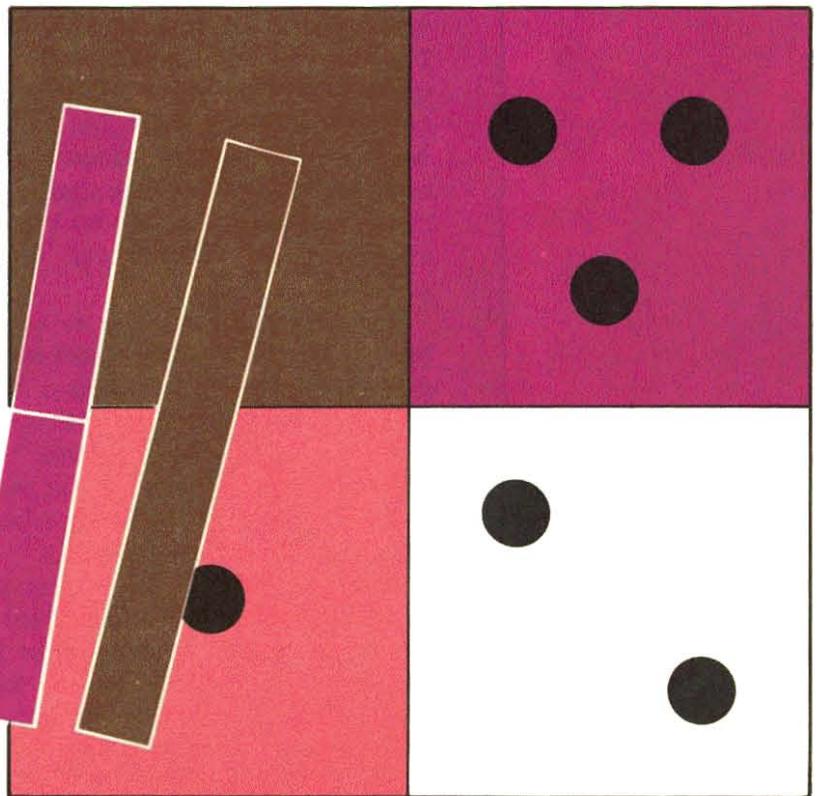
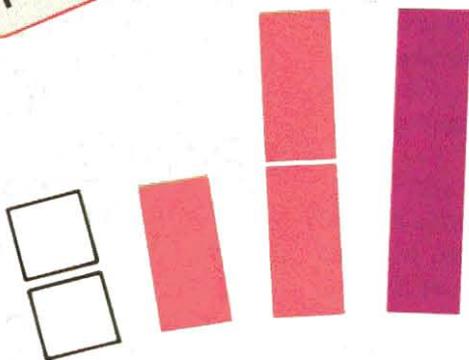


cuisenaire[®] REPORTER

Special Issue
Papy's Minicomputer



In this issue:

The Cuisenaire Reporter is a free newsletter offered to teachers. This year's Fall issue has been enlarged to bring you two special reports on Papy's Minicomputer.

FALL 1971

Editor's Comments

Innovative ideas in mathematics education come frequently from England and Belgium where models have been popular with teachers and pupils for many years.

This issue of the Cuisenaire Reporter contains two articles on Papy's Minicomputer. It is an excellent device for introducing arithmetic through both the binary and the decimal systems, and is based upon the color sequence of Cuisenaire® rods.

The first article in the Reporter is reprinted from Dr. Frédérique Papy's book, "Children and Mathematics" and appeared in the Spring 1970 issue of *Mathematics Teaching*. It shows pupils experimenting with the Papy Minicomputer to discover solutions to problems in addition and subtraction. Frédérique is an internationally distinguished mathematics educator from Belgium with much experience in teaching primary children. The minicomputer discussed in her book has been widely acclaimed as an excellent tool for developing math concepts.

The second article comes to us from Mrs. Adelyn Muller, Director of Mathematics for the Shawnee Mission Public Schools in Overland Park, Kansas. Mrs. Muller illustrates how Papy's Minicomputer can be successfully adapted using Cuisenaire® rods with sheets of paper sectioned into quarters. The back page of this Reporter provides a full-size sample which you may wish to use with your Cuisenaire® rods.

As a convenience to our readers, pads of 100 sheets have been printed in the four colors used with Papy's Minicomputer. Sets of pads may be purchased with an additional copy of the Cuisenaire Reporter for \$2.00. See our enclosed order form for further instructions.

Frédérique's book "Children and Mathematics" is available in English from the Cuisenaire Company of America, Inc. Further information on her book has been enclosed with this issue of the reporter.

Papy's Minicomputer

FREDÉRIQUE PAPY

Centre for the Pedagogy of Mathematics
Brussels

The method we have used to introduce children to mechanical and mental arithmetic employs the distinct advantages of the binary system over other positional systems, while at the same time taking account of the decimal environment in which we are immersed. It is thanks to the *Minicomputer Papy* that we have been able to achieve this.

Inspired by some work of Lemaitre, this variety of two-dimensional abacus uses the binary system on boards which are arranged according to the decimal system.

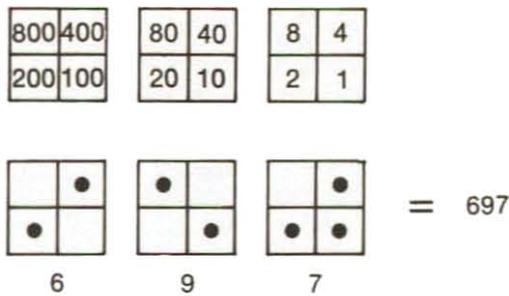


Fig. 1

The colors recall the red family of the set of Cuisenaire rods and help to make the four rules of the machine easily accessible.

2nd October, 1967

Frédérique hangs one of the display boards on the blackboard

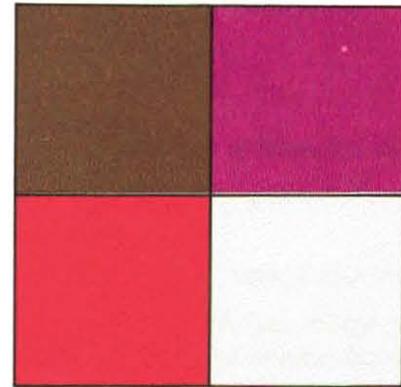


Fig. 2

— Oh, they're our colors! said Jean-Jacques, showing a white, red, purple and brown rod.

— Yes, indeed!

We will play a game together; you will use your rods and I will use the board and these counters.

Frédérique puts two black counters on the white square.

— They stay up by themselves! one child marvelled.

— Couple two white coaches together.

— It's the same as the red rod!

— A white-white train is the same length as a red coach.

Frédérique takes two counters from the white square and puts one on the red square.

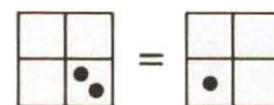


Fig. 3

— Two counters on the white square are equal to one on the red.

The children are quick to adopt the shorthand 'a red' for 'a counter on the red square' and state the rule 'two whites are equal to a red'.

— *Let us go on with the game!*

Frédérique puts two counters on the red square and the children put two red rods together.

— It is the same as the purple rod!

Frédérique takes the two counters from the red square and puts one on the purple square.

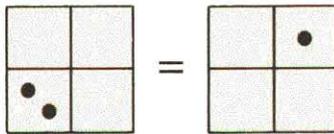


Fig. 4

— Two reds are equal to a purple.

Jean-Jacques is bursting with desire to move the counters and begs:

— Madame! Can I play?

Frédérique agrees and Jean-Jacques lifts the counter from the purple square and replaces it with two purple counters.

— It is beautiful!

— *Couple two purple coaches together.*

— It is the same as a brown rod.

Frédérique removes the purple counters and puts one on the brown square.

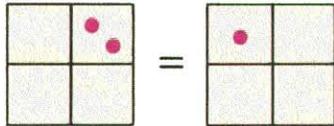


Fig. 5

— Two purples are equal to a brown.

5th October, 1967

— *A new game!*

Frédérique places a counter on the white square — 1
She places a second counter on the white square — 2

— I can play another way! says Carine. She lifts off the 2 counters from the white square and puts one on the red.

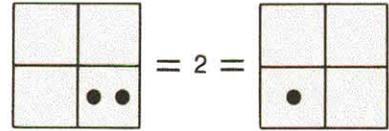


Fig. 6

— *Add 1 to the number 2*, Frédéric suggests, putting a new counter on the white square.

— It is 3.

— Like with rods, red-white

— or light green.

— *Add 1 to the number 3*, Frédéric continues, putting a new counter on the white square.

— It is 4.

— I can play another way, says Sylvie, who takes the two counters from the white square and puts one on the red.

— And another way, states Jean-Jacques, replacing the two counters on red by one on the purple.

— *Let us go on adding 1 . . .*

The recital continues and the numbers 5, 6, 7, 8, 9 appear on the Minicomputer.

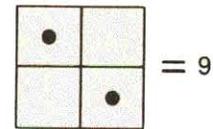


Fig. 7

— *Add 1 to the number 9.*

A child puts a counter on the white square.

— It is 10.

— I play in another way, says Anita, replacing the two counters on white by one counter on the red.

— It's the red-brown train.

— The orange rod.

— It's 10.

Frédérique hangs a second Minicomputer board on the blackboard, to the left of the first. She takes the two counters from the red and brown squares and puts a counter on the white square of the new board, saying simply

— *And this is still 10.*

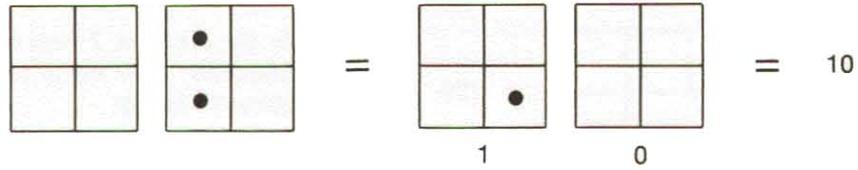


Fig. 8

— Jump! On to the second board, remarks Jean-Jacques, not in the least surprised.

Thus armed, our pupils are able to represent on the Minicomputer the number of elements in any set of counters put on square 1. The application (in reverse order) of the four fundamental rules on a small scale lets us keep a concrete link between a number in its written form and as represented on the machine.

13th October, 1967

Two small Minicomputer boards and a box of counters are on the desks in front of each child.

— How many counters do you have in your box?
 — 29 . . . 32 . . . 18 . . . 26 . . . 35

The children's boxes are not all filled with the same number!

— Put the box of counters on the white square of the first board. Play . . . and then write down the result.

The first extended piece of individual work on the Minicomputer: concentration, precise and quick movements: some children arrive at the result without a mistake.

Clumsy techniques, counters spilled, false moves; the others must be helped.

The second part of the lesson is played on the wall Minicomputer.

We start with the number 25 on the machine. We work all the counters back to the white square on the first board and count them: confirmation!

After a fortnight the demands of the class forced Frédérique to introduce a third board and to accept numbers over 100.

— If I put a counter on each square of the machine, will I make the number 100? asks Didier, who still has a machine with only two boards.

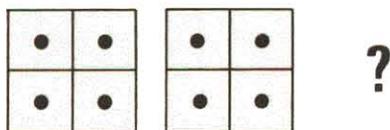


Fig. 9

Frédérique does not answer. She expects the problem to arise later in a different form and prefers to let the child's thinking follow its own course.

24th October, 1967

Didier returns to the attack.

— I want to make 100 on the machine.
 — All right . . . let us have your method!
 — 100 is twice 50, Didier goes on, showing:

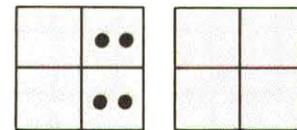


Fig. 10

— I can play!

He replaces the two counters on the white square by one counter on the red square and the two counters on the purple square by one on the brown.

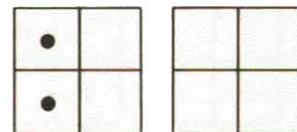


Fig. 11

— I want to play again! he says, taking the counters from the red and brown squares and putting a counter on the left side of the second board.



Fig. 12

And Didier demands his rights:

— I must have another board, Madame!

Frédérique gives him one. Didier triumphantly forms the number.

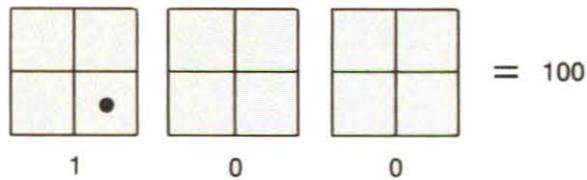


Fig. 13

Very excited, he shouts out the numbers as he shows them on the machine.

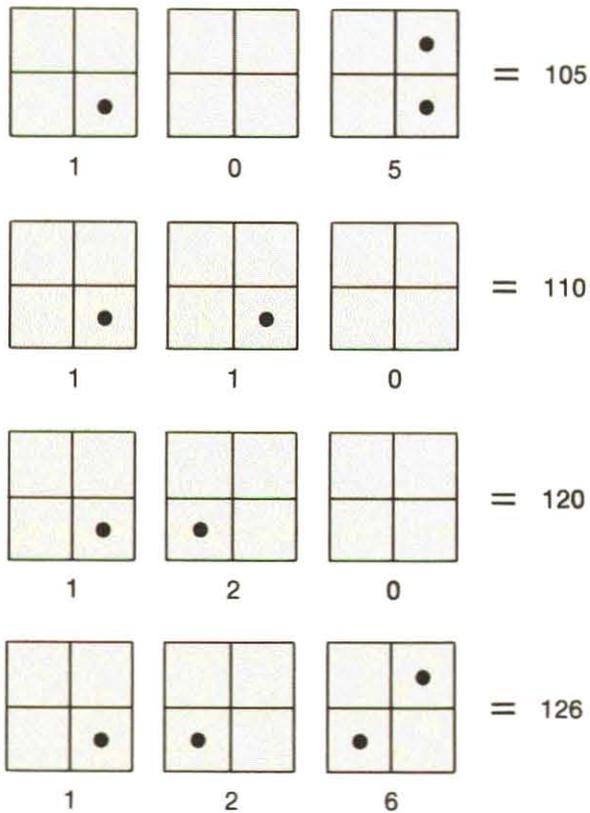


Fig. 14

Impressed, the class has shared in this discovery.

The addition of whole numbers is carried out automatically by 'playing' the machine. The same is true for *doubling* which is a primitive experience for children and fundamental to the Minicomputer.

The part played by arbitrary memorizations, so often a distasteful part of learning to compute, is reduced to a minimum. The addition of small numbers is carried out by means of intelligible rules: a purely binary system when the sum is less than 9 and a mixed decimal-binary system in other cases. From the beginning, therefore, the children are initiated into a positional system of numeration.

— In the car park I counted 75 Volkswagens and 49 Mercedes. How many cars are there altogether?
— Show 75 in red!

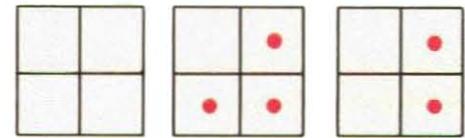


Fig. 15

— And 49 in purple!

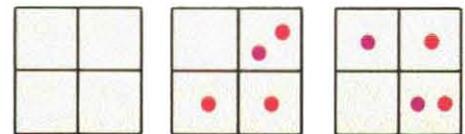


Fig. 16

— Can I play, Madame?

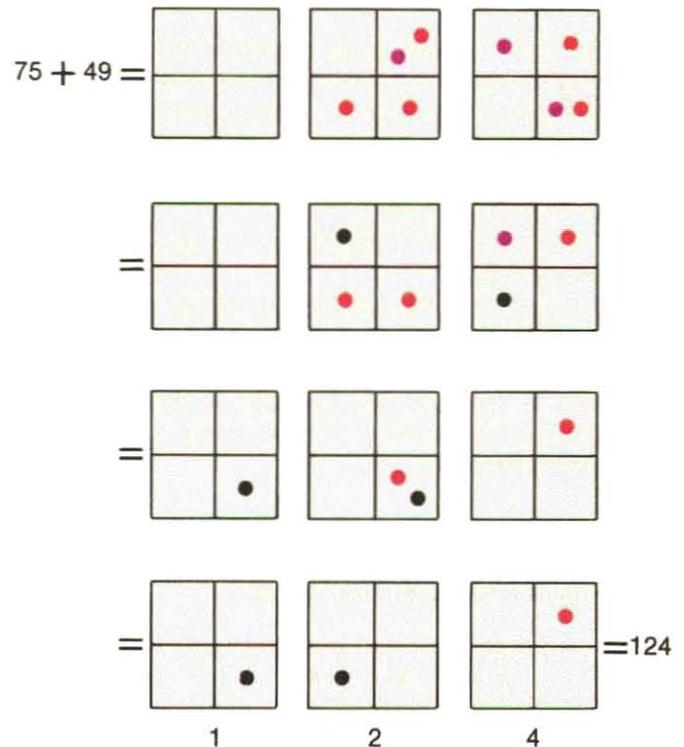


Fig. 17

The children make a note of the temperature in degrees centigrade (Celsius) each morning. At the start of the school year, in Brussels, these are always natural numbers, but the winter months force the use of negative numbers. From the *sixth month*, negative integers are part of the common knowledge of the children, having authentic status as numbers 'measuring' a 'size' which has been experienced.

The results of a sequence of two-person games are written with black and red numbers which are mutually destructive, since each point captured by one of the players 'kills' a point captured by the other. The addition of black and red numbers follows. From the given notation the children arrive at the additive group of integers.

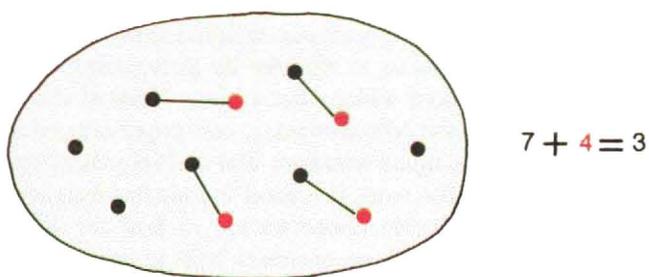


Fig. 18

Eventually, the writing is simplified and all the numbers are written in black, those originally red having a bar across the top. In effect this time we have the group Z , $+$, except for a very minor difference, 3 being used instead of -3 . We have known the advantages of the $\bar{3}$ notation for beginners for a long time since we have used it to simplify logarithmic calculations.

At the beginning we show the result of each game by putting black and red counters on a flat surface. A battle to the death gives the final score. The children spontaneously transfer the procedure to the Minicomputer.

— Let us calculate this to find how big it is:

$$100 + 23$$

Great excitement at all the desks!

— Do it on the machine!

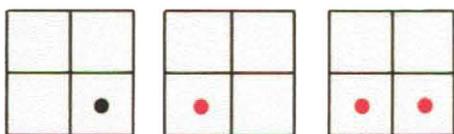
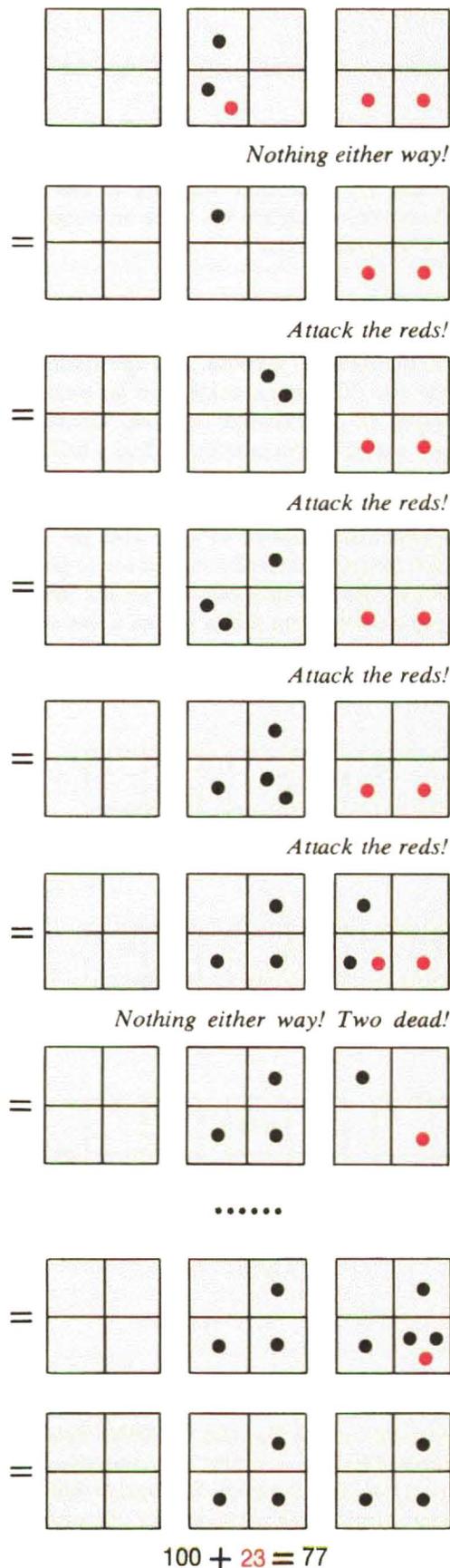


Fig. 19

Fig. 20 (opposite)



- Who will win?
- Black!
- Black soldiers! Attack the reds!

In our teaching, the function 'a half of' appears at the reciprocal of the function 'twice' which is a transformation of \mathbf{Z} , that is, a function mapping \mathbf{Z} into \mathbf{Z} . But some numbers exist which are not twice an integer. They do not have integral halves.

A bar of chocolate can be broken fairly into two pieces. A 100 franc note can be changed into two 50 franc notes. 100 is certainly an even number. But one franc can be changed into two 50 centime coins, and fortunately this always appeals to six-year-old children. Consequently it seems very natural to them to try to find a half of 1 on the machine.

It was because they wanted to write 100, got by doubling 50, that the pupils literally obliged me to give them the third board. We can now put 100 on the board and watch the way in which we find a half of it; we can start with 10 in the same way.

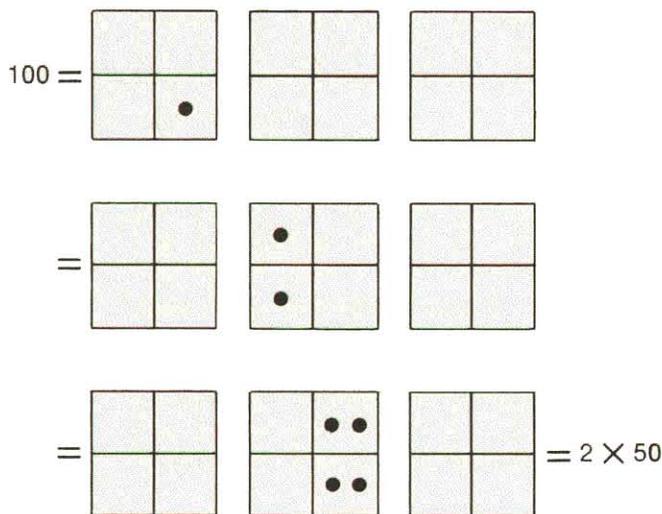


Fig. 21

How can the same rule be applied to finding a half of 1? The children ask for a new board, to the right of the others, and at once call it the 'tiny numbers board'. How shall we remember that it is the 'tiny numbers board'? By putting a red line between the boards which will later become the decimal point. So we can calculate a half of 1 and we write 0.5.

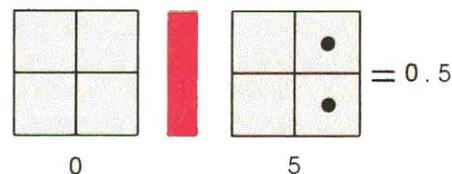


Fig. 22

The problem of dividing 100 francs fairly between three children introduces an exceptionally interesting situation. The first glimpse of a non-terminating decimal comes through in this remark of one child: 'We will still be here tomorrow morning . . .'

The interest which six-year-old children show in quite large numbers forces us to respond by giving them unmotivated calculations which offer various kinds of challenge. Thanks to the Minicomputer, our pupils can add and subtract three-figure numbers and multiply them by simple fractions. The work is a good foundation because it requires considerable concentration to find the best strategies each time. These exercises help to produce a harmonious cooperation between the intelligent human being and a true machine, which is what the Minicomputer is in the eyes of the pupils.

Bibliography

- G. Lemaitre, 'Comment calculer' in *Bulletin de l'Académie royale de Belgique: Classe des Sciences*, Brussels, 1954.
- G. Lemaitre, 'Pourquoi de nouveaux chiffres?' in *Revue des Questions Scientifiques*, July, 1955.
- G. Lemaitre, 'Le calcul élémentaire' in *Bulletin de l'Académie royale de Belgique: Classe des Sciences*, Brussels, 1956.
- G. Lemaitre, *Calculons sans fatigue*. E. Nauwelaerts, Louvain, 1954.
- Papy, *Minicomputer, Ivac*, Brussels, 1968. English version: Collier-Macmillan, New York (in press).
- Papy, *Mathématique moderne 1*. Didier Brussels-Paris-Montreal, 1963. (English version: Collier-Macmillan, New York, 1969.)
- Frédérique and Papy, *L'enfant et les graphes*. Didier, 1968. English version: Algonquin Publishing, Montreal, Canada, 1970.
- Frédérique, *Les enfants et la mathématique 1*. Didier, 1970. American version: Cuisenaire Co. of America, 12 Church St., New Rochelle, N. Y.

The accounts of lessons described in this article are extracted from the last reference and are reproduced here by kind permission of the editor.

An Adaptation of Papy's Minicomputer

ADELYN C. MULLER

Mme. Frédérique Papy, well-known mathematics educator from the Centre Belge de Pedagogie de la Mathematique in Brussels, was present at the Golden Jubilee Convention of the National Council of Teachers of Mathematics as the recipient of the Houghton-Mifflin Distinguished Lectureship in Mathematics Award, and at the General Session on April 3, 1970, she made an absorbing presentation entitled "Minicomputer: A Revolution in the Teaching of the First Numerical Concepts". Mme. Papy's thesis provided discussion topics for four follow-up sessions. (Excerpt NCTM Newsletter, May, 1970.)

Mme. Papy has returned to the United States several times since as the guest of the Comprehensive School Mathematics Program, sponsored by CEMREL at St. Louis. I observed several demonstration classes in June, 1970 at Carbondale, Illinois, the location of the Comprehensive School Mathematics Program. The following month during a workshop which I was conducting at Pierre, South Dakota, I devised the following adaptation of Mme. Papy's Minicomputer by using the Cuisenaire rods.

Several $8\frac{1}{2} \times 11$ " sheets of unlined paper which have been sectioned into quarters and a box of Cuisenaire rods are needed. This two dimensional device (abacus, if you like) uses the binary system on sheets of paper which are arranged according to the decimal system.

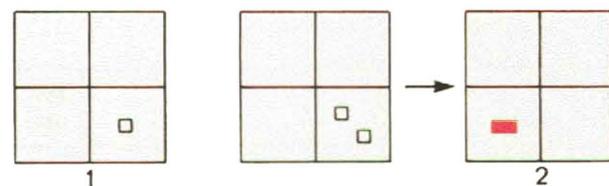
HUNDREDS	
800	400
200	100

TENS	
80	40
20	10

ONES	
8	4
2	1

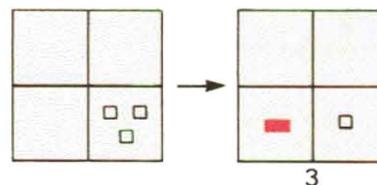
The difference at this stage in Mme. Papy's Minicomputer is that colored magnetic boards are used. In this

adaptation children have the sheets of paper (one, at first) and place rods of the correct value in the above assigned areas. (A sample sheet has been printed on the back of this issue. The colors on the sheet are not essential, but a helpful reminder to young students. The use of the dotted subdivision will be explained later in the article.) The one rule—no area may contain more than one rod and it must be the appropriate color, so changes must be made to accommodate this situation as it occurs during the operation of addition or subtraction. At first, single digit numbers are represented on the ONES sheet of paper:

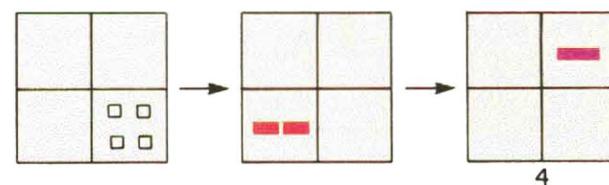


One White rod

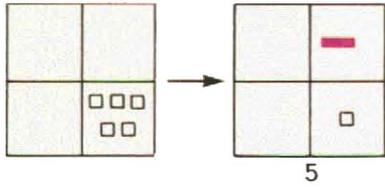
Two White rods must be changed to one Red rod



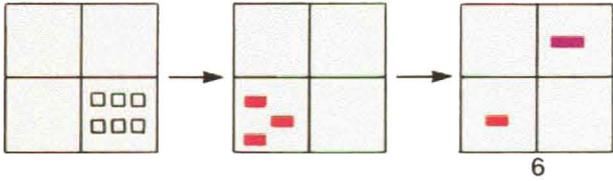
Three White rods must be changed to one Red and One White rod



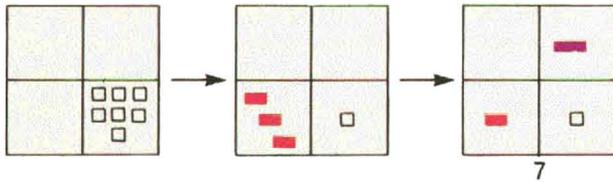
Four White rods must be changed to one purple rod



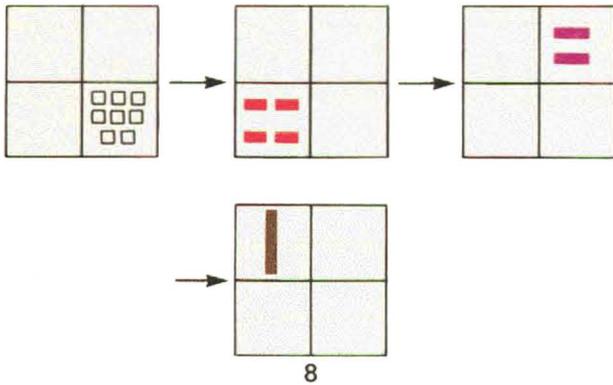
Five White rods must be changed to one Purple and One White rod



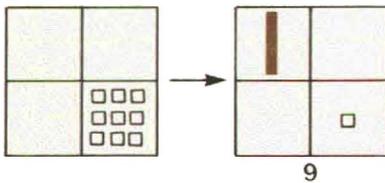
Six White rods must be changed to one Purple and one Red rod



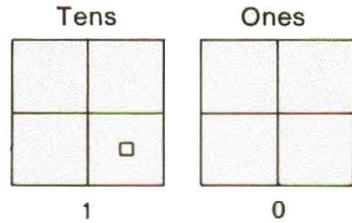
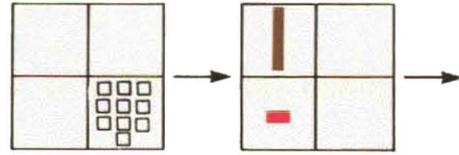
Seven White rods must be changed to one Purple, one Red and one White rod



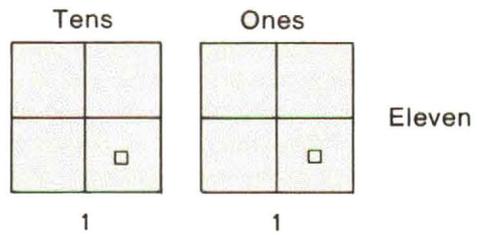
Eight White rods must be changed to one Brown rod



Nine White rods must be changed to one Brown and one White rod

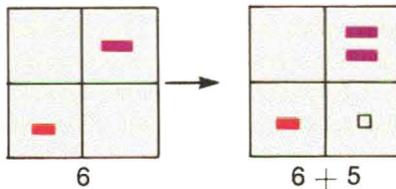


Ten as a Brown and a Red rod must be changed to a representation of one White rod on the tens sheet of paper. If number is two digits—one needs two sheets of paper. If three digits—three sheets, et cetera.

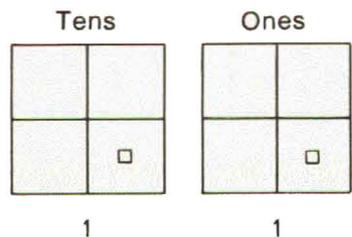
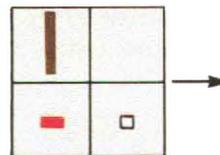


Now for addition: $6 + 5$

First place rods representing six, then place rods representing five (both on ONES sheet)

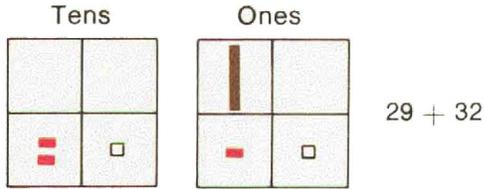
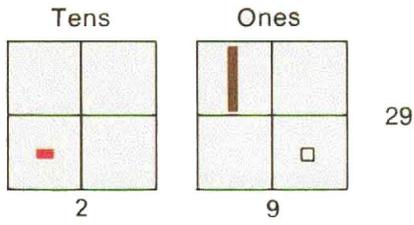


Now enforce rule:

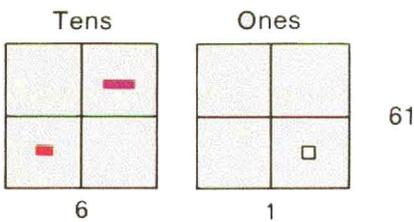
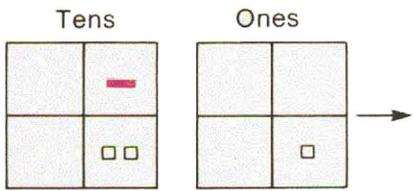
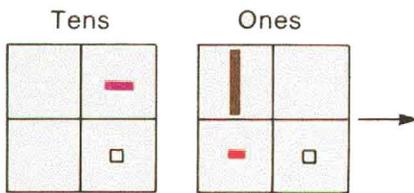


Read one ten, and one one, or eleven

Add: $29 + 32$



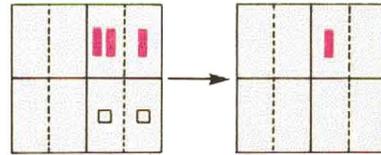
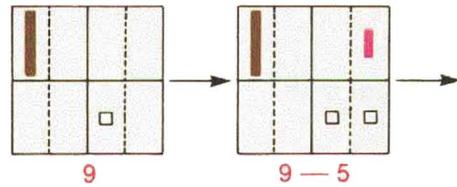
Now enforce rule:



Now for subtraction: (Work backwards)

Mme. Papy refers to the operation of subtraction as a battle between two armies with the larger number always on the attack toward the smaller.

The first number (subtrahend) is set up with rods on the left side of the dotted lines, and the number to be subtracted is set up to the right. The attack is always across the line.



Change the Brown rod to two Purple—one Purple on the left captures the Purple on the right and one White rod captures the other White rod—and all rods involved in capture are removed. So, the remaining rod on the ONES sheet is the Purple rod $9 - 5 = 4$

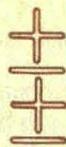
First graders and even some kindergarten students can handle such addition or subtraction:

$$\begin{array}{r} 2468 \\ +4987 \\ \hline \end{array} \qquad \begin{array}{r} 9630 \\ -2741 \\ \hline \end{array}$$

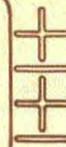
after progressing through the game in small steps. And the serendipity is—they think it is great fun!

Bibliography

- Papy, Minicomputer, Ivac, Brussels, 1968.
- Papy, Mme. Frédérique. Papy's Minicomputer, *Mathematics Teaching*, Spring, 1970.
- Whitney, Hassler. A Minicomputer for Primary Schools, *Mathematics Teaching*, Autumn, 1970.



This sample student's worksheet can be used with Papy's Minicomputer. The inside articles describe the successful use of the Minicomputer for introducing addition, subtraction and place value.



CUISENAIRE COMPANY OF AMERICA, INC.
12 Church Street, New Rochelle, N.Y. 10805

