STORIES BY FRÉDÉRIQUE

Storybook Set II

Singing Friends
The Little Donkey
Dancing Friends
I Am Not My Name
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Ages 8 – 12

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For additional information, contact:

McREL - CSMP

2550 S. Parker Rd., Suite 500

Aurora, CO 80014-1678

(303) 337-0990

http://www.mcrel.org/products/csmp

STORIES BY FRÉDÉRIQUE (SET II) INTRODUCTION

A CSMP Minipackage

A CSMP Minipackage presents a part of the CSMP curriculum through introductory lessons that can be taught by teachers with no prior CSMP training to students with no prior CSMP background. The purpose of each CSMP Minipackage is twofold:

- to introduce one or more of the nonverbal languages and instructional tools used in the CSMP elementary curriculum so that a teacher can pursue the possibility of implementing the entire program; and
- to provide mathematically rich activities that can be used immediately in the classroom.

This CSMP Minipackage introduces the Languages of Strings and Arrows, number line tools, and random devices, using a collection of storybooks. These delightful stories by Frédérique Papy capture the imagination of young students and cover a wide range of topics involving mathematics. Students easily become engaged in these stories: there are questions to answer, pictures to relate to the story, and new adventures to invent for story characters. Dot, string, and arrow pictures reinforce each story while teaching important mathematical concepts and fundamental properties of numbers. The stories are appropriate for individual or group reading, at home or school.

Six Storybooks and How to Use Them

This booklet gives lesson ideas for the six storybooks in Set II of *Stories by Frédérique*. The lesson presentations describe classroom use of the storybooks, but they may easily be adapted for use with an individual child or a small group of students. Although the order of use of the storybooks is not critical, you will find the content of the first couple stories easier than the last two. The storybook *Dancing Friends* introduces a finite arithmetic and is a predecessor to the Multiplication Modulo 10 lessons.

Sometime before using a storybook lesson, read the storybook on your own. Be sure to read the comment on the last page. While doing this, think about the various situations and how your students might react to them. Feel free to bring your imagination, your experience, and your general familiarity with the interests and concerns of your students to bear on the lesson. The lesson description simply provides some suggestions on how students might react and how you might prompt creative thinking.

Brief story descriptions for all storybooks in this collection are given here.

Singing Friends

Singing Friends introduces students to properties of 0 and ½ through fantasy. The numbers' behavior in the story closely parallels their dynamic roles in mathematics. 0, bold and self-confident, plays an important role in many mathematical games, and little ½, so quick to cry, struggles to find an identity among the solid whole numbers with their simple, but sometimes boring, dances.

The games in *Singing Friends* are of two kinds. At first the numbers are exuberant and their games suggest a first notion of infinity, as greater numbers dance right off the page in unending snake dances. Then, in a quieter moment, several friends sit together thoughtfully and ponder the meaning of their relationships to one another. Finally, in a game designed to include little ½, some numbers race toward 0 in a futile attempt to reach their leader, and this time smaller and ever smaller numbers form an infinite line of players.

The Little Donkey

The Little Donkey is a delightful story about how a little girl and her friend, a donkey, entertain themselves on a lazy afternoon. As children so often do, the little girl pretends that the pebbles she has so carefully collected are the people she knows. Then, with a stick, she draws arrows in the sand from one pebble to another to show the donkey the story of her family. As the donkey identifies the various family members, these two friends share their feelings about the people in their lives.

Dancing Friends

Dancing Friends is a story about a boy and ten of his friends who just happen to be numbers. They speak to each other, play together, and make up games. When they play after school and are free from the usual constraints placed upon them, they invent marvelous games and dance to the tune of vividly colored arrows.

The games of the numbers are not frivolous. There are rules to followed and, as a result, orderly patterns emerge as they dance. They observe similarities and differences as they go from one dance to another, and 0 discovers that it plays a very important role in the whole scheme.

I Am Not My Name

In *I Am Not My Name,* ½ weathers a serious identity crisis and 0 has some of its pomposity deflated. Much to 0's annoyance, a small group of numbers have been playing some games in its absence. During the course of these games, ½ suddenly becomes aware of the fact that there is another potential occupant of its place on the number line. In great distress, ½ seeks comfort in the company of a human friend, who is spending an afternoon chatting with 0. Through the discussion and questions that arise, the young reader is introduced to the subject of equivalent fractions in a clever way.

The Square Trap

Inspired by a suggestion from the probabilist Lennart Råde, *The Square Trap* provides an exciting introduction to the fascinating world of probability and statistics. The hero of the story is given a spinner, and he invites the numbers 0 and 1 over to help him make the most of his new toy. The two friends oblige by each inventing a game that can be played using the spinner.

Examples of the games being played are included in the narration of this story, but the bulk of experimentation is performed by the readers as the story progresses. Thus, students gain firsthand experience of the gradual build-up of statistical information and acquire a deeper appreciation of the probabilities at work in each situation is acquired.

The first game is a random walk on a square, and the second is a very special case of what is known as "the EHRENFEST model." However, even though these games are apparently very dissimilar, the reader takes part in the major discovery that they are in fact equivalent in the sense that moves in one game parallel moves in the other, and if one game ends, the other game always end on the same move.

Nabu Wins an Award

Nabu Wins an Award is a story that exhibits many important aspects of the structure of the system of numbers.

It is Nabu's tenth birthday. In his honor, his friends the numbers have organized a show of spectacular dances involving many numbers, some of whom Nabu has never even met before. One dance is like an earlier one danced backwards; other pairs of dances involve different dancers and different operations but have identical choreographies. The numbers 0 and 1 forget their usual rivalry in order to cooperate in the choreography of several ballets of great numerical interest. The evening reaches its climax when a special award is presented to Nabu.

For Further Information

Nonverbal languages and instructional tools are used extensively in the CSMP curriculum. This CMSP Minipackage provides a simple introduction to some of these languages and tools in story contexts. To preview CSMP's unique approach to mathematics at the elementary school level and for more in-depth use of the languages and tools of CSMP, other Minipackages such as *Relations and the Language of Arrows*, *A-Blocks String Game*, and *Minicomputer Games* are useful. A brief description of these Minipackages can be found on page 39 of this booklet. For more information, contact:

McREL – CSMP
2550 S. Parker Rd., Suite 500
Aurora, CO 80014-1678
(303) 337-0990

SINGING FRIENDS

Capsule Lesson Summary

Read and discuss the storybook *Singing Friends*. Find the numbers that can be named by inserting parentheses in expressions such as 3 x 5 + 2. Explore the possibilities that result when x and + are interchanged, or when 3 and 2 are interchanged.

Description of Lesson

Distribute copies of the storybook *Singing Friends* to student pairs, and let them look at the book until you are ready to begin.

Pages 2 and 3

As you read page 3, stop and ask a student to read the numbers: **1 3 5 7 10 14**

T: ***Do you know a number less than any of these?***

S: ***0.***

T: ***...less than 0?***

S: ***⁻5 (negative 5).***

T: ***Who can read the large numbers on this page?***

After someone reads the numbers correctly, ask,

T: ***Does anyone know a number greater than 65,375?***

Let several students respond and, for each correct suggestion, ask if anyone can write that number on the board.

Pages 4 and 5

Read these pages aloud or let a student do so.

T: ***What is your favorite number?***

Pages 6–9

Read these pages aloud (or let a student do so), and then ask students to complete the +1 snake dance on page 9.

T: ***What is the greatest number in this picture?*** (38)
If we drew one more arrow starting from 38, where would it end? (39)
If we drew ten more arrows starting from 38, what number would be at the end? (48)
If we drew 20 more arrows (from 38)...? (58)
If we drew 30 more arrows (from 38) ...? (68)

T: ***What is the least number in this picture?*** (0)
If we went back ten arrows, what number would we be at? ($^-10$)
If we went back 20 arrows (from 0), ...? ($^-20$)
If we went back 35 arrows (from 0) ...? ($^-35$)

Pages 10 and 11

Ask students to compare their +1 snake pictures on page 9 to the one on page 10. Then read page 11 aloud or let a student do so.

T: ***Why is 38 unhappy?***

Pages 12–15

Read page 12 aloud or let a student do so.

T: ***This +2 dance starts at 0. What do you notice about these numbers?***

S: ***They are even numbers.***

T: ***What kind of numbers would be in a +2 dance that starts at 1?***

S: ***Odd numbers.***

T: ***Turn the page and you'll see a +2 dance starting at 1.***

Read pages 14 and 15 aloud or let a student do so.

T: ***What are some of the numbers in the yard who are dancing in this +2 dance?***

Let several students answer. (Any odd number greater than 203 is dancing in the yard.)

T: ***What are some of the numbers in the yard who are dancing in the +2 dance shown on pages 12 and 13?***

Students may respond that any even number greater than 240 is dancing in the yard.

Pages 16 and 17

As you read page 16, write this expression on the board.

$$3 + 5 \times 2$$

T: ***What number is this? How do you know?***

S: ***16, because 3 + 5 = 8 and 8 x 2 = 16.***

S: ***13, because 5 x 2 = 10 and 3 + 10 = 13.***

T (after reading page 17)**:** ***Like you, the numbers 3 and 2 cannot decide whether this*** (point to the expression on the board) ***is 13 or 16.***

Pages 18 and 19

As you read page 18, write these expressions on the board. (Be sure to emphasize the spacing.)

$3 + 5 \quad \times 2$

$3 + \quad 5 \times 2$

T (reading page 19)**:** ***I smiled and drew two parentheses. I put them like this.***

Draw parentheses around "3 + 5" in the first expression.

$(3 + 5) \times 2$

T: ***How does that help?***

S: ***It shows that we calculate 3 + 5 first.***

T: ***3 + 5 = …?*** (8) ***And 8 x 2 = …?*** (16)

Complete the first number sentence.

$(3 + 5) \times 2 = 16$

T (reading)**:** ***"Now we name the number 16," agreed 3 and 2. I moved the parentheses.***

Draw parentheses around "5 x 2" in the second expression.

$3 + (5 \times 2)$

T: ***How does that help?***

S: ***It shows that we calculate 5 x 2 first.***

T: ***5 x 2 = …?*** (10) ***3 + 10 = …?*** (13)

On the board, complete the second number sentence.

$3 + (5 \times 2) = 13$

Pages 20 and 21

Under the two number sentences on the board, write this expression.

T (reading page 20)**:** ***"And what happens if we switch + and x ?" asked 5.***

$3 + 5 \times 2$

Switch the symbols on the board.

$3 \times 5 + 2$

T: ***What number could this be?***

S: ***17.***

T: ***Can you put in parentheses so we can see how you are looking at this expression?***

Let the student put in parentheses, check the solution with the class, and complete the number sentence.

$(3 \times 5) + 2 = 17$

Write the expression again, and let a student who sees it as 21 put in parentheses.

$3 \times (5 + 2) = 21$

Finish reading pages 20 and 21.

Vary the activity in the storybook by asking what happens if you

- switch 3 and 2 (in the original expression);
- switch 3 and 2, and switch x and + (in the original expression);
- switch 5 and 2 (in the original expression); or
- switch 5 and 2, and switch x and + (in the original expression).

Your board might look similar to this.

$(3 + 5) \times 2 = 16$	$(2 + 5) \times 3 = 21$	$(3 + 2) \times 5 = 25$
$3 + (5 \times 2) = 13$	$2 + (5 \times 3) = 17$	$3 + (2 \times 5) = 13$
$(3 \times 5) + 2 = 17$	$(2 \times 5) + 3 = 13$	$(3 \times 2) + 5 = 11$
$3 \times (5 + 2) = 21$	$2 \times (5 + 3) = 16$	$3 \times (2 + 5) = 21$

Students should notice the repetition of answers.

Pages 22–25

Read these pages aloud. Reproduce the picture on pages 24 and 25 on the board. Let students locate ½ and then draw a red arrow from 1 to ½.

Pages 26 and 27

Read these pages aloud.

T: ***What do you suggest we call this new dance?***

S: *½x.*

Pages 28–31

Finish reading the storybook with your class.

Writing Activity

Suggest students write to a friend about why they sometimes need to use parentheses in a number sentence.

Capsule Lesson Summary

Discuss grandparents and (possibly) make a class graph by collecting some information about grandparents. Use relational thinking to discuss an arrow picture as it tells about a little girl's family in *The Little Donkey* Storybook. The arrows are for "You are my mother" and "You are my father."

Description of Lesson

Exercise 1

Initiate a discussion of grandparents with the class. Students may enjoy telling about

- how many grandparents they have;
- where their grandparents live;
- what they like to do with their grandparents; and
- having great-grandparents.

Try to mention in the discussion that many children have two sets of grandparents, one on their mother's side of the family and one on their father's side of the family.

You may like to choose one particular characteristic about grandparents to make a class graph.

Exercise 2

Distribute copies of *The Little Donkey* Storybook to student pairs. Ask your students not to read ahead of the class during the lesson.

Pages 2–5

Read these pages with your class.

Pages 6 and 7

After reading page 6, draw the arrow picture from that page on the board. Add the key arrow to your picture, indicating that the blue arrows are for "You are my mother."

Read page 7 with your class and label dots for the donkey and for his mother. Remind students not to turn the page.

T: ***I'll show you where the little girl is in this picture.***

Label the lower left dot for the little girl.

T: ***Now can you tell for sure who some of the other dots are for?***

Take the opportunity to discuss how to distinguish a person's two sets of grandparents and to introduce the terms *maternal* and *paternal*. Use hints such as maternal—ma, and paternal—pa. Label dots for the little girl's mother, maternal grandmother, and great grandmother. You might use a student as an example in the discussion about maternal grandmother and great grandmother.

Perhaps some of your students will comment that the dot to the right of the little girl must be for either her brother or her sister, and that the dot to the left of the little girl's mother must be for either an aunt or an uncle. If so, when page 8 is read, note which choice is made for each dot.

Pages 8 and 9

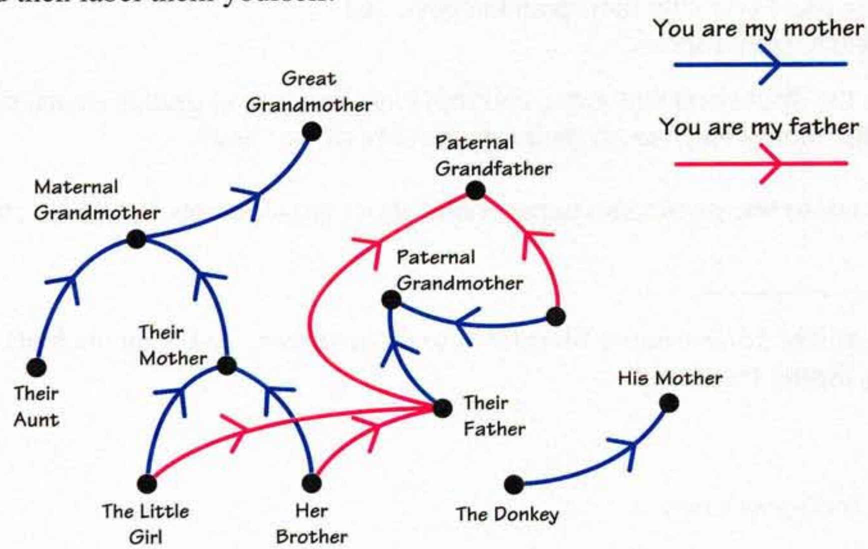
Read these pages with your class. Label the dots in the picture on the board for the girl's brother and for their aunt.

Pages 10 and 11

Read these pages with your class. Add the red arrows to your picture as shown on page 10 and add a key arrow indicating that the red arrows are for "You are my father." Let a student locate the dot for the little girl's father, and then label the dot yourself.

Pages 12 and 13

Read these pages with your class. Invite students to locate the dots for the little girl's paternal grandparents and then label them yourself.



There is still one dot that is unlabeled. Ask what it could be for. Your students should decide that it is for either a brother or a sister of the girl's father; i.e., an aunt or an uncle of the girl.

Pages 14 and 15

Finish reading the storybook with your class.

Reading Activity

Other books about relationships with grandparents may be of interest to your students; for example, *Kevin's Grandma* by Barbara Williams, *Annie and the Old One* by Miska Miles, *Song and Dance Man* by Karen Ackerman, or *Now One Foot, Now the Other* by Tomie De Paola.

Writing Activity

Suggest students draw an arrow picture of some members of their families. Different relationships can be pictured, but ask that pictures include a color key for the arrows.

Capsule Lesson Summary

Read the *Dancing Friends* Storybook and explore a new addition operation ⊕ invented for use with just ten whole numbers, 0 through 9.

Description of Lesson

You may like to pair students during this lesson. Distribute copies of the *Dancing Friends* Storybook and instruct students to follow along (without reading ahead) as you read this story. Students will need paper and colored pencils.

Pages 2 and 3

Read these pages aloud. When you come to the middle of page 3, read the column of addition facts on the left and ask how to extend this list. For example, the next three addition facts might be $8 + 6 = 14$; $9 + 6 = 15$; and $10 + 6 = 16$. Similarly, extend the list of multiplication facts on the right.

When you finish reading page 3, ask what other games the numbers might play. You may receive a wide variety of suggestions; accept all of them without comment.

Pages 4 and 5

Read these pages aloud or ask some students to do so. Some students may have difficulty reading $4 \div 8 = \frac{1}{2}$ and will need help.

Record the ten numbers on the board and keep the list on the board for the rest of the lesson.

0 1 2 3 4 5 6 7 8 9

T: ***Can you imagine a game that just these ten numbers could play?***

Pages 6 and 7

Read these pages aloud or ask some students to do so. Perhaps your students would enjoy reading the number sentences in unison. They may express curious surprise at the number sentences on page 7. Ask the class if they understand 0's game, and let some students try to explain it.

Pages 8 and 9

Read page 8 aloud and then ask students to read page 9 to themselves while you write some problems on the board.

$$6 \oplus 9 = \qquad 5 \oplus 5 \oplus 5 \oplus 5 =$$
$$8 \oplus 8 = \qquad 7 \oplus 7 \oplus 7 =$$
$$7 \oplus 7 = \qquad 9 \oplus 8 \oplus 6 =$$

After most students have finished reading page 9, ask,

T: ***Who can explain 0's game?***

S: ***0 doesn't add in the usual way.***

S: ***8 + 7 = 15, but in 0's game you just keep 5 (the ones digit).***

S: ***In 0's game you add in the usual way and then subtract 10.***

T (pointing to the problems on the board)**:** ***What would 0's answers be to these problems?***

Ask each student who gives an answer to explain it; for example:

S: $6 \oplus 9 = 5$.[†]

T: ***How did you get 5?***

S: ***6 + 9 = 15 and I kept only the 5.***

Continue until all the number sentences are completed.

$6 \oplus 9 = 5$	$5 \oplus 5 \oplus 5 \oplus 5 = 0$
$8 \oplus 8 = 6$	$7 \oplus 7 \oplus 7 = 1$
$7 \oplus 7 = 4$	$9 \oplus 8 \oplus 6 = 3$

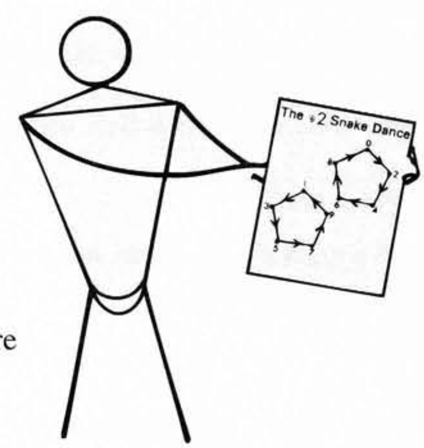
Pages 10 and 11

Read page 10 aloud or ask some students to do so.

T: ***Look at the ⊕2 dance. Why does it have two pieces?***

S: ***If you add 2 starting at 0, you only get even numbers. If you add 2 starting at 1, you only get odd numbers.***

Hold the page so that you can trace the arrows in the upper piece of the picture and so that students can see what you are doing. As you trace each arrow, state the corresponding number sentence.



Point to the lower piece of the arrow picture.

T: ***But suppose we start the ⊕2 snake dance at 1.***

As you did before, trace the arrows in the lower piece of the picture and state the corresponding number sentences.

T: ***What numbers are in this piece of the ⊕2 dance?***

S: ***The odd numbers from 1 to 9.***

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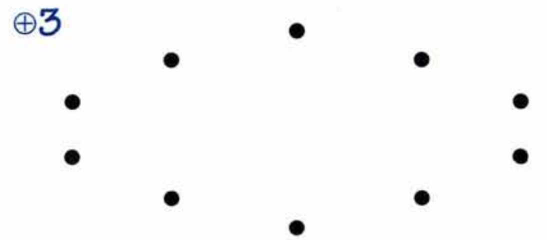
† Read ⊕ as "circle-plus."

Ask the class to think about what a ⊕3 snake dance would look like. If necessary, tell them not to turn the page yet. Allow a minute or two for comments and predictions. You may like to assign ten students the numbers from 0 to 9 (e.g., give them cards with digits written on them). Ask one of these students to come up to the front and then call for the next number by adding ⊕3 to the first. The student with this number should go to the front of the class and join hands with the first student. Continue calling the next number when ⊕3 is added until all the numbers 0–9 are in the dance.

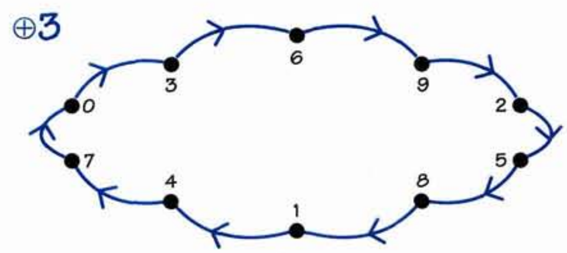
Draw this picture of dots for ten numbers on the board and ask students to copy it at their desks.

⊕3 Snake Dance



T: ***This picture has a dot for each of these ten numbers*** (point to the list of numbers 0–9 on the board or to the ten students) ***because they all can dance in the ⊕3 snake dance.***

⊕3 Snake Dance



Instruct students to label the dots and to draw arrows for ⊕3 in their pictures as you do so at the board. Use the ten students holding hands to direct how to label dots and draw arrows.

Pages 12 and 13

Students can compare their pictures to the one shown on page 13. If necessary, remind them that their pictures might look a little different but should still be similar. Suggest that they check their pictures by starting at 0 and following the arrows. They should meet the ten numbers in this order: 0, 3, 6, 9, 2, 5, 8, 1, 4, 7, and return to 0.

Before turning to page 14, ask students to think about what a ⊕5 dance would look like.

Pages 14 and 15

After you read page 14 aloud, hold up page 15 so you can trace the arrows in the ⊕5 dance and students can see what you are doing. As you trace each arrow, state the corresponding number sentence; for example, while tracing the ⊕5 arrow from 6 to 1 say, "6 + 5 = 11, so 6 ⊕ 5 = 1."

Students may like to observe that in the ⊕5 dance the ten numbers are in pairs. Relate this fact to the ones digit pattern in a +5 arrow road or counting by fives.

T: ***Before you turn the page, try to draw a ⊕7 dance and a ⊕8 dance on your paper.***

When students finish both pictures, let them go on reading the rest of the storybook with their partners. Suggest that they check their pictures against the ones on pages 16 and 18.

DANCING FRIENDS (LESSON TWO)

Capsule Lesson Summary

Review the story of *Dancing Friends* and the ⊕ (addition with ten number friends) operation. Solve some problems involving the operation ⊕ in this finite system. Draw arrow pictures for the relations ⊕3, ⊕6, and ⊕9, and observe that return arrows for these relations are, respectively, ⊕7, ⊕4, and ⊕1.

Description of Lesson

Exercise 1

Briefly review the story of *Dancing Friends*.

T: ***Do you remember the story of* Dancing Friends*?***

S: ***It was about ten numbers and a new game that 0 made up.***

T: ***What happened in the story?***
Why did 0 make up a new game for only ten numbers?

S: ***The boy in the story invited only the numbers 0 through 9, and there weren't enough numbers to play their usual addition, subtraction, multiplication, and division games.***

Write this list of numbers on the board so it can be referred to throughout the lesson.

0 1 2 3 4 5 6 7 8 9

T: ***Do you remember the*** ⊕ (read as "circle-plus") ***operation?***
What number is 6 ⊕ 7?

A student might add 6 and 7 in the usual way.

S: ***13.***

T: ***But remember only these numbers*** (point to the list) ***are playing the game.***

S: ***3.***

T: ***How did you get 3?***

S: ***6 + 7 = 13 and I just kept 3 (the ones digit).***

Complete a number sentence on the board and write other calculations (e.g., 8 ⊕ 1 and 8 ⊕ 6) under it.

T: ***What number is 8 ⊕ 1?*** (9)

T: ***What number is 8 ⊕ 6?*** (4)

S: ***8 + 6 = 14, so 8 ⊕ 6 = 4.***

$$6 \oplus 7 = 3$$
$$8 \oplus 1 = 9$$
$$8 \oplus 6 = 4$$

Continue this activity, asking your class to complete each of the number sentences below. (Answers are in boxes.)

$$4 \oplus 4 \oplus 4 \oplus 4 = \boxed{6} \qquad 7 \oplus 7 \oplus 7 = \boxed{1}$$
$$4 \oplus 4 \oplus 4 \oplus 4 \oplus 4 = \boxed{0} \qquad 7 \oplus 7 \oplus 7 \oplus 7 = \boxed{8}$$

Students will need paper and colored pencils for certain parts of the rest of this lesson.

T: ***I will put some problems on the board. Copy them on your paper, and then figure out which of these numbers*** (point to the list of whole numbers from 0 to 9) ***can be put in the boxes to make true number sentences.***

Write these problems on the board. (Answers are in boxes.)

$$7 \oplus \boxed{5} = 2 \qquad 2 \oplus \boxed{1} = 3$$
$$6 \oplus \boxed{9} = 5 \qquad 6 \oplus \boxed{5} = 1$$

Collectively check the above problems and then add this open sentence to the list.

$$\square \oplus \square = 2$$

T: ***The same number goes in both of these boxes. Which of these numbers*** (point to the list of whole numbers from 0 to 9) ***could we put in the boxes to get a true number sentence?***

S: ***6.***

T: ***6 + 6 = 12, so 6 ⊕ 6 = 2. That's right.***

S: ***1.***

T: ***1 + 1 = 2, so 1 ⊕ 1 = 2 also.***
Is there any other number we could put in the boxes?

S: ***No.***

Write this open sentence on the board.

$$\square \oplus \triangle = 5$$

T: ***These frames have different shapes, so we can put different numbers in them.***

S: ***Put 9 in one of them and 6 in the other.***

T: ***9 + 6 = 15, so 9 ⊕ 6 = 5.***

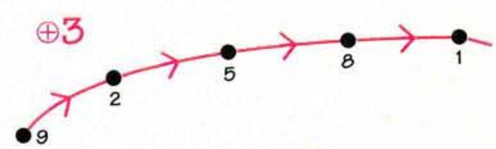
Continue until all of the pairs in this chart have been suggested and checked.

$$\square \oplus \triangle = 5$$

$\square$	$\triangle$
9	6
8	7
5	0
4	1
3	2

Exercise 2

Erase the board, except for the list of whole numbers from 0 to 9, and then draw this arrow road. Collectively label the dots, starting with 5 at the center dot.



Trace imaginary arrows as you ask the following questions. The first such arrow starts at 1.

T: ***What is the next number we would meet if we drew more ⊕3 arrows?*** **S:** ***4.***

T: ***1 ⊕ 3 = 4. And the next?*** **S:** ***7.***

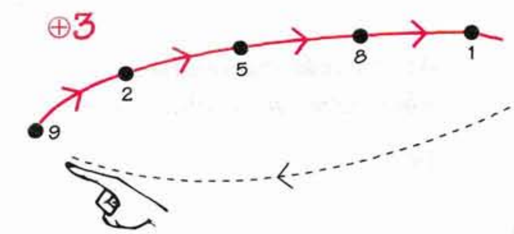
T: ***4 ⊕ 3 = 7. And the next?*** **S:** ***0.***

T: ***7 ⊕ 3 = 0. And the next?*** **S:** ***3.***

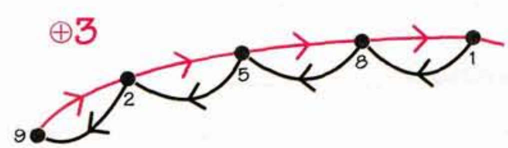
T: ***0 ⊕ 3 = 3. And the next?*** **S:** ***6.***

T: ***3 ⊕ 3 = 6. And the next?*** **S:** ***9.***

With a sweeping motion indicate that the road returns to 9 as you say, "6 ⊕ 3 = 9."



Draw return arrows for each ⊕3 arrow in the picture.



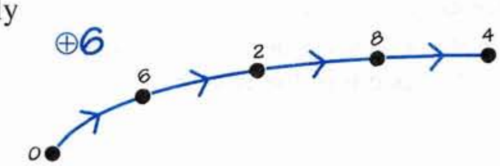
T: ***What could the green arrows be for?***

S: *⊖3.*

S: *⊕7.*

If students do not suggest ⊕7 on their own, write ⊕□ in green on the board and ask whether the green arrows could be for ⊕ some number. Check that the green arrows could be for ⊕7 by considering each number. Look at the starting number, follow the arrow saying, "⊕7," and check that the ending number is correct. Then write ⊕7 in green near the arrow road.

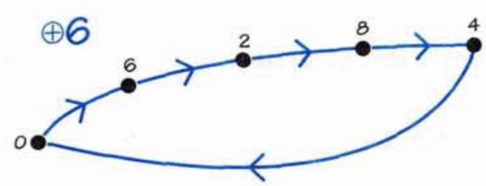
Draw this arrow road next to the ⊕3 arrow road. Collectively label the dots, starting with 2 at the center dot.



T: ***If we drew another ⊕6 arrow, what is the next number we would meet?***

S: ***0.***

Draw a blue arrow from 4 to 0.



T: ***Where is the number 3 in the ⊕6 snake dance?***

S: ***Only even numbers are in this piece.***

S: ***You need to start a new part of the ⊕6 snake dance with 3 in it.***

Draw arrows, starting at 3, as you ask the following questions.

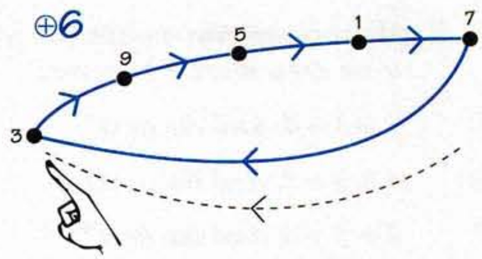
T: ***What is 3 ⊕ 6?*** **S:** *9.*

T: ***What is 9 ⊕ 6?*** **S:** *5.*

T: ***What is 5 ⊕ 6?*** **S:** *1.*

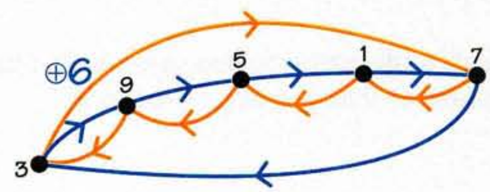
T: ***What is 1 ⊕ 6?*** **S:** *7.*

T: ***What is 7 ⊕ 6?*** **S:** *3.*



With a sweeping motion indicate that the road returns to 3.

Draw the return arrows in this part of the ⊕6 snake dance.



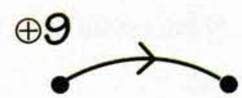
T: ***What could the yellow arrows be for? Try to label them circle-plus some number.***

S: *⊕4.*

Check that the yellow arrows could be for ⊕4 by considering each one. Then write ⊕4 in yellow near the arrow picture. Check also that the return arrows in the other part of the ⊕6 picture are for ⊕4.

Exercise 3

Erase the board and then draw a ⊕9 arrow.



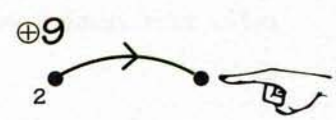
T: ***How can we label these dots?***

Let students make suggestions. Record correct number pairs in a chart, as shown here.

8	7
5	4
4	3
2	1
1	0
7	6
0	9
6	5
3	2
9	8

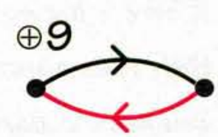
If your students do not find all the possible number pairs, you can help by calling attention to the left column of the list and asking whether any whole number from 0 to 9 is not included. Then ask if a ⊕9 arrow could start at that number. For example, suppose that the pair (2,1) is not in the list.

T: ***2 is not on this side*** (trace down the left column of the chart).
Could this ⊕9 arrow start at 2? (Yes)
If so, what number would be here? (1)



Repeat this procedure, if necessary, until all ten possible number pairs are listed in the chart.

Draw a return arrow in the picture.



T: ***What could the red arrow be for?***
Could it be for circle-plus some number? (⊕1)

Using the entries in the chart, check that the return arrow could be for ⊕1. (You will need to go from right to left in the chart to check the return arrow.) Then write ⊕1 in red near the arrow picture.

MULTIPLICATION MODULO 10 (LESSON ONE)

Capsule Lesson Summary

Review the addition with ten friends operation, ⊕, introduced in *Dancing Friends,* and introduce a multiplication operation, ⊗, for the ten friends. Draw an arrow picture for ⊗2 with all ten numbers—0, 1, 2, 3, 4, 5, 6, 7, 8, and 9—in it.

Description of Lesson

Exercise 1

During this exercise you may prefer to ask students to copy and solve the problems on their papers, before doing a class check. List the whole numbers from 0 to 9 on the board. Keep the list on the board for quick reference during the lesson.

0 1 2 3 4 5 6 7 8 9

Briefly review the operation ⊕ invented by the number 0 in the storybook *Dancing Friends.* Write several problems on the board and do them collectively. For example:

$$9 \oplus 9 = 8 \qquad 7 \oplus 5 = 2 \qquad 6 \oplus 6 = 2$$
$$3 \oplus 6 = 9 \qquad 4 \oplus 7 = 1 \qquad 5 \oplus 8 = 3$$

Ask which numbers can be put in the frames in each of the following number sentences. A table of solutions for the problem on the right is given below it. (The □ and the △ numbers can, of course, be reversed.)

$$\boxed{7} \oplus 3 = 0$$
$$8 \oplus \boxed{4} = 2$$
$$\boxed{9} \oplus 5 = 4$$

□ ⊕ △ = 1

□	△
1	0
2	9
3	8
4	7
5	6

T: ***The ten number friends 0 to 9 did ⊕ dances. Do you think they could invent a way to multiply, say ⊗, and do ⊗ dances?***

What number do you think 2 ⊗ 4 (read as "two circle-times four") ***is?***

S: *8.*

T: ***What number do you think 3 ⊗ 4 is?***

S: *2.*

T: ***How did you get 2?***

S: ***3 x 4 = 12, and we keep only the ones digit.***

Make sure the class understands this last example and then continue with other ⊗ facts.

$2 \otimes 4 = 8$	$4 \otimes 5 = 0$	$5 \otimes 3 = 5$
$3 \otimes 4 = 2$	$2 \otimes 9 = 8$	$4 \otimes 7 = 8$
$4 \otimes 4 = 6$	$3 \otimes 7 = 1$	$6 \otimes 4 = 4$

Write this open sentence on the board. $3 \otimes \square = 4$

T: ***3 ⊗ one of these numbers*** (point to the list 0 to 9) ***is 4. Which number could be in the box?***

S: ***8, because 3 x 8 = 24 and 4 is in the ones place.*** $3 \otimes \boxed{8} = 4$

T: ***If I change this number (4) to 7, which number could be in the box?***

S: ***9, because 3 x 9 = 27 which ends in 7.*** $3 \otimes \boxed{9} = 7$

T: ***If I change this number (7) to 1, which number could be in the box?***

S: ***7, because 3 x 7 = 21 which ends in 1.*** $3 \otimes \boxed{7} = 1$

Write this open sentence on the board. $\square \otimes 2 = 8$

T: ***Which number could be in the box?***

S: ***4, because 4 x 2 = 8.***

S: ***9, because 9 x 2 = 18 which ends in 8.***

T: ***If I change this number (8) to 6, which number could be in the box?***

S: ***3, because 3 x 2 = 6.*** $\square \otimes 2 = 6$

S: ***8, because 8 x 2 = 16 which ends in 6.***

Write this open sentence on the board. $5 \otimes \square = 0$

T: ***Which number could be in the box?***

Students should suggest that 0, 2, 4, 6, or 8 could be in the box. Observe that these are all of the even numbers in the list 0 to 9.

T: ***When we take 5 ⊗ some number, we can get 0 or we can get…?***

S: ***5.***

Change the open sentence to reflect this possibility. $5 \otimes \square = 5$

T: ***Now which number could be in the box?***

Students should suggest that 1, 3, 5, 7, or 9 could be in the box. Observe that these are all the odd numbers in the list 0 to 9.

Write this open sentence on the board.

$\square \otimes \triangle = 4$	
1	4
2	2
2	7
3	8
4	6
6	9
8	8

T: ***The number in the box may be different from or the same as the number in the triangle.***

Students should be able to find all the solutions. List them in a chart as they are suggested. (The □ and the △ numbers can, of course, be reversed.)

Exercise 2

Erase the board and then write ⊗2 in a color.

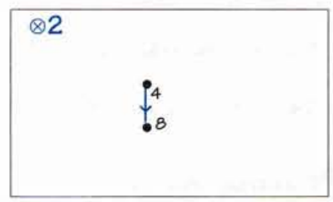
T: ***Let's put the ten number friends in a ⊗2 arrow picture.***
What number would you like to start with?
Remember, the ten dancing friends are the numbers 0 to 9.

The following is a sample dialogue, starting with the number 4. The illustrations on the right show how the picture on the board progressively develops.

S: *4.*

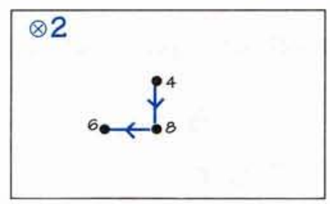
T: *4 ⊗ 2 = …?*

S: *8.*



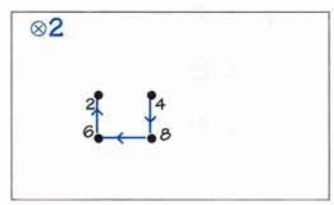
T: *8 ⊗ 2 = …?*

S: *6.*



T: *6 ⊗ 2 = …?*

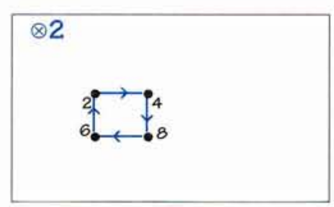
S: *2.*



T: *2 ⊗ 2 = …?*

S: *4.*

T: ***We already have a dot for 4.***

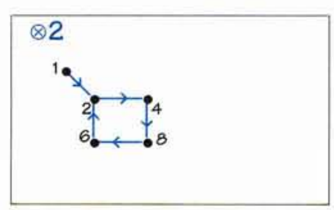


T: ***Where should we put 1 in this picture?***

S: ***Near 2.***

T: ***Why there?***

S: ***Because 1 ⊗ 2 = 2.***

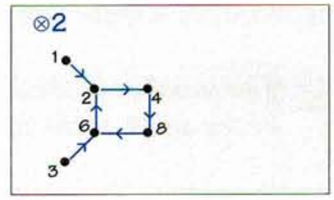


T: *What number friends are we missing?*

S: *3.*

T: *Where should we put 3?*

S: *Near 6, because 2 ⊗ 3 = 6.*

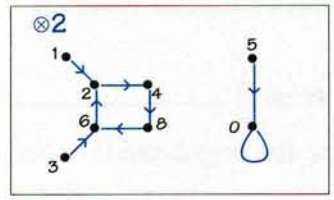


T: *Where should we put 5?*

S: *Near 0, because 5 ⊗ 2 = 0.*

S: *0 is not in the picture yet.*

S: *Draw a loop at 0 because 0 ⊗ 2 = 0.*



T: *What number friends are missing?*

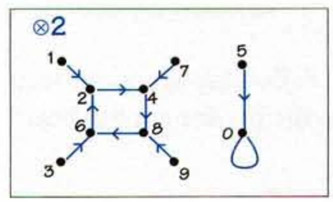
S: *7 and 9.*

T: *Where should we put 7?*

S: *Near 4, because 7 ⊗ 2 = 4.*

T: *Where should we put 9?*

S: *Near 8, because 9 ⊗ 2 = 8.*



Practice Activity

Use the following problems about ⊗ as additional practice for multiplication facts.

2 ⊗ 6 = ____	2 ⊗ 7 = ____
3 ⊗ 6 = ____	3 ⊗ 7 = ____
4 ⊗ 6 = ____	4 ⊗ 7 = ____
5 ⊗ 6 = ____	5 ⊗ 7 = ____
6 ⊗ 6 = ____	6 ⊗ 7 = ____

□ ⊗ △ = 2

Capsule Lesson Summary

Review the story of *Dancing Friends* and the operations ⊕ and ⊗. Also recreate the ⊗2 arrow picture with the ten numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Draw an arrow picture for ⊗3. Provide individual work on other similar arrow pictures.

Description of Lesson

Exercise 1

Begin the lesson by briefly recalling the story of *Dancing Friends.*

T: ***How many and which number friends were invited to play in the dancing games?***

S: ***Ten friends: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.***

List the whole numbers from 0 to 9 on the board.

0 1 2 3 4 5 6 7 8 9

Review the operation ⊕ by doing these problems. (Answers are in boxes.)

$5 \oplus 3 = \boxed{8}$ $5 \oplus 8 = \boxed{3}$ $\boxed{7} \oplus 4 = 1$

$7 \oplus 5 = \boxed{2}$ $6 \oplus \boxed{9} = 5$ $1 \oplus \boxed{9} = 0$

Continue by asking for pairs of numbers to go in the box and the triangle of this open sentence. A table of possible answers is given here. (Also, the □ and △ numbers can be reversed.)

□ ⊕	△ = 6
0	6
1	5
2	4
3	3
7	9
8	8

Review the operation ⊗ by doing problems like the following. (Answers are in boxes.) Continue by asking for pairs of numbers to go in the box and the triangle of this open sentence. A table of possible answers is given here. (Also, the □ and △ numbers can be reversed.)

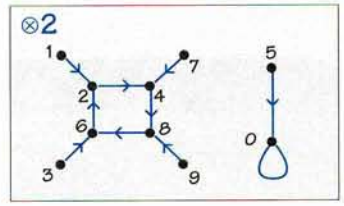
$3 \otimes 4 = \boxed{2}$ $2 \otimes \square = 6$ (Answers: 3 and 8)

$5 \otimes 4 = \boxed{0}$ $4 \otimes \square = 4$ (Answers: 1 and 6)

$4 \otimes 7 = \boxed{8}$ $\boxed{5} \otimes 3 = 5$

□ ⊗	△ = 0
0	1,..., 9
2	5
4	5
6	5
8	5

Exercise 2



T: ***Do you remember how we put the ten number friends in a ⊗2 picture?***

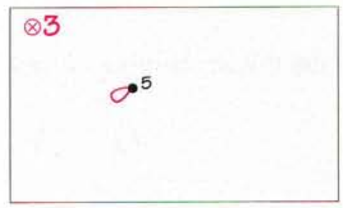
With students help, recreate the ⊗2 picture from the previous lesson.

Keep this picture on the board for the remainder of the lesson.

T: ***Let's put the ten number friends in a ⊗3 picture. What number would you like to put in first?***

The following is a sample dialogue, starting with the number 5. The illustrations on the right show how the picture on the board progressively develops.

Note: Careful location of the numbers in the picture will yield a more attractive final drawing (see the last illustration in the sequence).



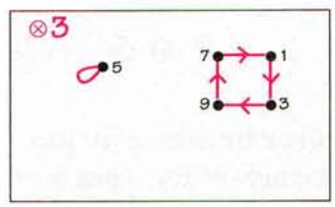
S: *5.*

T: *5 ⊗ 3 = ?*

S: *5.*

S: ***There's a loop at 5.***

T: ***What number would you like to put in the picture next?***



S: *7.*

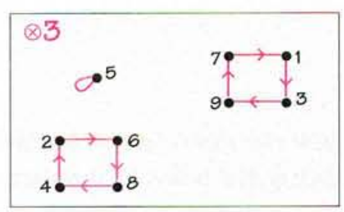
T: *7 ⊗ 3 = …?* (1)

1 ⊗ 3 = …? (3)

3 ⊗ 3 = ? (9)

9 ⊗ 3 = …? (7)

T: ***We have all the odd numbers in our picture; let's put in an even number.***



S: *2.*

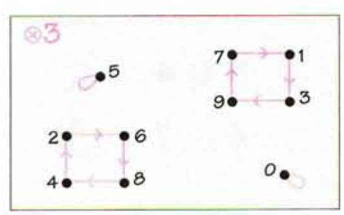
T: *2 ⊗ 3 = …?* (6)

6 ⊗ 3 = …? (8)

8 ⊗ 3 = …? (4)

4 ⊗ 3 = …? (2)

T: ***Are any of the ten number friends missing from our picture?***



S: ***0.***

T: ***0 ⊗ 3 = …?***

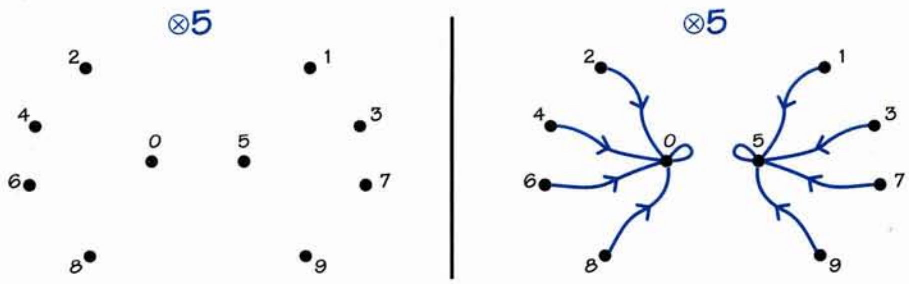
S: ***0.***

S: ***There's a loop at 0.***

Exercise 3

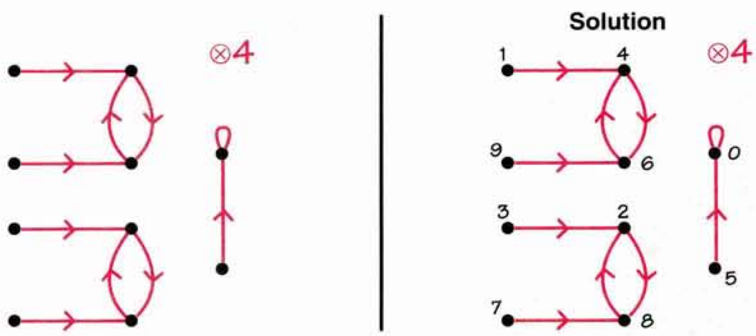
On the board, draw the picture on the left below. Observe with the class that all ten number friends 0 to 9 are here. Then ask students to copy the picture and draw all the possible ⊗5 arrows.

A completed picture is given on the right below.



Exercise 4

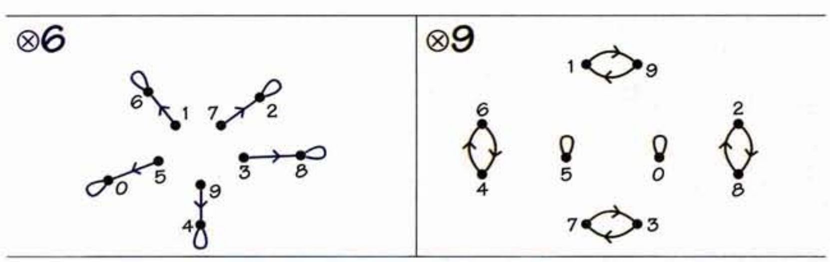
Draw this arrow picture for ⊗4 on the board. Ask students to copy it and label the dots with the ten numbers 0 to 9.



NOTE: Other solutions are possible, but 0 and 5 are fixed, and odd numbers must be on the left.

Exercise 5

Ask students to put the ten number friends in a ⊗6 arrow picture and/or in a ⊗9 arrow picture. Insist that all ten number friends be in the arrow pictures.



Extension Activity

Invite students to prepare class posters or a booklet with all the different ⊗ arrow pictures.

Writing Activity

Suggest students write to a friend describing one of the ⊗ arrow pictures and how to construct it.

I AM NOT MY NAME

Capsule Lesson Summary

Read and discuss the storybook *I Am Not My Name*. Find many different names for 0 and for ½, and perhaps for other numbers such as 1.

Description of Lesson

Distribute copies of the storybook *I Am Not My Name* and let students look at them until you are ready to begin the lesson.

On the board, draw a line segment and place two dots on it, one for the number 0 and one for the number 4.

Pages 2–7

Read these pages aloud or ask some students to do so.

T: ***Can you imagine a dance that the numbers 16, 8, 4, 2, 1, ¼, ⅛, 1/16, and so on could do?***

S: ***An "is greater than" dance.***

S: ***A 2x dance.***

S: ***A ½x dance.***

Pages 8 and 9

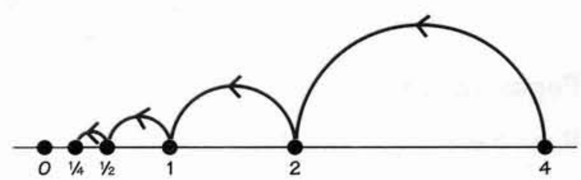
Read these pages aloud or ask some students to do so.

T (pointing to the number line on the board)**:** ***Where is 2 on this number line?***

When a student indicates the point halfway between 0 and 4, draw a dot there and label it 2. Emphasize that 2 is halfway between 0 and 4 on a number line. Draw a blue arrow from 4 to 2.

Continue in the same manner, asking these questions.

- Where is 1 on this number line?
- Where is ½ on this number line?
- Where is ¼ on this number line?



Put your finger halfway between 0 and ¼ on the number line as you ask,

T: ***What number is here?***

S: ***⅛.***

T: ***On the number line, what number is halfway between 0 and ⅛?***

S: ***1/16.***

T: ***On the number line, what number is halfway between 0 and $\frac{1}{16}$?***

S: $\frac{1}{32}$.

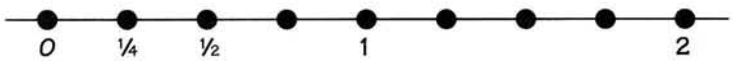
T: ***What could the blue arrows be for?***

S: $\frac{1}{2}$x.

Pages 10–13

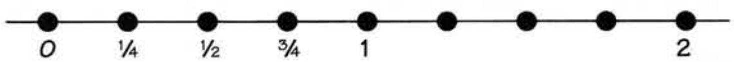
Read these pages aloud or ask some students to do so.

Erase the arrows in the picture on the board. Rescale the number line by repositioning the numbers $\frac{1}{4}$, $\frac{1}{2}$, 1, and 2, each one dot to the right. (4 is excluded from the part of the number line drawn.) Erase the unlabeled dot next to 0. Draw additional dots so that there are seven equally spaced dots between 0 and 2.



T: ***Where is $\frac{3}{4}$ on this number line?***

When a student suggests the dot halfway between $\frac{1}{2}$ and 1, label that dot $\frac{3}{4}$.



Pages 14 and 15

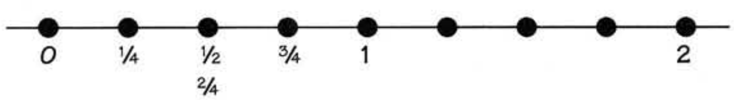
Read these pages aloud.

T: ***Why did $\frac{1}{2}$ begin to cry?***

S: ***The dot for $\frac{1}{2}$ is also the dot for $\frac{2}{4}$.***

Point to the appropriate dots as you say,

T: ***This dot is for $\frac{1}{4}$ and this dot is for $\frac{3}{4}$; so this dot*** (point to the dot labeled $\frac{1}{2}$) ***must be for $\frac{2}{4}$.***



Pages 16–21

Read these pages aloud.

Pages 22 and 23

Read these pages aloud. Discuss why 175 x 45 x 0 x 46 x 728 x 75 easily can be seen to be a name for 0.

T: ***What are some other names for 0?***

Here are some examples of other names for 0 that may be offered.

1,000,000 – 1,000,000	10 – 10 + 2 – 2
(5 x 50) – 250	100 – 1 – 99
16 + 16 – 32	50 + $^{-}25$ + $^{-}25$
1 + $^{-}1$	0 ÷ 2

T: ***How could the boy in the story have more than one name?***

S: ***Maybe he has a nickname as well as his real name.***

You might ask several students to give their full names and their nicknames.

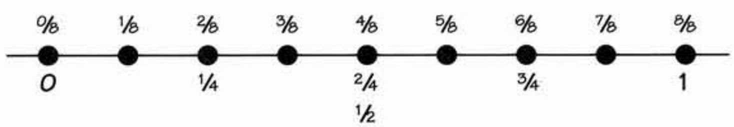
Pages 24 and 25

Read these pages aloud.

Rescale the number line on the board by repositioning the numbers $\frac{1}{4}$, $\frac{1}{2}$ ($\frac{2}{4}$), $\frac{3}{4}$, and 1 as in the next illustration. (2 is excluded from the part of the number line drawn.)

Ask someone to locate and label a dot for $\frac{1}{8}$.

Label some of the other dots in terms of eighths, as long as there is student interest. It may help students to consider $\frac{1}{2}$ first.



Pages 26 and 27

Read these pages aloud. With class assistance, draw and label dots for $\frac{9}{8}$ and $\frac{10}{8}$ on the number line.

Pages 28–31

Read these pages aloud. Ask questions such as the following:

T: ***What are some other names for $\frac{1}{2}$?***
What is a name for $\frac{1}{2}$ that is 2x some other number? (2 x $\frac{1}{4}$)
What is a name for $\frac{1}{2}$ that is 4x some number? (4 x $\frac{1}{8}$)
Do you remember a name for $\frac{1}{2}$ we used with money? (0.50 or 0.5)

Practice Activity

With your class, count by fourths—for example, $\frac{1}{4}$, $\frac{2}{4}$ ($\frac{1}{2}$), $\frac{3}{4}$, $\frac{4}{4}$ (1), $\frac{5}{4}$, $\frac{6}{4}$ ($1\frac{1}{2}$), …—and by eighths.

Capsule Lesson Summary

Introduce the storybook *The Square Trap* and play one of the two games presented in this story. Compile and examine the results. Calculate an average score for the results of many games.

Description of Lesson

Distribute copies of the storybook *The Square Trap* to student pairs, and let them look at the book until you are ready to begin.

Pages 1–6

Read these pages collectively, stopping for students' comments or questions. For example, you may ask students to comment on how and why spinners are used in games. Provide direction in reading the chart on page 4 as a record of the results pictured in the spiral on page 3.

On the board, draw a square as shown on page 5. Illustrate moving a checker on the square as suggested at the bottom of page 6.

Pages 7 and 8

Read page 7 and discuss what random devices could be used to simulate the spinner in playing the game. Choose or let students choose a random device to use, and then instruct student pairs to play the game several times.

With a pair of students, one student can use the random device while the other student moves a marker on the square (page 7), counting how many moves before the checker is trapped at **E**. Suggest that students create a chart like the one on page 8 to record how many moves in a game. Then they can switch roles and play another game.

Monitor students as they play the game, checking that each pair understands the game and is keeping accurate records. If you notice that a game is recorded with an odd number of moves, question how this could happen. Students should begin to notice that games do not end after an odd number of moves and that most games end rather quickly (after two or four moves).

When most pairs of students have completed about ten games, ask everyone to stop playing and to participate in discussing the results. Collect the results from all pairs of students in a class record on the board. For each column (2 moves, 4 moves, and so on), total the number of games from all pairs of students in which the checker was trapped after that number of moves. For example, a typical distribution of results for a class playing a total of 150 games might be as follows.

CHECKER TRAPPED AFTER										
1 move	2 moves	3 moves	4 moves	5 moves	6 moves	7 moves	8 moves	9 moves	10 moves	more than 10 moves
0	72	0	41	0	19	0	8	0	4	6

Encourage comments on the results.

S: ***Most games end after only two or four moves.***

S: ***The checker can only be trapped after an even number of moves.***

Ask students to explain why this is so, but do not expect a well-formulated reply.

S: ***The checker starts at S. After one move, it can be at the upper left or lower right corner, but not trapped. After two moves, it can be back at S or trapped. After three moves, it can be at the upper left or lower right corner, but not at S or trapped. After four moves, it can be at S or trapped. This can go on and on. After an even number of moves, the checker can be at S or trapped; after an odd number of moves, it can be at the upper left or lower right corners.***

Pages 9 and 10

Read and discuss these pages collectively. This discussion will be similar to the discussion on the class's results.

Pages 11 and 12

Read page 11 collectively. Use balancing or another method (for example, sum and divide by 10) to find the average (mean) number of moves for the ten games described on page 11. A class might use these steps in balancing the number of moves to find an average:

2	4	6	2	6	2	4	2	4	12
(+2)			(+2)		(+2)		(+2)		(−8)
4	4	6	4	6	4	4	4	4	4
		(−2)		(−2)		(+1)	(+1)	(+1)	(+1)
4	4	4	4	4	4	5	5	5	5

The average is between 4 and 5.

Note: To explain the storybook's statement that the average is 4.4, suggest that to share four moves among ten games one gives 0.4 to each game. Of course, a game never has 4.4 moves and this can only be understood as a theoretical average.

Read page 12 and again calculate the average number of moves (four) for the ten games described there. For example:

2	2	6	8	2	2	4	8	2	4
(+2)	(+2)		(−4)	(+2)	(+2)		(−4)		
4	4	6	4	4	4	4	4	2	4
		(−2)						(+2)	
4	4	4	4	4	4	4	4	4	4

Instruct each pair of students to use the scores from the games that they played to calculate the average number of moves for their games.

Collect the class data on the board. Fifteen pairs of students might generate these average scores:

4.2	3	3.6	3.8	4.2
4	4.4	2.8	5.2	3.8
4.6	3.6	5	4	3.4

Note: If you prefer, let students report averages as between 3 and 4 rather than 3.6. Students who have played more than ten games may have difficulty calculating their average number of moves as a decimal.

With the class data on the board, discuss the questions at the bottom of page 12.

T: ***Could an average score be 2?***

S: ***It's possible, but in that case all games would have to end after two moves.***

T: ***Could an average score be 1.5?***

S: ***No; the least number of moves is two, so the average could not be less than 2.***

Your students may observe that the average score is usually between 3 and 5, or close to 4.

Pages 13 and 14

Suggest students read these pages on their own or with their partners.

Capsule Lesson Summary

Review the story and the first game introduced in the storybook *The Square Trap*. Play the second game introduced in the storybook and compile results. Find that this second game is very much like the first game in the results and, in fact, the two games end at the same time when played simultaneously.

Description of Lesson

Pair students and distribute copies of the storybook *The Square Trap*. Review the story and the game introduced in the first half of this storybook. Let students recall some of the results and observations that were made in the first lesson.

Pages 15 and 16

Read page 15 collectively and stop to ask the class if they understand the new game. You may need again to have a discussion on simulating the spinner with another random device such as a coin, a die, or red and blue marbles. Let students comment on how long they think a typical game might take to play; that is, would most games end after only a few moves or after many moves?

Read page 16 collectively and act out this game at the board. Let students select the random device they wish to use.

Page 17

Prepare pairs of students to play the game. Each pair of students needs a piece of paper marked with a separation line down the center (a playing mat), a red checker marked 0 and a blue checker marked 1, a random device to simulate the spinner, and a chart like that shown on page 17.

Instruct each pair of students to play the game ten times and to record their results in the chart. Monitor students as they play the game, checking that each pair understands the game and is keeping accurate records. If you notice that a game is recorded as ending in an odd number of moves, question how this could happen.

As pairs of students complete playing the game ten times, ask them to recall how to find the average score (number of moves) for their games. Instruct them to calculate their average score and to save it for use in a collective discussion of results.

Collect results from all pairs of students in a class record on the board. For example, a typical distribution of results and average scores for a class playing 150 games might be as follows.

GAME ENDED AFTER										
1 move	2 moves	3 moves	4 moves	5 moves	6 moves	7 moves	8 moves	9 moves	10 moves	more than 10 moves
0	77	0	38	0	17	0	10	0	3	5

Average Scores

3.2	4.8	4.4	3.8	5
5.2	3.6	4	4.2	4.2
2.8	3	3.6	3.4	4

Note: If you prefer, let students report averages as between two whole numbers.

Direct a discussion of the class results. Students should comment that this game is similar to the game from the first half of the storybook. Examples of such similarities are the following:

- A game ends after an even number of moves.
- Most games end after only two or four moves.
- The average score is usually between 3 and 5.
- The average score cannot be less than 2.

Some students may wish to compare the results of the two games. Avoid drawing any conclusions at this time.

Page 18

Ask students to work with their partners to answer the questions at the top of the page. Perhaps you can suggest that they cover the answers at the bottom of the page with a piece of paper and then uncover the answers to check their own responses.

Pages 19–22

Read and discuss these pages. Students should see that the two games are much the same. The results from one game cannot be distinguished from the results of the other. The games end at the same time when played simultaneously.

Extension Activity

Ask students to look at the back page of the storybook. Invite them to make up games, similar to the ones in the storybook, that can be played with this spinner (or with three marbles: red, blue, and yellow). They might enjoy predicting the kind of results that such games would produce.

NABU WINS AN AWARD

Capsule Lesson Summary

Explore certain fundamental properties of the set of integers as well as the role that 0 and 1 play in that system.

The purpose of this storybook is to explore certain properties of the integer number system in a very open way, a way that should provide an opportunity for students to participate at their level of understanding. It will be up to you, the teacher, to direct the discussion and to capitalize on insights that students might have.

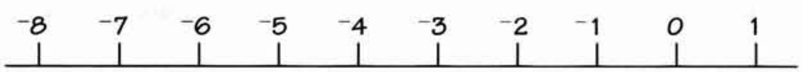
Below are some questions and students' comments that might be made during the presentation of the storybook. Feel free to adjust your presentation to the needs, abilities, and interests of your students. Above all, let students explore and enjoy this story of Nabu.

Description of Lesson

Distribute copies of the storybook *Nabu Wins an Award* and invite students to read the story aloud (or you can read it to the class).

Page 5

Draw this number line on the board.



T: ***Where is $^{-}\frac{1}{2}$ on the number line?***

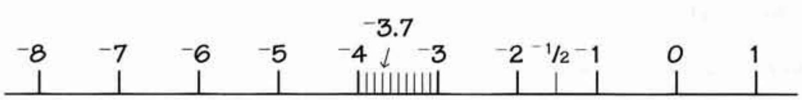
S: ***$^{-}\frac{1}{2}$ is halfway between $^{-}1$ and 0.***

Ask a student to locate $^{-}\frac{1}{2}$ on the number line.

T: ***Where is $^{-}3.7$ on the number line?***

S: ***Between $^{-}3$ and $^{-}4$. Divide the segment between $^{-}3$ and $^{-}4$ into ten smaller segments of equal length.***

Invite students to do the division and to locate $^{-}3.7$.



Pages 7 and 8

T: ***What do you think of the*** $\frac{1}{2}$x ***and*** 2x ***dances?***

S: ***They are very much alike, but*** $\frac{1}{2}$x ***goes one way and*** 2x ***goes the other way.***

T: ***Yes. The ½x dance is just the opposite of the 2x dance.***

S: ***They go on forever. There is no beginning number or ending number.***

Pages 11 and 12

T: ***What do you notice about the –2 and the +2 dances?***

S: ***There are no odd numbers in these dances.***

Note: Of course, these dances could have been made with just the odd numbers.

S: ***They are like the ½x and 2x dances—one goes in the opposite direction from the other.***

T: ***Yes. They, too, are opposites of each other.***

Page 15

T: ***If 15 were in this dance, who would be its partner?***

S: ***$\frac{1}{15}$.***

T: ***Why?***

S: ***$15 \times \frac{1}{15} = 1 = \frac{1}{15} \times 15$.***

Page 16

S: ***The dances look like flowers.***

S: ***The + dance looks just like the x dance, but each positive number dances with a negative number.***

T: ***With any negative number?***

S: ***No, with its opposite: 8 and $^{-}8$; $^{-}26$ and 26; and so on.***

T: ***Can 1 dance in 0's dance?***

S: ***Yes; 1 would dance with $^{-}1$.***

T: ***Can 0 dance in 1's dance?***

S: ***No; no number times zero equals one.***

Page 19

S: ***This dance looks like teardrops.***

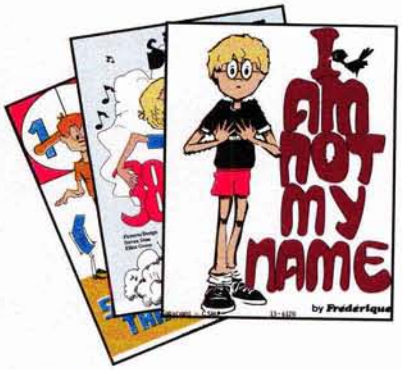
S: ***It's raining numbers.***

S: ***Each number dances with itself.***

T: ***Why?***

S: ***Because any number plus zero equals that number, and any number times one equals that number.***

Allow students as much time as they want to read and look at the pictures in the storybook.



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