} 13 \\ \times 97 \\ \hline \end{array}$                                    | $\begin{array}{r} 13 \\ \times 79 \\ \hline \end{array}$                                    |                                                          |                                                          |                                                          |                                                          |                                                          |  |  |
|   | $\begin{array}{r} \text{-----} \\ \text{-----} \\ \text{-----} \end{array}$                                        | $\begin{array}{r} \text{-----} \\ \text{-----} \\ \text{-----} \end{array}$                 | $\begin{array}{r} \text{-----} \\ \text{-----} \\ \text{-----} \end{array}$                 | $\begin{array}{r} \text{-----} \\ \text{-----} \\ \text{-----} \end{array}$                 |                                                          |                                                          |                                                          |                                                          |                                                          |  |  |

"From the list" grows up. The "common factor" now appears in the top of each entry box.

Now the advantages of knowing several ways to get each entry offers opportunities for short cuts and for checking.

Since the "addition" aspect of the activity has become more complex, a new format is introduced.

$$\begin{array}{r}
 13 \\
 \times 42 \\
 \hline
 26 \leftarrow A \\
 520 \leftarrow B \\
 \hline
 546 \leftarrow A + B
 \end{array}$$

Some students may want to demonstrate their ability to work with only list B or with neither list. They can fold under as much of the page as they like or leave the lists blank.

Others may have the same urge, but feel more comfortable with the list available for checking.

Others have a very unsure grasp on the basic "multiplication skills" involved and would make a discouraging number of errors. By using the lists, most

errors are eliminated, and they will increase their familiarity with facts they have not yet committed to memory.

In this way, all members of the group can focus on the mechanics of the algorithm without being unduly distracted or burdened . . . and each at his own level of development.

Students ought to save this page so that after they have completed the next page, they can compare the two pages.

AB

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NAME \_\_\_\_\_

|   |                        |                        |                        |                        |                        |                        |                        |                        |                        |
|---|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| A | $\frac{13}{\times 1}$  | $\frac{13}{\times 2}$  | $\frac{13}{\times 3}$  | $\frac{13}{\times 4}$  | $\frac{13}{\times 5}$  | $\frac{13}{\times 6}$  | $\frac{13}{\times 7}$  | $\frac{13}{\times 8}$  | $\frac{13}{\times 9}$  |
| B | $\frac{13}{\times 10}$ | $\frac{13}{\times 20}$ | $\frac{13}{\times 30}$ | $\frac{13}{\times 40}$ | $\frac{13}{\times 50}$ | $\frac{13}{\times 60}$ | $\frac{13}{\times 70}$ | $\frac{13}{\times 80}$ | $\frac{13}{\times 90}$ |

|                                                                                                                                         |                                                                                                   |                                                                                                    |                                                                                                    |
|-----------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad +A \\ \quad \quad \quad +B \\ \hline 546 \quad +A+B \end{array}$ | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 871 \end{array}$ | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 754 \end{array}$  | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 208 \end{array}$  |
| $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 1235 \end{array}$                                      | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 377 \end{array}$ | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 1079 \end{array}$ | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 1014 \end{array}$ |
| $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 1092 \end{array}$                                      | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 494 \end{array}$ | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 1261 \end{array}$ | $\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \\ \hline 1027 \end{array}$ |

A comparison with the previous page shows that all examples from that page are repeated here, but in their equivalent "division" mode.

The problem is to find two entries, one from each list whose sum is indicated already.

$$\begin{array}{r} 13 \\ \times \quad \quad \\ \hline \quad \quad \quad \leftarrow A \\ \quad \quad \quad \leftarrow B \\ \hline 546 \leftarrow A + B \end{array}$$

A little study of this problem leads to the strategy of beginning by finding the largest number in list B that is not larger than the final sum. (This is often referred to as "finding the trial divisor.")

In this case, the number from list B is 520:

$$\begin{array}{r} 13 \\ \times 40 \\ \hline \quad \quad \quad \leftarrow A \\ \quad \quad \quad \leftarrow B \\ \hline 546 \leftarrow A + B \end{array}$$

That entry is  $13 \times 40$  (or  $40 \times 13$ ), so "4" is written in the "tens place."

How many more to be accounted for? . . . The difference between 546 and 520—or 26. That number occurs in list A— $13 \times 2 = 26$  (or  $2 \times 13 = 26$ ), so the example is completed.

$$\begin{array}{r} 13 \\ \times 42 \\ \hline 26 \leftarrow A \\ 520 \leftarrow B \\ \hline 546 \leftarrow A + B \end{array}$$

Compare this procedure with the standard division algorithm:

$$\begin{array}{r} 42 \\ 13 \overline{) 546} \\ \underline{520} \\ 26 \\ \underline{26} \\ 0 \end{array} \rightarrow \begin{array}{r} 42 \\ 13 \overline{) 546} \\ \underline{520} \\ 26 \\ \underline{26} \\ 0 \end{array}$$

The steps and reasons for each are precisely those called for in the activities on this page.

If students compare this page with the previous page they will see that the only difference, example for example, is in the information that was given . . . and that is the difference between multiplication and division.

Multiplication: Given two factors, find the product.

Division: Given the product and one factor, find the other factor.

A

|                                                                 |                                                                 |                                                                 |                                                                 |                                                                 |                                                                 |                                                                 |                                                                 |                                                                 |
|-----------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------|
| $\begin{array}{r} \text{---} \\ \times 1 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 2 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 3 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 4 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 5 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 6 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 8 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 9 \\ \hline \end{array}$ |
|-----------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------|

B

|                                                                  |                                                                  |                                                                  |                                                                  |                                                                  |                                                                  |                                                                  |                                                                  |                                                                  |
|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|
| $\begin{array}{r} \text{---} \\ \times 10 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 20 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 30 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 40 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 50 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 60 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 70 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 80 \\ \hline \end{array}$ | $\begin{array}{r} \text{---} \\ \times 90 \\ \hline \end{array}$ |
|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|

|                                                                                                                                                                          |                                                                                                  |                                                                                                  |                                                                                                  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|
| $\begin{array}{r} \text{---} \\ \times 42 \\ \hline \end{array}$<br>$\leftarrow A$<br>$\text{---}$<br>$\leftarrow B$<br>$\text{---}$<br>$\leftarrow A+B$<br>$\text{---}$ | $\begin{array}{r} \text{---} \\ \times 67 \\ \hline \end{array}$<br>$\text{---}$<br>$\text{---}$ | $\begin{array}{r} \text{---} \\ \times 51 \\ \hline \end{array}$<br>$\text{---}$<br>$\text{---}$ | $\begin{array}{r} \text{---} \\ \times 16 \\ \hline \end{array}$<br>$\text{---}$<br>$\text{---}$ |
| $\begin{array}{r} \text{---} \\ \times 95 \\ \hline \end{array}$<br>$\text{---}$<br>$\text{---}$                                                                         | $\begin{array}{r} \text{---} \\ \times 89 \\ \hline \end{array}$<br>$\text{---}$<br>$\text{---}$ | $\begin{array}{r} \text{---} \\ \times 83 \\ \hline \end{array}$<br>$\text{---}$<br>$\text{---}$ | $\begin{array}{r} \text{---} \\ \times 79 \\ \hline \end{array}$<br>$\text{---}$<br>$\text{---}$ |
| $\begin{array}{r} \text{---} \\ \times 84 \\ \hline \end{array}$<br>$\text{---}$<br>$\text{---}$                                                                         | $\begin{array}{r} \text{---} \\ \times 38 \\ \hline \end{array}$<br>$\text{---}$<br>$\text{---}$ | $\begin{array}{r} \text{---} \\ \times 97 \\ \hline \end{array}$<br>$\text{---}$<br>$\text{---}$ | $\begin{array}{r} \text{---} \\ \times 78 \\ \hline \end{array}$<br>$\text{---}$<br>$\text{---}$ |

| NAME _____ |                        |                        |                        |                        |                        |                        |                        |                        |                        |
|------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| A          | $\overline{\times 1}$  | $\overline{\times 2}$  | $\overline{\times 3}$  | $\overline{\times 4}$  | $\overline{\times 5}$  | $\overline{\times 6}$  | $\overline{\times 7}$  | $\overline{\times 8}$  | $\overline{\times 9}$  |
| B          | $\overline{\times 10}$ | $\overline{\times 20}$ | $\overline{\times 30}$ | $\overline{\times 40}$ | $\overline{\times 50}$ | $\overline{\times 60}$ | $\overline{\times 70}$ | $\overline{\times 80}$ | $\overline{\times 90}$ |

|                                                                                                                    |                                                                                        |                                                                                        |                                                                                        |
|--------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|
| $\begin{array}{r} \overline{\times 42} \\ \hline \text{---} + A \\ \text{---} + B \\ \text{---} + A+B \end{array}$ | $\begin{array}{r} \overline{\times 67} \\ \hline \text{---} \\ \text{---} \end{array}$ | $\begin{array}{r} \overline{\times 51} \\ \hline \text{---} \\ \text{---} \end{array}$ | $\begin{array}{r} \overline{\times 16} \\ \hline \text{---} \\ \text{---} \end{array}$ |
| $\begin{array}{r} \overline{\times 95} \\ \hline \text{---} \\ \text{---} \end{array}$                             | $\begin{array}{r} \overline{\times 89} \\ \hline \text{---} \\ \text{---} \end{array}$ | $\begin{array}{r} \overline{\times 83} \\ \hline \text{---} \\ \text{---} \end{array}$ | $\begin{array}{r} \overline{\times 79} \\ \hline \text{---} \\ \text{---} \end{array}$ |
| $\begin{array}{r} \overline{\times 84} \\ \hline \text{---} \\ \text{---} \end{array}$                             | $\begin{array}{r} \overline{\times 38} \\ \hline \text{---} \\ \text{---} \end{array}$ | $\begin{array}{r} \overline{\times 97} \\ \hline \text{---} \\ \text{---} \end{array}$ | $\begin{array}{r} \overline{\times 78} \\ \hline \text{---} \\ \text{---} \end{array}$ |

The second table (B) is so easily made up by referring to the first table (A), it can become an easy bit of arithmetic. Multiplying by 10 is accomplished by including another "0" on the right side of the other factor. This has the effect on the number of "moving it over 1 place":  $1700 \times 10 = 17000$ .

Any 2-digit or 3-digit number can be selected as the missing factor (multiplicand).

$$\begin{array}{r}
 425 \\
 \times 42 \\
 \hline
 850 \quad \leftarrow \text{from table A} \\
 17000 \quad \leftarrow \text{from table B} \\
 \hline
 17850
 \end{array}$$

At some point, the question of using a single table can be explored. What problems arise?

In the example above, such "moving over" can be shown by omitting the "0" in the units place of all entries in table B. The example given above would then appear this way:

$$\begin{array}{r}
 425 \\
 \times 42 \\
 \hline
 850 \quad \leftarrow A \\
 1700 \quad \leftarrow B \\
 \hline
 17850
 \end{array}$$

and this is the standard form.

|   |            |            |            |            |            |            |            |            |            |
|---|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| A | <u>x 1</u> | <u>x 2</u> | <u>x 3</u> | <u>x 4</u> | <u>x 5</u> | <u>x 6</u> | <u>x 7</u> | <u>x 8</u> | <u>x 9</u> |
|---|------------|------------|------------|------------|------------|------------|------------|------------|------------|

|   |             |             |             |             |             |             |             |             |             |
|---|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| B | <u>x 10</u> | <u>x 20</u> | <u>x 30</u> | <u>x 40</u> | <u>x 50</u> | <u>x 60</u> | <u>x 70</u> | <u>x 80</u> | <u>x 90</u> |
|---|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|

|                                                                     |                                     |                                     |                                     |
|---------------------------------------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
| <u>x</u><br>-----<br>← A<br>-----<br>← B<br>-----<br>← A+B<br>----- | <u>x</u><br>-----<br>-----<br>----- | <u>x</u><br>-----<br>-----<br>----- | <u>x</u><br>-----<br>-----<br>----- |
| <u>x</u><br>-----<br>-----<br>-----                                 | <u>x</u><br>-----<br>-----<br>----- | <u>x</u><br>-----<br>-----<br>----- | <u>x</u><br>-----<br>-----<br>----- |
| <u>x</u><br>-----<br>-----<br>-----                                 | <u>x</u><br>-----<br>-----<br>----- | <u>x</u><br>-----<br>-----<br>----- | <u>x</u><br>-----<br>-----<br>----- |



| NAME _____ |             |             |             |             |             |             |             |             |             |
|------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| A          | <u>x 1</u>  | <u>x 2</u>  | <u>x 3</u>  | <u>x 4</u>  | <u>x 5</u>  | <u>x 6</u>  | <u>x 7</u>  | <u>x 8</u>  | <u>x 9</u>  |
|            |             |             |             |             |             |             |             |             |             |
| B          | <u>x 10</u> | <u>x 20</u> | <u>x 30</u> | <u>x 40</u> | <u>x 50</u> | <u>x 60</u> | <u>x 70</u> | <u>x 80</u> | <u>x 90</u> |
|            |             |             |             |             |             |             |             |             |             |

|                                                                                |                                         |                                         |                                         |
|--------------------------------------------------------------------------------|-----------------------------------------|-----------------------------------------|-----------------------------------------|
| <u>  x  </u><br>-----<br>← A<br><u>  x  </u><br>-----<br>← B<br>-----<br>← A+B | <u>  x  </u><br>-----<br>-----<br>----- | <u>  x  </u><br>-----<br>-----<br>----- | <u>  x  </u><br>-----<br>-----<br>----- |
| <u>  x  </u><br>-----<br>-----<br>-----                                        | <u>  x  </u><br>-----<br>-----<br>----- | <u>  x  </u><br>-----<br>-----<br>----- | <u>  x  </u><br>-----<br>-----<br>----- |
| <u>  x  </u><br>-----<br>-----<br>-----                                        | <u>  x  </u><br>-----<br>-----<br>----- | <u>  x  </u><br>-----<br>-----<br>----- | <u>  x  </u><br>-----<br>-----<br>----- |

This page can be particularized in the style of any of the previous pages.

Or children can make up their own example and make their own investigations.

An interesting missing factor (or multiplicand) to look at is 37. The interest develops from the facts that:

$$\begin{aligned}
 37 \times 3 &= 111 \\
 37 \times 6 &= 222 \\
 37 \times 9 &= 333 \\
 37 \times 12 &= 444 \\
 &\text{etc.}
 \end{aligned}$$

Is 489 a multiple of 37? Select the largest digit from each place:

$$\begin{array}{r}
 489 \\
 - 444 \\
 \hline
 45
 \end{array}$$

and since 45 is not divisible by 37, neither is 489. Further, 489 will have a remainder of 8 ( $45 - 37 = 8$ ) when divided by 37.

Also, since 444 is  $4 \times 3 \times 37$  or  $12 \times 37$ ,  $487 \div 37$  must be  $12 + 1$  or 13 with remainder 8.

Multiples of 37 remain interesting when they have more than 3 digits since

$$37 \times 30 = 1110$$

$$37 \times 60 = 2220$$

etc.

Consider  $7298 \div 37$ :

$$\begin{array}{r}
 7298 \\
 - 6660 \quad (3 \times 6 \times 10 \times 37 \text{ or } 180 \times 37) \\
 \hline
 638 \\
 - 555 \quad (5 \times 3 \times 37 \text{ or } 15 \times 37) \\
 \hline
 83 \\
 - 74 \quad (2 \times 37) \\
 \hline
 9
 \end{array}$$

So,  $7298 \div 37 = 180 + 15 + 2$  and remainder of 9.

The same approach can be used with larger numbers:

Is 487,923 divisible by 37?

$$\begin{array}{r}
 487,923 \\
 - 444,000 \quad (12,000 \times 37) \\
 \hline
 43,923 \\
 - 33,300 \quad (900 \times 37) \\
 \hline
 10,623 \\
 - 9,990 \quad (270 \times 37) \\
 \hline
 633 \\
 - 555 \quad (15 \times 37) \\
 \hline
 78 \\
 - 74 \quad (2 \times 37) \\
 \hline
 4 \quad \text{remainder.}
 \end{array}$$

Answer: No—there would be a remainder of 4.

But there is a shorter way to the answer.

|   |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |
|---|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|
| A | $\begin{array}{r} 27 \\ \times 1 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 2 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 3 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 4 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 5 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 6 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 8 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 9 \\ \hline \end{array}$ |
|---|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|

|   |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |
|---|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|
| B | $\begin{array}{r} 27 \\ \times 10 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 20 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 30 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 40 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 50 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 60 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 70 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 80 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 90 \\ \hline \end{array}$ |
|---|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|

$$\begin{array}{r} 27 \overline{) 1917} \\ B \rightarrow \end{array}$$

$$\begin{array}{r} 27 \overline{) 2214} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 459} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 1566} \\ \hline \end{array}$$

$$\begin{array}{r} A \rightarrow \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 1701} \\ \hline \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 756} \\ \hline \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 2538} \\ \hline \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 1053} \\ \hline \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 2565} \\ \hline \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 1539} \\ \hline \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 2322} \\ \hline \text{---} \\ \hline \end{array}$$

$$\begin{array}{r} 27 \overline{) 1323} \\ \hline \text{---} \\ \hline \end{array}$$

|   |                                                          | NAME _____                                               |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |  |  |
|---|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|--|--|
| A | $\begin{array}{r} 27 \\ \times 1 \\ \hline \end{array}$  | $\begin{array}{r} 27 \\ \times 2 \\ \hline \end{array}$  | $\begin{array}{r} 27 \\ \times 3 \\ \hline \end{array}$  | $\begin{array}{r} 27 \\ \times 4 \\ \hline \end{array}$  | $\begin{array}{r} 27 \\ \times 5 \\ \hline \end{array}$  | $\begin{array}{r} 27 \\ \times 6 \\ \hline \end{array}$  | $\begin{array}{r} 27 \\ \times 7 \\ \hline \end{array}$  | $\begin{array}{r} 27 \\ \times 8 \\ \hline \end{array}$  | $\begin{array}{r} 27 \\ \times 9 \\ \hline \end{array}$  |  |  |
|   |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |  |  |
| B | $\begin{array}{r} 27 \\ \times 10 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 20 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 30 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 40 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 50 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 60 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 70 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 80 \\ \hline \end{array}$ | $\begin{array}{r} 27 \\ \times 90 \\ \hline \end{array}$ |  |  |
|   |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |  |  |

|                                                                                                                          |                                                                         |                                                                         |                                                                         |
|--------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------|-------------------------------------------------------------------------|-------------------------------------------------------------------------|
| $\begin{array}{r} 27 \overline{)1917} \\ B \rightarrow \quad \quad \quad \\ A \rightarrow \quad \quad \quad \end{array}$ | $\begin{array}{r} 27 \overline{)2214} \\ \quad \quad \quad \end{array}$ | $\begin{array}{r} 27 \overline{)459} \\ \quad \quad \quad \end{array}$  | $\begin{array}{r} 27 \overline{)1566} \\ \quad \quad \quad \end{array}$ |
| $\begin{array}{r} 27 \overline{)1701} \\ \quad \quad \quad \end{array}$                                                  | $\begin{array}{r} 27 \overline{)756} \\ \quad \quad \quad \end{array}$  | $\begin{array}{r} 27 \overline{)2538} \\ \quad \quad \quad \end{array}$ | $\begin{array}{r} 27 \overline{)1053} \\ \quad \quad \quad \end{array}$ |
| $\begin{array}{r} 27 \overline{)2565} \\ \quad \quad \quad \end{array}$                                                  | $\begin{array}{r} 27 \overline{)1539} \\ \quad \quad \quad \end{array}$ | $\begin{array}{r} 27 \overline{)2322} \\ \quad \quad \quad \end{array}$ | $\begin{array}{r} 27 \overline{)1323} \\ \quad \quad \quad \end{array}$ |

The actual operations are the same as previous pages, but the format is changed—using the standard long-division notation.

As before, the first task is to complete the tables at the top of the page.

There are 4 entries that require a minimum of effort:

$$\begin{array}{r} 27 \\ \times 1 \\ \hline 27 \end{array} \quad \begin{array}{r} 27 \\ \times 2 \\ \hline 54 \end{array} \quad \begin{array}{r} 27 \\ \times 4 \\ \hline 108 \end{array} \quad \begin{array}{r} 27 \\ \times 8 \\ \hline 216 \end{array}$$

This is a familiar "doubling" series.

The first 2 entries can be "added," that entry doubled—and the two new entries added:

$$(27 + 54) \rightarrow \begin{array}{r} 27 \\ \times 3 \\ \hline 81 \end{array} \quad \begin{array}{r} 27 \\ \times 6 \\ \hline 162 \end{array} \quad \begin{array}{r} 27 \\ \times 9 \\ \hline 243 \end{array}$$

Leaving only  $27 \times 5$  and  $27 \times 7$ . The first one can be found by adding 27 to  $27 \times 4$ :  $108 + 27 = 135$ . The second is,  $27 \times 6 + 27$ ;  $162 + 27 = 189$ .

The second table can be completed as fast as the products can be written after glances at the first table.

And, of course, there are many alternatives to the shortcuts we took above.

The first problem asks: how many 27's in 1917? The second table quickly reveals a good approximation: there are seventy 27's in 1917 and a little more. We can write down that partial answer and find out how many more of those 1917 we must still account for

$$\begin{array}{r} \text{list B} \rightarrow \begin{array}{r} 27 \overline{)1917} \\ \underline{1890} \\ 27 \\ \underline{27} \end{array} \quad \text{or} \quad \begin{array}{r} 71 \\ \begin{array}{r} 70 \\ 1 \end{array} \end{array} \left. \vphantom{\begin{array}{r} 71 \\ 70 \\ 1 \end{array}} \right\} 71$$

and complete the example as shown with broken-line numbers. The first method is standard; the second may help as a brief transitional method of keeping track.

Notice that by the use of tables the children have worked out in advance, the need for the rather painful and inefficient "scaffold" or "ladder" algorism is avoided . . . to the great relief of teachers and parents, as well as children.

Because all necessary multiplications are carried out in advance, this activity focuses in an uncluttered way on the algorism itself: the possibility for careless or computational error is minimized. The children have a far greater opportunity to understand the process—the "way it works" and "why it works."

|   |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |
|---|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|
| A | $\begin{array}{r} 85 \\ \times 1 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 2 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 3 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 4 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 5 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 6 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 7 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 8 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 9 \\ \hline \end{array}$ |
|   |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |                                                         |

|   |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |
|---|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|
| B | $\begin{array}{r} 85 \\ \times 10 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 20 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 30 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 40 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 50 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 60 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 70 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 80 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 90 \\ \hline \end{array}$ |
|   |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |

|                                                                        |                                                                                  |                                                              |                                                                               |
|------------------------------------------------------------------------|----------------------------------------------------------------------------------|--------------------------------------------------------------|-------------------------------------------------------------------------------|
| $\begin{array}{r} 85 \\ \overline{)1955} \\ B \rightarrow \end{array}$ | $\begin{array}{r} 74 \\ 85 \overline{) \quad \quad \quad} \\ \hline \end{array}$ | $\begin{array}{r} 85 \overline{)3060} \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \overline{) \quad \quad \quad} \\ \hline \end{array}$ |
| $\begin{array}{r} \hline \\ A \rightarrow \end{array}$                 |                                                                                  |                                                              |                                                                               |

|                                                              |                                                                                  |                                                              |                                                                                  |
|--------------------------------------------------------------|----------------------------------------------------------------------------------|--------------------------------------------------------------|----------------------------------------------------------------------------------|
| $\begin{array}{r} 85 \overline{)4080} \\ \hline \end{array}$ | $\begin{array}{r} 96 \\ 85 \overline{) \quad \quad \quad} \\ \hline \end{array}$ | $\begin{array}{r} 85 \overline{)4420} \\ \hline \end{array}$ | $\begin{array}{r} 79 \\ 85 \overline{) \quad \quad \quad} \\ \hline \end{array}$ |
|                                                              |                                                                                  |                                                              |                                                                                  |

|                                                              |                                                                                  |                                                              |                                                                                  |
|--------------------------------------------------------------|----------------------------------------------------------------------------------|--------------------------------------------------------------|----------------------------------------------------------------------------------|
| $\begin{array}{r} 85 \overline{)7310} \\ \hline \end{array}$ | $\begin{array}{r} 78 \\ 85 \overline{) \quad \quad \quad} \\ \hline \end{array}$ | $\begin{array}{r} 85 \overline{)8245} \\ \hline \end{array}$ | $\begin{array}{r} 64 \\ 85 \overline{) \quad \quad \quad} \\ \hline \end{array}$ |
|                                                              |                                                                                  |                                                              |                                                                                  |

| NAME _____ |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |                                                          |
|------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|
| A          | $\begin{array}{r} 85 \\ \times 1 \\ \hline \end{array}$  | $\begin{array}{r} 85 \\ \times 2 \\ \hline \end{array}$  | $\begin{array}{r} 85 \\ \times 3 \\ \hline \end{array}$  | $\begin{array}{r} 85 \\ \times 4 \\ \hline \end{array}$  | $\begin{array}{r} 85 \\ \times 5 \\ \hline \end{array}$  | $\begin{array}{r} 85 \\ \times 6 \\ \hline \end{array}$  | $\begin{array}{r} 85 \\ \times 7 \\ \hline \end{array}$  | $\begin{array}{r} 85 \\ \times 8 \\ \hline \end{array}$  | $\begin{array}{r} 85 \\ \times 9 \\ \hline \end{array}$  |
| B          | $\begin{array}{r} 85 \\ \times 10 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 20 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 30 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 40 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 50 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 60 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 70 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 80 \\ \hline \end{array}$ | $\begin{array}{r} 85 \\ \times 90 \\ \hline \end{array}$ |

|                                                                                     |                                                                               |                                                               |                                                                               |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------|---------------------------------------------------------------|-------------------------------------------------------------------------------|
| $\begin{array}{r} 85 \overline{) 1955} \\ B+ \\ \hline \\ A+ \\ \hline \end{array}$ | $\begin{array}{r} 74 \\ 85 \overline{) \phantom{0000}} \\ \hline \end{array}$ | $\begin{array}{r} 85 \overline{) 3060} \\ \hline \end{array}$ | $\begin{array}{r} 85 \overline{) \phantom{0000}} \\ \hline \end{array}$       |
| $\begin{array}{r} 85 \overline{) 4080} \\ \hline \end{array}$                       | $\begin{array}{r} 96 \\ 85 \overline{) \phantom{0000}} \\ \hline \end{array}$ | $\begin{array}{r} 85 \overline{) 4420} \\ \hline \end{array}$ | $\begin{array}{r} 79 \\ 85 \overline{) \phantom{0000}} \\ \hline \end{array}$ |
| $\begin{array}{r} 85 \overline{) 7310} \\ \hline \end{array}$                       | $\begin{array}{r} 78 \\ 85 \overline{) \phantom{0000}} \\ \hline \end{array}$ | $\begin{array}{r} 85 \overline{) 8245} \\ \hline \end{array}$ | $\begin{array}{r} 64 \\ 85 \overline{) \phantom{0000}} \\ \hline \end{array}$ |

Two columns of the examples use the usual long-division format to ask "division" questions. The other columns use the same form but ask "multiplication" questions. Using the same forms and the same tables can you answer both kinds of questions?

$$85 \overline{) 1955} \quad \text{and} \quad 85 \overline{) 74}$$

If you put 1955 books on 85 shelves with the same number of books on each shelf, how many would there be on each shelf?

If you had 85 shelves and 74 books on each shelf, how many books would you have?

How can you use the same tables (multiples of 85) to answer these two different questions?

The flow of these two related algorithms and their close relationships are usually lost in a heavy sea of difficult and incidental computations.

If a child has difficulties with the question of

$$\begin{array}{r} 85 \\ \times 4 \\ \hline \end{array} \quad \text{and} \quad \begin{array}{r} 85 \\ \times 70 \\ \hline \end{array}$$

how could he understand the standard algorithm for finding the answer or product:

$$\begin{array}{r} 85 \\ \times 74 \\ \hline \end{array}$$

If a child has difficulty with the questions of

$$\begin{array}{r} 85 \\ \times 8 \end{array} \quad \text{and} \quad \begin{array}{r} 85 \\ \times 40 \end{array}$$

how could he understand the "why" and "how" of the standard algorithm for finding the answer or quotient.

$$85 \overline{) 4080}$$

which when completed in standard form is

$$\begin{array}{r} 48 \\ 85 \overline{) 4080} \\ \underline{340} \phantom{00} \\ 680 \phantom{00} \\ \underline{680} \phantom{00} \\ 0 \phantom{00} \end{array}$$

If the focus is on the algorithm, then remove every distraction and all complexity to let the central problem be the central task.

If our goal is friendliness with numbers, we will remove every possible threat that would distract any child from the particular we are looking at. When he has created necessary multiplication tables in advance, then, and only then for many children, does a division example become a reasonable problem.

A

|           |           |           |           |           |           |           |           |           |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| <u>x1</u> | <u>x2</u> | <u>x3</u> | <u>x4</u> | <u>x5</u> | <u>x6</u> | <u>x7</u> | <u>x8</u> | <u>x9</u> |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|

B

|            |            |            |            |            |            |            |            |            |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| <u>x10</u> | <u>x20</u> | <u>x30</u> | <u>x40</u> | <u>x50</u> | <u>x60</u> | <u>x70</u> | <u>x80</u> | <u>x90</u> |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|

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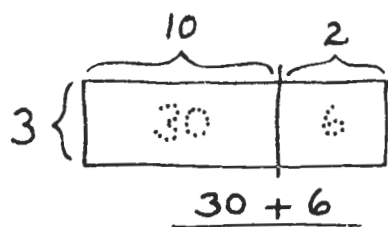
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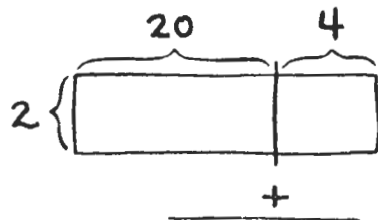
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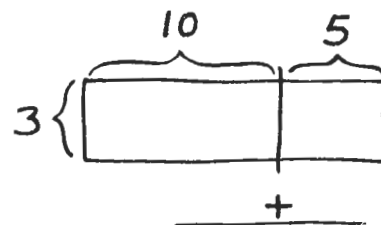


$$\begin{array}{r} 12 \\ \times 3 \\ \hline \end{array}$$

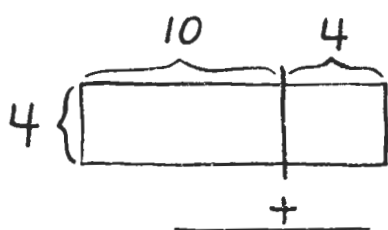
$$\begin{array}{r} 12 \\ 3 \overline{) 36} \end{array}$$



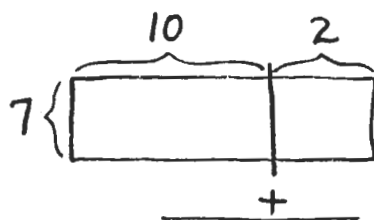
$$\begin{array}{r} \times \\ \hline \end{array}$$



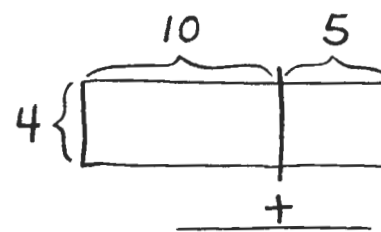
$$\begin{array}{r} \times \\ \hline \end{array}$$



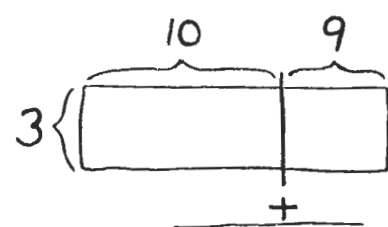
$$\begin{array}{r} \times \\ \hline \end{array}$$



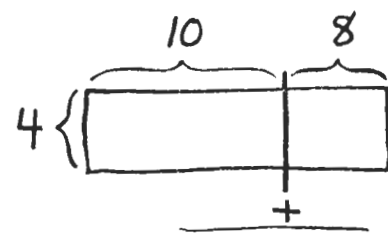
$$\begin{array}{r} \times \\ \hline \end{array}$$



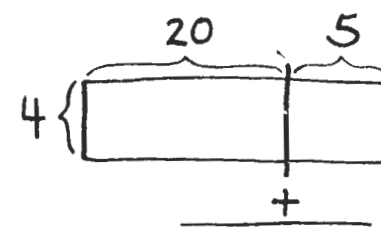
$$\begin{array}{r} \times \\ \hline \end{array}$$



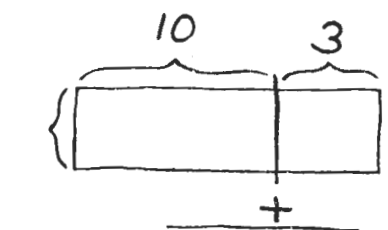
$$\begin{array}{r} \times \\ \hline \end{array}$$



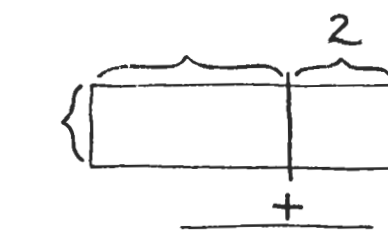
$$\begin{array}{r} \times \\ \hline \end{array}$$



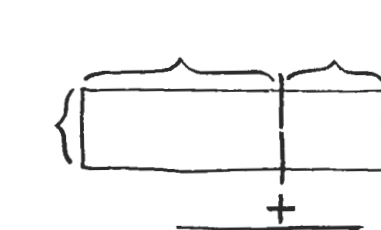
$$\begin{array}{r} \times \\ \hline \end{array}$$



$$\begin{array}{r} 13 \\ \times 8 \\ \hline \end{array}$$



$$\begin{array}{r} 12 \\ \times 9 \\ \hline \end{array}$$



$$\begin{array}{r} 18 \\ \times 6 \\ \hline \end{array}$$



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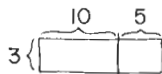
|                                                          |                                                          |                                                         |
|----------------------------------------------------------|----------------------------------------------------------|---------------------------------------------------------|
| $\begin{array}{r} 12 \\ \times 3 \\ \hline \end{array}$  | $\begin{array}{r} 20 \\ \times 2 \\ \hline \end{array}$  | $\begin{array}{r} 10 \\ \times 3 \\ \hline \end{array}$ |
| $\begin{array}{r} 10 \\ \times 4 \\ \hline \end{array}$  | $\begin{array}{r} 10 \\ \times 7 \\ \hline \end{array}$  | $\begin{array}{r} 10 \\ \times 4 \\ \hline \end{array}$ |
| $\begin{array}{r} 10 \\ \times 3 \\ \hline \end{array}$  | $\begin{array}{r} 10 \\ \times 4 \\ \hline \end{array}$  | $\begin{array}{r} 20 \\ \times 3 \\ \hline \end{array}$ |
| $\begin{array}{r} 10 \\ \times 13 \\ \hline \end{array}$ | $\begin{array}{r} 10 \\ \times 12 \\ \hline \end{array}$ | $\begin{array}{r} 18 \\ \times 6 \\ \hline \end{array}$ |

This page helps the learner build a useful technique for cutting some multiplication examples "down to size".

We have already established that:

$$\begin{array}{r} 15 \\ \times 3 \\ \hline \end{array}$$

in terms of real things means three rows with 15 counters in each row. As we look at such an array, we notice that there is a convenient way to break it up to count the total number of counters. We can see three rows of 10 and then 3 rows of 5, as in this diagram.



It is easy to count each group.  $3 \times 10$  is 30 and  $3 \times 5$  is 15, so there are  $30 + 15$ , or 45 counters in the array.

Essentially we have said that

$$3 \times 15 = 3(10 + 5) = 3 \times 10 + 3 \times 5 = 30 + 15 = 45$$

In formal terms, this is the "distributive principle of multiplication over addition", and it is one of the least obvious of the basic principles of arithmetic. We prefer to let children do many examples of the type on this page and have lots of experiences where they build some real referents for this principle before they ever hear the words "distributive principle". In fact at this level, if the need arises to give a name to this new "rule", children ought to think of a name that makes sense to them.

This principle is quite useful. Essentially it tells us: If we don't know the product of 15 and 3, we can "regroup" 15 in any way we want, multiply each new group by 3, and finally add the two results to find the product of 15 and 3. Because multiplication by 10 and multiples of 10 are usually easy, it is convenient to "regroup" the number into 10 or a multiple of 10 and some other number. (e.g.,  $18 = 10 + 8$ ,  $26 = 20 + 6$ ,  $32 = 30 + 2$ .)

Thus, to multiply

$$\begin{array}{r} 24 \\ \times 3 \\ \hline \end{array}$$

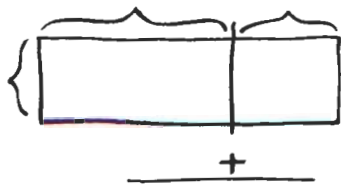
it is convenient to think:  $3 \times 20 = 60$ ,  $3 \times 4 = 12$ ,  $60 + 12 = 72$

so 
$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \end{array}$$

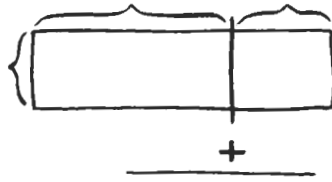
On this page there are diagrams and record-keeping devices. The learner records the number of counters in each group shown in the array and then writes the multiplication and division he sees illustrated by the array. The first example indicates how the record-keeping devices are used.

In the last row, there is a new twist to the examples. The multiplication fact is given and some bit of information about the array. The learner must figure the missing information.

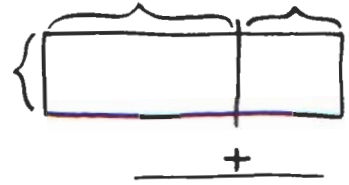
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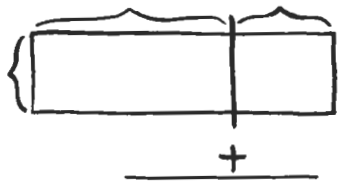
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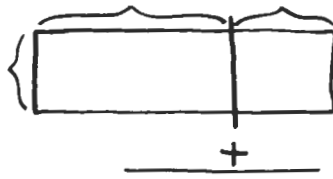
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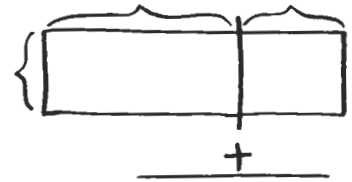
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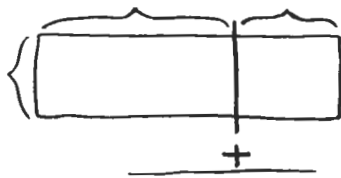
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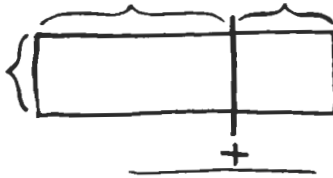
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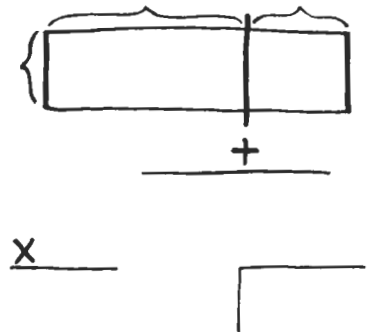
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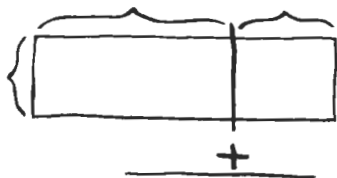
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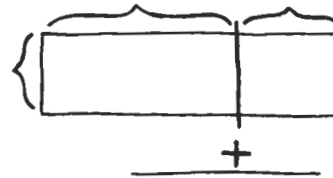
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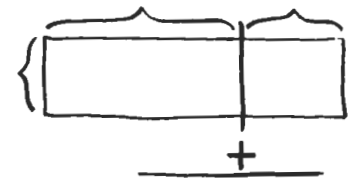
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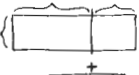
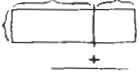
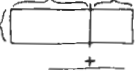
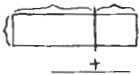
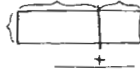
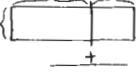
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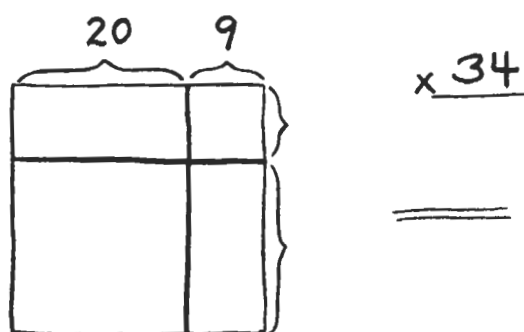
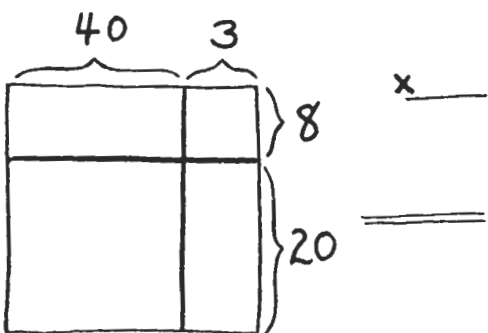
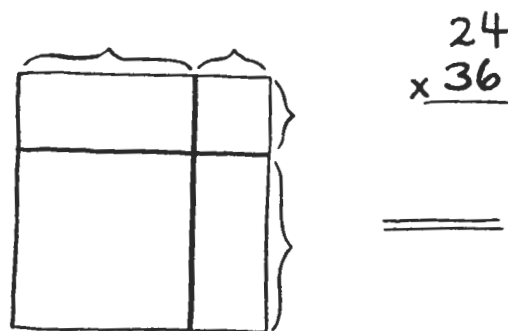
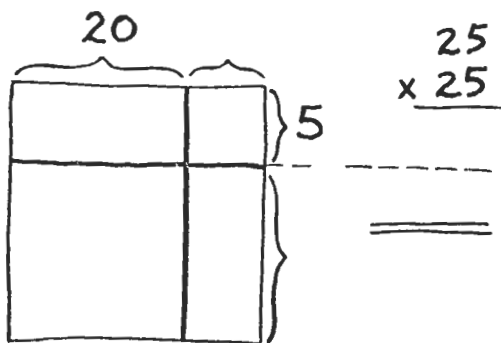
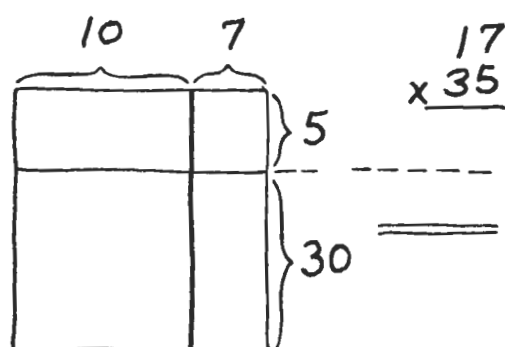
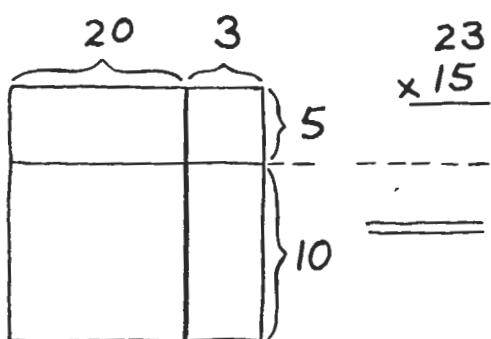
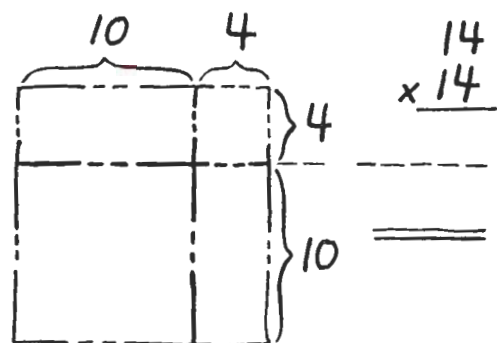
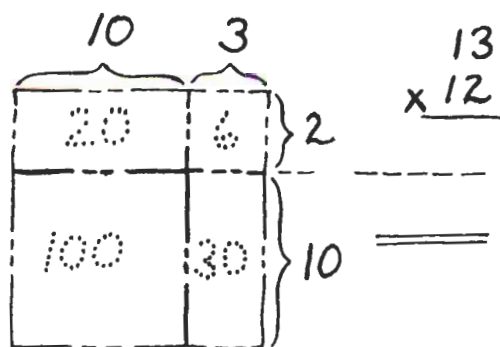
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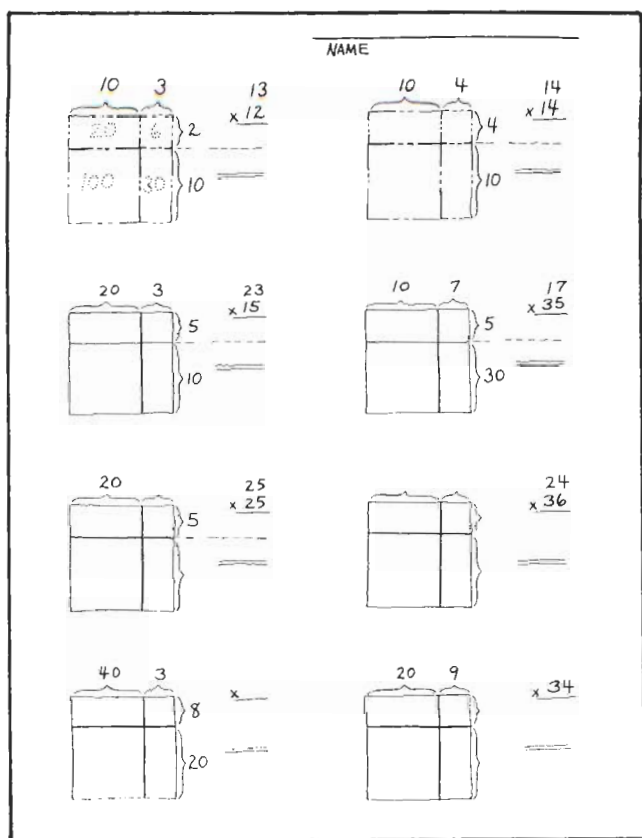


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| NAME _____                                                                        |                                                                                   |                                                                                   |
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| X _____                                                                           | X _____                                                                           | X _____                                                                           |
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| X _____                                                                           | X _____                                                                           | X _____                                                                           |
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| X _____                                                                           | X _____                                                                           | X _____                                                                           |
|  |  |  |
| X _____                                                                           | X _____                                                                           | X _____                                                                           |

This page is a continuation of activities on the previous page. Learners can make up their own examples and problems. It could also be particularized by the teacher.





On this page we extend the activities of the previous pages and use rough sketches to help the learner understand the multiplication algorithm more fully. We make use of our discovery that in multiplication a number can be "regrouped" in any way that makes the multiplication easier. Here, we essentially lead the learner to do that twice. For example, if we don't know how much  $13 \times 12$  is, we can think of the problem as  $13 \times (10 + 2)$ , or  $(13 \times 10) + (13 \times 2)$ .  $13 \times 10$  is easy. If we don't know  $13 \times 2$ , we can apply the new rule again.  $13 \times 2 = (10 + 3) \times 2$ , or  $(10 \times 2) + (3 \times 2)$ .

In the diagram, we have illustrated an array of counters in 12 rows with 13 in each row. The top part of the array illustrates  $13 \times 2$ . If we consider only that part of the array, we have an example like one on the previous page. We have considered  $13 \times 2$  as  $(10 + 3) \times 2$ , or  $(10 \times 2) + (3 \times 2)$ . There are 26 counters in this "part" of the diagram. This is recorded as a "partial" product on the dotted line:

$$\begin{array}{r} 13 \\ \times 12 \\ \hline 26 \end{array}$$

The lower part of the array illustrates  $13 \times 10$ . In this part, we have another example like those on the previous page.  $13 \times 10$  is shown as  $(10 + 3) \times 10$ , or  $(10 \times 10) + (10 \times 3)$ . The number of counters in this part of the diagram is 130, and is recorded on the double line.

$$\begin{array}{r} 13 \\ \times 12 \\ \hline 26 \\ \hline 130 \end{array}$$

We add the two "partial" products to get 156, our final result.

$$\begin{array}{r} 13 \\ \times 12 \\ \hline 26 \\ \hline 130 \\ \hline 156 \end{array}$$

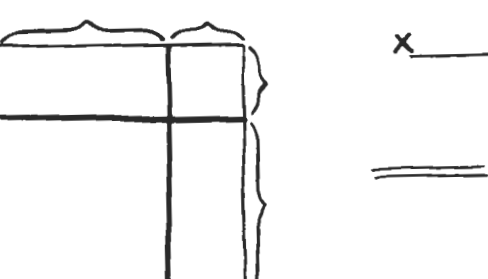
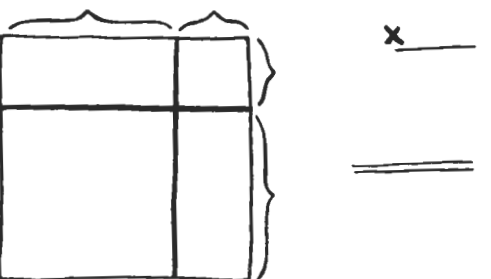
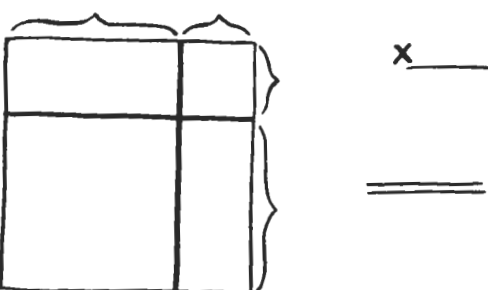
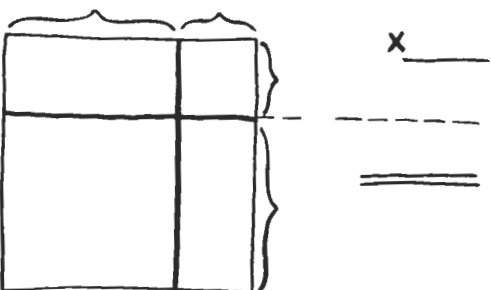
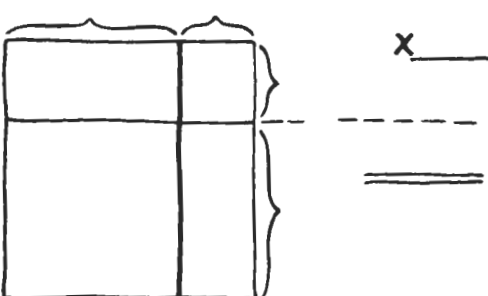
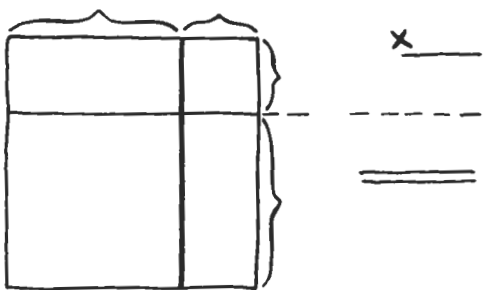
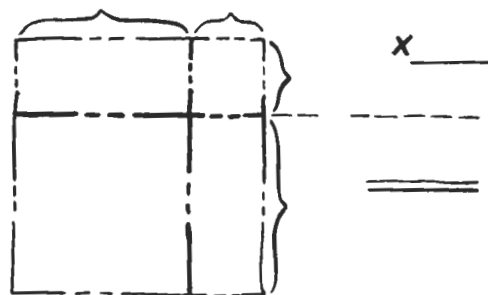
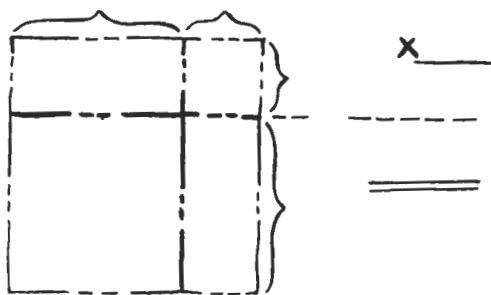
Children do not need such a logical explanation at this point. It is sufficient if they understand that, for example

$$\begin{array}{r} 13 \\ \times 12 \\ \hline \end{array}$$

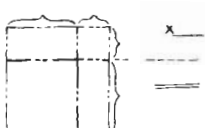
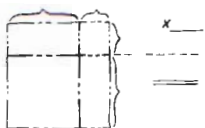
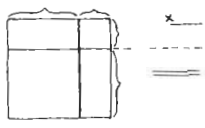
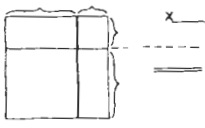
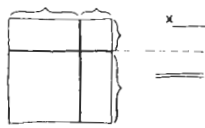
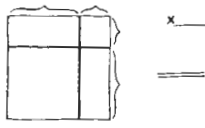
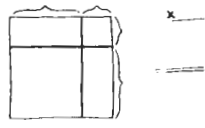
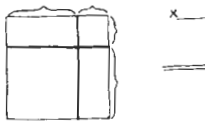
is the same thing as  $13 \times (2 + 10)$ , or  $(13 \times 2) + (13 \times 10)$ . This can best be explained using the diagram. Once they understand this, the partial products become simply the record of the result of counting the counters in each part.

After many experiences with examples of this type, children may begin to suggest that they could take a "shortcut" and not write a "0" at the end of the second partial product. But at that point they will understand where it came from and why such a shortcut can be used. Many children simply learn the rule that "you move over a place when you write the second partial product" and never have any notion that the number they write there is a "shorthand" for really writing that number times 10.

Some examples on this page require that the learner label the diagram, and some ask that he figure out what multiplication is represented by the diagram. It should be pointed out that each diagram should be considered separately. In some, a certain amount of space might represent 40 units, and in another, the same space might represent 20 units. The point is that we are not trying to make scale drawings. We want to make quick rough sketches to help with an understanding of the multiplication algorithm.



NAME \_\_\_\_\_

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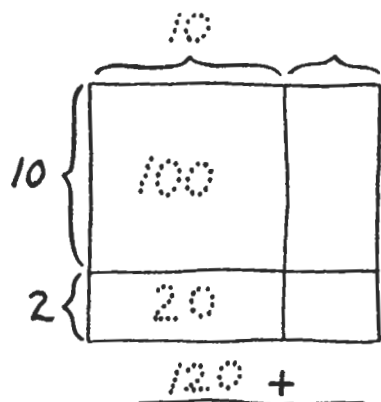
Learners can make up their own experiments on this page or teachers can particularize it.

You might want to try experimenting with providing different bits of information. For example:

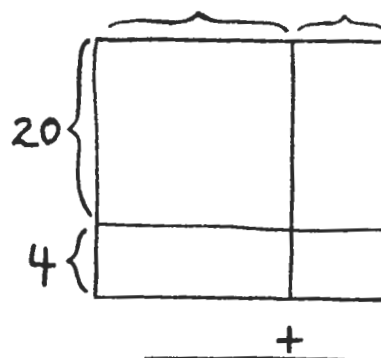
- (1) What multiplication would lead to the following situation?

|    |    |
|----|----|
| 80 | 12 |
|    | 30 |

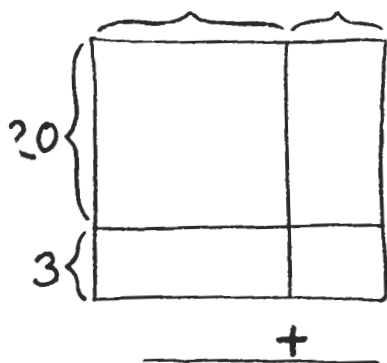
$\times$  \_\_\_\_\_  
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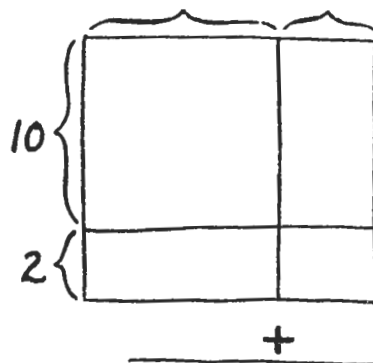
$$\begin{array}{r} 12 \overline{) 156} \\ \underline{120} \\ 36 \end{array}$$



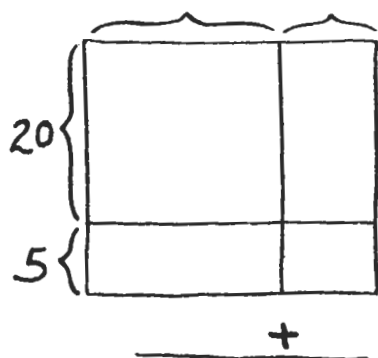
$$24 \overline{) 384}$$



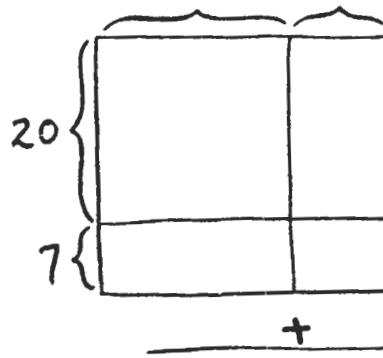
$$23 \overline{) 598}$$



$$12 \overline{) 444}$$

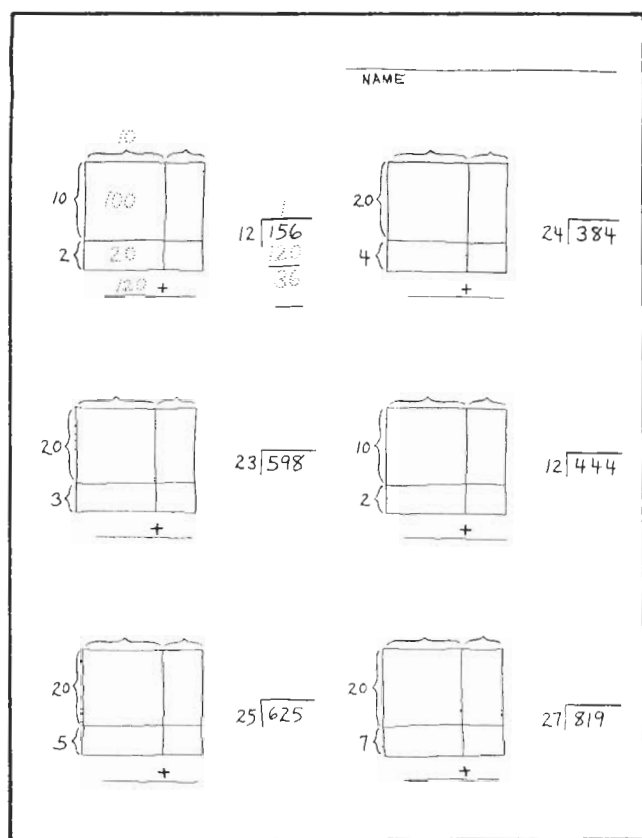


$$25 \overline{) 625}$$



$$27 \overline{) 837}$$



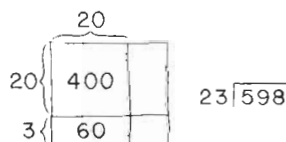


The activities on this page help the learner focus on the long division algorithm through the use of sketches. The essential question for division becomes "Given an array of a certain number of counters arranged in a certain number of rows, how many counters are there in each row?"

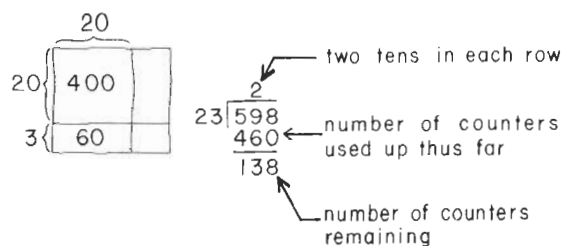
Consider the example in which 598 counters are arranged in 23 rows.

First we should decide how many tens (10-sticks or rods) could be put in each row.

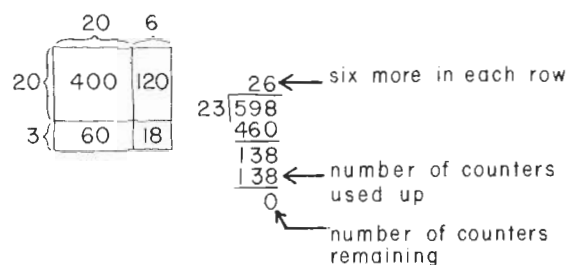
We could certainly put one ten in each row to start. But that only uses up 230 of the counters. If we put another ten in each row, we use 460 of the counters.



Putting another ten in each row would require another 230 counters, or 690 all together. We only have 598 counters, so we have to settle for two tens in each row. That means we have 138 counters still to arrange. We can record this partial result:



Now we must decide how many of the remaining counters go in each row. Five in each row would use up 115 . . . but six in each row would use up 138 . . . exactly the number we have. Thus:



Once again, we must recognize a different strategy used in division. In addition, subtraction and multiplication, the first step could be described as "taking care of the units first". Here are examples of such first steps:

$$\begin{array}{r} 1 \\ 14 \\ + 28 \\ \hline 2 \end{array} \quad \begin{array}{r} 2 \\ 34 \\ - 19 \\ \hline 5 \end{array} \quad \begin{array}{r} 2 \\ 18 \\ \times 3 \\ \hline 4 \end{array}$$

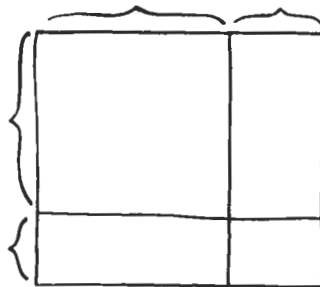
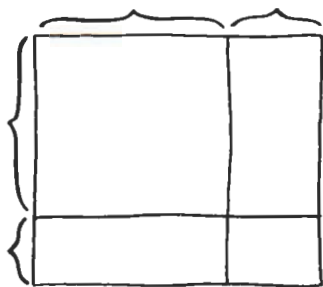
Compare this with the first step of our division strategy:

$$23 \overline{) 598} \quad 2$$

In addition, subtraction, and multiplication, we worry about loose beans first and make a trade so we can report the number of loose beans that will appear in our final report. In division, we worry first about how many sticks there will be in each row — and then, distribute the rest as loose beans.

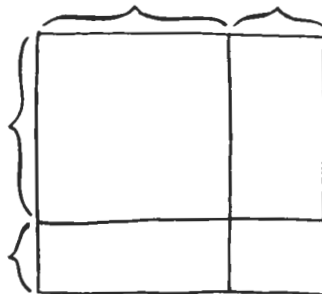
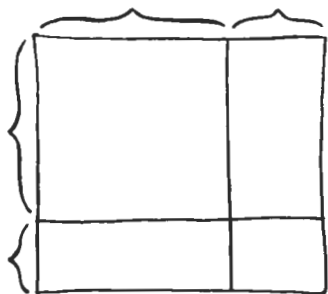
When such a fundamental shift in strategy occurs, we should afford time to discuss this shift with children.

This strategy of cutting a problem down to size is a most powerful mathematical technique. A tactic most engineers rely on in difficult problems is trial and error. They guess, and record results. Their next guess is influenced by the outcome of the first guess. Soon they begin trapping their elusive answer between "too much" and "too little".



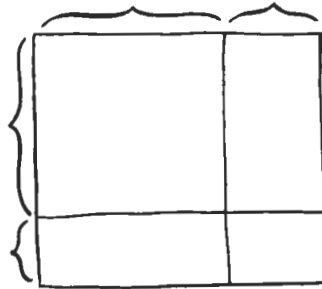
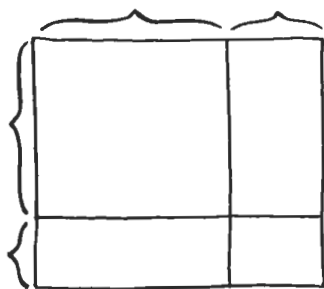
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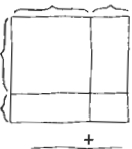
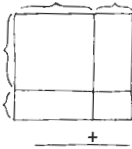
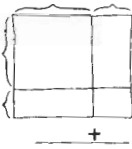
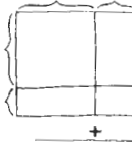
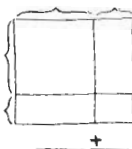
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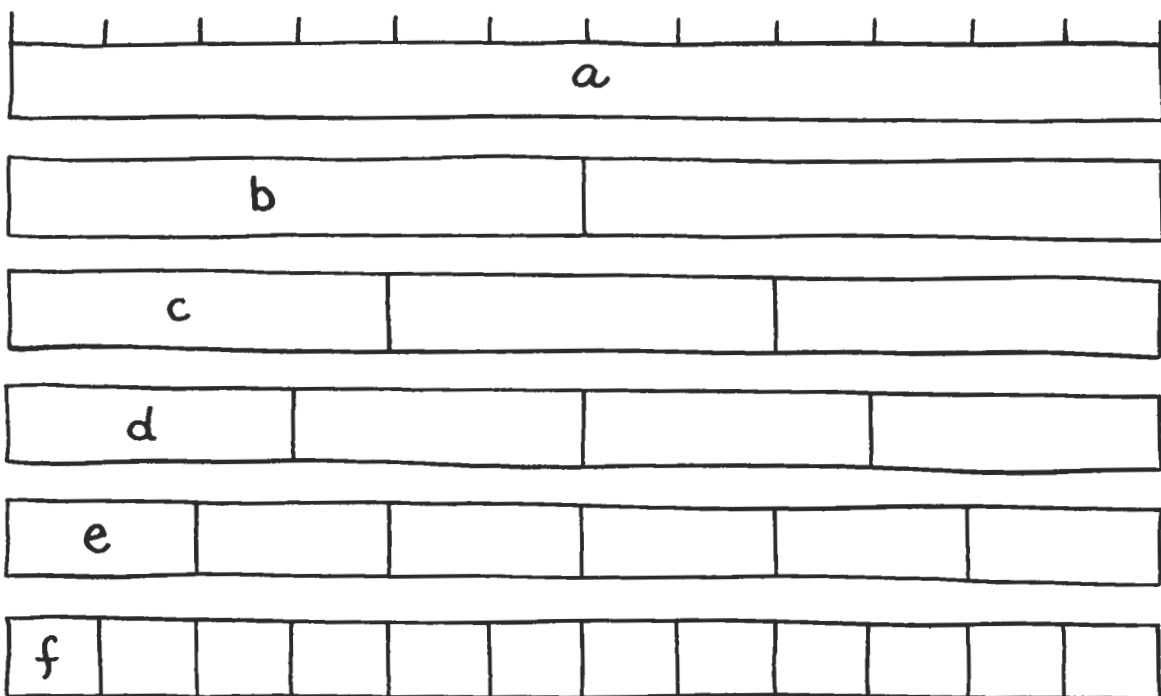
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NAME \_\_\_\_\_

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This is a general form for the type of activity on the previous page.



$$\frac{b + b}{+} = \underline{\quad}$$

$$\frac{2 \times b}{\times} = \underline{\quad}$$

$$\frac{a - b}{-} = \underline{\quad}$$

$$\frac{a \div 2}{\div} = \underline{\quad}$$

$$\frac{e + f}{+} = \underline{\quad}$$

$$\frac{b + e}{+} = \underline{\quad}$$

$$\frac{b \div 3}{\div} = \underline{\quad}$$

$$\frac{\frac{1}{2} \times c}{\times} = \underline{\quad}$$

$$\frac{2c \div 4}{\div} = \underline{\quad}$$

$$\frac{b + c}{+} = \underline{\quad}$$

$$\frac{f + 5f}{+} = \underline{\quad}$$

$$\frac{b + 2d}{+} = \underline{\quad}$$

$$\frac{a \div c}{\div} = \underline{\quad}$$

$$\frac{b \div d}{\div} = \underline{\quad}$$

$$\frac{d \div b}{\div} = \underline{\quad}$$

WHAT CAN YOU SEE? NAME \_\_\_\_\_

|                                                    |                                                                  |                                                     |
|----------------------------------------------------|------------------------------------------------------------------|-----------------------------------------------------|
| $\frac{b+b}{+} = \underline{\hspace{2cm}}$         | $\frac{2 \times b}{\times} = \underline{\hspace{2cm}}$           | $\frac{a-b}{-} = \underline{\hspace{2cm}}$          |
| $\frac{a \div 2}{\div} = \underline{\hspace{2cm}}$ | $\frac{e+f}{+} = \underline{\hspace{2cm}}$                       | $\frac{b+e}{+} = \underline{\hspace{2cm}}$          |
| $\frac{b \div 3}{\div} = \underline{\hspace{2cm}}$ | $\frac{\frac{1}{2} \times c}{\times} = \underline{\hspace{2cm}}$ | $\frac{2c \div 4}{\div} = \underline{\hspace{2cm}}$ |
| $\frac{b+c}{+} = \underline{\hspace{2cm}}$         | $\frac{f+5f}{+} = \underline{\hspace{2cm}}$                      | $\frac{b+2d}{+} = \underline{\hspace{2cm}}$         |
| $\frac{a \div c}{\div} = \underline{\hspace{2cm}}$ | $\frac{b \div d}{\div} = \underline{\hspace{2cm}}$               | $\frac{d \div b}{\div} = \underline{\hspace{2cm}}$  |

This page can be used in many different ways. We suggest one, but are certain you will invent many more.

- (1) Learners can consider the bar labelled "a" as a unit, and then figure out what fractional part of "a" is represented by the "b" bar, the "c" bar, etc. This could be determined by measuring or by comparing. One easy way to compare sizes is to simply count the number of bars of each size that it takes to make up the length of the "a" bar. There are two "b" bars in the length of one "a" bar, so "b" can be considered  $1/2$ . Likewise, "c" is  $1/3$ , "d" is  $1/4$ , "e" is  $1/6$ , and "f" is  $1/12$ .

Then we can begin a "double talk" approach to operations. In each problem, we can work with letters above the lines, and numbers below the lines. To solve

$$\begin{array}{r} e + f = \\ \hline + \quad = \end{array}$$

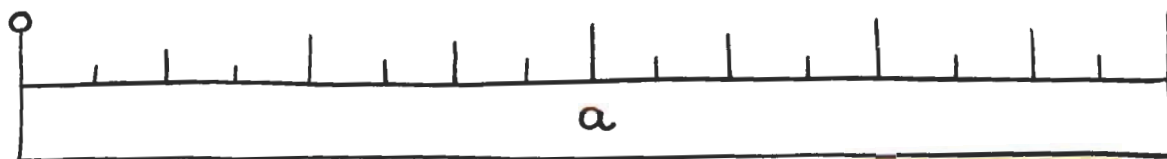
we can do several things. First, we could simply use our previous investigation and write fractional equivalents below the line.

$$\begin{array}{r} e + f = \\ \hline 1/6 + 1/12 = \end{array}$$

Then we could add  $1/6 + 1/12$  and get  $3/12$ , or  $1/4$ . Or, if we don't know how much  $1/6 + 1/12$  is, we could use another strategy. We might use a card, or folded piece of paper, and hold it so that an edge is directly on the vertical line at the end of the first "e" bar (so the first "e" bar is to the left of the card's edge). We could look down to the "f" row, and see that the edge of the card is at the end of two "f" blocks. To those two "f" blocks, which are equivalent to one "e" block I want to add one more "f", so I could move the card to the right one block. I would then show three "f" blocks, which I know is  $3/12$ . If we want to push toward fractions in lowest terms, we might have the rule that the final answer must be in terms of the largest blocks which have an end coinciding with the edge of the card. In this case, "d" does the trick. One "d" is the same length as 3 "f", and one "d" is  $1/4$ . Thus, I can record

$$\begin{array}{r} e + f = \\ \hline 1/6 + 1/12 = \end{array} \quad \begin{array}{r} d \\ \hline 1/4 \end{array}$$

Other examples are done in a similar way.



$$\frac{c + c}{+} = \underline{\hspace{2cm}}$$

$$\frac{a \div 8}{\div} = \underline{\hspace{2cm}}$$

$$\frac{2d + c}{+} = \underline{\hspace{2cm}}$$

$$\frac{c - e}{-} = \underline{\hspace{2cm}}$$

$$\frac{b \div 8}{\div} = \underline{\hspace{2cm}}$$

$$\frac{a - 3c}{-} = \underline{\hspace{2cm}}$$

$$\frac{2c - 3e}{-} = \underline{\hspace{2cm}}$$

$$\frac{3d + 3d}{+} = \underline{\hspace{2cm}}$$

$$\frac{7e - 3e}{-} = \underline{\hspace{2cm}}$$

$$\frac{c + d}{+} = \underline{\hspace{2cm}}$$

$$\frac{d + 3e}{+} = \underline{\hspace{2cm}}$$

$$\frac{a - 5d}{-} = \underline{\hspace{2cm}}$$

$$\frac{a \div 4}{\div} = \underline{\hspace{2cm}}$$

$$\frac{3d \div 2}{\div} = \underline{\hspace{2cm}}$$

$$\frac{b \div 2e}{\div} = \underline{\hspace{2cm}}$$

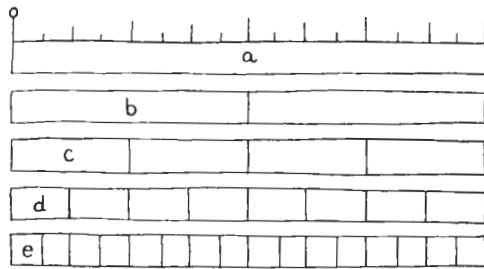
$$\frac{a \div c}{\div} = \underline{\hspace{2cm}}$$

$$\frac{3d \div 3e}{\div} = \underline{\hspace{2cm}}$$

$$\frac{5d \div c}{\div} = \underline{\hspace{2cm}}$$

WHAT CAN YOU SEE?

NAME \_\_\_\_\_



$$\frac{c+c}{+} = \underline{\quad} \quad \frac{a \div 8}{\div} = \underline{\quad} \quad \frac{2d+c}{+} = \underline{\quad}$$

$$\frac{c-e}{-} = \underline{\quad} \quad \frac{b \div 8}{\div} = \underline{\quad} \quad \frac{a-3c}{-} = \underline{\quad}$$

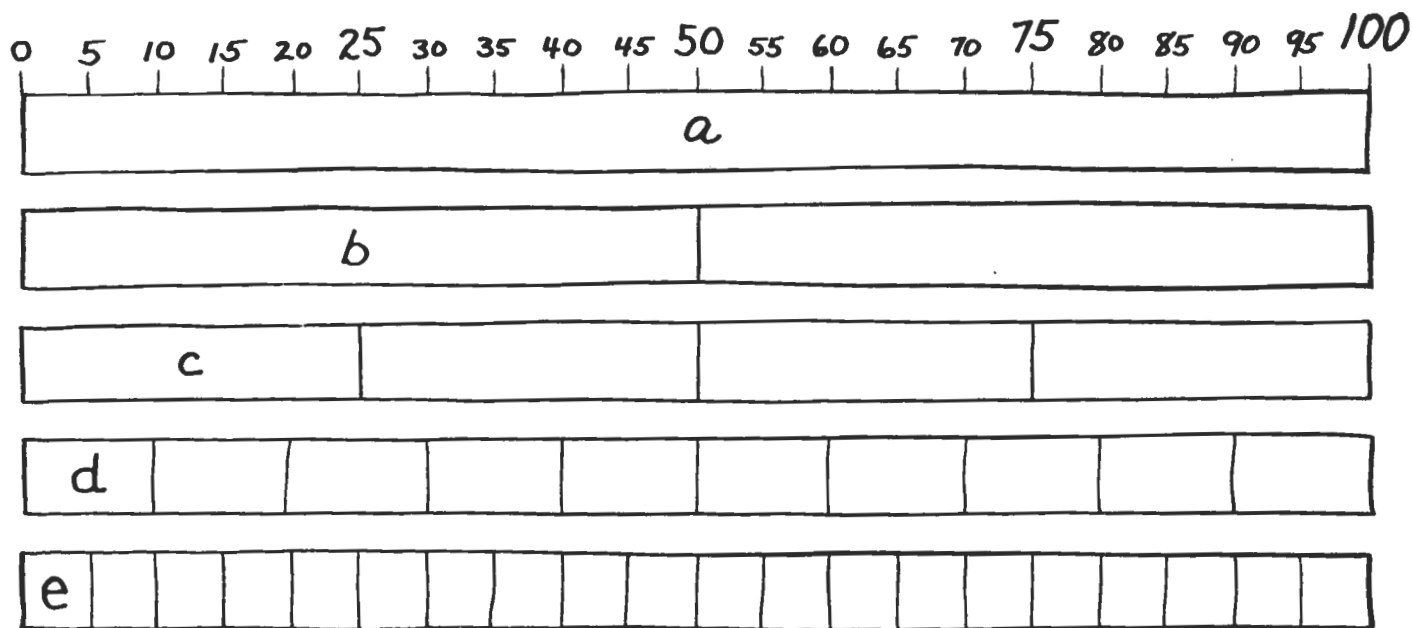
$$\frac{2c-3e}{-} = \underline{\quad} \quad \frac{3d+3d}{+} = \underline{\quad} \quad \frac{7e-3e}{-} = \underline{\quad}$$

$$\frac{c+d}{+} = \underline{\quad} \quad \frac{d+3e}{+} = \underline{\quad} \quad \frac{a-5d}{-} = \underline{\quad}$$

$$\frac{a \div 4}{\div} = \underline{\quad} \quad \frac{3d \div 2}{\div} = \underline{\quad} \quad \frac{b \div 2e}{\div} = \underline{\quad}$$

$$\frac{a \div c}{\div} = \underline{\quad} \quad \frac{3d \div 3e}{\div} = \underline{\quad} \quad \frac{5d \div c}{\div} = \underline{\quad}$$

This is a continuation of the activity on the previous page. Bars of different lengths are used.



$$\frac{a - c}{-} = \underline{\quad}$$

$$\frac{b + c}{+} = \underline{\quad}$$

$$\frac{3 \times c}{\times} = \underline{\quad}$$

$$\frac{b - 4e}{-} = \underline{\quad}$$

$$\frac{5 \times d}{\times} = \underline{\quad}$$

$$\frac{3d + 4e}{+} = \underline{\quad}$$

$$\frac{2c - 3e}{-} = \underline{\quad}$$

$$\frac{3c - b}{-} = \underline{\quad}$$

$$\frac{b + 2c}{+} = \underline{\quad}$$

$$\frac{a - 11e}{-} = \underline{\quad}$$

$$\frac{b - 2d}{-} = \underline{\quad}$$

$$\frac{c - e}{-} = \underline{\quad}$$

$$\frac{c \div e}{\div} = \underline{\quad}$$

$$\frac{b \div c}{\div} = \underline{\quad}$$

$$\frac{b \div d}{\div} = \underline{\quad}$$

$$\frac{c \div d}{\div} = \underline{\quad}$$

$$\frac{5d \div c}{\div} = \underline{\quad}$$

$$\frac{3c - 3e}{-} = \underline{\quad}$$



WHAT CAN YOU SEE? NAME \_\_\_\_\_

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100

a \_\_\_\_\_

b \_\_\_\_\_

c \_\_\_\_\_

d \_\_\_\_\_

e \_\_\_\_\_

$\frac{a-c}{-} = \underline{\hspace{2cm}}$        $\frac{b+c}{+} = \underline{\hspace{2cm}}$        $\frac{3 \times c}{\times} = \underline{\hspace{2cm}}$

$\frac{b-4e}{-} = \underline{\hspace{2cm}}$        $\frac{5 \times d}{\times} = \underline{\hspace{2cm}}$        $\frac{3d+4e}{+} = \underline{\hspace{2cm}}$

$\frac{2c-3e}{-} = \underline{\hspace{2cm}}$        $\frac{3c-b}{-} = \underline{\hspace{2cm}}$        $\frac{b+2c}{+} = \underline{\hspace{2cm}}$

$\frac{a-11e}{-} = \underline{\hspace{2cm}}$        $\frac{b-2d}{-} = \underline{\hspace{2cm}}$        $\frac{c-e}{-} = \underline{\hspace{2cm}}$

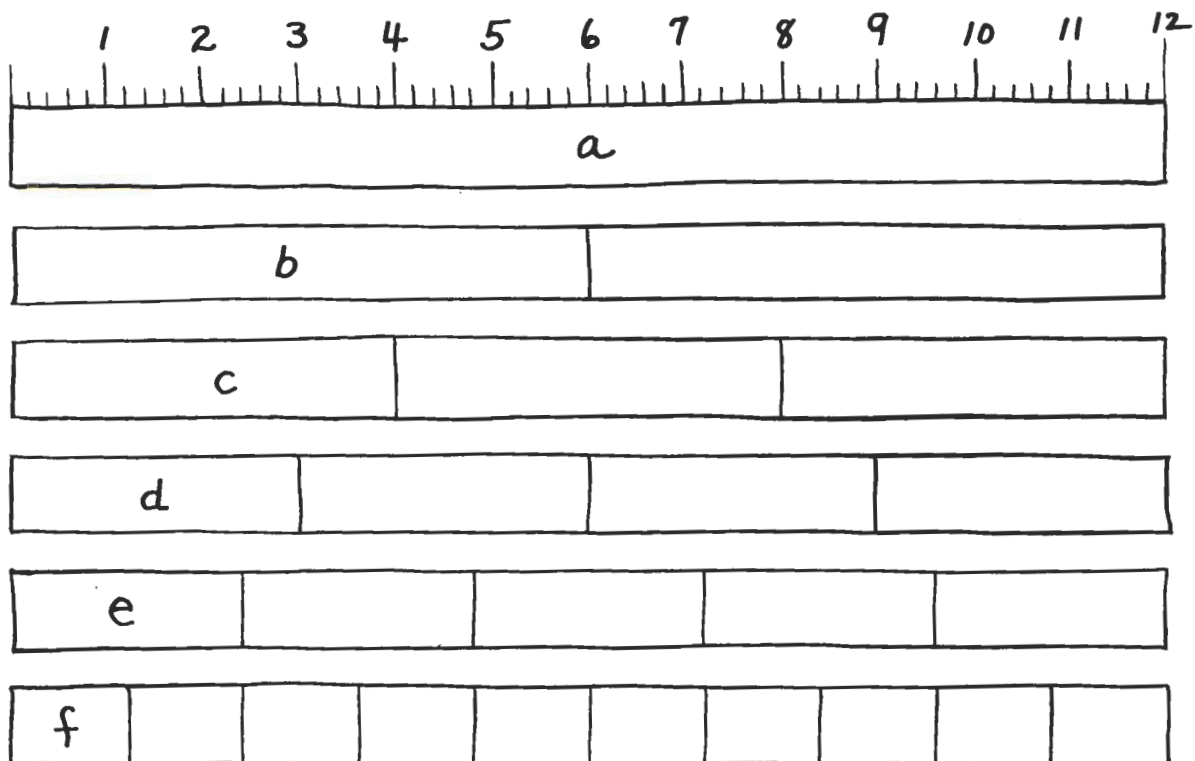
$\frac{c \div e}{\div} = \underline{\hspace{2cm}}$        $\frac{b \div c}{\div} = \underline{\hspace{2cm}}$        $\frac{b \div d}{\div} = \underline{\hspace{2cm}}$

$\frac{c \div d}{\div} = \underline{\hspace{2cm}}$        $\frac{5d \div c}{\div} = \underline{\hspace{2cm}}$        $\frac{3c-3e}{-} = \underline{\hspace{2cm}}$

This is a continuation of the activity on the previous pages. Bars of different lengths are used.

On this page, a scale is used at the top of bar "a", and you might want learners to "measure" the bars by using the card again and recording where the edge of the card cut the scale. (e.g.  $c = 25$ ,  $e = 5$ , etc.) The exercises could be done with whole numbers as well as with fractions.

By giving children exposure to a variety of systems of measurement and giving them opportunities to combine and compare lengths we are trying to build a "sense of measurement" which will facilitate the introduction of the metric or any other system of measurement.



$$\frac{2e + f}{+} = \underline{\quad}$$

$$\frac{b + d}{+} = \underline{\quad}$$

$$\frac{7f - e}{-} = \underline{\quad}$$

$$\frac{3e \div 2}{\div} = \underline{\quad}$$

$$\frac{a \div 3}{\div} = \underline{\quad}$$

$$\frac{2d - f}{-} = \underline{\quad}$$

$$\frac{5f + d}{+} = \underline{\quad}$$

$$\frac{a \div c}{\div} = \underline{\quad}$$

$$\frac{10f - c}{-} = \underline{\quad}$$

$$\frac{5f - d}{-} = \underline{\quad}$$

$$\frac{3f + e}{+} = \underline{\quad}$$

$$\frac{2d + 2e}{+} = \underline{\quad}$$

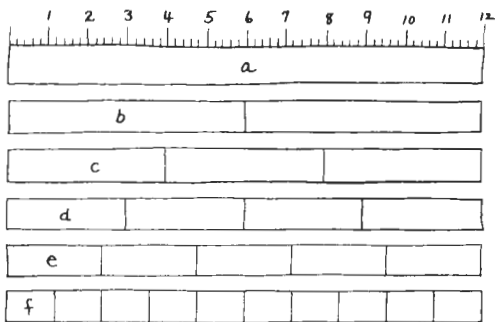
$$\frac{f \div e}{\div} = \underline{\quad}$$

$$\frac{3e \div 3f}{\div} = \underline{\quad}$$

$$\frac{a \div 2e}{\div} = \underline{\quad}$$

WHAT CAN YOU SEE?

NAME \_\_\_\_\_



$$\frac{2e + f}{+} = \quad \frac{b + d}{+} = \quad \frac{7f - e}{-} =$$

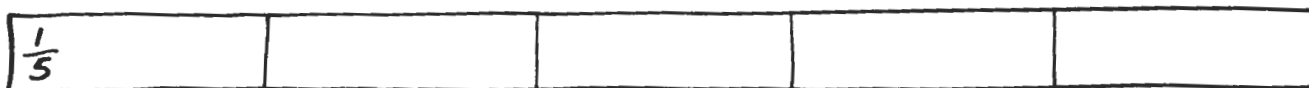
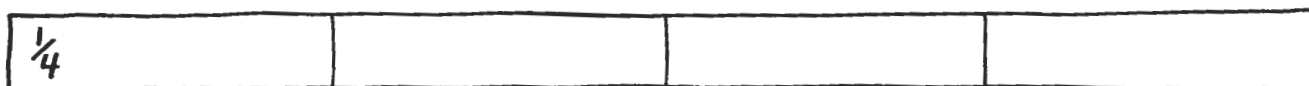
$$\frac{3e \div 2}{\div} = \quad \frac{a \div 3}{\div} = \quad \frac{2d - f}{-} =$$

$$\frac{3f + d}{+} = \quad \frac{a \div c}{\div} = \quad \frac{10f - c}{-} =$$

$$\frac{5f - d}{-} = \quad \frac{3f + e}{+} = \quad \frac{2d + 2e}{+} =$$

$$\frac{f \div e}{\div} = \quad \frac{3e \div 3f}{\div} = \quad \frac{a \div 2e}{\div} =$$

This activity is a continuation of those on the previous pages. A different measurement scale is given, and bars of different lengths are used.



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| $\frac{1}{3}$ |  |  |
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| $\frac{1}{4}$ |  |  |  |
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|---------------|--|--|--|--|
| $\frac{1}{5}$ |  |  |  |  |
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| $\frac{1}{10}$ |  |  |  |  |  |  |  |  |  |
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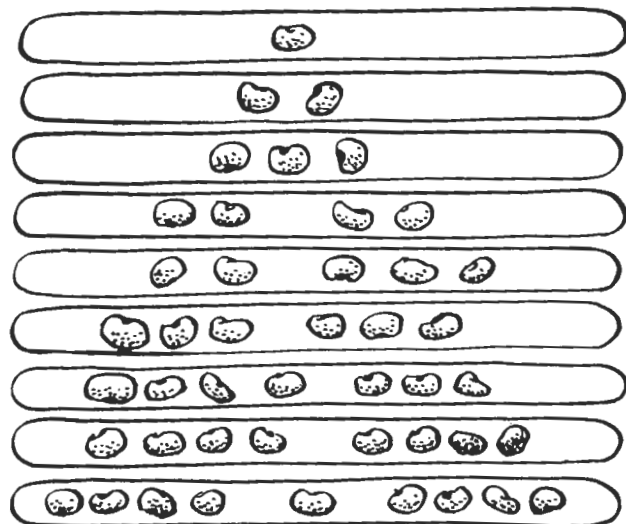
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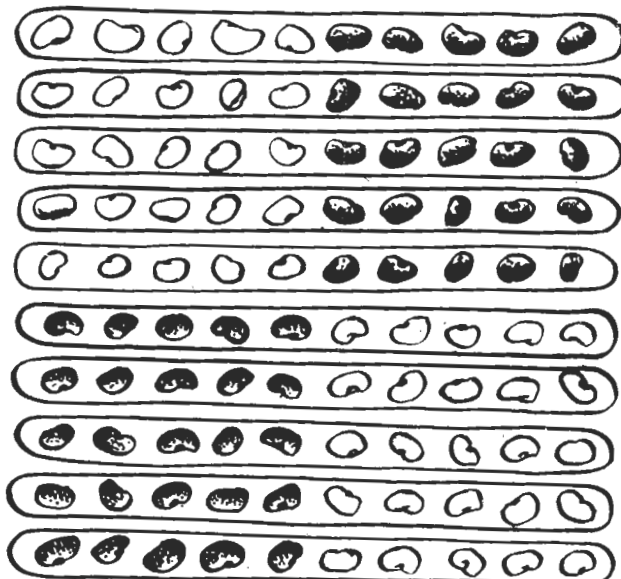
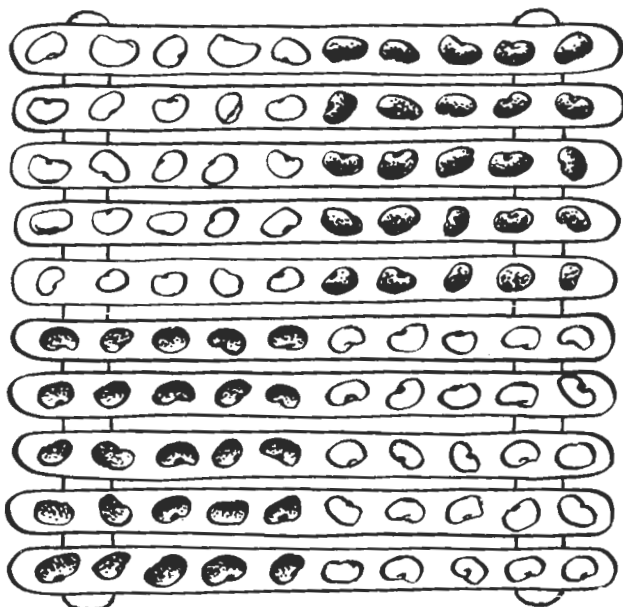
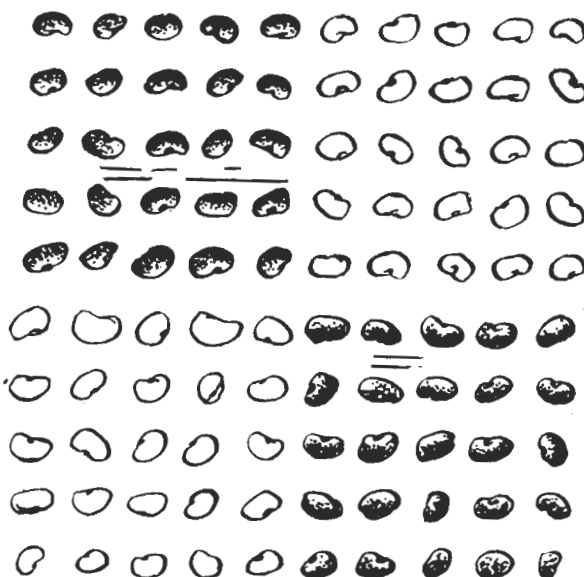
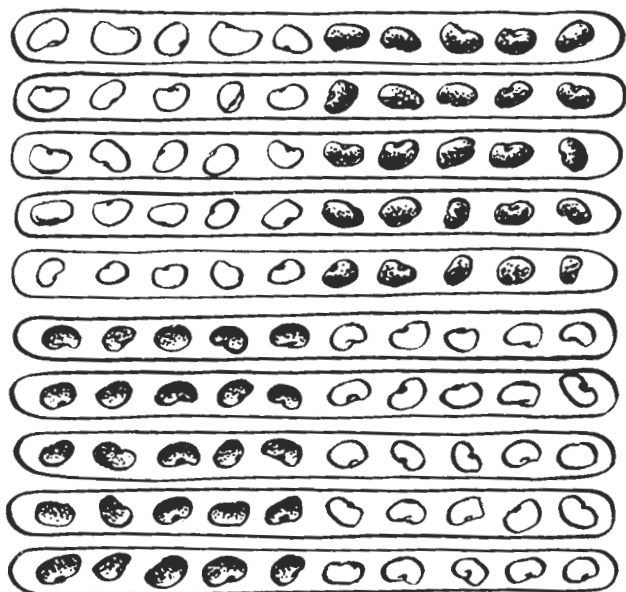
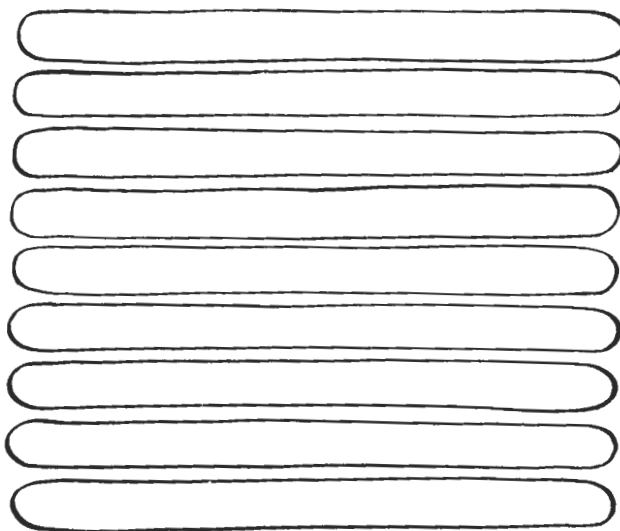
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|----|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 16 | $\frac{16}{1}$ | $\frac{16}{2}$ | $\frac{16}{3}$ | $\frac{16}{4}$ | $\frac{16}{5}$ | $\frac{16}{6}$ | $\frac{16}{7}$ | $\frac{16}{8}$ | $\frac{16}{9}$ | $\frac{16}{10}$ | $\frac{16}{11}$ | $\frac{16}{12}$ | $\frac{16}{13}$ | $\frac{16}{14}$ | $\frac{16}{15}$ | $\frac{16}{16}$ |
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| 14 | $\frac{14}{1}$ | $\frac{14}{2}$ | $\frac{14}{3}$ | $\frac{14}{4}$ | $\frac{14}{5}$ | $\frac{14}{6}$ | $\frac{14}{7}$ | $\frac{14}{8}$ | $\frac{14}{9}$ | $\frac{14}{10}$ | $\frac{14}{11}$ | $\frac{14}{12}$ | $\frac{14}{13}$ | $\frac{14}{14}$ | $\frac{14}{15}$ | $\frac{14}{16}$ |
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| 11 | $\frac{11}{1}$ | $\frac{11}{2}$ | $\frac{11}{3}$ | $\frac{11}{4}$ | $\frac{11}{5}$ | $\frac{11}{6}$ | $\frac{11}{7}$ | $\frac{11}{8}$ | $\frac{11}{9}$ | $\frac{11}{10}$ | $\frac{11}{11}$ | $\frac{11}{12}$ | $\frac{11}{13}$ | $\frac{11}{14}$ | $\frac{11}{15}$ | $\frac{11}{16}$ |
| 10 | $\frac{10}{1}$ | $\frac{10}{2}$ | $\frac{10}{3}$ | $\frac{10}{4}$ | $\frac{10}{5}$ | $\frac{10}{6}$ | $\frac{10}{7}$ | $\frac{10}{8}$ | $\frac{10}{9}$ | $\frac{10}{10}$ | $\frac{10}{11}$ | $\frac{10}{12}$ | $\frac{10}{13}$ | $\frac{10}{14}$ | $\frac{10}{15}$ | $\frac{10}{16}$ |
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| 2  | $\frac{2}{1}$  | $\frac{2}{2}$  | $\frac{2}{3}$  | $\frac{2}{4}$  | $\frac{2}{5}$  | $\frac{2}{6}$  | $\frac{2}{7}$  | $\frac{2}{8}$  | $\frac{2}{9}$  | $\frac{2}{10}$  | $\frac{2}{11}$  | $\frac{2}{12}$  | $\frac{2}{13}$  | $\frac{2}{14}$  | $\frac{2}{15}$  | $\frac{2}{16}$  |
| 1  | $\frac{1}{1}$  | $\frac{1}{2}$  | $\frac{1}{3}$  | $\frac{1}{4}$  | $\frac{1}{5}$  | $\frac{1}{6}$  | $\frac{1}{7}$  | $\frac{1}{8}$  | $\frac{1}{9}$  | $\frac{1}{10}$  | $\frac{1}{11}$  | $\frac{1}{12}$  | $\frac{1}{13}$  | $\frac{1}{14}$  | $\frac{1}{15}$  | $\frac{1}{16}$  |
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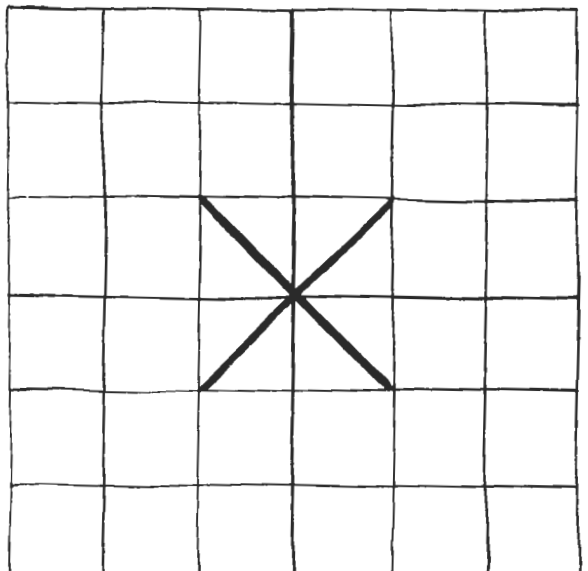
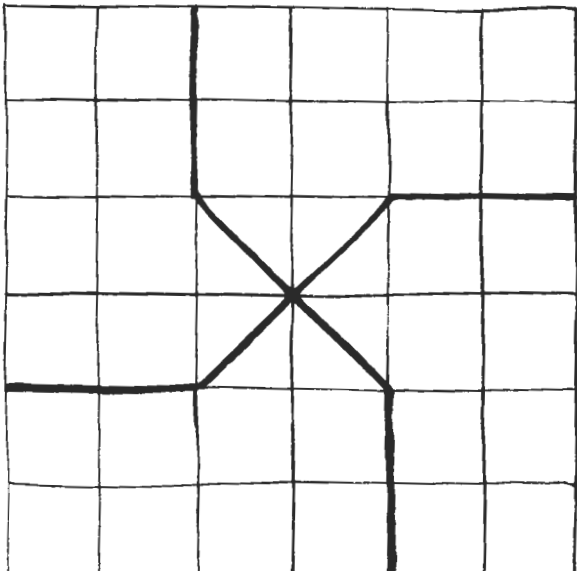
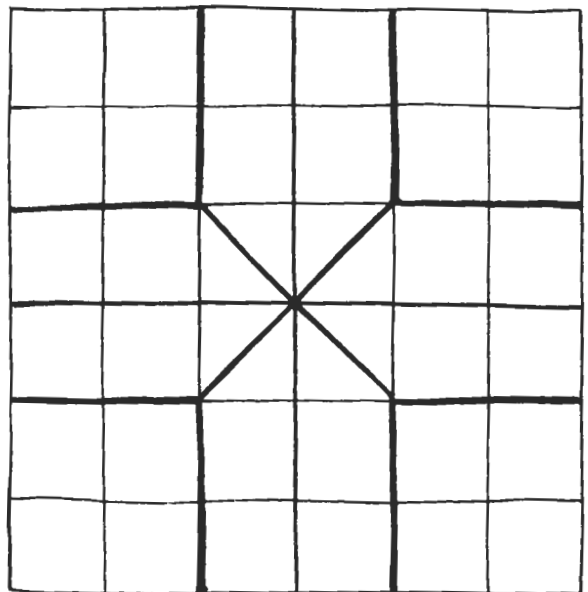
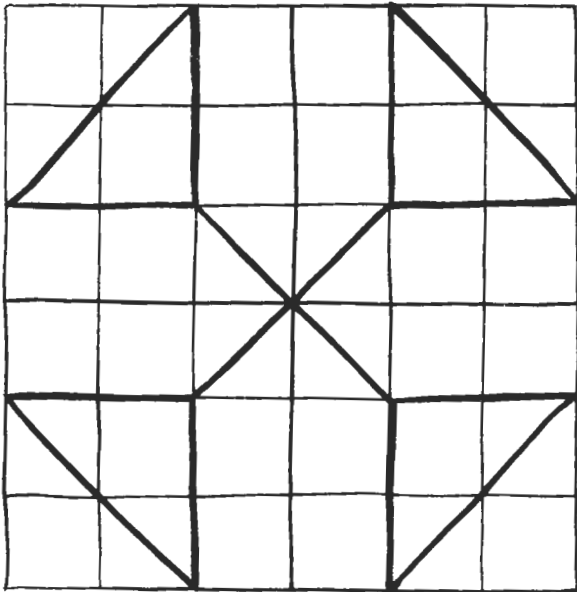
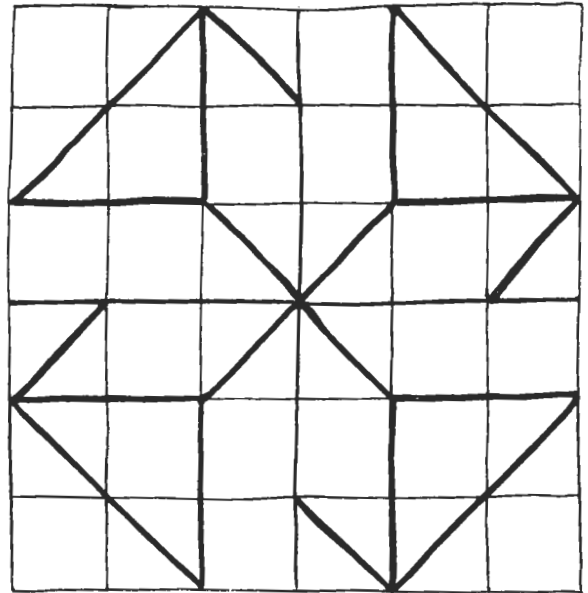
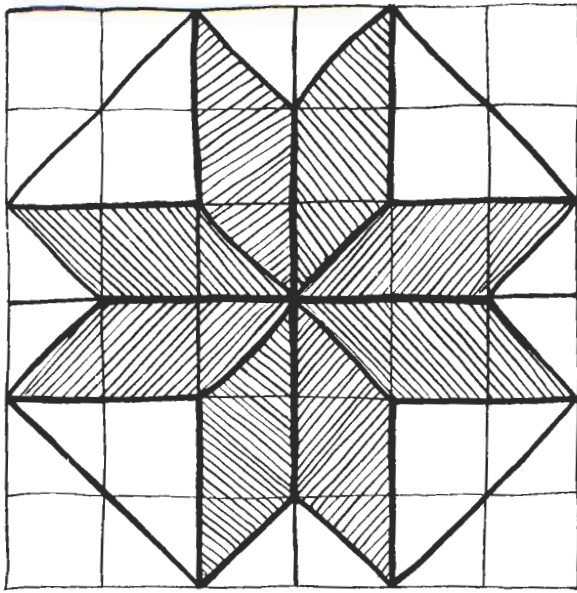


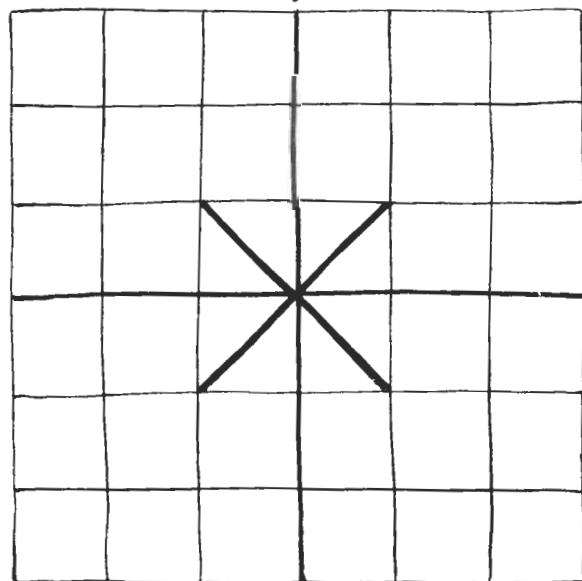
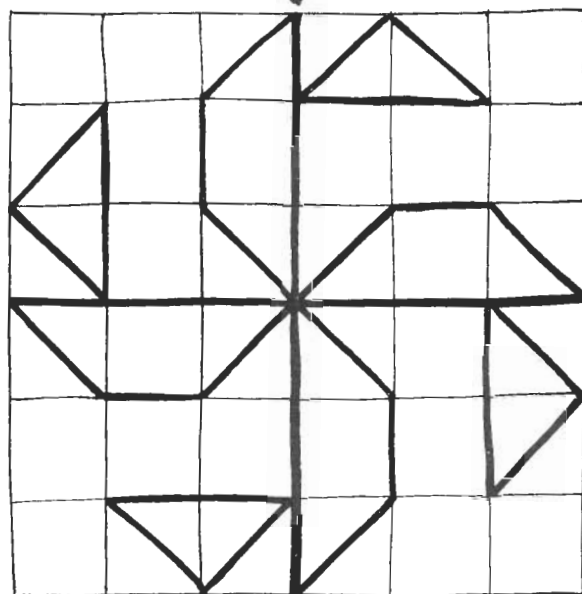
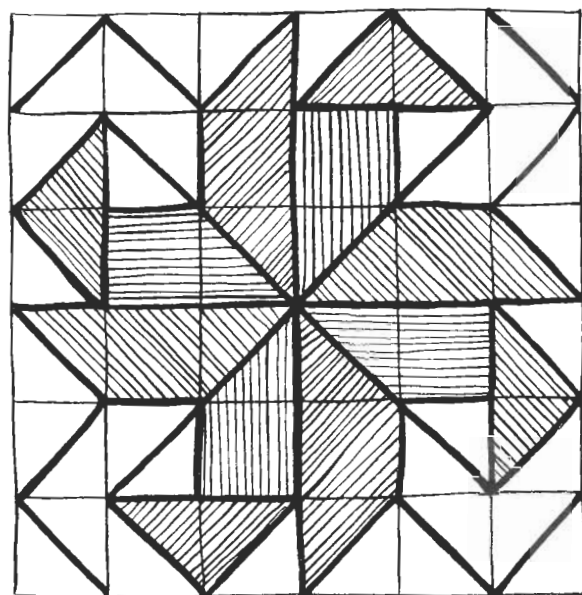
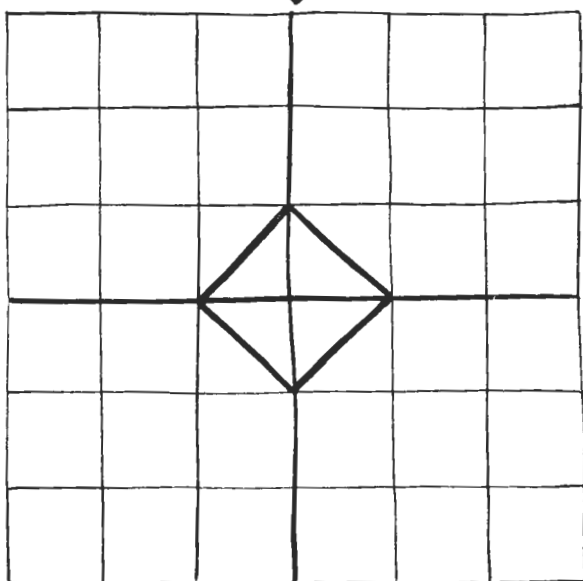
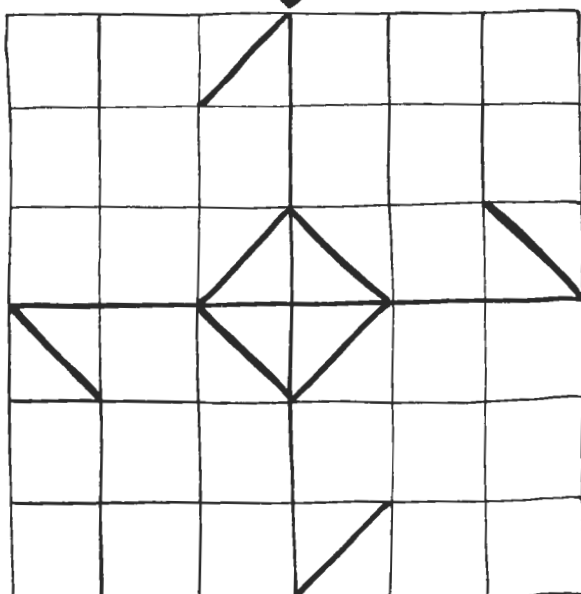
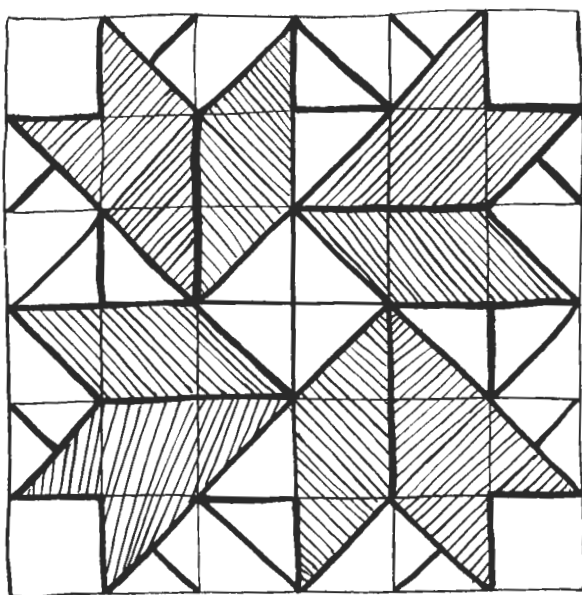
FOR USE IN MAKING OVERHEAD TRANSPARENCIES.

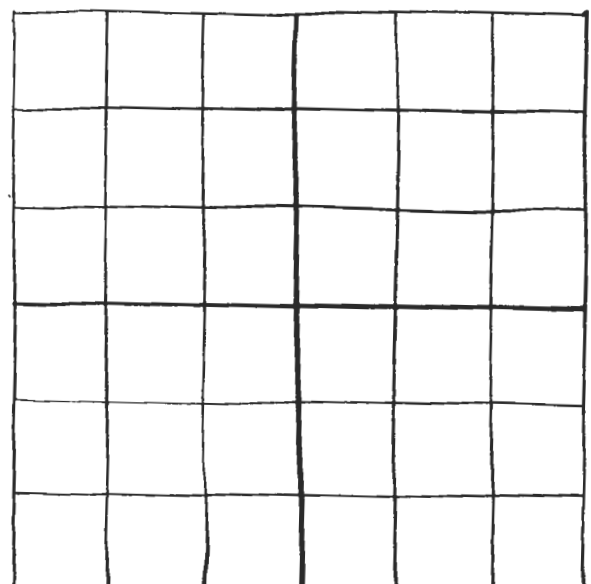
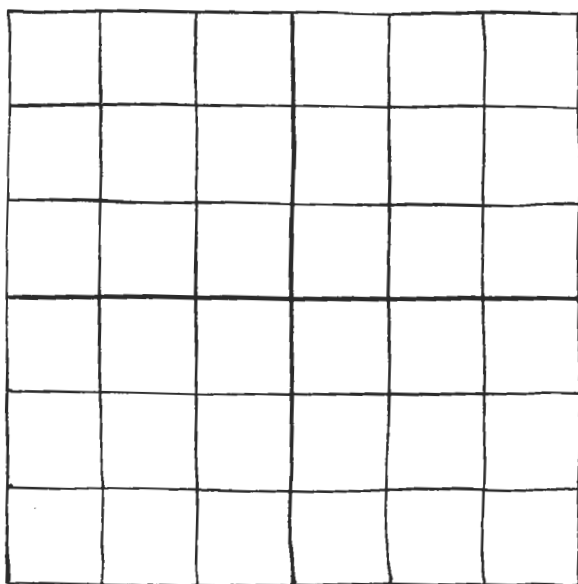
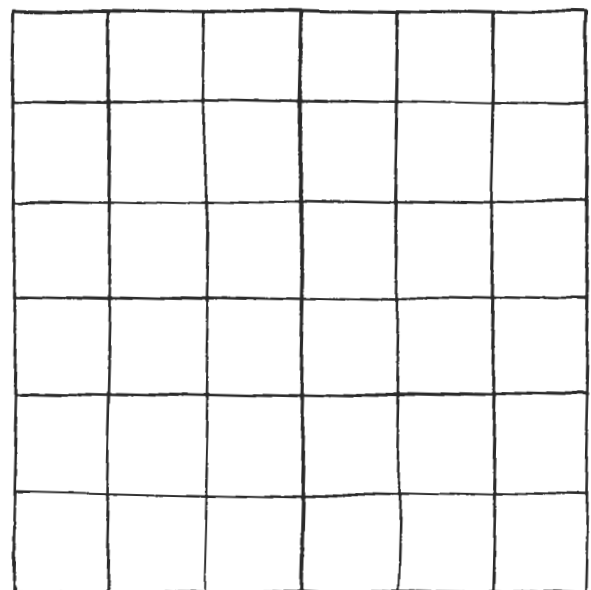
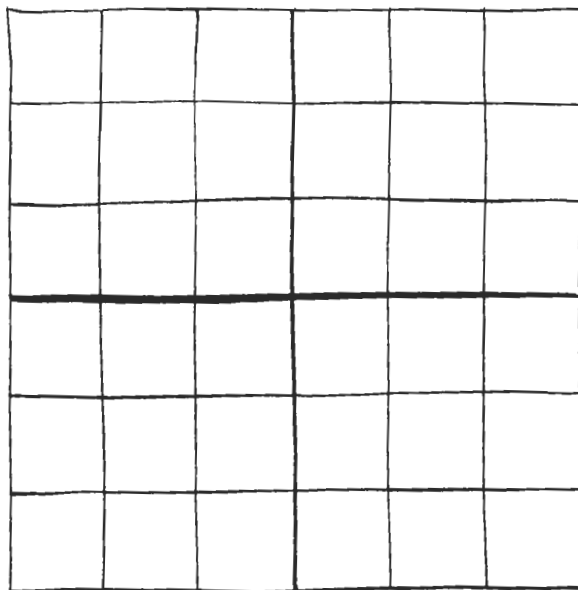
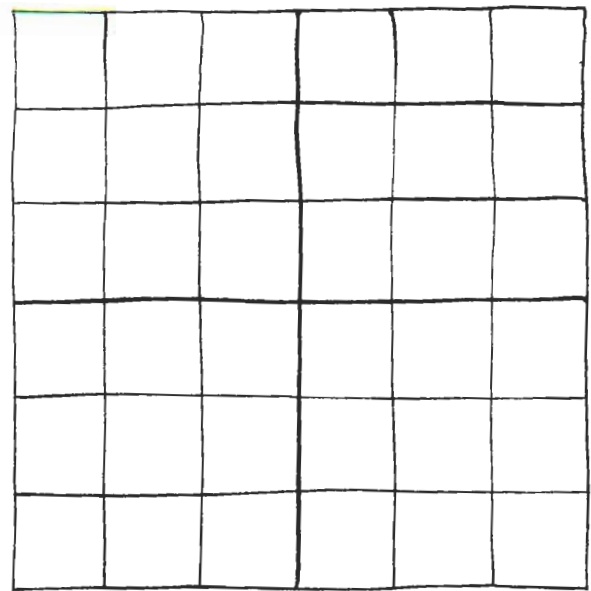
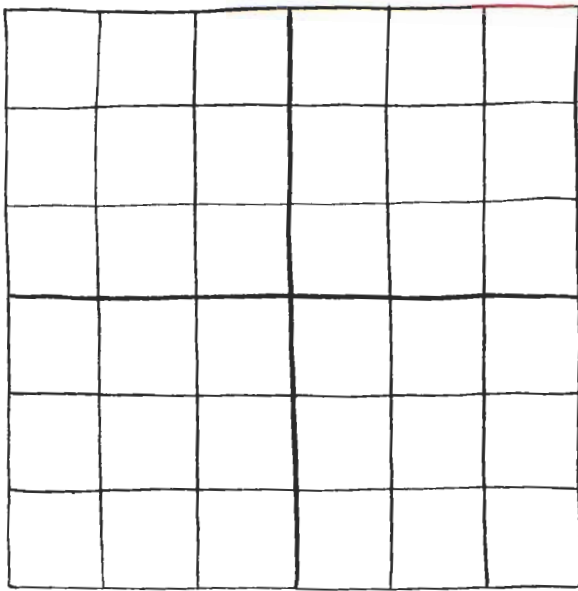


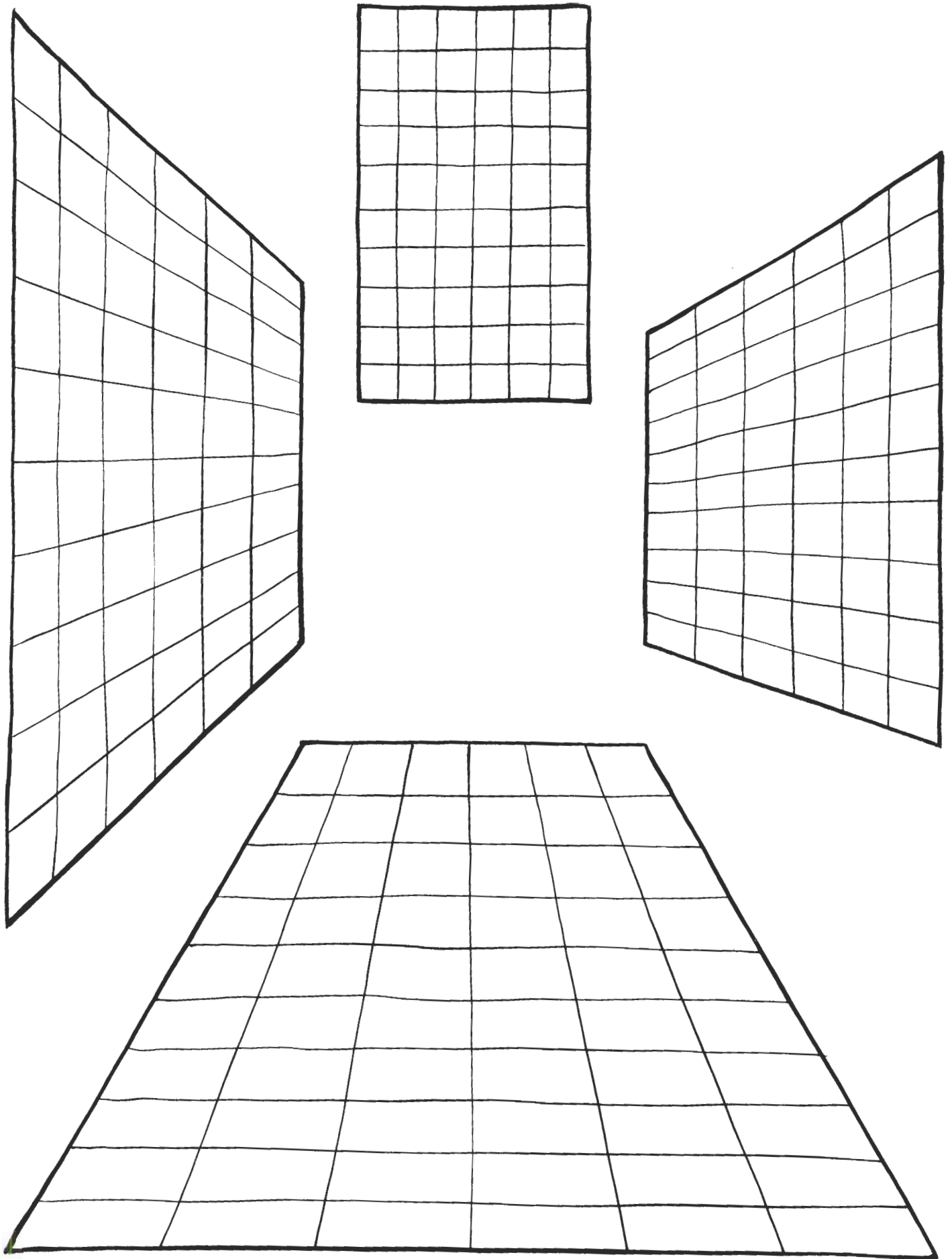
YOU MAKE YOUR OWN PATTERNS











(Note: Activities that are bracketed are related to each other)

1