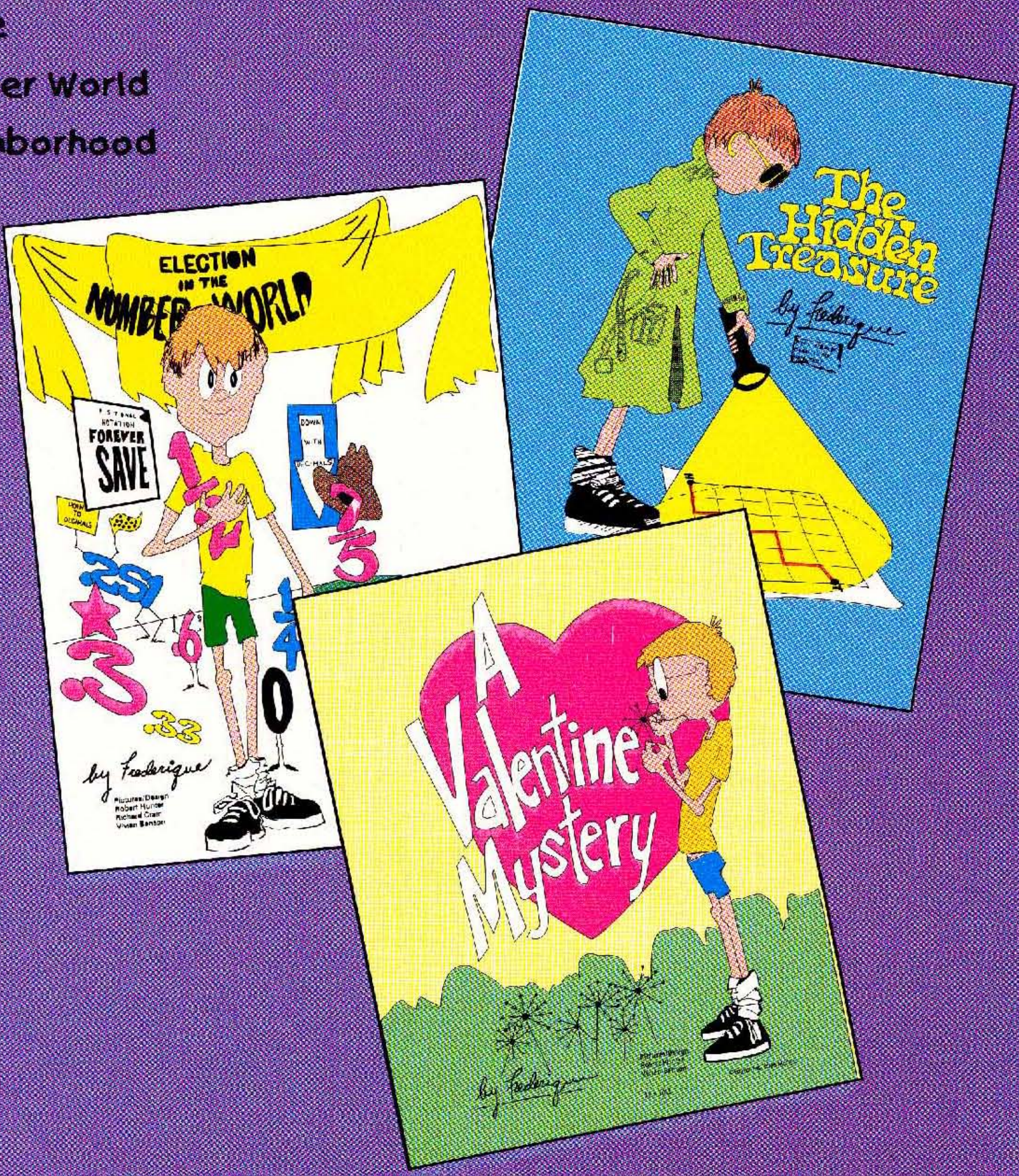
STORIES BY FRÉDÉRIQUE

Storybook Set III

Valentine Mystery
Hidden Treasure
ection in the Number World
/ery Strange Neighborhood



Ages 10 – 14

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A CSMP Minipackage

A CSMP Minipackage presents a part of the CSMP curriculum through introductory lessons that can be taught by teachers with no prior CSMP training to students with no prior CSMP background. The purpose of each CSMP Minipackage is twofold:

- to introduce one or more of the nonverbal languages and instructional tools used in the CSMP elementary curriculum so that a teacher can pursue the possibility of implementing the entire program; and
- to provide mathematically rich activities that can be used immediately in the classroom.

This CSMP Minipackage introduces the Languages of Strings and Arrows, grid displays, and number line tools, using a collection of storybooks. These delightful stories by Frédérique Papy capture the imagination of young students and cover a wide range of topics involving mathematics. Students easily become engaged in these stories: there are questions to answer, pictures to relate to the story, and new adventures to invent for story characters. Dot, string, and arrow pictures reinforce each story while teaching important mathematical concepts and fundamental properties of numbers. The stories are appropriate for individual or group reading, at home or school.

Four Storybooks and How to Use Them

This booklet gives lesson ideas for the four storybooks in Set III of *Stories by Frédérique*. The lesson presentations describe classroom use of the storybooks, but they may be adapted for use with an individual child or a small group of students. Although the order of use of the storybooks is not critical, you will find the number concepts in the first two storybooks only involve whole numbers, whereas the last two storybooks have ideas involving other sets of numbers. The storybook *A Very Strange Neighborhood* refers to the idea of repeating decimals introduced in *Election in the Number World*.

Sometime before using a storybook lesson, read the storybook on your own. Be sure to read the comment on the last page. While doing this, think about the various situations and how your students might react to them. Feel free to bring your imagination, your experience, and your general familiarity with the interests and concerns of your students to bear on the lesson. The lesson description simply provides some suggestions on how students might react and how you might prompt creative thinking.

Brief story descriptions for all storybooks in this collection are given here.

A Valentine Mystery

If each person is to receive ten Valentine cards and no person is to send more than one card, then you need at least ten times as many senders as receivers. That's obvious! Or is it? That's the question in *A Valentine Mystery*.

When Zero announces that this year in the World of Numbers they will celebrate Valentine's Day in a way that only numbers can, the little boy in the story is puzzled. However, as the story progresses and Zero gradually unfolds his grandiose scheme, the boy begins to suspect that perhaps there's something about the set of whole numbers that allows things to occur in the World of Numbers that could never happen to people!

Thus Zero's explanatory posters and the little boy's patient questioning give the reader a glimpse into the strange things that can occur in the realm of the infinite.

The Hidden Treasure

The principal character in *The Hidden Treasure* is a boy who has the misfortune to fall ill at Christmas time. Fortunately, for him, his grandmother is on hand to help him feel less sorry for himself. She gives him two interesting puzzles and a large box of colored pens to use figuring them out.

The first puzzle has to do with Spike, the famous spy, and the mysterious case of the stolen treasure. As part of his investigation, Spike has to travel along all the possible routes from his house to the place where the treasure is hidden. Even though all the roads in the area are one-way, there are still surprisingly many possibilities.

The second puzzle involves Spike's attempts to wind up his investigation by interviewing all ten suspects, four at a time. The reader discovers a very strong link between the two puzzles, and this greatly simplifies the situation.

Election in the Number World

In this story, the Numbers' Parliament has enacted legislation requiring the decimal way of writing numbers. This means, of course, that all the numbers have to register at the City Hall using their decimal names. Many numbers, such as $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$, are greatly distressed about this because they have grown up believing that their fractional names are perfectly adequate. To their relief, they discover that assuming their decimal names does not present as much upheaval as they had feared.

The situation is not so simple in the case of $\frac{1}{3}$, however. It is sent from office to office at the City Hall, and no one seems to be able to tell $\frac{1}{3}$ what its decimal name is. Fortunately, the Complaints Office is run by Zero, who carefully explains why $\frac{1}{3}$ is having such problems, and promptly provides it with a manageable decimal nickname.

A Very Strange Neighborhood

The numbers 7 and 8 live in a very strange neighborhood. No sooner does the hero of this story decide that two numbers live next door to each other than he is shown that, in fact, between their houses there are at least nine others. As the story proceeds, the reader begins to appreciate just how awesome this concept is; one feels quite dizzy just looking at the pictures of the street on which 7 and 8 live.

The story of this strange neighborhood continues with a tale of mystery radio signals. The police become involved in the search for the source of these signals, but it is Zero who eventually deciphers what they mean, and hence manages to locate a sad number, whose trouble is that it is irrational (in the mathematical sense, that is).

For Further Information

Nonverbal languages and the Papy Minicomputer are used extensively in the CSMP curriculum. This CMSP Minipackage provides a simple introduction to these languages and tools in story contexts. To preview CSMP's unique approach to mathematics at the elementary school level and for more in-depth use of the languages and tools of CSMP, other Minipackages such as *A-Blocks String Game* and *Minicomputer Games* are useful. A brief description of these Minipackages can be found on page 35 of this booklet. For more information, contact:

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A VALENTINE MYSTERY (LESSON ONE)

Capsule Lesson Summary

Begin the storybook *A Valentine Mystery* with a mysterious game played in the world of numbers in which each whole number sends exactly one valentine but receives ten. Discover that the rule for sending valentines involves dividing by 10. Determine which numbers send a valentine to a given number and which number receives a valentine from a given number.

Description of Lesson

Students should have unlined paper and colored pencils or crayons to use during the lesson.

Pages 1–4

Read pages 1 through 4 together; encourage class discussion of the game Zero invented for the numbers to play. Restate the rule of the game written in the red box on page 3.

Pages 5 and 6

Read pages 5 and 6 together. Draw a red arrow starting at 79 on the board.

T: ***Study the poster on page 5. See if you can tell to whom 79 sends a valentine.***

S: *7.*

Continue by asking to whom 709 and 4 037 send valentines. (Answers are in boxes.)

Draw a red arrow ending at 62 on the board.

T: ***From whom could 62 receive a valentine?***

S: ***623.***

T: ***Does 62 receive more than one valentine?***

S: ***Yes, 62 receives valentines from 620, 621, 622, 623, 624, 625, 626, 627, 628, and 629. 62 receives exactly ten valentines.***

T: ***Which numbers send valentines to 0?***

S: ***0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.***

Pages 7 and 8

Ask the class to read these pages and to study the posters.

T: ***On your paper, draw a poster for 827. When you finish the poster for 827, do one for 6 025 also.***

While students are working, draw this much of the posters on the board.

$827 \rightarrow$
$827 =$
$827 =$

$6\,025 \rightarrow$
$6\,025 =$
$6\,025 =$

When many students have finished their posters, invite students to complete the posters on the board.

$827 \rightarrow 82$
$827 = 820 + 7$
$827 = (82 \times 10) + 7$

$6\,025 \rightarrow 602$
$6\,025 = 6\,020 + 5$
$6\,025 = (602 \times 10) + 5$

Write this division problem on the board.

$$10\overline{)827}$$

T: ***What is the result of this division problem?***

S: ***82 with a remainder of 7.***

Write the solution on the board and note that 82 is the quotient.

quotient

$$\begin{array}{r} 82 \text{ R} = 7 \\ 10\overline{)827} \end{array}$$

T: ***The result of a division calculation is the quotient. In this calculation, the quotient is 82 and the remainder is 7.***

Pages 9 and 10

Ask the class to read pages 9 and 10 while you write these problems on the board.

$$10\overline{)2\,386} \qquad 10\overline{)7} \qquad 10\overline{)0}$$

T: ***Can you solve these division problems?***

S: ***For the first problem, the quotient is 238 and the remainder is 6.***

S: ***For the second problem, the quotient is 0 and the remainder is 7.***

S: ***For the third problem, the quotient is 0 and the remainder is 0.***

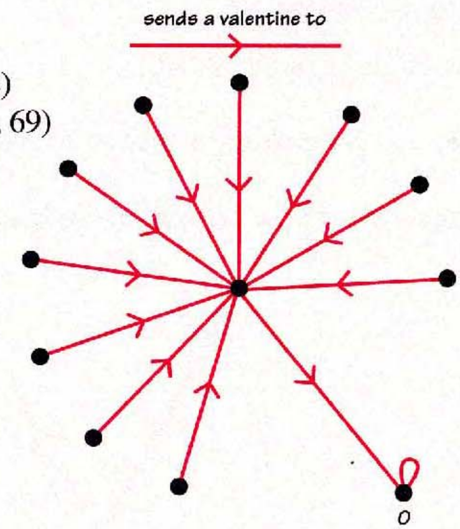
Record the solutions on the board.

Pages 11 and 12

Call on students to read these pages aloud. Note that the picture on page 12 is the completion of the poster on page 11.

T: ***On your paper, draw a flower like the one on page 12. Then put 0 at the ending dot of the longest arrow and label the other dots appropriately.***

As students work independently, draw this picture on the board.



T: ***What number could be at the center dot?*** (6, for example) ***Then from whom does 6 receive valentines?*** (60, 61, 62, …, 69) ***Does everyone have the same solution?***

S: ***No, I put 8 at the center dot. The other dots are for 80 through 89.***

T: ***How many different solutions are possible?***

S: ***Nine. The center dot could be for 1, 2, …, or 9 since each of these numbers sends a valentine to 0.***

Pages 13 and 14

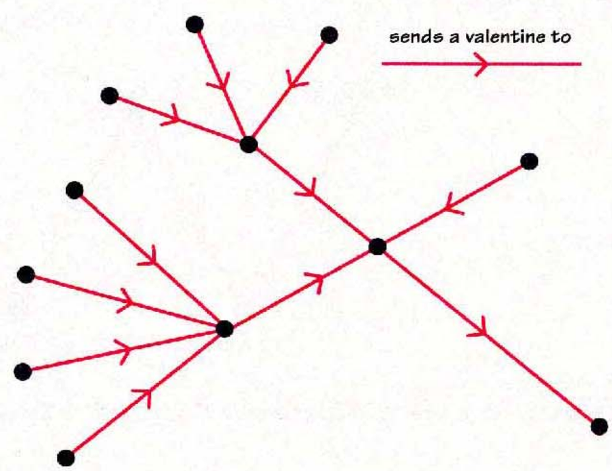
Call on students to read the pages aloud.

T: ***On page 14, label all of the dots that are centers of flowers and some of the other dots.***

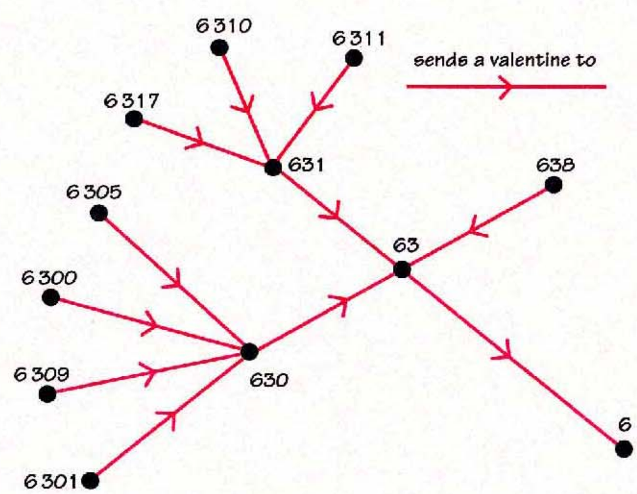
While students are working independently, draw this picture on the board.

After a few minutes, direct the class's attention to the picture on the board.

T: ***This is part of the picture on page 14. Who can label the dots?***



Invite students to label the dots. The class should observe that many solutions are possible. One possible solution in shown here.

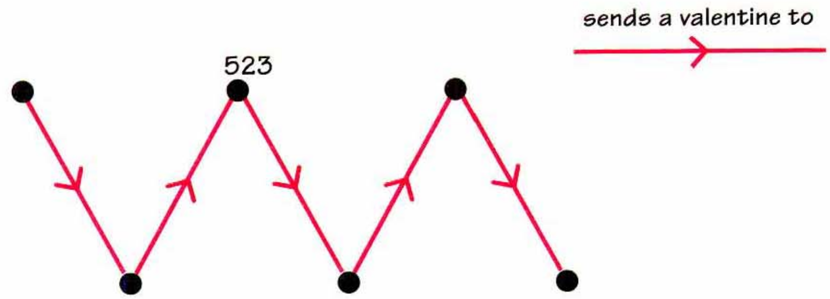


Pages 15 and 16

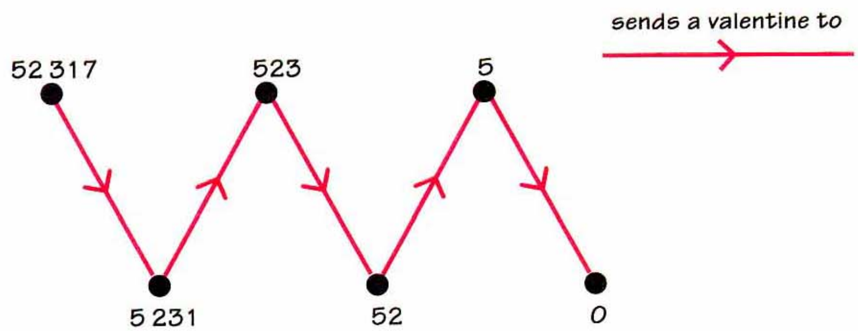
Read these pages aloud.

T: ***Now close your storybooks. I will put another problem on the board.***

Draw this arrow picture on the board, and ask students to copy it on their papers.



Invite students to label the other dots. The three dots on the right must be for 52, 5, and 0, respectively. There are many possibilities for the two dots on the left. The picture below shows one possible solution.



Collect students' copies of the storybook for use again in *A Valentine Mystery* (Lesson Two).

A VALENTINE MYSTERY (LESSON TWO)

Capsule Lesson Summary

Continue reading the storybook *A Valentine Mystery*. Find missing arrows for the relation "sends a valentine to" in an arrow picture. Discover that this game can be played with the whole numbers but not with people because there are infinitely many whole numbers.

Description of Lesson

Students should have unlined paper and colored pencils or crayons to use during the lesson. Have the copies of the storybook *A Valentine Mystery* ready to distribute to students in Exercise 3.

Exercise 1

Ask a student to tell the story of *A Valentine Mystery* read thus far.

Draw an arrow starting at 76 on the board.

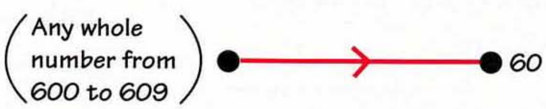
T: ***To whom does 76 send a valentine?***

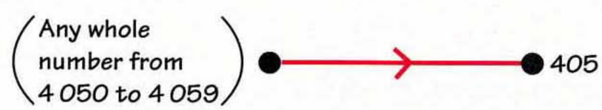
S: *7.*

T: ***Does 7 receive other valentines?***

S: ***Yes, 7 receives exactly ten valentines—one each from 70, 71, 72, 73, 74, 75, 76, 77, 78, and 79.***

Similarly, ask who sends valentines to 60 and to 405.

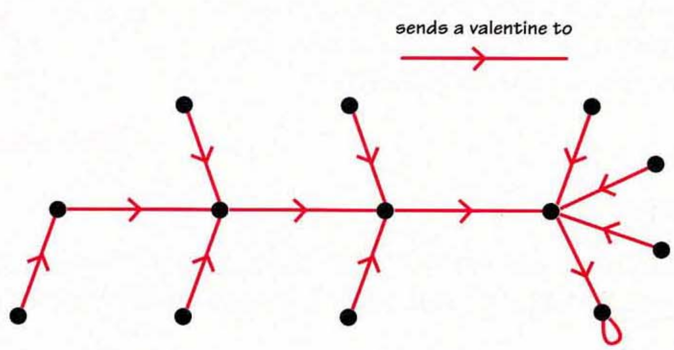




Repeat this activity using other numbers until students understand the rule for sending and receiving valentines.

Exercise 2

Draw this picture on the board and ask students to copy it on their papers.



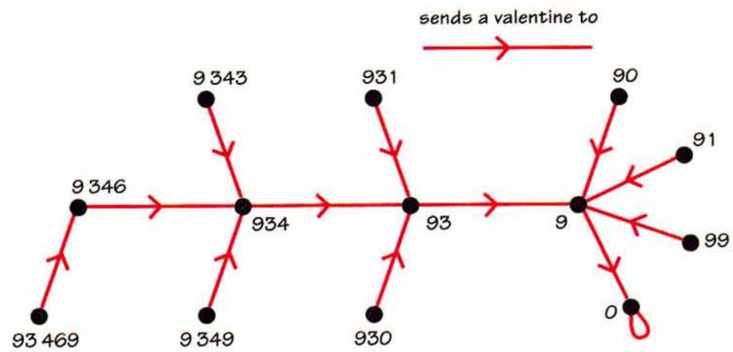
T: ***Who can label one dot?***

S: ***The dot with a loop must be 0 because 0 is the only number who sends a valentine to itself.***

Label the dot for 0 and then ask students to label the other dots. Let your students work independently for a few minutes, then label the dots in the picture on the board. Suppose a student chooses to put 9 at the start of the arrow ending at 0.

S: ***I put 9 here, but any number from 1 to 9 could be here because they all send valentines to 0.***

Invite students to label the other dots on the board. The class should notice that many solutions are possible. The picture here shows one possible solution.



Exercise 3

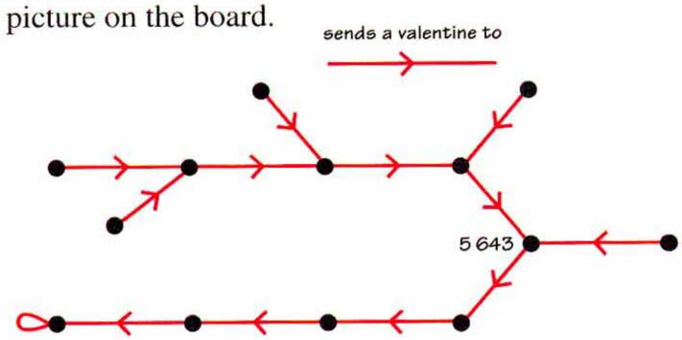
Distribute copies of the storybook *A Valentine Mystery*.

Pages 17 and 18

Call on students to read pages 17 and 18 aloud.

T: ***Label all of the center dots of flowers and some of the other dots on pages 17 and 18. Notice that one dot on page 18*** (far right) ***is already labeled*** (5 643).

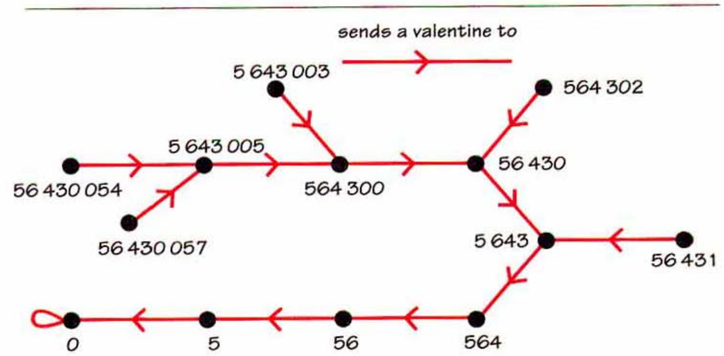
While students are working, draw this arrow picture on the board.



After several minutes, call the class's attention to the picture on the board.

T: ***This is part of the picture on pages 17 and 18. Who can label one of the dots?***

Invite students to label dots until they are all labeled. The numbers less than 5 643 must be 564, 56, 5, and 0. There are many possibilities for the other dots. Encourage students to read the numbers as they write them in the picture. One possible solution is shown here.



Pages 19–22

Read aloud and discuss these pages. Pages 19 and 20 give some hints on how to label the dots in the poster on pages 17 and 18. A full solution is on pages 21 and 22.

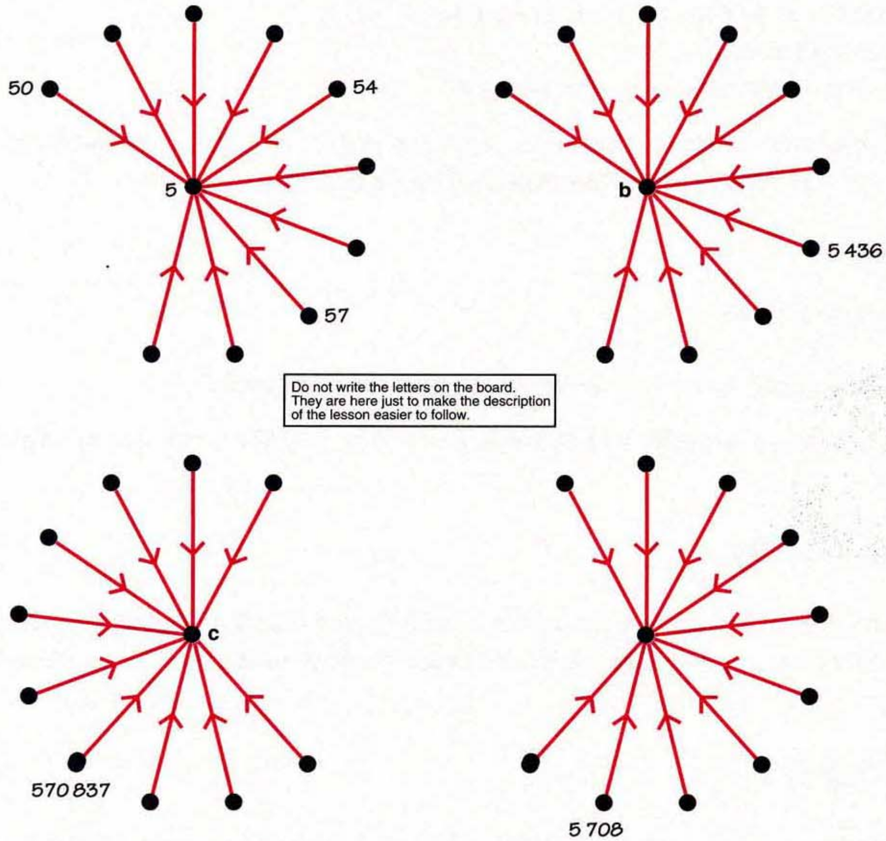
Pages 23 and 24

Call on students to read pages 23 and 24 aloud.

T: ***The flowers on pages 23 and 24 can be connected by red arrows. To help figure out where to draw the red arrows, label the center dot of each flower. I suggest that you begin on page 24 because the numbers there are smaller.***

Let students work on these pages for several minutes. If you have a large board or big piece of chart paper, copy the flowers on pages 23 and 24 to display.

Call students' attention to the top of page 24 (perhaps on your board copy).



T: ***Which number is this*** (point to **b**)***?***

S: ***543, because 5 436 sends a valentine to 543.***

T: ***Where could we draw a red arrow?***

S: ***From 543 to 54.***

Label the dot for 543 and draw a red arrow from 543 to 54.

T: ***Can someone label another center dot?***

S (pointing to **c**)**:** ***57 083 is here.***

S: ***We can draw a red arrow from 57 083 to 5 708.***

Continue in a similar manner until all of the missing red arrows have been drawn. Use the completed picture on pages 25 and 26 of the storybook as your answer key. Labeling the center dots of the flowers aids in finding the red arrows. Encourage your students to both write and read the numbers as they are put in the picture.

Pages 25–28

Tell students to look at the solution on pages 25 and 26 and then to read pages 25 through 28. Discuss the picture on pages 27 and 28.

T: ***Which number is in the center of the picture?*** (0)
Is the picture finished? (No)
Could we ever finish drawing this picture?

S: ***No. Each number receives valentines from ten different numbers, and then each of those numbers receives ten more valentines and so on forever.***

Pages 29 and 30

Read and discuss these pages.

T: ***Why can this game be played by numbers but not by people?***

S: ***There are a limited number of people in the world, but there are always more and greater numbers.***

Extension Activity

Some students may like to try to invent another similar game that the numbers could play. For example, ask students to describe a game where every number would send one valentine and receive 12.

Writing Activity

Suggest that students write a description of the valentine game and then explain to a friend how this game works with numbers, but not with people. Some students may compare this game to the idea behind chain letters.

Capsule Lesson Summary

As part of a detective story about a stolen diamond medallion, count the number of shortest routes along streets between two points of a city. In the process, construct part of Pascal's Triangle.

Description of Lesson

Note: Two lessons are based on the storybook *The Hidden Treasure*. In these lessons, we suggest you tell the story and present the problems. Then, after the problems have been solved, we suggest you ask students to read the storybook. By reading the storybook at that point, students review the methods used in class to solve the problem. You may wish to read the storybook yourself before presenting the two lessons.

Exercise 1

Display a copy of the grid map on the blackline following this lesson (page 18). Distribute copies of the grid map for student use.

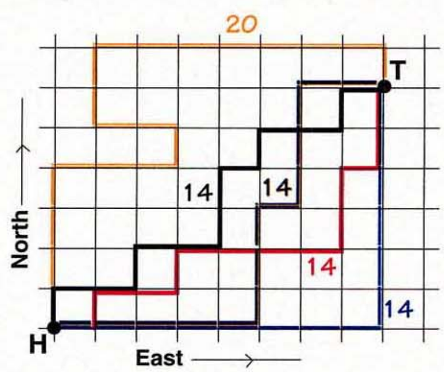
T: ***This is the street map of a town. Here at* H *is the house of a famous detective, Spike. Thieves recently stole a diamond medallion from his house. He has a lead and thinks that the thieves hid the medallion at* T*. Spike needs better evidence, so he decides to look for clues by exploring the routes the thieves might have taken from* H *to* T*. Spike knows that the thieves used a getaway car and, therefore, that they stayed on the streets. Who can trace a route that the thieves might have taken?***

Let students trace and draw several routes from **H** to **T**.

Tell students to draw three routes from **H** to **T** on their grids and to find the length of each route in blocks.

T: ***What is the length of a shortest route from* H *to* T*?***
(14 blocks)

Count the lengths of the routes already drawn on the board. Then invite students to draw several additional routes of length 14 from **H** to **T**. For example:



T: ***Did anyone find a route from* H *to* T *that is longer than 14 blocks?***

Invite students to draw one or two such routes on the board.

T: ***What do you notice about routes longer than 14 blocks? How can you tell, without counting, that the length of a route is more than 14 blocks?***

S: ***A route is longer than 14 blocks if it goes farther north or farther east than* T.**

S: ***A route is longer than 14 blocks if it ever goes west or south (moves away from* T.)**

Conclude that when Spike travels only north and east from **H** to **T**, his route will be exactly 14 blocks long.

T: ***Spike assumes that the thieves took a shortest route from* H *to* T*; that is, a route of length 14. He plans to explore all such routes. We found a few of these routes already. How many possible routes do you think there are for Spike to explore?***

Accept students' estimates, and record them on the board.

Let the class discuss methods for counting all the shortest routes from **H** to **T**. You may follow one or two suggestions, such as making a list or drawing a tree diagram, until these methods become too complicated. Lead the class to consider a method involving first trying to solve a simpler but similar problem.

T: ***Often in mathematics it is a good strategy to first solve some similar but easier problems. Let's try to solve some simpler problems; then maybe we can use their solutions to solve the original problem.***

Exercise 2

Display a clean copy of the grid map, and provide students with a clean copy as well. Refer to intersection points closer to **H**.

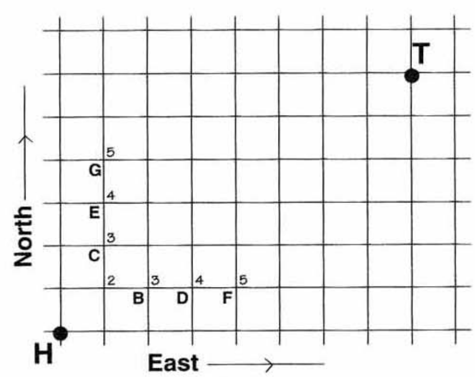
T: ***Before counting the number of routes from* H *to* T*, let's count the number of routes from* H *to some intersections closer to* H*. How many different routes are there from* H *to this point (diagonally opposite* H*).***

S: ***Two.***

Invite a student to trace the two routes, and put 2 near that corner.

In a similar manner, ask students to find the number of routes there are

- from **H** to **B** (Three)
- from **H** to **C** (Three)
- from **H** to **D** (Four)
- from **H** to **E** (Four)
- from **H** to **F** (Five)
- from **H** to **G** (Five)



Do not write the letters **B**, **C**, **D**, **E**, **F**, and **G** on the grid. They are here just to make the description of the lesson easier to follow.

Record the answers on the grid.

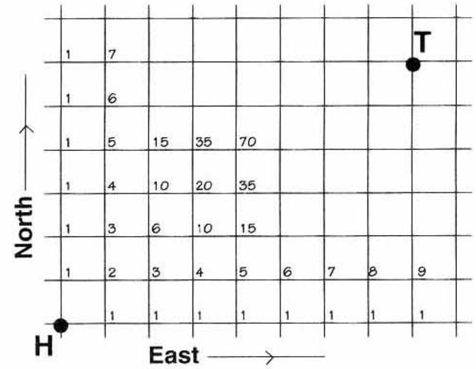
T: ***What patterns do you notice?***

Encourage students to identify and explain both the symmetry and the counting (2, 3, 4, 5) patterns. Extend the counting pattern by putting 6, 7, 8, and 9 by the appropriate corners.

T: ***Why are there ten routes from* H *to* V *and also ten routes from* H *to* W*?***

S: ***Because* V *is three blocks east and two blocks north of* H*, while* W *is three blocks north and two blocks east of* H*.***

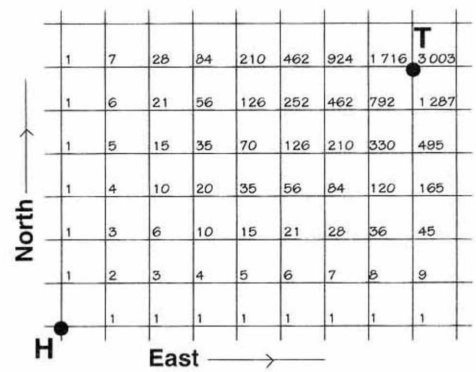
As necessary, discuss other corners until students easily use the pattern. A solution for part of the map is shown here.



At this time you may ask students if they wish to revise their estimates of the number of routes from **H** to **T**. Many of the original estimates may have been too low. Record new estimates.

Direct students to work individually or with partners to use the pattern discovered to find the number of routes from **H** to **T**. This will involve finding the number of routes to each intersection between **H** and **T**. Some students may want to use calculators to check their calculations or to speed up the process.

After a few minutes, invite students to put correct numbers of routes on the displayed grid so as to gradually find the number of routes from **H** to each corner. The complete solution is shown here. The class should find that there are 3 003 routes from **H** to **T**. Compare the solution (3 003) to students' estimates.



T: ***If* T *were five blocks east and four blocks north of* H*, how many shortest routes would Spike have to explore?***

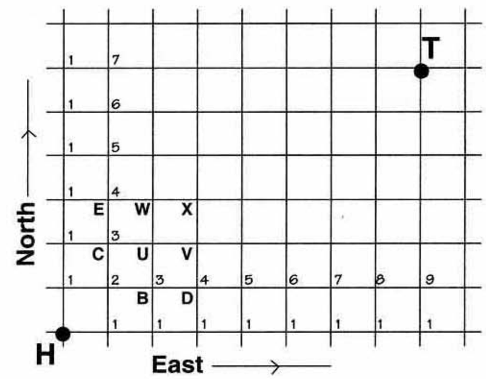
S: ***Only 126.***

Let a student find the intersection five blocks east and four blocks north of **H** to check that there are 126 shortest routes to that point.

In a similar manner, ask the number of routes to a few other intersections. Emphasize the following:

- that the class has found the number of routes not only from **H** to **T**, but also from **H** to many other intersections; and
- the number of routes from **H** to an intersection increases quickly as you get closer to **T**.

Also note that there is only one shortest route to each intersection directly north or directly east of **H**. As you label the intersections at the board, direct students to copy the numbers on their second copy of the grid.



Do not write the letters **B**, **C**, **D**, **E**, **U**, **V**, **W**, and **X** on the grid. They are here just to make the description of the lesson easier to follow.

T (pointing to **U**): ***How many routes are there from* H *to this corner?***

S: ***Six.***

There may be disagreement, so invite students to defend their answers by tracing all of the routes from **H** to **U**. Encourage students to be systematic so as to find all six routes without duplication. Put 6 near the corner.

T: ***Can anyone convince me that there are six routes from* H *to this corner (*U*) without having to trace the routes?***

Perhaps some students will notice that all of the routes from **H** to **U** must pass through **B** or **C**. Since there are three routes from **H** to **U** via **B** and three routes from **H** to **U** via **C**, there are six routes in all from **H** to **U**. If this explanation is not offered by students, use the following dialogue to lead them to this observation. The dialogue uses letters to indicate where you or students point on the grid.

T: ***How many routes from* H *to* U *pass through* B*?***

S: ***Three, because there are three routes from* H *to* B.**

T: ***How many routes from* H *to* U *pass through* C*?***

S: ***Three, because there are three routes from* H *to* C.**

T: ***Are there any routes from* H *to* U *that do not pass through either* B *or* C*?***

S: ***No.***

T: ***So there are six routes from* H *to* U*: the three routes through* B *and the three routes through* C.**

Direct students to work on their own or with partners to find the number of routes from **H** to **V** and from **H** to **W**. Check that before they start they copy all of the numbers from the displayed grid onto their grids.

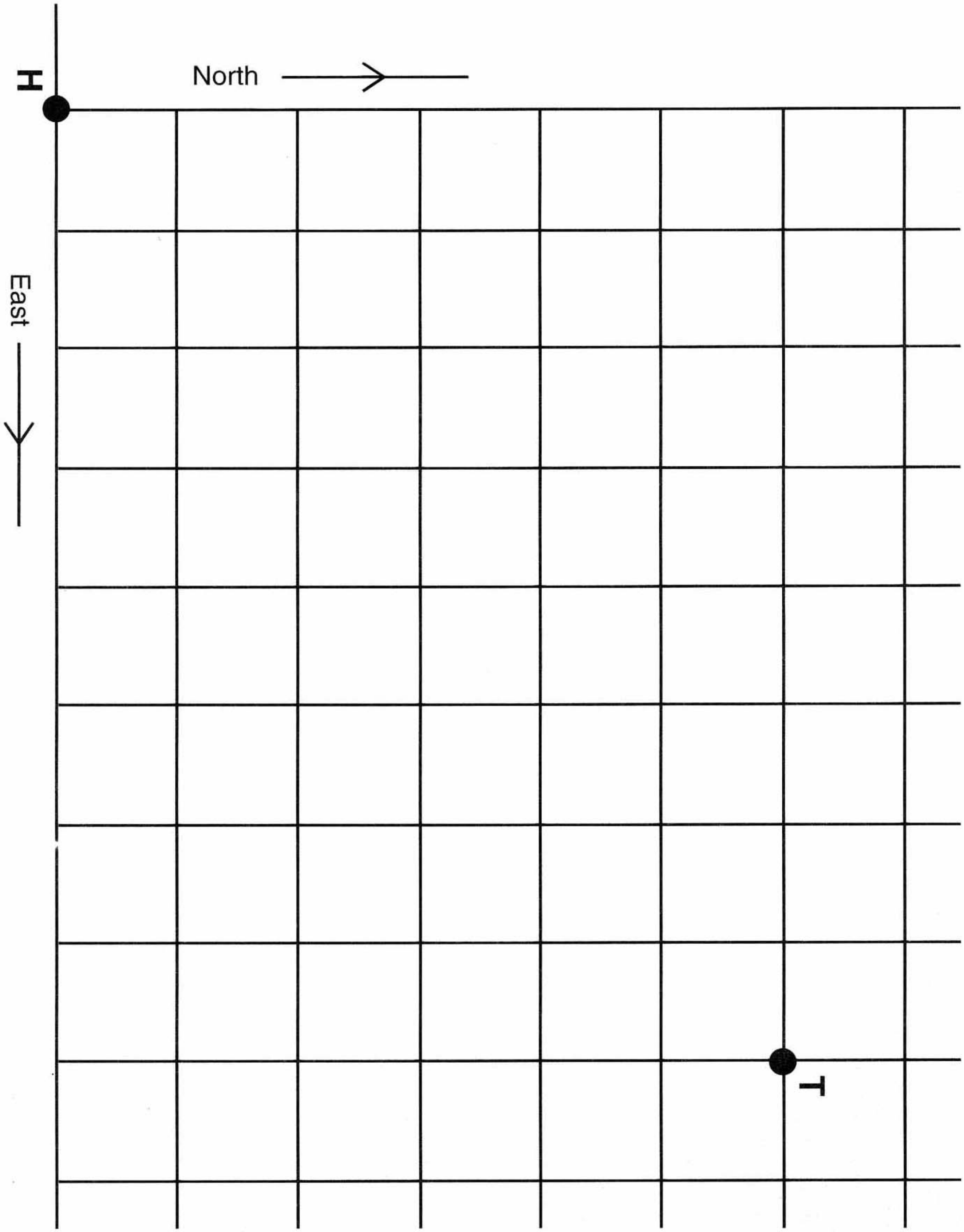
It is likely that some students will attempt to trace the routes and that they will have difficulty doing this. Suggest that they try to use the numbers of routes at corners **U** and **D**, or at **U** and **E**.

T: ***It's difficult to trace the routes and to be sure that all are counted but none are duplicated. Did anyone use a different method?***

S: ***All routes from* H *to* V *must pass through either* D *or* U*. There are four routes from* H *to* D *and six routes from* H *to* U*. Therefore, there are ten routes from* H *to* V *because 4 + 6 = 10.***

In a similar manner, lead the class to discover that there are ten routes from **H** to **W** (10 = 6 + 4) and there are 20 routes from **H** to **X** (20 = 10 + 10).

THE HIDDEN TREASURE - Blackline



Note: You may wish to mention that the array of numbers generated in this problem is part of Pascal's Triangle. Often Pascal's Triangle has the numbers arranged in this format:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
⋮

Pascal lived in the sixteenth century in France. He used this array of numbers to solve many problems in algebra, probability, and combinatorics. This arrangement of numbers was discovered many centuries earlier, but Pascal popularized it by writing a treatise on its patterns and its applications. Pascal's Triangle is still widely used in the above fields of mathematics.

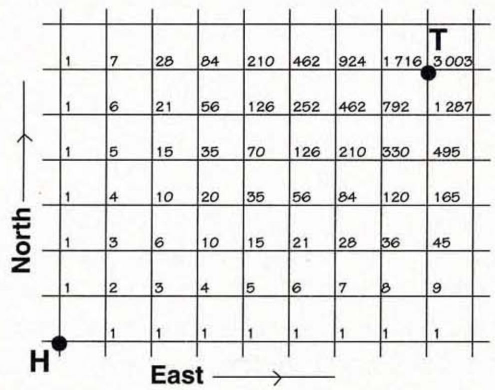
Capsule Lesson Summary

Review the story about a stolen diamond medallion and the problem concerning the number of routes between two locations in a city. Develop a 0–1 binary code for recording routes on a grid of streets. Use the code to determine the number of six-element subsets in a fourteen-element set.

Description of Lesson

Exercise 1

Display the grid with numbers of shortest routes from **H** to each intersection, generated in Lesson One.



Ask a student to recall the story about Spike, the detective who decided to search all of the shortest routes from his house, **H**, to the stolen treasure, **T**. Review briefly how it was determined that there were 3 003 such routes.

Point to an intersection.

T: ***What does this number tell us?***

S: ***There are that number of different shortest routes from* H *to that intersection.***

T: ***What patterns do you notice in this grid of numbers?***

Allow several minutes for students to locate and describe the patterns they observe, for example:

- 1, 1, 1, … in the first row and first column
- 1, 2, 3, … in the second row and second column
  +2 +3 +4 +5
- 1, 3, 6, 10, 15 … in the third row and third column
- the basic additive pattern for labeling intersections that was discovered in the previous lesson
- a line of symmetry along the diagonal with intersections having 2, 6, 20, 70, 252, and 924 shortest routes. That is, a point three blocks east and five blocks north has the same number of shortest routes as a point five blocks east and three blocks north of **H**.

Point to the intersection seven blocks east and four blocks north of **H**.

T: ***Where would 330 occur again if we extended the grid; i.e., where is the other corner with 330 shortest routes from* H*?***

S: ***Four blocks east and seven blocks north.***

Repeat this question a couple times with another point off the diagonal and with one on the diagonal to emphasize the last pattern noted above.

Exercise 2 ______________

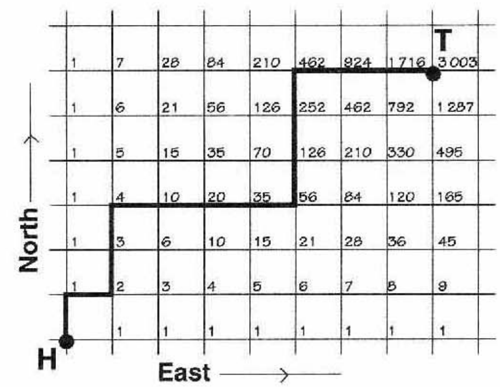
Write this code word on the board. **1 0 1 1 0 0 0 0 1 1 1 0 0 0**

T: ***Spike travels from his house at* H *to the treasure at* T *many times. He decides to write a secret code in his notebook for each route he takes. One day Spike drives from* H *to* T *and writes* 10110000111000 *in his notebook.***

Can anyone guess Spike's secret rule for writing code words? If you know the secret, keep it to yourself for the moment. Which route from* H *to* T *do you think Spike travels when he writes this code word?

Ask a student to trace a route but not to explain the rule yet. As soon as the student deviates from the correct path, let another student try. Spike is using two numbers, **1** and **0**, for the two directions, north and east, that he travels to go from **H** to **T**. Some students may suspect this coding but not know which number is for which direction. Spike uses **1** to indicate that he goes one block north and **0** to indicate that he goes one block east.

The correct route is shown here.



Continue until a student traces the correct route. You may need to help the class finds this route. If necessary, trace the first few blocks of the route as you read the corresponding numbers of the code word. Then ask a student to complete the route.

Repeat this exercise by writing these code words on the board and asking a student to trace the route for each code word.

1 1 0 0 0 0 1 0 1 0 1 1 0 0
0 0 0 1 0 0 0 1 0 1 0 1 1 1

As you slowly trace a route from **H** to **T**, invite a student to write the appropriate code word. Check the student's answer. Repeat the activity one or two more times.

T: ***Who can explain the code?***

S: ***Each* 1 *in the code word means to go one block north. Each* 0 *means to go one block east.***

T: ***Who can write a code word for another route from* H *to* T*?***

Let a student write a code word on the board, for example, **11000110010001**.

T: ***Do you think that this code word represents a route from* H *to* T*? Who can check it by tracing the route?***

Let another student start at **H** and trace the route indicated by the code word. Check whether the route ends at **T**. Repeat this activity one or two more times.

T: ***How can we check whether a code word represents a route from* H *to* T*?***

S: ***Trace the route.***

T: ***Can we check it without tracing the route?***

S: ***Yes, each code word must have 14 digits.***

S: ***There must be exactly six* 1*s and eight* 0*s in each code word, because* T *is six blocks north and eight blocks east of* H.**

Write these code words on the board.

1 0 1 0 1 0 1 0 1 0 1 0
0 0 0 0 1 1 1 1 0 0 0 0 1 1
1 1 0 1 1 0 1 1 0 1 1 0 1 0

T: ***Are each of these code words for a route from* H *to* T*?***

S: ***The first code word has only 12 digits. It needs two more* 0*s.***

T: ***The middle code word is for a route from* H *to* T *because it has six* 1*s and eight* 0*s.***

S: ***The last code word is not for a route from* H *to* T *because it has nine* 1*s and five* 0*s.***

T: ***If we traced the route indicated by the last code word, where would we finish?***

S: ***Nine blocks north and five blocks east of* H*; it's off our grid.***

T: ***How many code words could we write with six* 1*s and eight* 0*s?***

If necessary, give hints to lead to the following response.

S: ***3 003; because there is one code word for each route from* H *to* T*, and we know that there are 3 003 routes from* H *to* T.**

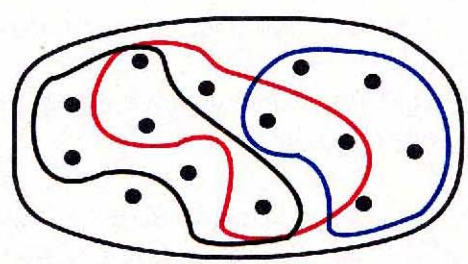
Students should observe that each code word with six **1**s and eight **0**s describes a different route from **H** to **T**. Also, for each route from **H** to **T**, there is exactly one such code word. There are 3 003 code words with exactly six **1**s and eight **0**s.

Exercise 3

On the board, draw a string with 14 dots inside it.

T: ***Spike learns that exactly six thieves stole the diamond medallion. He has 14 suspects and is sure that all six thieves are among his suspects. He feels that they will confess if he can interview all six thieves together. So he decides to interview the 14 suspects in groups of six.***

Draw a red string around six of the dots in the string picture. Then, invite students to draw strings around two different sets of six suspects. For example:

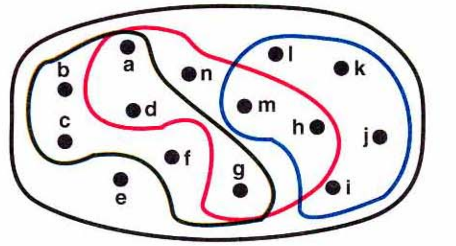


T: ***How many groups of six do you think there are for Spike to interview?***

Write students' estimates on the board. If many estimates are less than 10, let students draw a few more strings for groups of six suspects in order to suggest that there are many possible groups.

T: ***Spike decides to write a code word in his notebook for each group of six suspects that he interviews.***

First, label the dots with letters. Then write the code word for the red string on the board. Write a **1** for any dot inside the red string and a **0** for any dot outside the red string. Do not describe Spike's rule for writing code words yet; just write the word and let the class discover the rule.



T: ***Spike writes this code word in his notebook when he interviews the six suspects in the red string. Does anyone think they know his secret rule for writing code words? If you know his secret, keep it to yourself for the moment. On a piece of paper, write what you think the code word would be for the group of six suspects in the blue string.***

a	b	c	d	e	f	g	h	i	j	k	l	m	n
1	0	0	1	0	0	1	1	0	0	0	0	1	1

Check students' papers before letting a student write the code word for the blue string on the board. Continue by asking for the code word for the green string.

a	b	c	d	e	f	g	h	i	j	k	l	m	n
1	0	0	1	0	0	1	1	0	0	0	0	1	1
0	0	0	0	0	0	0	1	1	1	1	1	1	0
1	1	1	1	0	1	1	0	0	0	0	0	0	0

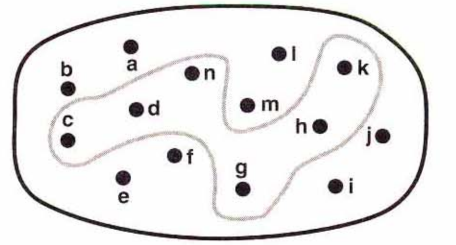
T: ***Who can write a code word for another group of six suspects?***

A student might suggest this:

a	b	c	d	e	f	g	h	i	j	k	l	m	n
0	0	1	1	0	0	1	1	0	0	1	0	0	1

T: ***Is this a correct code word for another group of six suspects?*** (Yes)

Invite a student to draw the string for the group. The string should include exactly the six dots for the six suspects marked with a **1**. This illustration shows a correct string for the preceding example.



T: ***Who can explain Spike's secret code?***

S: ***A* 1 *means that the suspect will be interviewed in this group of six.***
A* 0 *means that the suspect will not be interviewed with this group.

T: ***Without drawing a string, how can we tell if a code word represents a group of six suspects to be interviewed?***

S: ***The code word must have fourteen digits: six* 1*s and eight* 0*s.***

Erase all of the strings inside the original picture, leaving just the string with 14 dots inside the large white (black) string.

Invite a student to write another code word for a group of six suspects, and then ask another student to draw the corresponding string. Then reverse the problem by inviting one student to draw a string for six suspects and then asking another student to write the corresponding code word.

T: ***How many groups of six are there among the 14 suspects?***

S: ***3 003; since we know from the previous problem that there are 3 003 code words with exactly six* 1*s and eight* 0*s.***

S: ***Each route from* H *to* T *on Spike's map corresponds to a code word with six* 1*s and eight* 0*s. Each string for six suspects corresponds to a similar code word with six* 1*s and eight* 0*s. Since there are 3 003 routes from* H *to* T*, there are 3 003 groups of six suspects.***

S: ***The two problems have the same solution.***

Compare the answer (3 003) to the students' estimates.

Encourage students to read the storybook *The Hidden Treasure* on their own. The story reviews the two lessons about Spike, except that in the storybook the map of Spike's town is smaller (**T** is only six blocks east and four blocks north of **H**) and there are six thieves among only ten suspects. Thus, in the storybook there are 210 routes from **H** to **T** and there are 210 groups of six thieves among ten suspects. As the class reads the story, encourage students to comment on the similarities and the differences between the problem solved in class and the problem in the storybook.

Writing Activity

Suggest that students write about how the solution of one problem also solved a second problem in the storybook *The Hidden Treasure*.

ELECTION IN THE NUMBER WORLD

Capsule Lesson Summary

Introduce a relationship between the fractional and the decimal names for numbers through reading the storybook *Election in the Number World*.

Description of Lesson

Distribute copies of the storybook *Election in the Number World*.

If your students have never seen numbers written in the binary (base two) system, you may like to introduce it briefly. In the storybook the binary system is only mentioned on a couple pages.

Note: In binary the only digits used are **0** and **1**. The position values are powers of 2.

…	1	1	0	0	1	0	•					…
	$32 = 2^5$	$16 = 2^4$	$8 = 2^3$	$4 = 2^2$	$2 = 2^1$	$1 = 2^0$		$\frac{1}{2} = 2^{-1}$	$\frac{1}{4} = 2^{-2}$	$\frac{1}{8} = 2^{-3}$	$\frac{1}{16} = 2^{-4}$	

For example, the decimal (base ten) number 50 would be written **110010** in binary (base two).

Pages 1 and 2

Read pages 1 and 2, and write the following information on the board.

Fractional writing	Positional writing
$1\frac{2}{5} + 3\frac{1}{2}$	$1.4 + 3.5$

T: ***On page 2, Nabu is considering the positional way of writing numbers and the fractional way. I have written two calculations on the board, one with fractions and one with decimal writing. Which calculation is easier to do?***

Even without doing the calculations, students should agree that adding with the decimal writing of numbers is often easier than adding with fractions.

T: ***One advantage of a positional notation is that it is usually easier to add numbers with it.***

Pages 3–7

Read pages 3 through 7 together.

T: ***Look at the arrow road on page 7. What could the blue arrows be for?***

S: ***10x, because 10 x 0.1 = 1; 10 x 1 = 10; 10 x 10 = 100; and so on.***

T: ***Find 1 000 on the arrow road. Let's start at 1 000 and read the numbers in order as we follow blue arrows.***

Giving help when needed, invite students to read the numbers in order:

ten thousand, one hundred thousand;
one million (1,000,000), ten million, one hundred million;
one billion (1,000,000,000), ten billion, one hundred billion;
one trillion (1,000,000,000,000), ten trillion, one hundred trillion;
one quadrillion (1,000,000,000,000,000), and ten quadrillion.

Pages 8 and 9

Read page 8. Then write these expressions on the board.

decimal ________

binary ________
(or base two)

fractions ________

T: ***The numbers are going to vote on which method of writing numbers they prefer. Let's take a vote in this class. If we were planning to use only one of these ways of writing numbers, which would you vote for? Why?***

After a class discussion, take a class vote. Point out that the results from the numbers voting are on page 9. Ask students to read page 9.

T: ***Which system received the most votes?***

S: ***Decimal writing.***

T: ***Who can explain these results: 95%, 4.7%; and 0.3%?***

Encourage a discussion of the voting results. For example, the class may realize that in this situation 95% means 95 out of every 100 votes were for decimal writing.

Pages 10–21

As you and the class read pages 10 through 21, stop briefly to solve each problem by asking students to point to the answer in their storybooks.

Pages 22–31

Read pages 22 through 31 collectively. Briefly discuss the ideas as you wish. Students may like to find other fractions that behave like ⅓ and ⅐ when they try to write decimal names.

A VERY STRANGE NEIGHBORHOOD

Capsule Lesson Summary

While reading the storybook *A Very Strange Neighborhood*, explore the ordering and density of decimal numbers. Through a detective story about a lost number sending radio signals, introduce a new kind of number, an *irrational* number.[†]

Description of Lesson

Begin with a discussion of different kinds of numbers, asking students to describe or give examples. Students should mention whole numbers, negative integers, and rational numbers. Write students' examples on the board; for example:

$$17 \qquad {}^{-}82 \qquad \frac{5}{7} \qquad 8.2$$

If no suggestion uses a repeating decimal name such as $0.\overset{*}{3}$ or $5.1\overset{*}{\overline{462}}$,[††] ask,

T: ***Do you remember a name for ⅓ that we said was a kind of decimal "nickname"?***

S: ***0.333 … or $0.\overset{*}{3}$.***

Write this expression on the board.

$$5.1\overset{*}{\overline{462}}$$

T: ***What does this notation mean?***

S: ***The digits 4, 6, and 2 repeat. We could write 5.1462462462 ….***

T: ***You already know many different kinds of numbers. Today we will read a story about a new kind of number.***

Distribute copies of the storybook *A Very Strange Neighborhood*. Read the story with your class. Questions based on the mathematical ideas are suggested below. Use the questions carefully to enhance, not diminish, the spirit of the story. By keeping the discussions brief, you should finish the storybook in a reasonable time period. At a later time, you or your students may wish to discuss further some of the ideas.

Pages 1–4

Read pages 1 to 4 collectively.

T: ***Are there other numbers between 7 and 8?***

S: ***7.1, 7.2, 7.4, 7.6, and 7.9.***

[†] For your information, rational numbers are numbers which have fractional names; for example, ⅓, 2 (²⁄₁), and 5.02 (⁵⁰²⁄₁₀₀) are rational numbers. Irrational numbers do not have fractional names although they can be found on the (real) number line.

[††] This $*$-notation for repeating decimals was introduced in *Election in the Number World*. Other notations are common, and you may prefer to use a different way of indicating the repeating pattern for such decimals; for example, 0.333 … = $0.\overline{3}$.

Ask students to point to the unlabeled mark for each of these numbers on page 4.

T: ***Any others?***

S: ***7.06, 7.29, and 7.84.***

S: ***$7\frac{2}{3}$.***

T: ***Are there numbers between 7.7 and 7.8?***

S: ***Yes, 7.74 and 7.79.***

S: ***7.715.***

S: ***$7\frac{3}{4}$.***

Pages 5 and 6

Read pages 5 and 6 together.

T: ***On page 6 of your storybook, point to the mark on the number line for 7.73.***

Page 7

Read page 7 together.

T: ***Are there exactly nine numbers between 7.7 and 7.8?***

S: ***No; there are many more, for example, 7.727 and 7.70622.***

S: ***There are infinitely many numbers between 7.7 and 7.8.***

Page 8

Read page 8 together. Point out that the number line has been magnified again. Now only the numbers from 7.7 to 7.8 are shown.

Page 9

Read page 9 together.

T: ***Marks for many of the numbers between 7.7 and 7.8 are shown on this number line, but they are not labeled. What are some of the numbers at these marks?***

S: ***7.732.***

T: ***Where is 7.732 on the number line?***

S: ***Between 7.73 and 7.74; the second mark above 7.73.***

Let students point out other numbers between 7.7 and 7.8.

Pages 10 and 11

Read pages 10 and 11 together.

T: ***Does the story ever end?***

S: ***No, you can always find more numbers between 7 and 8.***

Pages 12–14

Read pages 12 to 14 collectively.

T: ***Look at the spiral on page 14 and try to figure out its rule. What would be the next number if we were to continue the spiral?***

S: ***7.888888.***

T: ***What numbers would come after 7.888888?***

S: ***Then 7.8888882, 7.8888884, 7.8888886, and so on.***

T: ***Can you explain the rule?***

There are several correct explanations, but "add 2 to the last digit" is not correct.

S: ***Start at 7. Add 0.2 four times; then add 0.02 four times; and so on.***

S: ***Add 2 to the last digit each time, except when the last digit is 8. Then add another decimal place by putting 2 at the end.***

Page 15

Read page 15 together.

T: ***What is the rule in the upper part of this snake dance?***

S: ***Put another 4 on the end of a number to get the next number.***

T: ***What is the rule in the lower part?***

S: ***Take off a 6 from the end of a number to get the next number.***

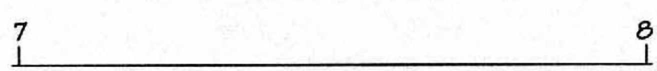
T: ***Is there a smallest number in this snake dance?***

S: ***Yes, 7.4.***

T: ***Is there a largest number?***

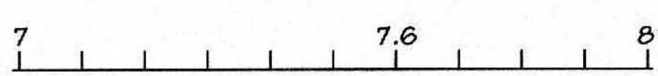
S: ***No. Although 7.666666666666 is the largest number shown on the page, the dance goes on.***

Draw this part of a number line on the board. A convenient length for the segment from 7 to 8 would be one meter.



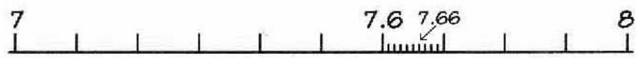
T: ***Who can point to where 7.6 is on this number line?***

Suggest that students use a meter stick to divide the segment from 7 to 8 into ten pieces of equal length.



T: ***Where is 7.66 on this number line?***

Again, suggest that students divide the segment from 7.6 to 7.7 into ten pieces of equal length.



T: ***Where is 7.666?***

S: ***Just to the right of 7.66.***

S: ***Between 7.66 and 7.67.***

T: ***Where is 7.6666?***

S: ***Between 7.666 and 7.667.***

T: ***These numbers in the snake dance, 7.6, 7.66, 7.666, and so on, are getting closer and closer to some number. That number has a fractional name. What number is it?***

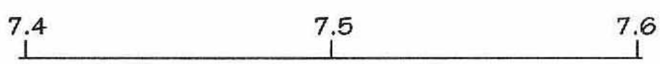
S: ***$7\frac{2}{3}$, because 0.666 … = $0.\overset{*}{6} = \frac{2}{3}$.***

If no one suggests $7\frac{2}{3}$, remind the class that $0.\overset{*}{3}$ = 0.333 … = $\frac{1}{3}$, and ask about a fraction for $0.\overset{*}{6}$. Lead the class to observe that 7.666 … = $7.\overset{*}{6} = 7\frac{2}{3}$.

T: ***If we start at 7.6 on the spiral and follow the arrows backwards, we get closer and closer to $7\frac{2}{3}$, but we never reach $7\frac{2}{3}$.***

Page 16

Read page 16 together, and draw this part of a number line on the board.



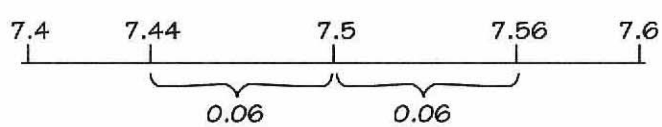
T: ***7.5 is exactly halfway between 7.4 and 7.6. Can you suggest two other numbers that 7.5 is exactly halfway between?***

S: ***7 and 8.***

S: ***7.3 and 7.7.***

S: ***7.44 and 7.56.***

Use the idea of distance from 7.5 to generate or confirm some answers; for example:



Depending on student interest, you may wish to challenge them to find pairs of numbers even closer to 7.5 which have 7.5 as their midpoint; for example, 7.49 and 7.51, 7.495 and 7.505, or 7.499 and 7.501.

Pages 17–20

Read pages 17 to 20 collectively.

T: ***So far in the story, you already know all of the numbers we have met. $7.\overset{*}{6}$ and 7.5000001 are somewhat unusual, but they are not new to you. Now we are going to meet a new kind of number.***

Pages 21 and 22

Read page 22.

T: ***This sad number is singing a strange song: "Bing, bang, bing, bing, bang; bing, bing, bing, bang; and so on." What number could be sending this message?***

Accept a few suggestions about what the sounds could mean.

Pages 23–26

Read pages 23 to 26 collectively. Refer to the last number, 7.5155155515555, on the red arrow road.

T: ***What would be the next number on the red arrow road?***

S: ***7.51551555155551.***

T: ***How can we get the next number?***

S: ***Put five 5s on the end of the new number.***

T: ***How can we get the next numbers?***

S: ***Put another 1 at the end.***

S: ***Then put six 5s.***

T: ***Who can explain the pattern?***

S: ***You alternate between putting on a 1 and putting on a group of 5s. Each time you put on 5s, you put on one more than the last time.***

T: ***And we could keep on going. Could we use the ∗-notation to write a shorthand name for the number we are heading towards?***

After a few attempts, students should conclude that it is impossible since no one part of the number is repeated; that is, one more 5 is used each time. Students might suggest some more elaborate abbreviations, such as $7.5\overline{155}^{+5}$. Accept these suggestions as possible notations, but mention that there is no standard notation for this type of number and that it would be impractical to invent a new notation for each non-repeating pattern we could imagine.

T: ***Now, can anyone explain the "bing-bang" song?***

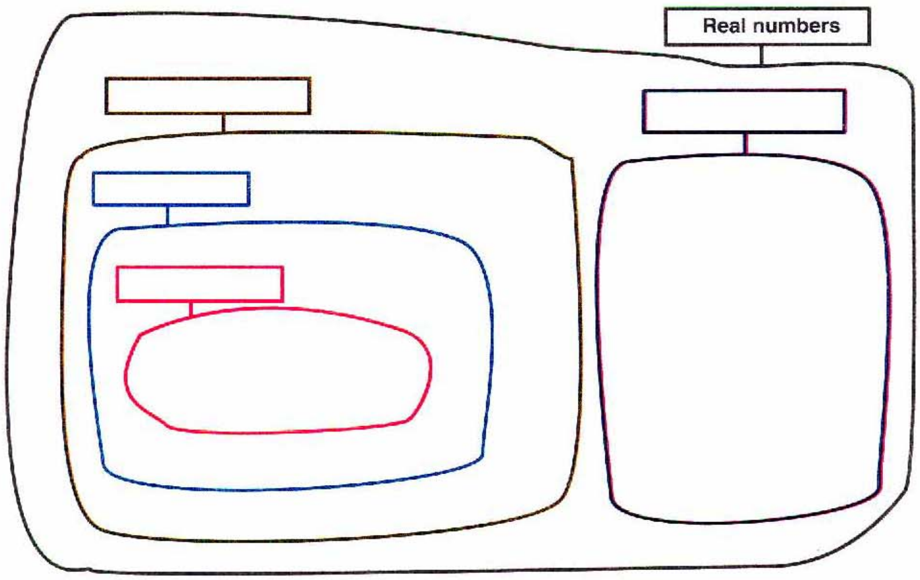
S: ***The number is trying to signal its name. Each bing is for a 5 and each bang is for a 1.***

Pages 27–30

Complete the story by reading pages 27 to 30 collectively. Let students comment on the story.

Optional Activity

Draw the following string picture on the board.



T (pointing to the black string)**:** ***All of the numbers you know about are called*** **real numbers.** ***When you were very young, you first learned about numbers like 0, 1, 2, 10, and 100. These are called*** **whole numbers.**

Label the red string **Whole numbers**, and put a few examples of whole numbers inside the red string.

T: ***Later on you learned about negative numbers like $^{-}1$, and $^{-}42$. When we combine these negative numbers and the whole numbers, we get a set of numbers called*** **integers.**

Label the blue string **Integers**, and put some examples of negative integers inside the blue string but outside the red string.

T: ***You next learned about fractions, decimal numbers like 6.02 and $^{-}8.5$, and a few unusual numbers like $7.\overline{6}$ and $5.1\overline{462}$. These numbers, along with the integers, are called*** **rational numbers.**

Label the green string **Rational numbers**, and put some examples of non-integer rational numbers inside the green string but outside the blue string. (See the next illustration.)

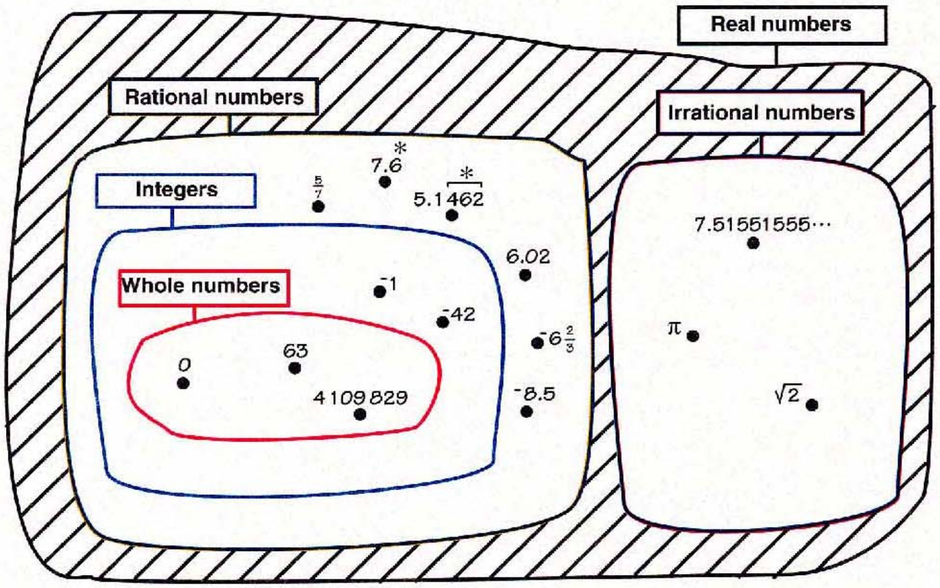
T (pointing to the purple string)**:** ***The number we met today, 7.515515551…, is an example of another kind of real number which is not a rational number. When we try to use decimal writing for this kind of number there is no end and no pattern that repeats. Numbers of this kind are called*** **irrational numbers.**

Label the purple string **Irrational numbers**, and put 7.515515551… inside the purple string.

T: ***There are infinitely many irrational numbers.***

If your students are familiar with some other irrational numbers, such as π or $\sqrt{2}$, put them in the picture, as well.

Hatch the indicated region of the string picture.



T (pointing to the hatching)**:** ***All real numbers are either rational numbers or irrational numbers. There are still numbers that are not real numbers,† however, you will not learn about these numbers for some time.***

†The *complex* numbers are one example of a set of numbers that contains numbers that are not real numbers.

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