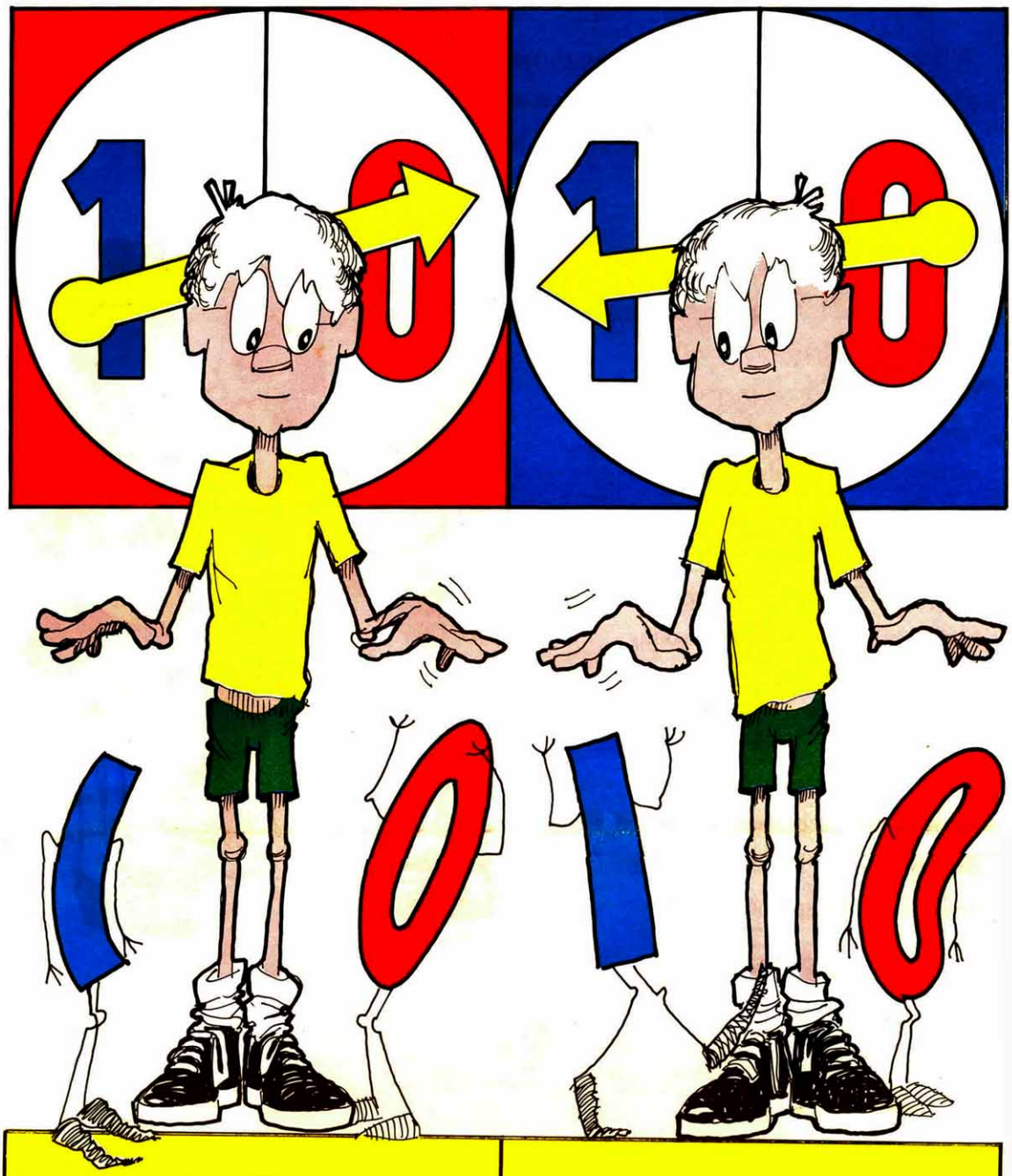


THE SQUARE TRAP

by Frederique

THE SQUARE TRAP

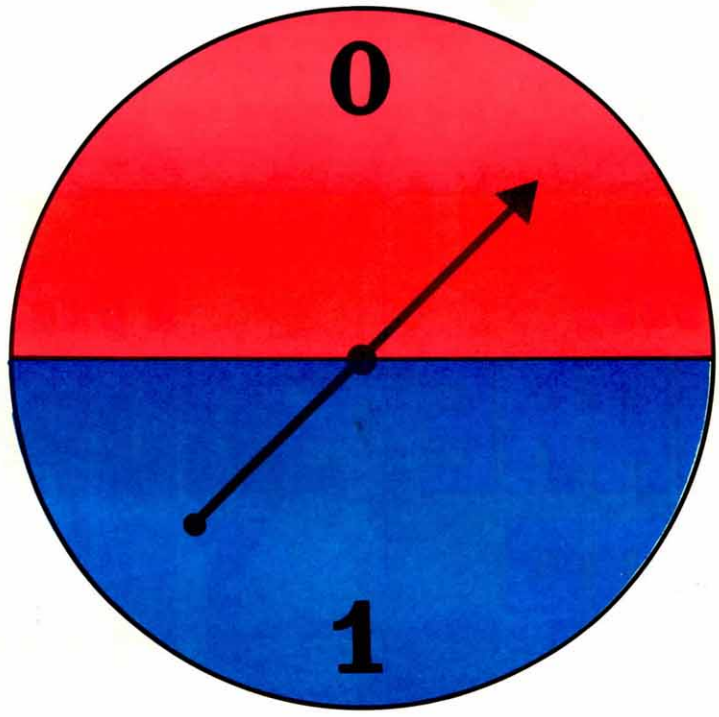
Pictures/Design
Robert Hunter
Vivian Benson

14-6385

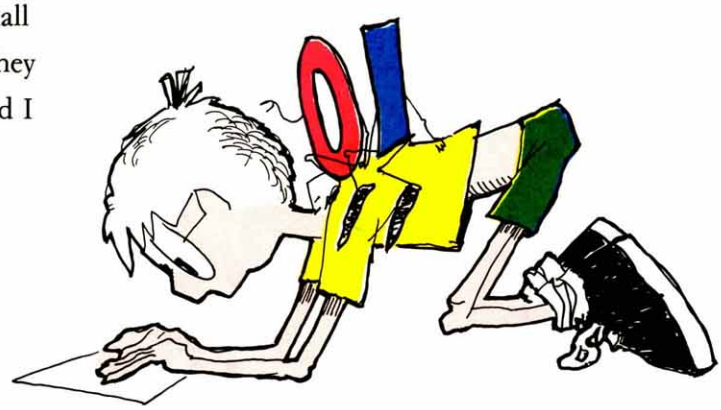
When I got home from school at the end of the afternoon, my grandmother and her little black poodle were waiting for me at the front door of our house.

"Your parents are very busy just now," she said. "They are working together and don't want to be disturbed. So I have prepared a new game for you that you can play with your friends, the numbers 0 and 1." Grandma gave me a mysterious box wrapped in colorful gift paper. I kissed her warmly, promised to play without making any noise, and called my friends.

0 and 1 came over to play right away. In my excitement I dropped the box twice. I finally opened it and found this spinner inside.



"What a marvelous toy!" said my friends happily. "With this spinner bearing both our names, we shall invent a lot of new games." They jumped onto my shoulders, and I began to spin the spinner. I stopped after 50 plays.



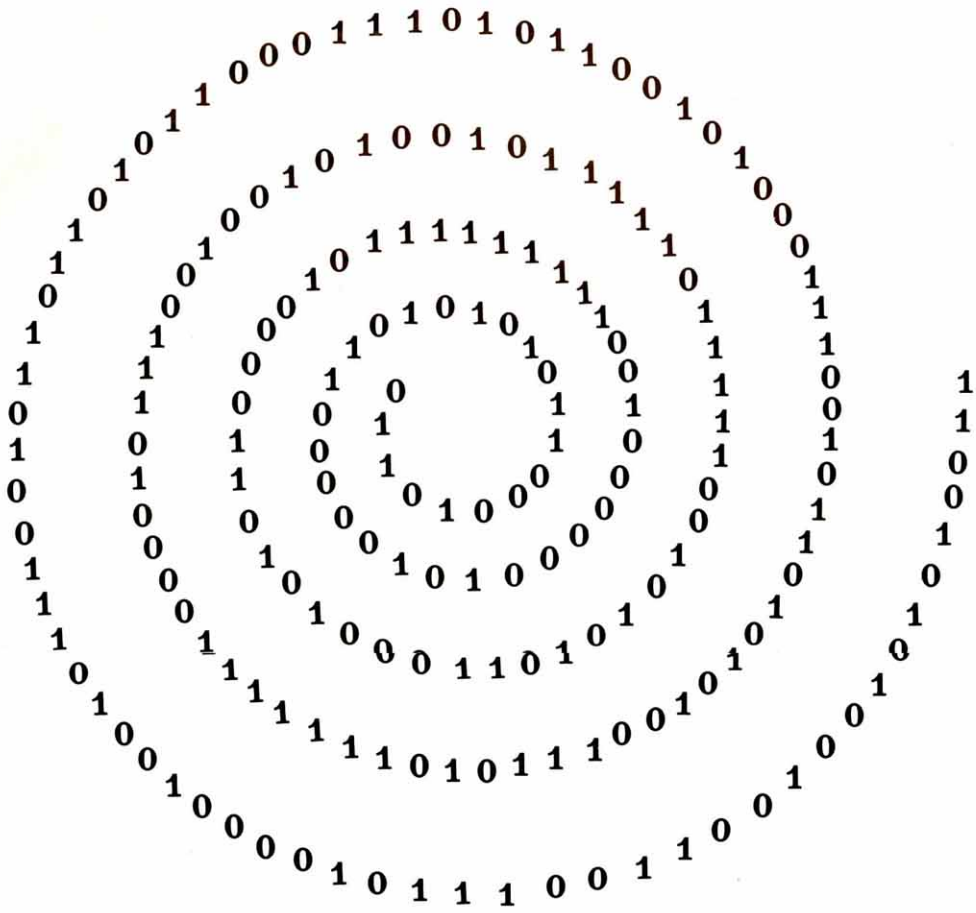
I had made this long string of digits.

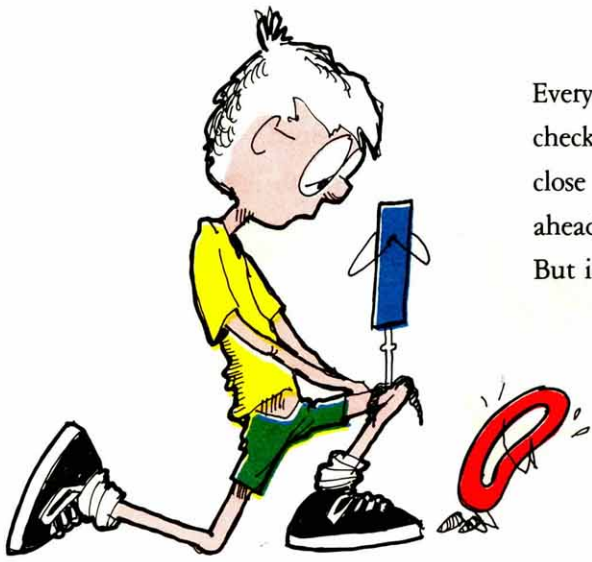
1 0 1 1 0 0 0 0 1 0 0 1 0 1 1 0 0 1 0 0 0 1 0 1 1 0 0 1 0 1 0 0 0 1 1 0 1 0 0 1 0 1 1 0 0 0 1 0 0 1 1 0 1 0 0 0

1 looked very disappointed, but 0 clapped. "I am the winner by 30 to 20," said 0 proudly.

"Please spin again," begged 1. "Perhaps I'll be luckier this time."

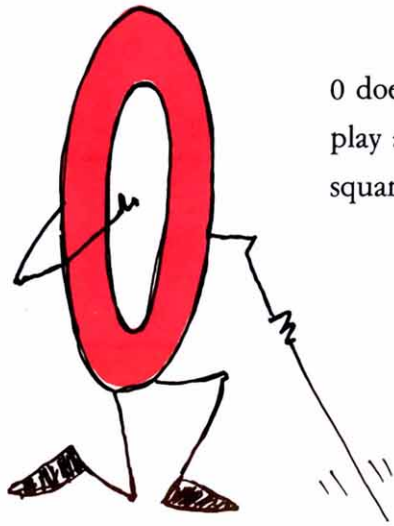
To please my friend, I spun 200 times in a row and got this interesting spiral of digits.



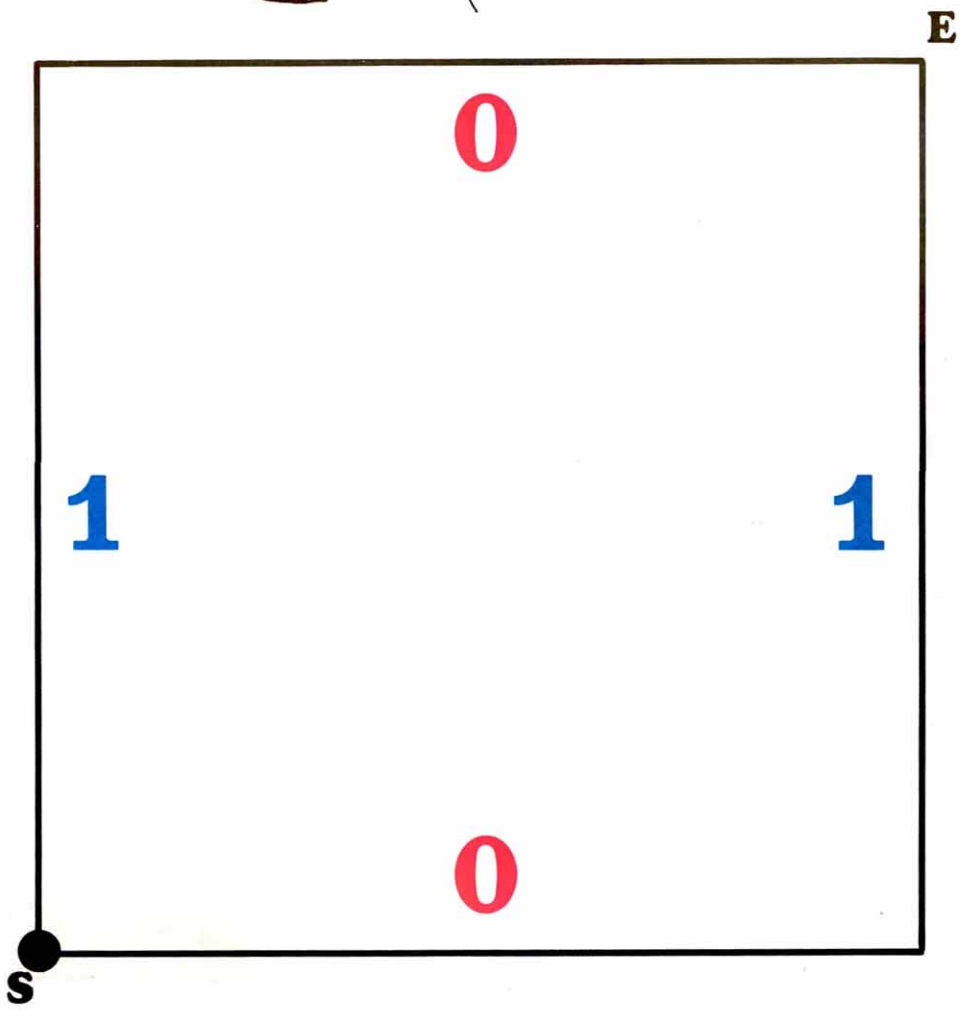


Every twentieth spin, 0 and 1 checked their results. It was a very close game. At the beginning, 0 was ahead. Halfway through, it was even. But in the end, 1 won by 103 to 97.

	For 0	For 1		For 0	For 1
From 0 to 20	10	10	After 20 Spins	10	10
From 21 to 40	13	7	After 40 Spins	23	17
From 41 to 60	10	10	After 60 Spins	33	27
From 61 to 80	6	14	After 80 Spins	39	41
From 81 to 100	11	9	After 100 Spins	50	50
From 101 to 120	7	13	After 120 Spins	57	63
From 121 to 140	11	9	After 140 Spins	68	72
From 141 to 160	8	12	After 160 Spins	76	84
From 161 to 180	11	9	After 180 Spins	87	93
From 181 to 200	10	10	After 200 Spins	97	103

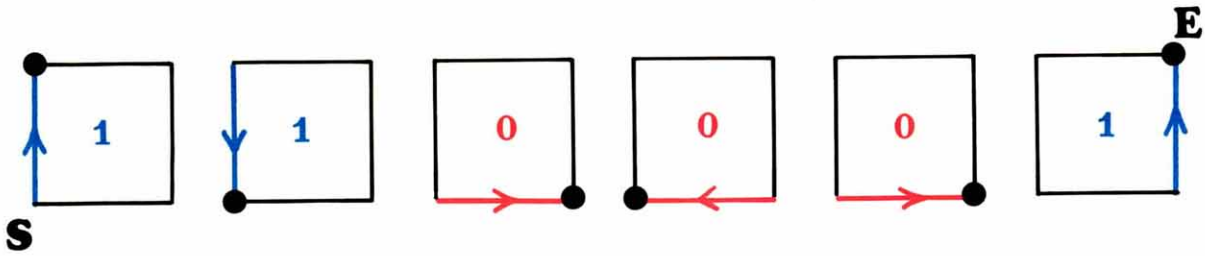


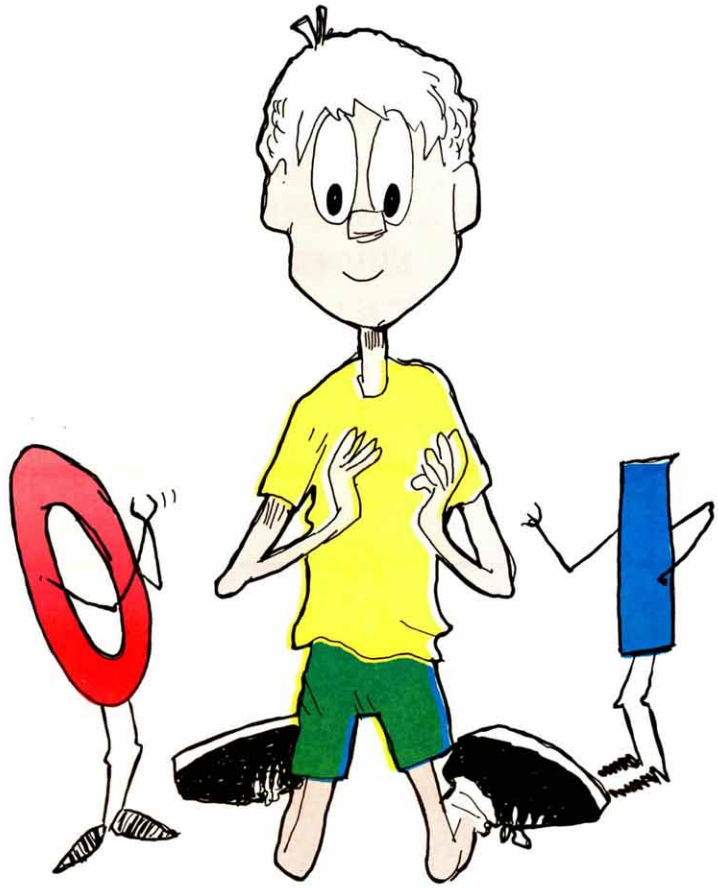
0 doesn't like losing and suggested we play a different game. 0 drew this square and put a checker at **S**.



0 explained the rules of the game. "Each time you spin **0**, you move the checker to a neighboring corner along a side marked **0**. Each time you spin **1**, you move it along one of the sides marked **1**. The game is over when the checker is trapped at **E**."

I spun and got the string **110001**.
The checker was trapped after six
moves, as you can see in this picture.



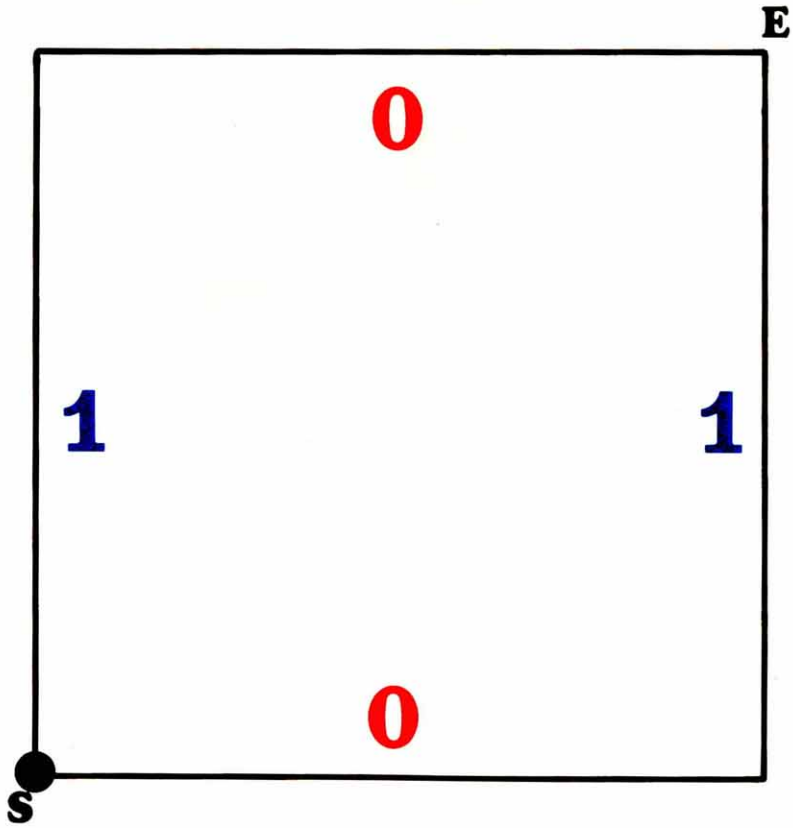


"Let's play again," suggested my
friends, getting very excited. I spun
again and got **00001110**. The checker was
trapped after eight moves.

WITH YOUR FINGER, FOLLOW THESE EIGHT MOVES ON
THE LARGE SQUARE ON PAGE 5.

BEFORE YOU CONTINUE READING THE STORY, WOULD YOU LIKE TO PLAY WITH US FOR A WHILE?

IF YOU HAVE NO SPINNER, YOU CAN USE A COIN AND MARK **0** ON ONE SIDE AND **1** ON THE OTHER SIDE, OR YOU CAN ROLL A DIE. WHEN AN EVEN NUMBER (2, 4, or 6) SHOWS, PRETEND YOU HAVE SPUN **0**. WHEN AN ODD NUMBER (1, 3, or 5) SHOWS, PRETEND YOU HAVE SPUN **1**.



PLAY!

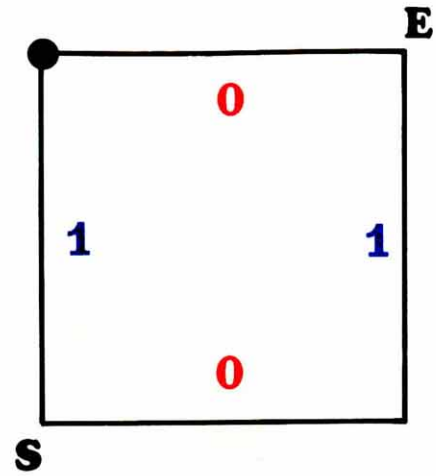
FOR EACH GAME, COUNT THE NUMBER OF MOVES YOU HAD TO MAKE BEFORE YOUR CHECKER WAS TRAPPED AND PUT A LITTLE MARK IN THE CHART BELOW. THEN PUT THE CHECKER BACK AT S AND PLAY AGAIN.

PLAY A LOT OF GAMES ONE AFTER ANOTHER. WHAT DO YOU NOTICE?

CHECKER TRAPPED AFTER

1 move	2 moves	3 moves	4 moves	5 moves	6 moves	7 moves	8 moves	9 moves	10 moves	more than 10 moves

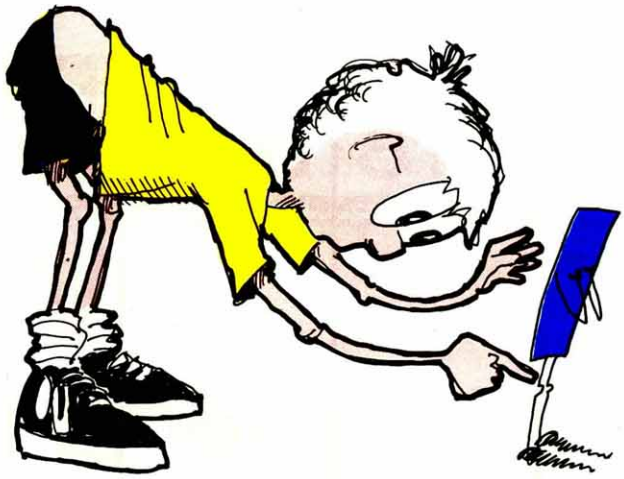
0 took the spinner, got **111100001**, and shouted, "My checker is trapped after nine moves."



I thought about that for a while.

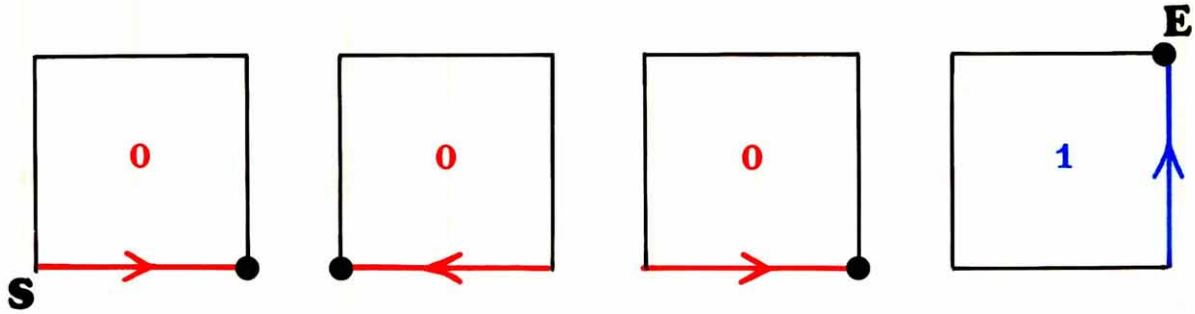
"That's impossible," I said. "After the first eight moves, your checker will be back at S again. After the last move, it will be here and the game won't be over."

"You are clever," observed 0. "I thought I would trick you."



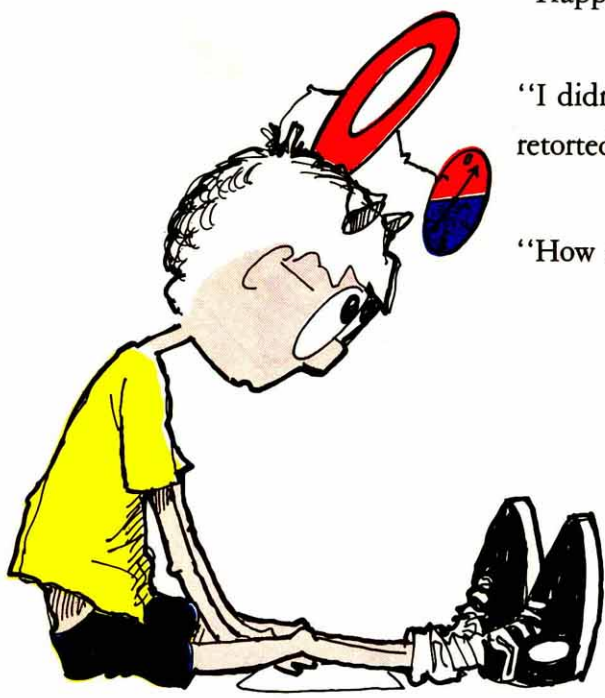
Then 1 took a turn to spin and got **000110**. "My checker is trapped after six moves."

"Yes," I said, "but your checker was already trapped after the first four moves, as you can see in this picture."



"You're right," agreed my friend.

0 took the spinner, jumped on top of my head, and played.



"Trapped after 13 moves," 0 said.

"I didn't see which string of digits you got," I retorted, "but in any case, you are wrong."

"How do you know that?" asked my friend.

"In order to trap the checker, you need exactly 2 or 4 or 6 or 8 or 10 or 12 or 14 or 16 or . . . moves. You need an even number of moves."

"It's easier to trap the checker than it is to trick you," observed 0.

1 hid under my chair.

"Trapped after ten moves," 1 whispered.

"That's possible," I said, "because ten is an even number. However, in order to check your answer, I would have to know at least the last two digits you got."

"0 and 1," answered my friend.

"O.K.," I said. "Then your checker is trapped."

"Yes, but it might be that 1's checker was already trapped after less than ten moves," added 0.

EXPLAIN WHY 0 IS RIGHT.

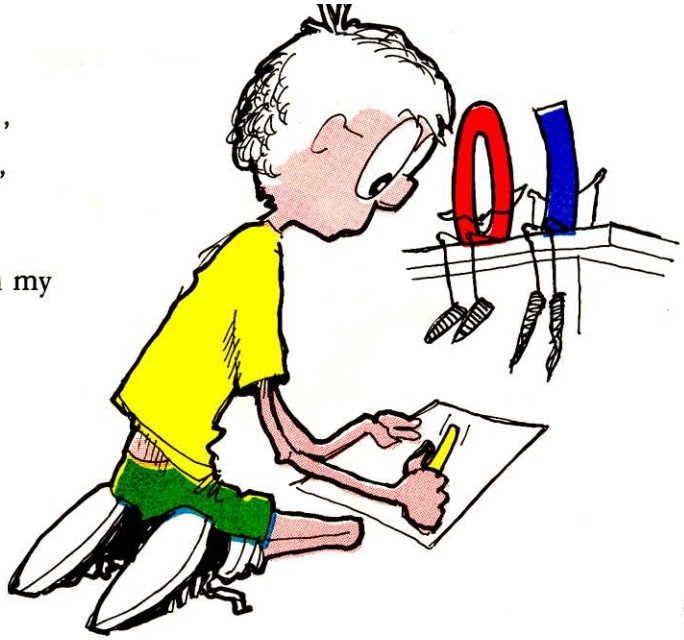
"Believe it or not," said 0 climbing down from my head, "but just now I had to spin 100 times before the checker was trapped."

I nodded. "That can happen. But it's very unusual."

0 spun and 1 moved the checker.

"Each time the checker is trapped," said 1, "I put it back at S again."

I wrote the results of their games in my notebook, writing from bottom to top.



game 1	game 2	game 3	game 4	game 5	game 6	game 7	game 8	game 9	game 10
1	1	1	0	0	1	0	0	1	1
0	1	1	1	0	0	0	1	1	1
	1	1	1	0	0	0	1	0	1
	0	1	1	0	0	1	0	1	1
		0	1	0	0	1	0	1	1
			0	1	0	1	0	0	1
				0	0	1	0	0	1
					0	1	0	0	1
						1	0	0	1
						0	0	0	1

"Your checker was trapped after 2, 4, 6, 2, 6, 2, 4, 2, 4, and 12 moves," I observed.

"I spun 44 times," said 0.

"And the checker was trapped 10 times," added 1.

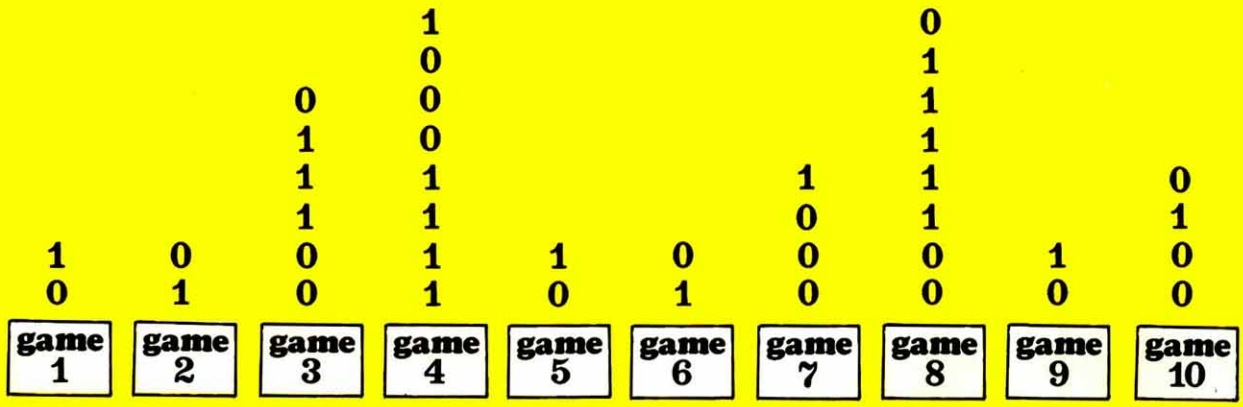
"Your average score is between 4 and 5," I said.

"Our average score is exactly 4.4," concluded 0, "because $10 \times 4.4 = 44$."

"Let's play again," suggested my friends.

"I spun 40 times and the checker was trapped 10 times as shown in this picture."

"Our new average score is exactly 4," concluded 0.



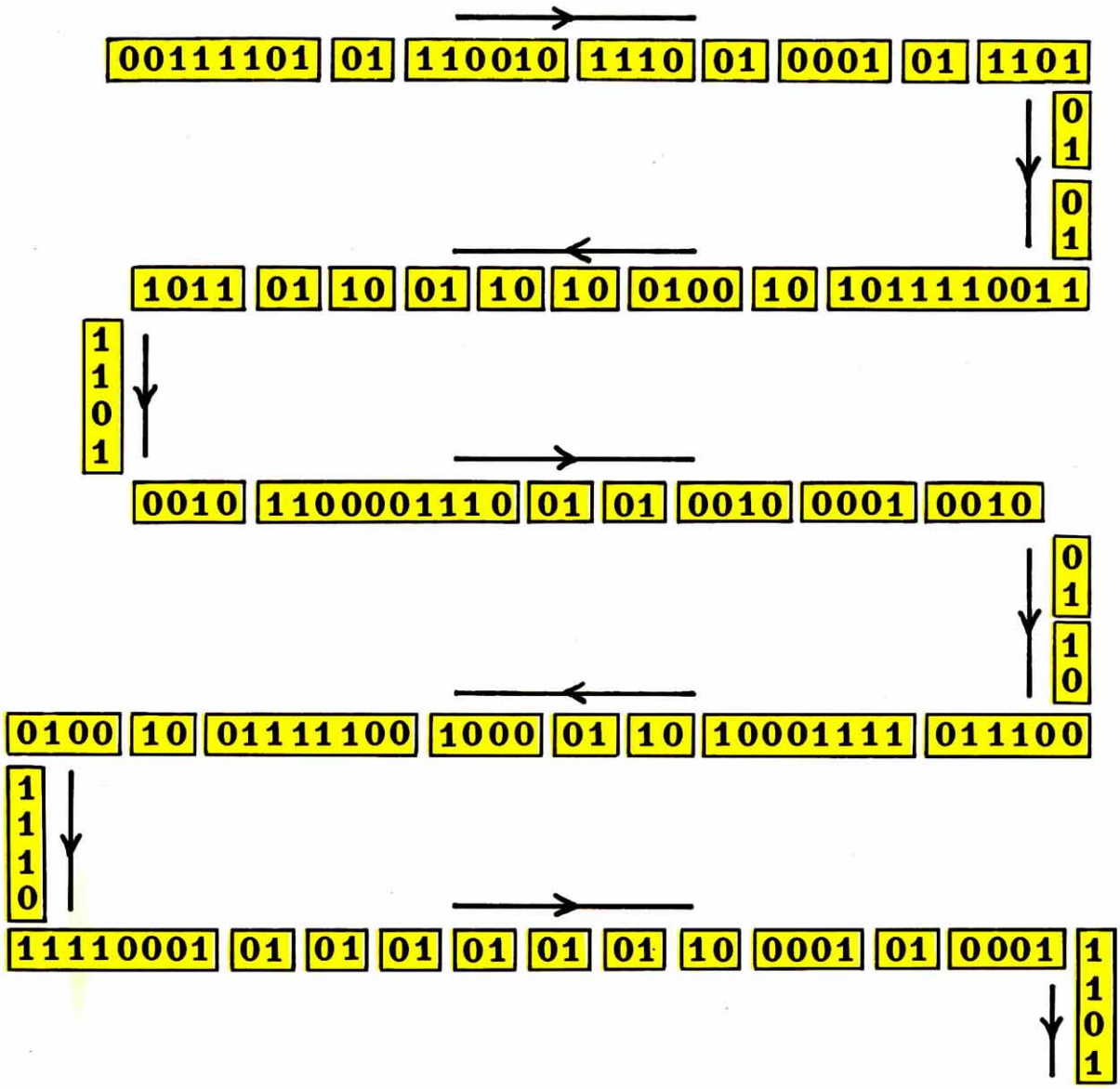
YOU PLAY TEN GAMES TOO, WRITING DOWN YOUR RESULTS.

WHAT IS YOUR AVERAGE SCORE?

COULD YOUR AVERAGE SCORE BE 2?

COULD YOUR AVERAGE SCORE BE 1.5?

My friends decided to play the game 50 times in a row. I checked their results on a large sheet of paper.



"In 26 of the games, the checker was trapped after two moves," counted 0.

"And 16 times after four moves," said 1.

"Two times after six moves, four times after eight moves, and two times after ten moves," I went on.

My friends were a little disappointed that ten was the largest number of moves we had taken.

"How many times did we spin altogether?" asked 0.

"That's easy to calculate." I wrote in my notebook:

$(26 \times 2) + (16 \times 4) + (2 \times 6) + (4 \times 8) + (2 \times 10) = 52 + 64 + 12 + 32 + 20 = 180$

"It took us 180 spins to play 50 games," I concluded.

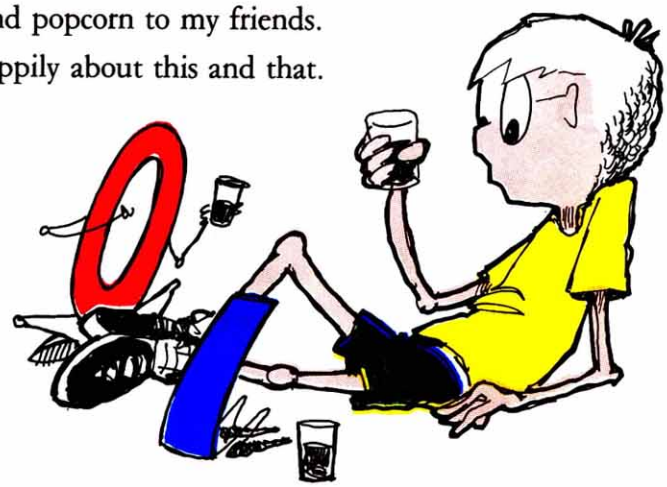
"What is our average score?" asked 1.

"$3 \times 50 = 150$; $4 \times 50 = 200$," calculated 0.

"Therefore, your average score is between 3 and 4," I observed.

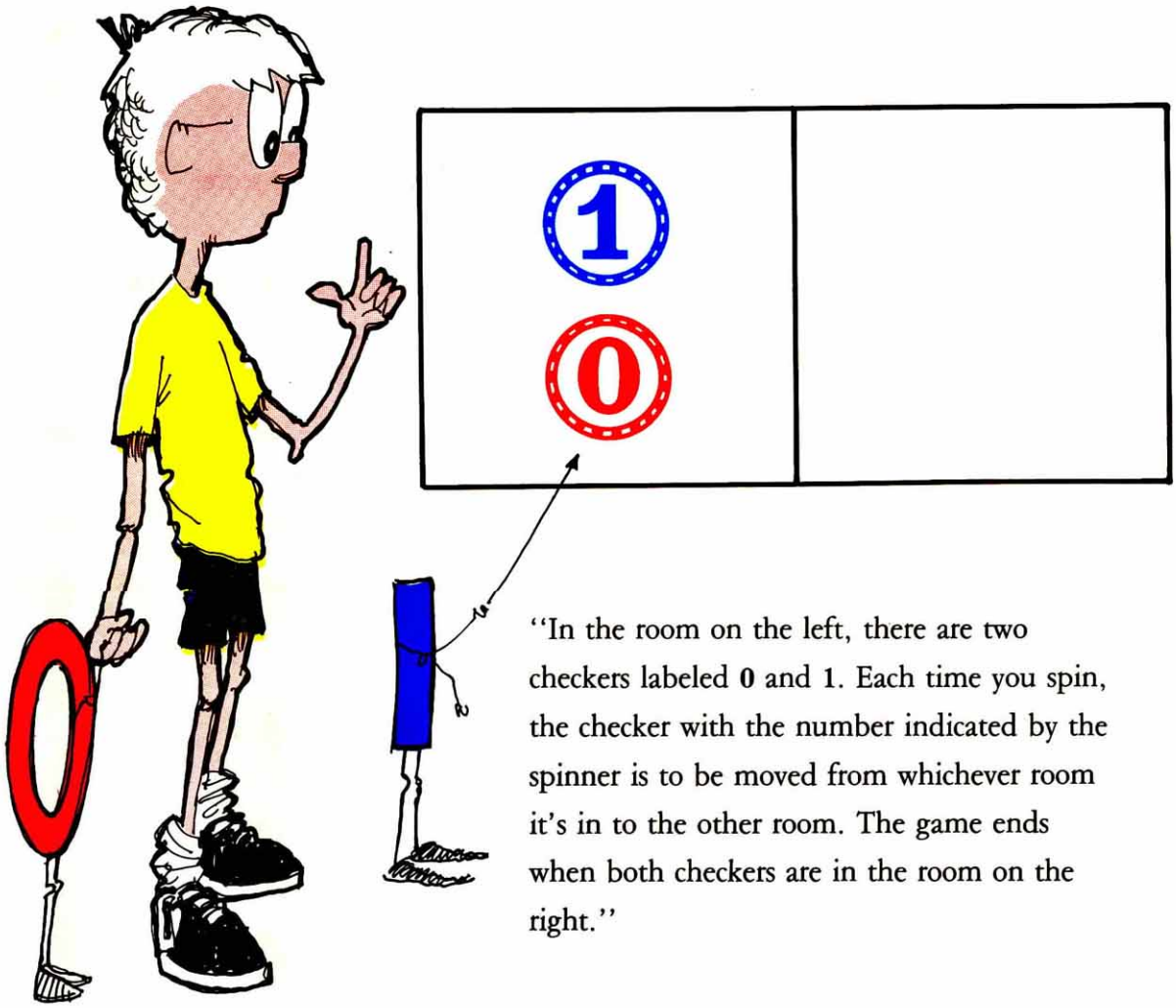
"It's about 3.5," said 1.

We relaxed for a while, and I offered orange juice and popcorn to my friends. We chatted happily about this and that.



IF YOU ENJOY THIS GAME, PLAY IT 50 TIMES IN A ROW AND COMPARE YOUR AVERAGE SCORE WITH OURS.

I drew this picture and said, "I have a new idea!"



"In the room on the left, there are two checkers labeled **0** and **1**. Each time you spin, the checker with the number indicated by the spinner is to be moved from whichever room it's in to the other room. The game ends when both checkers are in the room on the right."

"Let's start playing right away," I suggested. "I'll spin. 1, you move the checker with your name on it. 0, you move the checker with your name on it."

I spun and got **001110.**

0 moved the red checker.

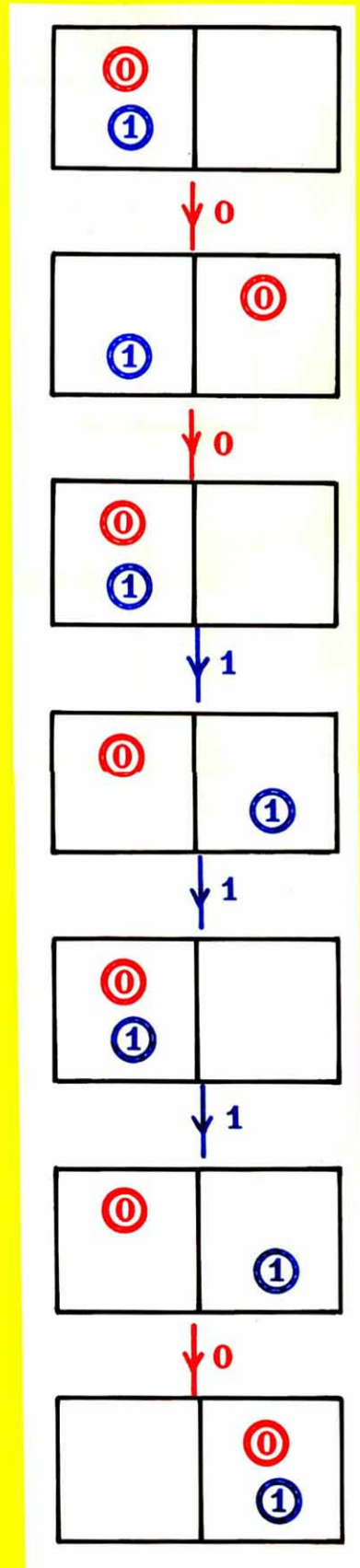
Again.

Now, it was 1's turn.

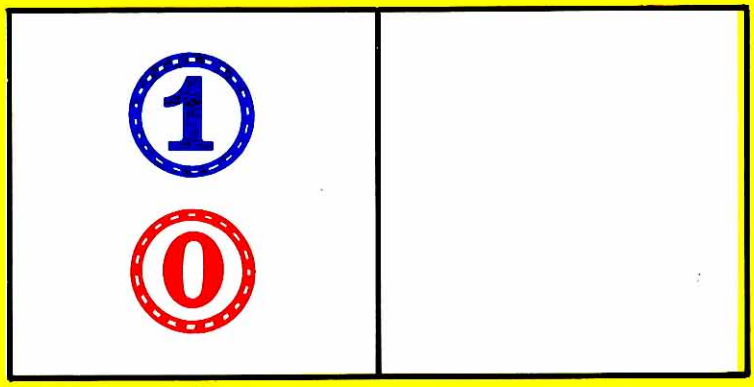
Again.

And again.

0 made the last move. Both checkers were in the room on the right. The game was over.



WOULD YOU LIKE TO PLAY OUR NEW GAME?
PUT THE TWO CHECKERS IN THE ROOM ON
THE LEFT.



SPIN AND MOVE THE CHECKERS.

AFTER HOW MANY SPINS DID YOUR GAME END?
INDICATE IT BY PUTTING A LITTLE MARK IN
THE CHART BELOW.

PLAY A LOT OF GAMES ONE AFTER ANOTHER.

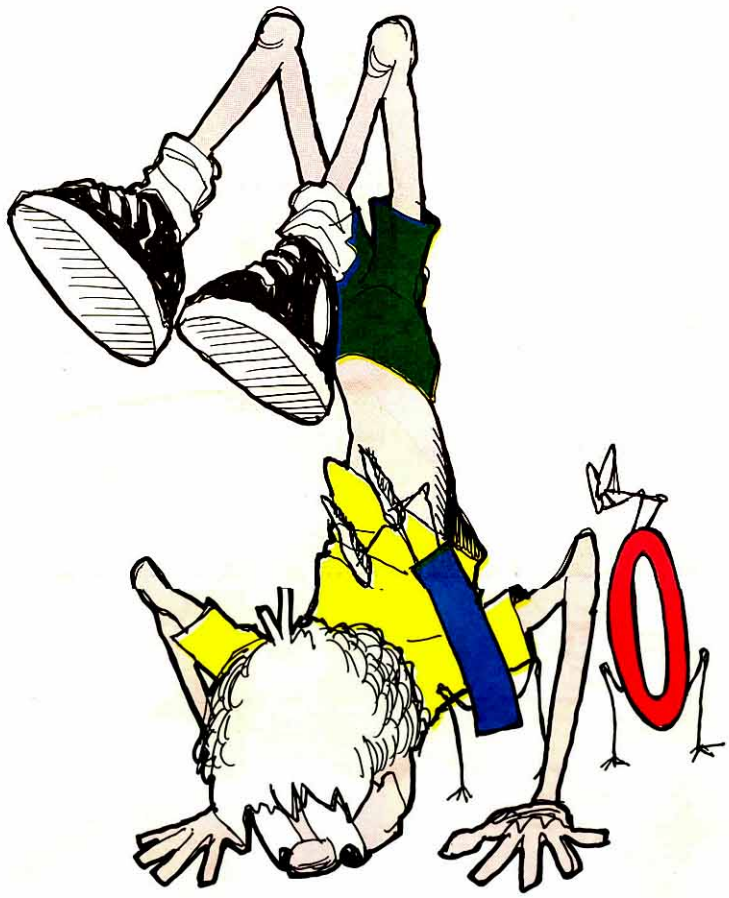
WHAT DO YOU NOTICE?

GAME ENDED AFTER

1 move	2 moves	3 moves	4 moves	5 moves	6 moves	7 moves	8 moves	9 moves	10 moves	more than 10 moves

TRY TO ANSWER THESE QUESTIONS:

1. I spun and got **11110.**
   Is my game over?
2. 0 spun and got **1100111101.**
   Is 0's game over?



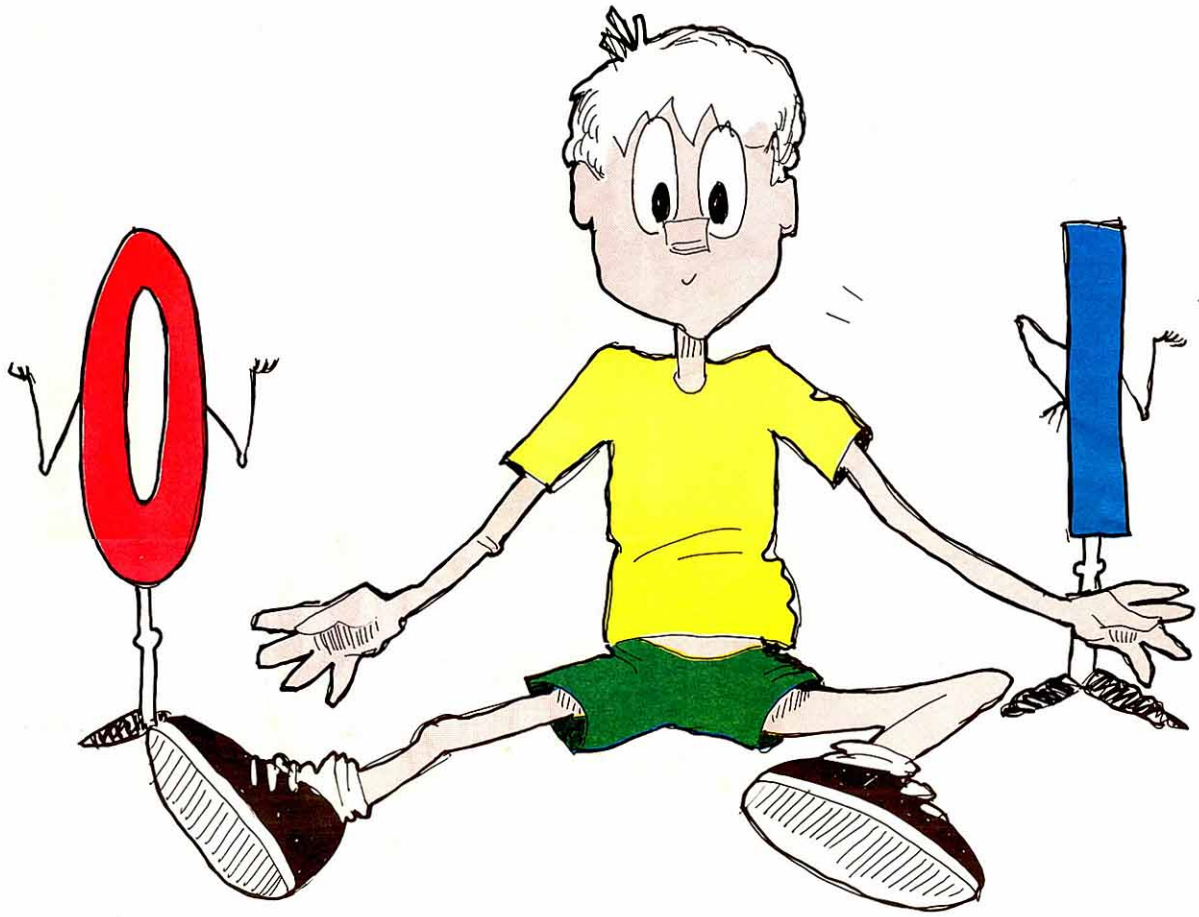
ANSWERS TO THE QUESTIONS:

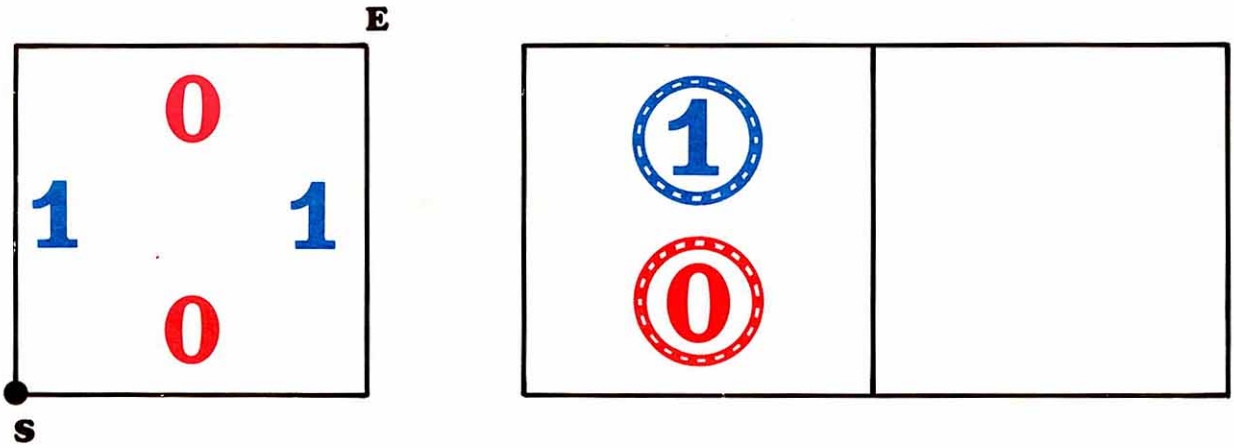
1. After **11110**, my game is not over because **0** and **1** are not both in the same room.

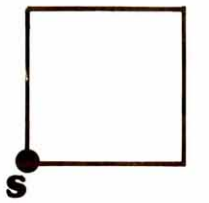
2. After **1100111101**, the two checkers are both in the room on the right and so 0's game is over.

0 and 1 stood in front of the
games they had invented.

"Let us play both games at the same
time," they suggested. "You spin and
we'll move the checkers. The winner
will be the first one to finish."

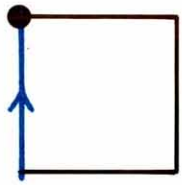


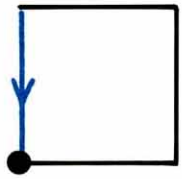




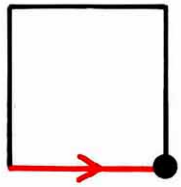
I spun and got **110001.**

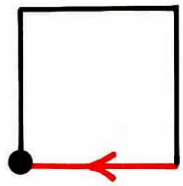
Strangely enough, my two friends shouted the same words at the same moment:



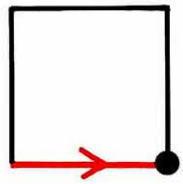


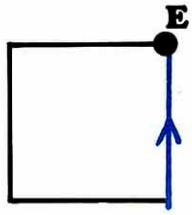
"The first two moves **11** don't change anything!"



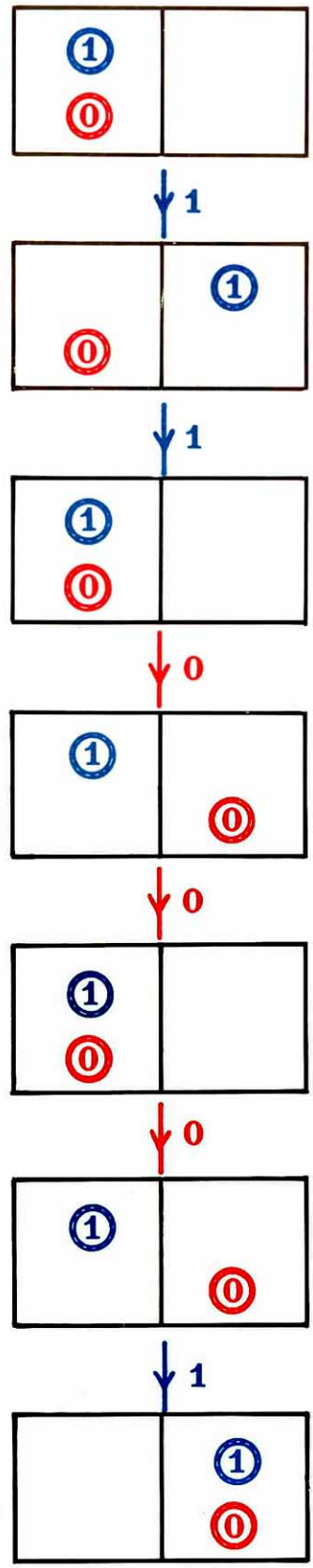


"After the next two moves **00**, we are back where we started again."





"The last two moves **01** end both games."



"We have a tie," said my friends,
looking very disappointed.
"Let's play again."

I spun and got **11110001.**

CAN YOU PREDICT WHAT HAPPENED? PLAY THE GAMES FOR 0 AND FOR 1. WHO IS THE WINNER?

"We have a tie again," sighed my friends. "That's strange."

DO YOU KNOW WHY THEY KEEP ENDING IN A TIE?

"I think that there will never be a winner," I said. "There is a strong link between your two games. Whenever one game ends, the other game ends on the same move."

"That's true," agreed 0, "and I can even explain why."

I listened carefully, and so did 1.

"Look at this string of digits,"
said 0. "Write them two by two and
try to figure out how the checkers are
moving in each game."

00 11 11 00 11 11 11 01

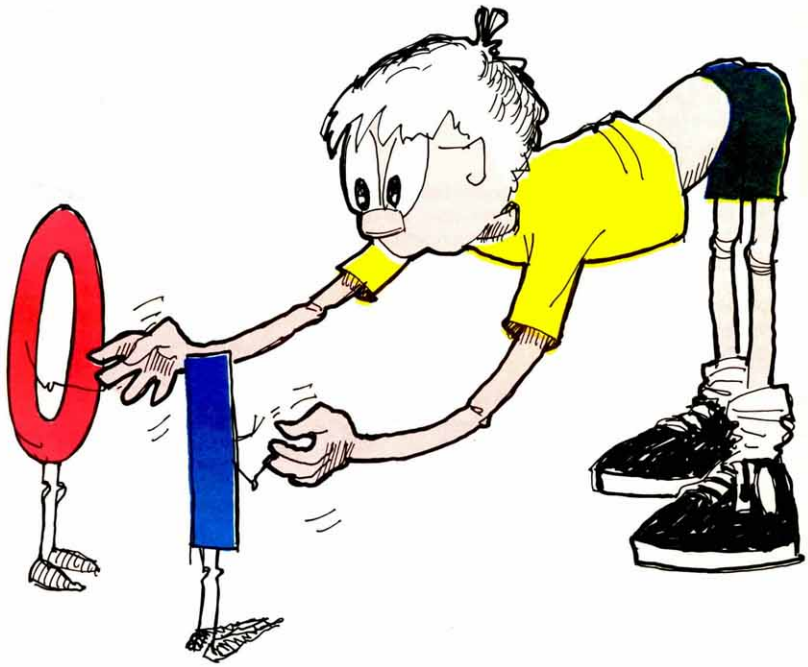
"Each time you get **01** or **10**, both games are over."

"Each time you get **00** or **11**, it means: no change."

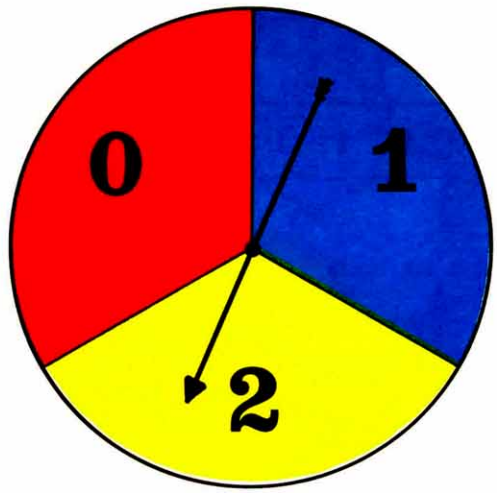
"Neither of you will ever beat the other," I concluded. "Whatever the string of digits may be, you will always have a tie."

"No competition and therefore, no fighting and no noise," observed 1.

"You have helped me keep my promise to Grandma," I said. "You are really good friends."



After 0 and 1 had left, I told Grandma about the games we had played with the marvelous spinner. She drew this picture...



and said, "Next Sunday you can invite your friend 2 as well. I am sure that this new spinner will give you a lot of interesting ideas."

HOW ABOUT YOU?

Inspired by a suggestion from the probabilist LENNART RÅDE, "The Square Trap" provides an exciting introduction to the fascinating world of probability and statistics. The hero of the story is given a spinner and he invites the numbers 0 and 1 over to help him make the most of his new toy. The two friends oblige by each inventing a game that can be played using the spinner.

Even though examples of the games being played are included in the narration of this story, the bulk of the experimentation is to be performed by the reader as he or she progresses through the book. Thus, firsthand experience is gained of the gradual build-up of statistical information, and a deeper appreciation is acquired of the probabilities at work in this situation.

The first game is a random walk on a square, and the second is a very special case of what is known as "the EHRENFEST model." However, even though these games are apparently so very dissimilar, the reader takes part in the major discovery that they are in fact equivalent in the sense that moves in one game parallel moves in the other, and if one game ends, the other game always ends on the same move.

Edward Martin