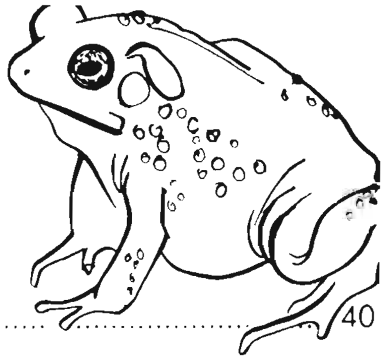
CDA matt
ROBERT W. WIRT

patterns & problems

(Jumping around in mathematics)

f

JUMPING AROUND IN MATHEMATICS



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"HELP!" . . . a famous problem about four "4"s.

How many whole numbers 0 thru 100 can you indicate using four "4"s and no other number?

Because it is such a long and hard problem, you will probably need help, lots of it.

Ask your friends to help. Work on it with them.

Ask other members of your family if they would like to help.

Perhaps we can help too. Here are some examples of expressions using four "4"s and no other numbers. What numbers do they indicate?

$(4+4)-(4+4)=$ ___

$(4+4)+(4+4)=$ ___

$(4\times 4)+(4+4)=$ ___

$(4\times 4)-(4+4)=$ ___

$(4\times 4)\div(4+4)=$ ___

$(4+4)\div(4+4)=$ ___

$(4\times 4)\times(4-4)=$ ___

$(4\div 4)+(4\div 4)=$ ___

$(4\times 4)-(4\div 4)=$ ___

$(4\div 4)\div(4\div 4)=$ ___

$(4\times 4)\div(4\div 4)=$ ___

$(4\times 4)+(4\div 4)=$ ___

That's a good start.

But there are many other forms you can use including the use of two "4"s to write 44.

$44+(4+4)=$ ___

$44-(4\div 4)=$ ___

$(44\div 4)-4=$ ___

$44+44=$ ___

$(44-4)\div 4=$ ___

$(44+4)\div 4=$ ___

$44\times(4\div 4)=$ ___

Mathematicians use a sign that will be very useful if you want to use it.

4! is read "factorial four" and it is shorthand meaning

$4\times 3\times 2\times 1$ or $\underline{24}$.

So we can write:

$4!+4+4+4=$ ___

$(4!+4)\times(4\div 4)=$ ___

$(4!\times 4)-(4\times 4)=$ ___

$4!+(44+4)=$ ___

$(4!-4)+44=$ ___

More Help

You may want to use another bit of shorthand from "decimal fractions".

.4 . . . is another way to write $\frac{4}{10}$

It requires only a single "4" and no other numbers.

$4 \div .4 =$ ____ is very much like asking: How many $.40's are there in $4.00?

$4.00 \div .40 =$ ____

Or, 4/10 of an hour is 24 minutes, and 4 hours is 4 × 60 minutes.

$240 \div 24 =$ ____

So $4 \div .4$ or $\frac{4}{.4}$ are ways to show 10 using two 4's in our "help problem".

$(4 \div .4) + (4 \times 4) =$ ____

$(4 \div .4) - (4 + 4) =$ ____

$(4 \div .4) + (4 + 4) =$ ____

$[(4 \div .4) + 4] \times 4 =$ ____

$(4 \div .4) \times (4 \div .4) =$ ____

$(4 \div .4) + (4 \div 4) =$ ____

"Decimal fractions" are a source of further help.

$4 - .4 =$ ____ is the same as saying

$4.00 - .40 =$ ____

and

$3.6 \div .4$ is very much like asking: How many $.40's are there in $3.60?

$3.60 \div .40 = 9$

and

$3.6 \div .4 = 9$

so

$(4 - .4) \div .4 = 9$

$[(4 - .4) \div .4] - 4 =$ ____

$[(4 - .4) \div .4] \times 4 =$ ____

$[(4 - .4) \div .4] + 4 =$ ____

a final bit of help:

$\sqrt{4} =$ ____ asks, "What is the square root of four . . . or, what number multiplied by itself equals 4?"

Since 2 × 2 = 4, we say

$\sqrt{4} = 2$

$\sqrt{4} + 4 + 4 + 4 =$ ____

$\sqrt{4} + 4! + (4 \div .4) =$ ____

Use all the "Help" you can get to find answers to these problems:

1. How many whole numbers 1 thru 100 can you indicate using four 4's and no other number?
2. How many can you indicate in at least 2 different ways?

0 = (4 x 4) x (4 − 4) =

1 = = 44 ÷ 44

2 = =

3 = =

4 = = 4 + [4 x (4 − 4)]

5 = =

6 = =

7 = = (44 ÷ 4) − 4

8 = = $\sqrt{4}+\sqrt{4}+\sqrt{4}+\sqrt{4}$

9 = =

10 = =

11 = =

12 = = 4! − (4 + 4 + 4)

13 = =

14 = =

15 = =

16 = =

17 = =

18 = =

19 = =

20 = =

21 = =

22 = =

23 = =

24 = =

25 = =

26 = =

27 = =

28= =
29= =
30= =
31 = =
32= =
33= =
34= =
35= =
36= =
37= =
38= =
39= =
40= =
41 = =
42= =
43= =
44= =
45= =
46= =
47= =
48= =
49= =
50= = $(4!\div 4)+44$
51 = =
52= =
53= =
54= =
55= =
56= =
57= =
58= =
59= =
60= =
61 = =
62= =
63= =
64= = $(4!+4!)+(4\times 4)$
65= =

66 = =

67 = =

68 = =

69 = =

70 = =

71 = =

72 = =

73 = =

74 = =

75 = =

76 = =

77 = =

78 = =

79 = =

80 = =

81 = =

82 = =

83 = =

84 = =

85 = =

86 = =

87 = =

88 = =

89 = =

90 = =

91 = =

92 = =

93 = =

94 = =

95 = =

96 = =

97 = =

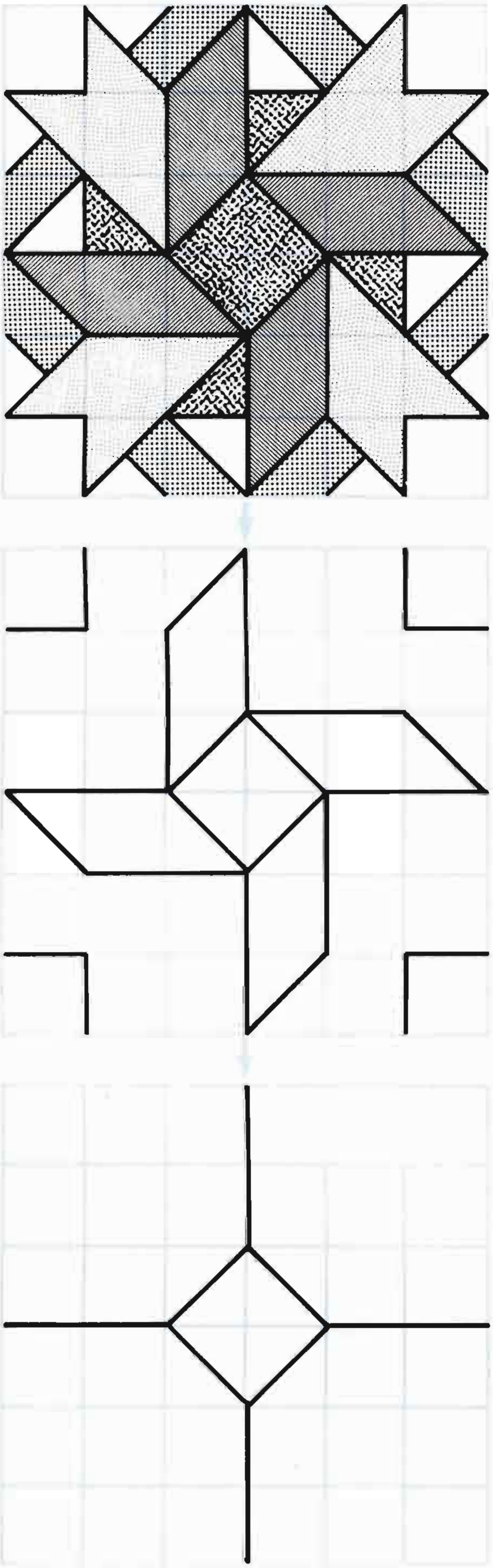
98 = =

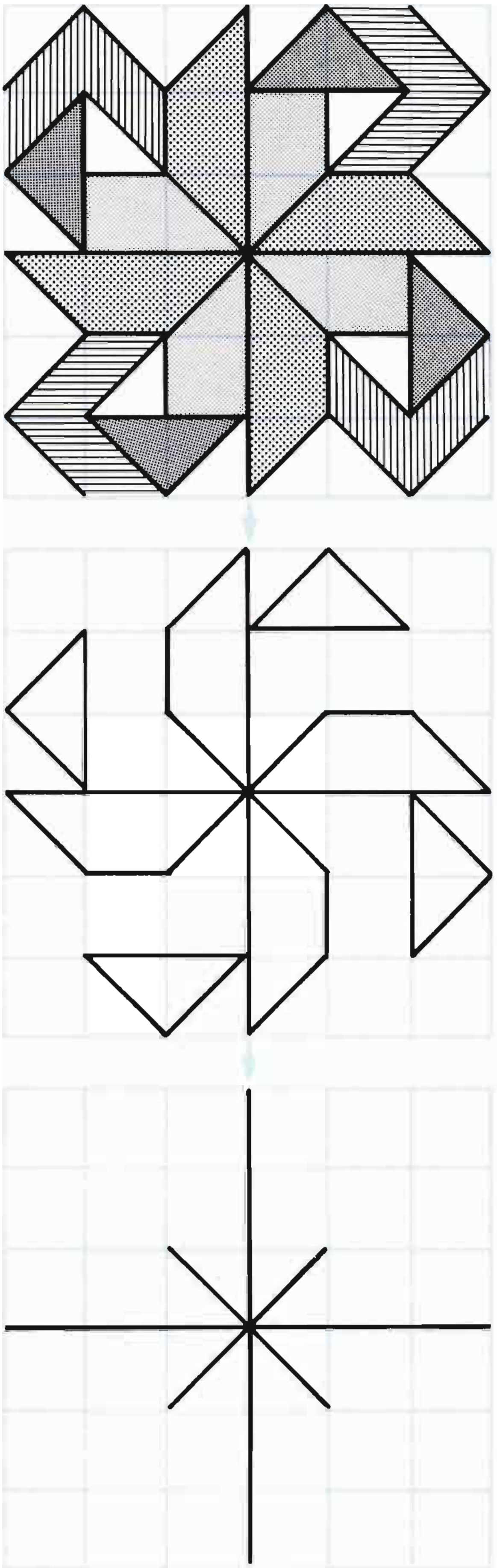
99 = =

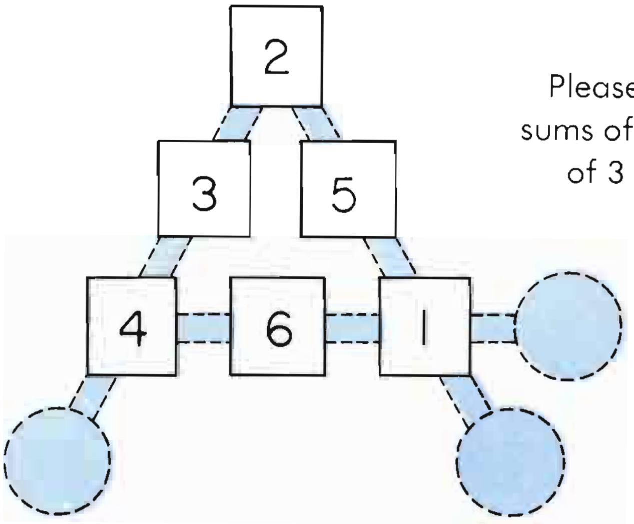
100 = = $(4 \div 4) \times (4 \div 4)$

At the time of this printing, the author knows of no group that has completed this problem.

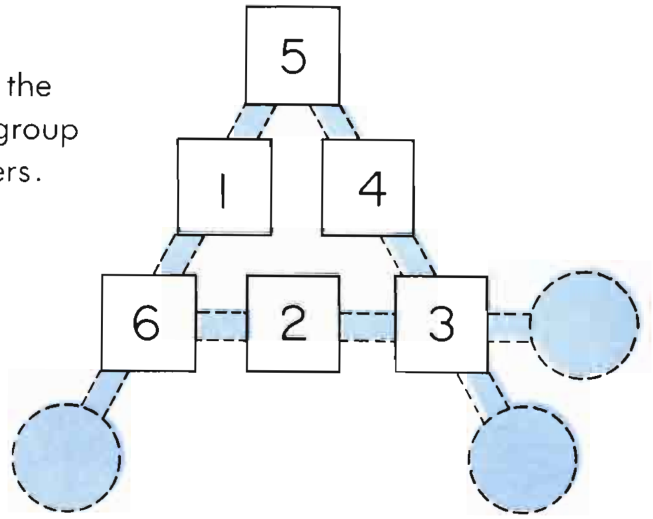
If you do, send him your solution and you will be credited in the next edition.

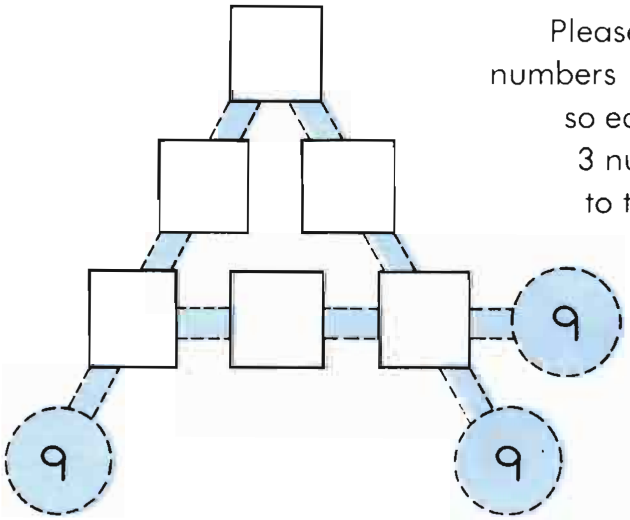




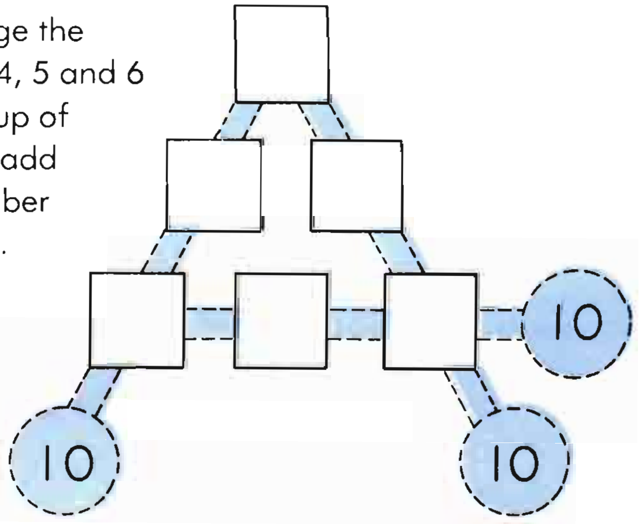


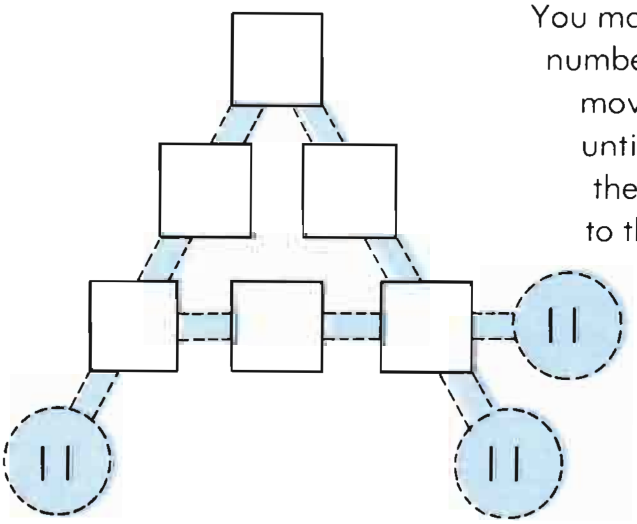
Please write the
sums of each group
of 3 numbers.



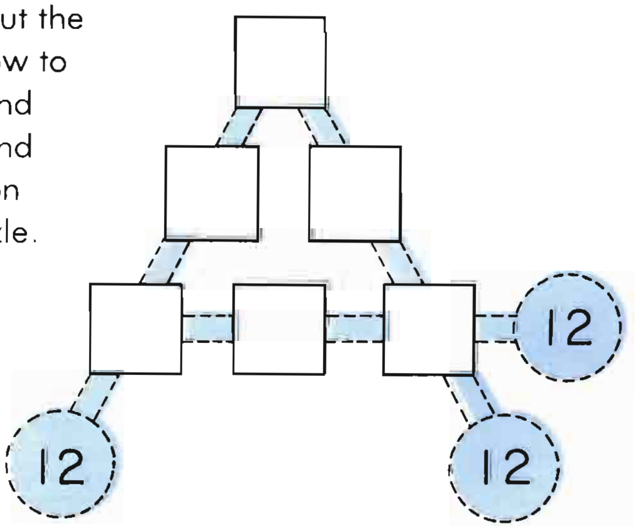


Please arrange the
numbers 1, 2, 3, 4, 5 and 6
so each group of
3 numbers add
to the number
shown.





You may cut out the
numbers below to
move around
until you find
the solution
to the puzzle.



1 2 3 4 5 6

How Tall Are You?

Please find out how tall each person in your class is .

You can mark it on the graph.

Now connect the points.

What do you see ?

The average height of the boys is ________ .

The average height of the girls is ________ .

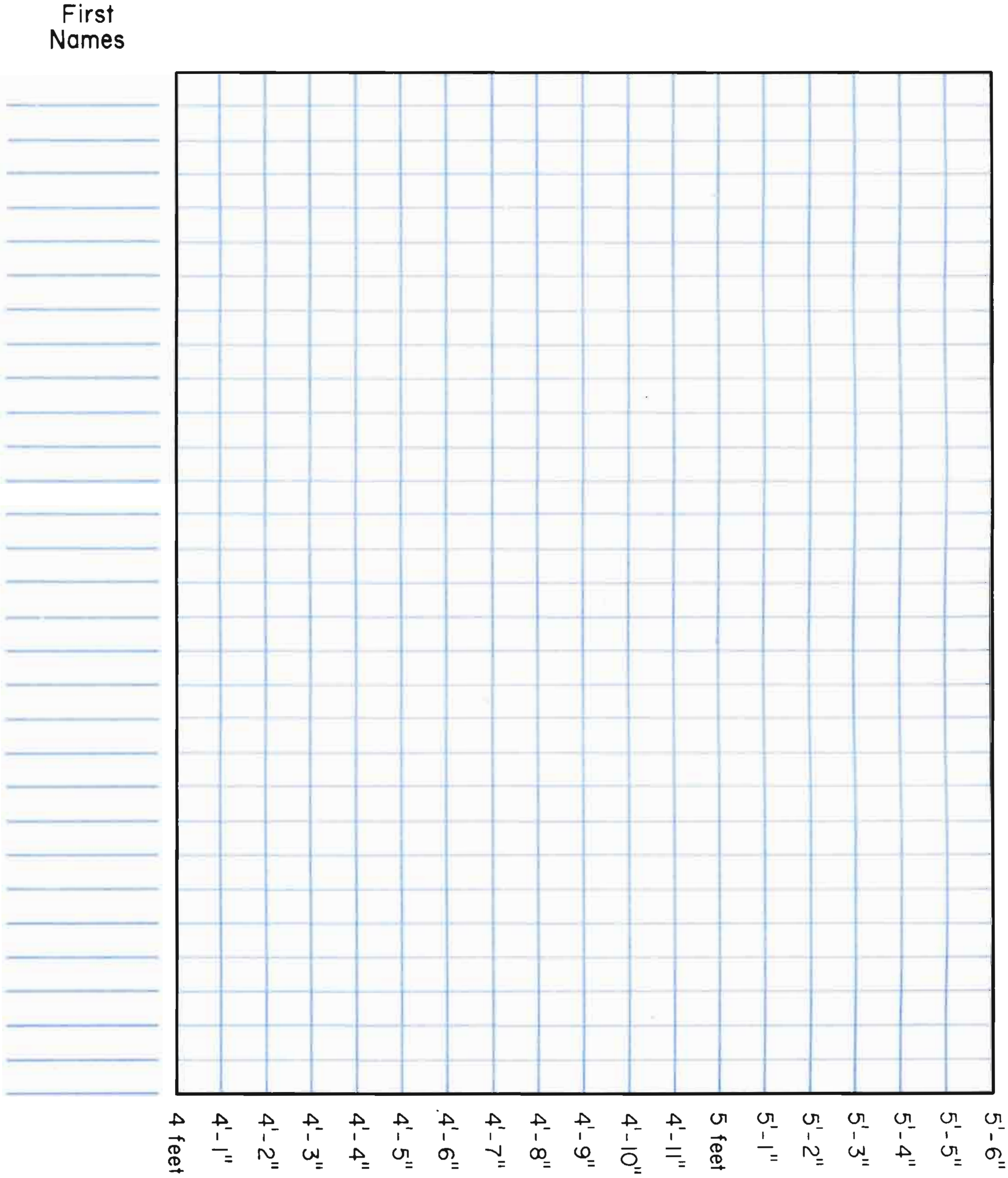
which girls have the average height ?

Which boys have the average height ?

The tallest person in the class is ____________________ .

The shortest person in the class is ____________________ .

CLASS HEIGHT



Pies cut with the help of a 6-piece pie cutter.

pies	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{5}{6}$	$1\frac{1}{3}$	$1\frac{1}{2}$
pieces	6							

$\frac{1}{3} + \frac{1}{6} = \frac{\quad}{\quad}$

\+ =

$\frac{1}{6} + \frac{1}{6} = \frac{\quad}{\quad}$

\+ =

$1\frac{1}{3} - \frac{2}{3} = \frac{\quad}{\quad}$

− =

$\frac{5}{6} - \frac{1}{6} = \frac{\quad}{\quad}$

− =

There are 36 inches in 1 yard.

yards	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{5}{6}$	$1\frac{1}{3}$	$1\frac{1}{2}$
inches	36							

$\frac{1}{3} + \frac{1}{6} = \frac{\quad}{\quad}$

\+ =

$\frac{1}{6} + \frac{1}{6} = \frac{\quad}{\quad}$

\+ =

$1\frac{1}{3} - \frac{2}{3} = \frac{\quad}{\quad}$

− =

$\frac{5}{6} - \frac{1}{6} = \frac{\quad}{\quad}$

− =

There are 60 minutes in 1 hour.

hours	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{5}{6}$	$1\frac{1}{3}$	$1\frac{1}{2}$
minutes	60							

$\frac{1}{3} + \frac{1}{6} = \frac{\quad}{\quad}$

\+ =

$\frac{1}{6} + \frac{1}{6} = \frac{\quad}{\quad}$

\+ =

$1\frac{1}{3} - \frac{2}{3} = \frac{\quad}{\quad}$

− =

$\frac{5}{6} - \frac{1}{6} = \frac{\quad}{\quad}$

− =

There are 16 ounces in 1 pound.

pounds	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$
ounces	16							

$\frac{3}{8}$	+	$\frac{1}{8}$	=	$\frac{}{}$
	+		=	

$\frac{3}{8}$	−	$\frac{1}{4}$	=	$\frac{}{}$
	−		=	

$\frac{3}{8}$	x	2	=	$\frac{}{}$
	x	2	=	

$\frac{3}{4}$	÷	2	=	$\frac{}{}$
	÷	2	=	

There are 24 hours in 1 day.

days	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$
hours								

$\frac{3}{8}$	+	$\frac{1}{8}$	=	$\frac{}{}$
	+		=	

$\frac{3}{8}$	−	$\frac{1}{4}$	=	$\frac{}{}$
	−		=	

$\frac{3}{8}$	x	2	=	$\frac{}{}$
	x	2	=	

$\frac{3}{4}$	÷	2	=	$\frac{}{}$
	÷	2	=	

There are 8 pints in 1 gallon.

gallons	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$
pints								

$\frac{3}{8}$	+	$\frac{1}{8}$	=	$\frac{}{}$
	+		=	

$\frac{3}{8}$	−	$\frac{1}{4}$	=	$\frac{}{}$
	−		=	

$\frac{3}{8}$	x	2	=	$\frac{}{}$
	x	2	=	

$\frac{3}{4}$	÷	2	=	$\frac{}{}$
	÷	2	=	

There are 12 eggs in 1 dozen.

dozens	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{5}{6}$	$\frac{5}{12}$	$\frac{7}{12}$
eggs								

$\frac{1}{6} + \frac{1}{3} = \frac{}{}$

$\quad + \quad =$

$\frac{7}{12} - \frac{5}{12} = \frac{}{}$

$\quad - \quad =$

$\frac{3}{4} - \frac{2}{3} = \frac{}{}$

$\quad - \quad =$

$\frac{1}{6} \times 3 = \frac{}{}$

$\quad \times 3 =$

There are 36 inches in 1 yard.

yards	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{5}{6}$	$\frac{5}{12}$	$\frac{7}{12}$
inches								

$\frac{1}{6} + \frac{1}{3} = \frac{}{}$

$\quad + \quad =$

$\frac{7}{12} - \frac{5}{12} = \frac{}{}$

$\quad - \quad =$

$\frac{3}{4} - \frac{2}{3} = \frac{}{}$

$\quad - \quad =$

$\frac{1}{6} \times 3 = \frac{}{}$

$\quad \times 3 =$

There are 24 hours in 1 day.

days	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{5}{6}$	$\frac{5}{12}$	$\frac{7}{12}$
hours								

$\frac{1}{6} + \frac{1}{3} = \frac{}{}$

$\quad + \quad =$

$\frac{7}{12} - \frac{5}{12} = \frac{}{}$

$\quad - \quad =$

$\frac{3}{4} - \frac{2}{3} = \frac{}{}$

$\quad - \quad =$

$\frac{1}{6} \times 3 = \frac{}{}$

$\quad \times 3 =$

There are 60 minutes in 1 hour.

hours	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{3}{4}$	$\frac{2}{5}$	$\frac{3}{10}$	$\frac{7}{10}$
minutes								

$\frac{1}{2} - \frac{2}{5} = \frac{\quad}{\quad}$

$\square - \square = \square$

$\frac{1}{4} + \frac{1}{4} = \frac{\quad}{\quad}$

$\square + \square = \square$

$\frac{1}{10} + \frac{1}{10} = \frac{\quad}{\quad}$

$\square + \square = \square$

$\frac{1}{10} + \frac{1}{5} = \frac{\quad}{\quad}$

$\square + \square = \square$

$\frac{1}{4} \times 2 = \frac{\quad}{\quad}$

$\square \times 2 = \square$

$\frac{1}{10} \times 2 = \frac{\quad}{\quad}$

$\square \times 2 = \square$

$\frac{1}{10} \times 3 = \frac{\quad}{\quad}$

$\square \times 3 = \square$

There are 100 cents in 1 dollar.

dollars	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{3}{4}$	$\frac{2}{5}$	$\frac{3}{10}$	$\frac{7}{10}$
cents								

$\frac{1}{2} - \frac{2}{5} = \frac{\quad}{\quad}$

$\square - \square = \square$

$\frac{1}{4} + \frac{1}{4} = \frac{\quad}{\quad}$

$\square + \square = \square$

$\frac{1}{10} + \frac{1}{10} = \frac{\quad}{\quad}$

$\square + \square = \square$

$\frac{1}{10} + \frac{1}{5} = \frac{\quad}{\quad}$

$\square + \square = \square$

$\frac{1}{4} \times 2 = \frac{\quad}{\quad}$

$\square \times 2 = \square$

$\frac{1}{10} \times 2 = \frac{\quad}{\quad}$

$\square \times 2 = \square$

$\frac{1}{10} \times 3 = \frac{\quad}{\quad}$

$\square \times 3 = \square$

From the List

$\frac{2}{4}$ $\frac{4}{20}$ $\frac{3}{24}$ $\frac{2}{6}$ $\frac{3}{15}$ $\frac{4}{6}$ $\frac{15}{24}$ $\frac{6}{24}$

$\frac{1}{7}$ $\frac{10}{16}$ $\frac{2}{12}$ $\frac{4}{32}$ $\frac{6}{9}$ $\frac{3}{12}$ $\frac{9}{15}$ $\frac{2}{10}$ $\frac{3}{6}$ $\frac{5}{6}$

$\frac{6}{8}$ $\frac{4}{16}$ $\frac{5}{10}$ $\frac{9}{12}$

$\frac{3}{13}$ $\frac{5}{25}$ $\frac{3}{7}$ $\frac{4}{5}$ $\frac{10}{15}$ $\frac{2}{9}$ $\frac{3}{9}$ $\frac{7}{21}$

$\frac{5}{15}$ $\frac{8}{12}$ $\frac{7}{11}$ $\frac{12}{20}$ $\frac{4}{12}$ $\frac{12}{16}$ $\frac{20}{32}$ $\frac{6}{18}$ $\frac{3}{18}$

$\frac{7}{28}$ $\frac{6}{12}$ $\frac{2}{16}$ $\frac{2}{8}$ $\frac{12}{18}$ $\frac{4}{24}$ $\frac{5}{20}$ $\frac{6}{10}$ $\frac{4}{8}$

Please fill the blanks below with fractions from the list above.

*$\frac{1}{2}$ = — = — = — = —

*$\frac{1}{3}$ = — = — = — = —

*$\frac{1}{4}$ = — = — = — = —

*$\frac{2}{3}$ = — = — = — = —

*$\frac{3}{4}$ = — = —

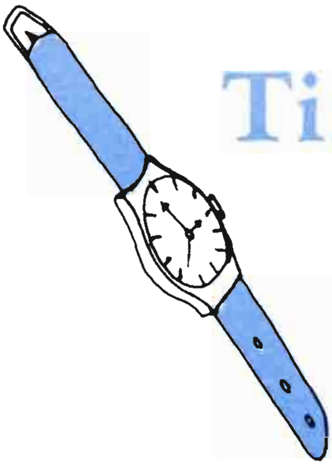
*$\frac{1}{5}$ = — = —

*$\frac{1}{6}$ = — = —

*$\frac{1}{8}$ = — = —

*$\frac{3}{5}$ = — = —

*$\frac{5}{8}$ = — = —



Time Your Heartbeat

How fast does your heart beat ?

I guess ___ beats per minute.

In fact it beats _____ times per minute.

After walking fast for 3 minutes: ______ beats per minute

After running fast for 1 minute: ______ beats

After doing 10 sit-ups: ______ beats

Now can you think of any activities (or inactivities) to make your heart beat faster or slower ?

__________________________ ______ beats

__________________________ ______ beats

__________________________ ______ beats

Why does the heart beat faster after running ?

U.S. DEPARTMENT OF COMMERCE
National Bureau of Standards
Washington, D.C. 20234

Letter Circular 1052
February 1974

All You Will Need to Know About Metric

(For Your Everyday Life)

Note: This chart may be reproduced

10

Metric is based on Decimal system

The metric system is simple to learn. For use in your everyday life you will need to learn only ten new units. You will also need to get used to a few new temperatures. There are even some metric units with which you are already familiar: those for time and electricity are the same as you use now.

BASIC UNITS

METER:	a little longer than a yard (about 1.1 yards)
LITER:	a little larger than a quart (about 1.06 quarts)
GRAM:	a little more than the weight of a paper clip

(comparative sizes are shown)

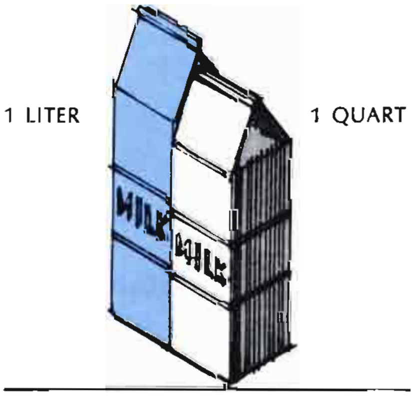
25 DEGREES FAHRENHEIT

25 DEGREES CELSIUS

COMMON PREFIXES
(to be used with basic units)

milli:	one-thousandth (0.001)
centi:	one-hundredth (0.01)
kilo:	one-thousand times (1000)

For example:
1000 millimeters = 1 meter
100 centimeters = 1 meter
1000 meters = 1 kilometer

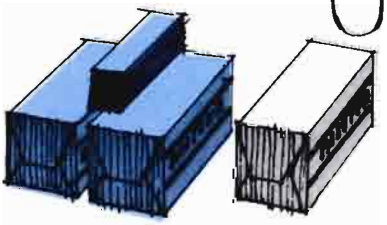


OTHER COMMONLY USED UNITS

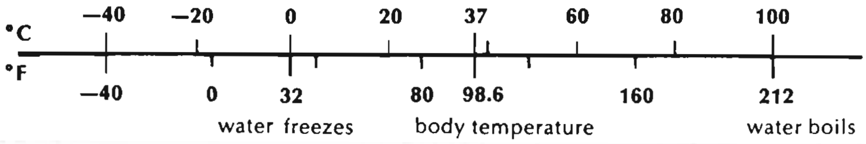
millimeter:	0.001 meter	diameter of paper clip wire
centimeter:	0.01 meter	a little more than the width of a paper clip (about 0.4 inch)
kilometer:	1000 meters	somewhat further than ½ mile (about 0.6 mile)
kilogram:	1000 grams	a little more than 2 pounds (about 2.2 pounds)
milliliter:	0.001 liter	five of them make a teaspoon

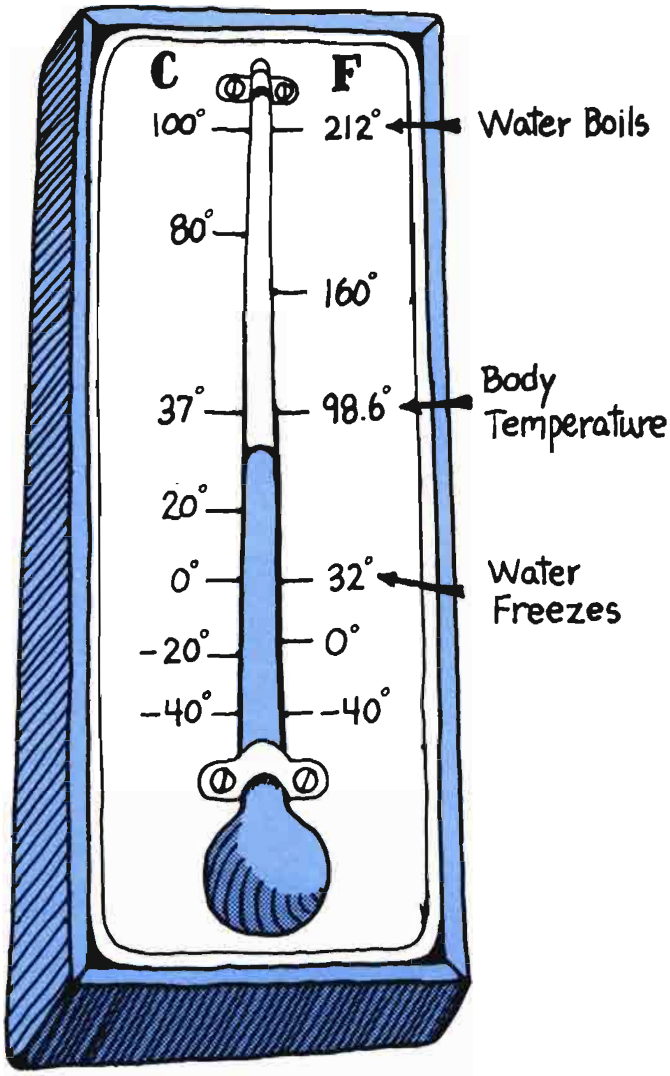
OTHER USEFUL UNITS

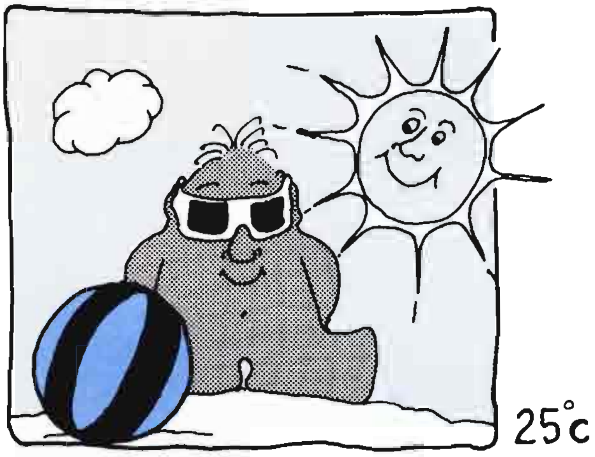
hectare:	about 2½ acres
tonne:	about one ton



TEMPERATURE
degrees Celsius are used

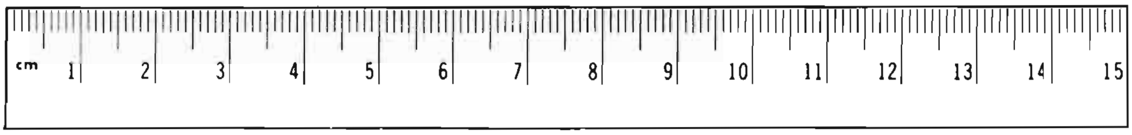




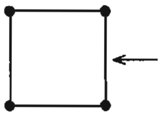


The meter was invented by the French in 1800. They took one ten-millionth of the distance between the North Pole and the Equator and called it the "meter".

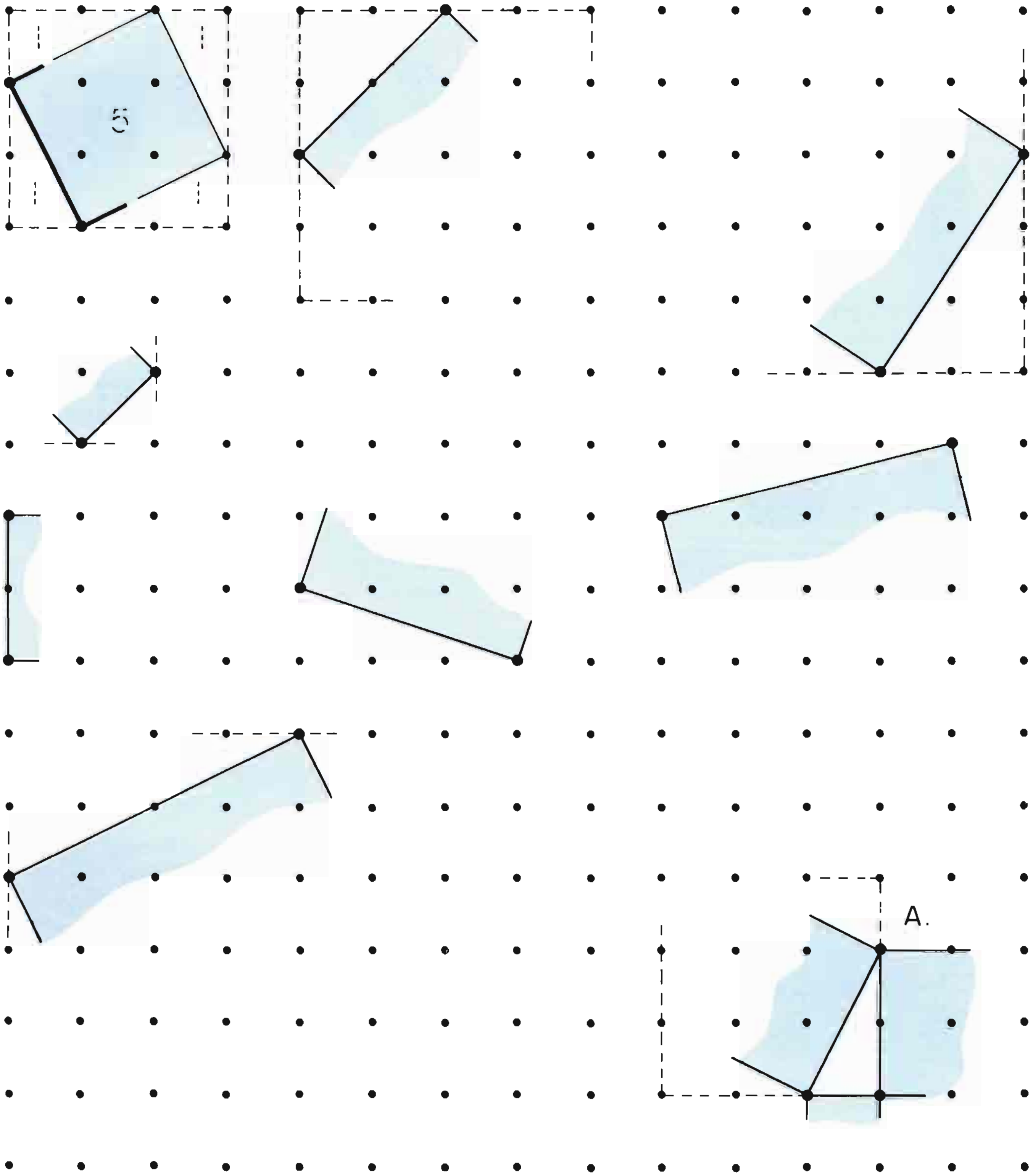
Check the accuracy of the opposite page by comparing the underlined items with paperclips in your classroom.



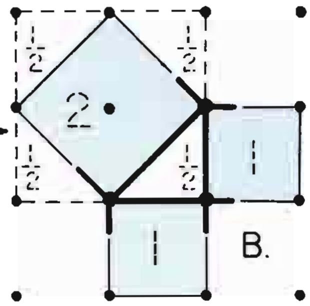
Please draw a square on each line by connecting points. What is the area of the square? It will often help to draw a larger square as shown in the example. Then show the area of each part of the sketch.



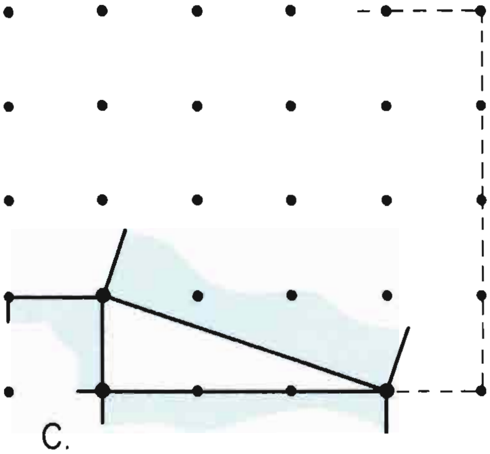
We will call this 1 unit of area for this page.



Here are more examples—They are very much like the last one on page 18. Another example is shown here.



B.



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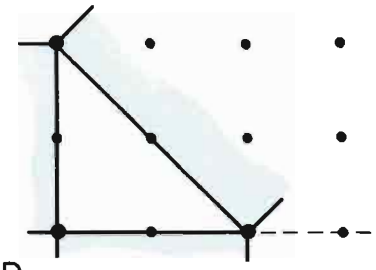

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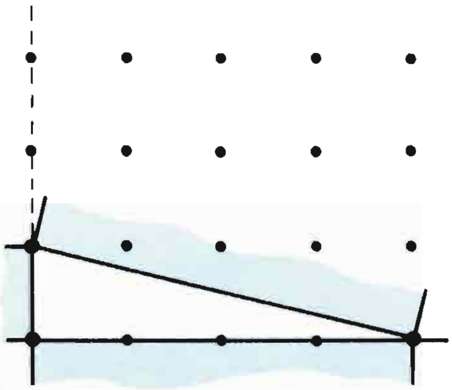

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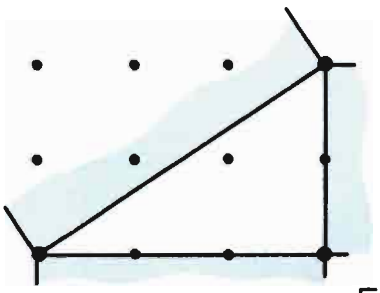
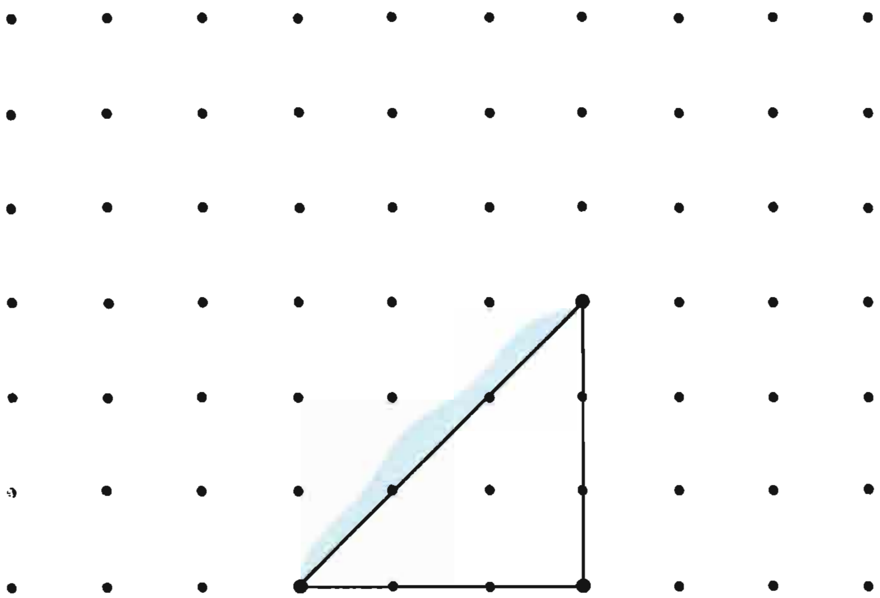


F.

Please try 2 more examples—on your own.

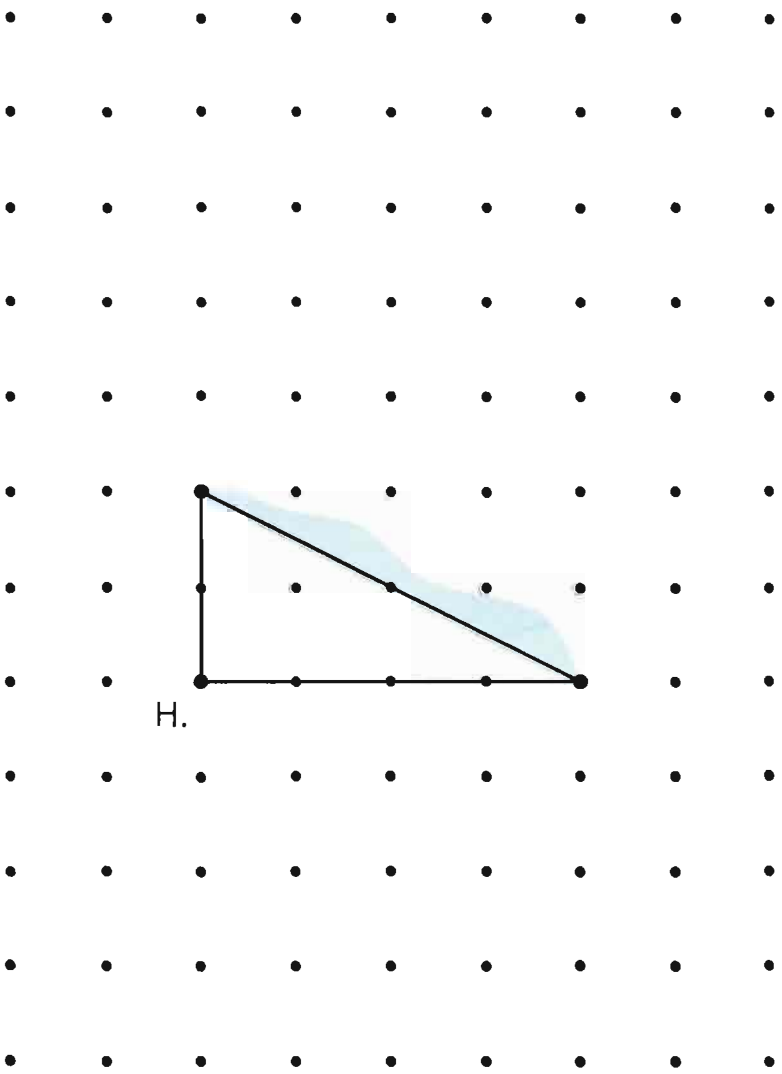


G.

All examples on these pages are "right triangles"— each has a square corner.

Please make a chart of your own results below—listing the area of the square on the longest side of the triangle (called the hypotenuse) in each sketch, and the squares on the other 2 sides.

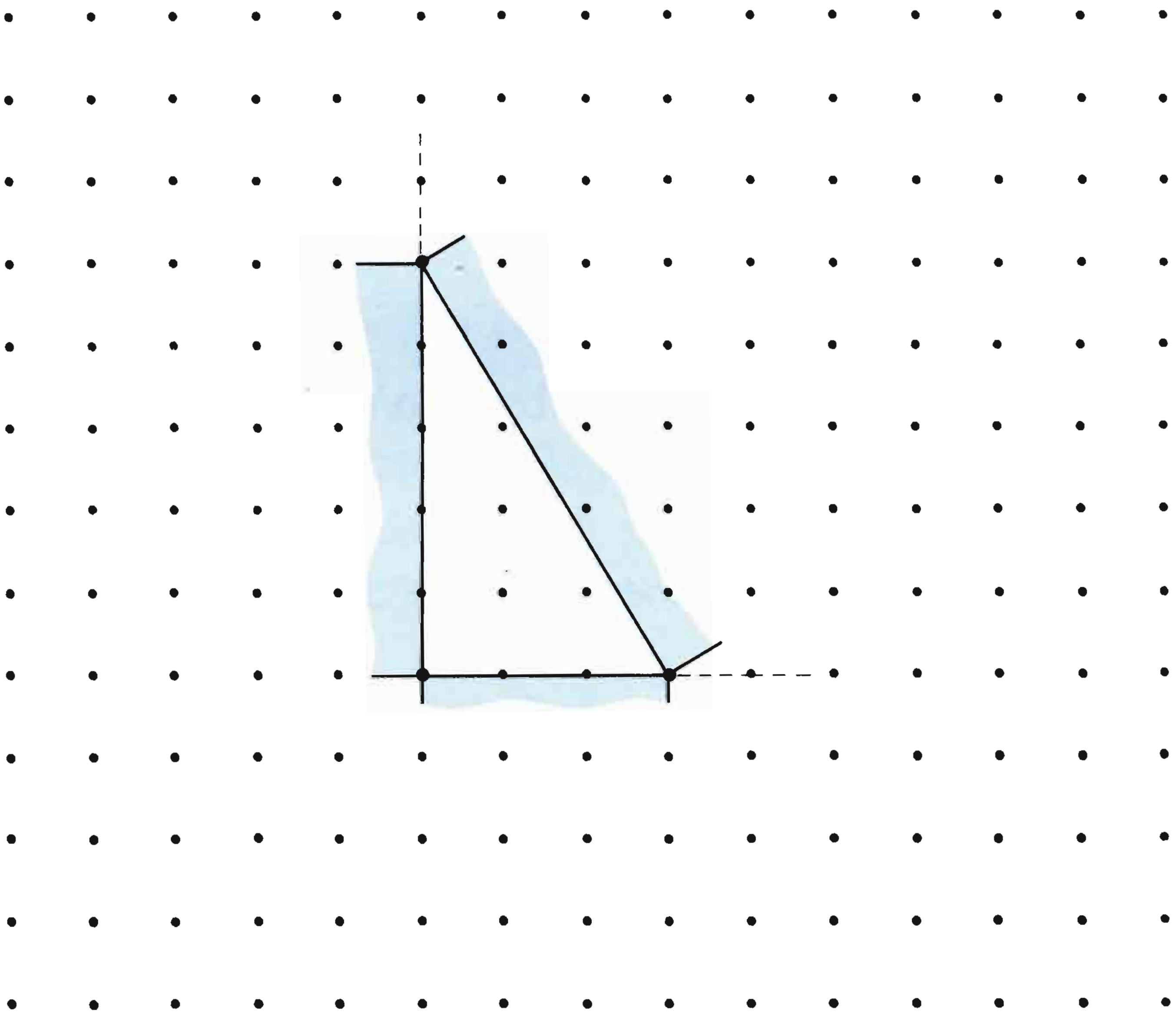
	area of square on longest side ↓	area of squares on other 2 sides ↓	sum of the areas on other 2 sides ↓
A	5	1 , 4	5
B	2	,	
C		,	
D		,	
E		,	
F		,	
G		,	
H		,	



H.

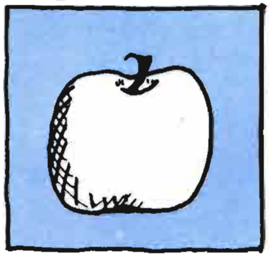
In each right triangle (a triangle with a square corner) we studied, the square on the longest side (called the hypotenuse) equalled the sum of the squares on the other 2 sides.

Please try one more example—a large one. Are the results similar?



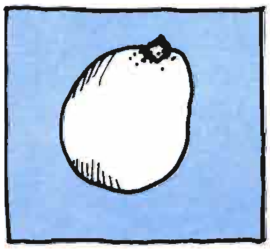
GUESS AND COUNT

An Apple



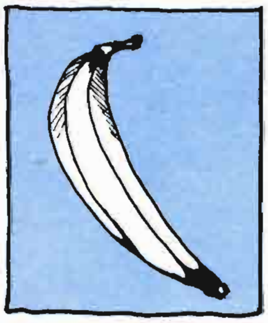
① I guess there are ___ seeds in an apple.
② Please eat it.
③ How many seeds are there ?

An Orange



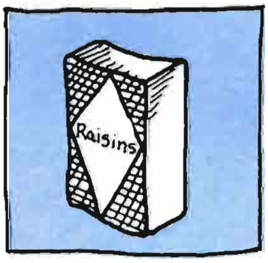
① I guess there are ___ sections in an orange.
② Please peel it and count the sections. ___
③ Please eat it.

A Banana

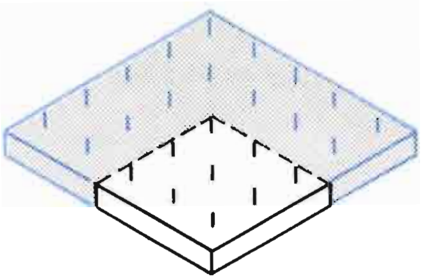


① I guess the banana is ___ cm long.
② Please bite off 1 cm at a time
until you think you are halfway.
③ The banana is now ___ cm long.

A Small Box of Raisins



① I guess there are _____ raisins in a box.
② After counting there are ____ raisins.
③ How close was your guess ? ________



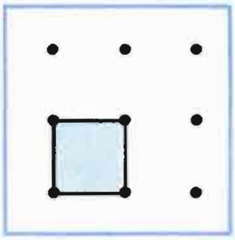
GEOBOARD

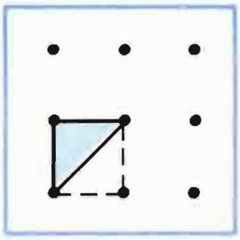
. . . a lot about the corner of a geoboard

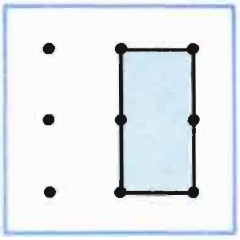
a 3 × 3 arrangement of 9 pegs or pins.

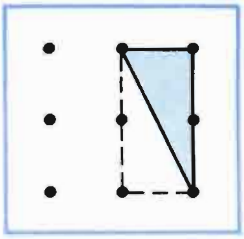
Let's agree that the smallest square 🟦 is 1 unit of area.

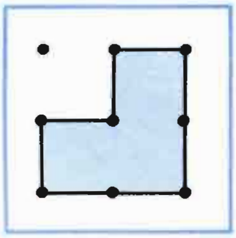
What can you say about the areas of the shapes below?

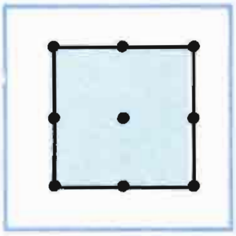


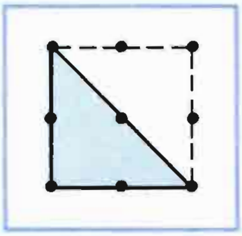


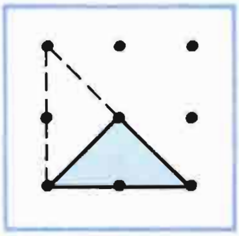


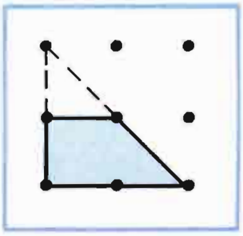


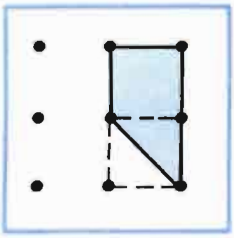


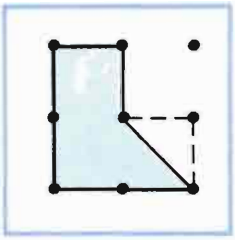


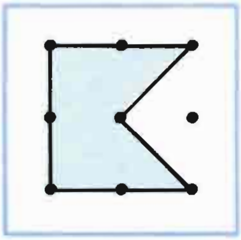


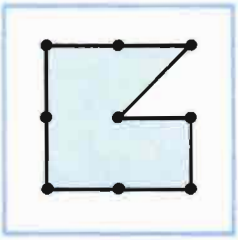


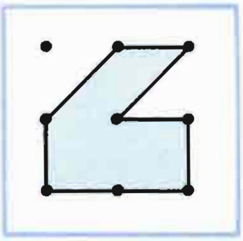


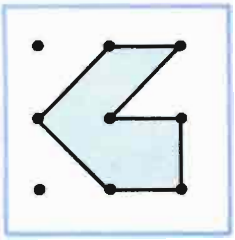


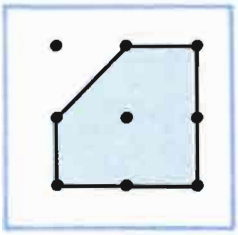


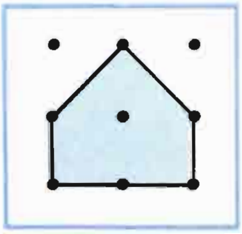


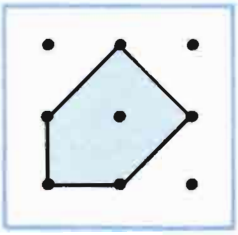


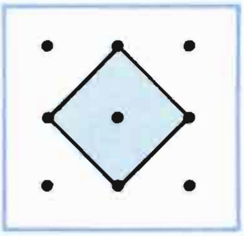


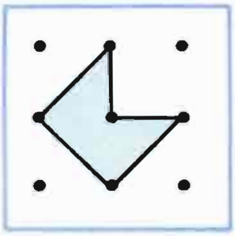








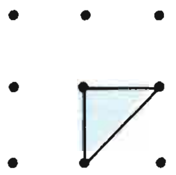




Carl's Claim

In Carl's Claim "different" means if both were cut out they would not fit on top of each other even if one of them is turned over.

Claim 1: "There are only 2 different 3-peggers with no pegs inside and they have the same area . . . if there is a peg inside, the area is 1 more. (There is only 1.)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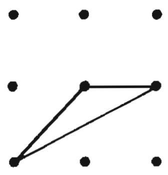
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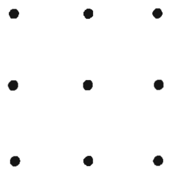
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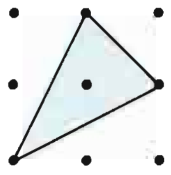
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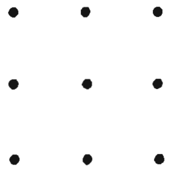
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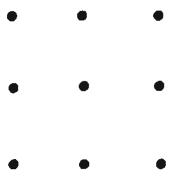
Claim 2: "There are only 6 different 4-peggers with no pegs inside and all have the same area . . . when there is a peg inside, the area is 1 more. (There are 3.)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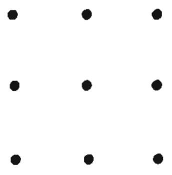
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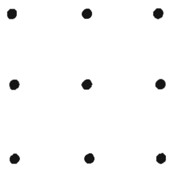
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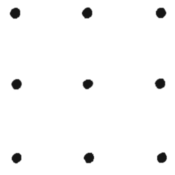
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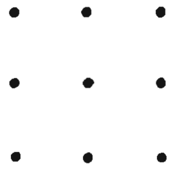
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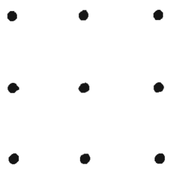
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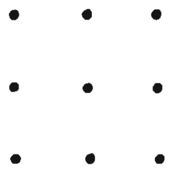
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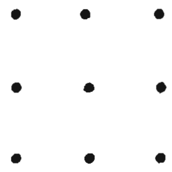
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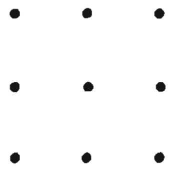
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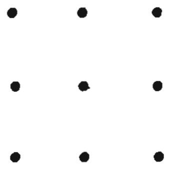
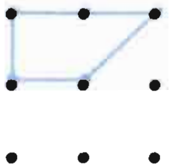

___( )

Claim 3: "There are 10 different 5-peggers with no pegs inside . . . and they all have the same area . . . When there is a peg inside, the area is 1 more. (There are 3.)

Five peggers:

DO YOU AGREE WITH CARL'S CLAIMS?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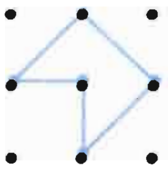
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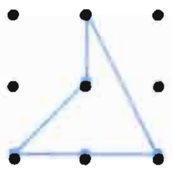
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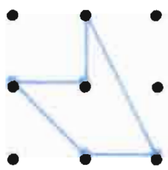
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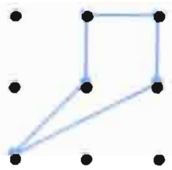
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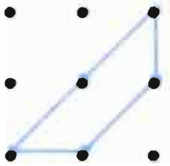
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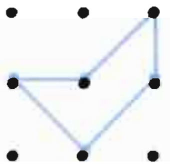
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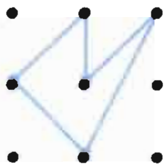
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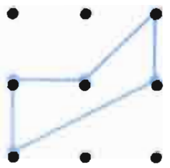
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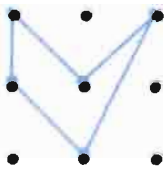
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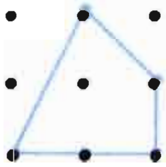
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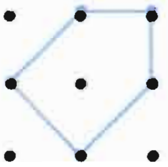
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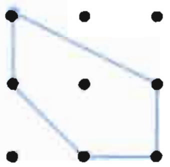
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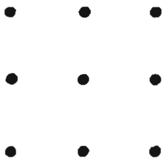
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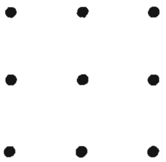
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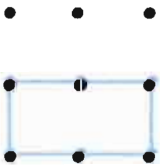
Six peggers: There are more than 10 with no peg inside.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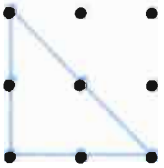
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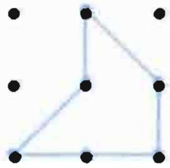
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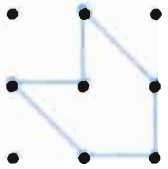
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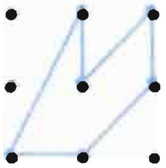
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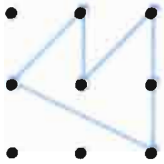
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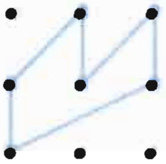
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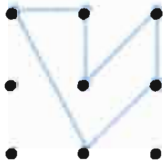
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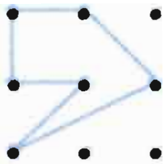
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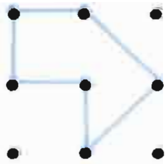
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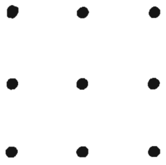
More investigating on your own.



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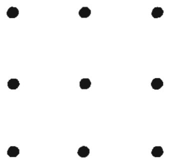

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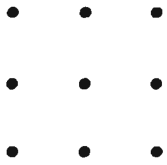

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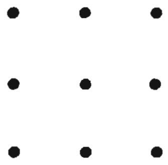

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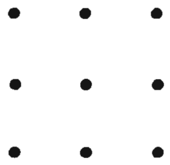

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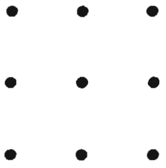

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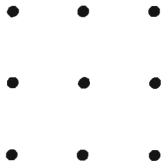

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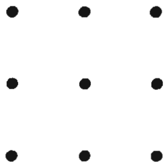

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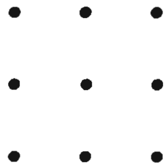

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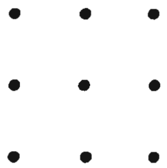

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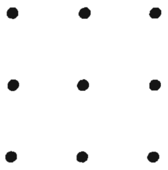

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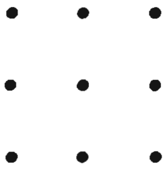

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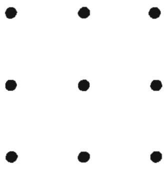

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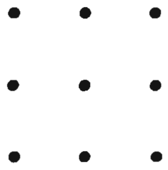

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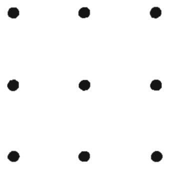

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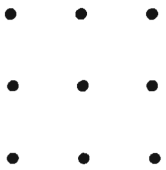

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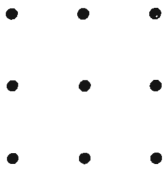

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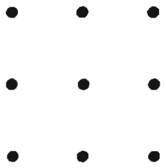

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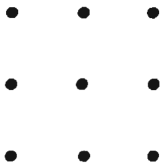
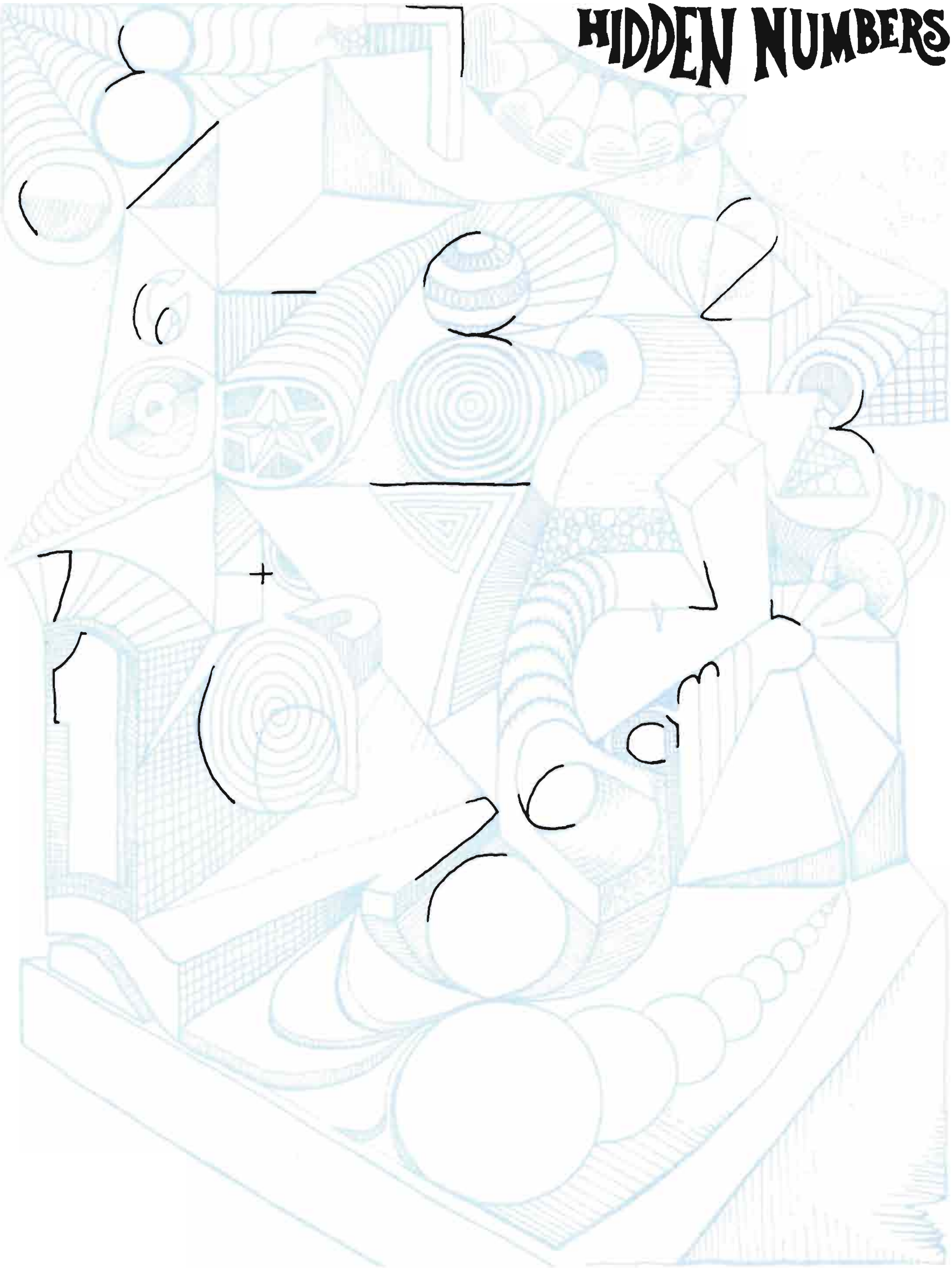


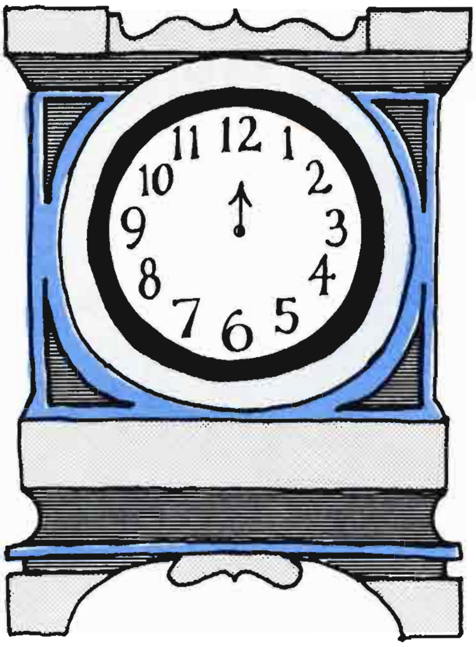
____(   )

Here is part of Carl's record. Please complete it as your own.

	No pegs inside		1 Peg inside	
	Number Diff.	Area	Number Diff.	Area
3 peggers	2	$\frac{1}{2}$	1	$1\frac{1}{2}$
4 peggers				
5 peggers				
6 peggers				
7 peggers				
8 peggers				
9 peggers			☒	

HIDDEN NUMBERS





TIME PASSES

HERE IS A CLOCK WITH ONLY AN HOUR HAND. (SMALL HAND)

EVERY TIME AN HOUR PASSES THE HAND MOVES CLOCKWISE TO THE NEXT NUMBER.

IF THE CLOCK STARTS ON 12 AND 8 HOURS PASS, WHAT NUMBER WILL THE HAND BE POINTING TO?

____

IF THE CLOCK STARTS ON 12 AND 20 HOURS PASS, WHAT NUMBER WILL THE HAND BE POINTING TO?

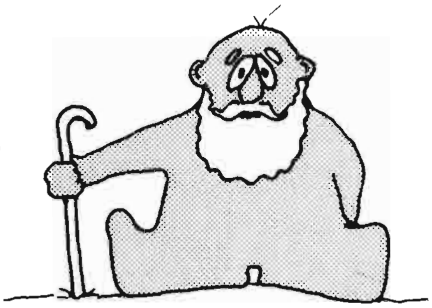
____

IF THE CLOCK STARTS ON 12 AND 7,464 HOURS PASS, WHAT NUMBER WILL THE HAND BE POINTING TO?

____

HOW MUCH OLDER WOULD YOU BE AFTER 7,464 HOURS?

____



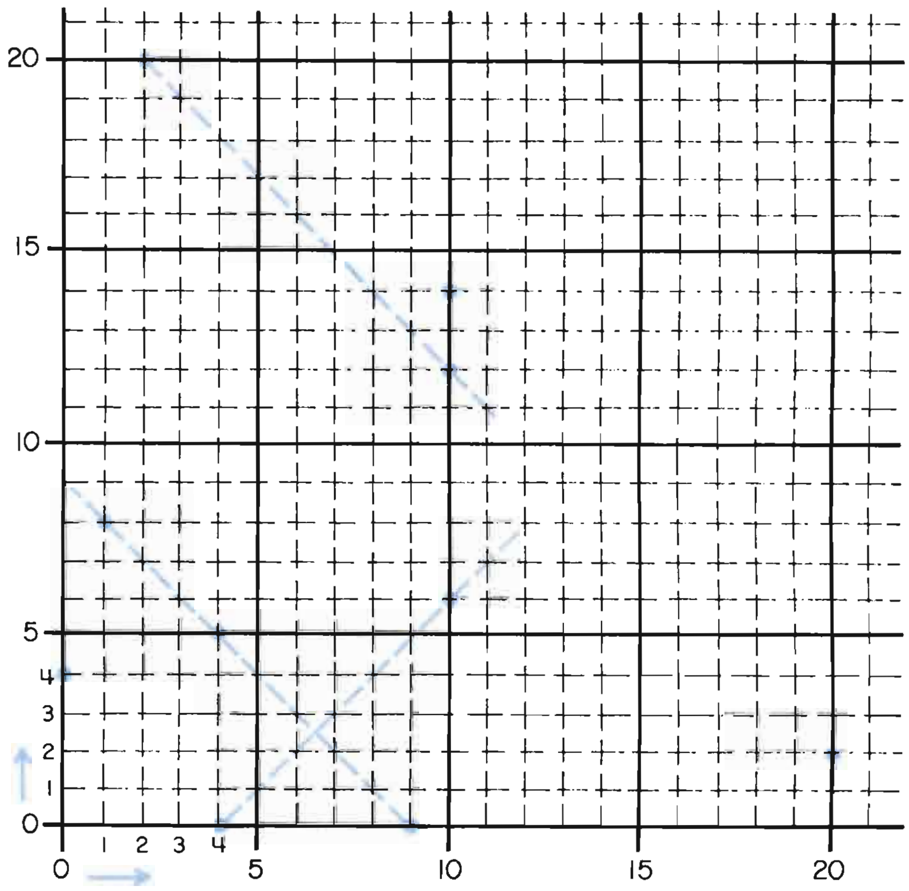
What's My Rule?

n	9-n
4	5
9	0
1	8
7	
3	
5	

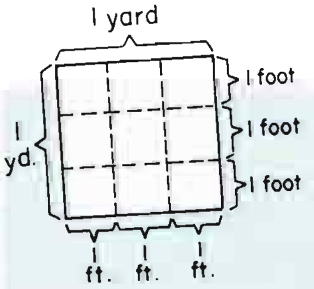
n	22-n
2	20
20	2
10	12
8	
15	

n	n+4
10	14
0	4
15	
6	

n	n-4
10	6
4	0
20	



There are _____square feet in 1 square yard.



square yards	1	3	4	2	6	8	9	7
square feet								

sq. yd.	5	10	20	30	40	30	1	29	20	2	18
sq. ft.											

sq. yd.	30	15	44	4	40	20	5	15	14	13	12
sq. ft.											

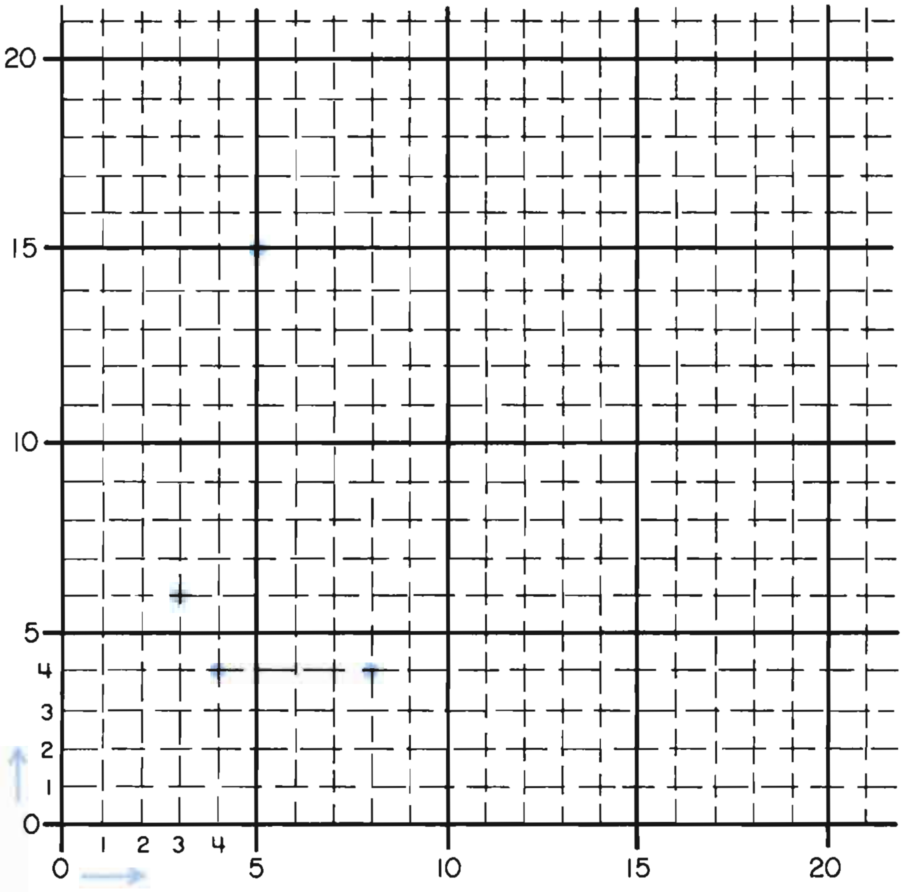
What's My Rule?

a	a
4	4
10	10
0	
17	
7	

a	2a
3	6
5	
0	
9	

a	3a
5	15
1	
3	
0	
6	

a	$\frac{1}{2}a$
8	4
6	
0	
20	



From the List! . . .

all numbers must be from the list, and no numbers may be repeated in any one statement.

2, 3, 5, 6, 8, 10, 13, 15, 17, 21, 24, 32

____ + ____ = ____

____ − ____ = ____

____ x ____ = ____

____ ÷ ____ = ____

____ + ____ = ____

____ − ____ = ____

____ x ____ = ____

____ ÷ ____ = ____

Please make up some of your own.

________ = ____

________ = ____

________ = ____

________ = ____

________ = ____

________ = ____

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36

____ + ____ = ____

____ − ____ = ____

____ x ____ = ____

____ ÷ ____ = ____

____ + ____ = ____

____ − ____ = ____

____ x ____ = ____

____ ÷ ____ = ____

Please make up some of your own.

________ = ____

________ = ____

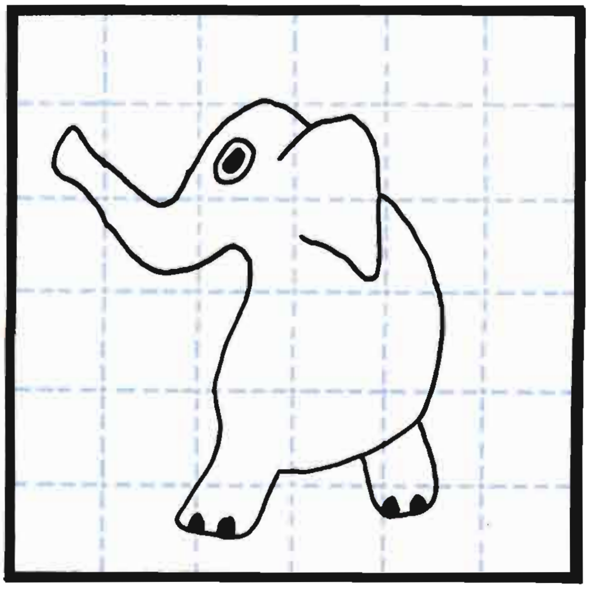
________ = ____

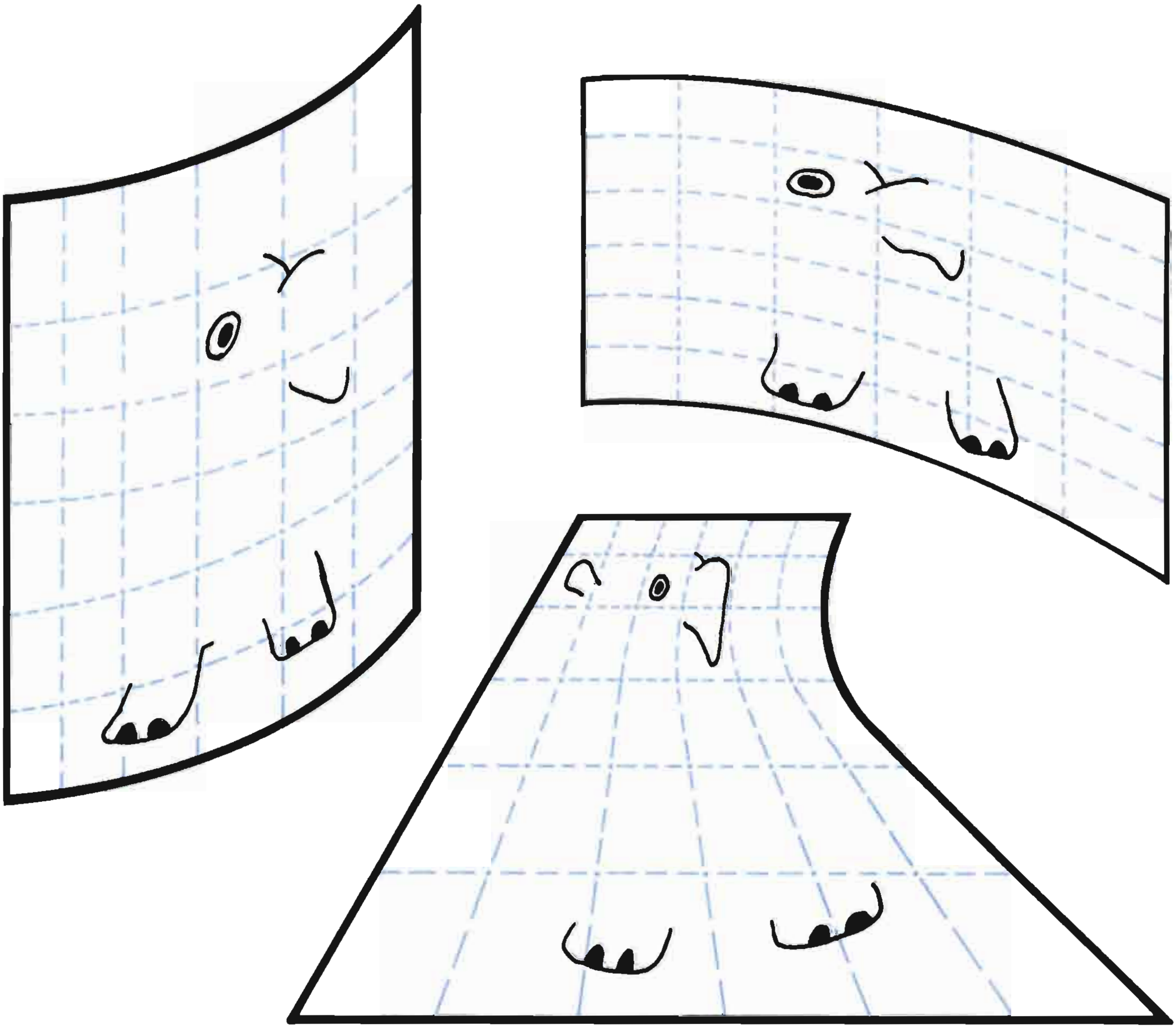
________ = ____

________ = ____

________ = ____

CHANGING SHAPES

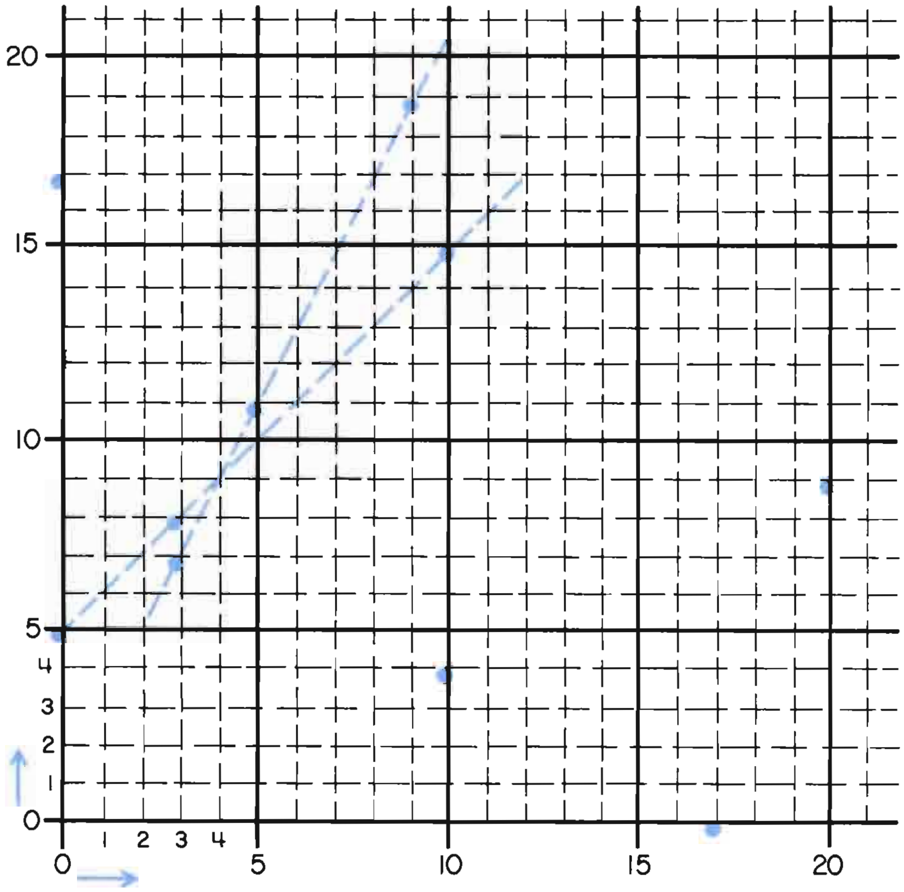




Please complete the tables and locate the points indicated. Then connect the points given by each table with a straight line.

→ x	↑ $x+5$
3	8
0	5
10	15
7	
5	
15	

→ x	↑ $2x+1$
3	7
5	11
9	19
7	
0	



→ x	↑ $\frac{1}{2}x-1$
10	4
20	9
2	
8	
18	

→ x	↑ $17-x$
17	0
0	17
10	
3	

From the List! . . . all numbers must be from the list, and no numbers may be repeated in any one statement.

3, 4, 6, 8, 9, 12, 15, 16, 18, 20, 21, 24

Please make up your own list.

____ + ____ = ____ ____ + ____ = ____ ____ + ____ = ____ ____ + ____ = ____

____ − ____ = ____ ____ − ____ = ____ ____ − ____ = ____ ____ − ____ = ____

____ x ____ = ____ ____ x ____ = ____ ____ x ____ = ____ ____ x ____ = ____

____ ÷ ____ = ____ ____ ÷ ____ = ____ ____ ÷ ____ = ____ ____ ÷ ____ = ____

Please make up some of your own. Please make up some of your own.

________ = ____ ________ = ____ ________ = ____ ________ = ____

________ = ____ ________ = ____ ________ = ____ ________ = ____

________ = ____ ________ = ____ ________ = ____ ________ = ____

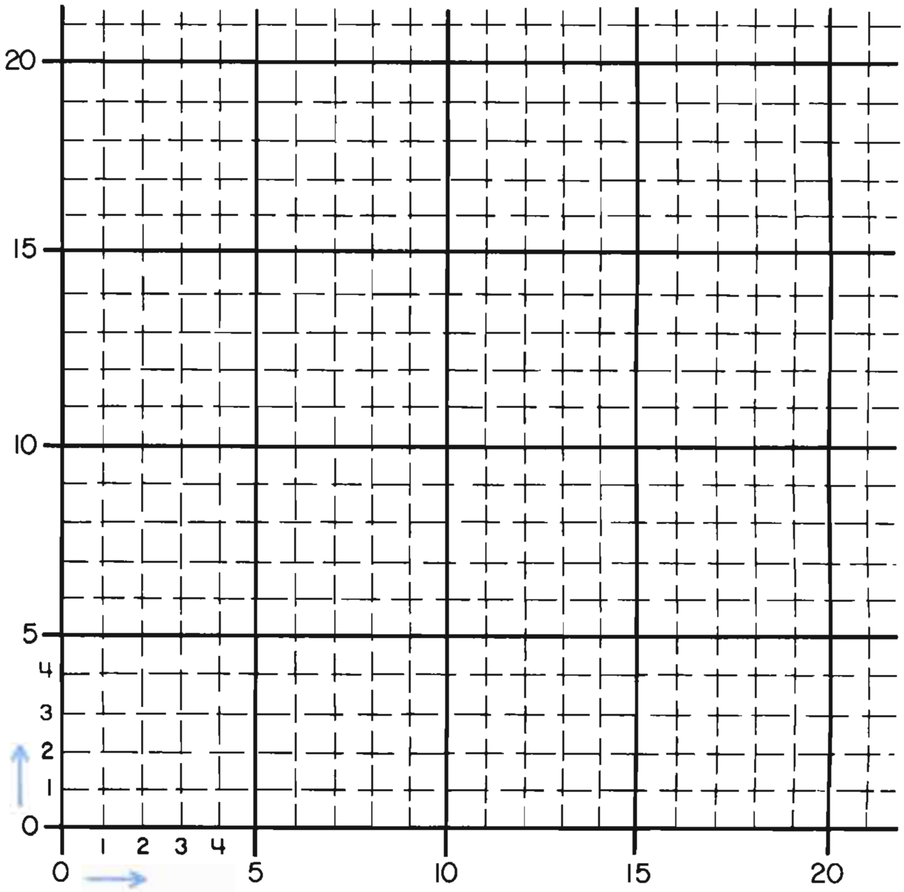
Please make up your own rules, and tables and graphs.

→	↑

→	↑

→	↑

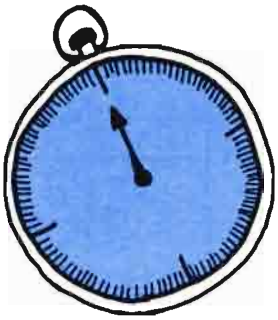
→	↑



weeks	0	1	2	3	4	5	6	7
days	0	7						

weeks	5	10	4	8	9	10	20	30	7	10	17
days											

weeks	6	12	4	30	34	23	3	20	8	30	38
days											



Time Yourself

Out on the playground, mark off 30 meters and pick a partner to time you.

Walk as fast as you can.

1st try ______ seconds

2nd " ______ "

3rd " ______ "

Run with your hands tied.

1st try ______ seconds

2nd " ______ "

3rd " ______ "

Now untie them and run.

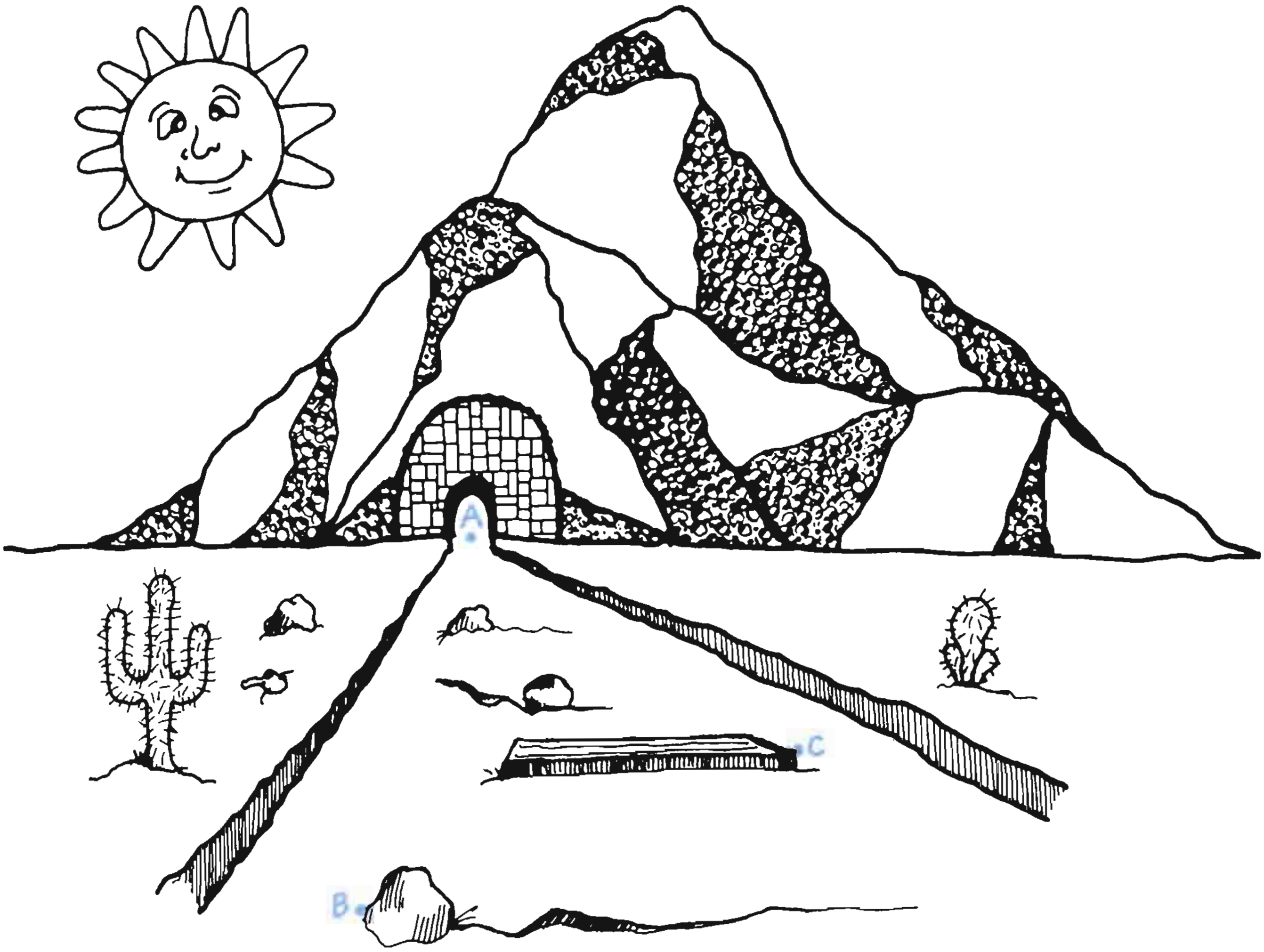
1st try ______ seconds

2nd " ______ "

3rd " ______ "

What happened when you untied your hands? ______________________________

OPTICAL ILLUSION



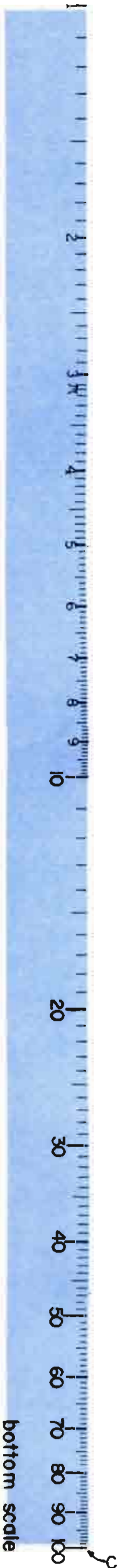
Draw a line from Ⓐ to Ⓑ.

Draw a line from Ⓐ to Ⓒ.

Which line looks longer?

Now measure the 2 lines.

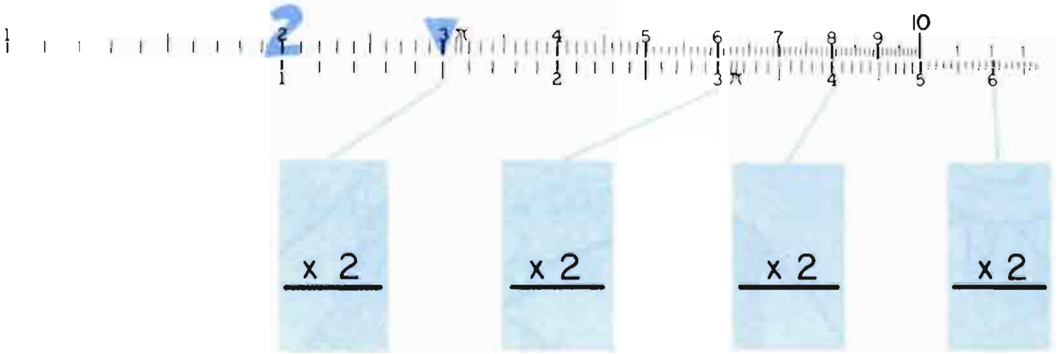
What did you discover?

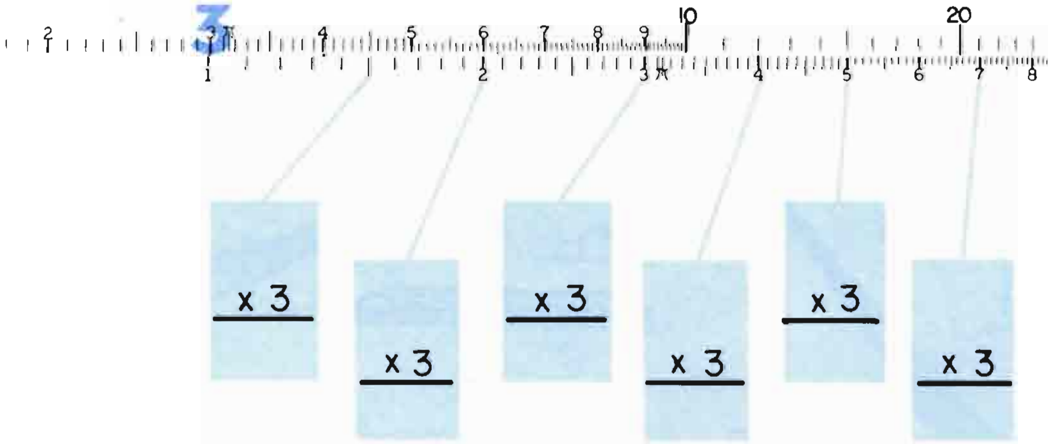


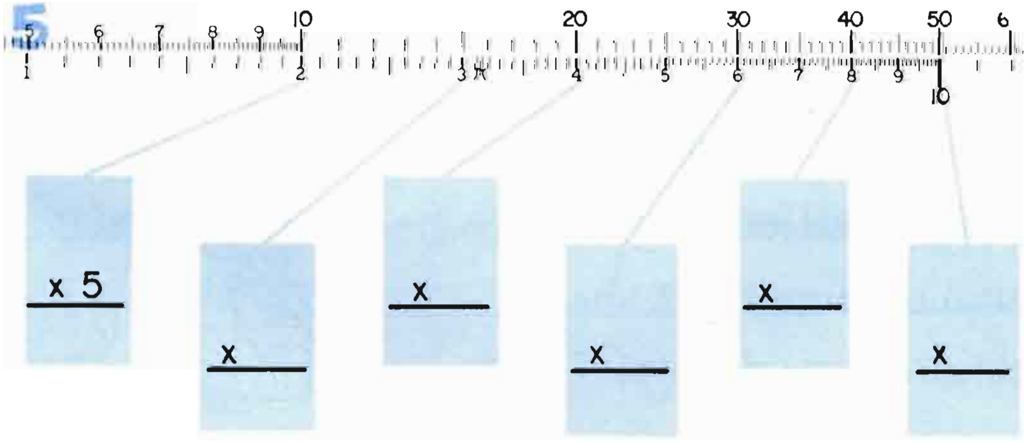
X and ÷

After completing this page cut out the scales.

You may use them as a portable "slide rule."







cut on this line

Make up your own ....

4

X 4 ______

X ______

X ______

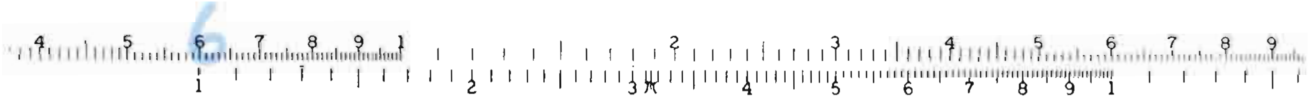
÷ ______

÷ ______

______

______

6



X 6 ______

÷ ______

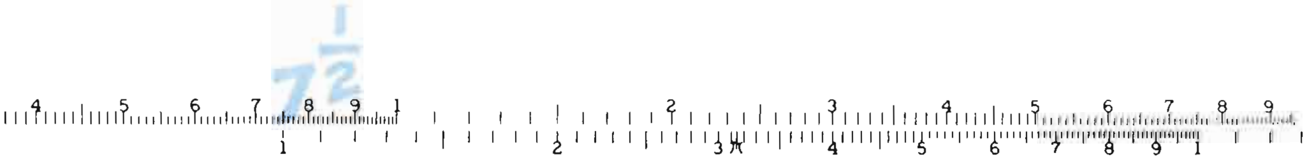
X ______

÷ ______

______

______

$7\frac{1}{2}$



X $7\frac{1}{2}$ ______

X ______

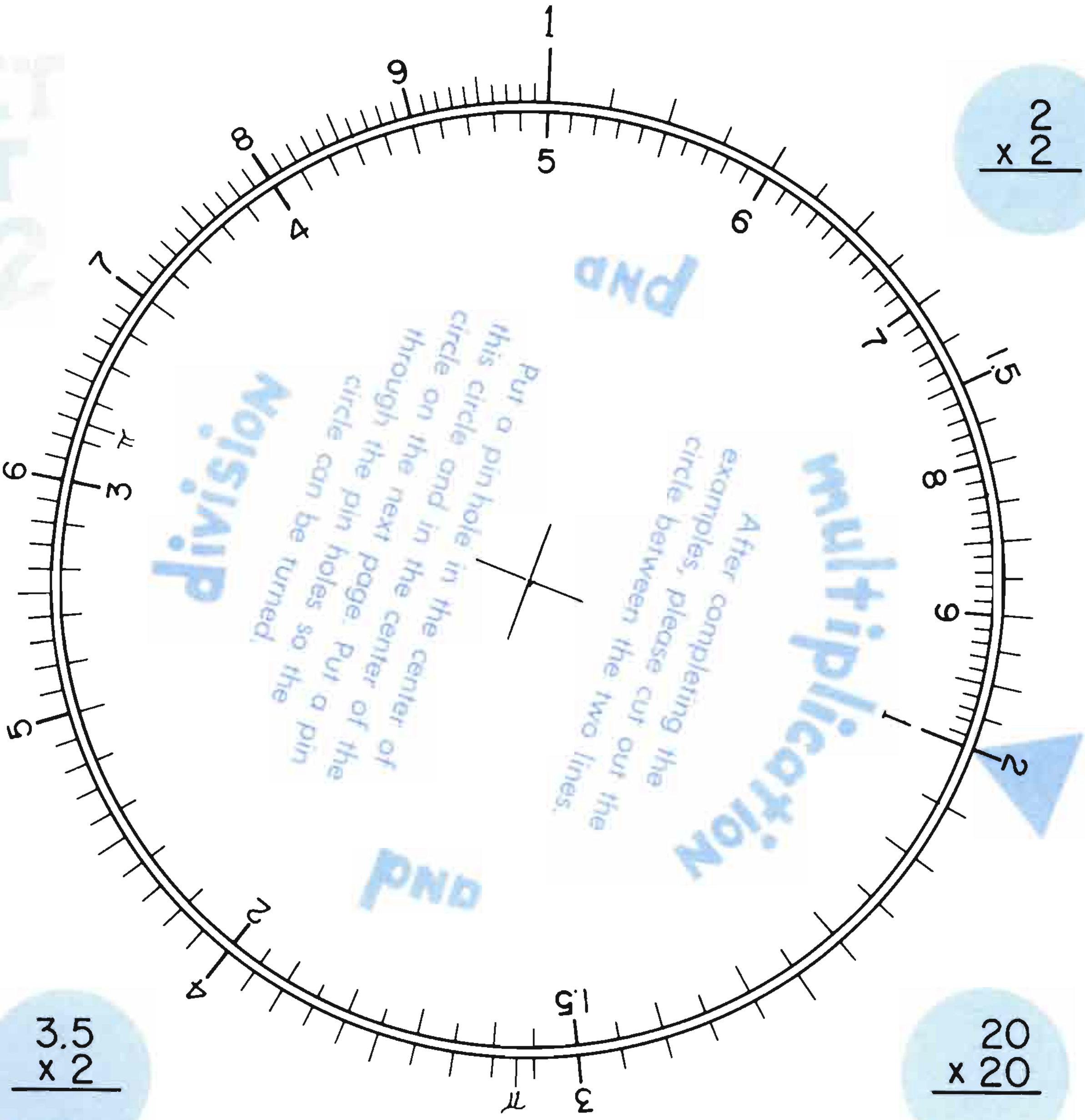
÷ ______

______

______

× and ÷

SET AT ×2 ÷2



After completing the examples, please cut out the circle between the two lines.

Put a pin hole in the center of this circle and in the center of the circle on the next page. Put a pin through the pin holes so the circle can be turned.

$$\begin{array}{r} 2 \\ \times\ 2 \\ \hline \end{array}$$

$$\begin{array}{r} 3.5 \\ \times\ 2 \\ \hline \end{array}$$

$$\begin{array}{r} 20 \\ \times\ 20 \\ \hline \end{array}$$

$$2\overline{)\ 8\ }$$

$$\begin{array}{r} 6 \\ \times\ 2 \\ \hline \end{array}$$

$$2\overline{)\ 9\ }$$

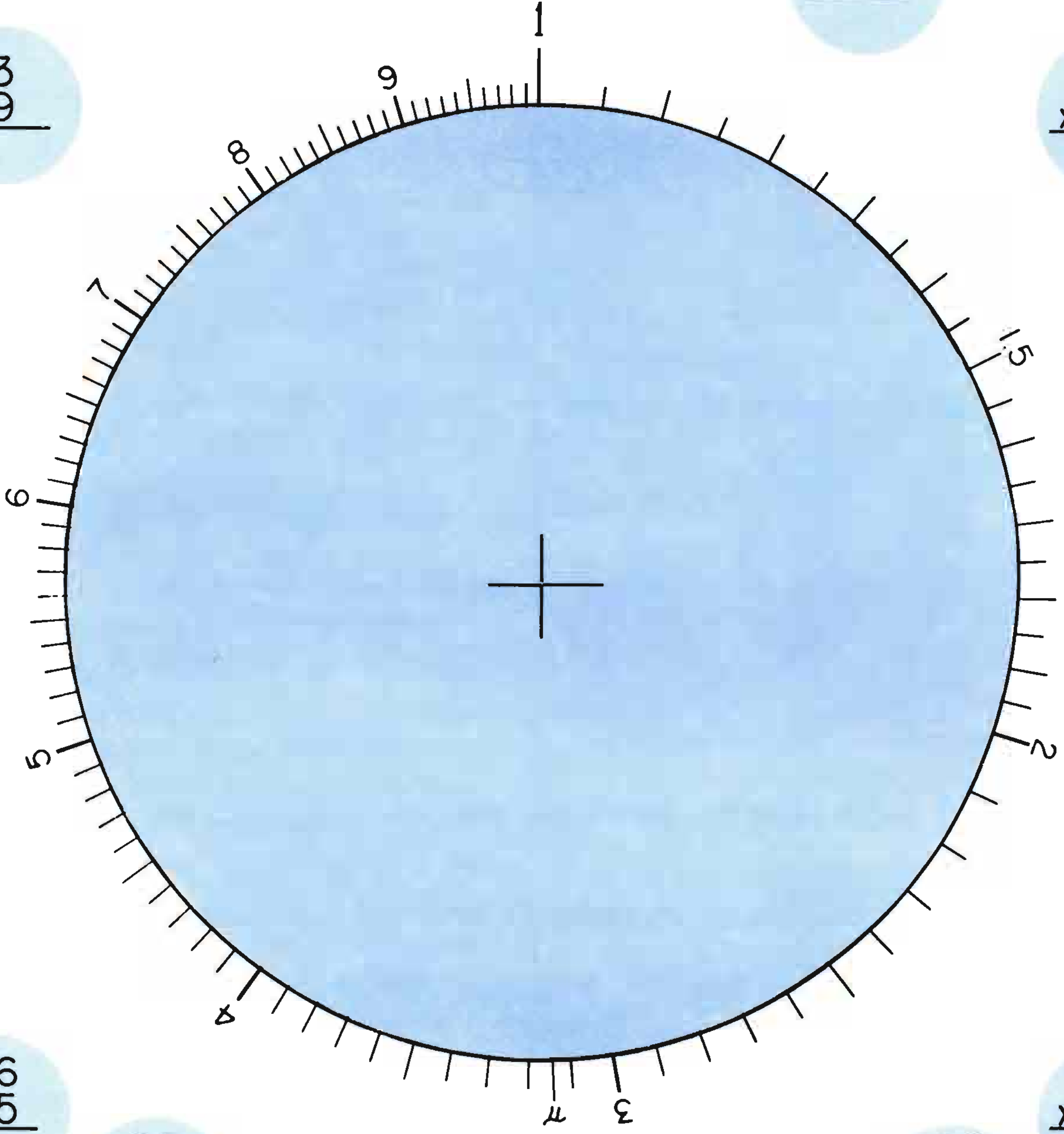
8
x 5

9
x 6

4
x 7

3
x 9

15
x 12



6
x 1.5

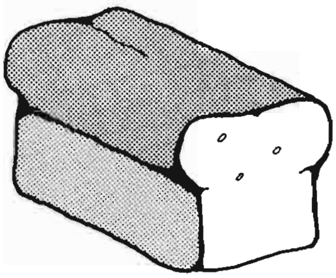
25
x 36

5
x 1.7

30
x 45

24 ⟌ 600

MEASURING MY PEANUT BUTTER AND JELLY SANDWICH



__________ oz. __________ oz. __________ oz.

Two slices of bread = ______ oz.

The perimeter of one slice = ______ cm

1 Tablespoon of jelly = ________ oz.

2 Tablespoons of peanut butter = ______ oz.

My peanut butter & jelly sandwich weighs ______ oz.

Please weigh different ingredients and sandwiches ...
What other discoveries can you make?

The longest loaf of bread was baked in New Zealand in 1969. It was 100 feet long.
How many meters is that? ________

TOP SECRET

Maria found a small piece of paper on her desk. It read:

ICTTRNAEPUEEHLMM

She read the message from her friend Priscilla, and wrote an answer.

FNLIOOLSOEIERBWY

As soon as she could, Maria passed the answer to her friend.

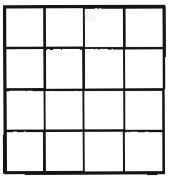
Later, Henry found this answer and spent all day trying to find out what it said.

Both Priscilla and Maria laughed about Henry because they knew he could never break their code.

Henry finally gave up. He offered the girls a trade!

"If you tell me about your code I promise never to steal your notes again."

Maria and Priscilla liked Henry and, after a long discussion gave in. They explained that the real secret was this:



"I don't get it," Henry admitted.

"If you will write a note with not more than 16 letters," Maria offered, "I'll put it in code for you."

Henry agreed. "Here is my message."

I WANT TO KNOW MORE

Maria wrote that in code.

ZMKNEWOAROTWUNTI

"I still don't get it," Henry complained.

When Maria had teased Henry enough, she showed him how she had first written his message.

I	W	A	N
T	T	O	K
N	O	W	M
O	R	E	Z
(4)	(3)	(2)	(1)

"I added a Z to fill out the little squares since you only used 15 letters.

"Then I wrote them from bottom to top in the right-hand column.

ZMKN

"Next I added to the second column from the right:

ZMKN EWOA

"And the same with the third and fourth columns."

Priscilla asked Henry, "Would you like to see the messages Maria and I passed to each other?"

"I sure would," Henry answered.

"Here they are."

ICTTRNAEPUEEHLMM

FNLIOOLSOEIERBWY

When Henry placed these letters in the squares as Maria had showed him, he got these results:

A

			T
			T
			C
		R	I

B

			N
			F

Now, rewriting the message left to right and top row to bottom row, we have —

A MEET ME AT LUNCH PRI

B YES I WILL BE ON ROOF

Maria asked Henry, "Do you want to know what we talked about on the roof?"

"Sure."

"We had to decide who we were going to let in on our code. Would you like to know what we decided?"

"Of course."

"We decided that you should be the first."

"Why me?" Henry asked.

"Because we need your help. Sometimes we want to send messages that are more than 16 letters long. Have you any ideas?"

Maria had another request. "I think Frank is close to solving our code. Could you help us make it harder to figure out?"

Henry's first suggestion was to use a larger grid. "Suppose we have a larger message such as:

'Can you go swimming with me after school?'

"That has 33 letters. Now, a 4 by 8 grid has 32 spaces. We could leave an 'm' out of swim'ing so it would fit like this (please complete it).

C	A	N	Y	O	U	G	O
							W
							L

Start here ↑

So, the coded message is

LTWOOFGGOANUHEIO

CMMY

(Please finish the message)

Priscilla wondered how the receiver could know what size grid to use. "Do we always have to use a 4 x 8?"

"How could we handle messages with more or less than 32 letters?" Maria asked.

Henry had an idea about shorter messages.

"We can fill up as many squares as we need to with 'Q' and 'Z' and then ignore them when we decode."

Several days later, Priscilla asked Maria and Henry to meet her on the roof. She was excited.

When they got together, Priscilla was anxious to tell the others her plan.

"We can include numbers in the code to tell the size of the grid and where to start and a 'Z' for Zig-Zag."

"What's a Zig-Zag? Show me what you mean?"

Priscilla already had a simple message.

OACTEI6SR(3)TE(Z)CE
AMEH5STTTEELERM

"You will notice 4 new strange parts of the message:

6, (3), (Z), 5

"The two numbers not in parentheses tell you the size of the grid . . . 5 by 6 (the small number) means 'rows', and the large the number in each row .

"The number in parentheses — (3) signals the starting point, and 'Z' means Zig-Zag.

"Let's decode it."

					C
E			E	A	
(Z)		T	O		
(6)	(5)	(4)	(3)	(2)	(1)

"The numbers in parentheses are starting points . . . (3) in this case.

"The 'Z' means to follow a pattern along a diagonal — up and to the right . . . as I have showed with 'O, A and C.'

"Then move to the square above (4) and move up the diagonal. After the diagonal starting above (6), keep moving up to fill in that corner.

"Then start above (1) and finally up the diagonal starting above (2).

"Can you read the message?"

Maria and Henry went to work. Later that day they gave Priscilla the coded message.

RSE4GANKXE(Z)NEBRI
RIUH8OTYEWTAAE(4)W

Priscilla wrote 4, 8, (4), (Z) and drew the grid below.

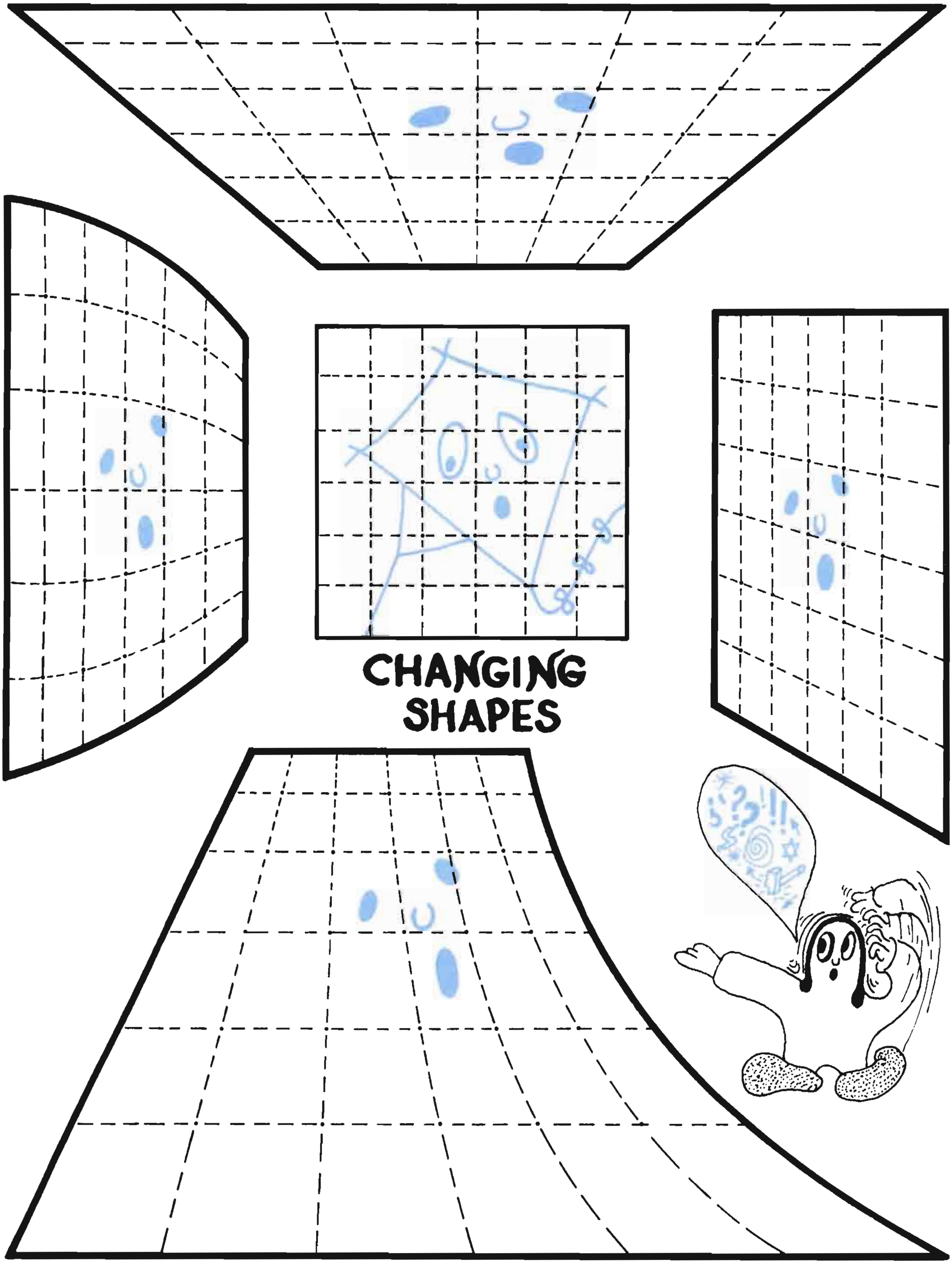
Can you unscramble the message?

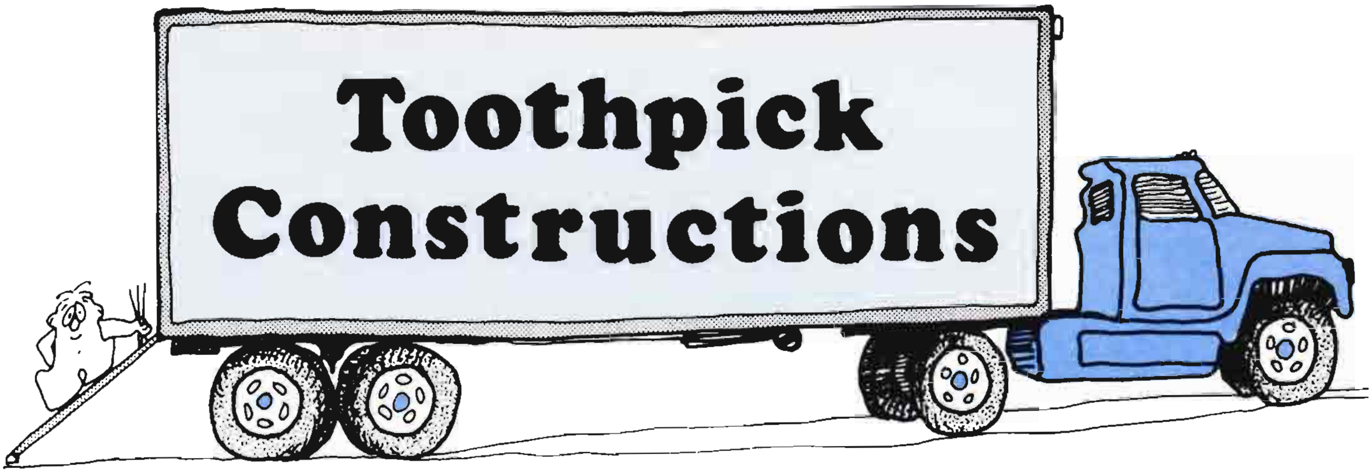
				R			
				(4)	(3)	(2)	(1)

In a few minutes, Priscilla smiled and told her friends "thanks."

"Of course, if there is no (Z) in the message, you work straight up each column and from the starting column to the left."

CHANGING SHAPES





Toothpick Constructions

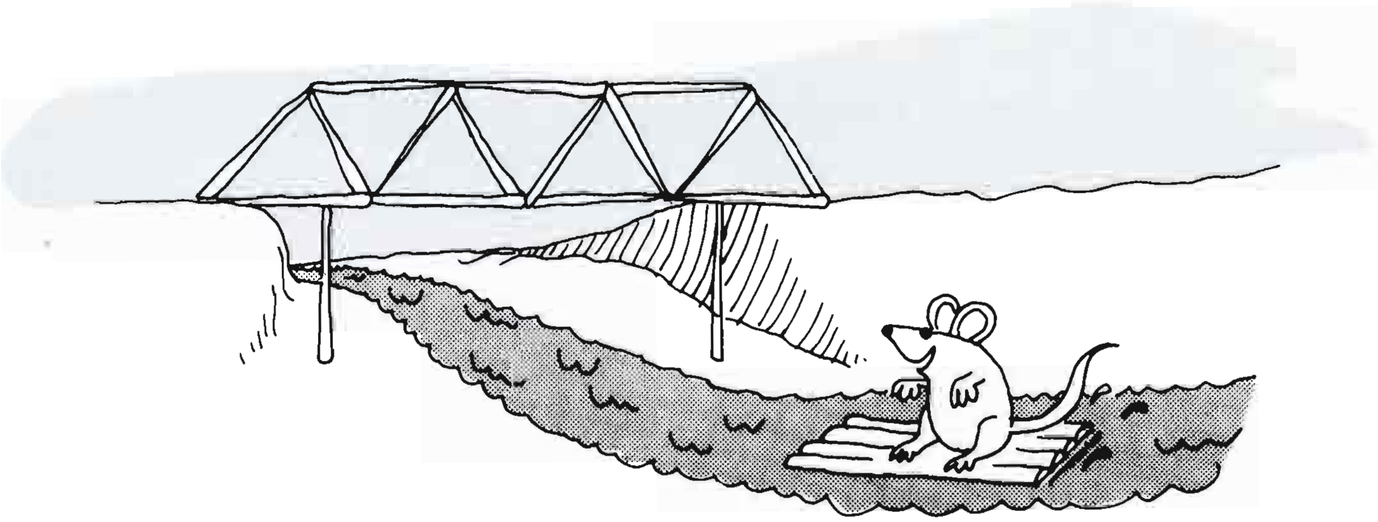
Form a group with 2 or 3 people, get some toothpicks and glue —

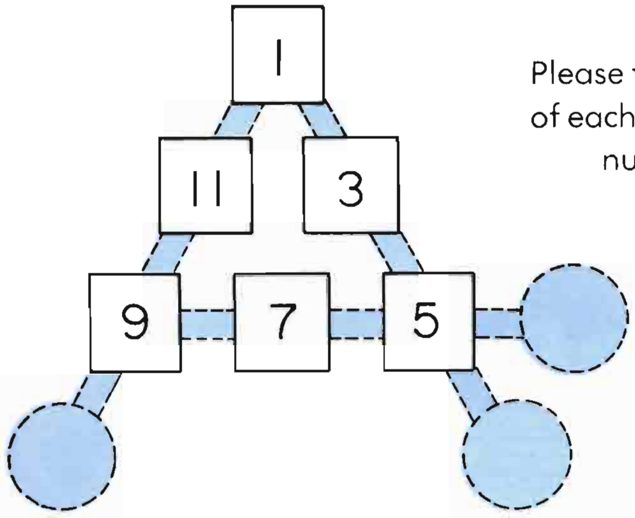
With the fewest toothpicks possible make a construction that will hold a regular brick about 2 inches above a surface.

With no more than 100 toothpicks build a construction at least 4 inches high that will hold a regular brick.

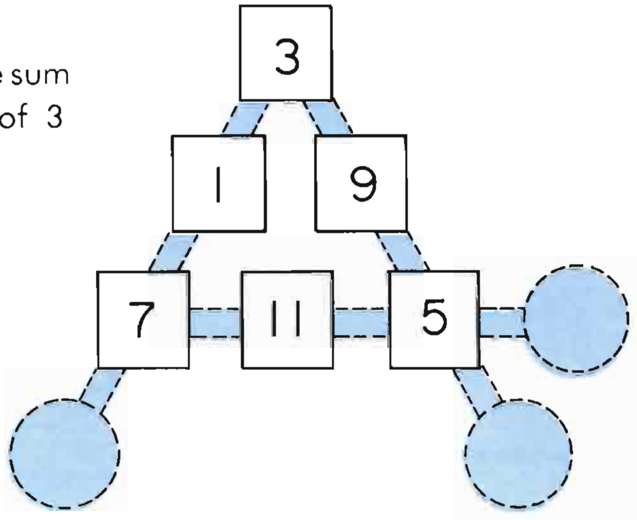
Build two constructions using 50 toothpicks in each. In one tower use only rectangles or squares; in the other use only triangles. Which is stronger?

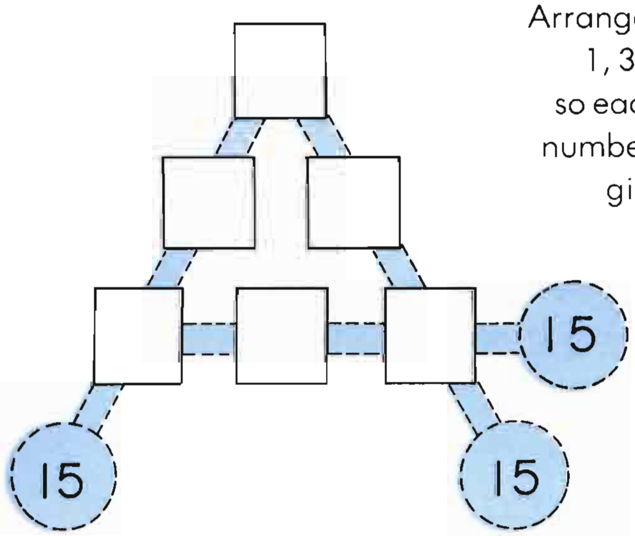
With no more than 100 toothpicks build a bridge to span an 8-inch river.



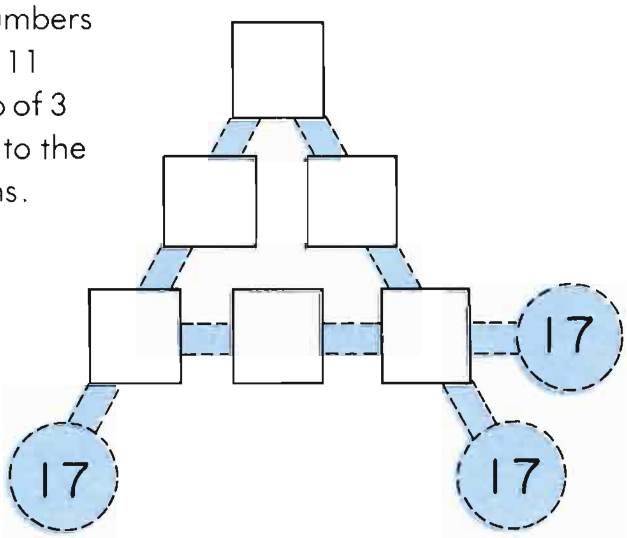


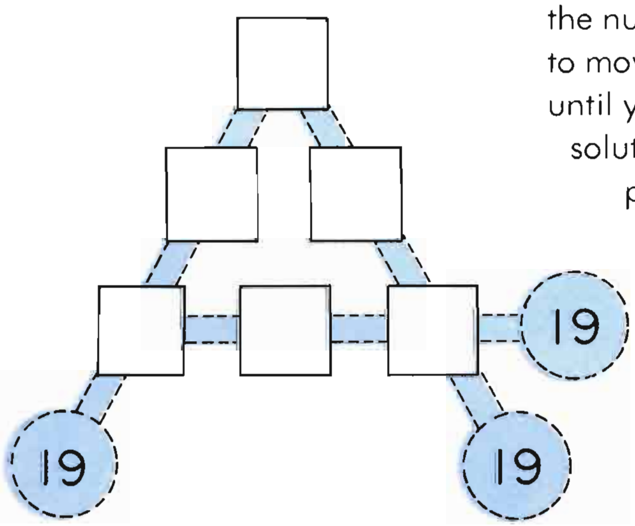
Please find the sum
of each group of 3
numbers.



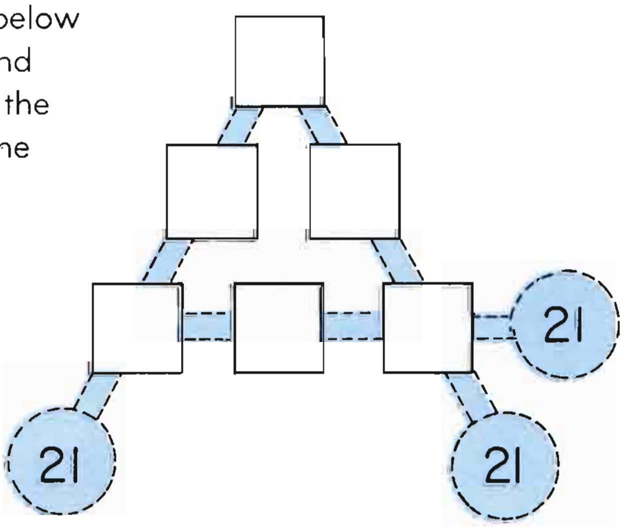


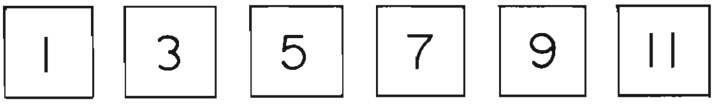
Arrange the numbers
1, 3, 5, 7, 9, 11
so each group of 3
numbers adds to the
given sums.





You may cut out
the numbers below
to move around
until you find the
solution to the
puzzle.





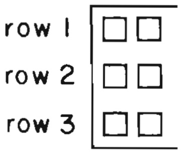
mrs. burg's left-over problem

"This is my mother," Ben told his class. "She is probably the best tile-setter in the world. And she also makes up problems. Mom, will you tell everyone about the problems you have invented?"

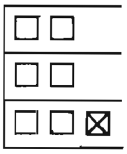
"Sure, Ben, but I don't know whether I'm the best tile-setter in the world: let's just say I don't know anyone who can set them straighter, faster or more evenly.

"But sometimes, when the job gets boring, I think about something else.

"Just the other day, I began wondering about setting rows of tiles. I asked myself: suppose I want to set tiles in even rows, with no tiles left over in any row, for instance:



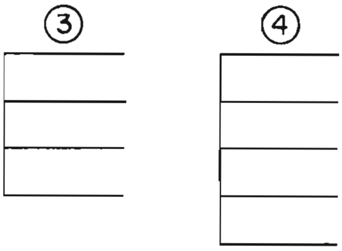
6 tiles, none left over

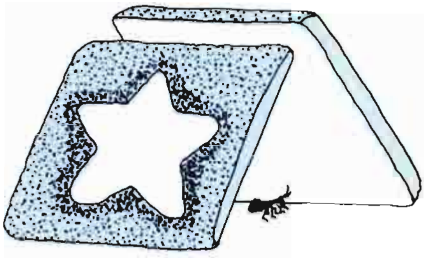


7 tiles, 1 left over

"Now, suppose I want to set two different arrangements of tiles, one with 3 even rows of tiles and the other with 4 even rows of tiles. But, and this is the problem, I want to use the same number of tiles in each of the arrangements. Also, I want to use the smallest number of tiles I can with no left-over tiles in any row in either arrangement.

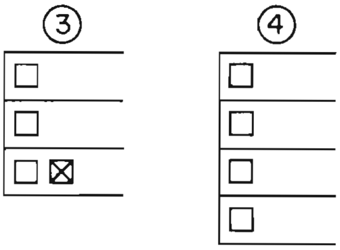
"Here's a sketch we can start with:



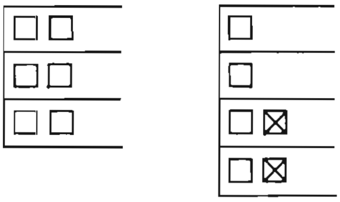


"You'll need at least 4 tiles to get the (4) row arrangement started," said Ruth.

"That's right," said Mrs. Burg. "With 4 tiles our arrangement would be":

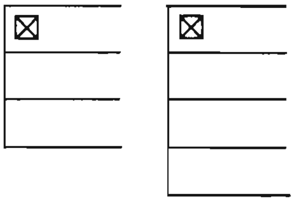


"The (4) row doesn't have any left-overs, but the (3) row needs 2 more tiles, so we'll have to add 2 more to the (3) row, but then the (4) row will have 2 left-overs," said Milt.



"Mrs. Burg, why don't we start from the beginning? That way maybe we'll see a pattern or something," said Tina.

"Good idea, let's start filling in the rows," said Mrs. Burg.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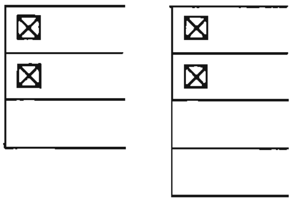
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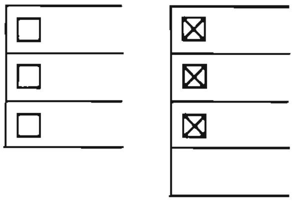
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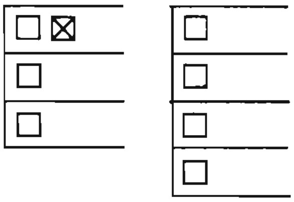
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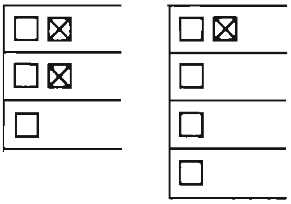
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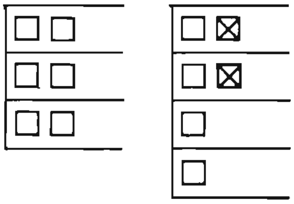
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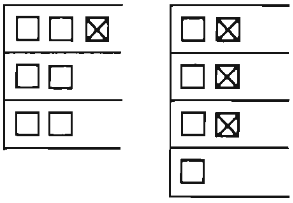
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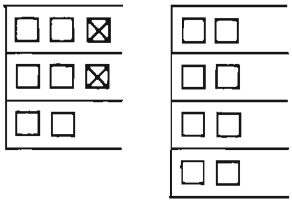
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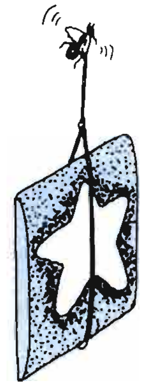

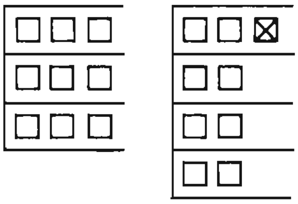
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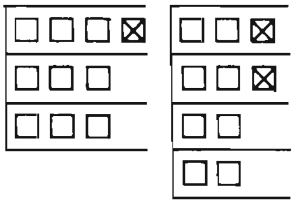
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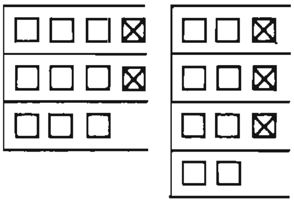

11

"Let's stop right here. Is there anybody who doesn't know what's next?" asked Mrs. Burg.

What's Next?

"That's easy. One more tile in each arrangement will do it. No left-overs in either (3) or (4)," said Michael.

"Does anybody want to make a prediction about when we'll get to the next 'no left-over' situation?" asked Mrs. Burg.

A Prediction

"I'll make a prediction. I just did a couple of arrangements and it looks like we're starting all over again. The left-overs with 13 and 14 tiles look like the same order as 1 and 2" said Millie.

"That's right. That's because 13 tiles is just like starting out again with 1 tile, 1 left-over in the (3) and (4) tile arrangements," said Mrs. Burg.

"Yup. And 14 tiles is like starting out with 2 tiles, 2 left-over in the (3) and (4) tile arrangements," said Thomas.

"Good. You kids are pretty smart. I brought this little table to show you that Millie's prediction is right. Here it is":

TABLE 1

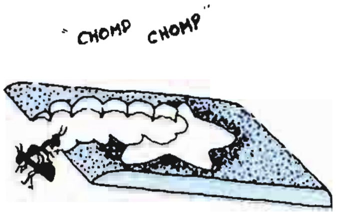
Number of tiles

	1	2	3	4	5	6	7	8	9	10	11	12
3 rows	1	2	0	1	2	0	1	2	0	1	2	0
4 rows	1	2	3	0	1	2	3	0	1	2	3	0

left over tiles

TABLE 2

	13	14	15	16	17	18	19	20	21	22	23	24
3 rows	1	2	0	1	2	0	1	2	0	1	2	0
4 rows	1	2	3	0	1	2	3	0	1	2	3	0



"At first, I thought I should keep on going, but I noticed that the results in both tables are identical . . . so I would learn nothing new by figuring out still another table: all I would do is change the numbers on top from 13 to 25, 14 to 26, etc., and nothing else."

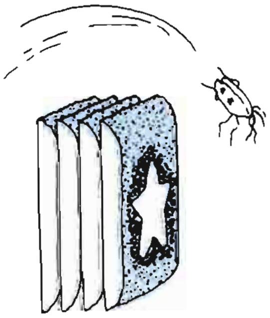
"What good is it to know that stuff, Mrs. Burg?" asked Wendy.

"Well, for one thing, I can do tricks with 'that stuff'," said Mrs. Burg.

"What kind of tricks?" asked Wendy.

"Before I tell you, you can think of left-overs as 'remainders', okay?" said Mrs. Burg.

"Arithmetic, yuk!" said Ralph.



"It's not so bad. Go on, Mrs. Burg," said Millie.

"Okay, I've got a secret number and I'll tell you this much about it: I'm thinking of a number from 1 to 12.

"I'll tell you what's left over (or the remainder) when I divide the number by 3 and 4. Then you can use my first chart to find the answer. When you divide the secret number by 3, the remainder is 2; when you divide it by 4, the remainder is 3."

$$3\overline{)\quad ?\quad}^{\text{R2}} \qquad 4\overline{)\quad ?\quad}^{\text{R3}}$$

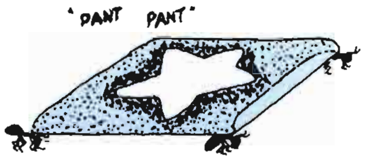
"That's easy. The answer is 11," said Martin.

"You're right, it is easy now that you have the table. Now, how about a harder one?"

"Sure, nothing to it," said Ruth.

"Okay, here we go," said Mrs. Burg. "Suppose I change from using 3 and 4 as rows (or divisors) and work with 3, 4 and 5?" asked Mrs. Burg.

÷3	÷4	÷5



"What is the smallest number of tiles I need in each arrangement so that in row arrangements of 3, 4 *and* 5 there will be no left-over tiles?"

"Hey, that's tougher. We haven't done it with 3 row arrangements," said Willie.

"Well, you could make your own table just like before, but let's see if you can play the 3-number trick without a table. Try this one. Here's all the information you need to know about my secret number:

$$3\overline{)\quad ?\quad}^{\text{R1}} \qquad 4\overline{)\quad ?\quad}^{\text{R2}} \qquad 5\overline{)\quad ?\quad}^{\text{R3}}$$

"What's my secret number?"

"Hey, I have an idea. Let's write down all the numbers with 1 as a remainder when divided by 3, all the numbers that leave 2 as a remainder when divided by 4 and all the numbers that leave 3 as a remainder when divided by 5.

"When we find a number that has 1,2 *and* 3 as a remainder, we'll have it," said Juanita.

"That's silly, Juanita," said David. "All we have to do is look at Mrs. Burg's chart to get all the remainders of numbers from 1 through 24. Look, the only time when both row (3) when divided by 3 and 4 has a remainder of 1 and row (4) has a remainder of 2 are with '10' and '22'. We can forget about the rest of those numbers."

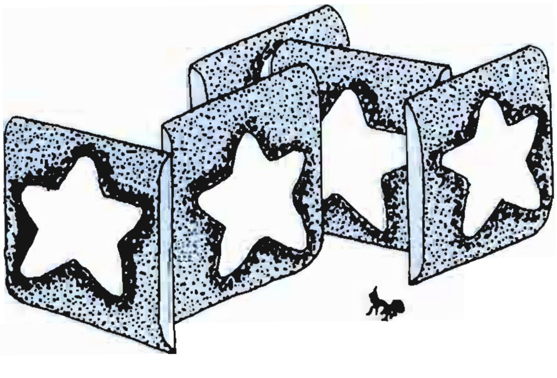
"That makes sense; and neither 10 nor 22 are the secret number because 10 ÷ 5 leaves 0 for a remainder and 22 ÷ 5 has 2 for a remainder," said Marge. "We need a number that has R^3 when divided by 5. So the secret number must be larger than 24."

"Well, let's get to work. 3 into 26 is 8, remainder 2 . . ."

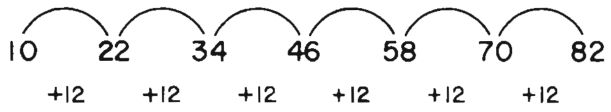
"Hold everything, everyone. Stop the show, I've got another idea," said David.

"Show off," said Wendy.

"Cut it out, Wendy," said Juanita.



David began, "Remember that once we went from 1 to 12 before, the order of the remainders began to repeat every 12. Since 10 and 22 were the only numbers that had remainders of 1 and 2 when divided by 3 and by 4 it's clear that the next numbers with that pattern of remainders will be 12 more on the list. Look:



"I get it, all we have to do is keep dividing *those* numbers by 15 and we'll find it. Pretty good, David," said Wendy.

"Thanks, Wendy," said David.

$$\overset{\text{R0}}{5\overline{)\,10}} \quad \overset{\text{R2}}{5\overline{)\,22}} \quad \overset{\text{R4}}{5\overline{)\,34}} \quad \overset{\text{R1}}{5\overline{)\,46}} \quad \overset{\text{R3}}{5\overline{)\,58}}$$

"We got it, Mrs. Burg. We found your secret number. It's 58," shouted the class.

$$\overset{\text{R1}}{3\overline{)\,58}} \quad \overset{\text{R2}}{4\overline{)\,58}} \quad \overset{\text{R3}}{5\overline{)\,58}}$$

"You're right, my secret number is 58. If you feel like it, try a couple more while I make a quick phone call," said Mrs. Burg.

A. $\overset{\text{R0}}{3\overline{)}}$ and $\overset{\text{R2}}{4\overline{)}}$ and $\overset{\text{R2}}{5\overline{)}}$

B. $\overset{\text{R1}}{3\overline{)}}$ and $\overset{\text{R2}}{4\overline{)}}$ and $\overset{\text{R1}}{5\overline{)}}$

C. $\overset{\text{R2}}{3\overline{)}}$ and $\overset{\text{R1}}{4\overline{)}}$ and $\overset{\text{R3}}{5\overline{)}}$

Mrs. Burg returned from making her call. The answers were on the board.

A. $\overset{\text{R0}}{3\overline{)}}$ and $\overset{\text{R2}}{4\overline{)}}$ and $\overset{\text{R2}}{5\overline{)}}$

David's short cut (÷ 5) $\underset{\text{R1}}{6}, \underset{\text{R3}}{18}, \underset{\text{R0}}{30}, \underset{\underline{\underline{\text{R2}}}}{\underline{\quad}}, \underline{\quad}, \underline{\quad}, \underline{\quad}$

B. $\overset{\text{R1}}{3\overline{)}}$ and $\overset{\text{R2}}{4\overline{)}}$ and $\overset{\text{R1}}{5\overline{)}}$

(÷ 5) $\underset{\text{R0}}{10}, \underset{\text{R2}}{22}, \underset{\text{R4}}{\underline{\quad}}, \underset{\underline{\underline{\text{R1}}}}{\underline{\quad}}, \underline{\quad}, \underline{\quad}$

C. $\overset{\text{R2}}{3\overline{)}}$ and $\overset{\text{R1}}{4\overline{)}}$ and $\overset{\text{R3}}{5\overline{)}}$

(÷ 5) $\underset{\text{R}}{\underline{\quad}}, \underset{\text{R}}{\underline{\quad}}, \underset{\text{R}}{\underline{\quad}}, \underset{\text{R}}{\underline{\quad}}, \underset{\text{R}}{\underline{\quad}}, \underset{\text{R}}{\underline{\quad}}$

(Please complete the lists above.)

Mrs. Burg's Talk

"You kids are really fast. You must have used Juanita's and David's plan to get done that fast. Now that you're done, I'll give each of you my special tables that include (3), (4) and (5)," said Mrs. Burg.

"Thanks a lot, Mrs. Burg," said Tina.

	1	2	3	4	5	6	7	8	9	10	11	12
÷3	1	2	0	1	2	0	1	2	0	1	2	0
÷4	1	2	3	0	1	2	3	0	1	2	3	0
÷5	1	2	3	4	0	1	2	3	4	0	1	2

	13	14	15	16	17	18	19	20	21	22	23	24
3	1	2	0	1	2	0	1	2	0	1	2	0
4	1	2	3	0	1	2	3	0	1	2	3	0
5	3	4	0	1	2	3	4	0	1	2	3	4

	25	26	27	28	29	30	31	32	33	34	35	36
3	1	2	0	1	2	0	1	2	0	1	2	0
4	1	2	3	0	1	2	3	0	1	2	3	0
5	0	1	2	3	4	0	1	2	3	4	0	1

	37	38	39	40	41	42	43	44	45	46	47	48
3	1	2	0	1	2	0	1	2	0	1	2	0
4	1	2	3	0	1	2	3	0	1	2	3	0
5	2	3	4	0	1	2	3	4	0	1	2	3

	49	50	51	52	53	54	55	56	57	58	59	60
3	1	2	0	1	2	0	1	2	0	1	2	0
4	1	2	3	0	1	2	3	0	1	2	3	0
5	4	0	1	2	3	4	0	1	2	3	4	0

"My mother also gave me enough of these little cards so everyone can have a set if you will promise to keep them secret," said Ben.

Everyone eagerly promised and got his set of cards.

"But how do we use them?"

"Here's how it works," Ben said as he drew a sketch on one side of the crowded chalk board:

0	0	1	2	3
0	60	45	30	15
1	36	21	6	51
2	12	57	42	27
3	48	33	18	3
4	24	9	54	39

1	0	1	2	3
0	40	25	10	55
1	16	1	46	31
2	52	37	22	7
3	28	13	58	43
4	4	49	34	19

2	0	1	2	3
0	20	5	50	35
1	56	41	26	11
2	32	17	2	47
3	8	53	38	23
4	44	29	14	59

"The Remainders on division by 3—0, 1 or 2—tells you which card to use by noticing the number in the upper left hand corner. The other numbers in the top row are remainders when you divide by 4 and tell you which column to look down. The other numbers on the left side—0, 1, 2, 3 and 4—refer to remainders when you divide by 5—or which row to use. Here's an example:"

$\overset{R1}{3\overline{)\quad\quad}}$ → use card with "1" in the upper left hand corner

and

$\overset{R2}{4\overline{)\quad\quad}}$ → Column "2"

and

$\overset{R3}{5\overline{)\quad\quad}}$ → row "3"

"And the secret number is 58. That was Mother's first secret number . . . and she's right—I knew it before she got to the door."

"Those cards sure save a lot of time," said David, "it's so much faster than the plan Juanita and I worked out."

"You two were headed fast down the right track . . . looking for shortcuts, for patterns and extending them. That's the way I started," said Mrs. Burg.

"I had an advantage. I had more time than you had to work at the problem and I tried a lot of plans to organize my results before I could make that set of 3 cards.

"I have another set of cards which I use when the remainders are given after division by 3, 5, and 7 . . . and it works for all numbers 1 through 105. I didn't bring a set with me.

"But, here is a blank for that set of 3 cards that you might like to complete," said Mrs. Burg.

÷3 (÷5)

	0	0	1	2	3	4
	0	105				
	1					
	2					
(÷7)	3					
	4					
	5					
	6					

÷3 (÷5)

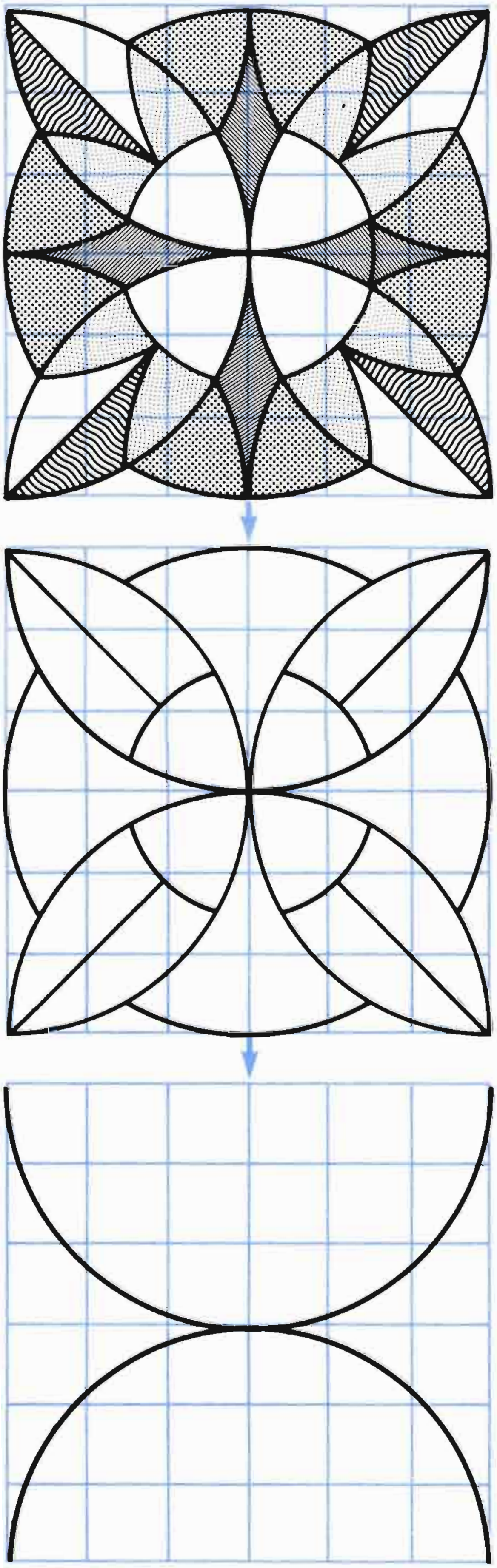
	1	0	1	2	3	4
	0	70				
	1					
	2					
(÷7)	3					
	4					
	5					
	6					

÷3 (÷5)

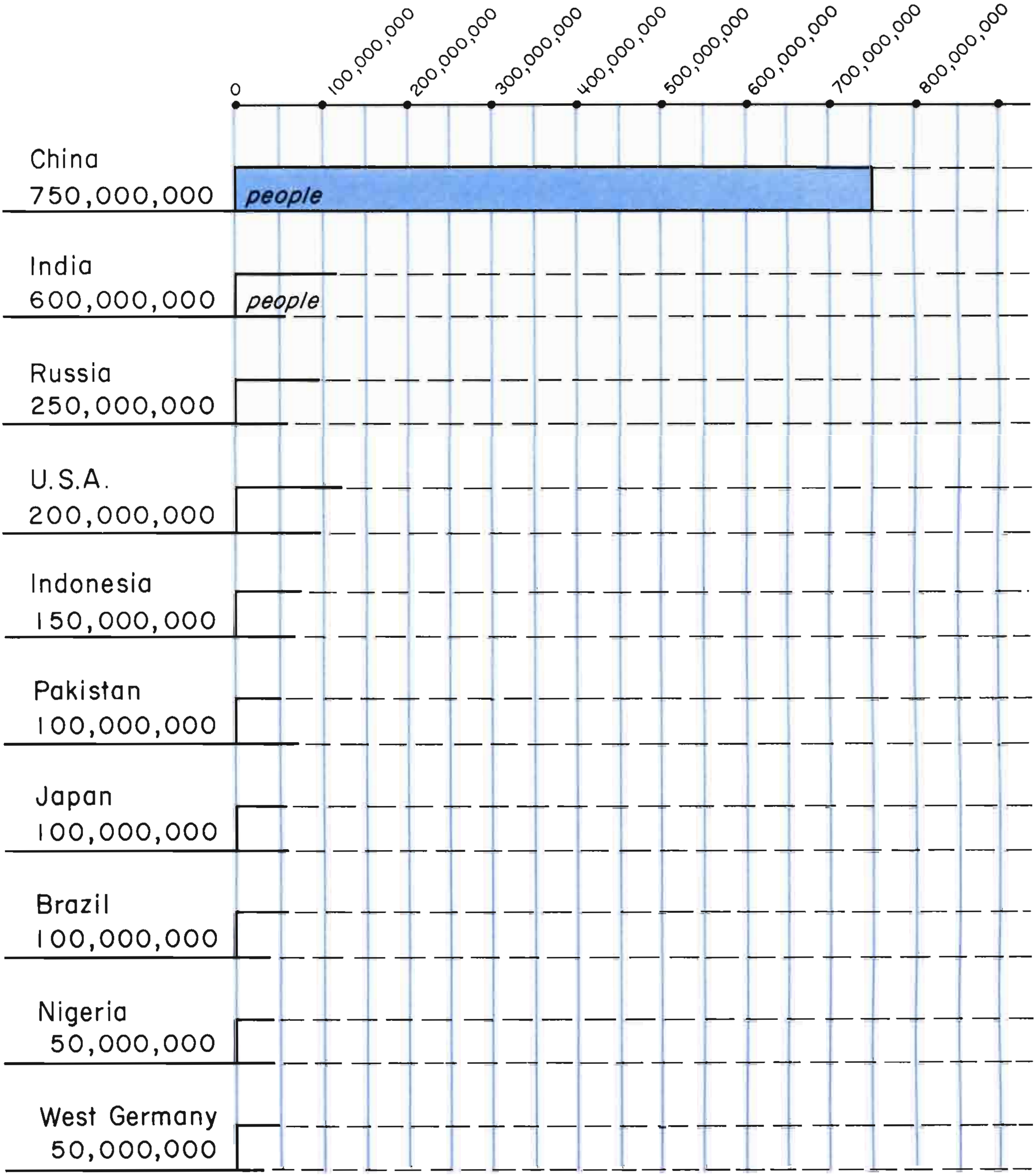
	2	0	1	2	3	4
	0	35				
	1					
	2					
(÷7)	3					
	4					
	5					
	6					

"I've got to go now. Good-bye and good luck."

"Good-bye, Mrs. Burg," everyone said.

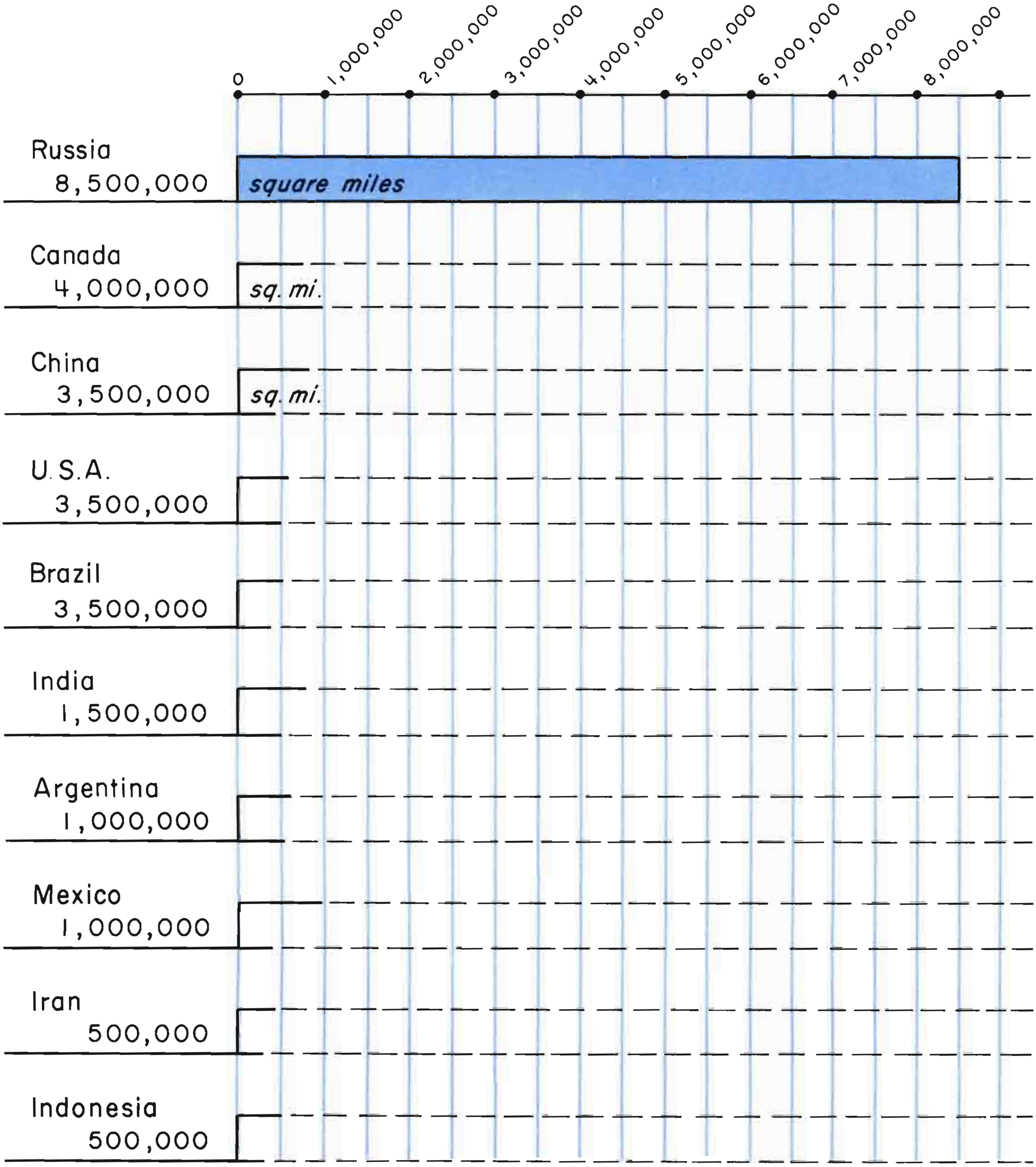


The 10 countries with the largest population are listed below.
Please color in a bar that gives a picture of the population of each.
(Populations are given to the nearest 50 million.)

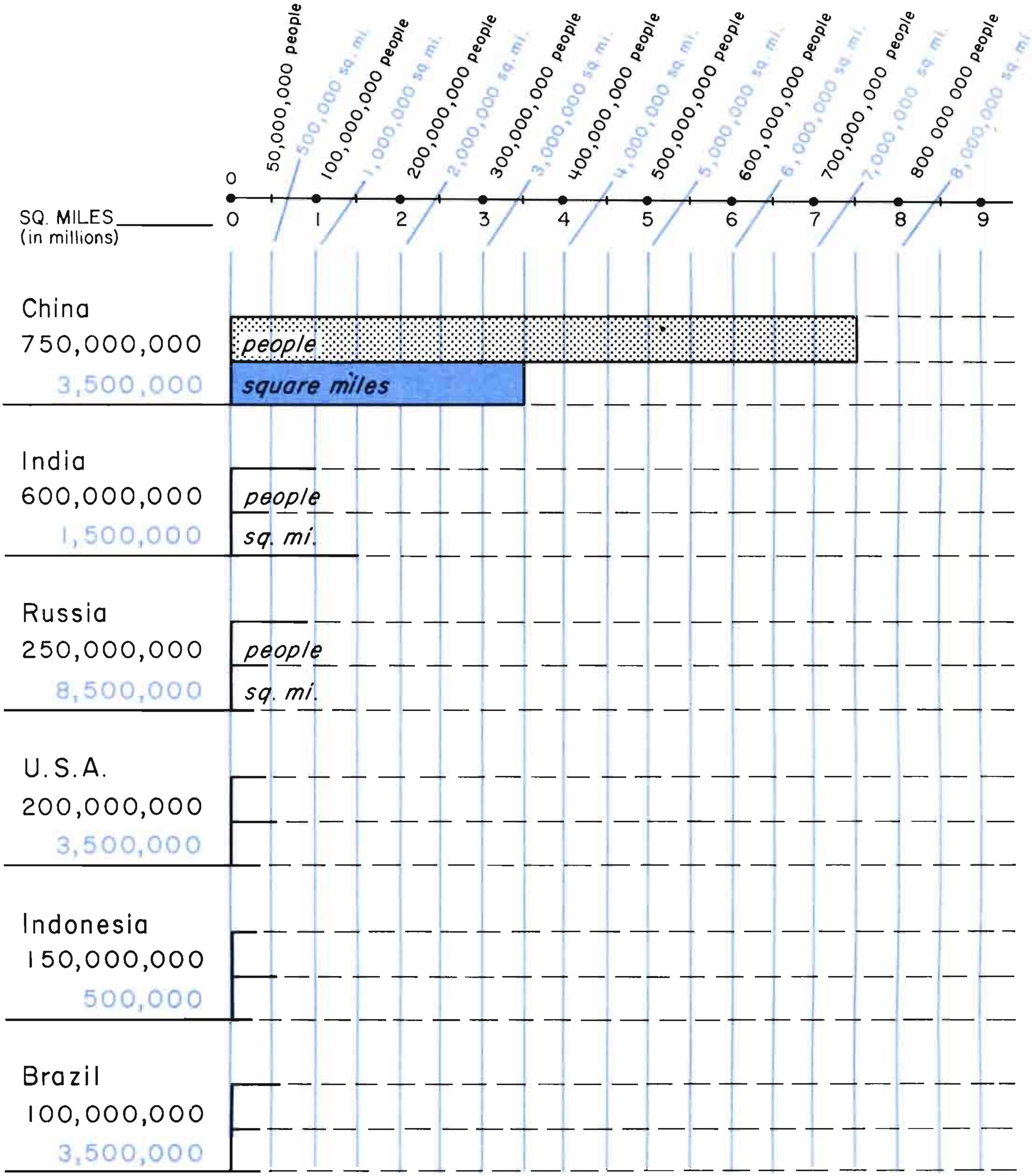


TEAMWORK

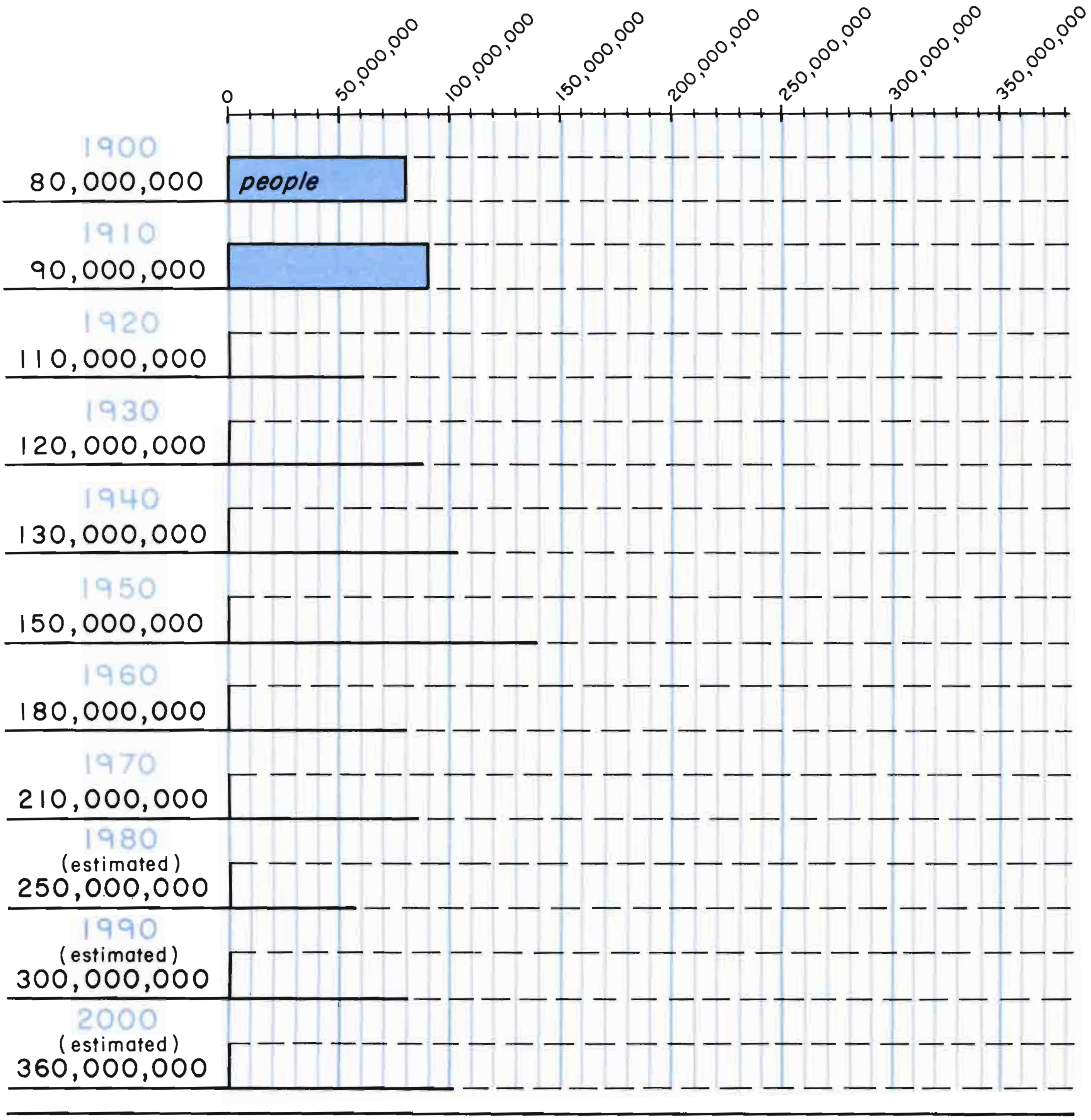
The 10 countries with the largest area are listed below. Please color in a bar that gives a picture of the area of each. (Areas are given to the nearest 500,000 square miles.)



Six countries appear on both lists. Please complete the graph below that shows both the number of people (one color) and the area in square miles (another color.) We will use the same approximate figures.



Please complete the graph below showing the way the population of the United States of America is growing. The figures through 1970 are from the U.S. Bureau of the Census. Each is given to the nearest 10 million.



JUMPING ROPE

How many times can you
jump rope in 1 minute? ____

1st Try ____

2nd try ____

3rd try ____

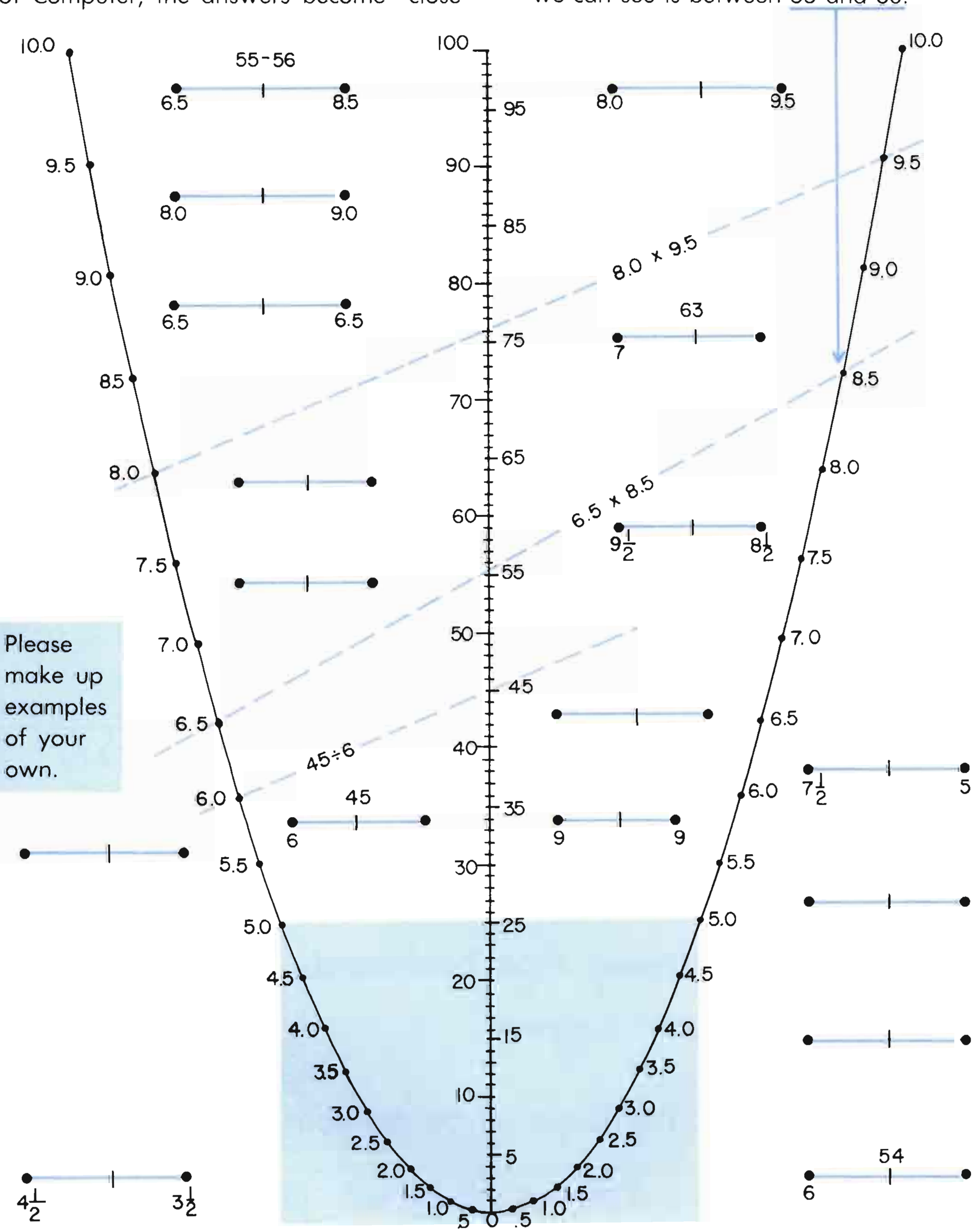
Now try jumping rope backwards and compare with the record above.

Can you try hopping or skipping rope?

See page 15 and try that activity with jumping rope.

Here is an Advanced Computer: Can you make it work?

As the scales become smaller in this kind of Computer, the answers become "close approximations". 6.5 x 8.5 = 55.25 but all we can see is between 55 and 60.

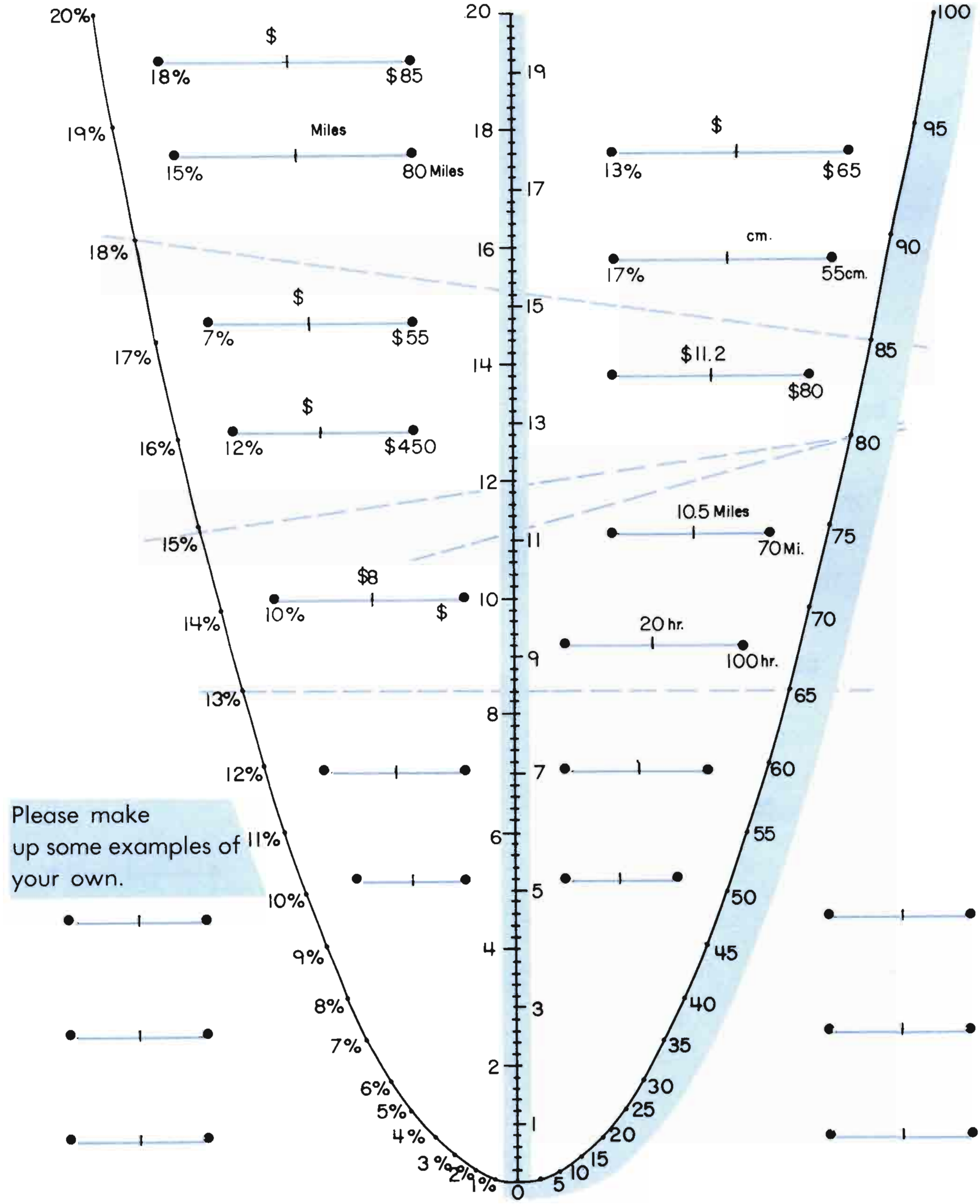


Please make up examples of your own.

Can we change the scales on our Advanced Computer?

Notice: each step on the center scale is .2

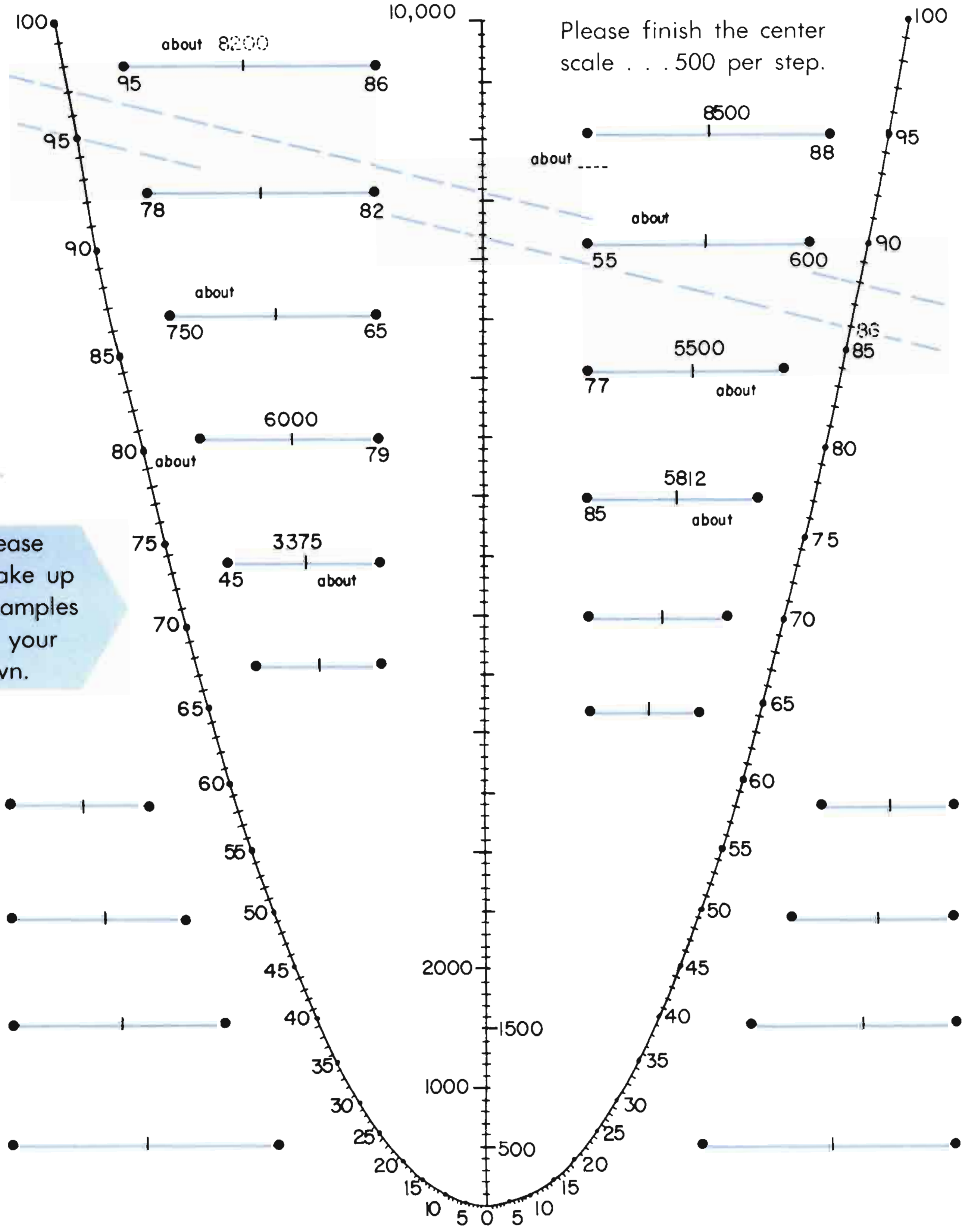
If you think "dollars" on one blue scale, then you think "dollars" on the other blue scale. If you think "miles" on one, think "miles" on the other.



Please make up some examples of your own.

Another change in scales — a more Advanced Computer

If you multiply the numbers on either side scale by 10, then multiply numbers on the center scale by 10. If you multiply both side scales by 10, then multiply the center scale by 100. Why?

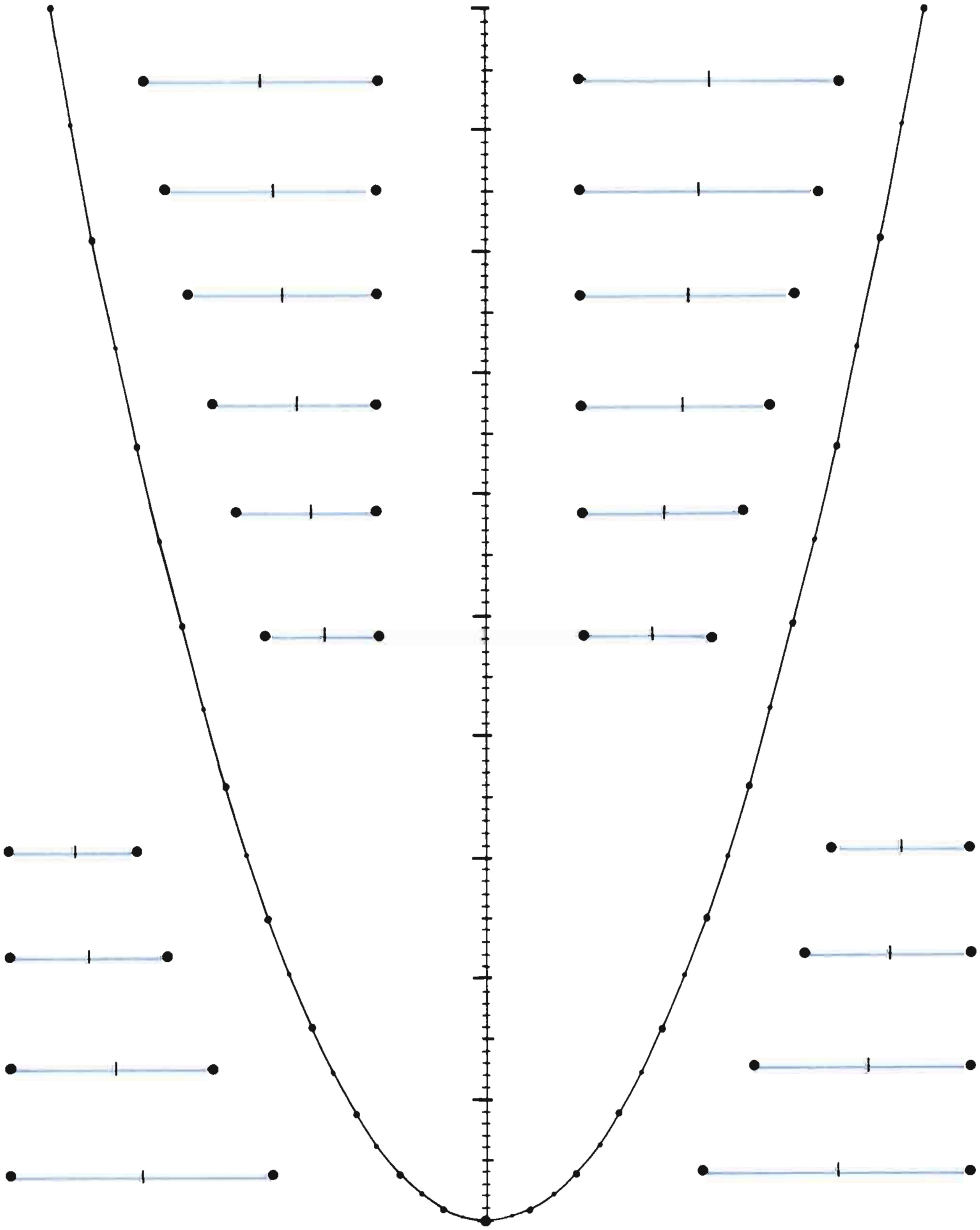


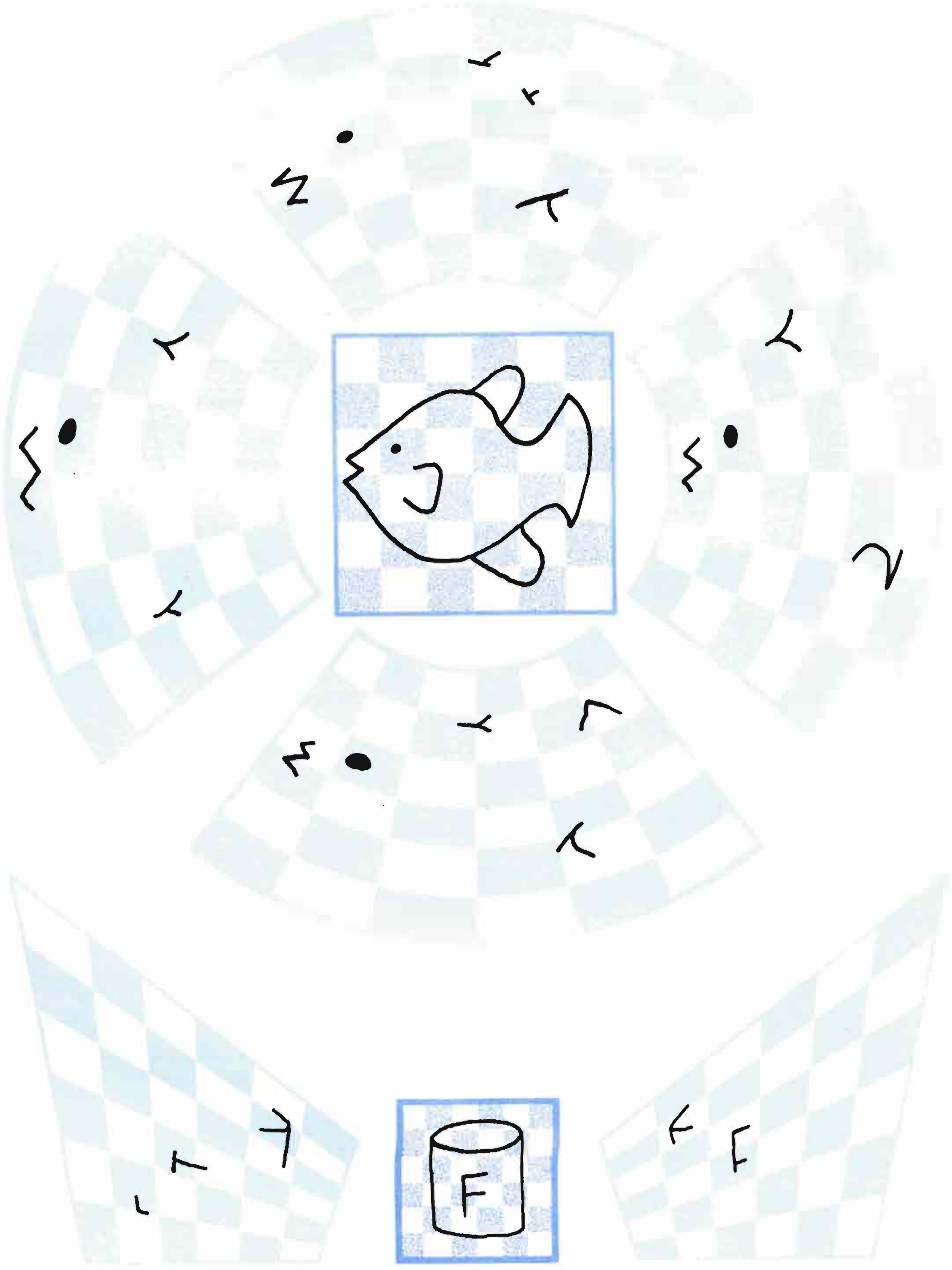
Please finish the center scale . . . 500 per step.

Please make up examples of your own.

Please build your own computer –

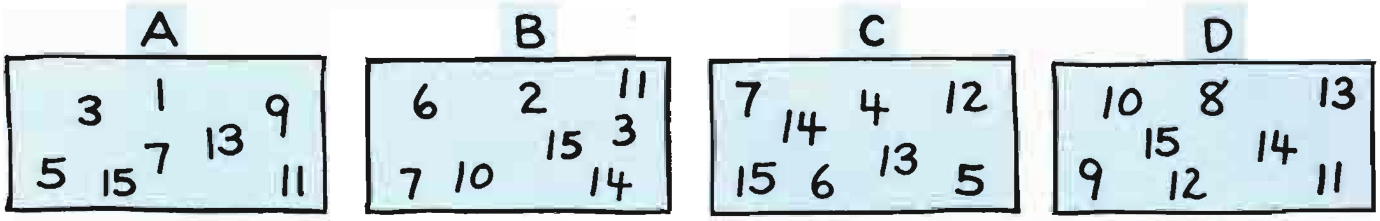
select your own scales and make up your own examples.





F

A Very Special Set of Cards



Do you see any patterns in these cards?

What about the numbers on A?
What about the numbers on B?
What about the numbers on C?
What about the numbers on D?

The largest number on any card is 15. Let's arrange the numbers on each card in order from smallest to largest, and look at the numbers 15 or less that are not on each card.

On A: 1 3 5 __ __ __ __ __

Not on A: 2 4 __ __ __ __ __

On B: 2 3 6 7 __ __ __ __

Not on B: 1 4 __ __ __ __ __

On C: 4 5 __ __ __ __ __ __

Not on C: 1 2 __ __ __ __ __

On D: 8 9 __ __ __ __ __ __

Not on D: 1 __ __ __ __ __ __

Patterns in the cards are now more easily noticed.

What number appears only on card A?____
What number appears only on card B?____
What number appears only on card C?____
What number appears only on card D?____

What number appears only

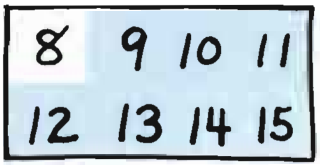
. . . on cards A and B?________
. . . on cards A and C?________
. . . on cards A and D?________
. . . on cards B and C?________
. . . on cards B and D?________
. . . on cards C and D?________

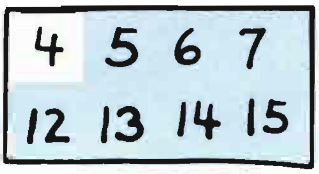
. . . on cards A, B and C?______
. . . on cards A, B and D?______
. . . on cards A, C and D?______
. . . on cards B, C and D?______

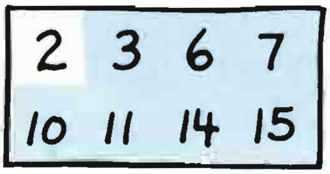
. . . on all cards? _____

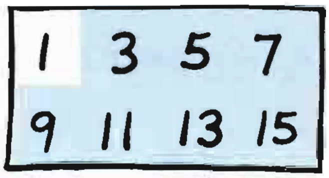
Look at all the numbers in this column.

Is there anything surprising about those numbers? (See next page)









On the last page, the numbers in the right-hand column given as answers to questions —

(1) . . are all different, and
(2) . . all numbers are there —
1 through 15.

The cards on this page show the numbers more in order.

Because numbers 1, 2, 4 and 8 appear only once and on different cards, let's call them the "1, 2, 4 and 8" cards.

Please show on which cards you find each of the numbers given below: —

Now you can play a

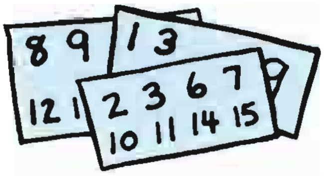
THINK-OF-A-NUMBER TRICK!

Make a set of 4 cards like the ones shown above.

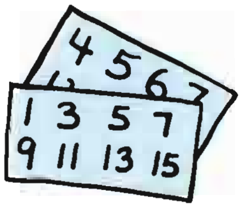
Ask someone to "think of a whole number less than 16" and remember it.

Ask him to hand you all the cards that have that number. You just look at the corners of the cards he gives you — and you can tell him his secret number.

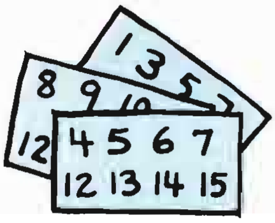
ON CARDS: —
1 → 1
2 → 2
3 → 2, 1
4 → ______
5 → ______
6 → ______
7 → 4, 2, 1
8 → ______
9 → ______
10 → ______
11 → ______
12 → ______
13 → ______
14 → ______
15 → ______



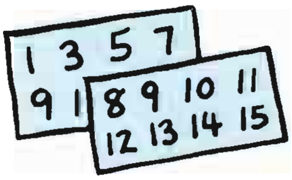
must be ____



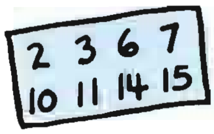
must be ____



must be ____



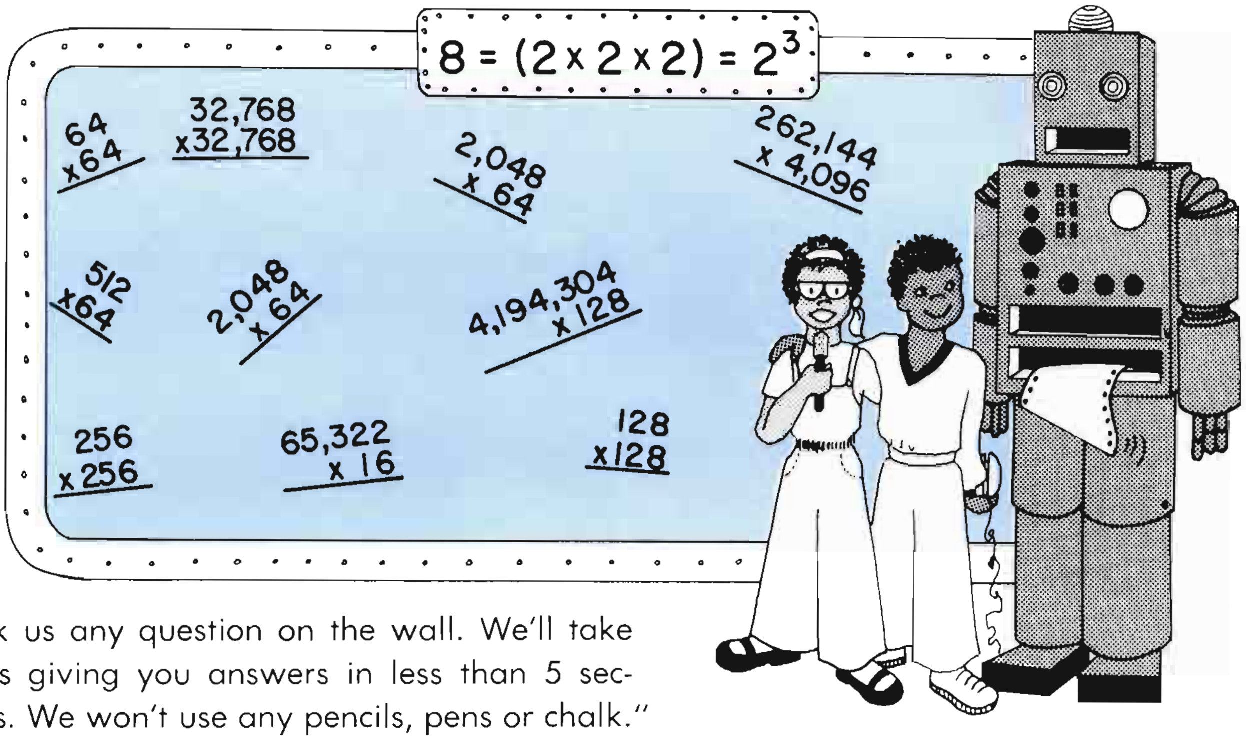
must be ____



must be ____

"it isn't on any card."

must be ____



"Ask us any question on the wall. We'll take turns giving you answers in less than 5 seconds. We won't use any pencils, pens or chalk."

A PROMISE

You will be able to answer all those questions as easily as Angie and Lenny before you finish this section.

It all started with that "allowance-problem arithmetic" that tricked Mrs. and Mr. Pencel.

1st week ___1 ¢
2nd week ____ ¢
3rd week ____ ¢
4th week ____ ¢
5th week ____ ¢

"Twice as much each week as the week before."

Beginning with 2, that list of numbers can be written differently:

2 = 2
4 = 2 x 2
8 = 2 x 2 x 2

16 = 2 x 2 x 2 x 2
32 = 2 x 2 x 2 x 2 x 2
64 = 2x2x2x2x2x2

Angie and Lenny found out that mathematics have a shorthand so they don't have to write such long sentences.

$4 = (2 \times 2) = 2^2$

They read that as "2 squared" or "2 to the 2nd power."

$8 = (2 \times 2 \times 2) = 2^3$
$16 = (2 \times 2 \times 2 \times 2) = 2^4$
$32 = (2 \times 2 \times 2 \times 2 \times 2) = 2^5$
$64 = 2^6$
$128 = 2^7$
256 = ____
512 = ____
1,024 = ____
______ = ____
______ = ____

A neat shorthand — but is it useful?

(see next page)

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, etc.

There is a surprise hidden in this list.

Select any pair of numbers in this list.
Multiply them.
The product is also in the list.

$$\begin{array}{r} 2 \\ \times 2 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ \times 4 \\ \hline \end{array} \quad \begin{array}{r} 64 \\ \times 2 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ \times 4 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ \times 8 \\ \hline \end{array} \quad \begin{array}{r} 16 \\ \times 4 \\ \hline \end{array}$$

On the previous page, we wrote all the numbers in this list in shorthand — except 1 and 2.

Let's repeat that shorthand table and look again at 1 and 2. We will write the shorthand under the number it stands for.

1	2	4	8	16	32
		2^2	2^3	2^4	2^5

The little raised numbers to show the number of 2's to be multiplied together are called "exponents".

Do you see a pattern in the exponents that could be extended to include new names for 1 and 2?

$$2^{0},\ 2^{1},\ 2^{2},\ 2^{3},\ 2^{4},\ 2^{5},\ \text{etc.}$$

They would lead us to saying "$1 = 2^0$ . . . one equals 2 to the zeroeth power", and "$2 = 2^1$ . . . two equals two to the first power."

While that may sound strange, let's see how it works out. If it causes trouble, we'll take another look.

Now, all numbers in the list at the top of the page can be written in shorthand.

$1 = 2^0$
$2 = 2^1$
$4 = 2^2$
$8 = 2^3$
$16 = 2^4$
$32 = 2^5$
$64 = 2^6$
$128 = 2^7$
$256 = 2^8$
$512 = 2^9$

Now we can write the arithmetic at the beginning of this page in shorthand.

$$\begin{array}{r} 2 \\ \times 2 \\ \hline 4 \end{array} \quad \begin{array}{r} 2^1 \\ \times 2^1 \\ \hline 2^2 \end{array} \qquad \begin{array}{r} 64 \\ \times 2 \\ \hline 128 \end{array} \quad \begin{array}{r} 2^6 \\ \times 2^1 \\ \hline 2^7 \end{array} \qquad \begin{array}{r} 4 \\ \times 4 \\ \hline 16 \end{array} \quad \begin{array}{r} 2^2 \\ \times 2^2 \\ \hline 2^4 \end{array}$$

$$\begin{array}{r} 8 \\ \times 4 \\ \hline 32 \end{array} \quad \begin{array}{r} 2^3 \\ \times 2^2 \\ \hline 2^5 \end{array} \qquad \begin{array}{r} 8 \\ \times 8 \\ \hline 64 \end{array} \quad \begin{array}{r} 2^3 \\ \times 2^3 \\ \hline 2^6 \end{array} \qquad \begin{array}{r} 16 \\ \times 4 \\ \hline 64 \end{array} \quad \begin{array}{r} 2^4 \\ \times 2^2 \\ \hline 2^6 \end{array}$$

Do you notice a pattern in the "exponents" that would help you in the following examples?

$$\begin{array}{r} 32 \\ \times 2 \\ \hline \end{array} \quad \begin{array}{r} 2^5 \\ \times 2^1 \\ \hline \end{array} \qquad \begin{array}{r} 64 \\ \times 1 \\ \hline \end{array} \quad \begin{array}{r} 2^6 \\ \times 2^0 \\ \hline \end{array} \qquad \begin{array}{r} 16 \\ \times 16 \\ \hline \end{array} \quad \begin{array}{r} 2^4 \\ \times 2^4 \\ \hline \end{array}$$

When Angie and Lenny noticed this, they found a new use for their "allowance arithmetic".

Allowance Arithmetic — in Shorthand.

Now you too can answer the questions on this page in less than 5 seconds . . . without using pencil, pen or chalk.

$2^{0} = 1$
$2^{1} = 2$
$2^{2} = 4$
$2^{3} = 8$
$2^{4} = 16$
$2^{5} = 32$
$2^{6} = 64$
$2^{7} = 128$
$2^{8} = 256$
$2^{9} = 512$
$2^{10} = 1,024$
$2^{11} = 2,048$
$2^{12} = 4,096$
$2^{13} = 8,192$
$2^{14} = 16,384$
$2^{15} = 32,768$
$2^{16} = 65,536$
$2^{17} = 131,072$
$2^{18} = 262,144$
$2^{19} = 524,288$
$2^{20} = 1,048,576$
$2^{21} = 2,097,152$
$2^{22} = 4,194,304$
$2^{23} = 8,388,608$
$2^{24} = 16,777,216$
$2^{25} = 33,554,432$
$2^{26} = 67,108,864$
$2^{27} = 134,217,728$
$2^{28} = 268,435,456$
$2^{29} = 536,870,912$
$2^{30} = 1,073,741,824$

16 (Look it up in the list — 2^4 — notice exponent — 4)
x 32 (Look it up in the list — 2^5 — notice exponent — 5)
512 (4 + 5 = 9 . . . what number is 2^9 in the list?)

$$\begin{array}{r} 128 \ \ (2^{7}) \\ \times 64 \ \ (2^{6}) \\ \hline \leftarrow(2^{13}) \end{array} \qquad \begin{array}{r} 16,384 \ (2^{14}) \\ \times 16 \ (2^{4}) \\ \hline \leftarrow(2^{\ \ }) \end{array} \qquad \begin{array}{r} 1,024 \ (2^{10}) \\ \times 1,024 \ (2^{10}) \\ \hline \leftarrow(2^{\ \ }) \end{array}$$

Notice how $1 = 2^{0}$ and $2 = 2^{1}$ fit in.

$$\begin{array}{r} 1 \ \ (2^{0}) \\ \times 1 \ \ (2^{0}) \\ \hline \leftarrow(2^{0}) \end{array} \qquad \begin{array}{r} 2 \ \ (2^{1}) \\ \times 1 \ \ (2^{0}) \\ \hline \leftarrow(2^{1}) \end{array} \qquad \begin{array}{r} 2 \ (2^{1}) \\ \times 2 \ (2^{1}) \\ \hline \leftarrow(2^{2}) \end{array} \qquad \begin{array}{r} 8 \ (2^{3}) \\ \times 1 \ (2^{0}) \\ \hline \leftarrow(2^{3}) \end{array}$$

Here are some examples on the wall on the first page of this section. We promised you could find answers as quickly as Angie and Lenny.

$$\begin{array}{r} 1,024 \\ \times 256 \\ \hline \end{array} \qquad \begin{array}{r} 2,048 \\ \times 64 \\ \hline \end{array} \qquad \begin{array}{r} 128 \\ \times 128 \\ \hline \end{array} \qquad \begin{array}{r} 8,192 \\ \times 16 \\ \hline \end{array}$$

$$\begin{array}{r} 65,322 \\ \times 16 \\ \hline \end{array} \qquad \begin{array}{r} 512 \\ \times 64 \\ \hline \end{array} \qquad \begin{array}{r} 256 \\ \times 256 \\ \hline \end{array}$$

$$\begin{array}{r} 1,048,576 \\ \times 32 \\ \hline \end{array} \qquad \begin{array}{r} 32,768 \\ \times 32,768 \\ \hline \end{array}$$

Later, Angie and Lenny realized they could use their "allowance arithmetic" in division examples, provided the divisor and dividend were both in the list and that the divisor is less than the dividend.

(quotient)
(divisor) ⟌ (dividend)
(dividend) ÷ (divisor) = (quotient)

32 ÷ 4 = 8
$2^5 \div 2^2 = 2^3$

64 ÷ 2 = ___
$2^6 \div 2^1 =$

128 ÷ 8 = ___
$2^7 \div 2^3 =$

128 ÷ 16 = ___
÷ =

128 ÷ 32 = ___
÷ =

128 ÷ 64 = ___
÷ =

Can you use the list to find answers without the second line?

64 ÷ 8 = ___
64 ÷ 4 = ___
128 ÷ 4 = ___

1,024 ÷ 32 = ___
512 ÷ 128 = ___
2,048 ÷ 64 = ___

16,384 ÷ 256 = ___
131,072 ÷ = ___
1,048,076 ÷ 1,024 = ___

$8\overline{)128}$ $4\overline{)512}$ $128\overline{)1024}$ $16\overline{)256}$ $512\overline{)8{,}192}$

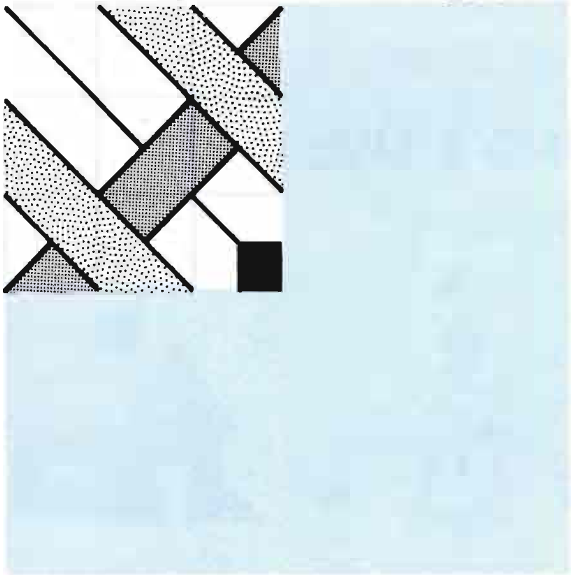
Don't you wish all numbers were in that "allowance arithmetic" list? . . . so difficult multiplication and division problems become simple addition and subtraction examples?

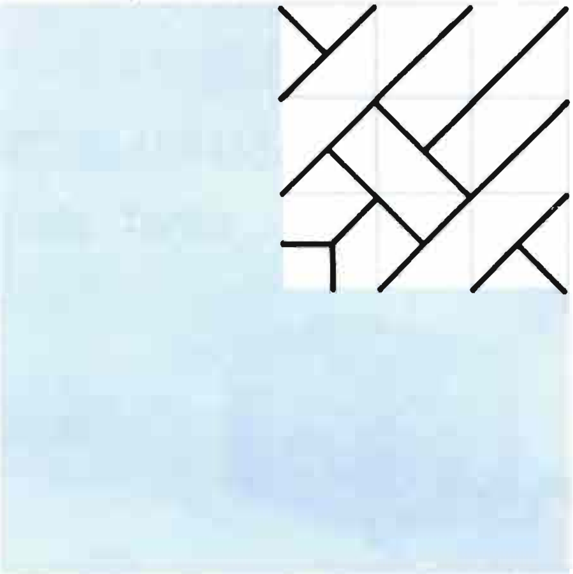
Good news!! There are such lists you can find called tables of "common logarithms."

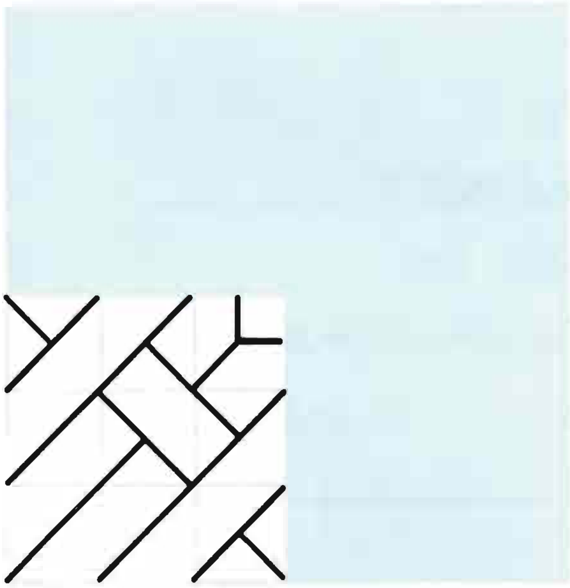
"Allowance Arithmetic" came from a plan in which Angie and Lenny were a little more greedy. Instead of "twice as much" they could have asked for "3 times as much."

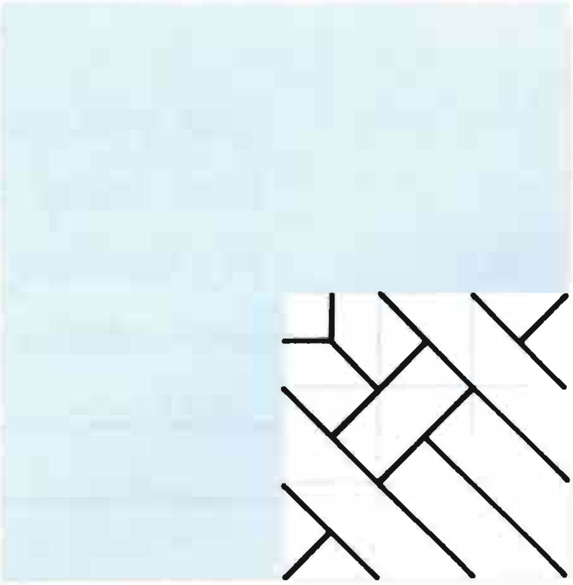
1	week — 1¢		$3^0 = 1$
2	week — 3¢		$3^1 = 3$
3	week — 9¢	and	$3^2 = 9$
4	week — 27¢		$3^3 = 27$
5	week — 81¢		$3^4 = 81$ etc.......

GEOMETRY for FUN





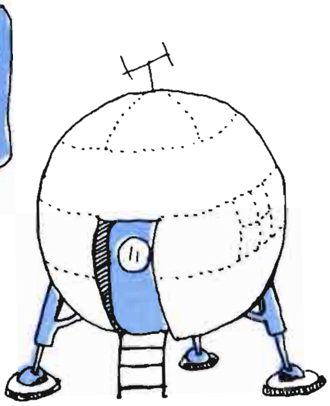




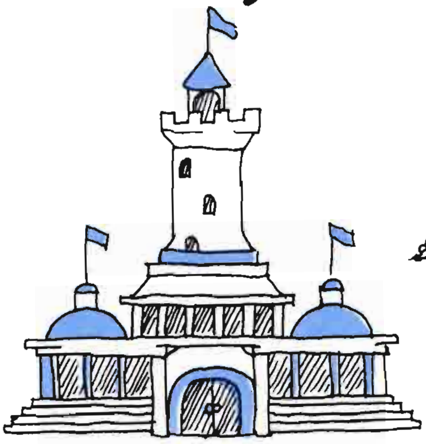
MEASURING THE BUILDING I LIVE IN

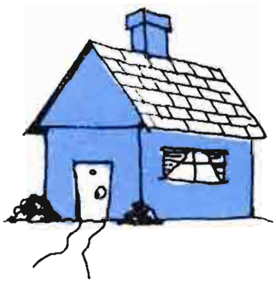
What shape is your building?

___ Square



___ Round





___ Combination

My building looks like this

Can you walk around it?

How many steps around is it? ________

My building is _____ feet tall.

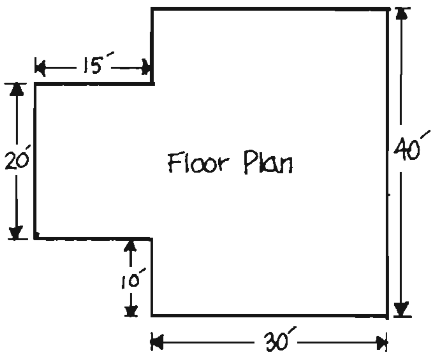
Tell some more measurement facts about your building.

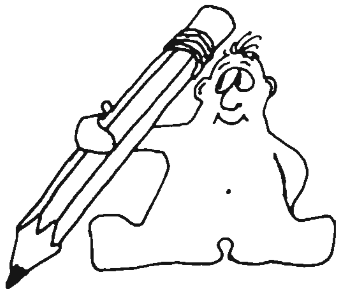
____

____

____

MEASURING THE BUILDING I LIVE IN





Measure the outside walls of your building and draw a sketch as accurately as you can. Record the measurements as in the sample sketch above.

(This might be difficult to do alone so pick a partner to help.)

WHAT IS THE PERIMETER OF YOUR BUILDING? ______ ft.

Can you find the area of the ground floor of your building? ______ sq.ft.

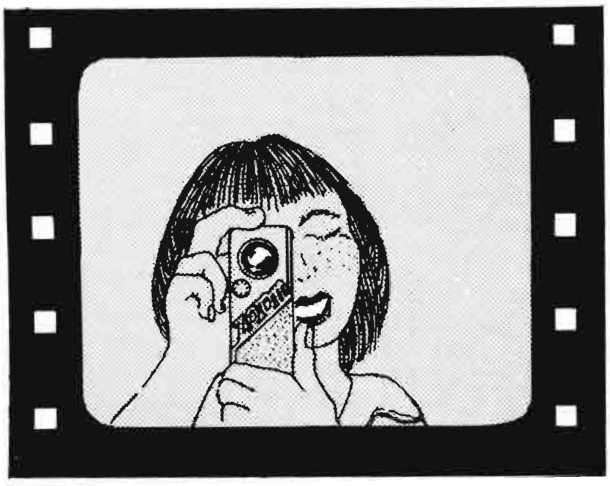
SQUARE NUMBERS

I. Gretchen's observations—

A "square number" is the product of a whole number multiplied by itself.

$1 \times 1 = 1$
$2 \times 2 = \underline{4}$
$3 \times 3 = 9$
$4 \times 4 = \underline{16}$
$5 \times 5 = 25$
$6 \times 6 = \underline{36}$
$7 \times 7 = 49$
$8 \times 8 = \underline{64}$

$9 \times 9 = 81$
$10 \times 10 = \underline{100}$
$11 \times 11 = 121$
$12 \times 12 = \underline{144}$ etc.



1, *4*, 9, *16*, 25, *36*, 49, *64*, 81, *100* . . . are all "square numbers".

So if "n" is any whole number, n × n or n^2 is a "square number".

A. All even square numbers underlined in the list are *multiples of 4*—so they can be divided evenly by 4, in other words, there will be no remainder when they are divided by 4.

$4 \div 4 = 1$
$16 \div 4 = 4$
$36 \div 4 = 9$
$64 \div 4 = 16$
$100 \div 4 = 25$

B. When the even square numbers in the list are divided by 4, the result is a square number—alternately odd and even.

$4 \div 4 = \underline{1}$ 1 is an *odd* square number.
$16 \div 4 = \underline{4}$ 4 is an *even* square number.
$36 \div 4 = \underline{9}$ 9 is an *odd* square number.
$64 \div 4 = \underline{16}$ is an *even* square number.

Question: Will those two statements, "A" and "B", be true about larger even numbers?

If we made the list of square numbers larger, we might find examples that wouldn't work—and prove that the statements "A" and "B" are not true for all *even square numbers*, but only for *some* of them.

But, if all the examples we have time and room to list follow these patterns, "A" and "B", we could still *never be sure that a still larger even square number we haven't yet listed wouldn't disprove statements "A" and "B". So we need to do something other than just keep looking, because, as we agreed, we could never look long enough to be sure.*

II. Gretchen takes another look at EVEN *numbers—All even numbers,* including even square numbers, can be written as the *product of 2 and some other whole number.*

$0 = 2 \times 0 \qquad 6 = 2 \times 3$
$2 = 2 \times 1 \qquad 8 = 2 \times 4$
$4 = 2 \times 2 \qquad 10 = 2 \times 5$ etc.

If we agree that "n"stands for the other whole number factor, then *all even numbers* can be written in the form of:

$2 \times n$ or $2n$
2 times some whole number

$6 = 2 \times 3$
$6 = 2 \times n$ or $2n$
$(n = 3)$

$8 = 2 \times 4$
$8 = 2 \times n$ or $2n$
$(n = 4)$

$16 = 2 \times 8$
$16 = 2 \times n$ or $2n$
$(n = 8)$

III. A look at EVEN SQUARE NUMBERS.

An "even square number" is the product of some even number multiplied by itself.

Remember: "n" is *any* number.

$2 \times n$ or $2n$ is an *even* number.

$n \times n$ or n^2 is a *square* number.

So, $(2 \times n) \times (2 \times n)$ is an *even square number.*

We can look at some particular *even square numbers* this way:

$$4 = 2 \times 2 = (2 \times 1) \times (2 \times 1)$$
$$16 = 4 \times 4 = (2 \times 2) \times (2 \times 2)$$
$$36 = 6 \times 6 = (2 \times 3) \times (2 \times 3)$$
$$64 = 8 \times 8 = (2 \times 4) \times (2 \times 4)$$
$$100 = 10 \times 10 = (2 \times 5) \times (2 \times 5)$$

The factors in these statements can be rearranged:

Factors Rearranged:

$$4 = (2 \times 1) \times (2 \times 1) = (2 \times 2) \times (1 \times 1) = 4 \times 1$$
$$16 = (2 \times 2) \times (2 \times 2) = (2 \times 2) \times (2 \times 2) = 4 \times 4$$
$$36 = (2 \times 3) \times (2 \times 3) = (2 \times 2) \times (3 \times 3) = 4 \times 9$$
$$64 = (2 \times 4) \times (2 \times 4) = (2 \times 2) \times (4 \times 4) = 4 \times 16$$
$$100 = (2 \times 5) \times (2 \times 5) = (2 \times 2) \times (5 \times 5) = 4 \times 25$$

So, EVEN SQUARE NUMBERS can be described in several ways, including:

$$\underline{2n \times 2n} = \underline{(2 \times n) \times (2 \times n)}$$

or

$$\underline{2n \times 2n} = \underline{(2 \times 2)(n \times n)} = \underline{4 \times (n \times n)} = \underline{4n^2}$$

Look back at statements, "A" and "B":

$$4 \times (n \times n) \text{ or } 4 \times n^2$$

is an even square number.

"A" *All even square numbers can be divided evenly by 4:*

$$4 \times (n \times n) \div 4 = n \times n = n^2$$

or

$$\frac{\cancel{4}}{\cancel{4}} \times n^2 = n^2$$

"B" After dividing by 4, *the result is* $n \times n$, or n^2.

$$1 \times (n \times n) = n \times n = n^2$$

n^2 is a whole number, "n", multiplied by itself, and that's the definition of a square number. So, we have *proved* that "A" and "B" are both *always true of all* even square numbers, from 4 to as far as you want to look.

PART II

Can we find a *square number* that is *twice as big as another square number?*

I. The Problem—
We will use a *large square to stand for the larger square number* and a *smaller diamond to stand for the smaller square number*. So the problem can be seen in this way:

Larger Sq. No. Smaller Sq. No.

□ = 2 × ◇

or

□ = ◇ + ◇

Just to get an idea of what we're doing, let's try a few guesses using larger and smaller square numbers:

$\boxed{9}$ = ◇4 + ◇4 or 2 x ◇ (not true)

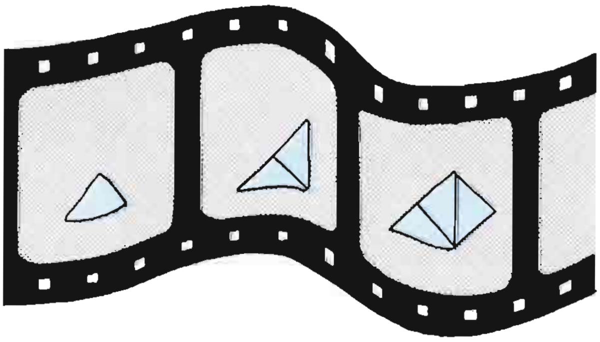
$\boxed{16}$ = ◇9 + ◇9 (not true)

$\boxed{36}$ = ◇16 + ◇16 (not true)

$\boxed{49}$ = ◇25 + ◇25 (not true)

(Can we complete this as a true statement?)

$\boxed{}$ = ◇ + ◇



II. Odd Squares and Even Squares:

NOTE: An *even square number* is an *even number* that is multiplied by itself

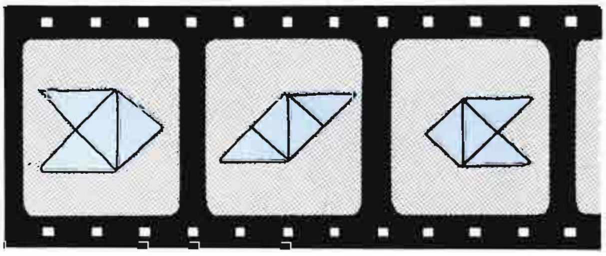
$4 \times 4 = 16 \quad 6 \times 6 = 36$ etc.

An *odd square number* is an *odd number* that is multiplied by itself.

$3 \times 3 = 9 \quad 5 \times 5 = 25$ etc.

NOTE: What I am going to say here is very easy, but we have to remember it for our problem:

An *odd number*, of course, *can not equal an even number;* also, an *odd square number cannot equal an even square number,* because an odd square number is the product of two odd numbers and an even square number is the product of two even numbers.



Now, let's look at all of the different combinations that are possible in this problem. They are:

A. odd $\boxed{}$ = odd ◇ + odd ◇ (?)

This is *impossible*, because the sum of two odd numbers (including odd square numbers) is an even number, and an even number *cannot* equal an odd number.

B. odd even even

□ = ◇ + ◇ (?)

This is *impossible*, because the sum of two even numbers (or even square numbers) is an even number, and an even number *cannot* equal an odd number.

C. even even even

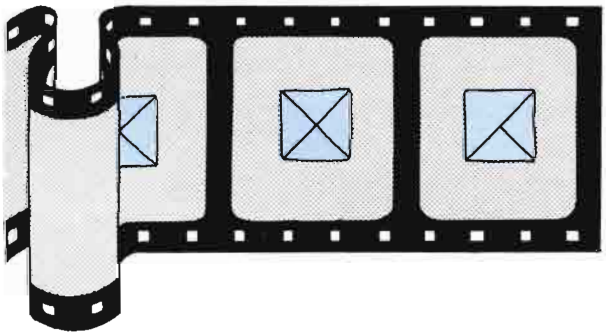
□ = ◇ + ◇ (?)

This *may be possible*, because the sum of two even numbers (or even square numbers) is an even number and an even number *can* equal an even number.

D. even odd odd

□ = ◇ + ◇ (?)

This *may be possible*, because the sum of two odd numbers (or odd square numbers) is an even number and an even number *can* be equal to an even number.



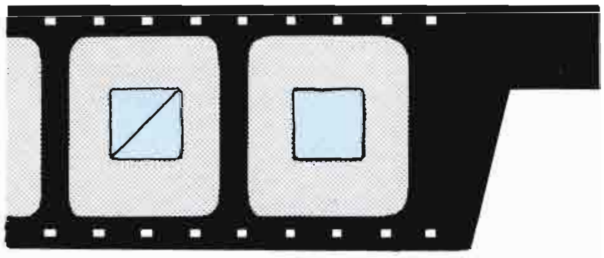
III. A look back at (C) EVEN = EVEN + EVEN

even even even

□ = ◇ + ◇

We can divide each of the square numbers by 4, because remember that:

1) all even square numbers can be divided by 4 with no remainder and
2) the result will, itself, be a square number, sometimes odd and sometimes even.



After we have divided each of the square numbers in our problem by 4, the resulting square numbers might both be even (but they might be odd).

144 ÷ 4 = 36   64 ÷ 4 = 16

If they are both even, then keep dividing both the larger and the smaller square number by 4 until . . .

Let's stop for a minute and try a few examples of divison of even square numbers by 4.

DIVIDE BY 4:

4 ÷ 4 = (1)
1

16÷4=4
4÷4=(1)
1

36÷4=(9)

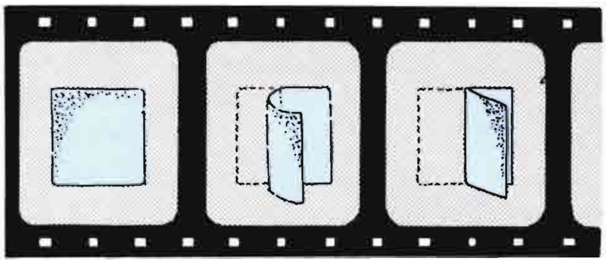
64÷4=16
16÷4=4
4÷4=(1)
1

100÷4=(25)
25

What do you see?

Every time we continue to divide an even square number by 4, sooner or later we finally get to an odd square number!

(Remember: 1 is an odd square number.)



So, in the case of our problem, "C", if we keep dividing by 4, sooner or later either the larger even square number or the smaller even square number will become *odd*; or both even square numbers may become *odd at the same time*.

But we have already found that:

B. $\overset{\text{odd}}{\square} = \overset{\text{even}}{\diamond} + \overset{\text{even}}{\diamond}$ is impossible

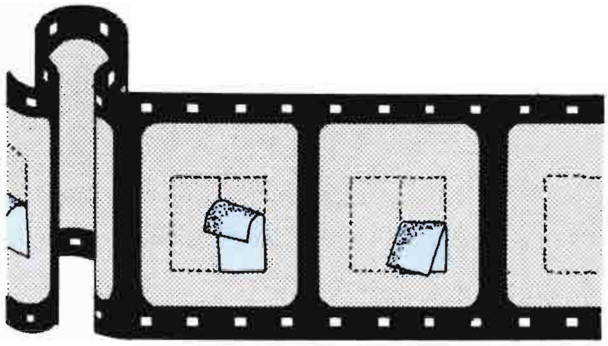
and that,

A. $\overset{\text{odd}}{\square} = \overset{\text{odd}}{\diamond} + \overset{\text{odd}}{\diamond}$ is impossible

but

$\overset{\text{even}}{\square} = \overset{\text{odd}}{\diamond} + \overset{\text{odd}}{\diamond}$ may be possible

So, this one result of dividing by 4 must be looked at further.



IV. In fact, the last result found above in (C) is exactly the same combination we find ready for us in (D);

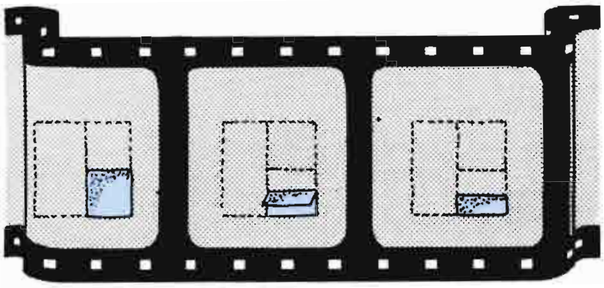
$\overset{\text{even}}{\square} = \overset{\text{odd}}{\diamond} + \overset{\text{odd}}{\diamond}$

So, if we can prove or disprove (D), we will also have proved or disproved the rest of (C) and we'll have completed the whole problem.

If we divide the numbers on one side of the "=" sign and if we divide the numbers on the other side of the "=" sign by the same number, both results will still be equal.

(÷ 2) 16 = 8 + 8
8 = 8
8 = 4 + 4

(÷2) 14 = 7 + 7
7 = 7
7 = 3½ + 3½



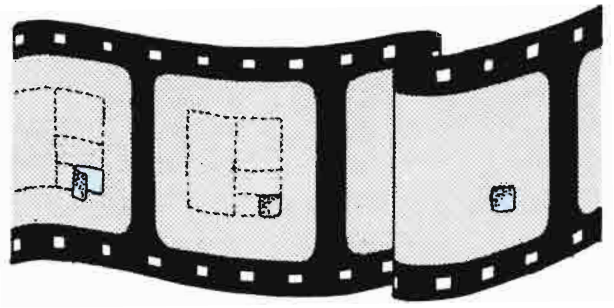
So, let's divide both the larger even square number and the sum of the odd square numbers in our problem by 2.

(a) Remember that the larger even square number can be divided by 4 with no remainder because it is "4 × (n × n)". So we can also divide it by 2, with no remainder, and *it will still be even*.

4 × (n × n) ÷ 2 = 2 × (n × n)

4 × (3 × 3) ÷ 2 = 2 × (3 × 3) (even)
4 × (2 × 2) ÷ 2 = 2 × (2 × 2) (even)
4 × (5 × 5) ÷ 2 = 2 × (5 × 5) (even)

$$\frac{4 \times (n \times n)}{2} = 2 \times (n \times n)$$



(b) But when we take half of the odd square numbers, this will leave only one of them—*still an odd number*.

$$\boxed{\text{even}} = \frac{\overset{\text{odd}}{\diamond} + \overset{\text{odd}}{\diamond}}{2} = \frac{2 \times \diamond}{2} = 1 \times \overset{\text{odd}}{\diamond} = \overset{\text{odd}}{\diamond} \ (?)$$

If (D) was, in fact, possible, then an even number would have to equal an odd number,

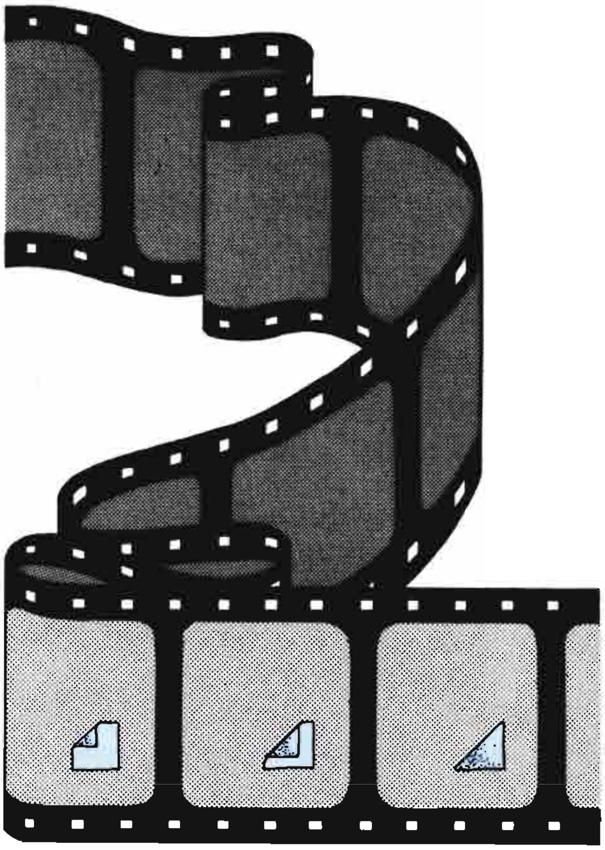
$$\overset{\text{even}}{\boxed{}} = \overset{\text{odd}}{\diamond}$$

which we have already said is clearly impossible. So, all combinations of odd and even square numbers for our problem

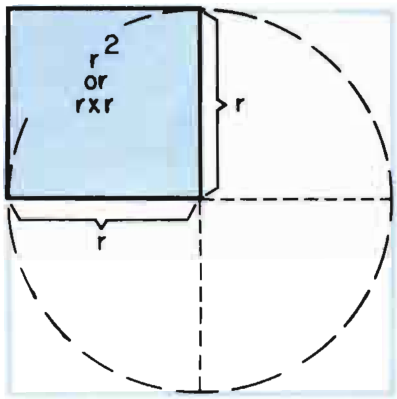
larger sq. no. smaller sq. no.
□ = 2 x ◇ or ◇ + ◇

have led to situations we know are not true. We have *proven* that there is *no* square number that is half as big as another square number.

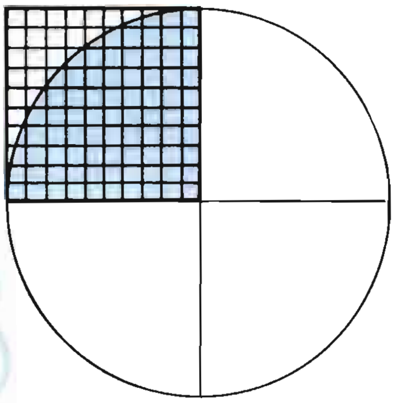
Q.E.D.

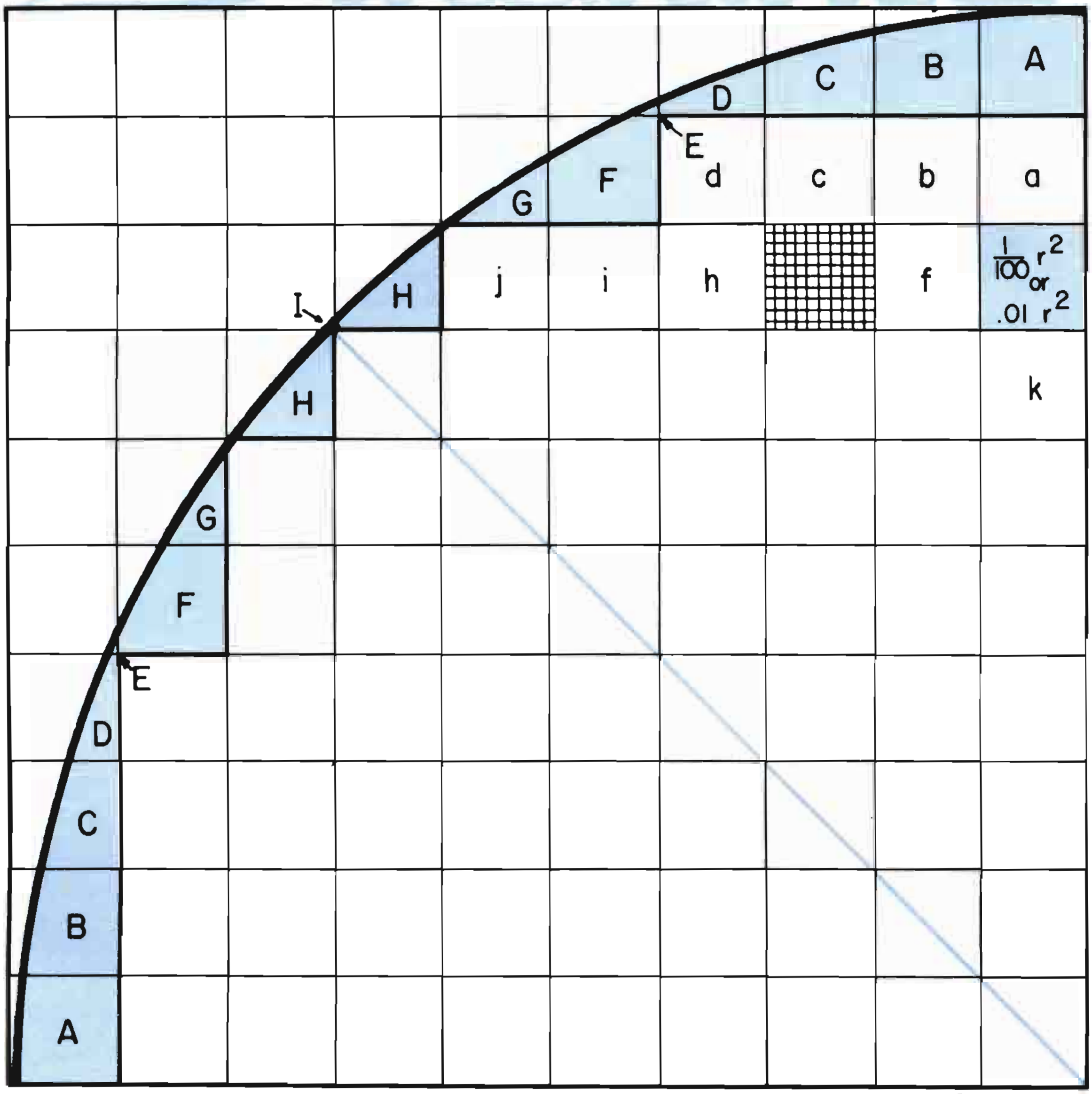


Area=___x "Radius Squared" (r^2)



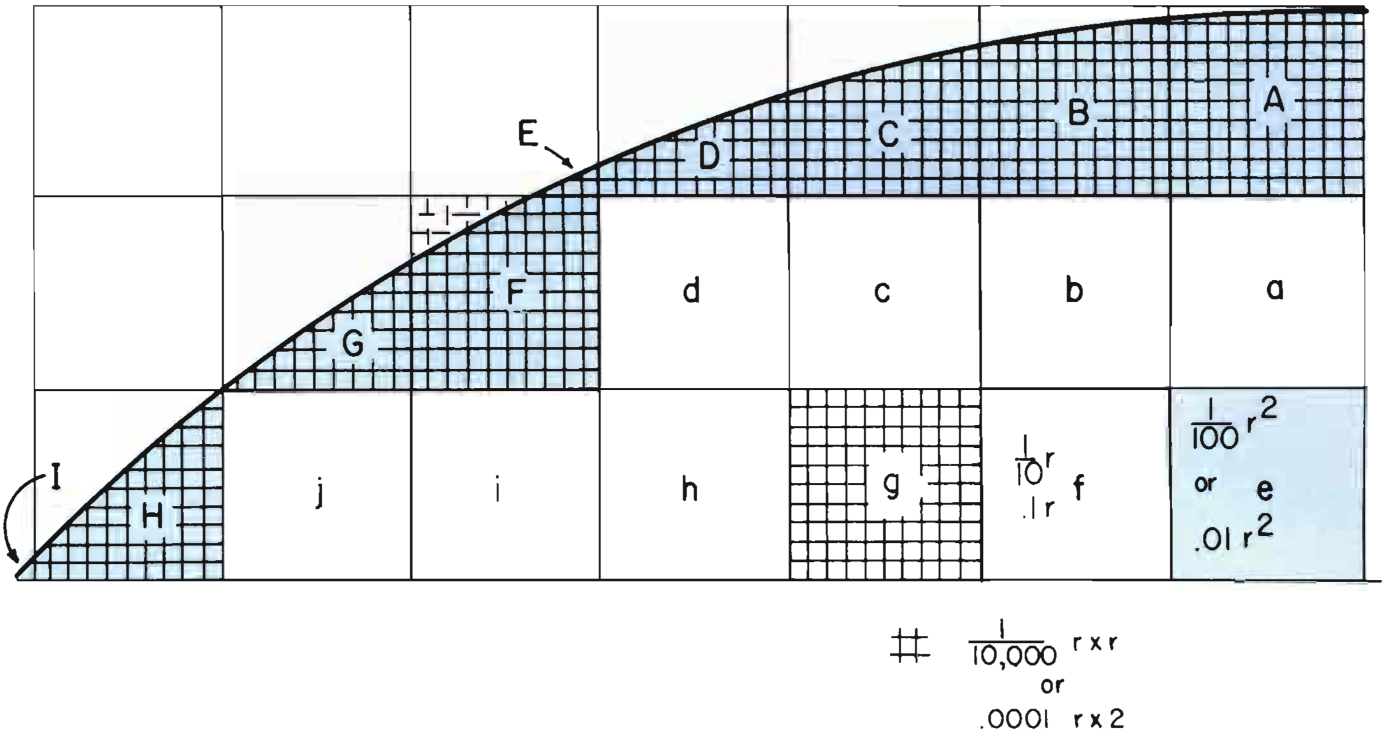
If "r^2" were cut into 100 little squares how many make up a quarter of the circle?





Does the sketch on the next page help with the problem? In r^2 there are 100x100 (10,000) very small squares — each is $\frac{1}{10,000}$ or .0001

This is another enlargement of a part of the sketch on the opposite page.



Each large square is .01 of r^2. There are 100 small squares in each large square.

How many large squares such as a, b, c, d, etc. are all inside the circle? ______ ; and each has 100 smaller squares; or a total of ________ smaller squares.

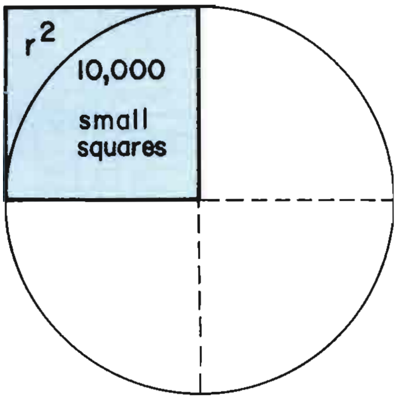
A diagonal in the sketch (up and to the left, shows that it cuts the shaded area in half. So we can consider A only once, and double our estimates. The same procedure can be followed with B, C, D, E, F, C, and H: area I will be counted only once.

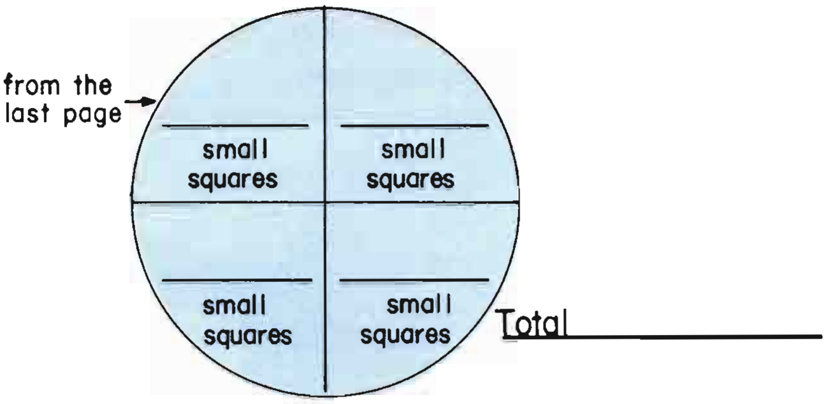
Please fill in the record on the right showing what you find.

In each shaded area labelled A, B, C, etc., how many small squares are there?

you will need to make estimates or careful guesses.

	squares shaded	twice as many
A		
B		
C		
D		
E		
F		
G		
H		
	I	
Total shaded area		
Total unshaded area		
Grand Total		





$\frac{\text{Total}}{10,000}$ = ________ ←(An approximation)

If our sketches had been drawn more accurately and our estimation done with more care, our results would have been closer to 3.1416 . . . and that is still only an "approximation."

This relationship between the radius squared and the area of a circle has been given a sign "π", the Greek letter (pronounced as we pronounce "pie").

(It cannot be written exactly as a decimal or common fraction no matter how many places nor how large a denominator.)

Area = πr^2

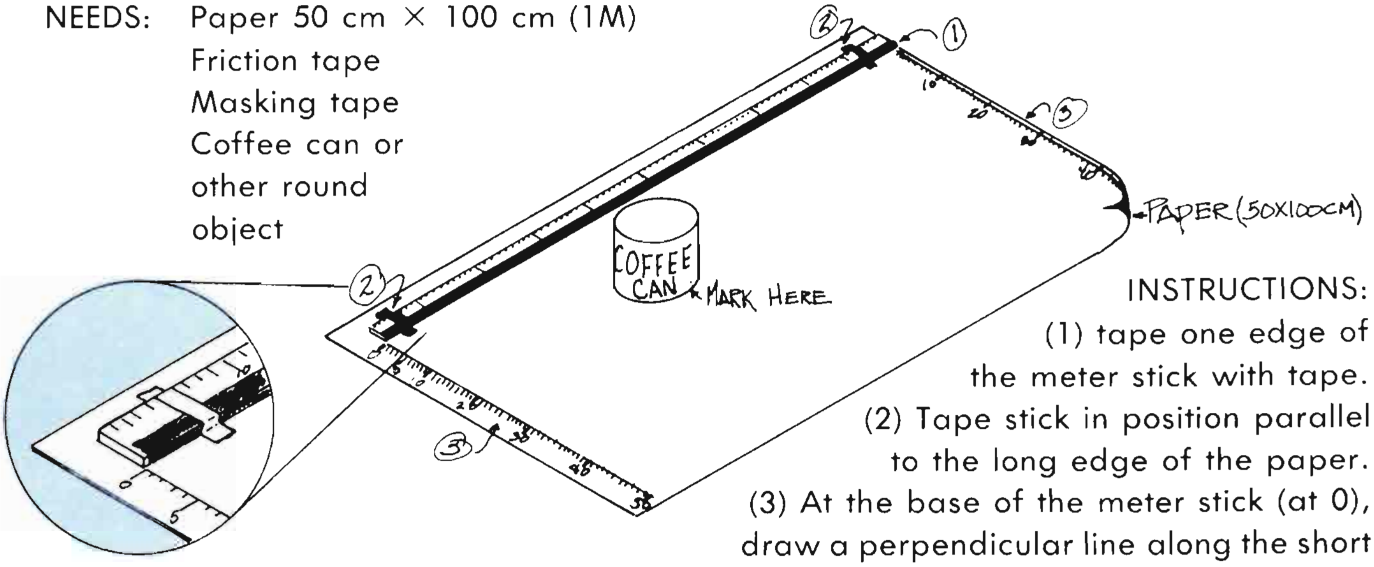
In these examples use 3.14 or $3^1/_7$ ($^{22}/_7$) as an approximation for π.

(This relationship comes up again: The circumference of a circle is π × its diameter).

	diameter	radius	r^2 radius squared	Area
a	ft.	10 ft.	sq. ft.	sq. ft.
b	m.	25 m.	sq. m.	sq. m.
c	100 feet	ft.	sq. ft.	sq. ft.
d	18 m.	m.	sq. m.	sq. m.
e	1 mile	mile	sq. miles	sq. miles

(Remember: your results will always be "approximations")

NEEDS: Paper 50 cm × 100 cm (1M)
Friction tape
Masking tape
Coffee can or
other round
object



INSTRUCTIONS:
(1) tape one edge of the meter stick with tape.
(2) Tape stick in position parallel to the long edge of the paper.
(3) At the base of the meter stick (at 0), draw a perpendicular line along the short edge and mark centimeters. Repeat at the other end.

DIAMETER and CIRCUMFERENCE

THE EXPERIMENT:

a. Place an object with a circular base in a position so that its diameter can be marked through the center on the base line (as shown below).

b. Do the same and make a mark on the other base line at the top.

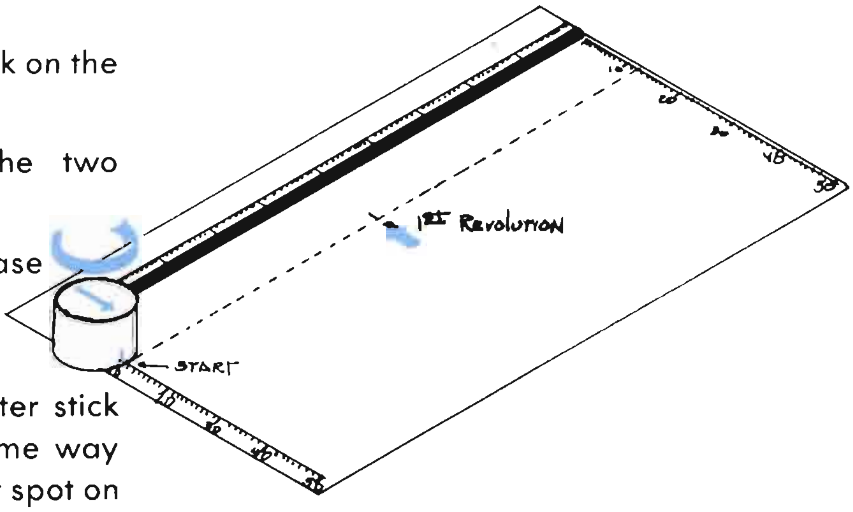
c. Draw a line connecting the two marks you made.

d. Place the object on the base line again so that it is in position as shown to start.

e. Roll the object along the meter stick until the object faces the same way as at the start, then mark that spot on the line.

f. Record the distance along the line.

HINT:
Put an arrow on the top of the can that you can watch as the can revolves.



TRY SOME OF THESE:

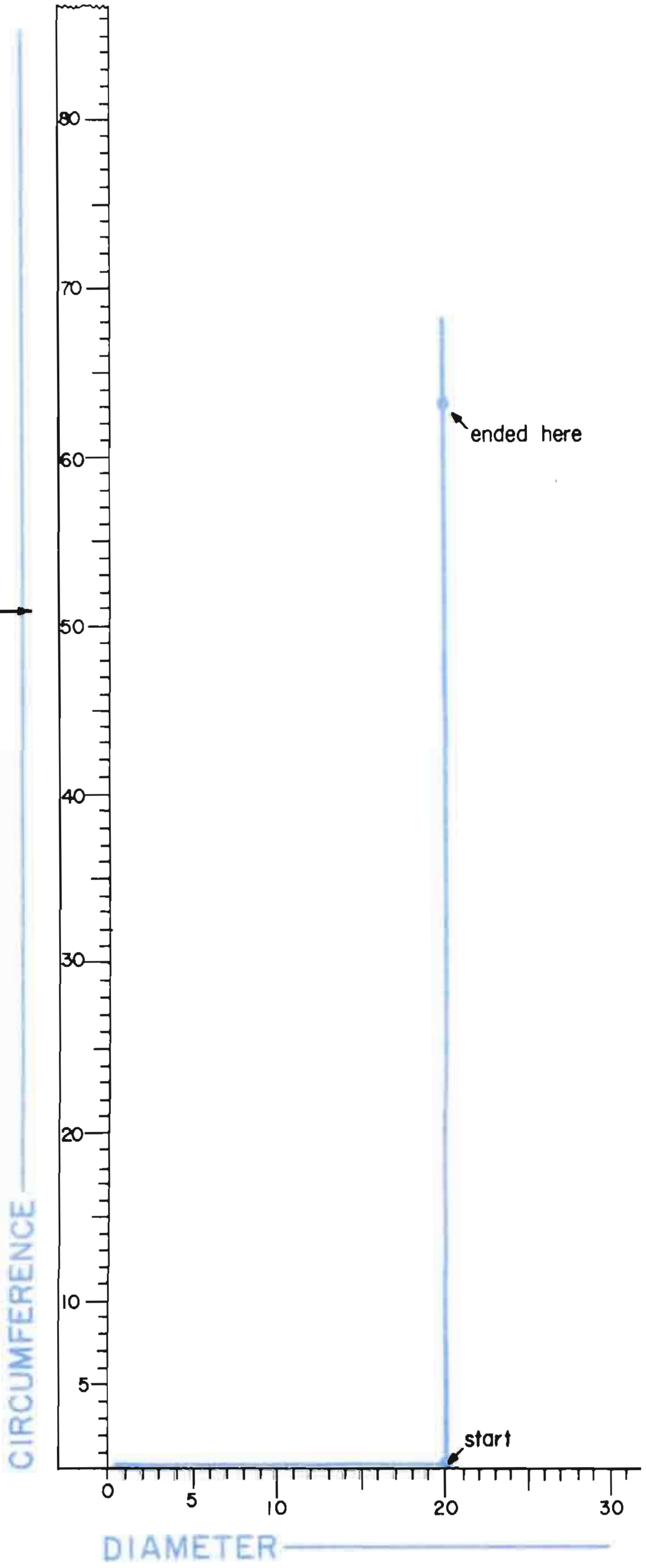
Soda bottle, baseball, spool of thread, frisbee—find some others on your own.

Repeat the experiment with objects of varying diameters, some quite small, others with diameters as much as 30 centimeters.

The "starting points" all lie along the "base line".

What can you say about the "end points"?

Please transfer the results from your experiment to the reduced sketch on this page.



Also, please show 6 of your results in the chart below, and find "circumference" divided by the "diameter" in each.

	Diameter	Circumference	C ÷ D
1			
2			
3			
4			
5			
6			

Total of 3rd Column

_______ ÷ 6 = _______

Total Average

This relationship is indicated by the symbol "π" a Greek letter (pronounced "pie").

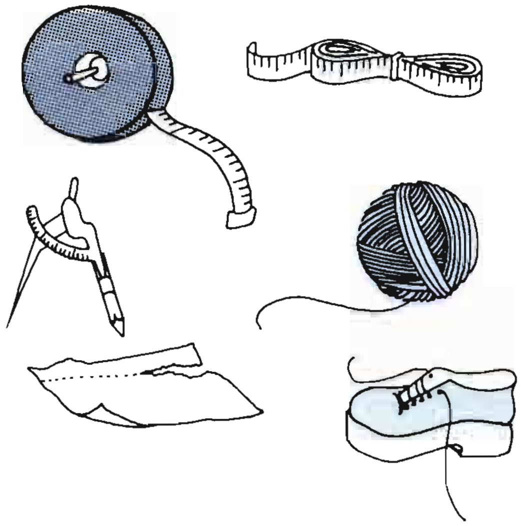
CIRCUMFERENCE ÷ DIAMETER = π

Please measure the distance around 10 objects, and find the diameter of each. Keep records in the table provided.

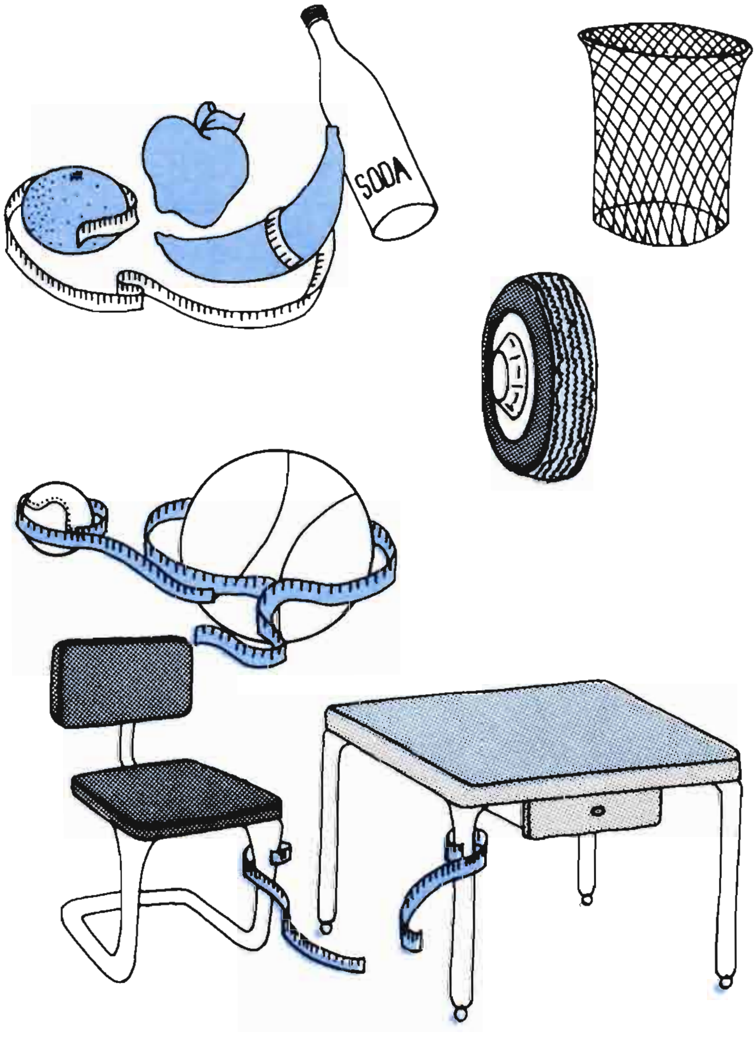
Then find the relationship between the circumference and diameter in each case: C ÷ D =

Choose your own methods. We will suggest a few:

USE THESE



TO MEASURE THESE...



Diameter	Circumference	C ÷ D

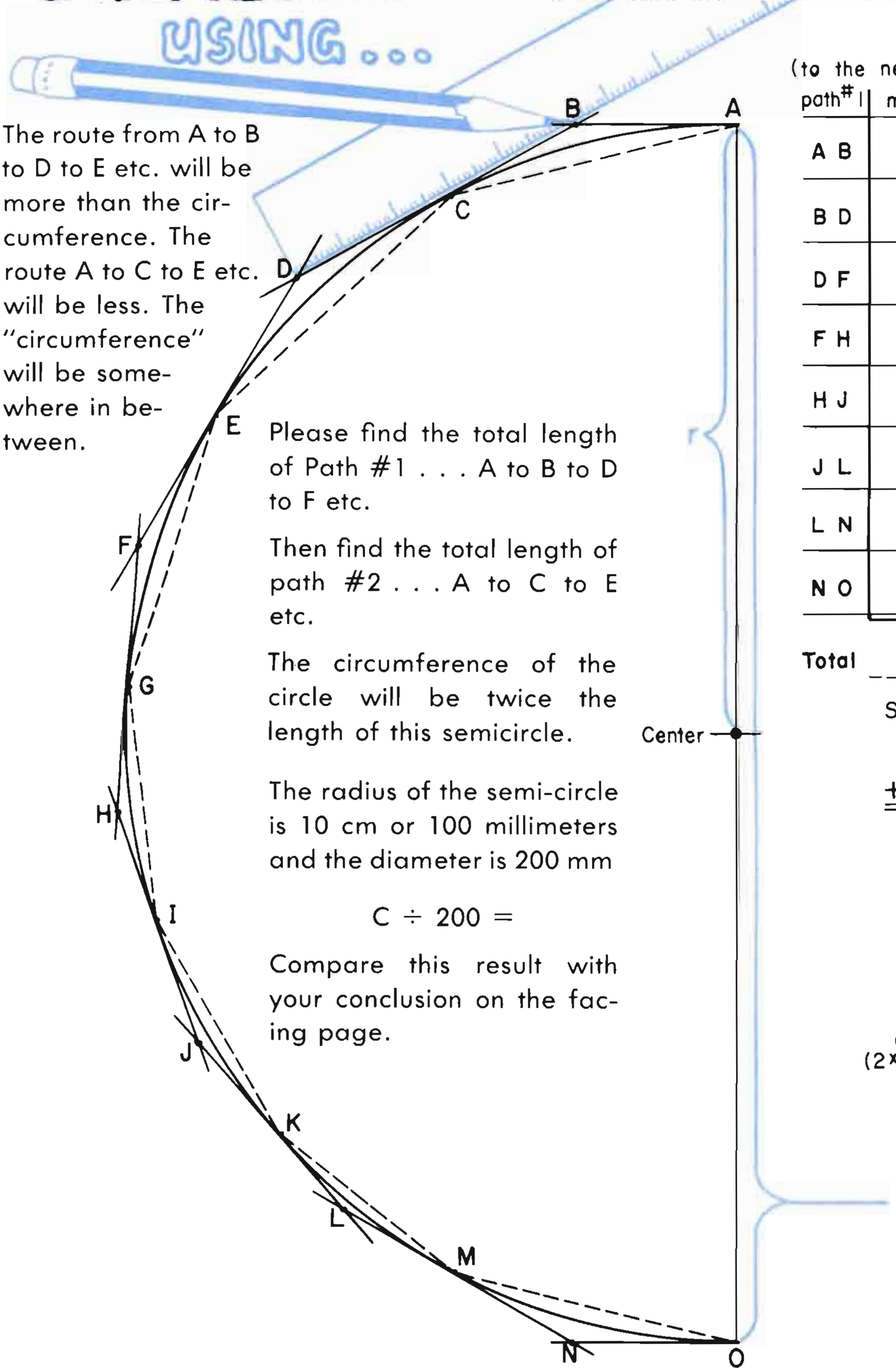
Total of 3^{rd} Column ________

________ ÷ 10 ________

Total Average

DIAMETER and CIRCUMFERENCE

USING . . .



The route from A to B to D to E etc. will be more than the circumference. The route A to C to E etc. will be less. The "circumference" will be somewhere in between.

Please find the total length of Path #1 . . . A to B to D to F etc.

Then find the total length of path #2 . . . A to C to E etc.

The circumference of the circle will be twice the length of this semicircle.

The radius of the semi-circle is 10 cm or 100 millimeters and the diameter is 200 mm

C ÷ 200 =

Compare this result with your conclusion on the facing page.

(to the nearest millimeter)

path# 1	mm
A B	
B D	
D F	
F H	
H J	
J L	
L N	
N O	

path#2	mm
A C	
C E	
E G	
G I	
I K	
K M	
M O	

Total ____ mm

Total ____ mm

Sum of Totals

______ mm

+ ______ mm

______ mm

Sum ÷ 2

______ mm

Circumference (C)
(2 × Length of semi-circle)

______ mm

Diameter (D)

______ mm

C ÷ D

______

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
2	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	
3	0	3	6	9	12	15										
4	0	4	8	12	16											
5	0	5	10	15												
6	0	6	12													
7	0	7														
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10	0	10														
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14	0	14														
15	0	15														
16	0	16														
17	0	17														
18	0	18														
19	0															
20	0															
21	0															
22	0															

IN SEARCH OF "TWICE ONLY NUMBERS"

A large "multiplication chart" was discovered on an island. It had been constructed by some group—perhaps before pocket computers were handy.
It showed the products of all pairs of whole number factors, 0 through 100—from 0 × 0 = 0 to 100 × 100 = 10,000.

You could pick any number 0 through 100 along the top, and any second number 0 through 100 along the side, then find the box where the column selected crossed the row selected and there it was—the product.

There were 101 columns and 101 rows—10,201 entries . . . and that's clear evidence it had to be done by some group. Even with a group of twenty working on it, you can find out for yourself that each member had to find lots of products. (About 500 each.)

So, perhaps several groups organized the work to complete such an enormous task. Or perhaps they found some short cuts.

(You might like to find out about how many digits had to be written . . . with products 0 through 10,000. Some products were one digit and 10,000 has five digits. It was a big job, anyway.)

"Look at all those times 0 had to be written as a factor," Jane said, "all across the top and all the way down the side–because 0 times any number equals 0 and any number times 0 equals 0."

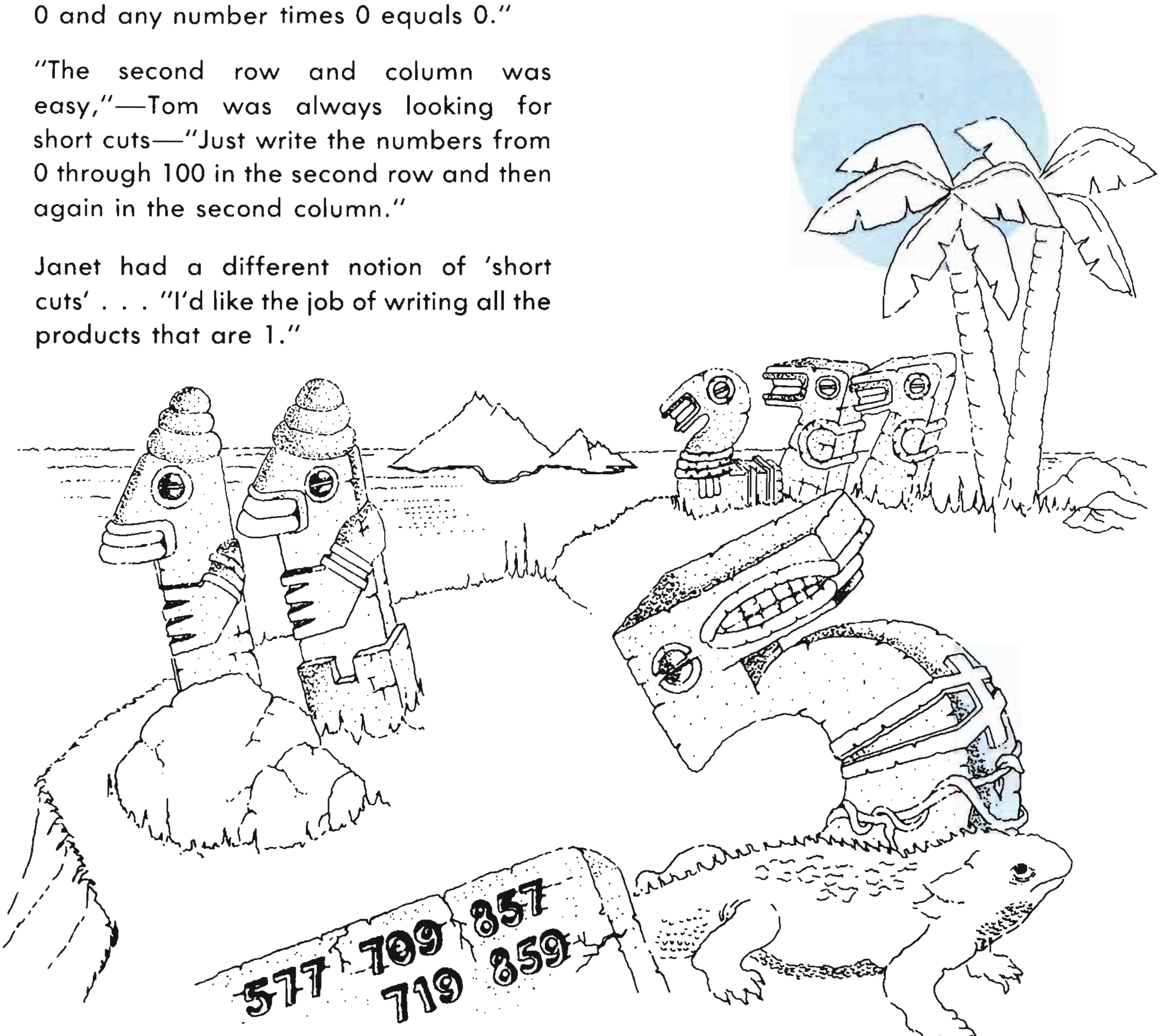
"The second row and column was easy,"—Tom was always looking for short cuts—"Just write the numbers from 0 through 100 in the second row and then again in the second column."

Janet had a different notion of 'short cuts' . . . "I'd like the job of writing all the products that are 1."

"Sure," taunted Sue, "it only appears once."

Carl wanted "2 for my job," since there are only two 2's as products in the whole chart.

"How many other 'twice only' numbers appear as products in the chart? . . .or is 2 the only one?" asked Wanda.



"No, 3 is up there only twice."

"Yes , but 4 appears 3 times—1 × 4 = 4, 2 × 2 = 4 and 4 × 1 = 4."

"But then 5 is a 'twice only' number."

"We've already found three 'twice only' numbers—2, 3 and 5. I think there must be a lot more."

"Well, all numbers larger than 1," Jefferey announced in his usual drawl, "are going to appear as products at least twice—once in the second row . . . 2, 3, 4, 5, 6, 7, etc., and once in the second column . . . 2, 3, 4, 5, 6, 7—that is, as far as 100."

Juanita was a dreamer: "But what Jefferey is saying would be true for all numbers if the chart went on beyond 100 × 100 . . . on and on and on . . . into the Great Beyond."

"How many of those 'twicers' will be 'twice only' like 2, 3 and 5?" was Wanda's question. "Let's make a list of 'twice only' numbers—numbers that will appear twice and only twice if the chart was enlarged to a million."

Harry was selected as recorder, and the list grew:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 . . .

"Oh, I get it, we're looking for all numbers that can only be written as the product of only two different numbers, themselves and one."

$$\boxed{\begin{array}{c}1 \times 2 = 2\\ 2 \times 1 = 2\end{array}} \qquad \boxed{\begin{array}{c}1 \times 3 = 3\\ 3 \times 1 = 3\end{array}}$$

$$\begin{array}{c}1 \times 4 = 4\\ 2 \times 2 = 4\\ 4 \times 1 = 4\end{array} \qquad \boxed{\begin{array}{c}1 \times 5 = 5\\ 5 \times 1 = 5\end{array}}$$

$$\begin{array}{c}1 \times 6 = 6\\ 2 \times 3 = 6\\ 3 \times 2 = 6\\ 6 \times 1 = 6\end{array} \qquad \boxed{\begin{array}{c}1 \times 7 = 7\\ 7 \times 1 = 7\end{array}}$$

$$\begin{array}{c}1 \times 8 = 8\\ 2 \times 4 = 8\\ 4 \times 2 = 8\\ 8 \times 1 = 8\end{array} \qquad \begin{array}{c}1 \times 9 = 9\\ 3 \times 3 = 9\\ 9 \times 1 = 9\end{array}$$

"The products with the boxes around them are 'twicers', and they are called 'prime numbers'."

Starry-eyed Juanita was back again: "And do prime numbers continue forever out into the Great Beyond? . . . primes forever?"

"Twice only" numbers turn up to be primes, but now Juanita has really raised a serious question. Do "twice only" numbers or "primes" go on forever? Is there a largest "twice only" number, a largest "prime"?

enter smart sam

At this point, I want to again call up my imaginary character, Smart Sam, who brings me a long list of numbers.

"These are all the prime numbers there are," he says, "and here is the largest of the list—the largest prime there is. The number is . . ."

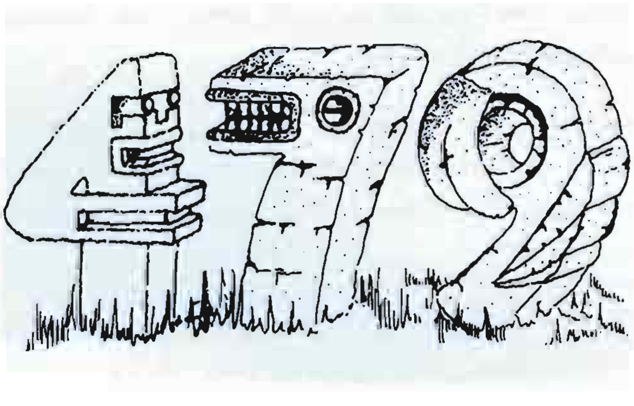
I break in: "No, don't tell me your number. Without knowing what your 'largest prime' is, I'm going to show you there is a larger one.

"I want you to think of a multiplication sign between each prime and its prime neighbor. So your list would begin like this:

2 × 3 × 5 × 7 × 11 × 13 . . . . . × YLP

"The dots (. . .) stand for all the rest of your list—with 'your largest prime', YLP, at the end of the list."

"You mean I have to carry out all those multiplications? . . . it would take me a week."



"No," I explain, "unless you want to. It will be enough if you write it down this way: (2 × 3 × 5 × 7 . . . × YLP)

"I sure would prefer to do that."

"Could you add 1 and then call it (2 × 3 × 5 × 7 . . . × YLP+ 1)?"

"Of course."

"All right, Sam, are all the following statements true?"

$$\begin{array}{rl} & 1 \times 17 \\ 2 & \overline{)\,2 \times 17} \end{array} \qquad \begin{array}{rl} & 2 \times \ 1 \\ 17 & \overline{)\,2 \times 17} \end{array}$$

$$\begin{array}{rl} & 1 \times 11 \\ 3 & \overline{)\,3 \times 11} \end{array} \qquad \begin{array}{rl} & 1 \times \ 1 \\ 11 & \overline{)\,1 \times 11} \end{array}$$

$$\begin{array}{rl} & 1 \times 3 \quad R1 \\ 5 & \overline{)\,5 \times 3 + 1} \end{array} \qquad \begin{array}{rl} & 1 \times 5 \quad R1 \\ 3 & \overline{)\,3 \times 5 + 1} \end{array}$$

$$\begin{array}{rl} & 4 \times 1 \quad R1 \\ 7 & \overline{)\,4 \times 7 + 1} \end{array} \qquad \begin{array}{rl} & 1 \times 7 \quad R1 \\ 4 & \overline{)\,4 \times 7 + 1} \end{array}$$

$$\begin{array}{rl} & 1 \times 3 \times 5 \\ 2 & \overline{)\,2 \times 3 \times 5} \end{array} \qquad \begin{array}{rl} & 2 \times 1 \times 5 \\ 3 & \overline{)\,2 \times 3 \times 5} \end{array}$$

$$\begin{array}{rl} & 2 \times 3 \times 1 \quad R1 \\ 5 & \overline{)\,2 \times 3 \times 5 + 1} \end{array} \qquad \begin{array}{rl} & 2 \times 1 \times 5 \quad R1 \\ 3 & \overline{)\,2 \times 3 \times 5 + 1} \end{array}$$

$(2 \times 3) + 1 = 7$

$(2 \times 3 \times 5) + 1 = 31$

$(2 \times 3 \times 5 \times 7) + 1 = 211$

The products of each of the examples above have become a "new" prime when we added 1.

$$\begin{array}{r} 3 \times 5 \times 7 \\ 2\overline{)2 \times 3 \times 5 \times 7} \end{array}$$

Check $2 \times 3 \times 5 \times 7 = 210$

$3 \times 5 \times 7 = 105$

$$\begin{array}{r} 105 \\ 2\overline{)210} \end{array}$$

and

$$\begin{array}{r} 2 \times 1 \times 5 \times 7 \quad ^{R1} \\ 3\overline{)2 \times 3 \times 5 \times 7 + 1} \end{array} \text{ or } 2 \times 5 \times 7 \quad ^{R1}$$

Check $2 \times 3 \times 5 \times 7 + 1 = 211$

$2 \times 5 \times 7 = 70$

$$\begin{array}{r} 70 \quad ^{R1} \\ 3\overline{)211} \end{array}$$

In the form below, the dotted lines indicate that for every factor written under the division sign, the same factor is written above the sign as in the examples above.

$$\begin{array}{r} 3 \times 5 \times 7 \times 11 \times 13 \quad ^{R1} \\ 2\overline{)2 \times 3 \times 5 \times 7 \times 11 \times 13 + 1} \end{array}$$

is the same as

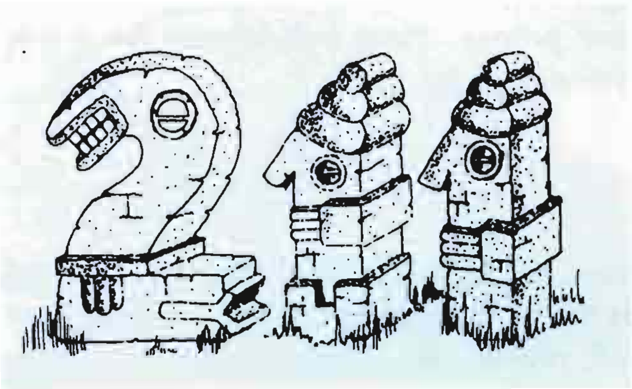
$$\begin{array}{r} 3 \times 5 \ldots 13 \quad ^{R1} \\ 2\overline{)2 \times 3 \times 5 \ldots 13 + 1} \end{array}$$

and

$$\begin{array}{r} 2 \times 5 \ldots 23 \quad ^{R1} \\ 3\overline{)3 \times 2 \times 5 \ldots 23 + 1} \end{array}$$

and

$$\begin{array}{r} 2 \times 3 \ldots 19 \quad ^{R1} \\ 5\overline{)5 \times 2 \times 3 \ldots 19 + 1} \end{array}$$

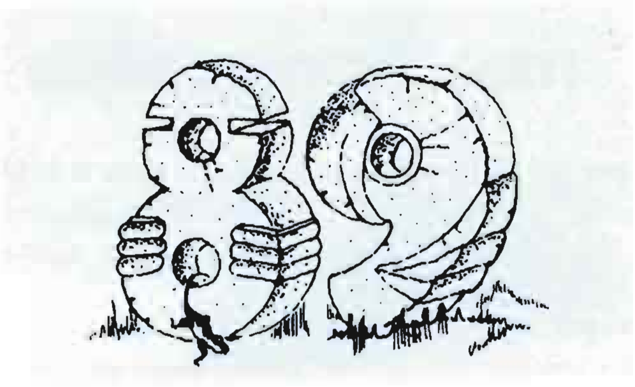


Suppose we let "YLP" stand for "your largest prime". We can indicate the product of all your primes . . . including "YLP" this way:

(product of primes)
$2 \times 3 \times 5 \times 7 \ldots \times \text{YLP}$

Next, we can select any of those primes to divide by, much as we have been doing and write the answer as we have been doing:

$$\begin{array}{r} 2 \times 5 \times 7 \ldots \text{YLP} \\ 3\overline{)3 \times 2 \times 5 \times 7 \ldots \text{YLP}} \end{array}$$



And there will never be a remainder. So each of the primes in the list will divide evenly into the number under the division sign. But, if we add 1 to the product of all your primes, there will always be a remainder of 1.

$$\begin{array}{l} \quad 2 \times 3 \times 7 \ldots \text{YLP} \qquad ^{R1} \\ 5\overline{)5 \times 2 \times 3 \times 7 \ldots \text{YLP} + 1} \end{array}$$

Now, any prime in your list could be used as the number to divide by . . . and there will always be a remainder . . . a remainder of 1.

So (2 × 3 × 5 × 7 . . . × YLP+ 1) is divisible with no remainder only by itself and 1—and is therefore prime. And every time you give me a list of all primes including "your largest prime" number, YLP, all I have to do is multiply all the primes and add 1 to the product. With this addition of 1 to your product, I will have a new "largest prime".*

"All right, I haven't found the largest prime yet, but I'll keep trying."

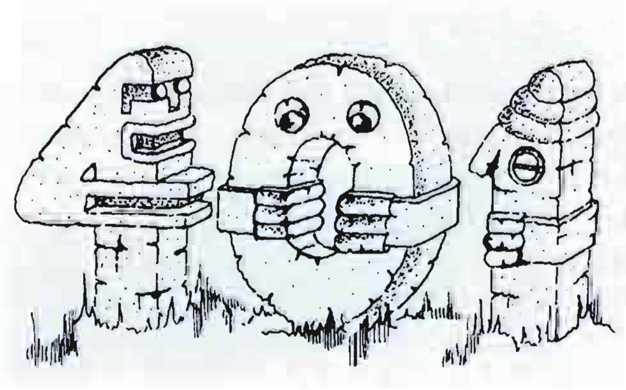
"Sam, remember, I didn't want to know what your largest prime is. If you find a new and larger prime and think it is the largest; bring it to me and I'll play you back the tape of this conversation . . . to prove there is still a larger prime.

"In fact, here is the tape—you can play it for yourself anytime you think you've found the largest prime."

Smart Sam took the tape. He shook his head as he walked away.

I didn't have the heart to tell him that Euclid, a famous Greek mathematician, proved there was no largest prime about 300 B.C. . . . and did it with arguments something like I did with Sam.

But then you already know Smart Sam is my favorite imaginary character, so he won't mind.



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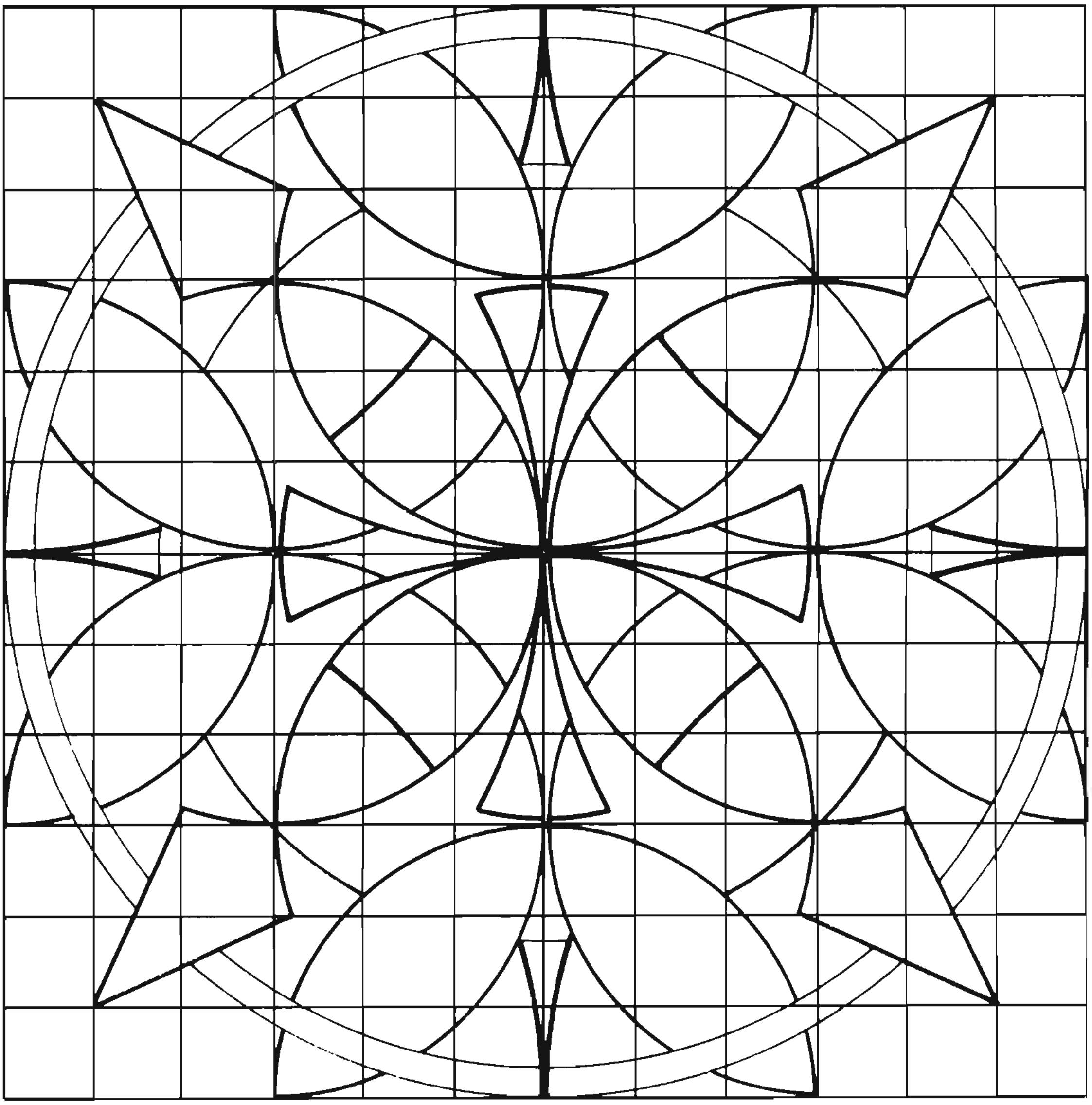
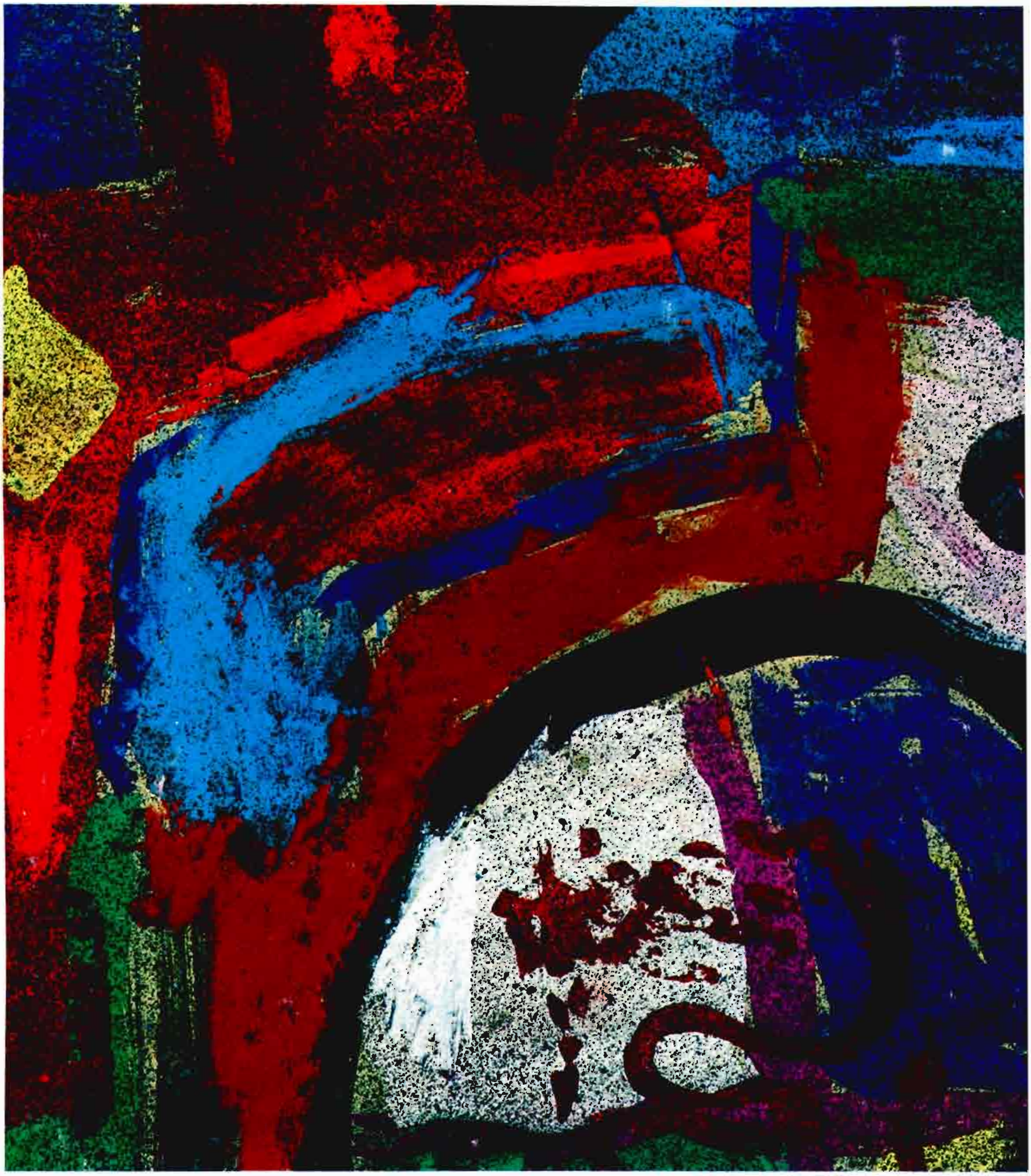
*Footnote: Unless, of course, that number 2 × 3 × 5 . . . × YLP+ 1 can be written as the product of other primes, all of which would have to be larger than any of the primes in the list.





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