Geometry & Measurement

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GEOMETRY & MEASUREMENT INTRODUCTION

Geometry is useful and interesting, an integral part of mathematics. Geometric thinking permeates mathematics and relates to most of the applications of mathematics. Geometric concepts provide a vehicle for exploring the rich connections between arithmetic concepts (such as numbers and calculations) and physical concepts (such as length, area, symmetry, and shape). Even as children experience arithmetic notions in their daily lives, they encounter geometric notions. Common opinion recognizes that each child should begin developing a number sense early. There is no sound argument to postpone the development of geometric sense. These are compelling reasons why the systematic development of geometric thinking should be a substantial component of an elementary school curriculum.

A benefit of early development of geometric thinking is that it provides opportunities for all learners. Children develop concepts at individual rates. We are careful to allow students a great deal of time to explore quantitative relationships among physical objects before studying arithmetic. In the same spirit, students should be allowed to explore geometric relationships through free play and guided experiences. Before studying geometric properties of lines and shapes, they should have free play with models and with drawing instruments, using both to create new figures and patterns. Before considering parallelism and perpendicularity as relations on geometric lines, they should have free play in creating patterns with parallel and perpendicular lines. Before learning length and area formulas, they should have informal experiences to establish and sharpen intuitive notions of length and area. These early experiences are aesthetically rewarding and just as stimulating and suggestive of underlying concepts as early experiences leading to arithmetic. Such experiences are a crucial first step in the development of a geometric sense.

As with most educational development occurring through informal experiences, the payoffs come unexpectedly and spasmodically. In the development of geometric thinking, the suggested sequence of activities is so varied, reaches into so many other facets of the program, and continues for so long a span that it would be nonsense to set *a priori* milestones of accomplishments.

Thus, it is important to allow students to move through the free exploration activities that precede the more substantive lessons without rushing them. These preliminary activities have implicit significance for geometric thinking and also provide aesthetic opportunities and explicit practice—they are not mere play. On the other hand, some activities appear to have important consequences or clearly relate geometry to other parts of the program. Here, your inclination might be to spend more time on these activities than is suggested, but try to remember that similar activities will continue to be experienced later in a slightly different context.

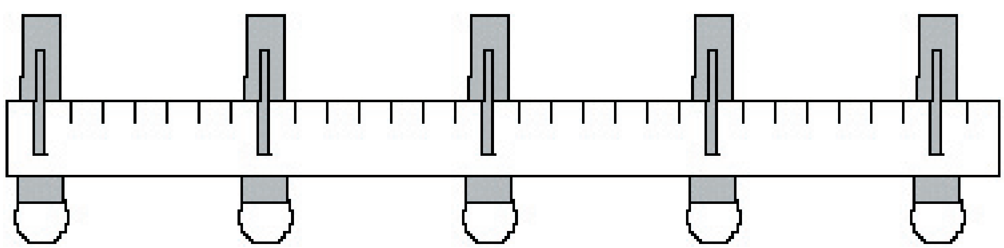
This strand also includes measurement activities, many that lead to the use of metric units, as well as topics not usually found in traditional programs at this level. Measurement activities eventually lead to notions of distance. Traditionally, teachers wait for a good deal of maturity in their students before introducing properties of figures, such as circles, that are defined in terms of distance. The reason is that Euclidean distance (distance "as the crow flies") requires sophisticated notions of "exact" measurement in terms of the real number system in order to be able to measure every line segment. But in the *CSMP* curriculum very young children (even in kindergarten) begin thinking about distance in the simplified setting of "taxi-geometry." Taxi-distance (as a taxicab travels) between points is measured in terms of whole numbers by easy counting processes. Experiences in taxi-geometry lead to some creative situations, yet do not require special technical skills or vocabulary.

Putting the emphasis on experience rather than on mastery frees you to engage in imaginative activities with children that you ordinarily might not attempt. For example, even when considering such commonplace topics as area and length, feel free to devise activities that help students recognize patterns, develop cognitive strategies, and relate the topic to others in the program, while suppressing the urgency to get the basic formulas. What is basic here, at this level, is that children are thinking, rather than doing tasks on command. The skills and mastery will come eventually, in their own time and place in the curriculum.

Note on Grids

A demonstration grid board is needed for several lessons, especially those on taxi-geometry. This should be a large square grid board at least 12 squares by 12 squares (with grid lines spaced 4 to 8 centimeters apart) on which you can draw with colored chalk or colored pens. A portable, commercially made, permanent grid chalkboard is ideal. However, if such a grid board is not available, try one of these alternatives.

- Printed grids on large sheets of paper are provided in classroom sets of materials for *CSMP* for the upper primary grades. Tape one of these sheets to the board, use thick felt-tipped pens for drawings, and discard the sheet after use. You may prefer to laminate or mount one of the sheets on cardboard and cover it with clear contact paper to provide a protective surface. Then use erasable markers or wax crayons for drawings. These grid sheets also are useful as graphing mats.
- For each lesson, carefully draw a grid on the chalkboard using an easy-to-make tool. Either (1) use a music staff drawer with chalk in the first, third, and fifth positions; or (2) clip five pen caps, each having the diameter to hold a piece of chalk tightly, to a ruler at intervals of 4–8 centimeters and secure them with tape.



- Make a grid transparency for use on an overhead projector and use erasable overhead markers. The Blackline section includes several size grids to use in making a grid transparency.

Capsule Lesson Summary

Find several ways to color a shape one-half blue and one-half red. Use the concepts of counting, correspondence, symmetry, and area.

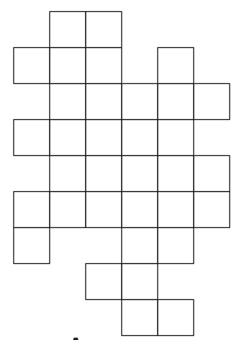
Materials

Teacher		Student	
	• *UPG-III Geometry Poster* #1, #2, and #3 • Red and blue markers		• Colored pencils • Worksheets G1(a) and (b)

Description of Lesson

Exercise 1

Put a copy of shape A from *UPG-III Geometry Poster* #1 on the board. Write coloring directions near the shape .



T: ***Let's color one-half of this shape red and one-half blue. How can we do it?***

S: ***Color the top part red and the bottom part blue.***

T: ***Remember, we want exactly one-half of the shape for each color.***

S: ***Count the squares and color one-half that number each color.***

S: ***Color one square blue, one red, one blue, one red, and so on.***

S: ***Color two squares blue, two red; two blue, two red; and so on.***

S: ***Color these three red and those three blue; this one red and that one blue; these five red and those five blue; and so on.***

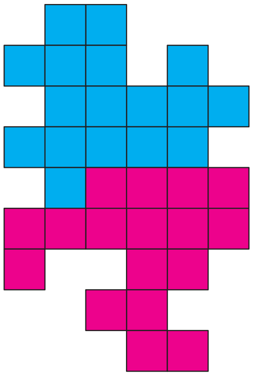
With these last two methods, you have to be careful that you always color both blue and red. To avoid confusion, choose a certain number of blue with the same number of red simultaneously, i.e. treat "two blue-two red" as a single choice. Also, when you consistently choose a number of squares at a time for one color, half the total number of squares must be a multiple of that number for the method to continue working; otherwise an adjustment needs to be made at the end.

Distribute copies of Worksheet G1(a) and colored pencils (or crayons). Shape **A** is on both sides of the worksheet for students wanting another chance at the coloring.

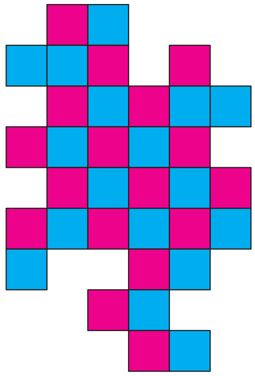
T: ***This worksheet has the same shape as on the board. Color one-half of it red and one-half of it blue using whatever method you prefer.***

As you observe the students' work, talk to them individually about the methods they are using. Worksheet G1(b) can be given at this time to faster students if there is a wide range in the amount of time your students need to color the shape on Worksheet G1(a). Otherwise, Worksheet G1(b) can be used at the end of the lesson.

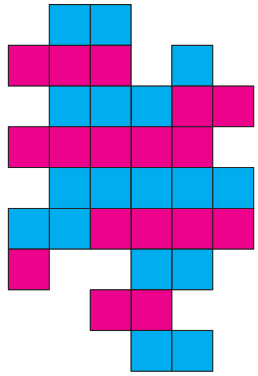
These are some possibilities.



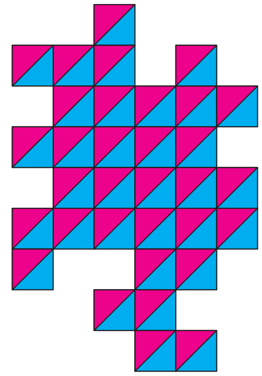
17 blue - 17 red



One blue, one red; one blue, one red; and so on.



Three red, three blue; one red, one blue; five red, five blue; and so on.



One-half of each square is red; the other half is blue.

Put one or two† more copies of shape **A** on the board. Choose two (or three) students with different methods to color the pictures on the board as on their worksheets. The rest of the class can continue coloring their pictures.

When most of the class has completed a picture or has indicated an approach, draw the class's attention to the colorings on the board.

Illustrate one method of deciding how to color the shape as in the following dialogue.

T: ***Some of you counted and found that there are 34 squares in this shape.***

Indicate this on the board.

34 □

T: ***Does anyone know what number $\frac{1}{2}$ x 34 (read as "one-half of thirty-four") is?*** (17)

You may like to suggest a calculation strategy by asking, "What number is $\frac{1}{2}$ x 30? … $\frac{1}{2}$ x 4? … $\frac{1}{2}$ x 34?"

Write the number sentence on the board.

$\frac{1}{2} \times 34 = 17$

T: ***How many red squares should there be in your picture?***

S: ***17.***

T: ***How many blue…?***

S: ***17.***

Verify with your class that there are 17 red squares and 17 blue squares in one of the colorings on the board.

Exercise 2

†An extra copy of shape A can be found on *UPG-III Geometry Poster* #3.

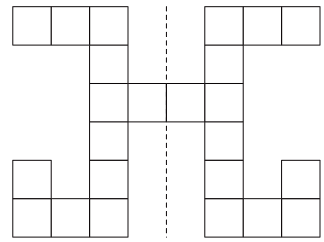
Put a copy of shape **B** from *UPG-III Geometry Poster* #2 on the board.

T: ***How can we color one-half of this shape blue and one-half red?***

Let a student demonstrate a method at the board.
Very likely someone will use the symmetry of the shape.

Do not draw the dotted line. It is here just to make the lesson easier to follow.

A student having indicated the line of symmetry might explain like this:



B

S: ***Color this side red and this side blue .***

T: ***Why does your method work?***

S: ***There are the same number of squares on each side.***

T: ***How many squares?***

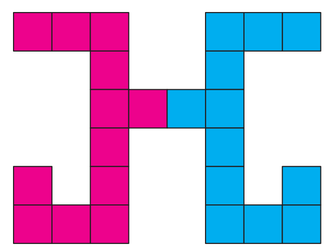
S: ***Twelve on each side.***

T: ***Can anyone explain this method in a different way?***

S: ***The shape is the same on both sides.***

Allow discussion of methods similar to those used in Exercise 1, but emphasize a discussion of symmetry. The line of symmetry is like a mirror; the shape is exactly the same on both sides.

Quickly color the shape in this manner.



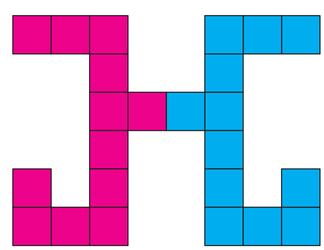
B

Leave shape B on the board and put a copy of shape **C** from *UPG-III Geometry Poster* #2 next to it.

T: ***Once again, how can we color one-half of the shape blue and one-half red?***

Let a student demonstrate a method at the board.

A student having indicated the dotted, separation line may explain like this:



B

S: ***Color this side red and this side blue.***

T: ***How do you know that would give us exactly one-half red and one-half blue?***

S: ***The sides are the same.***

T: ***Do both sides look exactly the same (like mirror images)?***

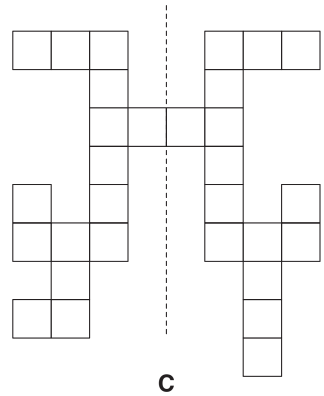
S: ***No. Part of this picture is the same as our last shape, but there are three extra squares on each side.***

Discuss that the separation line does cut the shape in half (same size on both sides), but it is not a line of symmetry because the two sides are not mirror images of each other.

Invite someone to point to these particular squares.

They are shaded here.

Do not draw the dotted line. It is here just to make the lesson easier to follow.



C

T: ***How many red squares were in the last shape (B)?***

S: ***Twelve.***

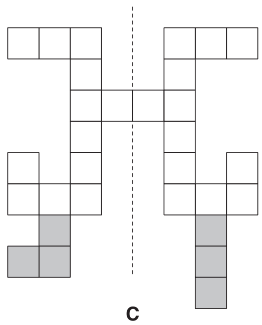
T: ***How many red squares should there be in this shape (C)?***

S: ***Fifteen, 12 + 3 = 15.***

T: ***How many blue squares …?***

S: ***Fifteen also; 12 + 3 = 15.***

Color the shape as in the diagram on the right.



C

Exercise 3

Take shapes ~~**B** and **C** off the b~~oard and put a copy of shape **D** from *UPG-III Geometry Poster #3* on the board.

T: ***Here is another shape for us to color one-half blue and one-half red.***

Let students come to the board and point to squares they would color red and to squares they would color blue. Students might suggest these colorings or similar ones.

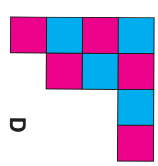
If so, count the squares with the students to show that these colorings are not correct. Ask for another method.

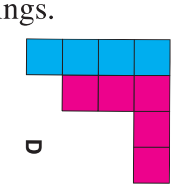
S: ***Color four red and four blue, and leave the other one uncolored.***

T: ***But the whole shape should be colored one-half blue and one-half red.***

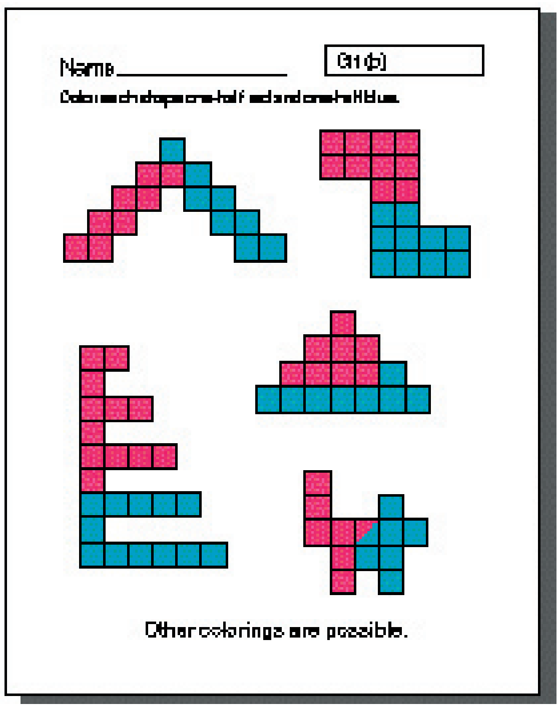
S: ***Cut the extra square into two halves.***

Let a student color the shape. Two possibilities are given below. You may want to use both copies of the shape to let students display two different colorings.





Worksheet G1(b) is available for individual work.


Name ____________ G1(b)

Color each shape one-half red and one-half blue.

Other colorings are possible.

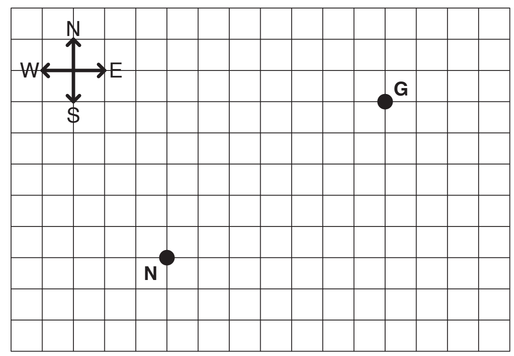
Capsule Lesson Summary

Introduce paths that follow the lines on a grid, and describe paths in terms of direction: north, south, east, and west. Measure the lengths of paths and look for shortest paths between two points. Introduce the notion of taxi-distance.

Materials

Teacher	• Grid board[†] • Colored chalk or markers	Student	• Colored pencils, pens, or crayons • Worksheets G2, G2* and **

Description of Lesson



Display a grid board. Draw and label two dots as shown here and on Worksheet G2. Indicate directions in an upper corner of the grid. Students should have a copy of the worksheet. Several times during the lesson, suggest that students work with a partner.

T: ***This is a map of a city. The lines are for streets. Starting at any corner, there are four directions to go: north, south, east, and west.***

Let students show how to move in each of these four directions starting from **N**.

T: ***Does anyone remember who lives here (at* N*)?***

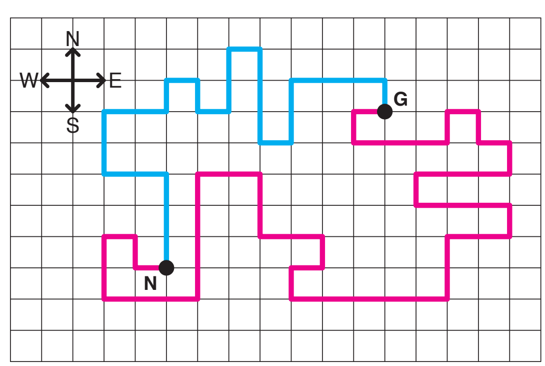
S: ***Nora.***

T: ***Who lives here (at* G*)?***

S: ***Nora's grandmother.***

Note: For entry classes, introduce the city as Nora's Neighborhood and tell the class that Nora lives at **N** and Nora's grandmother lives at **G**.

T: ***Nora likes to walk all around the city and she often stops at her grandmother's house to visit. Who can show us a nice long path from Nora's house to her grandmother's house? Remember, Nora only walks on sidewalks along the streets.***



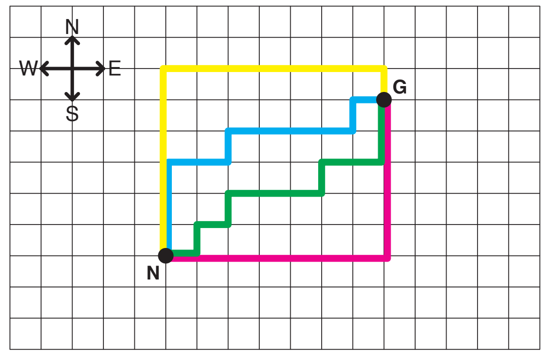
Call on volunteers to trace long paths from **N** to **G**. Do not draw them. The illustration here gives two examples of such paths.

Instruct students to draw a path from **N** to **G** on their worksheets. Then each student should describe his or her path to a partner (without letting the partner see the path) while the partner tries to duplicate it on his or her worksheet.

[†]See the "Note on Grids" section in the introduction to this strand.

T: ***This week is rainy in Nora's city, and Nora's grandmother isn't feeling very well. Nora visits her every day. Since Nora doesn't like to walk a long distance in the rain, she tries to find some short paths. Who can show us a short path from Nora's house to her grandmother's house?***

Invite several students to trace or describe short paths. There are many shortest paths, all of which have the same length. Accept any of these or any close to them in length. Draw the paths yourself, using a different color for each path. Students should draw a couple short paths on their worksheets.

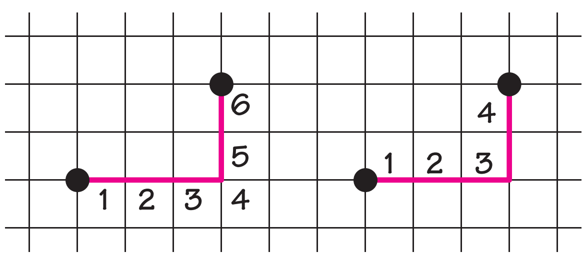


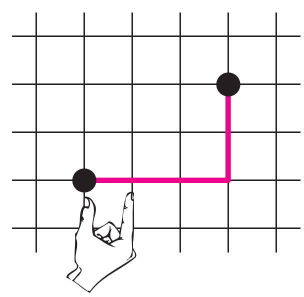
Very likely your students will indicate several shortest paths and one or two slightly longer paths. The paths in this illustration will be used for the lesson description. The red, blue, and green paths are shortest paths; the yellow is longer.

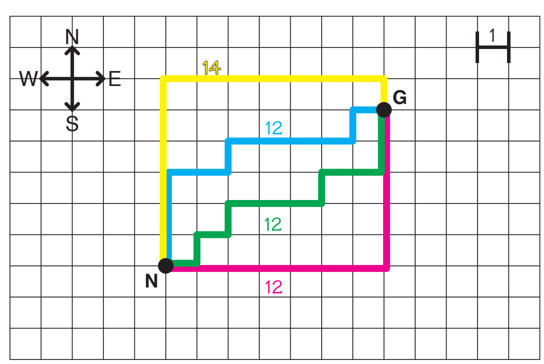
T: ***Which path is the shortest? How do you know?***

Use this opportunity to discuss how to measure the length of a path. The most obvious way is to count city blocks. Indicate the unit (1 block) on the grid, and call on students to measure different paths.

Note: Some very common mistakes can occur when students count the blocks in a path, especially at the corner turns. The next illustration shows two common counting errors. Both paths in the illustration are, in fact, five blocks long. To avoid counting mistakes, it may be helpful to instruct the students to show each block with their thumb and index finger as they count. This technique is illustrated below.







Write the length of each path in the same color as the path.

Erase any path that is not a shortest path; for example, the yellow one in the lesson description.

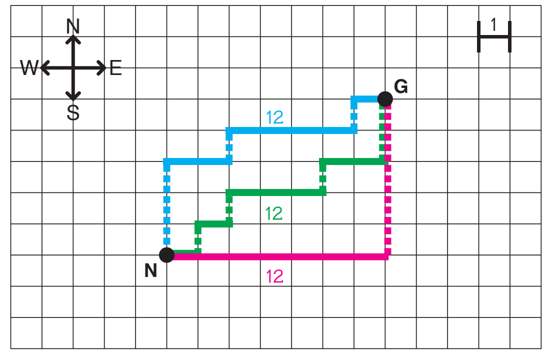
T: ***Why do all the paths on the board have the same length? Why are there so many shortest paths?***

Most likely students will have good ideas, but they might be difficult to verbalize. First let students discuss the question with their partners. Then encourage them to come up to the board; let them make gestures and use their own words to explain.

T: ***In which directions does Nora walk in all of these paths?***

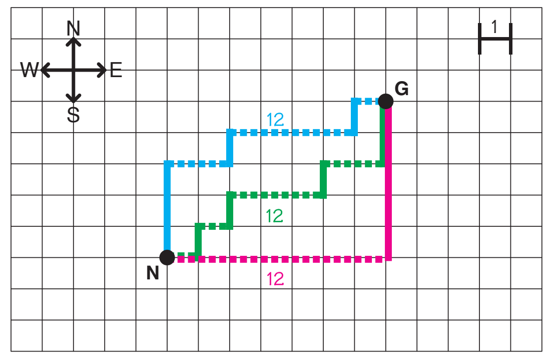
S: ***North and east.***

T: ***Let's look at how many blocks Nora walks north in each of these paths.***



Consider each path on the board. For each one, invite someone to count the blocks going north. Conclude Nora walks five blocks north in each one. The dotted lines in the next picture show the blocks on which she walks north for the particular paths in the lesson description.

T: ***Let's look at how many blocks Nora walks east in each of these paths.***



Again, let students count the blocks going east and conclude Nora walks seven blocks east.

T: ***For each of the shortest paths, Nora walks five blocks north and seven blocks east. 5 + 7 = …?***

S: ***12.***

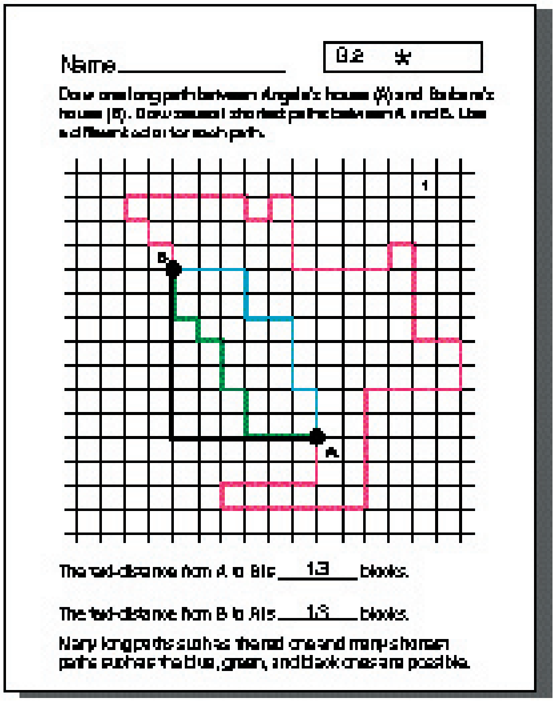
T: ***When Nora follows the streets, all the shortest paths from her house to her grandmother's house have exactly the same length: 12 blocks. We say that the taxi-distance from* N *to* G *is 12 blocks.***

Write this on the board. **taxi-distance from N to G = 12 blocks**

Worksheets G2* and ** are available for individual work.

Center Activity

Place maps in a center for students to use in planning routes between locations. Use the directions north, south, east and west to tell or write about the routes.

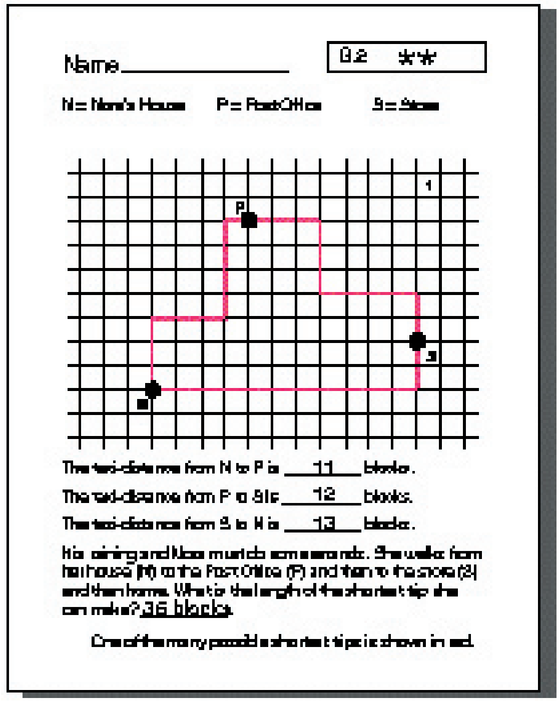

Name______________ G2 *

Draw one long path between Angela's house (A) and Barbara's house (B). Draw several shortest paths between A and B. Use a different color for each path.

The taxi-distance from A to B is ___13___ blocks.

The taxi-distance from B to A is ___13___ blocks.

Many long paths such as the red one and many shortest paths such as the blue, green, and black ones are possible.


Name______________ G2 **

N = Nora's House P = Post Office S = Store

The taxi-distance from N to P is ___11___ blocks.

The taxi-distance from P to S is ___12___ blocks.

The taxi-distance from S to N is ___13___ blocks.

It is raining and Nora must do some errands. She walks from her house (N) to the Post Office (P) and then to the store (S) and then home. What is the length of the shortest trip she can make? 36 blocks.

One of the many possible shortest trips is shown in red.

Capsule Lesson Summary

Construct rectangles for numbers using square tiles and cutting the rectangles out of grid paper. Make a display of all the rectangles and observe which numbers have only one rectangle, have two or three rectangles, have the most rectangles, are squares, and so on.

Materials

Teacher
- Grid paper
- Scissors
- Tape

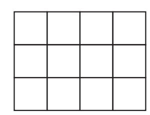
Student
- Square tiles
- Grid paper
- Scissors
- Tape

Advance Preparation: Use Blackline G3 to make grid paper that has the same size grid squares as most commercial square tiles (one-inch squares). Otherwise, choose a grid in the blacklines that has grid squares to match your square tiles. You may also make construction paper square tiles to match your grid paper.

Description of Lesson

Exercise 1

Allow students to work with a partner, and provide each pair with twelve square tiles. Direct the students to make a rectangle using all twelve square tiles. After a couple minutes, invite several pairs of students to describe their rectangles. As each rectangle is described, demonstrate how to cut it out of grid paper.



S: ***Our rectangle is three squares by four squares.***

T: ***Does it look like this or this?***

In the discussion, clarify what a rectangle is and include language of dimensions (3 by 4, or 4 by 3). Agree that the 3 by 4 and 4 by 3 rectangles are essentially the same, although they are positioned differently.

T: ***Did anyone make a different rectangle with all twelve squares?***

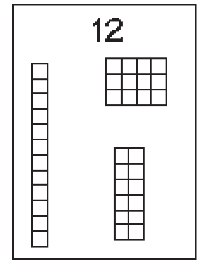
S: ***Our rectangle is two squares by six squares.***

With the class, find the three different rectangles for 12. For the 1 by 12 rectangle, show how to tape grids together. Observe that these rectangles all have area 12 little squares.

Note: There are, of course, other rectangles with area 12 little squares, but in this lesson only those that can be covered with whole square tiles will be considered.

T: ***I would like you to help me make rectangles for other numbers. In fact, let's make all the rectangles we can for the numbers from 1 to 27.***

Note: We selected 27 here assuming there are 26 students in the class and the number 12 has been completed. Each student then will be assigned a different number. On the board or on a long wall, prepare space to display all rectangles for numbers from 1 to 27. Display the three rectangles for 12.



Provide student pairs with grid paper (be sure the grid size is the same as the square tiles), scissors, and tape. Assign each pair two numbers and direct them to make all the rectangles for those numbers (determine the area in little squares), cutting them out of grid paper and displaying them. One way to do this is to write the numbers from 1 to 27 (except 12) on slips of paper and put them in a box; then let individual students choose a number randomly. You may prefer to arrange that no pair has two prime numbers. Student pairs who complete their job before others can work on other numbers, or check rectangles that are put in the display to be sure all possible rectangles have been found.

For your information, every number has a "1 by that number" rectangle. In addition, composite numbers have the following rectangles.

4: 2 by 2	14: 2 by 7	22: 2 by 11
6: 2 by 3	15: 3 by 5	24: 2 by 12; 3 by 8; 4 by 6
8: 2 by 4	16: 2 by 8; 4 by 4	25: 5 by 5
9: 3 by 3	18: 2 by 9; 3 by 6	26: 2 by 13
10: 2 by 5	20: 2 by 10; 4 by 5	27: 3 by 9
12: 2 by 6; 3 by 4	21: 3 by 7	28: 2 by 14; 4 by 7

Exercise 2

Ask the class to make some observations about the display of rectangles. You may start by just asking what students notice. Then, as appropriate, ask,

T: ***Which numbers have only one rectangle?***
Which numbers have squares?
Which numbers have a two by something rectangle?
Which number has the most rectangles?
Which numbers have exactly two (three) rectangles?

You may like to let students predict how many rectangles the next one or two numbers will have. For example, 29 has only one rectangle and 30 has four rectangles. Or, ask students to predict the next number to have a square among its rectangles. (36 has five rectangles including a 6 by 6 square.) Save the rectangles made during this lesson for use in Lesson N11.

Capsule Lesson Summary

Find the length in centimeters of zigzags by measuring the lengths of the parts and adding them together. Find a shorter path with the same end points as a given zigzag path both when there is an obstacle to circumvent and when there is not.

Materials

Teacher
- *UPG-III Geometry Posters* #4 and #5
- Centimeter tape measure
- Calculator

Student
- Paper
- Centimeter tape measure
- Straightedge
- Worksheets G4*, **, ***, and ****

Description of Lesson

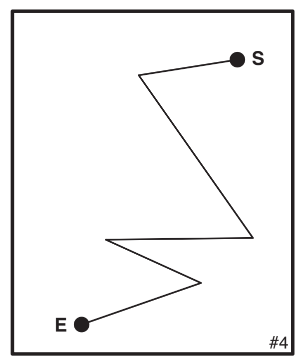
You may want to arrange for groups of students to do the measuring for Exercise 1 before the lesson begins.

Exercise 1

Organize the class in groups of two to four students and provide each group with a centimeter tape measure. Let each group select three or four things they would like to measure for length. Let each group report to the class what they measured, how they made the measurements, and their results.

Exercise 2

Tape *UPG-III Geometry Poster* #4 to the board and put a centimeter tape measure in an obvious place near the board.



If your class was introduced to the story of a bug walking a zigzag path to find some food last year (in *UPG-II* Lessons G4 and G8), recall that situation for the class. Otherwise, begin by introducing the bug.

T: ***Imagine that here*** (point to S) ***is a bug and here*** (point to E) ***is some food. This bug is very hungry and starts walking, hoping to get to the food he smells. The bug starts at*** S ***and walks this zigzag path*** (trace the path from S to E) ***to end at*** E. ***Did the bug get to the food?***

S: ***Yes.***

T: ***How can we find out how far the bug walked?***

S: ***Measure the path.***

T: ***What unit should we measure in?***

S: ***Centimeters.***

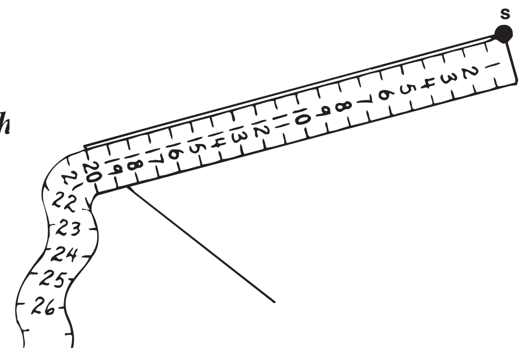
If necessary, suggest centimeters yourself, and write the unit on the board.

centimeter (cm)

T: ***Do you know the abbreviation for centimeter?***

S: ***Cm.***

Hold up a centimeter tape measure for the students to see.



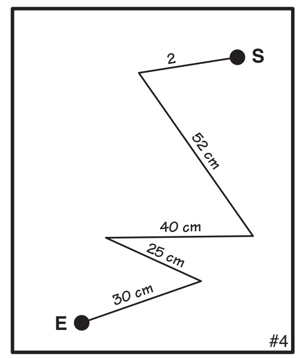
T: ***How can we use this tape to measure the bug's path***

S: ***Measure each piece and then add the numbers.***

T: ***Before we do the measuring, can you predict how long this path is?***

Allow students to make predictions. Then call on different students to measure each segment of the zigzag path. With each student, emphasize that they hold the end mark on the tape (0) at one end of the segment, and read the number of centimeters at the other end. Use this opportunity to give instructions, on how to use a tape measure to measure a segment.

When a segment's length is announced be sure to include the unit. Write the lengths next to the segments on the poster.



T: ***We have measured all of the pieces. Now, how do we find the length of the entire path?***

S: ***Add.***

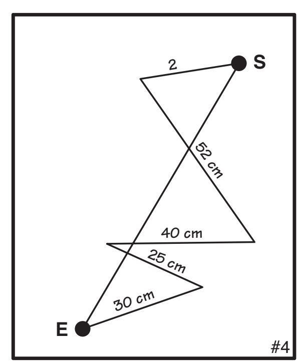
T: ***Tell me the numbers to add and I'll write them on the board.***

Write the numbers in column form as a student gives them. Instruct students to do the addition problem on their papers. Then invite a student to do the calculation at the board and to explain. You may like to let a student check the calculation with a calculator.

$$\begin{array}{r} 20 \\ 52 \\ 40 \\ 25 \\ +\ 30 \\ \hline 167 \end{array}$$

T: ***The bug walked 167 centimeters. Do you think there is a shorter path that the bug could have walked?***

Let students trace paths until someone traces the line segment from **S** to **E**. Draw that path.



Ask a student to measure the segment from **S** to **E**.

S: ***66 centimeters.***

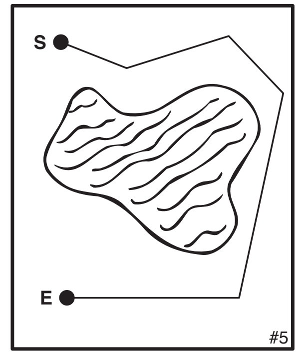
T: ***How much shorter is this path than the zigzag path that the bug walked? What calculation do we need to do?***

S: ***101 centimeters; 167 – 66 = 101 or 66 + 101 = 167.***

Erase the addition problem on the board and remove the poster.

Exercise 2

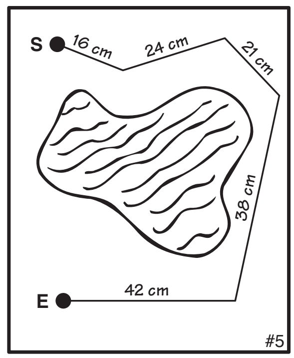
Tape *UPG-III Geometry Poster* #5 on the board.



T: ***This bug is still hungry and manages to find some more food. Only this time there is a big puddle of water in its way. Our bug cannot swim so it walks around the puddle.*** (Trace the zigzag from **S** to **E** with your finger.) ***About how long is this path? How can we find out how far the bug walked?***

Continue as in the previous exercise, letting students measure each segment of the bug's path.

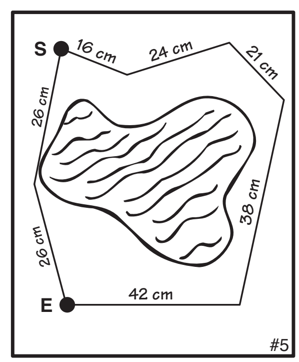
Write the appropriate addition problem, instruct students to do the calculation, and then call on a student to do the addition at the board. Again, you may let a student check the calculation with a calculator.



$$\begin{array}{r} 16 \\ 24 \\ 21 \\ 38 \\ +\ 42 \\ \hline 141 \end{array}$$ centimeter (cm)

Discuss the possibility of finding shorter paths from **S** to **E** and invite students to trace shorter paths. Draw one of them and let a student measure it. Encourage students to make observations about finding shorter paths, such as shorter paths usually pass close to the water.

For example:



T: ***26 + 26 = 52. This path is 52 cm and is shorter than the path the bug walked. How much shorter? What calculation do we need to do?***

S: ***89 cm. 141 – 52 = 89 or 52 + 89 = 141.***

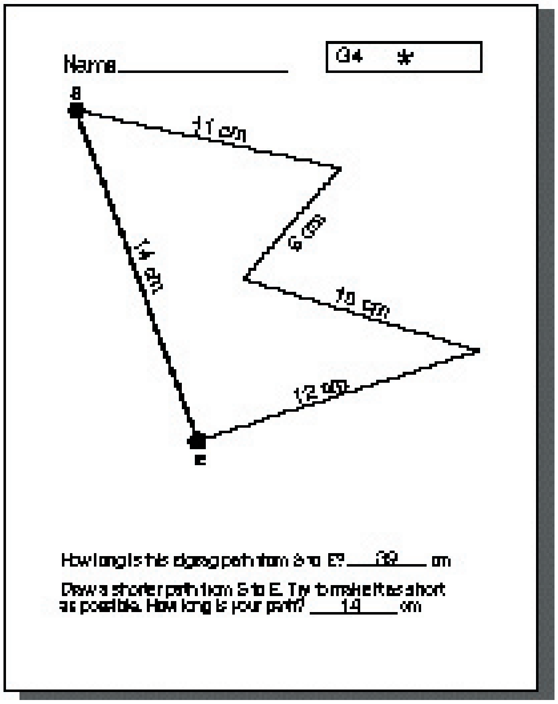
Worksheets G4*, **, ***, and **** are available for independent or group work. Students will need tape measures and straightedges.

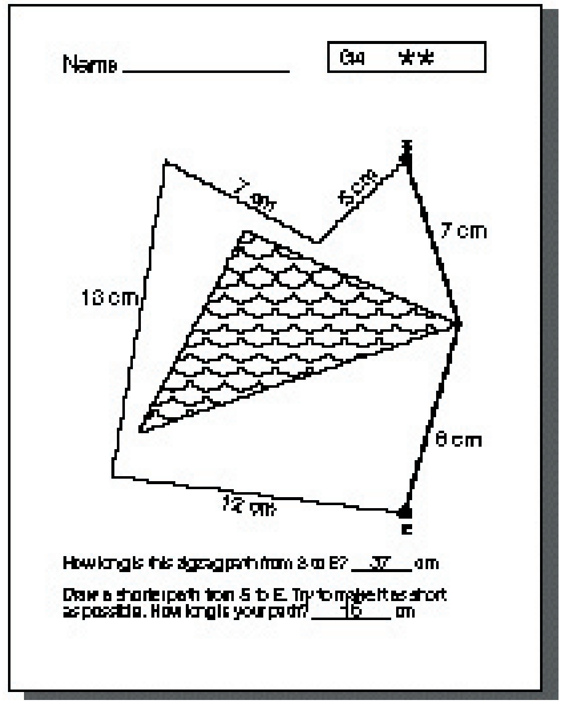
Writing Activity

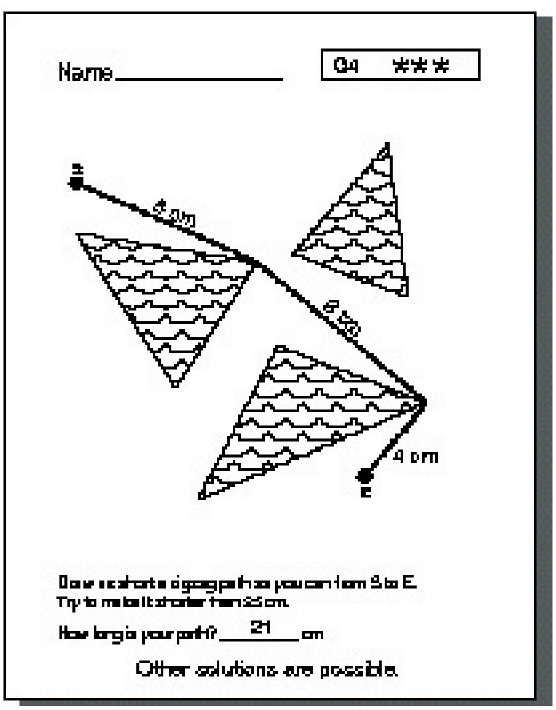
Suggest students choose one worksheet and write a note to the bug explaining how to find a shorter (shortest) path.

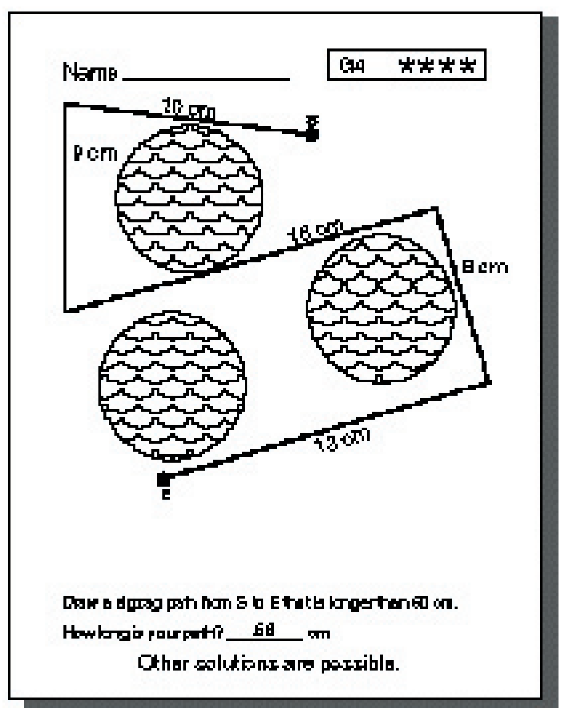
Home Activity

Suggest that parents/guardians find some things around the house (such as a table top, a book, or a window sill) to measure lengths in centimeters and in inches. Compare the number of centimeters to the number of inches for a particular object.


Name
G4 *
S
11 cm
6 cm
10 cm
12 cm
14 cm
E
How long is this zigzag path from S to E? 39 cm
Draw a shorter path from S to E. Try to make it as short as possible. How long is your path? 14 cm


Name
G4 **
S
7 cm
5 cm
7 cm
13 cm
8 cm
12 cm
E
How long is this zigzag path from S to E? 37 cm
Draw a shorter path from S to E. Try to make it as short as possible. How long is your path? 15 cm


Name
G4 ***
S
8 cm
9 cm
4 cm
E
Draw as short a zigzag path as you can from S to E. Try to make it shorter than 25 cm.
How long is your path? 21 cm
Other solutions are possible.


Name
G4 ****
10 cm
S
9 cm
16 cm
8 cm
13 cm
E
Draw a zigzag path from S to E that is longer than 50 cm.
How long is your path? 56 cm
Other solutions are possible.

G5 FRACTIONAL PARTS OF SHAPES #2

Capsule Lesson Summary

Divide a shape into halves, into thirds, and into fourths. Use concepts of counting, correspondence, symmetry, and area. Find the number of little squares in the whole shape and in one-half, one-third, and one-fourth of the shape. Divide another shape into thirds and observe congruence.

Materials

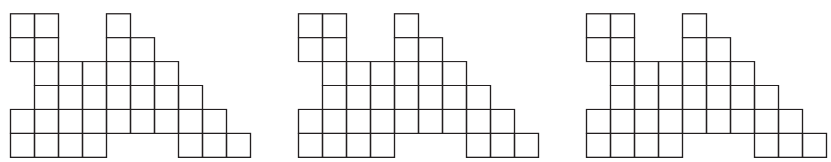
Teacher
- *UPG-III Geometry Posters* #6 and #7
- Tape
- Colored markers

Student
- Paper
- Worksheets G5* and **
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

Put three copies of shape **A** from *UPG-III Geometry Posters* #6 and #7 on the board.



A **A** **A**

T: ***I put three shapes on the board. What do you think of these shapes?***

Let the students comment freely.

T (pointing to the left-most copy)**:** ***I want this shape to be colored one-half red and one-half blue. How can we do it?***

S: ***Count the squares. Then color one-half of that many squares red and the others blue.***

T: ***How many squares are there in each of these shapes?***

S: ***36.***

T: ***What number is $^1/_2$ x 36 (read as "one-half of thirty-six")?***

S: ***18.***

T: ***Let's check.***

On the board, write the multiplication number sentence and a related addition problem. Students should copy these number sentences on their papers.

$$\tfrac{1}{2} \times 36 = 18 \qquad \begin{array}{r} 18 \\ +\ 18 \\ \hline \end{array}$$

T: ***What number is 18 + 18?***

S: ***36.***

If students offer the division fact 36 ÷ 2 = 18, write it on the board as well.

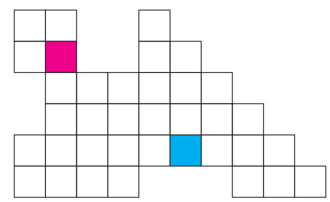
T: ***We could color 18 squares red and then color 18 squares blue. What is another way we could color one-half of this shape red and one-half blue?***

S: ***Color one square blue and one square red; one square blue and one square red; and so on.***

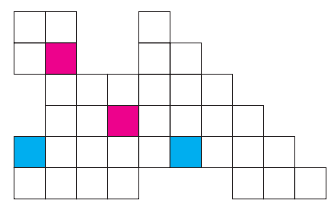
Students may suggest other ways of coloring the shape, but the two in the above dialogue are the ones in which we are most interested.

Choose two students to come to the board to color the shape furthest to the left. Give one student a red marker (or crayon) and the other a blue one. Encourage the rest of the class to count the red and the blue squares as the two students color them.

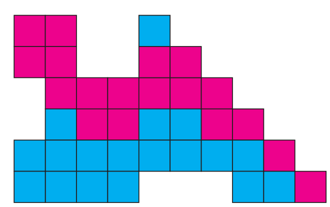
T: ***When I say go, I want each of you*** (look at the two students at the board) ***to color a square. … Go!***



T: ***Color another square. … . Go!***



Continue signaling the students to color another square until every square in the shape has been colored. This is one possible coloring.



$\frac{1}{2} \times 36 = 18$

Note: You might want to instruct the students to simply put colored X's or squiggles in the squares. This can keep the lesson from slowing down due to careful coloring.

T: ***How many red squares are there?*** (18)

Point to each red square as the students count them.

T: ***How many blue squares are there?*** (18)

Point to each blue square as the students count them.

Stand by the middle copy of shape **A**.

T: ***I want this shape to be colored one-third red, one-third blue, and one-third green. How can we do it?***

S: ***Divide the number of squares by 3 and color that many squares each color.***

T: ***We already know there are 36 squares in this shape. How many should be colored red? … blue? … green?***

S: ***12 of each color.***

T: ***How do you know?***

S: ***36 ÷ 3 = 12.***

S: ***12 + 12 + 12 = 36.***

S: ***3 x 12 = 36, so $^1/_3$ x 36 = 12.***

Write some of these number sentences on the board.

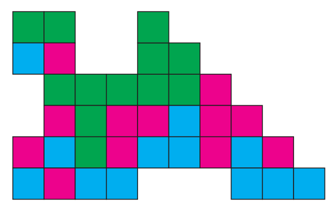
$$\begin{array}{r} 12 \\ 12 \\ +12 \\ \hline 36 \end{array} \qquad \begin{array}{r} 12 \\ \times\ 3 \\ \hline 36 \end{array} \qquad \begin{array}{c} 1/3 \times 36 = 12 \\ 36 \div 3 = 12 \end{array} \qquad \begin{array}{r} 12 \\ 3\overline{)36} \end{array}$$

Instruct students to write some of these number sentences on their papers.

T: ***We could color 12 squares red, then 12 squares blue, and then 12 squares green. What is another way we could color one-third of this shape red, one-third blue, and one-third green?***

S: ***Do what we did before, only this time let three people color squares.***

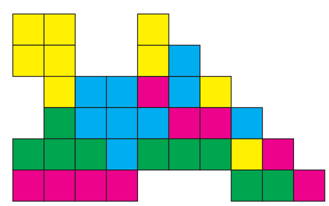
Choose three students and give them each a different color marker. Signal them to color squares so that three squares are colored each time until all the squares in the shape have been colored. This is one possible coloring.



$1/3 \times 36 = 12$

Point to all the squares of each color as the students count them to verify that there are twelve of each color.

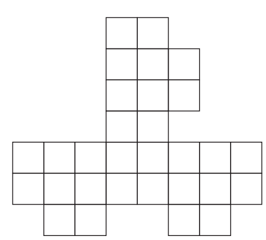
Repeat this activity with the remaining copy of shape **A**, except this time color one-fourth red, one-fourth blue, one-fourth green, and one-fourth yellow. You will need four students to come to the board. This is one possible coloring.



$1/4 \times 36 = 9$

Exercise 2

Tape shape **B** from *UPG-III Geometry Poster* #7 to the board. If necessary for space, remove the copies of shape **A** from the board.

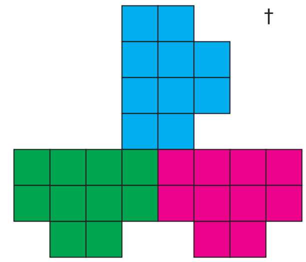


B

T: ***Look at this shape carefully. Let's color it so that it will be easy to see one-third red, one-third blue, and one-third green. How should we do it?***

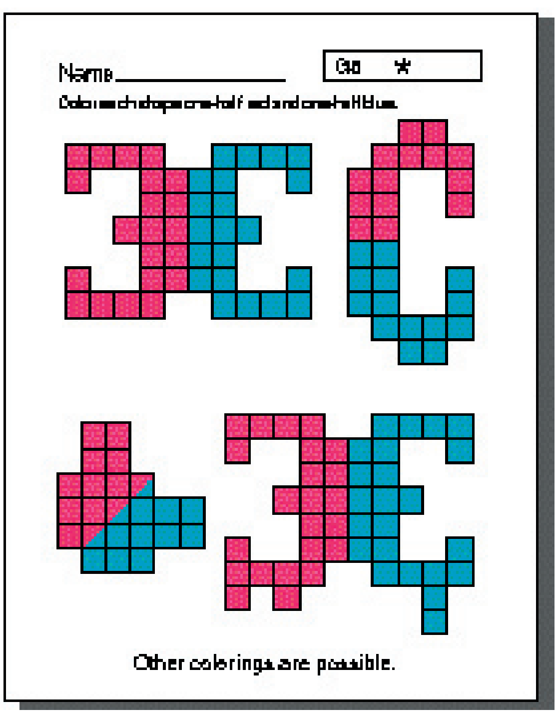
G5

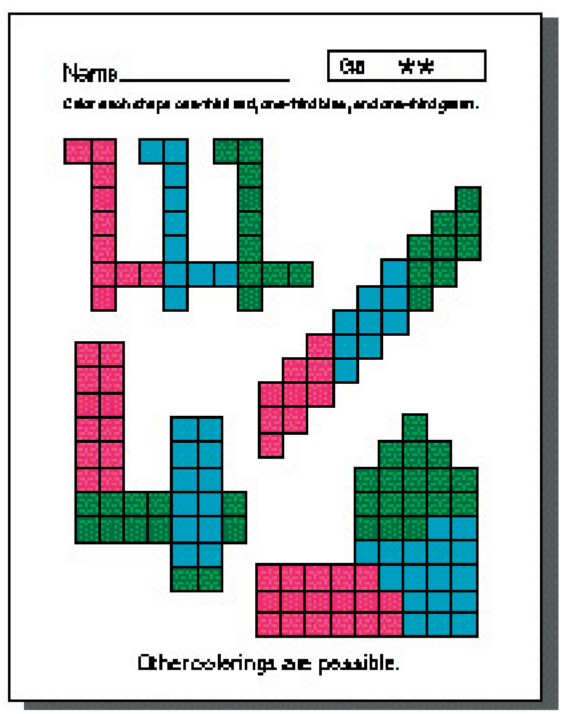
Let students come to the board and point to the pieces they would color red, blue, and green. If someone suggests letting three students color the squares as in the previous exercise, accept this as a good method but observe that someone not watching while the coloring was being done would probably need to count the squares of each color to check that there were an equal number of red, blue, and green. Then ask the class if they can think of another way to color the shape so that it would be easy to see one-third of each color at a glance.


[†]

When the coloring illustrated here is suggested, color the poster yourself. (Again, colored X's will suffice.) Discuss how the three colored pieces look exactly the same; in other words they are *congruent*.

Worksheets G5* and ** are available for individual work. Students will need colored pencils.





[†]The fact that there are congruent pieces is important, but which piece is colored a particular color is not.

G6 NINE-SQUARE AND SIXTEEN-SQUARE

Capsule Lesson Summary

Find the areas of shapes that can be made by combining two squares of different sizes where each square is composed of smaller 1 cm by 1 cm squares. If a new shape is created by overlapping the two shapes, you must completely cover a 1 cm square, or not cover it at all.

Materials

Teacher
- Nine-square and sixteen-square
- Overhead projector (optional)
- Tape
- Grid board
- Colored chalk
- Blackline G6

Student
- Squares and rectangles
- Colored pencils, pens, or crayons
- Scissors
- Worksheets G6(a) and (b)

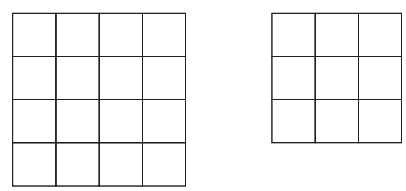
Advance Preparation: Use Blackline G6 to make squares and rectangles for students. The nine-square and sixteen-square will be used in Exercise 1. The rectangles will be used in Exercise 2 for further exploration. Prepare demonstration nine- and sixteen-squares. You may like to do this for use on an overhead projector, or you may like to make the squares larger for use on the board. Grid posters in the classroom set have 8 cm grid squares and can be used for this purpose.

Description of Lesson

Exercise 1

Display a grid board with grid size the same as the small squares in your demonstration nine- and sixteen-squares.

Hold up the two demonstration shapes.



A B

T: ***What shape are these?***

S: ***Squares.***

T : ***What is the area of this square*** (point to **B**)***?***

S: ***9.***

T: ***9 what?***

S: ***9 little squares.***

T: ***How did you get 9?***

S: ***I counted the little squares.***

S: ***3 + 3 + 3 = 9.***

S: ***3 x 3 = 9.***

Record the area of square **B** on the board.

T: ***What is the area of this square*** (point to **A**)***?***

S: ***16 little squares.***

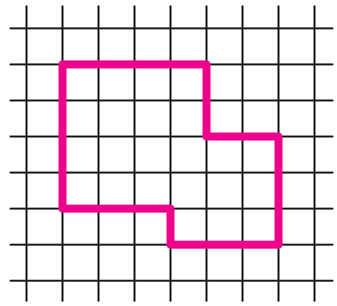
G6

T: ***How did you get 16?***

S: ***4 x 4 = 16.***

S: ***Half of the square is 8 (4 + 4 = 8) and 8 + 8 = 16.***

Record the area of square **A** on the board.



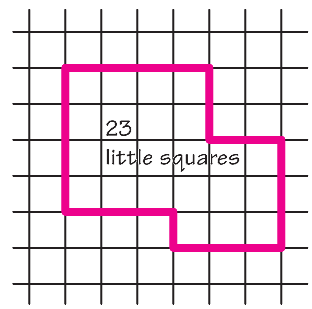
Place (tape) the demonstration shapes on the grid board, carefully overlapping the squares to get the shape in this illustration. Outline the border of the resulting shape in red.

Remove the demonstration shapes leaving only the outline on the grid. You may need to touch up the outline to make it clear.

T: ***What is the area of this new shape?***

S: ***23 little squares.***

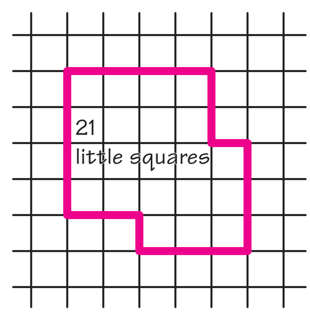
If there is disagreement, collectively add the number of squares in each row: 4 + 4 + 6 + 6 + 3 = 23. Indicate area 23 little squares inside the shape.



T: ***Why is the area only 23 little squares and not 25 little squares, the total area of the two squares (9 + 16 = 25).***

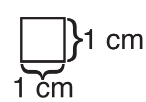
S: ***Because we overlapped the squares.***

Repeat this activity for another shape formed by overlapping the two squares, as shown here. Note for students that when you overlap the squares you must either completely cover a little square or not cover it at all.



Provide students, in pairs if you wish, with a nine-square and a sixteen-square, Worksheet G6(a), and red pencils. If necessary, ask students to cut out the squares carefully.

Indicate a grid square on the worksheet and a little square in either the nine-square or the sixteen-square.



T: ***Each little square on your paper is exactly 1 centimeter on a side.***

The area of one of these little squares is one square centimeter.
This shape (hold up a sixteen-square) ***has area 16 square centimeters. This one*** (hold up a nine-square) ***has area 9 square centimeters.***

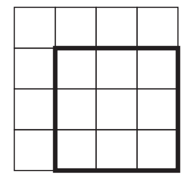
Point out that on Worksheet G6(a) the area of the shape similar to the first one you demonstrated at the beginning of this exercise is shown as 23 cm^2. Mention that cm^2 is an abbreviation for square centimeters.

Point to the second shape on the board (area 21 little squares).

T: ***I want you to make a similar shape with your nine-square and sixteen-square. Then outline the shape on your worksheet. Record its area inside the shape.***

Check to make sure the students are outlining the shape on grid lines. When most students have completed this task, ask,

T: ***What do you think is the smallest area of a shape you can make using these two squares?*** (16 cm^2)

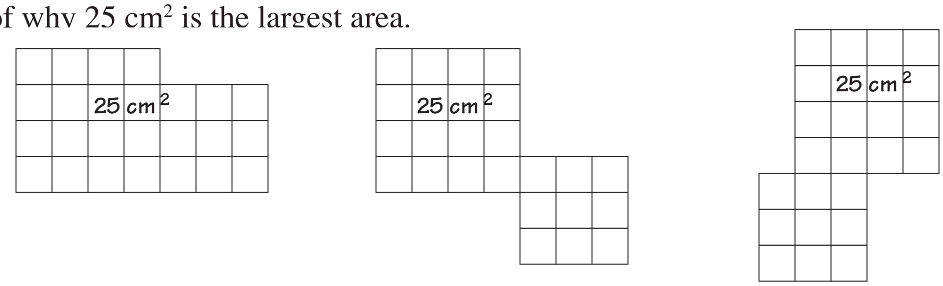


Let students experiment, and then call on a student to show a solution using the demonstration shapes. Solicit an explanation of why this is the smallest area.

Direct students to make and outline a similar shape on their worksheets, and then to record its area inside.

T: ***What is the largest area we could get?*** (25 cm^2)

Let students experiment, and then let several students come to the board, one at a time, to show various solutions. Three of several possible solutions are illustrated below. Again, ask for an explanation of why 25 cm^2 is the largest area.



Direct students to draw one of the shapes with area 25 cm^2 on their worksheets and to record the area inside.

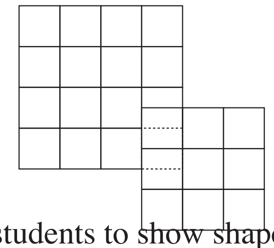
T: ***The smallest area we can get with the nine-square and the sixteen-square is 16 cm^2, and the largest area is 25 cm^2. We also found a shape with area 21 cm^2 and a shape with area 23 cm^2.***

Record the whole number areas from 16 cm^2 to 25 cm^2 on the board. Circle those that have already been found.

16 cm^2	21 cm^2
17 cm^2	22 cm^2
18 cm^2	23 cm^2
19 cm^2	24 cm^2
20 cm^2	25 cm^2

T: ***Do you think we could find shapes for any of these areas?*** (Indicate the areas that are not circled in the list on the board.) ***If you find such a shape, outline it on the back of your worksheet and record its area. Remember, when you cover part of a little square, you must cover all of it.***

Note: You may need to illustrate that your last comment prohibits overlapping the squares as shown here.



Encourage students to work with their partners. When several students or pairs of students have found a shape with a new area, circle the appropriate area on the board.

Shapes with areas 19 cm^2, 22 cm^2, and 24 cm^2 can be found. Invite some students to show shapes with these areas using the demonstration shapes. Sample solutions are shown on the Worksheet G6(a) answer key. Under the given restrictions, it is impossible to make shapes with areas 17 cm^2, 18 cm^2, and 20 cm^2 using the nine-square and the sixteen-square. Ask the students for explanations.

S: ***We can't get 17 cm^2 (18 cm^2) because we already have 16 cm^2 in the larger square and so we need just one (two) more little square(s) to get 17 cm^2 (18 cm^2). Using the nine-square, the fewest (other than none) we can get is three more squares.***

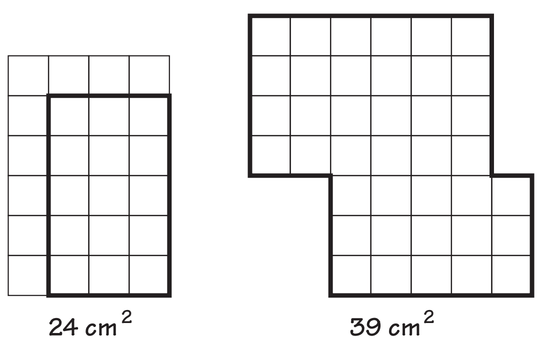
S: ***We can't get 20 cm^2 because we already have 16 cm^2 in the larger square and so we need four more little squares to get 20 cm^2. Using the nine-square, we can get three more or six more, but not four more little squares.***

Another approach is to look at how many little squares of the sixteen-square would need to be covered by the nine-square to get such areas.

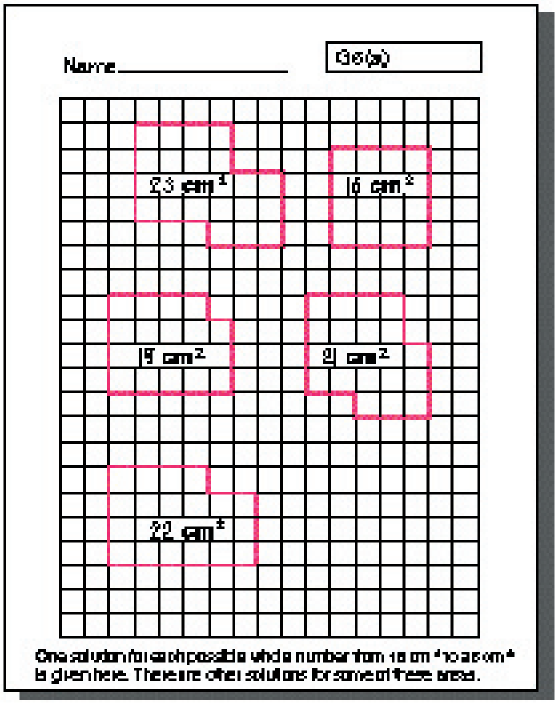
Exercise 2

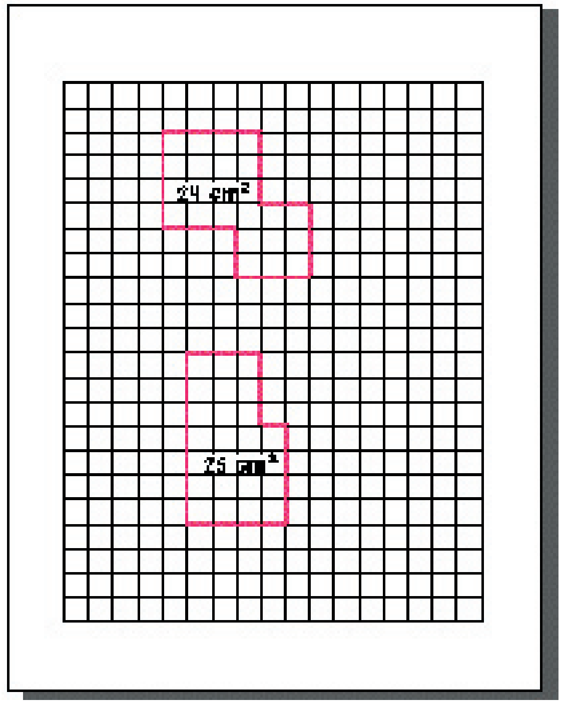
Repeat Exercise 1, finding all possible areas using the rectangles on the lower half of Blackline G6. This exercise can be conducted mostly as a student exploration with little or no demonstration. Students can record their findings on Worksheet G6(b).

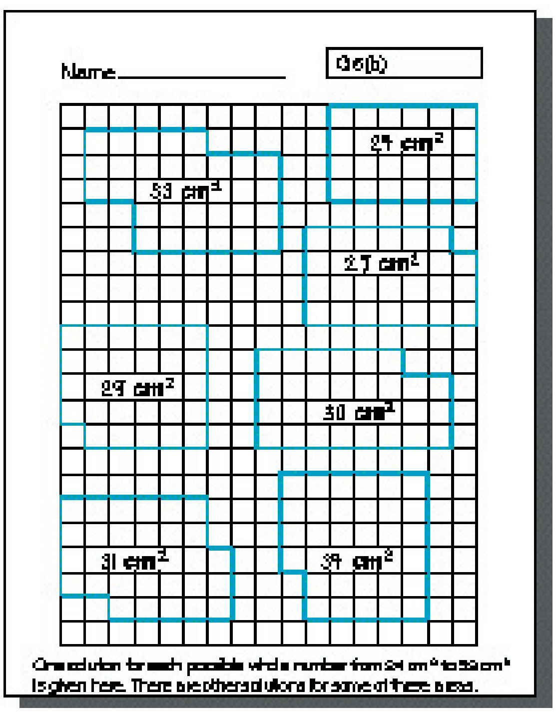
The smallest shape that can be made with these rectangles has an area of 24 cm^2. The largest shape has an area of 39 cm^2.

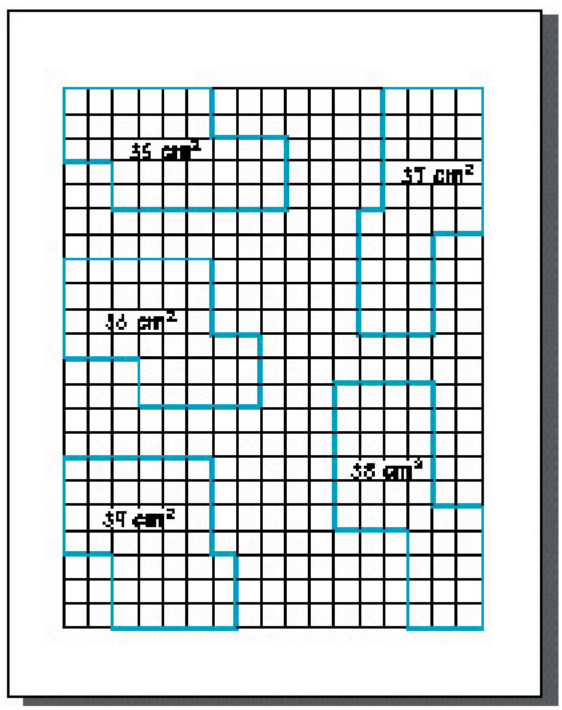


Shapes with whole number areas from 24 cm^2 to 39 cm^2 can be made with the exception of 25 cm^2, 26 cm^2, 28 cm^2, and 32 cm^2.


Name
G6(a)
23 cm²
16 cm²
19 cm²
21 cm²
22 cm²


24 cm²
25 cm²


Name
G6(b)
33 cm²
27 cm²
29 cm²
30 cm²
31 cm²


35 cm²
37 cm²
36 cm²
38 cm²
39 cm²

G7 WHERE SHALL WE MEET? #1

Capsule Lesson Summary

Draw and compare several shortest paths from Nora's house to her grandmother's house. Find places the same taxi-distance from a rollerskating rink and a movie theatre. Do related problems involving Nora's house and a friend's house.

Materials

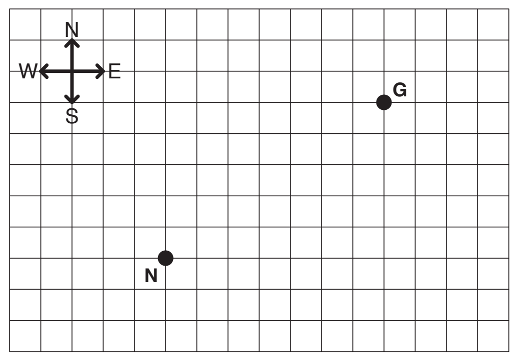
Teacher		Student	
	• Grid board • Colored chalk or pens		• Worksheets G7* and ** • Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

Note: This first exercise reviews Nora's neighborhood and the notion of taxi-distance; it should move quickly.

Display a grid board with directions indicated in an upper corner. Draw and label two dots as shown.



T: ***Remember this map of Nora's neighborhood? Who can tell us something about Nora?***

S: ***Nora likes to visit her grandmother.***

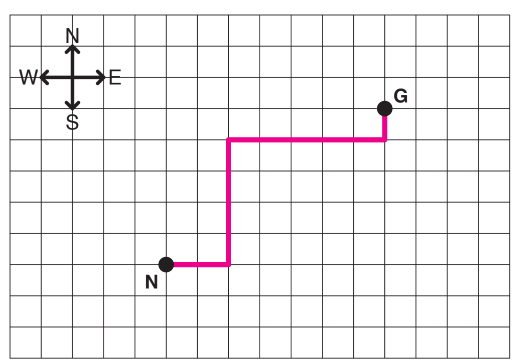
S: ***Nora sometimes takes long walks and sometimes short walks.***

S: ***Nora doesn't cut through people's yards. She follows the streets.***

Call on a student to trace a walk from **N** to **G**. Accept any path that follows the lines on the grid board. Then ask for a longer walk and possibly a shorter walk.

T: ***When it is raining, Nora finds a shortest route to her grandmother's house. Who can show us a shortest walk?***

Invite students to trace paths until a shortest path (in this case, one in which Nora walks only in the north and the east directions) is suggested, and then draw it on the board. Help students count how many blocks are in each path that is drawn.

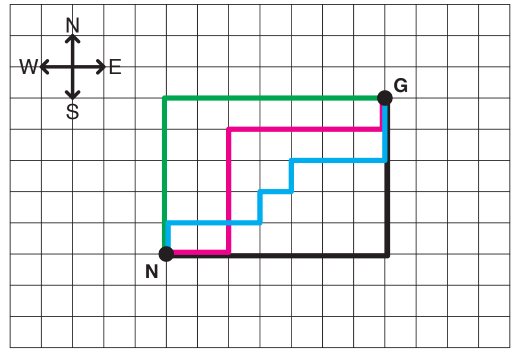


S: ***This walk is 12 blocks long.***

T: ***Does anyone think that Nora could find a shorter walk?***

G7

Students should observe that there are several shortest paths and all have a length of 12 blocks. But for students who are not yet convinced, continue asking students to trace paths until several shortest paths have been suggested and drawn on the grid board.



T: ***Who can explain why there are many shortest paths, each having a length of 12 blocks?***

A student might suggest looking at how many blocks Nora walks east and how many blocks north, but most likely you will need to initiate this discussion yourself.

T: ***Let's look at the red path. How many blocks would Nora walk east*** (indicate the east direction of the map)***?***

Ask the class to count the blocks of the red path in the east direction as you indicate the blocks, one at a time, using the "index finger and thumb" method (see Lesson G2).

In the same manner, count the blocks Nora would walk east in each path on the board. Conclude that on any shortest path Nora would walk seven blocks in the east direction.

T: ***Let's count the number of blocks Nora would walk north in each of these paths*** (indicate the north direction of the map)***.***

Ask a student to count the blocks Nora would walk north on one of the paths. Conclude there are five blocks in the north direction.

Write this information on the board.

7 blocks east
5 blocks north

T: ***How many blocks are there altogether in each of these paths?***

S: ***12 blocks.***

T: ***Does anyone remember what we call the length of shortest paths?***

S: ***Taxi-distance.***

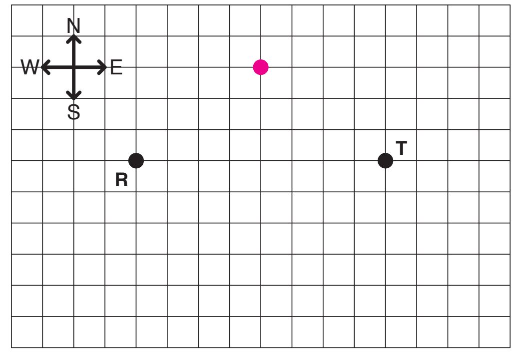
Write this information on the board to the side of the grid board.

taxi-distance from
N to G = 12 blocks

Exercise 2

Clear the grid board and then draw two dots labeled **R** and **T** as shown here.

T: ***Early one afternoon a friend calls Nora and suggests they meet at a place the same taxi-distance from the rollerskating rink*** (point to **R**) ***as from the movie theater*** (point to **T**)***. From there they can decide whether to go skating or to a movie. Shortly after Nora hangs up the telephone she becomes confused. She realizes that her friend could be referring to different places. Does anyone see a place the same taxi-distance from*** **R** ***and*** **T*****?***



Draw a dot in red as suggested by a student.

T: ***What is the taxi-distance from the rollerskating rink to this dot*** (tap the red dot)***?***

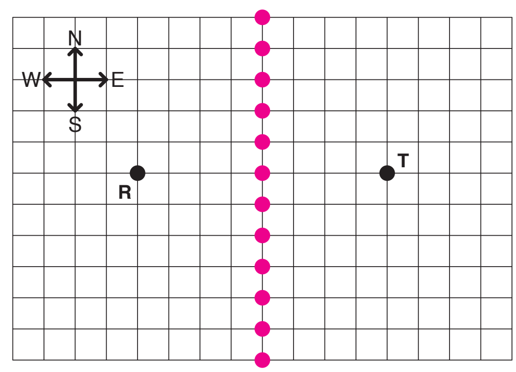
Let a student count the blocks. Make sure the student is considering a shortest path between the red dot and **R**.

S: ***7 blocks.***

T: ***What is the taxi-distance from the movie theater to this dot*** (tap the red dot)***?***

S: ***7 blocks.***

T: ***So if Nora were here*** (red dot)***, she would have the same distance to walk to* R *or to* T*.***



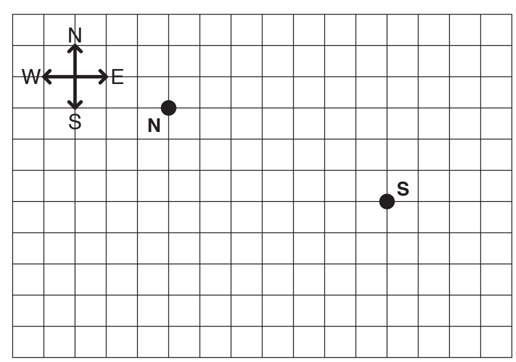
Continue asking for other places the same taxi-distance to **R** and to **T** until you have this picture. (Check the taxi-distance for several but not all of the suggested dots).

T: ***If this map were bigger, do you think we could draw more places the same taxi-distance from* R *and* T*?***

S: ***Yes, all the places in this line.***

Exercise 3

You may like to allow students to work in pairs for this exercise. Each student should have a copy of Worksheet G7*.

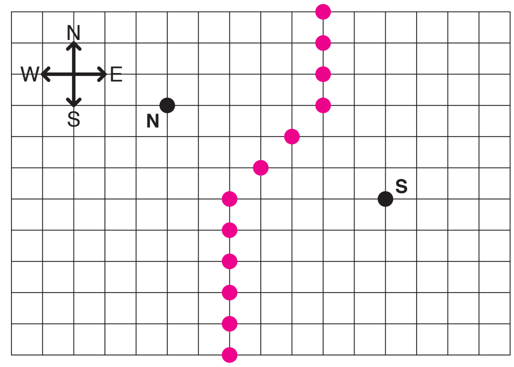


Clear the grid board and draw two dots labeled **N** and **S** as on the worksheet.

T: ***Nora has a friend Sally. Sally lives here*** (point to **S**) ***and Nora lives here*** (point to **N**).

On your worksheet, try to find places that are the same taxi-distance from Nora's house and from Sally's house. Use a red pencil to mark the places you find.

As you observe students' work, help those who cannot find any correct place by suggesting one, and then check with them that the taxi-distance from that dot to **N** is the same as the taxi-distance from that dot to **S**. Call on some students to transfer dots from their worksheets to the demonstration grid board. Suggest that they count blocks over and up (down) from either **N** or **S**.



A completed picture should look like this one.

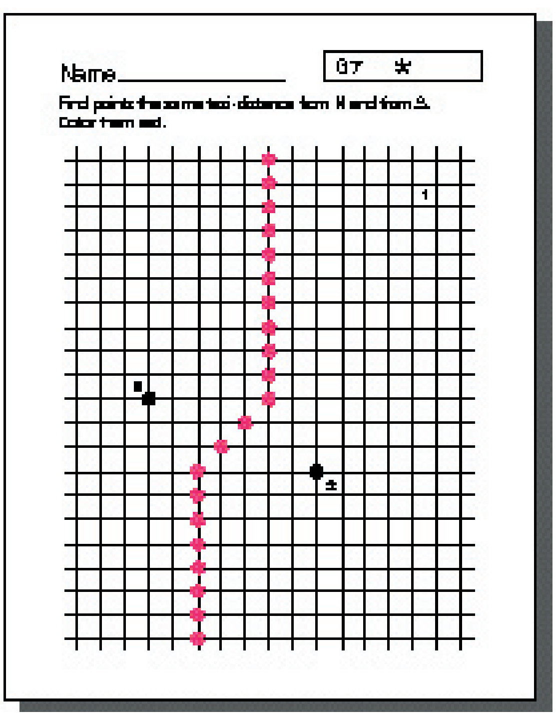
Afterward, check several of the red dots on the board by tapping each and asking, “How far is it from Nora’s house to this dot? … from Sally’s house to this dot?”

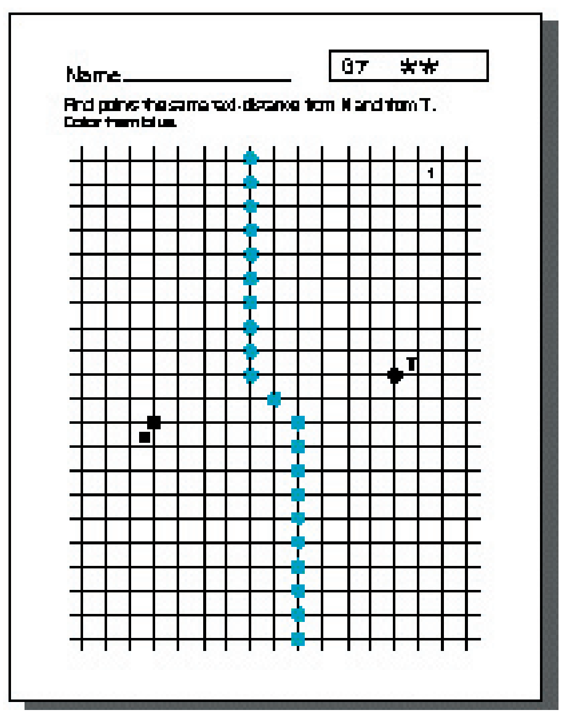
T: ***What would the picture look like if the map were bigger?***

S: ***We would get more dots going straight up and more going straight down.***

Refer students to Worksheet G7**, asking them to find the places the same taxi-distance from Nora’s house (**N**) and Tom’s house (**T**). Students can work individually or with a partner on this worksheet.

Encourage students to observe the patterns of dots the same taxi-distance from the two houses. They may want to compare the differences between the two worksheets and especially the difference between the problems in Exercise 2 and Exercise 3.





Capsule Lesson Summary

Review centimeter and meter units of length, and estimate the heights of some people in the room. Find things in the room that measure about 1 m, about 10 cm, and about 1 cm. Check estimates by measuring with a centimeter tape measure. Draw a zigzag with three specified lengths, and then calculate the length of the entire zigzag path.

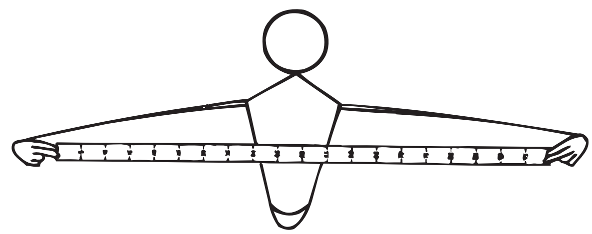
Materials

Teacher	• Centimeter tape measure • Meter stick or centimeter ruler	• Straightedge • Unlined paper
Student	• Centimeter tape measure	

Description of Lesson

Exercise 1

Write **1 m** on the board and display 1 meter with a tape measure or a meter stick.



T: ***I wrote the length of this tape measure (meter stick) on the board. Do you know what the m is for?***

S: ***Meter.***

T: ***Do you know how many centimeters that is?***

S: ***100 centimeters.***

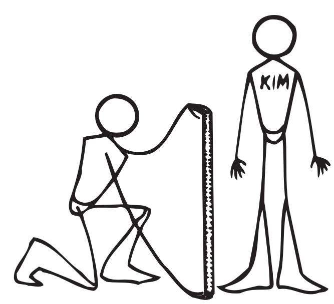
Record this equivalence on the board.

1 m = 100 cm

Ask one of your students to stand up. Suppose her name is Kim.

T: ***Do you think Kim is taller or shorter than 1 meter?***

Let students give their opinions, and then compare the length (1 m) of the tape measure (meter stick) to the height of the particular student (Kim).



T: ***Kim is taller than 1 meter. Do you think she is taller or shorter than 2 meters?***

S: ***Shorter.***

With a student's help, display a length of 2 meters using two tape measures (or two meter sticks) next to the particular student (Kim).

T: ***Do you think that there is anyone in the room shorter than 1 meter?***

Compare the length (1 m) of the tape measure (meter stick) to the height of any student suggested.

T: ***Do you think that there is anyone in the room taller than 2 meters?***

Perhaps some of your students will suggest that you or another adult in the room is taller than 2 meters. You may want to let the students compare your height to the length of 2 meters.

T: ***Look around the room and see if there is something that measures about 1 meter.***

Look at the difference between 1 meter and the actual measurement of each item suggested.

S: ***The cabinet.***

T: ***How do you want to measure the cabinet?***

S (indicating with a hand motion)**:** ***The width across.***

T (measuring the width of the cabinet)**:** ***106 centimeters. Is that longer or shorter than 1 meter?***

S: ***Longer.***

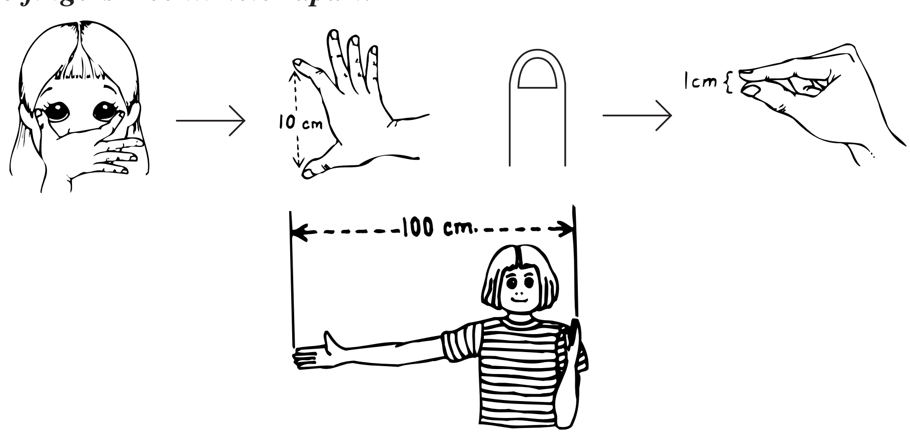
T: ***1 meter = 100 centimeters, so 106 centimeters is longer than 1 meter. How much longer?***

S: ***6 centimeters.***

Continue in this manner until five or six things have been measured. Commend students for good estimates.

Repeat this activity, looking for things in the room that measure about 10 centimeters in some direction and things that measure about 1 centimeter in some direction.

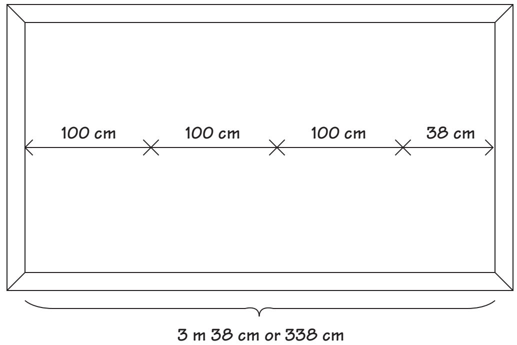
T: ***Everyone try this*****:** ***Hold your hands 1 meter apart…***
hold two fingers 10 centimeters apart …
hold two fingers 1 centimeter apart.



Check at least one of the three estimates with each student.

T: ***How long do you think the chalkboard is?***

Write estimates on the board, and then measure the length of the chalkboard in the usual way. Call on students to help you. For example, your chalkboard might be like the one shown below.



Acknowledge students who made close guesses, determining with the class how close each guess is.

Exercise 2

You may like students to work in pairs for this exercise.

Distribute blank paper, and centimeter tape measures or metric rulers. Students should have straightedges of some kind.

T: ***Draw a zigzag with three pieces on your paper. The first piece should be 25 centimeters; the second, 15 centimeters; and the third, 10 centimeters. There are many ways to draw such a zigzag.***

Write the measurements on the board.

25 cm
15 cm
10 cm

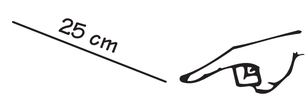
As students complete their zigzag, they can exchange papers with their partners to check each other's work.

After most of the students have drawn a zigzag, call the class's attention to the board.

T: ***Let's draw a zigzag with these measurements on the board. Who can help me draw the 25 centimeter piece?***

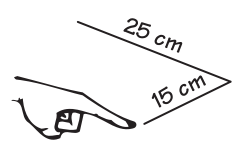
Ask the student who volunteers to show you the 25 cm mark on the meter stick and then to hold the stick in place while you draw the segment. Label it 25 cm.

T: ***The second piece is to be 15 centimeters. I'll start it at the end of the first piece*** (point to it)**.**



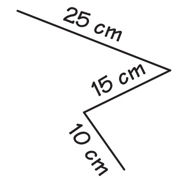
Again, call on a student to help you. Ask the student to show you the 15 cm mark on the meter stick and then to hold the stick in place while you draw the segment. Label it 15 cm.

T: ***The third piece is to be 10 centimeters. I'll start it at the end of the second piece*** (point to it)**.**



G8

As before, draw the third segment with a student's help.



T: ***What is the length of the zigzag*** (trace the whole thing)***?***

When a student suggests adding the lengths of the three pieces, write the addition problem on the board, and invite someone to do the calculation at the board while others do it on their papers.

25 cm + 15 cm + 10 cm = 50 cm

$$\begin{array}{r} 25\text{cm} \\ 15\text{ cm} \\ +10\text{ cm} \\ \hline 50\text{ cm} \end{array}$$

Assign two problems for individual work:

- Draw a zigzag with four parts.
  Make the total length of the zigzag 50 cm.
- Draw a zigzag with total length 1 meter (100 cm).

You may remind students that the zigzag on the board is 50 centimeters long but has only three pieces.

Note: This lesson may be repeated using English units (yards, feet, and inches), if you like.

Center Activity

Place measuring tapes or rulers, and items to be measured in a center for additional practice. Invite students to make a chart and add to it as they find things that are 1cm, 10 cm, or 100 cm/1 meter in length.

Home Activity

Suggest that students measure some things at home in centimeters, inches, or other units of your choice.

Capsule Lesson Summary

Find places that are the same taxi-distance from Nora's house and from her friend Alice's house. Do a related problem involving Nora's house and another friend's house, observing a different pattern in the solution.

Materials

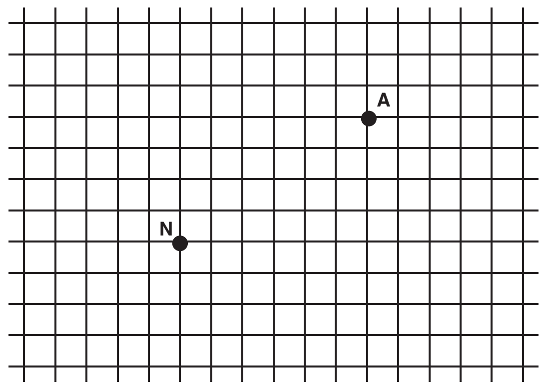
Teacher
- Grid board
- Colored markers
- Chalk or overhead pens

Student
- Worksheet G9
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

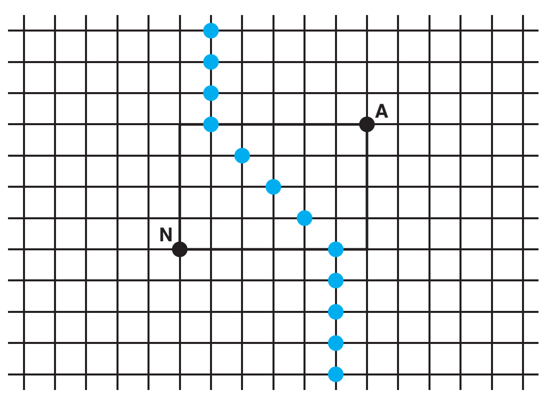
Display a grid board. Draw and label two dots as shown below.



T: ***A couple of weeks ago we looked for places where Nora and her friends could meet. Nora has another friend, Alice. Alice lives here*** (point to the dot labeled **A**) ***and Nora lives here*** (point to the dot labeled **N**)***. Who can show us a place where Nora and Alice could meet? Remember we are looking for places the same taxi-distance from Nora's house and from Alice's house.***

Let students draw dots in color for possible meeting places. Each time one student draws a dot in the picture, choose another student to check the taxi-distance from the suggested dot to Nora's house and then to Alice's house. Erase incorrect dots. After many dots have been drawn and the pattern is clear, let students plot the rest of the correct points.

A completed picture (within the limits of the grid) might look like this one.

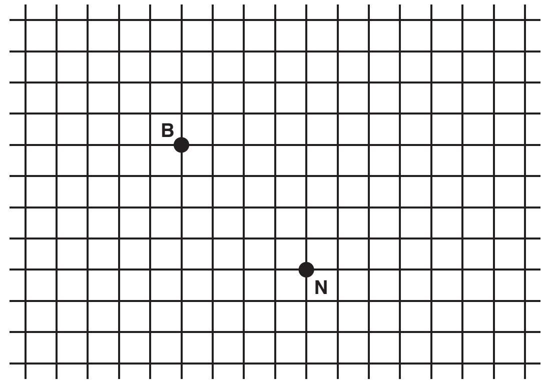


Note: A rectangle with **N** and **A** as opposite corners can be drawn in the picture to highlight the pattern, but should be drawn only after several dots have been placed.

Exercise 2

G9

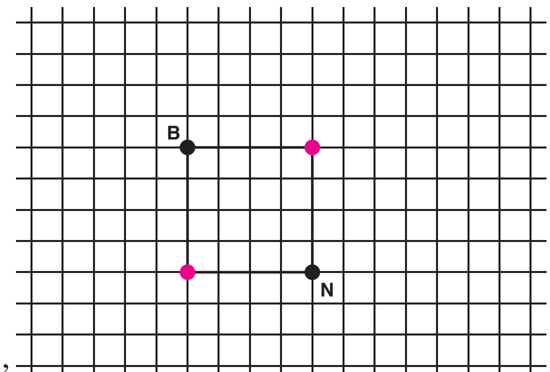
You may let students work with a partner on this exercise. Each student should have a copy of Worksheet G9. Clear the grid board and then draw two dots labeled **N** and **B** as on the worksheet.



T: ***Nora’s friend Barney lives here*** (point to the dot labeled **B**) ***and Nora lives here*** (point to the dot labeled **N**)***.***

On your worksheet, try to find places that are the same taxi-distance from Nora’s house and from Barney’s house. Draw red dots at those places.

Give help to students having difficulty and, as necessary, encourage others to find more dots. After several minutes, direct the class’s attention to the grid board. Draw a square and two dots as shown here.

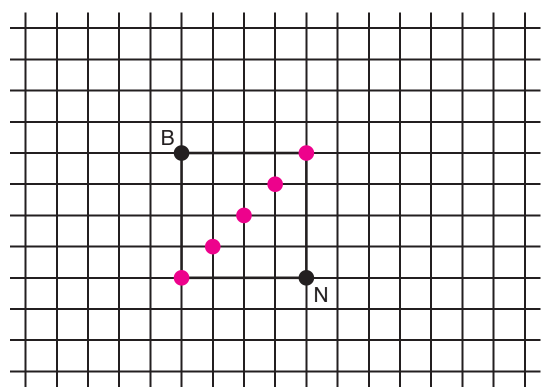


T (pointing to the red dots)**:** ***Almost everyone has found these two dots.***

Check the taxi-distance from one of the red dots to Nora’s house and then to Barney’s house (each will have a taxi-distance of 4 blocks). Students should check their papers and draw these dots if they do not have them.

T (tracing the square)**:** ***Are there any places inside this square that are the same taxi-distance from both of their houses?***

Let students point to and, if correct, draw the dots along the diagonal of the square.



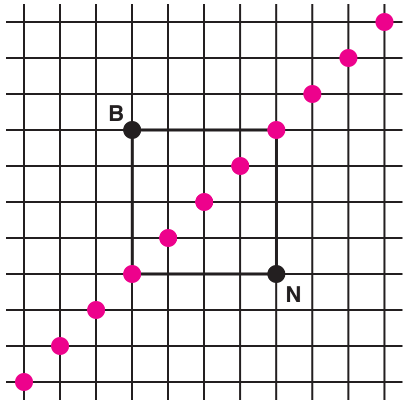
T: ***There are still more places the same taxi-distance from both houses. Try to find some of them.***

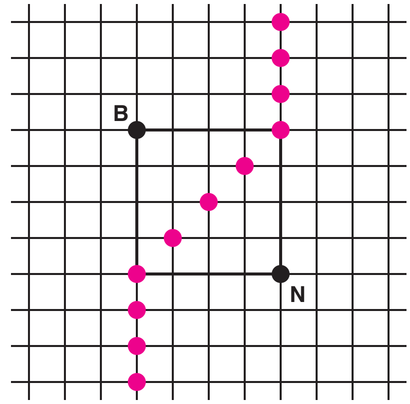
Students should continue working with their partners or independently for about 10 minutes. Then call the entire class’s attention back to the board.

T: ***You have found lots of places the same taxi-distance from Nora’s house and from Barney’s house. Let’s put all of the dots you found on the board.***

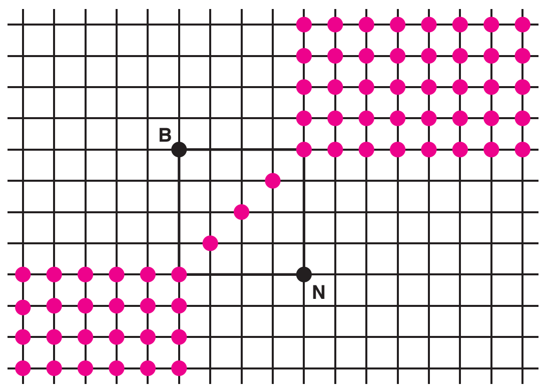
Let students draw dots in the picture, possibly several at a time. When several dots are suggested by one student, ask another student to check the taxi-distance of one of the dots. Since more and more dots are being added to the picture, put your finger on the dot being checked.

There are two patterns of dots that most students will discover, and very likely these dots will be the first ones suggested. They are shown below.

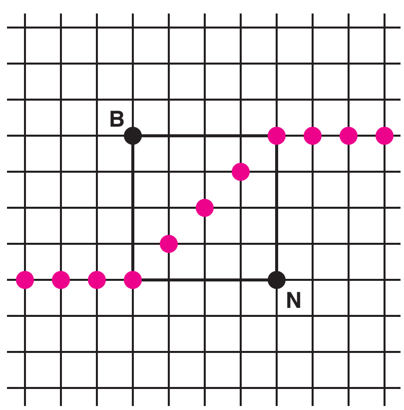


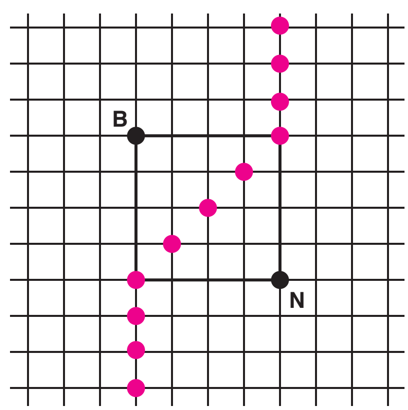


A completed picture (within the limits of the grid) is given here.



Encourage students to comment on any patterns they see. For example, some may observe that you get all the corners in two regions of the grid and that those regions have boundaries obtained by extending the sides of the black square with **B** and **N** as diagonal corners. The fact that **B** and **N** are corners of a square rather than a non-square rectangle leads to this extended pattern. Earlier experience lead to these two minimal patterns of dots equidistant from **B** and **N**.





With a square, all the dots inside the region bounded by these patterns are included.

Capsule Lesson Summary

Divide circular shapes into halves, fourths, eighths, and sixteenths. Use the resulting pictures to write equivalent fractions. Divide a rectangle into thirds and then into sixths. Use the resulting picture to write equivalent fractions.

Materials

Teacher
- Chalkboard compass (or any tool with which to draw a circle)
- Meter stick or straightedge
- Colored chalk
- Blacklines G10(a) and (b)

Student
- Straightedge
- Circle and rectangle cut-outs
- Worksheets G10*, **, ***, and ****
- Colored pencils, pens, or crayons

Advance Preparation: Use Blacklines G10(a) and (b) to make circle and rectangle cut-outs for students. You may also use Blackline G10(a) to make four circles for demonstration in Exercise 1, or draw four larger circles directly on the board.

Description of Lesson

Allow students to work in pairs during the lesson. Provide each student with two circle cut-outs and one rectangle cut-out.

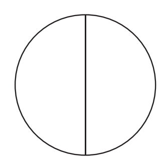
Exercise 1

Put four circles of the same size on the board. Refer the class to their circle cut-outs.

T: ***What shapes are these on the board?*** (Circles)

Suppose these circles are pizzas (cookies, and so on). We need to share one equally between two people. How can we divide this pizza (point to one circle) ***into two pieces of the same size?***

Give a student a meter stick to indicate where the circle can be divided. Draw the line segment yourself while the student holds the meter stick in position. Student pairs should do the same on one of their circles.



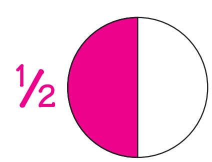
Note: You may like to let students fold a circle in half to find an accurate cutting line.

T: ***What part of the pizza does each of the two people get?***

S: ***One-half.***

T: ***Let's color one-half of this shape red.***

Point to the colored side of the circle and show a student how to write $\frac{1}{2}$ next to it.

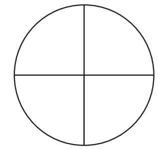


Point to a second circle.

T: ***Suppose we need to share this pizza equally among four people. Can we divide it into four pieces of the same size?***

G10

Let a student hold a meter stick in positions that indicate how the circle can be divided while you draw the appropriate line segments. Again, student pairs can do the same on one of their circles.

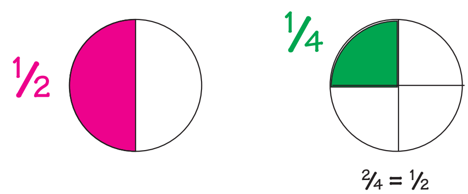


Note: You may like to let students fold the circle in half and then in half again to find accurate cutting lines.

T: ***What part of the pizza does each person get?***

S: ***One-fourth.***

T: ***Let's color one-fourth of this shape green.***



Refer to the circle you colored one-half red and then to the one-fourth section as you ask,

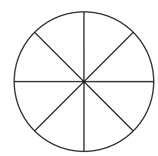
T: ***How many fourths make $^1/_2$?*** (Two)

Write $^2/_4 = {}^1/_2$ near the picture on the board.

Point to a third circle on the board.

T: ***We need to share this pizza equally among eight people. Can we divide it into eight pieces all the same size?***

Let a student hold a meter stick in positions to indicate how the circle can be divided while you draw the appropriate line segments. If necessary, refer to the circle divided into four pieces and suggest dividing this circle first into fourths, and then dividing each of those pieces in half. Again, student pairs should do the same on one of their circles.



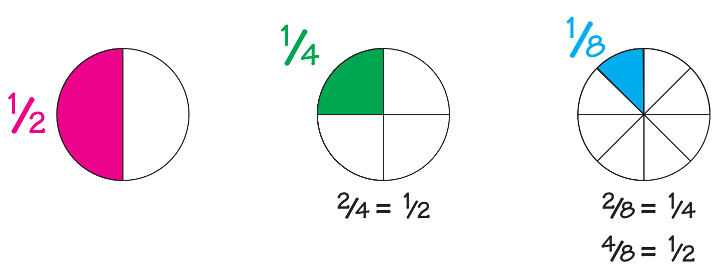
T: ***What part of the shape is each of these pieces?***

S: ***One-eighth.***

T: ***Let's color one-eighth of this shape blue.***

Refer to the circles you colored one-fourth green and one-half red, and then to the one-eighth section as you ask,

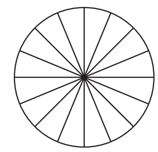
T: ***How many eighths make $^1/_4$?*** (Two)
How many eighths make $^1/_2$? (Four)



Refer to the fourth circle on the board.

T: ***We divided this pizza*** (point to the third circle) ***into eight pieces.***
How can we divide this pizza (point to the fourth circle) ***into sixteen pieces all the same size?***

S: ***First divide it into eight pieces and then divide each piece in half.***

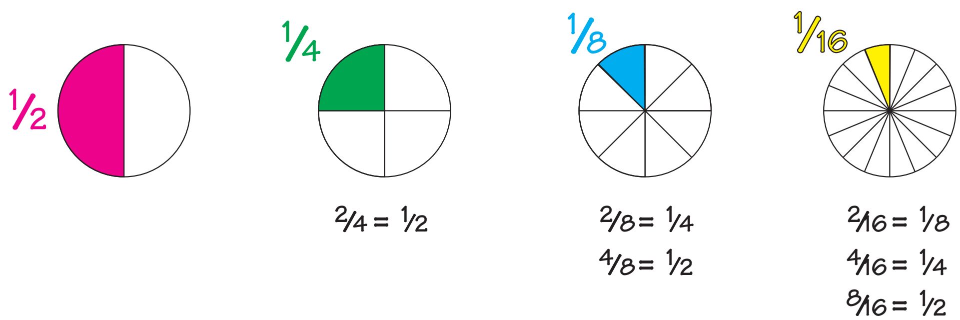


If no one suggest this, suggest it yourself and let a student hold the meter stick appropriately while you draw the line segments. Again, student pairs should do the same on one of their circles.

Ask someone to color one-sixteenth of the fourth circle yellow, and ask another to write $^1/_{16}$ on the board.

Point in turn to the one-eighth, one-fourth, and one-half sections as you ask,

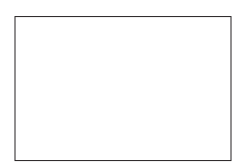
T: ***How many sixteenths make*** $^1/_8$***?*** (Two)
How many sixteenths make $^1/_4$***?*** (Four)
How many sixteenths make $^1/_2$***?*** (Eight)



Exercise 2

Draw a rectangle on the board and refer students to their rectangle cut-outs.

T: ***What shape is this?*** (Rectangle)
Suppose this rectangle is a cake and we need to divide it into three pieces all the same size. How can we do that?



Let a student hold a meter stick in positions that indicate a way to divide the shape while you draw the appropriate line segments. One possibility is illustrated here. Instruct student pairs to divide one of their rectangles in the same way.

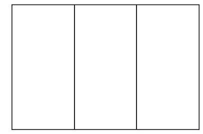
Note: If you have drawn a rectangle with side lengths easily divisible by 3 (for example, 36 cm by 24 cm) you may like to include some discussion of how to make this division fairly accurate. The students have rectangles that are 12 cm by 18 cm and they can be encouraged to make accurate divisions.

T: ***What part of the cake is each of these pieces?***

S: ***One-third.***

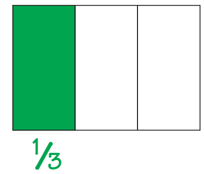
T: ***Let's color one-third of the shape green.***

Point to the colored part of the rectangle, and show students how to write $\frac{1}{3}$ next to it.



T: ***Suppose someone tells us we need to divide this cake into six pieces all the same size? Can we do it using the cut lines we already have drawn?***

S: ***Yes, divide each of the three pieces in half.***



Again, let a student indicate with a meter stick a way to divide the shape while you draw the appropriate line segment(s). One possibility is shown here.

Then call on a student to color one-sixth of the shape red, writing $\frac{1}{6}$ next to the red piece.

T: ***How many sixths make $\frac{1}{3}$?*** (Two)
How many sixths make $\frac{1}{2}$? (Three)

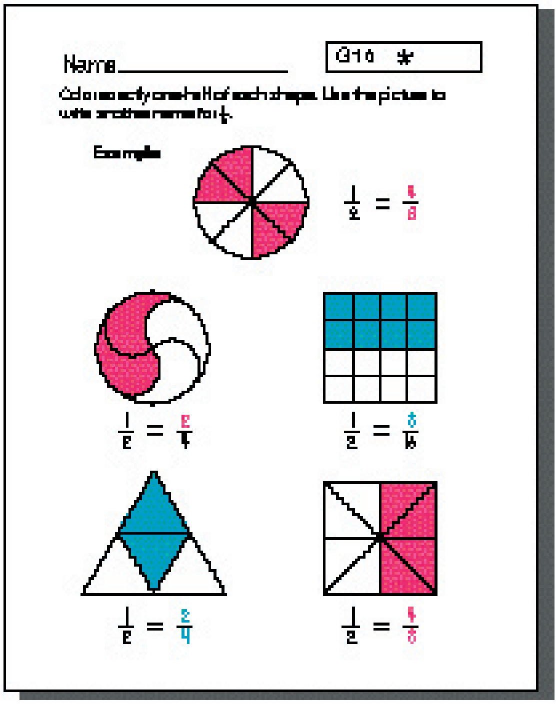
$2/6 = 1/3$
$3/6 = 1/2$

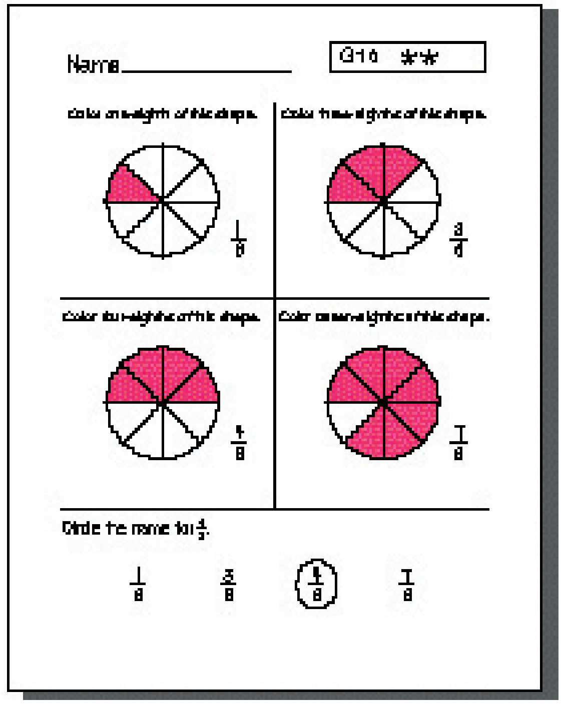
Write the appropriate equivalences on the board.

Worksheets G10*, **, ***, and **** are available for individual work.

Center Activity

Place commercial fraction models in a center for student exploration.


Name
G10 *
Example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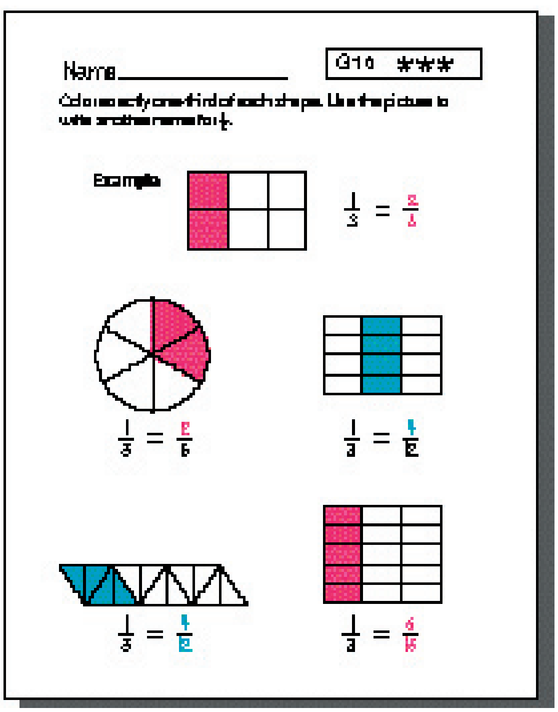
G10 **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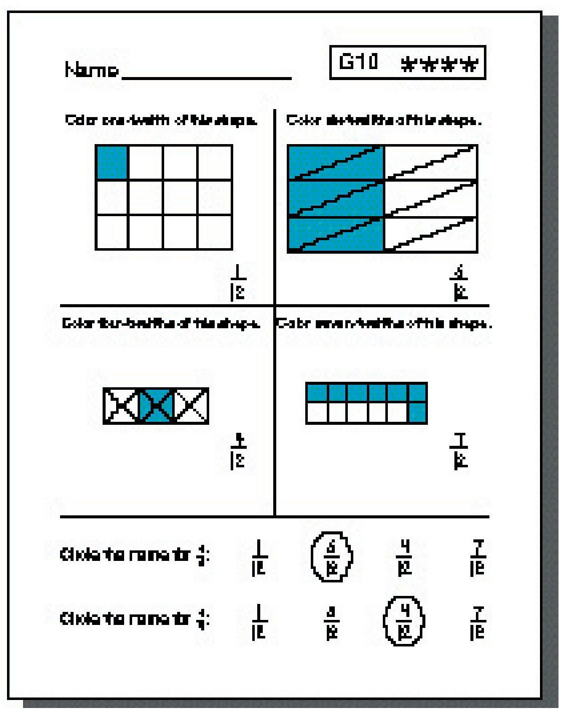
G10 ***
Example


Name
G10 ****

Capsule Lesson Summary

Find many ways to divide a four by four square in half on the geoboard. Introduce the concept of square corners, and find shapes with zero, one, two, three, four, five, and so on square corners.

Materials

Teacher	• Geoboard and bands • Blackline G11	**Student**	• Geoboard and bands • Geoboard paper • Square tile or A-block square • Colored pencils, pens, or crayons • Worksheet G11

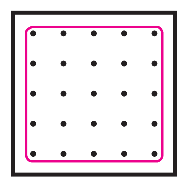
Advance Preparation: Use Blackline G11 to make geoboard paper for students. If available, use an overhead geoboard for demonstration and display of solutions to problems.

Description of Lesson

This would be a good lesson to let students work on with a partner, especially if your supply of geoboards does not equal the number of students. Provide each student or student pair with a geoboard and some bands. Allow a few minutes for free exploration, and use this time to discuss the proper use of the manipulative.

Exercise 1

Direct students to choose a big band and to put it on the geoboard making the biggest square they can. For example, with a five peg by five peg geoboard students should place a band to make this square.



T: ***How big is this square?***

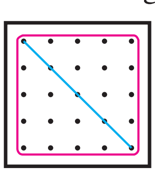
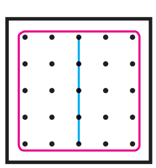
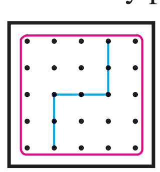
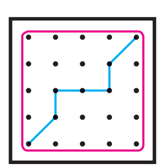
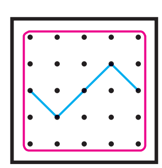
There may be some discussion about size—whether you mean its area or the length of its border. Agree that both are good notions of bigness, but then ask for its area. Lead the discussion to counting how many little squares  are in this biggest square.

S: ***16 little squares.***

T: ***Now I would like you to use another band to show how to divide this square in half, into two parts both the same size. Try to find several different ways to do it.***

You may like to provide students with geopaper on which to picture the ways they find to divide the square in half.

After awhile, call on students to show the class how they divided the square in half and to explain how they know it is a good division. Some of the many possibilities are shown below.

Exercise 2

G11

Instruct students to remove any dividing bands from their geoboards and to leave the square. Draw a square on the board.

T: ***What can you tell me about a square?***

S: ***A square has four sides all the same length.***

S: ***A square has four corners***

T: ***Are all the corners the same?***

S: ***Yes.***

T: ***We call corners that are like the corners of a square, square corners. Can you find some examples of square corners in our room?***

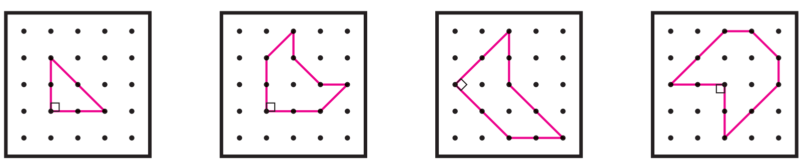
Let students suggest several examples such as the corners of a desk, the corners of a window, a corner where two walls come together, and so on.

T: ***Not all shapes have square corners. Try to put a shape on your geoboard that has a corner that is not a square corner.***

As students find such examples, let them show the class which corners are not square corners. You may like to let students use a square tile or an A-block square as a corner template to check whether or not a given corner is a square corner.

T: ***Try to put a shape on your geoboard that has just one square corner.***

Again, as students find examples, let them show the class and check each corner to be sure there is just one square corner. Try to generate a variety of examples, as illustrated below. Square corners are marked with a smaller square.



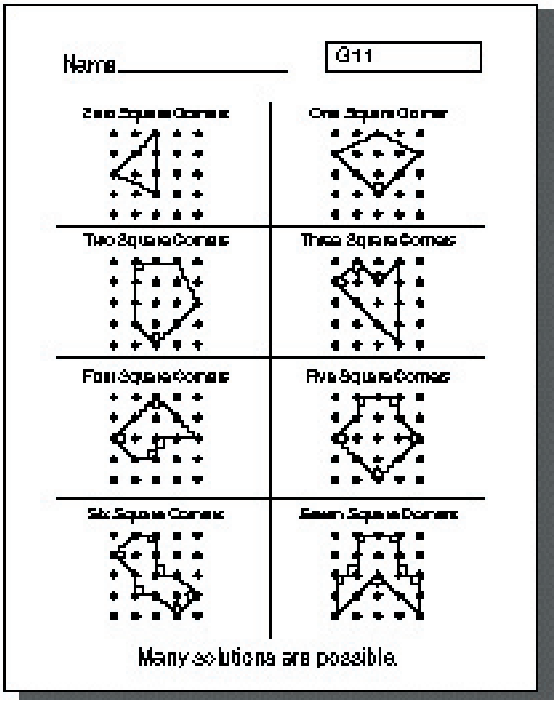
T: ***Now we have seen shapes with four square corners and with one square corner. Do you think we could put shapes on the geoboard that have two square corners, three square corners, or five square corners?***

Allowing students to work with a partner, give them a project to find shapes with zero, one, two, three, four, five, six, and so on square corners. Students can use Worksheet G11 to record their findings.

Note: You may like to ask that students find a different shape, not a square, with four square corners. The back side of the worksheet can be used to extend the project for students who want to continue.

Center Activity

Put geoboards and geoboard paper in a center for student explorations.



Name______________ G11

Zero Square Corners

One Square Corner

Two Square Corners

Three Square Corners

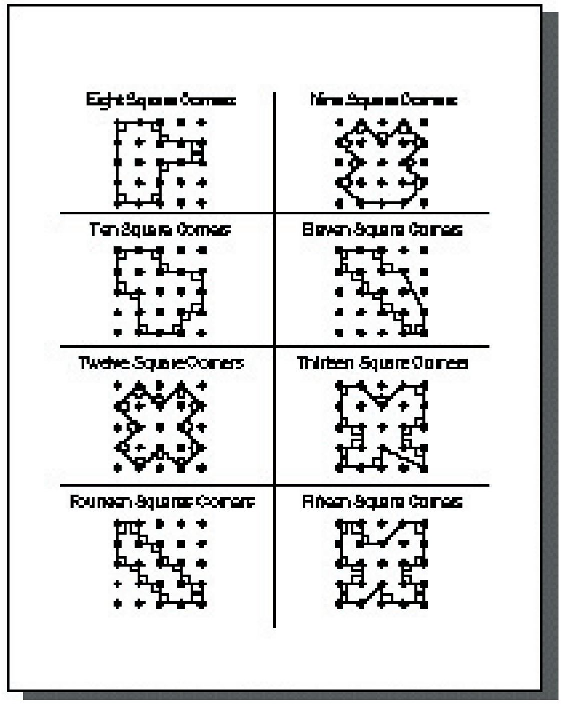
Four Square Corners

Five Square Corners

Six Square Corners

Seven Square Corners

Many solutions are possible.



Eight Square Corners

Nine Square Corners

Ten Square Corners

Eleven Square Corners

Twelve Square Corners

Thirteen Square Corners

Fourteen Square Corners

Fifteen Square Corners

Capsule Lesson Summary

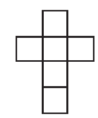
Make observations about the properties of a cube and introduce a map of a cube. Find many different maps of a cube. Consider what maps of other solids would look like.

Materials

Teacher
- Cubes
- Cardboard cube
- Cardboard box
- Can
- Blackline G3

Student
- Square tiles
- Grid paper
- Scissors
- Paper

Advance Preparation: Use Blackline G3 to make one-inch grid paper for students. This size grid matches most commercial square tiles. In any case, grid paper for students should have grid squares the same size as the square tiles. Make a cardboard cube using the map illustrated here (Blackline G12). Cut it out, fold along the lines indicated, and tape it along the edges.



Description of Lesson

Exercise 1

Display several cubes of various sizes and let students examine them.

T: ***What do you notice about all of these cubes.***

S: ***They are different sizes.***

S: ***They have square corners.***

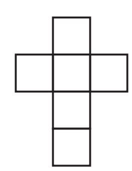
S: ***They all have square sides (faces).***

S: ***All the edges are the same length.***

T: ***How many faces does a cube have?*** (Six)
How many corners does a cube have? (Eight)
How many edges does a cube have? (Twelve)

Show the class your cardboard cube, and cut the tape to flatten it as you say,

T: ***Today we are going to make maps (or nets) of a cube. If we cut along some of the edges of a cube and flatten it out in one piece, the figure we get is called a map (or net).***



Demonstrate the reverse process of folding the map to form the cube as you say,

T: ***A map of a cube can be folded and taped to make a cube.***
What do you notice about this map of a cube?

S: ***It is made up of six squares.***

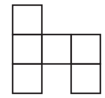
S: ***The squares touch along a side.***

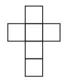
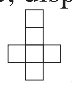
S: ***It has square corners.***

T: ***Do you think we could have a map of a cube that looks different from this one?***

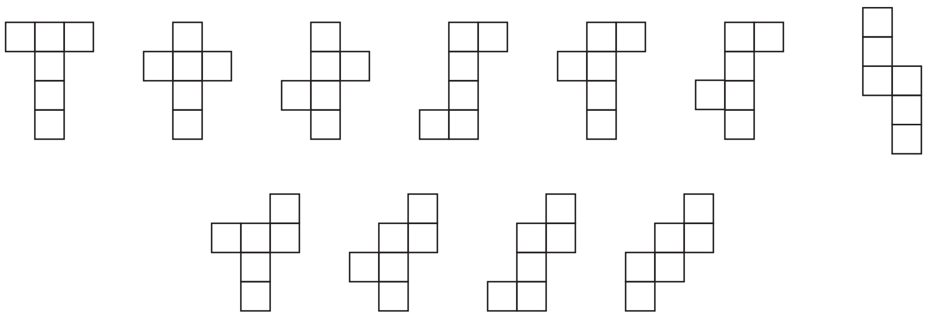
S: ***Maybe if you cut the cube along different edges.***

Organize the class into small cooperative groups, and provide each group with grid paper and six color tiles. Direct the groups to cut out as many different maps of a cube as they can find. Students can use the six color tiles to cover and then mark six faces for a cube. Each time they cut out a potential map of a cube, suggest that they fold the map to ensure that a cube can be made.

For example, this figure with six squares is not a map of a cube. 

As students begin to find several maps of a cube, display them on the board. Check with students for possible duplicate maps. For example,  and  are the same map (one is just flipped).

Tell the class you would like to get one copy of every different possible map of a cube on the board. The following illustration shows all the different possibilities.



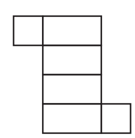
Do not expect your class to find all these different maps of a cube, but challenge them to find most of them. Some interesting discussions may result when students claim they have found a different map. You will first want them to verify that a map is indeed a map of a cube by folding it to form a cube, and then checking to see if it is really different from those already on the board. Is it different even after flipping or turning?

Exercise 2

Display a cardboard box (rectangular prism) and a can (cylinder).

T: ***What might the map of this box look like?***

Let students try to draw on their papers what they think a map of your box would look like. Then, with the class's help, cut along some of the edges of the box so it can be flattened out. Again, there are several possible maps of a given rectangular prism, and students' drawings may look different from this one.



T: ***What might a map of this can (cylinder) look like?***

Let students try to draw on their papers what they think a map of your can would look like. Although you will not be able to cut the can to flatten it out, you can make a full label (covering) for the can and show how it flattens out.

Extension Activity

Find other solid shapes for which students can make maps.

Home Activity

Let students take home a map of a cube to show their parents/guardians. Suggest that parents/guardians work with their child to flatten a solid such a rectangular prism (shoebox), cylinder (salt or oatmeal box), or cone.

Capsule Lesson Summary

Use the three pieces of a square puzzle to create a variety of shapes. Given a variety of shapes made from the pieces of a different square puzzle, use the puzzle pieces to make these shapes. Create a four-piece puzzle, and try to solve puzzles created by others.

Materials

Teacher
- Overhead projector
- Blacklines G13(a), (b), and (c)

Student
- Three-piece square puzzle
- Unlined paper
- Square card
- Scissors
- Envelope

Advance Preparation: Use Blacklines G13(a), (b), and (c) to make two copies each of the five different three-piece square puzzles. Each different puzzle can be made with a different color card stock. Use Blackline G13(c) to make square cards for students.

Description of Lesson

Organize the class into cooperative groups of two or three students.

Exercise 1

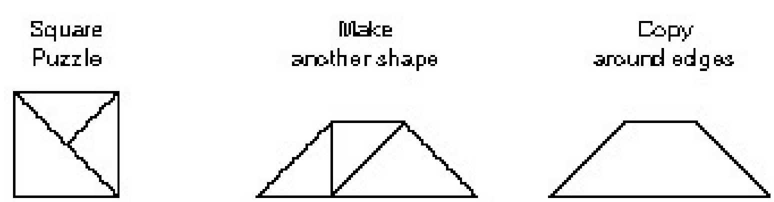
Each group should have one three-piece square puzzle, unlined paper, and a pencil for this first exercise.

T: ***Each group has a three-piece puzzle. Use the three pieces and try to make a square.***

When most of the groups have completed this task, invite one person from each group to show the class how they made a square with their three pieces. This can be done on an overhead projector.

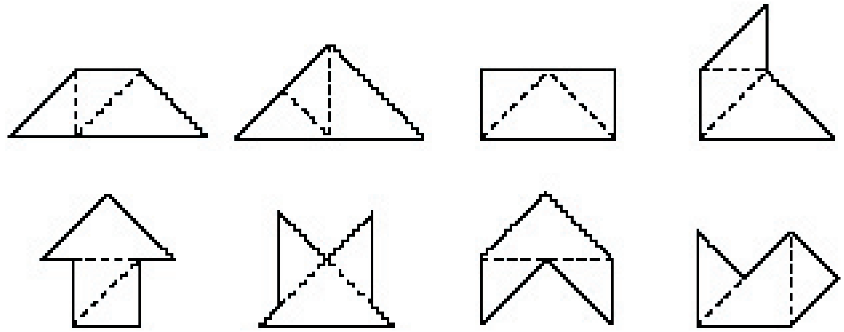
T: ***Now you are going to make other shapes with your three pieces.***

Instruct the groups to use the three pieces to make other shapes. Each time they make a new shape, tell them to carefully trace around the outside edges of the pieces on unlined paper to see just the shape—not how the three pieces are put together to make the shape. Model this activity with one example at the overhead.



Optional: You may like to suggest to the students that they give descriptive names to the shapes they make.

Each group should make between five and ten different shapes with their puzzle pieces. Encourage students to be fairly accurate in their drawings. For example, the group using the puzzle shown might make the shapes drawn here. The dotted lines show how the pieces are put together but should not be drawn on the students' papers.



Exercise 2

Arrange for each group to trade their puzzles and drawings with another group. For example, Group one gives its puzzle and drawings to Group two and vice versa. Then instruct the groups to attempt to make each other's shapes.

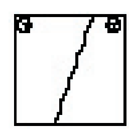
When the students in Group one make one of the shapes given to it by Group two (by covering the shape with puzzle pieces), suggest they indicate that they have found the shape by drawing it on another paper, this time using lines to indicate the puzzle pieces.

If this activity goes well, let the groups trade puzzles and shapes again, this time with a different group. In this way, each group can eventually work with three or four different puzzles.

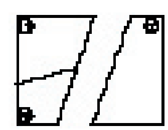
Exercise 3

Give each student a square card and scissors. Instruct the students to create their own puzzles as follows:

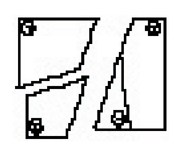
1. Start with the square and cut it into two pieces making a straight line cut. Check that you can put the two pieces back together to make a square.



2. Choose any one of the two pieces and cut it into two pieces with a straight line cut. Make sure you can put the three pieces back together to make a square.



3. Choose any one of the three pieces and cut it into two pieces with a straight line cut. Make sure you can put the four pieces back together to make a square.



T: ***Now you have made a four-piece square puzzle. Mix up the pieces and put all four pieces in an envelope. Write your name on the envelope. Now trade puzzles with a classmate. See if you can make a square with your classmate's puzzle.***

Let students attempt to solve several other classmates' puzzles. This is not always very easy. It can be made easier if the puzzle is made on a card having different colored sides (or by somehow distinguishing the sides).

Extension Activity

Any of the exercises in the lesson can be done with puzzles made from another original shape (rectangle, triangle, and so on.)

Writing Activity

Suggest that students write a description of their square puzzle to give to someone who cannot see the puzzle.

Home Activity

Allow students to take home their puzzle for family members to solve.

Capsule Lesson Summary

Use common food items to compare the sizes of single items, the number of items it takes to make a pound, and the size (volume) of a container needed to hold a pound.

Materials

Teacher
- Kitchen or diet scale
- Peanuts, fish crackers, potatoes, raisins, bread, carrots (or other similar items) in quantity of approximately one pound
- Paper bags of various sizes

Student
- Paper

Note: You may prefer to use things other than food items in this lesson. For example, you can choose four or five items such as plastic counters, cardboard checkers, styrofoam packing peanuts, Unifix® cubes, pennies, or centimeter cubes.

Description of Lesson

Show the class one of each item you have chosen for comparison.

T: ***Let's put these items in order from smallest to largest.***

Let the class discuss the problem and try to agree on an order. There should be some discussion of what aspect of size to use in the order—weight (mass), length around or height, or volume. There may be different orderings suggested for different aspects of size, and students may consider how they are different.

T: ***Now let's try to estimate how many of each item will weigh one pound.***

S: ***I think just a couple potatoes weigh one pound.***

S: ***There are a lot of raisins in one pound.***

Solicit from the class some specific estimates of how many of each item weigh one pound, and record the estimates on the board.

T: ***Before we count to find the number of items in a pound, let's try to decide what size bag will hold one pound of each item.***

Show the class the various sizes of paper bags, and ask them to select a bag to hold one pound of each item. Of course, the largest bag should be able to hold one pound of any of the items, but emphasize that you would like them to select bags that will be close to full with one pound of an item. That is, you do not want to have much extra space in the bags.

T: ***Should we use the same size bag for one pound of every item?***

Let students select bags and place them next to the items that are ordered from smallest to largest.

T: ***Are the bags in the same order as the size of the items? Why do you think that the bag for one pound of potatoes will not be the largest?***

S: ***One potato is heavy, but it takes a lot of fish crackers to make a pound.***

Call on several students to help weigh out one pound of each item, and let other students place the one pound of an item in the bag selected for that item to see how it fits. You may prefer to have students place pre-weighed containers with one pound of each item in the bags. Discuss the choice of bags and what students discovered.

T: ***Now let's count how many of each item there are in one pound.***

Organize students in cooperative groups to do the counting and to report on their findings. Compare the actual counts to the estimates.

End the lesson by discussing what size bags or other containers you would need to hold one pound of other kinds of things, such as feathers, pennies, socks, or paper clips. You may like students to think about various things that can be bought in one pound containers and consider the size of those containers.

Home Activity

Suggest to parents/guardians that they observe with their child how different things of similar weights come in different size packages.