

A p p e n d i x

Counting and Place Value

Count on Your Fingers African Style by Claudia Zaslavsky
Count Your Way Through... (series highlighting different countries) by Jim Haskins
Counting by Henry Pluckrose
How to Count Like a Martian by Glory St. John
Knots on a Counting Rope by Bill Martin, Jr. and John Archambault
Less Than Nothing is Really Something by Robert Fromes
One Hundred Hungry Ants by Elinor J. Pinczes
One Is Unique by Marnie Luce
Only One by Marc Harshman
Phoebe and the Hot Water Bottles by Terry Furchgott and Linda Dawson
17 Kings and 42 Elephants by Margaret Mahy
666 Jelly Beans! All That by Malcolm Weiss
Ten What? A Mystery Counting Book by Russell Hoban
Twelve Circus Rings, The by Seymour Chwast
Two of Everything by Lily Toy Hong
Two Ways to Count to Ten by Ruby Dee
When the King Rides By by Margaret Mahy
Zero Is Not Nothing by Mindel and Harry Sitomer
Zero: Is It Something? Is It Nothing? by Claudia Zaslavsky
Zero Is Something by Marnie Luce

Large Numbers

Anno's Mysterious Multiplying Jar by Mitsumasa Anno
Homer Price by Robert McCloskey
How Much is a Million? by David Schwartz
King's Chessboard, The by David Birch
Million Fish... More or Less, A by Patricia McKissack
People by Peter Spier
Six Dogs, Twenty-Three Cats, Forty-Five Mice, and One Hundred Sixteen Spiders
by Mary Chalmers
Something Absolutely Enormous by Margaret Wild
What Is beyond the Hill? by Ernest Ekker

Estimation

Counting on Frank by Rod Clement
Dinosaur Who Lived in My Backyard, The by B.G. Hennessey
Estimation by Charles Linn
How Many Is Many? by Illa Podendorf
Jelly Bean Contest, The by Kathy Darling
Really Eager and the Glorious Watermelon Contest by Richard Cheney

Measurement

Angles Are Easy as Pie by Robert Froman
Area by Jane Srivastava
Biggest Nose, The by Kathy Caple
Capacity by Henry Pluckrose

A

How Big Is a Foot? by Rolf Myller
King Kaid of India published by The Victorian Readers
Let's Talk about the Metric System by Joyce Lamm
Lineup Book, The by Marisabina Russo
Liter, The by William Shimek
Long, Short, High, Low, Thin, Wide by James Fey
Mr. Archimedes' Bath by Pamela Allen
Much Bigger Than Martin by Steven Kellogg
My Feet by Alik
Mysterious Tadpole, The by Steven Kellogg
Sizes by Gillian Youldon
Something Absolutely Enormous by Margaret Wild
Temperature and You by Betsy Maestro
Think Metric! by Franklyn Branley
Weighing and Balancing by Jane Srivastava

Position/Size

Can I Keep Him? by Steven Kellogg
Dinosaur Is the Biggest Animal That Ever Lived, The by Seymour Simon
Fast-Slow, High-Low by Peter Spier
King's Flower, The by Mitsumasa Anno
Much Bigger Than Martin by Steven Kellogg

Geometry

Circles by Mindel and Harry Sitomer
Cloak for the Dreamer, The by Aileen Friedman
Draw Me a Star by Eric Carle
Eating Fractions by Bruce McMillan
Eight Hands Round by Ann Whitford Paul
Ellipse by Mannis Charosh
Exploring Triangles: Paper-Folding Geometry by Jo Phillips
Fractions Are Parts of Things by J. Richard Dennis
Grandfather Tang's Story by Ann Tombert
Greedy Triangle, The by Marilyn Burns
Lines by Philip Yenawine
Look at Annette by Marion Walter
Look Closer! by Peter Ziebel
Mirror Puzzle Book, The by Marion Walter
Reflections by Ann Jonas
Right Angles: Paper-Folding Geometry by Jo Phillips
Round Trip by Ann Jonas
Shapes by Philip Yenawine
Shapes Game, The by Paul Rogers
Straight Line, Parallel Lines, Perpendicular Lines by Mannis Charosh
Tangram Magician, The by Lisa C. and Lee Ernst
Three-D, Two-D, One-D by David Adler
Topsy-Turvies by Mitsumasa Anno

What Is Symmetry? by Mindel and Harry Sitomer
What's That Shape by Kate Petty and Lisa Kopper
Wing on a Flea, The by Ed Emberley

Money

Alexander, Who Used to Be Rich Last Sunday by Judith Viorst
All Kinds of Money by David Adler
Chair for My Mother, A by Vera Williams
Dollars and Cents for Harriet by Betsy and Giulio Maestro
Four Dollars and Fifty Cents by Eric Kimmel
From Gold to Money by Ali Mitgutch
Gia and One Hundred Dollars Worth of Bubble Gum by Frank Asch
Go-Around Dollar, The by Barbara Johnston Adams
How the Second Grade Got \$8,205.50 to Visit the Statue of Liberty by Nathan Zimelman
If You Made a Million by David Schwartz
Lion in the Night, A by Pamela Allen
Money by Audrey Briers
Money by Benjamin Elkin
Pet for Mrs. Arbuckle, A by Gwenda Smyth

Time

Chicken Soup with Rice by Maurice Sendak
Clock Shop, The by Simon Henwood
How Did We Get Clocks and Calendars? by Susan Perry
I'm in Charge of Celebrations by Byrd Baylor
Minute Is a Minute, A by Barbara Neasi
Only Six More Days Left by Marisabina Russo
Ten-Alarm Campout, The by Cathy Warren
Time! by Jane Edmonds and Mark Sachner
Time by Henry Pluckrose
Time and Clocks by Herta Breiter
What Time Is It Around the World by Hans Baumann

Classification

Anno's Math Games by Mitsumasa Anno
Odds and Evens by Thomas O'Brien
Shoes by Elizabeth Winthrop
Venn Diagrams by Robert Froman

Addition, Subtraction, Multiplication, Division

Binary Numbers by Clyde Watson
Building Tables on Tables by John V. Trivett
Bunches and Bunches of Bunnies by Louise Mathews
Doorbell Rang, The by Pat Hutchins
Dozen Dizzy Dinosaurs, A published by Rigby Books
Each Orange Had 8 Slices by Paul Giganti, Jr.
Eating Fractions by Bruce McMillan
Goats in Boats published by Rigby Books

A

Grain of Rice, A by Helena Pittman
Great Carrot Mystery, The published by Rigby Books
Greatest Guessing Game, The by Robert Froman
Half-Birthday Party, The by Charlotte Pomerantz
King's Chessboard, The by David Birch
King's Commissioners, The by Aileen Friedman
Melisande by E. Nesbitt
Nice Mice, The published by Rigby Books
Number Families by Jane Srivastava
Number Ideas Through Pictures by Mannis Charosh
Our Sister's Surprise published by Rigby Books
Pixie's Toyshop, The published by Rigby Books
Sea Squares by Joy N. Hulme
10 for Dinner by Jo Ellen Bogart

Other

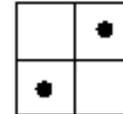
Anno's Hat Tricks by Akihiro Nozaki and Mitsumasa Anno
Averages by Jane Srivastava
Base Five by David Adler
Binary Numbers by Clyde Watson
Graph Games by Frédérique and Papy
Graphs by Dyno Lowenstein
How to Count Sheep Without Falling Asleep by Ralph Leighton and Carl Feynman
Logic for Space Age Kids by Lyn Butrick
Math for Smarty Pants by Marilyn Burns
MatheMagic: Magic, Puzzles, and Games with Numbers by Royal Heath
Mathematical Games for One or Two by Mannis Charosh
Numbers by Philip Carona
Pattern by Henry Pluckrose
Roman Numerals by David Adler
Sixes and Sevens by John Yeoman
Sneaky Square and 113 Other Math Activities for Kids, The by Richard Sharp and Seymour Metzner
Solomon Grundy, Born on One Day: A Finite Arithmetic Puzzle by Malcolm Weiss
Statistics by Jane Srivastava
What Do You Mean By Average? by Joel Schick
Yes-No, Stop-Go: Some Patterns in Mathematical Logic by Judith Gersting and Joseph Kuczkowski

STANDARD CONFIGURATION AND STANDARD MINICOMPUTER TRADES

B

A number can be represented by putting checkers[†] on the squares of the Minicomputer. Each square has a numerical (positional) value, so that the number represented by a checker depends on the square on which the checker is placed. If several checkers are put on the Minicomputer, the number shown is the sum of the numbers represented by the checkers. A number can be put on the Minicomputer in a variety of ways, but the representation that uses at most one checker on each square and uses checkers to represent a digit 9 or less on each board is usually the easiest to read. In such a case, the number is said to be in *standard configuration*. Standard configurations for the numbers 1–9 soon become as familiar to the students as the usual numerals, so that they no longer need to do any mental calculation for such configurations.

For example, this configuration is seen as a name for 6 without doing the calculation $4 + 2 = 6$.



Doing calculations on the Minicomputer involves putting one or more checkers on the various squares and making *trades* to get a specific configuration. In this context, a trade is an exchange of checkers in a configuration that does not change the number represented. Some trades are natural to the design of the Minicomputer. These are called *standard* or *usual* trades and are described in detail below. Among the advantages to establishing such a set of trades are that

- they become routine, and a large group of students with varying abilities can follow a calculation without needing to verify every trade that is made;
- they lead to standard configurations of numbers.

These are the standard Minicomputer trades.

$$\begin{array}{r} 1 + 1 = 2 \\ 10 + 10 = 20 \\ 100 + 100 = 200 \\ 1,000 + 1,000 = 2,000 \end{array}$$

Two checkers on the white square of a board
=

one checker on the red square of the same board.

$$\begin{array}{r} 2 = 1 + 1 \\ 20 = 10 + 10 \\ 200 = 100 + 100 \\ 2,000 = 1,000 + 1,000 \end{array}$$

One checker on the red square of a board
=

two checkers on the white square of the same board.

$$\begin{array}{r} 2 + 2 = 4 \\ 20 + 20 = 40 \\ 200 + 200 = 400 \\ 2,000 + 2,000 = 4,000 \end{array}$$

Two checkers on the red square of a board
=

one checker on the purple square of the same board.

$$\begin{array}{r} 4 = 2 + 2 \\ 40 = 20 + 20 \\ 400 = 200 + 200 \\ 4,000 = 2,000 + 2,000 \end{array}$$

One checker on the purple square of a board
=

two checkers on the red square of the same board.

[†]This Appendix refers only to whole numbers and, therefore, uses only regular (positive) checkers.

B

$$\begin{array}{rcl} 4 & + & 4 = 8 \\ 40 & + & 40 = 80 \\ 400 & + & 400 = 800 \\ 4,000 & + & 4,000 = 8,000 \end{array}$$

Two checkers on the purple square of a board
=
one checker on the brown square of the same board.

$$\begin{array}{rcl} 8 & = & 4 + 4 \\ 80 & = & 40 + 40 \\ 800 & = & 400 + 400 \\ 8,000 & = & 4,000 + 4,000 \end{array}$$

One checker on the brown square of a board
=
two checkers on the purple square of the same board.

$$\begin{array}{rcl} 8 & + & 2 = 10 \\ 80 & + & 20 = 100 \\ 800 & + & 200 = 1,000 \\ 8,000 & + & 2,000 = 10,000 \end{array}$$

One checker on each of the brown and red squares of a board
=
one checker on the white square of the next board to the left.

$$\begin{array}{rcl} 10 & = & 8 + 2 \\ 100 & = & 80 + 20 \\ 1,000 & = & 800 + 200 \\ 10,000 & = & 8,000 + 2,000 \end{array}$$

One checker on the white square of a board
=
one checker on each of the brown and red squares of the next board to the right. †

After many experiences with the Minicomputer, some students might suggest *non-standard* trades, for example, $8 + 4 = 10 + 2$. When students suggest such trades, ask them to convince the class that the trades are indeed correct. There will be a natural acceptance or rejection of non-standard trades by the students, sometimes depending on the situation.

Some non-standard trades will be introduced for specific purposes; for example, ten checkers on a square can be exchanged for one checker on the same color square of the next board to the left. Such trades serve to reinforce the support that the Minicomputer gives to decimal place value, or to provide insight into a certain kind of problem. They are useful in particular situations, but do not play the same routine role that the usual trades do.

†This provides the link that motivates the introduction of the decimal numbers. For example, one checker on the white square of the ones board equals one checker on each of the brown and red squares of a special board (the tenths board—or as it is often referred to in the lessons, the dimes board) located to the right of the ones board and separated from it by a special barrier.

MENTAL ARITHMETIC

C

Mental arithmetic activities are short, fast-moving question and answer sessions in which students are asked to calculate mentally, preferably without the aid of paper and pencil. Often a mental arithmetic session consists of several sequences in which the answers to one or more questions lead to the answer of another, more difficult question. For example:

$$\begin{aligned}2 \times 3 &= ? & (6) \\2 \times 10 &= ? & (20) \\2 \times 13 &= ? & (26)\end{aligned}$$

There are specific lessons that call for mental arithmetic; however, try to include such activities for five to ten minutes several times a week. Use mental arithmetic as a warm-up activity to a lesson, as a transition between two exercises in a lesson, as a conclusion to a lesson, or as a quick change of pace at any time during the day.

Why Is Mental Arithmetic Important?

- 1) It develops mental computational skills.
- 2) It provides an opportunity to involve and help students at all ability levels.
- 3) It creates an awareness of patterns and mathematical relationships.
- 4) It helps students to be able to recall arithmetic facts easily.
- 5) It provides an opportunity to keep alive and to reinforce concepts previously introduced.
- 6) It helps students to see that there is often more than one way to solve a problem.
- 7) It helps students construct their understanding of the arithmetic operations.

Hints for Mental Arithmetic Activities

- 1) Vary the level of difficulty of questions throughout a session.
- 2) Involve many students.
- 3) Follow a pattern for awhile and then start a new pattern.
- 4) Keep a brisk pace, although you should allow students time for thought.
- 5) Give extra attention to a student who shows signs of improvement.
- 6) Illustrate, whenever possible, the value of estimation in making calculations.
- 7) Occasionally ask a student to explain an answer.
- 8) Occasionally use the chalkboard to emphasize a pattern.
- 9) Occasionally ask students to whisper an answer to you so that many students will have an opportunity to answer.
- 10) Occasionally discuss the reasonableness of an answer.

Suggested Mental Arithmetic Activities

The following section contains specific examples that illustrate patterns in arithmetic. You should elaborate and expand upon ideas depending upon the abilities and interests of your students. As you work with your class on the regularly scheduled mathematics lessons, you might make a note of computational skills you would like to improve; then plan a short mental arithmetic activity that develops those skills. Do not expect every one of your students to be good at all of these suggested activities, even at the end of the year.

C

Counting Activities

- 1) Count by twos, threes, fives, or tens collectively. Refer to the 0–109 numeral chart as the class counts, if you wish.
- 2) Count by twos, threes, fives, or tens as a group activity with each student saying one number. Use a natural seating order in your class (such as going up and down rows) and ask each student in turn to say the number that comes next. If you are counting by threes, for example, the first student will say, “3”; the second will say, “6”; and so on. Repeat the counting but start with a different student.
- 3) Count by twos, threes, fives, or tens, starting at 1 or any number other than 0.
- 4) Count backward by twos, fives, or tens. Do this immediately following a similar forward count.
- 5) Play “Buzz.” The students count by ones starting at 0 but skip every fourth number by saying “buzz.” This should be done rapidly. This can be done as a whole class or small group activity with each student saying, in turn, the number that comes next or “buzz.”

More challenging variations of this activity involve

- skipping every sixth number rather than every fourth number;
- starting the counting at 2 and skipping every fourth number;
- counting by twos or fives and skipping every fourth number.

Addition and Subtraction

The first nine examples are lists of calculations to be done mentally. Each list suggests an arithmetic pattern that students should begin to use in doing such calculations.

- 1)

$3 + 4$	$8 + 2$	$20 + 20$	$2 + 2$
$3 + 5$	$8 + 3$	$20 + 21$	$3 + 3$
$3 + 6$	$8 + 4$	$20 + 22$	$4 + 4$
$3 + 7$	$8 + 5$	$20 + 23$	$5 + 5$
- 2)

$4 + 4$	$2 + 5$
$40 + 40$	$20 + 50$
$400 + 400$	$200 + 500$
$4,000 + 4,000$	$2,000 + 5,000$
$4,000,000 + 4,000,000$	$2,000,000 + 5,000,000$
- 3)

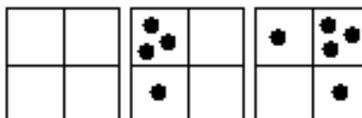
$10 + 5$	$10 + 6$	$30 + 6$
$5 + 10$	$20 + 6$	$31 + 6$
$10 + 7$	$30 + 6$	$40 + 5$
$7 + 10$	$40 + 6$	$43 + 5$
- 4)

$5 + 3$	$10 + 20$	$106 + 105$
$8 - 3$	$30 - 20$	$211 - 105$
$7 + 6$	$90 + 40$	$150 + 150$
$13 - 6$	$130 - 40$	$300 - 150$
- 5)

$10 - 4$	$40 - 1$	$20 - 10$
$10 - 5$	$40 - 2$	$20 - 11$
$10 - 6$	$40 - 3$	$20 - 12$
$10 - 7$	$40 - 4$	$20 - 13$

- 6) $4 + 10$ $7 + 10$ $6 + 10$
 $14 + 10$ $17 + 10$ $18 + 10$
 $24 + 10$ $117 + 10$ $33 + 10$
 $44 + 10$ $1,017 + 10$ $71 + 10$
- 7) $7 - 5$ $10 - 3$ $100 - 2$
 $17 - 5$ $20 - 3$ $200 - 2$
 $27 - 5$ $30 - 3$ $300 - 3$
 $37 - 5$ $40 - 3$ $400 - 3$
- 8) $2 + 4$ $10 + 18$ $10 + 13$
 $2 + 14$ $20 + 18$ $12 + 13$
 $2 + 24$ $30 + 18$ $10 + 14$
 $2 + 34$ $40 + 18$ $13 + 14$
- 9) $50 + 40$ $30 + 20$ $40 + 60$
 $7 + 2$ $5 + 7$ $9 + 4$
 $57 + 42$ $35 + 27$ $49 + 64$

- 10) Choose some numbers and ask the students to add 1 to each number. For example, if you say, "108," a student responds, "109." You can vary this activity by asking the students to add 2, 3, 5, or 10 to each of your numbers, or to subtract 1, 2, 3, or 10 from each of your numbers.
- 11) Conduct an activity similar to an exercise entitled "Transforming a Number" in Lesson N3. Put four to ten checkers on the ones and tens boards. Move one checker from some square (it makes no difference which square you choose) to another square. Ask the students how much more or less the new number is. For example, suppose this configuration is on the Minicomputer.



Move a checker from the 4-square to the 10-square. When the students tell you that the new number is more, ask them how much more this number is than the previous number. (In this case, 6 more.)

- 12) Ask the students to give you various names for a particular number. Encourage them to suggest names involving subtraction and also names involving addition of three numbers. For example, names for 10 include $6 + 4$; $20 - 10$; $4 + 4 + 2$; $11 + 1$; 2×5 ; $\frac{1}{2} \times 20$.

Multiplication

The nine examples here are lists of calculations to be done mentally. Each list suggests an arithmetic pattern which students should begin to use in doing such calculations.

- 1) $3 + 3$ $7 + 7$ $100 + 100$
 2×3 2×7 2×100
 $5 + 5$ $20 + 20$ $200 + 200$
 2×5 2×20 2×200

C

- | | | | | |
|----|--|--|--|------------------------------|
| 2) | 1 x 5
2 x 5
3 x 5
4 x 5 | 1 x 10
2 x 10
3 x 10
4 x 10 | 1 x 20
2 x 20
3 x 20
4 x 20 | |
| 4) | 2 x 3
2 x 33
2 x 333
2 x 3,333 | 3 x 3
3 x 33
3 x 333
3 x 3,333 | 2 x 5
2 x 55
2 x 555
2 x 5,555 | |
| 5) | 6 + 6
2 x 6
$\frac{1}{2}$ x 12 | 10 + 10
2 x 10
$\frac{1}{2}$ x 20 | 50 + 50
2 x 50
$\frac{1}{2}$ x 100 | |
| 6) | 2 x 10
2 x 3
2 x 13
2 x 60
2 x 4
2 x 64 | 2 x 40
2 x 2
2 x 42
2 x 100
2 x 7
2 x 107 | 2 x 20
2 x 6
2 x 26
2 x 200
2 x 30
2 x 1
2 x 231 | |
| 7) | 10 x 2
10 x 20
10 x 200 | 10 x 5
10 x 50
10 x 500 | 10 x 8
10 x 80
10 x 800 | |
| 8) | $\frac{1}{2}$ x 6
$\frac{1}{2}$ x 60
$\frac{1}{2}$ x 600
$\frac{1}{2}$ x 6,000
$\frac{1}{2}$ x 6,000,000 | $\frac{1}{2}$ x 10
$\frac{1}{2}$ x 100
$\frac{1}{2}$ x 1,000
$\frac{1}{2}$ x 10,000
$\frac{1}{2}$ x 10,000,000 | $\frac{1}{2}$ x 24
$\frac{1}{2}$ x 240
$\frac{1}{2}$ x 2,400
$\frac{1}{2}$ x 24,000
$\frac{1}{2}$ x 24,000,000 | |
| 9) | 2 x 13
$\frac{1}{2}$ x 26 | 2 x 34
$\frac{1}{2}$ x 68 | 2 x 35
$\frac{1}{2}$ x 70 | 3 x 13
$\frac{1}{3}$ x 39 |

Estimation

Ask the students to estimate a calculation and to explain their answers. For example:

T: *Is $218 + 157$ more than 300? How do you know?*

S: *Yes, because $200 + 100 = 300$.*

T: *Is it more than 500? How do you know?*

S: *No, because $300 + 200 = 500$. 218 is less than 300 and 157 is less than 200, so $218 + 157$ must be less than 500.*

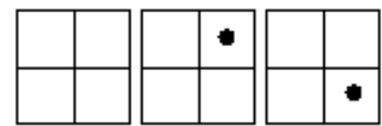
T: *Is it more than 400?*

S: *No, because $18 + 57$ is less than 100.*

During the lessons, students are often asked to estimate a number displayed on the Minicomputer. See Lesson N3 for an example.

Other Mental Arithmetic Activities

1) Use the Minicomputer to give visual clues for addition and subtraction problems. For example, suppose this configuration is on the Minicomputer.



T: *What number is on the Minicomputer?* (41)

Hold a checker over the 8-square.

T: *What number is 41 + 8?* (49)

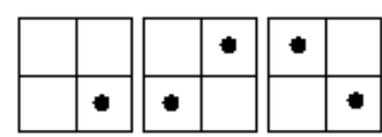
Hold a checker over the 10-square.

T: *What number is 41 + 10?* (51)

After a few such clues, hold a checker over the square only if the class has difficulty.

The next example involves subtraction.

T: *What number is on the Minicomputer?* (169)



Lift the checker temporarily from the 8-square.

T: *What number is 169 - 8?* (161)

Lift the checker temporarily from the 20-square.

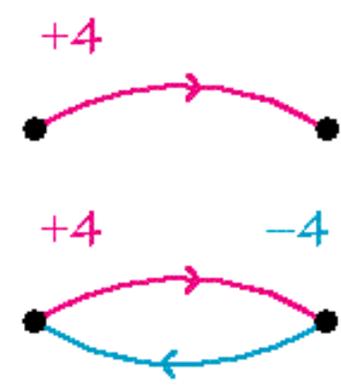
T: *What number is 169 - 20?* (149)

Lift the checker temporarily from the 20-square and the 1-square.

T: *What number is 169 - 21?* (148)

After a few such clues, lift the checkers only if the class has difficulty.

2) Use an arrow picture to pose mental arithmetic questions. For example, draw this arrow picture on the board.



Ask what number is at the right if the number at the left is, for example, 7. Repeat this activity several times and then draw the -4 return arrow.

T: *What could this blue return arrow be for?*

S: -4.

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Ask the students what number is at the left if the number at the right is, for example, 10. Repeat this activity several times.

There are many variations for this activity. For example:

- Use larger starting and ending numbers.
- Use functions other than $+4$ and -4 .
- Use two or more arrows of the same function.



- Use a composition of two functions.



Note: In this last variation, choose the ending number carefully when asking the class for the starting number. For example, if the ending number is 16, then $16 - 3 = 13$ and $\frac{1}{2} \times 13$ might be too difficult a calculation for most students, especially early in the year.

- 3) Play “Guess My Rule.” Choose a secret rule such as $+10$. Then, ask students to suggest a number to you; you respond with the number that is 10 more. Continue until the students discover your rule. For example:

S: 34.

T: 44.

S: 1.

T: 11.

S: 100.

T: 110.

S: 50.

T: 60.

T: *Does any one know my secret rule?*

S: $+10$.

T: *How do you know?*

S: *Each time you answered with a number that was 10 more than the number we gave you.*

Choose other secret rules such as $+3$, $+5$, -2 , -1 , and $2x$.

Calculator Activities

Use the calculator to develop and reinforce counting skills; to emphasize counting patterns; and to improve estimation skills.

- 1) Use one or more calculators in the class to support counting activities in which patterns are generated. For example, such an activity might be similar to the first clue in *Detective Story #2* from Lesson W15. Start with any whole number on the display of your calculator and press $\boxed{+}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ On the board, record the number that appears on the display of the calculator each time you press $\boxed{=}$. As soon as most of your students are able to predict the sequence quickly, you can abandon the calculator. Ask students to explain the pattern.

There are many variations of this activity, such as

- press $\boxed{+}$ $\boxed{5}$, (or $\boxed{\times}$ or $\boxed{4}$ or $\boxed{100}$) $\boxed{=}$ $\boxed{=}$ $\boxed{=}$...; or
- start with a large number and press $\boxed{-}$ $\boxed{5}$, (or $\boxed{\div}$ or $\boxed{100}$ or $\boxed{1000}$) $\boxed{=}$ $\boxed{=}$ $\boxed{=}$...

- 2) Play “Let’s Concentrate” using one or more calculators in the class. Ask students who have the calculators to put a number you specify on the display and then to hide the display with one hand. Give a sequence of two operations and instruct the students to perform them by pressing the appropriate keys without looking at the display. Then, ask what number is on the display, and let the students check to see if they are correct. Continue with the new number and another sequence of two operations. For example:

Start with 4 on the display.

- Press $\boxed{+}$ $\boxed{5}$ $\boxed{+}$ $\boxed{4}$ $\boxed{=}$. The resulting number is 9.
(Students check that 9 is on the display.)
- Press $\boxed{+}$ $\boxed{1}$ $\boxed{\times}$ $\boxed{4}$ $\boxed{=}$. The resulting number is 20.
(Students check that 20 is on the display.)
- Press $\boxed{-}$ $\boxed{5}$ $\boxed{+}$ $\boxed{1}$ $\boxed{=}$. The resulting number is 18.
(Students check that 18 is on the display.)

- 3) Play “Calculator Golf” with a small group of students, each one having a calculator. Ask the students to put a small whole number you specify on the display of their calculators. Then let the students take turns suggesting some numbers to add or subtract until a particular goal, such as 200, is reached.

A possible game in which 200 is the goal and four students are playing is described below.

Start with 12 on the display.		(12)
First player:	+ 9	(21)
Second player:	+72	(93)
Third player:	+17	(110)
Fourth player:	+7	(117)
First player:	+93	(210)
Second player:	-10	(200)

Some variations of this game include

- changing the starting number;
- making the goal a larger multiple of 100;
- allowing the use of $\boxed{+}$ some number between 0 and 10.

THE STRING GAME

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The String Game is used in many versions throughout the *Comprehensive School Mathematics Program*. It gives students an opportunity, in a game-like atmosphere, to become familiar with the language of strings, while at the same time it involves them in the kind of reasoning that will be developed and reinforced in various contexts throughout the program.

The String Game is first played with A-Blocks (shapes), but later, in the intermediate grades, variations that make use of numbers appear. This Appendix provides you with the necessary information and examples to enable you to play the game using A-Blocks, and suggests some of the various possibilities open to you when you play the game with shapes. There are several lessons in *UPG-III* that call for The String Game, but we hope you will not feel restricted to playing the game only at these times. It is most beneficial and enjoyable for students if you make a regular practice of playing the game about once every two weeks or whenever you have an extra 10–15 minutes during the course of the day.

Equipment

PLAYING BOARD

The equipment for this game may be most easily managed if you have a magnetic (magnet-sensitive) chalkboard available. Many permanently mounted chalkboards in classrooms are magnetic; you can test yours using a magnet. If your permanent chalkboards are not magnetic, try any portable chalkboard (dry erase board, and so on) that is available. If you do not have a magnetic chalkboard available, you can use your regular chalkboard.

TEAM BOARD

The Team Board is divided into regions as illustrated below.

(Attach a poster list of string cards here.)	
Team A	Team B

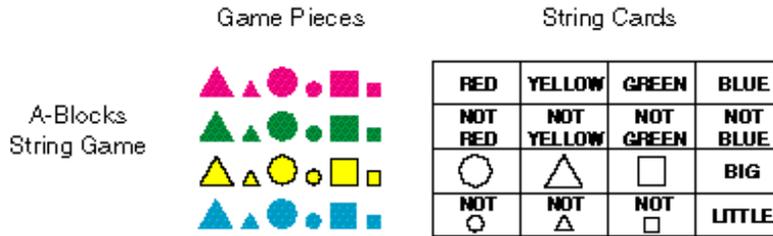
Note: The game may be played with three or four teams rather than two. In this case, create a team board with sections for more teams.

- Magnetic: If you have a large magnetic classroom chalkboard, you can draw the team board directly on a portion of it. However, if you have a relatively small (portable) magnetic chalkboard, you may need to obtain a sheet of metal (minimum size 60 cm by 80 cm) or locate a convenient metallic surface in the classroom, such as the side of a file cabinet, on which to put the team board. In such a case, draw the team board on a large sheet of (chart) paper and tape this paper to your metallic surface.
- Non-magnetic: If you do not have a magnetic chalkboard available for the playing board, your team board can be a large piece of poster board (minimum size 60 cm by 80 cm).

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GAME PIECES AND STRING CARDS

One set of game pieces and string cards is needed for each version of the game. A list of the string cards (the A-Blocks String Game poster) should be posted above the team board—it is a constant reminder during the game of the possible labels for the strings.



Game pieces, string cards, and the poster of the string cards can be found in the A-Blocks String Game kit.

- Magnetic:** You can magnetize the game pieces (A-blocks) by sticking a small piece of magnetic material to the back of each one. (Magnetic material is included in the A-Blocks String Game kit, or it is available in many stores, in the hobby or notions departments.) Similarly, you can magnetize string cards by sticking a small piece of magnetic material to the front of each card, taking care not to obscure what is written on it.
- Non-magnetic:** Game pieces can be attached to the team board using loops of masking tape stuck to the backs. A string card should have a loop of masking tape stuck to the front in such a way that what is written on the card is not obscured. With this type of equipment, be prepared to make necessary repairs by having masking tape on hand so that if a loop of tape loses its stickiness it can be replaced on the spot. As an alternative, use a small wad of a plastic caulking compound (Rope Caulk or Mortite, for example) in place of the loop of masking tape.

Preparation for the Game

Draw two (or three, depending on which variation you are using) large, overlapping strings on the playing board using two (or three) different colors. Next to each of these strings attach one string card facedown. Place the team board conveniently nearby. Randomly distribute the game pieces among the sections of the team board. Divide the class into teams using whatever method is acceptable to your class, and assign each team a section of the team board.

Before any student takes a turn, correctly place an equal number (at least one) of each team's game pieces in the string picture. This eliminates the necessity of beginning the game on the basis of pure guesswork. You can influence how long the game will take by the number of pieces you place in the string picture before the game begins.

Object of the Game

Each team tries to place all of its game pieces correctly (according to the facedown string cards) in the string picture. The winning team is the one that places all of its game pieces correctly and identifies the facedown cards correctly first.

Rules of the Game

- 1) The teams alternate making plays, and the members take turns within each team. A player comes to the board and selects a piece from his or her team's collection to place in one of the regions of the string picture.
- 2) You are the judge. If the piece is correctly placed, say yes. The piece then remains in the string picture and the player immediately has a second (bonus) turn (no player may have more than two consecutive turns). If the piece is incorrectly placed, say no. The player returns the piece to the team's unplayed collection and play passes to the next team.

As an aid in judging, prepare a crib sheet showing the correct position of each game piece or at least reminding you of what is on the facedown cards. If at any time you discover that you have made an error, say so immediately and rectify the mistake. Then, either move an incorrectly placed piece to its correct region or replace a correctly placed piece that has been removed.

- 3) When a team has correctly placed all of its pieces, the player who placed the last piece may then attempt to identify the string cards. If he or she is correct, the team wins. If a mistake is made (even if it is only in the case of one of the string cards), simply indicate that the identification is incorrect and let the game continue.
- 4) If a team has exhausted its stock of game pieces and the strings have not been identified, that team continues to attempt to identify the strings on its turn, while the other team(s) works to place its game pieces.

The A-Blocks String Game (Version A)

This simplest version of the game uses 24 A-blocks as game pieces and only nine string cards.

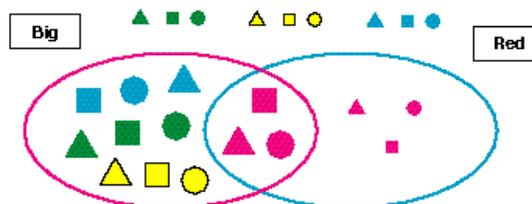


This list of string cards should be attached above the team board.

Below are several crib sheets for variations of the game with two and with three strings.

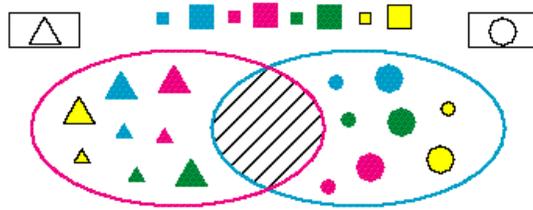
TWO STRINGS

Example 1: No empty regions



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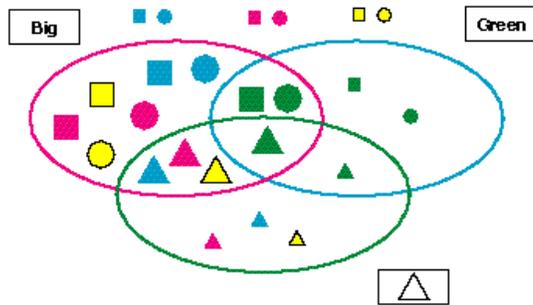
Example 2: One empty region



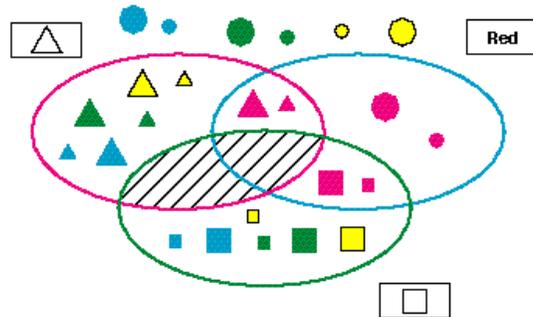
Note: We have indicated that the intersection of the strings is empty by “hatching” that region.

THREE STRINGS

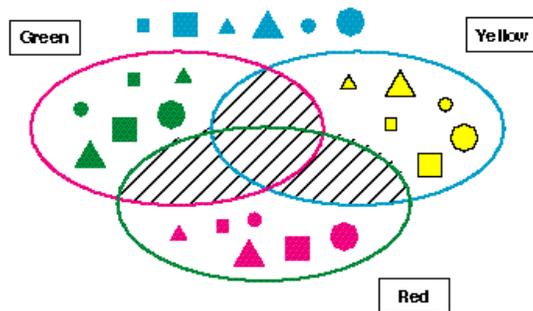
Example 1: No empty regions



Example 2: Two empty regions



Example 3: Four empty regions

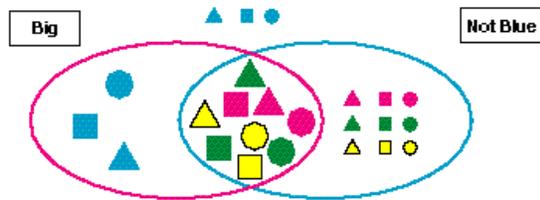


The A-Blocks String Game, Using Not-Cards (Version B)

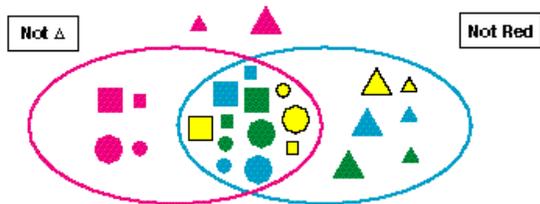
A more complicated version of the String Game with A-blocks uses all 16 of the string cards. A list of all 16 string cards should be posted above the team board.

Here are several crib sheets for this version of the game played with two strings.

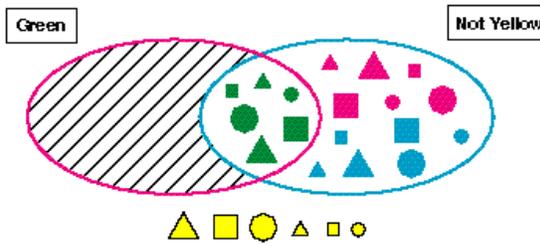
Example 1: No empty regions



Example 2: No empty regions

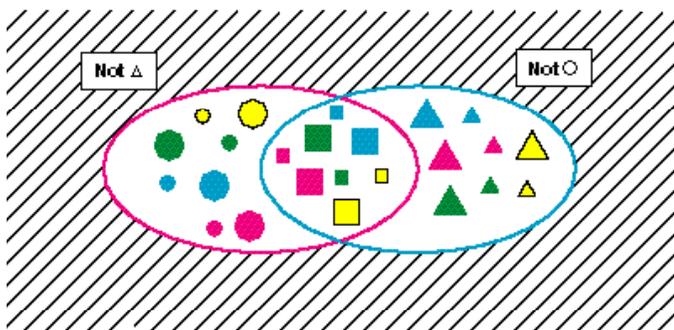


Example 3: One empty region



Example 4: One empty region

Note: By hatching the "outside" region of the diagram we mean to indicate that no game pieces can be placed there correctly. Strictly speaking, that region is not empty because, for example, the number 50 is in the outside region.



You should be warned that the three-string version of the game played with three strings is very difficult to judge without a crib sheet and is equally difficult to play. Hence, use it only when you think the two-string version is no longer challenging enough for the majority of your students.

