

# TWO BY TWO



by FREDERIQUE & PAPY

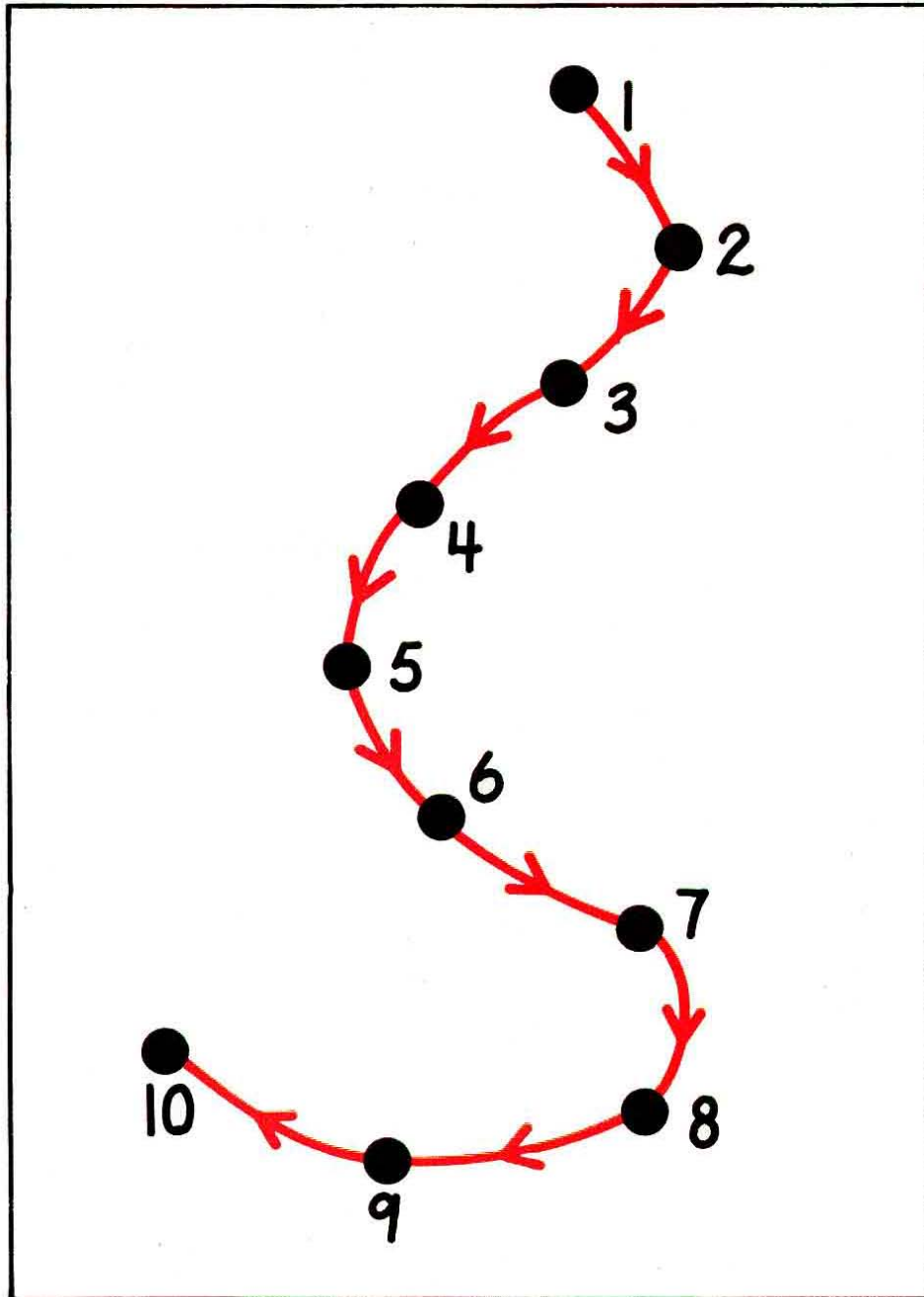
Design  
Marcelle Gober

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17-8746

My little friend Cedric Shirtsleeves likes numbers very much. Each rainy Sunday, I have to make up some new games for him.

WOULD YOU LIKE TO PLAY WITH US?



FOLLOW THIS RED SNAKE WITH YOUR FINGER AND COUNT THE DOTS.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10  
but I know many more numbers.

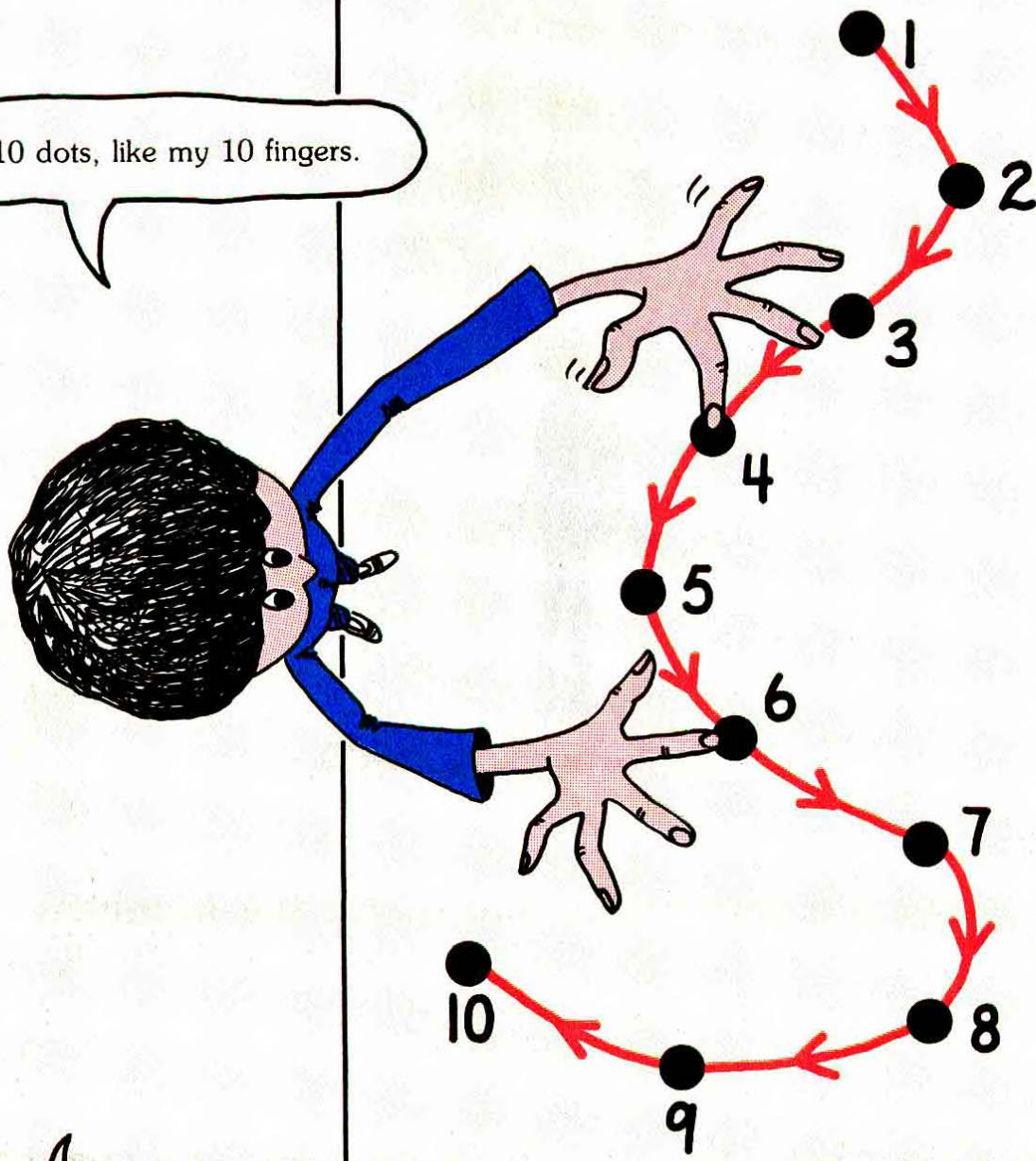


**Cedric draws a longer snake and writes the numbers up to 26.**

**“Cedric, your snake has exactly 26 dots and my snake only has 10.”**

“Try to touch the 10 dots of my snake with your 10 fingers, one finger on each dot.”

10 dots, like my 10 fingers.



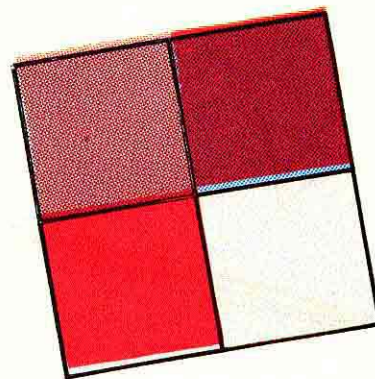
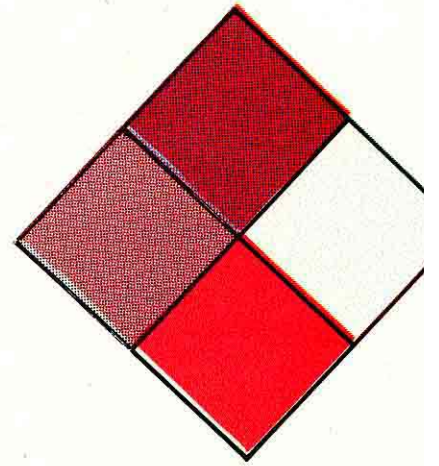
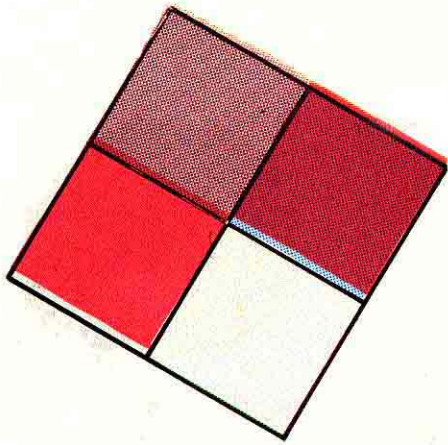
That's not very easy to do. My fingers are too short.

Still, I know many more numbers.  
27, 28, 29, 30, 31, 32, 33, 34, 35 . . .

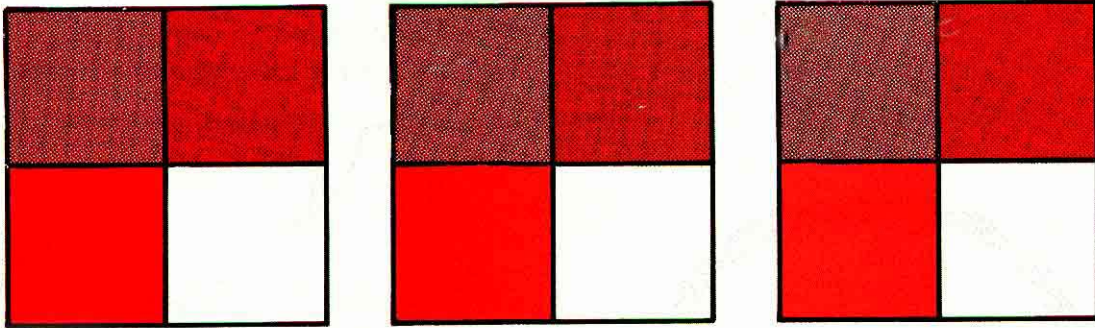


**“Let us play a new game.”**

**I put these colored boards on the table.**



**“First, you put the boards in front of you in this way.”**



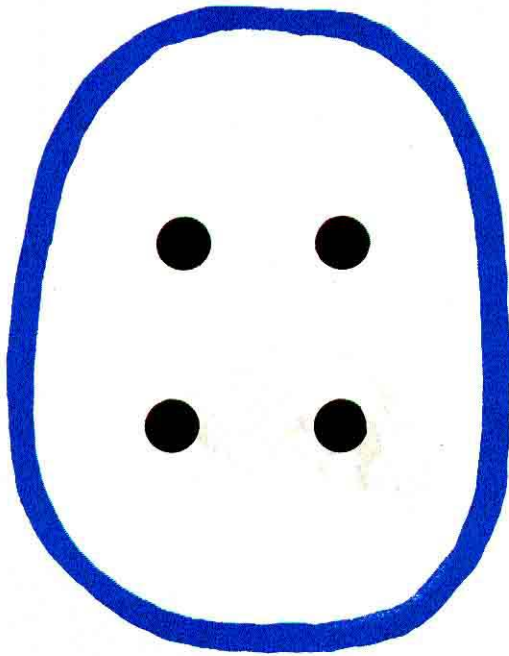
Four colors: brown, purple, red and white.  
They are all squares. How do we play?



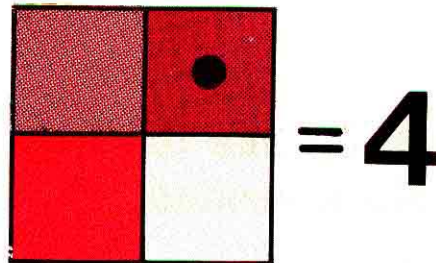
**“The colored squares are like the keys of a piano;  
you cannot change them around.”**

**“How many dots are inside this blue string?”**

4 dots. I don't need to count them.  
I see immediately that there are 4 dots.



**“Right. Also, one checker on the purple square of this board is 4.”**





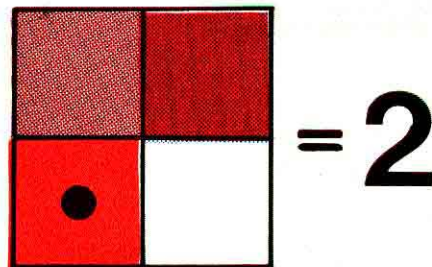


But you are making fun of me. You put only one checker on the board and yet you say it is 4.

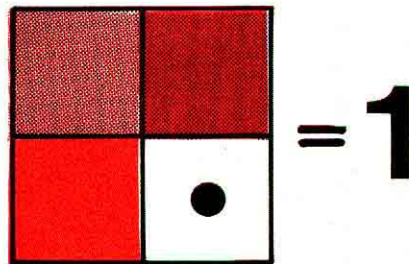
**“Yes, it is 4 because the checker is on the purple square.”**

Very strange!

**“And if I put the checker on the red square, it is 2;**



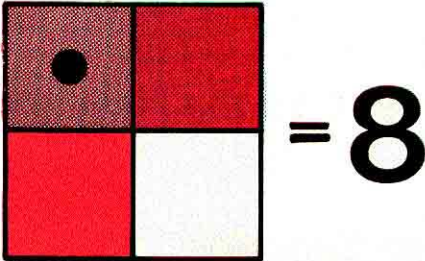
**and on the white square, it is 1.”**



And on the brown square, it is 3.



“No, on the brown square, it is 8.”

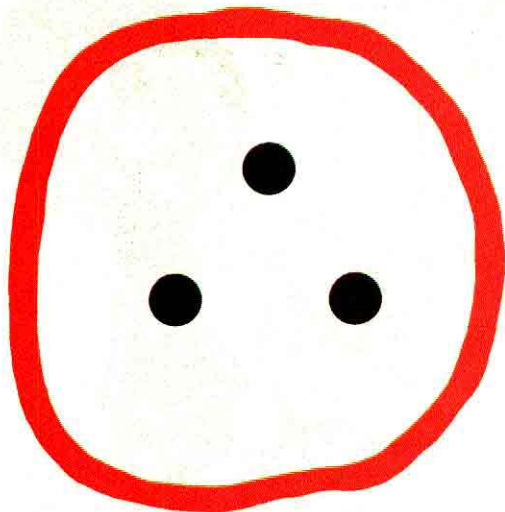


**Don't forget:**

- 1 → white
- 2 → red
- 4 → purple
- 8 → brown

I see, but where is 3??

**“You can easily find 3 by yourself, Mr. Shirtsleeves.  
How many dots are inside this red string?”**



3, obviously.

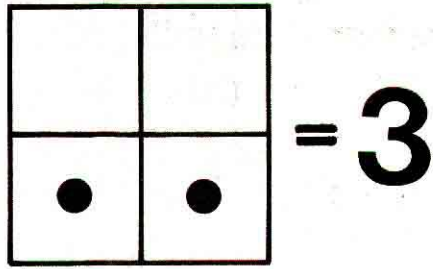


**“Right, and 3 equals 2 + 1.”**

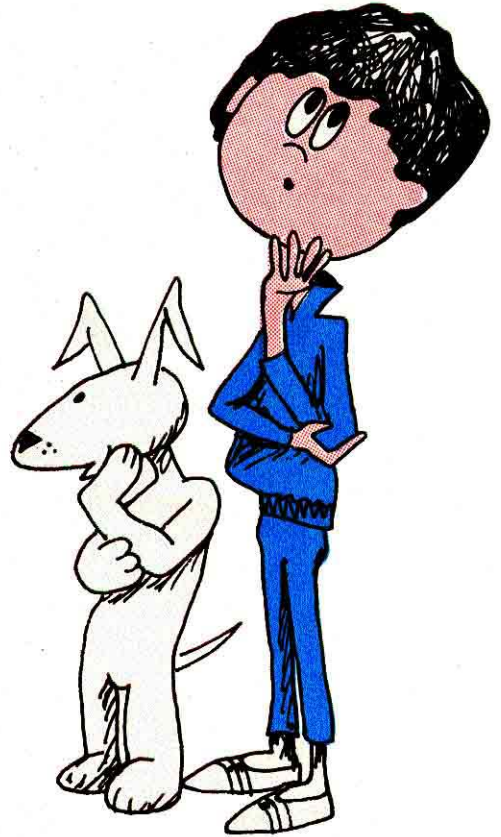
**Cedric thinks for a moment. Suddenly he shouts:**

I know how to do it! I need two checkers.

**CAN YOU SOLVE THIS PROBLEM TOO?  
TRY TO FIND THE ANSWER BEFORE  
READING THE NEXT PAGE.**



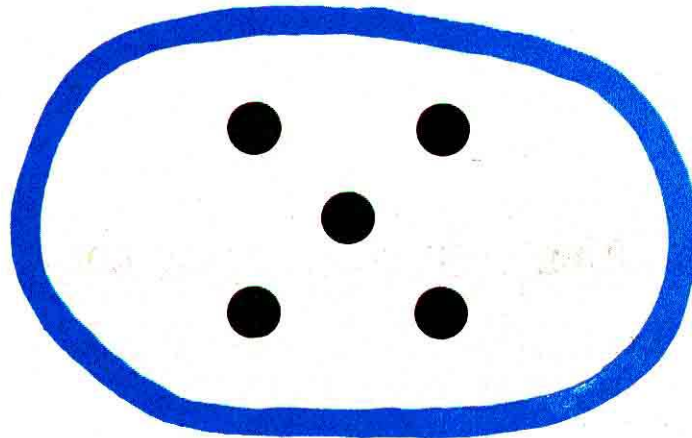
That's 3 because 3 is the same as 2 + 1.



**“Right!”**

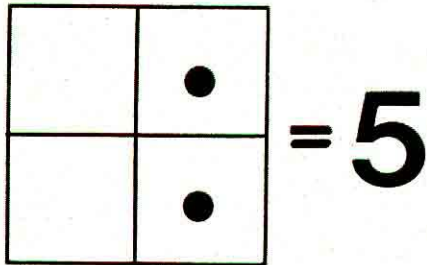
**“Can you show 5?”**

**“There are 5 dots inside this blue string?”**



**“Can you put 5 on our board?”**

**Cedric looks at his picture, then takes two checkers and puts them on the board like this, saying simply:**



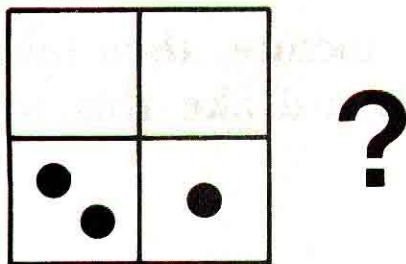
That's 5 because 5 is 4 + 1.



**“You’re becoming very good at my game.**

**I want to ask you a more difficult question.”**

“What number is this?”



Cedric is silent for a long time. He says to himself:

2 and 2 and 1 . . .

Looking at his right hand, he shouts:

It is 5 again!

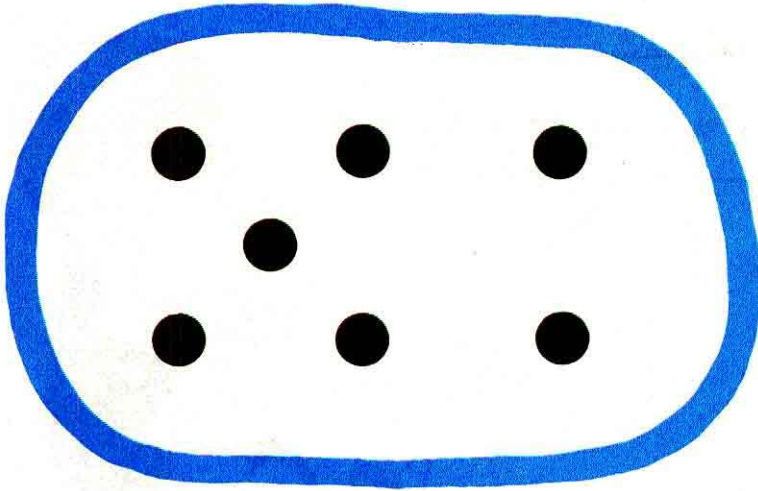


“Right, 5 equals  $2 + 2 + 1$ ; the same as  $4 + 1$ .”

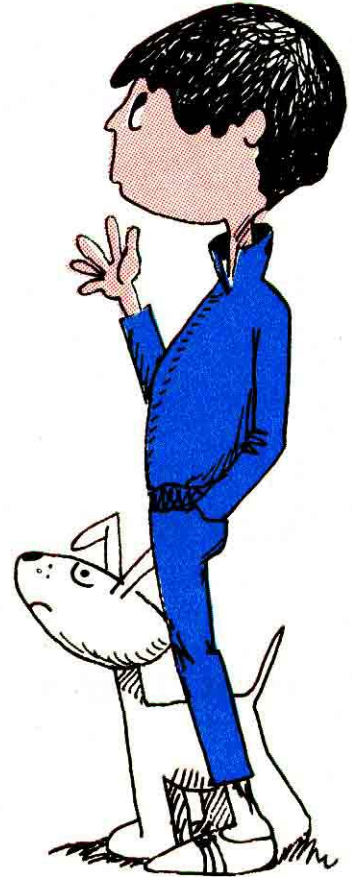
$$\begin{array}{|c|c|} \hline & \\ \hline \bullet & \bullet \\ \hline \end{array} = 5 = \begin{array}{|c|c|} \hline & \bullet \\ \hline & \bullet \\ \hline \end{array}$$

Now it is my turn to ask a question.  
How do you put 7 on the board?

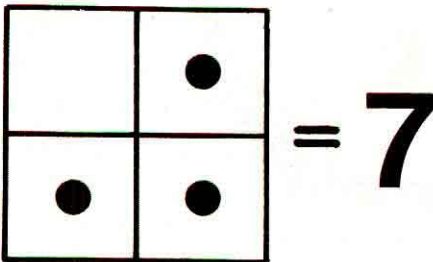
“First, I draw this picture.”



1, 2, 3, 4, 5, 6, 7.  
There are 7 dots, but I asked  
you to put 7 on our board.



“Look at my picture and you will find the answer  
by yourself.”



5 and 2, 2 and 5, 5 and 2 . . .  
I know how to do it! I need three checkers!

It is an interesting game!

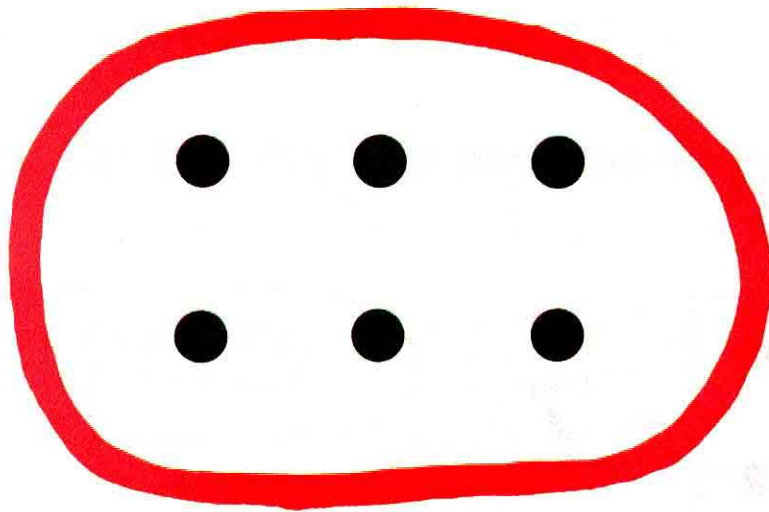
“How old are you, Cedric?”

Tomorrow is my birthday. I'll be 6.



“Can you put 6 on our board?”

First, Cedric draws 6 dots in a string.





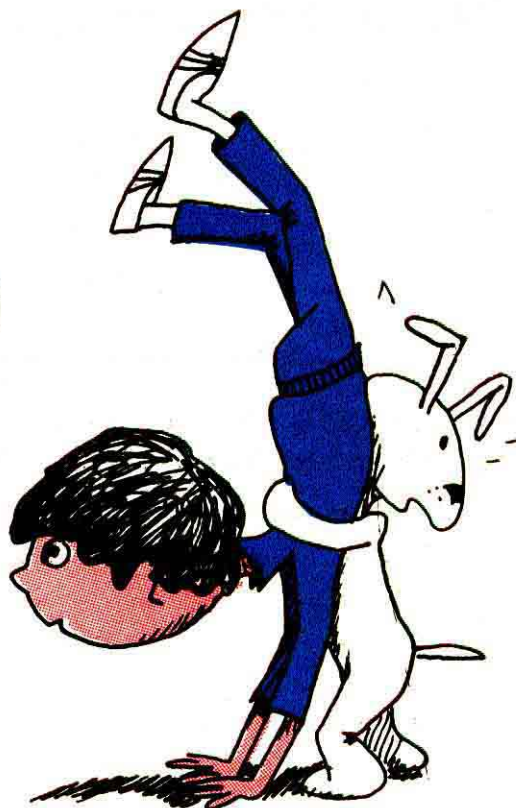
**"2 + 2 + 2"**

3 + 3

**"5 + 1 or 1 + 5"**

4 + 2 or 2 + 4

I know how to do it!  
I need two checkers!



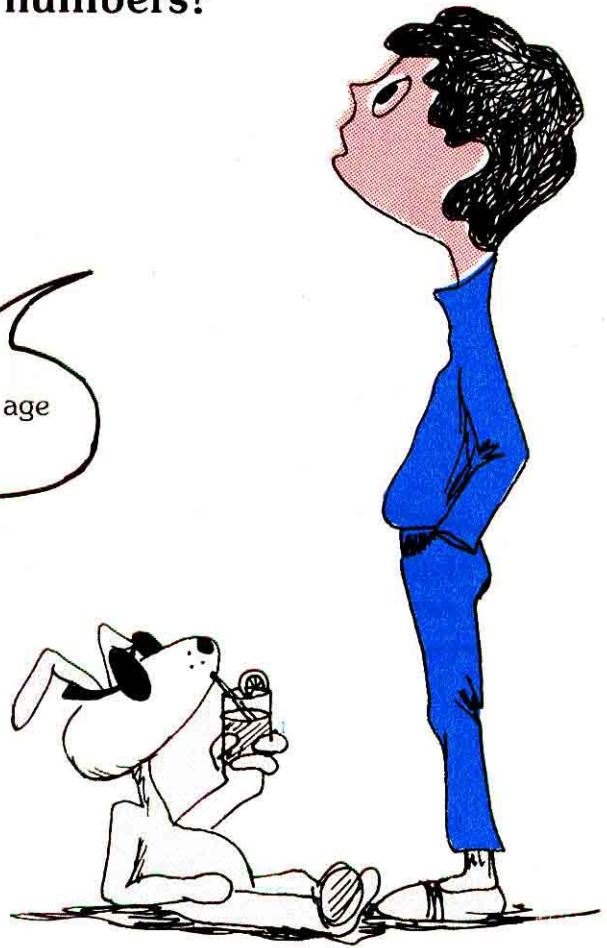
	●
●	

 = 6

I'm enjoying your game very much, but why did you put three boards on the table? So far, we played with only one board.

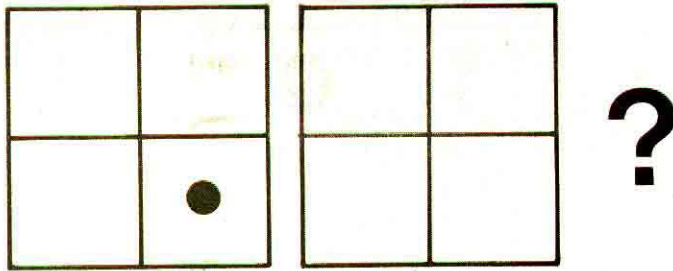
**“Do you know any very large numbers?”**

My house number is 15. My grandmother's age is 80 and her house number is 246.



**“Let's try to put those numbers on the boards.”**

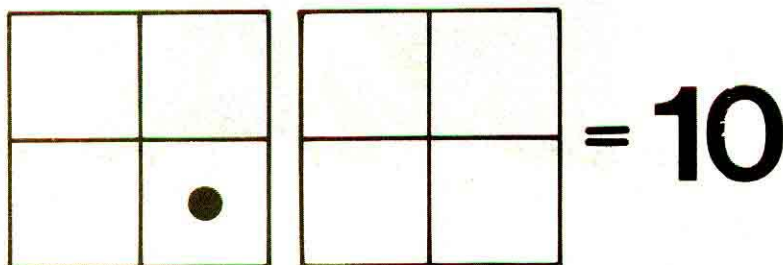
“What number is this?”

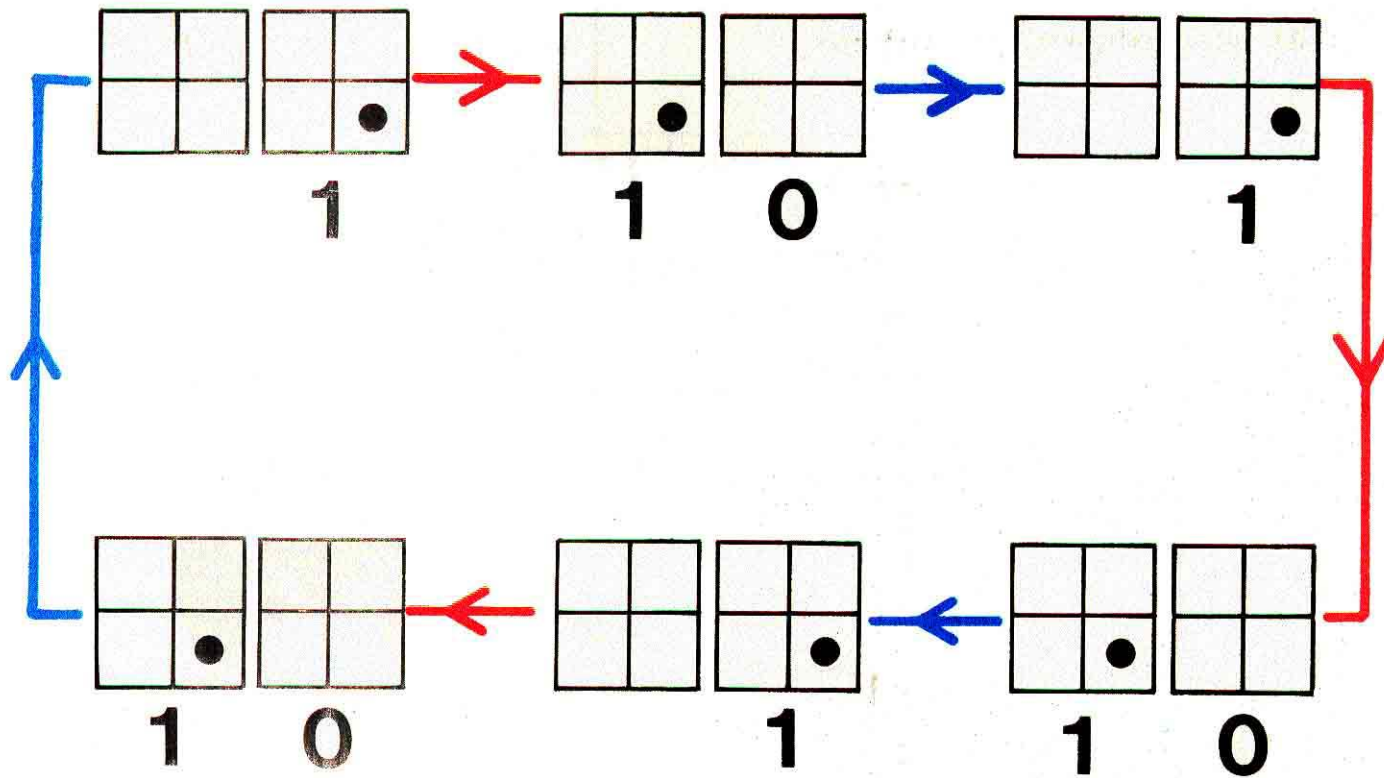


Obviously, it is 1 because you put one checker on the white square.



“Wait a minute! Not so fast! I put one checker on the white square of the second board. Now it is 10, like your 10 fingers.”

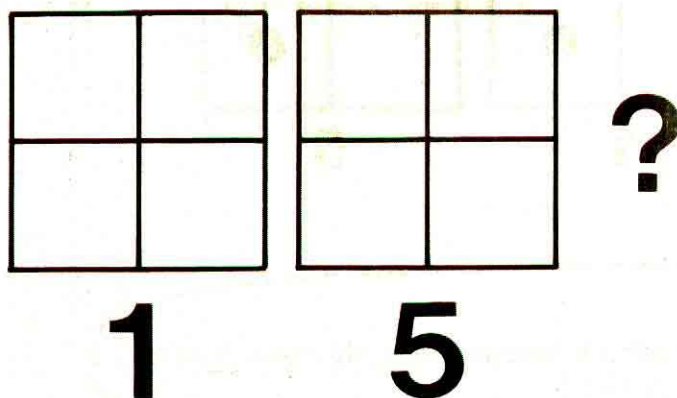




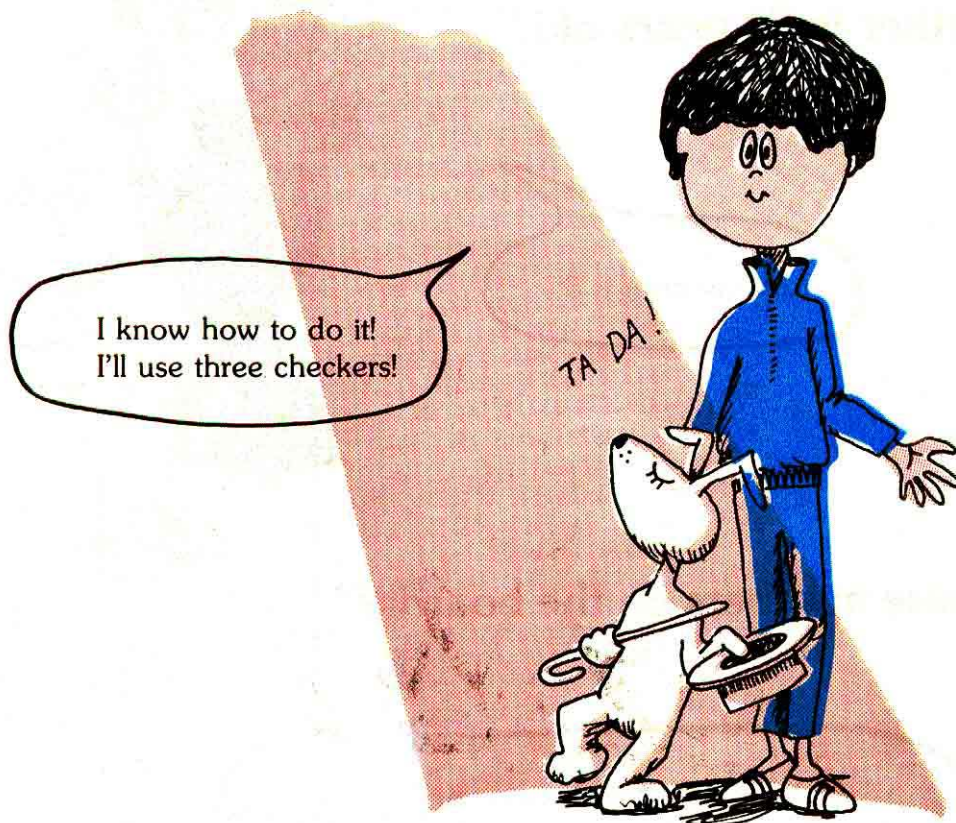
That's magic!



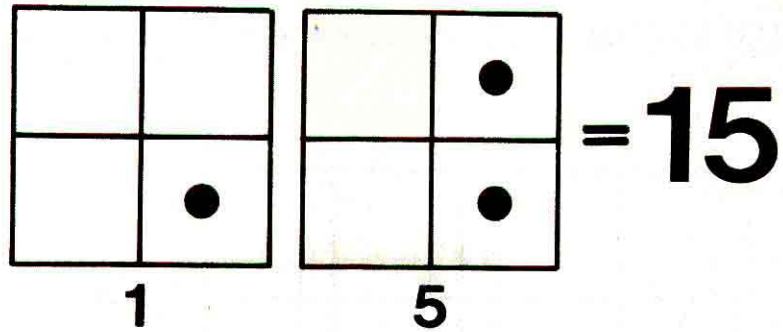
“Now, let’s try to put your house number on the boards.”



Cedric looks at the board for a long time.  
Suddenly he shouts:



**CAN YOU SOLVE THIS PROBLEM TOO?  
DO IT BEFORE TURNING THE PAGE.**



It is not difficult because I already know how to put on 10 and 5.

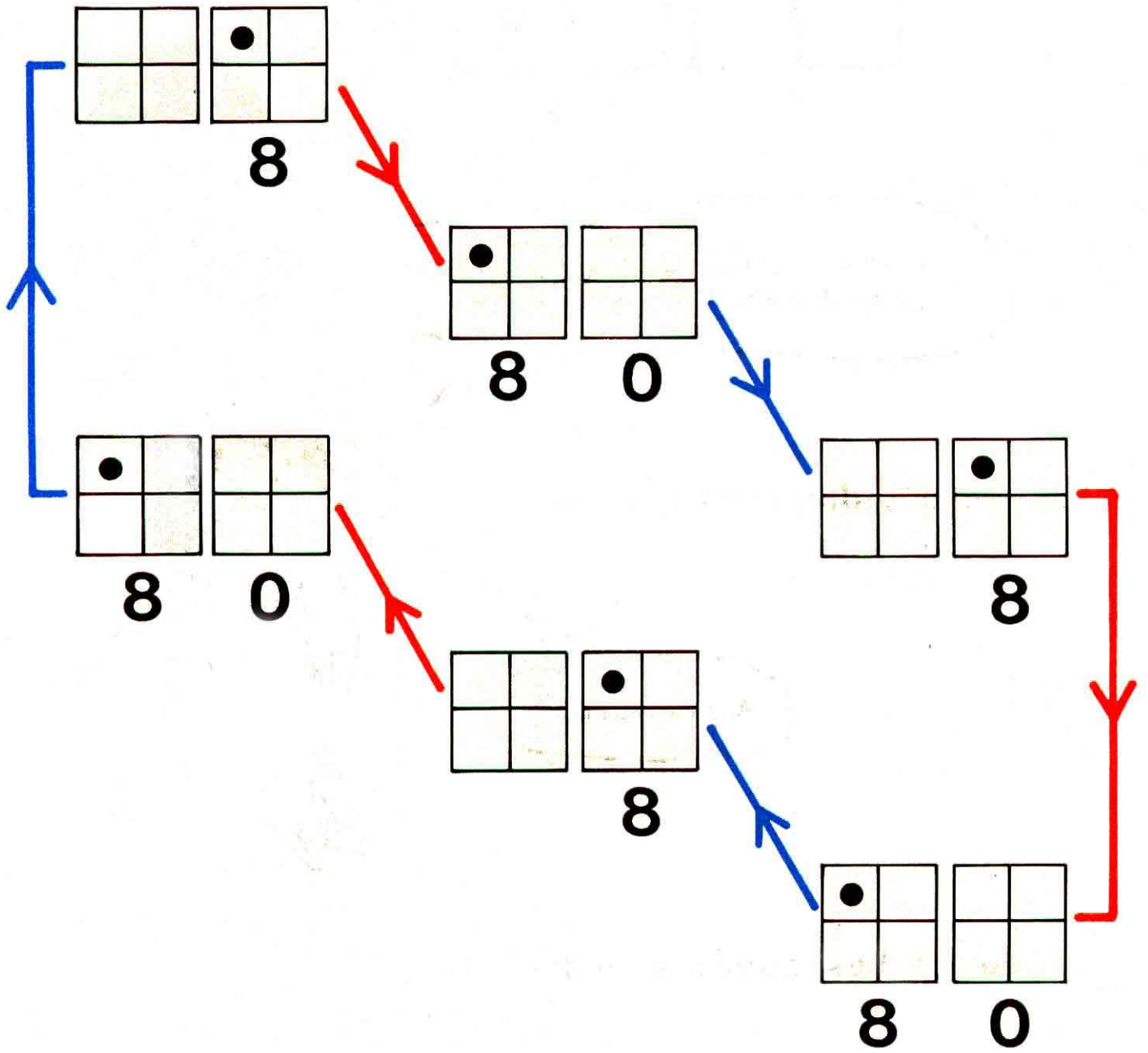
**“Your grandmother is 80 years old.”**

And my sister is 8.



**“Can you put these numbers on the boards?”**

8 → 80 → 8 → 80 . . .  
 That's easy! It is the same game as  
 1 → 10 → 1 → 10 . . .



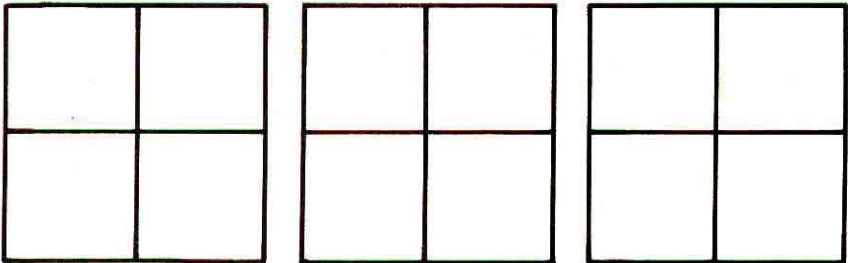
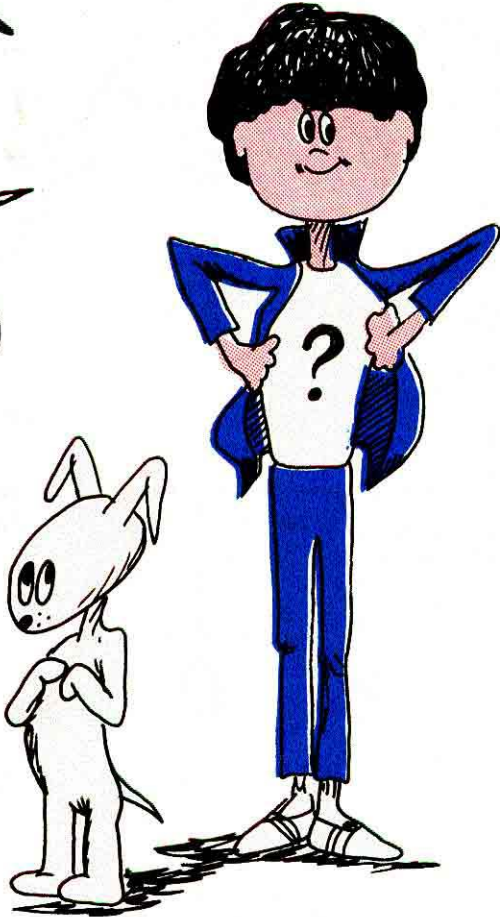
And now how about my grandmother's house number? That is 246.

**Cedric thinks hard.**

I think we should use the three boards.

**Cedric remains silent for a long time.**

I know how to do it!  
I'll use four checkers.

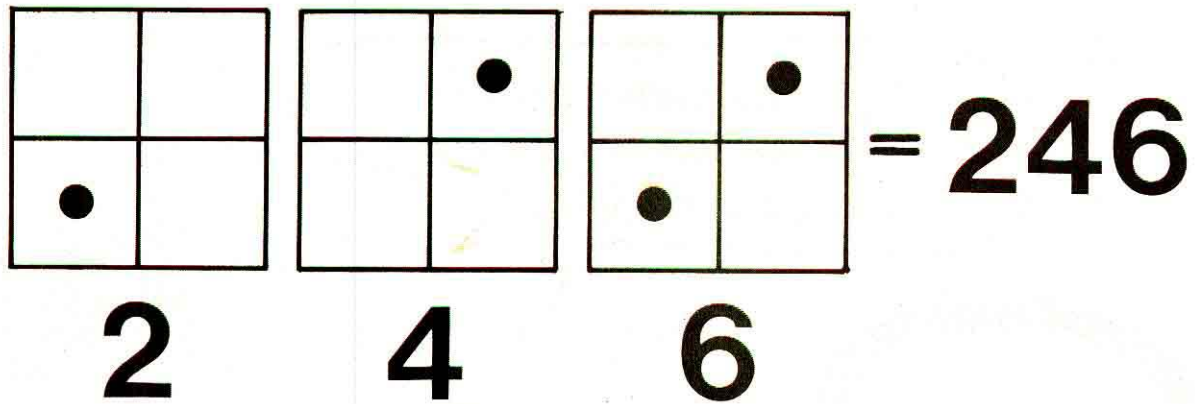


**2**

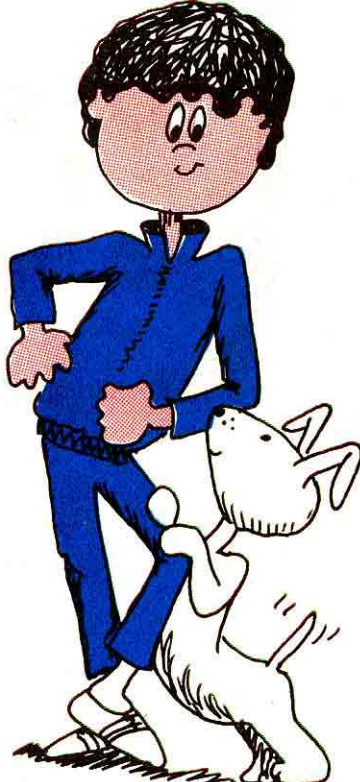
**4**

**6**





It is not difficult because I already know how to put 2, 4, and 6 on the boards.



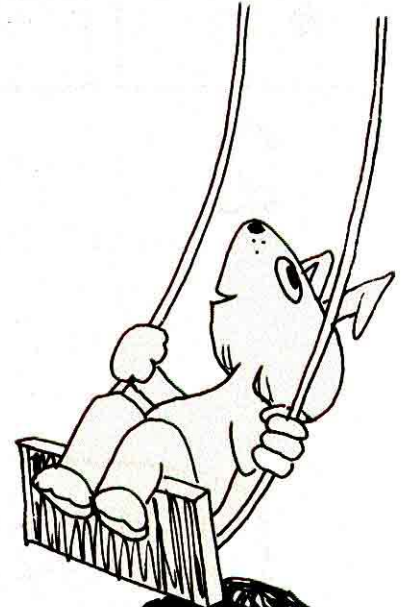
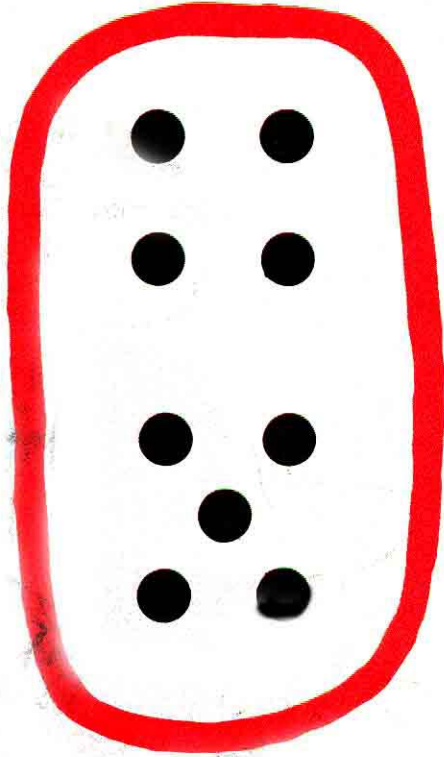
“So far, we have not met the number 9.”

First let's draw 9 dots in a string.

**YOU DO IT TOO, BEFORE READING THE NEXT PAGE . . . AND TRY TO PUT 9 ON THE BOARDS.**

I drew the dots in patterns, like in my dominoes game.

1, 2, 3, 4, 5, 6, 7, 8, 9.



**“Can you show 9 fingers?”**

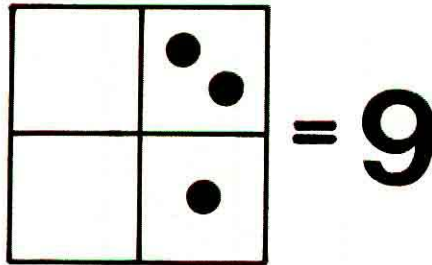
Easy! 1, 2, 3, 4, 5, 6, 7, 8, 9:  
all 5 fingers on one hand and 4 fingers  
of my other hand.



9 is the same as  $5 + 4$  or  $4 + 5$ .

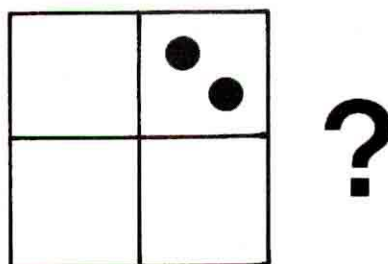
**“Can you put 9 on our board?”**

I know 4 and 5. So this is easy.  
I can use three checkers.

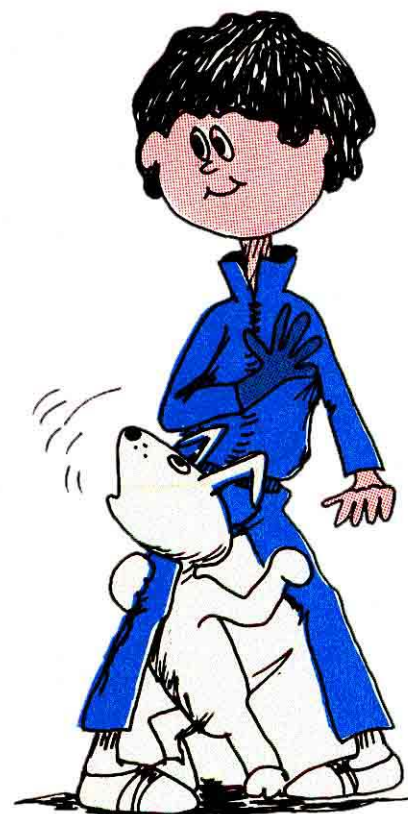


**“Right, but I have another solution using only two checkers.”**

“What number is this?”

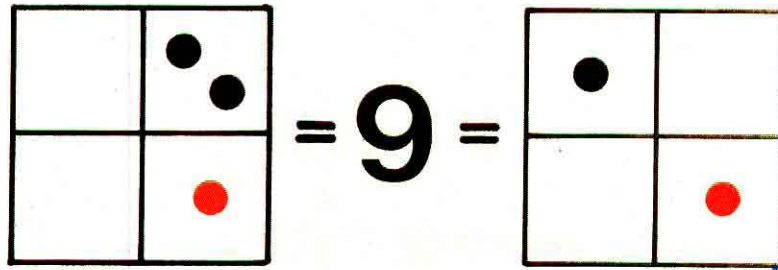


4 and 4.  
4 + 4 equals 8.



$$\begin{array}{|c|c|} \hline & \bullet \\ \hline & \bullet \\ \hline \end{array} = 8 = \begin{array}{|c|c|} \hline \bullet & \\ \hline & \\ \hline \end{array}$$

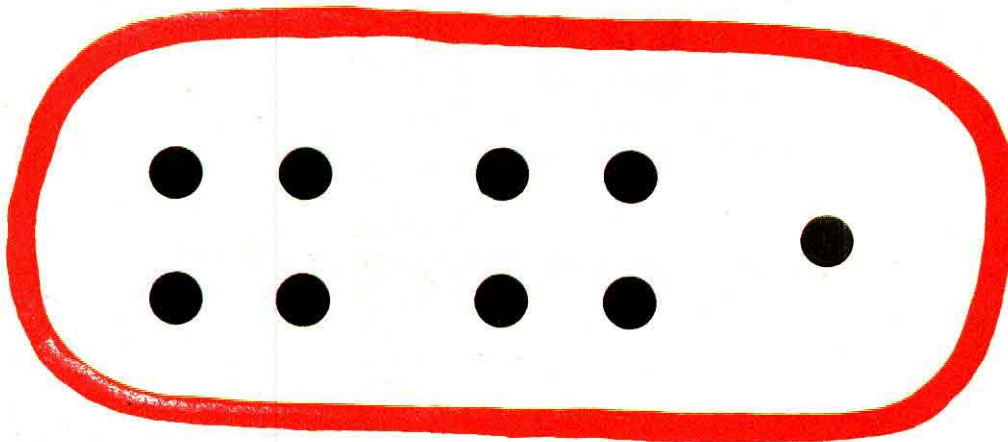
“Now I ask you again, how can you put on 9 with only two checkers?”



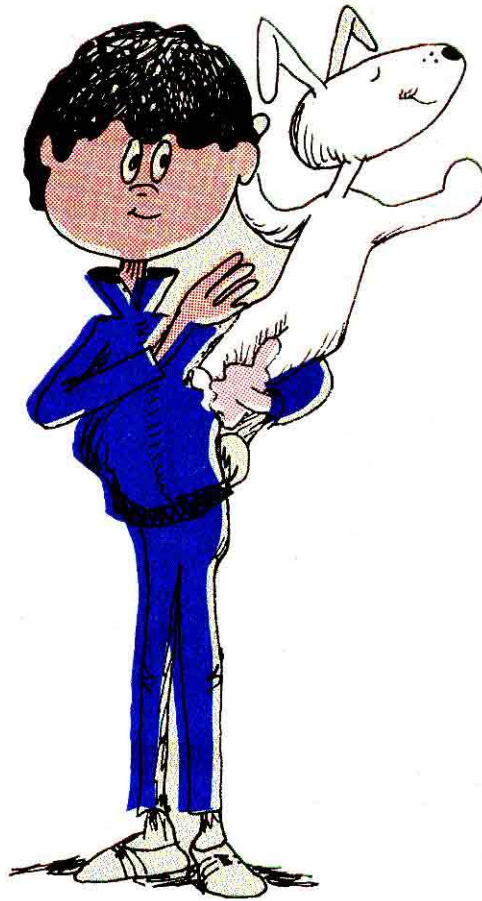
That's 9 because 4 plus 4 is 8 and 8 plus 1 is 9.



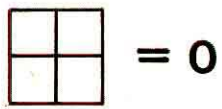
“Right! And 9 equals 8 + 1 as this picture shows.”



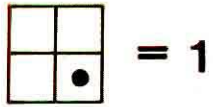
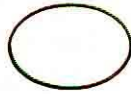
It is a nice game. Tomorrow I shall explain it to my friends.



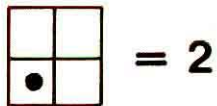
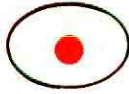
In order not to forget the game, Cedric writes the following information in his notebook.



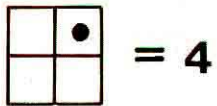
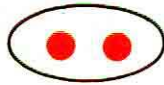
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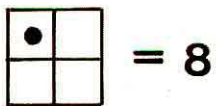
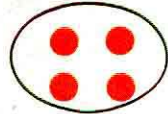
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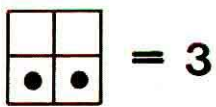
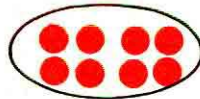
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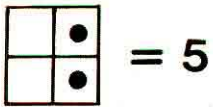
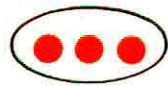
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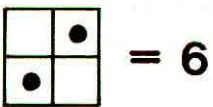
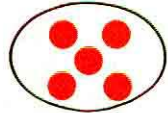
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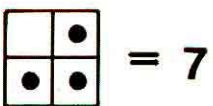
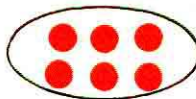
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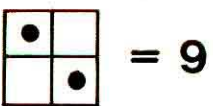
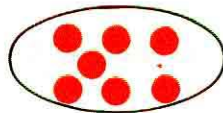
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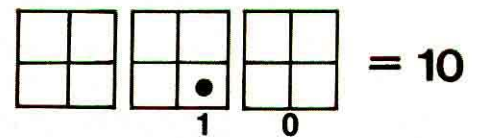
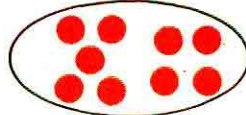
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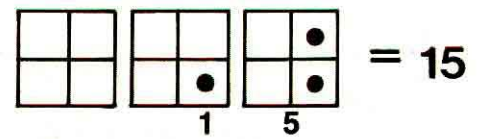
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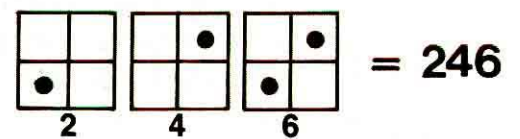
= 9



= 10



= 15



= 246

## "Two by Two"

Cedric Shirtsleeves learns an intriguing new game. He finds out how he can represent the numbers that he knows simply by using checkers and some colorful square boards. In fact the representation relies on an exceedingly simple positional system. For numbers less than ten, it depends on the very powerful notion of doubling. For larger numbers we find that it more closely approximates the usual way of writing numbers.

As the narrative unfolds we find that our insight into the "anatomy" of numbers is subtly deepened. With Cedric, we are constrained to think of numbers in many different ways until we discover how they can be shown by checkers on the colored squares.

This positional system is remarkable for the strong support it lends to learning the usual system of writing numbers. The positions of that system acquire a physical significance, and this "concreteness" brings the representation of numbers to life.

Edward Martin

## Stories by Frederique

### *Ages 5 to 8*

The Playful Numbers  
The Baby Is Born  
81 Roses  
One Out of Seven  
The Old Shoemaker  
I Am a Very Happy Boy  
The Little Dreamer  
Two by Two  
The Weird Story of 24  
Where's My Nose?  
The Happy Puppet  
The Magic Box  
Summer School in the Old Days

### *Ages 8 to 12*

The Little Donkey  
Singing Friends  
Dancing Friends  
I Am Not My Name  
The Living Lines  
The Square Trap  
Nabu Wins an Award

### *Ages 10 to 14*

The Hidden Treasure  
A Valentine Mystery  
Election in the Number World  
A Very Strange Neighborhood