

WORKBOOKS INTRODUCTION

There are many opportunities for the student to work individually during the course of the lessons described in the other content strands. In the Workbooks strand, however, it is this individualized work which becomes the chief end of the majority of lessons. The goal in this strand to provide the students with opportunities

- to review many of the ideas they have met in other content strands;
- to apply their acquired knowledge to new situations requiring various kinds of strategic thinking; and
- to learn how to read and use mathematics workbooks.

The following workbooks are provided:

- 7-12
- Catalog of Problems #1
- 8?-15?
- Catalog of Problems #2
- Fishing for Numbers, Part I

Each workbook contains problems of varying levels of difficulty. Approximately the first ten pages of each workbook are easy problems, the next ten to twelve pages are average level difficulty, and the last ten pages are more challenging problems. For each workbook we suggest that all students start work at the easiest level (i.e., on page 2) and then work through as many pages as they can handle during the two lessons scheduled for that workbook. We estimate that, in a typical class, about two-thirds of the students will correctly finish the first ten pages, about one-third will finish the first twenty pages, and a few will finish all or most of the workbook. These proportions will vary from class to class.

This guide contains an answer key for each workbook. The key follows an introduction to the workbook and a suggested collective lesson. The lesson either presents the workbook to the whole class or provides a warm-up activity on a problem similar to one found in the workbook.

The first lesson in this strand is based on The Number Line Game. The game provides a situation in which students try to locate mystery numbers on the number line. To do so, they must develop a strategy to find the number given a relative location for it on the number line. This game can be played often during the year even though it appears explicitly only once as a lesson.

Several lessons present detective stories in which there is a secret number and several clues that lead to it. The clues provide review of many ideas from the other content strands and also present students with new situations involving patterns and counting techniques. Each detective story is paired with an episode in the continuing story of Eli the Elephant. The Eli the Elephant episodes present the students with a model of the negative integers and extend some operations on whole numbers to the integers.

WORKBOOKS INTRODUCTION

Use of the Workbooks Strand for Evaluation Purposes

The workbooks provide an excellent instrument to assess the progress of your students on a regular basis. You may not feel it is necessary to check every page and problem for each student, but you should develop a procedure for checking students' work with which you are comfortable. This may include checking one or more specific pages, discussing some particular mistakes with individual students and letting them correct their work, or just looking carefully at a few pages to be sure the students have understood the general idea of the problems in that particular book.

In the Blacklines, you will find a record-keeping tool for each workbook to help you assess student progress in the various strands. This tool may also assist you in parent conferences and in filling out periodic progress reports.

Here are some important points to bear in mind for workbooks.

- Always read the introductory material for each workbook and give the short introductory collective lesson(s).
- All students should start at the beginning of each workbook and progress as far as they can.
- All students should begin a new title on the same day, even if some students have not finished work on the previous title.
- Not all students should be expected to complete a given workbook. Only some students will reach the most challenging problems. Other students may succeed only in doing the easiest problems, although you should not assume this automatically—surprises are not at all uncommon.

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Capsule Lesson Summary						
Locate numbers on the number line. Define the distance between two numbers using the number line. Introduce The Number Line Game.						
		Materials				
Teacher	• Number line	Student	• Worksheets W1*, **, ***, and ****			
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Description of Lesson

Exercise 1: Introduction to the Number Line

Draw a line on the board. Graduate it into intervals, about six centimeters in length. Locate 0 at one of the marks and locate the number 3, three marks to the right.



Point to marks on the number line asking students to label them. A sample dialogue follows:

- T: I have shown you where some of the numbers are on this number line. If 0 is here (point to a) and 3 is here (point to d), what number is here (point to c)?
- S: 2.
- T: What number is here (point to f)?
- S: 5.
- T: What number is here (point to i)? S: 9. a b e g h i j k bord write these letters on the board. They are here just to make the description of the lesson easier to follow

n

After each of these questions, let a student point to the appropriate mark and label it.

2 3

5

9



Point to the remaining marks between \mathbf{a} and \mathbf{k} , ask the class for the appropriate numbers, and label the marks yourself.

Some students may suggest erasing the line to the left of 0. If this happens, say, "There are many numbers that are less than 0. You will meet them another day."

T: The number line goes on and on in both directions. That is what the arrowheads mean. Point to where 20 would be on the line.

Some students may protest that 20 is not on the part of the line drawn, but most will point to the right-hand side of the picture.

T: Think of a very large number. Point to where it would be on this line.

Use your thumb and index finger to indicate one space.

T: The distance between 2 and 3 is 1.

Use your thumb and index finger to count two spaces.

T: The distance between 4 and 6 is 2. What is the distance between 9 and 12?

S: 3.

Ask for a volunteer to count the spaces. Do not insist that the students use the thumb and index finger method shown in the previous diagram as long as their methods are accurate. Continue this activity with the following or similar questions.

T: What is the distance between 9 and 4? (5) What is the distance between 5 and 12? (7)

Erase the numerals but not the line. Label one of the marks 28 and locate 34 six marks to the right.



T: If 28 is here (point to a) and 34 is here (point to c), what number is here (point to b)?

S: 30.

Let a student label the mark 30. Continue in this manner until all the marks between 28 and 34 are labeled.



- T: What are some numbers which will be to the left of 28 on this number line?
- S: 27.
- S: 15.



- What are some numbers which will be to the right of 34? T: S: 100. *40*. S: **T:** Where is 20 on this number line? S: To the left of 28. T: Where is 50 on this number line? S: To the right of 34. T: What are some of the numbers between 50 and 34? S: *40*.
- S: 36.

Lessons on the number line should be light and move briskly. Students will become accustomed to the number line as they use it in many activities.

Exercise 2_____

Leave the portion of the number line you have at the end of Exercise 1 on the chalkboard. To one side draw a large string and fit a red and a blue string snugly inside. You will be putting whole numbers (0, 1, 2, 3, ...) inside the strings with even numbers in the red string and odd numbers in the blue string. Of course, there are no whole numbers that are neither odd nor even.



T: This black string is for whole numbers. I have a rule for deciding which numbers go in the red string and which go in the blue string. See if you can guess my rule.

Note: It is appropriate for you to begin using the term *whole number* so that students will become familiar with it; however, do not expect your students to use the term. If a student asks for an explanation of whole numbers, simply reply that you are referring to the numbers 0, 1, 2, 3, and so on.

Invite students to suggest whole numbers for you to place in the picture. If a student suggests a negative number or a fraction, say simply that it is not a whole number. After you have placed several numbers with at least a couple in each of the red and blue strings, ask students to guess where you will place their suggested numbers. You may like to let students point to where you should put some numbers.

When it appears that many students know where to place odd and even numbers in the picture, ask if they can describe your rule.

- T: I think many of you know my rule. Who would like to explain it?
- S: The numbers in the red string end in 0, 2, 4, 6, or 8. The numbers in the blue string end in 1, 3, 5, 7, or 9.



- S: You can get the numbers in the red string if you start at 0 and count by twos. They are two apart on the number line.
- T: How about the numbers in the blue string?
- S: Every other number is in the blue string.
- S: You can get the numbers in the blue string if you start at 1 and count by twos.

Let students express their ideas in many ways. Some may remember about even and odd numbers and may use these names. You may find it helpful to choose several consecutive numbers such as 35, 36, 37, 38, 39, 40, and 41 to place in the picture. Rephrase the rule to relate even and odd to counting by twos, and demonstrate counting by twos on the number line.

T: The numbers in the red string are called even numbers. We get even numbers if we start at 0 and skip-count (count by twos). The numbers in the blue string are called odd numbers. We get odd numbers if we start at 1 and skip-count (count by twos).

Exercise 3: The Number Line Game

The Number Line Game is one which may be played frequently throughout the year whenever you finish a lesson early or have some extra time. A game probably will take only about five minutes. Several variations of the game are given at the end of this lesson. Use these variations when your class is ready for them.

T: I'm going to show you a game you can play on the number line. Some of you may have played it in first grade. I'm thinking of a number between 0 and 50. I'll show you where it is on the number line; my number is hidden in this box.

Draw a line on the board and draw a box for the secret number.



Note: Begin with the box near the center of your line. For convenience in locating numbers, you may wish to extend the line during the game.

The following is a description of a possible game. Suppose the secret number is 37.

- T: Guess what my secret number is.
- S: 20.
- T: I'll show you where 20 is on this number line. It might not be exactly in the right place, but I'll try to make it as close as possible.



- T: 20 is less than my number.
- S: 45.



T: My number is between 0 and 50. 60 cannot be my number.

Any time the guess is a number which is not between 0 and 50, do not write anything. Just remind the students that your number is between 0 and 50, and ask someone else to guess.

S: 25.

T:



Note: Although a guess may give no new information, such as 25 in this sample game, do not criticize or comment except to say that it is more or less than your number.

S:	40.						
T:	40 is more than my number.	\leftarrow					\rightarrow
S:	38	·	20	25	30	40	45
T:	38 is more than my number.	\leftarrow				38 	\rightarrow
S:	37.		20	25	30	40	45
T:	You discovered my secret number.	\leftarrow				38 	\rightarrow
	number.		20	25	30	37 40	45

Play the game a few more times with secret numbers between 0 and 50. Let the student who guesses your number in a game choose the secret number for the next game and whisper it to you. Then ask the student to keep the number secret and not to play in that game.

Worksheets W1*, **, ***, and **** are available for independent work. After allowing about ten minutes for students to work independently, you may want to do the **** worksheet as a collective activity with the class. Putting the exercises on an overhead projector may facilitate the discussion. Blackline W1 has blank number lines for this purpose, or to use to prepare additional practice.

Home Activity

Suggest that parents/guardians practice counting out loud with their child, both forward and backward. Start the count at numbers such as 18, 37, 82, or 179. The first time you send an activity home, you may want to send along the letter about home activities in general and/or the *CSMP* introduction letter. See the Home Activity Blacklines.

VARIATIONS OF THE NUMBER LINE GAME









Variation 1

When your class is comfortable with the game using numbers between 0 and 50, you can expand the range of possible numbers. Some possibilities are

- numbers between 0 and 100;
- numbers between 100 and 200;
- numbers between 0 and 200;
- numbers between 0 and 500;
- numbers between 500 and 1,000.

Variation 2

As the students become familiar with the number line game and negative numbers, you can expand the range of possible numbers to include negative numbers. Some possibilities are

- numbers between $\widehat{50}$ and 50;
- numbers between $\widehat{100}$ and 100.

Variation 3

As the students become familiar with numbers such as $2\frac{1}{2}$, occasionally you can choose a noninteger number as your secret number. It is best to limit the range of possible integers to those between 0 and 10 when you use this variation.

Variation 4

When you are using only whole numbers for your range of possible numbers, ask a student to choose a secret number and to judge the guesses of the other students. The student who chooses the secret number tells the class whether a guess is more or less than the secret number. You label a mark on the number line for each guess and, if necessary, help the student to judge. Remind student judges not to give additional clues.

W2 DETECTIVE STORY #1/ELI'S MAGIC PEANUTS #1

Capsule Lesson Summary

Solve a detective story for which the secret number is an odd number and where the other clues involve an arrow picture and the Minicomputer. Tell the story of Eli the Elephant who discovers magic peanuts. When a magic peanut meets a regular peanut, both disappear, much to Eli's confusion.

	Mate	erials
cher	 Coloredathater Minicomputer set 	Colored pencils, pens or crayonCalculatorMinicomputer set
Student	• Paper	• Worksheets W2* and **

Exercise 1: Detective Story

Before the lesson, write a large 21 on a slip of paper and fold the paper so that 21 is hidden.

Ask the class what a detective does. Guide the discussion to include the idea that a detective tries to uncover secrets by following clues. Tell the students that they are going to be detectives today, and they will be able to discover a secret number if they follow the clues.

Note: You may like to make the secret number something real for students to figure out such as "How old is my brother?" or "How many miles do I drive to school?" or "How many spots on my dog?"

Clue 1

T: The name of a secret number is written on this paper. Listen carefully to my story and you can discover what it is. The first clue I will give you is that the secret number will appear on the calculator display when we teach the calculator to count by twos starting at 1.

Remind or show students how to teach the calculator to count by twos.

- 1. Put on the starting number (1).
- 2. Press + 2.
- 3. Then press $\equiv \equiv \equiv$ and so on.

Observe which numbers appear and suggest that students write ten or more of these numbers on their papers in the order they appear. Encourage students to make observations and, if not mentioned, tell the class that the secret number is an odd number. Begin listing the positive odd numbers in order on the board with the students' assistance. Start with 1 and continue to at least 25. Mention that you could continue pressing \equiv on the calculator or writing odd numbers all day, but instead you will put three dots at the end of your list meaning "and so on."

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, ...

T: Now we know that the secret number is one of these numbers, but we do not know which one. What do we need?

S: Another clue.

Clue 2

Draw this arrow picture on the board and ask students to copy it on their papers.

T: The secret number is here (point to **s**) in this arrow picture. What are the red arrows for?

S: Is less than.



For example, suppose the class is not sure whether **s** could be 25 or not.

Trace the arrow from 16 to 25.

- S: 16 is less than 25.
- T: Is that true?
- S: Yes.

Trace the arrow from 16 to 24.

- S: 16 is less than 24.
- T: Is that true?
- S: Yes.

Trace the arrow from 25 to 24.

- S: 25 is less than 24.
- T: Is that true?
- S: *No.*
- T: So our secret number cannot be 25.

Erase 25 from the arrow picture and cross it out of the list of odd numbers.



S

16

is less than

As the class discovers which numbers in the list of odd numbers cannot be the secret number, cross them out. Continue until every odd number in your list has been considered. Ask if there are any other odd numbers which could be the secret number. Commend a student who concludes that none of the numbers indicated by the three dots at the end of the list could be the secret number because they are all more than 24. If necessary, supply this information yourself and cross out the three dots at the end of the list. Erase everything that has been crossed out.

X, X, X, X, X, X, X, 1X, 1X, 15, 17, 19, 21, 23, 25, 27, ----

Clue 3

Display two Minicomputer boards.

T: Now we know that the secret number is one of these numbers: 17, 19, 21, or 23. Listen carefully because I will give you only one more clue. The secret number can be put on the Minicomputer with exactly two checkers.

Ask the students to put the secret number on their desk Minicomputer and check some of their responses.

T: Do you know the secret number?

S: 21.

Invite a student to put 21 on the demonstration Minicomputer with two checkers. Ask how many checkers would be needed to put each of the numbers 17, 19, and 23 on the Minicomputer. Show the class that 21 is written on your paper.

Exercise 2: Eli's Magic Peanuts

Veteran students will have met Eli in the first grade. They can help you tell this story.

T: There is an elephant named Eli. Eli is always very hungry. What do you think is Eli's favorite food?

Accept some answers from the class and then tell them that Eli's favorite food is peanuts.

T: Eli likes peanuts so much that he always keeps a little bag of peanuts with him wherever he goes. One day as Eli was out walking, he found a plant that looked a lot like a peanut plant, but a little different than a regular peanut plant. Eli pulled up this strange plant and discovered five peanuts. Eli didn't know it, but the peanuts from this plant were magic peanuts! Eli gathered some of the magic peanuts and put them in his bag with the other peanuts. What do you suppose is so special about magic peanuts?

Allow students to discuss this briefly.

T: Let me show you what happens when Eli puts both regular peanuts and magic peanuts in his bag.

W-14

W2

Draw this picture on the board. If you prefer, use one color of magnetic checkers for the regular peanuts and \otimes checkers for magic peanuts.

- T: This is Eli's bag of peanuts with five regular peanuts in it. Now Eli also put some magic peanuts into his bag; here are the magic peanuts.
- T: How many magic peanuts did Eli put into his bag?
- S: Four.
- T: When Eli returned home, he was hungry from walking all day. He decided to eat some peanuts. When he opened his bag, he was very surprised. There was just one regular peanut in the bag. What do you think happened when Eli put both the regular and the magic peanuts in the bag?

Let students make suggestions and lead the discussion to the idea that when a regular and a magic peanut come together, they both disappear. Model this idea in the picture by pairing a regular peanut with a magic peanut and then removing them both from the picture.

If you use connecting lines to pair magic peanuts with regular ones, leave the picture on the board. Write an appropriate number sentence next to the picture.

- T: Five regular peanuts plus four magic peanuts is ...?
- S: One regular peanut.
- S: 1.
- T: Poor Eli was puzzled. He didn't know the secret of the magic peanuts and he couldn't imagine where his peanuts had gone! He was still hungry, so he went looking for more peanuts. This time, he found eight regular peanuts and put them into his bag.

Erase your previous picture and draw a new one.

T: Eli also found three magic peanuts and put them into his bag.

Add these three magic peanuts to your drawing.

T: When Eli returned home, what do you suppose he found when he opened his bag?

Ask several students to explain their answers; then choose volunteers to pair up regular peanuts with the magic ones. Write the number sentence next to the picture.

- T: Eight regular peanuts plus three magic peanuts is...?
- S: Five regular peanuts.





 $8 + \hat{3} = 5$



Do a couple more examples such as the ones shown here. Include one example in which Eli finds only magic peanuts when he opens the bag.



Worksheets W2* and ** are available for independent work. Tell the students to finish the calculations for each of Eli's bags of peanuts. The students should draw lines connecting regular and magic peanuts themselves. Ask students who finish quickly to write their own stories about Eli's peanuts.



This would be a good time to send home a letter to parents/guardians about negative numbers. Along with the letter, you can send a page of problems like one of the worksheets. Students should explain to their parents/guardians about Eli and magic peanuts and show them how to solve the problems. Blackline W2 has a sample letter.





Capsule Lesson Summary

Find many names for the number 12, using the Minicomputer to suggest some of them. Pose this problem: Is the number 12 on a +2 arrow road which contains the number 15 or on a +2 arrow road which contains the number 16? Begin the 7-12 Workbook. In the first section of this workbook, find the number 7 on each page. In the second section, find the number 12 on each page.

Materials

Teacher • MiniColopedechalk

Student • 7-12 Workbook



Call on several students to respond to each of these questions:

T: 12 is more than...? 12 is less than...?

Display three Minicomputer boards and ask someone to put 12 on the Minicomputer.



T: Who can put 12 on the Minicomputer in a different way?

Perhaps a student will suggest this configuration.

T: How do you see 12 on the Minicomputer?

S: As 8 + 4.

Write an appropriate number sentence on the board.

Continue asking for other ways to put 12 on the Minicomputer until you have at least four number sentences about 12 on the board.

Several examples are given below.



Put away the Minicomputer boards.



8 + 4 = 12

- T: What are some other names for 12?
- S: 15 3.
- S: (2 x 3) + 6.

Encourage students to suggest many names for 12 and record them on the board. If expressions involving subtraction or multiplication are not suggested, suggest some of these yourself.

T: You have found many names for the number 12. Can you find 12 in this arrow picture?

Draw this arrow picture on the board.

T: Do you think the number 12 is in this arrow picture? If you think it is, which road is it on?



Ask a few students to explain their choice of either the top or the bottom road. If many students believe that 12 is on the top road, collectively label the dots of this road and discover that 12 is skipped.

When your class has decided that 12 cannot be in the top road, direct their attention to the bottom road. Ask whether 12 would be to the right or to the left of 16, and let a few students explain their answers. Collectively label the dots until 12 has been found.

It is possible that your class will need to label only two dots to find 12, but they may wish to label the other dots. Make it clear that it is okay to label the other dots, but not necessary because your objective was only to find 12.

Distribute copies of the 7-12 Workbook.

Explain that the workbook has two sections. In the first section, students are to find the number 7 on each page. In the second section, they are to find the number 12 on each page.

Note: Once students have found and have labeled the dot for 7 (or 12 in the second section) on a page, they do not need to label any other dots on that page but may do so if they wish.

At the end of the class period, collect the workbooks for use in Lesson W4.

Capsule Lesson Summary

Play a cooperative game with the calculator in which you start with a number on the display, and try to reach a target number. Players can press \pm , \Box , or \boxtimes any number, followed by Ξ , to reach the target. Continue work in the 7-12 Workbook. (This is the second of two lessons using this workbook.)

Materials							
Teacher	• Overhead or class calculator	Student	• 7-12 Workbook				

Description of Lesson

Using an overhead calculator or a class calculator, put 67 on the display.

T: 67 is on the calculator and our target is 200. We are going to work together to reach the target. When you take a turn you can press ⊕, ⊡, or ⊠ any number you want and then ≡. Let's see how quickly we reach the target.

Call on students one at a time. After each turn, announce where you are and repeat the target as well as the rule. For example:

T: Anthony pressed ± □ □ □ = (read as "plus fifteen equals"). The number on the calculator is now 82 and our target is 200. The next person can press ±, □, or ⊠ any number and then □. Remember, we are trying to reach 200.

Continue this activity letting different students take turns until the class gets 200 (the target) on the calculator. If appropriate, you may like to keep track of how many turns it takes and challenge the class to reach a second target in fewer steps. For example, start with 56 on the display and set 175 as the target.

This calculator target game is one you may wish to use at other times when you finish a lesson early or have some extra time. You should find that students get better at estimating the difference between the starting number and the target and the games get very short. At other times you may like to give students similar target problems to work on with a partner.

Distribute the students' copies of the 7-12 Workbook and let them work independently for the remainder of the lesson. If some students finish the workbook early, suggest they make up their own problems to add to this workbook or for a similar workbook. At the end of the lesson, collect the workbooks for your review.

Assessment Activity

An individual student progress record for the workbook is available on Blackline W4(a). You may like to use this form to monitor student work.

Home Activity

If you choose to send workbooks home with students, you may want to include a letter to parents/guardians with this first workbook. Blackline W4(b) has a sample letter.



























































Exercise 1: Eli the Elephant_____

T: Who remembers Eli the Elephant?

Allow students to recall whatever they remember. Be sure to mention that Eli collects peanuts; he sometimes finds regular peanuts and sometimes magic peanuts. Review what happens when a magic peanut and a regular peanut meet.

T: Today Eli went walking in the park and found seven regular peanuts and four magic peanuts. He put all the peanuts in his bag. He plans to eat the peanuts tonight while he watches television. How many peanuts will he find when he opens his bag?

Draw a picture of the bag with regular and magic peanuts in it. Write a number sentence about the situation on the board.

- T: Seven regular peanuts plus four magic peanuts is...?
- S: Three regular peanuts.

Complete the number sentence. Display two Minicomputer boards.

- T: Let's show this story about Eli's peanuts on the Minicomputer. How many regular peanuts did Eli find?
- S: Seven.

Invite someone to put 7 on the Minicomputer. Show the class the negative checkers, \otimes .

- T: We can use negative, or magic, checkers to show how many magic peanuts Eli found in the park. How many magic peanuts did he find?
- S: Four.
- **T:** Show this on the Minicomputer using negative checkers.



 $7 + \hat{4} = ?$

Do not allow the students to remove any checkers from the Minicomputer.

T: What will happen?

S: The regular checker and the magic checker on the purple (4-) square will disappear.

Explain that four magic peanuts and four regular peanuts disappear and ask a volunteer to show the class what happens on the Minicomputer. The student should remove both checkers from the purple square.

- T: Seven regular peanuts plus four magic peanuts equals...?
- S: Three regular peanuts.

Erase the board and remove the checkers from the Minicomputer.

- T: Last week Eli found a lot of peanuts. One day he found 39 magic peanuts and 25 regular peanuts. When Eli opened his bag that night, what did he find?
- S: Magic peanuts.

Write a number sentence about this situation on the board.

If several students know the answer, ask them to write the result on their paper or whisper the number to you. Encourage students to estimate how many magic peanuts are in the bag.

- T: Who can put 39 on the Minicomputer with negative checkers? Who can put 25 on the Minicomputer with regular checkers?
- T: What happens?
- S: The checkers on the 1-square disappear, and the checkers on the 20-square disappear.

Invite students, one at a time, to remove the checkers from the 1-square and then from the 20-square. Encourage students to explain that one regular and one magic peanut disappear, and then twenty regular and twenty magic peanuts disappear.

- T: What could we do to make the number easier to read?
- S: Make a backward trade.

Do not expect the verbalization of this trade to be exact. If a student says, "8 = 4 + 4," accept it, but rephrase it by saying that eight magic peanuts is the same as four magic peanuts plus four magic peanuts.

If a student suggests that checkers not on the same square will disappear, for example, $\hat{8}$ and 4, remind the class that those checkers are for eight magic peanuts and four regular peanuts.

If no one suggests a backward trade, suggest it yourself.



	•	•

		\otimes	•
•	\otimes		 ●

39 + 25 = ?



Complete the number sentence on the board.

 $\widehat{39} \times 25 = \widehat{14}$

Repeat this exercise with $4\hat{6}$ + 22. Before continuing, erase the board and remove all the checkers from the Minicomputer.

Exercise 2: Detective Story

Allow students to work as math partners during this exercise.

Do a short review of even and odd numbers before beginning the detective story. Write a large 12 on a slip of paper and fold the paper so that 12 is hidden.

T: The name of a secret number is written on this paper. Listen carefully to this story and you can discover my secret number.

Clue 1

Display one Minicomputer board. Provide student pairs with one Minicomputer sheet and two checkers.

T: The secret number is on the ones board of the Minicomputer using exactly two checkers. You cannot see the checkers because they are invisible. What are some numbers it could be?

Direct student partners to cover one of the Minicomputer boards on their sheet with paper. Then ask them to work together to find possibilities for the secret number and to compile a list. After a few minutes begin a class list. Each time a number is suggested, ask a student to show how it can be put on the Minicomputer with two checkers. Record each number on the board as it is suggested, leaving space so that they can be recorded in order. Accept three or four suggestions from the students before continuing.

T: What is the least number it could be? (2) What is the greatest number it could be? (16) Let's finish our list of numbers that could be the secret number.

Continue until your list has all ten possibilities. Ask student partners to check that they, too, have this complete list of possible secret numbers.

2, 3, 4, 5, 6, 8, 9, 10, 12, 16

- **T:** Now we know that the secret number is one of these numbers, but we do not know which one. What do we need?
- S: Another clue.

Clue 2

Distribute copies of Worksheet W5 to the student partners.

T: The secret number is one of the numbers in this arrow picture. Which number could it be?

Allow the students to work on the arrow picture with their

partners. While they are working, draw the arrow picture on the board. When many of the students have completed the worksheet, complete the arrow picture on the board collectively.

Point to the list of numbers on the board from the first clue.

T: Which of these numbers cannot be the secret number? Help me cross them off our list.

Students should tell you to cross off 3 4 6 and 10 because these numbers are not in the arrow picture. Student partner 2, 3, 4, 5, 6, 8, 9, 10, 12, 16

+3

15

12

T: Now we know that the secret number could be 2, 5, 8, 9, 12, or 16. Here is another clue.

Draw this string picture on the board and instruct students to copy it on their papers.



T: Our secret number is in this string picture (point to **s**). Is the secret number an odd number? (No) Is it more than 8? (Yes) Which of these numbers could be the secret number?

Allow student partners a minute or two to put the possibilities (after Clue 2) into the string picture. When a number is suggested, use it to label the dot and then ask the class if that number could be there. Suppose a student suggests that the secret number could be 8.

T: Could this be 8? (Disagreement) Is 8 an odd number? (No) Then 8 must be outside the red string. Is 8 more than 8? (No) Then 8 must be outside the blue string. Where should 8 be in this string picture?

Let a volunteer point to the correct region.

T: 8 cannot be the secret number.

Put 8 correctly in the string picture and cross it off the list on the board.

For a correct suggestion, elicit class agreement that it could be the secret number and circle that number on your list. Continue until all of the numbers in the original list are either circled or crossed off. Erase those which have been crossed off. W-32
12, 16

Clue 4

T: The secret number is either 12 or 16. I will give you one more clue. The secret number can be put on the Minicomputer with three checkers, all on the same square.

Ask the student partners to write the secret number on a piece of paper and put it on their desk Minicomputers.

- T: Which is the secret number?
- S: 12.

Invite someone to put 12 on the Minicomputer with three checkers, all on the same square (three checkers on the 4-square). Reveal that 12 is written on your paper.

Capsule Lesson Summary

Decide who has more rocks in his collection—Derrick or Jenny—after various additions are made. Extend experiences from various content strands in the workbook *Catalog of Problems #1*.

Materials

Teacher	• None	 Student Catalog of Problems #1 Workbook Colored pencils, pens, or crayons
1		

Description of Lesson

Tell the class a progressive story to review < and >.

T: One day Derrick and Jenny each found three special rocks to bring to school for a science lesson.

Reco	ord this information on the board.	Derrick	Jenny	
T:	The next day Derrick found two more special rocks and Jenny found six more.	3	3	
Add	this information to what is on the board.	Derrick	Jenny	
T:	Who found the most rocks during those two days?	3 + 2	3+6	
S:	Jenny.			
	plete this number sentence on the board as you "Three plus two is less than three plus six."	Derrick 3 + 2 <	Jenny 3 + 6	

- T: Derrick and Jenny each displayed their rocks on a piece of cardboard to bring to class. How many rocks did Derrick put in his display?
- S: Five.
- T: How many rocks did Jenny put in her display?
- S: Nine.

Replace 3 + 2 with 5, and 3 + 6 with 9.

- Derrick Jenny 5 < 9
- T: The teacher was pleased with the work these children had done and gave each of them another special rock, a piece of quartz, for their collections. How can I show this in our number sentence (point to 5 < 9)?
- S: Add 1 to each side.

Ask a	student to write this on the board.	Derrick 5 + 1 <	Jenny 9 + 1
T:	Do we need to change this symbol (point to <)? Is 5 + 1 less than 9 + 1?	5+1 <	9 + 1
S:	Yes.		
T:	$5 + 1 = \dots ?$ (6) $9 + 1 = \dots ?$ (10)		
Repla	ace the expressions on the board with their sums.	_	Jenny
T:	Derrick's older brother gave Derrick ten new rocks for his collection. How can we show this in our number sentence?	6 <	10
S:	Add 10 to Derrick's side.		
Ask a	student to write this on the board.	Derrick 6 + 10 <	•
T:	Is this new number sentence true?		10
S:	No.		
T:	Who can fix it?		
	e someone to correct the statement on the board and to read the number sentence.	Derrick 6 + 10 >	-
T:	Now who has more rocks?		
S:	Derrick; he has sixteen.		
Repla	ace 6 + 10 with 16.	Derrick 16 >	Jenny 10

Distribute copies of the workbook *Catalog of Problems #1* and let the students begin working independently. If the questions you receive indicate that many students are having difficulty with a particular page, hold a collective discussion about that page.

Additional Practice

Teach students a two-person game involving addition facts and more-less-same comparisons.

Materials: Paper; pencil; die (or number cube 1–6)

Play:Each player rolls the die for a starting number.Each player rolls the die for a second number which they add to their starting number.Players record the results in a number sentence. For example:

	Аььеу		Roland
First roll + second roll:	4+2	<	6 + 1
Rewrite:	6	<	7

Continue until each player has rolled the die five times. With each play, record the new number, check the new number sentence, and rewrite.

Example:	Аььеу		Roland
	6+6	>	7 + 3 (three rolls)
	12	>	10 (rewrite)
	12 + 4	>	10 + 2 (four rolls)
	16	>	12 (rewrite)
	16 + 1	>	12 + 3 (five rolls)
	17	>	15 Abbey wins
Home Activity			

You may like to send a letter home about Goldy and students' use of the symbols <, >, and =. Blackline W6 is such a letter. Suggest to parents/guardians that they play a game similar to the one here with their child.

W7 CATALOG OF PROBLEMS #1 LESSON TWO



Description of Lesson

Use an overhead calculator or a classroom calculator for this warm-up activity.

T: I am going to teach the calculator a secret rule: plus some number or minus some number or times some number.

Prepare the calculator for a secret rule without letting the class see what keys you press. For example, teach the calculator +5 by pressing \Box \Box \Box \Box \Box \Box . Show the class that 0 is on the display.

T: Now the calculator is ready to use my secret rule. When you give the calculator a number and then press ≡, the calculator will use the secret rule on your number and show you the result.

Invite several students to put numbers on the calculator and press \equiv . Many students will guess the secret rule quickly, but suggest that they write the rule on their papers, not say it aloud. Tell them that some in the class may still need to test more numbers. When you believe most students know the secret rule, let someone tell the class.

S: The secret rule is +5. Each time we give the calculator a number, it adds 5 and tells us the result.

Test the rule with a few more numbers. Then let students predict the result for other entries before pressing \equiv . That is, use the calculator to do some mental arithmetic involving +5.

Repeat this activity a couple times with other rules. For example:

- -3 (Prepare the calculator by pressing $\exists \exists \equiv .$)
- 2x (Prepare the calculator by pressing $2 \times 0 \equiv$.)

Note: You can prepare the calculator to add, subtract, or multiply by any number *n* as follows:

- +n (Press n + n =.)
- -n (Press $\underline{n} \underline{n} \equiv$.)
- nx (Press $\square \times \bigcirc \equiv$.)

Check that your calculator has a constant feature that operates in this way. See "Role and Use of Calculators" in Section One of this manual for a description of the constant mode for calculators.

Most classes are ready at this time only for rules like + or - a one-digit number. However, this activity is one you may like to use at other times when you finish a lesson early or have some extra time.

Distribute students' copies of the workbook *Catalog of Problems #1* and let them work independently. At the end of the lesson, collect the workbooks for your review.

Assessment Activity

An individual student progress record for the *Catalog of Problems #1* Workbook is available on Blackline W7(a). You may like to use this form to monitor student work.

Home Activity

This is a good time to send a letter to parents/guardians about the use of calculators. Blackline W7(b) has a sample letter.













1	÷	2	<	6
4	٠	4	•	7
	۹		-	5 + 4
3	÷	3	5	5
12	÷	3	~	16
Ö	+	z	-	9 + 3
5	+	8	×	4 + 8
6	+	5	4	6 + 6
11	+	5	,	12 + 2

W-42





















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Minicomp	•	an arrow picture, a string picture, and the lephant and continue solving addition mbers.
	Mate	erials
Teacher	Colored chalkMinicomputer set	 Colored pencils, pens, or crayon Worksheets W8 (no star), W8*, **, ***, and ****
Student	• Paper	CalculatorMinicomputer set

Exercise 1: Detective Story

Write a large 60 on a slip of paper and fold it so that 60 is hidden.

T: The name of a secret number is written on this paper. Listen carefully to my clues and you can discover it.



Distribute copies of Worksheet W8 and draw the arrow picture from the worksheet on the board.

T: The secret number is one of the numbers in this arrow picture. Which numbers could it be?

Allow students (possibly working in pairs) a few minutes to label dots in their pictures. Make calculators available if students want to use them. When many students have completed the worksheet, invite them to help you label the dots in the picture on the board. After all the dots have been labeled, make a list of the numbers that could be the secret number. Ask students to make the same list on their papers.



44, 47, 50, 53, 56, 59, 60, 61, 64, 67, 70

T: We know that the secret number is one of these numbers but we don't know which one. We need another clue.

Clue 2

Erase the arrow picture and then draw this string picture on the board.

T (pointing to s): Here is the secret number. Is it less than 70? (Yes) Is it an odd number? (No)

Point to the list of numbers from the first clue.

T: Which of these numbers could be the secret number?

When a number is suggested, use it to label the dot and then ask the class if that number could be there. Suppose a student suggests that the secret number could be 67.

T: Could this be 67? (Disagreement) Is 67 less than 70? (Yes) Then 67 must be inside the blue string. Is 67 odd? (Yes) Then 67 must be inside the red string. Where should 67 be in this picture?

Let a student point to the correct region (the middle).

T: 67 cannot be the secret number.

Locate 67 correctly in the string picture and cross it off the list on the board. Students should also cross 67 off their lists.

For a correct suggestion, elicit class agreement that it could be the secret number and circle it on your list. Continue until many of the numbers in the original list are either circled or crossed off. Point to each unchecked number and guide your class in deciding if it could be the secret number. When all the numbers in the list have been checked, erase those that have been crossed off. You may ask students to rewrite these numbers, especially if their previous lists are hard to read.



T: The secret number is one of these numbers. I will give you one more clue.

Clue 3

Display two Minicomputer boards with this configuration. Ask students to put this number on their desk Minicomputers.



T: There are two checkers on the Minicomputer. This is not the secret number, but we can get the secret number with one more checker.

Ask students to put the secret number on their desk Minicomputers and to highlight it on their papers. Check to see if most students are understanding the clue.



T: Which is the secret number?

S: 60.

Invite a volunteer to place one more checker on the demonstration Minicomputer to show 60. Reveal that 60 is written on your paper.

Exercise 2: Eli the Elephant

Talk briefly with the students about Eli the Elephant until everyone remembers what happens when a regular and a magic peanut meet. Distribute Worksheets W8*, **, ***, and **** in turn. Emphasize that the pictures are there to help the students think about the number sentences, but that they do not have to draw connecting lines in every picture.

If some students finish these worksheets very quickly, ask them to write their own Eli the Elephant stories.



Name______WB **
compare.

$$2 + 4 = 6$$

 $5 + \hat{4} = 1$
 $\hat{3} + \hat{3} = \hat{3}$
 $4 + \hat{9} = \hat{5}$
 $\hat{5} + 6 = 1$
 $\hat{5} + \hat{6} = 1$
 $\hat{5} = \hat{5}$
 $\hat{5} + \hat{6} = 1$
 $\hat{5} = \hat{5}$
 $\hat{5} = \hat{5}$

We
 ###

 Complexe.

$$\widehat{2} + 3 = __1$$
 $\widehat{3} + \widehat{3} = __0$
 $10 + \widehat{1} = __9$
 $10 + \widehat{1} = __9$
 $4 + 3 = __7$
 $\widehat{9} + \widehat{3} = __1\widehat{2}$

Name______
Name______
complex.

$$22 + \widehat{2} = __{20}$$

$$\widehat{18} + 18 = __{9}$$

$$20 + \widehat{10} = __{10}$$

$$5 + \widehat{15} = __{1\overline{2}}$$

$$\widehat{1} + |00 = __{3\overline{2}}$$

$$\widehat{1,000} + 1,000 = __{9}$$

Capsule Lesson Summary

Locate a number name that does not belong in a list because it is the only one that is not for 8. Pose this problem: Is the number 8 on a +2 arrow road with the number 5 or on a +2 arrow road with the number 14? Once decided, locate the dot for 8 by extending one of the roads. Do pages 2 and 3 of the Workbook 8?-15? collectively. In the first section of this workbook, find the number 8 on each page or extend the picture to include 8. In the second section, find the number 15 or extend the picture to include 15.

Materials

Teacher Student

- Workbook 8?-15?
- Colored pencils, pens, or crayons

Description of Lesson

• Colored chalk

• Paper

Exercise 1_____

Put this list of number names on the board.

 7×1 2×4 10 - 2 5×5 18 - 10

T: One of these number names does not belong in my list. Can you tell which name does not belong?

Instruct students to copy the number names on their papers and to cross off the one they think does not belong. Encourage students, if necessary, to do the indicated calculations and write the result under the name in their list. Most students will agree that 5 + 5 does not belong because it is a name for 10 while all the others are names for 8.

Note: If some students have other reasons for thinking a name does not belong in the list, they are not incorrect. For example, someone may want to eliminate 2 x 4 because it is the only one with "x." Praise any such good reasoning, but encourage students to think about which numbers are being named.

Exercise 2_____

Draw these arrow roads on the board.

T: We are still looking for the number 8. Could 8 be in the top road? How do you know?

Someone may comment that 8 is an even number and that the numbers in the top picture will all be odd. If someone thinks 8 could be in the top picture, label the dots and observe that 8 is skipped.

- T: Is 8 in the bottom road?
- S: No, because there aren't enough dots there.



Ask whether more +2 arrows could be drawn in the bottom road so that 8 would be in the picture. Let one of the students extend the road. If a student just draws a dot and labels it 8, ask if there are any new arrows that can be drawn. Of course, since 10 is already in the picture, an arrow can be drawn from 8 to 10.

Distribute the workbooks. Look at several pages with your class and very briefly go over the directions. Stress that in the first section, they are to find 8 in every picture or to extend the picture to include 8; in the second section they are to find 15 in every picture or to extend the picture to include 15. Do pages 2 and 3 collectively. Ask the students to work on those pages while you copy the pictures on the board. On page 3, you will need to extend the picture to include 8, and there are many different ways to do that.

At the end of the class period, collect the workbooks for use in Lesson W10.

Capsule Lesson Summary

Pose this problem: Can the number 15 be found on a +3 arrow road containing the number 7 or on a +3 arrow road containing the number 30? Decide that 15 cannot be on the first road. Extend the second road to include 15. Continue working in the 8?-15? Workbook. (This is the second of two lessons using this workbook.)



Draw these arrow roads on the board.



T: We are looking for the number 15. Could 15 be on this road (trace the road starting at 7)?

If necessary, collectively label the dots on this road and discover that 15 is skipped.

T (tracing the road ending at 30): Could 15 be on this road?

Label dots until it is clear that 15 is not one of the dots.

- T: Did we skip over 15?
- S: *No.*

T: Can we extend the road by drawing more +3 arrows so that we do meet the number 15?

Let a student draw the appropriate arrow and label the dot for 15.



Distribute students' copies of the workbook 8?-15? and let them continue working in this booklet. If some students finish the workbook early, suggest they make up their own problems to add to this workbook or for a similar workbook. At the end of the lesson, collect the workbooks for your review.

Assessment Activity

An individual student progress record for the workbook is available on Blackline W10. You may like to use this form to monitor student work.

















































Capsule Lesson Summary

Solve a detective story with clues involving the Minicomputer and two different arrow pictures. Pose a problem about what Eli could have in his bag after putting in peanuts from three different bushes. Display the possibilities on the Minicomputer.

Materials

Colored chalkMinicomputer set

- Student Minicomputer set
 - Paper
 - Colored pencils, pens, or crayons

Description of Lesson

Allow students to work with a math partner during the lesson.

Exercise 1: Detective Story

Write a large 14 on a slip of paper and fold it so that 14 is hidden.

T: The name of a secret number is written on this paper. Listen carefully to my clues and you can discover it.



Teacher

Display one Minicomputer board. Give pairs of students one individual Minicomputer sheet (two boards) and three checkers.

T: The secret number is on the ones board of the Minicomputer using exactly three regular checkers. You cannot see the checkers because they are invisible.

Direct student pairs to cover one of the Minicomputer boards on their sheet with a paper. Then they should work together with three checkers on one Minicomputer board (the ones board) to find possibilities for the secret number. Students should write some possible numbers on their papers. Accept three or four suggestions from the students and check each suggestion on the Minicomputer. As they are suggested, make a list of numbers that could be the secret number, spacing them so that they will be recorded in order.

T: What is the least number it could be? (3) What is the greatest number it could be? (24) What other numbers could be the secret number?

Continue until all the possible numbers have been found.

3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 20, 24

Note: The greatest number that can be put on the ones board of the Minicomputer with three checkers is 24, all three checkers on the brown square.

The next greatest number is 20 with two checkers on the brown square and one checker on the purple square.

The next number is 18.



- T: Now we know that the secret number is one of these numbers, but we don't know which one. What do we need?
- S: Another clue.

Clue 2

Draw this arrow picture on the board and ask student pairs to copy it on their papers.



T: The secret number is in this arrow picture. Which numbers could it be?

Allow students to work together for a few minutes and then invite some students to label the dots in the picture on the board.

Remind the class of the first clue by pointing to the list of numbers on the board.

T: Which of these numbers cannot be the secret number? Tell me which ones to cross off our list.

T: Now we know that the secret number could be 4, 9, 14, or 24. Here is my last clue.

Clue 3

Erase the +5 arrow picture before drawing this one. Instruct student pairs to copy the picture.

T: Our secret number is here (point to s) in this arrow picture. Write the secret number at s in your picture.



Check the pictures of many student pairs and then ask a student to give the answer aloud.

S: 14.

Check that each of the remaining numbers in the list cannot be at \mathbf{s} in the picture. Then reveal that 14 is written on the paper.

Exercise 2: Eli the Elephant

T: This morning Eli the Elephant went for a walk. Along the way he found three peanut bushes. He put four peanuts from the first bush, one peanut from the second bush, and two peanuts from the third bush in his bag. How many peanuts do you think Eli has in his bag?

Allow the class to discuss this problem. Perhaps a student will suggest that some, or all, of the peanuts may have been magic peanuts; if necessary, suggest this yourself.

Display one Minicomputer board.

- T: How many peanuts from the first bush did Eli put in his bag?
- S: Four.
- T: Were they regular or magic peanuts?
- S: We don't know.

Note: It is possible that a student will suggest that regular and magic peanuts came from the same bush. This is an interesting idea, but the lesson will become very complicated if you accept this possibility. Assure the students that only magic peanuts grow on magic peanut plants and only regular peanuts grow on regular peanut plants.

T: We really don't know for sure what kind of peanuts they were, but let's suppose they were four regular peanuts.

Put a regular checker on the 4-square of the Minicomputer.

- T: How many peanuts from the second bush did Eli put in his bag?
- S: One.
- T: Was it a regular or a magic peanut?
- S: It could have been either kind.
- T: Let's suppose it was a regular peanut.

Put a regular checker on the 1-square of the Minicomputer.

- T: How many peanuts from the third bush did Eli put in his bag?
- S: Two.
- T: What kind of peanuts were they?

	•

•
•

- S: We don't know.
- T: Let's suppose they were two magic peanuts.

Put a negative checker on the 2-square of the Minicomputer.

- T: How many regular peanuts did Eli put in his bag altogether?
- S: Five.
- T: How many magic peanuts did Eli put in his bag altogether?
- S: Two.

Write $5 + \hat{2}$ on the board to the right of the Minicomputer.

- T: If Eli picked five regular peanuts and two magic peanuts, what will there be in Eli's bag?
- S: Three regular peanuts.

Complete the number sentence on the board.

 $5 + \widehat{2} = 3$

If your class is not certain that the number on the Minicomputer is 3, draw a picture of a bag with five regular and two magic peanuts. Ask a student to show which peanuts would disappear.

T: Eli could have three regular peanuts in his bag. Do you think Eli could have a different number of peanuts in his bag?

Allow students to work in pairs to find other possibilities and to keep a record of the other possibilities they find. They can use their Minicomputers or draw pictures of Eli's peanut bag. After a short while, let the student partners make suggestions about the kind of peanuts Eli could put in his bag. Represent each new possibility on the Minicomputer and in a number sentence. There are eight different possibilities (see below), but do not expect your students to find all of them.



Continue this activity as long as there is student interest. Emphasize that we are not sure which kind of peanuts grow on the three bushes so there are several possible answers to the question, How many peanuts does Eli have in his bag?

Home Activity

This is a good time to send a letter to parents/guardians about detective stories. Blackline W11 has a sample letter.

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Capsule Lesson Summary

Become familiar with the written form of number names while looking for patterns in the 0–109 numeral chart. Fill in missing numbers in a chart with portions blanked out. Extend experiences in various content strands in the Catalog of Problems #2 Workbook.

Materials

	maio	
acher Student	 0–¶091nokhuna1Wh2rt 0–109 numeral chart 	 Worksheet W12 <i>Catalog of Problems #2</i> Workbook Colored pencils, pens, or crayons
Advance	Preparation: Use Blackline W12 to make	student copies of the 0–109 numeral chart

Description of Lesson

Exercise 1

Teacher

You may want students to have a 0–109 numeral chart at their places for this first exercise.

T: I'm going to write some number names on the board. Don't read them aloud. After I write a name on the board, point to the number on the numeral chart.

In random order write zero through ten on the board and check that students point to the correct number on their numeral charts.

Next write in order (a list) ten, twenty, thirty, through ninety. After students locate a number on the chart, ask a student to write the numeral next to the word name. If your class 0-109 chart is laminated, you may also highlight these numbers with erasable marker.

ten	10	0	1	2	3	4	5	6	7	8	9
twenty	20	10	11	12	13	14	15	16	17	18	19
thirty	30	20	21	22	23	24	25	26	27	28	29
forty	40	30	31	32	33	34	35	36	37	38	39
fifty	50	40	41	42	43	44	45	46	47	48	49
sixty	60	50	51	52	53	54	55	56	57	58	59
seventy	70	60	61	62	63	64	65	66	67	68	69
eighty	80	70	71	72	73	74	75	76	77	78	79
ninety	90	80	81	82	83	84	85	86	87	88	89
		90	91	92	93	94	95	96	97	98	99
Do these w	vord names for the	100	101	102	103	104	105	106	107	108	109

- T: What do you notice?
- S: Six in sixty; seven in seventy.
- S: Twenty starts like two.

S: Counting by tens.

S: The numbers are all in the first column of the chart.

Repeat this activity by making another list of word names for numbers in order, counting by tens, starting at a number between one and nine. For example:

four	4										
fourteen	14										
twenty-four	24						_		_		
thirty-four	34	0	1	2	3	(4)	5	6	7	8	9
forty-four	44	10	11	12	13	(14)	15	16	17	18	19
fifty-four	54	20	21	22	23	24	25	26	27	28	29
sixty-four	65	30	31	32	33	34	35	36	37	38	39
seventy-four	74	40	41	42	43	44	45	46	47	48	49
eighty-four	84	50	51	52	53	54	55	56	57	58	59
ninety-four	94	60	61	62	63	64	65	66	67	68	69
innerg rour	01	70	71	72	73	74	75	76	77	78	79
Distribute Worksheet W1	2 and tell stu	80	81	82	83	84	85	86	87	88	89
over their 0–109 chart and			91	92	93	94	95	96	97	98	99
numbers in the puzzle (ur	nshaded box	100	101	102	103	104	105	106	107	108	109

Exercise 2

Distribute copies of the workbook *Catalog of Problems #2* and let the students begin working independently. If the questions you receive indicate that many students are having difficulty with a particular page, hold a collective discussion about that page.

At the end of the class period, collect the workbooks for your review and to use again in Lesson W13.

Home Activity

Send home a copy of the 0–109 numeral chart (Blackline W12) and suggest that parents/guardians use it to explore number patterns with their child.

	T					6			
	11	12				16			
		22	33			26			29
						36		38	30
40	41		48		46	46		48	
50	51			54		56		SB	
60	81		ഒ		66	66			
						76			
		82	83	64	85	86	W	88	89
						86			
						106			

Capsule Lesson Summary

Decide that a road between 20 and 32 using +4 and +1 arrows must start at 20 and then proceed to build it. Continue working in the workbook *Catalog of Problems #2*. (This is the second of two lessons using this workbook.)

		Materials	
Teacher	Colored chalk	Student	 <i>Catalog of Problems #2</i> Workbook Colored pencils, pens, or crayons

Description of Lesson

Draw two dots on the board, one dot to the right of the other with plenty of space between them.

- T: These dots are for numbers. Which of these numbers is greater?
- S: We don't know which is greater; the greater number could be in either place.

Label the dot on the right 20 and the dot on the left 32; then ask the question again.

T: We are going to build a road between these two numbers with red and blue arrows. Red arrows are for +4 and blue arrows are for +1. (Write the key on the board.)

• 20

• 20



T: Who would like to draw the first arrow?

Perhaps a student will suggest a +4 arrow starting at 32.



T (tracing the +4 arrow): Does 32 + 4 = 36? (Yes) Does this arrow help us build a road between 20 and 32? (No) Why not?

S: 20 is less than 32, but if you start at 32 and use only +4 and +1 arrows, the numbers will increase.

Do not expect students to be able to give precise explanations. Erase the arrow from 32 to 36 and the dot for 36; ask for another way to start the road. Continue asking students to draw arrows until a road is built between 20 and 32.

T: How many +1 arrows are in the road we built? (0, 4, 8, or 12) Could we build a shorter road (one with fewer arrows)?

If the road contains +1 arrows, some students should observe that they can replace every four +1 arrows with one +4 arrow. In any case, the students should conclude that the shortest road between 20 and 32 contains three +4 arrows. It is possible your arrow picture will look like this one.



T: What does the longest road between 20 and 32 look like?

S: Twelve +1 arrows.

Distribute students' copies of the workbook *Catalog of Problems #2* and let them continue working independently. At the end of the lesson, collect the workbooks for your review.

Assessment Activity

An individual student progress record for the workbook is available on Blackline W13. You may like to use this form to monitor student work.



















































W14 DETECTIVE STORY #5/ELI'S MAGIC PEANUTS #5

Capsule Lesson Summary

Solve a detective story with clues involving the calculator, a string picture, and the Minicomputer. Present an Eli the Elephant episode in which after collecting many peanuts, Eli arrives at his grandmother's house to find only one peanut in his bag. What could have happened? Discuss the possibilities.

		Materials	
Teacher	CalculatorColored chalkMinicomputer set	Student	 Calculator Paper Colored pencils, pens, or crayons Minicomputer set

Description of Lesson

Allow students to work with a partner during the lesson.

Exercise 1: Detective Story

Write a large 32 on a slip of paper and fold it so that 32 is hidden.

Clue 1

Distribute calculators to student pairs. If you have an overhead calculator, you may prefer to use it to introduce this clue and to let students help with pressing keys.

T: Today you must discover a secret number. The calculator will help us with the first clue. We will teach the calculator to count by fives starting at 2. The secret number will appear on the calculator's display.

Instruct students to turn on their calculators and check that 0 is on the display. (They may need to press \square a couple of times to get 0 on the display.) Then ask them to follow your instructions exactly.

T: Start with 0 on the display. Press 2. What number is on the display?

S: 2.

Record 2 on the board.

T: *Press* \pm $5 \equiv$ *. What number is on the display?*

S: 7.

- **T:** What did the calculator do?
- S: It added 5; 2 + 5 = 7.

Record 7 on the board.

- T: Press \equiv again. What number is on the display?
- S: 12.

T: What did the calculator do?

S: Added 5 again; 7 + 5 = 12.

Record 12 on the board.

- **T:** The calculator will continue to count by fives (add fives) as we press \equiv . Press \equiv one more time. What number is on the display?
- S: 17.

Continue your list on the board.

2, 7, 12, 17

T: Think about pressing \equiv three more times. What are the next three numbers we will get on the calculator?

Allow the class to predict these numbers. Then record them on the board as the students press \Box three more times and note them on the display. As soon as most of your students are able to predict the sequence quickly, you can abandon the calculator. Continue counting and recording until the class realizes this sequence goes on and on. Your list on the board should have several numbers greater than 50. You may like to ask the students to suggest a number that would be in the list greater than 100 or between 150 and 160.

2, 7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, 62, 67, 72, ...

Explain that the three dots mean that the list goes on and on.

- **T:** Our secret number is one of the numbers in this list. Do we know which one? What do we need?
- S: Another clue.

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Clue 2
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Draw this picture on the board and ask students to copy it on their papers.



T: The second clue is that the secret number is here (point to s). Is it an even number? (Yes) Is it more than 50? (No) Which of these numbers could be the secret number?

Suggest students put numbers from the list into their string pictures and find possibilities for the secret number. Each time a number is suggested, label the dot with that number and ask the class if the number is placed correctly. Suppose, for example, a student suggests 72.

T: Could this be 72? (Disagreement) Is 72 even? (Yes) Then it must be inside the red string. Is 72 more than 50? (Yes) Then it must be inside the blue string. Where should 72 be?

Ask a student to point to the correct region (the middle).

T: 72 cannot be the secret number.

Put 72 correctly in the string picture and cross 72 off the list on the board.

If a correct number is suggested, elicit class agreement and circle it on your list. Continue until many of the numbers in the original list are either circled or crossed off.

Point to the three dots.

- **T:** Could the secret number be any of the numbers we didn't write on the board? (No) How do you know?
- S: Because the secret number is not more than 50.

Cross off the three dots.

Point to each unchecked number and guide your class in deciding if it could be the secret number. When all the numbers have been checked, erase those that have been crossed off.

T: Now we know the secret number is one of these (point to the five remaining numbers).



Clue 3

Display two Minicomputer boards.

T: This will be the final clue. When you know which of these numbers is the secret number, write it on your paper. You cannot put the secret number on the Minicomputer board with only one or two checkers. At least three (regular) checkers are needed to put the secret number on the Minicomputer. Which is the secret number?

Look at (or listen to) the responses of several student partners. After most students realize that 32 is the secret number, ask a student to say the secret number aloud. Check the other numbers on the Minicomputer by letting students demonstrate that each of them can be put on the Minicomputer with one or two checkers. Reveal that 32 is written on your paper.

Exercise 2: Eli the Elephant

T: Today Eli the Elephant went to visit his grandmother. On the way to her house, he took a detour through a field where he had seen many peanut plants. Eli's grandmother loves peanuts, so Eli planned to bring her a bag full of peanuts as a surprise.

Eli spent a long time collecting peanuts. He was thinking how pleased his grandmother would be when he gave her the bag of peanuts.

Eli's grandmother met him at the gate and gave Eli a big hug. Eli held out the bag and said, "Grandmother, I've brought you a bag of peanuts." Eli's grandmother was very happy. She gave Eli a big elephant kiss and went to get a bowl for the peanuts. She shook the bag and out fell—one peanut! She searched inside the bag and then Eli searched inside the bag, but there were no more peanuts. Eli felt confused.

What do you think happened to the peanuts Eli picked?

Allow the students to comment. Very likely someone will suggest that the other peanuts disappeared. If a student suggests that Eli ate the other peanuts, or that there was a hole in the bag, or any other reasonable answer, accept it as a possibility.

T: Why would the peanuts disappear?

Guide this discussion until it is clear that a number of regular peanuts and the same number of magic peanuts would disappear.

T: Do we know how many peanuts Eli put in his bag?

S: No.

If students begin to give you suggestions of how many regular and magic peanuts Eli picked, record them on the board. If no one suggests any specific numbers, ask,

T: How many regular and how many magic peanuts could Eli have collected?

Each suggestion should be considered to see if all but one peanut would disappear. Write the corresponding number sentence for each suggestion. Some possibilities are:

$$10 + \hat{9} = 1$$

$$\widehat{10} + 9 = \hat{1}$$

$$59 + \widehat{58} = 1$$

$$58 + \widehat{59} = \hat{1}$$

$$\widehat{1,000} + 1,001 = 1$$

$$5 + 7 + \hat{3} + \hat{4} + 4 + \hat{2} + \hat{6} = 1$$

T: Poor Eli, he still doesn't know about magic peanuts. We will never know exactly how many peanuts Eli gathered that day for his grandmother.

Exercise 3_

T: Another day Eli put 68 regular and 45 magic peanuts in his bag. When Eli sat down to eat the peanuts, did he find regular or magic peanuts in his bag?

S: Regular.

Write 68 + 45 on the board. If some students know how many peanuts were in Eli's bag, let them write it on their papers.

T: Who can put 68 on the Minicomputer with regular checkers? Who can put 45 on the Minicomputer with negative checkers? What will happen?

	•	•	\otimes
•			\otimes

S: The checkers on the 40-square will disappear.

Ask a student to take these checkers off the Minicomputer. If someone suggests that checkers not on the same square will disappear, for example 8 and $\hat{4}$, remind the class that those checkers are for eight regular peanuts and four magic peanuts.

T: What could we do to make the number easier to read?

S: Make some trades.

This is one possible sequence of trades.



Interrupt the activity to ask these questions.

- T: What number do we have with these checkers (point to the regular checkers)?
- S: 24.
- T: What number do we have with this checker (point to the negative checker)?
- S: *î*.
- T: We have $24 + \hat{1}$ on the Minicomputer. Now some of you might know the number of peanuts Eli has, but let's continue to make some more trades.

Continue until all the negative checkers have been removed from the board; you may have to ask some leading questions or suggest the backward trade 4 = 2 + 2 yourself.



T: How many peanuts does Eli have in his bag?

S: 23 regular peanuts.

Complete the number sentence on the board.

$68 + \widehat{45} = 23$

Pose one or two other such problems for student pairs to solve on their desk Minicomputers. For example:

 $34 + \widehat{22}$ $55 + \widehat{33}$



Draw this picture on the board.



Tell the class that the problem is to put all the numbers shown in the string in their proper places in the arrow picture. Ask someone to suggest a number for the first dot on one of the roads. Then pursue that choice and see where it leads. If it leads to an incorrect situation, let the students discover this and discuss why it is wrong and what they could do. A student may suggest that the least number (1) must go at the beginning of one of the roads; ask the student to convince the rest of the class. The students might suggest crossing out the numbers inside the string after they have been placed in the arrow picture. This procedure is subject to timing since in a trial and error method, many mistakes may be made and new starts may be necessary. In the course of solving this problem collectively, the opportunity to discuss various strategies should arise, but allow ample opportunity for students to discover some good strategies themselves.

The completed arrow picture is given below.



Distribute copies of the workbook *Fishing for Numbers, Part I*. At the end of the lesson, collect the workbooks for your review and to use again in Lesson W16.

Capsule Lesson Summary

Put the numbers in a string picture in their proper places in a story. Continue working in the workbook *Fishing for Numbers, Part I*. (This is the second of two lessons using this workbook.)

		Materials	
Teacher	Colored chalk	Student	 Paper Fishing for Numbers, Part I Workbook

Advance Preparation: Write the story for the warm-up activity on the board or on an overhead transparency before starting the lesson.

Description of Lesson

Allow students to work with a partner for this warm-up activity.

Write this story and set of numbers on the board.

Jeff has a large family. There are _____ children in Jeff's family, _____ boys and _____ girls (more boys than girls). Jeff's house is quite big. It has _____ rooms, including _____ bedrooms.

At school, there are _____ children in Jeff's class. There are _____ children in the entire school.



Tell the class that the problem is to put all the numbers inside the string into the story so that it makes sense. Invite student partners to work on this problem for a few minutes.

When many students have finished the problem, conduct a collective discussion of the problem. As students tell the class how to fill in the blanks, ask them to explain their reasoning. A solution is given below.

Jeff has a large family. There are <u>7</u> children in Jeff's family, <u>4</u> boys and <u>3</u> girls (more boys than girls). Jeff's house is quite big. It has <u>13</u> rooms, including <u>5</u> bedrooms.

At school, there are $\underline{23}$ students in Jeff's class. There are $\underline{355}$ children in the entire school.

Distribute the students' copies of the workbook *Fishing for Numbers, Part I* and allow them to continue working independently. At the end of the lesson, collect the workbooks for your review.

Assessment Activity

An individual student progress record for the workbook is available on Blackline W16. You may like to use this form to monitor student work.

Home Activity

Prepare a story similar to the warm-up activity for students to do at home with family participation. For example:

Tory lives with her mother, grandmother, and younger brother. Tory is _____ years old. Tory's mother is _____ years old. Tory's grandmother is _____ years old. And Tory's brother is _____ years old. Tory can't wait for her next birthday, because she will be _____.



Fill in the blanks of the story so that it makes sense. Use all numbers in the string.



















































W17 DETECTIVE STORY #6/ELI'S MAGIC PEANUTS



Solve a detective story with clues involving an arrow picture, odd numbers, and positive/negative number facts. Introduce Clarence the Crafty Crocodile who plays a trick on Eli the Elephant in which Eli discovers some of his peanuts are missing. What could Clarence have done in Eli's bag?



Allow students to work in pairs during this lesson.

Exercise 1: Detective Story

Write a large 3 on a slip of paper and fold it so that 3 is hidden.

T: Today you are going to be detectives. You will have several clues which will help you discover a secret number.



Draw this arrow picture on the board and tell the class that the secret number is in the picture. You may want to instruct student partners to copy the arrow picture and to label the dots along with the class.



Point to 9 and trace the first red arrow as you say, " $9 + \hat{3} = ...$? Nine regular peanuts and three magic peanuts is...?" If your class is not certain that the second dot from the left is for 6, draw Eli's bag with nine regular peanuts and three magic peanuts. Ask a student to show which peanuts would disappear. Look at the remaining peanuts and conclude $9 + \hat{3} = 6$.

As necessary, refer to Eli the Elephant to help your students label the remaining dots. Always erase a peanut picture before you draw another one.



After all dots have been labeled, make a list of the possible numbers on the board and then erase the picture. $\hat{}$

9, 6, 3, 0, 3, 6, 9

Clue 2

T: *The second clue is that the secret number is odd. Which of these numbers is odd* (point to the list on the board)?

Consider each number in the list with the class and erase any numbers that are not odd.

T: The secret number could be 9, 3, $\hat{3}$, or $\hat{9}$. I will give you only one more clue.

Clue 3

Draw this string picture on the board.



T: The secret number is in this string picture. When you know which one is the secret number, write it on your paper.

After you have checked the responses of most students, do each of the calculations in the string picture. If necessary, draw peanut pictures to show that $\hat{5} + \hat{2} = \hat{7}$ and $5 + \hat{2} = 3$.

- T: What is the secret number?
- S: 3.

Reveal that 3 is written on the slip of paper.

Exercise 2: Eli the Elephant

T: One day Eli was walking through the jungle. There were 12 regular peanuts in his bag.

Draw this bag of peanuts on the board.



T: Eli walked a long time; he was very tired and decided to take a nap. As soon as Eli fell asleep, along came Clarence the Crafty Crocodile. Clarence saw that Eli was asleep and decided that it would be fun to play a trick on Eli. Clarence opened Eli's bag of peanuts and did something. Then he closed Eli's bag quietly and went away.

Eli felt hungry when he awoke, so he opened his peanut bag. Eli counted his peanuts and cried, "Oh no! There are only eight peanuts. Four peanuts are missing." What do you think Clarence the Crafty Crocodile did?

Allow the class to discuss this problem. Two possibilities should emerge: Clarence could have removed four peanuts from Eli's bag, or Clarence could have put four magic peanuts into the bag.

Note: There are other more complex possibilities which might be suggested. For example, Clarence could have added two magic peanuts and removed two regular peanuts. Accept such suggestions as correct, but for the purpose of this lesson, focus primarily on these two possibilities: Clarence removed four regular peanuts, or Clarence added four magic peanuts.

Illustrate each of these situations as it is suggested. Write the appropriate number sentence under each bag of peanuts. Arrange the board so that the two pictures are side by side.



Emphasize that you do not know what Clarence did when he opened Eli's bag, because removing four regular peanuts and adding four magic peanuts have exactly the same effect. Write this as a number sentence on the board.

- T: Another day when Eli was walking in the jungle he gathered 55 regular peanuts. He became very tired and decided to take a nap. Can you guess what happened while Eli was sleeping?
- S: Clarence the Crafty Crocodile came by and played another trick on Eli.
- **T:** Eli felt hungry when he awoke, so he opened his peanut bag. Eli counted his peanuts and found that 29 peanuts were missing. What do you think Clarence the Crafty Crocodile had done?
- S: Clarence could have removed 29 peanuts from Eli's bag.
- S: Clarence could have put 29 magic peanuts into Eli's bag.

As each possibility is suggested, write the appropriate number sentence on the board.

55 - 29 = ?55 + 29 = ?

T: How are we going to find out how many peanuts Eli has left in his bag?

If someone suggests drawing a picture of Eli's bag, accept this as a good but not very practical method, because you would have to draw so many peanuts. If no one suggests using the Minicomputer, suggest it yourself. Point to the number sentences on the board.

T: Which of these problems do you want to do first on the Minicomputer?

Suppose the students want to calculate 55 - 29 first. Invite someone to put 55 on the Minicomputer.



T: Can someone take 29 off the Minicomputer?

S: We do not have checkers in position for 29.

Let someone point to where the checkers for 29 would be.

T: Who can make a backward trade which will help us to calculate 55 – 29?

Ask a student to identify a trade before moving checkers. This will help to discourage trades which are not useful. Whenever a trade is made that puts a checker in a position for the subtraction, mention this to the class and indicate that they are getting closer to the goal.

Let the students make trades until 29 can be taken off the Minicomputer. A possible sequence of trades is shown below.



Complete the appropriate number sentence on the board and remove the checkers from the Minicomputer.

T: Now let's suppose Clarence put 29 magic peanuts into Eli's bag.

Invite a student to put 55 on the Minicomputer and another to put $\widehat{29}$ on the Minicomputer.

T: We want to get a regular and a negative checker on the same square so that we can remove them. Who can make a trade?

	•	\otimes	•
\otimes	•		⊗

Let students make trades until the checkers on the Minicomputer are all of the same kind. In this case, they will all be regular.

One sequence of trades is illustrated below. (Note: $1 + \hat{1} = 0$ has already been made.)



Complete the number sentence on the board.

55 - 29 = 2655 + 29 = 26

Emphasize that we do not know what Clarence did when he opened Eli's bag, because removing 29 regular peanuts and adding 29 magic peanuts have exactly the same effect. Write this as a number sentence.

$$55 - 29 = 26$$

 $55 + 29 = 26$
 $55 - 29 = 55 + 29$

T: Which calculation did you find easier on the Minicomputer?

Accept either response, letting students comment on why they thought one calculation was easier than the other.

Continue this activity with a similar situation in which Eli has 83 regular peanuts and later discovers that 57 peanuts are missing. Let student partners choose which calculation they will do, either 83 - 57 or 83 + 57, on their desk Minicomputer. It is not necessary to do both calculations.



Ask students, possibly working with their partners, to write and illustrate a story about Eli and his friend Clarence. The story should include an example of what happens with Eli's peanut bag. Stories can be used to explain this episode of the Eli story to parents/guardians.