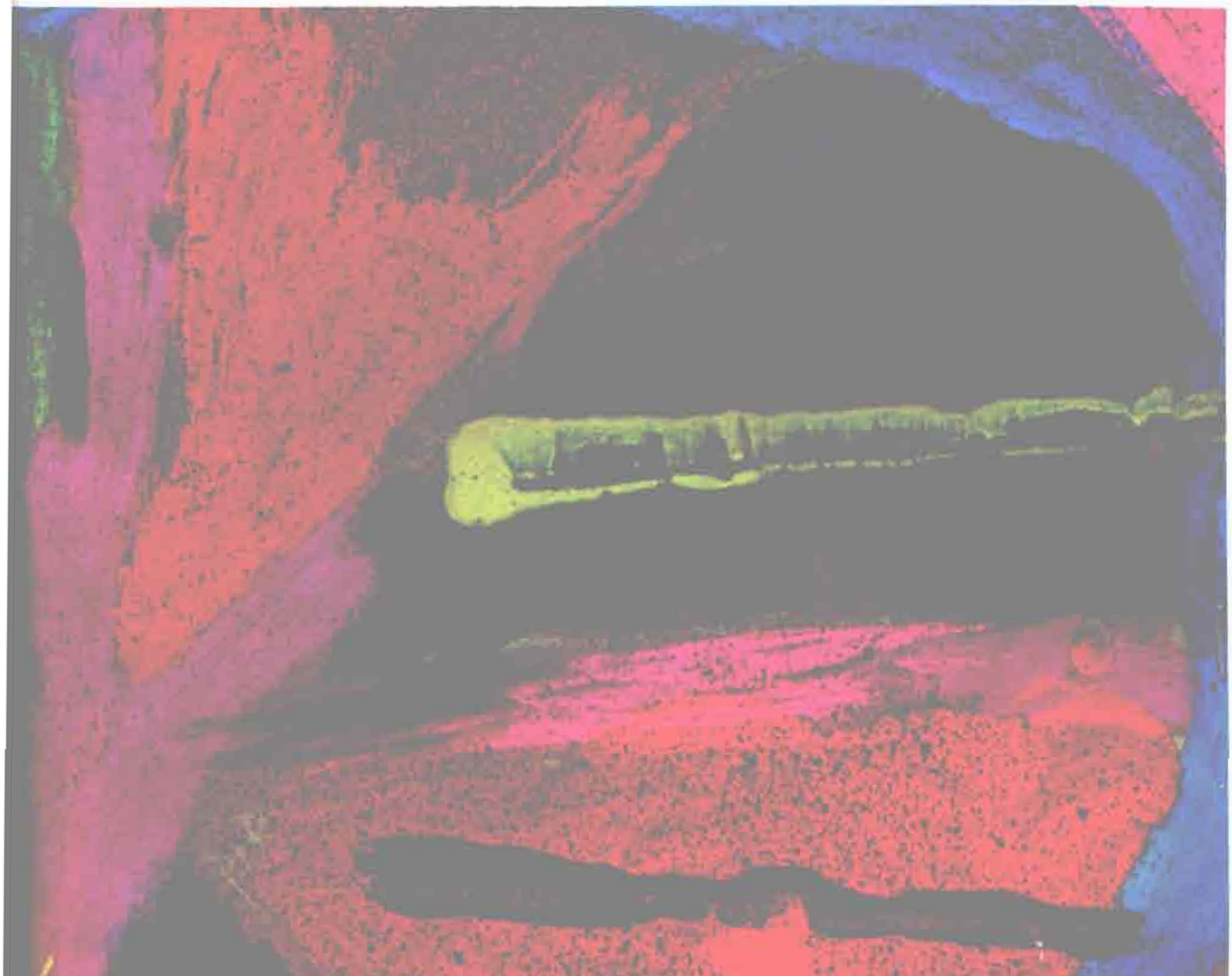

CDA math
ROBERT W. WIRTZ

answers & annotations

e

patterns & problems

(Jumping around in mathematics)

JUMPING AROUND IN MATHEMATICS

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The class of Pat Spencer at La Mesa School

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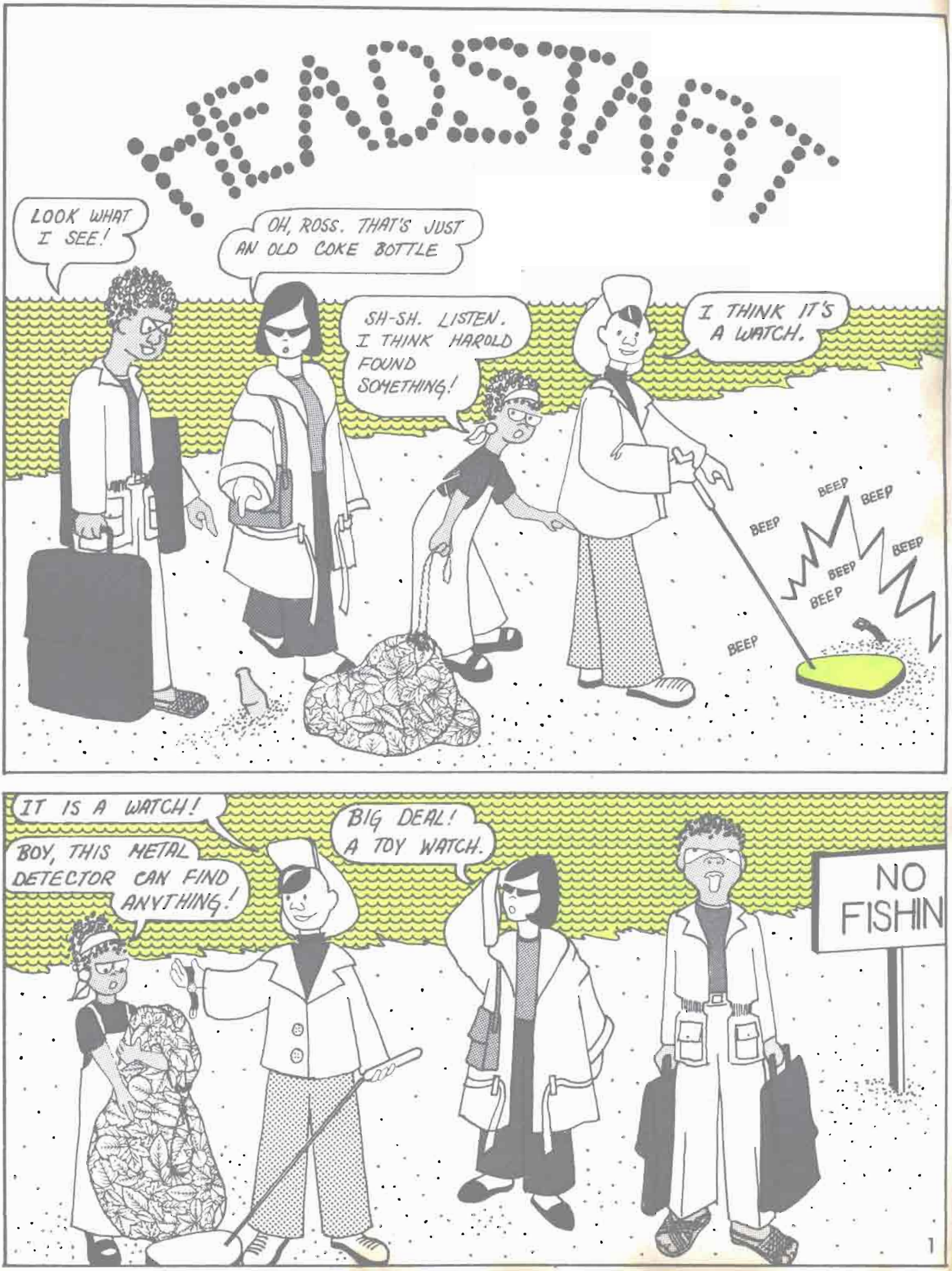
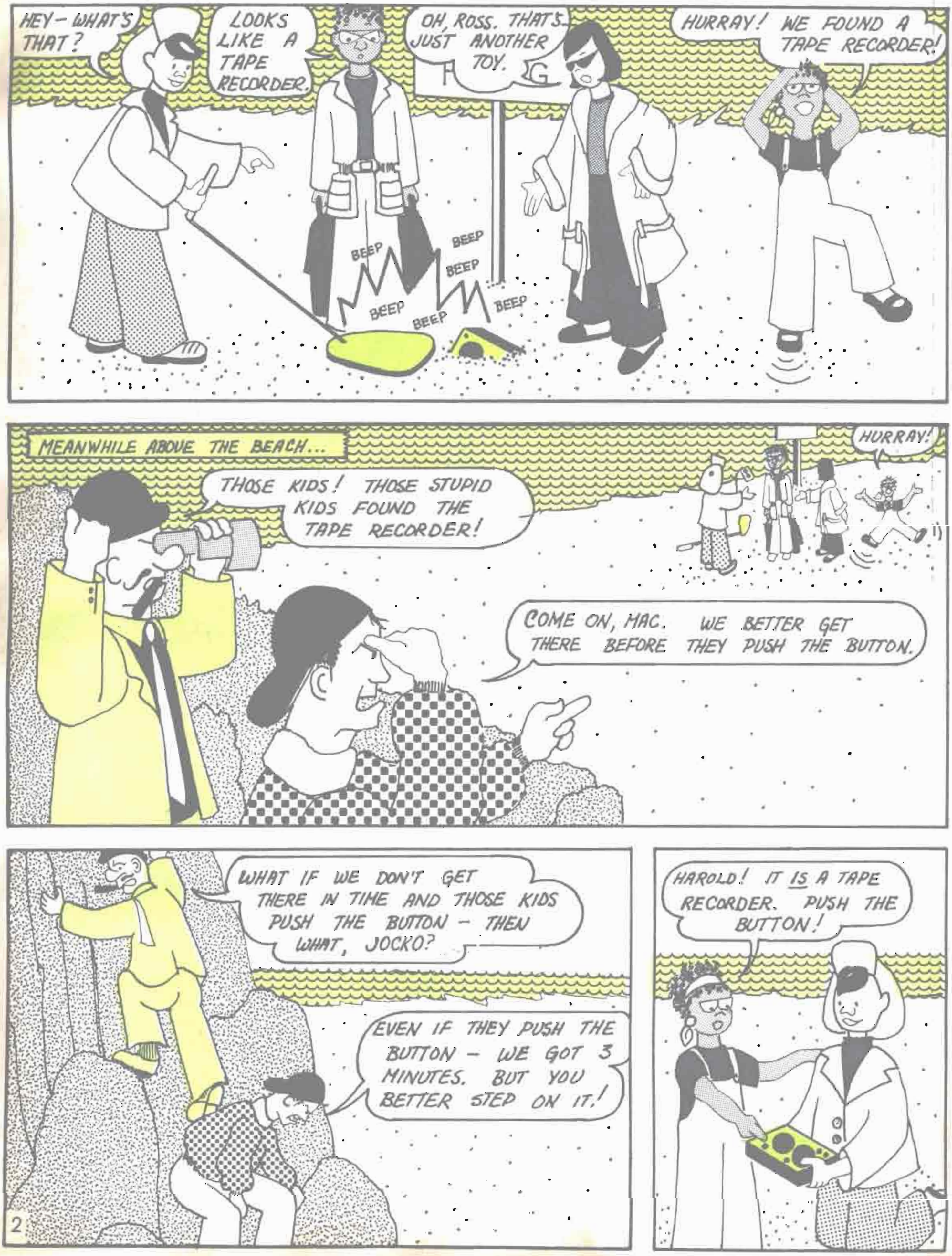
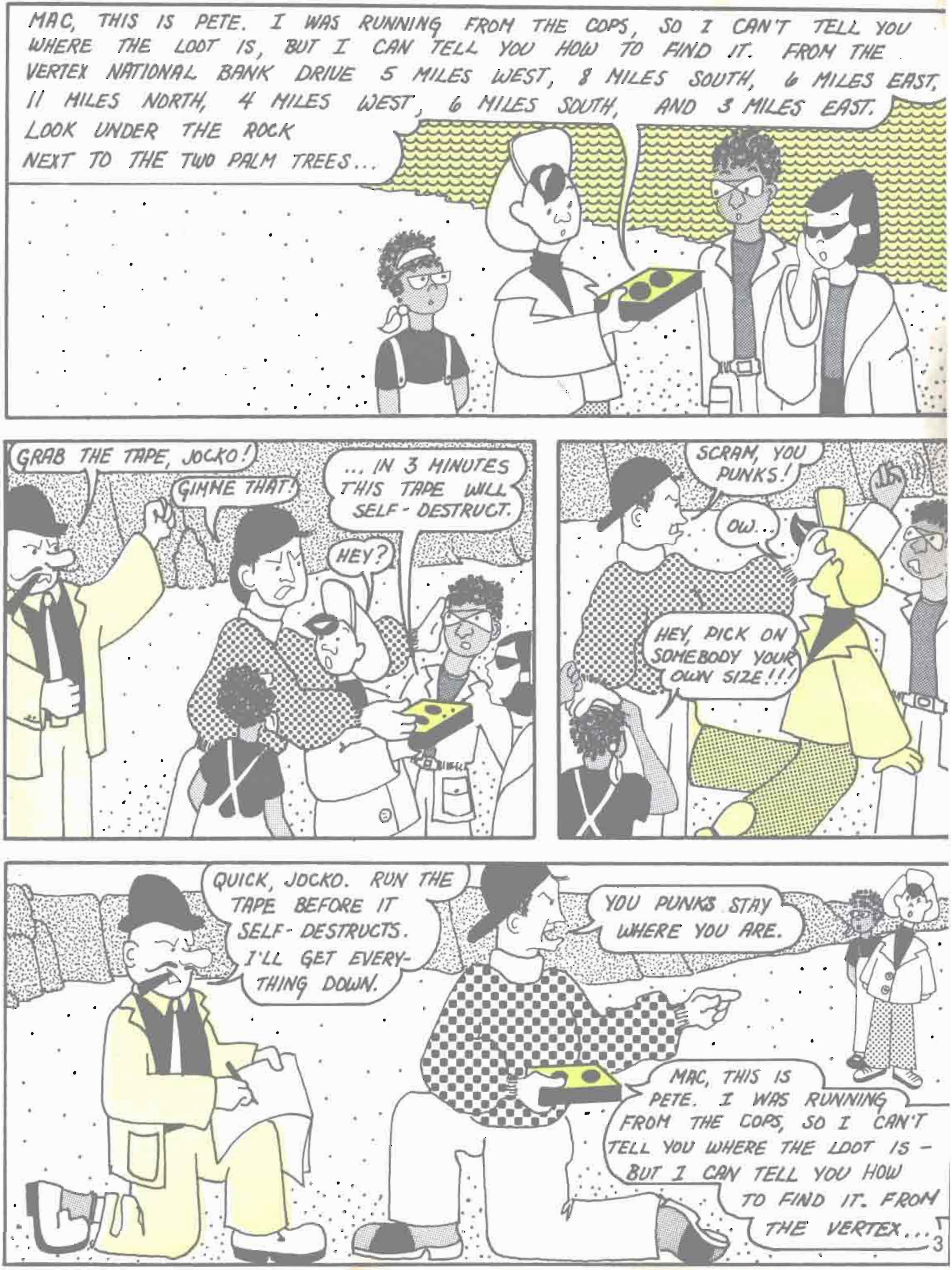
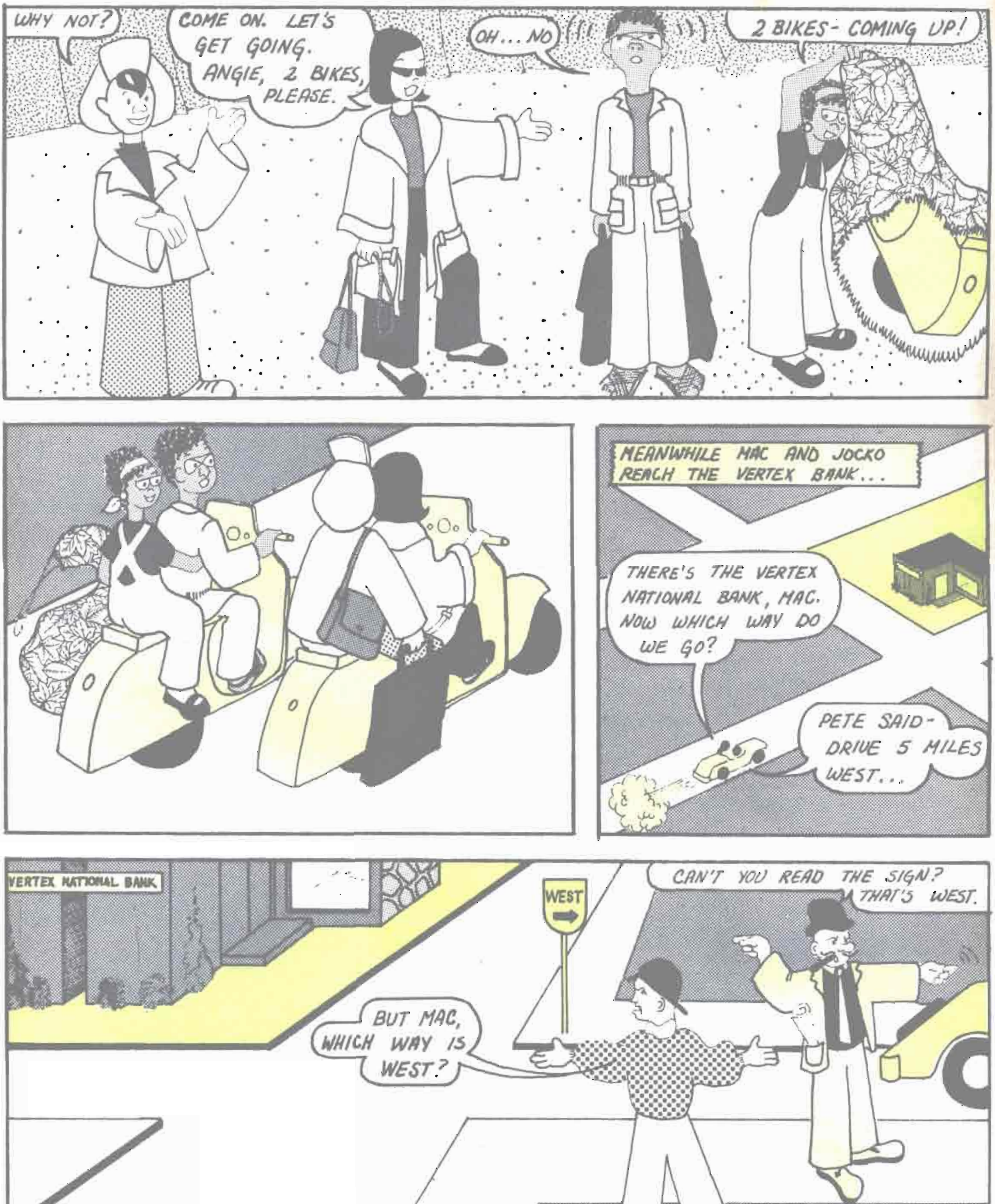
HEADSTART?

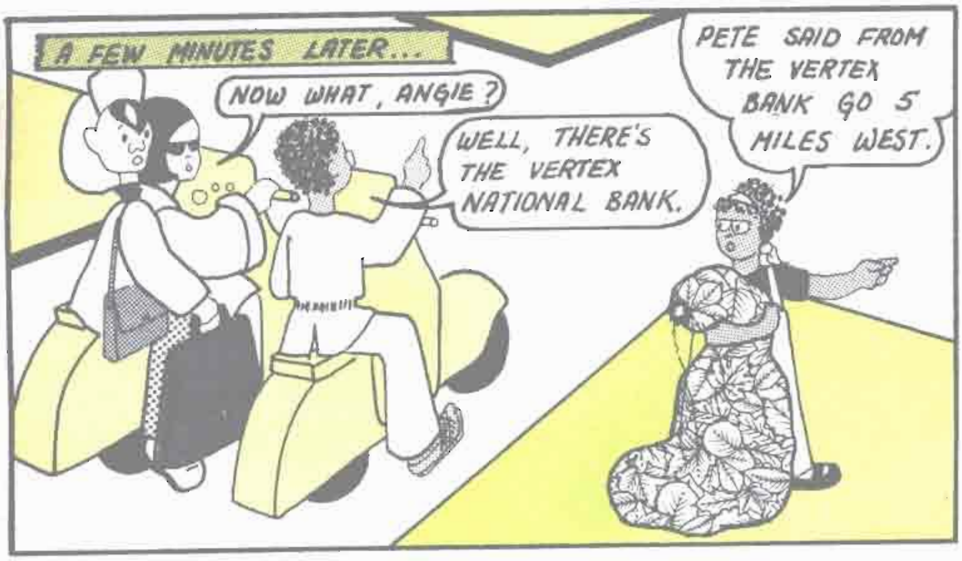

LOOK WHAT I SEE!
OH, ROSS. THAT'S JUST AN OLD COKE BOTTLE
SH-SH. LISTEN. I THINK HAROLD FOUND SOMETHING!
I THINK IT'S A WATCH.
BEEP BEEP BEEP BEEP BEEP BEEP BEEP
IT IS A WATCH!
BOY, THIS METAL DETECTOR CAN FIND ANYTHING!
BIG DEAL! A TOY WATCH.
NO FISHIN

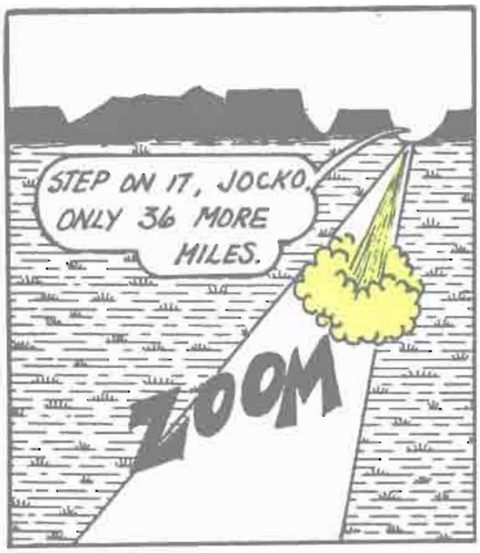

HEY — WHAT'S THAT?
LOOKS LIKE A TAPE RECORDER.
OH, ROSS. THAT'S JUST ANOTHER TOY.
HURRAY! WE FOUND A TAPE RECORDER!
BEEP BEEP BEEP BEEP BEEP BEEP
MEANWHILE ABOVE THE BEACH...
THOSE KIDS! THOSE STUPID KIDS FOUND THE TAPE RECORDER!
HURRAY!
COME ON, MAC. WE BETTER GET THERE BEFORE THEY PUSH THE BUTTON.
WHAT IF WE DON'T GET THERE IN TIME AND THOSE KIDS PUSH THE BUTTON — THEN WHAT, JOCKO?
EVEN IF THEY PUSH THE BUTTON — WE GOT 3 MINUTES. BUT YOU BETTER STEP ON IT!
HAROLD! IT IS A TAPE RECORDER. PUSH THE BUTTON!


MAC, THIS IS PETE. I WAS RUNNING FROM THE COPS, SO I CAN'T TELL YOU WHERE THE LOOT IS, BUT I CAN TELL YOU HOW TO FIND IT. FROM THE VERTEX NATIONAL BANK DRIVE 5 MILES WEST, 8 MILES SOUTH, 6 MILES EAST, 11 MILES NORTH, 4 MILES WEST, 6 MILES SOUTH, AND 3 MILES EAST. LOOK UNDER THE ROCK NEXT TO THE TWO PALM TREES...
GRAB THE TAPE, JOCKO!
GIMME THAT!
... IN 3 MINUTES THIS TAPE WILL SELF-DESTRUCT.
HEY?
SCRAM, YOU PUNKS!
OW...
HEY, PICK ON SOMEBODY YOUR OWN SIZE!!!
QUICK, JOCKO. RUN THE TAPE BEFORE IT SELF-DESTRUCTS. I'LL GET EVERYTHING DOWN.
YOU PUNKS STAY WHERE YOU ARE.
MAC, THIS IS PETE. I WAS RUNNING FROM THE COPS, SO I CAN'T TELL YOU WHERE THE LOOT IS - BUT I CAN TELL YOU HOW TO FIND IT. FROM THE VERTEX...

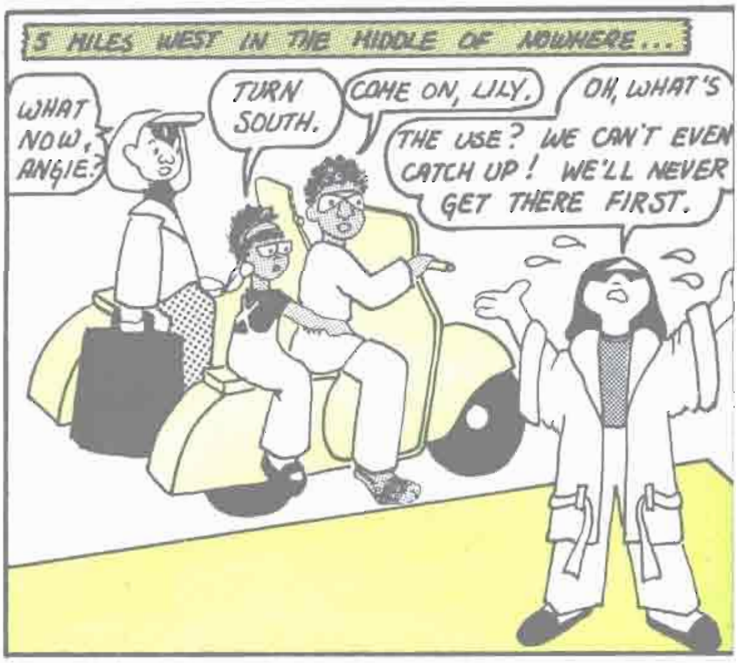

THAT'S IT. LET'S GO, JOCKO.
OK, MAC. I'LL DRIVE.
... IN 3 MINUTES- PFFT
ZOOM
THERE THEY GO. I GUESS THOSE GUYS MUST BE IN PETE'S GANG.
YEAH, TOO BAD YOU DIDN'T WRITE DOWN WHAT PETE SAID.
ME?
NEVER FEAR. ANGIE IS HERE.
FROM THE VERTEX NATIONAL BANK DRIVE 5 MILES WEST, 8 MILES ...
ANGIE, YOU'RE A GENIUS.
BIG DEAL! SO WHAT IF WE HAVE DIRECTIONS. THEY HAVE A HEADSTART.
WAIT A MINUTE. ARE WE GOING TO TRY TO FIND THE MONEY, TOO?


WHY NOT?
COME ON. LET'S GET GOING. ANGIE, 2 BIKES, PLEASE.
OH... NO
2 BIKES - COMING UP!
MEANWHILE MAC AND JOCKO REACH THE VERTEX BANK...
THERE'S THE VERTEX NATIONAL BANK, MAC. NOW WHICH WAY DO WE GO?
PETE SAID - DRIVE 5 MILES WEST...
VERTEX NATIONAL BANK
WEST
CAN'T YOU READ THE SIGN? THAT'S WEST.
BUT MAC, WHICH WAY IS WEST?


A FEW MINUTES LATER...
NOW WHAT, ANGIE?
WELL, THERE'S THE VERTEX NATIONAL BANK.
PETE SAID FROM THE VERTEX BANK GO 5 MILES WEST.

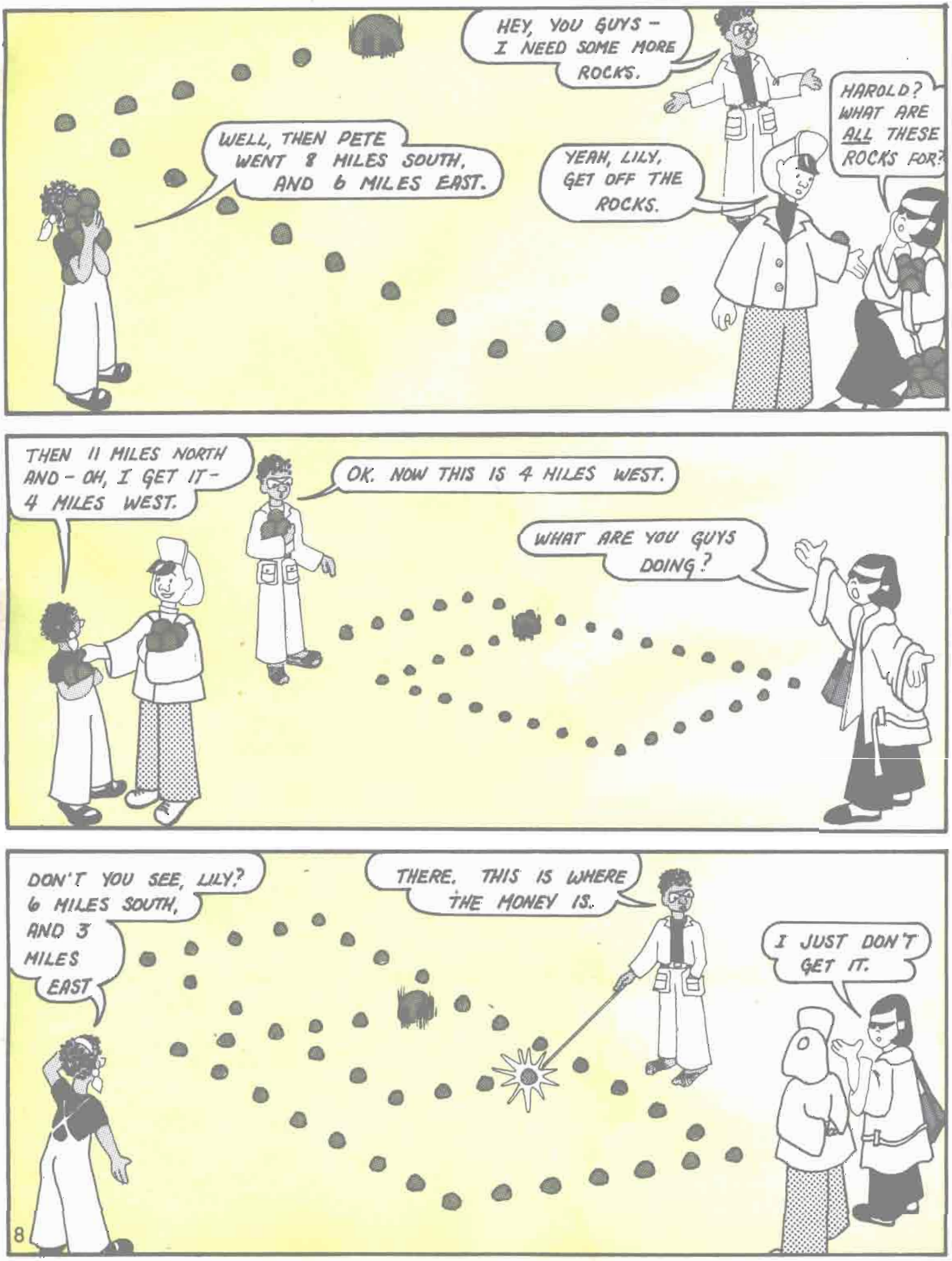

STEP ON IT, JOCKO, ONLY 36 MORE MILES.
ZOOM

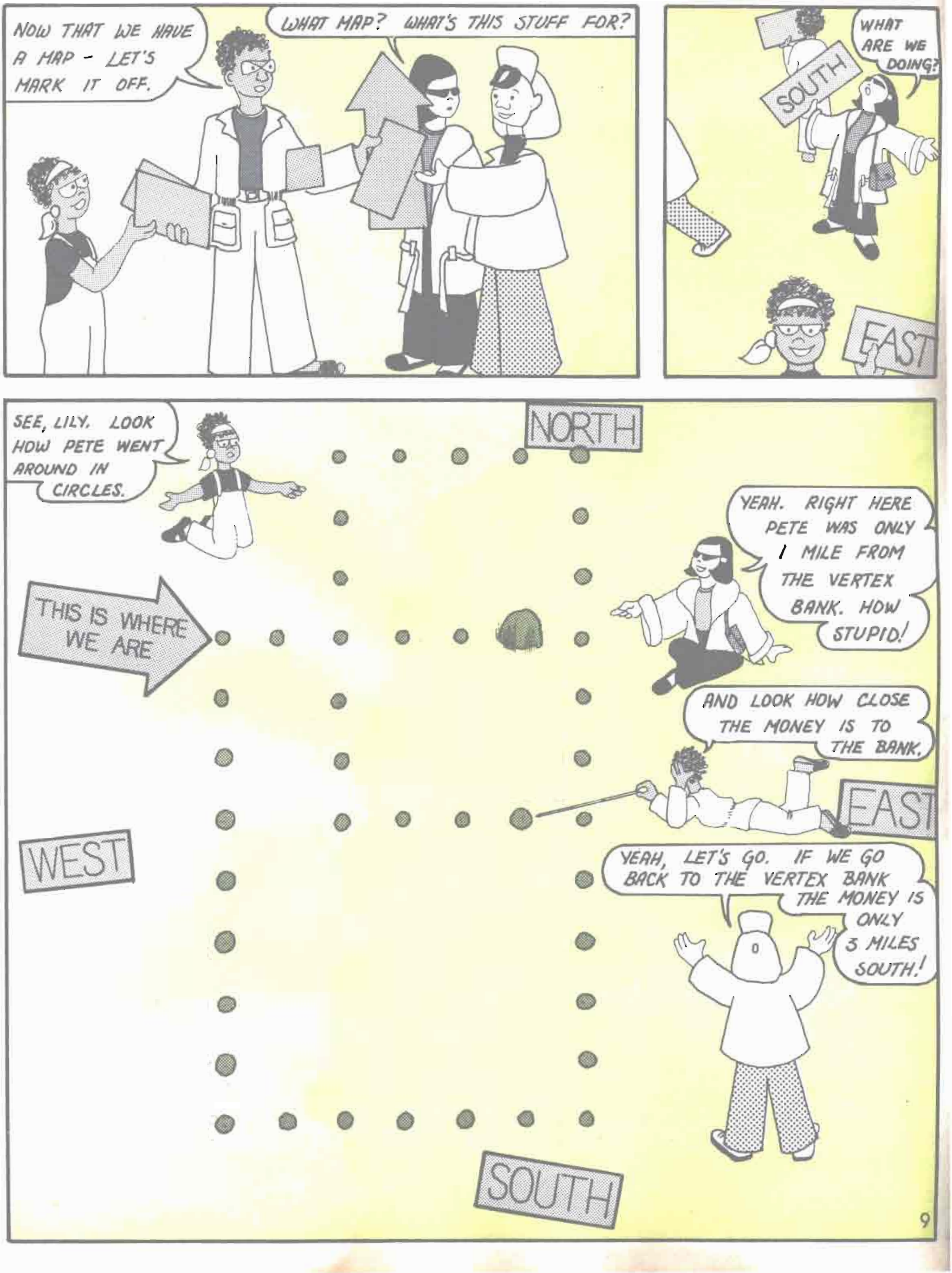

MAC, YOU THINK THAT THOSE KIDS MIGHT GIVE US ANY TROUBLE?
FORGET IT, JOCKO. THEY DIDN'T WRITE ANYTHING DOWN; THEY DON'T HAVE A CAR — AND WE HAD A HEADSTART. SO WHAT KIND OF TROUBLE CAN THEY GIVE US?

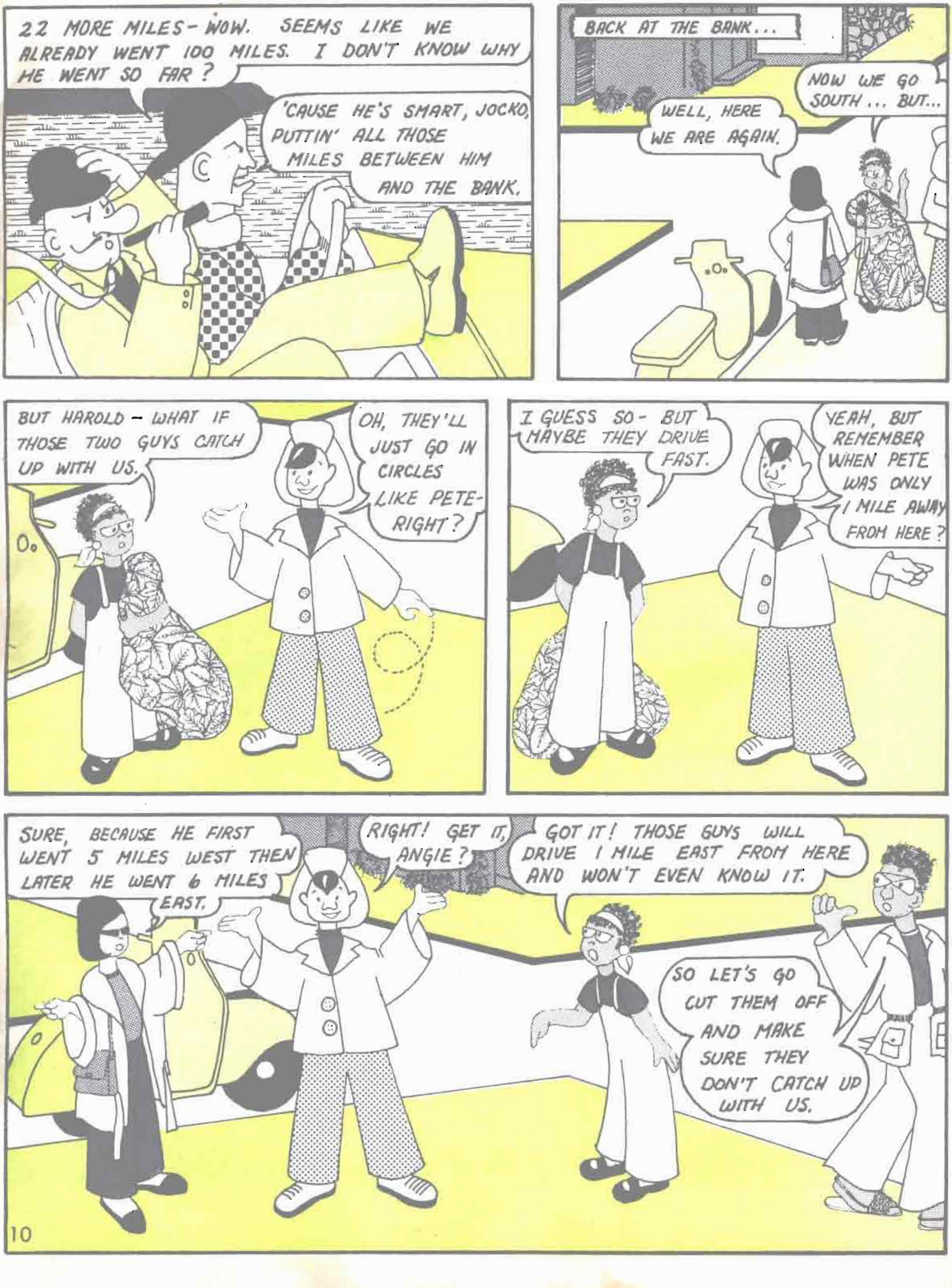

5 MILES WEST IN THE MIDDLE OF NOWHERE...
WHAT NOW, ANGIE?
TURN SOUTH.
COME ON, LILY.
OH, WHAT'S THE USE? WE CAN'T EVEN CATCH UP! WE'LL NEVER GET THERE FIRST.


ROSS, WHAT ARE YOU DOING? WE'LL NEVER CATCH UP!
WE DON'T HAVE ANY TIME TO PICK UP ROCKS!
I JUST HAD A GOOD IDEA. HELP ME GET SOME MORE ROCKS.

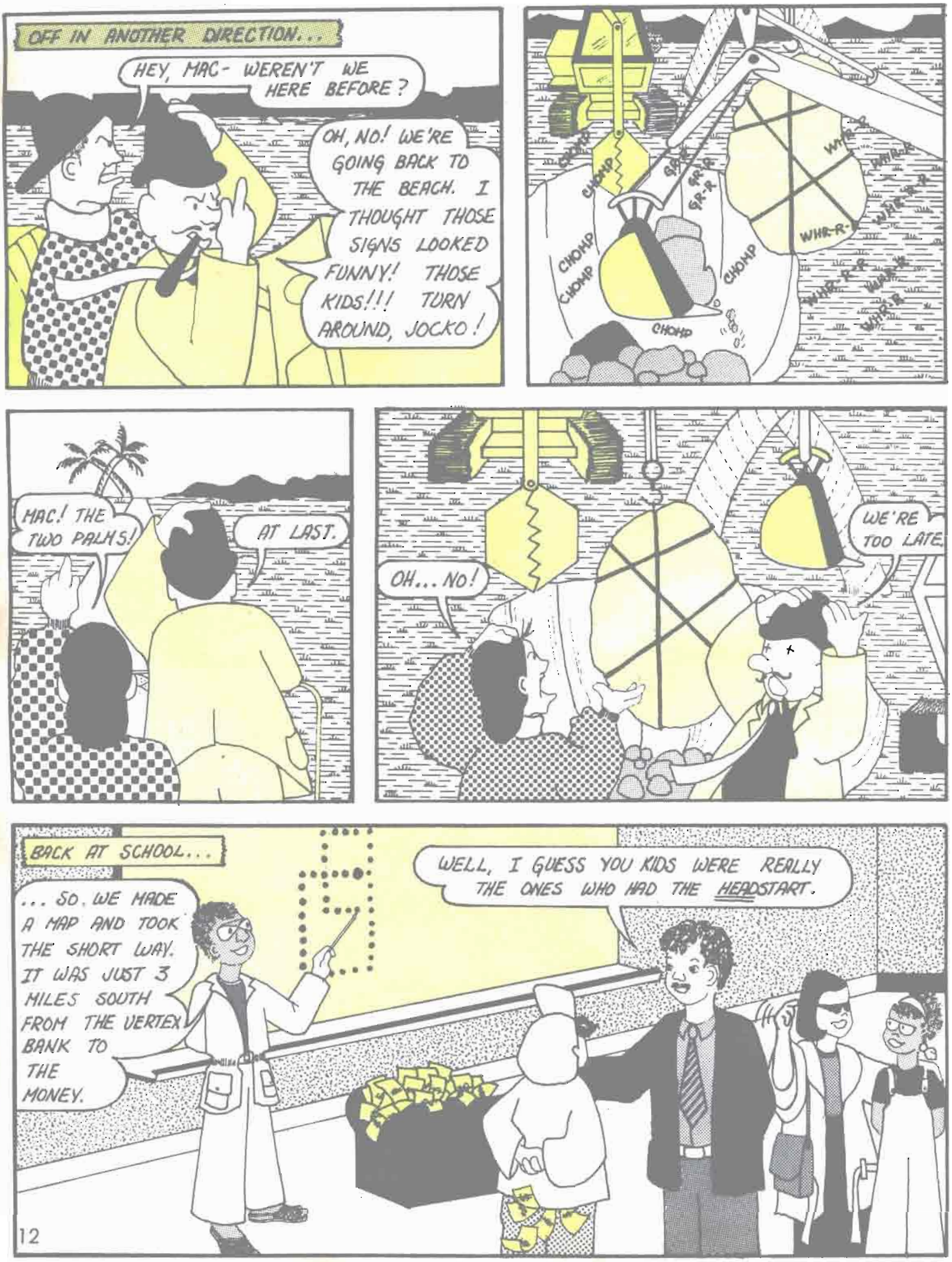

WAIT A SECOND, LILY.
SECOND'S UP.
OH, LILY
WELL, ROSS- NEED MORE ROCKS?
HE'S ALREADY GOT ROCKS IN HIS HEAD.
LILY, GIVE ROSS A CHANCE.
LOOK. THIS BIG ROCK WILL BE THE VERTEX NATIONAL BANK.
SEE? NOW I'LL PUT 5 ROCKS GOING WEST - LIKE 5 MILES. GET IT?
SURE, I GOT IT! WHAT'S NEXT ANGIE?
I DON'T GET IT.


HEY, YOU GUYS - I NEED SOME MORE ROCKS.
HAROLD? WHAT ARE ALL THESE ROCKS FOR?
WELL, THEN PETE WENT 8 MILES SOUTH, AND 6 MILES EAST.
YEAH, LILY, GET OFF THE ROCKS.
THEN 11 MILES NORTH AND - OH, I GET IT - 4 MILES WEST.
OK. NOW THIS IS 4 MILES WEST.
WHAT ARE YOU GUYS DOING?
DON'T YOU SEE, LILY? 6 MILES SOUTH, AND 3 MILES EAST
THERE. THIS IS WHERE THE MONEY IS.
I JUST DON'T GET IT.


NOW THAT WE HAVE A MAP - LET'S MARK IT OFF.
WHAT MAP? WHAT'S THIS STUFF FOR?
WHAT ARE WE DOING?
SOUTH
EAST
SEE, LILY. LOOK HOW PETE WENT AROUND IN CIRCLES.
NORTH
YEAH. RIGHT HERE PETE WAS ONLY 1 MILE FROM THE VERTEX BANK. HOW STUPID!
THIS IS WHERE WE ARE
AND LOOK HOW CLOSE THE MONEY IS TO THE BANK.
EAST
WEST
YEAH, LET'S GO. IF WE GO BACK TO THE VERTEX BANK THE MONEY IS ONLY 3 MILES SOUTH!
SOUTH


22 MORE MILES - WOW. SEEMS LIKE WE ALREADY WENT 100 MILES. I DON'T KNOW WHY HE WENT SO FAR?
'CAUSE HE'S SMART, JOCKO, PUTTIN' ALL THOSE MILES BETWEEN HIM AND THE BANK.
BACK AT THE BANK...
WELL, HERE WE ARE AGAIN.
NOW WE GO SOUTH ... BUT...
BUT HAROLD - WHAT IF THOSE TWO GUYS CATCH UP WITH US.
OH, THEY'LL JUST GO IN CIRCLES LIKE PETE - RIGHT?
I GUESS SO - BUT MAYBE THEY DRIVE FAST.
YEAH, BUT REMEMBER WHEN PETE WAS ONLY 1 MILE AWAY FROM HERE?
SURE, BECAUSE HE FIRST WENT 5 MILES WEST THEN LATER HE WENT 6 MILES EAST.
RIGHT! GET IT, ANGIE?
GOT IT! THOSE GUYS WILL DRIVE 1 MILE EAST FROM HERE AND WON'T EVEN KNOW IT.
SO LET'S GO CUT THEM OFF AND MAKE SURE THEY DON'T CATCH UP WITH US.

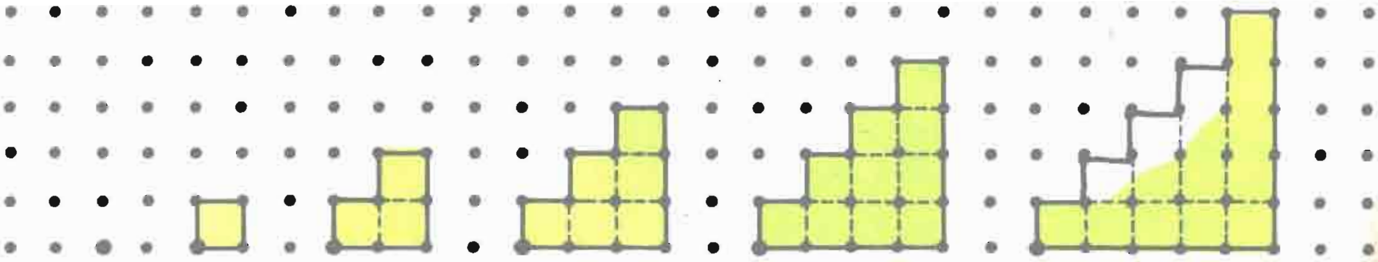

1 MILE EAST OF THE VERTEX BANK...
HERE, ROSS.
COME ON, ANGIE. WE DON'T HAVE ALL DAY.
OUR
T
I THINK I HEAR A CAR.
BIKE LANE
OUR
WE BETTER GET OUT OF HERE. MAYBE IT'S THOSE GUYS.
BIKE LANE
LEFT
TURN
DETOUR
DETOUR, STUPID!
OH, NO. WHAT DO WE DO?
IT WORKED!
HURRAY! HURRAY!
ZOOM
COME ON, LET'S GET BACK TO THE BANK.
3 MILES SOUTH OF THE BANK...
THERE'S THE TWO PALMS.
AND THERE'S THE ROCK.
LET'S HURRY UP AND GET TO WORK.


OFF IN ANOTHER DIRECTION...
HEY, MAC- WEREN'T WE HERE BEFORE?
OH, NO! WE'RE GOING BACK TO THE BEACH. I THOUGHT THOSE SIGNS LOOKED FUNNY! THOSE KIDS!!! TURN AROUND, JOCKO!
CHOMP
CHOMP
CHOMP
CHOMP
CHOMP
CHOMP
GR-R
GR-R
GR-R
WHR-R
WHR-R
WHR-R
WHR-R
WHR-R
WHR-R
WHR-R
MAC! THE TWO PALMS!
AT LAST.
OH... NO!
WE'RE TOO LATE.
BACK AT SCHOOL...
... SO, WE MADE A MAP AND TOOK THE SHORT WAY. IT WAS JUST 3 MILES SOUTH FROM THE VERTEX BANK TO THE MONEY.
WELL, I GUESS YOU KIDS WERE REALLY THE ONES WHO HAD THE HEADSTART.

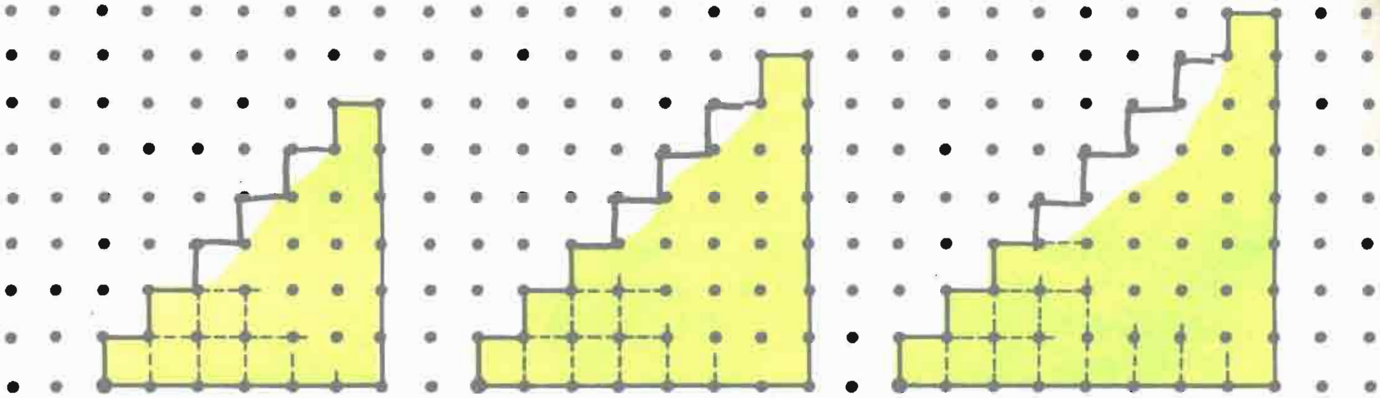
What's My Rule?

TRIANGULAR NUMBERS

Please complete the sketches below . . . the "Stair Step" numbers.



0 1 3 6 10 15



21 28 36

Please make a list of these numbers and extend it.
Then, on the line below, show the differences between neighbors.

Triangular Numbers	0	1	3	6	10	15	21	28	36	45	55
Rate of Growth (Differences)	1	2	3	4	5	6	7	8	9	10	

1 = 1

1 + 2 = 3

1 + 2 + 3 = 6

1 + 2 + 3 + 4 = 10

1 + 2 + 3 + 4 + 5 = 15

1 + 2 + 3 + 4 + 5 + 6 = 21

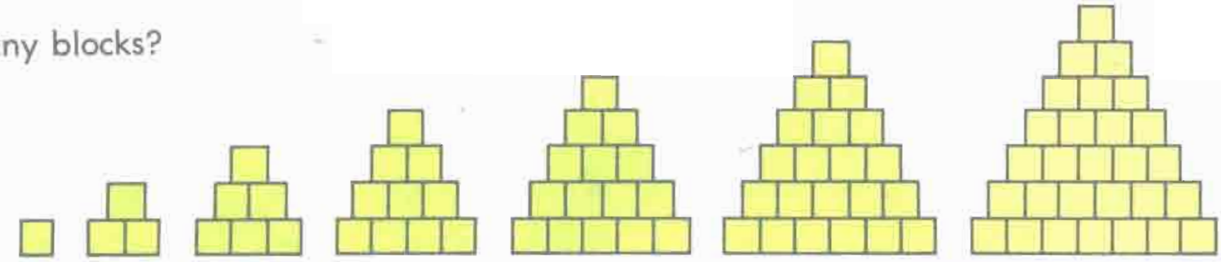
1 + 2 + 3 + 4 + 5 + 6 + 7 = 28

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45

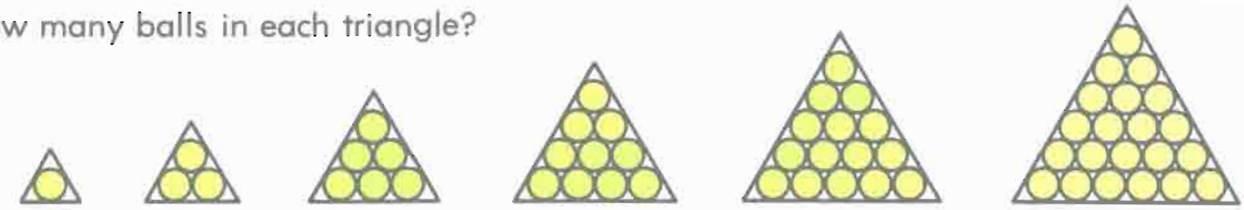
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55

How many blocks?



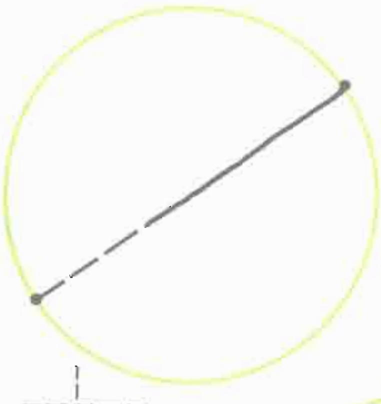
0 1 3 6 10 15 21 28

How many balls in each triangle?

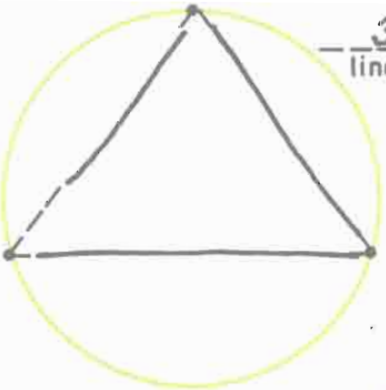


1 3 6 10 15 21

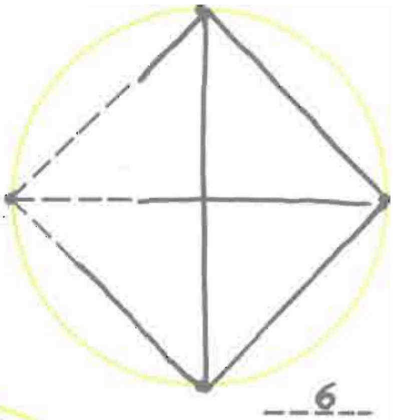
Problem: How many straight lines are needed so each point in the sketches below is connected to all other points?



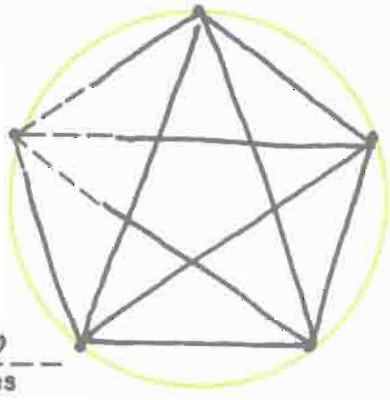
1 line



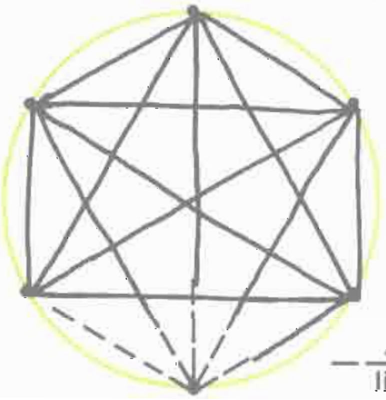
3 lines



6 lines



10 lines



15 lines

Another Problem:

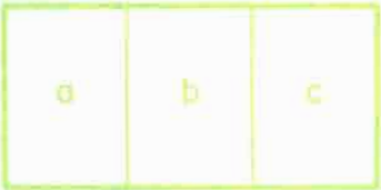
In each of the sketches below, how many rectangles or squares do you see?

a.

1 rectangle

a, b, ab

3 rectangles or squares

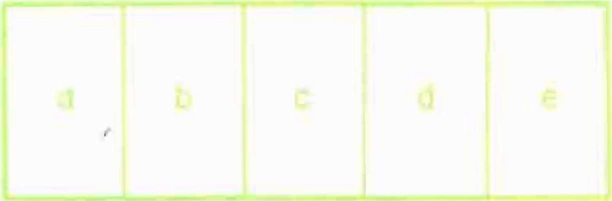


a, b, c
ab, bc, abc

6 rectangles or squares

a, b, c, d, ab, bc, cd,
abc, bcd, abcd

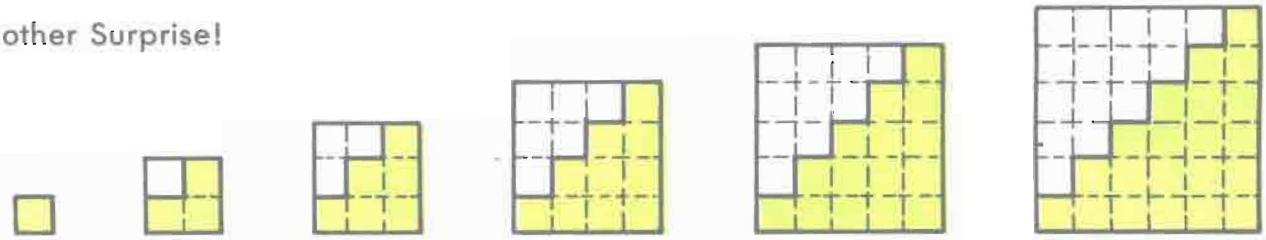
10 rectangles or squares



a, b, c, d, e, ab, bc,
cd, de, abcd, bcde,
abc, bcd, cde, abcde

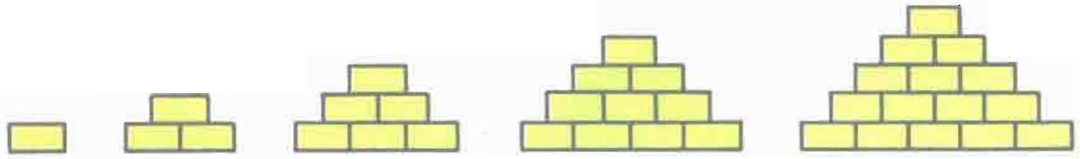
15 rectangles or squares

Another Surprise!



Δ no.	0	1	3	6	10	15
Δ No.	+ 1	+ 3	+ 6	+ 10	+ 15	+ 21
Sq. No.	1	4	9	16	25	36

It seems the families of "triangular" and "square" numbers are closely related.



0 1 3 6 10 15 21

Triangular Numbers	0	1	3	6	10	15	21	28	36	45
Rate of Growth (Differences)	1	2	3	4	5	6	7	8	9	

Sam Jones says that all whole numbers can be shown as the sum of 3 triangular numbers . . . if the list is extended. Do you believe this? Let's try the first 36 numbers.

0 + 0 + 1 = 1

0 + 1 + 1 = 2

1 + 1 + 1 = 3

0 + 3 + 1 = 4

1 + 1 + 3 = 5

3 + 3 + 0 = 6

3 + 3 + 1 = 7

1 + 1 + 6 = 8

3 + 3 + 3 = 9

6 + 3 + 1 = 10

0 + 1 + 10 = 11

0 + 6 + 6 = 12

0 + 3 + 10 = 13

1 + 3 + 10 = 14

0 + 0 + 15 = 15

0 + 6 + 10 = 16

1 + 1 + 15 = 17

0 + 3 + 15 = 18

1 + 3 + 15 = 19

0 + 10 + 10 = 20

1 + 10 + 10 = 21

0 + 1 + 21 = 22

1 + 1 + 21 = 23

0 + 3 + 21 = 24

1 + 3 + 21 = 25

6 + 10 + 10 = 26

0 + 6 + 21 = 27

1 + 6 + 21 = 28

0 + 1 + 28 = 29

1 + 1 + 28 = 30

0 + 3 + 28 = 31

1 + 3 + 28 = 32

6 + 6 + 21 = 33

0 + 6 + 28 = 34

1 + 6 + 28 = 35

6 + 15 + 15 = 36

Maria Gonzales says she found 8 different ways to write 40 as the sum of 4 triangular numbers.

36 + 3 + 1 + 0

28 + 10 + 1 + 1

28 + 6 + 6 + 0

28 + 6 + 3 + 3

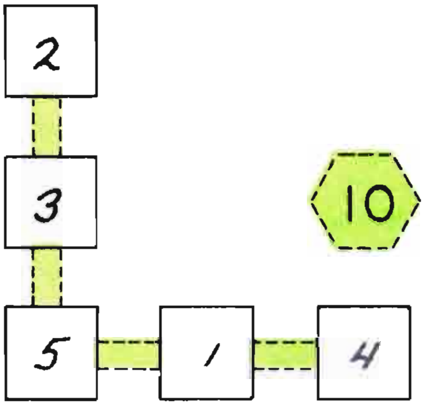
21 + 15 + 3 + 1

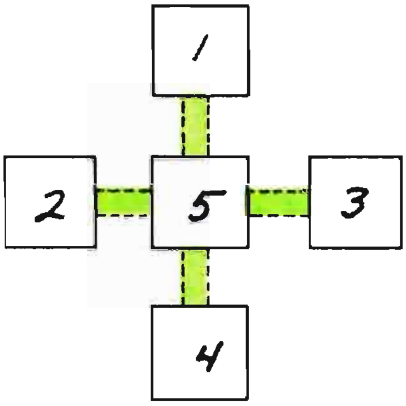
21 + 10 + 6 + 3

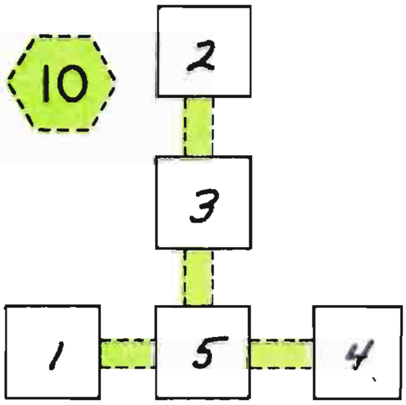
15 + 15 + 10 + 0

10 + 10 + 10 + 10

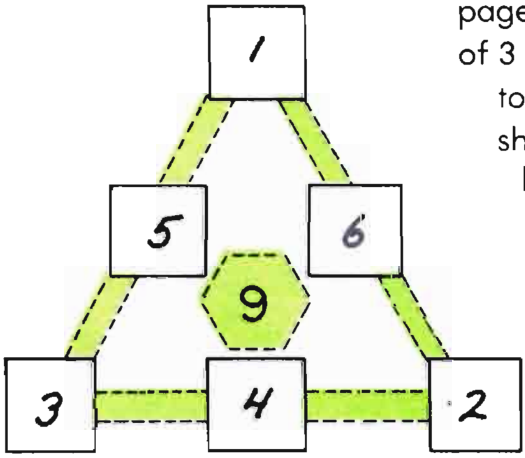
Arrangement of PUZZLES with Small Numbers

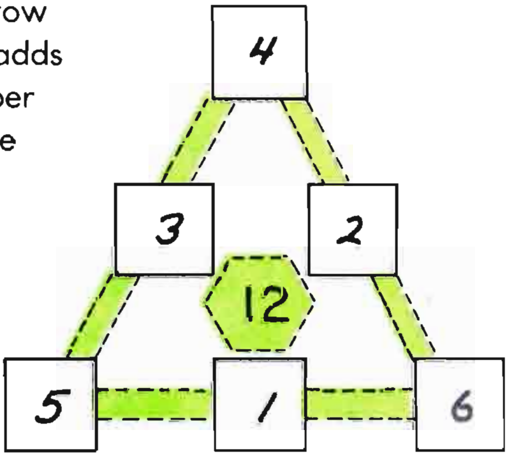


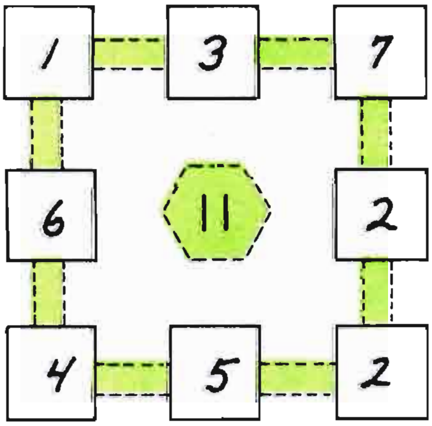


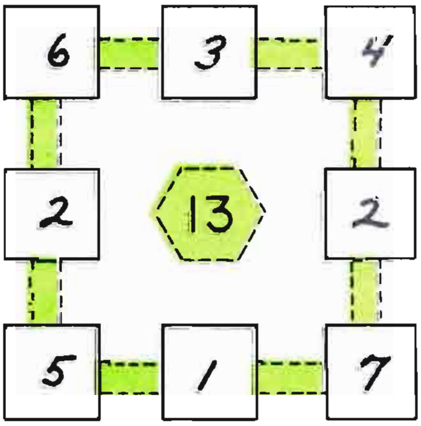


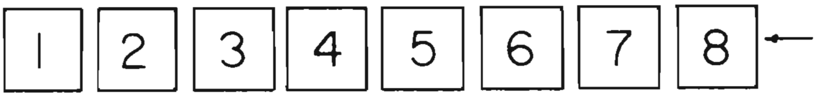
Please arrange the
numbers on this
page so each row
of 3 numbers adds
to the number
shown in the
hexagon.











Numbers like these can be cut out
and moved around until an answer
is found.

Sam's "Shortcuts"

"I make up examples for myself and think of many different ways to get right answers. Usually there is an easiest way for me. Now its become a habit. I always look for the shortest way before I start. I'll show you some of the different ways I think."

15 + 9 24	+10 -1	+5 +4	-1 +10
38 + 5 43	+2 +3	+10 -5	-5 +10

31 - 5 26	-1 -4	-10 +5	-11 +6

Make up your own different ways.

31 - 9 22	-10 +1	-11 +2	-1 -8

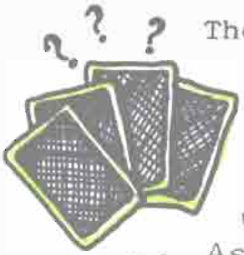
possible
answers

Lucy's "Lots of examples out of 3 doubles"

"Nat only got 9 examples out of 3 doubles. I can get as many as 27 examples out of 3 doubles but I'll only show you 21 of them. Of course, I can handle numbers larger than 100."

starters

1 + 1 2	3 + 3 6	6 + 6 12			
113 + 113 226	131 + 131 262	133 + 133 266	116 + 116 232		
136 + 136 272	161 + 161 322	111 + 111 222	136 + 136 272	363 + 363 726	333 + 333 666
331 + 331 662	631 + 631 1262	633 + 633 1266	316 + 316 632	663 + 663 1326	666 + 666 1332



The important clue here is to reduce the number of possibilities by half with each question. This

CARD GUESSING

can easily be done and will work with any group of 1 to 16 numbers. Asking if the number is more or less than 7 (in this case) reduces the possibilities by half. Then asking if it is odd or even reduces it in half once again. Now, asking if it is in the first or second half of the remaining numbers brings the possibilities to only 1 or 2 numbers. One more question and you've got it!

You need a deck of cards. Have a friend pick one card without showing it to you.

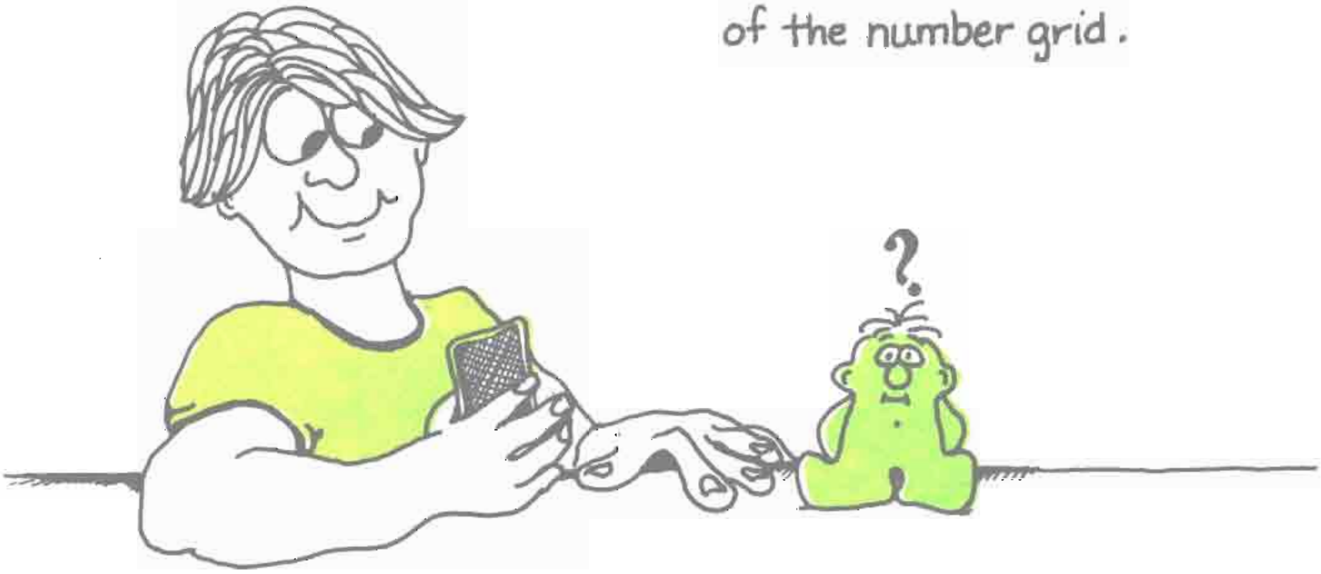
You only have 4 questions to guess the number. Use this grid to keep track of what you learn with each question.

ACE										JACK	QUEEN	KING
1	2	3	4	5	6	7	8	9	10	11	12	13

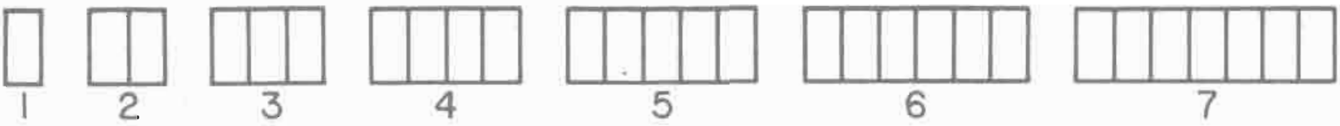
Clues for best questioning

Think about odd and even numbers.

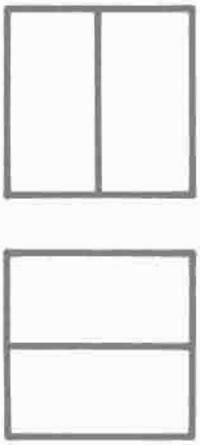
Use the first half or second half of the number grid.



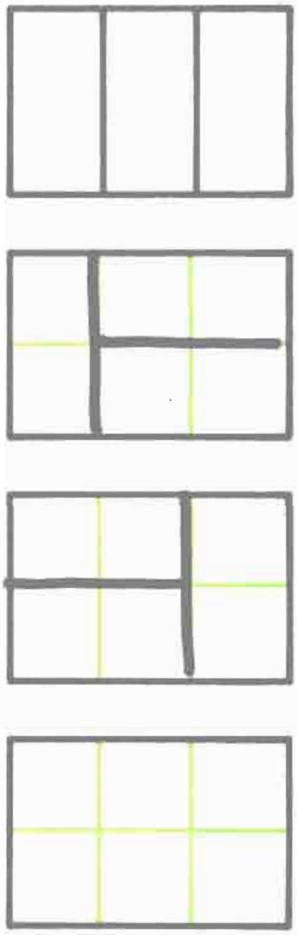
Building Domino Roads with different patterns.



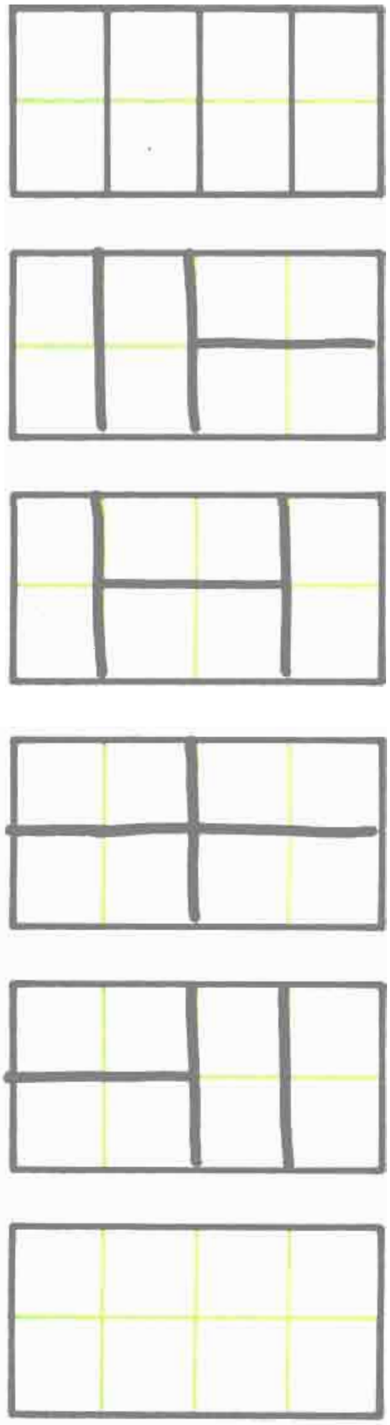
only 1 way



2 different ways



3 different ways



5 different ways

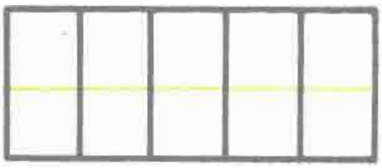
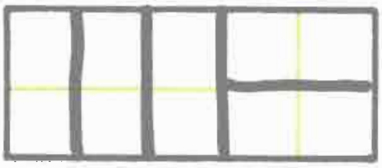
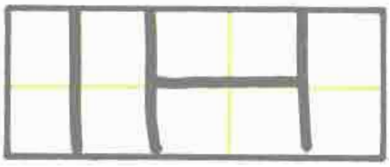
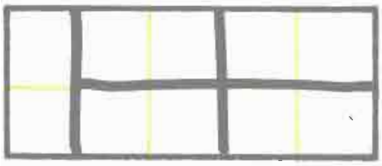
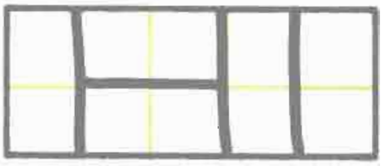
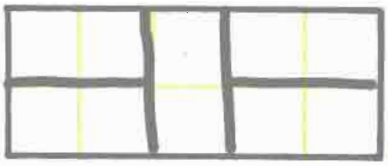
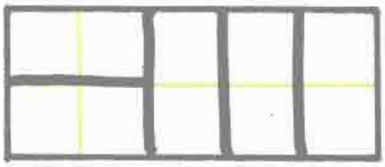
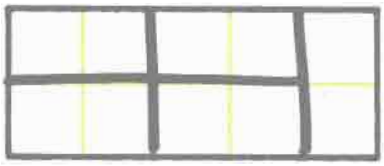
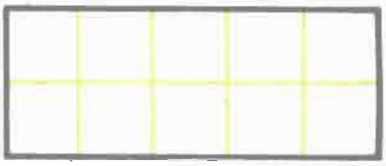
Please find different ways to build roads of each length.

There may not be as many different ways as there are sketches.

After the first three experiments, can you predict the outcome of the first example on the next page?

(Continued on next page)

How many ways to pave a 5-domino road?

_8_ different ways

Can you extend this series of numbers?

1 1 2 3 5 8 13 21 34 55 89 ...

This is the "Fibonacci" series first described in about 1200 A.D. by Leonardo of Pisa, surnamed Fibonacci.

During almost 1000 years — from about 400 to 1400 A.D. — there was almost no important mathematics and science created in the Western World.

The most important contribution Leonardo made was to introduce the decimal system of notation which is now used by almost everyone in the world.

You might look at just one of the many relationships in this series.

Consider any group of 3 terms, the product of the 1st and 3rd will be 1 more or 1 less than the middle term multiplied by itself. For example:

1, 2, 3 $1 \times 3 = 2 \times 2 - 1$

2, 3, 5 $2 \times 5 = 3 \times 3 + 1$

3, 5, 8 $3 \times 8 = 5 \times 5 - 1$

5, 8, 13 $5 \times 13 = 8 \times 8 + 1$

... and this is only one of many relationships in the Fibonacci series.

BIRTHDAY TRICKS

Starting with next month, my birthday will be in ____ months.

After 70 months I will be ____ yrs. old.

I was born ____ months ago.

AGE GAME

Multiply your age by 3.

Add 6 to the answer.

Divide the sum by 3.

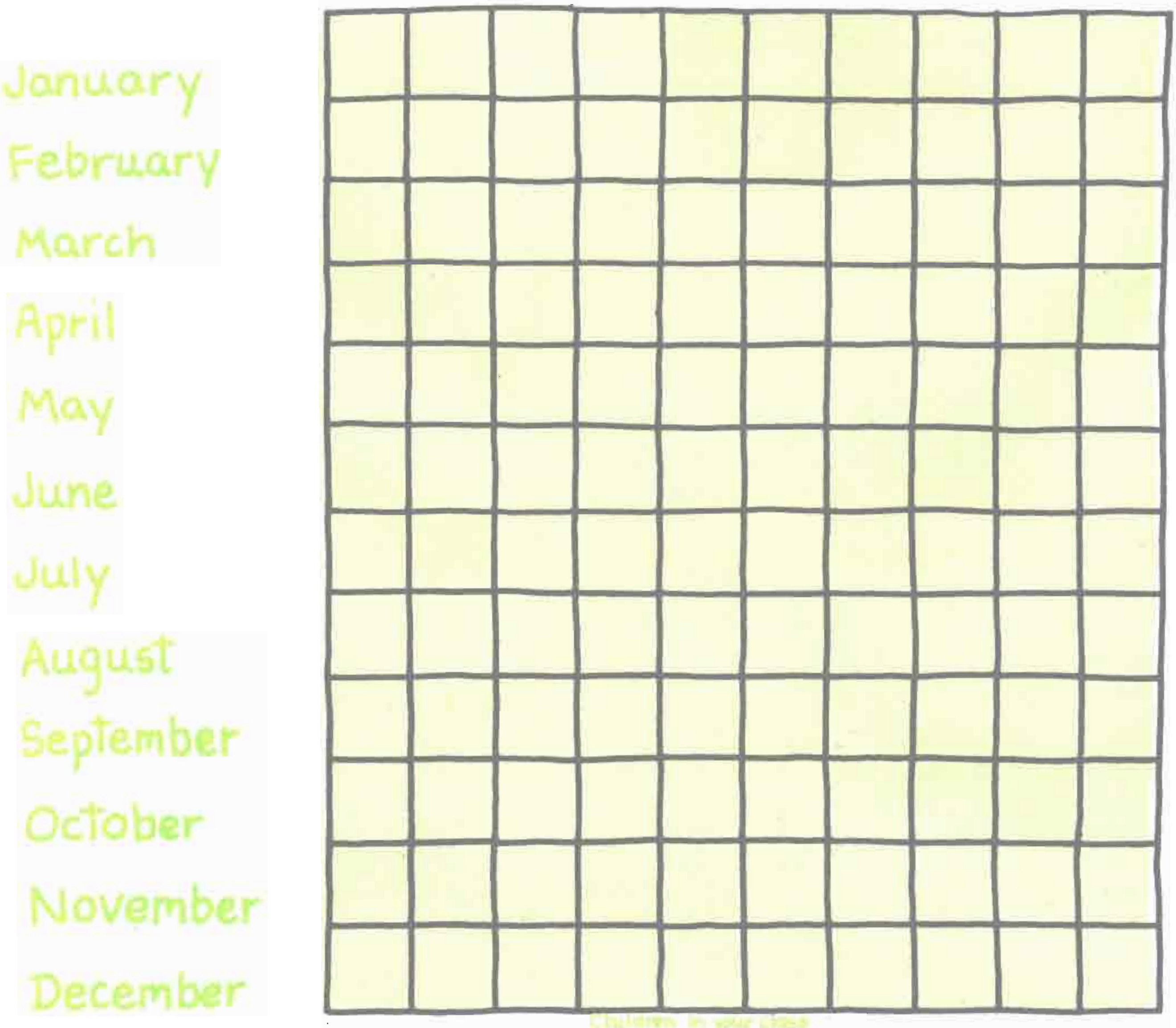
Subtract 2 and you will find your age.

How does this work?

BIRTHDAYS

You may want to suggest that each child make a tally sheet while collecting the information and then draw it on the graph. This can be the take-off point for lots more class graphing about things of interest to the children.

Please find out when everybody in your class has a birthday.



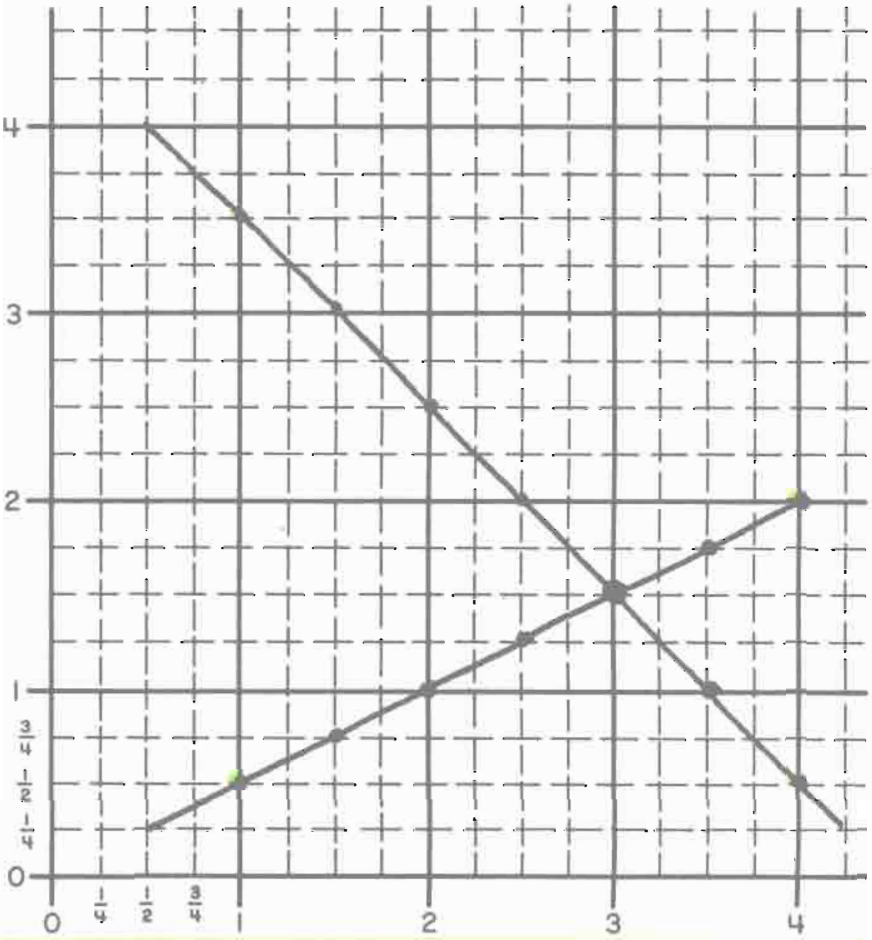
Which month has the most birthdays? ______

Please complete each table of examples and draw the graph of each.

Connect each set of points with a straight line.

→	↑
b	$\frac{1}{2}$b
1	$\frac{1}{2}$
4	2
3	$1\frac{1}{2}$
2	1
$2\frac{1}{2}$	$1\frac{1}{4}$
$1\frac{1}{2}$	$\frac{3}{4}$
$3\frac{1}{2}$	$1\frac{3}{4}$

→	↑
b	$4\frac{1}{2}$-b
1	$3\frac{1}{2}$
4	$\frac{1}{2}$
$2\frac{1}{2}$	2
2	$2\frac{1}{2}$
3	$1\frac{1}{2}$
$1\frac{1}{2}$	3
$3\frac{1}{2}$	1



The two lines cross when b = 3 .

Sam's "Shortcuts"

15
+ 8
23

+10	+5	-5
-2	+3	+13

25
+ 7
32

+5	+10	-5
+2	-3	+12

20
- 5
15

-10	+5	-1
+5	-10	-4

35
- 8
27

+2	-5	-10
-10	-3	+2

44
- 7
37

-4	-10	-14
-3	+3	+7

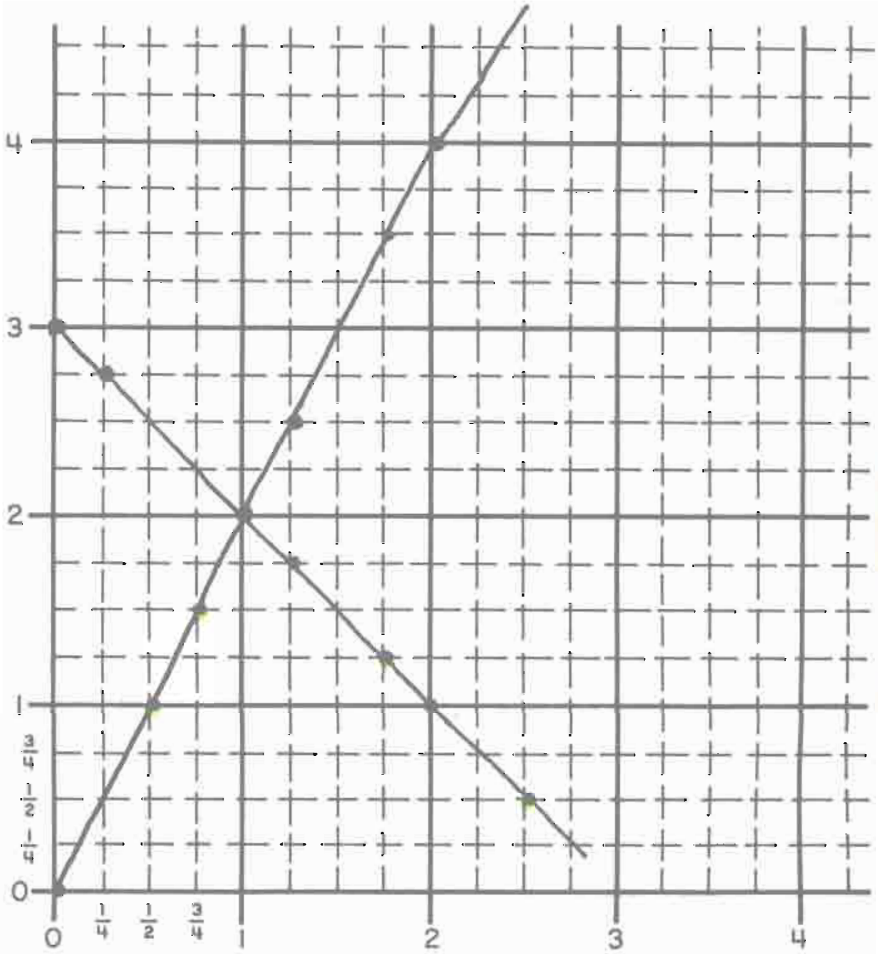
39
+ 5
44

+1	+10	-4
+4	-5	+9

Please find the points given in each table and connect each of the sets of points with a straight line.

→ n	↑ 2n
$\frac{3}{4}$	$1\frac{1}{2}$
$\frac{1}{2}$	1
2	4
1	2
$1\frac{1}{4}$	$2\frac{1}{2}$
0	0
$1\frac{3}{4}$	$3\frac{1}{2}$

→ n	↑ 3-n
$2\frac{1}{2}$	$\frac{1}{2}$
$1\frac{3}{4}$	$1\frac{1}{4}$
$\frac{1}{4}$	$2\frac{3}{4}$
2	1
1	2
$1\frac{1}{4}$	$1\frac{3}{4}$
0	3



The two lines cross when n = 1.

Lucy's "Lots of examples out of 3 doubles"

starters

$$\begin{array}{r} 3 \\ +3 \\ \hline 6 \end{array} \quad \begin{array}{r} 4 \\ +4 \\ \hline 8 \end{array} \quad \begin{array}{r} 5 \\ +5 \\ \hline 10 \end{array} \quad \begin{array}{r} 343 \\ +343 \\ \hline 686 \end{array} \quad \begin{array}{r} 433 \\ +433 \\ \hline 866 \end{array} \quad \begin{array}{r} 335 \\ +335 \\ \hline 670 \end{array} \quad \begin{array}{r} 533 \\ +533 \\ \hline 1066 \end{array}$$

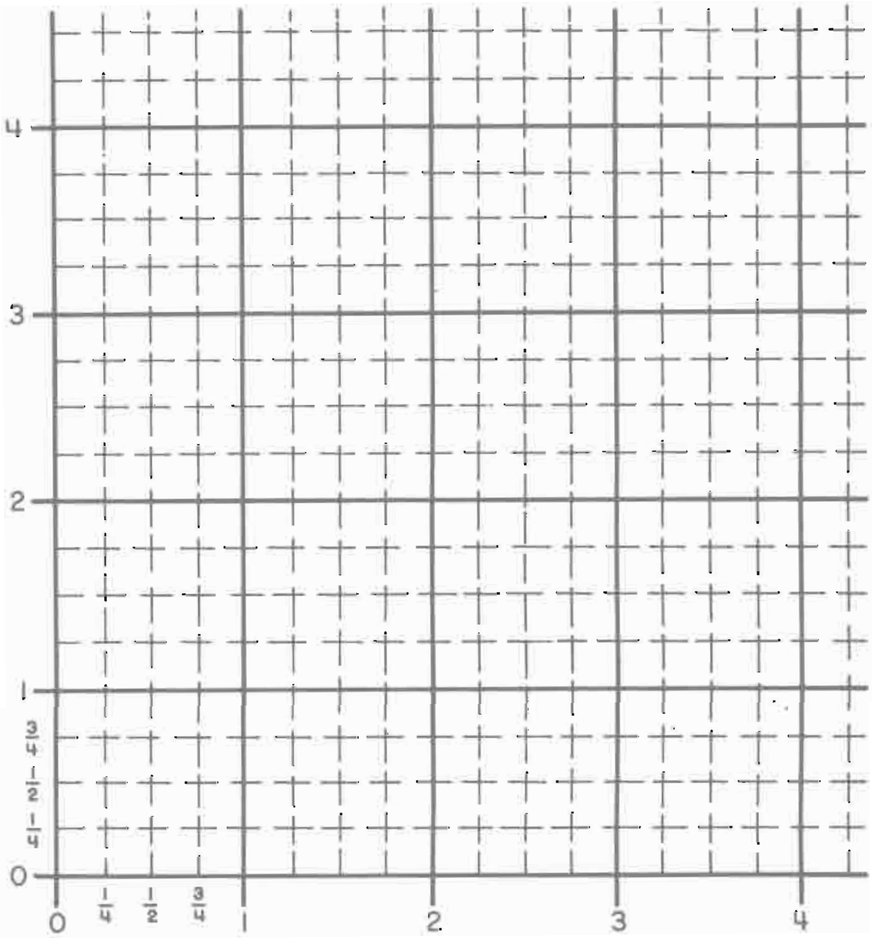
$$\begin{array}{r} 353 \\ +353 \\ \hline 706 \end{array} \quad \begin{array}{r} 445 \\ +445 \\ \hline 890 \end{array} \quad \begin{array}{r} 454 \\ +454 \\ \hline 908 \end{array} \quad \begin{array}{r} 544 \\ +544 \\ \hline 1088 \end{array} \quad \begin{array}{r} 333 \\ +333 \\ \hline 666 \end{array} \quad \begin{array}{r} 444 \\ +444 \\ \hline 888 \end{array}$$

$$\begin{array}{r} 345 \\ +345 \\ \hline 690 \end{array} \quad \begin{array}{r} 354 \\ +354 \\ \hline 708 \end{array} \quad \begin{array}{r} 355 \\ +355 \\ \hline 710 \end{array} \quad \begin{array}{r} 543 \\ +543 \\ \hline 1086 \end{array} \quad \begin{array}{r} 544 \\ +544 \\ \hline 1088 \end{array} \quad \begin{array}{r} 545 \\ +545 \\ \hline 1090 \end{array}$$

Please make up your own pair of rules and a table of examples for each. Then find the points indicated and connect each set with a straight line.

→ m	↑

→ m	↑



Do the lines cross? . . . If they do, they cross when m = ____ .

Lucy's "Lots of examples out of 3 doubles"

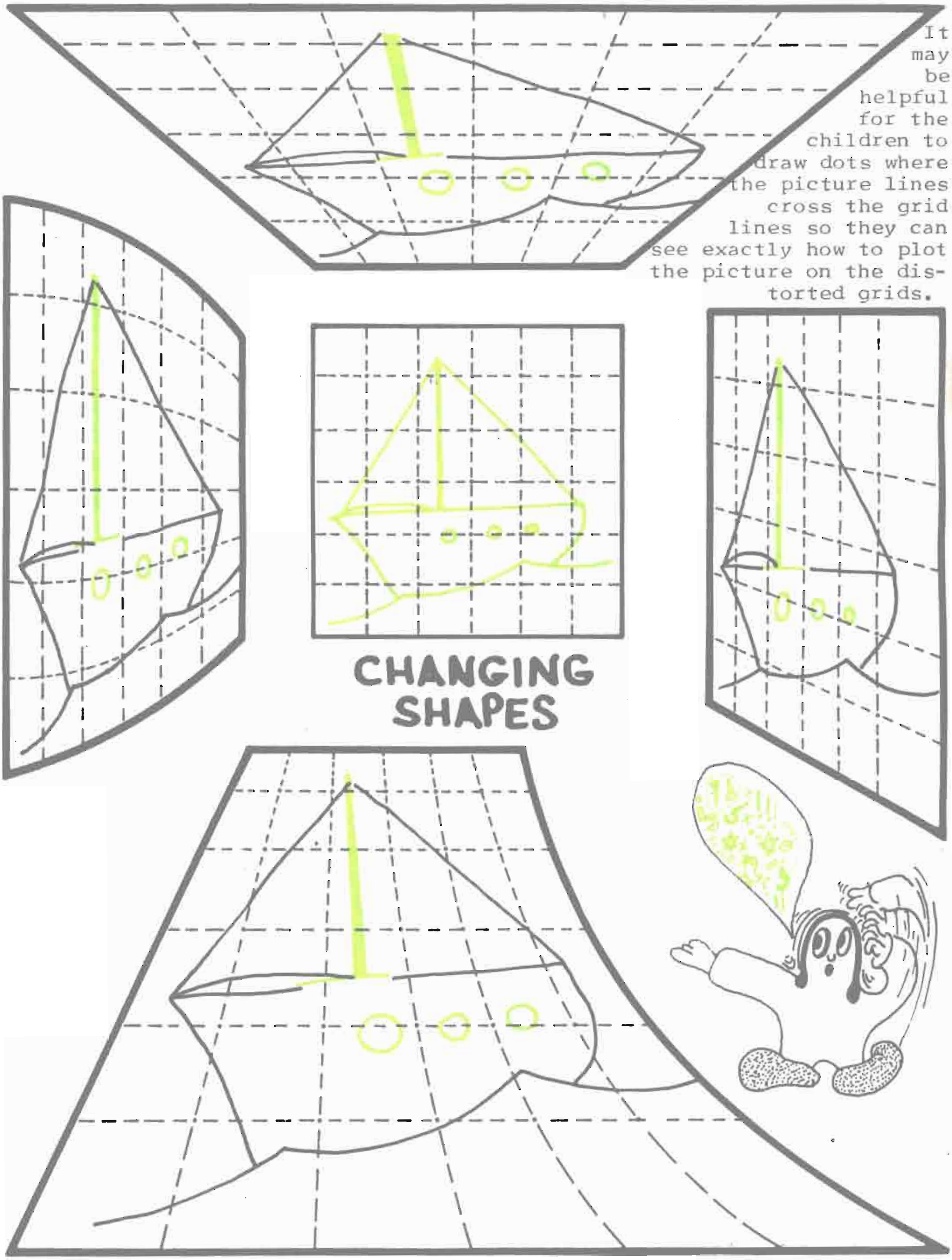
starters

$\begin{array}{r} 4 \\ +4 \\ \hline 8 \end{array}$ $\begin{array}{r} 7 \\ +7 \\ \hline 14 \end{array}$ $\begin{array}{r} 9 \\ +9 \\ \hline 18 \end{array}$

$\begin{array}{r} 449 \\ +449 \\ \hline 898 \end{array}$ $\begin{array}{r} 474 \\ +474 \\ \hline 948 \end{array}$ $\begin{array}{r} 477 \\ +477 \\ \hline 954 \end{array}$ $\begin{array}{r} 499 \\ +499 \\ \hline 998 \end{array}$

$\begin{array}{r} 744 \\ +744 \\ \hline 1488 \end{array}$ $\begin{array}{r} 747 \\ +747 \\ \hline 1494 \end{array}$ $\begin{array}{r} 749 \\ +749 \\ \hline 1498 \end{array}$ $\begin{array}{r} 774 \\ +774 \\ \hline 1548 \end{array}$ $\begin{array}{r} 777 \\ +777 \\ \hline 1554 \end{array}$ $\begin{array}{r} 779 \\ +779 \\ \hline 1558 \end{array}$

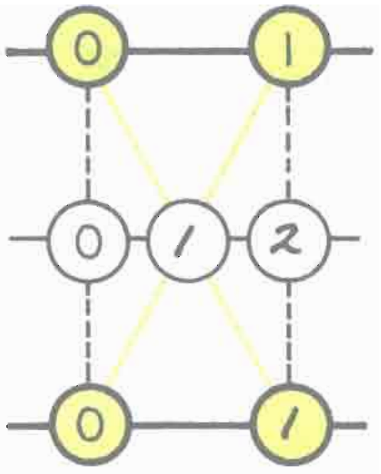
$\begin{array}{r} 794 \\ +794 \\ \hline 1588 \end{array}$ $\begin{array}{r} 797 \\ +797 \\ \hline 1594 \end{array}$ $\begin{array}{r} 799 \\ +799 \\ \hline 1598 \end{array}$ $\begin{array}{r} 944 \\ +944 \\ \hline 1888 \end{array}$ $\begin{array}{r} 947 \\ +947 \\ \hline 1894 \end{array}$ $\begin{array}{r} 949 \\ +949 \\ \hline 1898 \end{array}$



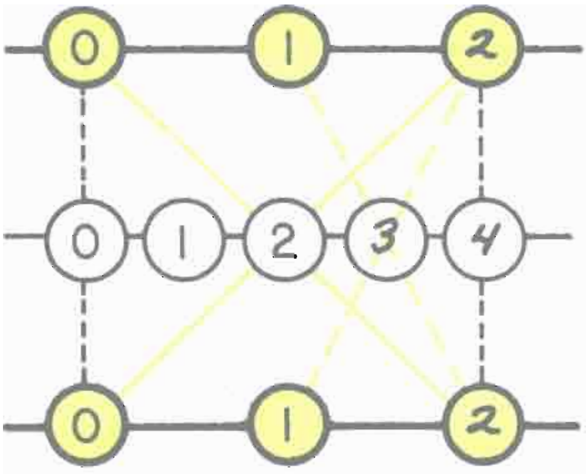
It may be helpful for the children to draw dots where the picture lines cross the grid lines so they can see exactly how to plot the picture on the distorted grids.

CHANGING SHAPES

Building a Mini-Adder

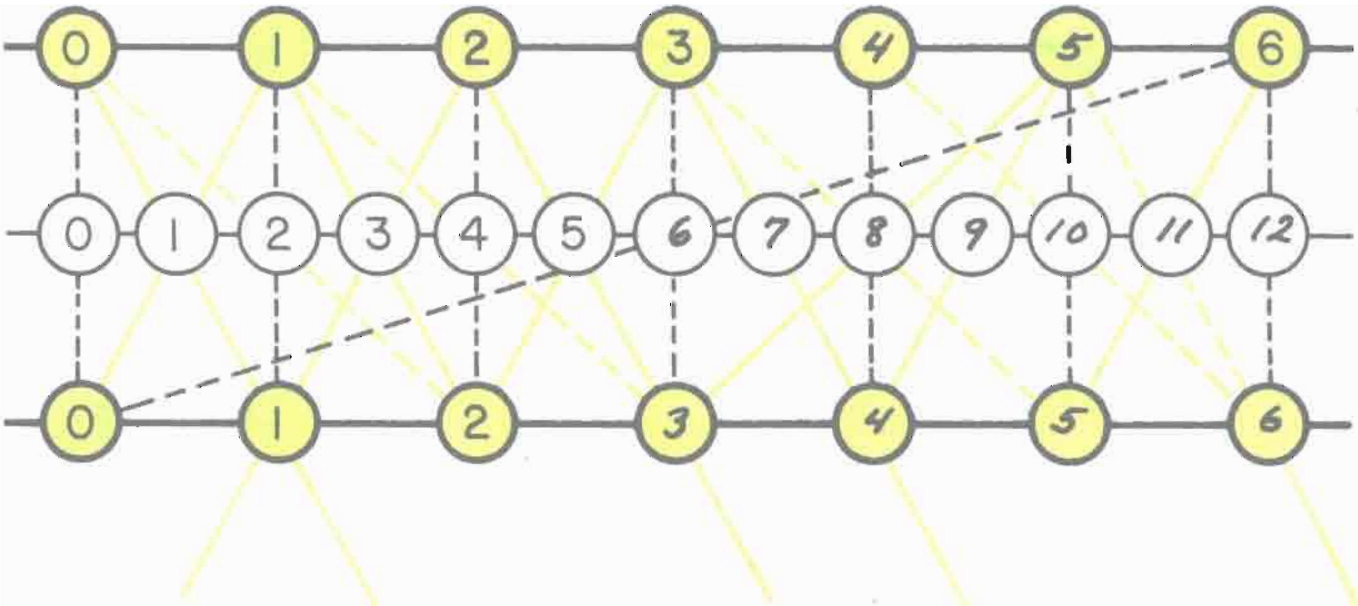


a "start" . . .



. . . an extension . . .

. . . and more extension



2 + 1 = 3

2 + 3 = 5

3 + 2 = 5

5+ 6 = 11

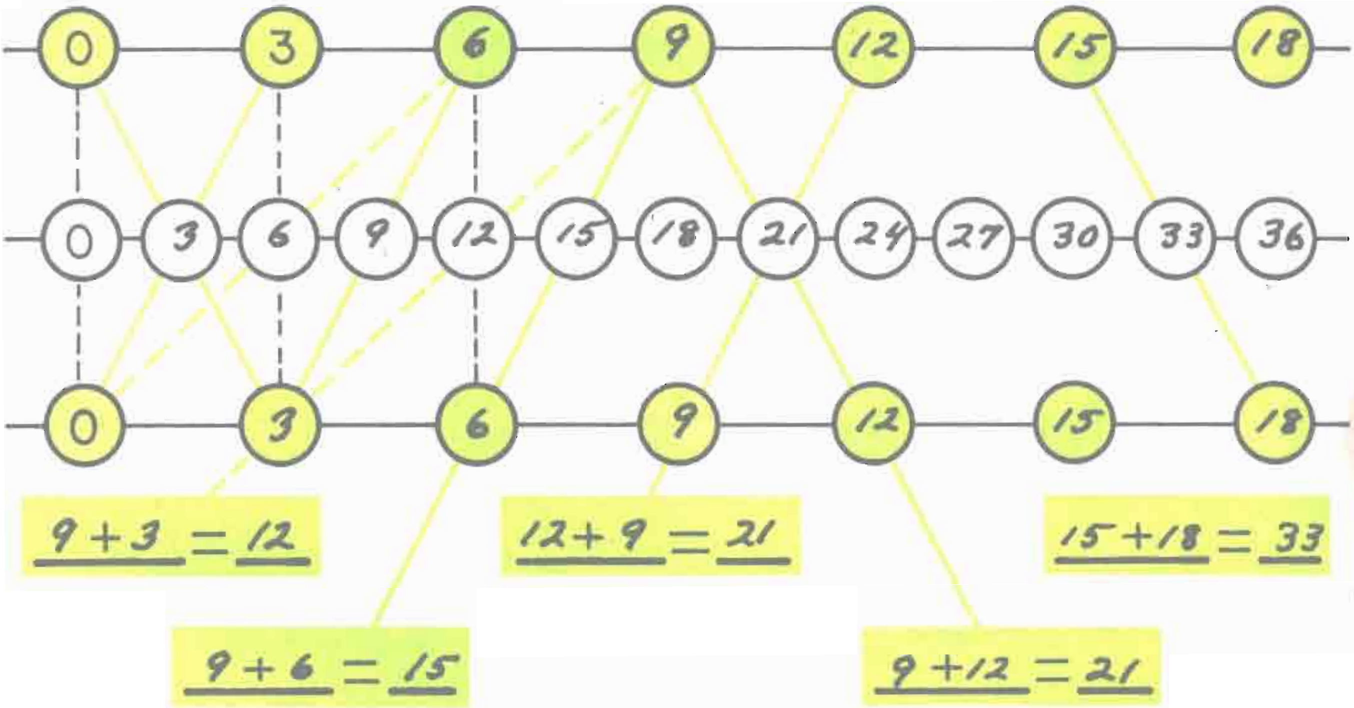
6 + 5 = 11

0 + 1 = 1

3+ 4= 7

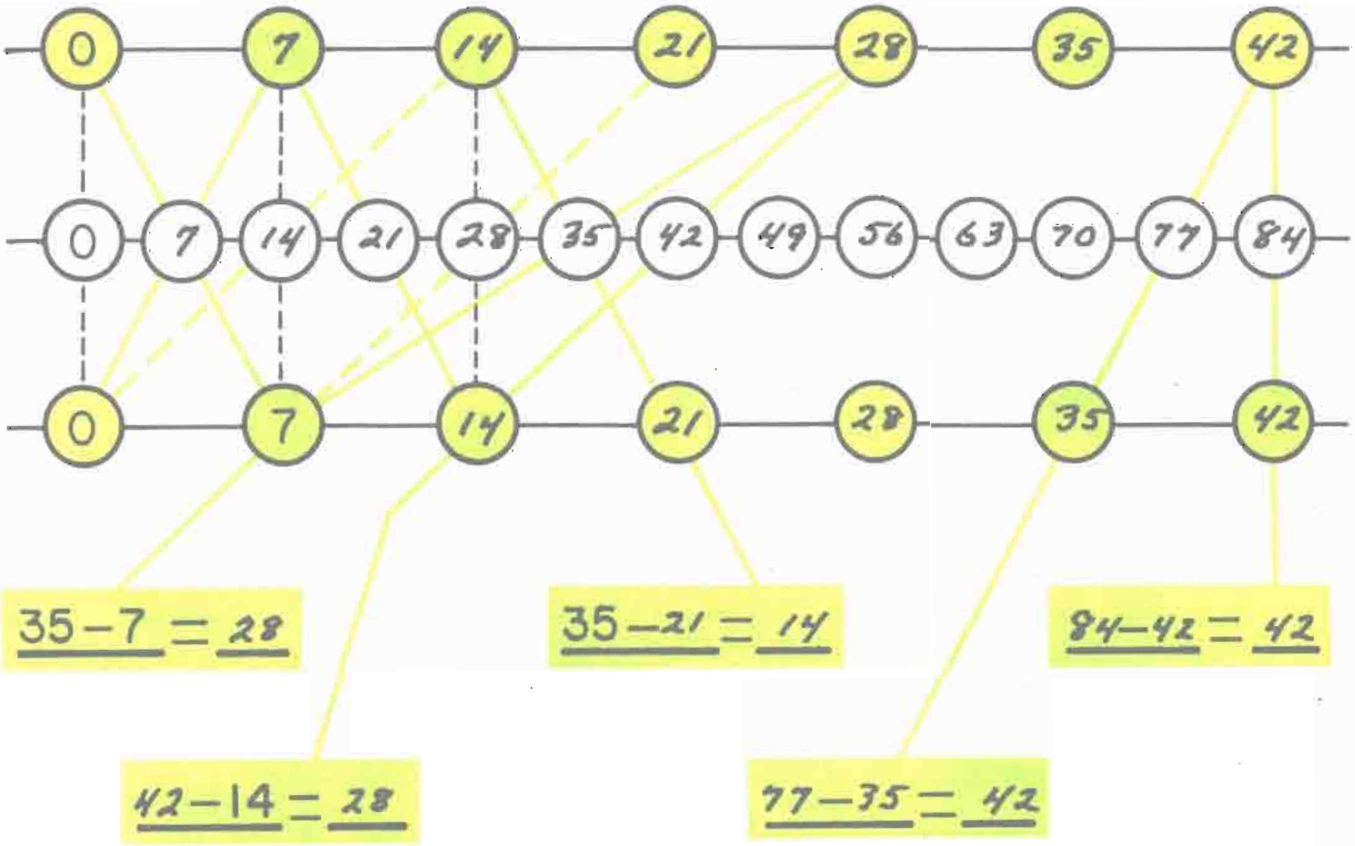
4 + 3 = 7

Varying the keys on the Mini-Adder

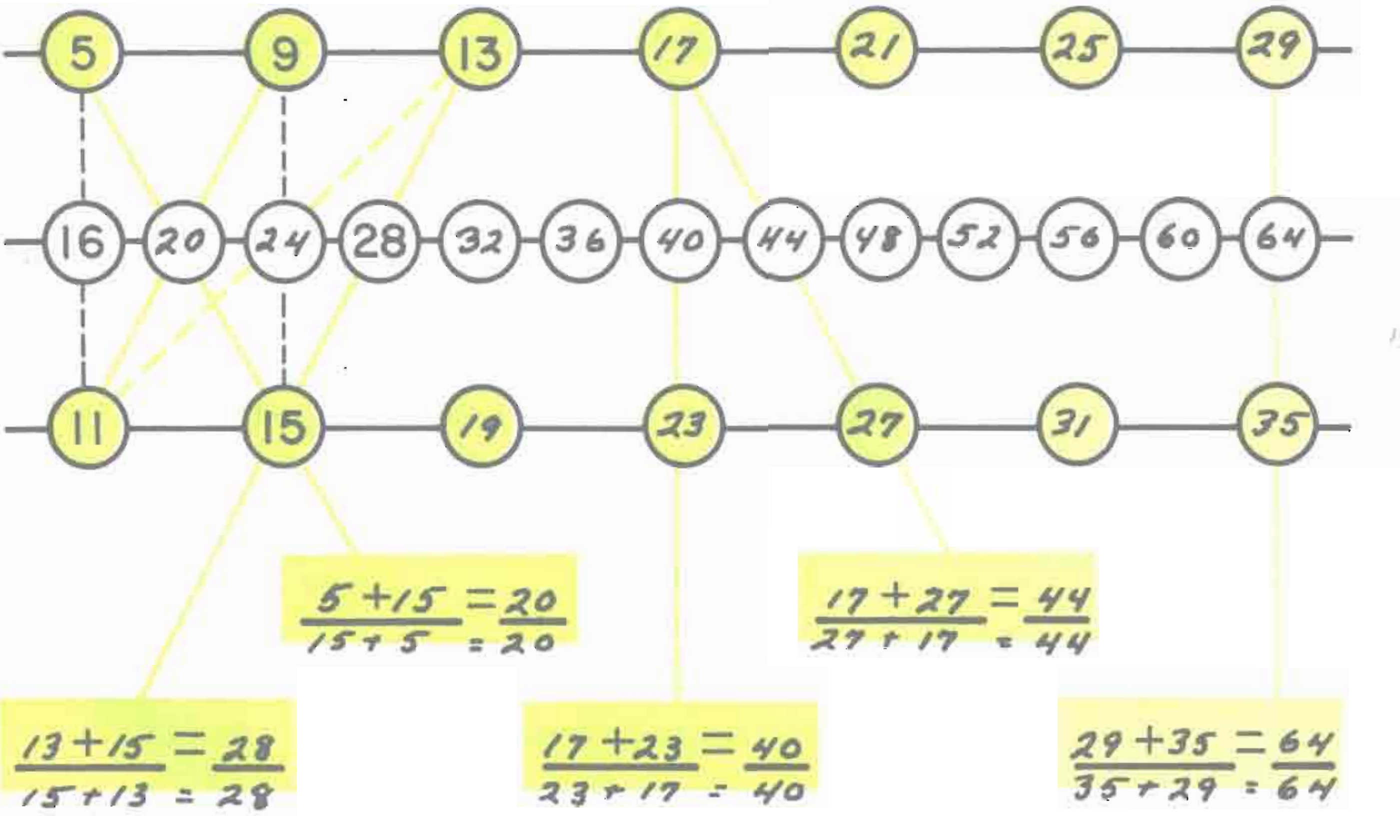


There are other possible combinations.

Subtraction on a Mini-Adder



More variations of keys on the Mini-Adder



$5 + 15 = 20$
$15 + 5 = 20$

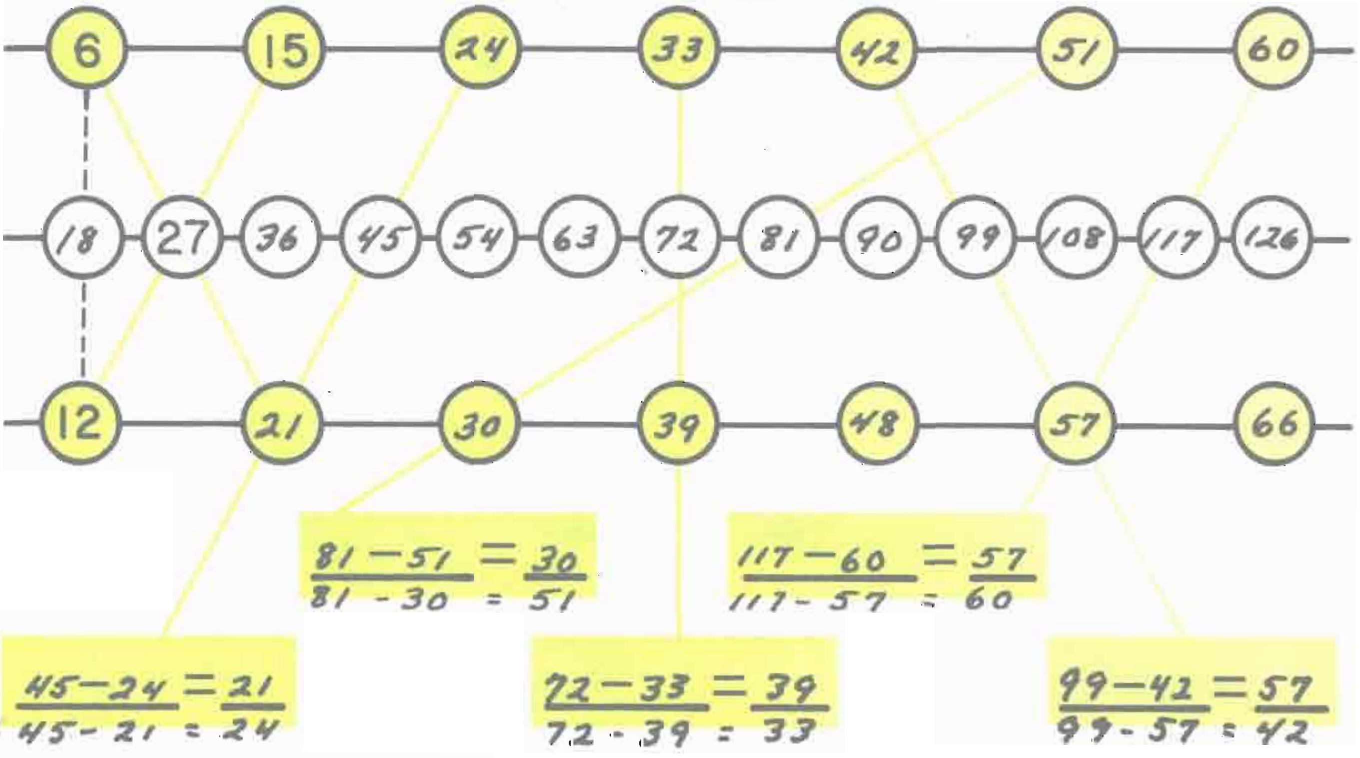
$17 + 27 = 44$
$27 + 17 = 44$

$13 + 15 = 28$
$15 + 13 = 28$

$17 + 23 = 40$
$23 + 17 = 40$

$29 + 35 = 64$
$35 + 29 = 64$

Subtraction on the Mini-Adder



$81 - 51 = 30$
$81 - 30 = 51$

$117 - 60 = 57$
$117 - 57 = 60$

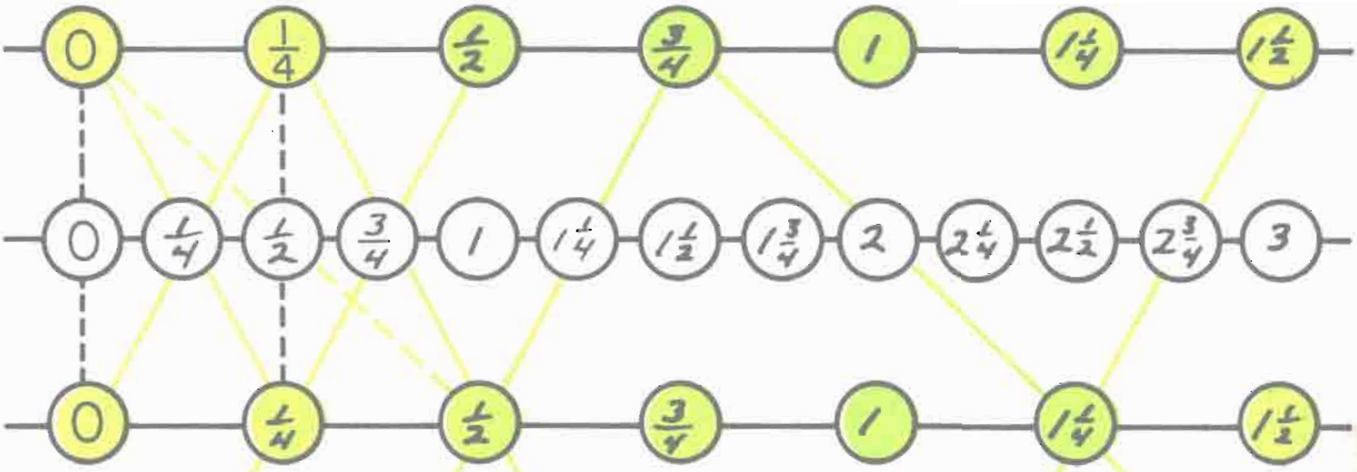
$45 - 24 = 21$
$45 - 21 = 24$

$72 - 33 = 39$
$72 - 39 = 33$

$99 - 42 = 57$
$99 - 57 = 42$

Adding and Subtracting Fractions on a Mini-Adder

You may want to think about coins.



$1\frac{1}{4} - \frac{3}{4} = \frac{1}{2}$ or

$1\frac{1}{4} - \frac{1}{2} = \frac{3}{4}$

$1\frac{1}{2} + 1\frac{1}{4} = 2\frac{3}{4}$

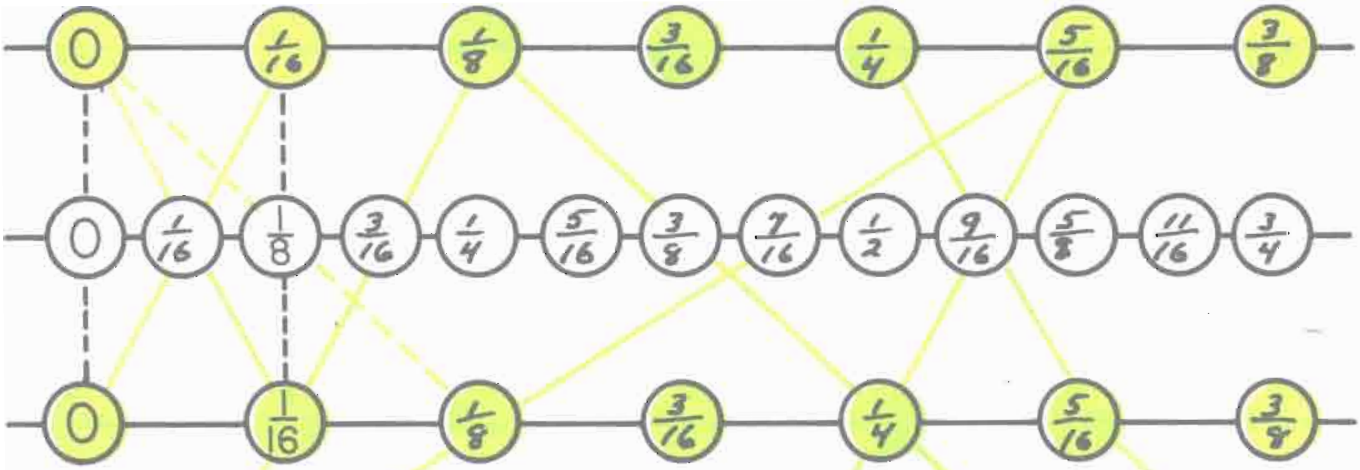
$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$\frac{3}{4} - \frac{1}{4} = \frac{1}{2}$ or

$\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$

$2 - 1\frac{1}{4} = \frac{3}{4}$ or

$2 - \frac{3}{4} = 1\frac{1}{4}$



$\frac{5}{16} + \frac{1}{8} = \frac{7}{16}$

$\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$

$\frac{3}{16} - \frac{1}{8} = \frac{1}{16}$

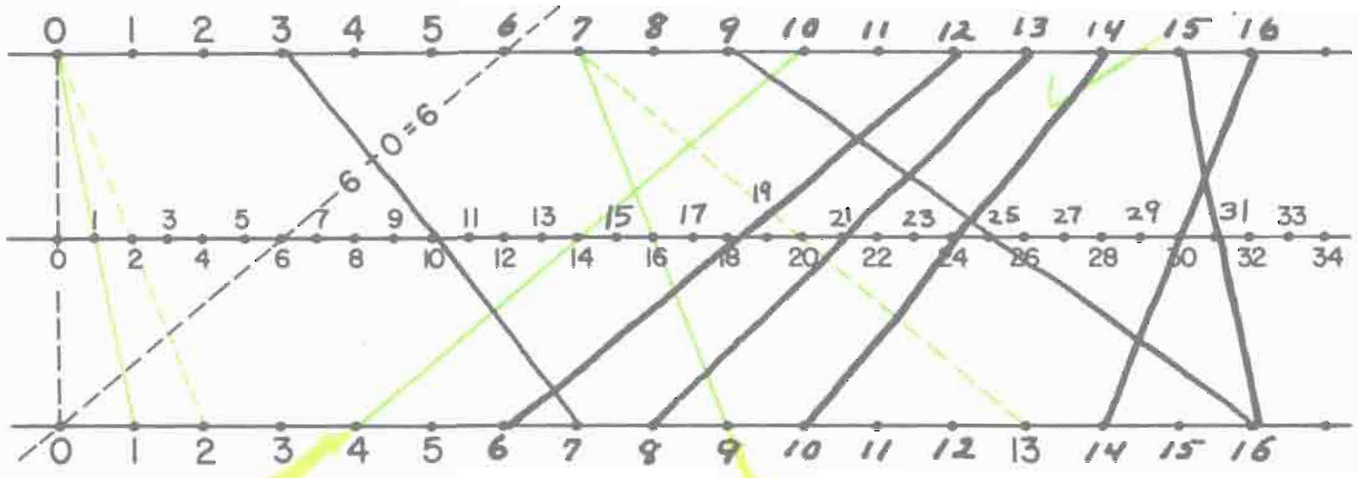
$\frac{9}{16} - \frac{1}{4} = \frac{5}{16}$

$\frac{9}{16} - \frac{1}{4} = \frac{5}{16}$

New Scales for the **Mini-Adder**

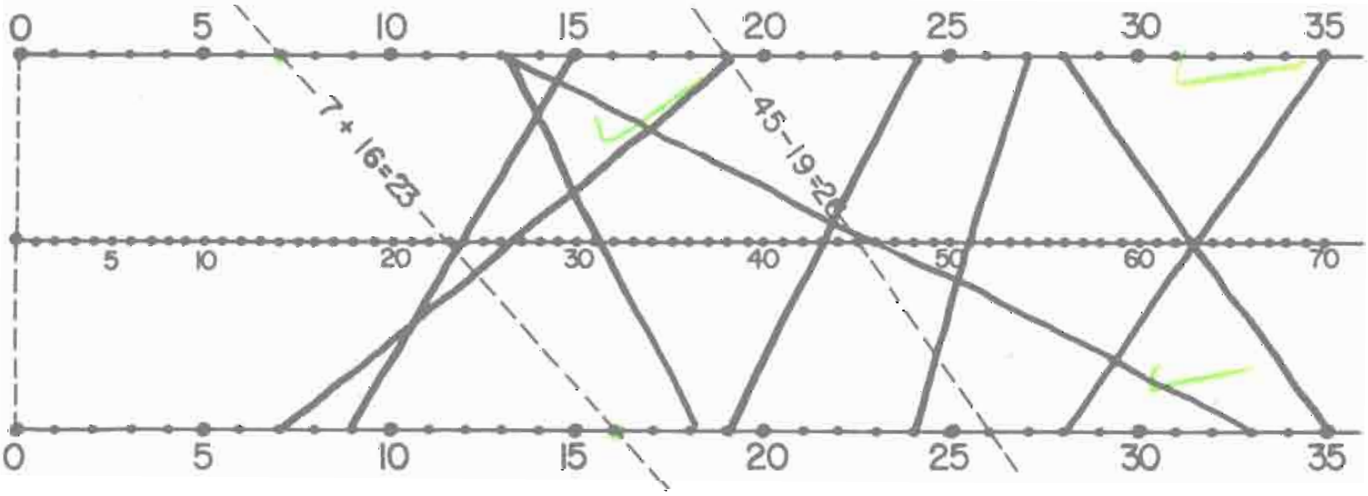
Lines may slant
to right or left.

Please complete the scales. Then draw lines suggested by the example.



10	12	13	15	16	10	24	25
+ 4	+ 6	+ 8	+16	- 7	- 3	-14	- 9
14	18	21	31	9	7	10	16

Pages 74 and 76 are extensions of this activity.



19	24	13	35	24	31	51	63
+ 7	+19	+33	+28	-15	-13	-27	-28
26	43	46	63	9	18	24	35

HIT THE TARGET

MEASURE OFF A DISTANCE 3 METERS LONG.

AT ONE END PUT A TARGET (WASTEBASKET, SMALL BOX, ETC.)

STAND AT THE OTHER END AND TRY TO HIT THE TARGET WITH A PAPER CLIP.

TRY IT 3 TIMES.
HOW CLOSE DID YOU COME? ________ cm

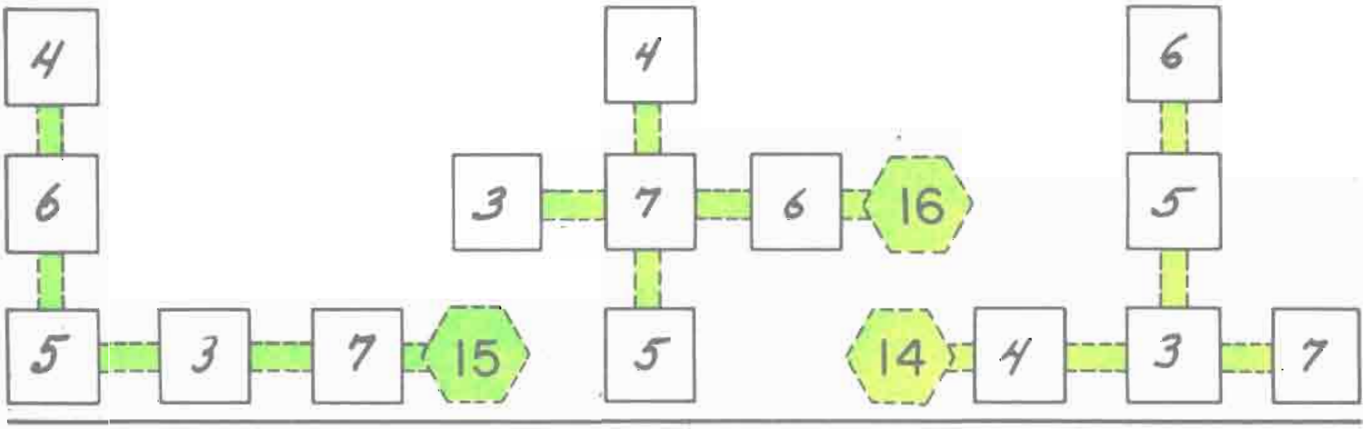
NOW TRY IT WITH THE OTHER HAND.
HOW CLOSE WERE YOU? ________ cm

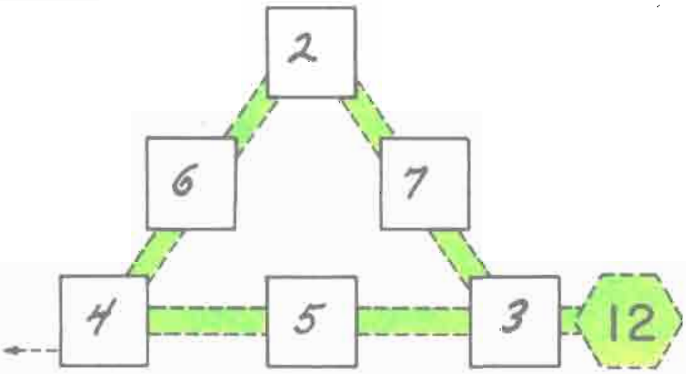
USE A HEAVIER OBJECT.
DID YOU COME CLOSER? ____________________

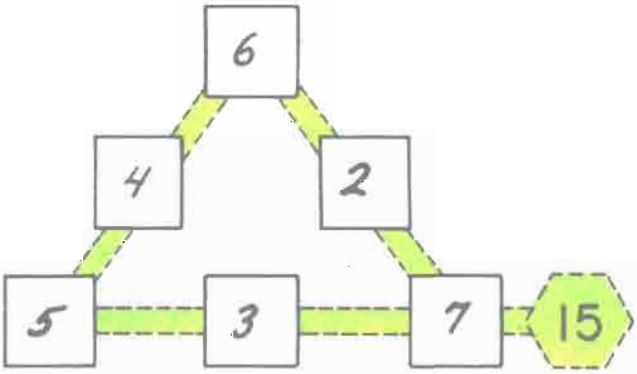
ARE YOU MORE ACCURATE WITH HEAVIER OR LIGHTER OBJECTS? ____________
HAVE OTHER PEOPLE TRY IT, AND SEE IF YOU GET THE SAME RESULTS.

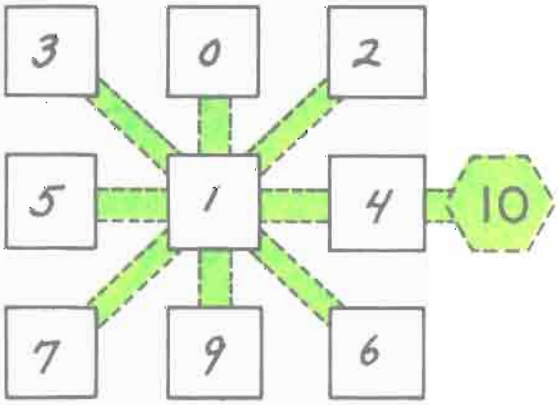
Arrangement PUZZLES with Small Numbers

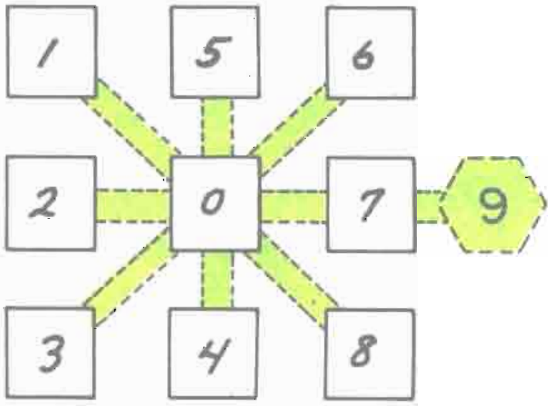
Please arrange different numbers in each example so each line of 3 numbers adds to the numbers given in the hexagon.











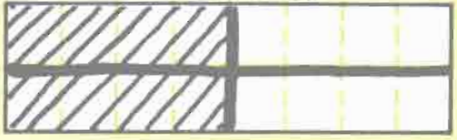
cut-outs may help 0 1 2 3 4 5 6 7 8 9 10

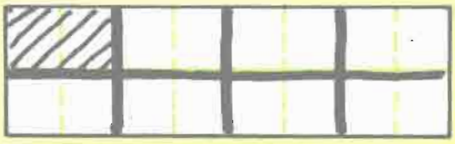
3 ← Then color in this many parts.

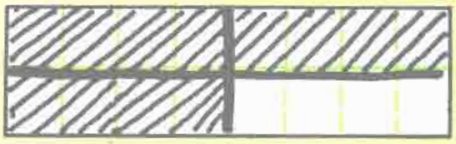
Please divide each pound → 4
into this many parts

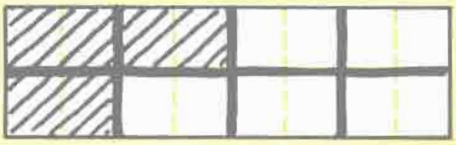
12 ← Report number of ounces colored in.

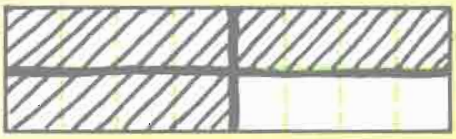
Fractional cardboard cut-outs may be useful on this page and those following.

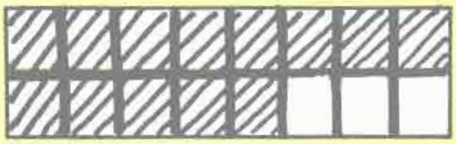
8 oz.

1/2 lb.

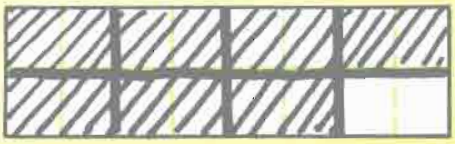
2 oz.

1/8 lb.

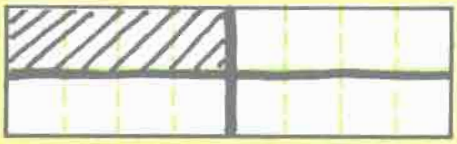
12 oz.

3/4 lb.

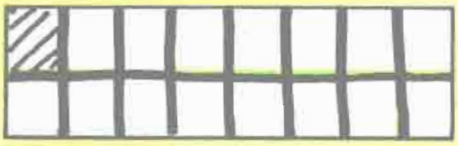
6 oz.

3/8 lb.

12 oz.

3/4 lb.

13 oz.

13/16 lb.

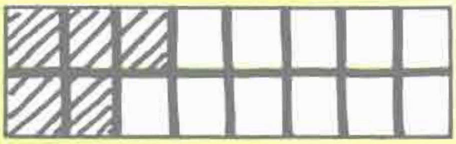
14 oz.

7/8 lb.

4 oz.

1/4 lb.

1 oz.

1/16 lb.

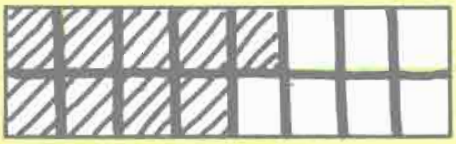
3 oz.

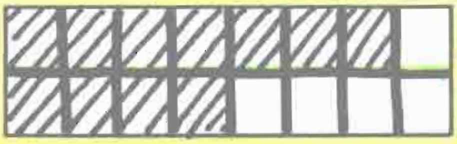
3/16 lb.

5 oz.

5/16 lb.

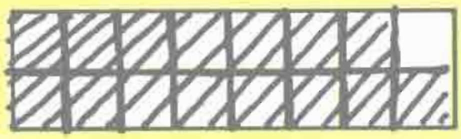
7 oz.

7/16 lb.

9 oz.

9/16 lb.

11 oz.

11/16 lb.

15 oz.

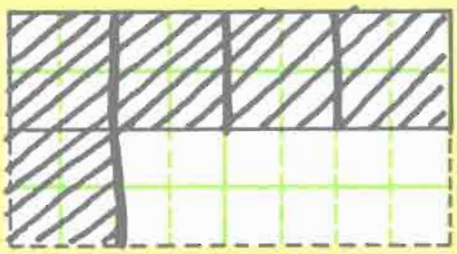


$\frac{15}{16}$ lb.

16 oz.

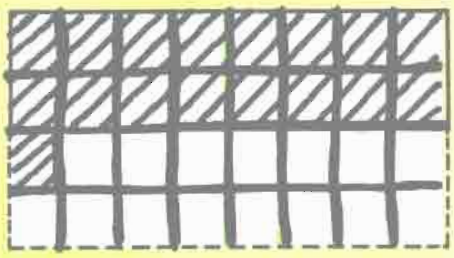
1 lb.

20 oz.



$1\frac{1}{4}$ lb.

17 oz.



$1\frac{1}{16}$ lb.

Please complete this chart:

pounds	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$
ounces	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

lb. $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$
oz. $4 + 8 = 12$

$\frac{1}{8} + \frac{3}{8} = \frac{1}{2}$
$2 + 6 = 8$

$\frac{5}{8} + \frac{1}{4} = \frac{7}{8}$
$10 + 4 = 14$

$\frac{1}{2} - \frac{1}{16} = \frac{7}{16}$
$8 - 1 = 7$

$\frac{3}{4} - \frac{1}{8} = \frac{5}{8}$
$12 - 2 = 10$

$\frac{7}{8} - \frac{3}{8} = \frac{1}{2}$
$14 - 6 = 8$

$\frac{1}{8} \times 2 = \frac{1}{4}$
$2 \times 2 = 4$

$\frac{3}{16} \times 2 = \frac{3}{8}$
$3 \times 2 = 6$

$\frac{15}{16} \div 3 = \frac{5}{16}$
$15 \div 3 = 5$

$\frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

$\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$

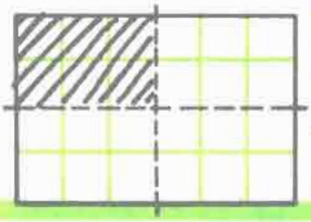
$\frac{1}{16} \times 4 = \frac{1}{4}$

There are 24 hours (hr.) in 1 day (da.)

Please divide each sketch into this many parts → $\frac{}{}$ ← Color in this many parts.

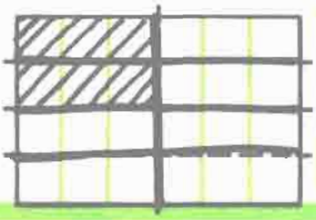
← No. of hours colored in.

Some "numbers of hours" can be colored in by different pairs of directions.



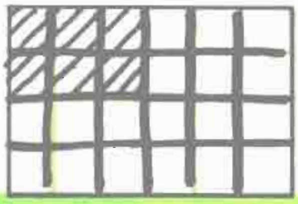
$\frac{1}{4}$ da.

6 hr.



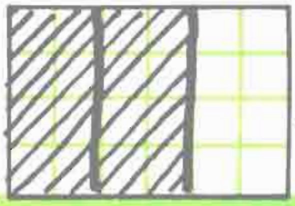
$\frac{2}{8}$ da.

6 hr.



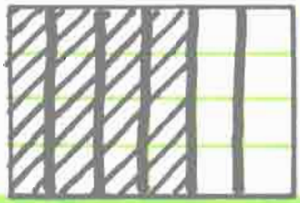
$\frac{6}{24}$ da.

6 hr.



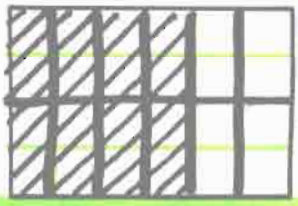
$\frac{2}{3}$

16



$\frac{4}{6}$

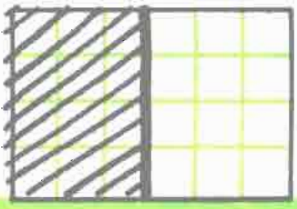
16



$\frac{8}{12}$

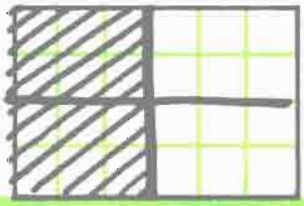
16

Please find different pairs of directions for each sketch.



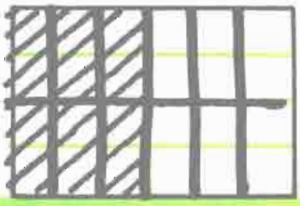
$\frac{1}{2}$

12



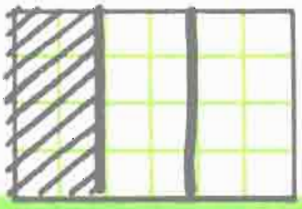
$\frac{2}{4}$

12



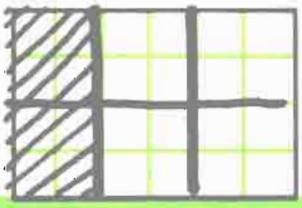
$\frac{6}{12}$

12



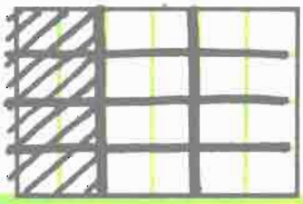
$\frac{1}{3}$

8



$\frac{2}{6}$

8



$\frac{4}{12}$

8

So we say:

$$\text{da.}\ \overset{*}{\frac{2}{3}} = \frac{4}{6} = \frac{8}{12} = \frac{16}{24}\ \text{and}\ \overset{*}{\frac{1}{2}} = \frac{2}{4} = \frac{6}{12}\ \text{and}\ \overset{*}{\frac{1}{3}} = \frac{2}{6} = \frac{4}{12}$$

$$\text{hr.}\ 16 = 16 = 16 = 16 \qquad 12 = 12 = 12 \qquad 8 = 8 = 8$$

* and the pair of directions with the smallest numbers is called the "simplest form."

Please fill in the blanks.

parts of a pound	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{7}{16}$	$\frac{9}{16}$	$\frac{11}{16}$
ounces	16	8	4	12	2	6	10	14	1	3	5	7	9	11

$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$8 + 4 = 12$

$\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$

$8 - 2 = 6$

$\frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

$12 - 4 = 8$

$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$4 + 4 = 8$

$\frac{1}{4} \times 2 = \frac{1}{2}$

$4 \times 2 = 8$

$\frac{1}{4} \div 2 = \frac{1}{8}$

$4 \div 2 = 2$

parts of a day	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{1}{12}$	$\frac{1}{6}$
hours	24	12	8	16	6	18	4	20	3	9	15	21	2	4

$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$12 + 6 = 18$

$\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$

$12 - 3 = 9$

$\frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

$18 - 6 = 12$

$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$6 + 6 = 12$

$\frac{1}{4} \times 2 = \frac{1}{2}$

$6 \times 2 = 12$

$\frac{1}{4} \div 2 = \frac{1}{8}$

$6 \div 2 = 3$

A MAGIC NUMBER GAME

Ask a friend to choose any
number between 10 and 100.

Now here's how to find that number! -

Tell your friend to:

ⓐ Write the figure in the ten's place
and double it.

ⓑ Add 1 to this product.

ⓒ Multiply your answer by 5.

ⓓ Take your original number:
what is the figure in the one's place? _____

ⓔ Add that figure to the answer from c.

ⓕ Now add 106 to this sum.

the key → subtract 111 from the result.

example: 78

ⓐ 7
×2
14

ⓑ +1
15

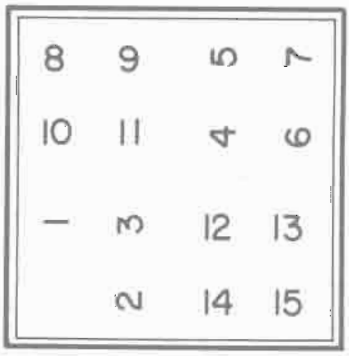
ⓒ ×5
75

ⓓ 8

ⓔ 75
+8
83

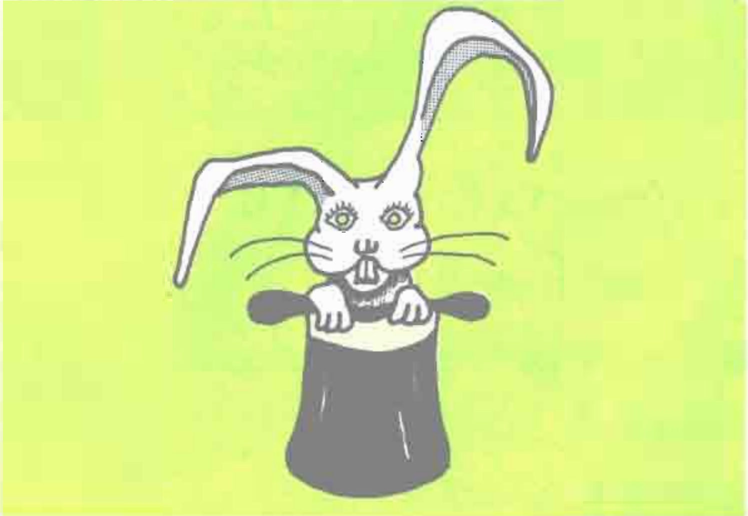
ⓕ +106
189
−111 the key
78

The square will be cut out and used on the following page.



8	9	5	7
10	11	4	6
1	3	12	13
	2	14	15

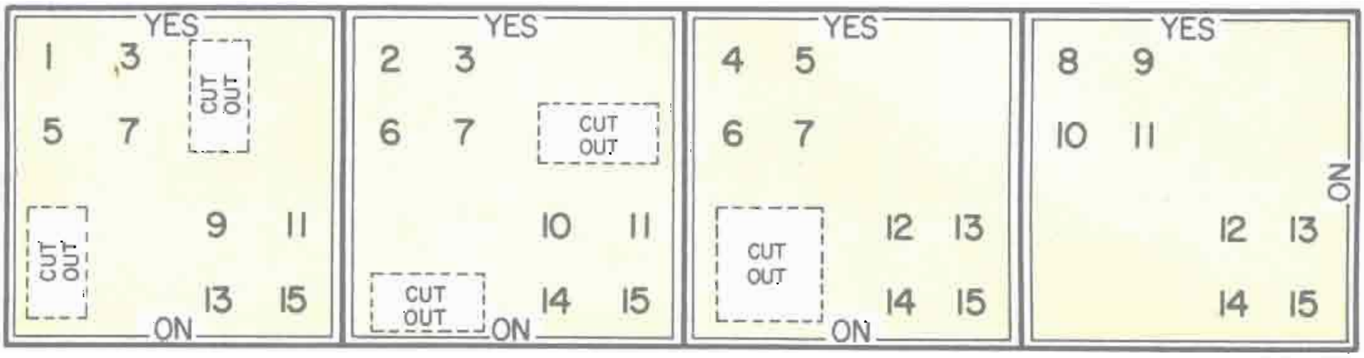
MAGIC
CARDS GAME



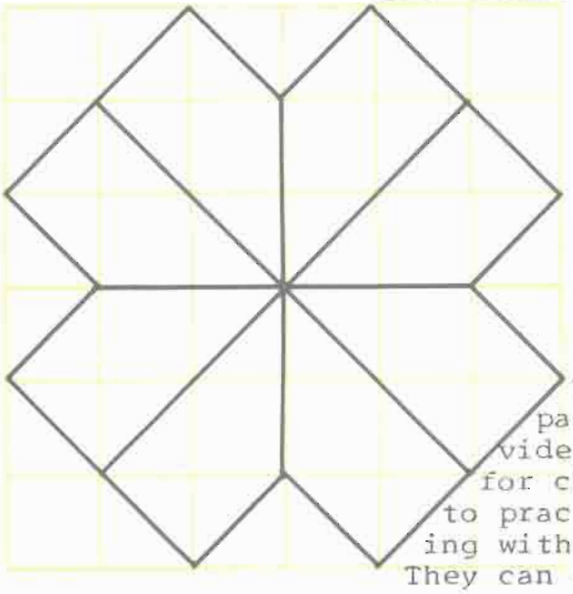
CUT OUT THE CARDS BELOW; THEN CUT OUT THE SPACES MARKED 'CUT OUT'. NOTICE THAT ONE CARD HAS NUMBERS WRITTEN ON THE BACK AS WELL AS THE FRONT.

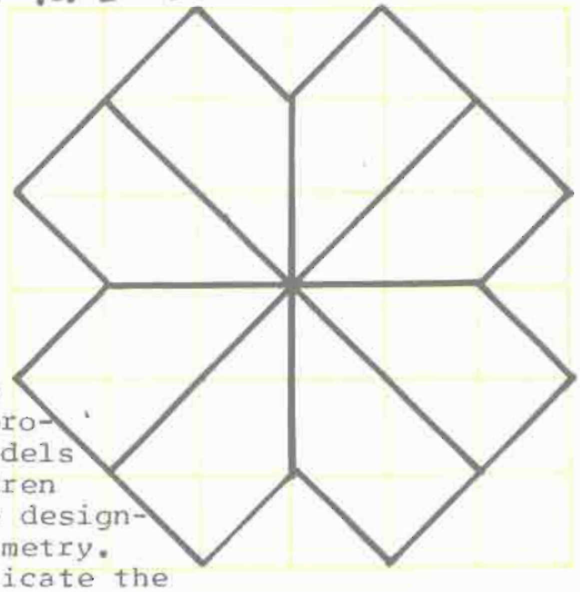
WHEN THE CARDS ARE READY, ASK SOMEONE TO THINK OF A NUMBER FROM 1 TO 15. SHOW THEM THE FRONT OF EACH CARD AND ASK THEM IF THEIR NUMBER IS ON THE CARD. IF THE ANSWER IS 'YES' PUT THE CARD ON THE TABLE WITH 'YES' AT THE TOP. IF IT'S 'NO', PUT 'NO' AT THE TOP.

STACK THE CARDS ONE OVER THE OTHER, WITH THE CARD THAT HAS NO HOLE IN IT ON TOP OF THE OTHERS. PICK UP THE CARDS AND TURN THE STACK OVER. THE CORRECT NUMBER WILL SHOW THROUGH THE WINDOW IN THE CARDS.

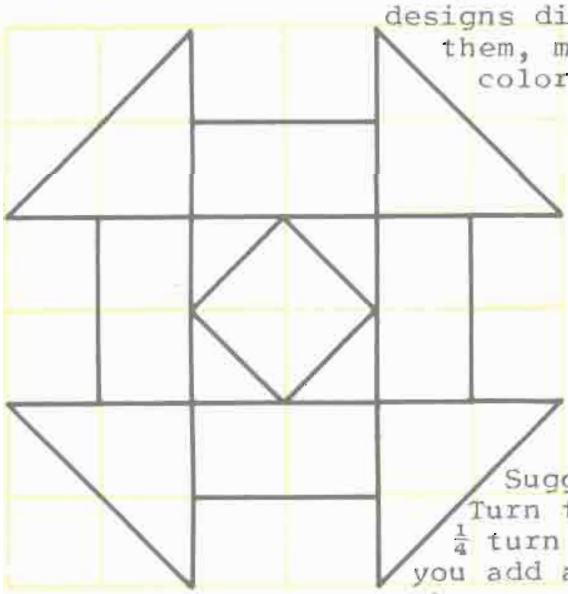


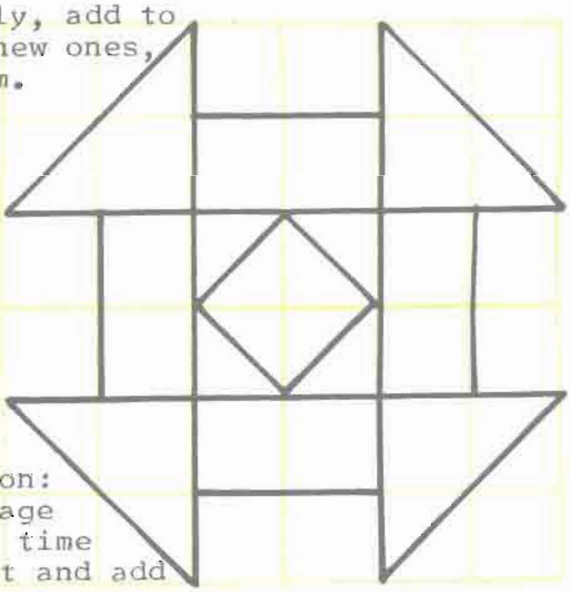
GEOMETRY for FUN



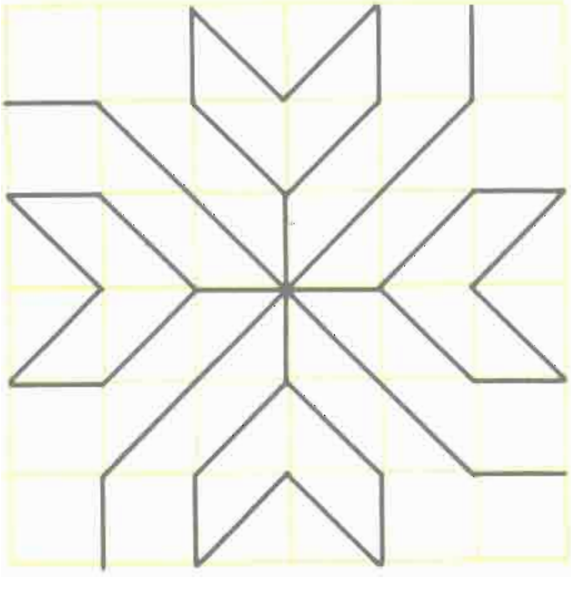


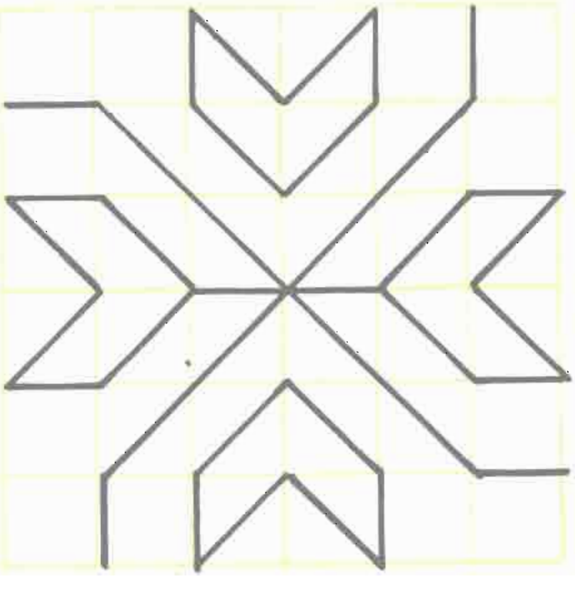
This page provides models for children to practice designing with geometry. They can duplicate the designs directly, add to them, make new ones, color them.





Suggestion:
Turn the page
$\frac{1}{4}$ turn each time
you add a part and add
the same to each part.





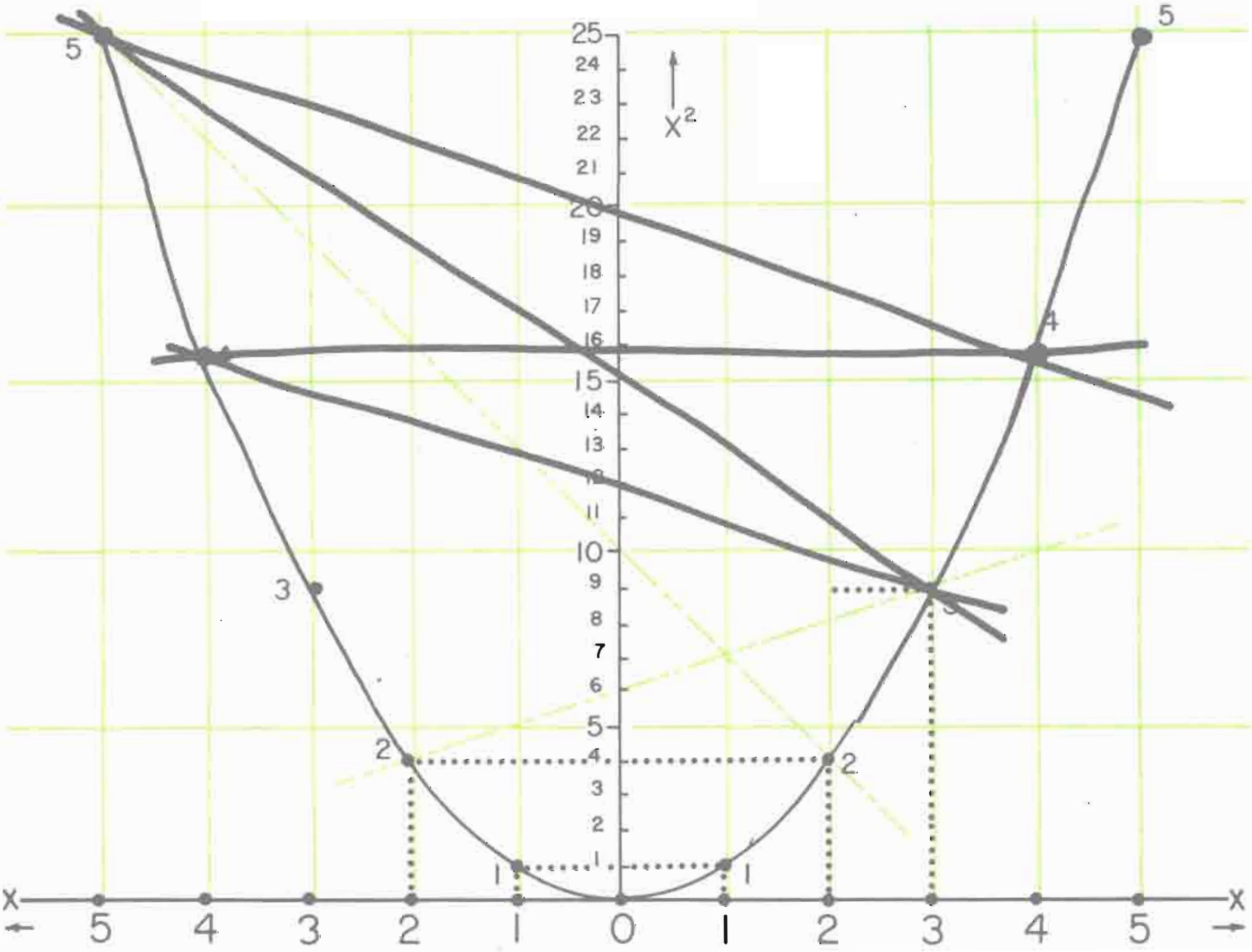
X and X^2 ↑

. . . or, building a mini-computer for multiplication and division

The machine begins with a graph of this table.

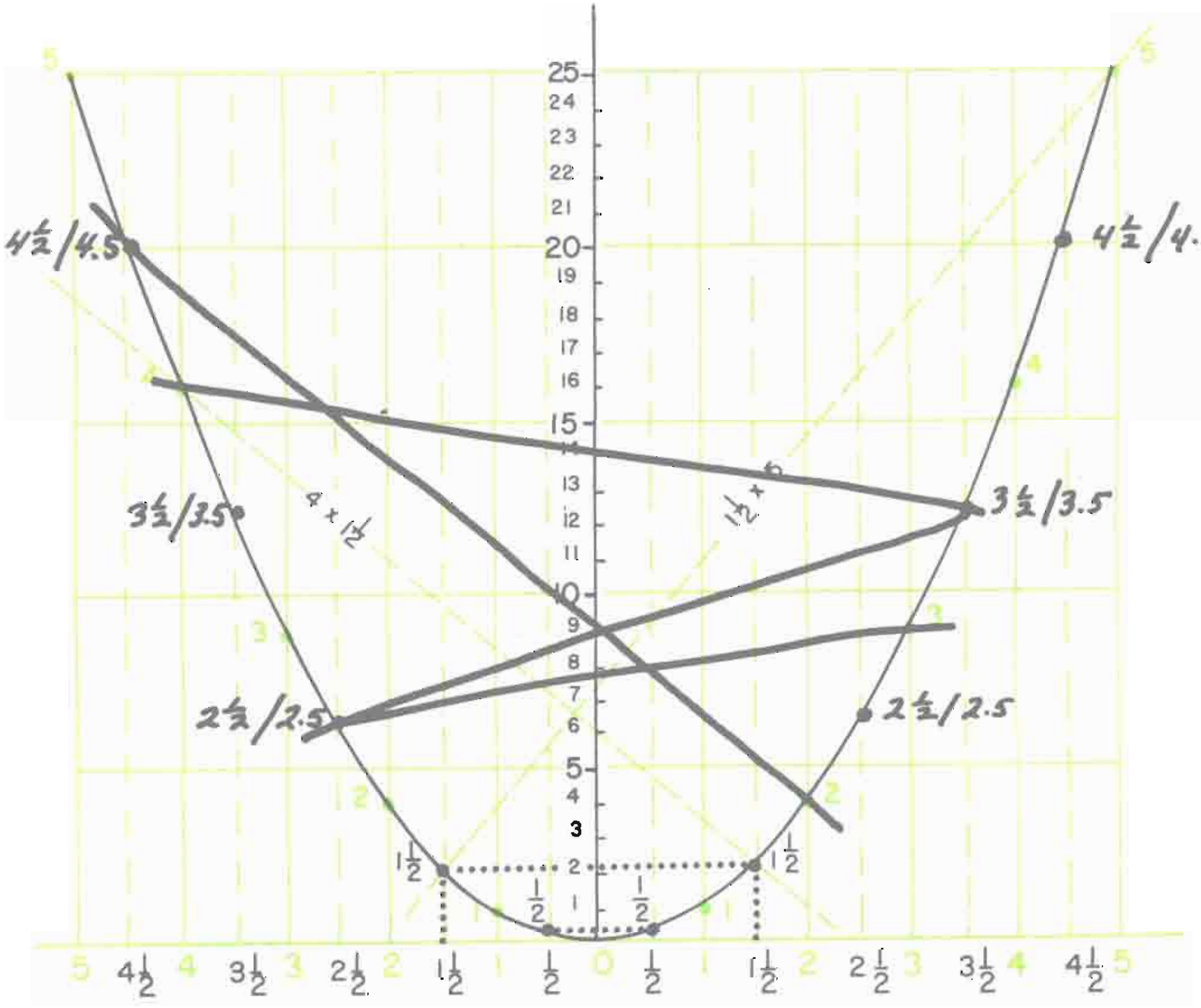
X ←→	0	1	2	3	4	5
↑ X × X or X^2 ↑	0	1	4	9	16	25
	0×0	1×1	2×2	3×3		

Can you complete this sketch?



Please complete the following table, and add the points they indicate.

← x →	.5 or ½	1.5 or 1½	2.5 or 2½	3.5 or 3½	4.5 or 4½
x^2 ↑	.25 or ¼	2.25/2¼	6.25/6¼	12.25/12¼	20.25/20¼



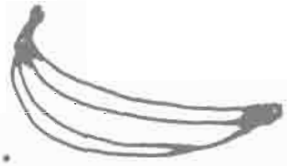
4	6	1½
1½	7½	5
2½	7½/7.5	3
4	14	3½
4½	9	2
2½	8¾/8.75	3½

Using the Mini Computer as a divisor

5	20	4
4	12	3
4½	18	4
5	17½	3½
4½	11¼	2½
4½	20¼	4½

MEASURING WITH FRUIT

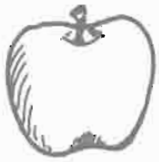
The children will need real fruit to complete this page.



There are ____ sides on a banana.

Do all bananas have the same number of sides? ____

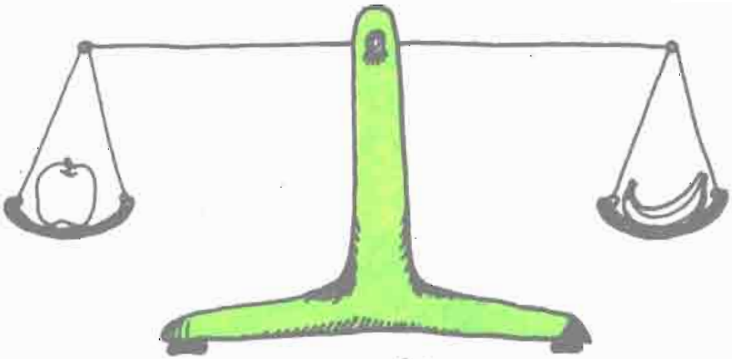
The banana I have is ____ cm long.



There are ____ seeds in an apple.

Do all apples have the same number of seeds? ____

My apple is ____ cm in circumference.



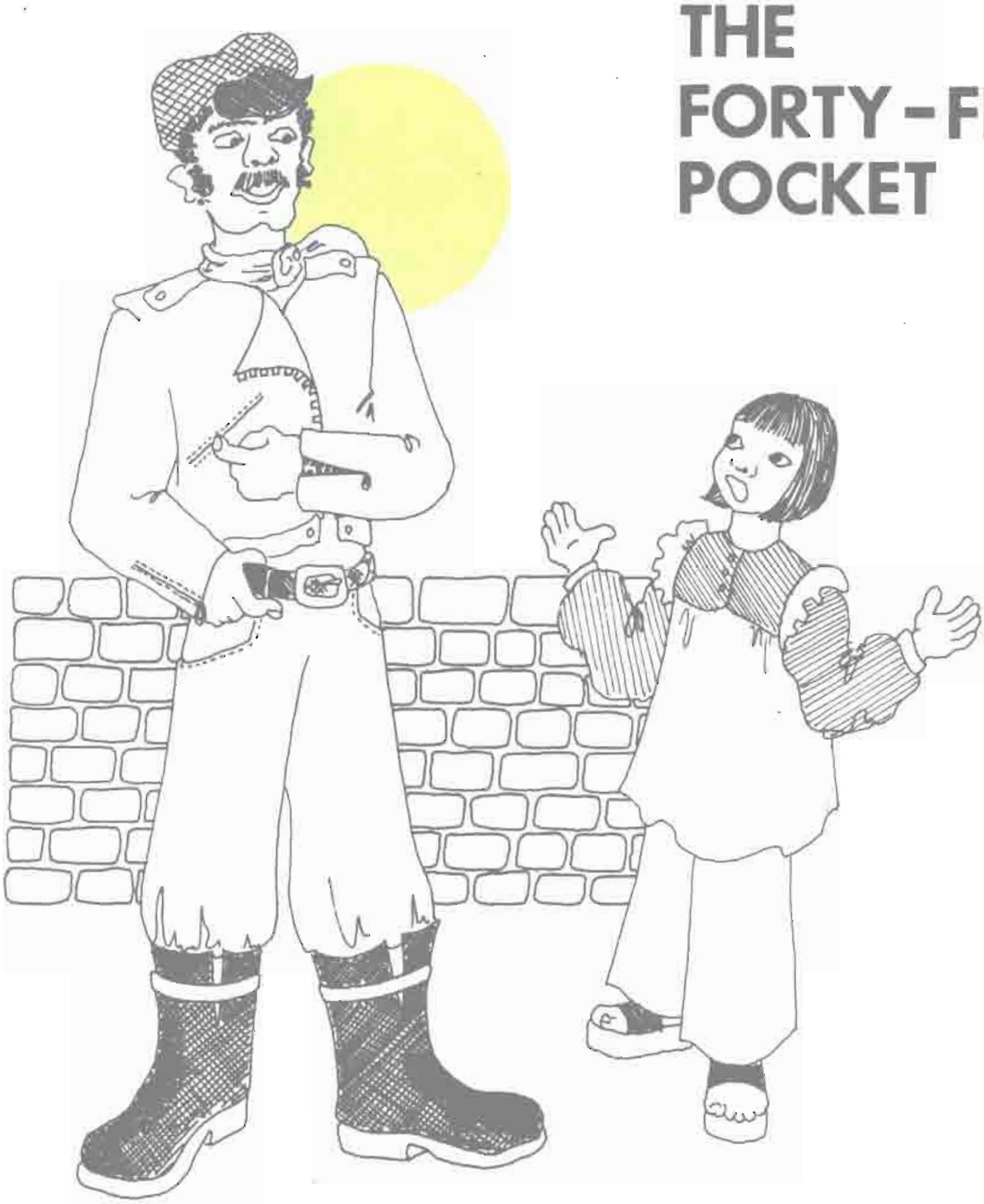
Compare an apple with a banana.

Which weighs more? ____________

Please bite off some of the apple or banana to equalize the weight.

What did you discover?

THE FORTY-FIFTH POCKET



Paula's Uncle Max was a teaser.

Once, when he came to visit, he said: "There is a quarter in my forty-fifth pocket. If you guess the right pocket, the quarter is yours."

"You haven't got 45 pockets."

"That's right; but it's in the pocket I call my 'forty-fifth pocket'."

"Is it in your shirt pocket?" Paula guessed.

"Wrong, but I'll give you another chance . . . after I tell you the way I number my pockets.

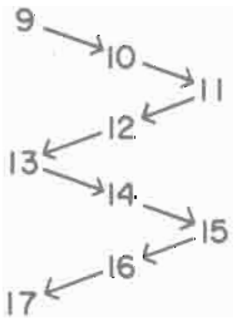
"My right hand pants pocket is 1; my shirt pocket is 2, and my left hand pants pocket is 3. Now would you like to try again to find my forty-fifth pocket?"

"But, you've only told me about pockets you call '1', '2' and '3'."

"You're right," Uncle Max agreed. "I'll tell you the rest. I go back to my shirt pocket for 4, then to my right pocket for 5 and I'm back where I began. Then the shirt pocket is 6, left is 7, shirt is 8, right 9, shirt is 10—etc.

"Now it's your turn to try again but you have only five seconds to find out which is the forty-fifth pocket."

Paula pointed to the pockets . . .



Uncle Max broke in: "Time's up . . . that's five seconds of counting; you started at 9 and only got to 17 . . . nowhere near 45."

"Okay; then I'll guess. You already told me it wasn't in your shirt pocket, so it must be in your right or left pants pocket."

"That's right. Now if you would rather have 15¢ than run the risk of being wrong, I'll give you 15¢ right now and we'll stop there. So take your pick: 15¢ for sure or make a guess—you may get 25¢ or you might get nothing. What do you say, Paula?"

Paula didn't answer. She just looked at her uncle. In her head she was counting "eighteen, nineteen, twenty" . . . and was already at 30 before her uncle guessed what she was doing.

"Your counting time was up a long time ago."

"Then I'll guess . . ." Paula began and then she stopped. As she had been counting those pockets she had noticed something that might be useful; it couldn't have been in his shirt pocket because that pocket was always called an even number . . . so her first guess that the forty-fifth pocket might be the shirt pocket was a very bad guess. But the right and left pockets were both called by odd numbers. If she only had more time.

"No, Uncle Max; I'd rather not guess or take the 15¢. Will you play this game again with me after dinner . . . and you can move the quarter into any pocket before we play again?"

"You mean I can move the quarter to my forty-eighth pocket if I want to . . . or my 113th pocket?"

"Right," Paula answered even though that 113 was a pretty big number . . . but then, she thought, Uncle Max wouldn't be that mean . . . or would he?

"It's a deal, Paula, after dinner; but this time there'll be no '15¢ for sure' . . . it'll be guess right or nothing."

Paula ran upstairs to her room and went to work with pencil and paper:

RP	SP	LP
1		
	2	
		3
	4	
5		
	6	
		7
	8	
9		
	10	
		11
	12	
13		
	14	
		15

She had been right; the shirt pocket is always "even" and the pants pockets are always "odd".

Next Paula noticed that the numbers called "right pocket" were 4 apart:

1 +4 5 +4 9 +4 13 +4 ___ +4 ___ +4 ___

"So, I can count from 1 by 4's; that'll be much faster." She wrote down the numbers to practice counting:

1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45

"See, it was in Uncle Max's right hand pocket," Paula said to herself, "and if he had said '39th pocket' it would be in the left hand pocket—because 39 is not even and it's not in my 4's list." And she went back to practicing . . .

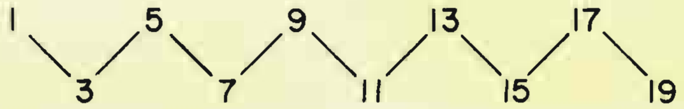
"1, 5, 9, 13, 17, etc."

Paula heard Uncle Max laughing in the kitchen—probably at one of his old jokes—and Paula thought, "113—he wouldn't do that . . . or would he?"

He just might, so Paula extended her list to practice . . .

"41, 45, 49, 53 . . ."

Paula stopped and looked at her list——back at the beginning. And she put down the odd numbers she was skipping.



(odd number for right hand pocket)

(odd number she skipped)

Then she saw something that might be useful. She wrote down the "4 times" table:

0, 4, 8, 12, etc.

And then she wrote the right hand list above and the left hand list below in this way:

Right	1	5	9	13	17	21	___	___	___
Shirt	0	4	8	12	16	20	___	___	___
Left		3	7	11	15	19	___	___	___

"I think I can win my quarter," she said to herself: "Let him try 113."

Right				113
Shirt	100	104	108	112
Left				111

"Right hand pocket."

Just then, Paula's father called her to dinner.

at dinner...

Thru dinner , Paula looked a little upset and seemed to be thinking about something.

"I think Paula's counting pockets while she eats," Uncle Max joked.

"Maybe you're right," Paula replied—but Uncle Max didn't really know what she was thinking about.

After dinner her uncle asked Paula: "Do you want to play the quarter game now?"

"Yes, but you know Uncle Max, I'm saving up for a speedometer for my bike and I only need 50¢ more. I've been practicing counting your pockets and I'm pretty fast. "You tell me the number of the pocket, and if I guess right, I get 50¢ . . . and if I'm wrong, I'll pay you 50¢ out of my savings."

"All right," Uncle Max said, "but only if I can cut your counting time to 5 seconds."

"OK; but I really could use more than 50¢. Suppose I asked you to put 3 half dollars in any of your pockets—all in one or any combination of pockets. If I guess all 3 right, then I keep them; but if I don't guess them all right, I get nothing . . . but I get 15 seconds counting time—one for each number."

Uncle Max got up. "Get ready, because your chances of guessing all three aren't very good."

"I'm ready; but please write your three numbers down so I don't have to remember them."

Uncle Max wrote them down, being careful Paula couldn't see them.

Finally: "Here they are . . .

23 41 99

and I'm counting seconds: one dinosaur, two dinosaurs, three dinosaurs . . ."

At "8 dinosaurs", Paula laughed and said, "you know Uncle Max, there's a different number in each pocket."

"What! What's that got to do with it? You're right, but you haven't told me how many there are in each pocket. Say——you're trying to get more counting time . . . 9 dinosaurs, 10 dinosaurs . . ."

Paula broke in again: "There are twice as many in your left hand pocket as in the right and two more in the left pocket than in your shirt pocket."

Uncle Max went right on counting . . . "14 dinosaurs, 15 dinosaurs . . . time's up. You lost!"

Paula laughed; "But I told you there were twice as many in your left hand pocket and 2 more than in your shirt pocket."

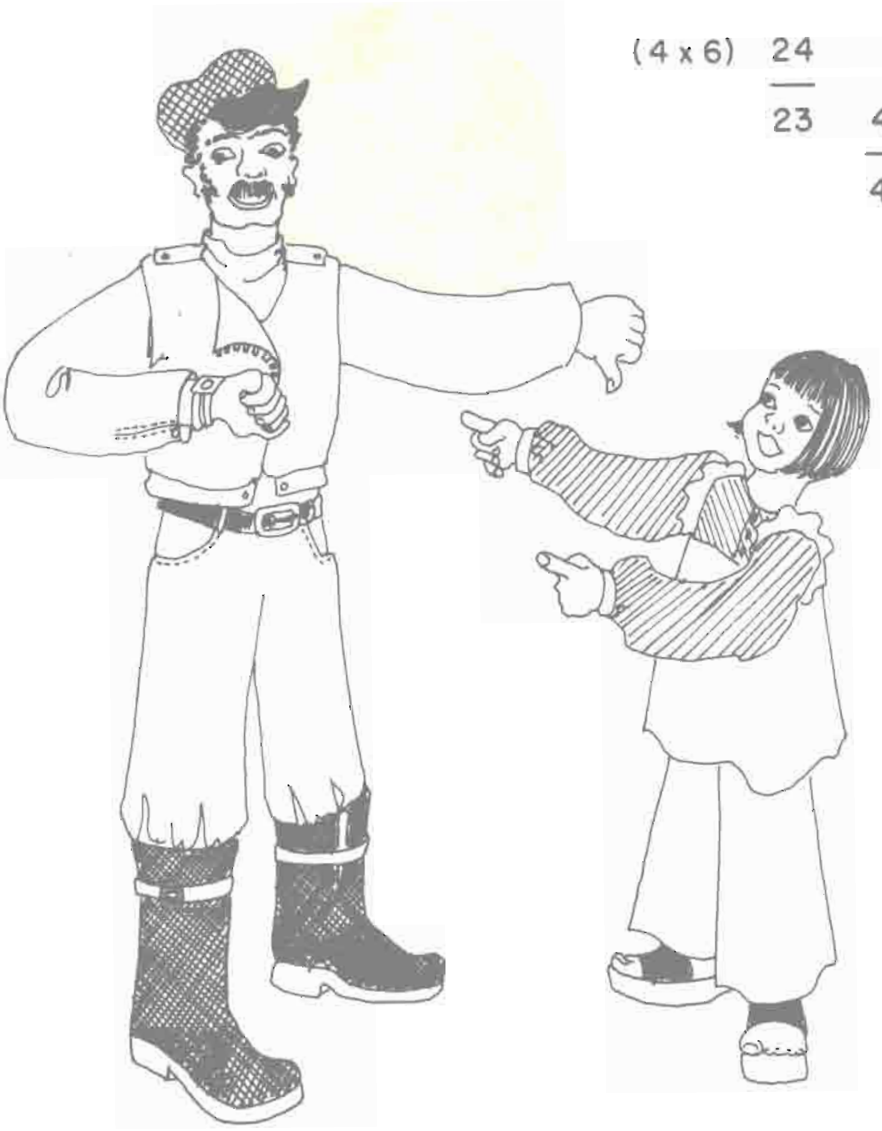
"But I could have 2 in my left hand pocket and 1 in my shirt pocket—that's two more, and that's not right."

"Yes, Uncle Max, but then your right hand pocket would be empty—and there wouldn't be twice as many in the left as in the right. So you have to have 2 in the left and 1 in the right pocket . . . and none in your shirt pocket."

Uncle Max was still not sure Paula had told him enough. "If you tell me how you did it, I'll pay up."

On the card with the three numbers Paula added some numbers:

$$\begin{array}{ccccc} (4 \times 6) & \underline{24} & & \underline{100} & (4 \times 25) \\ & 23 & \underline{41} & 99 & \\ & & 40\ (4 \times 10) & & \end{array}$$



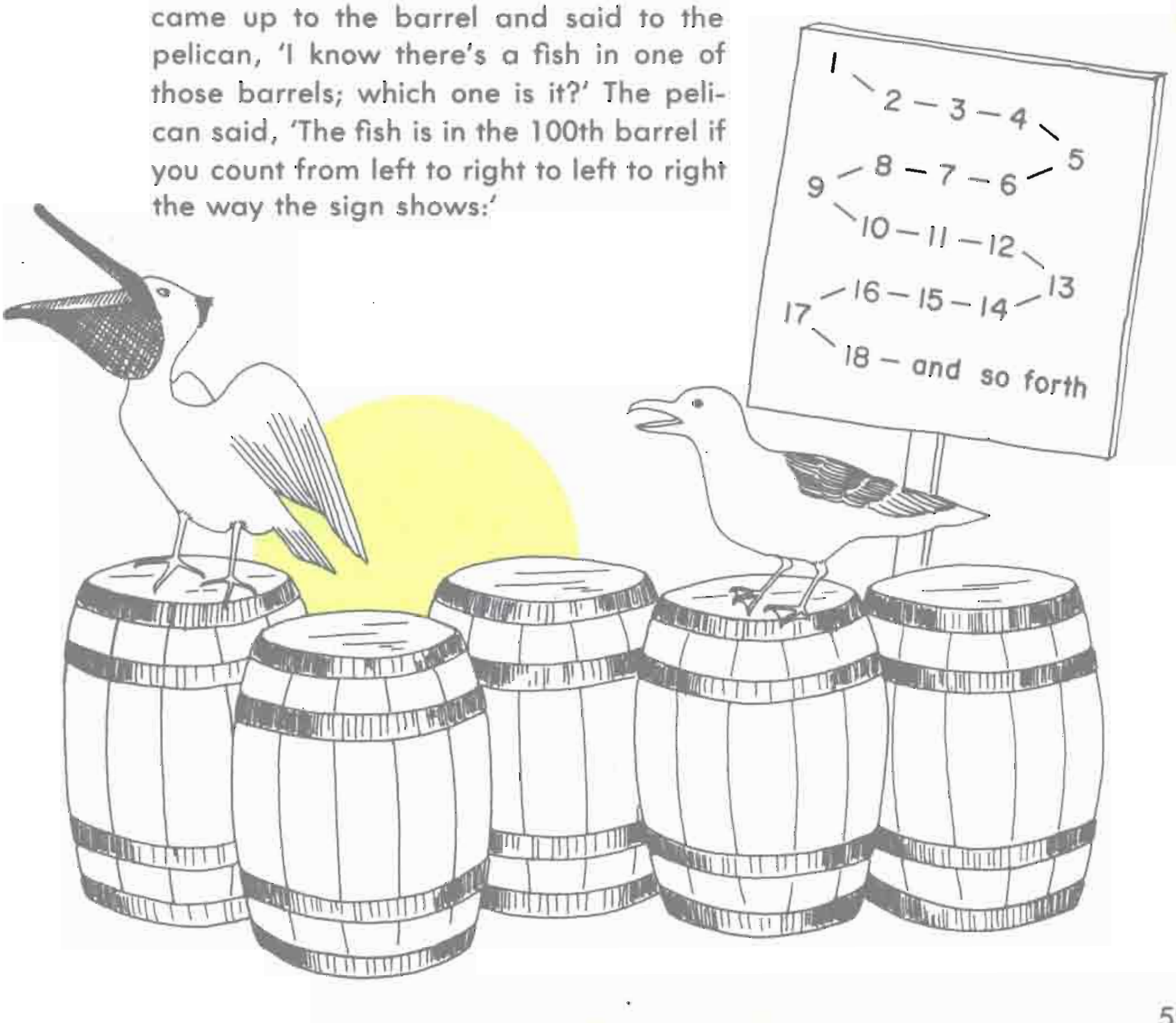
"Below the line is left and above the line is right."

Then Paula showed her Uncle her notes: and added some examples.

Right	1	5	9	13		41	
	0	4	8	12	24	40	100
Left		3	7	11	23		99

"You win," Uncle Max admitted. "But let me tell you about the pelican and the fish and the seagull.

"There were 5 barrels. Sitting on the barrel was a wise pelican. A seagull came up to the barrel and said to the pelican, 'I know there's a fish in one of those barrels; which one is it?' The pelican said, 'The fish is in the 100th barrel if you count from left to right to left to right the way the sign shows:'



"Now I've got to go, Paula. Maybe what you learned about my pockets could be useful in helping the seagull find the 100th barrel."

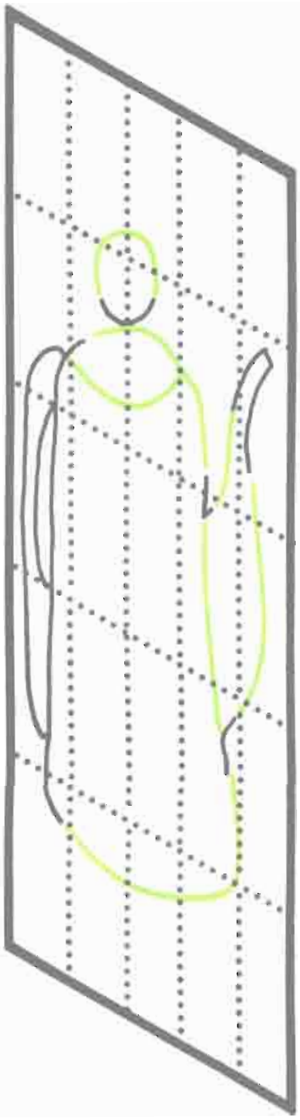
"You know, Uncle Max, if I could save enough, I could get a light for my bike. If I find out which is the 100th barrel, do you think . . ." said Paula putting the $1.50 in her pocket.

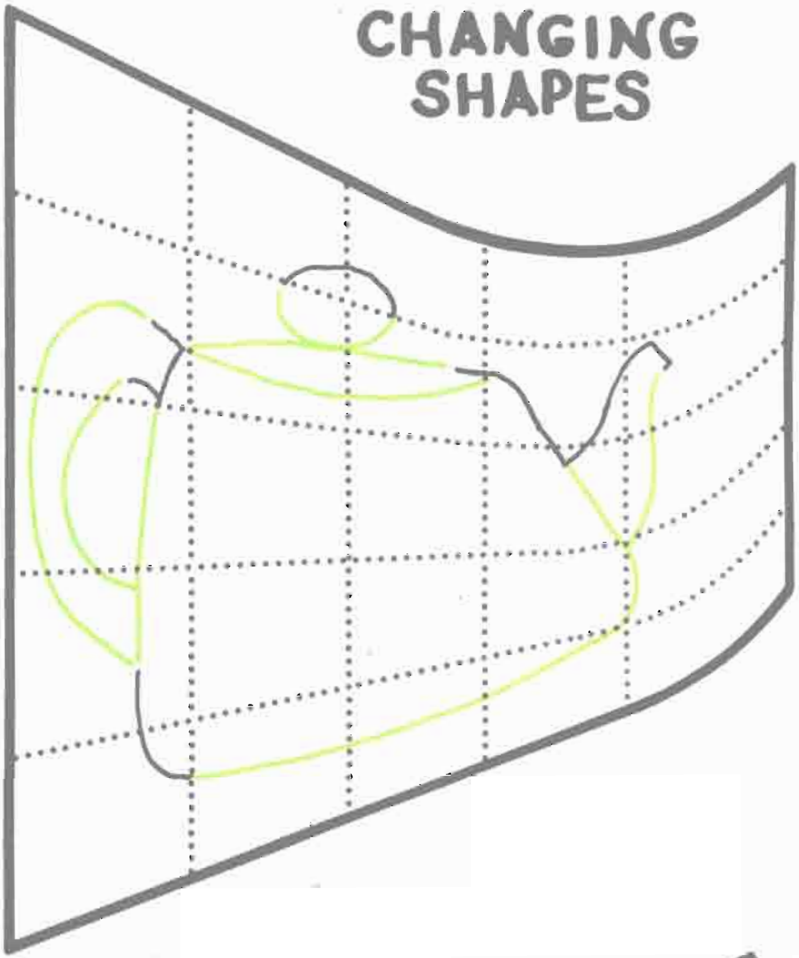
Paula's father saved Uncle Max: "Come on Max, you've just got time to get to the races."

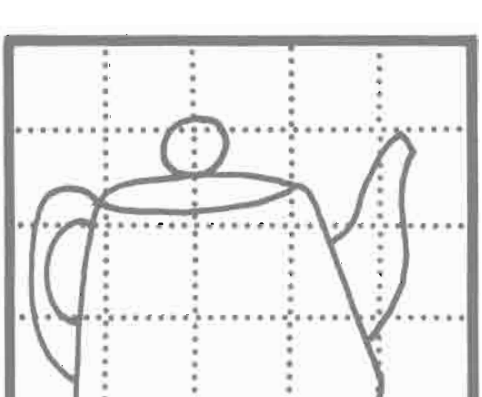
"Goodby, Paula."

"Come again, Uncle Max."

CHANGING SHAPES

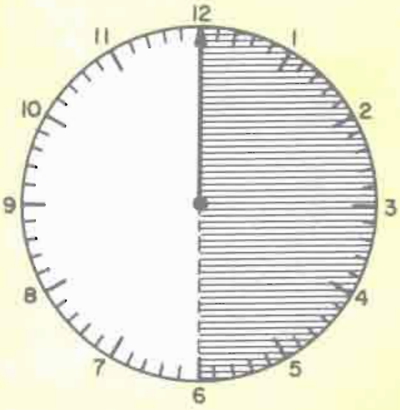




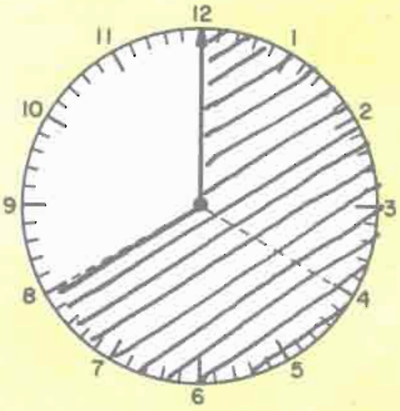


There are 60 minutes (min.) in 1 hour (hr.)

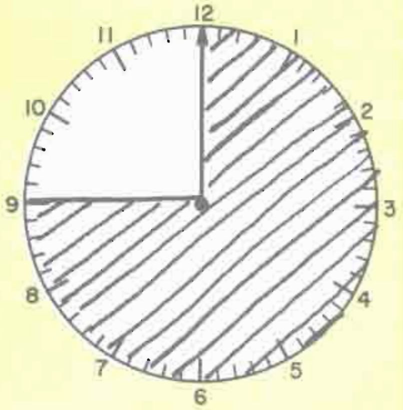
Please complete the shading.



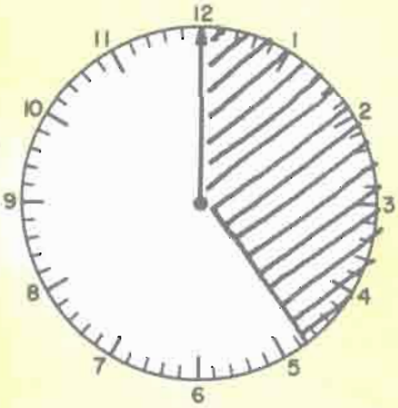
30 min. = $\frac{1}{2}$ hr.



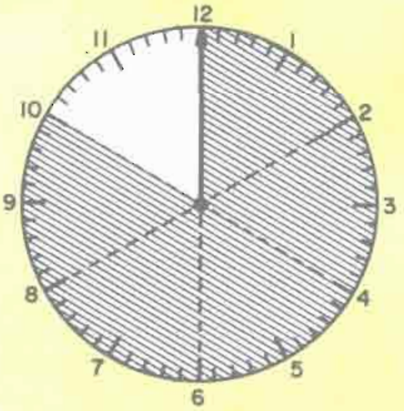
40 min. = $\frac{2}{3}$ hr.



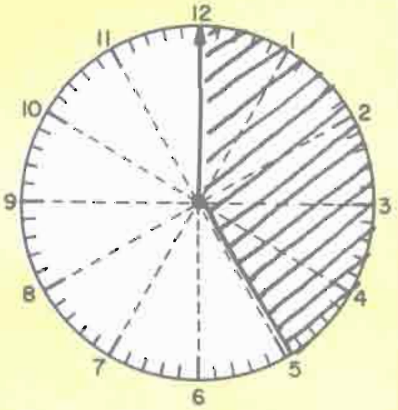
45 min. = $\frac{3}{4}$ hr.



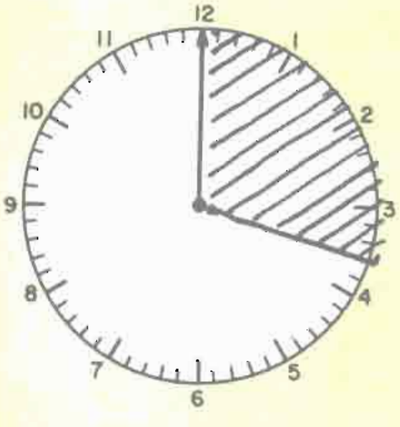
24 min. = $\frac{2}{5}$ hr.



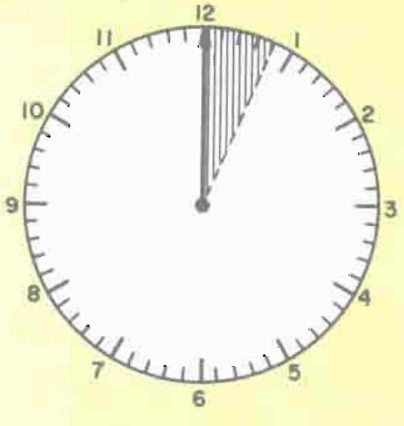
50 min. = $\frac{5}{6}$ hr.



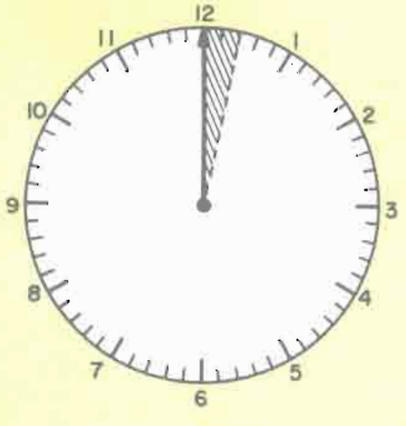
25 min. = $\frac{5}{12}$ hr.



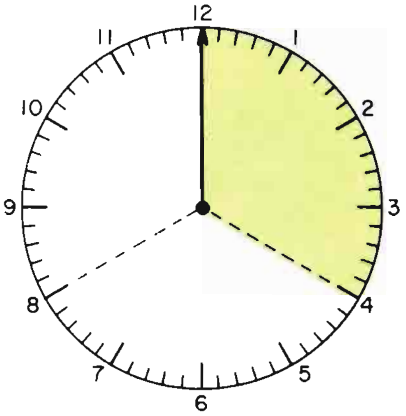
18 min. = $\frac{3}{10}$ hr.

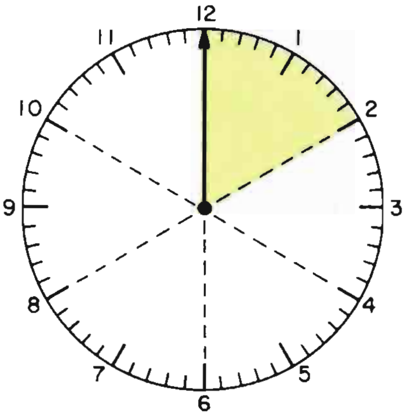


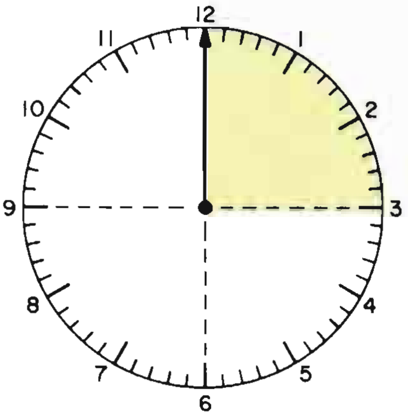
4 min. = $\frac{1}{15}$ hr.



2 min. = $\frac{1}{30}$ hr.







The sketches above may help you fill in the blanks.

parts of an hour $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ $\frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

minutes $30 + 15 = 45$ $20 + 20 = 40$ $20 - 10 = 10$

hr. $\frac{5}{6} - \frac{1}{3} = \frac{3}{6} = \frac{1}{2}$ $\frac{1}{3} \times 2 = \frac{2}{3}$ $\frac{2}{3} \div 2 = \frac{2}{6} = \frac{1}{3}$

min. $50 - 20 = 30$ $20 \times 2 = 40$ $40 \div 2 = 20$

Please make up examples of your own:

hr. — + — = — — + — = — — + — = —

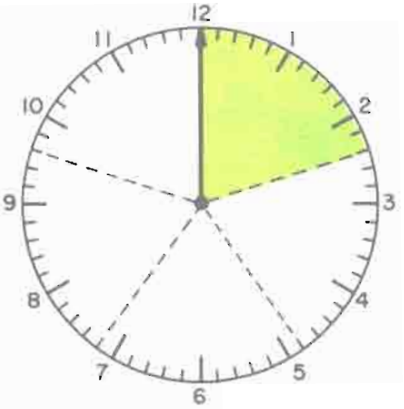
min. + = + = + =

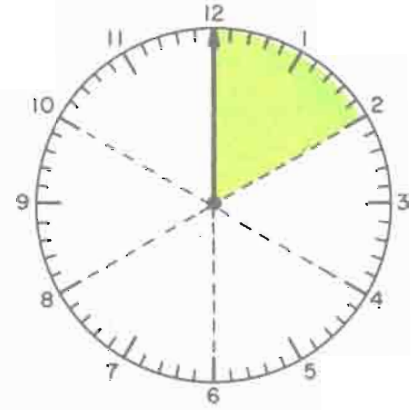
hr. — − — = — — − — = — — − — = —

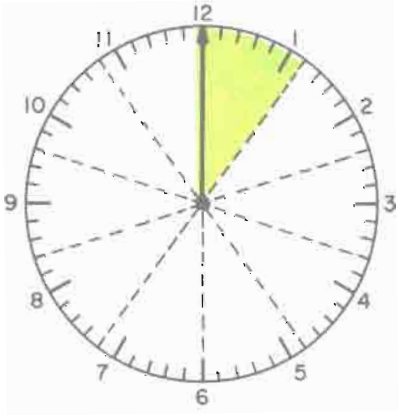
min. − = − = − =

hr. — x ___ = — — x ___ = — — ÷ ___ = —

min. x ___ = x ___ = ÷ ___ =







The sketches above may help you fill in the blanks.

hr. $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$
min. 12 + 36 = 48

hr. $\frac{1}{10} + \frac{1}{10} = \frac{2}{10} \div \frac{1}{5}$
min. 6 + 6 = 12

hr. $\frac{1}{5} - \frac{1}{10} = \frac{1}{10}$
min. 12 − 6 = 6

hr. $\frac{1}{2} - \frac{1}{6} = \frac{2}{6} \div \frac{1}{3}$
min. 30 − 10 = 20

hr. $\frac{1}{10} \times 2 = \frac{2}{10} \div \frac{1}{5}$
min. 6 x 2 = 12

hr. $\frac{1}{5} \div 2 = \frac{1}{10}$
min. 12 ÷ 2 = 6

Please make up examples of your own:

hr. — + — = —
min. + =

— + — = —
+ =

— + — = —
+ =

hr. — − — = —
min. − =

— − — = —
− =

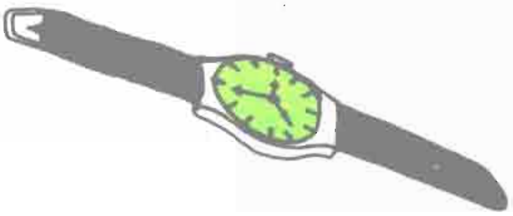
— − — = —
− =

hr. — x ___ = —
min. x ___ =

— x ___ = —
x ___ =

— ÷ ___ = —
÷ ___ =

HOW FAST CAN YOU TALK ?
(no mistakes)



"She sells seashells by the seashore."

Seconds

Say "Toy Boat" 10 times fast.

Seconds

"The sixth sick sheik's sixth sheep's sick."

Seconds

"How much wood could a woodchuck chuck if a woodchuck could chuck wood ?"

seconds

Make up a tongue twister and try it on your friends.

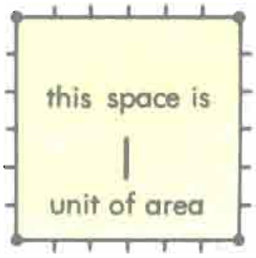
A CUT-OUT GAME

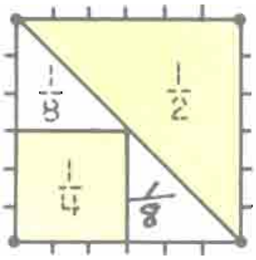
You can suggest to the children that they cut out pieces that have right angles. Then the person putting it back together cannot so quickly identify the outside corners (and thus the puzzle).

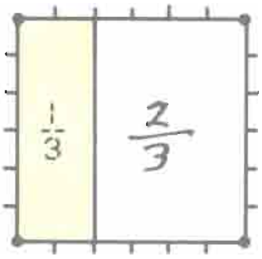
Cut out the space below into 5 pieces.
Ask 3 people to put the pieces together again.
Time them.

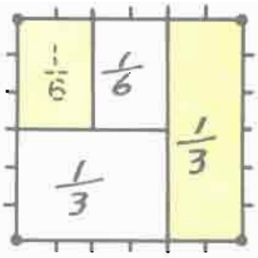
	name	name	name
1st try	Seconds	Seconds	Seconds
2nd try	Seconds	Seconds	Seconds
3rd try	Seconds	Seconds	Seconds

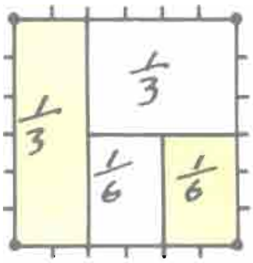
Please indicate the size of each area in the sketches below.

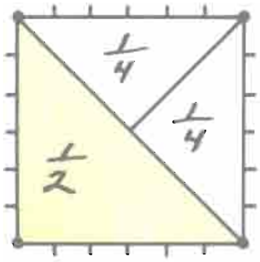


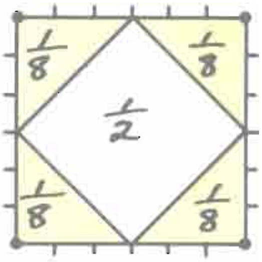


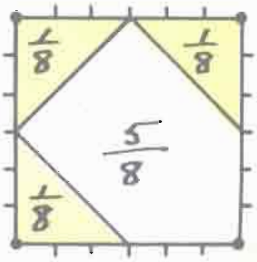


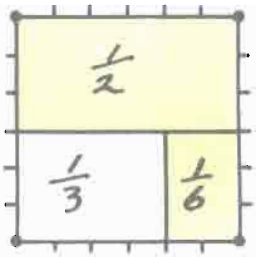


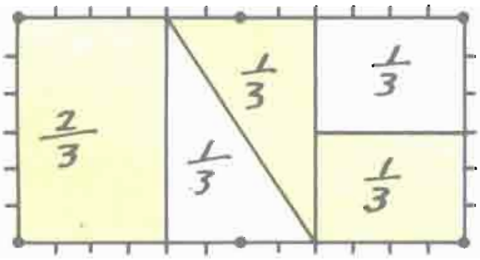


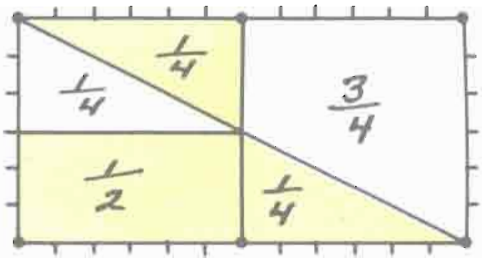


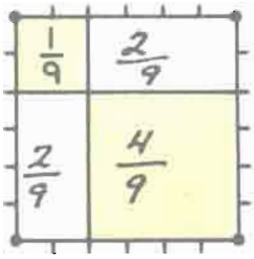


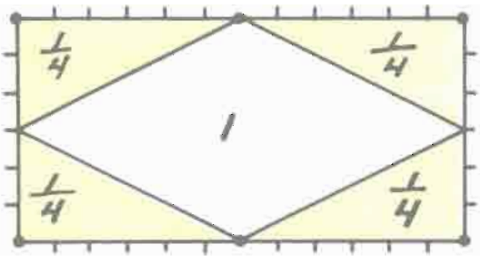


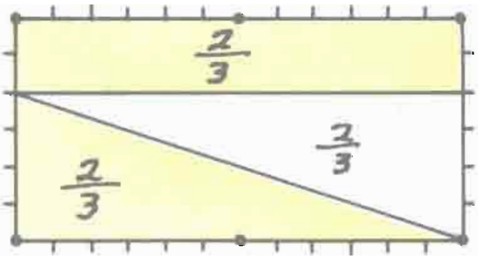


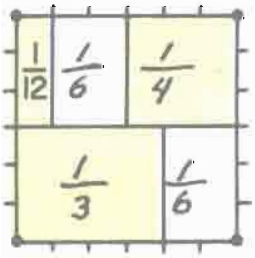


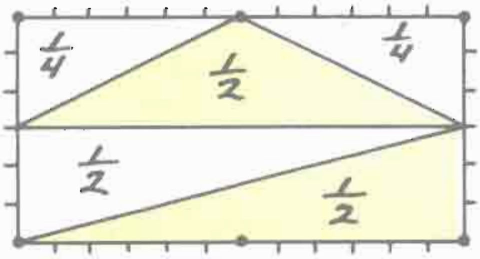


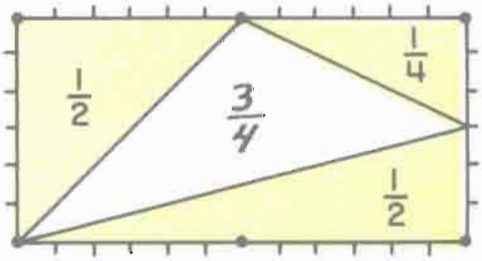


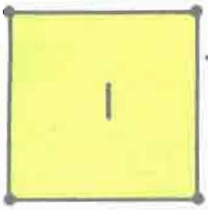






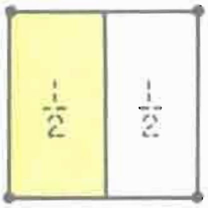


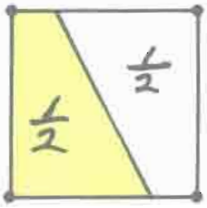


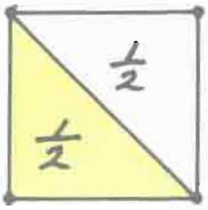


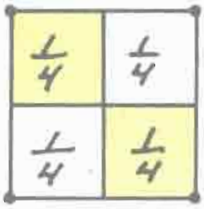
← this is 1 unit of area.

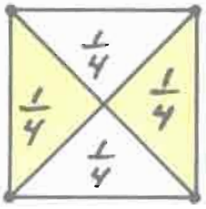
Please indicate the size of each area in the sketches below:

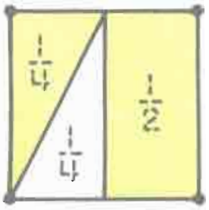


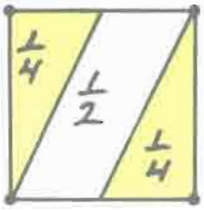


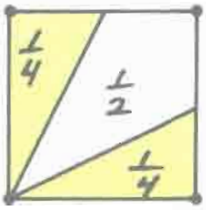


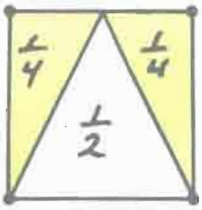


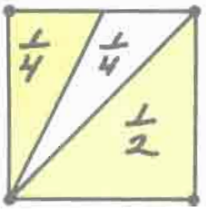


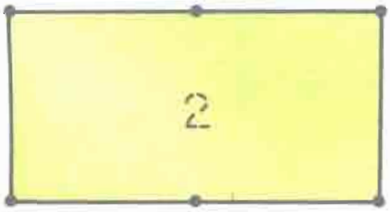


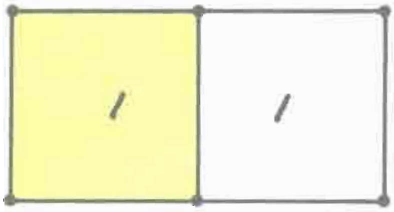


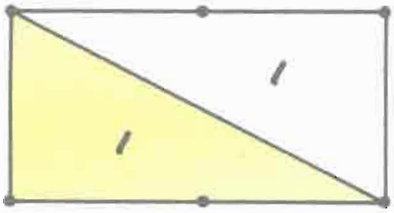


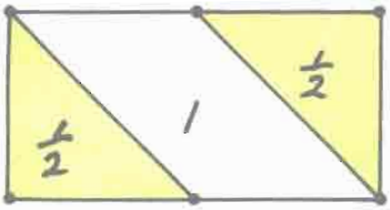


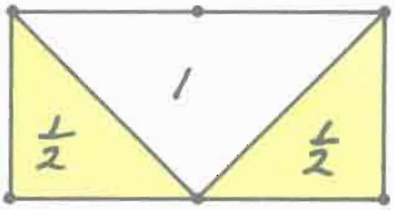


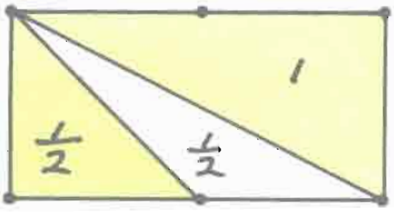


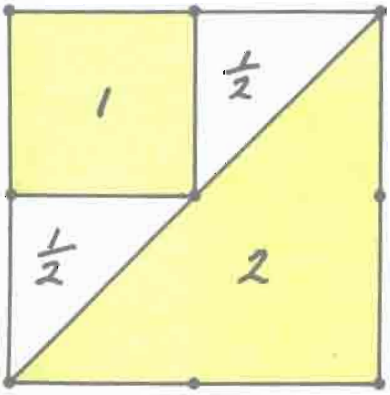


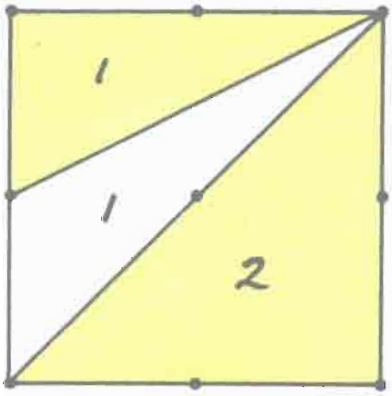


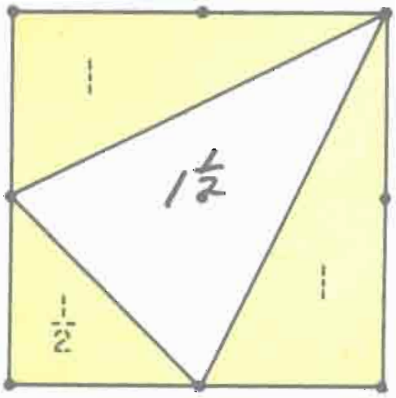






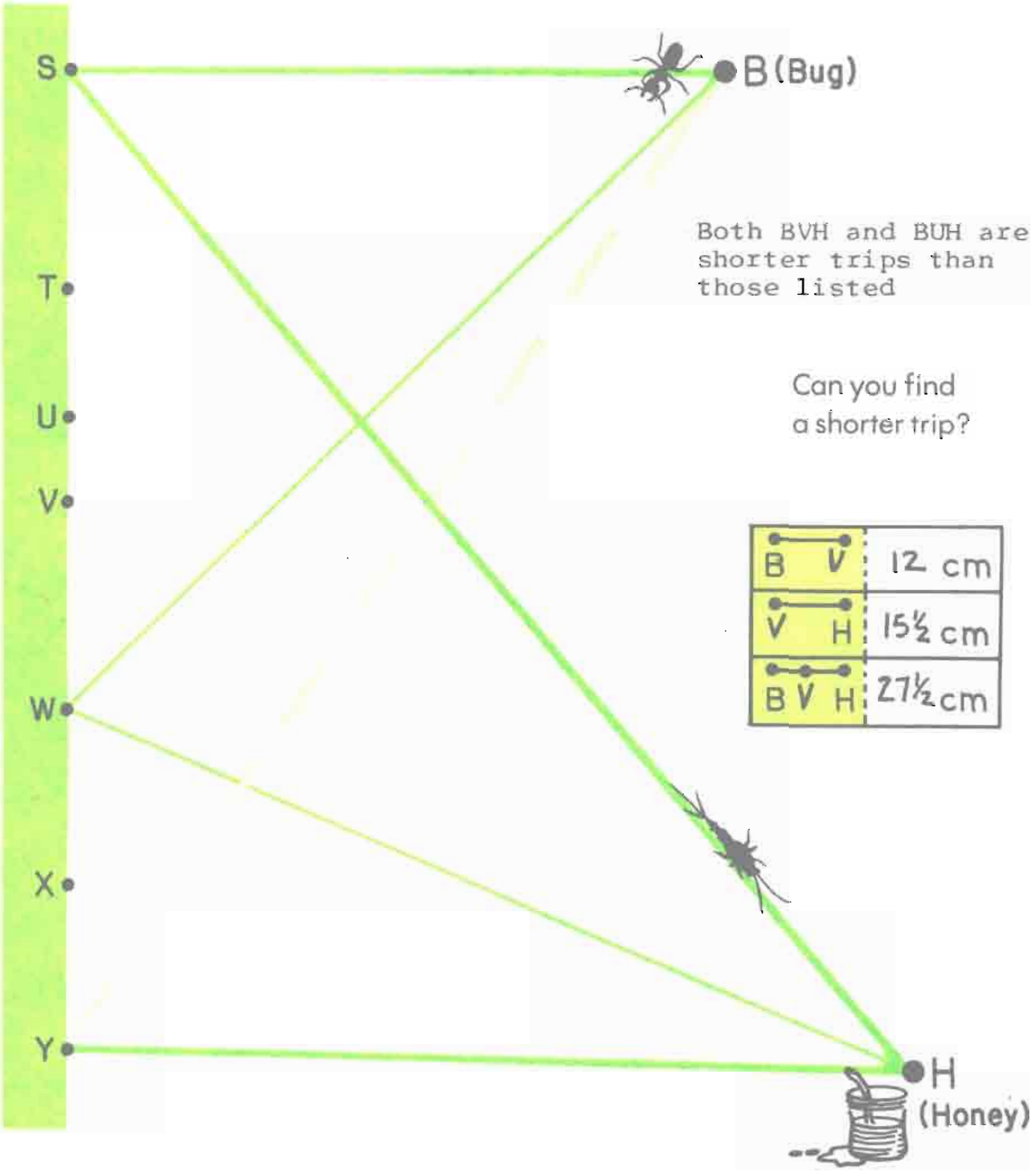






BUG-TRIP GEOMETRY

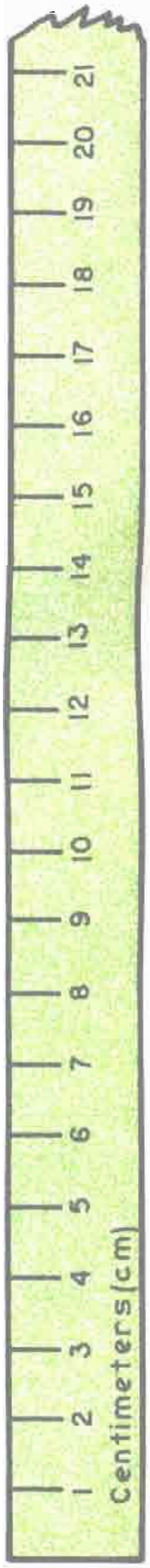
The bug must touch the wall before he can eat the honey.



Both BVH and BUH are shorter trips than those listed

Can you find a shorter trip?

B V	12 cm
V H	15½ cm
B V H	27½ cm



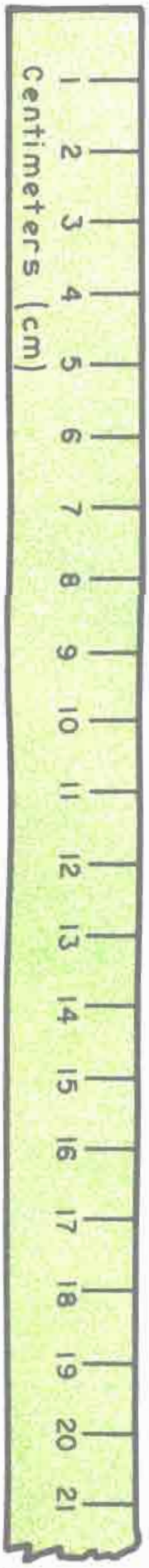
B S	10 cm
S H	20 cm
B S H	30 cm

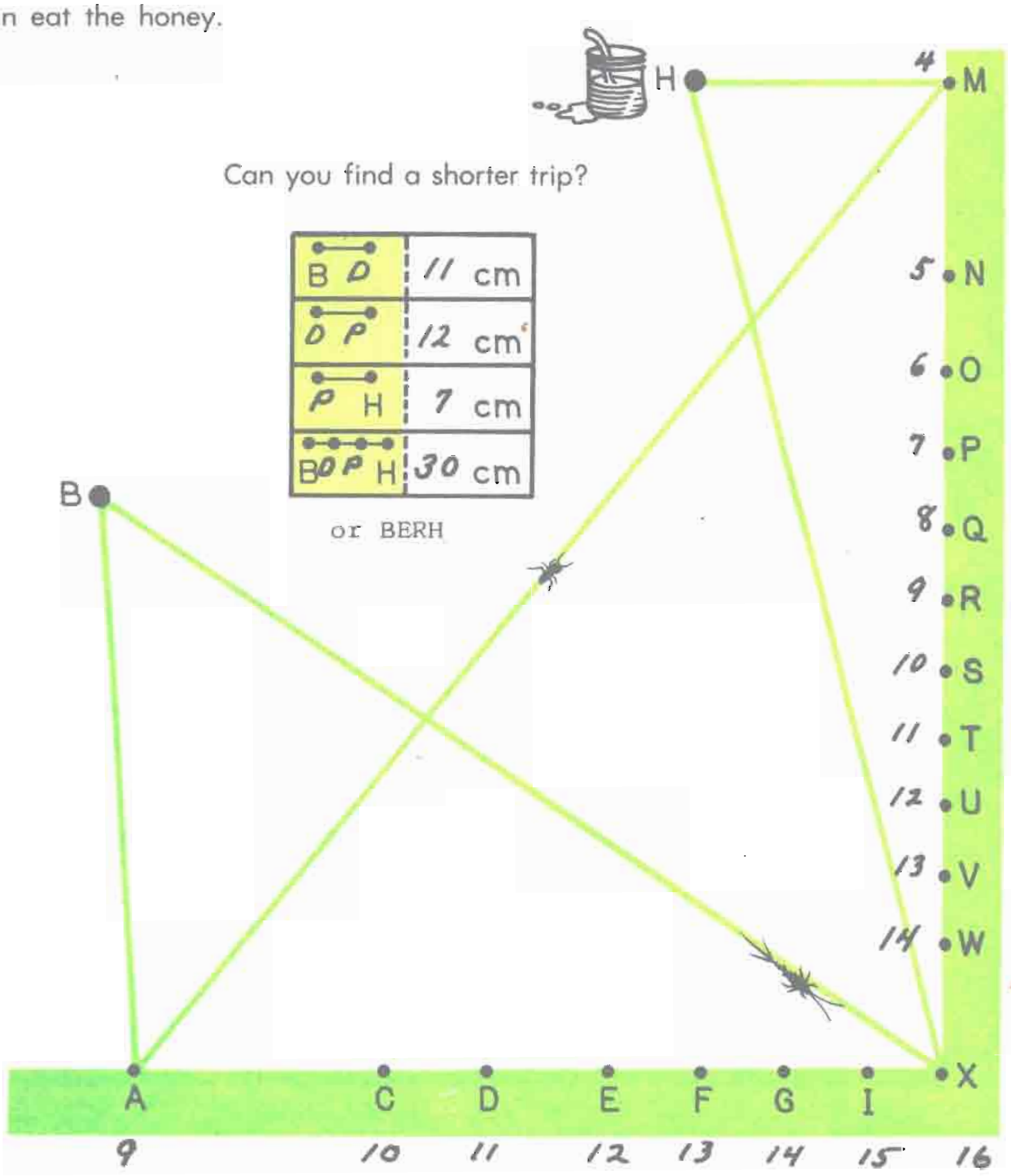
B W	14 cm
W H	14 cm
B W H	28 cm

B Y	18 cm
Y H	13 cm
B Y H	31 cm

BUG-TRIP GEOMETRY

The bug must touch both walls before he can eat the honey.





Can you find a shorter trip?

B D	11 cm
D P	12 cm
P H	7 cm
BDPH	30 cm

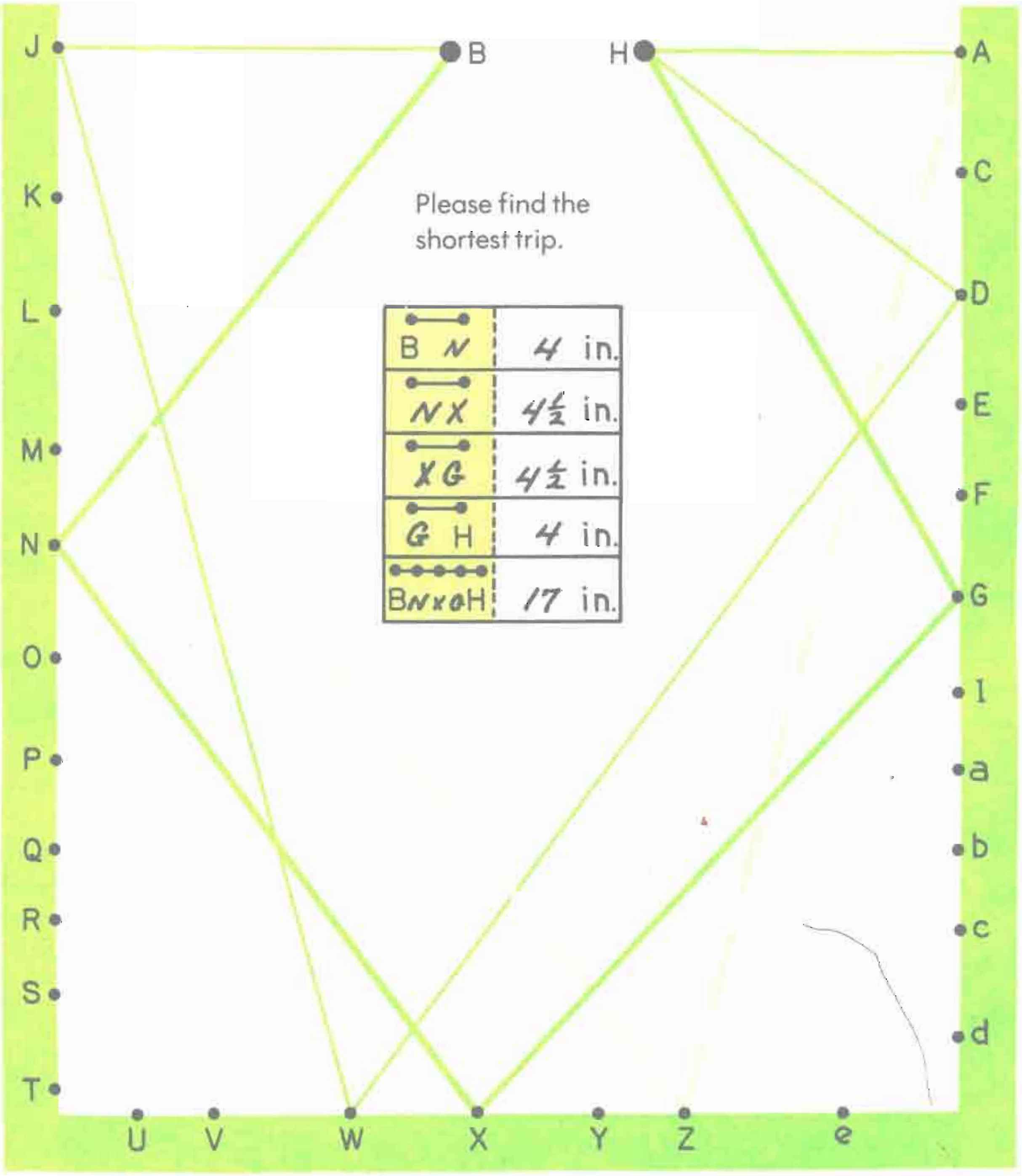
or BERH

B A	9 cm
A M	20 cm
M H	4 cm
BAMH	33 cm

B X	16 cm
X H	16 cm
BXH	32 cm

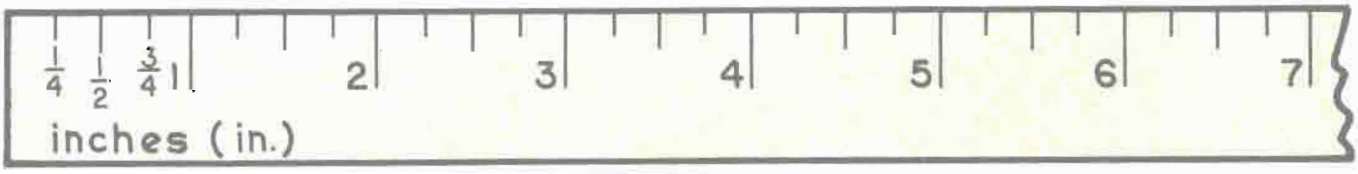
BUG-TRIP GEOMETRY

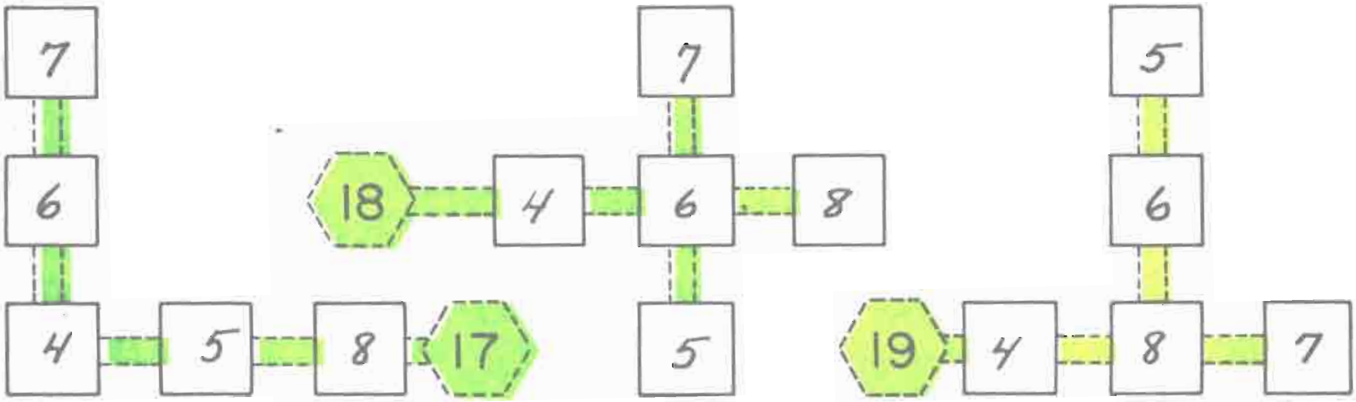
The bug must touch all three walls before he can eat the honey.



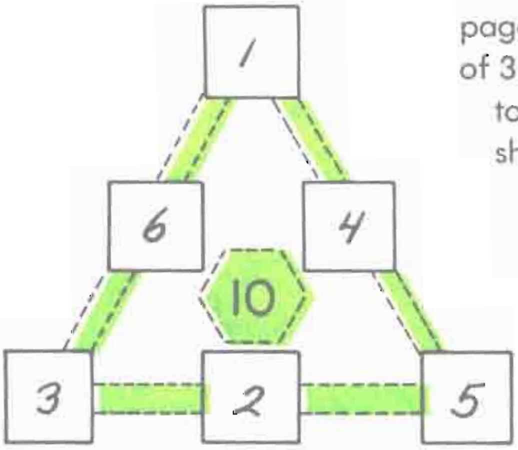
Please find the shortest trip.

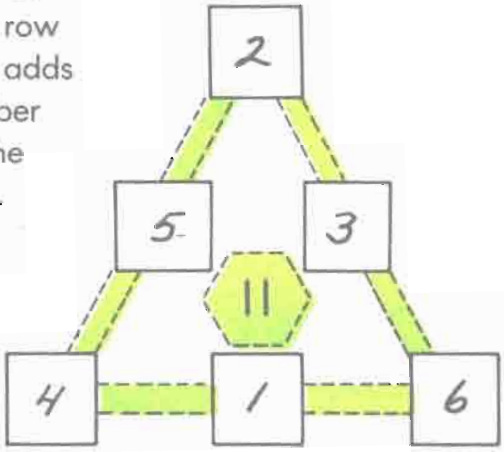
B N	4 in.
N X	4½ in.
X G	4½ in.
G H	4 in.
BNXGH	17 in.

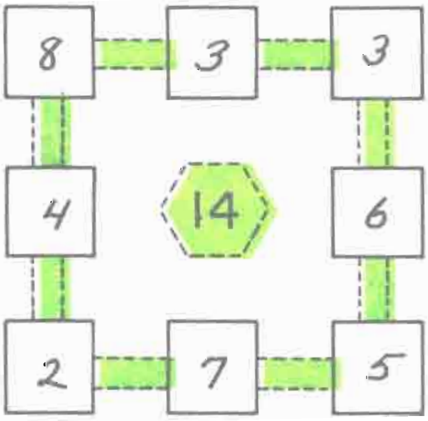


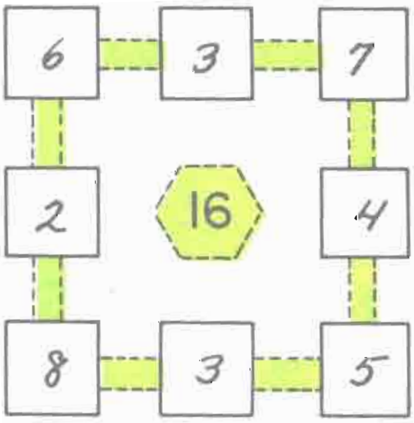


Please arrange the
numbers on this
page so each row
of 3 numbers adds
to the number
shown in the
hexagon.









Numbers like these can be cut out and moved around until an answer is found.

You will become famous as a mathematician if you can prove this statement:

"Every even number greater than 2 is the sum of two prime numbers."

This statement was written in 1742 by C. Goldbach (1690-1764).
Since then, no one has found an exception . . . and no one has proven there is no exception.

Don't think it's an easy task: Many famous mathematicians have tried to find an exception or to prove there are none.

Even numbers larger than 2 are

4, 6, 8, 10, 12, 14, 16, etc.

Prime numbers are

whole numbers that have only 2 different factors — 1 and the number itself:

2, 3, 5, 7, 11, 13, 17, 19, 23, etc.

Using only those prime numbers, can you complete the following as true statements:

2+2 = 4

3+3 = 6

3+5 = 8

5 +5 = 10

5 +7 = 12

3 +11 = 14

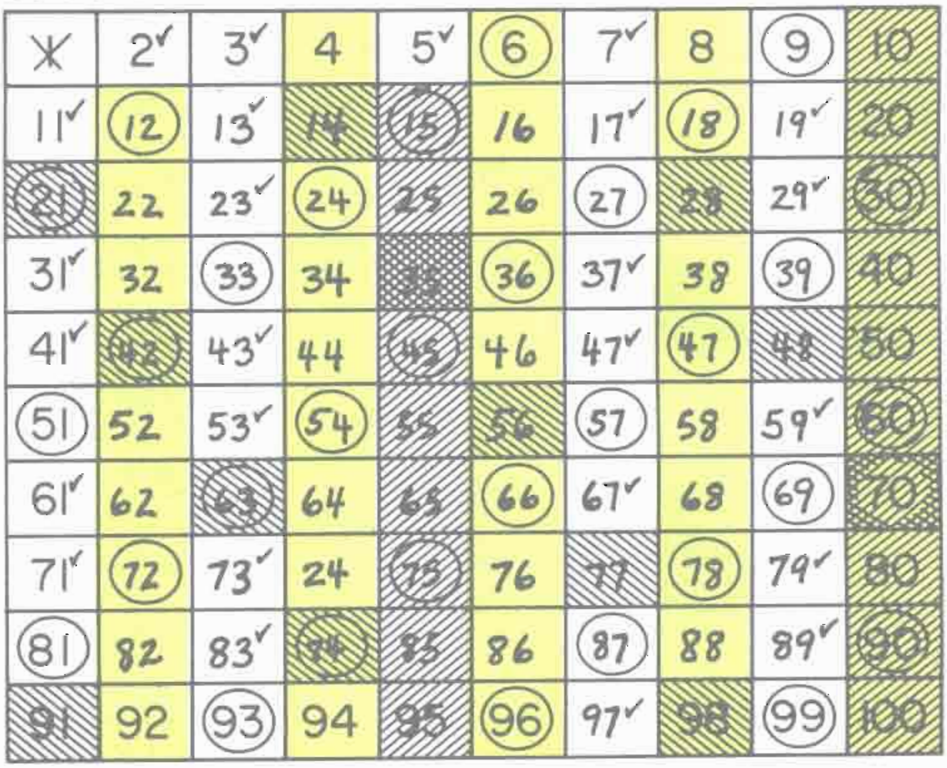
More than 2000 years ago, a Greek mathematician invented a "sieve" that would catch all prime numbers.

We can use his sieve on all numbers 100 or less, crossing out all multiples of

2, 3, 5 and 7

that are larger than 2, 3, 5 and 7.
(The number 1 is by definition not a prime.)
All the numbers left are primes.

Sieve of Eratosthenes



~~1~~	2✓	3✓	4	5✓	6	7✓	8	9	10
11✓	12	13✓	14	15	16	17✓	18	19✓	20
21	22	23✓	24	25	26	27	28	29✓	30
31✓	32	33	34	35	36	37✓	38	39	40
41✓	42	43✓	44	45	46	47✓	47	48	50
51	52	53✓	54	55	56	57	58	59✓	60
61✓	62	63	64	65	66	67✓	68	69	70
71✓	72	73✓	24	75	76	77	78	79✓	80
81	82	83✓	84	85	86	87	88	89✓	90
91	92	93	94	95	96	97✓	98	99	100

Primes less than 100 . . caught in Eratosthenes Sieve.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53

59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127....

Jim said he couldn't prove Goldbach's guess, "But I can prove every even number from 14 through 100 can be shown as the sum of 2 different pairs of primes — because I did it!"

He agreed that 3 + 11 and 11 + 3 are not "different pairs" but he found 2 different pairs for 14 = 7 + 7 = 3 + 11
Can you find 2 different pairs for all even numbers 16 through 100?

16 = 5 + 11 = 13 + 3
18 = 11 + 7 = 13 + 5
20 = 17 + 3 = 13 + 7
22 = 11 + 11 = 17 + 5
24 = 11 + 13 = 17 + 7
26 = 23 + 3 = 19 + 7
28 = 23 + 5 = 17 + 11
30 = 23 + 7 = 19 + 11
32 = 29 + 3 = 19 + 13
34 = 31 + 3 = 29 + 5
36 = 31 + 5 = 29 + 7
38 = 31 + 7 = 19 + 19
40 = 37 + 3 = 29 + 11
42 = 37 + 5 = 31 + 11
44 = 37 + 7 = 31 + 13

46 = 43 + 3 = 41 + 5
48 = 43 + 5 = 41 + 7
50 = 47 + 3 = 43 + 7
52 = 47 + 5 = 41 + 11
54 = 47 + 7 = 43 + 11
56 = 53 + 3 = 43 + 13
58 = 53 + 5 = 47 + 11
60 = 53 + 7 = 43 + 17
62 = 59 + 3 = 43 + 19
64 = 53 + 11 = 61 + 3
66 = 53 + 13 = 47 + 19
68 = 61 + 7 = 37 + 31
70 = 53 + 17 = 41 + 29
72 = 53 + 19 = 67 + 5
74 = 71 + 3 = 67 + 7

76 = 71 + 5 = 73 + 3

78 = 71 + 7 = 73 + 5

80 = 73 + 7 = 13 + 67

82 = 71 + 11 = 79 + 3

84 = 73 + 11 = 79 + 5

86 = 83 + 3 = 79 + 7

88 = 83 + 5 = 79 + 9

90 = 83 + 7 = 79 + 11

92 = 89 + 3 = 79 + 13

94 = 83 + 11 = 89 + 5

96 = 89 + 7 = 83 + 13

98 = 31 + 67 = 61 + 37

100 = 53 + 47 = 89 + 11

Q.E.D. — Quod Erat Demonstrandum (Which was to be demonstrated) — proved.

Jim made other claims:
"I found 9 different pairs for 90,
8 pairs for 84, and 7 pairs for 78 and 96."

Let's check his claims.

90 = 83 + 7
90 = 79 + 11
90 = 43 + 47
90 = 61 + 29
90 = 73 + 17
90 = 23 + 67
90 = 31 + 59
90 = 37 + 53
90 = 19 + 71

84 = 5 + 79
84 = 11 + 73
84 = 13 + 71
84 = 17 + 67
84 = 23 + 61
84 = 31 + 53
84 = 37 + 47
84 = 41 + 43

78 = 5 + 73
78 = 7 + 71
78 = 11 + 67
78 = 17 + 61
78 = 19 + 59
78 = 31 + 47
78 = 37 + 41

96 = 7 + 89
96 = 13 + 83
96 = 17 + 79
96 = 23 + 73
96 = 29 + 67
96 = 59 + 37
96 = 43 + 53

Also, "between 50 and 100 there is only 1 number that cannot be shown as the sum of 3 different pairs."

Can you find it? 68 = 7 + 61 = 31 + 37

"My studies," Jim concluded, "haven't proved Goldbach's guess, but I have a stronger feeling he was probably right!"

Do you aree with Jim?

Goldbach's guess was published in 1742.

Every even number 4 or larger can be shown as the sum of 2 primes!

For the next 185 years, mathematicians tried to find an exception or prove Goldbach was right . . . but without results.

In 1937, I. M. Vinogradov made the first significant progress. He proved that:

"Every 'sufficiently large' odd number is a sum of 3 odd primes."

What does "sufficiently large" mean?

Since "3" is the smallest odd prime, then what is the smallest odd number we could consider?

9 = 3 + 3 + 3

Let's consider odd numbers as sums of 3 odd primes.

Odd primes **3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71**

9 = 3 + 3 + 3
11 = 3 + 3 + 5
13 = 3 + 5 + 5
15 = 5 + 5 + 5
17 = 5 + 5 + 7
19 = 5 + 7 + 7
21 = 7 + 7 + 7
23 = 5 + 7 + 11
25 = 7 + 7 + 11
27 = 5 + 11 + 11
29 = 7 + 11 + 11
31 = 7 + 11 + 13

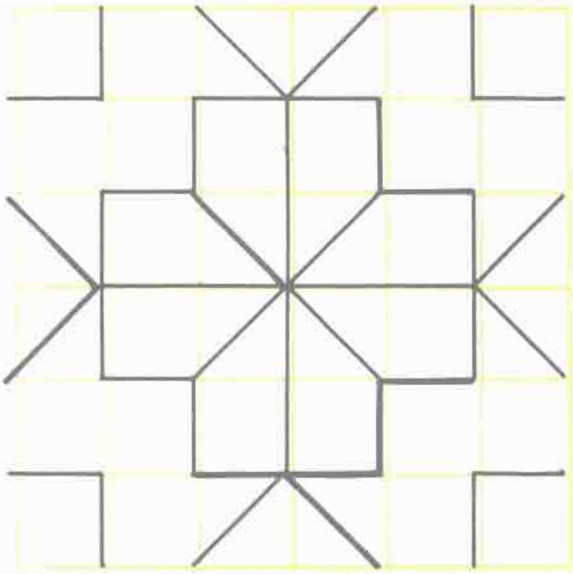
33 = 7 + 13 + 13
35 = 11 + 11 + 13
37 = 11 + 13 + 13
39 = 13 + 13 + 13
41 = 7 + 17 + 17
43 = 13 + 13 + 17
45 = 11 + 17 + 17
47 = 13 + 17 + 17
49 = 13 + 17 + 19
51 = 17 + 17 + 17
53 = 17 + 17 + 19
55 = 17 + 19 + 19

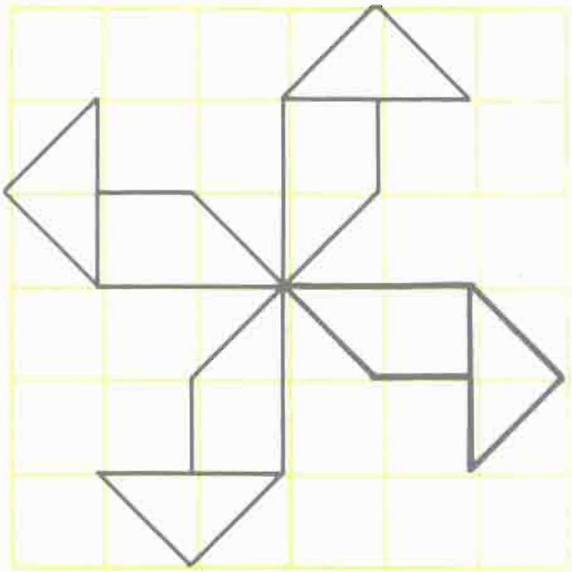
57 = 19 + 19 + 19
59 = 17 + 19 + 23
61 = 19 + 19 + 23
63 = 17 + 23 + 23
65 = 19 + 23 + 23
67 = 19 + 19 + 29
69 = 23 + 23 + 23
71 = 19 + 23 + 29
73 = 13 + 31 + 29
75 = 23 + 23 + 29
77 = 19 + 29 + 29
79 = 19 + 29 + 31

Perhaps you would like to extend this investigation, but remember,

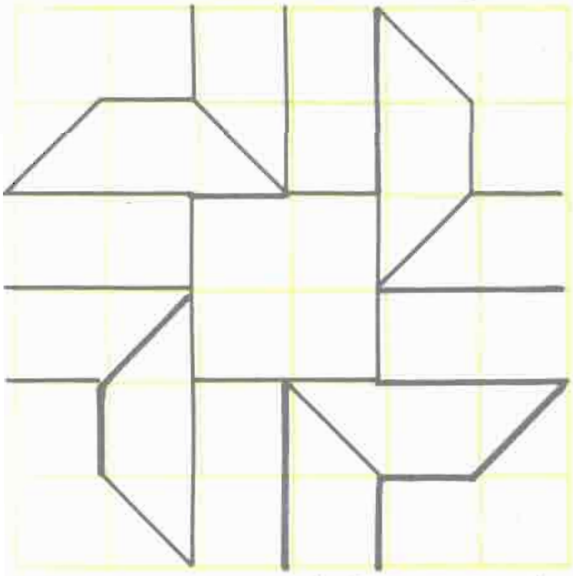
Vinogradov has proved you can never find an exception.

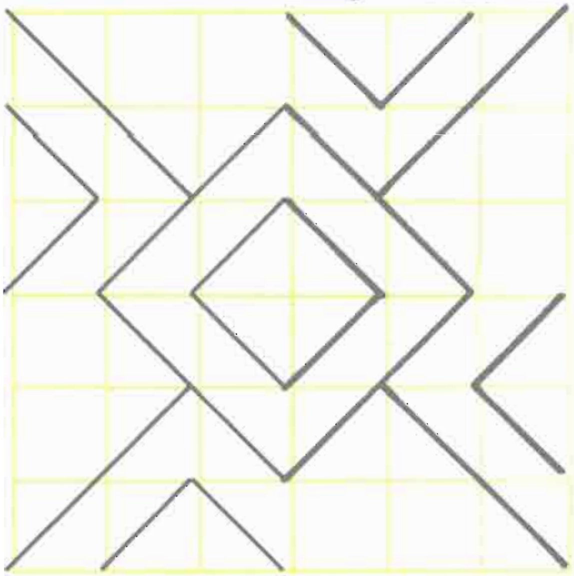
GEOMETRY for FUN

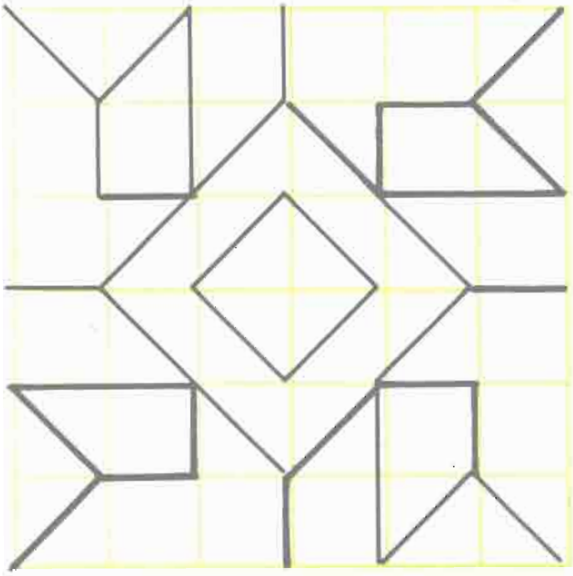


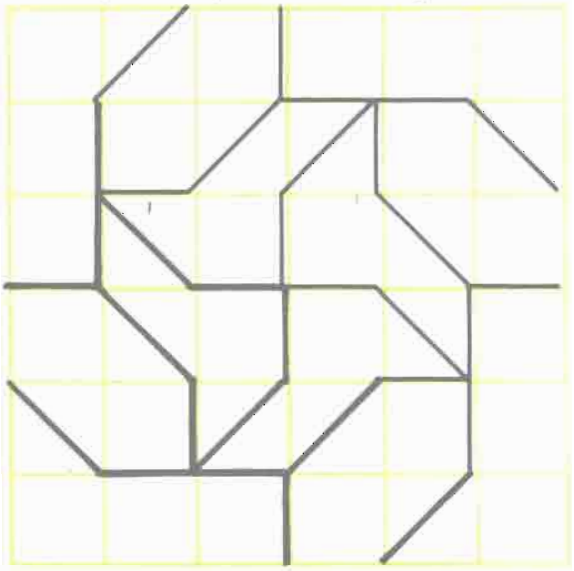


Portions of each design have been removed but enough clues have been left to complete each figure symmetrically.









U.S. DEPARTMENT OF COMMERCE
National Bureau of Standards
Washington, D.C. 20234

Letter Circular 1052
February 1974

All You Will Need to Know About Metric

(For Your Everyday Life)

Note: This chart may be reproduced

10

Metric is based on Decimal system

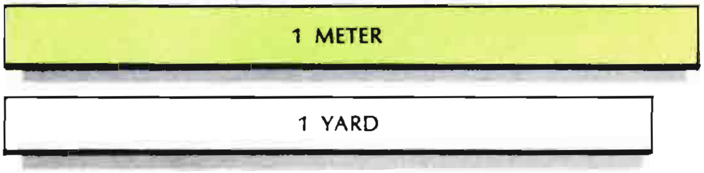
The metric system is simple to learn. For use in your everyday life you will need to learn only ten new units. You will also need to get used to a few new temperatures. There are even some metric units with which you are already familiar: those for time and electricity are the same as you use now.

The children can compare the accuracy of the underlined items with the paperclips in their classroom.

BASIC UNITS

METER:	a little longer than a yard (about 1.1 yards)
LITER:	a little larger than a quart (about 1.06 quarts)
GRAM:	a little more than the weight of a paper clip

(comparative sizes are shown)



25 DEGREES FAHRENHEIT

COMMON PREFIXES
(to be used with basic units)

milli:	one-thousandth (0.001)
centi:	one-hundredth (0.01)
kilo:	one-thousand times (1000)

For example:
1000 millimeters = 1 meter
100 centimeters = 1 meter
1000 meters = 1 kilometer

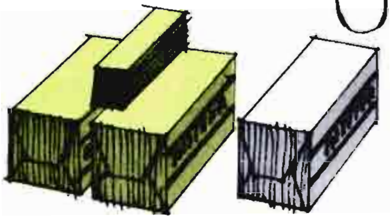
25 DEGREES CELSIUS

OTHER COMMONLY USED UNITS

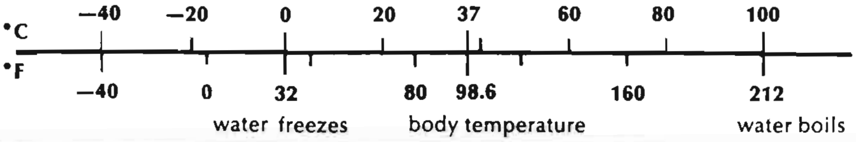
millimeter:	0.001 meter	diameter of paper clip wire
centimeter:	0.01 meter	a little more than the width of a paper clip (about 0.4 inch)
kilometer:	1000 meters	somewhat further than ½ mile (about 0.6 mile)
kilogram:	1000 grams	a little more than 2 pounds (about 2.2 pounds)
milliliter:	0.001 liter	five of them make a teaspoon

OTHER USEFUL UNITS

hectare:	about 2½ acres
tonne:	about one ton



TEMPERATURE
degrees Celsius are used



The children will need stopwatches or wristwatches with second hands.

Time Yourself Measuring Speeds

Out on the playground, mark off 30 meters and pick a partner to time you.

Walk as fast as you can.

1st try _____ seconds

2nd " _____ "

3rd " _____ "

Mark off 15 meters and try running sideways.

1st try _____ seconds

2nd " _____ "

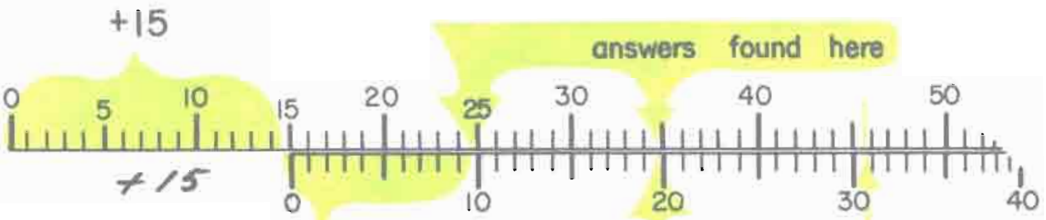
3rd " _____ "

Try skipping as fast as you can.

Try running with your hands tied.

A Slide Rule for Addition or Subtraction

After the page is completed cut off the bottom scale on the left-hand side of this page for use with the top scale.



10 + 15=25

or

15+10:25

20 + 15=35

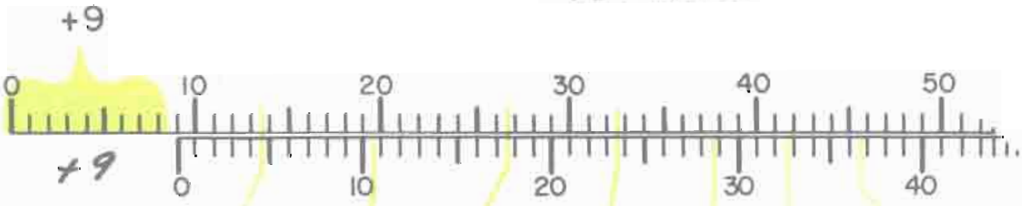
or

15+20:35

31 + 15=46

or

15+31: 46



$\begin{array}{r}9\\+5\\\hline 14\end{array}$ or $\begin{array}{r}5\\+9\\\hline 14\end{array}$

$\begin{array}{r}11\\+9\\\hline 20\end{array}$

$\begin{array}{r}9\\+11\\\hline 20\end{array}$ etc...

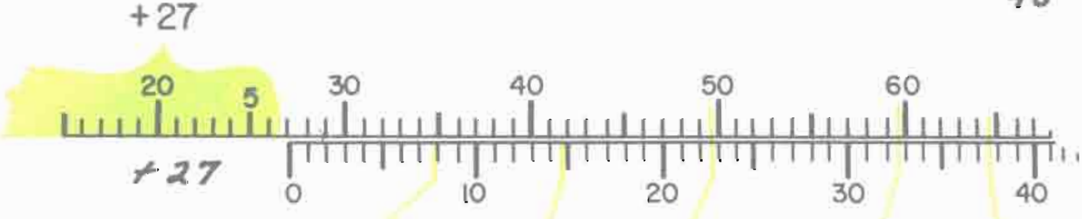
$\begin{array}{r}18\\+9\\\hline 27\end{array}$

$\begin{array}{r}24\\+9\\\hline 33\end{array}$

$\begin{array}{r}29\\+9\\\hline 38\end{array}$

$\begin{array}{r}33\\+9\\\hline 42\end{array}$

$\begin{array}{r}37\\+9\\\hline 46\end{array}$ or $\begin{array}{r}9\\+37\\\hline 46\end{array}$



$\begin{array}{r}8\\+27\\\hline 35\end{array}$ or $\begin{array}{r}27\\+8\\\hline 35\end{array}$

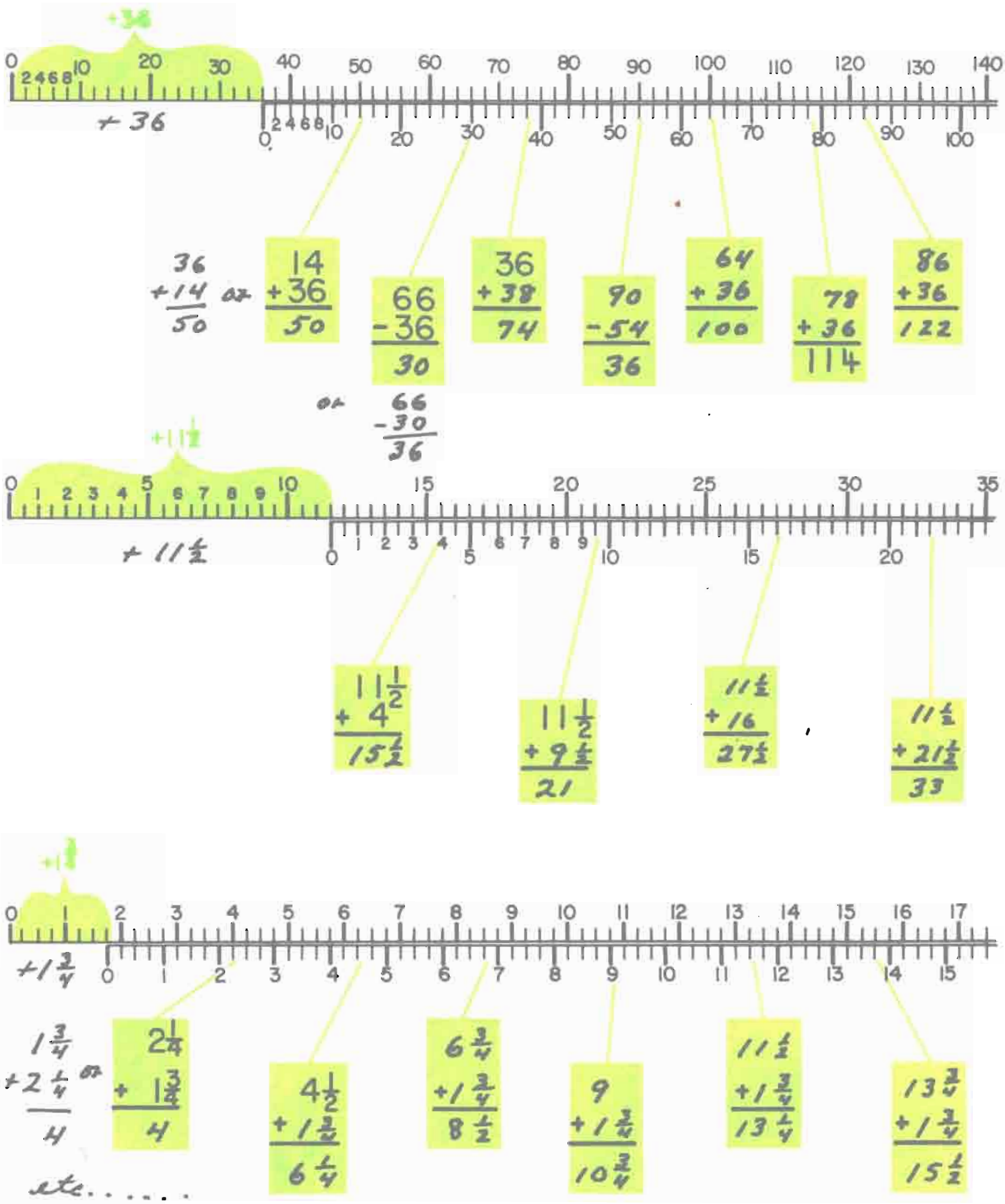
$\begin{array}{r}42\\-27\\\hline 15\end{array}$

$\begin{array}{r}50\\-27\\\hline 23\end{array}$ or $\begin{array}{r}50\\-23\\\hline 27\end{array}$

$\begin{array}{r}60\\-27\\\hline 33\end{array}$ or $\begin{array}{r}60\\-33\\\hline 27\end{array}$

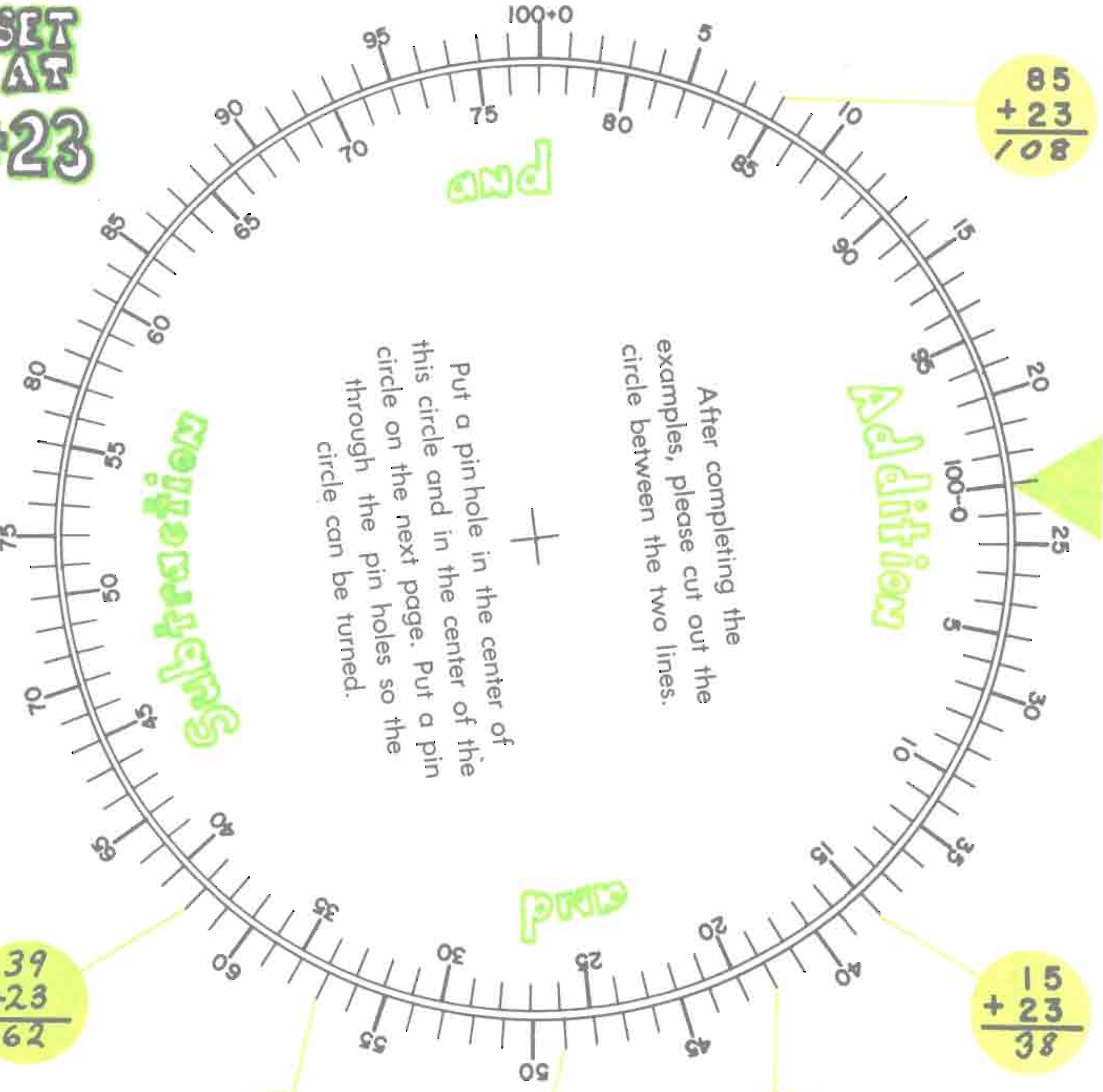
$\begin{array}{r}65\\-38\\\hline 27\end{array}$ or $\begin{array}{r}65\\-27\\\hline 38\end{array}$

←cut on this line


+36
0 2 4 6 8 10 20 30 40 50 60 70 80 90 100 110 120 130 140
0 2 4 6 8 10 20 30 40 50 60 70 80 90 100
+ 36
36 +14 50 or
14 +36 50
66 -36 30
or 66 -30 36
36 +38 74
90 -54 36
64 +36 100
78 +36 114
86 +36 122
+11½
0 1 2 3 4 5 6 7 8 9 10 15 20 25 30 35
0 1 2 3 4 5 6 7 8 9 10 15 20
+ 11½
11½ + 4 15½
11½ + 9½ 21
11½ + 16 27½
11½ + 21½ 33
+1¾
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
+1¾
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
1¾ +2¼ 4 or
2¼ + 1¾ 4
4½ + 1¾ 6¼
6¾ +1¾ 8½
9 + 1¾ 10¾
11½ +1¾ 13¼
13¾ +1¾ 15½
etc.......

A SLIDE RULE
IN A CIRCLE

SET AT +23



After completing the examples, please cut out the circle between the two lines.

Put a pin hole in the center of this circle and in the center of the next page. Put a pin through the pin holes so the circle can be turned.

85
+23
108

15
+ 23
38

19
+ 23
42

26
+23
49

34
+23
57

39
+23
62

With this Circular Slide Rule you can find the sums of any 2-digit numbers . . . but you must know whether the sum is more than or less than 100.

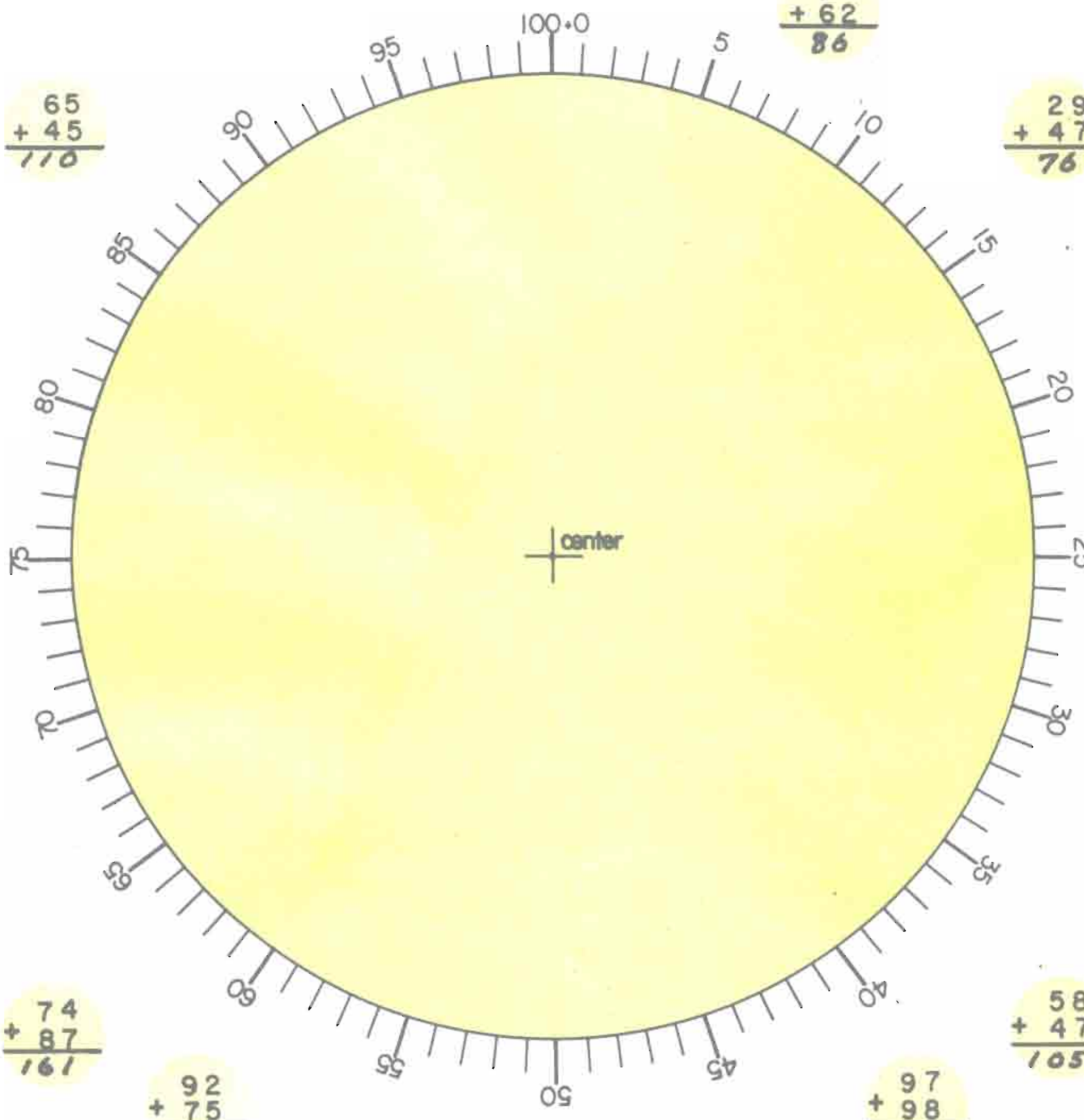
13
+ 17
30

36
+ 36
72

24
+ 62
86

65
+ 45
110

29
+ 47
76



74
+ 87
161

58
+ 47
105

92
+ 75
167

97
+ 98
195

(Your slide rule will last longer if you mount this sheet on heavy cardboard and the movable center scale on heavier paper or cardboard.)

After you finish the next page, you can tear out this sheet and have your own portable slide rule.

MEASURING THE BUILDING I LIVE IN

I live in a:

____ house
____ apartment
____ igloo
____ boat
____ tent
____ barn
____ duplex
____ bus
____ cave

☞ Can you walk around it? ______
How many steps around is it? ______

☞ If you stacked 10 of your buildings on top of each other, how tall would they be? ______

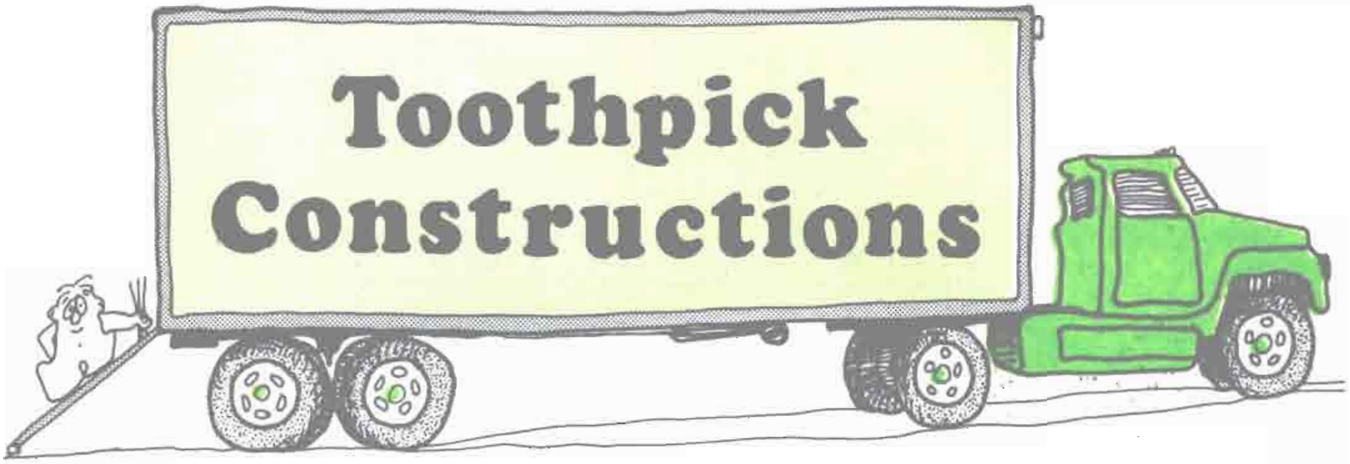
☞ Tell some more interesting facts about your building, such as how many windows, doors inside and outside, etc.

____

____

The Sears Tower in Chicago is the tallest inhabited building in the world. It is 1454 feet tall. (From the 1974 edition of The Guinness Book of Records, page 279.)

Toothpick Constructions



FORM A GROUP WITH 2 OR 3 PEOPLE AND GET 50 TOOTHPICKS OR POPSICLE STICKS AND GLUE.

NOW ....PICK A PROJECT!

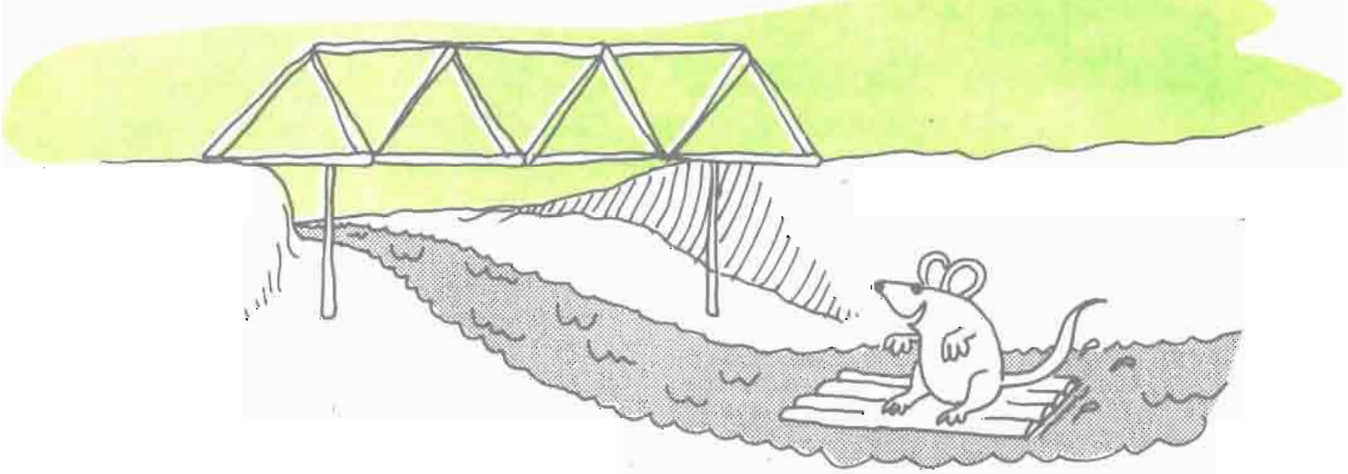
MAKE A CONSTRUCTION STRONG ENOUGH TO HOLD A BRICK AT LEAST 1" ABOVE THE TABLE.

MAKE THE TALLEST CONSTRUCTION THAT WILL STAND UP.

MAKE THE CONSTRUCTION THAT IS BIGGEST AROUND AT THE BOTTOM AND 3" HIGH.

MAKE A CONSTRUCTION TO HOLD A BASEBALL, AS HIGH AS POSSIBLE.

MAKE A BRIDGE TO SPAN AN 8" RIVER.



8	1	6
3	5	7
4	9	2

4	3	8
9	5	1
2	7	6

6	7	2
1	5	9
8	3	4

4	9	2
3	5	7
8	1	6

8	3	4
1	5	9
6	7	2

2	9	4
7	5	3
6	1	8

6	1	8
7	5	3
2	9	4

2	7	6
9	5	1
4	3	8

Mr. Magic has rearranged the numbers 1 thru 9 into 8 different "magic squares".

A "magic square" is an arrangement of numbers in such a way that the sum of the numbers in all rows, all columns and all diagonals is the same number.

In Mr. Magic's "magic squares", all rows, columns, and diagonals add to 15. He shows all ways this can be done with numbers 1 thru 9.

But there are many other "magic squares". Can you complete the following so they are all magic?

5	0	7
6	4	2
1	8	3

5	5	2
1	4	7
6	3	3

5	4	6
6	5	4
4	6	5

7	8	9
10	8	6
7	8	9

16	2	12
6	10	14
8	18	4

3	17	7
13	9	5
11	1	15

16	1	10
3	9	15
8	17	2

10	45	20
35	25	15
30	5	40

28	21	26
23	25	27
24	29	22

80	10	60
30	50	70
40	90	20

1	1	1
1	1	1
1	1	1

0	0	0
0	0	0
0	0	0

Looking at the previous page:

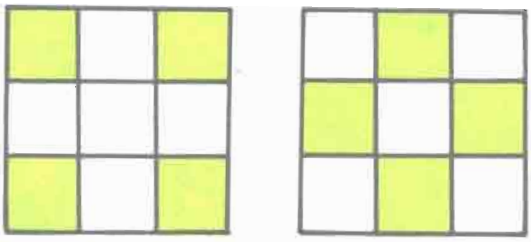
What do you notice that's true of all these?

In each, all rows, columns and diagonals add to the same number which we will call the "magic number". In Mr. Magic's 8 squares that number is 15.

Is there any connection between the magic number and the number in the center?

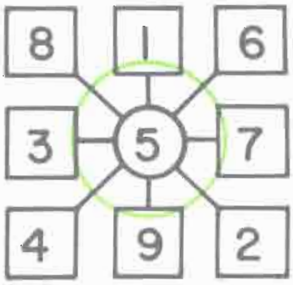
In Mr. Magic's square—15 is the magic number and 5 is the center number.

In each square, add the 4 corners and then add the 4 side numbers:



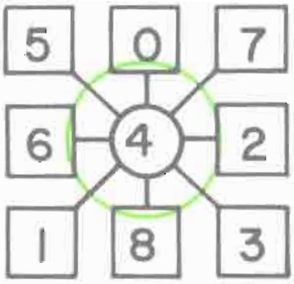
Do these sums have any relationship to the "magic number" in each example?

Consider the pairs of numbers at the ends of each row, column and diagonal that include the center number.



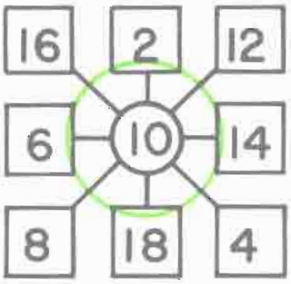
8	1	6
3	5	7
4	9	2

ends add to 10



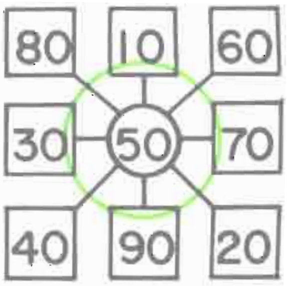
5	0	7
6	4	2
1	8	3

ends add to 8



16	2	12
6	10	14
8	18	4

ends add to 20



80	10	60
30	50	70
40	90	20

ends add to 100

What connection is there between those sums and the center number?

Please use what you have found out to complete the following as "magic squares"?

5	0	4
2	3	4
2	6	1

2	1	3
3	2	1
1	3	2

8	1	6
3	5	7
4	9	2

15	1	11
5	9	13
7	17	3

Ben's Guess

"I think all the multiplication tables can be arranged in "magic squares"

Let's test Ben's "guess".

32 H	4 A	24 F
12 C	20 E	28 G
16 D	36 I	8 B

40 H	5 A	30 F
15 C	25 E	35 G
20 D	45 I	10 B

48 H	6 A	36 F
18 C	30 E	42 G
24 D	54 I	12 B

56 H	7 A	42 F
21 C	35 E	43 G
28 D	63 I	14 B

4	8	12	16	20	24	38	32	36
A	B	C	D	E	F	G	H	I
5	10	15	20	25	30	35	40	45
A	B	C	D	E	F	G	H	I
6	12	18	24	30	36	42	48	54
A	B	C	D	E	F	G	H	I
7	14	21	28	35	42	49	56	63
A	B	C	D	E	F	G	H	I

64 H	8 A	48 F
24 C	40 E	56 G
32 D	72 I	16 B

72 H	9 A	54 F
27 C	45 E	63 G
36 D	81 I	18 B

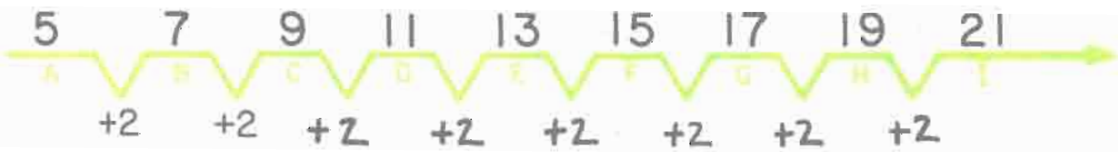
80 H	10 A	60 F
30 C	50 E	70 G
40 D	90 I	20 B

8	16	24	32	40	48	56	64	72
A	B	C	D	E	F	G	H	I
9	18	27	36	45	54	63	72	81
A	B	C	D	E	F	G	H	I
10	20	30	40	50	60	70	80	90
A	B	C	D	E	F	G	H	I

Joanne's Discoveries

"I can begin with any number at A and then keep adding any other number to get the rest of the list — if I use the same number each time. Here's the way I think of it:"

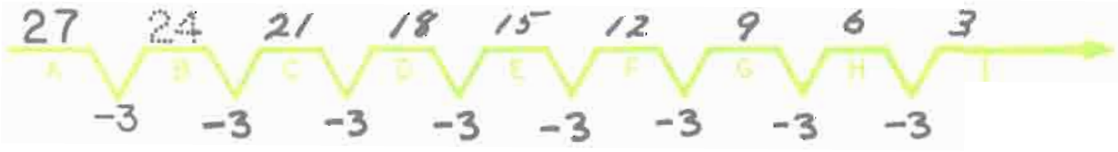
28 H	7 A	22 F
13 C	19 E	25 G
16 D	31 I	10 B



19 H	5 A	15 F
9 C	13 E	17 G
11 D	21 I	7 B

"Further, I find that it will work backwards if you start with a large enough number — and keep subtracting some numbers to find the next one in the list. Here are two examples.

4 H	11 A	6 F
9 C	7 E	5 G
8 D	3 I	10 B



6 H	27 A	12 F
21 C	15 E	9 G
18 D	3 I	24 B

Pick a number:

Pick a number to add or subtract each time:

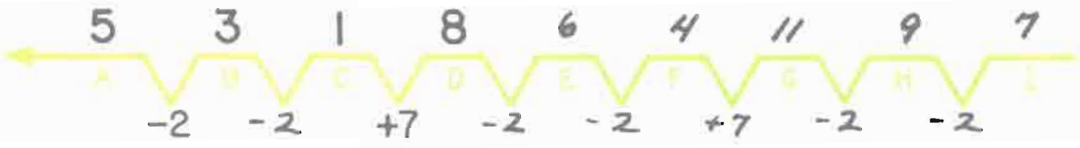
H	A	F
C	E	G
D	I	B

Ben Pushes Joanne's Discoveries Further

"I used Joanne's way to keep track as I looked at 'magic squares' that don't follow the pattern we have been using."

5 H	0 A	7 F
6 C	4 E	2 G
1 D	8 I	3 B

9 H	5 A	4 F
1 C	6 E	11 G
8 D	7 I	3 B

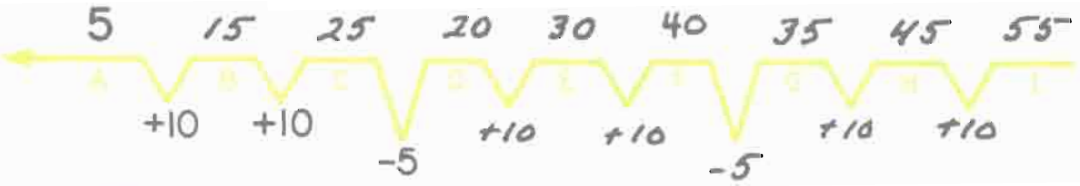


After I saw this pattern, I changed the form to show more clearly what I had noticed.

7 H	8 A	9 F
10 C	8 E	6 G
7 D	8 I	9 B

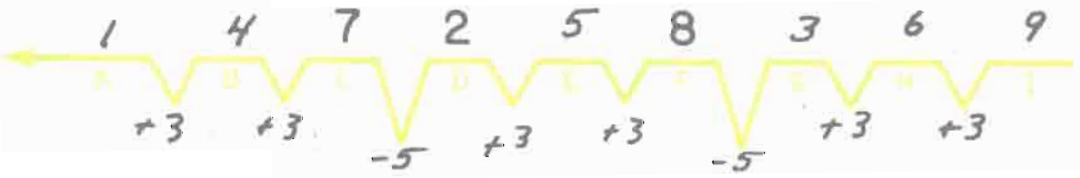
28 H	3 A	20 F
9 C	17 E	25 G
14 D	31 I	6 B

45 H	5 A	40 F
25 C	30 E	35 G
20 D	55 I	15 B



A Puzzle

6 H	1 A	8 F
7 C	5 E	3 G
2 D	9 I	4 B



Now you know more about 3 by 3 "magic squares" than most other people do.

You can make "magic squares" of the puzzles on this page using the discoveries of Ben and Joanne.

We will show the letter pattern only once.

Please complete as "magic squares"

10 H	1 A	7 F
3 C	6 E	9 G
5 D	11 I	2 B

1 2 3 5 6 7 9 10 11
A B C D E F G H I
+1 +1 +2 +1 +1 +2 +1 +1

10 15 20 13 18 23 16 21 26
+5 +5 -7 +5 +5 -7 +5 +5

21	10	23
20	18	16
13	26	15

12	27	6
9	15	21
24	3	18

27 18 9 24 15 6 21 12 3
-9 -9 +15 -9 -9 +15 -9 -9

1 11 21 2 12 22 3 13 23
+10 +10 -19 +10 +10 -19 +10 +10

13	1	22
21	12	3
2	23	11

8	28	24
36	20	4
16	12	32

28 32 36 16 20 24 4 8 12
+4 +4 -20 +4 +4 -20 +4 +4

20 18 16 13 11 9 6 4 2
-2 -2 -3 -2 -2 -3 -2 -2

4	20	9
16	11	6
13	2	18

10	11	6
5	9	13
12	7	8

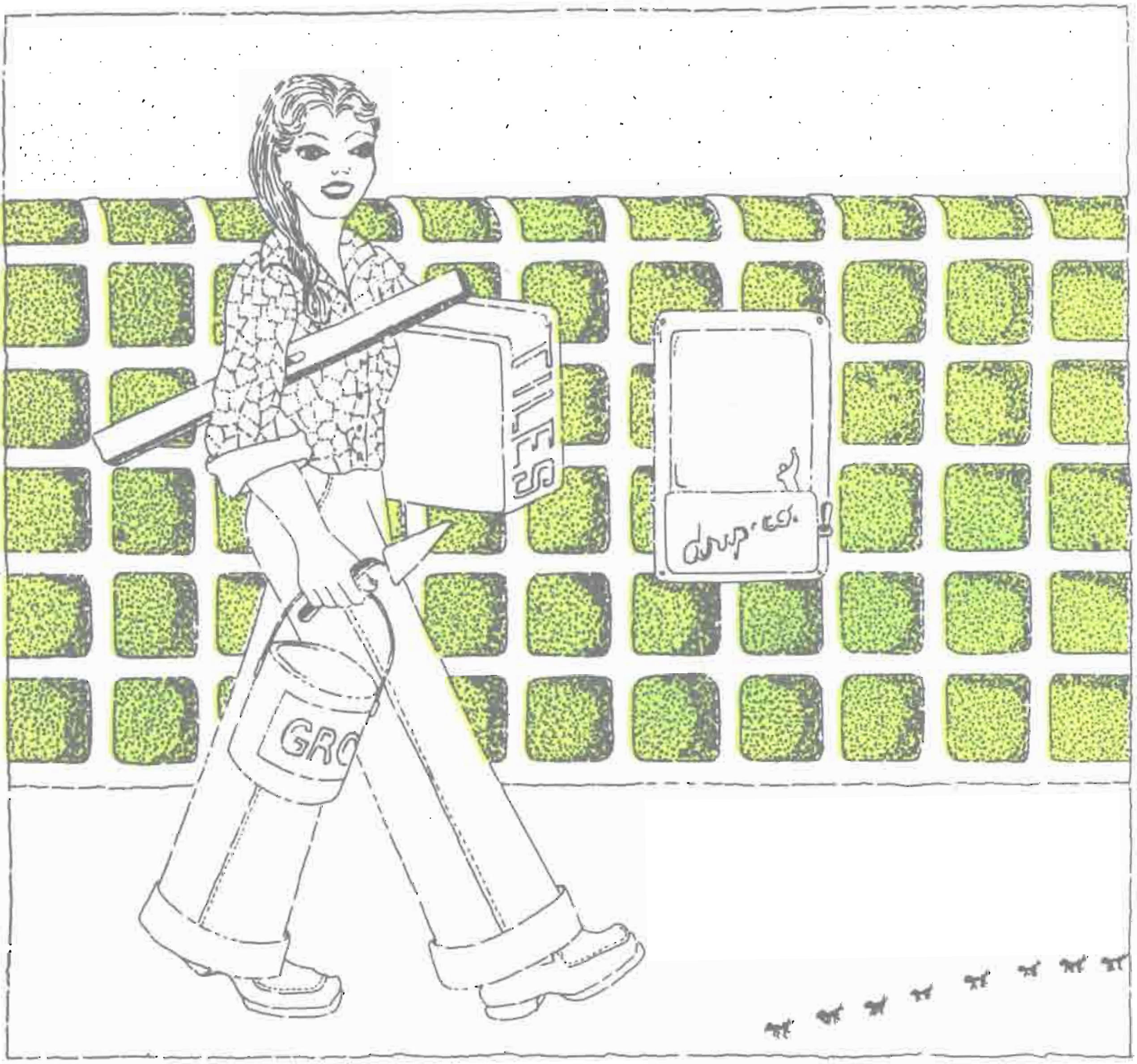
11 8 5 12 9 6 13 10 7
-3 -3 +7 -3 -3 +7 -3 -3

20 18 16 13 11 9 6 4 2
-2 -2 -3 -2 -2 -3 -2 -2

4	20	9
16	11	6
13	2	18

20	45	10
15	25	35
40	5	30

45 30 15 40 25 10 35 20 5
-15 -15 +25 -15 -15 +25 -15 -15



from tiles to tricks

Theresa told her teacher that her mother, Mrs. Burg, had made up some interesting tricks and would like to show them to her class.

So, no one was surprised when Mrs. Burg came to class one morning, but no one expected her to be carrying a large box full of tiles.

"What do you do with all those tiles?" asked Sarah.

"Didn't you know I was a tile-setter? . . . They call me 'Tanya the Tile-setter.' Let me show you a trick."

"Great!" said Bert.

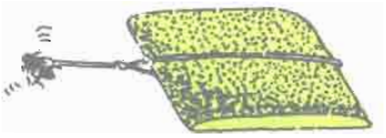
"I'm going to look out the window. Someone pick a number from 1 through 12. Write it on the board twice. First, divide it by 3 and then divide it by 4. Write down the "remainder". And erase everything else.

"I'll show you. If someone picked the secret number 7, this arithmetic would be written on the board . . .

$$\begin{array}{r} 2^{\text{R1}} \\ 3\overline{)\,7} \end{array} \quad \text{and} \quad \begin{array}{r} 1^{\text{R3}} \\ 4\overline{)\,7} \end{array}$$

"Then, the secret number and the whole number part of the answer would be erased so it would look like this:

$$\begin{array}{r} \text{R1} \\ 3\overline{)} \end{array} \quad \text{and} \quad \begin{array}{r} \text{R3} \\ 4\overline{)} \end{array}$$



"Finally, I'd turn around . . . and tell you the secret number.

"Remember: pick the secret number from the whole numbers 1 through 12. I'll write the list on the board.

12	9	6	3
4	1	10	7
8	5	2	11

"Did I forget any?"

"No, but why did you write them in that mixed up way?" Laurie asked.

"Because I wanted them to be in order," Mrs. Burg explained.

"Mrs. Burg, do you call that being in order?" asked Michael.

"Maybe you'll understand my order later. Let's see what happens. Bill, will you please pick a number from the list and do the arithmetic? Remember—divide your secret number by 3 and then divide it by 4."

While Mrs. Burg looked out the window, Bill wrote:

$$\begin{array}{r} 3\ \text{R2} \\ 3\overline{)\,11} \end{array} \quad \text{and} \quad \begin{array}{r} 2\ \text{R3} \\ 4\overline{)\,11} \end{array}$$

"Okay, I'm ready," said Bill.

"Before I turn around, please erase the secret number and the whole number part of both answers."

Bill followed directions. "It's done."

$$\begin{array}{r} \text{R2} \\ 3\overline{)\quad} \end{array} \quad \text{and} \quad \begin{array}{r} \text{R3} \\ 4\overline{)\quad} \end{array}$$

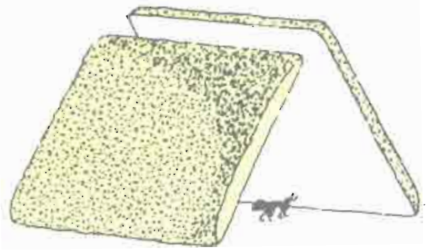
Mrs. Burg turned around and looked quickly at the board—"Your secret number is 11!"

"Right!" Then Bill went on: "You have a funny idea of 'order', but you sure must know something I don't know!"

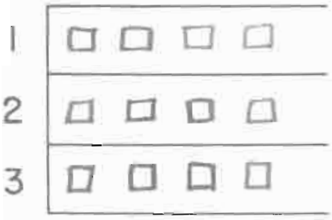
"How do you do it?" Tom asked.

"Believe it or not," Mrs. Burg answered, "the secret is in that order you think is strange."

12	9	6	3
4	1	10	7
8	5	2	11

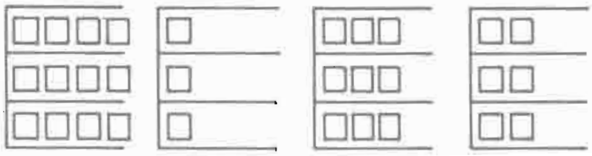


"I'd like you all to make some experiments with these little square tiles. Let's start with just 12 each. We're going to put them in rows, so you should draw a diagram so there is space for 3 rows of tiles:



☐ (a tile)

"How many different numbers of tiles can you put in the diagram so there are the *same number in each row?*



Total 12 3 9 6

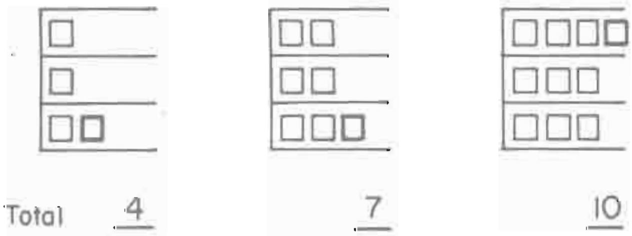
"We could describe the arrangements this way:

$\begin{array}{r} 4 \\ 3\overline{)12} \end{array}$	$\begin{array}{r} 1 \\ 3\overline{)3} \end{array}$	$\begin{array}{r} 3 \\ 3\overline{)9} \end{array}$	$\begin{array}{r} 2 \\ 3\overline{)6} \end{array}$
3 rows - 4 tiles in each row	3 rows - 1 tile in each row	3 rows - 3 tiles in each row	3 rows - 2 tiles in each row

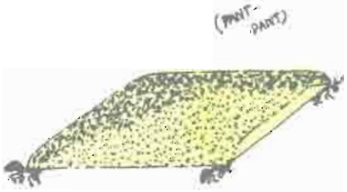
Another Experiment

"Let's try another experiment. How many different numbers of tiles can you arrange so there are the same number in each row with 1 tile left over? . . . Remember 12 is the largest number of tiles you can use."

The answers were quick in coming, and sketches were made:



Theresa couldn't keep quiet any longer. She had promised her Mother she would not give any of the secrets away too soon . . . "I know another way."



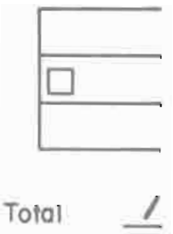
"All right, Theresa, what is it?"

"None in each row and 1 left over."

"That isn't fair," said Kathy.

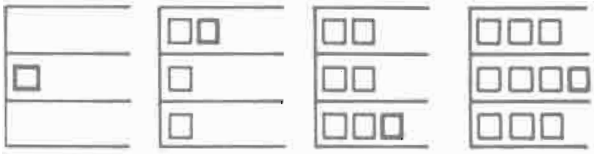
Connie blurted out, "You can't do that."

Theresa said: "Look at my tiles; I did it."



"Well," Mrs. Burg broke in, "Theresa wouldn't be right if we had a rule that there had to be at least one in each row . . . but let's not have that rule.

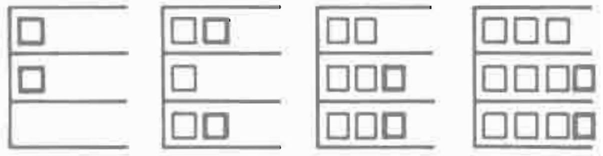
"So, we can describe our arrangements this way:



$\begin{array}{r} 0^{R1} \\ 3\overline{)1} \end{array}$ $\begin{array}{r} 1^{R1} \\ 3\overline{)4} \end{array}$ $\begin{array}{r} 2^{R1} \\ 3\overline{)7} \end{array}$ $\begin{array}{r} 3^{R1} \\ 3\overline{)10} \end{array}$

"Now, let's do still another experiment. Let's look for arrangements that have 2 left over."

Everyone went to work. Soon the results were in. The sketches and reports looked like this:



Total 2 5 8 11

$\overset{0\;R2}{3\overline{)2}}$ $\overset{1\;R2}{3\overline{)5}}$ $\overset{2\;R2}{3\overline{)8}}$ $\overset{3\;R2}{3\overline{)11}}$

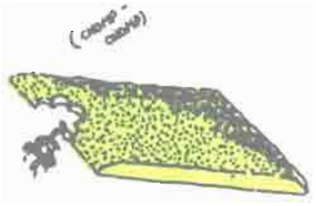
"Let's collect the results of our experiments and look at them together . . . and put one more thing in the first reports. We can show there were none left over this way:

Experiment 1. $\overset{4\;RO}{3\overline{)12}}$ $\overset{1\;RO}{3\overline{)3}}$ $\overset{3\;RO}{3\overline{)9}}$ $\overset{2\;RO}{3\overline{)6}}$

Exp. 2. $\overset{0\;RI}{3\overline{)1}}$ $\overset{1\;R1}{3\overline{)4}}$ $\overset{2\;R1}{3\overline{)7}}$ $\overset{3\;R1}{3\overline{)10}}$

Exp. 3. $\overset{0\;R2}{3\overline{)2}}$ $\overset{1\;R2}{3\overline{)5}}$ $\overset{2\;R2}{3\overline{)8}}$ $\overset{3\;R2}{3\overline{)11}}$

(Please fill in the missing numbers.)



"Now look at this summary and at my first arrangement of numbers 1 through 12:

12	9	6	3
4	1	10	7
8	5	2	11

"The number of tiles used in each experiment were:

Exp. 1.	12	3	6	9
Exp. 2.	1	4	7	10
Exp. 3.	2	5	8	11

Part of Mrs. Burg's "order" was clear. All the numbers when divided by 3 have—

(top row) a Remainder of 0
or (middle row) a Remainder of 1
or (bottom row) a Remainder of 2

Margie said, "We could rearrange our numbers in each row so they are in the same order as Mrs. Burg's—if there is any reason to do it."

"You can be sure I do have a reason," Mrs. Burg said. "Look at the columns in my arrangement."

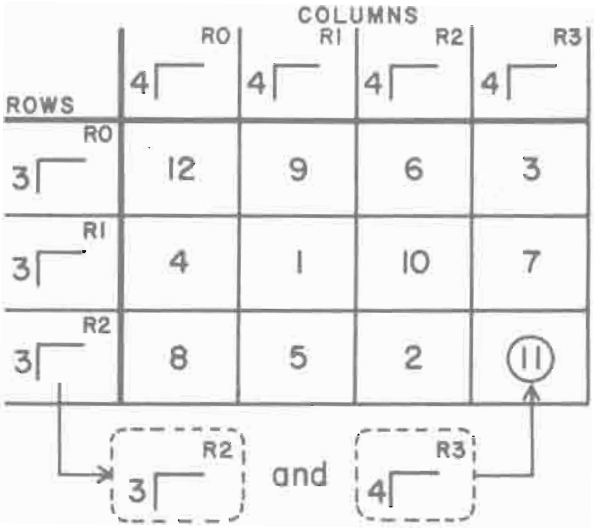
÷ 4

		a	b	c	d
	R0	12	9	6	3
÷3	R1	4	1	10	7
	R2	8	5	2	11

"What do you see about the numbers in column 'a'? . . .

Please fill in the answers in the following examples.

a	b	c	d
$\overset{3\,R0}{4\overline{)12}}$	$\overset{2\,R1}{4\overline{)9}}$	$\overset{1\,R2}{4\overline{)6}}$	$\overset{0\,R3}{4\overline{)3}}$
$\overset{1\,R0}{4\overline{)4}}$	$\overset{0\,R1}{4\overline{)1}}$	$\overset{2\,R2}{4\overline{)10}}$	$\overset{1\,R3}{4\overline{)7}}$
$\overset{2\,R0}{4\overline{)8}}$	$\overset{1\,R1}{4\overline{)5}}$	$\overset{0\,R2}{4\overline{)2}}$	$\overset{2\,R3}{4\overline{)11}}$

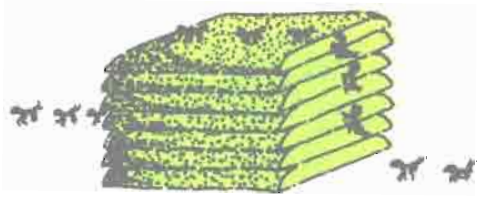


ROWS	R0 $4\overline{)\ \ }$	R1 $4\overline{)\ \ }$	R2 $4\overline{)\ \ }$	R3 $4\overline{)\ \ }$
R0 $3\overline{)\ \ }$	12	9	6	3
R1 $3\overline{)\ \ }$	4	1	10	7
R2 $3\overline{)\ \ }$	8	5	2	⑪

The Secret

"You see," Mrs. Burg explained, "there is a very clear pattern to my chart. Do you see how I did my trick now?"

"I get it," said Nancy. "You found Bill's secret number by finding a number that is in *both* row 'R^2' and that is also in column 'R^3' (when dividing by 4)—the number is 11."



"Can you use my secret order to find the secret number in each of these examples?" asked Mrs. Burg.

$\begin{array}{r}0\ \text{R1}\\3\overline{)1}\end{array}$ and $\begin{array}{r}0\ \text{R1}\\4\overline{)1}\end{array}$ is 1

$\begin{array}{r}1\ \text{R0}\\3\overline{)3}\end{array}$ and $\begin{array}{r}0\ \text{R3}\\4\overline{)3}\end{array}$ is 3

$\begin{array}{r}2\ \text{R2}\\3\overline{)8}\end{array}$ and $\begin{array}{r}2\ \text{R0}\\4\overline{)8}\end{array}$ is 8

$\begin{array}{r}4\ \text{R0}\\3\overline{)12}\end{array}$ and $\begin{array}{r}3\ \text{R0}\\4\overline{)12}\end{array}$ is 12

$\begin{array}{r}3\ \text{R1}\\3\overline{)10}\end{array}$ and $\begin{array}{r}2\ \text{R2}\\4\overline{)10}\end{array}$ is 10

$\begin{array}{r}1\ \text{R2}\\3\overline{)5}\end{array}$ and $\begin{array}{r}1\ \text{R1}\\4\overline{)5}\end{array}$ is 5

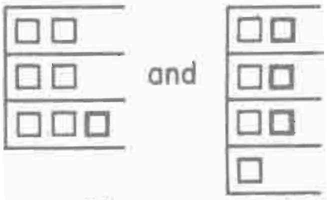
"I don't usually let anyone see the sketch I just showed you," said Mrs. Burg. "I keep a little card in my purse that I can glance at to find the secret numbers. Perhaps you would like to make a card of your own and try the trick on your friends."

Everyone thanked Mrs. Burg for showing them how she invented the trick.

"You see," Mrs. Burg explained, "since my job is setting tile, sometimes I do the same thing so often I can do it without thinking. So, I begin looking for something else about arranging tiles that is interesting . . . something most people wouldn't notice.

"By the way," said Mrs. Burg, "there's usually a lot of work behind 'tricks' . . . a lot of preparation.

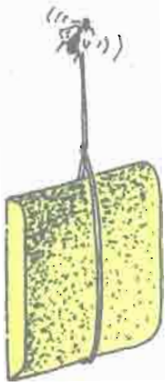
"I used to think about a given number of tiles, like 7 tiles, for example, and arrange them first in 3 rows and then in 4 rows . . . and I used arithmetic to record the result:



$\begin{array}{r}2\ \text{R1}\\3\overline{)7}\end{array}$ $\qquad$ $\begin{array}{r}1\ \text{R3}\\4\overline{)7}\end{array}$

"Then, I wrote the results on a slip of paper:

$\boxed{\begin{array}{r}2\ \text{R1}\\3\overline{)7}\end{array}\ \vdots\ \begin{array}{r}1\ \text{R3}\\4\overline{)7}\end{array}}$ and $\boxed{\begin{array}{r}3\ \text{R0}\\3\overline{)9}\end{array}\ \vdots\ \begin{array}{r}2\ \text{R1}\\4\overline{)9}\end{array}}$



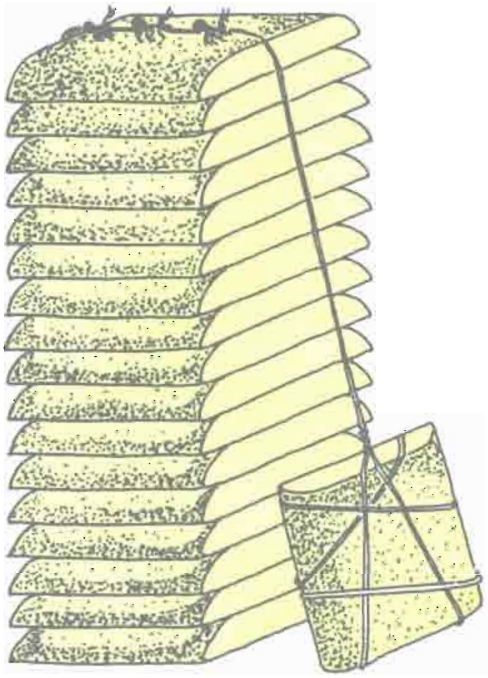
"I did this with all numbers of tiles, 1 through 12.

"I thought of this 'trick' when I found out that pairs of remainders were all different: so if I knew the remainder of any number, 1 through 12 when divided by 3 and by 4, I could find out the number.

"So, later I organized all the slips into this table:

Remainder when divided by 4

Remainder when divided by 3	R1		R2		R3		R0	
R1	R1 1 $3\overline{)1}$	R1 1 $4\overline{)1}$	R1 3 $3\overline{)10}$	R2 2 $4\overline{)10}$	R1 2 $3\overline{)7}$	R3 1 $4\overline{)7}$	R1 1 $3\overline{)4}$	R0 0 $4\overline{)4}$
R2	R2 1 $3\overline{)5}$	R1 1 $4\overline{)5}$	R2 0 $3\overline{)2}$	R2 0 $4\overline{)2}$	R2 3 $3\overline{)11}$	R3 2 $4\overline{)11}$	R2 2 $3\overline{)8}$	R0 2 $4\overline{)8}$
R0	R0 3 $3\overline{)9}$	R1 2 $4\overline{)9}$	R0 2 $3\overline{)6}$	R2 1 $4\overline{)6}$	R0 1 $3\overline{)3}$	R3 1 $4\overline{)3}$	R0 4 $3\overline{)12}$	R0 3 $4\overline{)12}$



"To make things easier, I wrote:

$4\overline{)\quad}$

$3\overline{)\quad}$

	R1	R2	R3	R0
R1	1	10	7	4
R2	5	2	11	8
R0	9	6	3	12

"I'll leave these spare tiles with you. Maybe you can invent a trick."

more tile tricks.....

Theresa walked into the hall with her Mother. "Can I tell some of my friends about 4's and 7's?" she asked.

"Sure, but don't tell them about 3's, 4's and 5's all together. I'm going to tell Ben's class about that trick."

"OK, we'll start with 4's and 7's. If they want to, can we try 5's and 8's?"

"Sure, if they want. Help them with any of our 2-number tricks!"

****

Later, Theresa talked with her friends about learning tricks using other numbers. They decided to start with 4's and 7's.

"We'll need to arrange numbers according to their remainder when divided by 7 . . . and then their remainder when divided by 4."

"How many different remainders can we have when dividing by 7?"

"The remainders can only be 0 through 6," said Mark, " . . . and, when dividing by 4, they can only be 0 through 3."

"We can name the columns and rows the way Mrs. Burg did," said Ellen.

		(÷7) COLUMNS						
		R0	R1	R2	R3	R4	R5	R6
ROWS (÷4)	R0	28	8	16	24	4	12	20
	R1	21	1	9	17	25	5	13
	R2	14	22	2	10	18	26	6
	R3	7	15	23	3	11	19	27

The group agreed on a plan. Each of Theresa's friends would try some examples and find where different numbers belong. For example:

$$\begin{array}{r} 4^{R2} \\ 4\overline{)18} \end{array} \quad \text{and} \quad \begin{array}{r} 2^{R4} \\ 7\overline{)18} \end{array}$$

So, 18, belongs in Row R^2 and column R^4.

They each took several numbers from 1 through 28 to find where they should go in the 4's and 7's chart. They divided each number by 4 and then by 7 to find the remainders—and then wrote that number in the proper row and column.

It was slow going.

Then Bill thought of an idea. "We can get the numbers in the proper row easy enough—just writing them in order, row by row:

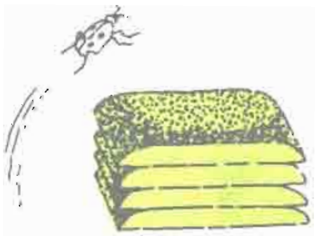
÷ 7 COLUMNS

	R0	R1	R2	R3	R4	R5	R6
÷ 4 ROWS	28	1	2	3	4	5	6
	7	8	9	10	11	12	13
	14	15	16	17	18	19	20
	21	22	23	24	25	26	27

'Then, we can work down *each column*, dividing each number by 4 and putting it in the right column.

(÷ 7) R0

28 ÷ 4 → R0
7 ÷ 4 → R3
14 ÷ 4 → R2
21 ÷ 4 → R1



"So, we can rearrange the first column:

R^0 (÷7)

	(÷7) R0
R0	28
R1	21
R2	14
R3	7

→

1st COLUMN (÷7)

	R0
R0	28
R1	21
R2	14
R3	7

"Now we can take Column R^1 (÷ 7)

(÷7) R1

1 ÷ 4 → R1
8 ÷ 4 → R0
15 ÷ 4 → R3
22 ÷ 4 → R2

So we can rearrange the second column:

	R0	R1
R0	28	8
R1	21	1
R2	14	22
R3	7	15

Some liked Bill's idea—others went on with their way of doing each number at a time.

When you have completed the chart suggested by Theresa's mother, you can let anyone select a number from 1 through 28—tell them to keep it secret——tell them to divide by 4 and by 7 and show you only the remainder.

And by checking the chart you can call out secret numbers in a few seconds. Try finding these secret numbers:

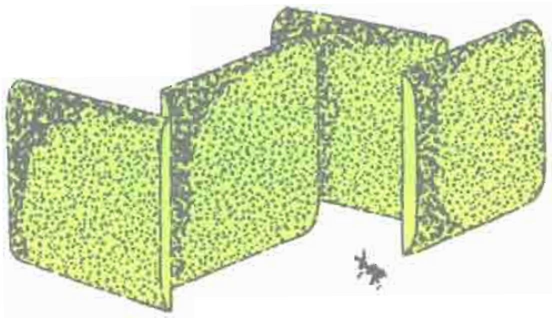
$\overset{6\ \text{R1}}{4\overline{)25}}$ and $\overset{3\ \text{R4}}{7\overline{)25}}$

$\overset{4\ \text{R1}}{4\overline{)17}}$ and $\overset{2\ \text{R3}}{7\overline{)17}}$

$\overset{1\ \text{R1}}{4\overline{)5}}$ and $\overset{0\ \text{R5}}{7\overline{)5}}$

$\overset{5\ \text{R2}}{4\overline{)22}}$ and $\overset{3\ \text{R1}}{7\overline{)22}}$

$\overset{0\ \text{R3}}{4\overline{)3}}$ and $\overset{0\ \text{R3}}{7\overline{)3}}$



Here is a chart worked out by Theresa and her friends to find out secret numbers 1 through 56 when given remainders after dividing by 7 and 8:

		(÷ 8)							
		R0	R1	R2	R3	R4	R5	R6	R7
	R0	56	49	42	35	28	21	14	7
	R1	8	1	50	43	36	29	22	15
	R2	16	9	2	51	44	37	30	23
(÷ 7)	R3	24	17	10	3	52	45	38	31
	R4	32	25	18	11	4	53	46	39
	R5	40	33	26	19	12	5	54	47
	R6	48	41	34	27	20	13	6	55

You can practice on the following examples before showing your friends how easily you can find their secret numbers.

$\overset{2\ \text{R4}}{7\overline{)18}}$ and $\overset{2\ \text{R2}}{8\overline{)18}}$

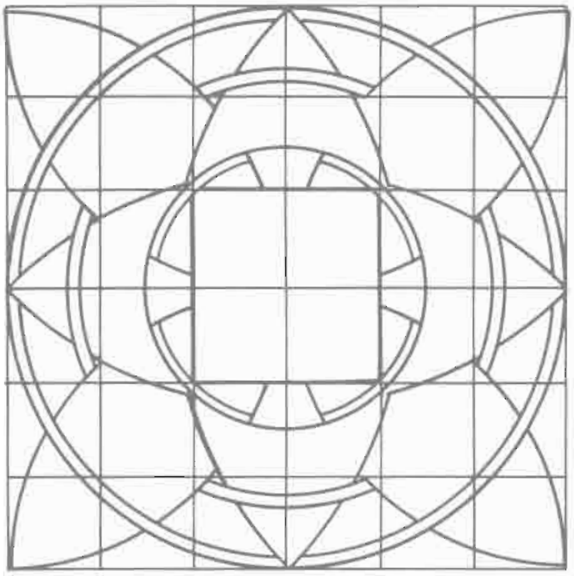
$\overset{4\ \text{R1}}{7\overline{)29}}$ and $\overset{3\ \text{R5}}{8\overline{)29}}$

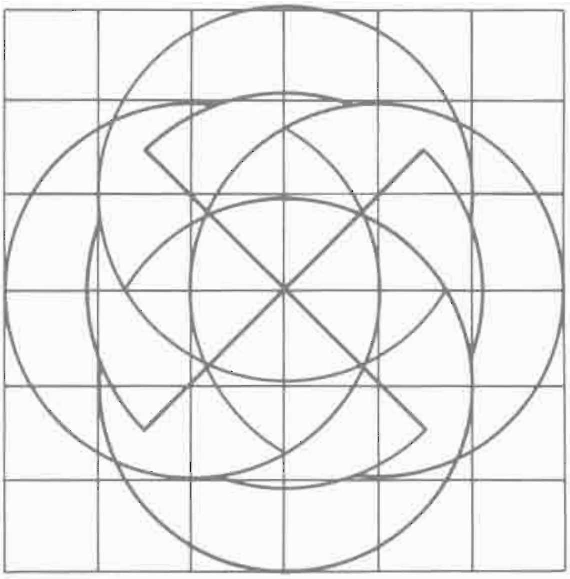
$\overset{6\ \text{R3}}{7\overline{)45}}$ and $\overset{5\ \text{R5}}{8\overline{)45}}$

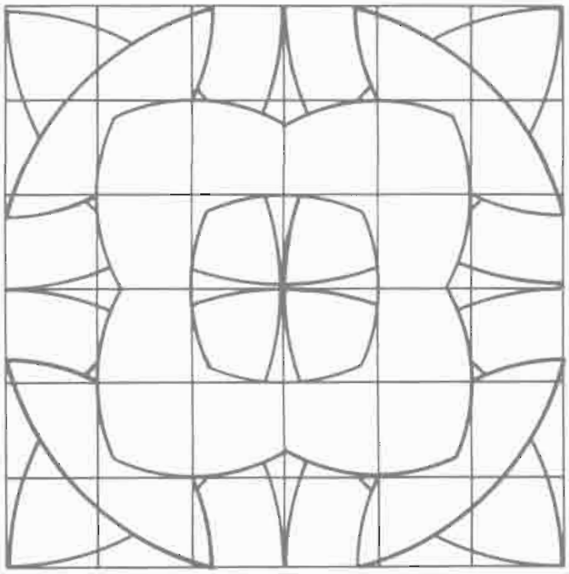
$\overset{4\ \text{R3}}{7\overline{)31}}$ and $\overset{3\ \text{R7}}{8\overline{)31}}$

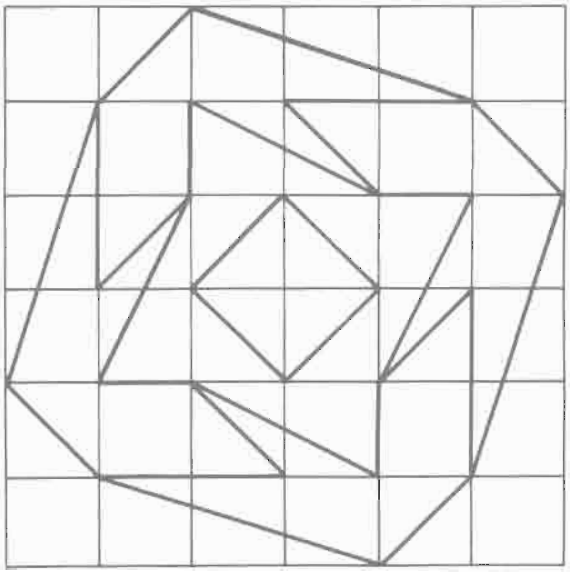
$\overset{6\ \text{R4}}{7\overline{)46}}$ and $\overset{5\ \text{R6}}{8\overline{)46}}$

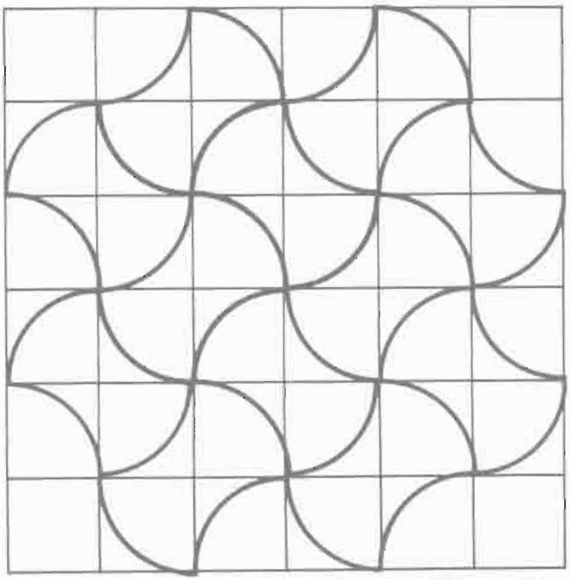
$\overset{7\ \text{R6}}{7\overline{)55}}$ and $\overset{6\ \text{R7}}{8\overline{)55}}$

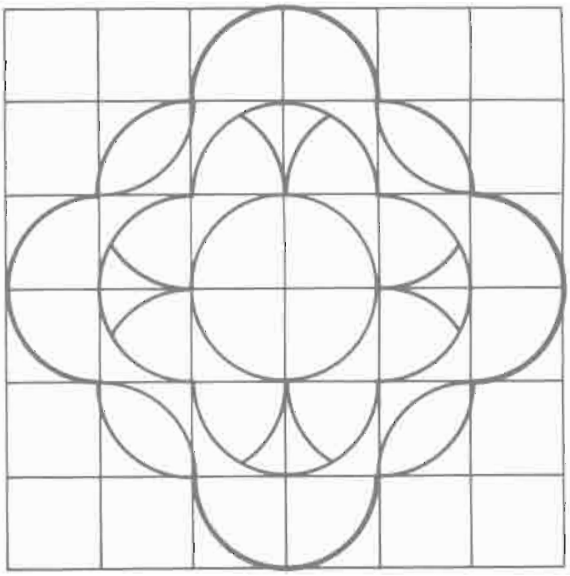












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