CSMP Mathematics for the Upper Primary Grades

A Supplement for Third Grade Entry Classes

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The Third Grade Entry Program

A Supplement for Third Grade Entry Classes is designed for third grade classes using *CSMP* for the first time. The "veteran" third grade program is contained in two volumes, *CSMP Mathematics for the Upper Primary Grades, Part III* and *Part IV* (*UPG-III* and *UPG-IV*). This supplement contains fifteen introductory lessons, drawn from *UPG-I* and *UPG-II*. These lessons acquaint students with the Minicomputer, the languages of strings and arrows, and negative numbers, thus providing necessary background for future lessons in the third grade program. A modified first semester schedule is included.

Notes to the Teacher

3RD GRADE ENTRY NOTES TO THE TEACHER

How to Use the Third Grade Entry Schedule

As a teacher of a third grade entry class, you will be teaching lessons from both this booklet and from the *UPG-III* teacher's guide. The schedule contained in this booklet shows you how they integrate. Follow this schedule rather than the one in the teacher's guide. Pay close attention to the letters preceding a lesson number. Lesson numbers beginning with **E** can be found in this booklet. For example, Lesson **EN1** is the first supplement lesson to the World of Numbers (**N**) strand; its description begins on page 1 of this booklet.

strand abbreviation (in this case, World of Numbers strand)

lesson number within the strand

a supplement lesson
for entry classes

EN1

If there is not an **E** before a lesson number, then the description of that lesson can be found in the *UPG-III* teacher's guide.

Materials

All the materials you will need can be found in your classroom set, as well as a few you will not need because the lessons requiring those materials are not scheduled for entry classes.

The worksheets that your class will need for a few lessons in the third grade entry program can be found in blackline form following the lessons. Parent letters and home activities can be found in the Blackline section of the *UPG-III* teacher's guide.

A Note on the Subtraction Algorithm

CSMP introduces a subtraction algorithm called "Nick's method" in the second semester of third grade. However, there are many earlier lessons devoted to finding various other ways to do subtraction calculations. Your students might come to the third grade having seen a subtraction algorithm, perhaps involving "borrowing"; if so, accept the algorithm as a valid method but do not promote it as the only method. After being introduced to "Nick's method," your students may begin to show a preference for an algorithm. Help students individually to gain facility with their preferences.

Suggested Schedule

Master Schedule for CSMP Mathematics for Third Grade Entry Classes

Week	World of Numbers	Languages of Strings & Arrows	World of Numbers	Workbooks	Geometry and Measurement
1	EN1 Minicomputer Introduction #1 EN-1	EL1 Sending Letters EL-1	EN2 The Functions +3 and −3 EN-9	EW1 #1 Detective Story #1/ Eli's Magic Peanuts EW-1	G1 Fractional Parts of Shapes#1 G-3
2	EN3 Minicomputer Introduction #2 EN-13	EL2 Introduction to A-Blocks EL-5	EN4 Arrow Roads EN-17	EW2 Eli's Magic Peanuts #2/ Detective Story #2 EW-7	G2 Taxi-Distance G-9
3	EN5 Minicomputer Introduction #3 EN-21	EL3 String Game with A-Blocks EL-11	EN6 The Functions +5 and −5 EN-25	EW3 Detective Story #3/ Eli's Magic Peanuts #3 EW-25	G3 Rectangles G-13
4	EN7 Minicomputer Introduction #4 EN-25	EL4 Multiples EL-15	N2 Introductory Arrow Problems #1 N-11	W2 Which Road? W-7	EN8 The Functions 2x and ½x EN-37
5	N3 Minicomputer Introduction #2 N-17	L1 String Games #1 L-3	N4 Roads #1 N-21	W3 The Empire State Building W-13	G4 Length #1 G-15
6	N5 Subtraction and Addition Problems N-25	L2 L-9 Composition Games #1	N6 The Function +10 and Composition N-29	W4 #1 Festival of Problems (Lesson One) W-19	G5 Fractional Parts of Shapes #2 G-19
7	N7 Multiplication Problems #1 N-33	L3 Two Lists L-15	N8 Roads #2 N-37	W5 #1 Festival of Problems (Lesson Two) W-23	N11 Multiplication Table N-55
8	N10 Introductory Arrow Problems #2 N-49	L4 Multiples of 3 and 4 L-21	N12 The Number Line Game N-59	W6 Fishing for Numbers, Part III (Lesson One) W-33	G6 Nine-Square and Sixteen-Square G-23
9	N13 Multiplication Problems #2 N-67	L6 String Games #2 L-31	N14 Roads #3 N-71	W7 Fishing for Numbers, Part III (Lesson Two) W-35	G7 Where Shall We Meet? #1 G-29

Week	World of Numbers	Languages of Strings & Arrows	World of Numbers	Workbooks	Geometry and Measurement
10	N15 Doubling and Halving on the Minicomputer N-75	L7 Composition Games #2 L-35	N16 The Road Map of Nevada N-79	W11 Rollerskating 37 (Lesson One) W-59	G8 Length #2 G-33
11	N17 Subtraction on the Minicomputer N-85	L8 Marble Game L-39	N18 The Functions 2x and $\frac{1}{2}$ x N-89	W12 Rollerskating 37 (Lesson Two) W-63	G9 Where Shall We Meet? #2 G-37
12	N20 +4 Arrow Roads N-99	L9 Three Strings L-45	N21 3x and $\frac{1}{3}$ x on the Minicomputer N-105	W10 Road Map W-53	G10 Fractional Parts of Shapes#3 G-41
13	N22 Subtraction and Composition #1 N-109	L10 A Detective Story L-51	N23 Subtraction Problems N-115	W13 Festival of Problems #2 (Lesson One) W-71	N24 The Functions 3x and $\frac{1}{3}$ N-119
14	N25 Multiplication Problems #3 N-125	L12 A Game with Two Cubes L-59	N26 Subtraction and Composition #2 N-131	W14 Festival of Problems #2 (Lesson Two) W-75	G11 Geoboard Problems G-47
15	N27 Introduction to Decimal Numbers #1 N-135	L13 String Games #3 L-63	N28 The Function 10x N-141	W15 Detective Story #2 W-85	G12 Maps of a Cube G-51
16	N29 Introduction to Decimal Numbers #2 N-147	L15 Changing Seats L-71	N30 Assorted Shopping Problems N-153	W16 20?-100? (Lesson One) W-91	G13 Square Puzzles G-55
17	N31 10x on the Minicomputer N-155	L16 Counting Problems L-77	N32 Subtraction and Composition #3 N-159	W17 20?-100? (Lesson Two) W-95	Adjustment Day
18	N34 Road Map of Virginia N-167	L17 Singing Friends L-83	N35 Multiplication and Composition N-171	W18 Who are Tic and Tac? W-105	G14 How Much is a Pound? G-59

EN Lessons

EN1 MINICOMPUTER INTRODUCTION #1

Capsule Lesson Summary

Introduce the values of the squares on the Minicomputer. Use the Minicomputer to represent numbers less than 20. Sometimes represent the same number in different ways. Model some trades and represent numbers less than 100. Introduce the hundreds board and extend trades and representations of numbers to include hundreds.

Materials

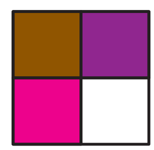
Teacher		Student	
	• Minicomputer set[†] • Base-10 blocks or other place-value manipulative		• None

Description of Lesson

Exercise 1

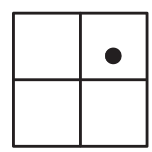
Note: If students are familiar with Cuisenaire Rods (C-rods), you may like to display a staircase of C-rods. Then, in addition to other representations of numbers, you can relate the square values on the Minicomputer to corresponding color C-rods.

Display a Minicomputer board.



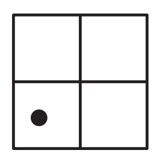
T: ***Show me four fingers. Trace a 4 on your desk. That's one way to write 4. I'm going to show you how to put 4 on the Minicomputer.***

Point to the Minicomputer and put one checker on the purple square.



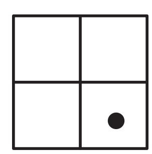
T: ***This is the Minicomputer. We put numbers on the Minicomputer using checkers. This is the number 4 on the Minicomputer.***

Move the checker to the red square.



T: ***This is the number 2 on the Minicomputer. Show me two fingers. Trace a 2 on your desk.***

Move the checker to the white square.

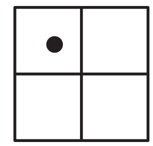


T: ***This is the number 1 on the Minicomputer. Show me one finger. Trace a 1 on your desk.***

Review the configurations for 4, 2, 1 and then again for 1, 2, 4 on the Minicomputer. Do this a couple times letting students tell you the numbers.

[†]A teacher's Minicomputer set consists of four demonstration Minicomputer boards and a sufficient number of demonstration Minicomputer checkers.

Move the checker to the brown square.



T: ***What number do you think this is?***

Some students may guess 3 or 5.

T: ***No, it is not 3 (or 5).***

Pause. Review again 1, 2, 4 and then pause as you put the checker on the brown square. After a moment a student may suggest 8. If not, simply say,

T: ***This is 8.***

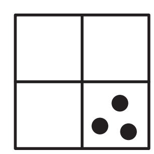
Move the checker quickly from one square to another and ask the class to call out each number. In doing this, follow the doubling pattern: 1, 2, 4, 8. Visually suggest the doubling pattern by putting two checkers on the white square and saying, “1 plus 1 is 2.” Then take off the two checkers and put one checker on the red square. Repeat for “2 plus 2 is 4” and “4 plus 4 is 8.” You may also ask the students to show 1 plus 1 is 2 with fingers, and so on.

Remove the checkers from the Minicomputer.

T: ***Who can put 3 on the Minicomputer?***

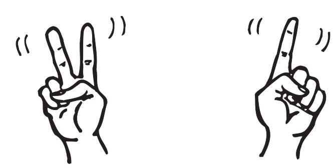
Ask volunteers to tell you first how many checkers they will need. (Two and three are both correct answers.) Let a student put 3 on the Minicomputer.

A student might put three checkers on the white square.

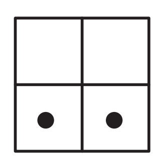


If a student places the checkers this way, lift the checkers one by one and say, “1 + 1 + 1 = 3.” Remove the checkers from the Minicomputer. Ask if someone can put 3 on the Minicomputer in another way.

If your students need a hint, alternately wave two fingers on one hand and one finger on the other hand saying, “2 + 1 = 3.”



By now someone should be able to put this configuration for 3 on the Minicomputer. After a student places the checkers in this way, lift each checker as you mention its value and then replace it quickly on the Minicomputer.



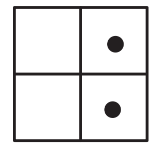
T: ***What number is on the red square?*** (2) ***What number is on the white square?*** (1) ***2 + 1 = 3. 1 + 2 = 3.***

Remove the checkers from the Minicomputer.

T: ***Who can put 5 on the Minicomputer?***

Ask volunteers to tell you how many checkers they will need. (Two, three, four, and five are all correct answers.) Ask a volunteer to place the checkers on the Minicomputer. Add the numbers on the various squares out loud as you did for 3. Whenever more than two checkers are used, ask for another way until you get the standard configuration.

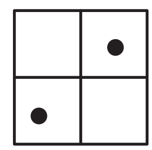
When the checkers are in this position, lift each checker as you mention its value and replace it quickly on the Minicomputer.



T: ***What number is on the purple square?*** (4)
… on the white square? (1)
4 + 1 = 5. 1 + 4 = 5.

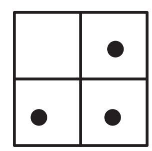
Remove the checkers from the Minicomputer.

T: ***Can anyone put 6 on the Minicomputer?***



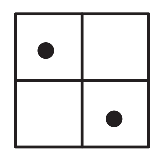
Accept all correct answers. If more than two checkers are used, ask for another way until you get the standard configuration.

T: ***Can anyone put 7 on the Minicomputer?***



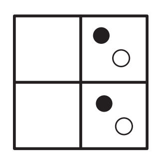
Accept all correct answers. If more than three checkers are used, ask for another way until you get the standard configuration.

T: ***Can anyone put 9 on the Minicomputer?***



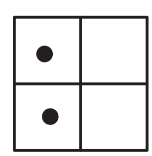
Accept all correct answers. If more than two checkers are used, ask for another way until you get the standard configuration.

Put on 5 and ask what number it is. Then put on 5 again and ask what number 5 + 5 is. (10)



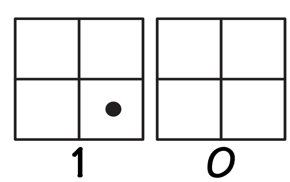
T: ***Can anyone put 10 on the Minicomputer in another way?***

At some moment someone will probably suggest this configuration.



T: ***There is a way to put 10 on the Minicomputer using only one checker, but we will need another board.***

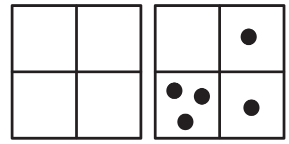
Display a second board (the tens board) to the left of the first board and place one checker on the white square of the tens board. As you do this say, “8 + 2 = 10.” Write 1 below (or above) the tens board and 0 below the ones board.



At this point you may like to use base-10 blocks or some other place-value manipulative to model a trade of 10 ones for 1 ten and 0 ones.

Exercise 2

Put some checkers on the ones board very gradually. For example:



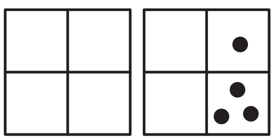
- place three checkers on the red square, pause;
- then one checker on the purple square, pause again;
- then one checker on the white square.

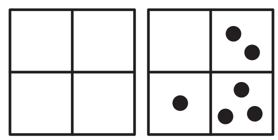
This gives your students a chance to calculate mentally.

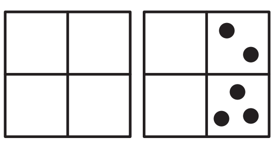
T: ***What number is on the Minicomputer?*** (11)

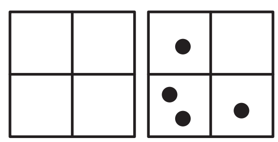
No explanation is necessary if everybody gets the right number. If someone gives the wrong number, you might use this procedure: Cover part of the board with a piece of paper to focus the class's attention on certain checkers, and gradually uncover the full set of checkers.

Repeat this activity with other examples; some possibilities are suggested below.

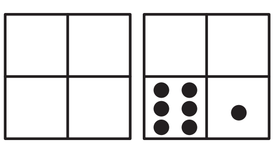
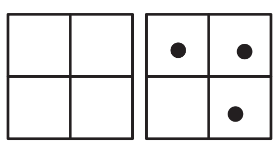
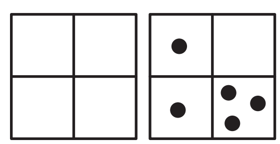
 = 7

 = 13

 = 11

 = 13

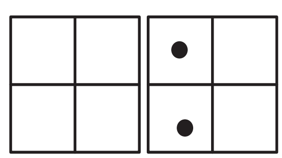
Note: It is interesting to present the same number several times, each time with a different number of checkers. For instance, 13 can be represented as

 or  or 

and so on.

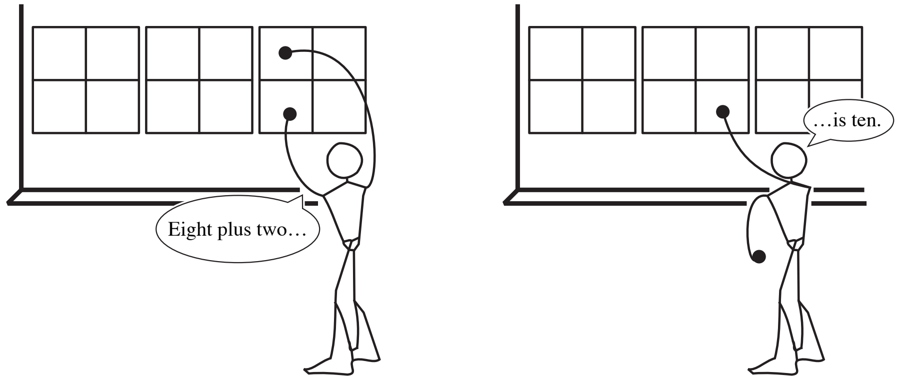
Exercise 3

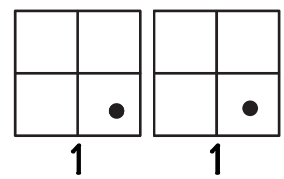
Put one checker on the 8-square and one on the 2-square.



T: ***What number is on the Minicomputer?*** (10)

Pick up the checker on the brown square with one hand and the checker on the red square with the other. Then put one of these checkers on the 10-square and take the other checker away (put it in the chalk tray). As you are making the trade, say, "8 + 2 = 10."





T: ***This is a way to put 10 on the Minicomputer with just one checker. With one more checker, can you put 11 on the Minicomputer?***

Ask students to remove the checker from the ones board.

T: ***With one more checker, can you put on 12?*** (Yes)
Can you put on 14? (Yes)
Can you put on 17? (No)
How many more checkers do you need? (At least three more)
Can you put on 18? (Yes)
19? (No)

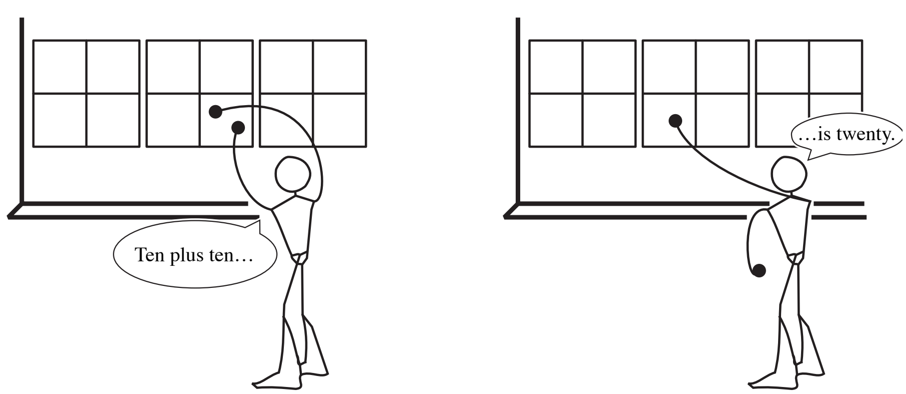
Note: The question, How many checkers do you need for 17? has many answers. For the standard configuration you need four checkers, but it is possible to put 17 on the Minicomputer with three checkers: 8 + 8 + 1 = 17. In this case, however, one checker is given on the 10-square.

In some cases, you may request a student to model a number as well with base-10 blocks or other place-value manipulatives; for example, model 14 as 1 ten and 4 ones.

Ask someone to put 20 on the Minicomputer using two checkers. If no one volunteers, put two checkers on the 10-square.

T: ***There is a way to put 20 on the Minicomputer with one checker.***

Make the trade and say, "10 + 10 = 20."

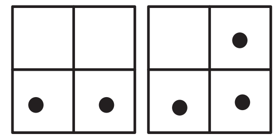


Remove the checkers and continue with other numbers.

T: ***Who can put 25 on the Minicomputer? 27? 30?***

Again you may invite students to model 25 as 2 tens and 5 ones with base-10 blocks or other place-value manipulatives.

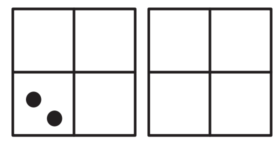
Put this configuration on the Minicomputer.



T: ***What number is this?***

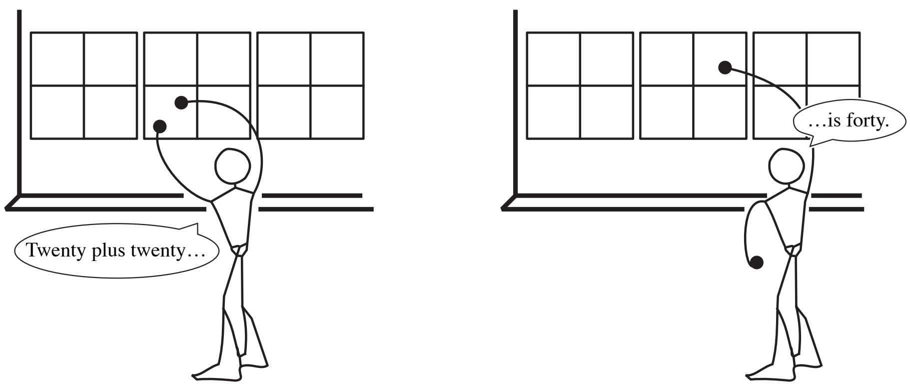
When you receive the correct answer, write 3 below the tens boards and 7 below the ones board.

Put two checkers on the 20-square.



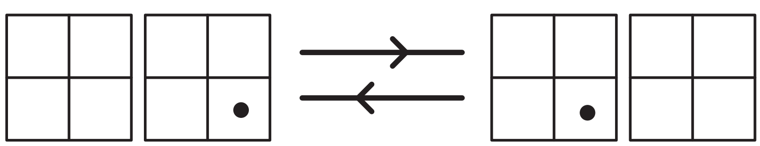
T: ***What number is this?*** (40)

Make the trade yourself and say, “20 + 20 = 40.”



Remove the checker from the 40-square.

Move a checker back and forth very quickly from the 1-square to the 10-square. Each time you move the checker, ask the class which number is on the Minicomputer: one—ten; one—ten; and so on.

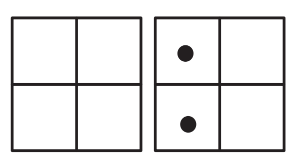


Repeat this with 2 and 20; 4 and 40; 8 and 80. You may use this as an introduction to the 80-square.

T: ***Who can put 47 on the Minicomputer? 53? 96? 32?***

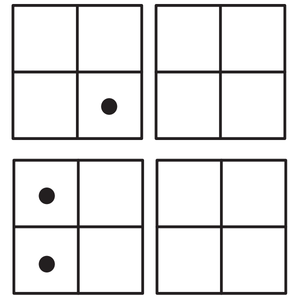
Exercise 4

Put 10 on the Minicomputer as 8 + 2.



T: ***What number is this?*** (10)

Who can put 10 on the Minicomputer with one checker?

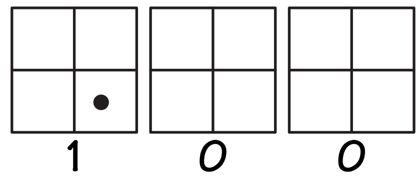


Put 100 on the Minicomputer as 80 + 20.

T: ***What number is this?*** (100)
How do you know? (80 + 20 = 100)

T: ***There is a way to put 100 on the Minicomputer with just one checker, but we need another board.***

Display the hundreds board and make the trade yourself as you say, "80 + 20 = 100." Write 100 below the Minicomputer.



You may want to model a trade of 10 tens for 1 hundred with your place-value manipulative.

T: ***Who can put 105 on the Minicomputer? 126? 157?***

Occasionally, you may wish to ask a student to write a numeral below the Minicomputer while other students model the number with their place-value manipulatives.

Move checkers quickly from one board to another to show the following:

1, 10, 100	8, 80, 800
2, 20, 200	5, 50, 500
4, 40, 400	3, 30, 300

Center Activity

Put two Minicomputer boards and checkers in a center. Let pairs of students practice putting on numbers and reading each other's numbers on the Minicomputer. Task cards made with a number on one side and its standard Minicomputer configuration on the other can be used by individual students.

EN2 THE FUNCTIONS +3 AND –3

Capsule Lesson Summary

Explore ways to put a number on the calculator when the keys for that number cannot be used. Teach a calculator to count by ones and by threes. Label the dots and draw the return arrows in a +3 arrow picture. Relate number sentences such as 7 + 3 = 10 and 10 – 3 = 7.

Materials

Teacher	• Overhead calculator • Colored chalk • Number line	Student	• Calculator • Paper

Note: Paper for students can be scratchpads, notebooks, slates, or whatever serves your classroom management and record-keeping purposes.

Description of Lesson

Display an overhead calculator (if available) and distribute calculators to individual students. Depending on your students' experience with calculators, review the parts and some features of the calculator. In particular, students should be able to turn on the calculator, enter numbers, read the display, clear the calculator, use the calculator for simple addition and subtraction problems, and use the [=] key to display the result.

Exercise 1

Tell the students you want them to pretend that a particular number key on the calculator is broken. Then ask them to display that number without using the broken key. For example:

T: ***Suppose the [8] key is broken. Try to display 8 without using the [8] key.***

S: ***Press [5] [+] [3] [=].***

S: ***Press [9] [–] [1] [=].***

S: ***Press [2] [×] [4] [=].***

Allow students to find several solutions and demonstrate their solutions on the overhead calculator.

Repeat this exercise a couple times with other broken keys.

T: ***Suppose both the [2] and the [5] keys are broken. Try to display 25.***

S: ***Press [1] [4] [+] [1] [1] [=].***

S: ***Press [9] [+] [8] [+] [7] [+] [1] [=].***

S: ***Press [3] [1] [–] [6] [=].***

S: ***Press [1] [0] [0] [÷] [4] [=].***

Again, allow students to find several solutions and demonstrate their solutions on the overhead calculator. You might not expect the same variety given here, but be open to many different solutions and encourage students' experimentation.

Exercise 2

Review (or introduce) with students how to teach a calculator to count. You may want to let students describe the counting process first. Demonstrate with the overhead calculator while students use their calculators.

T: ***We teach a calculator to count by ones by***
(1) putting on the starting number;
(2) pressing $\boxed{+}$ $\boxed{1}$***; and then***
(3) pressing $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ***and so on.***

Let students spend a few minutes making their calculators count up to "big" numbers.

T: ***How do you suppose we can teach a calculator to count by threes?***

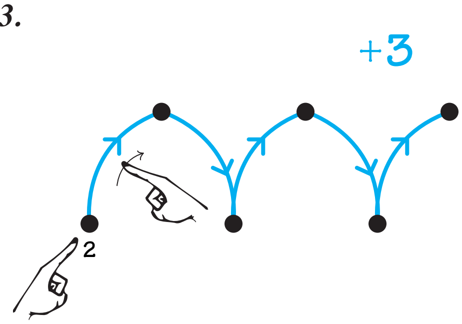
S: ***Put on the starting number.***
Press $\boxed{+}$ $\boxed{3}$.
Then press $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ***and so on.***

Spend a few minutes exploring the counting calculator. Some students may want to make their calculators count by other numbers as well.

Exercise 3

Draw the arrow picture below on the board. Put your left forefinger on the dot for 2.

T: ***This dot is for the number 2. The blue arrow is for +3.***



Trace the blue arrow starting at 2 with your right forefinger in the direction of the arrowhead as you say,

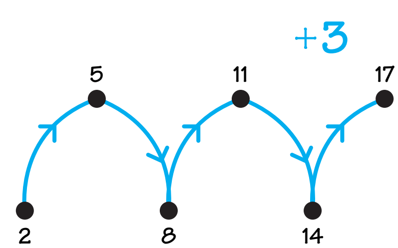
T: ***2 + 3. What number is this?*** (5)

Label the second dot 5. Put your left forefinger on the dot for 5 and trace the blue arrow starting at 5 with your right forefinger as you say,

T: ***5 + 3. What number is this?*** (8)

Label the third dot 8. Point to the unlabeled dots.

T: ***What are these numbers?***



Invite students to point to the dots as they announce the numbers. Continue until all the dots are labeled. Occasionally, you may want to write a number sentence corresponding to an arrow on the board; for example, 8 + 3 = 11.

T: ***Could we go on with more +3 arrows?*** (Yes)

Do not draw more arrows. You can trace more arrows if you like.

T: ***If I keep drawing arrows, do you think we will ever meet the number 20?*** (Yes)

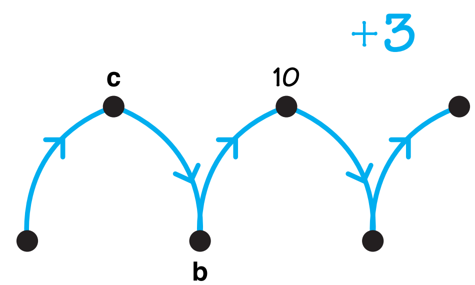
If several students respond correctly but many are uncertain, trace an imaginary arrow and ask the students which number comes next in the picture. Ask if you will ever meet the number 25. (No) How about 30? (No) Encourage students to explain why they think yes or no to such questions.

At this time you may like to let students use their calculators, counting by threes, to follow the arrow picture and to help answer such questions. Students may also use the number line to explain answers to questions about an extended arrow picture. That is, the arrow picture starts at 2 and makes "jumps" of 3 (+3 arrows). It lands on 20 but jumps over 25 and 30.

Exercise 4

Erase the numerals but not the dots or arrows.
Label the fourth dot from the left 10.

Point to the dot labeled **b** in the illustration.

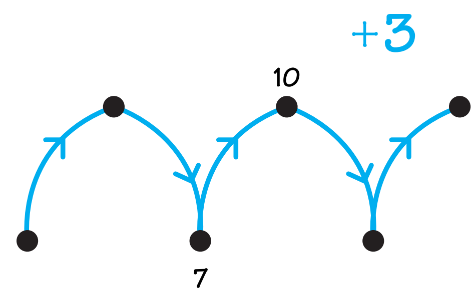


Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: ***What number is here?***

Invite students to whisper their answers to you or to write on the paper for you to check†. Then ask a student to answer aloud.

Label the dot.



T: ***How do you know this is 7?***

S: ***7 + 3 = 10.***

Write the number sentence on the board.

T: ***Does someone have another way to see that this is 7?***

S: ***10 – 3 = 7.***

It may be necessary to give the 10 – 3 = 7 observation yourself.
Write the number sentence under 7 + 3 = 10 on the board.

7 + 3 = 10
10 – 3 = 7

T: ***We also know that this number is 7 because 10 – 3 = 7. We could draw a red arrow for –3.***

Put your left forefinger on the dot for 10. Trace an arrow (starting at 10 and ending at 7) with your right forefinger as you say, "10 – 3 = 7."

Draw the return arrow in red from 10 to 7 and write –3 in red near the arrow picture.

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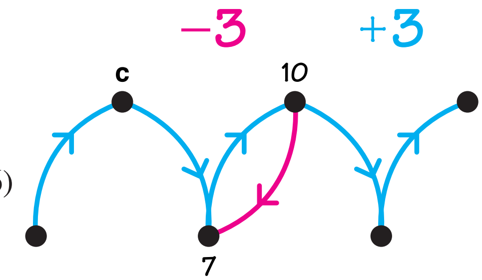
†Often in☐
respond. If a☐
answer is correct because you want students to continue to think about the situation.

Point to the dot labeled **c** in this illustration.



Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

T: ***What number is here?***

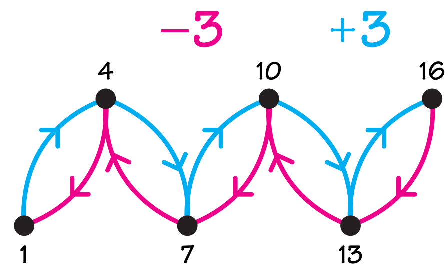
S: ***4, because 7 – 3 = 4 (or 4 + 3 = 7).***

T: ***What are the other numbers in the picture?*** (1, 13, 16)

Quickly label the remaining dots.

T: ***Could we draw more –3 arrows in this picture?*** (Yes)

Ask students to draw the red arrows. After each red arrow is drawn, point to its starting number and trace the arrow to its ending number while you say, for example, "13 – 3 = 10." Continue until the arrow picture is complete.



T: ***Could we use the calculator to check our work?***

S: ***Yes, we could make it count backward by threes from 16 to see if we meet each dot on the arrow road.***

T: ***Tell me how to do this.***

S: ***Put 16 on the display. Press*** $\boxed{-}$ $\boxed{3}$ $\boxed{=}$***. It will show 13. Continue to press*** $\boxed{=}$ ***and check each number.***

Home Activity

Suggest to parents/guardians that they ask their child to show them how to teach a calculator to count forward and backward by threes. With such a counting calculator they can predict which number will come next or which number they will see after pressing $\boxed{=}$ two more times.

Send a letter explaining home activities along with the first home activity you send to parents/guardians. A sample letter can be found at the front of the Home Activities sections of the *UPG-III* Blacklines.

EN3 MINICOMPUTER INTRODUCTION #2

Capsule Lesson Summary

Review the value of the squares on the Minicomputer. Practice reading numbers on the Minicomputer. Identify some Minicomputer trades and practice making trades.

Materials

Teacher
- Minicomputer set
- Base-10 blocks or other place-value manipulatives

Student
- Individual Minicomputer set†

- Base-10 blocks or other place-value manipulatives
- Calculator
- Paper
- Worksheets EN3*, **, ***, and ****

Advance Preparation: Use the blacklines following this lesson to make copies of student worksheets.

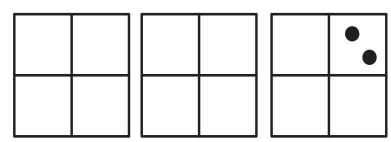
Description of Lesson

Exercise 1

Display three Minicomputer boards and review the values of the squares as in Lesson EN1. Also review trades on the ones board; i.e., 1 + 1 = 2, 2 + 2 = 4 and 4 + 4 = 8.

Exercise 2

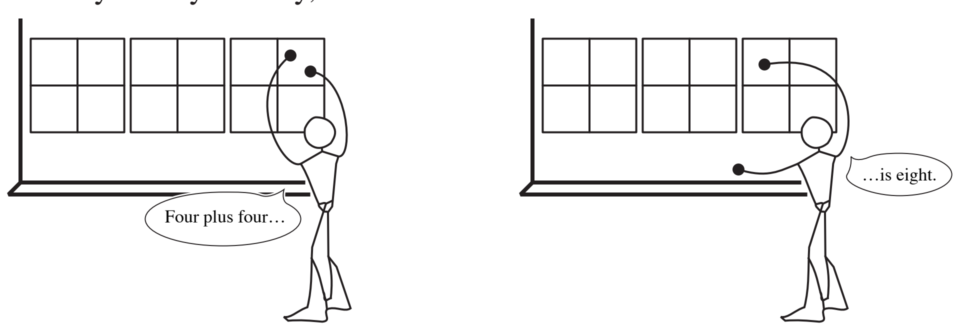
Put two checkers on the 4-square of the Minicomputer.



T: ***What number is this?*** (8)

T: ***Can someone put 8 on the Minicomputer with just one checker?***

Repeat the trade very clearly and say, "4 + 4 = 8 and 8 = 4 + 4."

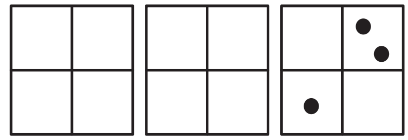


Similarly, demonstrate the following trades:

$$40 + 40 = 80 \quad \text{and} \quad 80 = 40 + 40$$
$$100 + 100 = 200 \quad \text{and} \quad 200 = 100 + 100$$
$$200 + 200 = 400 \quad \text{and} \quad 400 = 200 + 200$$
$$8 + 2 = 10 \quad \text{and} \quad 10 = 8 + 2$$

†A student's Minicomputer set consists of two sheets of Minicomputer boards (two boards per sheet) and cardboard checkers.

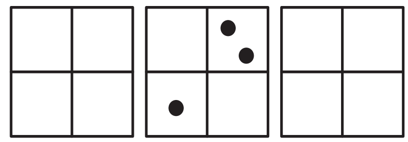
Put this configuration on the Minicomputer.



T: ***What number is this?*** (10)

Invite students to make trades until the standard configuration for 10 is on the Minicomputer. Students should name trades before and as they move checkers.

Repeat the exercise with this configuration.



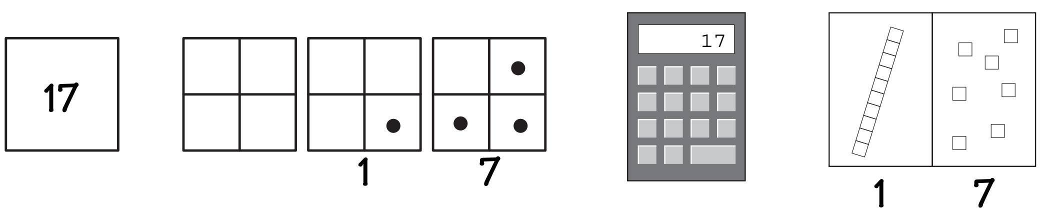
T: ***What number is this?*** (100)
What do you notice?

Encourage the students to comment on this new situation and to explain why the number is 100 before asking for volunteers to make trades.

Exercise 3

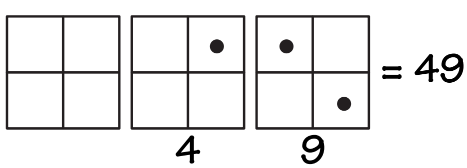
Place students in groups of four with a Minicomputer set, base-10 blocks or other place-value manipulative, a calculator, and paper and pencil. This activity should move quickly. Choose numbers appropriate for your class and encourage all students to participate.

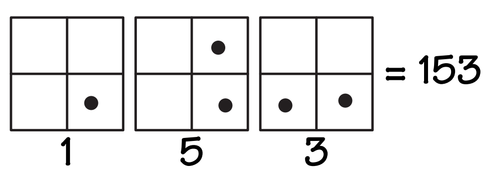
T: ***Each group is going to show the number 17 in several ways. One person will write 17 on paper, another person will put 17 on the Minicomputer, still another person will put 17 on the calculator, and finally one person will show 17 with the base-10 blocks.***

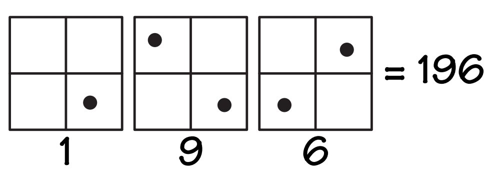


Check to see that each group has 17 represented in all four ways. Then let some students share and explain. Continue by directing students to switch jobs within their groups.

Repeat this activity with other numbers such as those illustrated (on the Minicomputer) below.

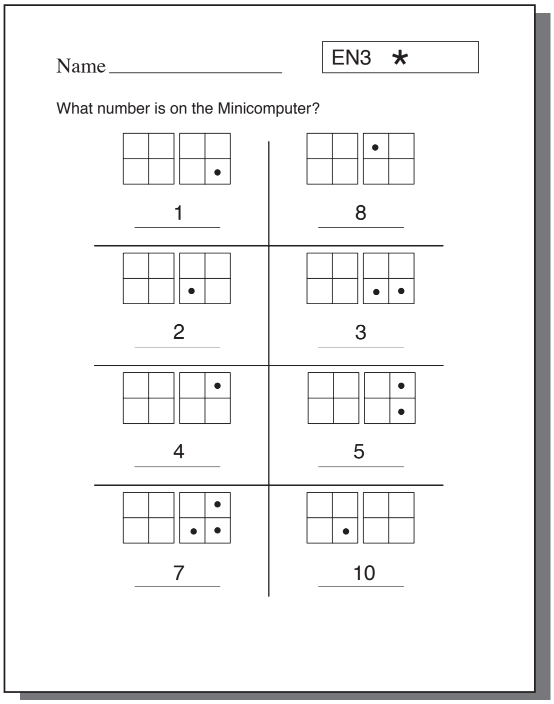






You may like to change the activity so that instead of reading a number to the groups, you put the number on the Minicomputer and the groups read the number. The person with the Minicomputer can copy the configuration you display.

Assessment Activity


Name

EN3 *

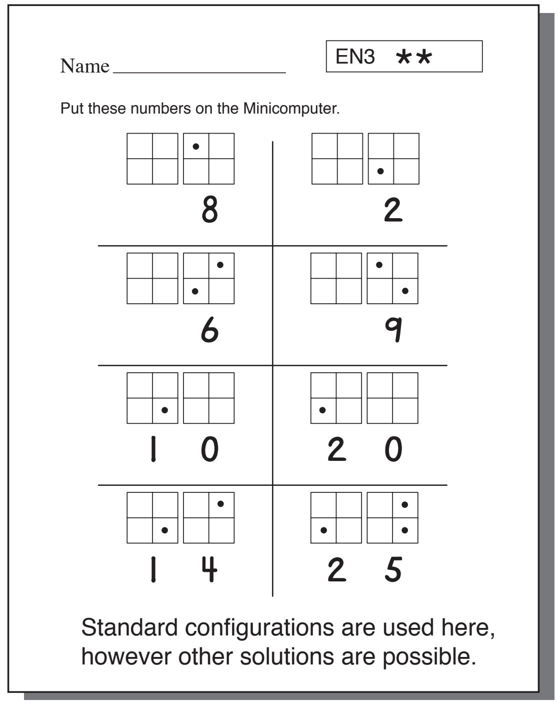
What number is on the Minicomputer?

1 8

2 3

4 5

7 10


Name

EN3 **

Put these numbers on the Minicomputer.

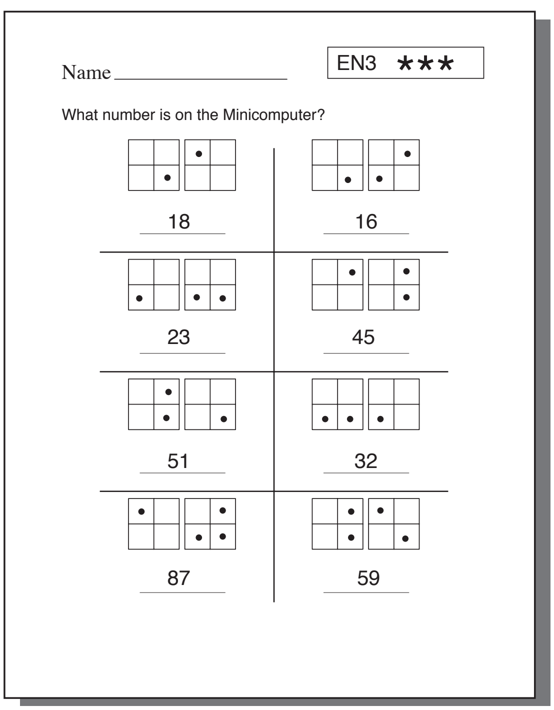
8 2

6 9

1 0 2 0

1 4 2 5

Standard configurations are used here, however other solutions are possible.


Name

EN3 ***

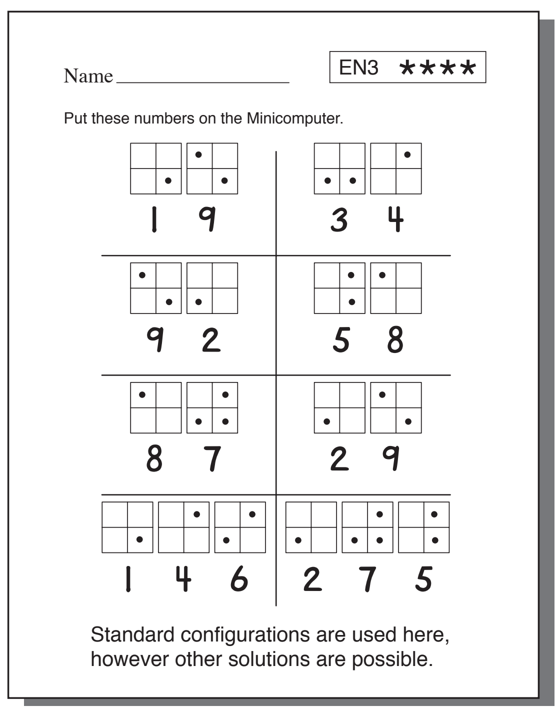
What number is on the Minicomputer?

18 16

23 45

51 32

87 59


Name

EN3 ****

Put these numbers on the Minicomputer.

1 9 3 4

9 2 5 8

8 7 2 9

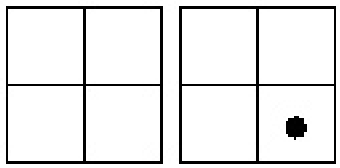
1 4 6 2 7 5

Standard configurations are used here, however other solutions are possible.

Name ______________________

EN3 *

What number is on the Minicomputer?



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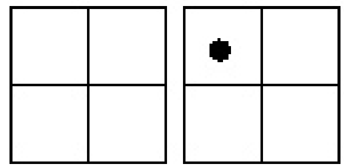

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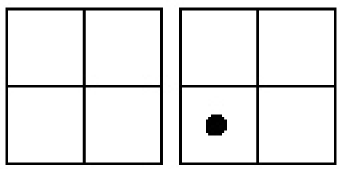

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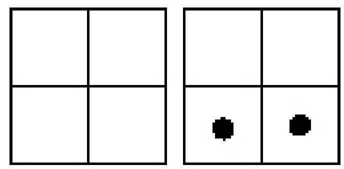

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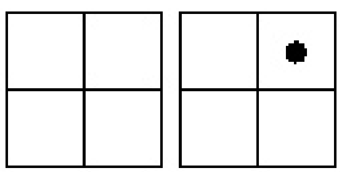

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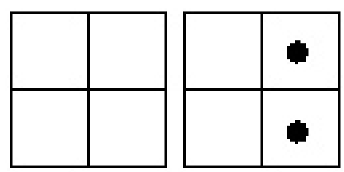

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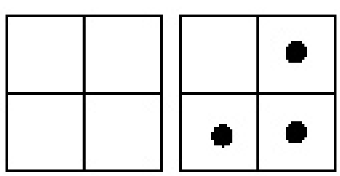

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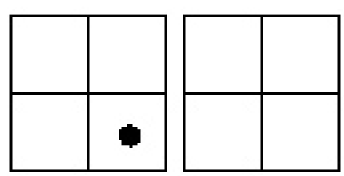


____

Name ______________________

EN3 **

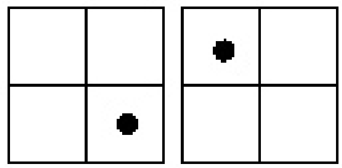
Put these numbers on the Minicomputer.

8	2
6	9
1 0	2 0
1 4	2 5

Name ____________________

EN3 ★★★

What number is on the Minicomputer?



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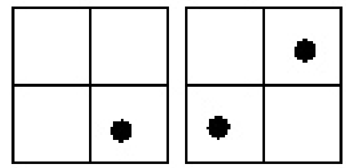

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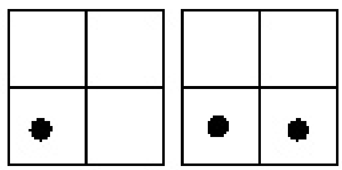

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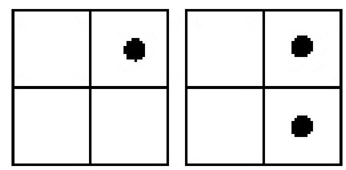

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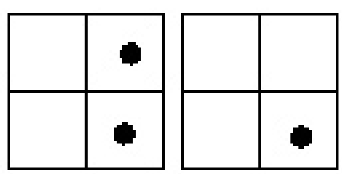

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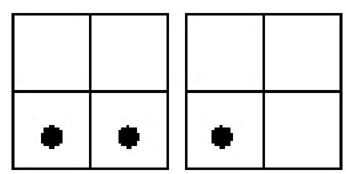

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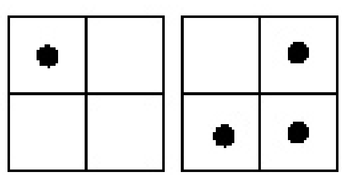

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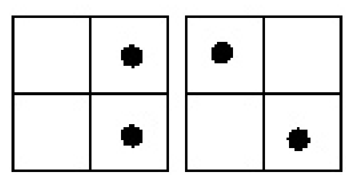

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Name ____________________

EN3 ★★★★

Put these numbers on the Minicomputer.

1 9	3 4
9 2	5 8
8 7	2 9
1 4 6	2 7 5

Capsule Lesson Summary

Build an arrow road from one number to another using two types of arrows.

Materials

Teacher		Student	
	• Colored chalk		• Colored pencils, pens, or crayons • Unlined paper • Unifix® cubes

Description of Lesson

Exercise 1

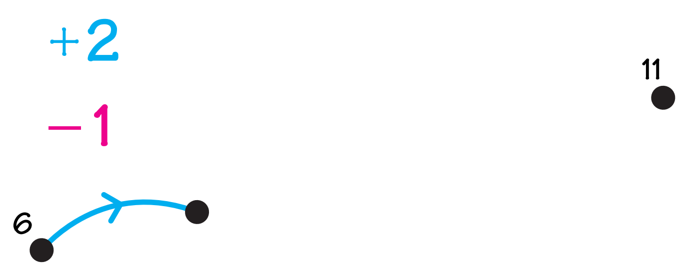
Draw two dots well-spaced on the board. Label one of the dots 6 and the other 11.

T: ***Today we are going to build arrow roads. Let's begin by building a road from 6 to 11 using blue arrows for +2 and red arrows for –1.***

Write +2 in blue and –1 in red on the board. You might refer to these as the "key" or "color code" for this road.

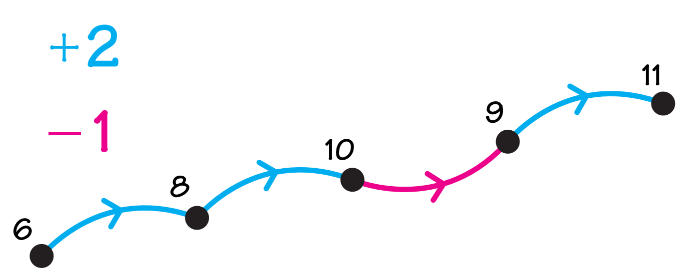
T: ***Which kind of arrow would you like to start with?***

Take whatever suggestion is made (for example, +2) and draw the first arrow yourself. Point to the ending dot of this arrow.



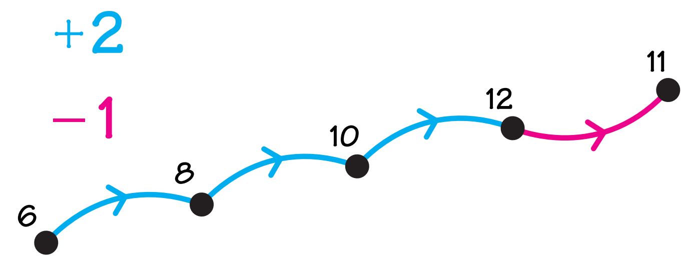
T: ***Which number is here?*** (8, if the arrow is blue; 5, if the arrow is red)

Ask students to complete the road one arrow at a time. One of the many possible roads is shown below.



Note: If a student is concentrating on a color pattern for the arrows, you might encourage him or her to keep in mind what the target number is. In this particular example, alternating +2 and –1 arrows will build a road from 6 to 11 using seven arrows (four +2 and three –1) or ten arrows (five –1 and five +2). There is nothing wrong with such a road, but you might ask if it is possible to build a road using fewer arrows. In this case, there is a shorter road as illustrated.

There is no rule against "overshooting" the target number. For example, here is a perfectly acceptable road.



Construct two or three different roads from 6 to 11 with suggestions from the class. The purpose of this collective exercise is to show what an arrow road is and that many different solutions are possible.

Exercise 2

Distribute paper and colored pencils, and put the following information on the board.

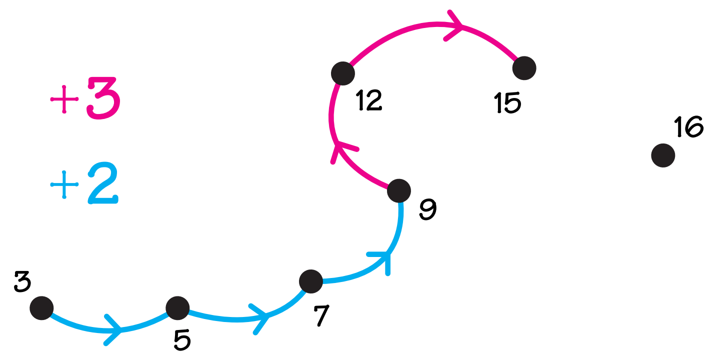
Ask the students to copy what is on the board, and to build a road from 6 to 21 using +2 and +1 arrows. Students who draw cramped pictures should be encouraged to use the whole sheet of paper.

This might be a good problem for students to work with a partner. To support the activity with a manipulative, give the student partners Unifix® cubes grouped in ones (red cubes) and twos (blue cubes). Start with a tower of six cubes. One student then either adds one red cube or two blue cubes to the tower while the partner records the addition with a corresponding arrow. The end product should be a tower of 21 cubes and an arrow road from 6 to 21.

As the rest of the class continues to work, ask a couple of students with different numbers of arrows in their roads to copy their pictures on the board. When most of the students have finished, direct the class's attention to the examples on the board. Check each picture on the board with the class to see that it is correct. Point out that there are many correct solutions to this problem. Afterward, ask which road on the board has the fewest arrows and determine if anyone has a road with even fewer arrows.

Repeat this activity, building a road from 3 to 16 using +3 and +2 arrows.

Note: A student might draw this road and discover that one can not reach 16 from 15 using only +3 and +2 arrows.



In such a case, you may need to suggest changing the color for the last arrow. In the preceding picture, changing the arrow starting at 12 from red (+3) to blue (+2) would enable the student to complete a road with one additional blue arrow.

If there is time remaining, the students can build one or more of the following:

1) Build an arrow road from 17 to 30 using +1 (red) and +2 (blue) arrows.
2) Build an arrow road from 8 to 19 using +3 (red) and –2 (blue) arrows.
3) Build an arrow road from 10 back to 10 using +3 (red) and –2 (blue) arrows.
4) Build an arrow road from 27 back to 27 using +3 (red) and –2 (blue) arrows.

Building these arrow roads can be made into a game for partners. For example, in Road 1 the students start by drawing and labeling a dot for 17. Then they take turns choosing a +1 or +2 arrow to put into the arrow road picture. Each time a student draws an arrow he or she also labels the ending dot. The next arrow, of course, must start at that ending dot. The student who puts in the arrow that ends at 30 (the target number) wins.

EN5 MINICOMPUTER INTRODUCTION #3

Capsule Lesson Summary

Represent a sum on the Minicomputer by the use of checkers of two different colors. Explore the effect of adding a board to the right. Practice making trades on the Minicomputer and present a situation where a backward trade is needed to get the standard configuration of a number. Find many ways to represent a given number on the Minicomputer.

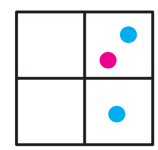
Materials

Teacher		Student	
	• Minicomputer set		• Individual Minicomputer set

Description of Lesson

Exercise 1

Display one Minicomputer board with this configuration of checkers. (Use two different colors for the checkers.)



T: ***What number is on the Minicomputer?***

S: ***9.***

T: ***How do you see 9?***

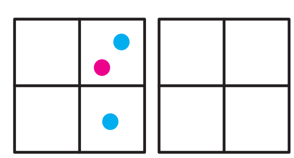
Let students make suggestions. If no one suggests that 9 can be seen as 4 (in red) plus 5 (in blue), ask,

T: ***What number do you see in red?*** (4)
What number do you see in blue? (5)

Write 4 + 5 = 9 on the chalkboard.

T: ***Suppose we counted some things, red cars and blue cars, for example. We can show this on the Minicomputer. Then we can put words in the number sentence and read, "Four cars plus five cars equals nine cars."***

Accept a couple more such examples and write corresponding sentences on the board. Before continuing, erase these sentences leaving only 4 + 5 = 9 on the board. Without removing the checkers, add a second Minicomputer board to the right.



T: ***Now, what number is on the Minicomputer?***

S: ***90.***

T: ***How do you see 90?***

S: ***40 in red plus 50 in blue.***

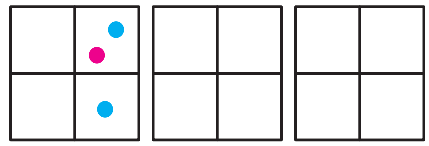
Write 40 + 50 = 90 on the chalkboard below 4 + 5 = 9.

T: ***What things could the 40 and 50 be for?***

S: ***40 red marbles and 50 blue marbles.,***

Write 40 marbles + 50 marbles = 90 marbles on the board. Accept a couple of other suggestions, and write the corresponding sentences on the board. Erase these sentences before continuing.

Add a third board to the right.



T: ***What number is on the Minicomputer now?***

S: ***900.***

T: ***How do you see 900?***

S: ***400 in red plus 500 in blue.***

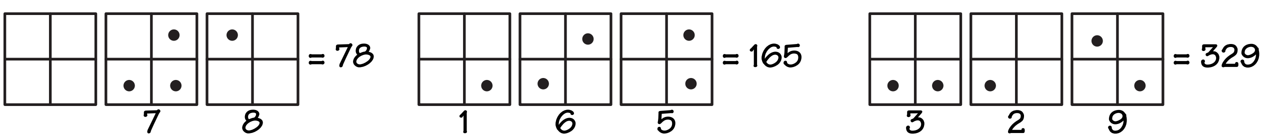
Write 400 + 500 = 900 on the chalkboard below 40 + 50 = 90. Again, allow the class to suggest what things the 400 and 500 could be for and write the corresponding sentences.

If your class seems to enjoy this exercise, you may go on adding boards and asking students to write the calculations:

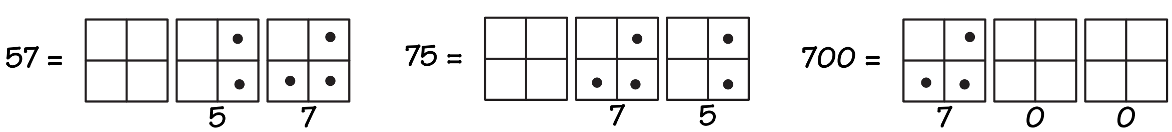
$$4 + 5 = 9$$
$$40 + 50 = 90$$
$$400 + 500 = 900$$
$$4{,}000 + 5{,}000 = 9{,}000$$

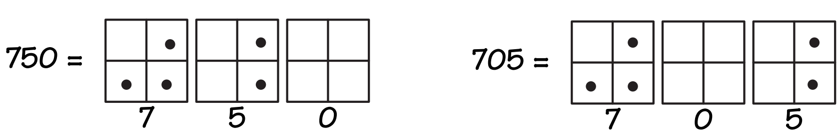
Exercise 2

Display three Minicomputer boards. Ask the students to read various numbers as you put them on the boards and to write the numerals, centering each digit below the appropriate board. Occasionally, write the numeral to the right of the boards as well.



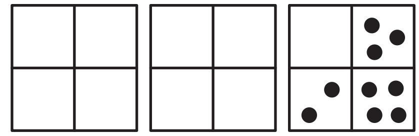
Vary the activity by asking the students to put various numbers on the Minicomputer and, again, to write the numerals below the boards. A good sequence of numbers to ask for is suggested here. Make your request by writing the numeral to the left of the boards.





Exercise 3

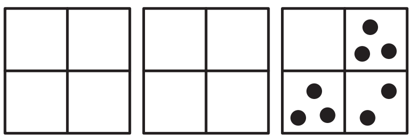
Display three Minicomputer boards. Put checkers on the Minicomputer gradually, allowing the students to calculate the number mentally.



T: ***What number is on the Minicomputer?*** (20)
Can you put 20 on the Minicomputer with one less checker (that is, with eight checkers)?

If the students do not respond, suggest that they make a trade. Any forward trade will solve the problem. For example:

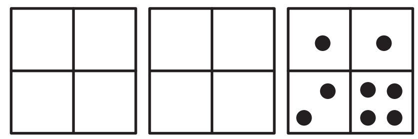
Trade A: 1 + 1 = 2



= 20

or

Trade B: 4 + 4 = 8

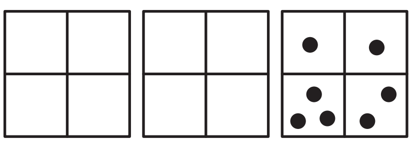


= 20

Other solutions are also possible.

T: ***This is 20 with eight checkers. Can you put 20 on the Minicomputer with seven checkers?***

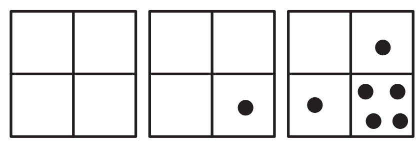
After Trade A (above)
trade: 4 + 4 = 8



= 20

or

After Trade B (above)
trade: 8 + 2 = 10



= 20

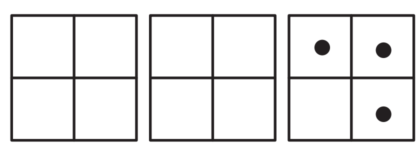
Other solutions are also possible.

T: ***Can you put 20 on the Minicomputer with one less checker (that is, with six checkers)?***

Do not overextend this activity, but return to similar exercises from time to time to review trades on the Minicomputer.

Exercise 4

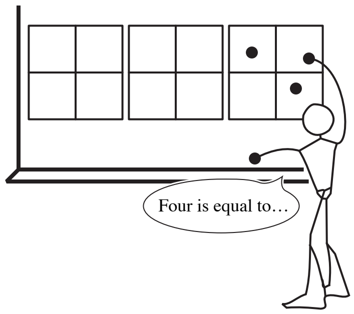
Put this configuration on the Minicomputer.

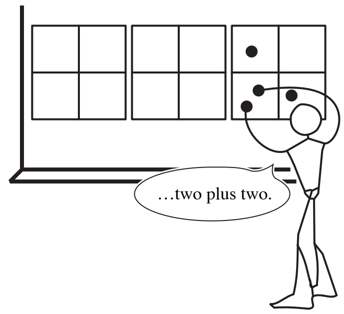


T: ***What number is this?*** (13)
Without changing the number, I want to get a checker on the red square so we can make the jump to the tens board. What trade should we make?[†]

If no one suggests a 4 = 2 + 2 trade, mention it yourself and make the trade.

[†]It is possible that a student will suggest an 8 + 4 = 10 + 2 trade. Although such trades are not emphasized, they should not be disc□
indicate that it is correct, but that you would like to see how to use the 4 = 2 + 2 trade.

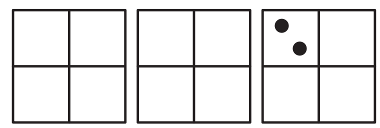




T: ***Who can make a trade so we will have a checker on the tens board?***

Ask the students to name the trade they intend to make before and as they move checkers. Do not accept the 2 + 2 = 4 trade since you have asked for a trade which results in a checker on the tens board.

Repeat the exercise with this configuration.



Note: Backward trades may be difficult for your students at this time. There will be many opportunities to work on these in the future.

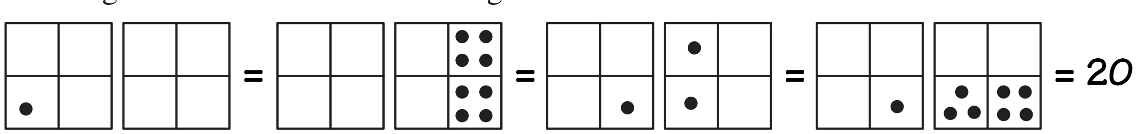
Exercise 5

Distribute individual Minicomputers and checkers to pairs of students. Each pair will need two Minicomputer boards (one sheet) and at least twenty positive checkers.

T: ***There are many ways to put 20 on the Minicomputer.***
Try to show some of them.

To be enjoyable, this exercise must move quickly. As students discover different configurations for 20, let them put their solutions on the demonstration Minicomputer. Do not attempt to get all the possible solutions; there are too many of them.

Possible configurations include the following:



If there is time remaining, you can repeat this exercise with other numbers (for example, 50, 100, 24, 35, 80, and so on).

Capsule Lesson Summary

Practice counting by fives, and relate counting by fives to money and telling time. Collectively, complete an arrow picture with +5 arrows and another with –5 arrows, making use of return arrows. Individually, draw another arrow picture using +5 and –5 arrows. Observe a digits pattern when adding fives to a number.

Materials

Teacher
- Colored chalk
- Calculator
- Number line

Student
- Colored pencils, pens, or crayons
- Unlined paper

Description of Lesson

Exercise 1

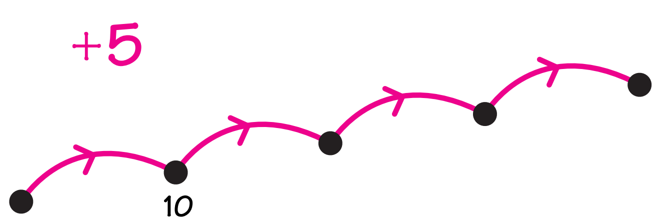
Start the lesson with some practice in counting by fives.

- Ask five children to stand and, one at a time, to hold up each hand, counting by fives to 50.
- Count how much money is in a collection of nickels.
- Look at a clock and count the minutes in an hour by fives.
- Ask students to tell the class how many cars or bikes there are in their families while you keep a running total on the board with tally marks ( ). Count by fives to find out how many cars or bikes the class's families have.

Note: Blackline L14 (*UPG-III*) is a clockface to use for counting the minutes in an hour (or telling time in five-minute intervals).

Exercise 2

Draw this arrow picture on the board.



T: ***Where is the least number in this arrow picture?***
Who can come to the board and point to the dot for the least number?
What is the least number in this picture? Show me with your fingers.

S (showing five fingers)**:** ***5.***

T: ***Who can point to the dot for the greatest number. What number is it?*** (25)

Instruct students to write the number on paper for you to check or allow them to whisper the number to you. Then ask someone to give the answer aloud.

Point to the dot to the right of 10.

T: ***What number is here? How do you know?***

S: ***15, because 10 + 5 = 15.***

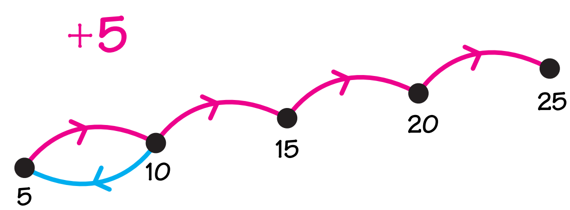
Invite a student to label this dot 15. Continue to label the other two dots to the right in this way. Then point to the first dot (on the left).

T: ***What number is here? How do you know?***

S: ***5, because 10 – 5 = 5 (or 5 + 5 = 10).***

Solicit both explanations and write the two number sentences on the board.

Draw a return arrow in blue from 10 to 5.



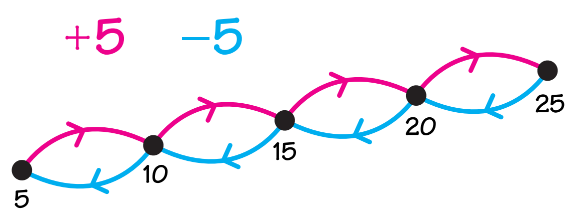
T: ***What could the blue arrow be for?***

S: ***–5.***

Write –5 in blue near the arrow picture. As you trace the arrow from 10 to 5, say,

T: ***10 – 5 = 5. Could we draw more –5 arrows in this picture?***

Complete the picture quickly with your class.



T: ***If we go on drawing +5 arrows (to the righ***
will we ever meet 31? (No)
Will we meet 50? (Yes) ***63?*** (No)
How do you know?

Accept any response that is reasonably correct. It may be difficult for students to verbalize their ideas. The number line may help to explain answers to questions about an extended arrow picture. That is, the arrow picture starts at 5 and makes "jumps" of 5 (+5 arrows). It lands on 50 but jumps over 31 and 63.

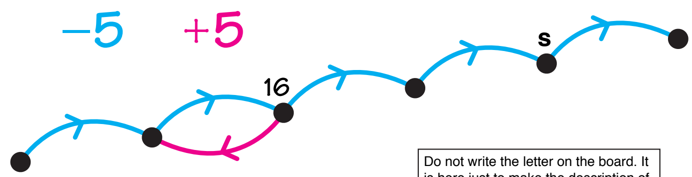
You may like to remind students of the counting calculator. First, teach the calculator to count by fives by

(1) putting on the starting number;
(2) pressing $\boxed{+}$ $\boxed{5}$; and then
(3) pressing $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on.

Then use the counting calculator to follow the +5 arrows and to check which numbers an extended arrow picture would meet.

Exercise 3

Erase the board and draw this arrow picture.



Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

T: ***What are the blue arrows for?*** (–5)
Where is the least number in this arrow picture?
Where is the greatest number in this arrow picture?

Point to the dot labeled **s** in the illustration.

T: ***What number is here?*** (6)

Instruct students to write the number on paper or take whispers. Then ask someone to answer aloud.

T: ***Who would like to label a dot?***

Occasionally, ask students to explain how they know which numbers correspond to particular dots. Encourage students to use return arrows when they are useful and to draw any missing return arrows to complete the picture.

T: ***If we go on drawing more +5 arrows (to the left), do you think we will ever meet 50?*** (No) ***How do you know?***
What do you notice about the numbers on the arrow road?

S: ***They all have the digit 1 or 6 in the ones place.***

Encourage students to observe this pattern in the numbers the arrow road would meet.

Again, you may like to use the counting calculator (count by fives starting at 1) to check which numbers an extended arrow picture would meet.

Erase the board and distribute unlined paper. Ask the students to draw their own arrow pictures using +5 arrows. Allow them to begin with a number of their own choosing and encourage them to continue as long as possible. Students who quickly finish a rather extensive picture can be asked to draw all the –5 arrows in their pictures.

Home Activity

Suggest that parents/guardians find opportunities to count by fives with their child.

EN7 MINICOMPUTER INTRODUCTION #4

Capsule Lesson Summary

Use the Minicomputer to solve addition, subtraction, and multiplication problems.

Materials

Teacher
- Minicomputer set

Student
- Paper
- Individual Minicomputer set

Description of Lesson

Exercise 1: Addition Problems

Write this problem on the board.

28 + 16 =

T: ***Can you think of a story problem for which we need to do this calculation?***

Allow students to compose (write) a story problem with a math partner. Accept a couple example story problems, but don't allow anyone to give an answer to the calculation at this moment. You may like to give students manipulatives to help them compose problems.

When students respond to the following estimation questions, ask them to explain their answers.

Note: Some students may know 28 + 16 = 44. Accept this knowledge but discuss why you may want an estimate rather than an exact answer. Estimates are sometimes what we use to compare numbers or to check that our results are reasonable.

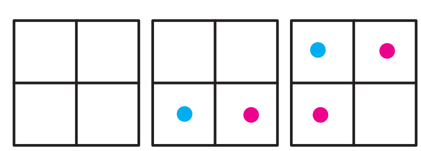
T: ***Do you think 28 + 16 is more or less than 20?*** (More)
More or less than 30? (More) ***More or less than 50?*** (Less)
We know that 28 + 16 is between 30 and 50. Would it be more or less than 40?

If the students disagree, say, "You don't know if it is more or less than 40, but you do know it is between 30 and 50." If the class is certain that 28 + 16 is more than 40, say, "Now you know it is between 40 and 50."

T: ***What number is 28 + 16?***

Accept several guesses and list them on the board. You can insist that guesses be between 30 and 50, (or between 40 and 50 if this has been determined) and so remind any student who gives an inappropriate guess.

T: ***Who would like to put 28 on the Minicomputer using blue checkers?***
Who would like to put 16 on the Minicomputer using red checkers?



Invite students to make trades so that the number will be easier to read. When all the trades have been made, call on a student to write 44 below the Minicomputer and conclude that 28 + 16 = 44.

Point to the list of estimates on the board. Acknowledge anyone who correctly predicted the answer. If no one guessed correctly, determine which estimate was the best. Indicate whether certain estimates are more or less than the sum. For example, if you had 42 and 45 as estimates, you would indicate that 42 is less than 44, and that 45 is more than 44. However, 45 is the best estimate because it is only 1 more than 44.

40
$42 < 44$
$45 > 44$
46

Note: Use the symbols > and < throughout the year whenever you have made a list of estimates and wish to decide which is the best estimate. These symbols are introduced in the *CSMP* first grade program. Most entry students will have been introduced to these symbols in other math programs. There is a short supplemental exercise following this lesson for classes with little or no experience with these symbols.

Repeat this exercise with 23 + 48 = 71.

Exercise 2: Subtraction Problems

First pose a story problem for subtraction. For example:

T: ***One day I opened a package from a friend and found 37 bright orange rocks. I gave 14 of them away. How many did I have to keep? What do you think I should do with them?***

Write this problem on the board.

37 – 14 =

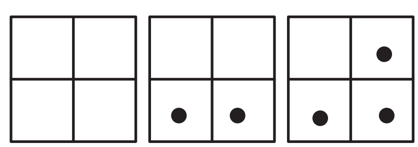
When students respond to these estimation questions, ask them to explain their answers.

T: ***Do you think 37 – 14 is more or less than 30?*** (Less)
More or less than 20? (More) ***We know that 37 – 14 is between 20 and 30. What number do you think it is?***

Accept several guesses and list them on the board. If you get a guess that is not between 20 and 30, remind the class that they already know 37 – 14 is between 20 and 30 and do not record that guess.

T: ***Who can put 37 on the Minicomputer?***
What should we do next?

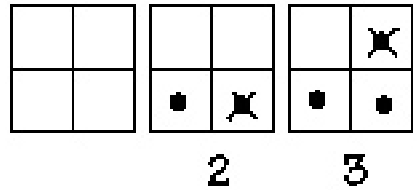
S: ***Take away 14.***



Note: When students are asked to put numbers on the Minicomputer, they may or may not use standard configurations for the numbers. The lesson description assumes that standard configurations of numbers are put on the Minicomputer. Adjust the lesson depending upon which configurations your students display.

A student should remove a checker from the 10-square and a checker from the 4-square.

An "X" through a checker indicates that the checker has been removed from the Minicomputer.



Ask a student to write 23 below the Minicomputer and conclude that 37 – 14 = 23.

Repeat this exercise with 53 – 42 = 11 and 76 – 34 = 42. As in Exercise 1, give math partners a few minutes to compose (write) a story problem for which you need to do this calculation. Let students share some of their stories.

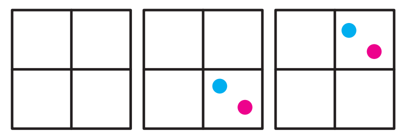
Exercise 3: Multiplication Problems

Write this problem on the board.

2 x 14 =

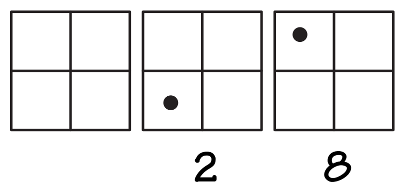
Ask each group to compose (write) a story problem for which you need to do this calculation. You may need to remind the students that 2 x 14 means 14 + 14. Let several groups share their story problems with the class.

Display three Minicomputer boards and ask,



T: ***Who would like to put 14 on the Minicomputer with blue checkers?***
Who would like to put 14 on the Minicomputer again with red checkers?

What number is on the Minicomputer? (2 x 14, or 28)
Who can make some trades so it will be easier to read?



Conclude that 2 x 14 = 28.

Repeat this exercise with one or two more multiplication problems such as 2 x 23 and 3 x 12.

Writing Activity

Ask students to write an addition, subtraction, or multiplication story problem of their choice and to also write how to solve their problem. Allow them to use manipulatives, pictures, or the Minicomputer.

EN7

INTRODUCTION TO < and > (for entry classes)

Tell the following story making it as imaginative as possible.

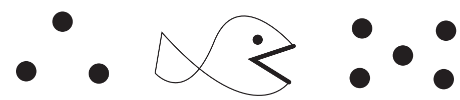
T: ***Goldy is a very large fish who lives in a large lake with smaller fish. The smaller fish swim together in "schools." Being such a large fish, Goldy gets very hungry and eats the small fish.***

Draw this picture of dots on the chalkboard.

T: ***The dots are for fish. How many fish are in this school*** (point to the dots on the left)***?*** (Three) ***How many are in this school*** (point to the dots on the right)***?*** (Five) ***If Goldy swims toward one school of fish, the other school swims away. So, if you were Goldy and very hungry, which school would you go after?***

Direct the responses so that the school with more fish is always chosen. Students should compare the numbers of fish (dots) in the two schools.

T: ***We'll show that Goldy goes after the school with five fish.***

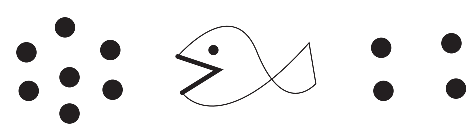


Draw Goldy between the schools of fish. Goldy's mouth should stand out.

Now draw this picture.

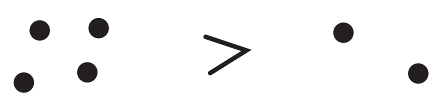
T: ***How many fish are in this school*** (point to the dots on the left)***?*** (Seven)
How many are in this school (point to the dots on the right)***?*** (Four)
Who would like to show us which school of fish Goldy will go after?

Draw Goldy, making sure the open mouth suggests that Goldy is ready to eat the larger school.



After a few examples, tell the students that from now on you are just going to draw Goldy's mouth.

Do several examples like the following, each time inviting a student to draw in Goldy's mouth.



Announce that, from now on, instead of drawing dots for fish, you will write a numeral to show how many fish there are in each school. For example, write 7 and 5 with space between them.

$$7 > 5$$

T: ***How many fish are in this school*** (point to 7)***?*** (Seven)
How many fish are in this school (point to 5)***?*** (Five)
Who can draw Goldy's mouth to show the school of fish Goldy would go after?
Yes, Goldy goes after the school with more fish. 7 is more than 5.

Do several examples asking students to read the number sentence each time. Then introduce = in an example comparing 4 and 3 + 1.

T: ***Goldy can't decide which school to go after, so we write this:***

$$4 = 3 + 1$$

EN8 THE FUNCTIONS 2X AND $\frac{1}{2}$X

Capsule Lesson Summary

Conduct a mental arithmetic activity involving 2x and $\frac{1}{2}$x. Relate the functions 2x and $\frac{1}{2}$x through the use of a simple arrow diagram: the arrow for one is the return arrow for the other. Label the dots in a 2x arrow picture with the help of the Minicomputer, and then draw the return arrows.

Materials

Teacher
- Colored chalk
- Minicomputer set
- Calculator
- Base-10 blocks (optional)

Student
- Unlined paper
- Colored pencils
- Minicomputer set (optional)

Description of Lesson

Exercise 1: Mental Arithmetic

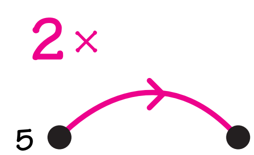
Conduct a brisk mental arithmetic activity with facts similar to the following:

10 + 10	50 + 50	30 + 30	300 + 300	25 + 25
2 x 10	2 x 50	2 x 30	2 x 300	2 x 25
$\frac{1}{2}$ x 20	$\frac{1}{2}$ x 100	$\frac{1}{2}$ x 60	$\frac{1}{2}$ x 600	$\frac{1}{2}$ x 50
20 ÷ 2	100 ÷ 2	60 ÷ 2	600 ÷ 2	50 ÷ 2

Note: $\frac{1}{2}$x should be read "one half of "; for example, $\frac{1}{2}$ x 20 is read "one half of twenty."

Exercise 2

Draw this arrow picture on the board.



Point to the dot on the right.

T: ***What number is this?***

S: ***10.***

Label the dot 10 and draw a return arrow from 10 to 5 in blue.

T: ***What could this blue arrow be for?***

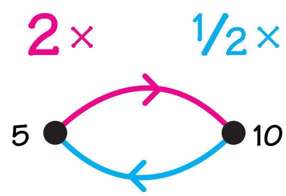
S: ***$\frac{1}{2}$x.***

S: ***÷ 2.***

If a student suggests that the blue arrow could be for –5 (or + $\widehat{5}$), agree that 10 – 5 = 5 (or that 10 +$\widehat{5}$ = 5), but then ask what else the blue arrow could be for.

EN8

Write $^1/_2$x in blue near the arrow picture. You may want to observe that $^1/_2$x and ÷ 2 are the same.



T: ***The blue arrow is for $^1/_2$x; $^1/_2$x is the opposite of 2x. What number is $^1/_2$ x 10?***

S: ***5.***

Erase 5 and 10. Point to the dot on the left.

T: ***If this number were 3, what number would be here*** (point to the dot on the right)***?***

S: ***6***

T (tracing the 2x arrow)**:** ***Yes, 2 x 3 = 6 and*** (trace the $^1/_2$x arrow) ***$^1/_2$ x 6 = 3.***

Continue in this manner with several other starting numbers at the left dot such as 7, 20, 500, and 1,000.

Point to the dot on the right.

T: ***If this number were 8, what number would be here*** (point to the dot on the left)***?***

S: ***4.***

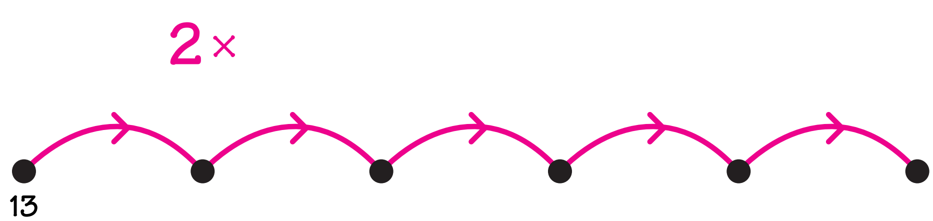
T (tracing the $^1/_2$x arrow)**:** ***Yes, $^1/_2$ x 8 = 4 and*** (trace the 2x arrow) ***2 x 4 = 8.***

Continue with several other starting numbers at the right dot such as 10, 16, 20, 80, and 100.

Alternating dots, assign a number to one of the dots and ask what the other number in the picture would be. Choose numbers appropriate to the abilities of your students.

Exercise 3

Draw this arrow picture on the board and ask students to copy it on their papers. Direct students to add to their pictures as is done on the board.



T: ***I drew this picture to tell you about my friend Winona. Winona was on a game show called "Double Up" for a week. The first day she started with \$13 and each day she could double her money.***

Observe that the first dot in the picture is for 13 (starting amount) and the red arrows are for 2x (double). Trace the 2x arrow from 13 to the second dot.

S: ***On the first day Winona doubled her money. What number is 2 x 13?***

S: ***26.***

T: ***How do we calculate 2 x 13?***

S: ***13 + 13 = 26.***

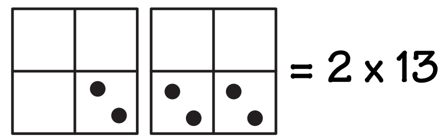
S: ***2 x 10 = 20 and 2 x 3 = 6, so 2 x 13 = 26.***

S: ***Use the Minicomputer.***

S: ***Use a calculator.***

Use one or two suggestions made by students to calculate 2 x 13. You may like to model the calculation with base-10 blocks.

If students or you choose to use the Minicomputer, guide the discussion until it is clear that 13 should be put on the Minicomputer twice; then ask two students to each put 13 on the Minicomputer.

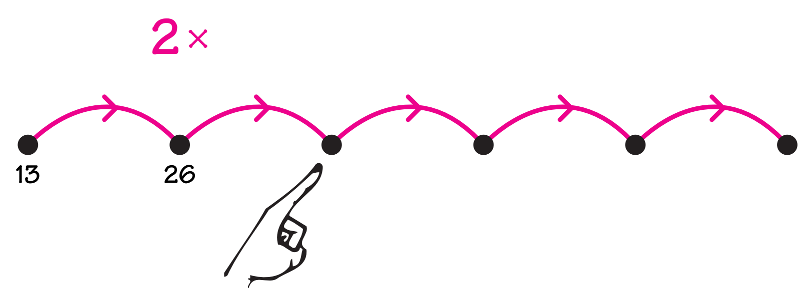


T: ***What should we do to make the number easier to read?***

S: ***Make some trades.***

Invite students to make trades until the standard configuration for 26 is obtained.

Label the second dot 26 and point to the third dot.



On the second day Winona doubled her money again.

T: ***What number is 2 x 26? Can you predict or give an estimate?***

Allow some discussion of estimation strategies or how to find an exact solution.

T: ***How can we calculate 2 x 26?***

S: ***2 x 20 = 40 and 2 x 6 = 12, so 2 x 26 = 40 + 12 = 52.***

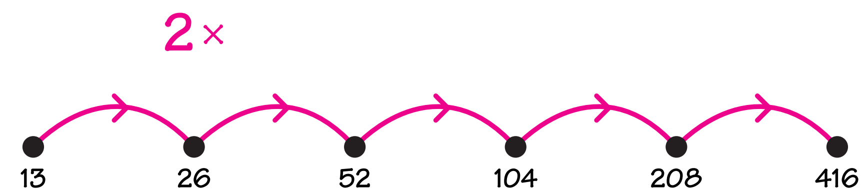
S: ***On the calculator.***

S: ***On the Minicomputer; put on 26 two times.***

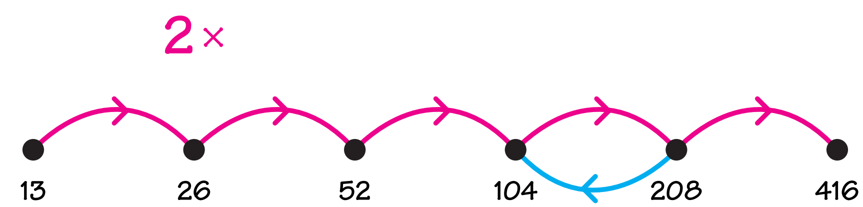
Again, use students' suggestions or model with base-10 blocks. If you or students choose to use the Minicomputer, ask two students to each put 26 on the Minicomputer. Then invite students to make trades until the standard configuration for 52 is obtained.

EN8

Continue in this manner until all the dots are labeled.



Draw a blue return arrow from 208 to 104.



T: ***What could this blue arrow be for?***

S: ***$^1/_2$x.***

S: ***÷ 2.***

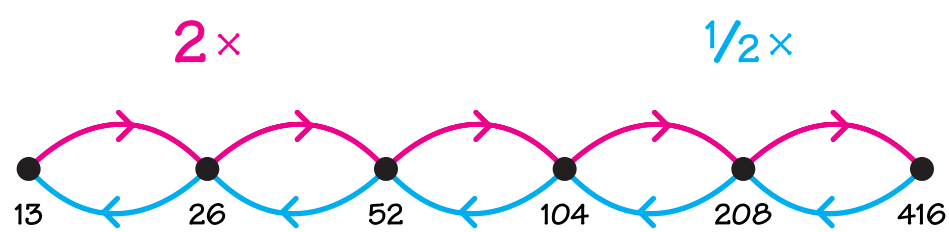
Write $^1/_2$x in blue near the arrow picture.

T (tracing the blue arrow)**:** ***$^1/_2$ x 208 = 104.***

Write $^1/_2$ x 208 = 104 on the board.

T: ***Could we draw some more $^1/_2$x arrows?***

Invite students to draw the remaining $^1/_2$x arrows. For each one record the appropriate number sentence on the board.



Extension Activity

Invite students to extend their arrow pictures with one or two more 2x arrows (days on the "Double Up" game show) and to label the ending dots.

Writing Activity

Suggest students write another story for their 2x arrow picture.

EL Lessons

Capsule Lesson Summary

Present an arrow picture where the dots are for children and the arrows are for the relation "sent a letter to." Collectively, discuss some of the many observations that can be made about this picture. Individually, draw similar arrow pictures.

Materials

Teacher	• Colored chalk	**Student**	• Unlined paper • Colored pencils, pens, or crayons

Description of Lesson

Begin the lesson with a brief exchange about writing letters to friends.

T: ***I have a friend, Edie, who just started at another school. She gave me an arrow picture that she had drawn herself. Edie's picture is very interesting and I would like to show it to you. I will draw it on the board so everyone can see.***

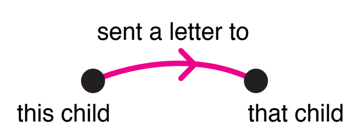
Draw eighteen well-spaced dots on the board. Suggest that the class count along as you draw the dots.

T: ***These are the children in Edie's class. How many children are there in Edie's class?***

Call on one or two students to check the count, pointing to each dot as they count. The class should agree that there are eighteen children in Edie's class.

T: ***Edie told me that during the summer vacation some of the children in her class wrote letters to other children in the class. She drew red arrows to show this.***

On another part of the board, draw this picture. Ask a student to read the message of the red arrow as you point to the starting dot, trace the arrow, and then point to the ending dot.



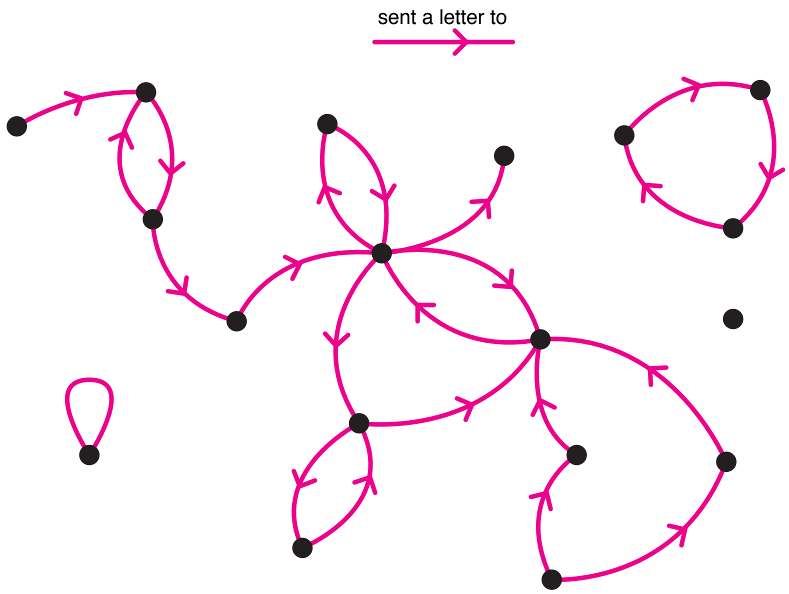
S: ***This child sent a letter to that child.***

Erase this picture and draw a key arrow near the eighteen dots. (See the next illustration.)

T: ***Edie's picture has many red arrows so it will take me a couple minutes to finish the drawing on the board. I would like to surprise you, so everyone cover your eyes while I complete the picture.***

EL1

T: ***You may open your eyes now. Look at the picture on the board. Think carefully about this picture for a moment without saying anything.***



After about a minute, ask the class what they think about Edie's picture. Let students comment spontaneously, even if their remarks are not directly related to the meaning of the arrows. For example, the following comments might be made.

S: ***It's a very complicated picture.***

S: ***There are lots of arrows.***

S: ***It looks like a dancer.***

S: ***I see a child who sent a lot of letters.***

S: ***I see a child who received a lot of letters.***

S: ***I see someone who didn't receive any letters.***

When comments like the last three are made, ask students to explain how they know. Be sure that arrows starting at a dot are counted as letters sent and that arrows ending at a dot are counted as letters received.

Let the discussion continue as long as new observations are being made. When the students stop making comments, ask some specific questions. For example:

T: ***Edie told me that her friend Andrew sent the most letters. Where is Andrew? How many letters did he send?*** (Four)

Ask a student to point to the dot for Andrew and to trace the four arrows for the letters Andrew sent. Label the dot.

T: ***Did Andrew receive any letters?***

S: ***Yes, three.***

Invite a student to trace the three arrows for letters Andrew received.

T: ***Can you find a child who sent just one letter and received just one letter?***

There are many, so allow several students to point to different dots at which just one red arrow starts and just one red arrow ends.

T: ***Edie told me that her friend Jenna received the most letters. Where is Jenna? How many letters did she receive?*** (Four)

Ask a student to point to the dot for Jenna and to trace the four arrows for the letters Jenna received. Label the dot.

T (pointing to the dot with a loop)**:** ***What do you think about this child?***

S: ***That child sent a letter to herself (himself).***

T: ***Find some children who exchanged letters, that is, sent letters to each other.***

There are several such pairs of children so allow students to find as many as they can.

T: ***Edie told me that there is a new student in her class. This child sent no letters and received no letters. Where is the new student in this picture?***

Ask a student to point to the appropriate dot.

T: ***In the story told by Edie's picture, how many letters were sent in all?*** (22) ***How many letters were received?*** (22)

If you could be one of the children in Edie's class, who would you like to be? Explain why.

Students will react differently to such a question; encourage them to express themselves.

Distribute unlined paper and colored pencils. Ask the students to draw a picture for their own story about sending letters. One way to get students started is to ask them to draw a dot for themselves and label it, and then to draw dots for some of their friends. Now ask them to draw some red arrows as they tell a story about sending letters. Encourage students to write something about their pictures, such as how many letters were sent and received or how many letters a particular person sent and received.

You will receive many different pictures. Show some of them to the whole class and let other students comment briefly. You may wish to display some pictures on a bulletin board.

Capsule Lesson Summary

Introduce the A-blocks and describe each piece with three attributes: color, shape, and size. Find pieces that differ in one way, two ways, and three ways. Place pieces in a string picture and find pieces that could be in a certain region of the string picture.

Materials

Teacher		Student	
	• A-Block String Game kit • Colored chalk • Box		• Paper • Colored pens, pencils, or crayons

Advance Preparation: Before this lesson begins, you will need to prepare some of the materials from the A-Block String Game kit. These materials will be used again in all future lessons on A-blocks and The String Game, so keep them together in the envelope with the kit.

Punch out one set of shapes (24 pieces). If you have a magnetic board, magnetize each piece by sticking a small piece of the magnetic material to the back. If you do not have a magnetic board, have loops of masking tape ready to stick to the back of each piece. Masking tape loses its stickiness quickly, so be prepared to reinforce the pieces with new loops of tape during the lesson.

Description of Lesson

Put the 24 A-block pieces in a box the size of a shoe box. Sort them so that you can locate any given piece quickly.

Exercise 1

T: ***In this box I have some cardboard pieces in different shapes, colors, and sizes. They are called A-blocks. There are three different shapes. What shapes do you think the pieces have?***

S: ***Squares.***

S: ***Triangles.***

S: ***Circles.***

These responses usually come immediately. If not, tell students the three shapes involved. As each shape is mentioned, write the word on the chalkboard.

T: ***There are four different colors. What colors do you think the pieces have?***

S: ***Red.***

S: ***Blue.***

S: ***Green.***

S: ***Yellow.***

As each color is mentioned, write the word on the chalkboard.

T: ***There are two different sizes. What should we call them?***

S: ***Big and little.***

Write these words on the chalkboard.

Tell the class it will be their job to get the pieces out of the box. Call on students, one at a time, to describe to you which piece they want to see. As the pieces are described and taken out of the box, put them on the board for all to see.

T: ***You tell me which piece you want, and I will take it out of the box for you.***

S: ***A square.***

T: ***I have many squares. Which one do you want?***

S: ***Any of them.***

Hold up a square, for example, a big red square.

T: ***Here's a square. I will put this piece back in the box. Now who can tell me which piece it was?***

S: ***A square.***

T (holding up a green square)**:** ***This one?***

S: ***No, a red square.***

T (holding up the little red square)**:** ***This one?***

S: ***No. It was a big red square.***

T (holding up the big red square)**:** ***Was it this piece?***

S: ***Yes.***

Put the piece on the board.

T: ***Someone else ask me for a piece.***

S: ***A big triangle.***

T: ***Yes, I have several big triangles. Which one do you want?***

S: ***A big blue triangle.***

Take out the big blue triangle and put it on the board.

T: ***Very good. Sam told me the size, the color, and the shape of the piece he wanted to see.***

S: ***A big yellow triangle.***

Put the big yellow triangle on the board and continue asking for pieces. If someone describes a piece already on the board, point to it and ask for one you have not yet seen. If the activity begins to drag, ask a student to describe all the rest of the triangles and put them up as they are described. In this way, you can get all the triangular pieces on the chalkboard.

T: ***How many triangles do we have?***

S: ***8.***

T: ***How many circles do you think there are?***

S: ***8.***

Take out all (the rest of) the circles and put them on the board.

T: ***What else could be in the box?***

S: ***Just some squares.***

T: ***Which ones?***

Take them out as they are described and put them on the board.

T: ***How many pieces are there altogether?***

S: ***24.***

T: ***Can you explain?***

S: ***I counted them.***

S: ***8 triangles, 8 circles, and 8 squares make 24 pieces in all.***

Write this calculation on the board.

$$8 + 8 + 8 = 24$$

T: ***Can anyone tell us another number sentence for this?***

Accept any number sentence for 24, but write only 3 x 8 = 24 on the board. If necessary, suggest this multiplication sentence yourself.

$$3 \times 8 = 24$$

Exercise 2

Take the big yellow circle and put it on a separate section of the board.

T: ***Who can show me a piece that is different from this piece in just one way?***

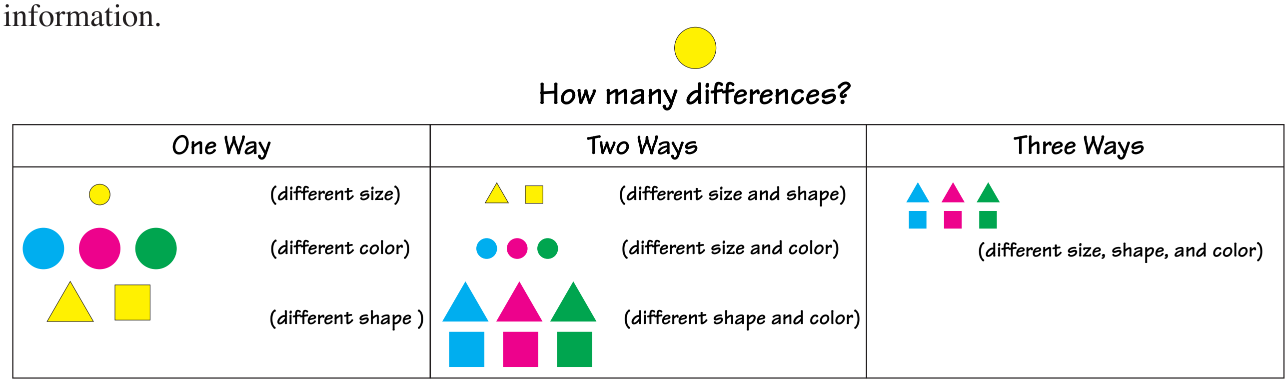
Suppose a student, Jason, chooses a small green circle.

T: ***Well, Jason, your piece is a different color but it is also a different size, so it is different from my piece in two ways.***

Ask the student to make another choice. Suppose Jason selects a small yellow circle.

T: ***Is Jason's piece a different color?*** (No)
Is it a different shape? (No)
Is it a different size? (Yes)
So it is different in just one way.

Organize a table or graph for "How many differences?" on the board. (See the illustration below.) Invite students to place pieces in the graph. Occasionally ask for a piece that goes in one column of the graph. Your graph need not be organized within the columns; the picture here is for your information.



When the graph is complete (all the pieces have been placed) make some observations comparing the number of pieces in each column.

Exercise 3

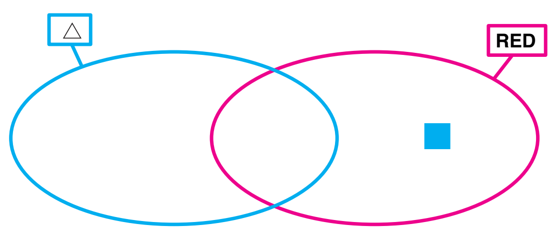
Put all the pieces back in the box. Draw two overlapping strings in two colors and label them as in the next illustration.

T: ***The blue string is for triangles. The red string is for red pieces.***

Hold up the large red square.

T: ***Who can put this piece in the picture?***

Call on a student to put the piece in the picture and ask the class for agreement or disagreement. If the piece is incorrectly placed, ask for an explanation and call on another student to replace the piece. If the piece is correctly placed, ask,



T: ***Why do you think Kristy put it inside the red string?***

S: ***Because the red string is for red pieces.***

T: ***Why did Kristy put it outside the blue string?***

S: ***Because the blue string is for triangles, not squares.***

T: ***Yes, you must think about both strings.***

Continue in this way, asking students to place pieces in the picture. Not every piece needs to be discussed, but try to get at least one piece in every region. Be sure the students understand the two questions that help decide where a piece goes: Is it red? Is it a triangle?

Sometimes vary the activity by asking,

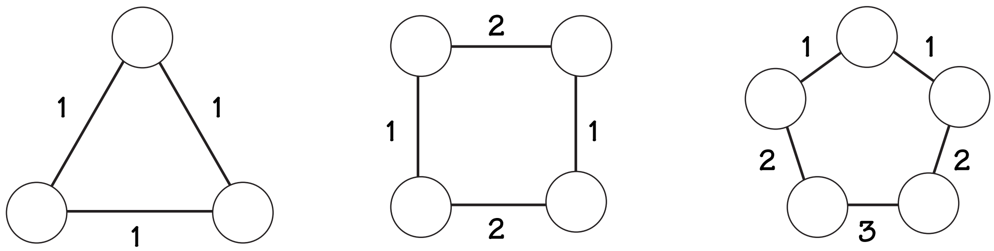
T: ***Can you find a piece that belongs here?***
(or)
I'll put this piece here. Is it correct?

You can also vary the activity by asking a student to describe a piece still in the box and then inviting another student to place it correctly.

Continue until all students have had an opportunity to participate.

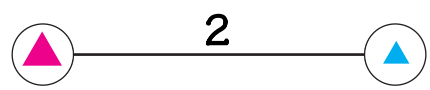
Center Activity

Prepare some task cards with A-block puzzles. For example:



Directions: Use one set of A-blocks. Place pieces in the circles of a puzzle so that the number on each connection shows how many differences the pieces being connected can have.

Here are two differences (color and size).



EL3 STRING GAME WITH A-BLOCKS

Capsule Lesson Summary

Review the different A-block pieces as to shape, color, and size by taking them out of a box, one at a time, as they are described. Place the A-blocks correctly into a two-string picture or a three-string picture. Play The String Game with hidden labels.

Materials

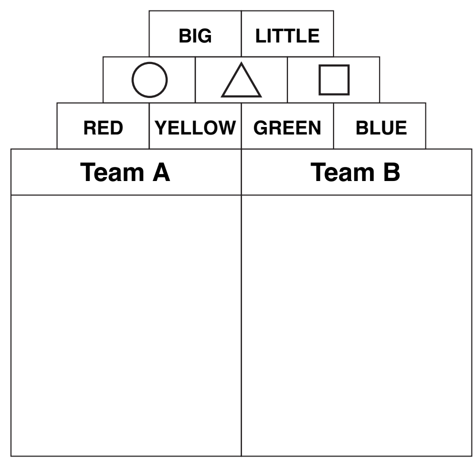
Teacher
- A-Block String Game kit
- Box
- Colored chalk

Student
- None

Advance Preparation: Before this lesson begins, prepare a team board as pictured below either directly on your chalkboard, on the side of a file cabinet, or on another easily accessible board. Be sure this board is metallic if you have magnetized the A-block pieces. Post a list of attributes above the team board. This list can be prepared using one of the posters in the A-Block String Game kit.

Description of Lesson

Put the 24 A-block pieces in a box about the size of a shoe box. Sort them so that you can locate any given one quickly. Divide the class into two teams. Try to arrange that the teams are roughly equal in ability. You may want to let the teams choose their own names rather than A and B.



Note: If you prefer, this lesson can be taught with more than two teams. For example, you might use three or four cooperative groups as teams. Prepare the team board accordingly.

Exercise 1

Briefly discuss the different attributes of A-block pieces: size, shape, and color. Refer to the list of attributes above the team board.

T: ***Your first task will be to get all the pieces out of my box and onto this board*** (point to the team board)***. We will take turns; someone on Team A will tell me a piece to put on Team A's side of the board, and then someone from Team B will tell me a piece to put on Team B's side. Remember, you must describe a piece that is still in the box so I know exactly which piece you want me to put on your side of the board. Team A goes first.***

S: ***Big red circle.***

Put the big red circle on Team A's side of the board.

T: ***Very good. Kevin told me the size, color, and shape of the piece he wanted. Now it is Team B's turn.***

S: ***Little red circle.***

Put the little red circle on Team B's side of the board.

T: ***Very good. Angela described a piece that is still in the box and I can put that piece on her team's side of the board.***

Continue in this way, keeping a brisk pace, until all the A-block pieces are on the board. If a student describes a piece that is already on the team board, point out that the piece has been chosen and ask the student to choose a different piece, one that is still in the box. Try to call on weaker students early in the game so that there are many choices open to them. Encourage all students to be thinking about a piece they will ask for when you call on them.

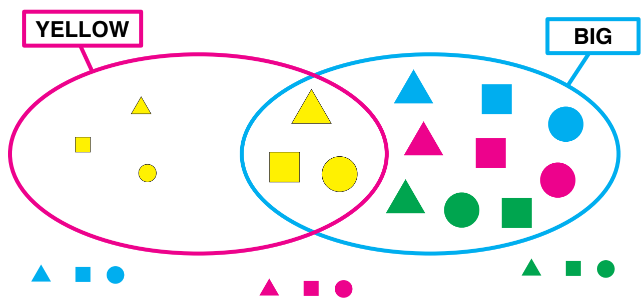
Exercise 2

Note: If this exercise seems too easy for your class, use the alternative Exercise 2.

Draw two different-colored, overlapping strings on the board. Label them as in the next illustration.

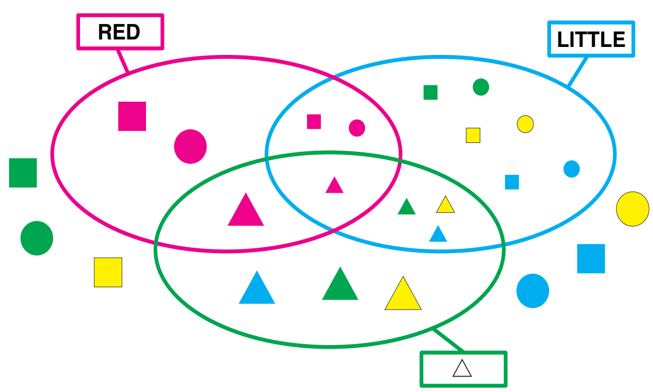
T: ***Now the teams will take turns . Each of you will try to place a piece from your team's side of the board in this string picture. If you put it in its correct place, I will say yes and it stays there. If you do not, I will say no and you must return it to your team's side of the board. The first team to get all of its pieces in the picture is the winner.***

Play the game, alternating teams, and alternating turns among the members of each team. When all the pieces are in the picture, your board will look like this.



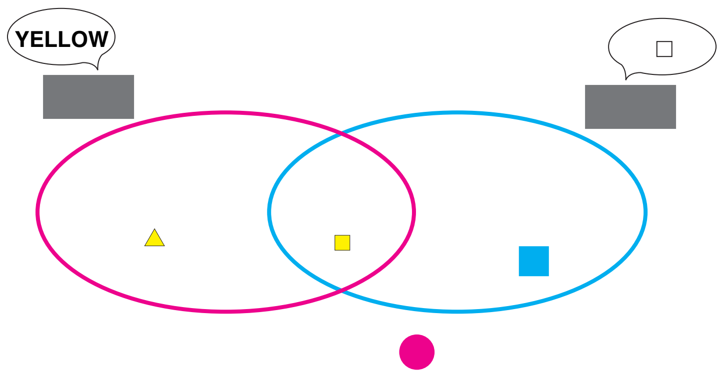
Exercise 2 (alternative)

Draw three different-colored, overlapping strings on the board. Label the strings as in the next illustration. Give directions as in Exercise 2 above. When all the pieces are in the picture, the board will look like this.



Exercise 3

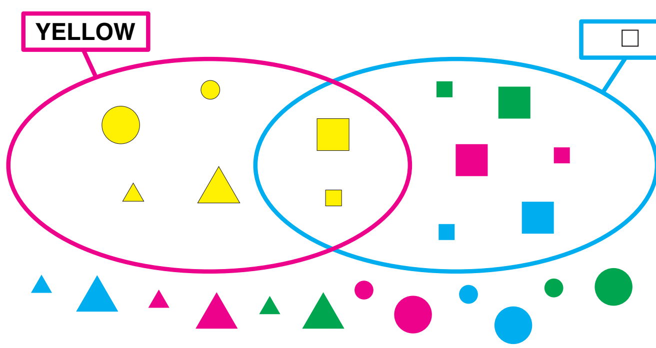
Prepare to play The String Game in the usual way.† Distribute game pieces to the two or more sides of the team board (Team A and Team B). Then, set up the playing board as shown below, but this time place the string cards face down (bubbles indicate what is hidden on the cards). Place three or four pieces correctly. Or you may want to allow the teams to each select one or two pieces for you to place correctly in the picture; these then serve as starting clues.



Remember that the rules of the game call for you to be the judge. If a piece is correctly placed, say yes, and immediately instruct the player to try to place a second piece (bonus turn). No player should have more than two consecutive turns. If a piece is incorrectly placed, say no, and ask the player to return the piece to his or her team's side of the team board.

For this game, the player who correctly places the last piece from his or her team's side of the team board may then attempt to identify each of the string cards. A team wins by being first to get all of its pieces in the string picture and to identify the strings.

The following picture shows correct placement for all the game pieces. You can use this picture as a crib sheet while the game is being played.



†See Appendix D on The String Game for a description of equipment and preparation for the game played in the usual way.

Capsule Lesson Summary

Play a type of Guess My Rule game where the players decide that a given string is for multiples of 3. Locate many integers, positive and negative, in the string picture. Label some dots in a double spiral arrow picture and find that the arrows could be for +3. Then observe that the numbers on the spiral are the same as those inside the string. Discuss other multiples, for example, multiples of 5 and of 2. Locate numbers in a two-string picture with one string for multiples of 2 and the other for multiples of 5.

Materials

Teacher
- Colored chalk
- Calculator

Student
- Paper
- Colored pencils, pens, or crayons
- Calculator

Description of Lesson

Exercise 1

Draw a large string on the board.

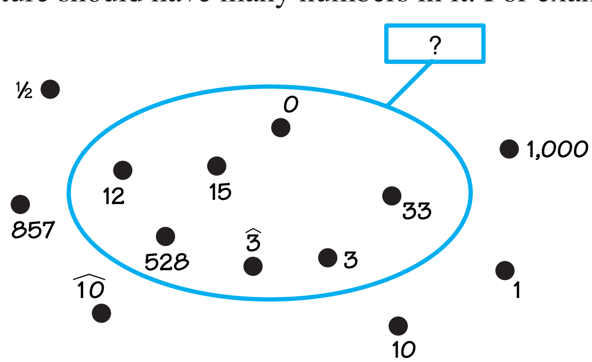
Note: The string is for multiples of 3, but do not announce this or use the word *multiples* until later in the lesson.

Tell the class that you are not going to tell them yet what the string is for except that it is for some numbers. They may choose numbers and you will place them correctly in the picture, either inside the string or outside the string. When they think they know what the string is for, you will give them numbers to place to see if they are right.

Give each student a chance to choose a number for you to place or to try to place a given number in the string. That is, go around the room and ask each student, in turn, either to give you a number to put in the picture or to place a number in the string.

Note: For your information only, an integer is a multiple of 3 if and only if the sum of its digits is a multiple of 3.

After a short while your picture should have many numbers in it. For example:



Some students may be very anxious to guess what the string is for and you may take some guesses. If students guess that the string is for even numbers, odd numbers, more than 10, less than 1,000, or something else that can easily be seen to be impossible, ask the class to say why the guess is incorrect. For example:

S: ***The blue string cannot be for even numbers, because 15 is inside and 15 is odd.***

S: ***The blue string cannot be for less than 1,000 because 10 is outside and 10 is less than 1,000.***

Although your students may be unfamiliar with the term *multiples of 3*, they should soon begin to notice that the numbers inside the string have something to do with counting by threes. When this occurs, invite students to predict where certain numbers will go in the picture. You may like to use a calculator to observe numbers you get counting by threes starting at 0. That is, start with 0 on the display and press $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on.

T: ***Where should I put 30?***

S: ***Inside the string, because you say 30 when you count by threes.***

Invite students to count by threes starting at 0 and ending at 30.

T: ***Where should I put 37?***

S: ***Outside the string, because from 30 we count next 33, 36, 39, and skip over 37.***

T: ***Where should I put 300?***

S: ***Inside the string, because if you count by threes 100 times you'll get to 300.***

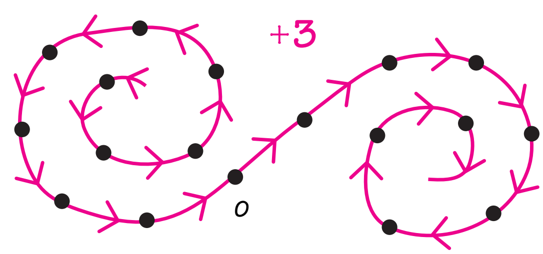
If you already have some negative numbers in the picture, also discuss counting backward by threes and put some more negative numbers in the picture. Use the calculator to count backward by threes by pressing $\boxed{-}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on. Otherwise, wait for the next exercise to discuss negative numbers.

T: ***There is a name for the numbers inside this string. They are called multiples of 3.***

Write this term inside the label for the string. Leave your picture on the board; you will refer back to it in Exercise 2.

Exercise 2

Draw a double spiral arrow picture on the board and label the middle dot 0.



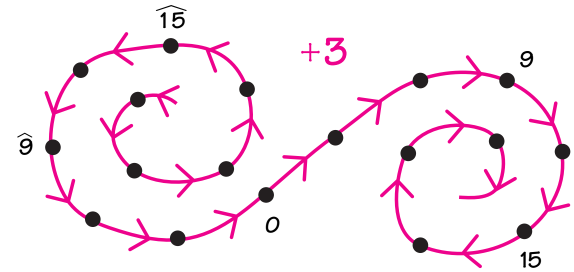
T: ***What do you think about this picture?***

Let the class react. Perhaps they will mention its symmetry, that 0 is in the center, that it could go on and on in both directions, and so on. Allow a minute or two for such observations before continuing, but do not insist that all of these things are mentioned.

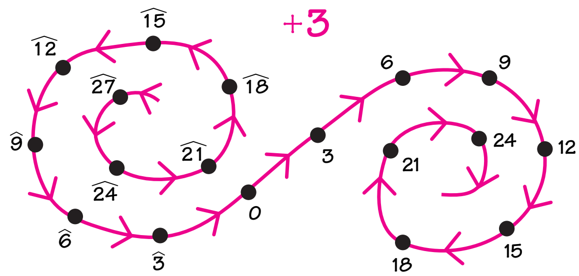
T: ***Let's put some more numbers in the picture.***

Label dots for 9, $\widehat{9}$, 15, and $\widehat{15}$.

T: ***What could the red arrows be for?***



Allow the students to experiment with some possibilities. For example, if a student suggests +9, observe that in that case 9 would be at the end of the arrow starting at 0, not where it is. Perhaps a student will suggest "is less than"; if so, observe that this is a possibility, but ask for another possibility. When a student suggests +3, check to find that indeed the red arrows could be for +3, and write the key near the picture. If no one has suggested +3, put in some more numbers, for example, 6 and $\widehat{6}$, and ask the question again. After it is established that the red arrows are for +3, invite students to label the other dots.



T: ***What do you notice about the numbers in this picture?***

S: ***They are alike on both sides; 3 and $\widehat{3}$, 6 and $\widehat{6}$, 9 and $\widehat{9}$, and so on.***

S: ***They are the numbers we say when we count by threes, forward and backward.***

S: ***They are the numbers we see on the calculator when we make it count by threes (forward or backward) starting at 0.***

S: ***They are the numbers inside the string, the multiples of 3.***

If this last observation is not made, call the class's attention back to the string picture and ask specifically which numbers in the string picture would be in the arrow picture. (The numbers inside the string.)

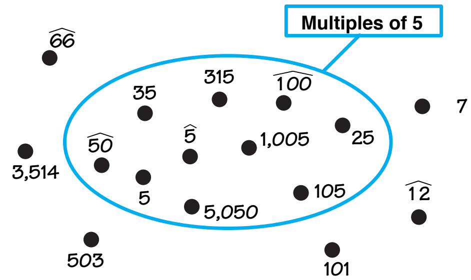
Continue by asking students to predict whether certain numbers are multiples of 3; that is, whether they would be in the double spiral for +3. Include some large numbers such as 300, 333, 900, 906, and 3,000. You may again like to use the calculator to follow the arrow picture and to check students' predictions of multiples of 3.

At this point, if there are no negative numbers in the string picture, direct the class to observe that all the numbers in the arrow picture should be inside the string, including the negative numbers.

Exercise 3

Repeat Exercise 1 for multiples of 5. The students should quite quickly guess that your new string is for multiples of 5 and perhaps they will even use that expression.

After the class discovers that your string is for multiples of 5, let students put in some numbers themselves. Encourage them to include both positive and negative numbers, and some large numbers.



T: ***If I were to draw an arrow picture as before to include all the multiples of 5, what kind of arrows would I use?***

S: *+5.*

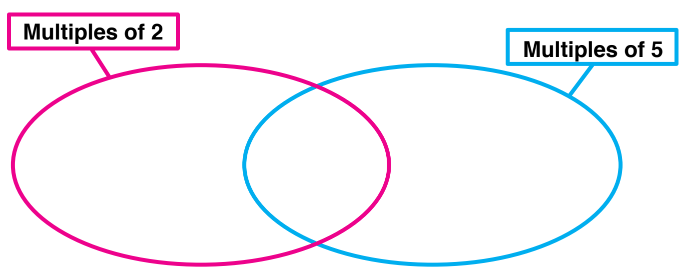
While you erase the board and prepare for Exercise 4, ask students to make lists on their papers of the following:

- multiples of 2 from 0 to 20
- multiples of 5 from 0 to 50
- multiples of 3 from 0 to 30
- multiples of 10 from 0 to 100

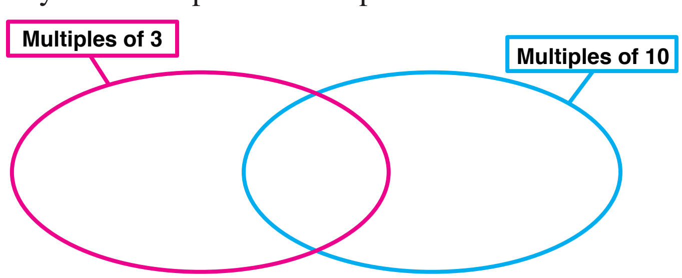
Exercise 4

During this exercise, allow students to work with a partner and use a calculator.

Draw this picture on the board and instruct students to draw the picture on their papers. Check with the class that everyone understands that multiples of 2 are even numbers. Then direct the students to put many numbers in the picture.



Student pairs that are ready for a new problem can put numbers in the following picture.



Home Activity

Suggest that parents/guardians find opportunities to count by twos, threes, fives, and tens with their child.

EW Lessons

EW1 DETECTIVE STORY #1/ELI'S MAGIC PEANUTS #1 EW1

Capsule Lesson Summary

Solve a detective story for which the secret number is an odd number, and where the other clues involve an arrow picture and the Minicomputer. Tell the story of Eli the Elephant who found magic peanuts. When a magic peanut meets a regular peanut, both disappear, much to Eli's confusion.

Materials

Teacher
- Colored chalk
- Minicomputer set
- Calculator

Student
- Paper
- Colored pencils, pens, or crayons
- Calculator
- Minicomputer set
- Worksheets EW1* and **

Advance Preparation: Use the blacklines following this lesson to make copies of student worksheets.

Description of Lesson

Exercise 1: Detective Story

Before the lesson, write a large 21 on a slip of paper and fold the paper so that 21 is hidden.

Ask the class what a detective does. Guide the discussion to include the idea that a detective tries to uncover secrets by following clues. Tell the students that they are going to be detectives today, and they will be able to discover a secret number if they follow the clues.

Note: You may like to make the secret number something real for students to figure out such as "How old is my brother?" or "How many miles do I drive to school?" or "How many spots on my dog?"

Clue 1

T: ***The name of a secret number is written on this paper. Listen carefully to my story and you can discover what it is. The first clue I will give you is that the secret number will appear on the calculator display when we teach the calculator to count by twos starting at 1.***

Remind or show students how to teach the calculator to count by twos.

1. Put on the starting number (1).
2. Press $\boxed{+}$ $\boxed{2}$.
3. Then press $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on.

Observe which numbers appear and suggest that students write ten or more of these numbers on their papers in the order they appear. Encourage students to make observations and, if not mentioned, tell the class that the secret number is an odd number. Begin listing the positive odd numbers in order on the board with the students' assistance. Start with 1 and continue to at least 25. Mention that you could continue pressing $\boxed{=}$ on the calculator or writing odd numbers all day, but instead you will put three dots at the end of your list meaning "and so on."

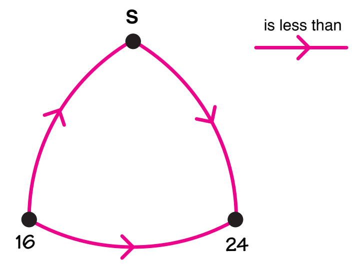
1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, ...

T: ***Now we know that the secret number is one of these numbers, but we do not know which one. What do we need?***

S: ***Another clue.***

Clue 2

Draw this arrow picture on the board and ask students to copy it on their papers.



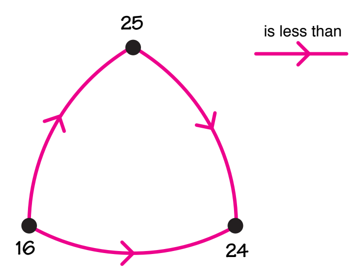
T: ***The secret number is here*** (point to **s**) ***in this arrow picture. What are the red arrows for?***

S: ***Is less than.***

Point to the dot labeled **s** and remind the class that the secret number is an odd number. Direct students to check possible secret numbers at **s** in their pictures. Choose an odd number and ask if it could be the secret number. If the class disagrees on whether a number could be the secret number or not, put that number in the arrow picture and ask a student to read the number sentence told by each arrow and its starting and ending numbers.

For example, suppose the class is not sure whether **s** could be 25 or not.

Trace the arrow from 16 to 25.



S: ***16 is less than 25.***

T: ***Is that true?***

S: ***Yes.***

Trace the arrow from 16 to 24.

S: ***16 is less than 24.***

T: ***Is that true?***

S: ***Yes.***

Trace the arrow from 25 to 24.

S: ***25 is less than 24.***

T: ***Is that true?***

S: ***No.***

T: ***So our secret number cannot be 25.***

Erase 25 from the arrow picture and cross it out of the list of odd numbers.

As the class discovers which numbers in the list of odd numbers cannot be the secret number, cross them out. Continue until every odd number in your list has been considered. Ask if there are any other odd numbers which could be the secret number. Commend a student who concludes that none of the numbers indicated by the three dots at the end of the list could be the secret number because they are all more than 24. If necessary, supply this information yourself and cross out the three dots at the end of the list. Erase everything that has been crossed out.

~~1~~, ~~3~~, ~~5~~, ~~7~~, ~~9~~, ~~11~~, ~~13~~, ~~15~~, 17, 19, 21, 23, ~~25~~, ~~27~~, ~~...~~

Clue 3

Display two Minicomputer boards.

T: ***Now we know that the secret number is one of these numbers: 17, 19, 21, or 23. Listen carefully because I will give you only one more clue. The secret number can be put on the Minicomputer with exactly two checkers.***

Ask the students to put the secret number on their desk Minicomputer and check some of their responses.

T: ***Do you know the secret number?***

S: ***21.***

Invite a student to put 21 on the demonstration Minicomputer with two checkers. Ask how many checkers would be needed to put each of the numbers 17, 19, and 23 on the Minicomputer. Show the class that 21 is written on your paper.

Exercise 2: Eli's Magic Peanuts

T: ***There is an elephant named Eli. Eli is always very hungry. What do you think is Eli's favorite food?***

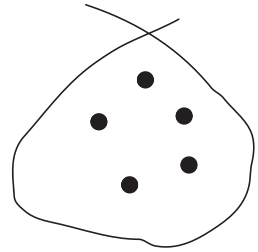
Accept some answers from the class and then tell them that Eli's favorite food is peanuts.

T: ***Eli likes peanuts so much that he always keeps a little bag of peanuts with him wherever he goes. One day as Eli was out walking, he found a plant that looked a lot like a peanut plant, but a little different than a regular peanut plant. Eli pulled up this strange plant and found some peanuts. Eli didn't know it, but the peanuts from this plant were magic peanuts! Eli gathered some of the magic peanuts and put them in his bag with the other peanuts. What do you suppose is so special about magic peanuts?***

Allow students to discuss this briefly.

T: ***Let me show you what happens when Eli puts both regular peanuts and magic peanuts in his bag.***

EW1

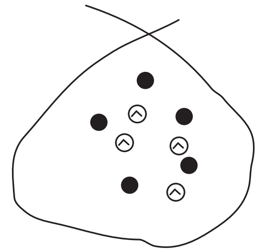


Draw this picture on the board. If you prefer, use one color of magnetic checkers for the regular peanuts and $\hat{\bigcirc}$ checkers for magic peanuts.

T: ***This is Eli's bag of peanuts with five regular peanuts in it. Now Eli also put some magic peanuts into his bag; here are the magic peanuts.***

T: ***How many magic peanuts did Eli put into his bag?***

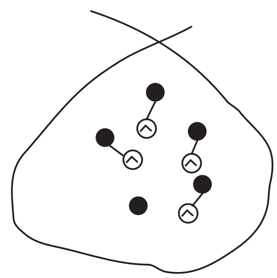
S: ***Four.***



T: ***When Eli returned home, he was hungry from walking all day. He decided to eat some peanuts. When he opened his bag, he was very surprised. There was just one regular peanut in the bag. What do you think happened when Eli put both the regular and the magic peanuts in the bag?***

Let students make suggestions and lead the discussion to the idea that when a regular and a magic peanut come together, they both disappear. Model this idea in the picture by pairing a regular peanut with a magic peanut and then removing them both from the picture.

If you use connecting lines to pair magic peanuts with regular ones, leave the picture on the board. Write an appropriate number sentence next to the picture.



5 + b4 = 1

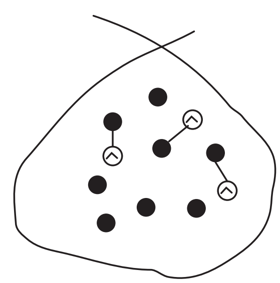
T: ***Five regular peanuts plus four magic peanuts is …?***

S: ***One regular peanut.***

S: ***1.***

T: ***Poor Eli was puzzled. He didn't know the secret of the magic peanuts and he couldn't imagine where his peanuts had gone! He was still hungry, so he went looking for more peanuts. This time, he found eight regular peanuts and put them into his bag.***

Erase your previous picture and draw a new one.



T: ***Eli also found three magic peanuts and put them into his bag.***

Add these three magic peanuts to your drawing.

T: ***When Eli returned home, what do you suppose he found when he opened his bag?***

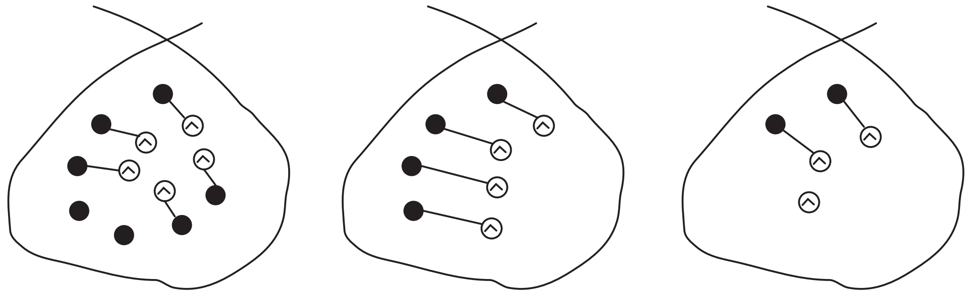
Ask several students to explain their answers; then choose volunteers to pair up regular peanuts with the magic ones. Write the number sentence next to the picture.

T: ***Eight regular peanuts plus three magic peanuts is…?***

$$8 + \hat{3} = 5$$

S: ***Five regular peanuts.***

Do a couple more examples such as the ones shown here. Include one example in which Eli finds only magic peanuts when he opens the bag.

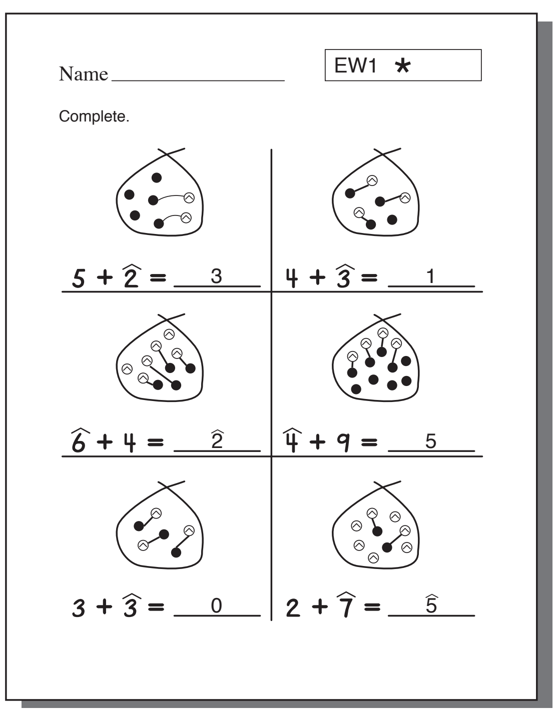


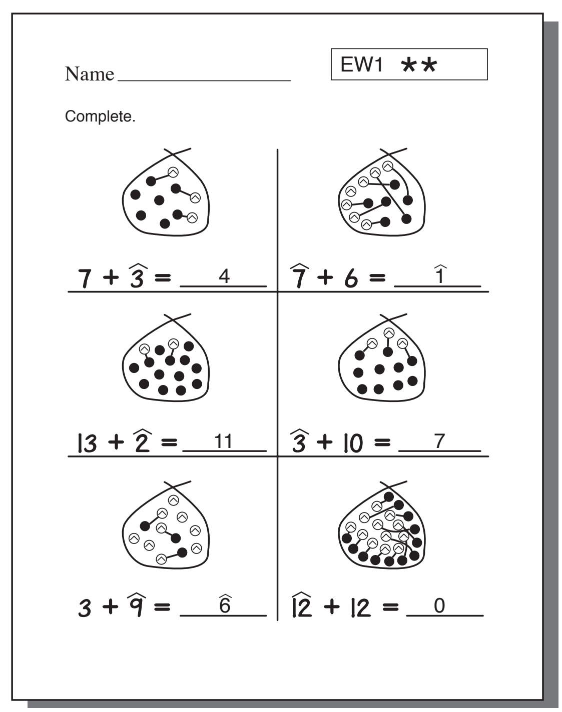
7 + b5 = 2　　　4 + b4 = 0　　　2 + b3 = b1

Worksheets EW1* and ** are available for independent work. Tell the students to finish the calculations for each of Eli's bags of peanuts. The students should draw lines connecting regular and magic peanuts themselves. Ask students who finish quickly to write their own stories about Eli's peanuts.

Home Activity

This would be a good time to send home a letter to parents/guardians about negative numbers. Along with the letter, you can send a page of problems like one of the worksheets. Students should explain to their parents/guardians about Eli and magic peanuts, and show them how to solve the problems. Blackline N9 in *UPG-III* has a sample letter.

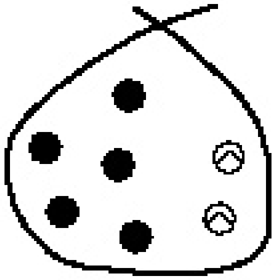




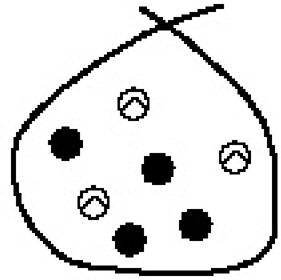
Name ____________________

EW1 *

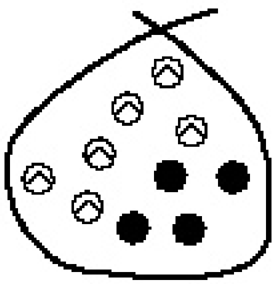
Complete.



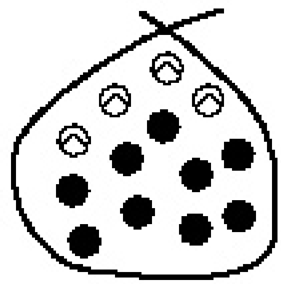
$5 + \hat{2} =$ ________



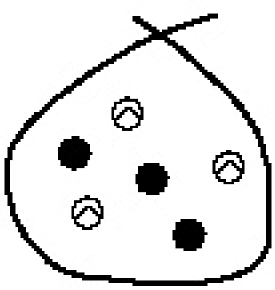
$4 + \hat{3} =$ ________



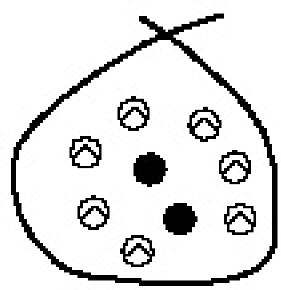
$\hat{6} + 4 =$ ________



$\hat{4} + 9 =$ ________



$3 + \hat{3} =$ ________

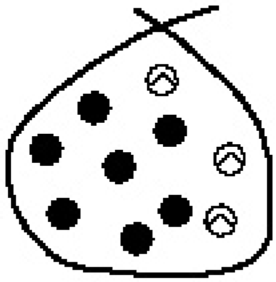


$2 + \hat{7} =$ ________

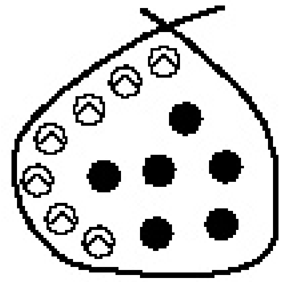
Name ____________________

EW1 **

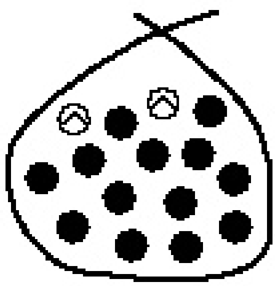
Complete.



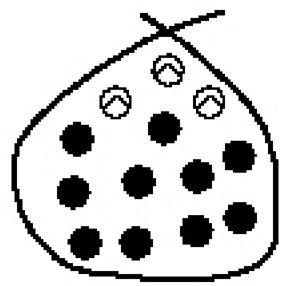
$7 + \hat{3} =$ ________



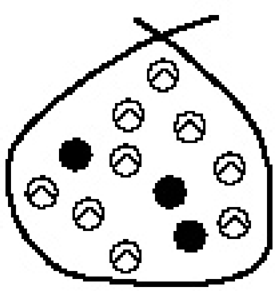
$\hat{7} + 6 =$ ________



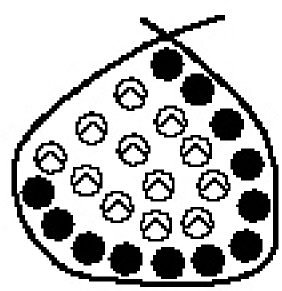
$13 + \hat{2} =$ ________



$\hat{3} + 10 =$ ________



$3 + \hat{9} =$ ________



$\hat{12} + 12 =$ ________

EW2 ELI'S MAGIC PEANUTS #2/DETECTIVE STORY #2

Capsule Lesson Summary

Introduce negative checkers to represent the number of magic peanuts in Eli's bag. Solve a detective story with clues involving the calculator, a string picture, and the Minicomputer.

Materials

Teacher
- Minicomputer set
- Colored chalk

Student
- Paper
- Minicomputer sheet

Description of Lesson

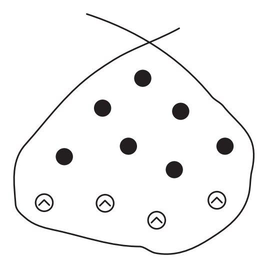
Exercise 1: Eli the Elephant

T: ***Who remembers Eli the Elephant?***

Allow students to recall whatever they remember. Be sure to mention that Eli collects peanuts; he sometimes finds regular peanuts and sometimes magic peanuts. Review what happens when a magic peanut and a regular peanut meet.

T: ***Today Eli went walking in the park and found seven regular peanuts and four magic peanuts. He put all the peanuts in his bag. He plans to eat the peanuts tonight while he watches television. How many peanuts will he find when he opens his bag?***

Draw a picture of the bag with regular and magic peanuts in it. Write a number sentence about the situation on the board.



$7 + \hat{4} = ?$

T: ***Seven regular peanuts plus four magic peanuts is…?***

S: ***Three regular peanuts.***

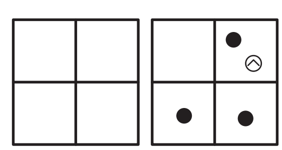
Complete the number sentence. Display two Minicomputer boards.

T: ***Let's show this story about Eli's peanuts on the Minicomputer. How many regular peanuts did Eli find?***

S: ***Seven.***

Invite someone to put 7 on the Minicomputer. Show the class the negative checkers, .

T: ***We can use negative, or magic, checkers to show how many magic peanuts Eli found in the park. How many magic peanuts did he find?***



S: ***Four.***

T: ***Show this on the Minicomputer using negative checkers.***

Do not allow the students to remove any checkers from the Minicomputer.

EW2

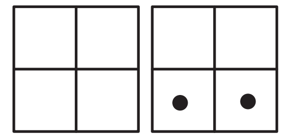
T: ***What will happen?***

S: ***The regular checker and the magic checker on the purple (4-) square will disappear.***

Explain that four magic peanuts and four regular peanuts disappear, and ask a volunteer to show the class what happens on the Minicomputer. The student should remove both checkers from the purple square.

T: ***Seven regular peanuts plus four magic peanuts equals…?***

S: ***Three regular peanuts.***



Erase the board and remove the checkers from the Minicomputer.

T: ***Last week Eli found a lot of peanuts. One day he found 39 magic peanuts and 25 regular peanuts. When Eli opened his bag that night, what did he find?***

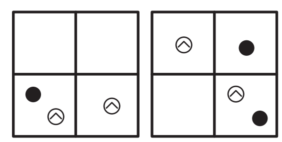
S: ***Magic peanuts.***

Write a number sentence about this situation on the board.

$$\widehat{39} + 25 = ?$$

If several students know the answer, ask them to write the result on their paper or whisper the number to you. Encourage students to estimate how many magic peanuts are in the bag.

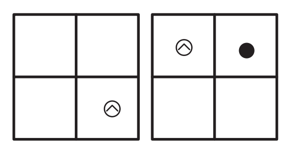
T: ***Who can put $\widehat{39}$ on the Minicomputer with negative checkers? Who can put 25 on the Minicomputer with regular checkers?***



T: ***What happens?***

S: ***The checkers on the 1-square disappear, and the checkers on the 20-square disappear.***

Invite students, one at a time, to remove the checkers from the 1-square and then from the 20-square. Encourage students to explain that one regular and one magic peanut disappear, and then twenty regular and twenty magic peanuts disappear.



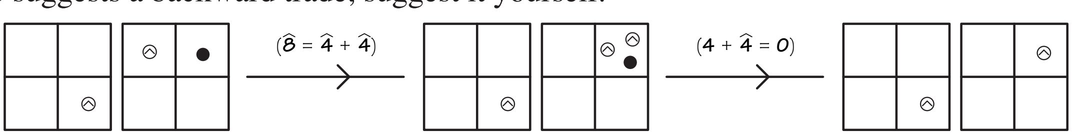
T: ***What could we do to make the number easier to read?***

S: ***Make a backward trade.***

Do not expect the verbalization of this trade to be exact. If a student says, "8 = 4 + 4," accept it, but rephrase it by saying that eight magic peanuts is the same as four magic peanuts plus four magic peanuts.

If a student suggests that checkers not on the same square will disappear, for example, $\widehat{8}$ and 4, remind the class that those checkers are for eight magic peanuts and four regular peanuts.

If no one suggests a backward trade, suggest it yourself.



Exercise 2: Detective Story

Allow students to work with a partner during this exercise.

Write a large 32 on a slip of paper and fold it so that 32 is hidden.

Clue 1

Distribute calculators to student pairs. If you have an overhead calculator, you may prefer to use it to introduce this clue and to let students help with pressing keys.

T: ***Today you must discover a secret number. The calculator will help us with the first clue. We will teach the calculator to count by fives starting at 2. The secret number will appear on the calculator's display.***

Instruct students to turn on their calculators and check that 0 is on the display. (They may need to press C a couple of times to get 0 on the display.) Then ask them to follow your instructions exactly.

T: ***Start with 0 on the display. Press 2. What number is on the display?***

S: ***2.***

Record 2 on the board.

T: ***Press + 5 =. What number is on the display?***

S: ***7.***

T: ***What did the calculator do?***

S: ***It added 5; 2 + 5 = 7.***

Record 7 on the board.

T: ***Press = again. What number is on the display?***

S: ***12.***

T: ***What did the calculator do?***

S: ***Added 5 again; 7 + 5 = 12.***

Record 12 on the board.

T: ***The calculator will continue to count by fives (add fives) as we press =. Press = one more time. What number is on the display?***

S: ***17.***

Continue your list on the board. 2, 7, 12, 17

T: ***Think about pressing = three more times. What are the next three numbers we will get on the calculator?***

Allow the class to predict these numbers. Then record them on the board as the students press ⊟ three more times and note them on the display. As soon as most of your students are able to predict the sequence quickly, you can abandon the calculator. Continue counting and recording until the class realizes this sequence goes on and on. Your list on the board should have several numbers greater than 50. You may like to ask the students to suggest a number that would be in the list that is greater than 100, or between 150 and 160.

2, 7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, 62, 67, 72, ...

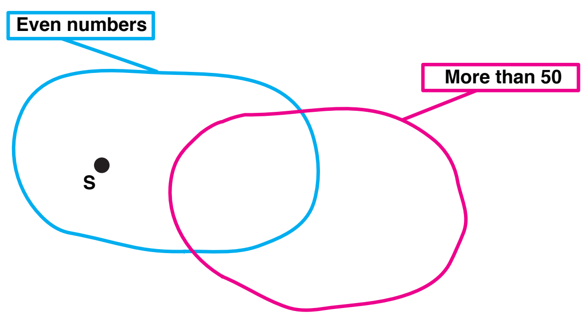
Explain that the three dots mean that the list goes on and on.

T: ***Our secret number is one of the numbers in this list. Do we know which one? What do we need?***

S: ***Another clue.***

Clue 2

Draw this picture on the board and ask students to copy it on their papers.



T: ***The second clue is that the secret number is here*** (point to **s**)***. Is it an even number?*** (Yes) ***Is it more than 50?*** (No) ***Which of these numbers could be the secret number?***

Suggest students put numbers from the list into their string pictures and find possibilities for the secret number. Each time a number is suggested, label the dot with that number and ask the class if the number is placed correctly. Suppose, for example, a student suggests 72.

T: ***Could this be 72?*** (Disagreement) ***Is 72 even?*** (Yes) ***Then it must be inside the red string. Is 72 more than 50?*** (Yes) ***Then it must be inside the blue string. Where should 72 be?***

Ask a student to point to the correct region (the middle).

T: ***72 cannot be the secret number.***

Put 72 correctly in the string picture and cross 72 off the list on the board.

If a correct number is suggested, elicit class agreement and circle it on your list. Continue until many of the numbers in the original list are either circled or crossed off.

Point to the three dots.

T: ***Could the secret number be any of the numbers we didn't write on the board?*** (No) ***How do you know?***

S: ***Because the secret number is not more than 50.***

Cross off the three dots.

Point to each unchecked number and guide your class in deciding if it could be the secret number. When all the numbers have been checked, erase those that have been crossed off.

T: ***Now we know the secret number is one of these*** (point to the five remaining numbers)**.**

2, 12, 22, 32, 42

Clue 3

Display two Minicomputer boards.

T: ***This will be the final clue. When you know which of these numbers is the secret number, write it on your paper. You cannot put the secret number on the Minicomputer board with only one or two checkers. At least three (regular) checkers are needed to put the secret number on the Minicomputer. Which is the secret number?***

Look at (or listen to) the responses of several student partners. After most students realize that 32 is the secret number, ask a student to say the secret number aloud. Check the other numbers on the Minicomputer by letting students demonstrate that each of them can be put on the Minicomputer with one or two checkers. Reveal that 32 is written on your paper.

Exercise 3: Eli the Elephant

T: ***Today Eli the Elephant went to visit his grandmother. On the way to her house, he took a detour through a field where he had seen many peanut plants. Eli's grandmother loves peanuts, so Eli planned to bring her a bag full of peanuts as a surprise.***

Eli spent a long time collecting peanuts. He was thinking how pleased his grandmother would be when he gave her the bag of peanuts.

Eli's grandmother met him at the gate and gave Eli a big hug. Eli held out the bag and said, "Grandmother, I've brought you a bag of peanuts." Eli's grandmother was very happy. She gave Eli a big elephant kiss and went to get a bowl for the peanuts. She shook the bag and out fell—one peanut! She searched inside the bag and then Eli searched inside the bag, but there were no more peanuts. Eli felt confused.

What do you think happened to the peanuts Eli picked?

Allow the students to comment. Very likely someone will suggest that the other peanuts disappeared. If a student suggests that Eli ate the other peanuts, or that there was a hole in the bag, or any other reasonable answer, accept it as a possibility.

T: ***Why would the peanuts disappear?***

EW2

Guide this discussion until it is clear that a number of regular peanuts and the same number of magic peanuts would disappear.

T: ***Do we know how many peanuts Eli put in his bag?***

S: ***No.***

If students begin to give you suggestions of how many regular and magic peanuts Eli picked, record them on the board. If no one suggests any specific numbers, ask,

T: ***How many regular and how many magic peanuts could Eli have collected?***

Each suggestion should be considered to see if all but one peanut would disappear. Write the corresponding number sentence for each suggestion. Some possibilities are:

$$10 + \widehat{9} = 1$$

$$\widehat{10} + 9 = \widehat{1}$$

$$59 + \widehat{58} = 1$$

$$58 + \widehat{59} = \widehat{1}$$

$$\widehat{1{,}000} + 1{,}001 = 1$$

$$5 + 7 + \widehat{3} + \widehat{4} + 4 + \widehat{2} + \widehat{6} = 1$$

T: ***Poor Eli, he still doesn't know about magic peanuts. We will never know exactly how many peanuts Eli gathered that day for his grandmother.***

EW3 DETECTIVE STORY #3/ELI'S MAGIC PEANUTS #3

Capsule Lesson Summary

Solve a detective story with clues involving an arrow picture, odd numbers, and positive/negative number facts. Introduce Clarence the Crafty Crocodile who plays a trick on Eli the Elephant in which Eli discovers some of his peanuts are missing. What could Clarence have done in Eli's bag?

Materials

Teacher
- Colored chalk
- Minicomputer set

Student
- Paper

- Colored pencils, pens, or crayons
- Minicomputer set

Description of Lesson

Allow students to work in pairs during this lesson.

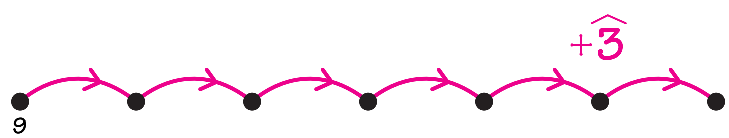
Exercise 1: Detective Story

Write a large 3 on a slip of paper and fold it so that 3 is hidden.

T: ***Today you are going to be detectives. You will have several clues which will help you discover a secret number.***

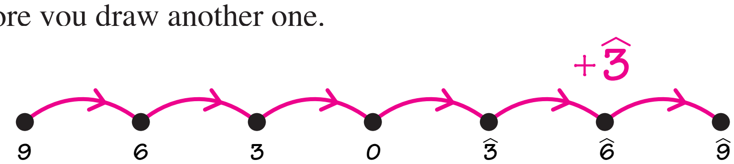
Clue 1

Draw this arrow picture on the board and tell the class that the secret number is in the picture. You may want to instruct student partners to copy the arrow picture and to label the dots along with the class.



Point to 9 and trace the first red arrow as you say, "$9 + \hat{3} = \ldots$? Nine regular peanuts and three magic peanuts is…?" If your class is not certain that the second dot from the left is for 6, draw Eli's bag with nine regular peanuts and three magic peanuts. Ask a student to show which peanuts would disappear. Look at the remaining peanuts and conclude $9 + \hat{3} = 6$.

As necessary, refer to Eli the Elephant to help your students label the remaining dots. Always erase a peanut picture before vou draw another one.



After all dots have been labeled, make a list of the possible numbers on the board and then erase the picture.

$$9,\ 6,\ 3,\ 0,\ \hat{3},\ \hat{6},\ \hat{9}$$

Clue 2

T: ***The second clue is that the secret number is odd. Which of these numbers is odd*** (point to the list on the board)***?***

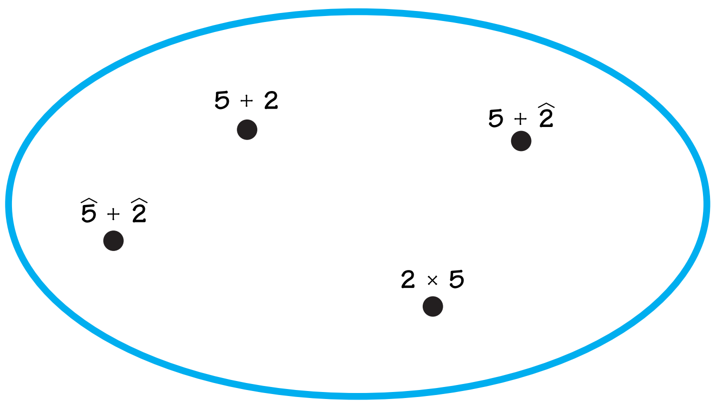
Consider each number in the list with the class and erase any numbers that are not odd.

$$9,\ 3,\ \widehat{3},\ \widehat{9}$$

T: ***The secret number could be 9, 3, $\widehat{3}$, or $\widehat{9}$. I will give you only one more clue.***

Clue 3

Draw this string picture on the board.



T: ***The secret number is in this string picture. When you know which one is the secret number, write it on your paper.***

After you have checked the responses of most students, do each of the calculations in the string picture. If necessary, draw peanut pictures to show that $\widehat{5} + \widehat{2} = \widehat{7}$ and $5 + \widehat{2} = 3$.

T: ***What is the secret number?***

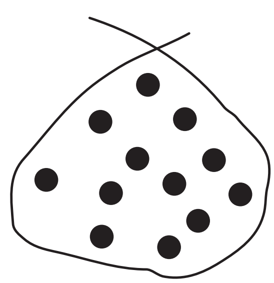
S: ***3.***

Reveal that 3 is written on the slip of paper.

Exercise 2: Eli the Elephant

T: ***One day Eli was walking through the jungle. There were 12 regular peanuts in his bag.***

Draw this bag of peanuts on the board.



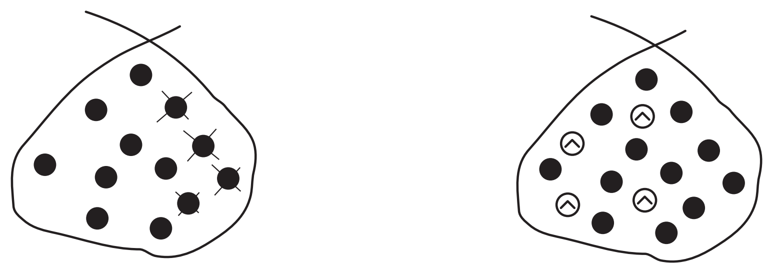
T: ***Eli walked a long time; he was very tired and decided to take a nap. As soon as Eli fell asleep, along came Clarence the Crafty Crocodile. Clarence saw that Eli was asleep and decided that it would be fun to play a trick on Eli. Clarence opened Eli's bag of peanuts and did something. Then he closed Eli's bag quietly and went away.***

Eli felt hungry when he awoke, so he opened his peanut bag. Eli counted his peanuts and cried, "Oh no! There are only eight peanuts. Four peanuts are missing." What do you think Clarence the Crafty Crocodile did?

Allow the class to discuss this problem. Two possibilities should emerge: Clarence could have removed four peanuts from Eli's bag, or Clarence could have put four magic peanuts into the bag.

Note: There are other more complex possibilities which might be suggested. For example, Clarence could have added two magic peanuts and removed two regular peanuts. Accept such suggestions as correct, but for the purpose of this lesson, focus primarily on these two possibilities: Clarence removed four regular peanuts, or Clarence added four magic peanuts.

Illustrate each of these situations as it is suggested. Write the appropriate number sentence under each bag of peanuts. Arrange the board so that the two pictures are side by side.



$12 - 4 = 8$ $\qquad\qquad$ $12 + \widehat{4} = 8$

Emphasize that you do not know what Clarence did when he opened Eli's bag, because removing four regular peanuts and adding four magic peanuts have exactly the same effect. Write this as a number sentence on the board.

$$12 + \widehat{4} = 12 - 4$$

T: ***Another day when Eli was walking in the jungle he gathered 55 regular peanuts. He became very tired and decided to take a nap. Can you guess what happened while Eli was sleeping?***

S: ***Clarence the Crafty Crocodile came by and played another trick on Eli.***

T: ***Eli felt hungry when he awoke, so he opened his peanut bag. Eli counted his peanuts and found that 29 peanuts were missing. What do you think Clarence the Crafty Crocodile had done?***

S: ***Clarence could have removed 29 peanuts from Eli's bag.***

S: ***Clarence could have put 29 magic peanuts into Eli's bag.***

As each possibility is suggested, write the appropriate number sentence on the board.

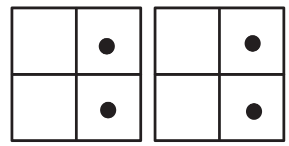
$$55 - 29 = ?$$

$$55 + \widehat{29} = ?$$

T: ***How are we going to find out how many peanuts Eli has left in his bag?***

If someone suggests drawing a picture of Eli’s bag, accept this as a good but not very practical method, because you would have to draw so many peanuts. If no one suggests using the Minicomputer, suggest it yourself. Point to the number sentences on the board.

T: ***Which of these problems do you want to do first on the Minicomputer?***



Suppose the students want to calculate 55 – 29 first. Invite someone to put 55 on the Minicomputer.

T: ***Can someone take 29 off the Minicomputer?***

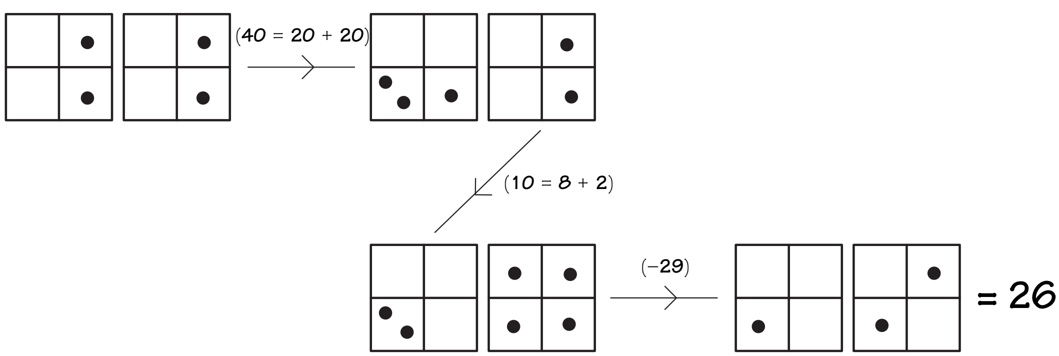
S: ***We do not have checkers in position for 29.***

Let someone point to where the checkers for 29 would be.

T: ***Who can make a backward trade that will help us to calculate 55 – 29?***

Ask a student to identify a trade before moving checkers. This will help to discourage trades that are not useful. Whenever a trade is made that puts a checker in a position for the subtraction, mention this to the class and indicate that they are getting closer to the goal.

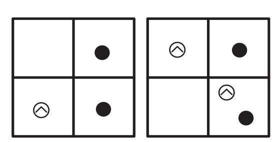
Let the students make trades until 29 can be taken off the Minicomputer. A possible sequence of trades is shown below.



Complete the appropriate number sentence on the board and remove the checkers from the Minicomputer.

55 – 29 = 26

T: ***Now let’s suppose Clarence put 29 magic peanuts into Eli’s bag.***

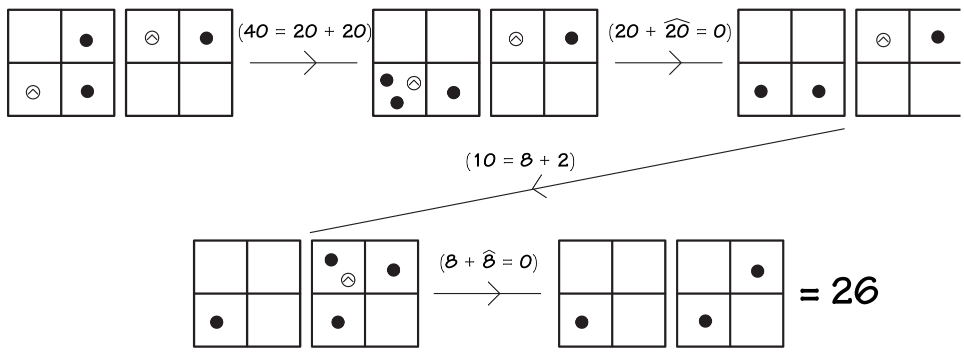


Invite a student to put 55 on the Minicomputer and another to put $\widehat{29}$ on the Minicomputer.

T: ***We want to get a regular and a negative checker on the same square so that we can remove them. Who can make a trade?***

Let students make trades until the checkers on the Minicomputer are all of the same kind. In this case, they will all be regular.

One sequence of trades is illustrated below. (Note: $1 + \hat{1} = 0$ has already been made.)



Complete the number sentence on the board.

$$55 - 29 = 26$$

$$55 + \widehat{29} = 26$$

Emphasize that we do not know what Clarence did when he opened Eli's bag, because removing 29 regular peanuts and adding 29 magic peanuts have exactly the same effect. Write this as a number sentence.

$$55 - 29 = 26$$

$$55 + \widehat{29} = 26$$

$$55 - 29 = 55 + \widehat{29}$$

T: ***Which calculation did you find easier on the Minicomputer?***

Accept either response, letting students comment on why they thought one calculation was easier than the other.

Continue this activity with a similar situation in which Eli has 83 regular peanuts and later discovers that 57 peanuts are missing. Let student partners choose which calculation they will do, either $83 - 57$ or $83 + \widehat{57}$, on their desk Minicomputer. It is not necessary to do both calculations.

Writing Activity

Ask students, possibly working with their partners, to write and illustrate a story about Eli and his friend Clarence. The story should include an example of what happens with Eli's peanut bag. Stories can be used to explain this episode of the Eli story to parents/guardians.